NON-SYMBOLIC AND SYMBOLIC NUMERICAL COGNITION: A CROSS-CULTURAL PERSPECTIVE

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DEDICATED TO

To my mother "*Zarina*"

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ABSTRACT

Present study hinges upon a very critical question that is whether approximate number system plays foundational role in symbolic math or not? More specifically in current research it has been tried to explore the causal relationship between non-symbolic and symbolic numerical cognition through a brief training paradigm. Research evidence of past decades has shed light on the relationship between non-symbolic and symbolic numerical cognition through neuroscience, neuropsychological, correlational and indirect research evidences. However there was no research evidence specifying the causal relationship between the two directly. To bridge this gap present study was carried out in an effort to disentangle this relationship through training study with first grade children who are at the very first step of connecting these two systems through class mathematics learning.

This research study has been divided in two phases. Phase 1 of the study comprises of four experiments (i.e; experiment 1: N= 48; experiment 2, N=48; experiment 3, N=24; experiment 4, N= 24) conducted with American first grade children. Phase 2 of study comprises of two experiments conducted with Pakistani first grade children (experiment 1, N= 48; experiment 2, N =72). In both, phase1 and 2 children were trained with different training conditions (non-symbolic approximate addition, brightness comparison, line length addition and non-symbolic approximate comparison) and were post tested on symbolic addition (in experiment 1,3,4 of phase 1 and experiment 1 of phase 2), sentence completion task(experiment2 of phase1) and number line placement (experiment 2 of phase 2). Results across different experiments of both phases of study revealed that training with non-symbolic approximate addition and non-symbolic approximate comparison give the children advantage to perform better on symbolic math and number line placement task as compare to control conditions in terms of speed and accuracy. Research evidence indicates that non-symbolic numbers played foundational role in enhancing children performance on symbolic addition, number line placement and that this effect was specific to the domain of mathematics.

Furthermore, training effect got replicated and extended with Pakistani sample belonging to a totally different cultural context. Results indicate that longitudinal training with non-symbolic approximate numbers might be helpful to improve children symbolic math and might also be helpful for children with math learning difficulties.

Chapter I

INTRODUCTION

Human beings have developed rapidly and progressed in each aspect of life using numbers. Humans use numbers frequently in daily life in terms of checking time, date, temperature, weather, money, using phone, computers, and in all kind of measurements like length, width, height, weight, etc. Effective numerical abilities are needed to live a functional life and to do daily work related tasks. All the technological advancement through which man have made his life enriched is resting at some level at numbers. This ability has given advantage to mankind to uncover nature's gifts.

From very early in life, infants and children keep track of different things and engage their number system in playing games, counting candies, counting staircase, etc. Most developed form of numbers in humans is complex mathematics through which humans became capable of carrying out large calculation to make buildings, to run businesses, to go through economic growth, to develop machines to make life more sophisticated and advanced. Even humans can estimate and calculate about universe, e.g., distance of earth from different planets, speed of their movement etc. In all these human endeavors, sophisticated mathematical skills play a crucial role.

Humans are only living being capable of discovering planets, estimate and travel in the space, and learn about galaxies. No other nonhuman animals have done such a progress. Question arises how humans got this opportunity to be so smart to carry out these complex calculations than any other non-human animals?

Nature has endowed human beings the capacities to unravel the secrets of universe. One of these capacities is their “number sense” that humans share with other nonhuman animals.

According to Dantzig (1954), “Man even in the lower stages of development, possesses a faculty which, for want of a better name, I shall call Number Sense. This faculty permits him to recognize that something has changed in a small collection when, without his direct knowledge, an object has been removed from or added to the collection”. Furthermore he states that, “Arithmetic is the foundation of all mathematics, pure or applied. It is the most useful of all the sciences, and there is, probably, no other branches of human knowledge which is more widely spread among the masses”.

Research shows that mathematical abilities are closely linked to logical reasoning abilities unless logicality is achieved through a domain specific route in mathematics. Education in mathematics improves general reasoning skills (Morsanyi & Szucs, 2014).

Human beings have been endowed with number sense that they share phylogenetically with other living beings. From very early in life infants and children can process numbers approximately. These quantitative capabilities get sophisticated ontogenetically with increased exposure to environment, increased interaction with people, with the help of language and education. In educated cultures, children start learning number words, and count with help of fingers in initial stages and this learning is not just matter of external forces rather they already have “number sense”, upon that they map on their later learning.

Core Systems of Number

Several research evidence supports two-core system view of numerical cognition (Ansari, Lyons, Van Eimeren, & Xu, 2007; Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Hyde & Spelke, 2009; 2011; Lipton & Spelke, 2004).

Numerical concepts emerge out of evolutionarily ancient cognitive systems termed as “systems of core knowledge”. These systems are innate, intuitive, useful and each system is present early in development independent of language or education. One system compares and combines approximate cardinal values of sets and where as other processes small number of objects. Human beings productively combine both systems with acquisition of language. There are five signatures of infant’s numerical representation that are following. First, ability to discriminate two numbers depends upon the ratio between them; second, same ratio limit applies for different type of arrays of sounds, or dots; third, ability to discriminate, order, add two successively presented numbers and compare the sum to the third number; accuracy of comparison and addition is subject to same ratio limit as discrimination; fourth, numerical discrimination is impaired or abolished when arrays are presented under conditions that favor the attentive selection and tracking of individual objects; and fifth, infants spontaneously relate changes in number to changes in a different quantitative variable, line length (Spelke, 2011).

Core system 1: Primitive number representations/ approximate number system (ANS). Wealth of research evidence suggests that there is a primitive non-verbal numerical system that represents the approximate numerical magnitude of collection of objects (sets) through which quantities are processed approximately. This system is imprecise, called approximate number system/analogue magnitude being shared by humans and non-human animals (Feigenson et al., 2004; Gallistel, 1990).

ANS is engaged by showing quickly bunch of dots or sequence of sounds for very short time so that subject could not count exactly. Discrimination of numerical quantities through ANS is ratio dependent and this ratio limit is identical for stimuli from different modalities. Underlying representations are in analogue magnitude (Feigenson et al., 2004; Dehaene, 1992), so according to Brannon, Jordan, and Jones (2010), "in this format numerosity is represented as a mental magnitude that is proportional to the quantity it represents; consequently discrimination obeys Weber law. Weber Law states that $\Delta I/I = C$, where ΔI is the increase in stimulus intensity to a stimulus of intensity I that is required to produce a detectable change in intensity and C is constant". Analogue magnitude representation involves Weber law: threshold of discriminating two stimuli increases linearly with stimulus intensity and in case of number, discrimination of two numbers depends upon ratio of two numbers (Dehaene, 2003).

A wealth of research reveals that even infants can discriminate between arrays of visual elements on the basis of number (e.g., Brannon, 2002; Xu, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005). This ability is present from birth, persists over the lifespan, and is common to a wide variety of non-human animals Feigenson, Dehaene, & Spelke, 2004; Izard, Sann, Spelke, & Steri, 2009). Studies in infants, preschool children, and non-human primates reveal that the ANS supports computations as diverse as numerical discrimination, ordinal comparison, addition, and subtraction (Brannon & Terrace, 1998; Cantlon & Brannon, 2006a, 2006b, 2007; Gilmore, McCarthy, & Spelke, 2010; McCrink & Wynn, 2004). Nevertheless, the ANS represents number imprecisely: Precision in the mental representations of number decreases as number increases, and comparison of two numbers is possible only when they differ by a sufficient ratio (Halberda, Mazzocco, & Feigenson, 2008). The signature ratio dependent imprecision of the ANS stands in stark contrast to the exact meaning and precision associated with the symbolic number system that is acquired in early childhood and is used to learn and perform higher symbolic mathematical computations (For reviews see Carey, 2009; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006).

Approximate number system (ANS) in animals. Evidence from different research studies shows that animals, such as pigeons (Olthof & Roberts, 2000), rats, and monkeys (Brannon & Terrace, 2000; Olthof, Iden, & Roberts, 1997), measure, count and remember quantities (Dehaene, 2011; Dehaene, Dehaene-Lambertz & Cohen, 1998; Gallistel & Gelman, 2000).

Research carried out by Rugani, Regolin, and Vallortigara (2010) showed that newborn chicks are sensitive to number vs. continuous physical extent of artificial objects. Fish can represent and use numerical information for discriminating small quantities (Agrillo, Dadda, Serena, Piffer, & Bisazza, 2009).

Moreover salamanders (Uler, Jaeger, Guidry & Martin, 2003), pigeons (Roberts, 1995), have been reported to make quantity discrimination. Lemurs were trained to respond to two visual arrays in numerically ascending order. Lemurs successfully ordered the novel values with above chance accuracy (Merritt, MacLean, Crawford, & Brannon, 2011).

Chimpanzee can do arithmetic with simple fractions and even they can combine two fractions. As presented with one quarter apple and one half glass and they were given choice between one full disc or three-quarters disc and chimpanzee chose by internal calculation the latter (half glass and three quarters disc) more often (Dehaene, 2011).

Animal arithmetic abilities are quite primitive even after considerable training as compare to human child who can spontaneously count up to 10 before age three. However animals have the ability to apprehend numerical quantities, to memorize, to compare and to add approximately (Dehaene, 2011).

Approximate number system in infants, children and adults. Research has shown that newborn infants can discriminate the quantities by ratio of 3 (4 vs. 12, 6 vs. 18) across different modalities, but could not discriminate the quantities differed by ratio of 2 (4 vs. 8) objects (Izard, Sann, Spelke, & Steri, 2009). However, six month old can discriminate numbers differed by ratio of 2:1 but they are unable to discriminate numerosities of 3:2 ratio (Lipton & Spelke, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005).

At 9-10 month of age ratio even drops to 3:2 (Lipton & Spelke, 2003; Xu & Arriaga, 2007). Numerical Acuity of approximate number system increases throughout childhood and adult like acuity is gained late in development. 3 year old can discriminate quantities differed by ratio of 4:3, 6 year old can discriminate by ratio 6:5 and adults by 11:10 (Halberda & Feigenson, 2008).

Young children (Barth, Beckman, & Spelke, 2008) and adults were able to perform approximate arithmetic on non-symbolic stimuli cross modally with signatures of non-symbolic number representation (Barth, Kanwisher, & Spelke, 2003; Barth et al., 2006).

Core system 2: Parallel individuation system/ exact small numbers representations. There is a second system for precise representation of distinct objects simultaneously also called object-tracking system (OTS). 12-14 months old infants have been shown to represent exact number of arrays 1, 2, and 3 but failed to represent 4 (Feigenson & Carey, 2003, 2005). 6-7.5 months old have been reported to process small number (1-3) and large numbers (8-32) differently through event related potentials (ERPs) suggesting evidence for two separate systems (Hyde & Spelke, 2010).

Jevons (1871) conducted ingenious experiment to determine how many objects the human mind could count by instantaneous and apparently single act of attention. He had concluded that power of mind is limited to less than five items at a time.

Starkey and Cooper (1980) showed through habituation-recovery of looking time method that infants are capable of discriminating representing and remembering small number of items. 16-30 weeks old babies were habituated to test their perception and representation of specific small number of items. They tested on small number condition, (2-3 and 3-2 items) and large number condition (4-6 and 6-4) items. Subjects were tested by showing them repetitively two large black dots spread horizontally until their looking time decreased, indicating habituation. Later slides were changed from 2 to 3 dots and babies stared to fixate longer on these new unexpected images. Results showed that dishabituation occurred for small number of items but not for large number condition. Findings indicate that a perceptual enumeration process called subitizing, present in 2 year olds, probably underlie this capacity.

Starkey, Spelke, and Gelman (1983), showed through a preferential looking time paradigm that 7-month-old infants can match the number of objects in spatial display to the number of sounds in a temporal sequence. Subjects were shown two photographic displays presented side by side; one display showing 2 objects and seconds display showing three objects. While infants watched 2 object or 3-object display, they heard two or three drumbeats from center location. Infants attended preferentially longer to a visible object that corresponded to accompanying sound than the non-corresponding display.

Humans are capable to go beyond that limit of 3, 4 objects and can precisely process the quantities beyond limit of 3, 4 with the help of language and education (Pica, Lemer, Izard, & Dehaene, 2004).

Emergence of symbolic mathematics. Both core systems of number has their own limits: one system allows just distinct objects representation but cannot represent larger number and other represents larger sets but not precisely rather just approximately. According to Spelke (2011), children overcome limits of both systems by using number words when they learn number words in natural language expression and counting. At 2 years of age children learn 10 or so words of counting list. Initially these words have little numerical meaning but at some point in 3 years of age they learn the cardinal meaning of words (one designate a single item and all other number words represent plurality of items). Language of number words and counting provides system of symbols for combining two core systems of number.

Combination of both systems is crucial for acquisition of symbolic number representation (Feigenson et al., 2004; Spelke & Kinzler, 2007), however Le Corr & Carey, (2007) discussed different perspectives on acquisition of verbal counting principles.

Symbolic numbers. Symbolic numbers can be presented as number symbols e.g., Arabic numbers, like 1, 2 3 or number words like one, two, three, etc.

Humans start learning symbolic numbers through certain rules like one to one correspondence with the help of language and education. Later in development they start learning operation like addition, subtraction, multiplication and division through school education.

Wynn, (1992) conducted a study with 2-3 year olds to investigate how and when children's understanding of the meaning of number words develops, whether children knowledge of number words come at once or they acquire different aspects of meaning of number words at different stages. Children were tested on four tasks; the give a number, how many, color control task and the point-to-x task. Results revealed that children learn the number words sequentially up to 2 or 3 and then acquire the cardinal meaning of larger number words in combination with cardinal word principle. Although children know that the number words refer to specific numerosities at a very early stage of counting but still they take long time to learn how the counting system represent number.

Zebian and Ansari (2011) investigated relationship between symbolic and non-symbolic numerical processing by comparing two groups on measures of symbolic (two single digit Hindu-Arabic numerals) and non-symbolic comparison (two arrays of squares). Comparison of symbolic and non-symbolic magnitude processing was investigated by comparing two groups of adults (of same socio economic status and culture); highly literate (HL, attended school for more than 10 years) and illiterate/ minimally literate (ML, had some rudimentary symbolic recognition and conceptualization skills, had attended 1 year of schooling, could track smaller quantities). Results suggested that ML group was not different in processing of nonsymbolic numerical magnitude processing than HL group, however ML group performance was substantially different that HL group on symbolic number processing. These findings suggest that symbolic and non-symbolic numerical magnitude processing is differently affected by literacy. Non-symbolic numerical magnitude processing is not affected by literacy where as symbolic processing is modulate by literacy.

Recent research evidence suggests that there is a functional relationship between approximate number processing and exact symbolic mathematics. Research shows that symbolic math arises in part from reuse of approximate number system (Dehaene, 2005; Hubbard, Diester, Cantlon, Ansari, Van Opstal, & Troiani, 2008).

Links between the ANS and symbolic mathematics[1] Despite the differences between the approximate number system and later acquired symbolic numbers and mathematics, three lines of evidence suggest a functional link between them.

1. First, tasks involving purely symbolic numbers and exact arithmetic reveal signatures of nonsymbolic, approximate number representations (see Piazza, 2010 for a review). For example, when adults or older children are asked to determine which of two symbolic numbers is larger, their performance depends on the numerical distance between the numbers to be compared (e.g., Dehaene & Akhavein, 1995; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Moyer & Landauer, 1967; Temple & Posner, 1998). Similarly, speed of processing a symbolic number depends on its numerical distance from a covertly presented, antecedent numerical prime (e.g., Van Opstal, Gevers, De Moor, & Verguts, 2008). Finally, in adults and older children, overlapping parietal brain regions are activated during processing of number in both symbolic and non-symbolic number formats, and these regions show similar release from adaptation to numerical changes independent of the format of

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[1] Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief non-symbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition, 131*(1), 92-107.

presentation (symbolic or nonsymbolic) (see Dehaene, Piazza, Pinel, & Cohen, 2003; Piazza, 2010; Piazza, Pinel, Bihan, & Dehaene, 2007).

2. Second, individual differences in ANS acuity correlate with mathematics achievement scores (e.g., Bugden & Ansari, 2011; DeWind & Brannon, 2012; Gilmore et al., 2010; Halberda et al., 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; but see Lyons & Beilock, 2011). Several studies show concurrent or retrospective correlations between ANS acuity and mathematics achievement scores (e.g., Halberda et al., 2008; Libertus et al., 2011, 2012; Lourenco et al., 2012). For example, individual differences in the acuity of approximate, non-symbolic number comparisons, tested at 14 years, were significantly associated with past mathematics achievement scores as far back as kindergarten (Halberda et al., 2008). In these correlational studies, it is unclear whether individual differences in ANS acuity play a causal role in creating individual differences in mathematics development, whether symbolic mathematics development causes changes in ANS acuity (e.g., Piazza, Pica, Izard, Spelke, & Dehaene, 2013), or whether a third, mediating factor, such as differences in the facility of operations on number symbols (e.g., Lyons & Beilock, 2011) or differences in aspects of executive function (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013) explain the relationship. Other studies show that individual differences in ANS acuity predict future mathematics achievement even after controlling for variables like general intelligence, verbal abilities, age (e.g., Gilmore et al.,

2010; Libertus, Feigenson, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011), and even when non-symbolic numerical processing is measured in infancy (Starr, Libertus, & Brannon, 2013). These studies, however, do not show that individual differences in ANS acuity cause the later changes in mathematics performance, because both the earlier differences in ANS acuity and the later differences in school mathematics learning could depend on one or more additional common factors.

3. Third, recent work suggests that practice with or training of the ANS, either alone or together with training of symbolic numbers, leads to gains in symbolic mathematics performance (Park & Brannon, 2013; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Dehaene, Dubois, & Fayol, 2009; Wilson, Dehaene, Pinel, Revkin, Cohen, & Cohen, 2006; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). One line of work showed that children who practiced a variety of symbolic number skills related to the ANS, including games involving approximate numerical comparisons, verbal counting, and mapping numbers to space, showed improvement on symbolic number tasks (Räsänen et al., 2009; Wilson, Dehaene et al., 2006; Wilson, Revkin et al., 2006; Wilson et al., 2009).

From this work, however, it is unclear which aspects of the training – targeted practice with the ANS, explicit practice mapping the ANS to symbols, symbolic number practice alone, or something else – contributed to the observed gains. More recently, Park and Brannon (2013) showed that several days of training on a non-symbolic approximate numerical addition task led to improvements in ANS acuity and symbolic mathematics performance in adults. Individual differences in ANS acuity change, although modest, correlated with individual differences in change on the symbolic arithmetic measures. Similar improvements were not seen in control groups with no training task, in a non-numerical, factual knowledge-training task, or in adults who practiced a symbolic number ordering task. These results provide the strongest evidence to date of a causal and specialized relationship between the ANS and symbolic mathematics. However, it is unclear whether such training depends on a mature mapping between the symbolic number system and the ANS or whether such training would also improve symbolic mathematics in children who are still acquiring mathematics skill and ANS precision. It is unclear whether engagement of the ANS, the cognitive operations involved (including comparison and addition), magnitude representations in general, or something else contributed to the improvements in symbolic arithmetic.

To date, however, most of the evidence suggesting a role for the ANS in symbolic mathematics is indirect, and the mechanism(s) driving this relationship are not well understood (e.g., Bugden & Ansari, 2011; Gilmore, McCarthy, & Spelke, 2010; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008; Holloway & Ansari, 2009; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; but see Lyons & Beilock, 2011; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, Defever, Maertens, & Reynvoet, 2014).

Gilmore, McCarthy, & Spelke, (2007) gave 5-6 year old children (who had mastered verbal counting and were on threshold for learning arithmetic algorithms for manipulating numerical symbols) symbolic addition and subtraction problems to solve with approximate solution. Children solved these arithmetic problems by drawing upon non-symbolic approximate number system. To probe it further, they gave the children problems requiring exact solution and children failed to do exact arithmetic, which showed that approximate arithmetic performance does not depend on knowledge of exact number. Children solved these arithmetic problems with signatures of non-symbolic arithmetic system (ratio effect on their accuracy, addition performance as accurate as comparison, and subtraction performance less accurate than comparison). Findings indicate that children recruit non-symbolic number knowledge when they confront new approximate symbolic number problems.

Similarly research evidence shows that children map onto non-symbolic number system when they are required to solve symbolic approximate problems with signatures of the non-symbolic number system (Barth, La Mont, Lipton, & Spelke, 2005; Mundy & Gilmore, 2009).

Theoretical Background

Theories of the relationship between the ANS and mathematics. Several theories have been proposed to explain the link between the ANS and symbolic mathematics[2]. One view is that symbolic mathematics depends specifically on the

[2] Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief non-symbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition, 131*(1), 92-107.

ANS (e.g., Barth, Beckmann, & Spelke, 2008; Barth, La Mont, Lipton, & Spelke, 2005; Barth et al., 2006; Dehaene, 2011; Gilmore et al., 2010; Nieder & Dehaene, 2009; Park & Brannon, 2013). In addition to the correlational studies and training studies cited above, further research consistent with this position comes from neuropsychological and trans-cranial magnetic stimulation research showing that damage or impairment of parietal brain regions thought to underlie the ANS alters the ability to performance symbolic numerical computations (e.g., Cappelletti, Barth, Fregni, Spelke, & Pascual-Leone, 2007; see Dehaene et al., 2003 for a review). Similarly, individuals with dyscalculia, a mathematics-specific learning disability, show poor ANS acuity (e.g., Butterworth, 2010; Piazza et al., 2010; Price, Holloway, Vesterinen, Rasanen, & Ansari, 2007).

Alternatively, the relationship between performance on tasks involving the ANS and on tests of symbolic mathematics may reflect a broader underlying relationship between symbolic mathematics and magnitude representations (see Lourenco et al., 2012). On this view, a generalized magnitude system underlies the representation of all magnitudes regardless of dimension ((physical size, number, duration, etc.) For reviews see Lourenco & Longo, 2011; Walsh, 2003). The hypothesis of a generalized magnitude system is supported by evidence showing overlap at the behavioral, cortical, and neuronal level between magnitude domains (e.g., Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Henik & Tzelgov, 1982; Lourenco & Longo, 2010, 2011; Tudusciuc & Nieder, 2007). Thus, individual differences in the generalized magnitude system (which includes number), rather than the ANS specifically, may be linked with individual differences in symbolic mathematics. Some evidence for this position comes from research with children showing that spatial magnitudes promote earlier understanding of higher numerical concepts (e.g., Mix, Levine, & Huttenlocher, 1999; Gunderson, Ramirez, Beilock, & Levine, 2012). Other evidence with adults shows individual differences in both discrimination of spatial extent and discrimination of number correlate with higher mathematics performance. However, further analysis of these results revealed that differences in spatial discrimination were uniquely associated with performance in the domain of geometry, whereas differences in numerical discrimination were uniquely associated with performance of symbolic arithmetic, suggesting a more specific role for the ANS in mathematical reasoning (Lourenco et al., 2012).

On a third family of views, the relationship between the symbolic and non symbolic number is mediated by other general cognitive operations or abilities common to both tasks Fuhs & McNeil, 2013; Gilmore et al., 2013; Holloway & Ansari, 2008; Lyons & Beilock, 2009, 2011). Several recent studies, for example, provide evidence that the relationship between number comparison and mathematics achievement could be explained by variation in general inhibitory ability, rather than ANS acuity Fuhs & McNeil, 2013; Gilmore et al., 2013). Other studies have found that domain-general cognitive operations, like the ability to compare one quantity to another, account for a significant portion of individual variation on non-symbolic number tasks (Holloway & Ansari, 2008). In one study, for example, the relationship between performance on a symbolic and a non-symbolic numerical task was mediated by symbol-ordering operations (Lyons & Beilock, 2009).

These studies suggest that the relationship between the ANS and mathematics may be mediated by more general-purpose cognitive operations, such as ordering, comparison, or addition, common to both symbolic and non-symbolic tasks, or more domain general cognitive abilities such as inhibitory or executive control.

In sum, previous work shows clear correlations between performance on tasks that involve the ANS and symbolic mathematics performance (e.g., Gilmore et al., 2010; Halberda et al., 2008; Libertus et al., 2011; Lourenco et al., 2012) and some evidence of a causal relationship between ANS training and symbolic mathematics performance in adults (Park & Brannon, 2013). However, the mechanisms responsible for this relationship remain unclear and are highly debated. Furthermore, it is unclear from previous research if symbolic mathematics is dependent on the ANS in children, without years of associations between the symbolic and non-symbolic systems. We addressed these questions by assigning children to participate in one of several training conditions, each aimed at engaging a particular mechanism hypothesized to explain the relationship between the ANS and mathematics, and then subsequently tested the groups on exact, symbolic arithmetic performance. If the ANS contributes to the cognitive mechanisms responsible for symbolic arithmetic in children, then engaging the ANS may enhance children's subsequent symbolic arithmetic performance.

Research evidence regarding brain areas involved in number, size, brightness and space from neuropsychological and neuroimaging studies suggests that for magnitudes of different kinds, there is both activation overlap and segregation in the brain regions involved in processing different dimensions of magnitude. Common effects of magnitude processing are found in region along right IPS (Intra Parietal Sulcus) and processing of discrete numerical magnitudes involves region of left anterior IPS (Cohen Kadosh et al., 2005; Dormal & Pesenti, 2009; Kucian et al., 2011; Pinel, Piazza, Le Bihan, & Dehaene, 2004; Vogel, Grabner, Schneider, Siegler, & Ansari, 2013).

Neuropsychological evidence. Neuropsychological research suggests strong relationship between ANS and symbolic numerical representation. Horizontal segment of intraparietal sulcus (hiPS) has been observed activated in various number processing tasks, in notation independent format suggesting an abstract coding of numerical magnitudes (Piazza, Pinel, Bihan, & Dehaene, 2007).

Comparison of both symbolic and non-symbolic were impaired after repetitive Transcranial Magnetic Stimulation (rTMS) to left IPS but enhanced by rTMS to the right IPS. A signature effect of numerical distance was found (greater impairment when comparing numerosities of similar magnitude (Cappelletti et al., 2007).

Dehaene, Piazza, Pinel & Cohen, (2003) reviewed and contrasted neuroimaging studies to investigate how the parietal activations reported by various studies relate to one another in cortical space focusing three regions, *horizontal segment of intraparietal Sulcus* (HIPS), *Angular gyrus* (AG), *Bilateral posterior superior parietal lobe* (PSPL). They proposed that HIPS is systematically activated in processing and representation of both symbolic and non-symbolic numerical magnitudes. Left AG area in connection with other left hemispheric perisylvian area, support manipulation of number in verbal form. PSPL system support attentional orientation on the mental number line. According to Stevenson and Stigler (1992) HIPS, AG PSPL are systematically activated in different subjects from different countries and with different educational strategies.

Cantlon, Brannon, Carter, and Pelphery (2006) conducted a neuroimaging study through FMRI adaptation paradigm on adults and 4 year old children to investigate whether an early developing neural basis for human numerical processing is essential for understanding cognitive origin of uniquely human capacity for math and whether the neural locus of non-symbolic numerical activity in adults show continuity in function over development? Result provided evidence that there is an important neurobiological link between symbolic and non-symbolic numerical cognition in adults. Study revealed that IPS is recruited for non-symbolic numerical processing early in development before the start of formal schooling.

Correlational. Research conducted by Halberda, Mazzocco, and Feigenson (2008) has shown that there are large individual differences in non-verbal approximation abilities of 14 year olds and these individual differences correlate with their past scores on standardized math achievement tests extending back to kindergarten. This correlation remains significant when controlling for individual differences in other cognitive and performance factors. Moreover, individual performances in the mathematics achievement of kindergarten children are related to individual differences in the acuity of their evolutionarily ancient unlearned approximate number sense.

Children's performance on non-symbolic arithmetic predicted their mathematics achievement and was related to their mastery of number words and symbols (Gilmore, McCarthy, & Spelke, 2010).

Mazzocco, Feigenson, and Halberda (2011) investigated whether precision of approximate number system (ANS) measured prior to entering in schools predicts later school mathematics or not. They measured children’s performance on ANS at 3-4 years of age through a non-symbolic comparison task and tested two year later same children’s performance on mathematics. Results revealed that ANS selectively predicts performance on school mathematics later at 6 years of age. Results on others tasks showed that this association is not explained by general full-scale IQ. It appears specific to mathematics, since no such association emerged for ANS precision and measures of expressive vocabulary (i.e., WASI), perceptual organization (i.e., Block Design, Matrix Reasoning), or non-numerical lexical retrieval (i.e., RAN Colors and Letters).

Non-symbolic and symbolic numerical magnitude representations have been shown to be related specifically to standardized math achievement tests (Booth & Siegler, 2006; 2008; Halberda et al., 2008; Holloway & Ansari, 2008; Laski & Siegler, 2007).

Vanbinst, Ghesquière, and De Smedt (2012), investigated numerical magnitude representations and individual differences in arithmetic strategy use with 8 year, 10 month old third grade children. Specifically they investigated whether numerical magnitude representations per se or its access through symbolic digit is important for math achievement. Result revealed that symbolic but not non-symbolic numerical magnitudes skills are correlated with individual differences in math. Children who can better access magnitude representations from symbolic numbers were better in retrieving more facts from memory and were faster in retrieving facts and using strategies after controlling for intellectual ability and general math achievement.

Bonny and Lourenco (2013) conducted study with children 3-5 year old and tested them on non-symbolic number discrimination task, standardized math achievement task and standardized vocabulary task. Their results support previous researches by showing that ANS precision was correlated, significant predictor of math competence and this relation was nonlinear after controlling for vocabulary performance.

Nosworthy, Bugden, Archibald, Evans, and Ansari (2013) conducted a study and demonstrated a relationship between performance on a basic magnitude processing task and individual differences in math achievement. In a two-minute paper and pencil test of symbolic and non-symbolic numerical magnitude processing of 1-3 grade children's performance on non-symbolic items correlated with their arithmetic skills. A significant positive relationship was found between Math fluency, calculation and accuracy with which participants solved symbolic items, non-symbolic items and on magnitude comparison task.

Mejias and Schiltz (2013) investigated numerical magnitude representations of second grade (4-5 years old) and third grade (5-6 years old) children using symbolic and non-symbolic output formats at two points in kindergarten. Results revealed that approximate numerical representations were linked to exact number competence in young children before the start of formal math education. In second and third grade the non-symbolic exact numerical task, correlated with non-symbolic estimation task but non-symbolic task did not correlated with symbolic estimation task in either group.

Dyscalculia research. Low math achievement gives rise to disadvantages to not only to the person and society but also to the nations at large. As person with low math achievement has less chances to be functional and productive for himself/herself and for society. Research has pointed out one important cause of low math achievement also called dyscalculia is having impairment in approximate number system.

Rousselle and Noel (2007) carried out a study with second grade children with math difficulties (MD), math and reading difficulties (MD/RD) and normally achieving (NA) children. Results revealed that children with math difficulties (MD) were slow, less accurate than NA when comparing Arabic digits (symbolic number magnitude), but they were able to compare non-symbolic number magnitude comparison task as well as NA children showing a ratio effect. Results showed that low accuracy of MD children indicate that they were slower to access number magnitude from symbols and a core deficit in MD is difficulty in relating numerical symbols to their meaning. Finding suggests that children with MD do not have problem in processing in number rather in accessing semantic information from numerical symbols.

De Smedt and Gilmore (2011) carried out research study with first grade, low achieving (LA), mathematics learning disabilities (MLD) and normally achieving (NA) children. Results showed that LA, and MLD children demonstrated impairment on task required accessing the magnitude representation from symbols as compare to NA children. LA children showed better performance than MLD. Where as, no group's differences were found on task involving non-symbolic numbers. This evidence suggests that processing the symbolic numbers requires accessing/recruiting the non-symbolic numbers and if they cannot map on in this direction children show impairments in math.

Mazzocco, Feigenson, and Halberda (2011) carried out research with 9^{th} graders 14-15 year old adolescents with math learning disabilities (MLD), typical achieving (TA), high achieving (HA) and low achieving (LA) compared their performance on psychophysical assessment of approximate number system measuring weber fraction (w) and on mapping of approximate number system and number words while controlling for domain general cognitive abilities. Results have shown that MLD had significantly high w (low ANS acuity) as compare to TA, HA, LA groups and significant poor mapping of ANS and number words relative to TA, HA, LA groups suggesting that poor ANS acuity underlies MLD and low math achievement Although this finding contrast with previous studies showing no difference between typically achieving and MLD children on non-symbolic numbers but this particular study has compared children performance on ANS through psychophysical assessment so it might be a more sensitive measure of children performance.

Piazza et al. (2010) has investigated the relationship between number sense and dyscalculia. Study has compared normally developing kindergarten, school age children and adults with dyscalculia. Subjects were matched in terms of age and IQ on a psychophysical task. Results revealed that number acuity was severely impaired in dyscalculic children as compared to normally developing children and their poor numerical acuity was reflected in symbolic number comparison task as well.

So these results show that number sense is specifically linked to symbolic number processing as dyscalculic children's impaired performance was exhibited compared to normally developing children.

Training evidence. Käser et al., 2013 conducted a study involving children with difficulties in learning mathematics. Children completed 6-12 weeks computer training of 20 minutes per day for 5 days. They evaluated effects of training using neuropsychological tests. Results showed that children benefitted significantly from the training regarding number representation, arithmetic operations and reported that training improved their mathematical abilities.

Number line placement and symbolic math. Siegler and Booth (2004) conducted a study with Kindergarten, first and second grade children and gave them 0-100 number line placement task, and math achievement test. Children predominantly relied on logarithmic (kindergarten), mixture of logarithmic and linear representations (first graders) to predominantly linear representations (second graders). Accuracy of their estimates on number line task correlated with math achievement at all three grade levels.

Booth and Siegler, (2006) gave 0-100 number line problems to kindergarten, 1st, 2nd, 3rd to solve in experiment 1 and 0-1000 number line problems to 2nd graders to solve in experiment 2. Children were given four types of estimation problems: computational, numerosity, measurement and number line problems. Results showed children's increasing reliance on linear representation of numbers and decreasing reliance on logarithmic representations. Furthermore, all types of estimations skills were positively related to math achievement test scores.

Numerical representation progress from logarithmically increasing function to linearly increasing function in children (Laski & Siegler, 2007; Siegler & Booth, 2004). Thomson and Siegler (2010) reported that children's more linear magnitude representation are closely related to their memory of the number approximated. As children who generally used linear representations of numerical magnitudes recall number better than those who use logarithmic representations.

Numerical (dot arrays) and non-numerical quantities (length magnitude) are represented in highly overlapping areas of the horizontal intraparietal sulcus (IPS) has been confirmed in single-cell recording studies. Monkeys were trained to discriminate either different numerosities or different line lengths; IPS neurons responded to length, numerical magnitude or both, indicating that there is both discrete (numerical) and continuous (non-numerical) coding of magnitude in the IPS, even at the single-cell level. Furthermore, these results suggest that there is no clear topographical segregation between populations of neurons that respond to either discrete or continuous magnitude (Ansari, 2008).

Individual differences in estimation ability are strongly related to general measures of mathematical proficiency, such as achievement test scores, and to arithmetic, numerical categorization, and numerical magnitude comparison (Booth & Siegler, 2008; Laski & Siegler, 2007). Moreover, early estimation skills predict later mathematics success (Chard et al., 2005; Jordan, Kaplan, Olah, & Locuniak, 2006).

Cross-cultural context. Pica, Lemer, Izard, and Dehaene, (2004) investigated numerical cognition in Amazonian culture in native speakers of Munduruku´ (An Amazonian language). Munduruku is language of Tupi family in Para state of Brazil. These people lack number words and have number words only for 1 through 5. Study involved monolingual adults and children without instruction and comparison group of more bilingual and educated French participants). Initially they tested them for competence for numbers in the absence of a well-developed language for number. Participants were shown displays of 1 to 15 dots in randomized order, and were asked in their native language to say how many dots were present. Participants relied on approximate quantifiers (some, many or a small quantity) and did not used numbers words to refer to precise quantity except they used words for 1 and 2. They were also tested on number comparison task and Munduruku participants responded far above chance level in all groups. They were also tested on non-symbolic approximate addition task (independent of language) to and again all group of participants performed above chance level. At the end they gave Munduruku exact subtraction task. Participants were asked to predict the outcome of a subtraction of a set of dots from an initial set comprising one to eight items. All groups performed, much worse than the French controls and their failure was not result of misunderstanding of the instructions, because they performed better than chance when the initial number was below 4 rather Munduruku appear to lack, a procedure for fast apprehension of exact numbers beyond 3 or 4. These studies shows that people belonging to different culture performed equally well on approximate number system tasks. However, they failed when tested for exact numbers due to the fact that exact numbers system was not well established so does their language for number words. However if they would have well established number words for exact number beyond 4; they might have performed equally well. It has important implication for learning language and number symbolic number systems for such populations and overall generally given the utmost importance of symbolic number system in human development.

Research studies conducted with Pirahã tribe, who use a ‘‘one-two-many’’ system of counting, were able to use analogue magnitude estimation but showed limited ability to enumerate exact quantities when set size exceeds two or three items (Gordon, 2004). Whereas, Pirahã speakers without any linguistic method for expressing exact quantity were able to perform exact matches with large numbers of object perfectly but were inaccurate on task involving memory (Frank, Gibson, Fedorenko, & Everett, 2008).

Despite of the cultural additions and interventions on numerical abilities, there is wealth of research evidence suggesting that ANS might serve as building block for symbolic formal math. ANS acuity (measured through comparison of dot arrays) correlate with measures of symbolic math in adults (Dewind & Brannon, 2012; Halberda et al., 2012; Halberda, Mazzocco, & Feignson, 2008; Lyons & Beilock, 2011).

According to Piazza, Pica, Izard, Spelke, and Dehaene (2013), number sense is rough in cultures without symbols for exact numbers and it is more precise in people who are introduced to the concepts of exact number and calculation. Research evidence suggests that culture and education have important effect on basic number skills.

Researchers conducted the study to disentangle the effects of maturation and of education on ANS acuity. Study involved two groups: a group of Munduruku children and adults that included participants, who have received no education and those who had received some years of schooling. They gave one group of participants varying from no schooling to several years of schooling (38 participants, 21 males and 17 females, ages 4-63) number comparison task and to other groups also varying from no schooling to several years of schooling (33 participants, 20 males 13 females, ages 4-67) size comparison task. Results indicated that education significantly enhances acuity of the ANS and this relationship is independent of maturation thus suggesting that education plays significant role in sharpening the sense of approximate numerical acuity. Effects of education on ANS are not generic effect of schooling rather it is specific effect of numeracy instruction.

Gap in Previous Research

Previous research findings shows the evidence that tasks involving purely symbolic numbers and exact arithmetic reveals signatures of non-symbolic, approximate number representations (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Moyer & Landauer, 1967; Piazza, 2010; Van Opstal et al., 2008). In adults and children overlapping brain areas (e.g., IPS) are activated during processing of symbolic and non-symbolic numbers independent of notation (Piazza, 2010; Dehaene, Piazza, Pinel & Cohen, 2003; Piazza, Pinel, Bihan, & Dehaene, 2007). Further more research evidence shows that individual difference in ANS acuity correlated with mathematics achievement (Bugden & Ansari, 2011; DeWind & Brannon, 2012; Gilmore et al., 2010; Halberda et al., 2008; Halberda et al., 2012; Libertus et al., 2011, 2012; Lourenco et al., 2012).

However, it is unclear whether it is due to individual differences in ANS acuity playing causal role in creating individual differences in mathematics development or is it symbolic math development that causes changes in ANS acuity or whether a third factor such as differences in facility of operations on number symbols or differences in executive function explain this relationship? Studies showing that individual differences in ANS acuity predicts future mathematics achievement (Gilmore et al., 2010; Libertus, Feigensen, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011) does not show that individual differences in ANS acuity cause later changes in math performance because both earlier differences in ANS acuity and later differences in math learning could depend on one or more additional common factors. Practice or training studies are also not conclusive in their findings.

Present study was carried out to elucidate these questions and to clarify the mechanism responsible for this relationship.

The present study addresses the limitations directly in several important ways to provide new insights in to numerical cognition. In Phase 1, the relationship between ANS and symbolic math was directly addressed. Above mentioned questions were addressed by assigning children to participate in one of the several training conditions (each aimed at engaging a particular mechanism) hypothesized to explain the relationship between ANS and the mathematics and then tested the children on exact symbolic arithmetic. If ANS contributes to the cognitive mechanisms responsible for symbolic arithmetic in children, then engaging the ANS might enhance children's symbolic arithmetic performance.

Furthermore, to investigate whether engagement of the ANS enhances subsequent cognitive performance more generally, we compared the effects of one numerical and one non-numerical training task on children's performance within and outside the domain of mathematics.

In Phase 2, these possibilities were tested by first, attempting to directly replicate the co-activation effect found in Phase 1 with children showing enhanced symbolic addition performance without change in the ANS itself. Next, it was tested whether this effect is associated with change in symbolic number representation, an aspect of training not yet tested. Finally, to assess the generalizability of such training routines, all studies were conducted in a novel, non-Western population (Pakistan) where access to technology and cultural values related to mathematics education differ substantially from those populations sampled in phase 1.

Rationale of Present Study

Researcher got particularly interested to work on this particular topic as a result of previous research experience, which showed that math is full of problem solving opportunities as well as very important subject with long term effects on individuals' life. Furthermore researcher got the chance to work at Lab for developmental studies at Harvard University USA and it gave very focused platform to work in this particular research area under the supervision of world-renowned developmental psychologist, Prof. Elizabeth Seplke who had decades of experience in working with numbers.

Extensive research data in last few decades have been reported showing evidence for a specific relationship between non-symbolic and symbolic numerical representations. Studies (e.g., De Smedt & Gilmore, 2011; Halberda, Mazocco, & Feigenson, 2008) have shown that non-symbolic representations are specifically correlated and predictor of later math achievement. But no study has yet tested directly the causal relationship between symbolic and non-symbolic numerical cognition. Present study focused specifically on the causal role of non-symbolic in symbolic number processing. It would be interesting to figure out the potential role of non-symbolic numbers in the development of later math abilities.

Study was carried out under experimental paradigm with first grade children from USA and Pakistan. First grade children were chosen for two reasons. First of all, it is the very basic stage where children start learning operations on numbers like addition, subtraction, multiplication and division. Secondly, it is the very basic stage where they start mapping between two systems formally.

It has great implications not only for the field of cognitive science but also in cognitively psychology and above all for educational improvements. It would be a great milestone particularly for children suffering from low math achievements and children with dyscalculia might be trained and get benefits in improving their number processing skills. Since math is very basic knowledge to survive and grow in environment and it's a great human endeavor through which humans are capable of making progress in every aspect of life.

Above all, children learning math in developing countries like Pakistan will be benefited from the possible advantages of non-symbolic training. It might give chance to these children improving their number related abilities at a rapid pace from early years so that they could perform better and could uncover the great human capacity of learning and processing of numbers through most effective ways.

Purpose of cross-cultural perspective is two fold. First of all, it would be very interesting to see whether priming with non-symbolic numbers drive same effect as reported in phase 1. Secondly, keeping in view huge differences between American and Pakistani children (in terms of environmental stimulation, economic differences, exposure to media and technology, differences in the level and quality of learning) it would be very informative how these factors influence children numerical capacities. Since symbolic numbers development is very much effected by cultural learning patterns and language and it in turn effect approximate number system. Moreover, how same training might drive effects in two populations?

There are no training studies conducted in Pakistan on the role of ANS in symbolic math and number line placement. To bridge this gap this study was carried out to see role of ANS in symbolic math with Pakistani first grade children.

Chapter II

METHOD

Major Research Question

Major research question for present research was to investigate whether training the children with non-symbolic numbers (approximate addition or comparison of arrays of dots) will give them any advantage in solving and processing symbolic numbers (addition problems) as compare to the control group? Non-symbolic numbers have been shown through wealth of research evidence to have a specific relation to symbolic number processing so it might improve mapping between approximate number systems and symbolic system.

It was assumed that if non-symbolic number system plays the foundational role in acquisition of symbolic numbers, then children trained in non-symbolic addition / comparison should show advantage in solving symbolic addition problems as compare to children trained with brightness comparison and line length addition.

In phase 1 of research children were tested from Massachusetts, USA under different training conditions and posttest measures. In Phase 2, a replication and extension of phase1 was conducted. A cross cultural perspective was given to see whether the training effect seen in USA children would be reflected in Pakistani children or not? To investigate cross-cultural perspective in phase 2, experiments were conducted on Pakistani sample keeping the training procedure same as in USA children (in experiment 1 of phase 2).

Instrument's Details

Children participated in one of four training conditions: a non-symbolic numerical addition task, brightness comparison task, a line-length (or area) addition task, or a non-symbolic number comparison task. Each condition targeted the engagement of a particular non-symbolic magnitude skill hypothesized to play a role in symbolic mathematics. In all these conditions, children practiced adding or comparing approximate, non-symbolic magnitudes. During (after 50 training trails and immediately after the training task (after additional 10 trails), children were asked to complete a symbolic addition test worksheet to assess the effects of the training task on the speed and accuracy of symbolic mathematics. Finally, at the end of the experiment, children's approximate numerical acuity was measured (Halberda et al., 2008).

Training tasks. The training tasks comprised of 3 sets of training trials in total 8 trials, accompanied by directions to play the game followed by 60 test trials (30 trials of ratio 7:4, 30 trials of ratio 7:5). Children attempted first easy trails (7:4 ratio) and then difficult trials (7:5 ratio) but across the subjects within each category trials were presented randomly.

Training tasks involved two experimental conditions (Non-symbolic approximate addition and comparison) and two control condition tasks (Brightness comparison and approximate line addition task). Key independent variable is practice of non-symbolic stimuli involving approximate number system and dependent variable is symbolic addition problems to see effect of practice on symbolic number processing.

1- Non-Symbolic Approximate Addition Task. One condition involved numerical addition of non-symbolic dot arrays (see Barth et al., 2005, 2006; Gilmore et al., 2010). In this condition, children were asked to estimate the numerical sum of two sequentially presented arrays of dots (addends) and judge whether an outcome array was more or less numerous than the actual sum. Previous research has shown that performance on this task correlates with mathematics achievement scores in young elementary school children (see Gilmore, McCarthy, & Spelke, 2010). Furthermore, a recent training experiment with adults showed that practice with this task improved symbolic arithmetic (Park & Brannon, 2013). In addition to requiring the engagement of the ANS, this task may require transformational operations at the core of symbolic arithmetic concepts, making it an ideal task to engage cognitive mechanisms in common with those used for symbolic mathematics (Barth et al., 2005; Gilmore et al., 2010). If the ANS and/or the arithmetic operations involved in non-symbolic addition overlap with those used in symbolic arithmetic, then we might observe enhanced performance on symbolic addition in children who first practice non-symbolic addition compared to children who practice tasks involving other quantities or other operations.

Non-symbolic approximate addition have been employed as adding two sets of arrays of dots and comparing their sum to third Foil) array and analyzing whether third set is more or less than the addition of previous two sets. Non-symbolic addition addends arrays were white dots ranged from 7-56 dots in each array on black background comprising of easy and difficult ratios (Appendix A & B).

Procedure of gameplay. The experiment was carried out in context of a computer game in which two gender specific cartoon characters (one bad, one good) like to play with marbles (dots) and colored blob for brightness comparison condition. Children were told in the start of game that first it is experimenter's turn to play the game while describing the directions to play the game and then it will be their turn to play the game themselves. Initial two set of training tasks trials (without occluder) were played by experimenter, while explaining the corresponding game to children. Whereas for third set of training trials children were told now it's their turn to play the game and the test trials themselves. Experimenter encouraged the children to ask any question regarding the game and answered the questions.

To play the game they were introduced with three keys on the separately attached keyboard, one key showing the small picture of good person second key showing the picture of bad person and 3rd key (space bar) for starting the game and hitting for each next trial. Children played the game in the following way.

Practice trials set 1. In first set of training trials (1 for good person, one for bad person), children saw a set of marbles appeared on the left side of screen and then another set of marbles appeared on right side of screen while listening to the story and instructions explained by the experimenter. Then both set moved towards the center of screen, merging and resulting in the third set of marbles that was either less or more than addition of first two sets. Experimenter showed surprise looking at marbles e.g in case of more and saying now there are more marbles, I wonder why that happened” and guessed saying “ I think good person must have put more marbles” and pressed the corresponding key (with picture of good person). Feedback beep indicated whether response was right or not, in case of right response experimenter verbally confirmed that it was good person.

In case of less marbles, experimenter surprisingly saw the screen saying, “now there are less marbles I wonder why that happened, I think bad person must have played with marbles and took away marbles and pressed the corresponding key. Through the feedback beep it was confirmed that it was right answer or not. So in this way children got the idea that if the third set is less than the combination of first two sets, it is due to bad person and if third set is more than the combination of first two sets it must be good person.

Practice trials set 2. Following the almost same directions and sequence of activities as in training set 1 (1 for good person, one for bad person), children were introduced with a yellow occluder and two sets of marbles moving behind the occluder. When the occluder disappeared they saw third set and guessed who had played with marbles?

Practice trials set 3. In third set of training (2 for good person, 2 for bad person) children were told that now they will be playing the game on their own. After the third set of training trials they were introduced with picture of a cylinder on a laminated sheet that was representing their progress in the game. So as they were making progress in the games, experimenter use to tell them by filling that cylinder with a colored marker in 3 or 4 steps so that children keep track of their progress in the game. Children attempted the 50 test trials themselves. After each trial they got feedback through beep and the nature of feedback itself indicated whether their guess was right or not.

Practice trials timings. On training trials without occluder, children in experimental group saw a black blank screen for 1000ms until experimenter hit the space bar and they saw first array on left side of screen of white dots on black background for 4000 ms after hitting the space bar. Then they saw second array of dots appeared on right side of the screen for 3000 ms. Then both set of dots moved in the center of the screen and visible for 3000ms. After that some more dots got added in the previous two sets making them more for 6000ms. Children saw the resulting set of more dots for infinite time to respond.

Timing of training trials of non-symbolic approximate addition task. In test trials, children saw a yellow occluder on black background screen for 500 milliseconds (ms). Then they saw first array of dots for 1000 ms on left side and then this array of dots moved towards middle of screen, behind the yellow occluder taking 500 ms. It paused for 300ms, showing occluder then they saw second array of dots for 1000ms on the right side of the screen and then this array of dots moved in the middle behind the screen in 500ms. There was pause showing yellow occluder for some time

1250 ms. Then the screen disappeared and they saw third array of dots for 5000ms, until response and were required to guess, whether this array of dots is less or more than previous two being added together. If they think that there were fewer dots, they were instructed to hit the bad person's key or if they think that third array is more so they were instructed to hit good person's key. Once, children answered by hitting the key either correctly or incorrectly, they got the feedback through a beep for 500ms, indicating the accuracy or inaccuracy of their response. Children had to hit the space bar for each next trial so that hey could play the game according to their own pace (See Figure 1).

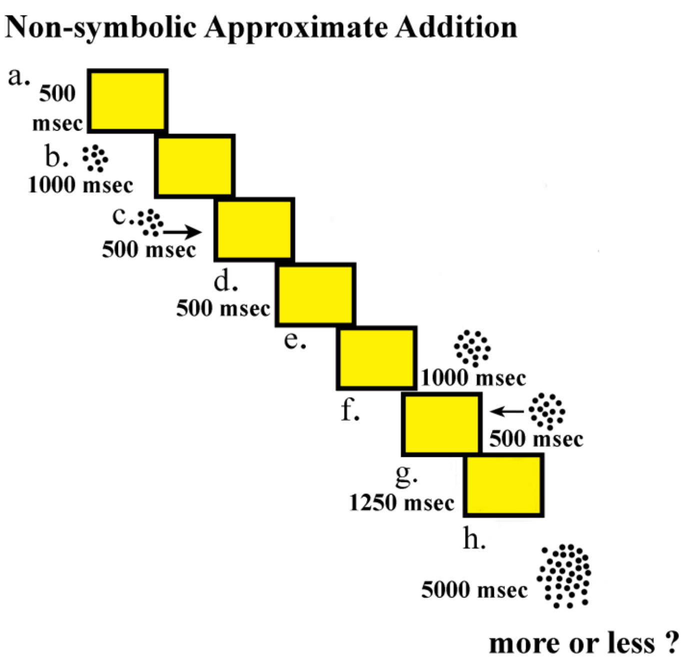

Figure 1. Schematic depiction of Non-symbolic approximate addition training task

Non-numerical stimulus controls. Two sets of numerical stimuli were created for each numerical value used in the non-symbolic training problems. One set of numerical arrays was equated on intensive parameters (individual item size and average inter-item spacing) across all numbers used and varied on the extensive parameters (total occupied area and total luminance). The other set was equated, on average, by number in the extensive parameters, but varied in the intensive parameters by number. For each numerical value in each set, 5 different numerical arrays were produced that varied in position and spacing of individual dots on the screen. Such controls have been employed in other studies of numerical cognition (Hyde & Spelke, 2009, 2011; Hyde & Wood, 2011; Izard et al., 2008; Piazza et al., 2004, 2007). For each numerical array needed in the numerical training programs (addend 1, addend 2, and foil array for the non-symbolic approximate addition condition; array 1 and comparison array for numerical comparison condition), the program randomly chose either the set equated on extensive parameters or the set equated on intensive parameters. Furthermore, the program randomly chose 1 of the 5 images produced for each numerical value. Thus, there was no systematic relationship between images chosen in the set, for each component of each trial, and each trial was very likely a different combination of images for each subject in the study.

2- Brightness Comparison Task. A second condition involved comparing the brightness magnitude of two objects. Cognitive and neural overlap between representations of numerical magnitudes and brightness magnitudes has been highly debated (see Lourenco & Longo, 2011; Walsh, 2003). Some evidence suggests brightness to be included with space and number in the generalized magnitude system (e.g., Cohen Kadosh & Henik, 2006a, 2006b, 2006c), whereas other evidence suggests it may be distinct (e.g., Pinel, Piazza, Le Bihan, & Dehaene, 2004). If previously observed associations between the ANS and symbolic mathematics development are due to commonalities in processing and comparing magnitudes in general, then no differences should be observed in symbolic arithmetic performance between the children in any of the training conditions. On the other hand, if approximate number or length representations play a functional role in symbolic arithmetic, then better performance may be seen in conditions where the ANS or length is engaged than in cases where brightness is engaged.

Brightness comparison have been employed as comparing the brightness of initial blob and then the resulting circle's (after shrinking of blob) brightness to analyze whether circle's is brighter or darker than that of the blob's brightness.

Practice trials Set 1. In first set of training trials, children were introduced with two gender specific cartoon characters (one good one bad) and experimenter gave the direction to play the game. They saw a colorful blob appeared on the screen and then the blob started shrinking first from left side and then from right side eventually turning in to a circle. Then the color of the circle changed either brighter or darker.

Like in non-symbolic training instruction, experimenter in this game ascribed dark color to the bad character and brighter color to the good person following same directions.

Practice trials set 2. In the second set of training trials they were introduced with a yellow rectangle occluder, the colored blob appeared behind the occluder visible from sides of screen and shrink from both sides and turned into circle. When the yellow occluder disappeared the circle appeared either of darker or brighter color than the blob. Children had to guess who have played with the circle either bad person or good person. They got feedback of their response through beep and experimenter confirmation, and the nature of beep itself indicated whether their response was right or not.

Practice trials set 3. Children were told that now they will be playing the game themselves, so followed by the third set of training trials (as in non-symbolic training condition), they got introduce with the cylinder showing their progress in the game and children attempted test trials of game.

***Timings of practice trails of brightness comparison task*.** Children in the control group saw a black blank screen for 12000ms followed by a colorful blob like an ellipse for 8000ms. Then the blob started shrinking first from left side and then from right side on successive steps, each step taking 200ms. Blob eventually turned into a circle and children saw it for 5000ms. Then the saw that circle changed its color either lighter or darker and to respond for infinite time.

***Brightness comparison training task*.** In test trials, children saw a yellow occluder on black background screen for 500 ms. Then they saw colorful blob of 600 by 40 pixels behind the occluder for 2000ms. Then the blob started shrinking, first from left side taking 4 successive steps each side taking 200 ms so in total it took 800ms to shrink from left side and hide behind the screen. Sizes of the shrinking blobs were following, 500 by 48 pixels (1), 400 by 60 (2), 300 by 80 (3), 250 by 96 pixels (4). Then there was intermission of 600ms showing occluder and right side of blob. After that blob started shrinking from right side taking 4 successive steps each comprising 200ms, so right side of blob took in total 800ms to shrunk and hide behind the screen. Sizes of shrinking blobs were same as on left side. Blob was completely behind the yellow occluder and children saw occluder for 500ms. Then yellow occluder disappeared, and children saw a circle of 155 by 155 pixels of either light or darker color that remained on the screen for 5000 ms for response. They were required to guess whether the color of the circle is lighter or darker than the blob by hitting the key. They were instructed to hit the bad person’s key if they think that circle is darker than before or hit good person’s key if they think that circle is lighter than before. Children got the feedback with beep for 500ms (See Figure 2).

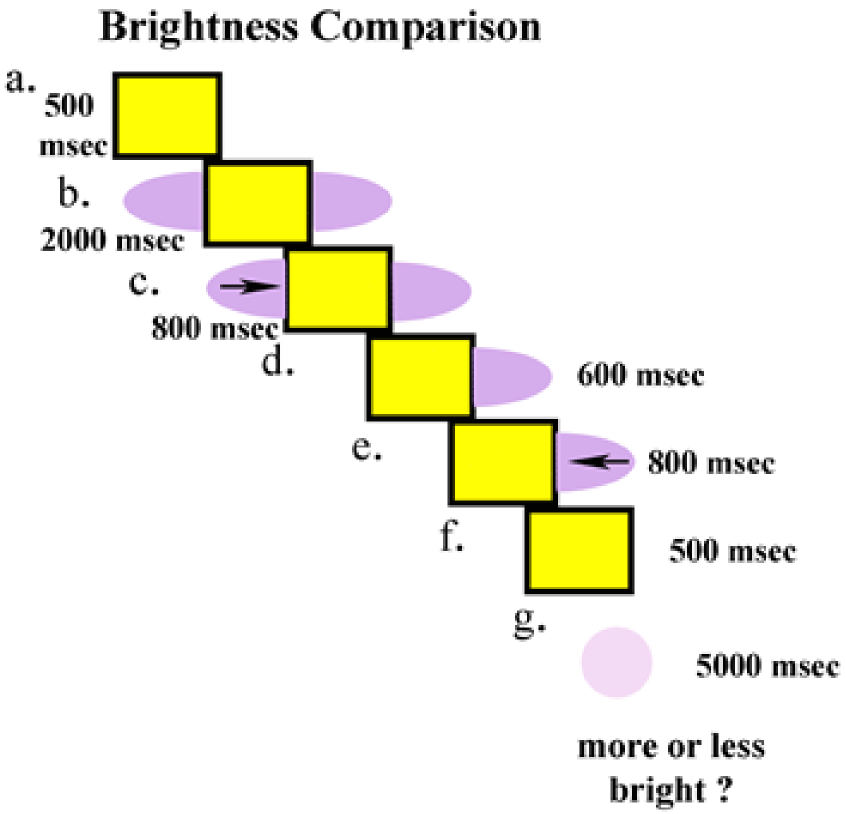

Figure 2. Schematic depiction of Brightness comparison training task

Brightness comparison task's generation of stimuli. Eight, equally spaced degrees of brightness were created (ranging from dark to light) in Adobe Photoshop by changing the brightness scale from 30-100 (brightness values were the default values used in the Photoshop scale). Array with brightness 30,40, 50, 60, 70, 80, 90, and 100 (saturation as 30 across all color arrays) were created for different colors.

Hue and saturation were kept constant so that difficulty of comparisons would be based on relative brightness rather than changes in color. The brightness values of 50, 60, and 70 were used as the standard values for the object presented at the start of brightness training problems. Comparison (or test objects) ranged in brightness from 30-100.

Ratio was manipulated approximately in a similar way as in numerical and other non-numerical magnitude training conditions as relationship between the brightness value of the initial object and the test object. Easier problems involved comparisons of 30 or 90 to 60 and 40 or 100 to 70; harder problems involved comparison of 30 or 70 to 50 and 40 and 80 to 60.

Non-Symbolic addition problems were equal to brightness comparison in these terms. There were five blocks of non-symbolic addition problems, where a number was once a comparison number and second time that number was split to make two addends out of it. Similarly in brightness comparison there were five colors and from each color four-brightness comparison problems (2 lighter and 2 darker color) were generated, by taking two arrays as blob and four arrays as comparison colors, all based on same color but with different brightness. No comparison array of color was repeated as addends in non-symbolic numbers except the comparison number or blob in both conditions. Two color array served as comparison color (e.g., blob1 and blob2 in color condition) and two numbers (e.g., 24 and 42 in above table) served as comparison number. So these problems in both condition was tried to be equal on all levels.

In easy brightness comparison condition each color array with 60 and 70 Brightness was chosen to present as a blob and on both sides of that array comparison color was chosen with brightness difference of 30.

Where as in difficult brightness comparison condition array with brightness 50, 60 and 70 was chosen as blobs and their comparison color array with brightness difference 20 on each side across colors chosen colors for difficult comparisons.

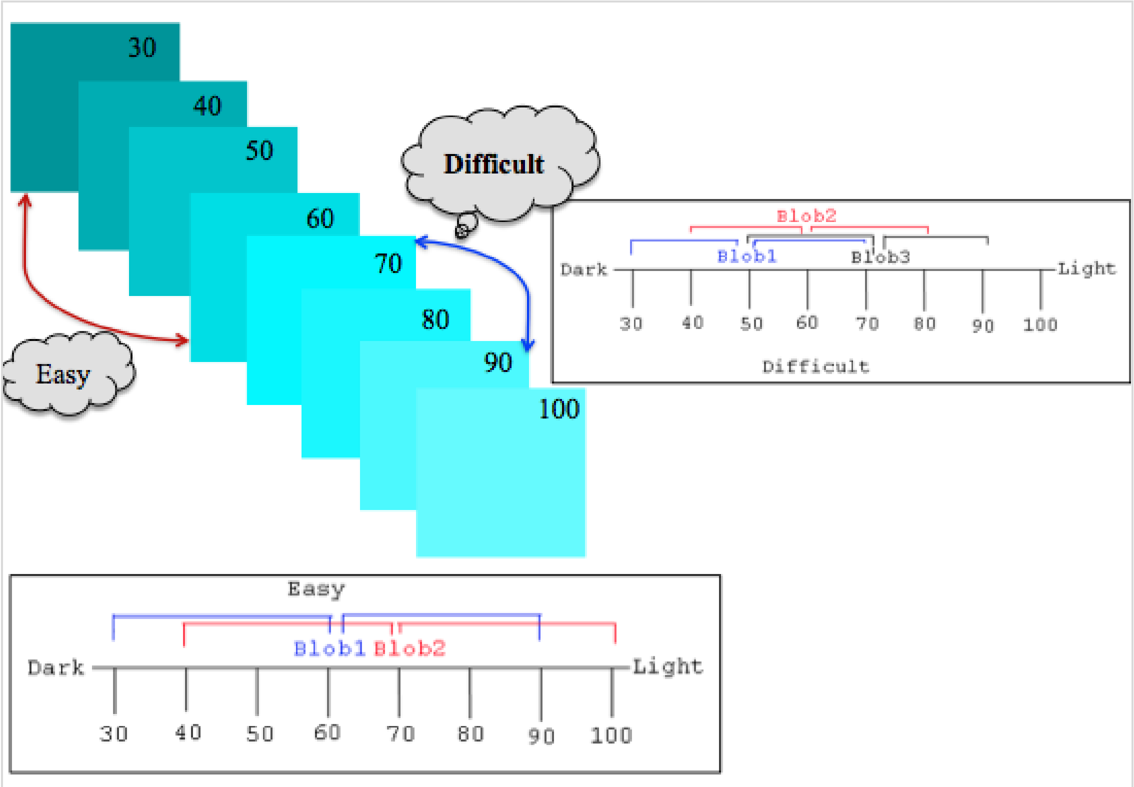

Figure 3. Schematic depiction of generation of stimuli for Brightness comparison training task

Sizes of the blobs. Brightness comparison task stimulus was a colored blob that changes it shape and turn into a circle. The blob changed it shape and turned into a circle by shrinking on successive steps from left and then from right side to make it comparable to presentation of non-symbolic stimuli. The blob and successive blobs sizes were as following (first number is width and second number is height). First blob of 600 by 40 pixels, second blob of 500 by 48 pixels, third blob of 400 by 60 pixels, fourth blob of 300 by 80 pixels, fifth blob of 250 by 96 pixels and circular blob of 155 by 155 pixels.

To display these blobs as shrinking from left and right they were cut in equal half and aligned properly. Same pattern was followed to create blobs and circles for other colors (See Figure 3).

3- Line Length Addition Task. This condition involved addition of line lengths (i.e. spatial extent). This condition was equal to the non-symbolic numerical condition in terms of timing, difficulty, and cognitive demands, but involved the addition of spatial magnitudes rather than numerical magnitudes. This condition was motivated by the generalized magnitude system hypothesis (Lourenco & Longo, 2011; Walsh, 2003), as well as by recent findings of a relationship between spatial magnitude representation and achievement in mathematics (Lourenco et al., 2012). If generalized magnitude representations drive the link between symbolic mathematics and performance on tasks involving the ANS, then practice adding lines (non-symbolic addition of lengths) may enhance subsequent symbolic arithmetic as much as practice adding arrays of dots (non-symbolic addition of numbers).

Line length addition was used as addition of two vertical line lengths on top of each other and then comparing this total length to the third vertical line. Subject's job was to analyze whether the third line is longer or shorter than the line resulted as addition of previous two lines (See Appendix C & D for details of numbers used for easy and difficult ratio).

Timing of line length addition task. The line-addition training task was similar to the non-symbolic addition task, except dot arrays were replaced by single vertical line segments of different lengths. Three sets of practice trials of line length task and their timing was similar to the non-symbolic addition task. In each test trial children saw a yellow occluder for 500ms. Then first line segment appeared on left side of screen for 1000ms and moved behind yellow occluder taking 500ms. Then there was pause of 300ms. Children saw second line segment on right side of screen for 1000ms and moved behind yellow occluder taking 500ms. There was pause for 1250ms showing yellow occluder. Subjects were instructed to add these segments together and compare their sum (height) to a third line segment (test line) that appeared after the occluder disappeared. Yellow occluder disappeared and children saw third line for 5000ms and were required to guess whether this third line is longer or shorter than the previous two lines being together on to of each other. The ratio of height of the sum of the line lengths to height of the test line was varied by the same ratios as those used in the non-symbolic approximate addition training task (7:4 and 7:5) (See Figure 4).

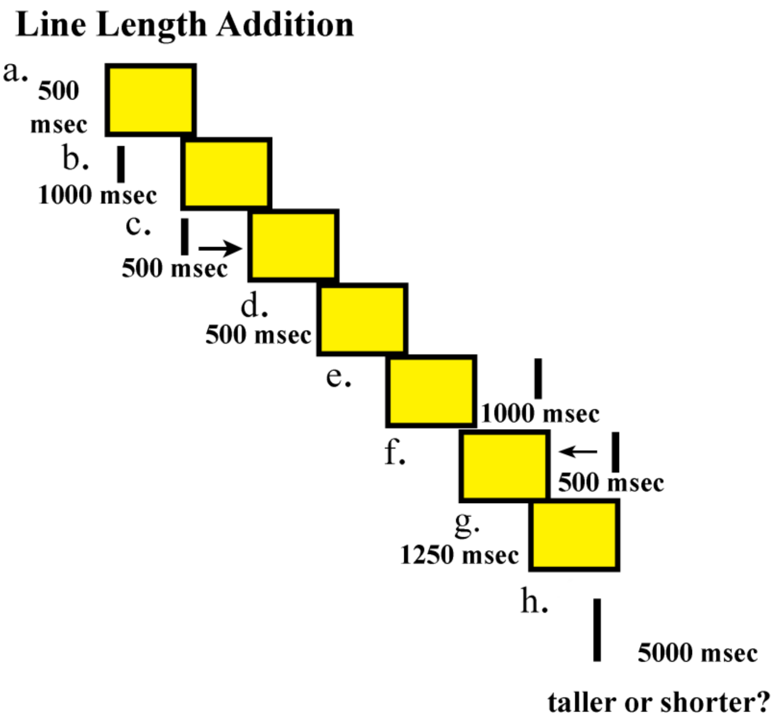

Figure 4. Schematic depiction of line length addition training task

4- Non-Symbolic Numerical Comparison. A fourth condition involved approximate, non-symbolic numerical comparison. In this condition, subjects saw two sequentially presented arrays of dots and had to judge whether the second array was more or less numerous than the first (see Figure. 4). As reviewed above, emerging work suggests that the ability to compare arrays of objects on the basis of number correlates with mathematics achievement scores in a variety of contexts (e.g., Budgen & Ansari, 2011; Halberda et al., 2008; Lourenco et al., 2012). If the ANS alone plays a functional role in symbolic arithmetic, rather than co-activation of the ANS and cognitive arithmetic computations as in the non-symbolic numerical addition condition, then performance on symbolic arithmetic problems may be enhanced in children who previously engaged the ANS through comparison or addition, relative to children who receive other the non-numerical training conditions.

Non-symbolic comparison involved comparing two sets of arrays of dots to judge whether the second set is more or less than the previous set (See Appendix E for details about numbers used for creating easy and difficult ratios).

Timing of non-symbolic comparison task. Non-symbolic comparison task had similar set of practice trials following similar timing as in non-symbolic addition task. In each test trial children saw a yellow occlude for 500ms on a black background. Then first array of white dots appeared on black background for 1000ms and moved behind yellow occlude taking 500ms. After that yellow occluder remained on screen for 1250ms. Then occlude disappeared and second array of white dots appeared on screen for 5000 ms for response. Children were required to guess whether this array is more or less than the first one. Children responded through hitting keys for more or less and got feedback tone indicating whether their response was right or not (See Figure 5).

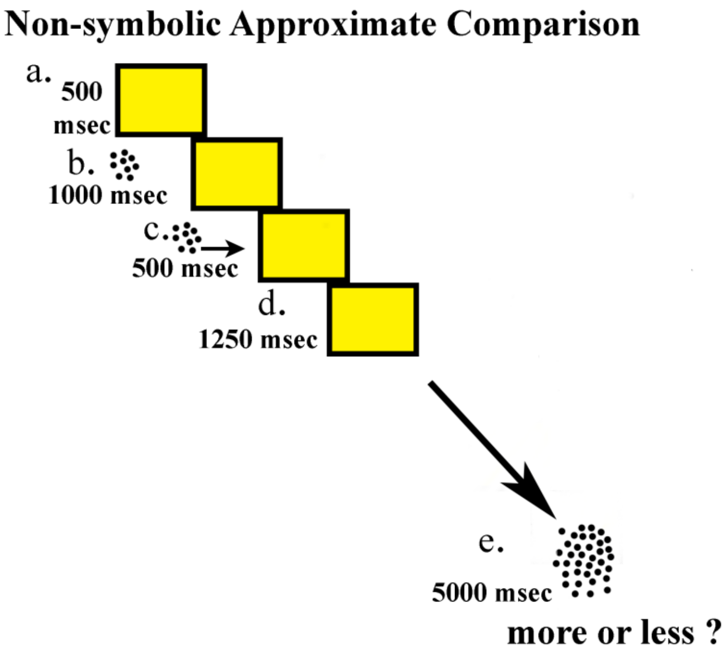

Figure 5. Schematic depiction of Non-symbolic comparison training task

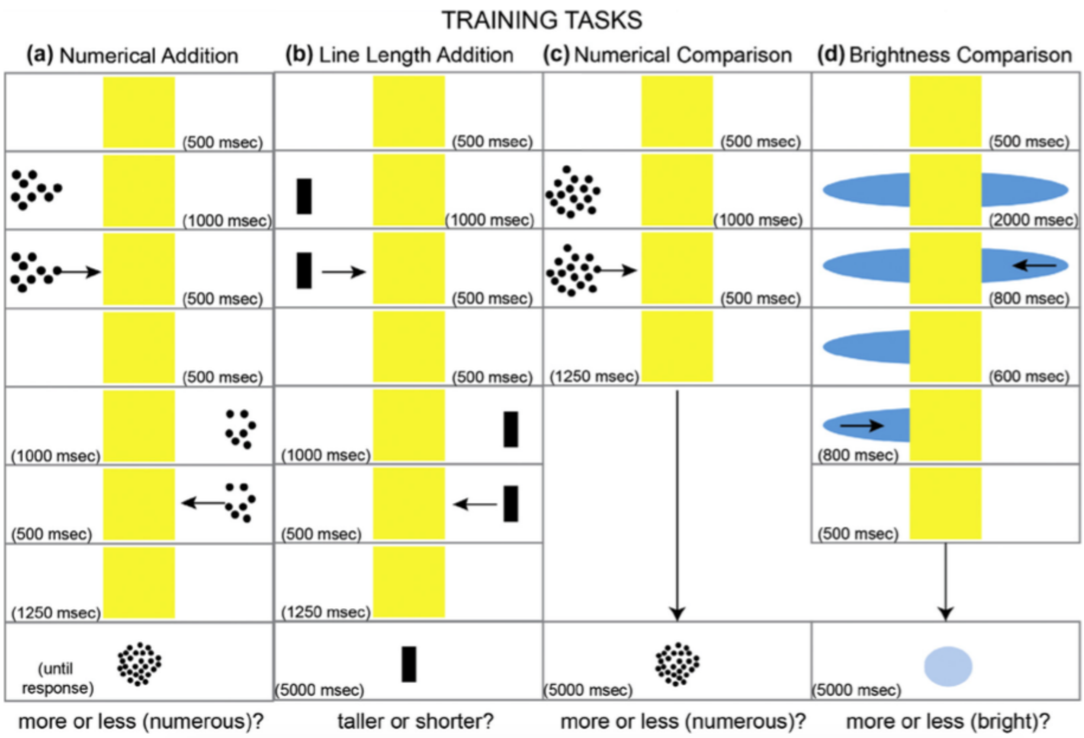

Figure 6. Schematic depiction of training tasks

Gameplay for Training Conditions and Cartoon Characters Used

The training tasks were presented in the context of a game on 16 inches screen (See Appendix F for script of training instructions) where either a good cartoon character surreptitiously added more dots (or made the display lighter, or made the line segment taller) or a bad cartoon character surreptitiously stole items (or made the display darker or made the line segment shorter) from behind the occluder, resulting in a final set with more or less numerous (or bright or tall) than the expected outcome. Characters A, B, C and D from Figure.7 were used for Phase 1 USA sample. Where as C, D, E, and F were used for Phase 2 Pakistani children keeping in view children familiarity with characters. A, B and E, F were used for boys and B, C were used for girls.

Bad Cartoon	Good Carton
A.  **Darth Vader**	B.  **Yoda**
C.  **Witch**	D.  **Princess**
E.  **Germander**	F.  **Captain Safeguard**

Figure 7. Cartoon characters used for gameplay for Training tasks

Apparatus

All training tasks were performed on a laptop computer with a cover on keyboard showing three response keys. Children sat approximately 86.36 cm from the computer screen. Training tasks were programmed using E-prime software (Psychological Software Tools, Pittsburgh, PA), which recorded reaction time and accuracy. Symbolic arithmetic test problems and sentences with blanks were presented on paper and completed with a pencil. The time to complete each page of symbolic addition problems was recorded by the experimenter with a stopwatch. Experiment with each subject was video recorded.

Symbolic Addition Problems (Used in Experiment 1 and 3 of Phase1 & Experiment 1 of Phase2)

There were four set of symbolic addition problems increasing in difficulty level on each next set 1-4, each set comprising 10 addition problems so in total 40 problems. Children were given a sample problem to solve before solving each set. These problems were formulated by reviewing first grade children math books, and across the 4 sets difficulty level was increased systematically by manipulating the size and distance of addends. First set of problem’s addends were mostly single digits considered to be easy e.g., 9 + 3, 14 + 2 total of addend below 20, at approximately same level of difficulty as children were attempting in first grade.

Second set of problems were slightly difficult with larger digits and mostly two digits being added to other single digit e.g., 16 + 3, 14 + 8, total of addend below 25; third set of difficult problems comprised of mostly both addends two digit, e.g 19 + 6, 17 + 13 total of addends below 30.

Fourth set, of very difficult problems comprised of problems with two digit addends e.g 19 + 18, 46 + 38, total of addends below 100. Each set's 10 addition problems were combination of approximately equal number of easy, slightly difficult and more difficult problems to keep variability. These problems were given to children by arranging from easy to difficult order. These problems were presented to children on paper showing each addition problem in a box with instruction that they will solve these problems by adding (See Appendix G & H). They were instructed to solve the problem and write down the answer under the each addition question. Experimenter recorded the reaction time for each set separately with stopwatch and scored each correctly answered problem as 1 calculating total for accuracy.

Panamath Task for ANS Acuity Assessment (Used in all Experiments of both Phase 1 and 2)

Children's numerical acuity was assessed through the Panamath task (See Halberda et al., 2008, www.panamath.org).

Children were shown two characters, Grover and big bird on the sides of computer screen. One blue (Grover) attached to right and one yellow (Big Bird) attached to the left side of screen. Before starting the game children were introduced with the game and they were given directions about playing the game including the usage of keys to hit for their responses. On each trial children saw two separate boxes of equal size on the screen one of yellow character and the other of the blue character. Two arrays of colorful dots, yellow dots in box of yellow character and blue dots in the box of blue character appeared ranging from 4-15 dots across trials. Each trial required children to guess which set of dots are more (yellow or blue) and to hit the corresponding key of character on the keyboard. There were three keys visible on the keyboard, one yellow for Big bird), other blue for Grover) and third key (space bar) to start the game and keep hitting for every next trials. There were six training trials and 60 test trials. It took approximately 10 minutes to complete the game. This task comprise of 6 training trials followed by 60 test trials presented on fixed trial order based on 4 ratios. There are 15 trials for each ratio. Ratio bin are 2:1, 3:2, 4:3, 6:5. Comparison array of dots appeared for 2000ms on the screen. Children responded through the key, and got feedback by beeps (ping) corresponding to right and (basso) to wrong response.

Based on accuracy at each ratio, the Panamath software generated a psychophysical model of performance and an estimate of numerical acuity (a Weber fraction). Details regarding the freely available software, the task, or the calculation of a Weber fraction can be found at www.panamath.org.

Chapter-III

PHASE 1

Experiment 1

Objective

Experiment 1 investigates the causal effects of non-symbolic numerical operations on symbolic numerical processing, by means of a training paradigm. It is to investigate whether subjects trained with non-symbolic approximate addition will demonstrate an advantage in solving symbolic addition problems in terms of speed and accuracy as compare to children trained with brightness comparison?

Method

Participants. A total of 48 first grade children, 24 in the non-symbolic approximate addition training group (11 girls, 13 boys, *M* age = 6 year 311 days, *SD* = 73.48 days) 24 in the brightness comparison training group (11 girls, 13 boys, *M* age = 6 years 332 days, *SD* = 94.27 days) participated in the study. Subjects were quasi-randomly assigned to experimental groups in order to equate the two groups by gender and age. An additional 13 children participated in the study but were excluded from main data analysis because of, children not completed the study (9), system error (1), experimenter error (2), and sequence of experimental activities was messed up (1). Children were recruited from Greater Boston area of Massachusetts, USA. Study was approved by committee on use of human subjects (CUHS) of Harvard University. All children and their parents gave written consent for participation in the study and were compensated for their participation in the study (See IRB approval in Appendix J).

Stimuli and display. All the training tasks were presented on laptop with the uniform instruction (Instruction can be found in Annexure section). Training tasks and posttest measures were following. Further details about tasks have been mentioned in detail in method chapter.

1. Non-symbolic addition task
2. Brightness comparison task
3. Four sets of symbolic addition problems (Appendix G)

Correctly answered addition problems were scored as 1 and time to complete each test set was recorded with a stopwatch.

4. Panamath task for ANS acuity assessment.

Design and Procedure. All the tasks and their sequence were same in both groups except the nature of training task itself. Experimenter introduced the children to the experimental activities in context of a computer game based on the corresponding training task for each group. Children in both training groups got 8 training trials with the instruction, directions given by experimenter followed by 50 test trials, (30 trials of ratio 7:4 or easy brightness comparison, 30 trials of ratio 7:5 or difficult brightness comparison). After that they solved two sets of symbolic addition problems, each set comprising 10 problems. Then they were given the choice of either to take a short break or continuing the experiment. After following their preference of break, children attempted 10 more test trials of the training task and then solved last two sets of symbolic addition problems, each set comprising 10 problems. Children's time to solve each math sets were recorded with a stopwatch and wrote down. At the end they played another game based on Panamath task (see Halberda et al., 2008) to assess the approximate number system acuity (See Figure 8).

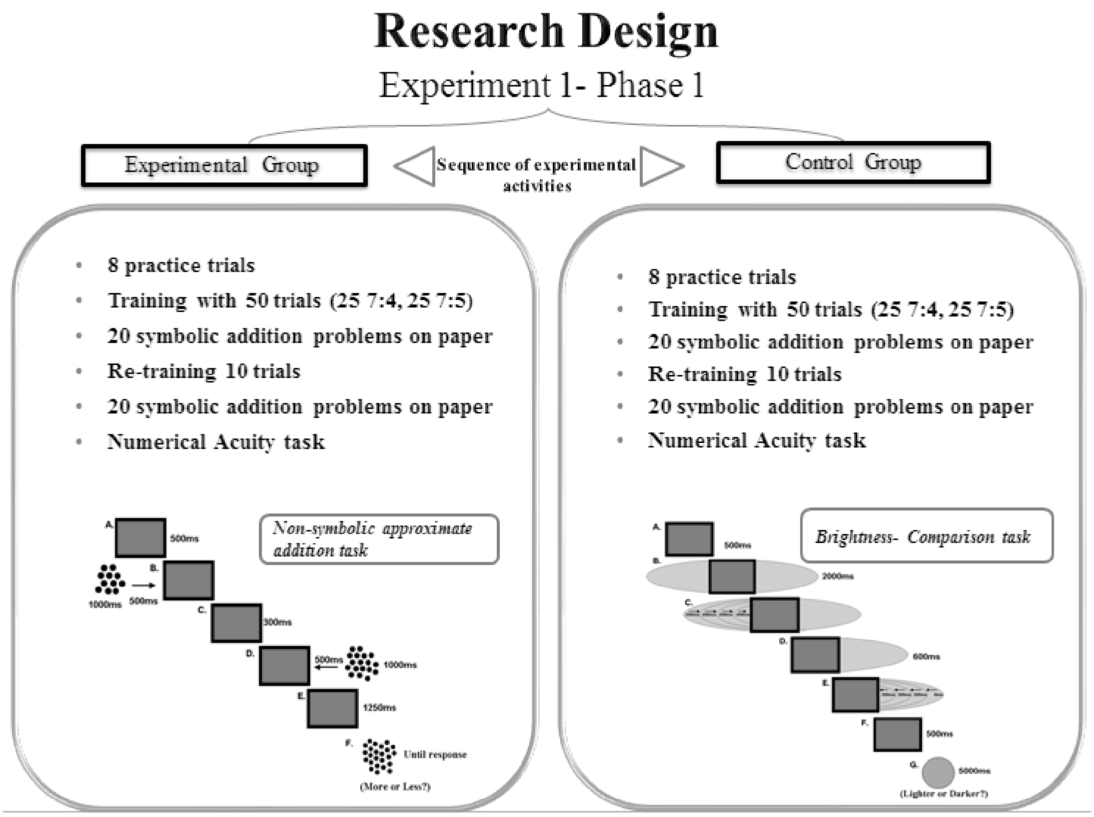

Figure 8. Research design of Experiment 1(Phase 1)

Lack of a response time limit in non-symbolic numerical addition condition only. As outlined in Figure.2 non-symbolic numerical addition impose a 5000 millisecond response time limit like the other three conditions. However, response time was infinite in the non-symbolic addition task in experiment 1 only.

The lack of a response time limit as of 5000ms for the final stimulus in the addition condition was, in fact, an error in the code found after the experiments had been ran. It went unnoticed in the running of the experiments because children overwhelmingly responded quicker than 5 seconds in all conditions (including the addition condition). In fact, an examination of the mean reaction time for all conditions was well below 3000 msec. Furthermore, the same principles were applied to the analysis of all conditions (all response times over 5 seconds in any condition were thrown out and counted as incorrect). In the end, it appears that this timing difference made no theoretical difference, as the numerical comparison condition, with the 5-second stimulus presentation limit, produced comparable enhancements in subsequent symbolic math performance to that of children in the addition condition without the 5-second limit.

Results

There were few children who had not attempted all the four sets of symbolic addition. Among those children, there were 2 children in control group who had not solved 3^{rd} set and 5 had not solved 4^{th} set of symbolic addition problems. In the experimental group, 2 children had not solved 3^{rd} set of symbolic addition and 5 had not solved 4^{th} set of symbolic addition Problems. To carry out the analysis, average accuracy and reaction time on corresponding set was calculated for each group and entered in the children missing data set of their own group.

Participant's factors (age, Weber fraction (w). Children in the both training conditions did not differ in mean age or weber fraction.

Table 1

t-test results comparing experimental and control group on age (in experiment 1 Phase 1)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	6.85 years	73.48	.845	46	=. 402	0.24918
Brightness Comparison	24	6.90 years	94.27				

Table 1 depicts that experimental and control groups were not significantly different on age.

Table 2

t-test results comparing experimental and control group on Weber Fraction (w), (in experiment 1 Phase 1)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	.17	.11	.003	46	=. 998	.00088
Brightness Comparison	24	.17	.08				

Table 2 shows that there was no significant difference on number sense acuity (weber Fraction, *w*) results between children trained with non-symbolic addition and brightness comparison.

Training task performance. A Mixed Factor ANOVA on training task mean percent reaction time with the within subjects factor of Ratio (2 levels: ratio 7:4 and ratio 7:5) and the between subjects factor of Training Condition (non-symbolic approximate addition vs. brightness comparison) was carried out.

Table 3

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5) and Training condition* (*Non-symbolic addition vs. brightness comparison group on training task reaction time)* (*in experiment 1 Phase1)*

Variables	*df*	*F*	η_p^2	*p*
Ratio	1,46	5.256	.103	<.05
Training Condition	1, 46	21.767	.321	<.001
Ratio * Training Condition	1, 46	.289	.006	= .594

Table 3 shows a significant main effect of Ratio, a significant main effect of Training Condition. There was no interaction between Ratio and Training Condition.

On average, difficult problems took longer than easier problems and those in the brightness comparison condition completed their training problems significantly faster than those in the non-symbolic approximate addition condition (See Figure 9).

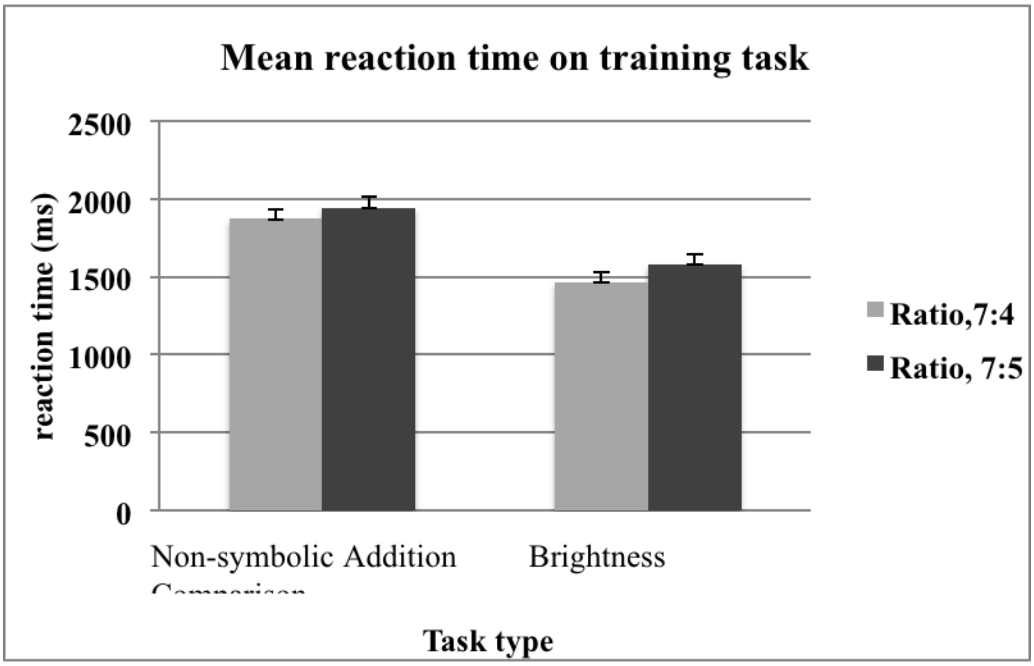

Figure 9. Average reaction time (in milliseconds) over ratio in each condition in experiment 1 (Phase 1)

Table 4

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5) and Training condition* (*Non-symbolic addition vs. brightness comparison group) on training task accuracy* (*in experiment 1 Phase1)*

Variables	*df*	*F*	$\eta_p 2$	*p*
Ratio	1,46	22.972	.333	<.001
Training Condition	1, 46	35.419	.435	<.001
Ratio * Training Condition	1, 46	4.663	.092	<.05

Table 4 depicts a significant main effect of Ratio, a significant main effect of Training Condition, and a significant interaction of Ratio and Training Condition.

Post hoc analysis of the interaction revealed that, on average, children were more accurate for the 7:4 (non-symbolic addition: *M* = 84.44, *SD* = 15.28; brightness comparison: *M*= 97.64, SD=3.47, *t* (46) = 4.125, *p* = .001) than for 7:5 (non-symbolic addition: *M* = 75.28, *SD* = 12.74; brightness comparison: 94.17, *t* (46) = 6.774, *p* = .001) and that children in the brightness comparison training group (*M* = 95.90, *SD* = 3.40) were generally more accurate on training problems than children in the non-symbolic numerical addition group (*M* = 79.86, *SD* = 12.76), *t* (46) = 5.951, *p* < .000) (see Figure 10).

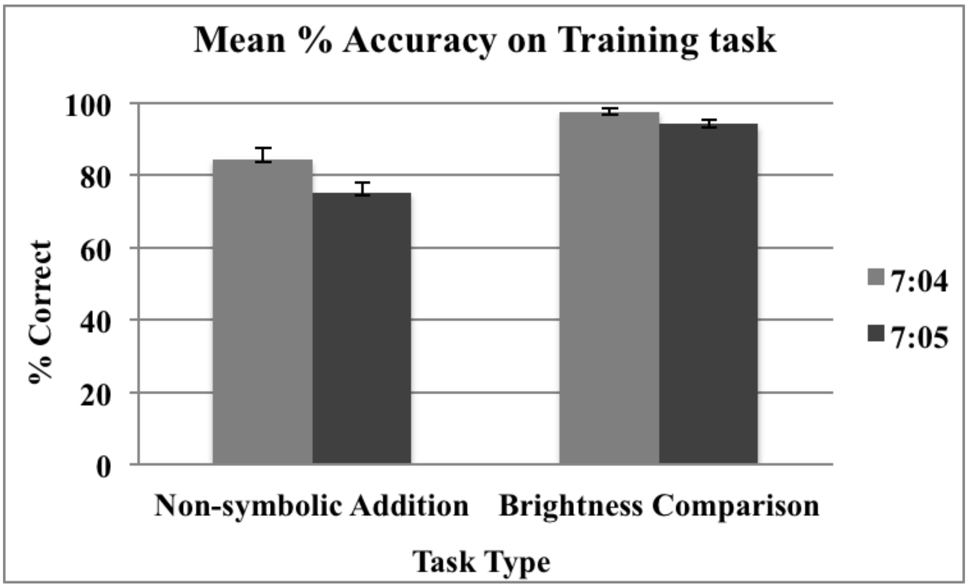

Figure 10. Average task accuracy (expressed as percent correct) for each condition in experiment 1 (Phase 1)

Exact symbolic addition. A mixed factor ANOVA on the time to complete exact symbolic addition test problems with the between-subjects factor of Training Condition (brightness comparison vs. non-symbolic addition) and the within-subjects factors of Difficulty (4 levels) revealed a significant main effect of Training Condition and a significant main effect of Difficulty. There was no significant interaction effect of difficulty and condition.

Table 5

Mixed factor ANOVA of condition (*Non-symbolic addition vs. brightness comparison group) and difficulty* (*1, 2, 3, 4) on time to solve symbolic addition* (*in experiment 1 Phase 1)*

Variables	*df*	*F*	$\eta_p 2$	*p*
Training Condition	1,46	4.885	.096	<.05
Difficulty	3, 138	45.409	.497	<. 001
Training Condition*Difficulty	3, 138	.540	.012	= .656

Table 5 shows that training condition had significant effect on children's symbolic addition performance. As children trained with non-symbolic approximate addition completed exact symbolic addition test sets faster than those trained on brightness magnitude comparisons. On average, all children were slower to complete more difficult problems (See Figure 11).

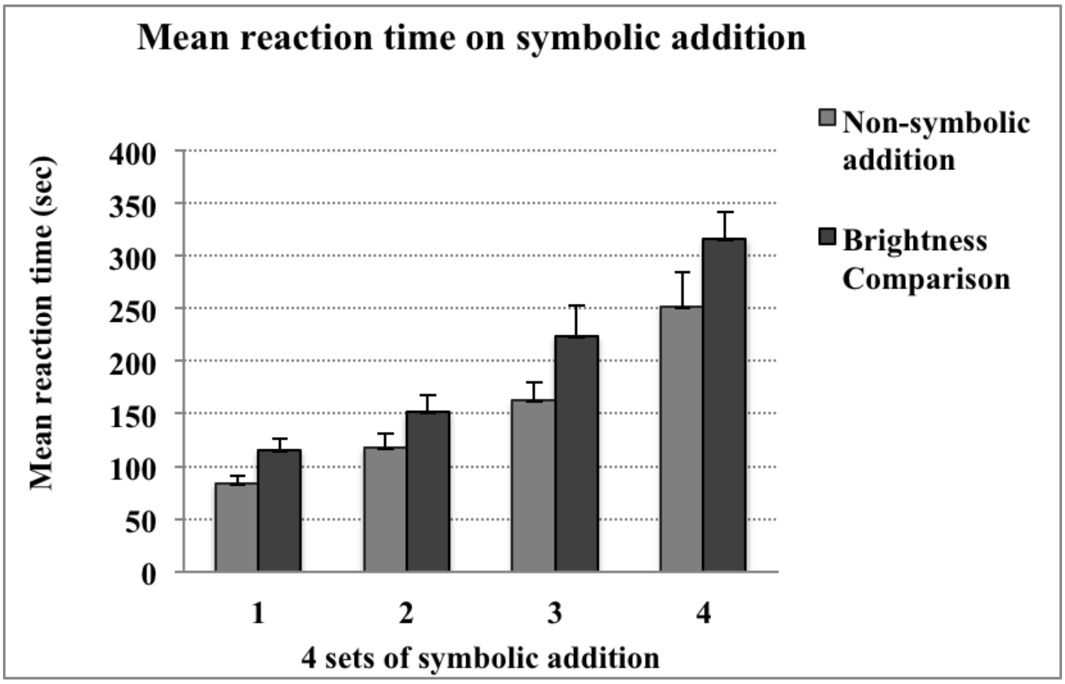

Figure 11. Average speed of test completion (in seconds) for each condition in experiment 1 (Phase 1)

A similar ANOVA on exact symbolic addition accuracy with the between subjects factor of Training Condition (brightness comparison vs. non-symbolic addition) and the within-subjects factor of Difficulty (4 levels) was carried out.

Table 6

Mixed factor ANOVA of condition (*Non-symbolic addition vs. brightness comparison group) and difficulty* (*1, 2, 3, 4) on symbolic addition accuracy* (*in experiment 1 Phase1)*

Variables	*df*	*F*	η_p^2	*p*
Training Condition	1,46	.027	.001	=. 871
Difficulty	3, 138	42.576	.481	<. 001
Training Condition*Difficulty	3, 138	1.089	.023	= .356

Table 6 shows a significant main effect of Difficulty. However, no significant differences were found on accuracy between Training Conditions nor was the interaction significant between difficulty and Training Condition. Although difficulty influenced accuracy of children, Training Condition did not influence accuracy on these problem sets.

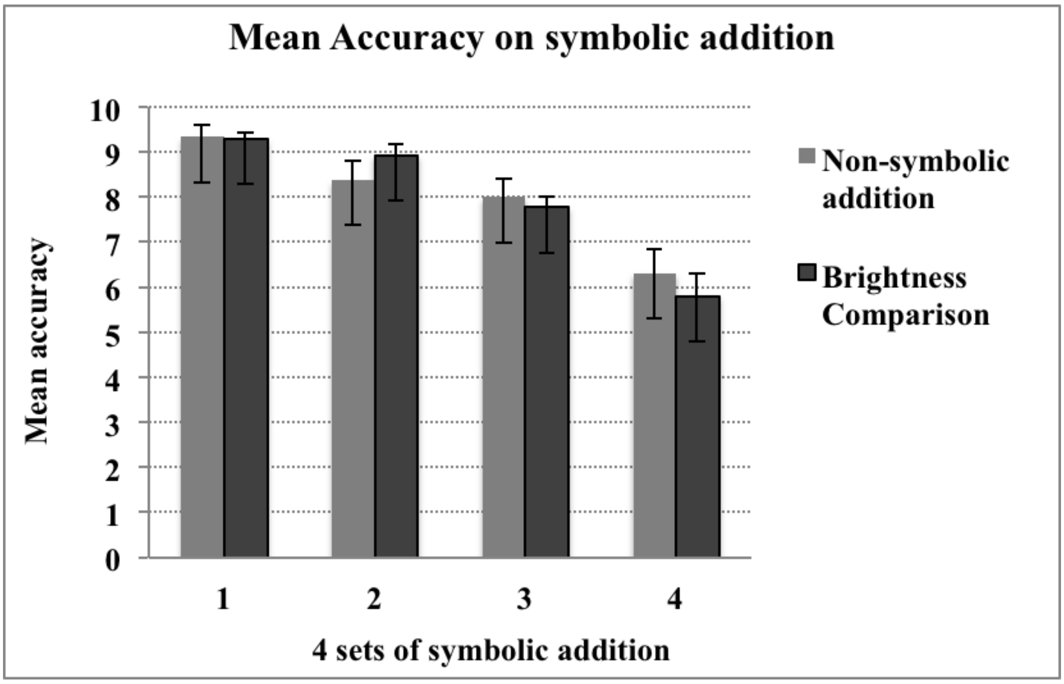

Figure 12. Average test accuracy (expressed as correct out of 10) for each condition in experiment 1 (Phase 1)

Speed accuracy tradeoff is evident as children took more time on harder problems and were less accurate on harder problems.

Further analysis. The critical main effect of Training Condition on speed remained significant even after effects of training task reaction time. A mixed Measure ANOVA was carried out with the within subject factor of Difficulty and between subject factor of Training Condition (Non-symbolic addition vs. Brightness Comparison) and reaction time on the training task performance as covariate. There was significant main effect of Training Condition F (1, 45) = 12.185, $p < .005$, ηp2 =

.213. A similar analysis on accuracy revealed no main effect of training condition on accuracy F (1, 45) = 1.260, p = .268, ηp2 =. 027.

Conclusion

Children in the non-symbolic approximate addition training group performed significantly better on symbolic addition problems in terms of speed as compared to brightness comparison training group. However, there were no differences in the performance of both groups on approximate number acuity as shown by their performance on weber fraction. These results suggest that children trained with non-symbolic approximate numbers showed advantage on symbolic addition in terms of speed. What are the specific factors in non-symbolic addition task that were driving this effect and whether it is specific to mathematics are yet additional questions to be investigated. So experiment 2 was conducted to figure out the possible factors contributing in better symbolic addition performance after non-symbolic addition training and whether it was specific to the domain of mathematics or not?

Experiment 2

Objective

To investigate whether engagement of the approximate number system enhances subsequent cognitive performance more generally or is the effect specific to the domain of mathematics? Moreover, it was to explore whether experiment 2 also controls for any effects due to greater difficulty of non-symbolic addition training. If this more challenging task enhanced performance in general, then it might enhance performance with sentences as well as addition in experiment 2. So to rule out this possibility experiment 2 was carried out. So that it could be more clearly distinguished whether training with non-symbolic addition would have an effect only on symbolic addition or on both tasks (symbolic addition and sentences with blanks).

A new set of harder exact symbolic addition test problems and an equally difficult test task involving sentence completion was created. Harder math problems were used in experiment 2 because of a small trend towards a difference between the two conditions in experiment 1 was seen as the difficulty increases (See Table 6). If the enhancement effect is specific to mathematics, then training on the non-symbolic approximate addition task should only enhance performance on the symbolic arithmetic problems and not on the sentence completion problems. On the other hand, if more general motivational or reasoning factors are driving the enhancement, improved performance should be observed on both the symbolic math problems and the sentence completion problems in the group of children performing the approximate addition training but not in children performing a control-training task (brightness control).

Method

Participants. A total of 48 new group of first grade children, age between 7 and 8 years, 24 in number training group (12 boys and 12 girls, mean age = *M* age = 7 years 204 days) 24 in the brightness-training group (12 boys and 12 girls, mean age = 7 years 196 days) were included in final data of study. An additional 12 children participated in the study but were excluded from main data analysis for not completing the study (7), reported developmental/language delays (1), reported reading delays (1), not being a native English speaker (1), kid took total time of study even beyond the approved time of study and also more than all children who participated in study (1), and because age did not allow appropriate counterbalancing between groups (1). Children were recruited from Greater Boston area of Massachusetts, USA through phone, email, and flyers and it was ensured that their first language is English and parents speak English at home. Moreover before starting the experiment, parents were asked about the language of child to ensure that English is first language of all children participating in the experiment. Study was approved by committee on use of human subjects (CUHS) of Harvard University. All children and their parents gave written consent for participation in the study and were compensated for their participation in the study (See IRB approval in Appendix J).

Stimuli and Display. Apparatus, instructions/directions and presentation of training task for both groups were the same as in experiment 1 comprising 8 training trials followed by 60 test trials following the order of presenting the trials of training task from easy to difficult. Training tasks and posttest measures were following.

1. Non-symbolic addition task
2. Brightness comparison task
3. Two sets of symbolic addition problems

Exact, symbolic addition test stimuli were comprised new and old problems from the previous experiments (see Appendix I). In experiment 2 children were presented with 2 set of symbolic addition problems each comprising 10 problems as a posttest. These addition problems were much harder than experiment 1. In both sets 16 problems, 8 in set 1, and 8 in set 2 were repeated from 40 symbolic addition problems presented in experiment. 2 problems were new in each set of experiment 2. The problems repeated from experiment 1 were selected in this way, 48 children's accuracy on each problem of experiment 1 was calculated and the problems included in experiment 2 were those where children accuracy was not 100%. So by choosing difficult problems from experiment 1 and adding 2 more difficult problems made these addition problems harder than those in experiment 1.

4. Two sets of sentence completion problems

Sentence completion problems (see Appendix H) were developed from basic vocabulary words of 1^{st}-4^{th} grade. The sentence included a blank with the first letter of the vocabulary word to be guessed. Children were to use the context of the sentence and the first letter of the word to guess the correct answer.

Children were presented 2 sets of sentences with blanks, as a posttest. Sentences, with a blank given at the end of each sentence were presented to the children and were instructed to read the sentence carefully and guess one word to fill the blanks. These sentences were matched to symbolic addition problems in terms of reaction time and difficulty level by piloting. Each set of sentences comprised of 10 problems. Children responses on this task were recorded in terms of accuracy and reaction time.

Examples of Sentences with Blanks.

Easy: Planes land at the A__________.

Hard: Animals that are raised in captivity live in C________________.

Scoring. Correctly answered addition problems and blanks filled with the vocabulary word of interest were scored as 1 and time to complete the test was recorded on each set of test problems with a stopwatch.

5. *Panamath task for ANS acuity assessment.*

Design and Procedure. Subjects were quasi-randomly assigned to one of the two training conditions to equate age, gender, and test order. Children in both groups attempted 8 training trials followed by 60 test trials of the corresponding training task (30 trials of 7:4 or easy brightness comparison, 30 trials of 7:5 or difficult brightness comparison). Children were post tested on 2 sets of symbolic addition problems and 2 sets of sentences with blanks after the training trials and test order of solving sentences or math problems was counterbalanced across both groups. Half of the children in both groups first solved two sets of symbolic addition problems after training and then 2 sets of sentences with blanks and other half received the post test in reverse order counterbalancing the order of post test.

Among these 24 children (12 girls, 12 boys) in non-symbolic approximate addition training group 7 girls and 5 boys first solved sentences (and then symbolic addition problems) and 7 boys and 5 girls first solved symbolic addition problems (and then sentences with blanks). Whereas, among brightness comparison 24 children (12 girls, 12 boys) 6 girls and 6 boys first solved sentences (and later symbolic addition problems) and 6 girls, 6 boys first solved symbolic addition problems (and then sentences with blanks). So overall, among 24 children in each group (non-symbolic approximate addition and brightness comparison), 12 of them solved sentences first and 12 of them solved symbolic addition problems first, counter balancing the order of test and gender.

Followed by 8 training trials of training task, both groups attempted 24 test trials of training task. Then half the children in both groups solved first set of math problems and half of them solved first set of sentences. Children again attempted 12 test trials of their assigned task and half of them solved second set of math and half of them solved second set of sentences. Type of test task presented first (sentences or math) was counterbalanced across participants. At this point they were given choice for either to take a short break or continue the experiment. After the break children attempted 12 test trials and half of them received math set1 (if they had done previously sent set 1, 2) and half of them received sentences set 1 (if they had done math set1, 2). Then children attempted last 12 trials of training task and half of them attempted second set of math (if they had done first sent set 1,2 before the break) and half of them attempted second set of sent (if they had done math 1, 2 before the break). Those participants, who completed math problems first during the first block of testing, completed sentence completion problems first during this second block of testing and visa versa.

At the end of the session, children were tested for acuity of the approximate number system using the Panamath task following the same direction and method as in Experiment 1 (See Figure 13).

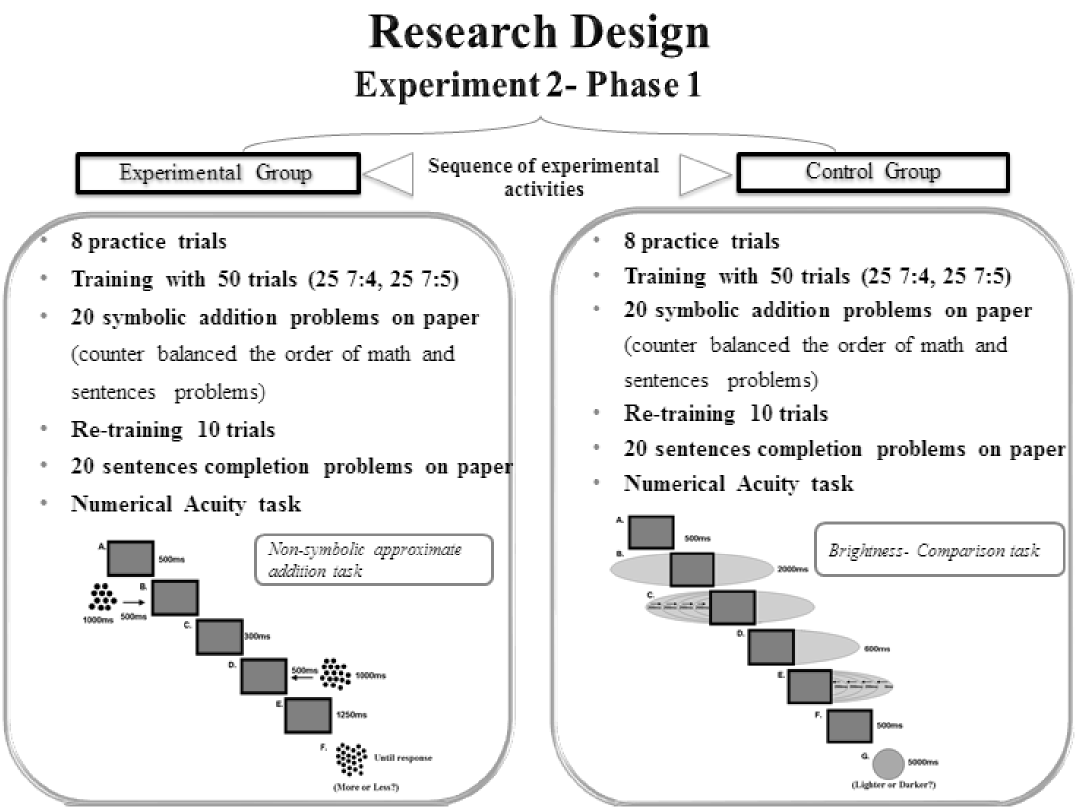

Figure 13. Research design of experiment 2

Results

Participant's factors (age, Weber fraction). Training groups were compared on age and Weber fraction (w). An independent sample t test was conducted to compare both groups on age and weber fraction.

Table 7

t-test results comparing experimental and control group on age (*in experiment 2, Phase1)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	7 years 204 days	87.77	- .299	46	= .766	-0.08817
Brightness Comparison	24	7 years 196 days	100.25				

Table 7 shows that there was no significant difference on age between brightness comparison and non-symbolic addition group.

Table 8

t-test results comparing experimental and control group on Weber Fraction (*w*), (*in experiment 2, Phase1)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	.17	.07	- .304	46	= .763	-0.08964
Brightness Comparison	24	.18	.07				

Table 8 shows that there was no significant difference in scores for weber fraction between brightness comparison and non-symbolic addition group.

Training task performance. A mixed measures ANOVA was conducted on reaction time by taking Ratio as within subject and Condition as between subject factor revealed main effect of Training Condition, and Ratio.

Table 9

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5) and Training condition* (*Non-symbolic addition vs. brightness comparison group) on training task reaction time),* (*in experiment 2, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Ratio	1,46	39.676	.463	< .001
Training Condition	1, 46	45.549	.498	<.001
Ratio * Training Condition	1, 46	.656	.014	= .422

Table 9 shows that there was significant main effect of Ratio and Training Condition. However, there was no interaction of Ratio and Training Condition.

Difficulty of training, regardless of condition, affected both reaction time *F* (1, 46) = 39.676, $p < .001$, ηp2 = .463 and accuracy *F* (1, 46) = 77.826, $p < .001$, ηp2 = .629. However, children performed significantly better on the brightness comparison task than the non-symbolic approximate addition task (See table 11). Specifically, the children in the brightness comparison group were faster *F* (1, 46) = 45.549, $p < .001$, ηp2 = .498 and more accurate *F* (1, 46) = 46.000, $p < .001$, ηp2 = .500 than the approximate addition training group. Post hoc analysis of the interaction on training accuracy between Difficulty and Training Condition *F* (1, 46) = 13.099, $p < .001$, ηp2 = .222 revealed that the difference in accuracy between training groups, with the brightness group showing better accuracy, was more pronounced on more difficult (7:5), brightness, *M* = 92.92, *SD* = 5.59, non-symbolic, *M* = 75.69, *SD* = 11.27, *t* (46) = 6.707, $p < .001$ compared to less difficult (7:4), (brightness: *M*= 99.30, *SD* = 1.70, non-symbolic, *M* = 90.97, *SD* = 9.03), *t* (46) = 4.441, $p < .001$ training problems.

Table 10

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5) and Training condition* (*Non-symbolic addition vs. brightness comparison group) on training task accuracy),* (*in experiment 2, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Ratio	1,46	77.826	.629	< .001
Training Condition	1, 46	46.000	.500	< .001
Ratio * Training Condition	1, 46	13.099	.222	< .005

Table 10 shows that ANOVA on accuracy revealed main effect of Training Condition and Ratio and interaction between Ratio and Training Condition.

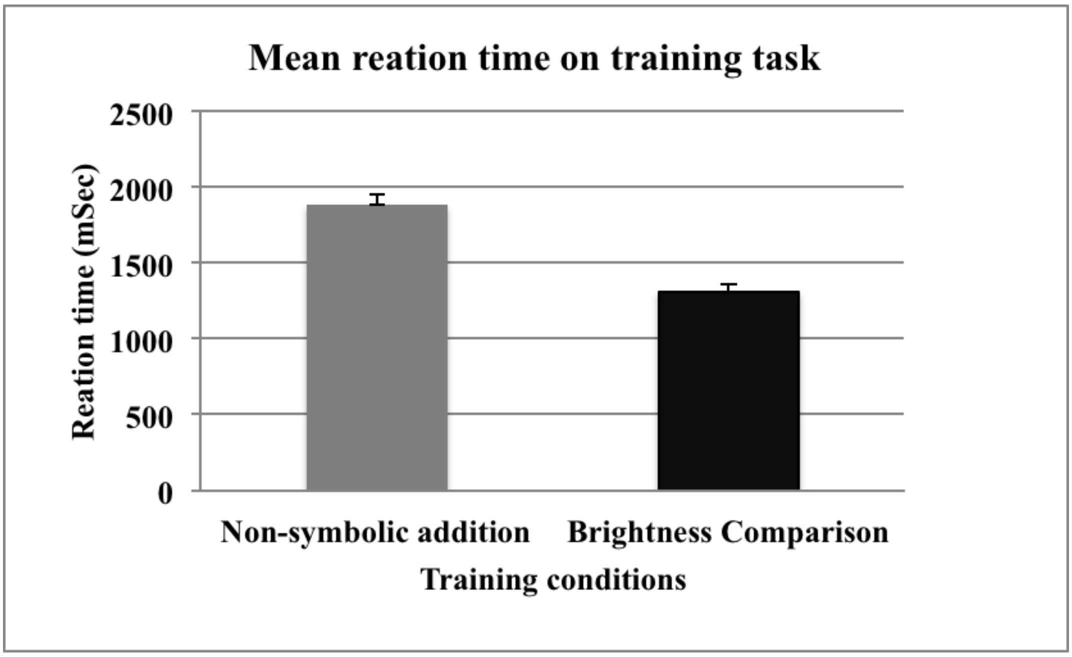

Figure 14. Average reaction time (in milliseconds) each condition in experiment 2 (Phase 1)

Table 11

t-test comparing experimental and control group on training task reaction time, (*in experiment 2, Phase1)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	1884.78	335.27	- 6.776	46	< .001	-1.99813
Brightness Comparison	24	1305.78	250.67				

Table 11 shows that there was significant difference on reaction time on training task between brightness comparison training group and non-symbolic addition training group.

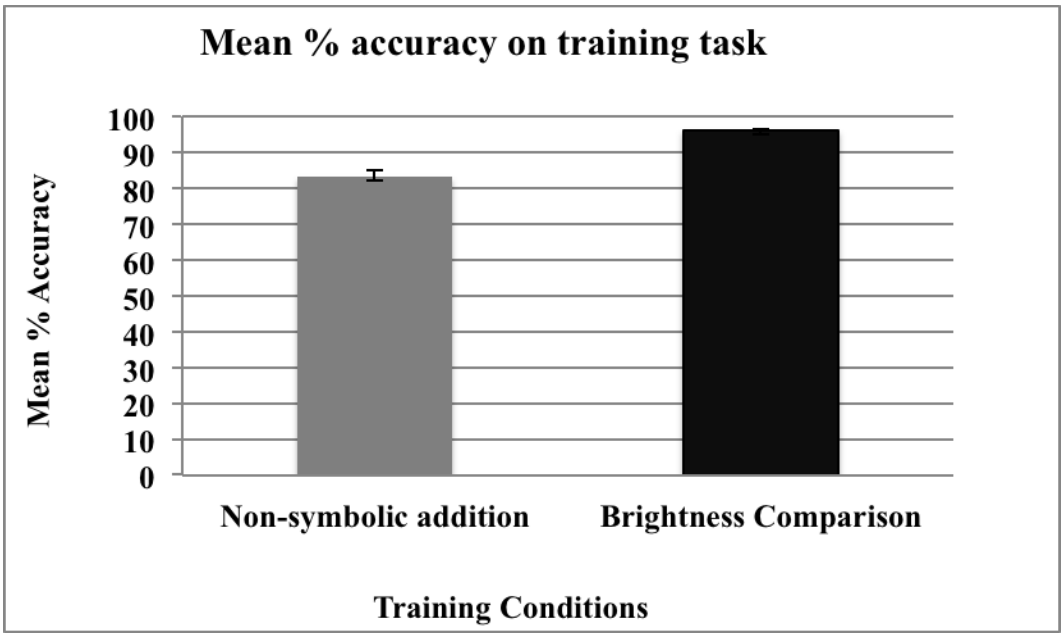

Figure 15. Average task accuracy (expressed as percent correct) for each condition in experiment 2 (Phase 1)

Table 12

t-test comparing experimental and control group on training task accuracy, (in experiment 2, Phase1)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	83.33	8.69	6.782	46	< .001	1.9999
Brightness Comparison	24	96.11	3.09				

Table 12 shows that there was significant difference on accuracy on training task between brightness comparison training group and non-symbolic addition training group.

Symbolic addition performance. An ANOVA on accuracy on test problems with the between subjects factor of Training Condition (brightness comparison vs. non-symbolic addition) and the within subjects factors of Test Type (math vs. sentences) and Difficulty revealed a main effect of Test Type, Difficulty, and Training Condition and a significant interaction between Test Type and Training Condition. There was no significant interaction of difficulty and condition. Neither there was any significant interaction effect of difficulty and test type, and training condition.

Table 13

Mixed Factor ANOVA of Training Condition (*Non-symbolic addition vs. brightness comparison group), Difficulty* (*easy vs. hard) and Test Type* (*math vs. sentences) on accuracy,* (*in experiment 2, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Test Type	1,46	10.185	.181	<.005
Difficulty	1, 46	52.399	.533	<.001
Training Condition	1,46	4.938	.097	<.05
Test Type * Training Condition	1, 46	5.086	.100	<.05
Difficulty * Training Condition	1,46	.040	.001	= .843

Difficulty* Test type * Training Condition	1, 46	.698	.015	.408

Table 13 presents that post hoc independent samples t-tests revealed the interaction between Test Type and Training Condition could be explained by the fact that children who received the non-symbolic approximate addition training task were significantly more accurate on subsequent math test of exact, symbolic arithmetic ($t(46) = -2.814$, $p < .01$), whereas there was no difference between training groups on performance of sentence completion tests ($t(46) = -.725$, $p = .472$). Further analysis revealed test order had no significant main effect or interaction with Test Type or Condition on accuracy and enhanced performance on test addition problems for the non-symbolic approximate addition training group relative to the brightness training group could not be explained by a differential strategy for speed vs. accuracy between groups. Overall, less accuracy was seen on more difficult problems (See figure 16).

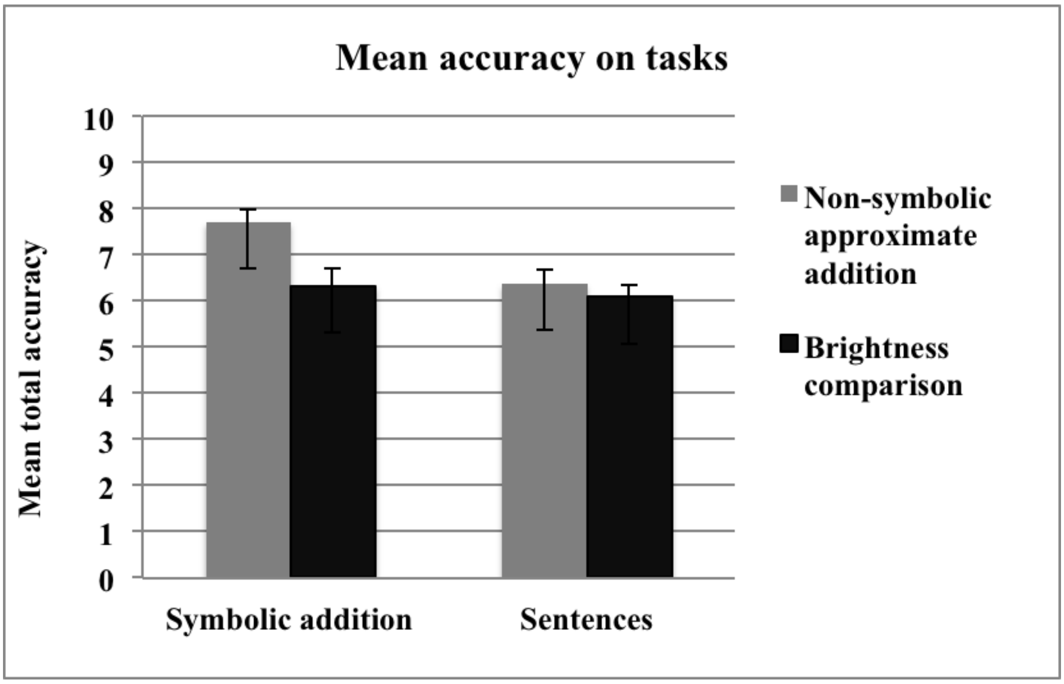

Figure 16. Average task accuracy (expressed as correct out of 10 problems) for each condition in experiment 2 (Phase 1)

Table 14

Mixed Factor ANOVA of Training condition (*Non-symbolic addition vs. brightness comparison group), Difficulty* (*easy vs. hard) and Test Type* (*math vs. sentences) on time to complete test problems,* (*in experiment 2, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Training Condition	1,46	1.361	.029	.249
Test Type	1,46	4.269	.085	< .05
Difficulty	1,46	1.608	.034	= .211
Test Type * Difficulty	1, 46	12.182	.209	< .005
Test Type * Training Condition	1,46	.007	.000	= .932
Difficulty * Training Condition	1,46	.147	.003	= .703
Difficulty * Test Type * Training Condition	1,46	.463	.010	.500

Table 14 presents an ANOVA on time to complete test problems with the between-subjects factor of Training Condition (brightness comparison vs. non-symbolic addition) and the within-subjects factor of Test Type (math vs. sentences) and Difficulty (2 levels). Results revealed a main effect of Test Type and an interaction between Type and Difficulty. There was no significant interaction of task and condition. There was no significant effect of condition. There was no significant main effect of difficulty. There was no significant interaction effect of difficulty and condition. There was no significant interaction of difficulty, test type and condition.

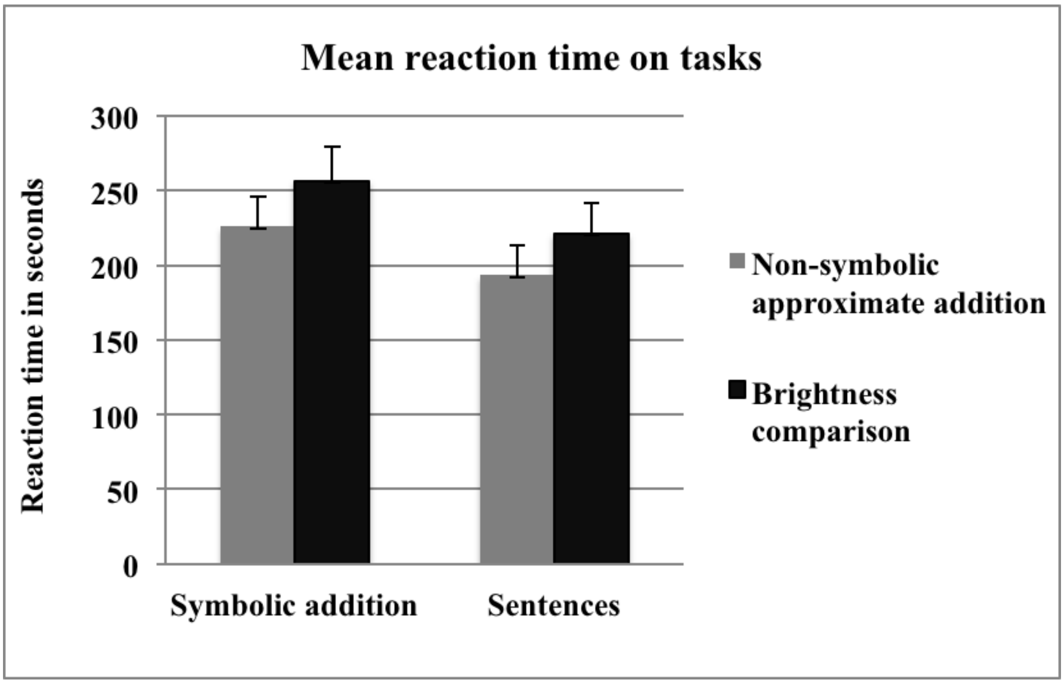

Figure 17. Average speed of test completion (in seconds) for each condition in experiment 2 (Phase 1)

On average, math test problems took longer to complete than sentence test problems. Post hoc test revealed that the interaction could be explained by the fact that that the Difficulty had a significant effect on time to complete math test problems F (1, 47) = 7.586, $p < .01$, ηp2 = .139 but not time to complete sentence completion problems F (1, 47) = 2.771, $p = .103$, ηp2 = .056.

Order effects. We analyzed accuracy with a mixed measure ANOVA of Test Type (math vs. sentences) Training Condition (Brightness vs. Non-symbolic addition) and Test Order (did math first or sentences first). There was a significant main effect of condition. There was no significant main effect of test order nor any interaction effect of condition and test order.

Table 15

Mixed Factor ANOVA of Training condition (*Non-symbolic addition vs. brightness comparison group), Test Type* (*math vs. sentences) and Test Order* (*Did Math first or sentences first) on test problems accuracy,* (*in experiment 2, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Training Condition	1, 44	4.801	.098	< .05
Test Order	1, 44	.457	.010	= .502
Test Order * Training Condition	1, 44	.264	.006	= .610
Test Type	1, 44	10.128	.187	<.005
Test Type * Training Condition	1, 44	5.058	.103	< .05
Test Type * Test Order	1, 44	.794	.018	= .378
Test Type * Training Condition * Test Order	1, 44	.953	.021	= .334

Table 15 shows that children in non-symbolic addition group performed more accurately than brightness comparison group irrespective of test order.

There was significant main effect of Test Type, significant interaction of Test Type and Training Condition. Neither there was any significant interaction of test type and test order, nor of test type, Training condition and test order.

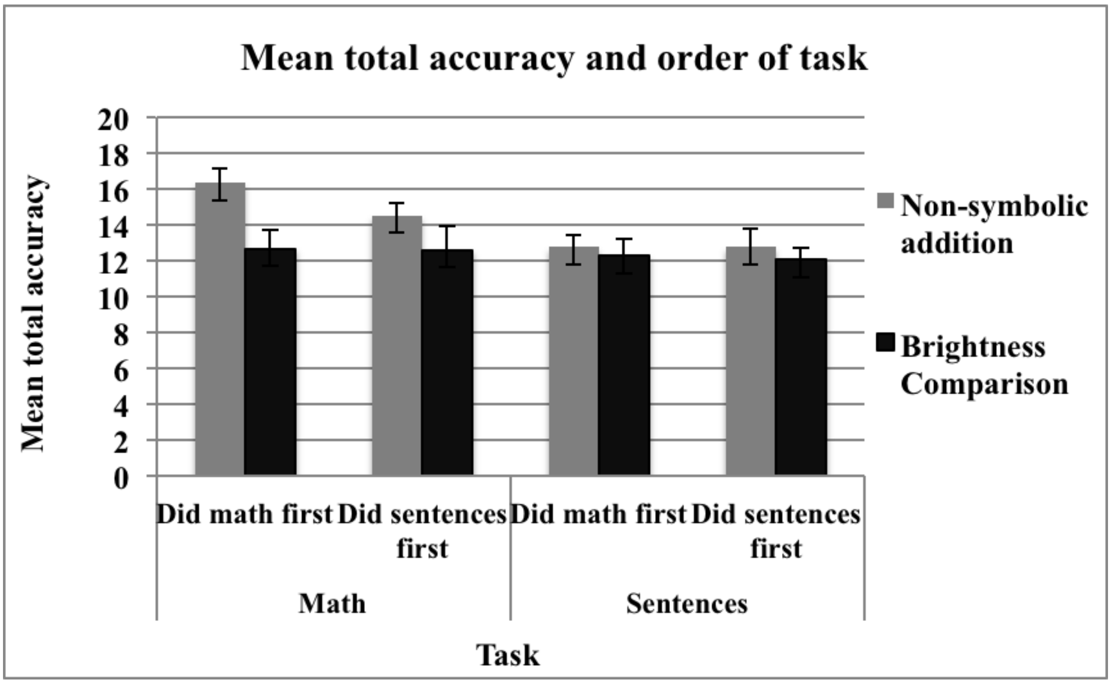

Figure 18. Average test accuracy over test order (expressed out of 20 symbolic addition and 20 sentences problems) for each condition in experiment 2 (Phase 1)

Alternative accounts. One possibility is that differences in performance during the training conditions influenced performance on subsequent test problems. To test this possibility we added training performance as a covariate to the ANOVAs conducted above. Accounting for average reaction time as a covariate did not reduce the interaction of interest between Test Type and Training Condition on test accuracy F (1, 45) = 7.072, p < .05, η_p^2 = .136. However, accounting for accuracy (percent correct) as a covariate did eliminate the interaction of interest between Test Type and Training Condition on test accuracy F (1, 45) = 1.046, p = .312, η_p^2 = .023. Further analysis of the relationship between training accuracy and test accuracy revealed that training accuracy marginally affected test performance of the children in the non-symbolic approximate addition training condition F (1, 22) = 4.297, p = .050, η_p^2 2 = .163) but not at all for children in the brightness training condition F (1, 22) = .382, p = .543, η_p^2 2 = .017).

Conclusion

Results of experiment 2 showed that children in non-symbolic approximate addition training group (experimental group) solved symbolic addition problems significantly more accurately than children in brightness comparison training group (control group) and there was no difference in ANS acuity.

These results were in contrast to experiment 1 and increased difficulty level of test problems and different sample seems to be the possible explanation. These results suggest that non-symbolic numbers were driving children to perform better on symbolic addition, and they showed no performance difference on language task (sentences). While performance was enhanced on exact, symbolic arithmetic after engaging the approximate number system, no differences in performance was observed between training groups on a task in the linguistic domain. We did, however, observe that accounting for training accuracy as a covariate eliminated the interaction of interest. Analysis of the training accuracy revealed that, overall, training accuracy was lower for the approximate addition training group compared to the brightness control group and through post hoc analysis the influence of training accuracy on test accuracy revealed that influence of training accuracy on test performance was restricted to those in the non-symbolic approximate addition training condition. One potential explanation is that the difficulty associated with the non-symbolic approximate addition training task, rather than the conceptual content of it, drove enhancement on subsequent symbolic addition test problems. So to figure it out experiment 3 was conducted.

Experiment 3

Objective

Results of experiment 1 and experiment 2 have shown that children trained with non-symbolic approximate addition task performed significantly faster (experiment 1), and accurate (experiment 2) on symbolic addition as compared to children trained with brightness comparison.

It gives rise to few further interesting questions, which needed to be investigated but more prominently it was to explore

1. Whether non-symbolic numbers itself was driving the effect or was it that somehow children were activating symbolic numbers in their cognitive processing by looking at non-symbolic number?
2. Whether it was due to non-symbolic numbers itself specifically or due to the addition aspect of non-symbolic numbers training?
3. Whether greater difficulty of non-symbolic approximate addition is driving the above mentioned effects as compared to the control task?

To answer this question, a non-numerical addition task was needed so that it could answer both questions. If children trained with this task would perform similarly as those tested in experiment 1 experimental group. Then it would be the addition aspect of training that might be warming up children's brain to process better symbolic addition. If children would perform more like brightness comparison trained group, then it would indicate towards the non-symbolic numbers role in processing better the symbolic numbers. So as extensive research evidence shows that processing of number and space association is specifically linked (Kucian et al., 2011).

To determine whether differences in difficulty between training tasks, the process of addition, and/or the engagement of the approximate number system drove the effects in Experiment 1, we created a third training condition involving the addition of line lengths that was equated in difficulty with the original approximate arithmetic training task in terms of reaction time and accuracy. The line addition task involved the same magnitudes of difficulty (7:4 ratio comparisons and 7:5 ratio comparisons), timing, and total number of trials as in the non-symbolic addition task.

Method

Participants. Subjects were 24 first grade children from Greater Boston area of Massachusetts, USA. 11 girls, 13 boys, (*M* age = 6 years 311 days, *SD* = 77 days) were included in final data analysis.

An additional 5 children participated in the study but were excluded from main data analysis for, not completing the study (4), and because of the required number of children was already included in main data keeping mean age and gender equal in both groups (1). Children were scheduled through Harvard lab for developmental studies database by phone and email. Study was approved by committee on use of human subjects (CUHS) of Harvard University. All children and their parents gave written consent for participation in the study and were compensated for their participation in the study (See IRB approval in Appendix J).

Stimuli and Display. Following tasks were used for experiment 3.

1. Line length addition task

Line task was similar to non-symbolic number task in all aspects except the fact that it was non-numerical. All the instructions, display and parameters were same as in non-symbolic addition task (See details in method chapter training task 3).

2. Symbolic addition problems (same as in experiment 1)
3. Panamath task

The procedure and materials for exact symbolic addition testing and approximate number acuity testing were identical to that used in Experiment 1.

Design and procedure. All the procedures, instructions and order of the experimental activities were same as in the experiment 1. Children played the computer game based on 8 training trials followed by 50 test trials of assigned training task (25 of 7:4, 25 of 7:5). After that children solved 2 sets of symbolic addition problems, each comprising 10 problems and before each set they did a sample problem. After children's choice of taking a break or not, they attempted 10 test trials of assigned training task and solved 2 more sets of symbolic addition problems, each comprising 10 problems. At the end kid played Panamath task based on all same parameters and procedures as in experiment 1.

Results

Participant factors. There were no significant differences in age or in approximate number system acuity between non-symbolic approximate addition group and line length addition group.

Table 16

t-test comparing experimental and control group on mean age, (*in experiment 3, Phase1)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	6 years 311 days	73.48	.035	46	= .973	0.01032
Line length Addition	24	6 years 311 days	77				

Table 16 shows that there were no significant differences on age between non-symbolic approximate addition group and line length addition group.

Table 17

t-test comparing experimental and control group on Weber Fraction (w) (in experiment 3, Phase1)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	.17	.11	-1.116	46	= .270	0.32909
Line length Addition	24	.21	.12				

Table 17 shows that there were no significant differences in approximate number system acuity between non-symbolic approximate addition group and line length addition group.

Training task performance. An ANOVA on comparing training performance of the new line length addition group to the training performance of the non-symbolic approximate addition group of Experiment 1 with the within-subjects repeated factor of Ratio (2 levels: 7:4, 7:5) and the between-subjects factor of Training Condition revealed a significant main effect of Ratio on accuracy F (1, 46) = 39.632, p < .001, ηp2 = .463 but no significant differences of Ratio on reaction time F (1, 46) = 2.168, p = .148, ηp2 = .045, of Training Condition on reaction time F (1, 46) = 1.647, p = .206**,** ηp2 = .035 and no interaction of Ratio and Training Condition F (1, 46) = .251, p .618, ηp2 = .005 or of Training Condition on accuracy F (1, 46) = .017, p = .898**,** ηp2 = .000**,** and no interaction of Training Condition and Ratio F (1, 46) = .075, p =

.786, ηp2 = .002 suggesting we were able to effectively equate performance on the new line addition task with the previous non-symbolic addition task.

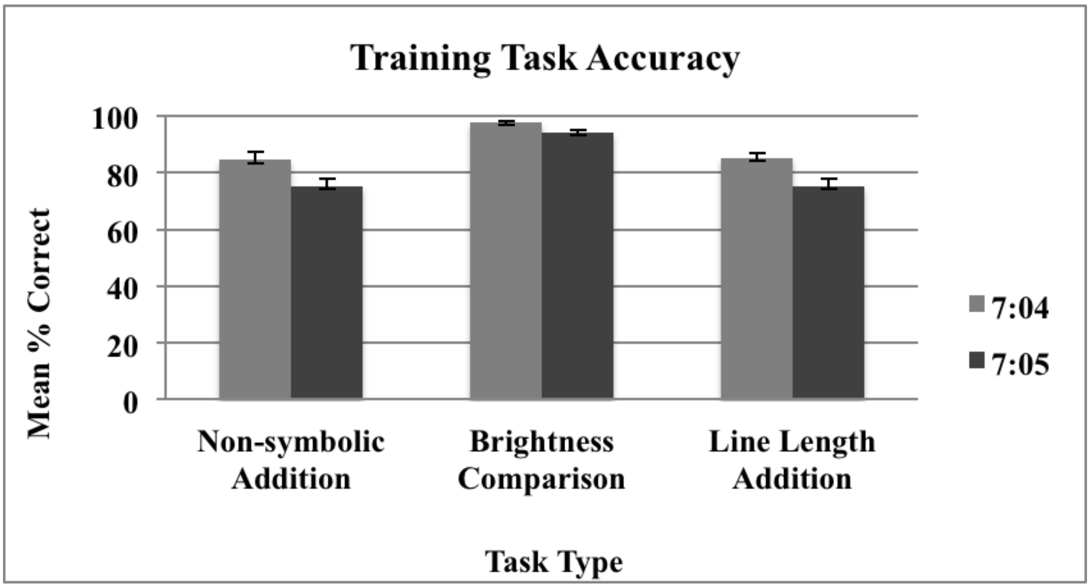

Figure 19. Average task accuracy (expressed as percent correct) over ratio for each condition in experiment 3 (Phase 1)

An ANOVA on comparing reaction time on training performance of the new line length addition group to the training performance of the Brightness Comparison group of Experiment 1 with the within-subjects repeated factor of Ratio (2 levels: 7:4, 7:5) and the between-subjects factor of Training Condition revealed a significant main effect of Ratio F (1, 46) = 5.289, p < .05, ηp2 = .103, main effect of Training Condition, (1, 46) = 9.128, p < .005, η^2 = .166, but no significant interaction of Ratio and Training Condition F (1, 46) = 1.499, p = .227, ηp2 = .032. A Similar analysis on accuracy with the within-subjects repeated factor of Ratio (2 levels: 7:4, 7:5) and the between-subjects factor of Training Condition revealed significant main effect of Training Condition F (1, 46) = 57.944, p < .001, ηp2 = .557 and of Ratio F (1, 46) =

40.083, $p < .001$, ηp2 = .466 and interaction of Ratio and Training Condition F (1, 46) = 9.410, $p < .005$, ηp2 = .170.

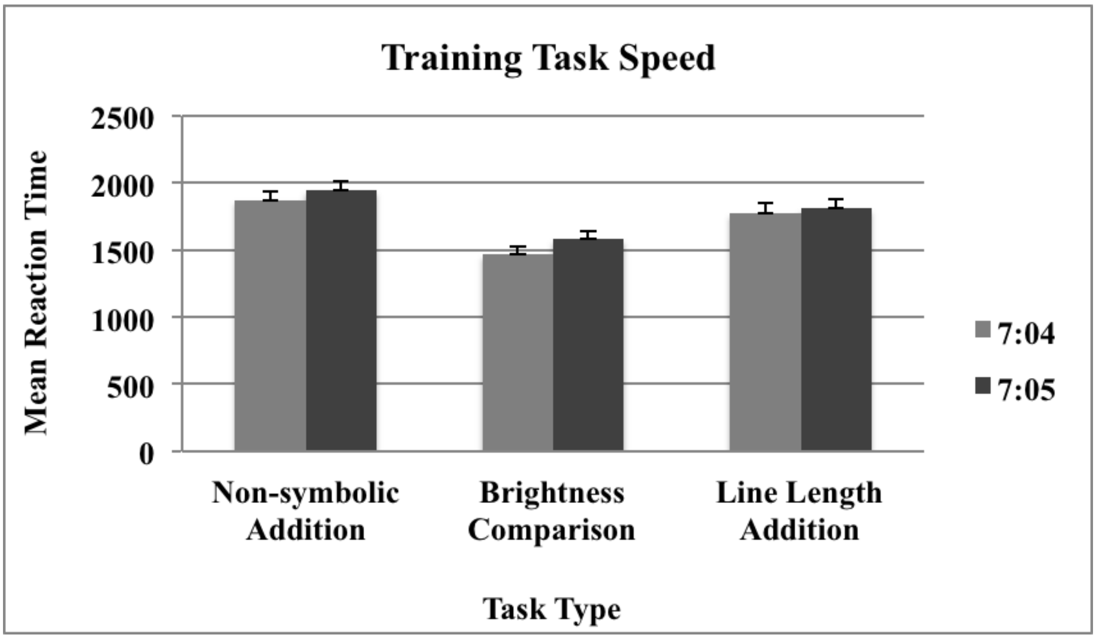

Figure 20. Average reaction time (in milliseconds) over ratio in each condition in experiment 3 (Phase 1)

Exact symbolic addition test performance. An ANOVA on the time to complete the exact, symbolic addition test sets with the within-subjects factor of Difficulty (4 levels) and the between-subjects factor of Training Condition (approximate numerical addition or line length addition) revealed a significant main effect of Difficulty F (3, 138) = 40.703, $p < .001$, ηp2 = .469 and a significant main effect of Training Condition F (1, 46) = 4.084, $p < .05$, ηp2 = .082 (see Figures 2 and 3). There was no significant interaction between Training condition and Difficulty F (3, 138) = .330, $p = .804$, ηp2 = .007.

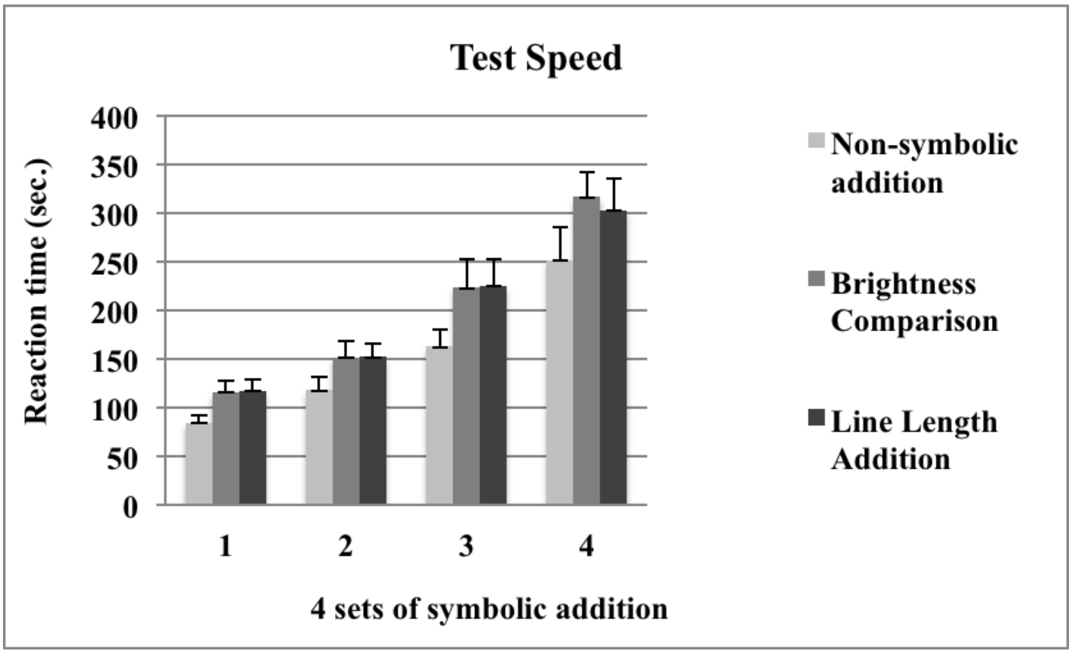

Figure 21. Average speed of test completion (in seconds) for each condition in experiment 3 (Phase 1)

A similar ANOVA on accuracy with the same factors revealed only a main effect of Difficulty of test problems on accuracy F (3, 138) = 37.430, p < .001, ηp2 = .449. There was no significant main effect of Training Condition, F (1, 46) = 1.898, p = .175, ηp2 = .040, nor any interaction of Training Condition and Difficulty F (3, 138) = 1.184, p = .318, ηp2 = .025.

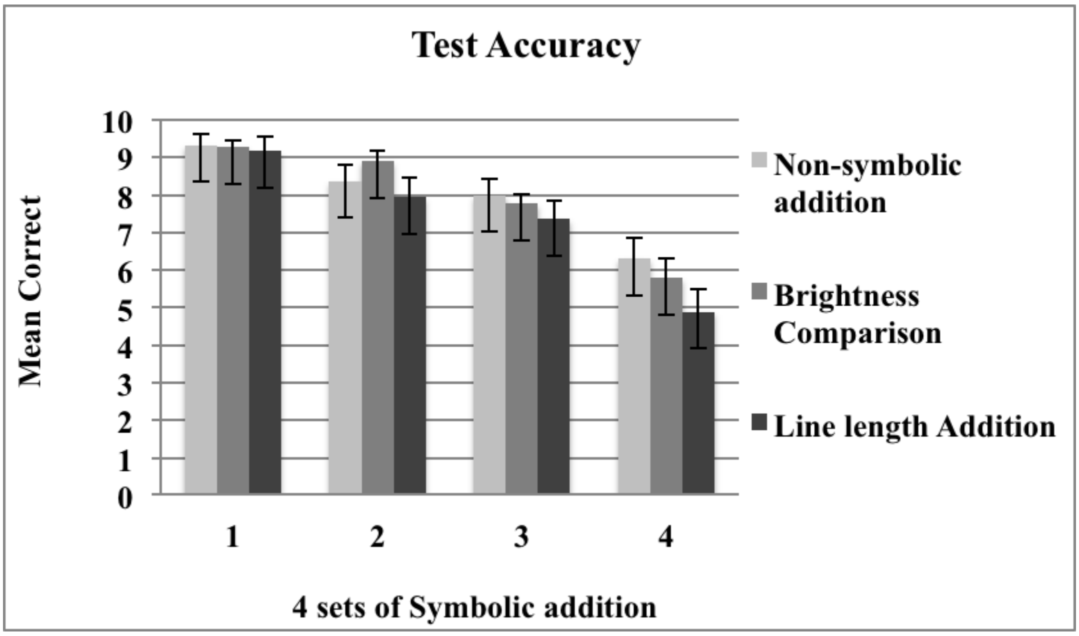

Figure 22. Average test accuracy (expressed as out of 10 problems) for each condition in experiment 3 (Phase 1)

An ANOVA on the time to complete the exact, symbolic addition test sets comparing children in the brightness training condition of Experiment 1 with children in the line length training condition of the current experiment revealed a significant main effect of Difficulty F (3, 138) = 48.200, p = < .001, ηp2 = .512, but no significant differences between Training Conditions F (1,46) = .011, p = .916, ηp2 = .000 or interaction of Training Condition and Difficulty, F (3, 138) = .088, p = .966, ηp2 = .002 (see Figures 2 and 3). A similar ANOVA on accuracy with the same factors revealed only a main effect of Difficulty of test problems on accuracy F (3, 138) = 53.885, p < .001, ηp2 = .539 and no main effect of Training Condition F (1, 46) = 1.957, p = .169, ηp2 = .041 and of Training Condition and Difficulty F (3, 138) = .776, p = .509, ηp2 = .017.

Overall, subjects were slower to complete and less accurate on more difficult sets of problems. Children performing the line length addition training task subsequently completed the exact, symbolic addition test problems significantly slower than children who completed the approximate numerical addition training (Experiment 1) but no different from children who completed the brightness training condition (Experiment 1). These differences in time to complete symbolic addition test sets between training groups were not due to differences in performance on the training task nor were they due to a speed-accuracy tradeoff difference between the groups.

Further Analysis. The critical main effect of Training Condition (between non-symbolic addition and line length addition) on speed remained significant even after effects of training task reaction time F (1, 45) = 11.586, $p < .005$, $\eta^2 = .205$.

Conclusion

Enhancement of exact symbolic arithmetic performance in children trained with non-symbolic approximate arithmetic problems could not be explained by differences in difficulty between the experimental (non-symbolic numerical addition) and control training task (line length addition), as the advantage in time to complete test problem sets remained for those trained on the non-symbolic numerical addition problems after equating performance on the new control task (line length addition). Furthermore, the enhancements seen in those of the non-symbolic numerical addition relative to the other groups cannot be explained by simply engaging the addition process, as those trained in a non-numerical line length addition task did not subsequently show the same enhancements. So this experiment was helpful to disentangle whether it's the addition aspect of training or varying difficulty level that was enabling the children trained with non-symbolic approximate addition to outperform their counter parts. It was really interesting to further explore the possible link and mechanism between non-symbolic and symbolic numerical cognition carrying many implications for cognitive science, numerical cognition and mathematics learning. Educational interventions can also be developed following the effectiveness of approximate number system training.

Experiment 4

Objective

In this experiment we investigated whether the enhancement of exact symbolic arithmetic was due to performance of arithmetic over numerical representations or engagement of the numerical representations more generally. To distinguish between these explanations, we devised a training task involving non-symbolic approximate numerical comparison and compared performance on the same exact symbolic arithmetic problems of those trained on the new numerical comparison training task to the previous results of Experiments 1 and 3.

Purpose of this task was to further rule out the possibility whether children would still be performing better on symbolic addition problems, if trained with non-symbolic comparison or not. If it would be the case that non-symbolic numbers are driving children's better performance on symbolic addition, then children trained with non-symbolic comparison should also show advantage on their performance on symbolic addition problems.

Method

Participants. Twenty-four first grade children (11 females, *M* age = 6 years 355 days, *SD* = 67 days) were included in final dataset. Another 4 children participated in the experiment but were excluded from main data analysis due to participants not following directions regarding the sequence of tasks (1) and not completing the study (3). Study was approved by committee on use of human subjects (CUHS) of Harvard University. All children and their parents gave written consent for participation in the study and were compensated for their participation in the study (See IRB approval in Appendix J).

Stimuli and display. Stimuli were similar to those used in the non-symbolic approximate addition task of Experiment 1 in terms of construction and presentation. Training task in this experiment was non-symbolic approximate comparison instead of non-symbolic addition. Tasks were following.

1. Non-symbolic approximate comparison task
2. Symbolic addition problems
3. Panamath task

Design and procedure. All the procedures, instructions and order of the experimental activities were same as in the experiment 1 and 3. Children played the computer game based on 8 training trials followed by 50 test trials of assigned training task (25 of 7:4, 25 of 7:5). After that children solved 2 sets of symbolic addition problems, each comprising 10 problems and before each set they did a sample problem. After children's choice of taking a break or not, they attempted 10 test trials of assigned training task and solved 2 more sets of symbolic addition problems, each comprising 10 problems. At the end kid played Panamath task based on all same parameters and procedures as in experiment 1 and 3 (See Figure 23).

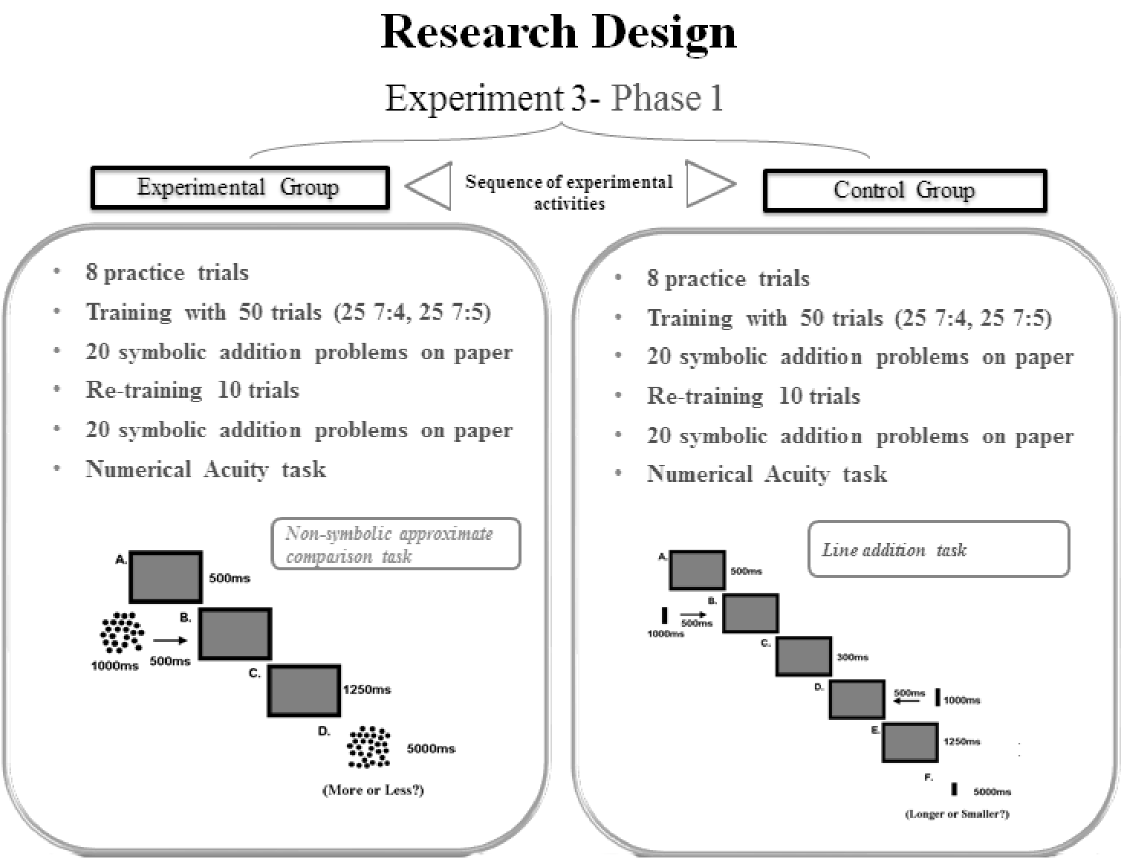

Figure 23. Research design of experiment 3

Results

Participant factors. A One way ANOVA was carried out to compare the different training groups on age and numerical acuity. The children in the different conditions did not differ in average age *F* (3, 95) = 1.697, *p* = .173: numerical addition, *M* = 6 years, 311 days, *SD* = 73 days; line addition *M* = 6 years 311 days, *SD* = 77 days; numerical comparison *M* = 6 years 355 days, *SD* = 67 days; brightness comparison *M* = 6 years, 332 days, *SD* = 94 days) or approximate numerical acuity *F* (3, 95) = 0.766, *p* = .516: numerical addition *M* = .17, *SD* = .11; line addition *M*

= .21, *SD* = .12; numerical comparison *M* = .18, *SD* = .08; brightness comparison *M* = .17, *SD* = .08).

Training task performance. Training task performance was analyzed by separate mixed-factor ANOVAs on average reaction time and accuracy with the within-subjects factors of Ratio (2 levels), and the between-subjects factor of Training Condition (4 levels: non-symbolic numerical addition, line length addition, non-symbolic numerical comparison, brightness comparison). Analysis revealed significant main effect of Training Condition on reaction time *F* (3, 92) = 7.081, *p* < .001, ηp2 = .188, and Ratio *F* (1, 92) = 12.821, *p* < .005, ηp2 = .122 however there was no interaction of Ratio and Training Condition *F* (3, 92) = .961, *p* = .414, ηp2 = .030.

Further post hoc analysis of main effect of Training Condition on speed revealed significantly faster performance on the brightness comparison task compared to all other tasks (brightness vs. non-symbolic addition: *t* (46) = - 4.665, *p* <. 001; brightness vs. line length addition: *t* (46) = -3.021, *p* <. 005; brightness vs. non-symbolic comparison: *t* (46) = -3.351, *p* < .005. No other significant differences were seen in speed of the different tasks (non-symbolic addition vs. line length addition: *t* (46) = 1.283, *p* = .206; non-symbolic addition vs. non-symbolic comparison: *t* (46) = 1.286, *p* = .205; line length addition vs. non-symbolic comparison: *t* (46) = -.084, *p* =. 933.

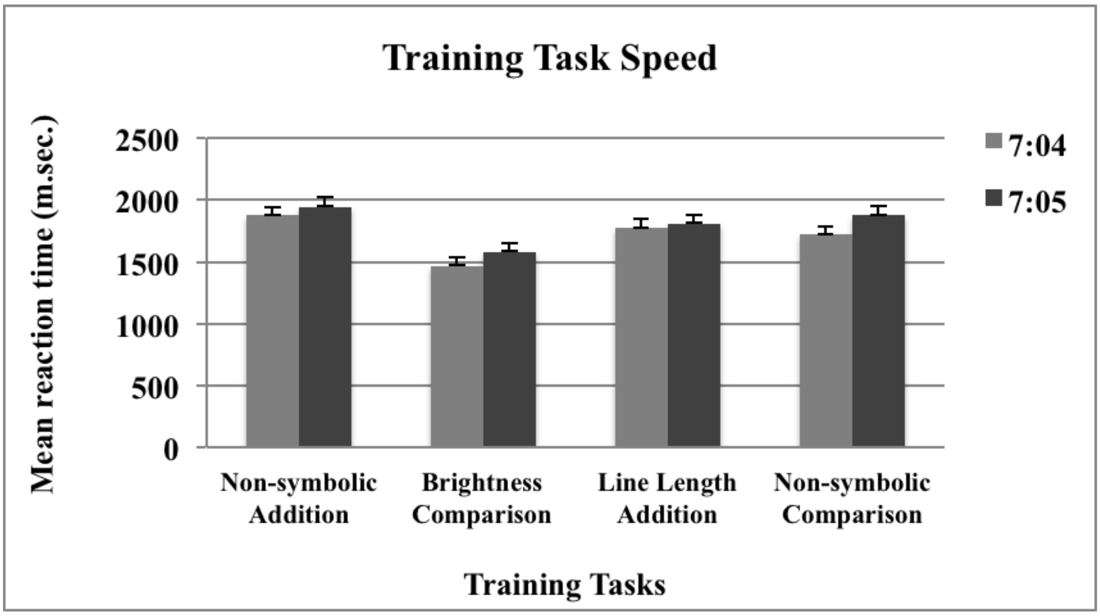

Figure 24. Average reaction time (in milliseconds) over ratio in each condition in experiment 4 (Phase 1)

A similar analysis on the measure of training task accuracy revealed main effects of Training Condition F (3, 92) = 17.233, $p < .001$, ηp2 = .360 and Ratio F (3, 92) = 69.934, $p < .001$, ηp2 = .432, however no significant interaction of Ratio and Training Condition, F (3, 92) = 2.518, $p = .063$, ηp2 = .076.

In addition, post hoc pairwise comparisons of accuracy revealed that subjects in the brightness condition were more accurate than all other groups (brightness comparison vs. non-symbolic addition: t (46) = 5.951, $p < .000$; brightness comparison vs. line length addition: t (46) = 7.612, $p < .001$; brightness comparison vs. non-symbolic comparison: t (46) = 5.811, $p < .001$; and (non-symbolic addition vs. line length: t (46) = -.129, $p = .898$; non-symbolic addition vs. non-symbolic comparison: t (46) = -2.291, $p < .05$; line length addition vs. non-symbolic comparison: t (46) = -2.659, $p < .05$.

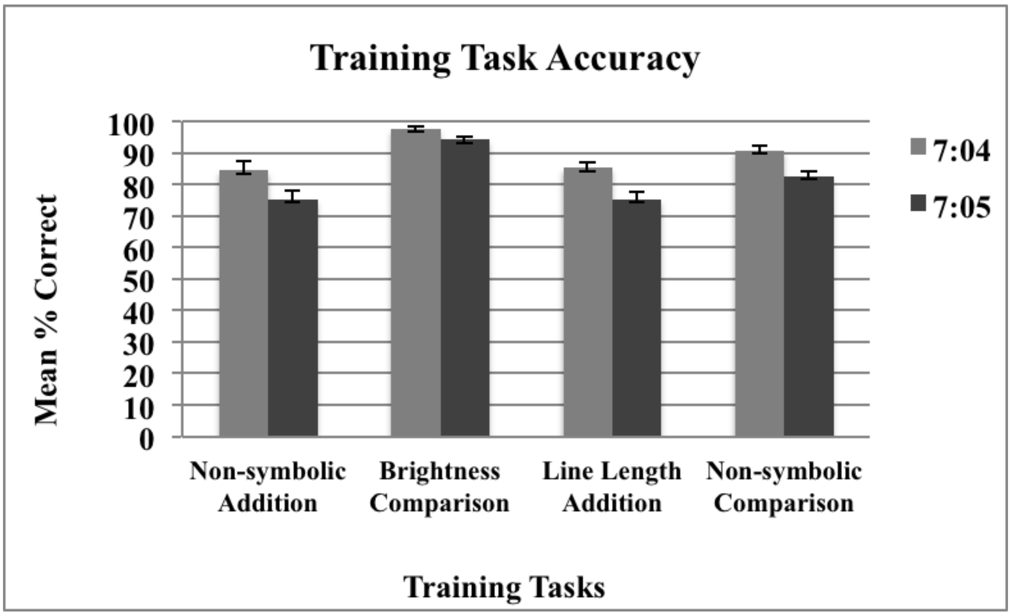

Figure 25. Average task accuracy (expressed as percent correct) over ratio for each condition in experiment 4 (Phase 1)

Exact, symbolic addition test performance. An ANOVA on the time taken by children to complete each page of the written arithmetic test problems of symbolic addition test sets with the within-subjects factor of Difficulty (4 levels) and the between-subjects factor of Training Conditions (approximate numerical addition, Line Length addition, approximate numerical comparison and brightness comparison) was carried out.

Table 18

Mixed Factor ANOVA of Training condition (*Non-symbolic addition, brightness comparison, line length addition, non-symbolic comparison), Difficulty* (*1, 2, 3, 4) on time to complete symbolic addition test sets* (*in experiment 4, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Training Condition	3, 92	3.503	.103	< .05
Difficulty	3, 276	102.920	.528	< .001
Difficulty * Training Condition	9, 276	.391	.013	.939

Table 18 showed significant main effect of Training Condition, main effect of Difficulty, however there was no significant interaction of Difficulty and Training Condition.

Pairwise post hoc analysis revealed that children in non-symbolic approximate numerical addition and non-symbolic approximate numerical comparison condition completed symbolic arithmetic problems faster than children in non-numerical conditions (non-symbolic numerical addition vs. brightness comparison: t (46) = 2.210, p <. 05; non-symbolic numerical addition vs. line length addition: t (46) = -2.021, p <. 05; non-symbolic comparison vs. brightness comparison t (46) = 2.644, p < .05; non-symbolic comparison vs. line length addition, t (46) = 2.388, p <.05. No differences in speed on symbolic addition test sets were observed between non-numerical condition (brightness comparison vs. line length addition t (46) = .106, p =

.961 or between numerical condition (nonsymbolic numerical addition vs. non-symbolic numerical comparison t (46) = .049, p = .961.

Across the four sets of symbolic addition problems, children trained with non-symbolic approximate addition and comparison took were faster than brightness comparison and line length addition condition.

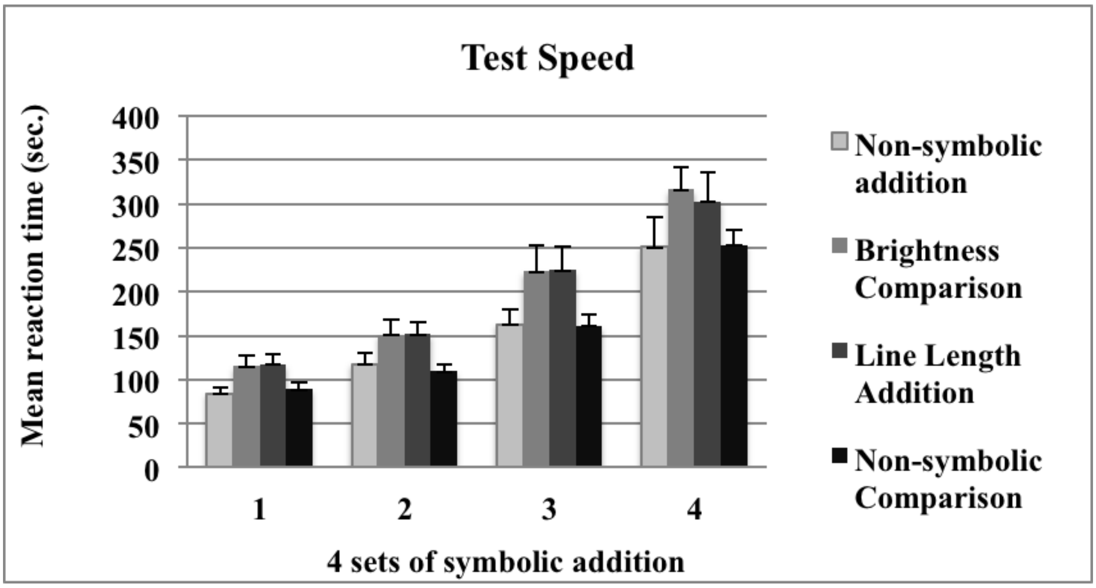

Figure 26. Average speed of test completion (in seconds) for each condition in experiment 4 (Phase 1)

A similar ANOVA on accuracy on the symbolic addition test sets with the within-subjects factor of Difficulty (4 levels) and the between-subjects factor of Training Conditions (approximate numerical addition, Line Length addition, approximate numerical comparison and brightness comparison) was carried out.

Table 19

Mixed Factor ANOVA of Training condition (*Non-symbolic addition, brightness comparison, line length addition, non-symbolic comparison), Difficulty* (*1, 2, 3, 4) on symbolic addition test sets accuracy* (*in experiment 4, Phase1)*

Variables	*df*	*F*	ηp2	*p*
Training Condition	3, 92	2.850	.085	< .05
Difficulty	3, 276	81.558	.470	< .001
Difficulty * Training Condition	9, 276	.981	.031	.456

Table 19 shows a significant main effect of training condition. There was significant main effect of Difficulty F (3, 276) = 81.558, p < .001, ηp2 = .470 however no interaction of Difficulty and Training Condition F (9, 276) = .981, p = .456, ηp2 = .031.

Post hoc pairwise analysis revealed only pair of groups showing a difference on accuracy (non-symbolic comparison vs. line length addition (t (46) = -2.782, p < .05) and no difference on accuracy between conditions (non-symbolic approximate addition vs. brightness comparison, t (46) = -.163, p = .871; non-symbolic approximate addition vs. line length addition t (46) = 1.378, p = .175; non-symbolic comparison vs. brightness comparison t (46) = -1.852, p = .071.

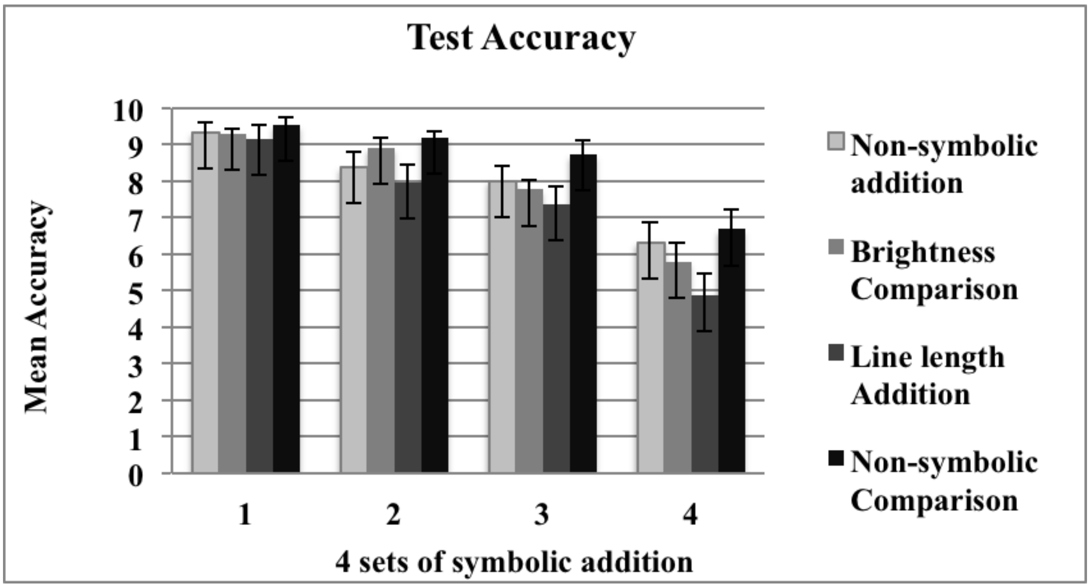

Figure 27. Average test accuracy (expressed out of 10 problems) for each condition in experiment 4 (Phase 1)

That is, although difficulty affected test performance in all training groups, those in the number comparison condition completed exact, symbolic addition test sets faster than either those in the brightness comparison condition or the line length addition condition. Furthermore, in this case, those in the approximate number comparison condition also were more accurate on exact, symbolic addition problems compared to those in the line addition condition. These results could not be explained by a speed-accuracy tradeoff. The speed–accuracy tradeoff refers to the phenomenon where, at a given level of stimulus discriminability, decision makers may produce faster responses but make more errors (Pachella, 1974).

Further analysis. We tested for the practice effect after controlling for effects of training task reaction time and accuracy. The critical main effect of Training Condition on speed remained significant even after effects of training task reaction time F (3,91) = 9.724, $p < .001$, ηp2 = .243, and accuracy F (3, 91) = 2.834, $p < .05$, ηp2 = .085 were accounted for as covariates. Thus, Training Condition had an effect on the time and accuracy it took participants to complete exact symbolic addition problems that cannot be explained by differences in performance, attention to, or engagement with the different training tasks.

***Regression analysis with age as predictor of symbolic addition accuracy and reaction time* (*Non-symbolic comparison*).** Age of children trained with non-symbolic comparison did not significantly predicted reaction time to be faster on symbolic addition, b = -.079, t (94) = -.771, p = .433. Age did not explained significant proportion of variance in reaction time scores, R^2 = .006, F (1, 94)= .595, p = .443. Age of children significantly predicted accuracy on symbolic addition, b = .277, t (94) = 2.800, $p < .05$. Age also predicted significant proportion of variance in accuracy scores, R^2 = .077, F (1, 94) = 7.841, $p < .005$.

Conclusion

Those participants trained on approximate number comparison subsequently completed exact symbolic addition problems faster than those trained on brightness magnitude comparison (Experiment 1) and line length comparison (Experiment 2).

However, no differences in time to complete exact symbolic arithmetic questions were seen between those trained on approximate numerical addition (Experiment 1) and those trained on approximate number comparison. These results suggest that engagement of the approximate number system in general, rather than arithmetic computation over number representations specifically, is driving enhancement of subsequently performed exact, symbolic arithmetic.

Results (of experiment 1, 2, 3 and 4 together)

Participant factors. The children in the different conditions did not differ in average age F (3, 95) = 1.697, p = .173: numerical addition, M = 6 years, 311 days, SD = 73 days; line addition M = 6 years 311 days, SD = 77 days; numerical comparison M = 6 years 355 days, SD = 67 days; brightness comparison M = 6 years, 332 days, SD = 94 days) or approximate numerical acuity F (3, 95) = 0.766, p = .516: numerical addition M = .17, SD = .11; addition M = .21, SD = .12; numerical comparison M = .18, SD = .08; brightness comparison M = .17, SD = .08).

Training task performance. The analysis of the average reaction time during each training task revealed main effects of Ratio F (1, 92) = 4.197, p < . 05, $\eta_p^{2\,=}$ .044, Time F (1, 92) = 19.385, p < .001, η_p^2 = .174, and Experimental Condition F (3, 92) = 7.222, p < .001, η_p^2 = .191, and an interaction between Ratio and Time F (1, 92) = 5.078, p < .05, η_p^2 = .052. Regardless of condition, subjects were faster on the second half compared to the first half of the training trials F (1, 95) = 19.297, p < .001 (See

Figure 28). Further analysis of the interaction between Ratio and Time revealed ratio differences averaged across all conditions emerged only on the second half of training problems $t(95) = -3.054$, $p < .005$, with longer average response times to problems involving close ratios compared to problems involving far ratios (see Figure 29). Further post hoc analysis of the main effect of Training Condition on speed revealed significantly faster performance on the brightness comparison task compared to all other tasks (brightness vs. numerical addition: $t(46) = -4.750$, $p < .001$; brightness vs. line addition: $t(46) = -2.919$, $p < .01$; bright- ness vs. number comparison: $t\ (46) = -3.312$, $p < .005$) (numerical addition: $M = 1951$ ms, $SD = 284$ ms, Range = 1416–2542 ms; line addition: $M = 1826$ ms, SD 346 ms, 1140–2764 m; number comparison: $M = 1835$ ms, $SD = 294$ ms, Range = 1111–2313 ms; brightness comparison: $M = 1555$ ms, $SD = 292$, Range = 895–2040 ms) (see Figure 28). No other significant differences were seen in speed of the different tasks (all other ps > .17).

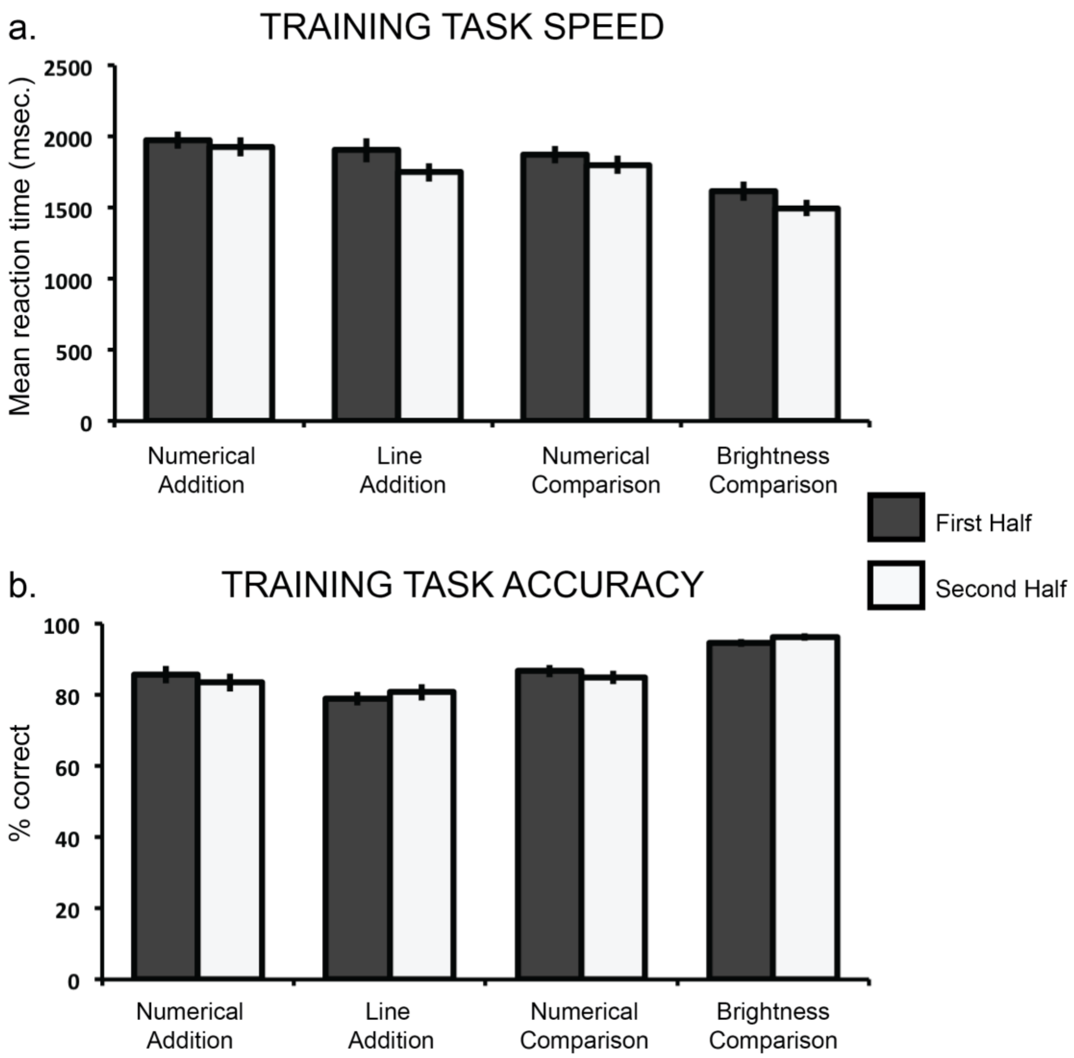

Figure 28. Average training task performance over time in Experiment 4. a) Average reaction time (in milliseconds) for each condition. b) Average task accuracy (expressed as percent correct) for each condition.

On the measure of training task accuracy, the analysis revealed main effects of Ratio F (1, 92) = 57.859, p < .001, η_p^2 = .386) and Training Condition F (3, 92) = 14.764, p < .001, η_p^2 =.325. No main effects of Time or interactions between factors were observed. Participants were less accurate on problems involving closer ratios, regardless of the experimental task. In addition, post hoc pairwise comparisons of accuracy revealed that subjects in the brightness condition were more accurate than all other groups (brightness vs. numerical addition: $t(46) = 4.546$, $p < .001$; brightness vs. line addition: $t(46) = 7.530$, $p < .001$; brightness vs. number comparison: $t(46) = 5.723$, $p < .001$), and the numerical comparison group was more accurate than the line addition group (line addition vs. number comparison: t $(46) = -2.436$, $p < .05$). However, neither the line addition group nor the numerical comparison group differed significantly from the numerical addition group in accuracy (numerical addition vs. line addition: t $(46) = 1.596$, $p = .117$; numerical addition vs. numerical comparison: $t(46) = -.440$, $p = .662$).

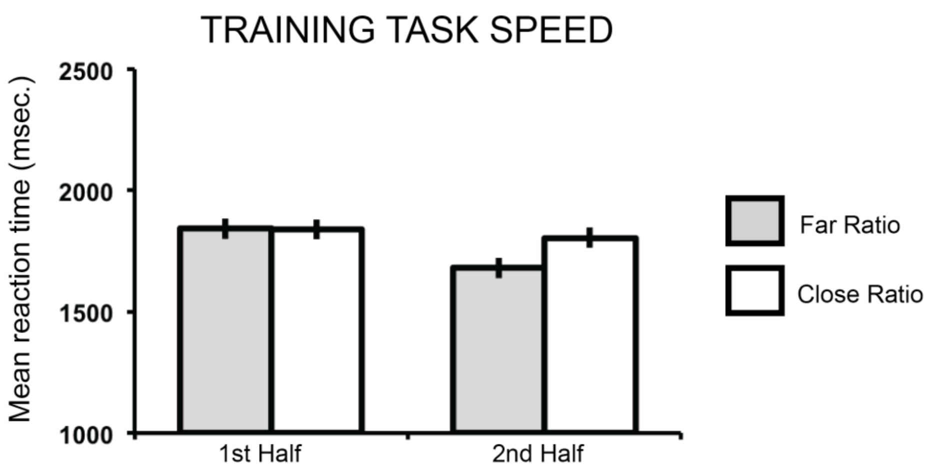

Figure 29. Effects of ratio on average training performance over time in Experiment 4

In sum, the analysis of performance on the four tasks of numerical addition, line length addition, numerical comparison, and brightness comparison suggests that subjects improved in speed in a ratio-dependent manner over the course of each task, independent of the actual training condition. Furthermore, those completing the brightness comparison task performed better than those in the other groups: they were both faster and more accurate. On the other hand, no differences on any of the performance measures were observed between the numerical addition group and the numerical comparison group or between the numerical addition group and the line-length addition group.

Exact symbolic arithmetic test performance. The analysis of the average time taken by children to complete each page of the written arithmetic test problems revealed a main effect of Training Condition F (1, 95) =3.366, p <.05) (see Figure 30). Pairwise post hoc analysis revealed that children in the numerical addition and numerical comparison conditions completed symbolic arithmetic problems faster than children in the non- numerical conditions (numerical addition vs. brightness comparison: t (46) = - 2.176, p < .05; numerical addition vs. line addition: t (46) = - 2.030, p < .05; brightness vs. number comparison: t(46) = 2.527, p < .05; line addition vs. number comparison: t(46) = 2.327, p < .05) . No differences in speed on symbolic arithmetic tests were observed between non-numerical conditions (brightness comparison vs. line addition: t (46) = .049, p = .961) or between numerical conditions (numerical addition vs. number comparison: t (46) = .032, p = .975) *F*igure 30).

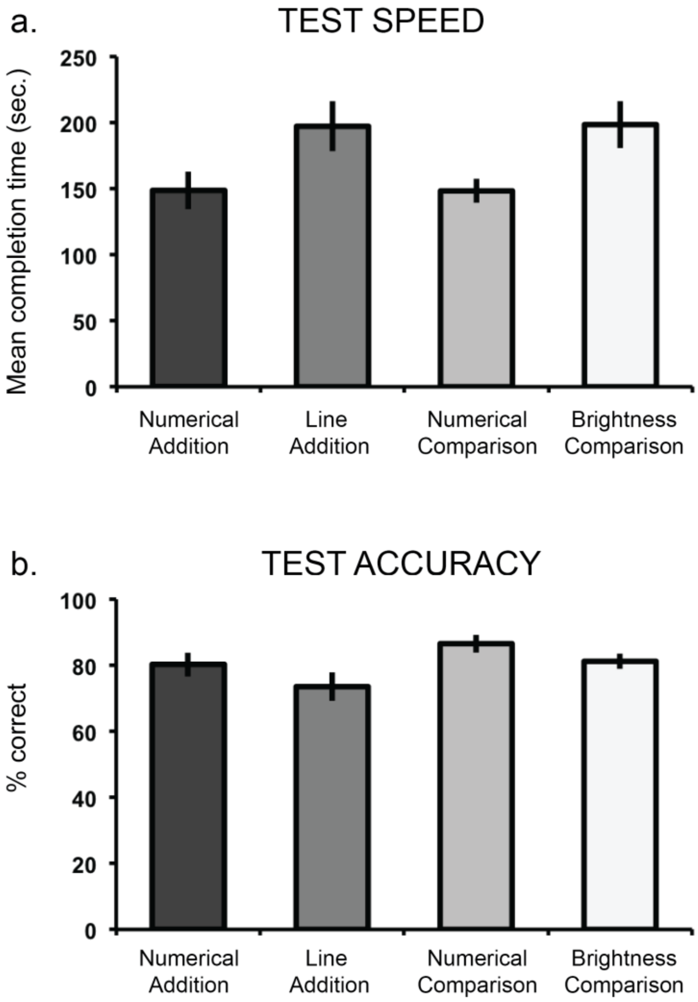

Figure 30. Average symbolic arithmetic test performance in Experiment 4. a) Average speed of test completion (in seconds) for each condition b) Average test accuracy (expressed as percent correct) for each condition.

The analysis of performance accuracy on the symbolic arithmetic test revealed a marginally significant main effect of Training Condition F (1, 95) = 2.598, p = .057). How- ever, post hoc pairwise comparisons revealed that the only pair of groups showing a difference in accuracy was the line-length addition group and the numerical comparison group pair, with the numerical comparison group subsequently performing more accurately on the symbolic arithmetic problems t (46) = -2.576, $p < .05$) .

Further analyses. An alternative account of the differing effects of the different training conditions on arithmetic tests appeals not to their differences in content but the extent to which they presented problems that were challenging or engaging. Two aspects of the findings presented above cast doubt on such an account. First, differences in training task performance did not consistently predict the effects of the different training conditions on subsequent test problems. For example, reaction time and accuracy on training problems were not different from each other in the numerical addition and line-length addition conditions, yet those in the numerical addition condition performed significantly faster on subsequent test problems compared to those in the line-length addition condition. Second, no differences were observed in the extent of learning on the different training tasks (i.e., the change in performance from the first half to the second half of the session), suggesting that participants where equally engaged or attentive in their given task. Nevertheless, additional analyses were undertaken to address this alter- native account further. We tested for the practice effect after controlling for effects of training task reaction time and accuracy. The critical main effect of Training Condition on speed remained significant even after effects of training task reaction time F (3, 91) = 8.680, $p < .001$) and accuracy F (3, 91) = 4.285, $p < .01$) were accounted for as covariates. Thus, Training Condition had an effect on the time it took participants to complete exact symbolic addition problems that cannot be explained by differences in performance, attention to, or engagement with the different training tasks.

Conclusion of experiment 1, 2, 3 and 4 (Phase 1)

Results of all the experiments have shown that children trained with non-symbolic approximate addition and non-symbolic approximate comparison as compare to brightness comparison and line addition performed significantly faster (experiment 1) and more accurate (experiment 2). In experiment 2 children were post tested on language task as well to control for general cognitive abilities and results showed that training of non-symbolic numbers gave advantage children to perform better on symbolic math and there was no difference on children's general cognitive abilities. Still we cannot conclusively say that whether it is specifically non-symbolic numbers driving this effect or is it due to non-symbolic numbers activating symbolic numbers and thus driving this effect? To answer this question further research is needed.

Future researchers can prime one group of children with symbolic numbers and one with non-symbolic and see which one drives children better performance on symbolic addition to disentangle it further.

Discussion

Results from experiments shed light on the significant role of priming/ training of analogue magnitudes on symbolic number processing. Through first experiment, it has been shown that if arithmetic problems are in the range of children solving abilities, children trained with non-symbolic numbers turned out to be faster than control group. As the difficulty level increases within each set of symbolic addition problems, we see a trend of experimental group being more accurate on difficult problems as compare to control group, although not significant.

However, in experiment 2, both set of symbolic addition problems were harder than the problems, children solved in experiment 1. Results of experiment 2 showed that if children are trained in non-symbolic arithmetic and solving the problems beyond their level, they turned out to be significantly more accurate that control group. Taken together both experiments findings suggest that experiences operating on non-symbolic magnitudes played a significant role in children's processing of symbolic numbers.

Results of experiments 3 and 4 also follow same pattern of results as we can see that children I experiment 4 (trained with non-symbolic comparison) performed significantly better than children trained with line addition training. It adds up to previous findings and shows that it is not "addition" as an operation that is driving the effect rather it is non-symbolic training that is driving the effect. Although children trained with non-symbolic comparison were older and ahead in first grade level than children participated in line addition group.

So may be children of experiment 3 were ahead of children tested in study 2 in grade 1 and may be more advance in math learning that might have gave advantage to them to perform better.

Results might be effected by the fact that control group was trained in bright vs. dark comparison as compare to experimental group trained in non-symbolic arithmetic thus experimental group going through more processing based on numerical task. We can see this effect on children's reaction time and accuracy on training task in both experiments. Where children trained in bright vs dark comparison are taking less time and showing higher accuracy than experimental group.

Another possibility is that results might be due to number-space association and not specific to non-symbolic number training. As study carried out by Booth & Siegler (2008) demonstrated that children's number line estimates were positively correlated and were predictive of learning of answers to novel addition problems.

Overall there was no significant difference in weber fraction among groups tested under different trainings, which support the important role of approximate number training in enhancement of symbolic number processing and different research evidence have already shown important link between approximate number system and math ability (see for reviews, Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011).

These results are less conclusive due to above mentioned factors but still promising in terms of showing an effect of non-symbolic addition training on symbolic arithmetic processing as compare to control group.

Results of Phase-1 suggest that non-symbolic numerical magnitudes play significant role in solving symbolic problems. Further research studies are needed to rule out the possibility of number-space association. Moreover, future studies can be designed in a way so that children performance on each problem could be monitored separately for looking more closely their responses in terms of reaction time and accuracy. So that children performance on easy and difficult problems could be analyzed separately.

Although children in this experiment got non-symbolic number training for short time in one day visit so training the children for little bit longer time for more days might demonstrate more positive results in children symbolic number processing.

Chapter-IV

PHASE-II

Despite of all the research evidence mentioned above, it is not known, whether training would be effective in different cultural context like Pakistan? There are no training studies conducted on ANS role in symbolic math and number line placement in Pakistan. A developing country in which children have less educational facilities, lack of economic resources, low or no exposure to practice computers, and less technology exposure as IPhone's or some other gadgets as compared to American children. No studies have yet been conducted in Pakistan to see how children from this part of world would perform on symbolic arithmetic when they will be trained with brief non-symbolic addition as compare to control conditions? So to bridge this gap in literature present study was aimed at replicating experiment 1, like the training study already conducted with US sample (Hyde, Khanum, & Spelke, 2014) with another group of children from Pakistan. Moreover, further extension was carried out in terms of experiment 2 by involving number line placement task after different training conditions. So overall aim of the phase 2 was to replicate and see the effectiveness of non-symbolic approximate training with Pakistani first grade children.

A great deal of work indicates towards foundational role of ANS in later math (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011; Star, Libertus, & Brannon, 2013). Two experiments were conducted in Pakistan as a replication and extension of phase 1 to probe this relationship focusing the question, whether training ANS would be helpful for first grade children to perform better on symbolic math or not?

Keeping in view results of all experiments of phase 1, it has been shown that if children are primed/ trained with non-symbolic addition or non-symbolic comparison as compare to control conditions (like color comparison or line addition), they showed the advantage on solving symbolic math problems. To see effectiveness of non-symbolic training on symbolic number processing in a different group of children in Pakistan, it is intended to carry out research in Pakistan along same lines and that might show some interesting findings from a different population.

Primarily experiments are intended to be carried out following the same procedures and material as done with American population. There are following reasons to do that. First of all, as we already know that children from Boston, Massachusetts showed advantage in their performance on symbolic addition as a result of non-symbolic addition, so it would be more useful to keep al the procedures, instruction and tasks same. Secondly, if children will show advantage similarly as American group of children showed, then further steps can be taken to proceed beyond these experiments.

Although there are huge differences in both populations in terms of language, technological advancement, exposure, learning environment, facilities like local library etc. Since children from Boston, Massachusetts had exposure to technologically advanced equipment like computers, I phones, games machines or even in general exposure to environment. Whereas, children in Pakistan does not have that much first hand exposure to advanced technology and environment is not that much stimulating in case of Pakistani population. But despite all these differences, it would be interesting to figure out whether Pakistani children would show an advantage on their symbolic addition performance when trained with non-symbolic numbers or not.

Keeping in view the results of initial experiment further experiments will alsobe conducted following the same procedures and materials.

Experiment 1

Objective

Through training paradigm, in experiment 1 it has been tried to explore whether training the first grade children with non-symbolic approximate addition as compared to control condition will enhance their performance on exact symbolic math or not? Purpose of this replication is to explore whether children even belonging to totally different culture would perform more or less similar as in experiments conducted in USA.

All the design and procedures were kept similar as in experiment 1 and 3, to see possibly same effect as a result of training. So children gender, age and all other factors were also preferably kept similar in order to compare two population and their responses. All the instructions were translated and were given to children in their native language, Urdu.

Participants

In the first experiment 63 first grade children of age 6-7 years from public schools of Islamabad participated, and 48 were included in main data. An additional group of 15 children participated in the experiment but they were excluded from main data analysis because of, not completing the study (10), equipment malfunction (2), kid did not followed the appropriate sequence of the study (1), and experimenter error (2).

Eleven girls and 13 boys were assigned to experimental group (mean age, 6 years 172.20833 days) and 11 girls, 13 boys were assigned to control group (mean age, 6 years 183.667 days).

National Institute of Psychology at Quaid-i-Azam University, Islamabad Pakistan and Federal Directorate of Education Islamabad Pakistan approved the study for ethical considerations (See Appendix J). Principal and head mistress of primary section school, teachers, parents through school administration and children's gave consent for data collection and children were compensated. Experiment was conducted in the school setting keeping in view children's comfort to perform the experiment.

Data was collected from first grade children of Islamabad Model College for Boys, F7/3, Islamabad Model College for Girls F6/2 (street 25) and Islamabad College for Girls (Post Graduate) F6/2.

Stimuli and display (same as in method: chapter 2)

1. Non-symbolic approximate addition training task (as in experiment 1)
2. Line length addition (as in experiment 3)
3. Symbolic addition task (as in experiment 1 & 3)

Children solved four sets of symbolic addition problems on paper one set at a time with increasing difficulty level from 1-4 and each set comprising 10 addition problems (40 in total). Children were given a sample problem to solve before each set. Researcher recorded time to complete each set with stopwatch. Each correctly solved problem was scored as 1.

4. Panamath task (as in experiment1)

Design and Procedure

Children were introduced to experiment in context of computer games and solving math problems on paper by experimenter in their native language: Urdu. Children were quasi randomly assigned to experimental and control condition to equate for age and gender in both groups. Experiment with each child was conducted during school time in a quite room. Children were trained with 50 trials of training task (25 trials of 7:4 and 25 of 7:5) on computer and later they solved 2 sets of exact symbolic addition on paper. Children were then retrained with 10 training trials of corresponding training task and then they solved last two sets of symbolic addition on paper. At the end they played Panamath game on computer (See Figure 31).

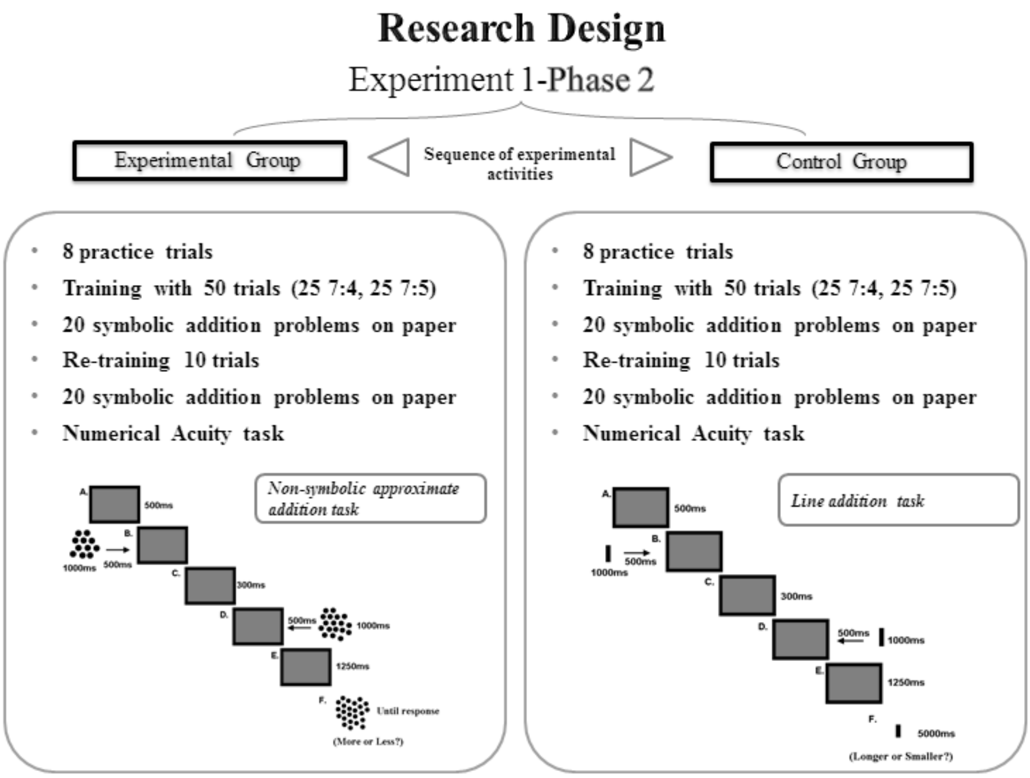

Figure 31. Research design of experiment 1 Phase 2

Results

ANOVAs were used to compare the groups on age, numerical acuity, training task performance, and test performance. Training task performance was analyzed by separate mixed-factor ANOVAs on average reaction time and accuracy with the within-subjects factors of Ratio (2 levels), Time First half vs. second half), and the between-subjects factor of Training Condition (2 levels: numerical addition, line addition). Test performance (speed and accuracy) was computed by averaging responses across completed test sets. Test performance was analyzed using ANOVAs on average time to complete test sets (speed) and accuracy, with the between-subjects factor of Training Condition (2 levels).

Majority of children completed all four test sets, numerical addition = 19, line addition = 23. However, in those children that did not complete all sets, the average score on the completed tests sets was used (5 children in the non-symbolic addition condition and 1 child of line length training group completed 3 out of 4 test sets).

Participant factors. There were no significant differences between the two training groups in average age *F* (1, 46) = .209, *p* =. 650: non-symbolic approximate addition group *M* = 6 years, 172 days, *SD* = 86.90, line length addition group *M* = 6 years, 184 days, *SD* = 86.68) or in approximate numerical acuity *F* (1, 46) = .001, *p* = .975: Non-symbolic approximate addition group *M* = .19, *SD* = .09, line length addition group *M* = .19, *SD* weber = .08).

Training task performance.

Table 20

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5), Time First half and second half) and Training condition* (*Non-symbolic addition vs. Line length Addition) on training task reaction time),* (*in experiment 1, Phase 2)*

Variables	*df*	*F*	η_p^2	*p*
Time	1, 46	26.958	.369	< .001
Ratio	1,46	7.554	.141	= .009
Training Condition	1, 46	5.277	.103	< .05
Ratio * Time	1, 46	10.215	.182	<.005

Table 20 shows analysis of mean reaction time on training trials and revealed a main effect of Time, Ratio, and Experimental Condition, and interaction between

Ratio and Time. Regardless of condition subject were faster on second half of training trials *F* (1, 46) = 2.697, p = .107) compared to first half *F* (1,46) = 7.316, p <. 05).

Participants in the line length condition performed training trials faster than those in the non-symbolic addition training condition t (46) = 2.264, $p < .05$, (Non-symbolic addition: *M* = 1914.94, *SD* = 317.87, range 1322.27 – 2533.66; Line length addition: *M* = 1730.21, *SD* = 242.49, range 1370.65 – 2295.21). Post-hoc paired sample t-tests revealed that the interaction between Ratio and Time could be explained by a significant difference between ratio conditions in the first half t (47) = 3.435, $p < .005$, but not the second half of training trials t (47)= .463, p = .646, with longer response time to problems involving (7:4) ratio compared to problems involving (7:5) in the first half.

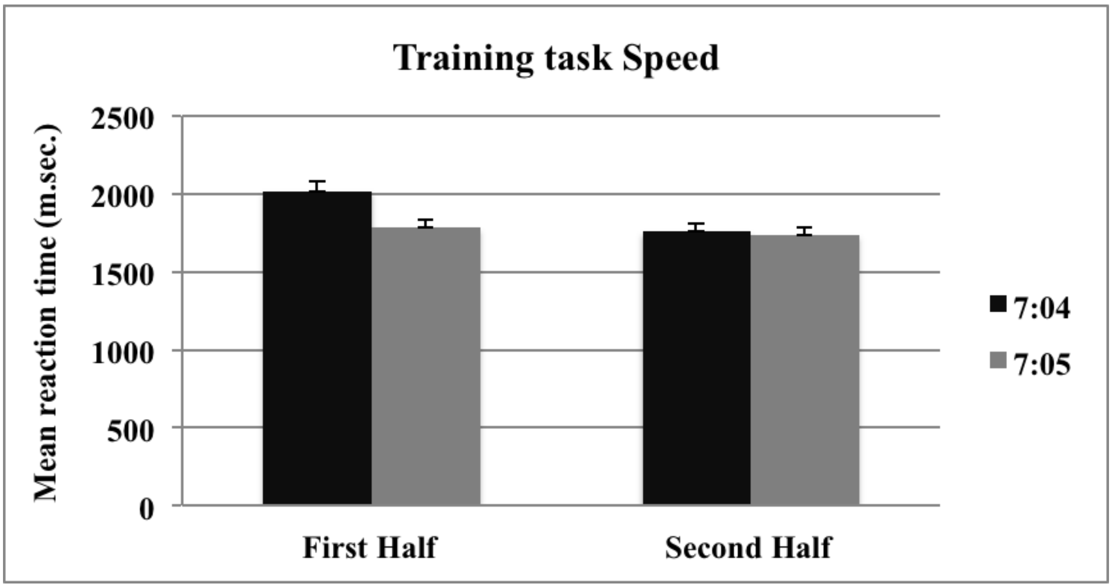

Figure 32. Effects of ratio on average training performance over time in experiment in experiment 1 (Phase 2)

Table 21

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5) and Training condition* (*Non-symbolic addition vs. line length addition) on training task accuracy),* (*in experiment 1, Phase 2)*

Variables	*df*	*F*	η_p^2	*p*
Time	1, 46	5.223	.102	< .05
Ratio	1,46	52.934	.535	< .001
Training Condition	1, 46	.028	.001	= .869
Time * Training Condition	1, 46	9.209	.167	< .005

Table 21 shows that the analysis of training task accuracy revealed main effect of Time and Ratio, but no main effect of Experimental Condition and an interaction between Time and Experimental Condition.

Post hoc analysis revealed that the interaction between Time and Training Condition resulted from a difference in accuracy between the two training conditions during the second half of training trials F (1, 46)= 5.263, $p < .05$, but not during the first half of the training trials F (1,46) = 3.458, $p = .069$.

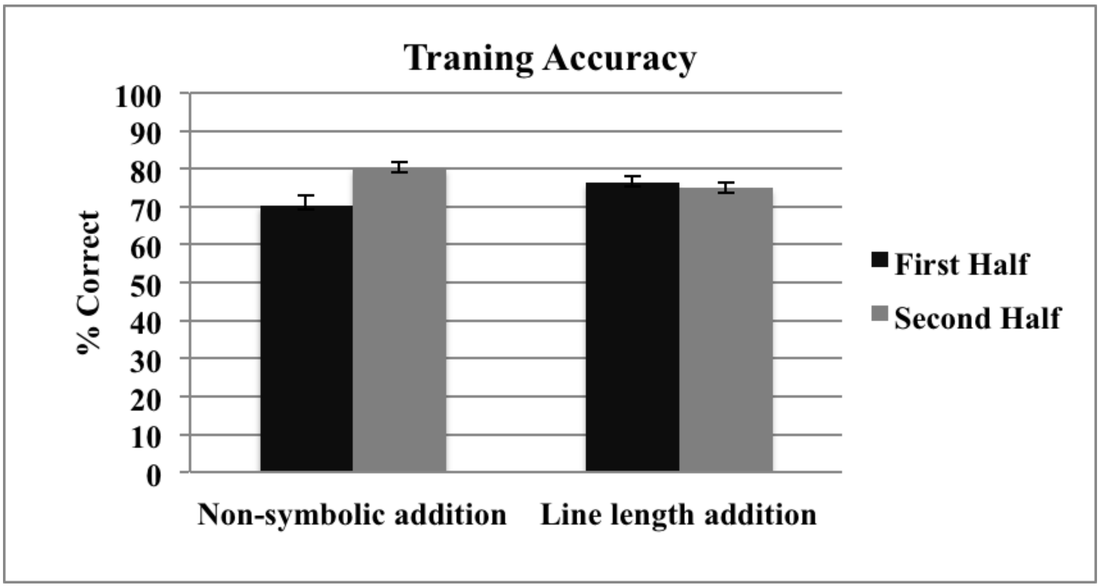

Figure 33. Average task accuracy (expressed as percent correct) over time for each condition in experiment 1 (Phase 2)

Table 22

t-test comparing experimental and control group on training task Ratio (7:4) accuracy, (in experiment 1, Phase 2)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	84.44	10.15	.269	46	= .789	0.07932
Line length addition	24	83.61	11.29				

Table 22 shows that accuracy results on 7:4 revealed no significant difference between non-symbolic addition, and line length addition condition.

Table 23

t-test comparing experimental and control group on training task Ratio (7:5) accuracy, (in experiment 1, Phase 2)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	75.83	12.56	2.861	46	< .05	0.84366
Line length Addition	24	66.11	10.93				

Table 23 shows that on 7:5 accuracy there was significant difference between non-symbolic addition and line length addition condition.

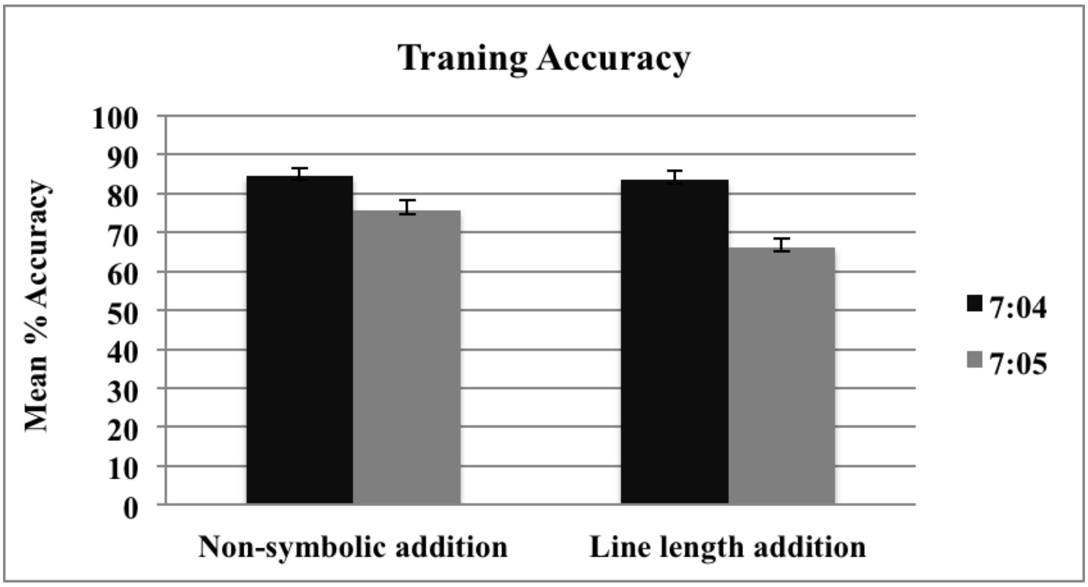

Figure 34. Average task accuracy (expressed as percent correct) over ratio for each condition in experiment 1 (Phase 2)

Exact symbolic addition test performance. The analysis of the mean time to complete each set of the written symbolic addition test problems through Mixed

Factor ANOVA on time to complete symbolic addition problems as within subject factor of difficulty (1, 2, 3, 4) and between subject factor of training condition (non-symbolic approximate addition vs. Line length addition) was carried out.

Table 24

Mixed Factor ANOVA of Difficulty (*1, 2, 3, 4) and Training condition* (*Non-symbolic addition vs. line length addition) on time to complete symbolic addition problems,* (*in experiment 1, Phase 2)*

Variables	*df*	*F*	η_p^2	*p*
Training Condition	1, 46	5.418	.105	< .05
Difficulty	3,138	70.372	.605	< .001
Difficulty * Training Condition	3,138	3.190	.065	< .05

Table 24 shows significant main effect of condition, significant main effect of difficulty and significant interaction of difficulty and condition. Children performed significantly faster on easier problems and slower to solve harder problems. In other words effect of condition was strongest at the most difficult level.

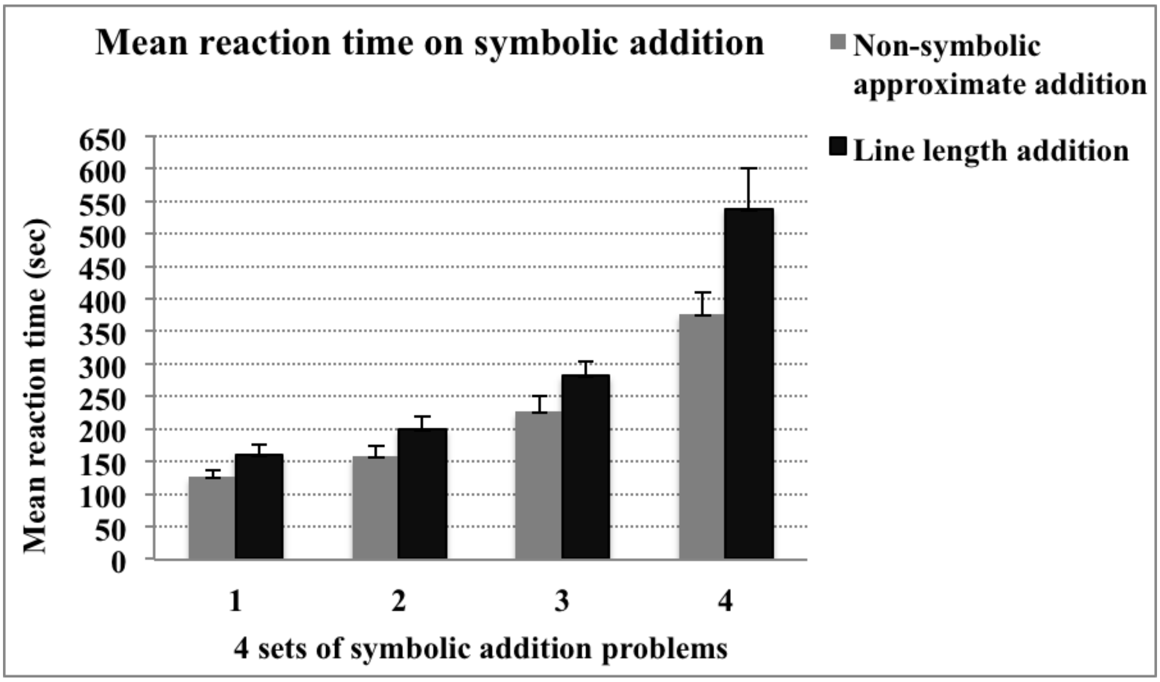

Figure 35. Average speed of test completion (in seconds) for each condition in experiment 1 (Phase 2)

Table 25

t-test comparing experimental and control group on time to complete symbolic addition, (*in experiment 1, Phase 2)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	221.83	90.89	-2.328	46	< .05	-0.68649
Line length addition	24	294.72	123.58				

Table 25 shows that children in non-symbolic addition training group performed significantly faster on symbolic addition test problems than children in line length addition training group.

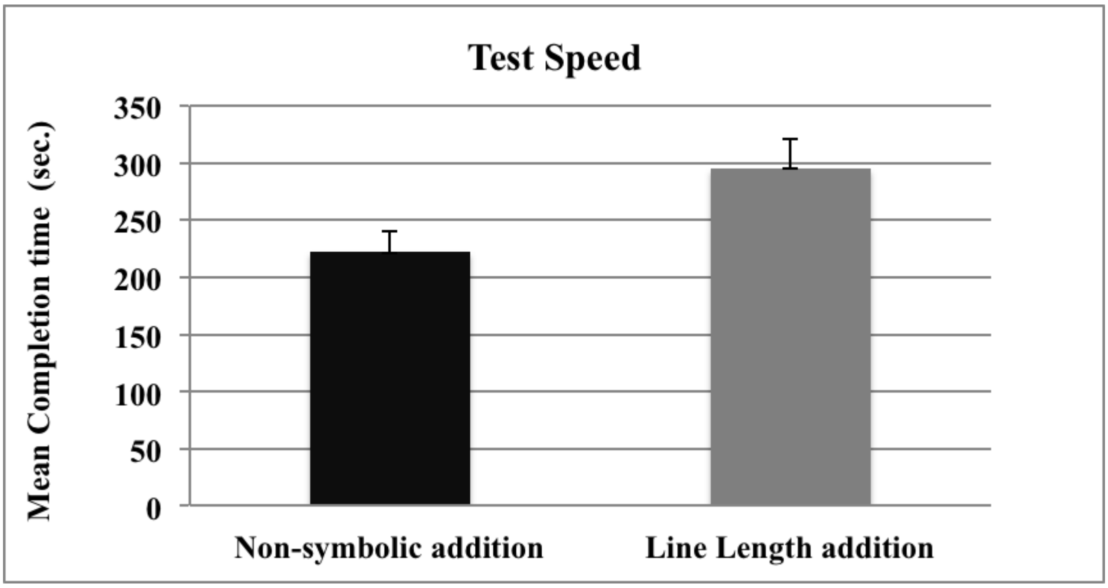

Figure 36. Average speed of test completion (in seconds) over all for each condition in experiment 1 (Phase 2)

A Mixed Factor ANOVA on measures ANOVA on accuracy on symbolic addition problems as within subject factor of difficulty (1, 2, 3, 4) and between subject factor of training condition (non-symbolic approximate addition vs. Line length addition) was carried out.

Table 26

Mixed Factor ANOVA of Difficulty (*1, 2, 3, 4) and Training condition* (*Non-symbolic addition vs. line length addition) on symbolic addition accuracy,* (*in experiment 1, Phase 2)*

Variables	*df*	*F*	η_p^2	*p*
Training Condition	1, 46	2.702	.055	= .107
Difficulty	3,138	122.577	.727	< .001
Difficulty * Training Condition	3,138	.315	.007	= .814

Table 26 shows no significant main effect of condition. However, there was significant main effect of difficulty, but no significant interaction of difficulty and condition. Children's were more accurate on easier math problems as compared to harder math problems.

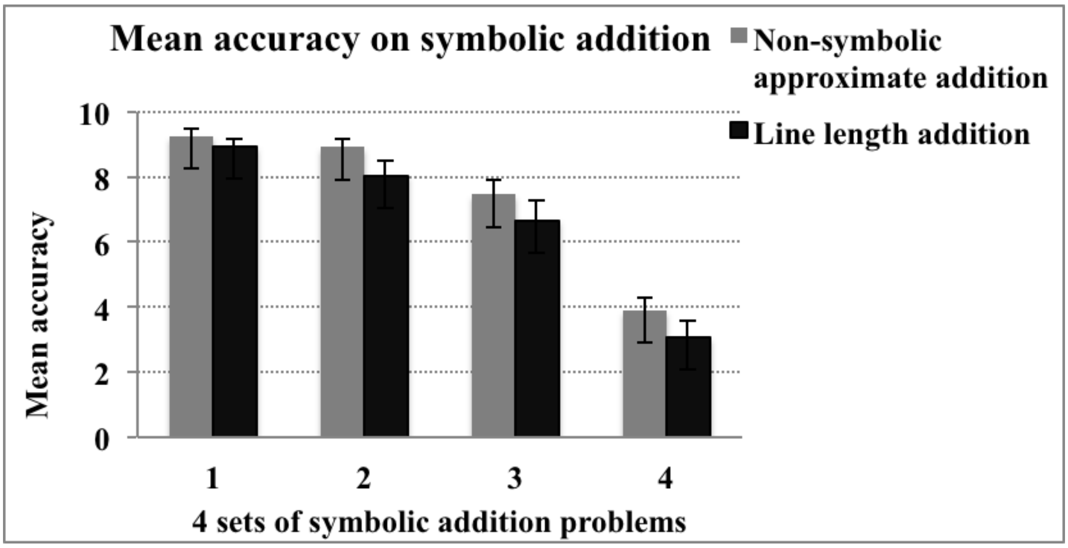

Figure 37. Average test accuracy (expressed out of 10 problems) for each condition in experiment 1 (Phase 2)

The analysis on performance accuracy on symbolic addition test revealed no significant main effect of Condition, *F* (1,46) = 2.704, *p* =. 107.

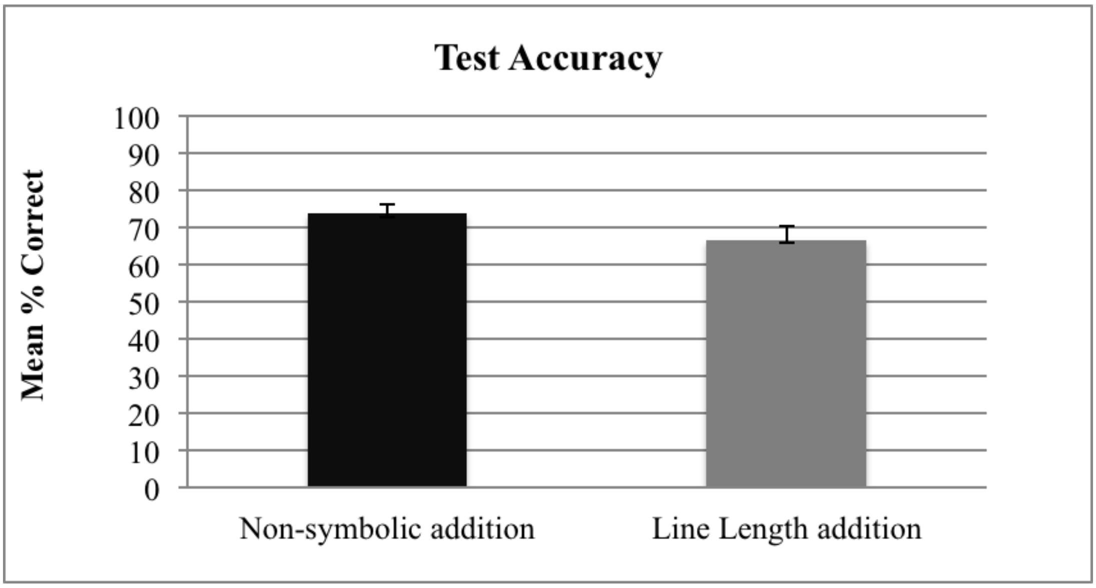

Figure 38. Average test accuracy (expressed as percent correct) over all for each condition in experiment 1 (Phase 2)

Further analysis. Differences were seen between training groups on both training task reaction time and time to complete exact symbolic addition task. In an attempt to rule out the possibility that significant differences in time to complete the symbolic arithmetic test sets between groups were due to differences in performance on the training task (reaction time) rather than the experimental manipulation of training condition (non-symbolic addition vs. line length addition), we analyzed the effects of training condition on symbolic math performance with time to complete the training task as a covariate.

To see whether reaction time during training, rather than content of training, could account for differences in test performance between groups, we entered reaction time to the training problems as a covariate in a univariate ANOVA with factor of Training Condition (line length or numerical addition) on performance speed to symbolic addition as dependent variable. Results revealed that the main effect of Training Condition, F (1,45) = 8.486, p <. 05, η_p^2 = .159 remained significant even after accounting for reaction time on training trials as a covariate.

In other words, the experimental manipulation of training condition had a significant effect on the time it took to participants to complete exact symbolic addition problems even after accounting for differences in reaction time and accuracy on the immediately previous training task.

Conclusion

Results showed that children who briefly practiced non-symbolic, approximate addition were faster to complete subsequent exact, symbolic addition tests than were children who briefly practiced a control condition (line addition).

The results show that enhancement of exact symbolic arithmetic performance in children who were trained on non-symbolic approximate arithmetic problems could not be explained by differences in difficulty between the experimental (non-symbolic numerical addition) and control training task (line length addition), as the advantage in time to complete test problem sets remained for those who were trained on the non-symbolic numerical addition problems after equating performance on the control task (line length addition). Furthermore, the enhancement seen in those of the non-symbolic numerical addition relative to control group cannot be explained by simply engaging the arithmetic process, as children trained in a non-numerical line length addition task did not subsequently show the same enhancements.

Results showed that training replicated the same pattern of results as for American Children in Hyde, Khanum, and Spelke (2014). Although, data for American children was collected in lab setting.

As such, these results obtained in a primary school sample in Pakistan replicate the same effect observed previously in a sample of upper middle-class children in the U.S (Hyde et al., 2014) to suggest a specific, causal effect of approximate number system engagement on symbolic addition.

Other studies have shown that training the approximate number system changes exact, symbolic arithmetic performance in adults (e.g., Park & Brannon, 2013; Park & Brannon, in press) and in children as well (Hyde et al., 2014). As evidence of this, we observe no differences in approximate number acuity between groups despite significant differences in symbolic addition speed. Thus, it does not seem that the mechanism driving enhanced performance in task is change (permanent or temporary) in non-symbolic approximate number representation. Several potential mechanisms still remain unanswered. It could be that our task of approximate number addition engages both symbolic and non-symbolic numbers, and changes (temporarily or, less likely, permanently) symbolic number representation but not non-symbolic number representations. It could be that no changes in representation of symbolic or non-symbolic number occur, but that simple engagement or co-activation of a common neural mechanism produces the enhancement.

Here we test between the representational co-activation hypothesis and the symbolic representational change hypothesis by using replacing our symbolic addition tests with a test of symbolic number representation, the symbolic number line placement test.

Experiment 2

Objective

Results of experiment 2 of phase 1 (data collected from USA), has shown that non-symbolic addition practices enhance symbolic addition but not a non-mathematical task, like vocabulary. So to explore whether non–symbolic addition task might enhance performance on another symbolic number task. For this, purpose another symbolic number task was included as the dependent variable in experiment 2 number line placement task such as number line task. Furthermore, it was to explore whether non-symbolic addition leads to more linear number line placements because ANS representations are ratio limited. Previous research evidence by Siegler and Booth (2004), Booth and Siegler (2006), and Ashcraft and Moore (2012), suggests log to linear shift on 1-100 scale of elementary school children.

Research evidence shows that first grade children's number line estimates on 1-100 scale were better fit by linear function than by logarithmic function (Booth & Siegler, 2006). Research evidence indicates that children in first grade were in transition from logarithmic to linear representation of magnitudes. Beginning with second grade linear values show better fit (Ashcraft & Moore, 2012).

Given the important role played by ANS in symbolic number representation we have also investigated in experiment 2, whether training the children of same age group (as of experiment 1) with non symbolic approximate addition as compared to control group would enhance their performance on number line placement task or not, that requires children to translate the symbolic numbers on a spatial line bounded by 0-100.

It is important to see the effect of non-symbolic approximate addition on number line task given the number line task's significant association with mathematics education and it may help children's symbolic number representations. Further more it was to explore whether non-symbolic approximate addition leads to more linear number placements because ANS representations are ratio limited (See signatures of number sense).

So to explore these possibilities symbolic number placement task used by Siegler and Booth, 2004 and Booth and Siegler, 2006 was used as a post test measure after training the children with experimental (Non-symbolic addition) and control tasks (Line addition, and brightness comparison) following the procedures as in experiment 1 of Phase 2.

Method

Participants. Participants were 95 children who had just passed first grade and belonged to Islamabad Pakistan area. Of those who participated in the main experiment,72 children were included in main data where as 25 children's were excluded from main data because of, children did not completed the study (16), system got stuck in the middle of study and electrical problems causing the computer to crash (6), and for failing to follow the instructions (1).

Of these children, 24 were quasi randomly assigned to non-symbolic approximate addition training group (13 boys 11 girls, Mean age = 6 years, 162.25 days, *SD* = 102.67), 24 to line length addition training group (13 boys 11 girls, Mean age = 6 years, 138.9 days, *SD* = 81.06) and 24 to brightness comparison training group (13 boys and 11 girls, Mean age = 6 years, 168.33 days, *SD* = 99.66) to equate for age and gender.

National Institute of Psychology at Quaid-i-Azam University, Islamabad Pakistan and Federal Directorate of Education Islamabad Pakistan approved the study for ethical considerations (See Appendix J). Principal and head mistress of primary section school, teachers, parents through school administration and children's gave consent for data collection and children were compensated. Experiment was conducted in the school setting keeping in view children's comfort to perform the experiment.

Data was collected from Islamabad Model College for Girls (IMCG) F6/2, Street 25 Islamabad, Islamabad Model College for Boys (IMCB), F7/3 Islamabad and from Islamabad College for Boys (ICG) G6/3 Islamabad.

Stimuli and display.

1. Non-symbolic approximate addition (same as in experiment 1)
2. Line length addition (same as in experiment 1)
3. Brightness comparison (same as in Method chapter)

4. Number Line placement task

The number line placement task was used to assess children's symbolic number representation after training (see for details Siegler & Booth, 2004). Previously it has been used by Ashcraft and Moore (2012), Dehaene (2003), Opfer and DeVries (2008), Gallistel and Gelman (1992), Opfer et al. (2011), Rips (2013), Siegler and Booth (2004), and Siegler and Opfer (2003).

Administration of the number line task followed the instructions given by Booth and Siegler (2006). Specifically, in this task child saw a horizontal line (23 cm long) on paper bounded by the Arabic digit 0 on the left end and 100 on the right end. Each problem was presented on a separate piece of paper and contained an additional Arabic digit on the top of the page. Children were asked to draw a mark at the position on horizontal line where they thought the test digit (printed on the top of the page) belonged. Child received a succession of test pages with test numerals between 1-99 to place on the line. Specifically, children where given each of the following numbers twice for a total of 48 test problems: 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90 and 96. Test speed was recorded via a handheld stopwatch, which started when children started each stack of 12 pages and was stopped right after children use to complete last page of stack. A researcher recorded the time it took for the child to make her/his response using a handheld stopwatch. Placement accuracy calculated offline after the experimental procedure was complete.

5. Panamath Task (same as in Method chapter)

The ANS acuity of each participant was assessed using Panamath computer game (see *Method chapter for details).*

Research Design

Children were trained with non-symbolic approximate addition task in experimental group as compare to the two control groups who were trained with line length addition and brightness comparison tasks, each with task 60 trails followed by 8 training trials. All the three groups were post tested on number line placement task. At the end children's attempted Panamath task.

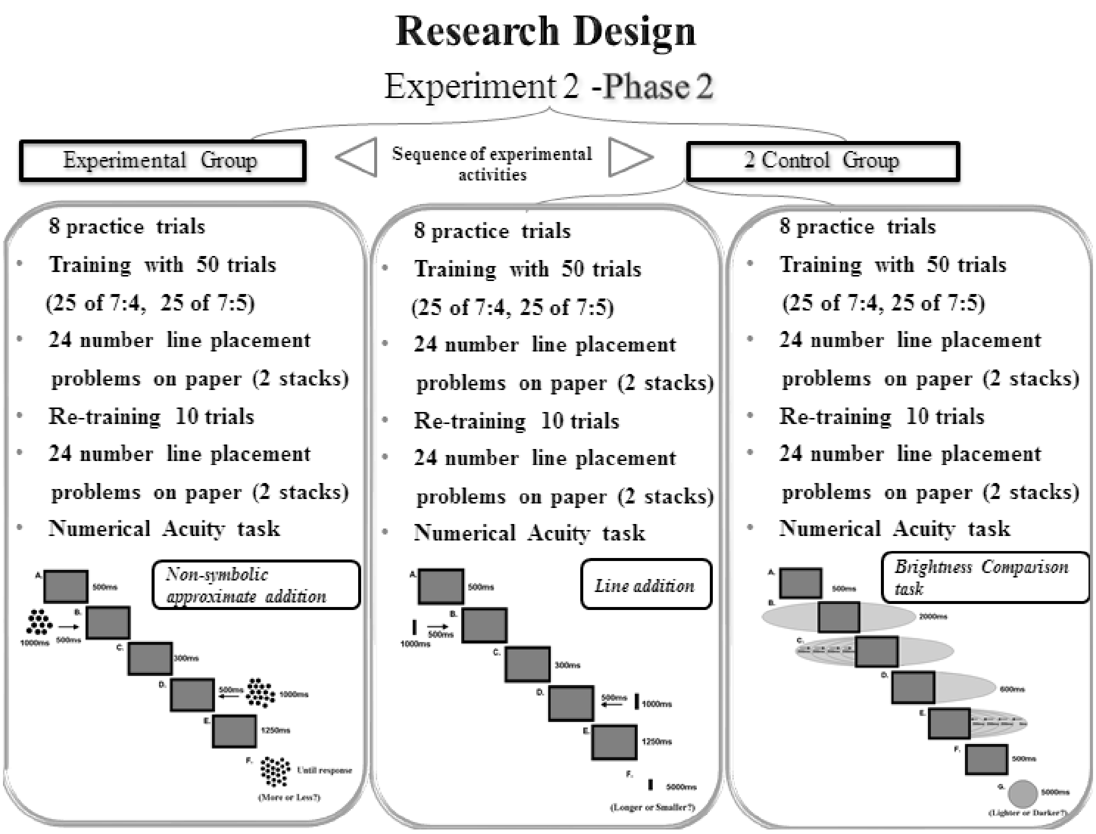

Figure 39. Research design of experiment 2

Procedure and Design

Experiment was conducted individually with each participant in a noise free room at school in their native Urdu language. Children in all three conditions were trained with the assigned training task following same instructions, procedures and parameters as in phase 1 experiments. Children played the computer game based on the corresponding training task's 8 practice trials followed by 50 test trials of training task.

Then they attempted first and second stack (each stack of 12 pages) of number line placement, to place the number mentioned on page on the given 0-100 line and researcher recorded reaction time on each stack with the stopwatch. Then children attempted 10 more training trails of training task and were given 3^{rd} and 4rth stack to place the above-mentioned numbers on the line and reaction time was recorded through stopwatch. Children's % absolute error for each sheet was calculated through formula given in Booth and Siegler, 2006. At the end children played Panamath game (see Figure 39).

Results

Participant factors. Children in different training conditions did not differ in mean age *F* (2, 69) = .642, *p* = .529: non-symbolic approximate addition group *M* = 6 years 162 days, *SD* = 102.67, brightness comparison group *M* = 6 years 168 days, *SD*

= 99.66, line length addition group *M* = 6 years 138 days, *SD* = 81.06) and groups were matched for gender (13 male, 11 female in each group).

There was no significant difference in different training conditions on approximate numerical acuity (weber fraction (*w*), F (2, 69) = .822, p = .444: non-symbolic approximate addition trained group (*M* = .19, *SD* = .14): line length addition trained group (*M* = .23, *SD* = .09) and brightness comparison trained group (*M* = .20, *SD* = .10).

Training task performance.

Table 27

t-test comparing *experimental and control group on training task reaction time* (*in experiment 2, Phase 2)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	1896.16	255.54	-.610	46	.545	-0.17988
Line length Addition	24	1945.81	306.39				

Table 27 shows that there was no significant difference on training task reaction time between non-symbolic training group and line length addition training group.

Table 28

t-test comparing experimental and control group on training task reaction time (in experiment 2, Phase 2)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	1896.16	255.54	-1.080	46	.286	-0.31847
Brightness Comparison	24	1983.53	303.12				

Table 28 shows that there was no significant difference on training task reaction time between non-symbolic training group and brightness comparison training group.

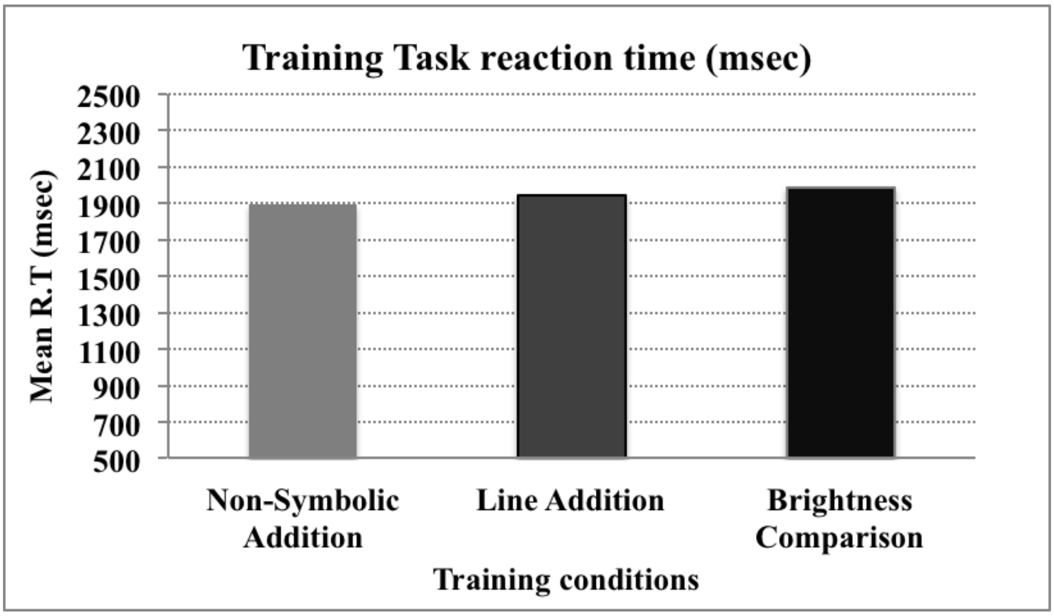

Figure 40. Average reaction time (in milliseconds) in each condition in experiment 2 (Phase 2)

The analysis of average reaction time to training problems revealed a main effect of Time *F* (1, 69) = 17.756, *p* < .001, η_p^2 = .205, and interaction effect of Time and Ratio F (1,69) = 20.601, $p < .001$, $\eta_p^2 = .230$. Paired sample t-test revealed that interaction could be explained by the fact that larger differences were observed between ratio conditions on first half (t (71) = 3.573, $p < .005$) compared to second half of training trials (t (71) = -2.398, p <. 05).

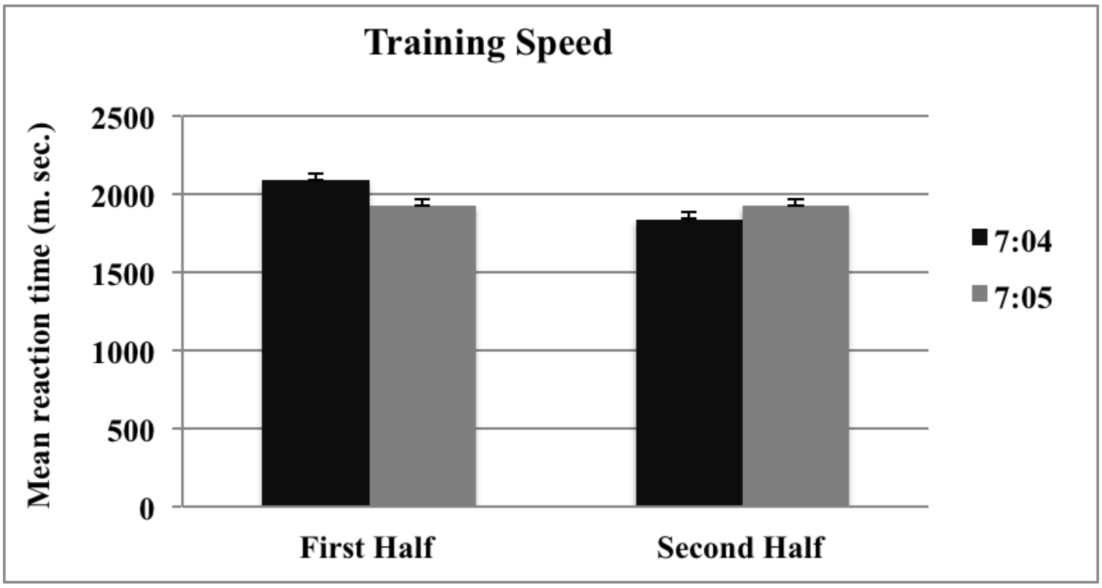

Figure 41. Effects of ratio on average training performance over time in experiment in experiment 2 (Phase 2)

Table 29

Mixed Factor ANOVA of Time First half, second half), Ratio (*7:4, 7:5) and Training condition* (*Non-symbolic addition, line length addition, brightness comparison group), on training trials accuracy* (*in experiment 2, Phase 2)*

Variables	*df*	*F*	η_p^2	*p*
Time	1, 69	5.123	.069	< .05
Ratio	1, 69	38.262	.357	< .001
Training Condition	1, 69	11.929	.257	< .001
Ratio * Training Condition	2, 69	7.667	.182	< .005
Time * Ratio	1, 69	7.885	.103	< .01

Table 29 shows main effects of Time, Ratio, and Training Condition, and interactions between Ratio and Training Condition and Time and Ratio.

Post hoc analysis revealed that the interaction between Ratio and Condition resulted from larger difference between training conditions on 7:5 F (2, 69) = 17.320, p < .001 compared to 7:4 F (2, 69) = 3.330, p < .05. Paired sample t-test revealed that the interaction between Time and Ratio could be explained by the fact that larger differences were observed between ratio conditions on the first half t (71) = 2.082, p < .05 compared to the second half of training trials t (71) = 6.560, p < .001.

Interaction between Ratio and Training condition could be explained by the fact that no significant difference was observed neither on 7:4 accuracy between non-symbolic addition (M = 80.83, SD = 6.97) and line length addition (M = 77.22, SD = 7.07), t (46)= 1.783, p =. 081, between non-symbolic addition and brightness comparison (M = 84.72, SD = 14.34), t (46), -1.195, p = .238 nor on 7:5 accuracy between non-symbolic addition (M = 66.25, SD = 10.23) and line length addition (M

= 66.89, *SD* = 7.90), *t* (46) = -1.000, *p* = .323. However on 7:5 accuracy there was significant difference between non-symbolic addition and brightness comparison (*M* = 82.92, *SD* = 12.90), *t* (46) = -4.959, *p* <. 001.

Figure 42. Effects of ratio on average training accuracy performance over time in experiment 2 (Phase 2).

Analysis on symbolic number task.

Mean % Absolute Error and reaction time. Children's estimation accuracy was calculated through scales of estimate like percentage of absolute error (PE). This was calculated using following formula (Siegler & Booth, 2004).

$$PE = \left| \frac{\text{estimate} - \text{target number}}{\text{Scales of estimate}} \right| \times 100$$

Accuracy of number line placements. An analysis of accuracy on number line placement task with the between subjects factor of training condition (numerical addition, brightness comparison, line addition) revealed a main effect of training condition $F(2, 69) = 4.65$, $p = .013$, $\eta p2 = .12$. Post-hoc pairwise comparisons revealed greater proportion of error in the line length addition group (*M* = .1434, se = .0136) compared to the numerical addition group (*M* = .0884, se = .0092; t (46) = 3.35, $p < .005$), with the brightness comparison group patterning in-between the other two conditions (*M* = .1174, se = .0148, ps > .10).

Speed of number line placements. There was no significant differences between experimental training groups in total time to complete number line placement problems (line *M* = 961.3 seconds, line *SD* = 623.5 seconds; number *M* = 740.7 seconds, number *SD* = 334.4 seconds; brightness *M* = 885.3 seconds, brightness *SD* = 609 seconds; line vs. number: t (46) = 1.53, $p = .134$; number vs. brightness: t (46) = -1.02, $p = .313$; line vs. brightness: t (46) = 0.43, $p = .671$). As there were no differences in time to complete number line placement task between groups so different performance on number line placement task accuracy between training groups observed cannot be explained by a speed-accuracy trade-off.

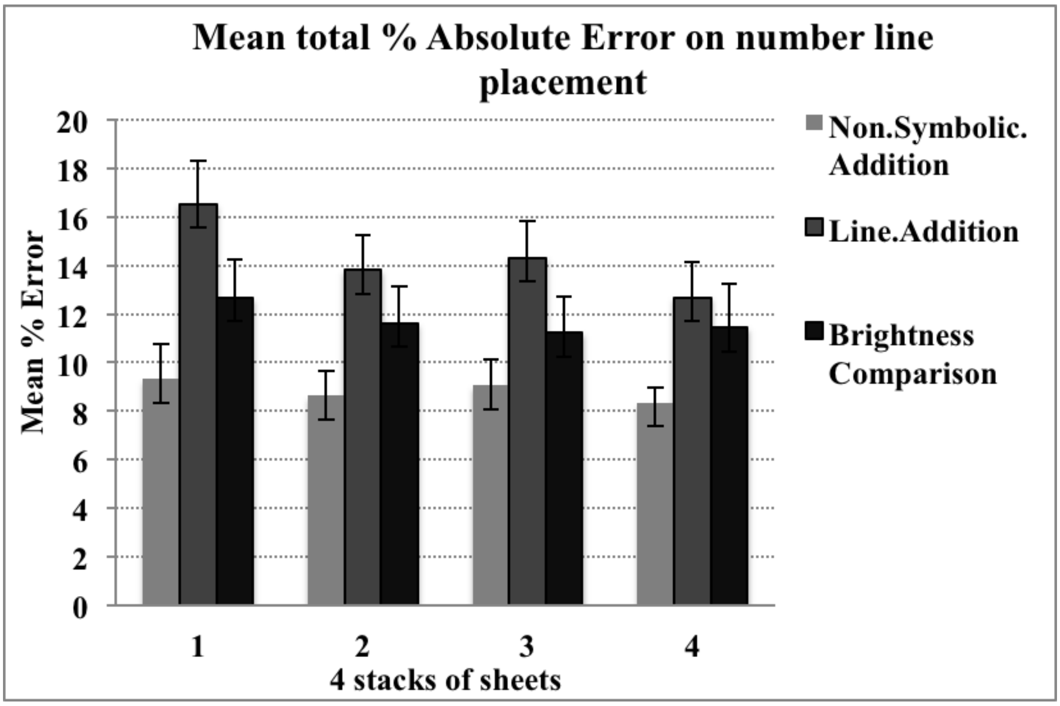

Figure 43. Average test accuracy (expressed as percent error) for each condition in experiment 2 (Phase 2)

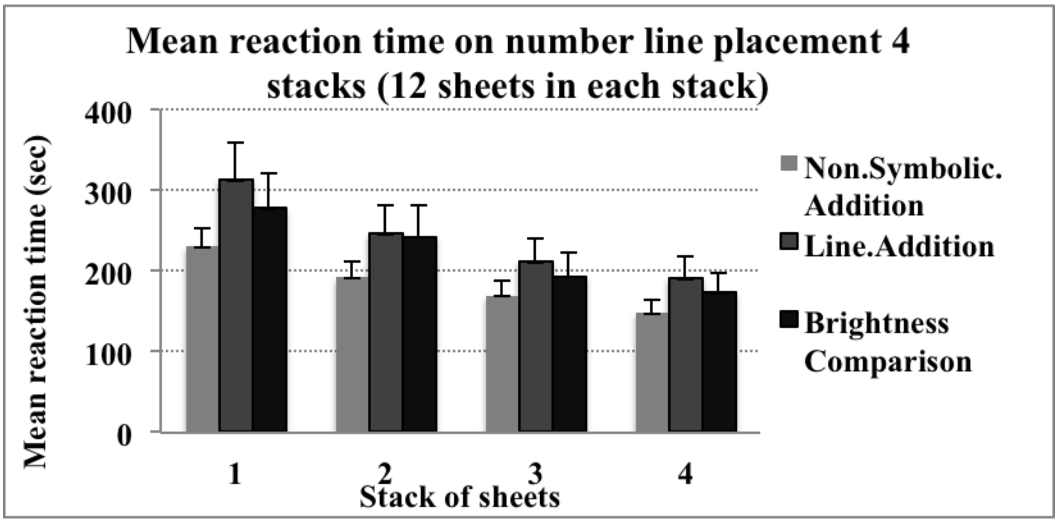

Figure 44. Average speed of test completion (in seconds) for each condition in experiment 2 (Phase 2)

Further analysis. Main effect of experimental condition on mean % absolute error on number line placement task remained significant even after accounting the effects of experimental condition tasks. A Mixed Factor ANOVA of between subject factor of condition (non-symbolic vs. line length addition) and within subject factor of Mean% absolute error on 4 stacks of number line placement with covariate of training task accuracy indicated that there was significant main effect of condition, F (1, 45) = 10.958, $p < .005$, $\eta_p^2 =$ .196 and no significant main effect of training task accuracy F (1, 45) = .006, $p = .939$, $\eta_p^2 =$ .000.

One way ANOVA was carried by taking condition as a factor and linear estimate (Gallistel & Gelman, 1992) reported that young children's number line estimations did follow a linear shape, but linear fit of their placements was reduced because of children's difficulty with accurately placing larger numbers on the number line. More recent accounts, however, state that prior to becoming linear, of each child as dependent. Children distribute numbers logarithmically across the number line and shift toward linear distributions when they get older (Ashcraft & Moore, 2012; Dehaene, 2003; Opfer & DeVries, 2008; Opfer et al., 2011; Rips, 2013; Siegler & Booth, 2004; Siegler & Opfer, 2003). Result revealed that there was no significant difference between the group F (2, 69) = 2.523, $p = .088$. However further analysis revealed that there was significant difference between Non symbolic addition group ($M = .85$, $SD = .20$) and Line length addition ($M = .69$, $SD = .27$), t (46) = 2.314, $p < .05$. However no difference between non-symbolic addition group ($M = .85$, $SD = .20$) and brightness comparison group ($M = .75$, $SD = .26$) was observed, t (46) =1.368, $p = .178$.

To see whether accuracy during training, rather than training could account for differences in test performance between groups, we entered accuracy on the training problems as a covariate in a univariate ANOVA with factor of Training Condition (non-symbolic addition, line length addition or brightness comparison) on performance accuracy (Mean % absolute error) on number line task as dependent variable. Results revealed that the main effect of Training Condition F (2, 68) = 4.581, p < .05, η_p^2 = .119 remained significant even after accounting for accuracy on training trials as a covariate. However no significant effect of training was observed F (1,68) = .004, p = .949, η_p^2 = .000.

Group analysis on number line placement task. Since children were given 24 number line placement problems twice (in total 48) so for group analysis following steps were carried out. Median of each child's 2 estimate on each number was calculated. Estimates and actual value of 0 cannot be modeled using certain regression models so, 1 was added to both actual and median estimated quantities before analysis following tutorial given by Siegler and Opfer (2003).

Number line placement estimation. As shown in the Figure.1, Non-symbolic approximate addition trained children's number line estimates were better fit by the linear function (R^2 = .98) than by logarithmic function (R^2 = .82), t (47) = 7.706, p <. 001).

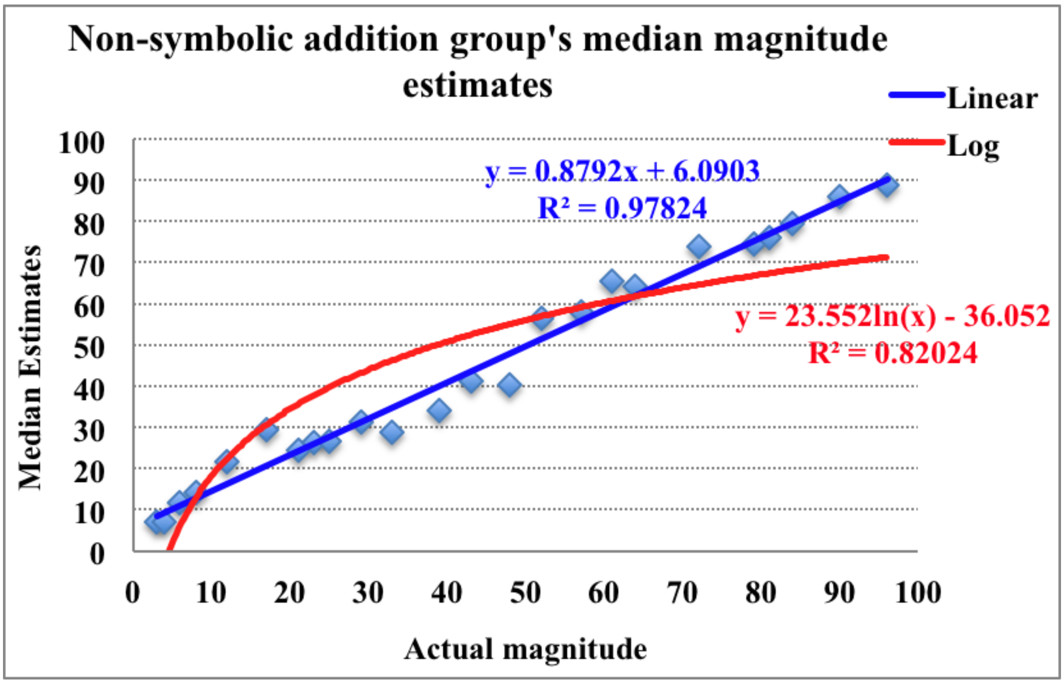

Figure 45. Median magnitude estimates on number line placement task of non-symbolic addition group in experiment 2 (Phase 2)

Line length addition trained children's number line estimates were better fit by linear function ($R^2 = .98$) than by logarithmic function ($R^2 = .86$), t (47) = 5.586, $p <. 001$.

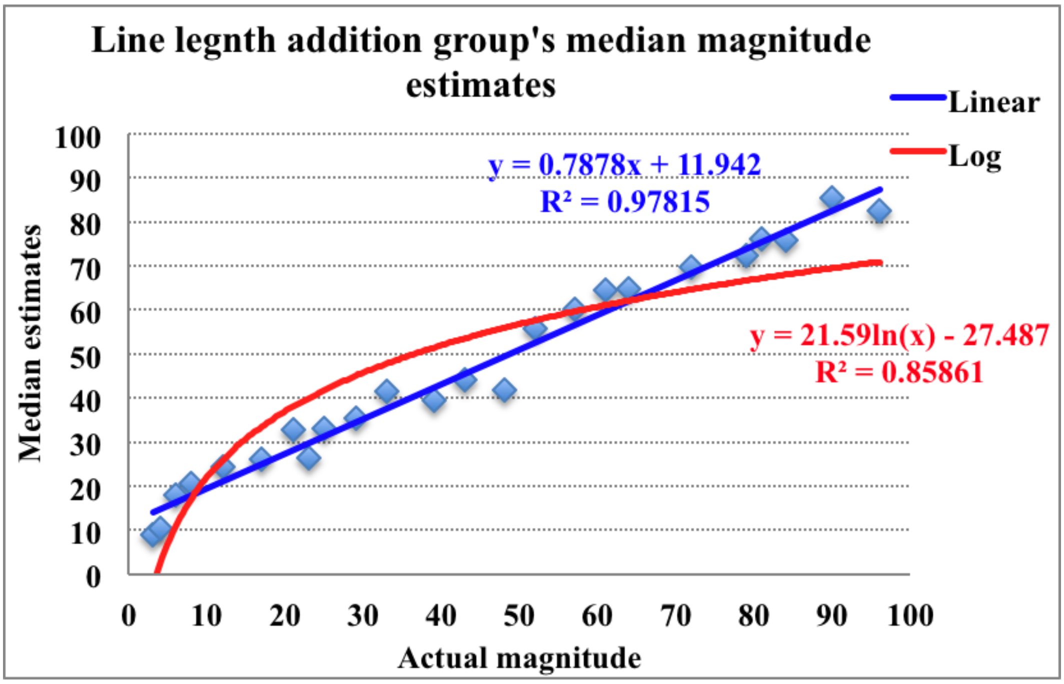

Figure 46. Median magnitude estimates on number line placement task of line length addition group in experiment 2 (Phase 2)

Brightness comparison trained children's estimates on number line placement task were better fit by linear function, (R^2 = .98), than by logarithmic function (R^2 = .85), t (47) = 3.510, p <. 005. Overall these results show that first grade children's estimates were better fit by linear function.

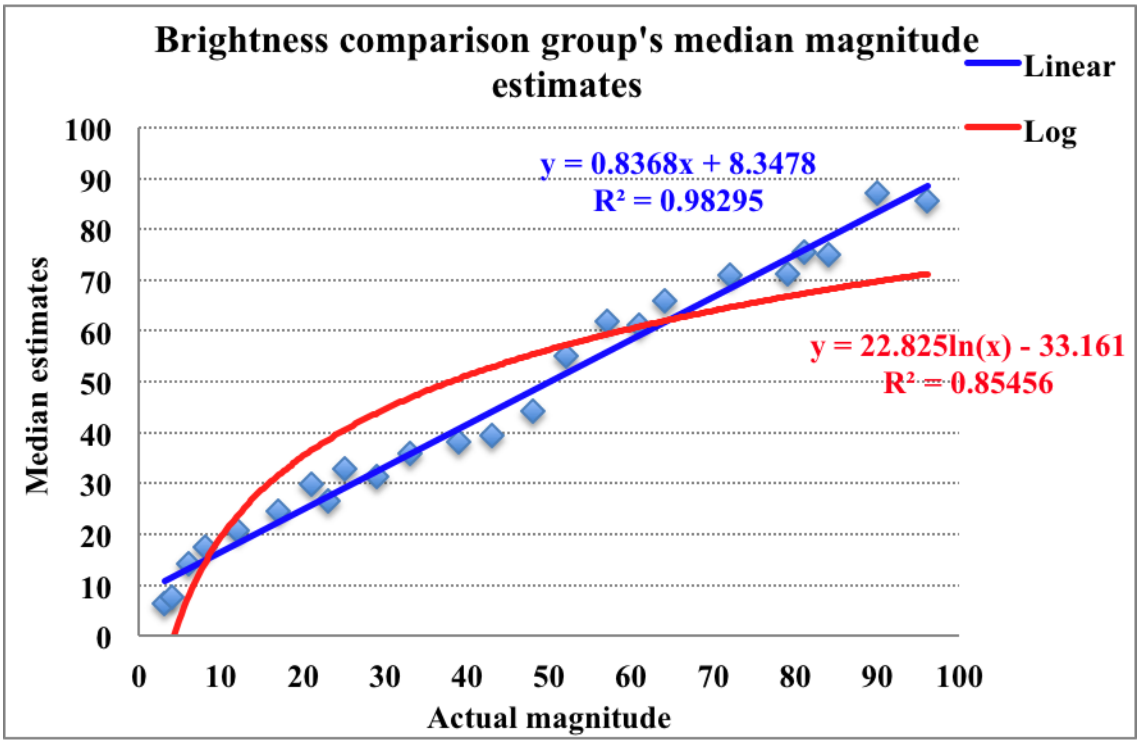

Figure 47. Median magnitude estimates on number line placement task of brightness comparison group in experiment 2 (Phase 2)

Individual analysis. Regression analysis was carried out on individual children data. The best fitting model between linear and logarithmic was attributed to each child, whenever significant (e.g., the child was attributed a logarithmic representation for a given interval if the highest R^2 was logarithmic). Children representation was considered as linear, logarithmic or none. If both values logarithmic or linear failed to reach significance, the child was classified as not having a representation for the considered interval.

Table 30

Type of representation as Function of task (*in experiment 2, Phase 2)*

	Type of Representation		
Task	Linear	Logarithmic	None
Non-symbolic addition	87.50 %	8.33 %	4.17 %
Line length addition	70.83 %	29.17 %	
Brightness comparison	87.50 %	4.17 %	8.33 %

Table 30 shows that linear function provided the best fit for 87.50 % of non-symbolic approximate addition trained children, logarithmic function for 8.33% and no representation for both function as none 4.17% [Figure 8]. Linear function provided best fit for 70.83% of line length addition trained children, and logarithmic for 29.17%. Linear function provided the best fit for 87.50% of brightness comparison trained children, logarithmic function for 4.17% and no representation for both function 8.33%.

These findings show that for all the three groups first grade children's estimates were better fit by linear function than by logarithmic function. Previous research evidence by Siegler and Booth (2004), Booth and Siegler (2006), and Ashcraft and Moore (2012), suggests log to linear shift on 1-100 scale of elementary school children.

Research evidence shows that first grade children aged = 6.8 years, number line estimates 1-100 scale were better fit by linear function (R^2 = .96) than by logarithmic function (R^2 = .89), (Booth & Siegler, 2006). Research evidence indicates that children in first grade were in transition from logarithmic to linear representation of magnitudes. Beginning with second grade linear values show better fit (Ashcraft & Moore, 2012).

Weber fraction. No differences in the weber fraction were observed in a between training groups *F* (2,69) = 0.853, p > .43; Line Length Addition Group *M* = .23, *SD* = .09; Approximate Numerical Addition *M* = .19, *SD* = .14; Brightness Comparison *M* = .20, *SD* = .10).

Conclusion

Results of experiment 2 revealed that non-symbolic addition training did not improve linearity significantly above and beyond any of the other conditions. Line length training actually hurt linearity estimates on the number line task, suggesting a further dissociation between number and length magnitude representations. As marginal differences have appeared in the linearity of line placement between the non-symbolic addition-training group and the two non-numerical magnitude control groups. Thus, based on this data alone, it is unclear whether ANS training shown to improve exact, symbolic addition also produces changes in symbolic number representation.

Discussion Phase 2

Results of both experiments reveal that children trained with non-symbolic approximate addition showed advantage on symbolic addition performance on reaction time as well as on number line placement task accuracy as compared to children trained with line length addition and brightness comparison.

Results of experiment 1 conducted in socially, economically, culturally and technologically different populations suggests that training the ANS of children from any educated background of children might improve their performance on symbolic math. These findings are unique in their own way, since American children (Hyde, Khanum, & Spelke, 2014) living in technological advance environment and Pakistani children having less exposure to latest technology, and learning opportunity performed equally well on symbolic addition task after training. Furthermore results across participants of both populations show that children trained with non-symbolic addition performed more accurately although not significantly.

Children in experiment 1 were faster in solving symbolic addition math as compare to children trained with Line length addition. However no significant difference was found on their performance in accuracy on symbolic addition. These results suggest that non-symbolic approximate addition training may benefit children to perform better in mathematics. Although these results go beyond correlational evidence that non-verbal approximate system is correlated with children's math performance (Gilmore et al., 2007, 2010; Halberda et al., 2008; Libertus et al., 2012; Park & Brannon, 2013).

These results are not due to addition aspect of the training task, if it would have been the case then children performance on symbolic addition problems should have been equal or closer to equal on accuracy and reaction time performance on symbolic addition. Whereas children trained with non-symbolic approximate addition training condition had still performed better, although not significant on accuracy as compare to children trained with Line length addition training condition. Results suggest that it might be non-symbolic numbers itself driving this effect of better performance on symbolic addition.

Results of experiment 2 also support this effect. As children trained with non-symbolic approximate addition training were more accurate on number line placement as compare to line length addition or brightness comparison training condition. Although it was speculated that spatial addition training might enhance children performance on number line placement (Kucian et al., 2011) but results suggested that only non-symbolic approximate addition training drove this effect. Training children with line length addition did not made them to perform better on number line placement task. Children trained with brightness comparison and line length addition training performed almost equally on number line placement task accuracy which supports the finding that it is the non-symbolic numbers itself enabling children to perform better on in both experiments as compare to control groups.

These findings are inline with the recent findings of phase 1 with same experimental and control conditions (Hyde, Khanum & Spelke, 2014) that also reported that children trained non-symbolic approximate number performed better on symbolic addition as compare to control groups. Their findings suggest that subsequent performance on reading task of comparable difficulty to math problems showed no enhanced performance, thus supporting the conclusion that training effect was specific to math. Research evidence by Park and Brannon (2013) suggests that non-symbolic approximate training improves math performance of adults and children.

These findings suggest that ANS serve as a cognitive foundation to learn formal mathematics. Findings go beyond correlational or indirect evidence and provide further support for causal role of ANS in learning formal mathematics.

Cross-cultural perspective of American (Experiment 1) and Pakistani (Experiment 1) Children on Non-Symbolic Addition and line Addition conditions

A great deal of work indicates towards foundational role of ANS in later math. Two experiments were conducted cross culturally in USA and Pakistan to probe this relationship focusing the question, whether training ANS would be helpful for first grade children to perform better on symbolic math or not? Children were trained with two types of training task, non-symbolic approximate addition task for experimental group) and line addition task *F*or control group). Experiment 1 mentioned below is involving exactly same participants and experimental conditions mentioned in experiment 1 (Non-symbolic approximate addition) and experiment 3 (line length addition) of Phase 1 of American sample. Experiment 2 mentioned below is exactly same experiment mentioned as experiment 1 in phase 2 from Pakistani sample. Here, both experiments have been mentioned just to draw a cross-cultural perspective (between American and Pakistani sample) involving same training conditions (non-symbolic approximate and line length addition).

Results (Experiment 1 and 3 of phase 1)

In non-symbolic addition training condition, 2 children had not solved 3rd set and 5 had not solved 4th set of symbolic addition problems. In line addition training condition, 4 children had not solved 2nd set, 2 had not solved 3rd set, and 6 had not solved 4rth set of symbolic addition problems. To carry out the analysis, average accuracy and reaction time on corresponding set was calculated for each group and entered in the children missing data set of their own group.

Participant factors. There was no significant difference in age between non-symbolic approximate addition group and line addition.

Table 31

t-test comparing experimental and control group on mean age (*in experiment 1, Cross-cultural perspective)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	6 years 311 days	73	. 035	46	= .973	0.01032
Line Addition	24	6 years 311 days	77				

There was no significant difference in weber fraction (*w)* between line addition (*M*= .21, *SD*= .12) and non-symbolic approximate addition (*M*=. 17, *SD*=. 11), *t* (46)= -1.116, *p* =. 270.

Table 32

t-test comparing experimental and control group on Weber Fraction (w), (in experiment 1, Cross-cultural perspective)

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	.17	.11	- 1.116	46	= .270	-0.32909
Line Addition	24	.21	.12				

Training task performance. There was no significant difference in training task mean % accuracy between line addition (*M*= 80.28, *SD* = 9.46) and non-symbolic approximate addition (*M* = 79.66, *SD* =12.76), *t* (46) = -.129, *p* =. 898. There was no significant difference in training task reaction time between line addition (*M* = 1791.73, *SD* = 334.44) and non-symbolic approximate addition (*M*= 1908.98, *SD* = 296.52), *t* (46)= 1.285, *p* =. 205.

Ratio differences. An ANOVA on comparing training performance of the line length addition group to the training performance of the non-symbolic approximate addition group with the within-subjects repeated factor of difficulty (7:4,7:5) and the between-subjects factor of Training Condition (Non-symbolic approximate addition vs. line length addition) revealed a significant main effect of difficulty on accuracy $F(1, 46) = 39.632$, $p < .001$, $\eta_p^2 = .463$ but nonsignificant differences of difficulty on reaction time F (1, 46) = 2.168, $p = .148$, $\eta_p^2 = .045$), of training condition on reaction time F (1, 46) = 1.647, $p = .206$, $\eta_p^2 = .035$ or of training condition on accuracy F (1, 46) = .017, $p = .897$, $\eta_p^2 =.000$**,** suggesting we were able to effectively equate performance on the line addition task with the non-symbolic addition task.

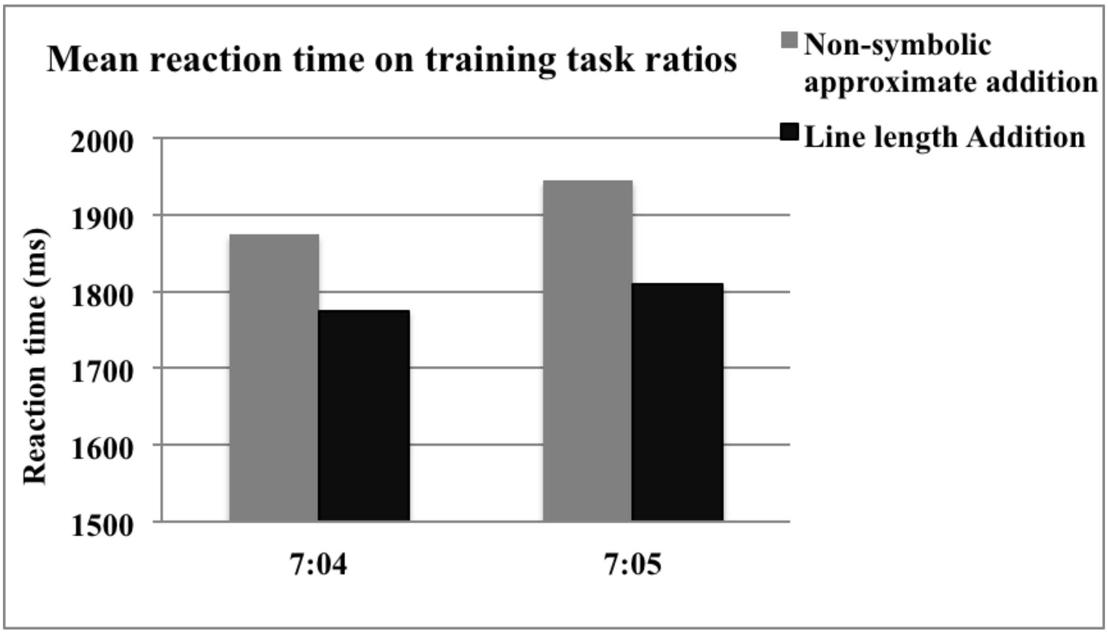

Figure 48. Average reaction time (in milliseconds) over ratio in each condition in experiment 1 (Cross-cultural perspective)

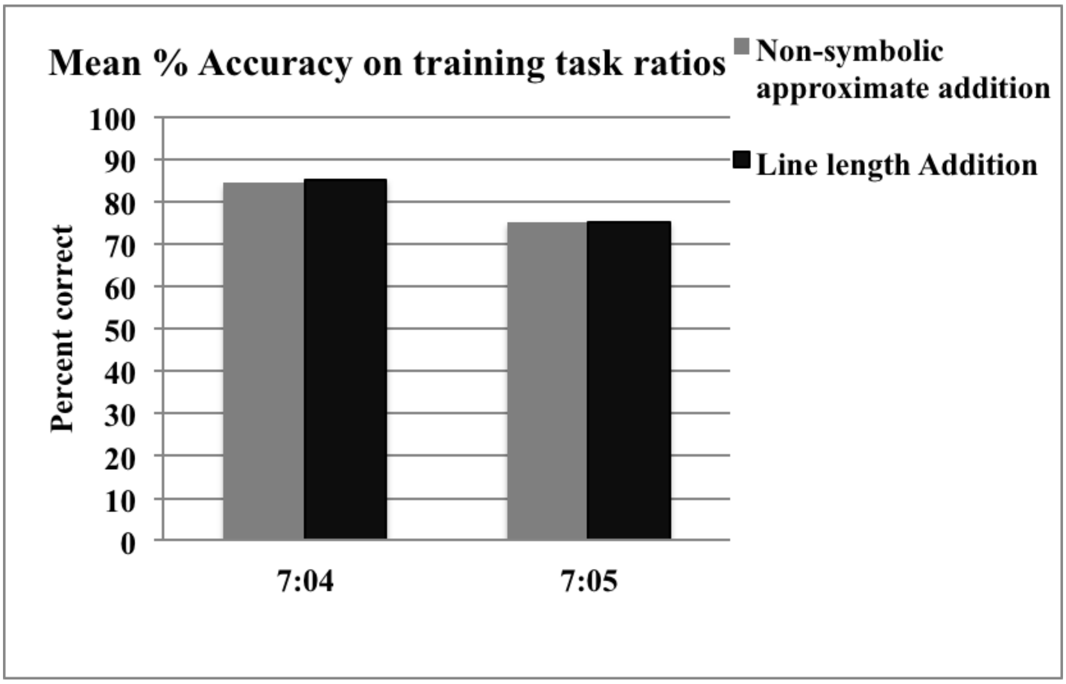

Figure 49. Average task accuracy (expressed as percent correct) over ratio for each condition in experiment 1 (Cross-cultural perspective)

Exact symbolic addition test performance. An ANOVA on the time to complete the exact, symbolic addition test sets with the within-subjects factor of Difficulty (4 levels) and the between-subjects factor of Training Condition (approximate numerical addition or line length addition) revealed a significant main effect of Difficulty F (3, 138) = 40.703, p < .001, η_p^2 = .469 and a significant main effect of Training Condition F (1, 46) = 4.084, p < .05, η_p^2 = .082. A similar ANOVA on accuracy with the same factors revealed only a main effect of Difficulty of test problems on accuracy F (3, 138) = 37.430, p < .001, η_p^2 = .449 and no significant main effect of condition F (1, 46) = 1.898, p =. 175, η_p^2 = .040.

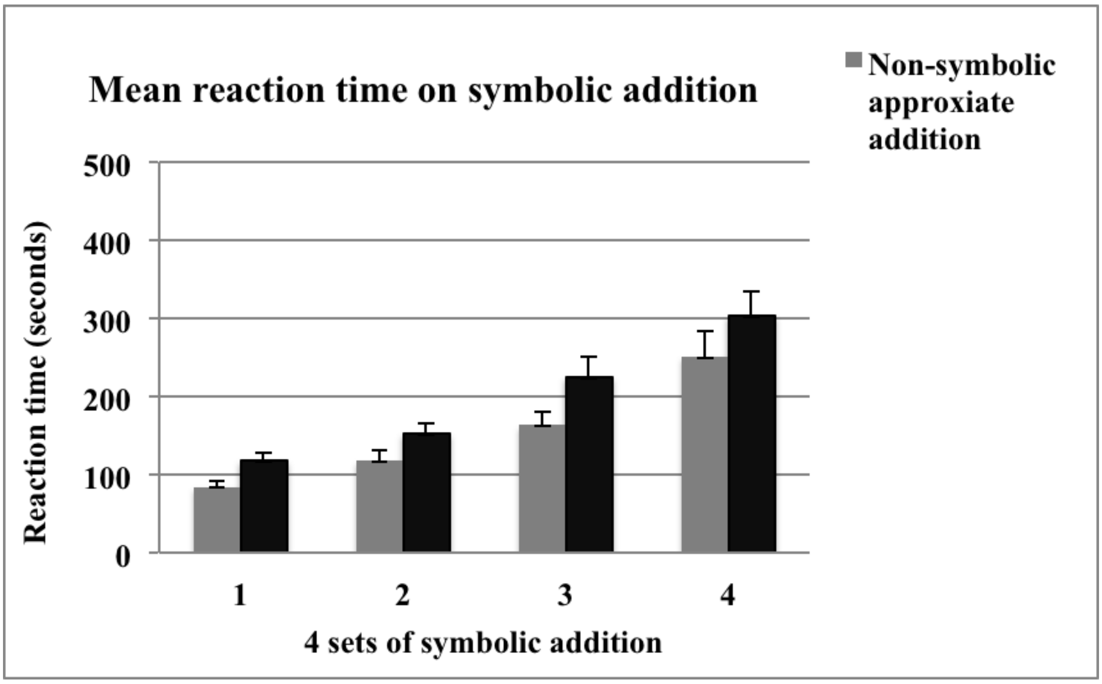

Figure 50. Average speed of test completion (in seconds) for each condition in experiment 1 (Cross-cultural perspective)

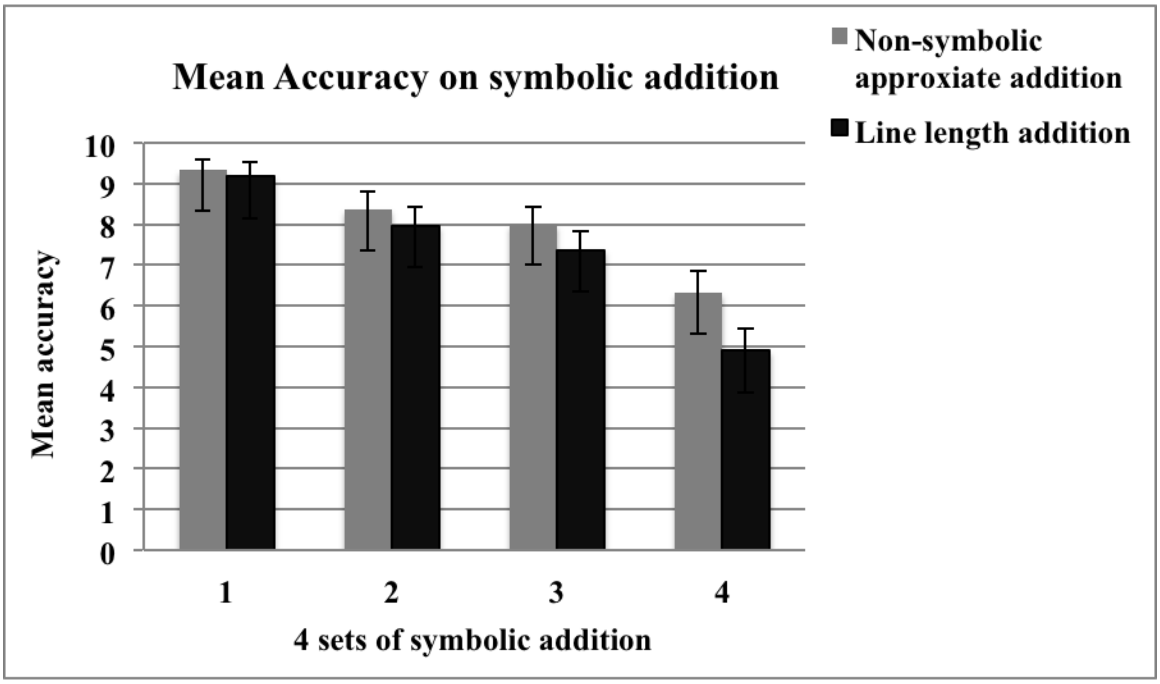

Figure 51. Average task accuracy (expressed as out of 10 problems) for each condition in experiment 1 (Cross-cultural perspective)

Overall, subjects were slower to complete and less accurate on more difficult sets of problems. Children performing the line length addition-training task subsequently completed the exact, symbolic addition test problems significantly slower than children who completed the approximate numerical addition training. These differences in time to complete symbolic addition test sets between training groups were not due to differences in performance on the training task nor were they due to a speed-accuracy tradeoff difference between the groups.

Results (experiment 1 of phase 2)

Participant's factors. There were no significant differences between the two training groups in average age *F* (1, 46) = .209, *p* =. 650: non-symbolic approximate addition group *M* = 6 years, 172 days, *SD* = 86.90, line length addition group *M* = 6 years, 184 days, *SD* = 86.68) or in approximate numerical acuity *F* (1, 46) = .001, *p* = .975: Non-symbolic approximate addition group *M* = .19, *SD* = .09, line length addition group *M* = .19, *SD* = .08).

Training task performance. Analysis of mean reaction time on training trials revealed a main effect of Time, Ratio, Training Condition, and an interaction between Ratio and Time. Regardless of condition subject were faster on second half of training trials *F* (1, 46) = 2.697, *p* = .107 compared to first half *F* (1,46) = 7.316, *p* <. 05.

Table 33

Mixed Factor ANOVA of Ratio (*2 levels: ratio 7:4 and ratio 7:5), Time* (*first half, second half) and Training condition* (*Non-symbolic addition vs. line length addition group) on training task reaction time,* (*in experiment 2, Cross-cultural perspective)*

Variables	*df*	*F*	η_p^2	*p*
Time	1,46	26.958	.369	< .001
Ratio	1, 46	7.554	.141	= .009
Training Condition	1, 46	5.277	.103	< .05
Ratio * Time	1, 46	10.215	.182	< .005

In general, participants in the line length condition performed training trials faster than those in the non-symbolic addition training condition *t* (46) = 2.264, *p* <

.05, (Non-symbolic addition: *M* = 1914.94, *SD* = 317.87, range 1322.27 – 2533.66; Line length addition: *M* = 1730.21, *SD* = 242.49, range 1370.65 – 2295.21). Post-hoc paired sample t-tests revealed that the interaction between Ratio and Time could be explained by a significant difference between Ratio conditions in the first half t (47) = 3.435, $p < .005$, but not the second half of training trials t (47)= .463, $p = .646$, with longer response time to problems involving (7:4) ratio compared to problems involving (7:5) in the first half.

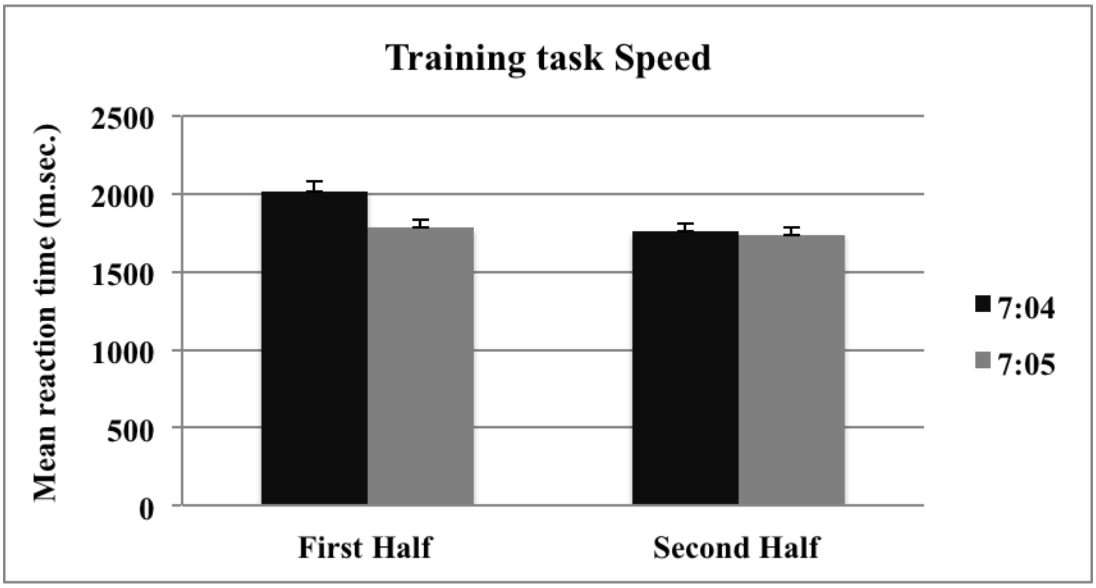

Figure 52. Effects of ratio on average training performance over time in experiment in experiment 2 (Cross-cultural perspective)

The analysis of training task accuracy revealed a main effect of Time, Ratio and an interaction between Time and Training Condition, but no main effect of Training Condition. Post hoc analysis revealed that the interaction between Time and Training Condition resulted from a difference in accuracy between the two training conditions during the second half of training trials F (1, 46)= 5.263, $p < .05$, but not during the first half of the training trials F (1,46) = 3.458, $p = .069$.

Table 34

Mixed Factor ANOVA of Ratio (2 levels: ratio 7:4 and ratio 7:5), Time (first half, second half) and Training condition (Non-symbolic addition vs. line length addition group) on training task accuracy (in experiment 2, Cross-cultural perspective)

Variables	*df*	*F*	η_p^2	*p*
Time	1,46	5.223	.102	< .05
Ratio	1, 46	52.934	.535	< .001
Training Condition	1, 46	.028	.001	= .869
Time * Training Condition	1, 46	9.209	.167	< .005

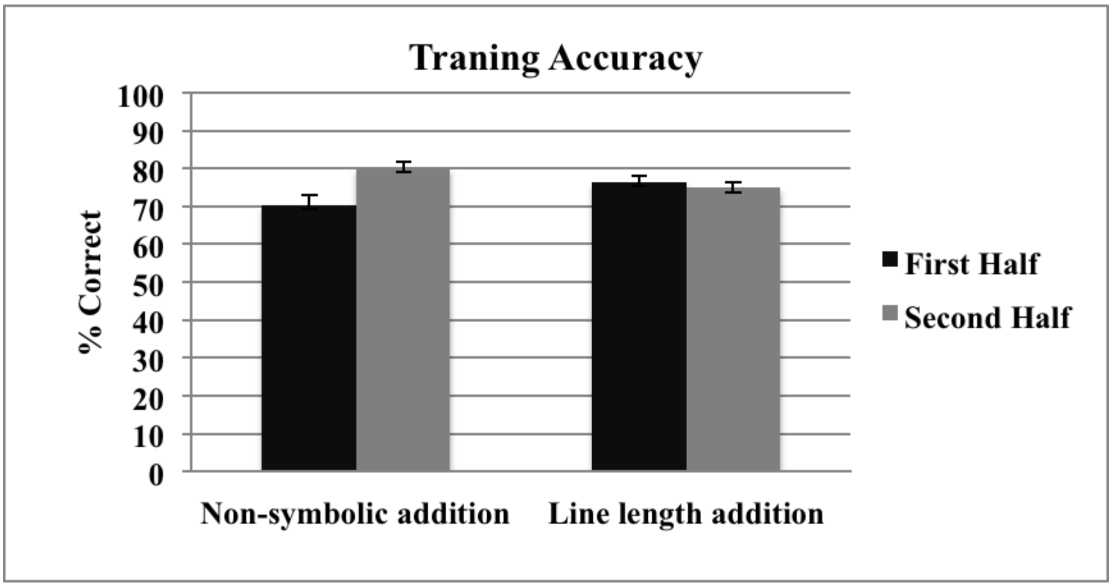

Figure 53. Average task accuracy (expressed as percent correct) over time for each condition in experiment 2 (Cross-cultural perspective)

There was no significant difference on mean % accuracy of training task between non-symbolic approximate addition training (M = 75.21, SD = 8.30) and line addition training (M = 75.56, SD = 5.99), t (46)= .166, p = .869.

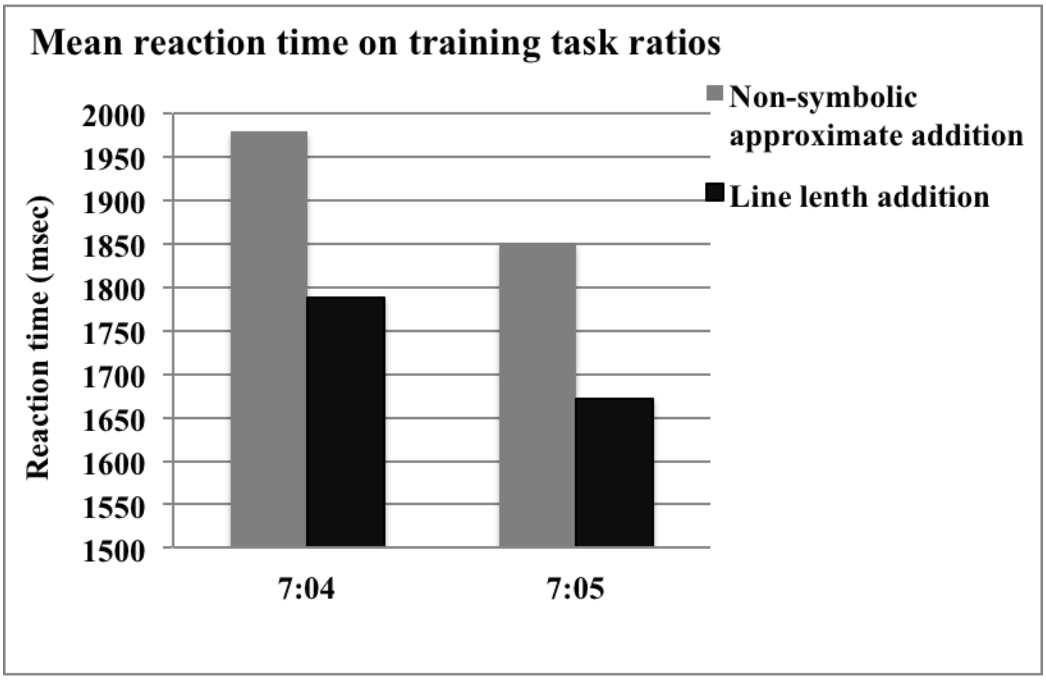

Figure 54. Average reaction time (in milliseconds) over ratio in each condition in experiment 2 (Cross-cultural perspective)

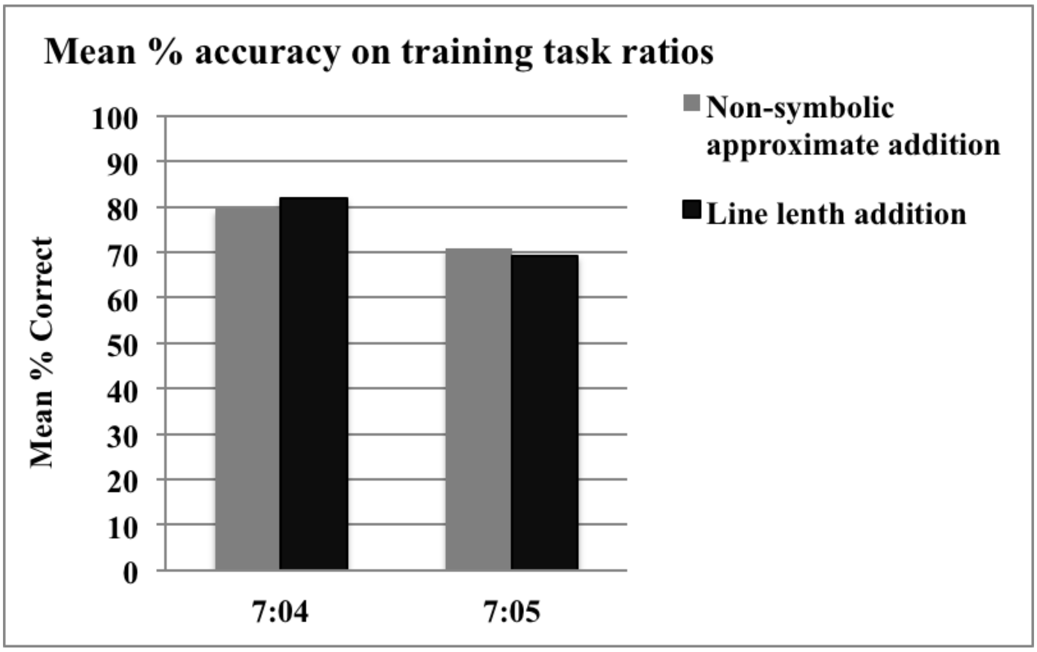

Figure 55. Average task accuracy (expressed as percent correct) over ratio for each condition in experiment 2 (Cross-cultural perspective)

Exact symbolic addition test performance. An ANOVA on the time to complete the exact symbolic addition test sets with the within subject factor of difficulty (4 levels) and between subject factor of training condition (non-symbolic approximate addition or Line length addition) showed significant main effect of difficulty, significant main effect of training condition and significant interaction of difficulty and condition.

Table 35

Mixed Factor ANOVA of Difficulty (*1, 2, 3, 4) and Training condition* (*Non-symbolic addition vs. line length addition) on time to complete symbolic addition problems* (*in experiment 2, Cross-cultural perspective)*

Variables	*df*	*F*	η_p^2	*p*
Training Condition	1, 46	5.418	.105	< .05
Difficulty	3,138	70.372	.605	< .001
Difficulty * Training Condition	3,138	3.190	.065	< .05

Where children in non-symbolic addition training group performed significantly faster on symbolic addition test problems than children in line length addition training group.

Table 36

t-test comparing experimental and control group on time to complete symbolic addition (*in experiment 2, Cross-cultural perspective)*

Group	*N*	*M*	*SD*	*t*	*df*	*p*	Cohen's *d*
Non-symbolic Addition	24	221.83	90.89	-2.328	46	< .05	-0.68649
Line length addition	24	294.72	123.58				

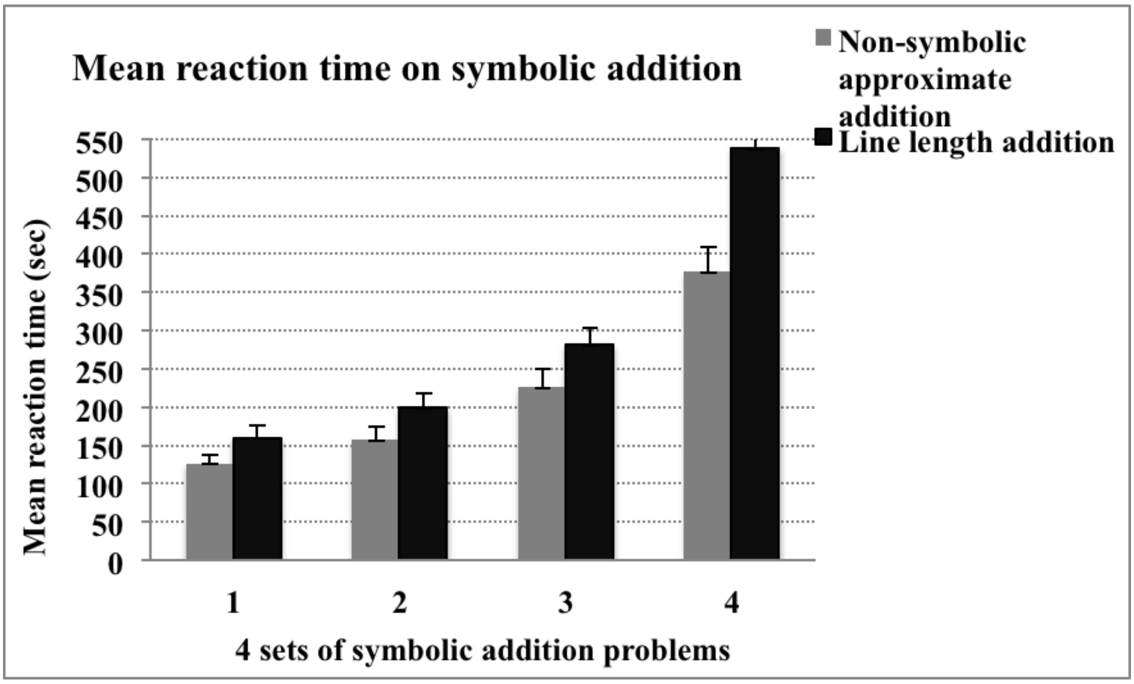

Figure 56. Average speed of test completion (in seconds) for each condition in experiment 2 (Cross-cultural perspective)

A similar ANOVA on accuracy with the same factors revealed only a significant main effect of difficulty of test problems on accuracy and no significant main effect of condition and no significant interaction of difficulty and condition.

Table 37

Mixed Factor ANOVA of Difficulty (*1, 2, 3, 4) and Training condition* (*Non-symbolic addition vs. line length addition) on symbolic addition accuracy* (*in experiment 2, Cross-cultural perspective)*

Variables	*df*	*F*	η_p^2	*p*
Training Condition	1, 46	2.702	.055	= .107
Difficulty	3,138	122.577	.727	< .001
Difficulty * Training Condition	3,138	.315	.007	= .814

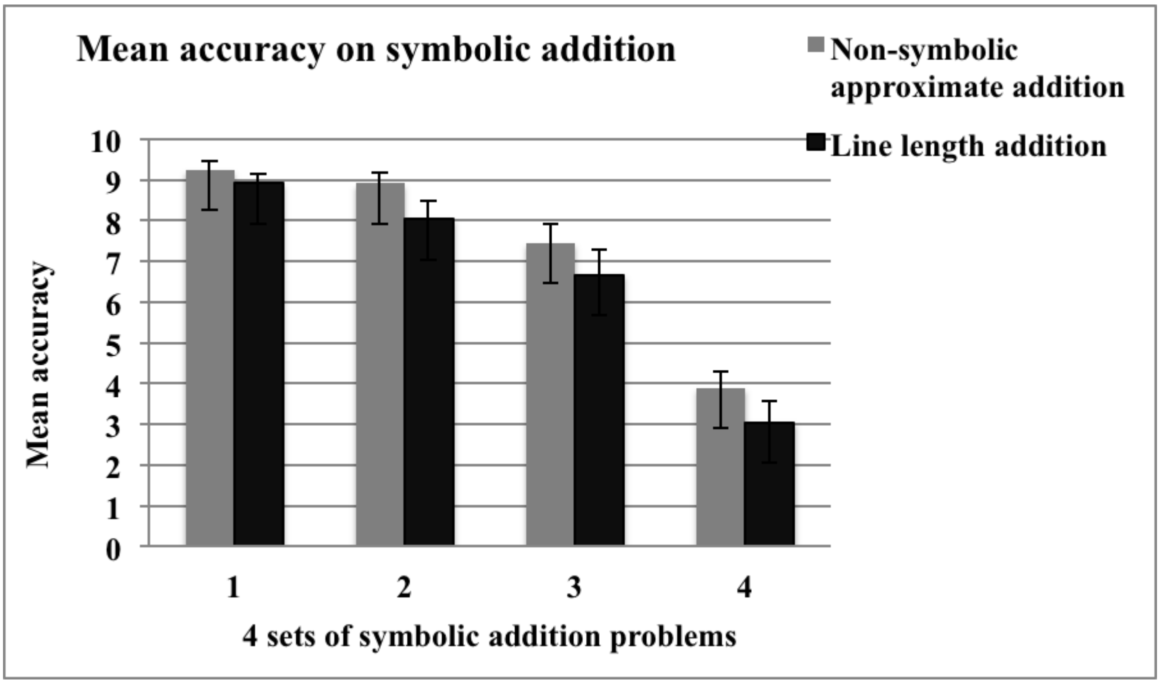

Figure 57. Average test accuracy (expressed out of 10 problems) for each condition in experiment 2 (Cross-cultural perspective)

Exact symbolic addition test performance with training task as a covariate. Differences were seen between training groups on both training task performance (reaction time) and exact symbolic addition test performance. In an attempt to rule out the possibility that significant differences in time to complete the symbolic arithmetic test sets between groups were due to differences in performance on the training task (reaction time) rather than the experimental manipulation of training condition (non-symbolic addition vs. line addition), we analyzed the effects of training condition on symbolic math performance with time to complete the training task or reaction time on the training task as a covariate.

A Mixed Factor ANOVA on reaction time on symbolic addition with the within subject factor of difficulty (1,2,34) and between subject factor of condition (non-symbolic approximate addition vs. Line addition) and training task reaction time was entered as a covariate. This analysis showed that the critical main effect of Training Condition remained after accounting for training reaction time. There was significant main effect of condition, F (1, 45)= 8.380, p <. 05, η_p^2= .157, however there was no significant main effect of training task reaction time, F (1,45) = 3.905, p =. 054, η_p^2 = .080. There was no significant main effect of difficulty, F (3, 135) = .747, p = .526, η_p^2 = .016. However there was significant interaction effect between difficulty and condition, F (3, 135)= 3.356, p < .05, η_p^2 = .069. There was no significant interaction between difficulty and training task reaction time, F (3, 135) = .500, p =. 683, η_p^2 = .011.

A Mixed Factor ANOVA on accuracy of symbolic addition with the within subject factor of difficulty (1, 2, 3, 4) and between subject factor of condition (non-symbolic approximate addition vs. Line addition) and training task accuracy as a covariate was carried out. There was no significant main effect of condition *F* (1, 45) = 2.739, *p* = .105, η_p^2 = .057, and training task accuracy, *F* (1, 45) = .596, *p* =. 444, η_p^2 = .013.

There was no significant effect of difficulty *F* (3, 135) = 1.437, *p* =. 235, η_p^2 = .031, interaction between difficulty and training task accuracy *F* (3, 135) = .173, *p* = .914, η_p^2 = .004, and between difficulty and condition, *F* (3, 135) = .318, *p* =. 812, η_p^2 = .007.

In other words, the experimental manipulation of training condition had a significant effect on the time it took to participants to complete exact symbolic addition problems even after accounting for differences in reaction time and accuracy on the immediately previous training task.

Conclusion

Our results show that children who briefly practiced non-symbolic, approximate addition were faster to complete subsequent exact, symbolic addition tests than were children who briefly practiced a control condition (line addition). As such, these results obtained in a primary school sample in Pakistan replicate the same effect observed previously in a sample of upper middle-class children in the U.S (Hyde et al., 2014). An alternative account of our data is that slight differences in difficulty between the experimental (non-symbolic numerical addition) and control (line length addition) training tasks, rather than the actual content of the tasks, can explain the effects on symbolic addition tests. We believe this is not likely the case given that the effect of training on test held after accounting for reaction time in the training portion as a covariate in our analysis of its effects on symbolic addition tests. Nevertheless, the mechanism(s) driving our effect remain elusive.

Other studies have shown that changing the approximate number system changes exact, symbolic arithmetic performance in adults (e.g., Park & Brannon, 2013). We find cross-culturally that simply engaging the approximate number system enhances subsequent symbolic arithmetic performance (Hyde et al., 2014). We observe no differences in approximate number acuity between groups. Thus, it does not seem that the mechanism driving enhanced performance in our task is change (permanent or temporary) non-symbolic approximate number representation. Several potential mechanisms still remain. It could be that our task of approximate number addition engages both symbolic and non-symbolic numbers, and changes (temporarily) symbolic number representation but not non-symbolic number representations. It could also be that no changes in representation occur, but that simple engagement or priming of a common neural mechanism produces the enhancement.

Chapter V

DISCUSSION

Current study was designed to probe the causal link between non-symbolic and symbolic numerical cognition through training paradigm. It was to explore directly whether non-symbolic numerical training give advantage to the children to perform better on symbolic math or not? Specifically children were trained in different training conditions such as non-symbolic numerical (non-symbolic approximate addition, non-symbolic approximate comparison) and non-numerical training conditions (line length addition and brightness comparison) and were subsequently tested on symbolic addition, sentences with blanks and number line placement task. All training conditions involved very brief one-shot evaluation.

In phase 1, Experiment 1 was carried out with four conditions and experiment 2 with two conditions. In phase 2, experiment 1 was conducted with two conditions and experiment 2 with three conditions and then comparative analysis from both phase 1 and phase 2 is discussed.

Here is summary of results from phase 1 children in U.S.A. Experiment 1. (1) Non-symbolic approximate addition and non-symbolic approximate comparison are faster than brightness comparison and line length addition (2) Non-symbolic approximate addition is similar in accuracy to brightness comparison and line length (3) Non-symbolic approximate comparison is more accurate than line length addition. Experiment 2 with harder problems. (1) non-symbolic approximate addition is similar to brightness comparison in reaction time. (2) non-symbolic approximate addition is more accurate than brightness comparison (but not difference on the sentence task).

Here is a summary of results from phase 2 children in Pakistan Experiment 1. (1).Non-symbolic approximate addition is faster than line length addition. It is a replication of results with American children. (2) non-symbolic approximate addition is similar to line length addition for accuracy. This also replicates a result with American children.

Experiment 2, with number line placement as the dependent variable (1) Non symbolic approximate addition is similar in reaction time to brightness comparison and line length addition. (2). Non-symbolic approximate addition more accurate than both brightness comparison and line length addition.

These results with line replacement resemble those in phase 1, experiment 2 with the more difficult symbolic addition problems.

Results of phase 1 experiments shed light on the significant role of priming/ training of analogue magnitudes on symbolic number processing. Through first experiment, it has been shown that if arithmetic problems are in the range of children solving abilities (means problems are based on the difficulty level these children are already carrying out in their classroom practice), children trained with non-symbolic numbers turned out to be faster than control group. The findings of Experiment 1 provide evidence that the ANS plays a functional role in symbolic arithmetic. As the difficulty level increases with in each set of symbolic addition problems, a trend of experimental group can be seen of being more accurate on difficult problems as compare to control group, although not significant.

In experiment 2, children who first practiced a non-symbolic approximate addition task subsequently performed more accurately on exact, symbolic addition problems than did children who practiced a control task involving brightness magnitude comparison. Their greater accuracy was achieved with no loss in speed. However, both sets of symbolic addition problems were harder than the problems, children solved in experiment 1. Results of experiment 2 showed that if children are trained in non-symbolic arithmetic and solving the problems beyond their level (as these problems were difficult than they were carrying out in their classroom practice), they turned out to be significantly more accurate that control group. Taken together both experiments findings suggest that experiences operating on non-symbolic magnitudes played a significant role in children's processing of symbolic numbers.

The benefits of ANS engagement were limited to performance on problems in the domain of mathematics, as children trained on non-symbolic addition performed more accurately only on the test of exact, symbolic addition, not the sentence completion test. Thus, the observed effects are likely explained by a specialized relationship between the ANS and symbolic mathematics, rather than by mediating factors such as effects of practice on children's general motivation or cognitive engagement, as such mechanisms would likely generalize to enhanced performance on cognitive tasks more broadly (including the sentence completion task). Finally, it appears that simply activating symbolic number representations in our brief paradigm is not sufficient to prime better performance on subsequent symbolic addition, as the presentation (presenting children with symbolic addition to solve) of symbolic numbers on the first symbolic addition test led to no enhancement of performance on the second symbolic addition test. These findings suggest that the present effects of the ANS on symbolic arithmetic do not simply depend on co-activation of symbolic number representations.

Results of experiments conducted under phase 1 and 2 demonstrate that brief practice on a non-symbolic approximate numerical task enhances the performance of 6–8 year old children on a subsequent test of exact, symbolic arithmetic. Several possible explanations have been proposed to explain the mechanism underlying the observed effects. The pattern of data obtained across the different conditions indicates that these results are not due to engagement of a generalized magnitude system, engagement of common cognitive operations (such as comparison or addition), or difficulty differences between the training tasks.

Rather, data provide evidence that symbolic arithmetic draws on at least some overlapping cognitive and/or neural structures used to represent approximate number. The pattern of data obtained across two different test conditions in Experiment 2, indicates that the enhancing effects of approximate number representations are limited to the domain of symbolic mathematics or number, as comparable enhancements were not observed in children's performance of the sentence completion task. This dissociation also provides evidence that participants who practiced approximate number tasks were not simply more motivated, focused, or engaged than those assigned to a training task involving other quantities or variables, and that numerical comparisons did not prime general cognitive abilities to a greater extent than did other tasks.

Current data argue against symbolic number representations underlying the observed effect. First, based on observed performance, it appears that children used the ANS to solve the non-symbolic addition and comparison training tasks. This claim is supported by evidence of two well-established signatures of the ANS in data: the ratio effect and the equality of comparison and addition performance (Barth et al., 2005, 2008; Gilmore et al., 2007; Izard, Dehaene-Lambertz, & Dehaene, 2008; Pica et al., 2004). Children were slower and less accurate on problems where the actual answer and outcome were closer in ratio compared to problems where the ratio between answer and outcome were more distant. Children also showed equal performance on numerical comparison and addition. In contrast, if exact symbolic comparison and addition strategies had been used, numerical comparison should have been easier than numerical addition, as the comparison involves only two numbers, not combining two numbers to compare to a third. Moreover, no children were noted to have used verbal counting or called out verbal numbers during the task; if such strategies were being used, they were being done covertly. Second, the design of the task employed established procedures to discourage the use of symbolic numbers to answer the questions (see Ballinger & Barth, 2007; Barth et al., 2006, 2008). The numerical arrays were presented too quickly to be enumerated exactly (1 second) and large numbers were used (average sum/outcome = 34; range for sum/outcomes = 16–56; average addend = 17; range for addends 7–40, 43) to discourage rapid identification, serial enumeration, or memorized answers to addition problems.

Third, previous work suggests that this type of task can be performed without symbolic arithmetic knowledge (monkeys: Cantlon & Brannon, 2007; preschool children: Gilmore et al., 2007) and the use of a symbolic number strategy does not facilitate performance (e.g., Ballinger & Barth, 2007; Barth et al., 2008; Gilmore et al., 2007). Fourth, Park and Brannon (2013) showed that training on a task involving ordering symbolic number does not lead to as significant gains in symbolic arithmetic as a training task engaging the ANS. Consistent with their findings, the participants in both conditions of Experiment 2 engaged symbolic numbers during the first block of symbolic addition test problems, but this engagement did not yield improvements on the second set of test problems. In fact, subjects in Experiment 2 performed worse on the second set of symbolic addition problems, regardless of training condition. These findings cast doubt on the possibility that symbolic number engagement over non-symbolic numerical arrays, rather than the ANS itself, drives the observed enhancements seen in the numerical training conditions of our experiments. While the possibility that symbolic number representations were co-activated with ANS representations cannot be entirely ruled out, as results, design, and previous research all suggest that the ANS rather than symbolic number representations was used to solve the tasks and likely drives the observed effect. Future research using the method of Experiment 2 with different symbolic tests as outcome measures may add further insight into this issue.

Children who practiced either a non-symbolic approximate numerical comparison or numerical addition task were faster to complete subsequent exact, symbolic addition test problems than were children who performed comparable tasks involving non-numerical magnitudes (length, brightness).

The results show that enhancement of exact symbolic arithmetic performance in children who were trained on non-symbolic approximate arithmetic problems could not be explained by differences in difficulty between the experimental (non-symbolic numerical addition) and control training task (line length addition), as the advantage in time to complete test problem sets remained for those that trained on the non-symbolic numerical addition problems after equating performance on the control task (line length addition). Furthermore, the enhancement seen in those of the non-symbolic numerical addition relative to control group cannot be explained by simply engaging the arithmetic process, as children trained in a non-numerical line length addition task did not subsequently show the same enhancements.

While one of these training tasks was easier than the others (brightness comparison), but results do not appear to be due to differences in the general difficulty of the training tasks in which the different groups of children engaged, because differential test performance was seen between numerical and non-numerical tasks of equal difficulty (e.g., line-length addition and numerical addition), and because entering performance on the four training tasks as a covariate over all tasks did not eliminate the critical main effect of training condition. Results does not appear to depend on differential levels of learning during the training phase, as participants improved in speed over time on the initial experimental task regardless of condition.

Two established signatures of the ANS in performance on the two training tasks involving numerical magnitudes were observed. First, reaction time was a function of the ratio between the two numbers to be compared (sum vs. foil or first array vs. second array) (Barth et al., 2005, 2008; Izard, Dehaene-Lambertz &

Dehaene, 2008; Pica, Lemer, Izard, & Dehaene, 2004). Second, no significant differences were observed in performance between the numerical comparison and the numerical addition tasks (Barth et al., 2006; Gilmore et al., 2007). These results provide strong evidence that subjects used the ANS to solve experimental tasks involving non-symbolic numerical magnitude.

Experimental design and analyses provide evidence against several alternative hypotheses related to the relationship between the ANS and symbolic arithmetic. First, it does not appear from present data that a generalized magnitude system (Walsh, 2003), rather than a number-specific system (Dehaene, 2011), explains the relationship between the ANS and symbolic arithmetic (Lourenco et al., 2012), as the experimental conditions that involved non-numerical magnitudes did not lead to better subsequent performance compared to the experimental conditions involving nonsymbolic numerical magnitudes. Results of experiments 3 and 4 follow same pattern of results as children in experiment 4 (trained with non-symbolic comparison) performed significantly better than children trained with line addition training. It adds up to previous findings and shows that it is not “addition” as an operation that is driving the effect rather it is non-symbolic training that is driving the effect. Second, it does not appear that common cognitive operations inherent in symbolic and non-symbolic tasks (Holloway & Ansari, 2008; Lyons & Beilock, 2009), rather than the ANS in particular, are responsible for correlations between the ANS and symbolic mathematics, as participants showed enhanced performance on symbolic arithmetic after practicing comparison or addition of numerical magnitudes but not after practicing tasks involving the same cognitive operations (ordering, comparison, and/or addition) over nonnumerical magnitudes. Similarly, results can not be explained as an easier arithmetic exercise ‘‘warming-up’’ or priming more difficult symbolic arithmetic (e.g., Fuchs et al., 2013), as practicing non-symbolic numerical comparison worked equally as well as practicing non-symbolic addition to improve subsequent symbolic arithmetic.

Findings of current study provide some evidence against the claim that the inhibitory demands of tasks involving the ANS drive correlations with symbolic mathematics (Fuhs & McNeil, 2013; Gilmore et al., 2013). It is possible, as some have argued, that non-symbolic numerical tasks engage executive function (EF) to a greater extent than do non-symbolic spatial or brightness tasks, because they require children to inhibit responses to continuous variables that are anti-correlated with number on some trials in order to respond correctly. Under this view, greater commonalities in EF engagement between the numerical training tasks and the symbolic arithmetic test, rather than specific overlap in the ANS and symbolic mathematics, might explain better subsequent symbolic arithmetic performance in the numerical training groups compared to the non-numerical training groups. For several reasons, this is not likely the case in present dataset. First, unlike previous studies reporting that the relationship between the ANS and symbolic mathematics is mediated by inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013), stimulus controls were used where continuous properties could not be reliably used to solve the tasks because they were not systematically related to the answer. The non-numerical continuous properties of each numerical array within each trial and between trials in our study were randomly chosen, in contrast to previous work where non-numerical properties of each numerical arrays within a problem were reliability and systematically related to the answer on a given trial (either all positively or all negatively correlated with number, although the direction of the relationship was manipulated across problems). Second, if the numerical tasks required substantially more inhibitory processes than other non-numerical tasks, this would likely be reflected in behavioral performance. However, the approximate numerical addition task was no harder than the line addition task, suggesting no substantial differences in the inhibitory control required, yet significant differences were observed in subsequent symbolic addition test performance. Third, exercising executive function appears to deplete rather than enhance performance on subsequent tasks also involving EF (Baumeister, Bratslavsky, Muraven, & Tice, 1998; Hagger, Wood, Stiff, & Chatzisarantis, 2010; Hofmann, Schmeichel, & Baddeley, 2012; Powell & Carey, 2013; Schmeichel, 2007). Given the temporal structure of experiments, with ANS training and symbolic mathematics testing occurring in immediate succession, a common role for EF in both tasks would be predicted to lead to impairment rather than to enhancement of symbolic arithmetic performance. Some may argue that visuospatial working memory is differentially engaged between numerical and nonnumerical training tasks and could mediate the observed relationship between approximate numerical training tasks and symbolic math performance. Most of the arguments provided against the idea of inhibitory control mediating the effect, apply equally well against a differential working memory account. Specifically, substantial differences in working memory between training conditions should have been evident in training task performance, but equal performance was observed between the numerical conditions and the non-numerical line length addition condition, for example. Contrary to the obtained test results, it is likely that a training task that taxed the working memory system would lead to worse rather than better performance on a subsequent task. Finally, the numerical addition task clearly should tax working memory more than the numerical comparison task, yet these two tasks had equal effects on children's subsequent symbolic arithmetic performance. Nevertheless, further research should investigate the role of EF and working memory more directly in children's ANS practice and symbolic arithmetic performance.

Finally, current study's results run contrary to the suggestion that non-symbolic numerical addition is a better task for improving symbolic mathematics than numerical comparison (Gilmore et al., 2010; Park & Brannon, 2013), at least under conditions of brief exercise and immediate testing.

Practice of numerical comparison and numerical addition produced similar effects in experiments. The scope of the observed practice effect, however, remains unclear. One possibility is that the practice effect is specific to the domain of number or mathematics. Alternatively, engaging the ANS may have more general effects on motivation, reasoning, or cognition that would translate to an entirely different cognitive task outside the domain of number or magnitude. In a second experiment, we tested this hypothesis by extending the rationale and method of Experiment 1 to include a cognitive test in the domain of reading.

Overall there was no significant difference in weber fraction among groups tested under different trainings, which support the important role of approximate number training in enhancement of symbolic number processing and different research evidence have already shown important link between approximate number system and math ability (see for reviews, Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011).

Several aspects of the phase 2 experiments contribute to our understanding of the reliability and generalizability of the effects of non-symbolic number on symbolic mathematics. Phase 2 experiments provides further evidence for the hypothesis that the approximate number system is causally related to symbolic arithmetic performance, as children that engaged approximate number through training outperformed children who engaged continuous magnitude representations on exact, symbolic arithmetic tests. This study used the same materials and procedure, thus directly replicating the recent finding of the same effect. This attests to the legitimacy of the effect as robust and replicable.

Second, the current data suggest that relationships between approximate number and symbolic mathematics in children hold outside of the laboratory. The current study was conducted in a large public school. Differences in motivation and pressure to perform could likely differ between such laboratory-based studies and studies conducted during the school day. Nevertheless, we observed similar effects on both groups suggesting that the effect generalizes from laboratory to more practical (educational) settings.

Third, the effects of engagement of approximate number on symbolic arithmetic performance were observed in a group of Pakistani children, whose education, curriculum and culture vary greatly from the upper-middle class children tested in previous studies of approximate number training in the United States. For example, Pakistan has one of the lowest literacy rates in the world (Unesco, 2012) and a low level of public investment in education (2.2 % GNP), particular in primary school (Ali & Siddiqui, 2013; PSLM, 2013). Although these numbers are improving, particular in Islamabad where testing was conducted, such statistics suggest less emphasis on cultural and resources available for education than in the U.S where children have been previously tested. The effects of approximate number practice on symbolic addition held despite vast differences in the cultural emphasis on education and access to technology, suggesting that the relationship between the approximate number system and mathematics likely generalizes to a variety of cultural and socio-economic settings. Relatedly, children in Pakistan have much less access to technology like the computers used for testing than do children in the U.S (Khan, 2004; Gulati, 2008). As such, our method does not depend on having substantial experience with computers, as was more likely to be the case with U.S. children than Pakistani children.

It should be noted that data the current data from Pakistan suggests comparable weber fractions and levels of performance on symbolic addition to those obtained in a previous U.S sample of children. Despite these vast cultural differences, early mathematics abilities may be similar. Any differences that arise, then, might be contributed to cultural differences with relation to education.

The replicability and generalizability of the influence of non-symbolic approximate number on symbolic arithmetic shown in Experiment 1 has conceptual and practical implications. From a practical perspective, it is likely that computer-based interventions based on our method could be applied internationally to serve a range of populations and cultures including those with less access to technology. From a theoretical perspective, these results increase confidence the idea that there is a strong link between early developing, non-symbolic numerical intuitions and symbolic mathematics acquired in school.

The implications of Experiment 2 results are promising but less clear. Previous work suggests that training on a number line placement task improved symbolic mathematics (Kucian et al., 2011). In our study, we observed that the non-symbolic approximate addition training enhances symbolic arithmetic and may also have an effect on number line placement. The effects of our approximate number training on symbolic number line placement, however, were only marginal significant and, thus, not robust enough to conclude differences between training groups. The pattern of results, however, suggests that future studies should track the effects of ANS training on ANS precision as well as the potential changes in symbolic number representation.

In sum, the present findings move beyond the findings of correlational studies (Gilmore et al., 2010; Halberda et al., 2008; Libertus et al., 2011; Mazzocco et al., 2011; Mundy & Gilmore, 2009) and build on recent training experiments (Park & Brannon, 2013) to provide experimental evidence that exercising the primitive system of approximate number representation can enhance both the speed and the accuracy of children’s performance of symbolic mathematics.

Questions, Limitations and Suggestions

Present study raised a number of questions regarding the nature of the observed effect and had some limitations mentioned below. Few suggestions have been mentioned keeping in view the results of study.

1. One most intricate question was that it could not rule out the possibility that symbolic number representations were co-activated with ANS representations. Future research using the method of Experiment 2 (language task) with different symbolic tests as outcome measures may add further insight into this issue.
2. The developmental origins of the relationship between the ANS and symbolic number remain unclear. ANS acuity is associated with facility at symbolic mathematics across the lifespan, from infants (Starr et al., 2013) to preschool children (Halberda et al., 2008) to octogenerians (Halberda et al., 2012). Experimental studies in children (current study) and adults (Park & Brannon, 2013) seem to suggest that practice or training with the ANS enhances symbolic mathematics. Our results show that the functional and causal link between ANS activation and symbolic arithmetic performance does not require a lengthy history of education in symbolic mathematics, as it occurs in children who are only in their second year of formal schooling and participants in most previous studies have had at least some working knowledge of symbolic number and formed initial mappings between symbolic number representations and the ANS. It is unclear if earlier interventions (such as

those in infants or toddlers) centered on engaging and exercising the ANS, would lead to better mathematics outcomes later in life.

3. It also is unclear if later interventions on participants whose manipulations of number systems are fully automatic (e.g., Bugden & Ansari, 2011; Girelli, Lucangeli, & Butterworth, 2000) would show the same immediate effects found in the present experiments. On one view, both initial learning and mature performance of symbolic mathematical computations such as arithmetic depend on the ANS (Dehaene & Cohen, 1997; Isaacs, Edmonds, Lucas, & Gadian, 2001; Lee, 2000; Levy, Reis, & Grafman, 1999; Molko et al., 2003; Takayama, Sugishita, Akiguchi, & Kimura, 1994), which plays an obligatory role in exact symbolic numerical representations and arithmetic operations. On a different view, the ANS and symbolic number representations become linked because they are repeatedly associated with one another over the course of children's learning of number symbols; thus, the ANS plays a habitual rather than obligatory role in symbolic mathematics performance (e.g., Lyons & Beilock, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). On a third view, symbolic mathematics performance may depend on the ANS at early points in learning, but its influence may decline or become more habitual once symbolic arithmetic skills are fully automatic.

4. Another open question concerns the symmetry or asymmetry of the causal relationship between the symbolic and non-symbolic number systems. Although the present experiments tested only for a relationship in one direction, and showed that exercising the ANS can enhance symbolic number

processing, it is possible that causal effects operate in the reverse direction as well. Consistent with the latter possibility, the Munduruku of the Brazilian Amazon provide suggestive evidence of an effect of symbolic number training on the acuity of the ANS (Piazza et al., 2013). The Munduruku language has a limited numerical vocabulary and no formal symbolic number system.

However, some Munduruku have learned the Portuguese numerical language and some have studied symbolic arithmetic in school. Individual differences among the Munduruku in ANS acuity are associated with both of these factors (Piazza et al., 2013).

5. Finally, the depth and temporal extent of the effects of ANS activation on symbolic number processing are not known. Recent work shows that extended, intense practice with the ANS through an approximate addition task can change both ANS acuity and symbolic mathematics ability and extent of ANS acuity change in individual participants correlates with individual increases symbolic arithmetic (Park & Brannon, 2013). No significant differences in ANS acuity were observed between children in the different training conditions of our study, casting doubt on the possibility that the mechanism of symbolic mathematics enhancement in our study was an ANS acuity change. Instead, it appears that simply preceding symbolic arithmetic with focused engagement of the ANS was sufficient to produce the effects on symbolic arithmetic. It is speculated that present effects arose through engagement of common cognitive mechanisms in the two tasks. Because the present research involved very brief practice and immediate testing, it is not known whether the

effects on symbolic arithmetic reported here are momentary or enduring. Future work should contrast the extent and duration of symbolic mathematics outcomes after tasks involving engagement of, compared to change in, the ANS.

6. These results were obtained in children who were just beginning formal schooling, and it is possible that neither younger nor older children would show the same training effects that were found in this study. So conducting similar experiments with pre-school children might further clarify the nature of relationship between non-symbolic and symbolic numerical cognition.
7. Further research studies are needed to rule out the possibility of number-space association. Moreover, future studies can be designed in a way so that children performance on each problem could be monitored separately for looking more closely their responses in terms of reaction time and accuracy. So that children performance on easy and difficult problems could be analyzed separately.
8. Although children in this experiment got non-symbolic number training for short time in one day visit so training the children for little bit longer time for more days might demonstrate more positive results in children symbolic number processing.

Implications, Scope and Recommendations

Regardless of the answers to these questions, current studies provide evidence for a causal relationship between non-symbolic approximate number and exact, symbolic arithmetic by children, and they move beyond previous work to delineate the specificity of this relationship. The fact that a single session of practice on an approximate number task can improve both the speed with which children solve easier symbolic mathematics problems, and the accuracy with which they solve harder mathematics problems, raises important possibilities for future educational research. In particular, it is possible that exercises engaging the ANS will provide a way not only to speed up mathematics performance in an immediately following test but also to boost performance of school mathematics in a more enduring way. In light of the importance of mathematics both in the elementary school curriculum and in diverse disciplines and professions, this possibility deserves to be tested.

The scope of the observed practice effect however, remains unclear keeping in view certain aspects of the study.

One possibility is that the practice effect is specific to the domain of number or mathematics as participants performed more accurately only on the test of exact, symbolic addition, not the sentence completion test (Experiment 2 of phase 1).

An alternative account is that participants were engaging symbolic number representations jointly with ANS representations in the numerical addition training task. Symbolic number representations may have primed symbolic arithmetic, and the role of the ANS representations may simply have been to activate number symbols.

Future research interventions can be developed intended to improve math-learning difficulties – Dyscalculia (Kucian et al., 2011; Räsänen et al., 2009; Wilson et al., 2006) and to improve deficits in number sense (Butterworth, 1999; Gersten & Chard, 1999; Wilson & Dehaene, 2007). Children with low numerical competence can be trained even before their acquisition of symbolic-number knowledge and then expect improved symbolic-math fluency later in development.

Present study has important insights for practitioners, teachers, learners for early math curriculum and educational intervention.

This research study is particularly useful for educational institutions. Higher Education commission of Pakistan and directorate of education can be very helpful in disseminating the findings to schools and to the general public in large in the form of books. So that utilization of this research for Pakistani children can be achieved by coordinating with higher authorities.

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Appendix-A

Easy non-symbolic approximate addition values used for training task in experiments 1 and 2.

7:4 RATIO					
Addend 1	Addend 2	Actual Sum	Foil	Correct Response	Ratio
13	43	56	32	Less	0.57
26	30	56	32	Less	0.57
21	11	32	56	More	0.57
15	17	32	56	More	0.57
40	9	49	28	Less	0.57
22	27	49	28	Less	0.57
18	10	28	49	More	0.57
13	15	28	49	More	0.57
30	12	42	24	Less	0.57
19	23	42	24	Less	0.57
16	8	24	42	More	0.57
12	12	24	42	More	0.57
24	11	35	20	Less	0.57
16	19	35	20	Less	0.57
13	7	20	35	More	0.57
9	11	20	35	More	0.57
19	9	28	16	Less	0.57
13	15	28	16	Less	0.57
9	7	16	28	More	0.57
8	8	16	28	More	0.57

Appendix-B

Difficult non-symbolic approximate addition values used for training task in experiments 1 and 2.

7:5 RATIO					
Addend1	Addend2	Sum	Foil	Correct Response	Ratio
40	16	56	40	Less	0.71
28	28	56	40	Less	0.71
27	13	40	56	More	0.71
21	19	40	56	More	0.71
35	14	49	35	Less	0.71
23	26	49	35	Less	0.71
23	12	35	49	More	0.71
17	18	35	49	More	0.71
30	12	42	30	Less	0.71
20	22	42	30	Less	0.71
20	10	30	42	More	0.71
15	15	30	42	More	0.71
25	10	35	25	Less	0.71
16	19	35	25	Less	0.71
17	8	25	35	More	0.71
11	14	25	35	More	0.71
20	8	28	20	Less	0.71
13	15	28	20	Less	0.71
13	7	20	28	More	0.71
10	10	20	28	More	0.71

Appendix-C

Easy line length addition values (in pixels) used in training of Experiment 3

7:4 RATIO					
Addend 1	Addend 2	Actual Sum	Foil	Correct Response	Ratio
84	84	168	96	Less	0.57
85	83	168	96	Less	0.57
41	55	96	168	More	0.57
60	36	96	168	More	0.57
68	79	147	84	Less	0.57
75	72	147	84	Less	0.57
35	49	84	147	More	0.57
60	24	84	147	More	0.57
61	65	126	72	Less	0.57
70	56	126	72	Less	0.57
32	40	72	126	More	0.57
47	25	72	126	More	0.57
33	72	105	60	Less	0.57
55	50	105	60	Less	0.57
30	30	60	105	More	0.57
35	25	60	105	More	0.57
40	44	84	48	Less	0.57
55	29	84	48	Less	0.57
20	28	48	84	More	0.57
26	22	48	84	More	0.57

Appendix-D

Difficult line length addition values (in pixels) used in training of Experiment 3

7:5 RATIO					
Addend1	Addend2	Sum	Foil	Correct Response	Ratio
84	84	168	120	Less	0.71
85	83	168	120	Less	0.71
59	61	120	168	More	0.71
71	49	120	168	More	0.71
68	79	147	105	Less	0.71
82	65	147	105	Less	0.71
40	65	105	147	More	0.71
55	50	105	147	More	0.71
57	69	126	90	Less	0.71
73	53	126	90	Less	0.71
30	60	90	126	More	0.71
50	40	90	126	More	0.71
47	58	105	75	Less	0.71
68	37	105	75	Less	0.71
35	40	75	105	More	0.71
45	30	75	105	More	0.71
42	42	84	60	Less	0.71
50	34	84	60	Less	0.71
28	32	60	84	More	0.71
30	30	60	84	More	0.71

Appendix-E

Approximate number comparison values used in training of Experiment 4

Number 1	Number 2	Correct Response	Ratio
56	32	Less	0.57
32	56	More	0.57
49	28	Less	0.57
28	49	More	0.57
42	24	Less	0.57
24	42	More	0.57
35	20	Less	0.57
20	35	More	0.57
28	16	Less	0.57
16	28	More	0.57
56	40	Less	0.71
40	56	More	0.71
49	35	Less	0.71
35	49	More	0.71
42	30	Less	0.71
30	42	More	0.71
35	25	Less	0.71
25	35	More	0.71
28	20	Less	0.71
20	28	More	0.71

Appendix-F

Script for Non-symbolic approximate addition

<u>Note.</u> Script was modified for Non-symbolic approximate comparison as comparison of arrays instead of addition and for Line length addition task as addition of vertical lines instead of arrays of dots).

<u>Phase 1-*First set of training trial without Occluder*</u>

Clip1: Let's play a game; in this game there is a bad witch and good princess, and they both like to play with marbles.

CLIP 2: Look, here is a good princess

Clip 3: Look, here are some marbles, and here are some more marbles, and now all the marbles are together.

Clip 4: Look, now there are more marbles! I wonder why that happened.

Clip 5: I think the princess must have put more marbles to the pile. Let's see if she did. (then the experimenter should hit the princess response button, and princess should dance around)

Clip 6: Yes it was the princess.

CLIP 7: Look, here is the bad witch.

Clip 8: Look here are some marbles, and here are some more marbles, and now all the marbles are together.

Clip 9: Look, now some of the marbles went away! I wonder why that happened.

Clip 10: I think the witch must have taken some marbles from the pile. Let's see if she did. (then experimenter should hit the witch button, and witch should dance around)

Clip 11: Yes, it was the witch.

Phase 2: *Training trials with Occuluder*

CLIP 1: Now the bad witch and good princess are playing with the marbles behind a screen.

Clip 2: See, look, here is a screen. A pile of marbles will go behind the screen and then some more marbles will also go behind the screen. Then either the bad witch or good princess will play with the marbles and then we will see, what happened.

CLIP 3: Look, here are some marbles, and here are some more marbles. Now all the marbles are behind the screen.

Clip 4: What happened? (Let the child respond, then give feedback saying).

Clip 5.a: That's right, there are more marbles. Who do you think played with the marbles: the princess or the witch?

Clip 5.b:(If child guessed wrong) No, there are more marbles than before. Who do you think played with the marbles: the princess or the witch?

Clip 6: Let's find out. (Then child should hit whichever button corresponds to his guess. If he guessed right princess should appear and dance around while saying …)

That's right! The princess played with the marbles. She put in more marbles.

Clip 6.b:(If child guessed wrong) (he should see witch fall over and hear)

No the witch did not play with the marbles and take any away. The princess played with the marbles. She put in more marbles

Clip 7: Look, here are some marbles, and here are some more marbles. Now all the marbles are behind the screen.

Clip 8: What happened? (Let the child respond, then give feedback saying).

Clip 9.a: That's right, there are less marbles. Who do you think played with them: the princess or the witch?

Clip 9.b:(If child guessed wrong) No, there are less marbles than before. Who do you think played with them: the princess or the witch?

Clip 10.a: Let's find out. (Then child should hit whichever button corresponds to his guess. If he guessed right witch should appear and dance around while saying …)

That's right! The witch played with the marbles. She take marbles away.

Clip 10.b :(If child guessed wrong) (he should see princess fall over and hear)

No the princess did not play with the marbles and put more. The witch played with the marbles. She takes marbles away.

Phase 3:*Children Required to Respond*

CLIP 1: Now it's your turn. (Have the marbles go behind the screen, and then have the screen disappear and reveal the outcome (just the marbles, no princess or witch, say...).

Clip2: who do you think was playing with marbles? (Have the character whose button the child hit appear. If the child was correct, have the character dance around and say…)

2.a:"you got it right!! it was the princess who put in more marbles.

2.b:"you got it right!! it was the witch who took away some marbles.

(If the child was wrong, have the character fall over and the other character appear, and say….)

2.c:"No, it wasn't the witch, it was the princess, who put in more marbles"

2.d:"No, it wasn't the princess, it was the witch, who took away some marbles"

Phase 4: *Test Trials with abbreviated feedback*

(After the screen rises: *say nothing)*

Positive feedback: (Randomly intermix these feedback words across the trials). (For each animate a different dance by Princess/ Witch)

Clip 1: Right,

Clip2: you got it right,

Clip3: Hooray

Negative Feedback: (Randomly intermix these feedback words across the trials). (For each animate a different dance by Princess/ Witch)

Clip 1: “No, not her”,

Clip 2:“ Sorry that’s wrong”

Clip 3: “Ooops..no”

B. Script for Brightness Comparison task

Phase 1-*First set of training trial without Occluder*

Clip1: Let's play a game; in this game there is a bad witch and good princess, and they both like to play with a blob.

CLIP 2: Look, here is a good princess

Clip 3: Look, here is a blob that can turn into circle. See, look how it turns into a circle?

Clip 4: Look, now the circle got lighter! I wonder why that happened.

Clip 5: I think the princess must have made the circle lighter. Let's see if she did. (then the experimenter should hit the princess response button, and princess should dance around)

Clip 6: Yes it was the princess.

CLIP 7: Look, here is the bad witch.

Clip 8: Look, here is a blob that can turn into circle. See, look how it turns into a circle?

Clip 9: Look, now the circle got darker! I wonder why that happened.

Clip 10: I think the witch must have made the circle darker. Let's see if she did. (then experimenter should hit the witch button, and witch should dance around)

Clip 11: Yes, it was the witch.

Phase 2: ***Training trials with Occuluder***

CLIP 1: Now the bad witch and good princess are playing with the blob behind a screen.

Clip 2: See, look, here is a screen. A blob will turn into a circle and then go behind the screen. Then either the bad witch or good princess will play with the circle and then we will see, what happened.

CLIP 3: Look, here is a blob that turns into a circle and then go behind the screen.

Clip 4: What happened? (Let the child respond, then give feedback saying).

Clip 5.a: That's right, the circle got lighter. Who do you think played with the circle: the princess or the witch?

Clip 5.b:(If child guessed wrong) No, the circle got lighter than before. Who do you think played with the circle: the princess or the witch?

Clip 6: Let's find out. (Then child should hit whichever button corresponds to his guess. If he guessed right princess should appear and dance around while saying …)

That's right! The princess played with the circle. She makes the circle lighter.

Clip 6.b:(If child guessed wrong) (he should see witch fall over and hear)

No the witch did not play with the circle and make it darker. The princess played with the circle. She makes the circle lighter.

CLIP 7: Look, here is a blob that turns into a circle and then go behind the screen.

Clip 8: What happened? (Let the child respond, then give feedback saying).

Clip 9.a: That’s right, the circle got darker. Who do you think played with the circle: the princess or the witch?

Clip 9.b:(If child guessed wrong) No, the circle got darker than before. Who do you think played with the circle: the princess or the witch?

Clip 10.a: Let’s find out. (Then child should hit whichever button corresponds to his guess. If he guessed right witch should appear and dance around while saying …)

That’s right! The witch played with the circle. She makes the circle darker.

Clip 10.b :(If child guessed wrong) (he should see princess fall over and hear)

No the princess did not play with the circle and make it lighter. The witch played with the circle. She makes the circle darker.

<u>Phase 3:*Children Required to Respond*</u>

CLIP 1: Now it’s your turn. (Have the bar/ circle go behind the screen, and then have the screen disappear and reveal the outcome (just the circle, no princess or witch, say...).

Clip2: who do you think was playing with circle? (Have the character whose button the child hit appear. If the child was correct, have the character dance around and say…)

2.a:"you got it right!! it was the princess who made the circle lighter.

2.b:"you got it right!! it was the witch who made the circle darker.

(If the child was wrong, have the character fall over and the other character appear, and say….)

2.c:"No, it wasn't the witch, it was the princess, who made the circle lighter"

2.d:"No, it wasn't the princess, it was the witch, who made the circle darker"

Phase 4: *Test Trials with abbreviated feedback*

(After the screen rises: *say nothing)*

Positive feedback: (Randomly intermix these feedback words across the trials). (For each animate a different dance by Princess/ Witch)

Clip 1: Right,

Clip2: you got it right,

Clip3: Hooray

Negative Feedback: (Randomly intermix these feedback words across the trials). (For each animate a different dance by Princess/ Witch)

Clip 1: “No, not her”,

Clip 2:“ Sorry that’s wrong”

Clip 3: “Ooops..no”

Appendix G

Symbolic addition problem sets 1-4 for experiments 1, 3, 4 (phase1), and experiment 1 (Phase 2).

Set 1. Solve the problems by adding

12 **+ 3**	**14** **+ 2**
9 **+ 3**	**11** **+ 4**
8 **+ 6**	**7** **+ 4**
6 **+ 5**	**13** **+ 3**
7 **+ 7**	**9** **+ 6**

Set 2. Solve the problems by adding

16 + 3	17 + 3
15 + 5	15 + 3
8 + 8	12 + 8
9 + 7	13 + 6
9 + 8	15 + 6

Set 3. Solve the problems by adding

18 **+ 4**	**19** **+ 6**
15 **+ 9**	**17** **+ 5**
19 **+ 9**	**16** **+ 14**
17 **+ 13**	**15** **+ 12**
16 **+ 8**	**14** **+ 14**

Set 4. Solve the problems by adding

20 + 15	17 + 14

20
+ 15

17
+ 14

18
+ 16

19
+ 18

17
+ 17

37
+ 28

46
+ 38

58
+ 23

25
+ 13

64
+ 36

Appendix-H

Symbolic addition problems sets 1-2 used for experiment 2 (Phase1).

Set 1. Solve the problems by adding

6 + 5	14 + 8
8 + 6	13 + 6
15 + 6	12 + 7
14 + 14	13 + 11
46 + 38	878 + 47

Set 2. Solve the problems by adding

8 **+ 8**	**19** **+ 6**
9 **+ 6**	**16** **+ 8**
16 **+ 14**	**15** **+ 12**
17 **+ 17**	**19** **+ 18**
77 **+ 65**	**987** **+ 79**

Appendix-I

Sentence completion problem sets 1-2 for experiment 2 (Phase 1).

Set 1. Please fill in the blanks by completing the one word

1. Planes land at the A___________.
2. Bees live together in a H____________.
3. A square has four corners and four S_____________
4. We keep our pants around our waist with a B_____________.
5. The birds all gathered together in a single F__________.
6. To enter a movie theater you first need to purchase a T________.
7. To draw straight lines you need a pencil and a R____________.
8. The Porcupine has defensive Q____________.
9. Poisonous mushrooms are bad to eat because they are T_______.
10. Lavender is a shade of P_______________.

Set 2. Please fill in the blanks by completing the one word

1. A blizzard is a very large snow S____________.
2. Animals will starve if they cannot find any F____________.
3. Pirates use maps to look for buried T____________.
4. If an animal has a fatal disease, it will D___________.
5. No matter how hard Jenny tried to find her missing lunch box, it was impossible to L____________.
6. A chameleon uses camouflage to blend in with its environment by changing its skin C___________.
7. Jimmy limped because he had injured his F___________.
8. Animals that are raised in captivity live in C_____________.
9. The brave soldier who never won battles and never gave up was V____________.
10. Amy felt so fatigued that she went to S____________.

Appendix-J

Approval forms of Committee on Use of Human Subjects in research (CUHS) Harvard University USA, Federal Directorate of Education (FDE) Pakistan and National Institute of Psychology (NIP) at Quaid-i-Azam University, Islamabad, Pakistan

HARVARD UNIVERSITY
COMMITTEE ON THE USE OF HUMAN SUBJECTS IN RESEARCH

Federal Wide Assurance (FWA) 00004837
IRB Identification # 00000109

JAMES C. BECK
Chair

RACHEL KREBS
Research Officer

ROOM 248
1414 Massachusetts Avenue
CAMBRIDGE, MASSACHUSETTS 02138
617-496-1185

Application Number: F12152-148

REPORT OF COMMITTEE ACTION

Investigator: Elizabeth Spelke and Susan Carey
Project Title: Sources of mathematical thinking

Funding Source: NSF

ACTION TAKEN: Approved as amended

TYPE OF REVIEW: Expedited   Review Date: 12/17/2010

Conditions, comments, etc.:

The approval covers amendment of the protocol to add Dr Saeeda Khanum's study.

Period of approval begins 12/17/2010 and expires 3/27/2011

IMPORTANT:

1. The investigator **must** submit a **Study Closing Form** when the project is complete.
2. If the project will extend beyond approval period (including the continuing use of identifiable data or identifiable human materials), a **Renewal Application must be submitted by:** 2/13/2011
3. Study Closing and Renewal Application Forms are available at http://cuhs.harvard.edu.
4. Please see additional conditions for which you are responsible on next page ...

INVESTIGATORS ARE RESPONSIBLE FOR THE FOLLOWING:

1. Procedural changes or amendments must be reported to the Committee in advance. No changes may be made without Committee approval except to eliminate apparent immediate hazards to the subject. Minor changes may be approvable by expedited review; major changes may require action at an assembled Committee meeting.

2. Continuation of subject participation beyond the approval period requires renewal of approval by separate application. It is the investigator's responsibility to submit renewal requests in a timely fashion.

3. Should there be reason to think that a subject is suffering or has suffered any harm, anticipated or not, as a result of participation, the investigator must suspend the research and report to the Committee. The research shall not resume without Committee approval.

4. Expedited approvals are granted with the understanding that the Committee may impose additional conditions after review at a convened meeting.

5. Approval confirms that the project as proposed is not in conflict with the Committee's rules and regulations, but it does not imply endorsement or sponsorship by the University. Although investigators may indicate their position at Harvard, they shall not represent that the research is sponsored by the University or a department within the University except by explicit arrangement with appropriate administrative authorities.

for the Committee,

Rachel Krebs
Research Officer

Date: 12/17/2010

cc: Saeeda Khanum
Mccaila Ingold-Smith

HARVARD UNIVERSITY
COMMITTEE ON THE USE OF HUMAN SUBJECTS IN RESEARCH

Federal Wide Assurance (FWA) 00004837
IRB Identification # 00000109

JAMES C. BECK
Chair

RACHEL KREBS
Research Officer

ROOM 248
1414 Massachusetts Avenue
CAMBRIDGE, MASSACHUSETTS 02138
617-496-1185

REPORT OF COMMITTEE ACTION

Application Number: F12152-150

Investigator: Elizabeth Spelke and Susan Carey
Project Title: Sources of mathematical thinking

Funding Source: NSF

ACTION TAKEN: Approved as amended

TYPE OF REVIEW: Expedited   Review Date: 4/21/2011

Conditions, comments, etc.:

The approval covers modification of the protocol to revise the post-test task of Saeeda Khanum's study.

Period of approval begins 4/21/2011 and expires 3/27/2012

IMPORTANT:

1. The investigator **must** submit a **Study Closing Form** when the project is complete.
2. If the project will extend beyond approval period (including the continuing use of identifiable data or identifiable human materials), a **Renewal Application must be submitted by:** 2/14/2012
3. Study Closing and Renewal Application Forms are available at http://cuhs.harvard.edu.
4. Please see additional conditions for which you are responsible on next page ...

INVESTIGATORS ARE RESPONSIBLE FOR THE FOLLOWING:

1. Procedural changes or amendments must be reported to the Committee in advance. No changes may be made without Committee approval except to eliminate apparent immediate hazards to the subject. Minor changes may be approvable by expedited review; major changes may require action at an assembled Committee meeting.

2. Continuation of subject participation beyond the approval period requires renewal of approval by separate application. It is the investigator's responsibility to submit renewal requests in a timely fashion.

3. Should there be reason to think that a subject is suffering or has suffered any harm, anticipated or not, as a result of participation, the investigator must suspend the research and report to the Committee. The research shall not resume without Committee approval.

4. Expedited approvals are granted with the understanding that the Committee may impose additional conditions after review at a convened meeting.

5. Approval confirms that the project as proposed is not in conflict with the Committee's rules and regulations, but it does not imply endorsement or sponsorship by the University. Although investigators may indicate their position at Harvard, they shall not represent that the research is sponsored by the University or a department within the University except by explicit arrangement with appropriate administrative authorities.

for the Committee,

Rachel Krebs
Research Officer

Date: 4/21/2011

cc: Konika Banerjee
Saeeda Khanum

HARVARD UNIVERSITY
COMMITTEE ON THE USE OF HUMAN SUBJECTS IN RESEARCH

Federal Wide Assurance (FWA) 00004837
IRB Identification # 00000109

MATTHEW NOCK
Chair

EMIKO SAITO
Research Officer

ROOM 231
1414 Massachusetts Avenue
CAMBRIDGE, MASSACHUSETTS 02138
617-496-2618

Application Number: F12152-152

REPORT OF COMMITTEE ACTION

Investigator: Elizabeth Spelke and Susan Carey
Project Title: Sources of mathematical thinking

Funding Source: NSF

ACTION TAKEN: Approved as amended

TYPE OF REVIEW: Expedited Review Date: 12/14/2011

Conditions, comments, etc.:

This modification adds a follow-up session to F15152-150 and adds ERP recordings.

Period of approval begins 12/14/2011 and expires 3/27/2012

IMPORTANT:

1. The investigator **must** submit a **Study Closing Form** when the project is complete.
2. If the project will extend beyond approval period (including the continuing use of identifiable data or identifiable human materials), a **Renewal Application must be submitted by:** 1/27/2012
3. Study Closing and Renewal Application Forms are available at http://cuhs.harvard.edu.
4. Please see additional conditions for which you are responsible on next page ...

INVESTIGATORS ARE RESPONSIBLE FOR THE FOLLOWING:

1. Procedural changes or amendments must be reported to the Committee in advance. No changes may be made without Committee approval except to eliminate apparent immediate hazards to the subject. Minor changes may be approvable by expedited review; major changes may require action at an assembled Committee meeting.

2. Continuation of subject participation beyond the approval period requires renewal of approval by separate application. It is the investigator's responsibility to submit renewal requests in a timely fashion.

3. Should there be reason to think that a subject is suffering or has suffered any harm, anticipated or not, as a result of participation, the investigator must suspend the research and report to the Committee. The research shall not resume without Committee approval.

4. Expedited approvals are granted with the understanding that the Committee may impose additional conditions after review at a convened meeting.

5. Approval confirms that the project as proposed is not in conflict with the Committee's rules and regulations, but it does not imply endorsement or sponsorship by the University. Although investigators may indicate their position at Harvard, they shall not represent that the research is sponsored by the University or a department within the University except by explicit arrangement with appropriate administrative authorities.

for the Committee,

Emiko Saito
Research Officer

Date: 12/14/2011

cc:

Ellyn Schmidt
Saeeda Khanum

HARVARD UNIVERSITY
COMMITTEE ON THE USE OF HUMAN SUBJECTS IN RESEARCH

Federal Wide Assurance (FWA) 00004837
IRB Identification # 00000109

MATTHEW NOCK
Chair

EMIKO SAITO
Research Officer

ROOM 231
1414 Massachusetts Avenue
CAMBRIDGE, MASSACHUSETTS 02138
617-496-2618

Application Number: F12152-154

REPORT OF COMMITTEE ACTION

Investigator: Elizabeth Spelke and Susan Carey
Project Title: Sources of mathematical thinking

Funding Source: NSF

ACTION TAKEN: Approved as submitted

TYPE OF REVIEW: Expedited   Review Date: 2/3/2012

Conditions, comments, etc.:

This modification, from, Saeeda Khanum, adds a new condition where children would be asked to add non-symbolic quantities in the form of lines, instead of dots.

Period of approval begins 2/3/2012 and expires 3/27/2012

IMPORTANT:

1. The investigator **must** submit a **Study Closing Form** when the project is complete.
2. If the project will extend beyond approval period (including the continuing use of identifiable data or identifiable human materials), a **Renewal Application must be submitted by:** 1/27/2012
3. Study Closing and Renewal Application Forms are available at http://cuhs.harvard.edu.
4. Please see additional conditions for which you are responsible on next page ...

INVESTIGATORS ARE RESPONSIBLE FOR THE FOLLOWING:

1. Procedural changes or amendments must be reported to the Committee in advance. No changes may be made without Committee approval except to eliminate apparent immediate hazards to the subject. Minor changes may be approvable by expedited review; major changes may require action at an assembled Committee meeting.

2. Continuation of subject participation beyond the approval period requires renewal of approval by separate application. It is the investigator's responsibility to submit renewal requests in a timely fashion.

3. Should there be reason to think that a subject is suffering or has suffered any harm, anticipated or not, as a result of participation, the investigator must suspend the research and report to the Committee. The research shall not resume without Committee approval.

4. Expedited approvals are granted with the understanding that the Committee may impose additional conditions after review at a convened meeting.

5. Approval confirms that the project as proposed is not in conflict with the Committee's rules and regulations, but it does not imply endorsement or sponsorship by the University. Although investigators may indicate their position at Harvard, they shall not represent that the research is sponsored by the University or a department within the University except by explicit arrangement with appropriate administrative authorities.

for the Committee,

Emiko Saito
Research Officer

Date: 2/3/2012

cc:

Ellyn Schmidt
Saeeda Khanum