In the Name of Allah, The Most beneficiary,
The Most Gracious, The Most Merciful

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Theoretical investigation of unsteady forced bio-convection slip flow of an exponentially stretching sheet.

By

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Preface

The transfer of heat is the elementary aspect of the great machine made applications. The heat transfer depends upon the thermal conductivity of the working fluids as compared to the capability of thermal devices and systems. In conventional heat transfer, transfer of heat in the fluids are obtained by liquefying nanoparticles which are nanometer sized particles (1 - 100) nm. The structure of nanofluid particles is formed by nitride ceramics, metals, and semi conductor. Nanofluid is found to be a good medium for thermal conductivity. In the base fluids, the existence of the nanoparticles increases the efficiency and thermal conductivity of the fluid.

An innovative technique for improving heat transfer by using ultra fine solid particles in the fluids has been used extensively during the last several years, and one of it, is nanofluid. The term nanofluid was used by Choi [1]. Das et al. [2] exposed sundary applications of nanofluids. The application of nanofluid in a convective boundary layer flow were studied by Buongiorno [3]. Brownian diffusion and the thermophoresis described the behavior of nanofluids. In Buongiorno model, we studied that the velocity of flow is the sum of the base fluid velocity to the slip velocity. Wang and Fan [4] studied that nanofluid have four scales which are the molecular, the micro, the macro, and the megascale. Zhang et al. [5] analyzed the MHD radiative flow of a nanofluid past a surface that have variable heat flux and chemical reaction. Haq et al. [6] described the stagnation point flow of a nanofluid past a stretching sheet with slip effects and thermal radiation. Makinde and Aziz [7] observed the convective boundary condition with boundary layer flow of nanofluid over a stretching sheet. Nadeem and Lee [8] analyzed the boundary layer flow past a stretching sheet in the presence of nanofluid. Nazar et al. [9] investigated the unsteady boundary layer flow of a nanofluid past a stretching sheet induced by impulsive motion. Das et al. [10] investigated entropy analysis of unsteady magneto hydrodynamics nanofluid flow past an accelerating stretching sheet with convective boundary condition. Nadeem and Haq [11] elaborated the effect of thermal radiation for magnetohydrodynamics (MHD) boundary layer flow of a nanofluid over a stretching sheet with convective boundary conditions. Nadeem and Lee [12] presented the steady boundary layer flow of nanofluid over an exponential stretching surface. Turkyilmazoglu [13] illustrated the heat and mass transfer analysis for MHD flow of viscous nanofluid with slip effect. He analyzed the closed form solutions of velocity, temperature, and concentration profiles. Ramzan et al. [14] studied unsteady second grade MHD flow of nanofluid induced by vertical sheet with thermal radiation and mixed convection. Ramzan and Ashraf [15] analyzed three-dimensional flow of an elastico-viscous nanofluid with chemical reaction and magnetic field effects.

Micropolar fluids are fluids with microstructures. They belong to the nonsymmetric stress tensor. Micropolar fluid consist of rigid randomly oriented or spherical particles. They have their own spins and microrotations, suspended in viscous medium. Miccropolar fluids are polar fluids, which have some microsize effects such as rotation and shrinking etc. Physical examples of micropolar flows are, blood flow, bubby liquids, liquid crystals and so on. A latest discovery for micropolar fluid is to combine nanofluid with bioconvection development was studied by Xu and Pop [16]. Aziz et al. [17] studied theoretically the natural bio - convection boundary layer flow of a nanofluid. Agarwal et al. [18] investigated the solution of finite element flow and transfer of heat of a micropolar fluid past a stretching sheet. Hassanian and Gorla et al. [19] and few other explored the steady boundary layer flow of a micropolar flow drive by permeable and non-permeable sheets. Eringen [20] proposed the theory of a micropolar fluids. El - Aziz et al. [21] investigated that the micropolar fluids can preserve rotation with individual motion. They support stress and body moments are effects by spin inertia. Nadeem et al. [22] studied axisymmetric stagnation flow of a micropolar nanofluid in a moving cylinder. Balram and Sastry [23] studied free convection flow of a micropolar fluids in a parallel plate vertical channel. Lok et al. [24] carried out the steady two-dimensional asymmetric stagnation point flow of a micropolar fluid

Bio-convection arrangement are mostly occur due to the upswimming of microorganism (very small living organism such as bacteria that are visible under a microscope) that are a little dense. By the upswimming the upper portion of the fluid layers of suspensions become to dense. Due to the gathering of microorganisms it become unstable. The microorganisms fall down to cause bio - convection. In geophysical phenomena thermo - bio - convection plays an important role e.g. the phenomena thermophiles in which hot springs immigrate by motile microorganism i.e. heat loving microorganisms. Another application is in the field of microbial increased oil recovery. In oil-bearing, layers are added microorganisms and nutrients to arrange permeability deviation. Nield and Kuznetsov [25] reported that microorganisms may contribute toward the development in bio-micro-systems, they show a significant aspect in mixing and increase in mass movement. Uddin et al. [26] recently studied bio-convection flow of a nanofluid past a moving plate in the presence of stefan blowing and influence of multiple slip. Uddin et al. [27] analyzed the MHD free convective boundary layer flow of Newtonian heating boundary condition with nanofluid over a flat vertical plate. Khan et al. [28] analyzed the boundary layer flow of a nanofluid consist of gyrotactic microorganisms over a vertical plate with magnetic field. Kuznetsov [29] is reported the onset of thermo - bio - convection in a shallow fluid, saturated porous layer heated from below in a suspension of oxytactic microorgansims. Zaimi et al. [30] studied the stagnation flow of a nanofluid over a stretching sheet in the existence of gyrotactic microorganisms. Beg et al. [31] investigated boundary layer bio - convective non - Newtonian nanofluid flow from a horizontal flat plate in a porous medium. Kuznetsov [32] studied the interaction of oxytactic microorganisms in a shallow horizontal layer of finite depth. Mandal and Mukhopadhyay [33] carried out the heat transfer effects on boundary layer flow embedded in a porous medium with variable heat flux. Here the flow generation is caused due to an exponential stretching of sheet. It appears that Wang [34] indicated an exact similarity solution for the steady three-dimensional flow of a viscous and incompressible fluid due to a stretching flat surface. Pop and Merkin[35] analyzed an unsteady three-dimensional free convection flow near a general stagnation point placed in a fluid-saturated porous medium. However, only one few studies are available in the literature which discussed a three-dimensional boundary-layer flow over an exponentially stretching surface. Suction/injection (blowing) of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction coefficient, whereas injection acts in the opposite manner. The mechanism of suction has also importance in several engineering activities such as in the design of thrust bearing, thermal oil recovery, and radial diffusers [36]. The bio-convection parameters have significant effects on flow, mass and heat transfer, and motile microorganism density number

Contents of this dissertation consist of three chapters. Chapter first is related to some basic and conceptual definitions of the fluid, basic governing equations and laws. In chapter two we have reviewed the paper of "Nur Amalina Abd. Latiff" i.e. "Unsteady forced bioconvection slip flow of a micropolar nanofluid from a stretching/shrinking sheet". Convergent series solutions are developed via homotopy analysis method. The results for velocity, micro rotation, temperature, concentration, and microorganism are established and analyzed. Chapter three is my extension work in which we have investigated the unsteady forced bio-convection slip flow of an exponentially stretching sheet. The analysis is carried out for the three dimensional unsteady, laminar, and an incompressible flow of a micropolar nanofluid. The governing partial differential equations of the flow are converted into non-linear ordinary differential equations using similarity transformation. These equations are solved numerically by using shooting/bvp4c method. The effects of different parameters are studied graphically. Further, the graphical behavior of skin friction, Nusselt number, microorganism flux is presented.

This thesis is dedicated to the ideal personalities of my life,

My Parents

Thank's for your love, encouragement and constant support while i was far away from home during my studies.

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MUHAMMAD NAVEED KHAN

Contents

Chapter 1

Basic definitions

This chapter consists of some fundamental definitions, concepts, and laws, which are helpful in understanding of analysis presented in the successive chapters.

1.1 Fluid

Fluid is a substance or material which cannot bear a shear force under the static condition. No matter how small stress applied on a substance. Also fluid is a material which deform constantly due to shear stress.

1.2 Flow

A substance or material goes under deformation in the presence of different forces. The phenomenon in which the deformation exceeds continuously without limit then it is called flow.

1.3 Fluid mechanics

It is that category of applied mechanics that concerns with the behavior of the fluids in motion or in rest. Thus this branch of science deals with static, kinematic, and dynamic aspects of fluids. It has different branches.

1.3.1 Fluid statics

In this branch of fluid mechanics, we study the fluids at rest.

1.3.2 Fluid dynamic

In this branch of fluid mechanics, we deals the behavior of fluids in motion when the pressure forces are considered.

1.3.3 Fluid kinematic

In this branch of fluid mechanics, we deals the behavior of fluids in motion when the pressure forces are not considered.

1.4 Viscosity

It is the internal property of fluid that occurs due to the motion of inter connected layers of fluid. It resist of a fluid to motion when different forces acting upon it. Mathematically, we can write it as,

$$\text{Viscosity}\,(\mu) = \frac{\text{Shear stress}}{\text{Shear strain rate}}. \tag{1.1}$$

The viscosity unit is $kg/m.s$ and dimension is $\left[\frac{M}{LT}\right]$.

1.4.1 Kinematic viscosity (υ)

The ratio between dynamic viscosity μ to the fluid density ρ is said to be kinematic viscosity of fluid. Mathematically, we can write it as,

$$\upsilon = \frac{\mu}{\rho}. \tag{1.2}$$

The unit of kinematic viscosity in SI system is m^2/s and its dimension $\left[\frac{L^2}{T}\right]$.

1.5 Stress

The force which deform the body when the force acting upon the unit surface area is called stress. The SI unit is $kg/m.s^2$ and its dimension $\left[\frac{M}{LT^2}\right]$. Further stress has two types shear stress (the forces acting parallel to the unit surface area is called shear stress) and normal stress (on a unit surface area the forces acting vertically to the unit surface area is called normal stress).

1.6 Strain

It is the deformation of an object caused by the applied forces. Strain is a dimensionless quantity.

1.7 Flow

A substance or material goes under deformation in the present of different forces. The phenomenon in which the deformation exceeds continuously without any limit then it is called flow.

1.7.1 Laminar flow

In such type of flows, the fluid particles possess definite path during flow and they do not disturb each others. If we observe the smoke rising from a cigarette. For the first few centimeters the flow is certainly laminar and after a short interval of time the smoke becomes turbulent.

1.7.2 Turbulent flow

In turbulent flow particles of fluid overlap each other and posses random motion with out any specific paths. The turbulence occurred due to convection can be understood by considering that we heat up water in a pot through an electric cooker. Now let wait for a few mints, we will see that from the bottom of water bubbles are twitch to rise to the surface due to which the motion of water molecules become very difficult or complicated or in simple words it is turbulent. This is occurred through due to convection.

1.7.3 Compressible flow

A flow in which fluid density varies throughout the flow. Mathematically,

$$\rho = \rho(x, y, z, t). \tag{1.3}$$

1.7.4 Incompressible flow

The density of the fluid remains unchanged during the fluid motion then the flow is known as incompressible. Mathematically,

$$\rho \neq \rho(x, y, z, t) \text{ or } \rho = \text{Constant},$$

or

$$\frac{d\rho}{dt} = 0. \tag{1.4}$$

where $\frac{d}{dt}$ represents the total derivative (i.e., the sum of local and convective derivatives).

1.7.5 Steady flow

The flow field in which the fluid properties do not depends up on time is called steady flow. This flow should be uniform or non uniform. Mathematically,.

$$\frac{\partial \xi}{\partial t} = 0, \tag{1.5}$$

here ξ denotes any property of fluid (i.e. velocity, density etc.).

1.7.6 Unsteady flow

The flow field in which the fluid properties depends upon the time at every point, such type of flow is called unsteady flow. Mathematically,

$$\frac{\partial \xi}{\partial t} \neq 0, \tag{1.6}$$

where ξ denotes any fluid property (i.e. density, velocity etc.).

1.8 Fluids Classification

1.8.1 Ideal fluids

The viscosity of fluids, near to be zero are known as ideal fluid. Ideal fluid does not exist physically.

1.8.2 Real fluids

Fluids with non-zero viscosity i.e., $\mu \neq 0$ are known as real fluids or viscous fluids. Subcategorized the real fluids into Newtonian and non-Newtonian.

1.8.3 Newtonian fluids

The fluids that obey Newton's law of viscosity are called Newtonian fluids i.e. their is a linearly proportionality between shear stress and deformation rate. Keep in mind that viscosity is constant for each Newtonian fluid at a given temperature and pressure. Examples of Newtonian fluids are air, water, sugar solutions, glycerin, light-hydrocarbon oils, Kerosene etc. Mathematically,

$$\boldsymbol{\tau}_{xy} = \mu \frac{dv}{dy}. \tag{1.7}$$

1.8.4 Non-Newtonian fluids

Non-Newtonian fluids are categorized as those fluids which do not obey Newton's law of viscosity. Remember that viscosity for such fluids changed but it varies with the application of shear stress. For such fluids we can write

$$\boldsymbol{\tau}_{yx} \propto \left(\frac{dv}{dy}\right)^n, \ n \neq 1, \tag{1.8}$$

or

$$\boldsymbol{\tau}_{yx} = \eta \frac{dv}{dy}, \ \eta = k \left(\frac{dv}{dy}\right)^{n-1}. \tag{1.9}$$

where η, k, and n is the viscosity, consistency index, and index of the behavior of a flow respectively. Above equation become to the Newton's law of viscosity for $n = 1$. Types of such fluids are paints, flour dough, ketchup, polymer solutions, tooth paste and blood etc.

1.9 Heat flow Mechanism

We know that the heat transfers from one system to another system due to temperature difference between two systems. Here we discuss that transfer of heat takes place by three ways.

1.9.1 Conduction

The transfer of heat from one object to another object due to the only molecules collision which are in contact and not due to the transfer of molecules is called conduction. Mathematically, we write

$$q = -kA(\frac{\Delta T}{\Delta \eta}). \tag{1.10}$$

here the temperature gradient is $\frac{\Delta T}{\Delta \eta}$ along the direction of area A, and thermal conductivity constant is k. Its unit is $W/m.K$.

1.9.2 Convection

If the energy is transfer between two systems due to the motion of fluid then it is called convection. Mathematically,

$$q = \xi A(T_f - T_0). \tag{1.11}$$

Here ξ represent the coefficient of convective heat transfer, A represent area of heat transfer process, T_f is regarded as system temperature and T_0 denote the reference temperature.

1.9.3 Natural convection

This type of heat transfer occurs by the temperature differences, that effect the density and thus buoyancy of the fluid. Natural convection can only occur, when there is gravitational field. It is also known as free convection.

1.9.4 Forced convection

The appliancy of transfer of heat, in which motion of fluid is caused by an extraneous source like a fan and pump etc. Forced convection usually used to enhance the exchange rate of heat.

1.9.5 Mixed convection

Mixed convection flow take place when both forced and natural convection processes simultaneously and significantly contribute to heat transfer.

1.9.6 Radiation

It is that process, which heat is transferred directly by electromagnetic radiations due to involvement of waves and particles, i.e. waves and photons. As a result, the radiative heat transfer can flow through vacuum. The exchange of heat between objects occurs due to radiation. Heat from the sun is the best example of radiation.

In gases and liquids, heat is transfer through radiation and convection but in solids radiation is almost negligible and convection does not occurs. Thus for solid materials conduction play major role in the transfer of heat. Mathematically,

$$q = \epsilon \sigma_{SB} \bar{A} \left(\Delta T\right)^4 . \tag{1.12}$$

Here q, ϵ, σ_{SB} is radiative heat transfer, emissivity of the system, and Stephan-Boltzmann constant respectively, the value of σ_{SB} is $5.6697 \times 10^{-8} W m^{-2} K^{-4}$, the area convoluted in the radiative heat transferred is $\bar{A}$ the fourth or higher power difference between two systems is the temperature of $(\Delta T)^4$.

1.10 Boundary layer

The layer adjacent to the solid surface, where the viscosity effects are dominant is termed as boundary layer. In determining the flow field, the viscous effects are considering into account, which have significant role on fluid motion. This approach helps us to reduce the equations.

1.11 Thermal conductivity

It is the heat conducting capability of a material. The thermal conductivity of a substance may depend on temperature. Mathematically, it is evaluated in terms of Fourier's law for heat conduction and defined as follows

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}. \tag{1.13}$$

here $\frac{dQ}{dt}$ is the rate of heat transfer, k is the thermal conductivity, and A is the area.

Now we define the thermal conductivity from the Fourier's law as " transfer of heat through unit thickness of a material is per unit area and temperature difference".

The thermal conductivity unit and dimension is $kg.m/s^3.K$ and $\left[\frac{ML}{T^3\theta}\right]$ respectively.

1.12 Thermal diffusivity

We define the thermal diffusivity of a material as, "thermal conductivity of a material divided by the product of density and specific heat at constant pressure".

Mathematically, it can be expressed as

$$\alpha = \frac{k}{\rho c_p}. \tag{1.14}$$

The unit of thermal diffusivity in SI system is m^2/s and dimension is $\left[\frac{L^2}{T}\right]$.

1.13 Nanofluids

Nanofluids are fluids which are restricted in nanometer structure. These fluids are a combination of nanoparticles (including oxides, metal, carbides or carbon nanotubes) and base fluids (including ethylene, oil, glycol, water etc.). These fluids give higher thermal performance due to their higher thermal conductivity. Applications of nanofluids in specific areas are mention below,

(1) Environmental nanofluids.

(2) Chemical nanofluids.

(3) Medical and pharmaceutical nanofluids.

1.14 Micropolar fluids

The fluids that have a microstructure is called micropolar fluids. Micopolar fluids have nonsymmetric stress tensor. Micropolar fluids may depict fluids that have of rigid, randomly oriented particles which consist their own spin as well as micro-rotations, suspended in a viscous medium, also the fluid particles deformation is neglected. Physical examples can be seen in blood flow, bubbly liquid etc.

1.15 Bio-convection

Bio-convection phenomena is the motion of small organisms in a fluid, especially free swimming zooplankton in water. Bio-convection patterns usually analyzed in the laboratory in shallow suspensions of randomly, and average upwardly swimming microorganism which are denser than water.

1.16 Dimensionless numbers

1.16.1 Prandtl number

It is used to measures the ratio of viscous rate to thermal diffusivity rate. It is dimensionless quantity. Mathematically, it can be written as,

$$\mathrm{Pr} = \frac{\text{Viscous diffusion rate}}{\text{Thermal diffusion rate}}, \tag{1.16}$$

$$\mathrm{Pr} = \frac{\upsilon}{\alpha} = \frac{\mu c_p}{k}. \tag{1.17}$$

where υ is the kinematic viscosity or momentum diffusivity, α is the thermal diffusivity, c_p is the specific heat capacity with unit in SI system as $J/kg\ K$, and here k is the thermal conductivity having unit in SI system are W/m. Physical significance of Prandtl number is that, it gives the relative thickness of thermal boundary layer and velocity. For small Pr heat diffuses hurriedly as related to momentum.

1.16.2 Reynolds number

The Reynolds number used to check the behavior of fluid either it is laminar or turbulent. It expressed as Re. It is the ratio of inertial to viscous force. Inertial forces act upon all masses in a non-inertial frame of reference while viscous force is the internal resistance of a fluid to flow.

Mathematically, we have

$$\mathrm{Re} = \frac{\text{Inertial force}}{\text{Viscous force}},$$

$$\mathrm{Re} = \frac{\rho \upsilon^2/x}{\mu \upsilon/x^2}, \qquad \text{or} \qquad \mathrm{Re} = \frac{\rho \upsilon x}{\mu}. \tag{1.15}$$

Here υ is the velocity of fluid, x is the characteristic length and μ is the kinematic viscosity. When Reynolds number is small then viscous forces are dominant which characterize by the laminar flow while turbulent flow occurs at high Reynolds number due to the dominant inertial forces.

1.16.3 Nusselt number

Nusselt number is basically the dimensionless heat transfer coefficient that defines measure of the ratio of convective transfer of heat to conductive transfer of heat through the boundary. Mathematically,

$$Nu_x = \frac{\text{Convective heat transfer coefficient}}{\text{Conductive heat transfer coefficient}}, \tag{1.18}$$

Now heat transfer by convection is $(h\Delta T)$ and by conduction is $(k\Delta T/x)$. So Nusselt number becomes

$$Nu_x = \frac{h\Delta T}{k\Delta T/x} = \frac{hx}{k}. \tag{1.19}$$

Here h, x, and k are convective heat transfer, characteristic length, and thermal conductivity of the fluid respectively.

1.16.4 Skin friction

Certain amount of friction acts on the fluid when it transfers through a surface, such kind of friction is known as skin friction. Mathematically,

$$C_f = \frac{j_w}{\rho U^2/2}. \tag{1.20}$$

Where j_w, ρ, and U are the shear stress at the wall, the density, and free-stream velocity respectively.

1.17 Some basic equations

1.17.1 Equation continuity

The law of conservation mass is used to derived continuity equation. Mathematically, it can be written as

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\left(\rho \mathbf{V}\right) = 0, \tag{1.21}$$

here ρ is fluid density, $\mathbf{V}$ the velocity field and t is the time. If the fluid is an incompressible then the continuity equation for such fluids can be obtained as

$$\boldsymbol{\nabla} \cdot \mathbf{V} = 0. \tag{1.22}$$

1.17.2 Equation of motion

The law of conservation momentum state as,

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div} \boldsymbol{\tau} + \mathbf{b}, \tag{1.23}$$

here $\boldsymbol{\tau}$ is Cauchy stress tensor that is different for different fluid, $\mathbf{b}$ denotes the body force and the material derivative is represented by d/dt, material derivative defined as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \left(\boldsymbol{\nabla}.\mathbf{V}\right), \tag{1.24}$$

for viscous incompressible fluid the Cauchy stress tensor $\boldsymbol{\tau}$ can be defined as

$$\boldsymbol{\tau} = -\, p\mathbf{I} + \mu\mathbf{A}_1, \tag{1.25}$$

here pressure is represented by p, $\mathbf{I}$ is the identity tensor, μ denotes dynamic viscosity of the fluid and $\mathbf{A}_1$ represented the first Rivlin-Ericksen tensor. i.e.,

$$\mathbf{A}_1 = (\mathrm{grad}\,\mathbf{V}) + (\mathrm{grad}\,\mathbf{V})^{\bigstar}, \tag{1.26}$$

the $\bigstar$ in superscript is the representation of the matrix transpose. For two dimensional velocity field, the strain tensor can be defined as,

$$\mathbf{L} = \mathrm{grad}\,\mathbf{V} = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.27}$$

Substituting Eqs.(1.26) and (1.27) in Eq. (1.25), we get

$$\boldsymbol{\tau} = \begin{bmatrix} 2\mu u_x - p & \mu\left(v_x + u_y\right) & 0 \\ \mu\left(u_x + v_y\right) & 2\mu v_y - p & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{1.28}$$

1.17.3 Energy equation

The energy equation for nanofluid with viscous dissipation effects can be expressed as

$$(\rho C_p)_f \left(\frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}\right) T = k\boldsymbol{\nabla}^2 T + \boldsymbol{\tau}.\mathbf{L} + (\rho C_p)_s \left[D_B \boldsymbol{\nabla} C.\boldsymbol{\nabla} T + \frac{D_T}{T_m}\boldsymbol{\nabla} T.\boldsymbol{\nabla} T\right]. \tag{1.29}$$

where $(C_p)_f$, $(C_p)_s$, ρ_f, and ρ_s are represents the base fluid specific heat, the specific heat of material, base fluid density, and density of nanoparticles respectively, also thermal conductivity is k, the rate of strain tensor is denoted by $\mathbf{L}$, T is the temperature of the fluid, D_B represents the Brownian motion coefficient, thermophoretic diffusion coefficient is D_T, and T_m is the mean temperature.

Also, the dissipation term can be computed as

$$\boldsymbol{\tau}.\mathbf{L} = tr\left(\boldsymbol{\tau}\mathbf{L}\right) = \tau_{xx} + \tau_{xy}\left(u_y + v_x\right) + \tau_{yy}. \tag{1.30}$$

1.17.4 Concentration equation

The concentration equation for nanofluid can be defined as

$$\left(\frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}\right) C = D_B \boldsymbol{\nabla}^2 C + \frac{D_T}{T_m} \boldsymbol{\nabla}^2 T. \tag{1.31}$$

here C is the concentration of nanoparticles, D_B represents the Brownian motion coefficient, D_T denotes the coefficient of diffusivity and T_m is the mean temperature.

1.17.5 Micropolar equation

The generalization of momentum equations for micropolar fluid is defined as

$$\rho \frac{d\mathbf{V}}{dt} = -\boldsymbol{\nabla}\mathbf{p} + (\mu + \kappa)\boldsymbol{\nabla}^2 V + \kappa(\boldsymbol{\nabla} \times \mathbf{N}) + \mathbf{J} \times \mathbf{B}, \tag{1.32}$$

$$\rho j \frac{D\mathbf{N}}{Dt} = \gamma \nabla^2 \mathbf{N} - \kappa(2\mathbf{N} - \boldsymbol{\nabla} \times \mathbf{V}). \tag{1.33}$$

1.17.6 Microorganism equation

The microorganism equation can be defined as

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{j} = \mathbf{0}, \tag{1.34}$$

where

$$\begin{aligned} j &= n\mathbf{v} + n\widetilde{\mathbf{v}} - D_m \boldsymbol{\nabla}\mathbf{n}, \\ \widetilde{\mathbf{v}} &= \frac{bW_c}{\Delta C} \boldsymbol{\nabla}\mathbf{C}. \end{aligned} \tag{1.35}$$

here D_m diffusivity coefficient of microorganism.

1.18 Homotopy analysis method

We have solved the non-linear problems through homotopy analysis method (HAM), as suggested by Liao [22]. To the elementary concept of homotopy analysis method, we study the successive differential equation

$$N\left[\zeta\left(x\right)\right]=0, \tag{1.36}$$

here N is a operator which is non-linear, $\zeta\left(x\right)$ is the unknown function, where independent variable is x. The equation in zeroth-order form is

$$\left(1-p\right)\mathcal{L}\left[\widehat{\zeta}\left(x;p\right)-u_{0}\left(x\right)\right]=p\hbar N\left[\widehat{\zeta}\left(x;p\right)\right], \tag{1.37}$$

in above $\zeta_0\left(x\right)$ indicates the initial approximation, $\mathcal{L}$ is the auxiliary linear operator, $p\in[0,1]$ is an embedding parameter, $\hbar$ is an subsidiary parameter and $\widehat{\zeta}\left(x;p\right)$ is function of x & p which is unknown.

For $p=0$, $p=1$, respectively one has

$$\widehat{\zeta}\left(x;0\right)=\zeta_{0}\left(x\right), \qquad \& \qquad \widehat{\zeta}\left(x;1\right)=\zeta\left(x\right), \tag{1.38}$$

when range of p from 0 to 1, the solution of $\widehat{\zeta}\left(x;p\right)$ turn from initial $\zeta_0\left(x\right)$ to the final solution $\zeta\left(x\right)$. By using Taylor series one can write

$$\widehat{\zeta}\left(x;p\right)=\zeta_{0}\left(x\right)+\sum_{m=1}^{\infty}\zeta_{m}\left(x\right)p^{m}, \qquad \zeta_{m}\left(x\right)=\left.\frac{1}{m!}\frac{\partial^{m}\widehat{\zeta}\left(x;p\right)}{\partial p^{m}}\right|_{p=0}. \tag{1.39}$$

when $p=1$, we get

$$\zeta\left(x\right)=\zeta_{0}\left(x\right)+\sum_{m=1}^{\infty}\zeta_{m}\left(x\right), \tag{1.40}$$

To obtain m-th order deformation equation, differentiating m-time equation w.r.t p, dividing both side of equation by $m!$ and substituting $p=0$, we have

$$\mathcal{L}\left[\zeta_{m}\left(x\right)-\chi_{m}\zeta_{m-1}\left(x\right)\right]=\hbar\mathcal{R}_{m}\left(x\right), \tag{1.41}$$

$$\mathcal{R}_m(x) = \frac{1}{(m-1)!} \frac{\partial^m N\left[\widehat{\zeta}(x;p)\right]}{\partial p^m}\bigg|_{p=0}. \tag{1.42}$$

with

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}. \tag{1.43}$$

Chapter 2

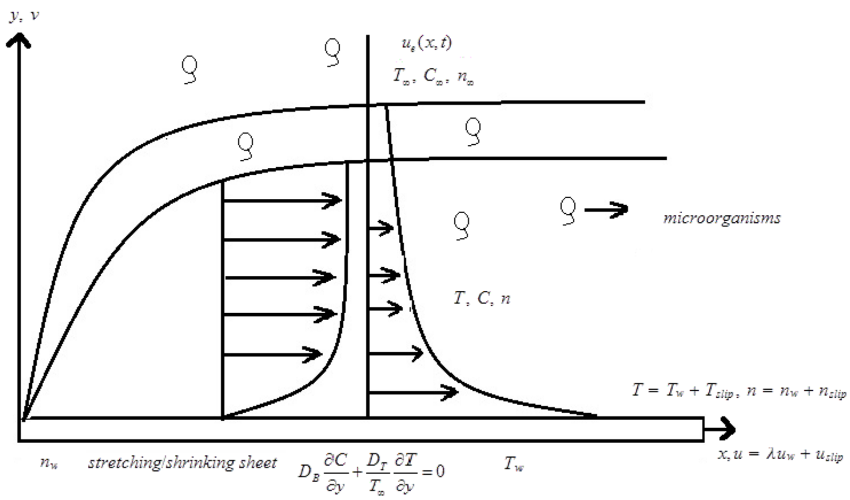
Unsteady forced bio-convection slip flow of a micropolar nanofluid from a stretching/shrinking sheet

2.1 Introduction

The analytical reasoning of an unsteady forced bio-convection viscous micropolar nanofluid past a stretching/shrinking sheet is incorporated. The analysis is performed in the presence of micro-organisms. By aid of convenient transformations to transfer the partial differential equations in the set of coupled non-linear ordinary differential equations. These equations are tackled analytically by adopting homotopy analysis method. The results of the determining parameters for the dimensionless velocity, temperature, micro-rotation, nanoparticle volume fraction as well as microorganism are investigated. Now delicately presented the results of a coefficient of local skin friction, the rate of heat and the rate of microorganism transfer graphically. The results shows that the value of Nusselt number and the skin friction are declined, while the value of microorganism number is rises, also the slip parameters for the velocity, thermal energy and microorganism are increased consequentially. The conclusions relate to past papers, and shows that the results are quite similar to each other. The present investigation are important in improving achievement of microbial fuel cells.

Homotopy analysis method was introduced by Liao (1995) and has been used for highly non-linear equations. The influence of different parameters such as, δ_ν, δ_T, δ_n, Δ and λ are investigated graphically, other parameters remain constant for each graph. Further, the graphical behavior of Nusselt number and skin friction is analyzed.

2.2 Mathematical formulation



Figure(2.1) : Physical model for idealized flow problem.

Consider two dimensional unsteady viscous flow containing microorganisms in a micropolar nanofluid over a static, vertical and non porous stretching/shrinking sheet. The geometry of the problem is represented in the Fig (2.1) in which the velocity of boundary layer is denoted by (i) and nanoparticle volume fraction, temperature, and microorganisms are denoted by (ii). The velocity field u and v in the directions of x and y respectively. Inside the boundary layer T represents the fluid temperature, C denotes the nanoparticle volume fraction, and n is the density of motile micro-organisms, whereas T_w, C_w, n_w represented the fluid temperature, the nanoparticle volume friction, and the density of motile micro-organisms at the wall respectively, while away from the wall they are denoted by T_∞, C_∞, n_∞ respectively. The surface of plate having multiple slip and zero mass flux conditions. By considering above supposition the required equation of the motion, energy, nanoparticle and microorganisms in the form,

$$\boldsymbol{\nabla}\cdot\mathbf{V}=0, \tag{2.1}$$

$$\rho\frac{d\mathbf{V}}{dt}=\operatorname{div}\boldsymbol{\tau}+\rho\mathbf{b}, \tag{2.2}$$

$$\boldsymbol{\tau}=-p\mathbf{I}+\mu\mathbf{A}_1. \tag{2.3}$$

The identity tensor is expressed by $\mathbf{I}$, p is pressure, the dynamic viscosity is μ, the first Rivlin-Ericksen tensor is $\mathbf{A}_1$. Mathematically $\mathbf{A}_1$ is expressed as

$$\mathbf{A}_1=\boldsymbol{\nabla}\cdot\mathbf{V}+(\boldsymbol{\nabla}\cdot\mathbf{V})^T. \tag{2.4}$$

Therefore after simplification, we get required momentum equation in the form of partial differentional equation along with micropolar defined as,

$$\rho\frac{D\mathbf{V}}{Dt}=-\boldsymbol{\nabla}\mathbf{p}+(\mu+\kappa)\boldsymbol{\nabla}^2\mathbf{V}+\kappa(\boldsymbol{\nabla}\times\mathbf{N}), \tag{2.5}$$

$$\rho j\frac{D\mathbf{N}}{Dt}=\gamma\boldsymbol{\nabla}^2\mathbf{N}-\kappa(2\mathbf{N}-\boldsymbol{\nabla}\times\mathbf{V}), \tag{2.6}$$

the energy equation for the present problem is,

$$\frac{DT}{Dt}=\alpha\nabla^2T+(\tau)\left[D_B\boldsymbol{\nabla}T\cdot\nabla C+(\frac{D_T}{T_\infty})\boldsymbol{\nabla}T\cdot\boldsymbol{\nabla}T\right], \tag{2.7}$$

the concentration equation for the present problem is,

$$\frac{DC}{Dt}=D_B\nabla^2C+(\frac{D_T}{T_\infty})\nabla^2T, \tag{2.8}$$

the microorganism equation for the present problem is,

$$\frac{Dn}{Dt}+\frac{\widetilde{b}W_c}{\nabla C}\left[\nabla n\cdot\boldsymbol{\nabla}C\right]=D_m(\boldsymbol{\nabla}^2n). \tag{2.9}$$

In this model at the wall the fluid velocity, temperature, and nanoparticle volume fraction all are varying with respect to the coordinates x, and t are given below,

$u_w(x,t) = \frac{ax}{1-\alpha_0 t}, \quad T_w = T_\infty + \frac{bx}{(1-\alpha_0 t)^2}, \quad C_w = C_\infty + \frac{b_1 x}{(1-\alpha_0 t)^2}$ and $\quad n_w = n_\infty + \frac{b_2 x}{(1-\alpha_0 t)^2}$.

Now both α_0 and a are positive constant have dimension per unit time. b, b_1, and b_2 are the constants with the dimensions temperature, nanoparticle volume fraction and microorganisms respectively.

The velocity field for the flow problem is

$$\mathbf{V} = [u(x,y,t)\,,\ v(x,y,t)\,,\ 0]\,. \tag{2.10}$$

Using these assumption and imposing the boundary layer approximation, the component form of above equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.11}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = K_1\left(\frac{\partial u_e}{\partial t} + u_e\frac{\partial u_e}{\partial x}\right) + \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y}, \tag{2.12}$$

$$\rho j\left(\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\frac{\partial^2 N}{\partial y^2} - k\left(2N + \frac{\partial u}{\partial y}\right), \tag{2.13}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 + \tau D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}, \tag{2.14}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{2.15}$$

$$\frac{\partial n}{\partial t} + u\frac{\partial n}{\partial x} + v\frac{\partial n}{\partial y} = D_m\frac{\partial^2 n}{\partial y^2} - \frac{\widetilde{b}W_c}{C_w - C_\infty}\left[\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)\right]. \tag{2.16}$$

Here, $\frac{D}{Dt}$ is material derivative with respect to time, ρ is the density of nanofluid, the microrotation vector is $\mathbf{N}$ which is normal to x, y plane, $\mathbf{V}$ is the velocity vector, α is the diffusivity constant of thermal fluid, γ is the viscosity of spin gradient, κ is the coefficient of micro - rotation viscosity, j is the density of micro - inertia, τ is the ratio between heat capacity of nanoparticle to the fluid, Brownian diffusion coefficient is D_B, D_T is thermophoretic diffusion coefficient, chemotaxis constant is $\widetilde{b}$, maximum cell speed is W_c and D_m is the diffusivity coefficient of the microorganism.

The concerned boundary conditions for the problem are assumed to be in the form,

$$u = \lambda u_w(x,t) + \nu N_1(x,t)\left(\frac{\partial u}{\partial y}\right),\ v = 0,\ N = 0, T = T_w(x,t) + D_1(x,t)\left(\frac{\partial T}{\partial y}\right),$$

$$D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0,\ n = n_w(x,t) + E_1(x,t)\left(\frac{\partial n}{\partial y}\right),\ \text{at } y \longrightarrow 0, \tag{2.17}$$

$$u = K_1 u_e(x,t),\ N \longrightarrow 0,\ T \longrightarrow T_\infty,\ C \longrightarrow C_\infty,\ n \longrightarrow n_\infty,\ at\ y \longrightarrow \infty.$$

Here, we define velocity, thermal and microorganism factors as, $N_1(x,t) = (N_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, $D_1(x,t) = (D_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, and $E_1(x,t) = (E_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, respectively. Here we assume that mass flux is zero i.e., $D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0$, Now we introduce specified dimensionless variables as assumed by,

$$\begin{aligned}
\eta &= y\sqrt{\frac{a}{\nu(1-\alpha_0 t)}},\ \psi = x f(\eta)\sqrt{\frac{a\nu}{(1-\alpha_0 t)}},\\
N &= x h(\eta)\sqrt{\frac{a^3}{\nu(1-\alpha_0 t)^3}},\\
u_e &= \frac{ax}{(1-\alpha_0 t)},\ T = T_\infty + \frac{bx}{(1-\alpha_0 t)^2}\theta(\eta),\\
C &= C_\infty + \frac{b_1 x}{(1-\alpha_0 t)^2}\phi(\eta),\ n = n_\infty + \frac{b_2 x}{(1-\alpha_0 t)^2}\chi(\eta),\\
\theta &= \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},\ \chi = \frac{n}{n_w}.
\end{aligned} \tag{2.18}$$

Here η is similarity variable, stream function is $\psi(x,y,t)$ and $f(\eta)$, $h(\eta)$, $\theta(\eta)$, $\phi(\eta)$, and $\chi(\eta)$ are the dimensionless variable for the linear velocity, micro-rotation, temperature, nanoparticle volume fraction, and microorganisms respectively. Now velocity components u and v are defined as,

$$u = \frac{\partial \psi}{\partial y} = \frac{a f'(\eta)}{(1-\alpha_0 t)}x, \quad v = -\frac{\partial \psi}{\partial x} = -f(\eta)\sqrt{\frac{a\nu}{(1-\alpha_0 t)}}x. \tag{2.19}$$

Here prime represented differentiation with respect to η.

Using similarity variables transformation above systems of equations are,

$$(1+\Delta)f''' - \frac{A}{2}(2f' + \eta f'') - (f')^2 + f f'' + \Delta h' + K_1(1 + A) = 0, \tag{2.20}$$

$$\lambda_0 h'' - \frac{A}{2}(3h + \eta h') - f'h + fh' - \Delta B(f'' + 2h) = 0, \tag{2.21}$$

$$\frac{1}{\Pr}\theta'' - \frac{A}{2}(4\theta + \eta\theta') - f'\theta + f\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0, \tag{2.22}$$

$$\frac{1}{Sc}\phi'' - \frac{A}{2}(4\phi + \eta\phi') - f'\phi + f\phi' + \frac{Nt}{Nb}\frac{1}{Sc}\theta'' = 0, \tag{2.23}$$

$$\frac{1}{Sb}\chi'' - \frac{A}{2}(4\chi + \eta\chi') - f'\chi + f\chi' - \frac{Pe}{Sb}(\chi\phi'' + \chi'\phi') = 0. \tag{2.24}$$

The relevant conditions on the boundary are,

$$\begin{aligned} f(\eta) &= 0,\ f'(\eta) = \lambda + \delta_\nu f''(\eta),\ h(\eta) = 0,\ \theta(\eta) = 1 + \delta_T\theta'(\eta),\ Nb\phi'(\eta) + Nt\theta'(\eta) = 0, \\ \chi(\eta) &= 1 + \delta_n\chi'(\eta), \quad \text{when} \quad \eta \longrightarrow 0, \\ f'(\eta) &= K_1,\ h(\eta) = 0,\ \theta(\eta) = 0,\ \phi(\eta) = 0,\ \chi(\eta) = 0 \quad \text{when } \eta \to \infty. \end{aligned} \tag{2.25}$$

In the above system of non-linear ordinary differential equations, the parameters defined as micropolar parameter $\Delta = \frac{\kappa}{\mu}$, $A = \frac{\alpha_0}{a}$ is the parameter of unsteadiness, (spin gradient viscosity parameter) is $\lambda_0 = \frac{\gamma}{j\mu}$, B is the parameter of micro - inertia density i.e. ($B = \frac{\nu(1-\alpha_0 t)}{ja}$), $Nb = \frac{\tau D_B \Delta C}{\nu}$ (Brownian motion), $Nt = \frac{\tau D_B \Delta T}{\nu T_\infty}$ (thermosphoresis), $Sc = \frac{\nu}{D_B}$ (Schmidt number), $Sb = \frac{\nu}{D_m}$ (bio-convection Schmidt number), $Pe = \frac{\widetilde{b} W_c D_m}{\nu^2}$ (bio-convection Peclet number) and $\Pr = \frac{\upsilon}{\alpha}$ (Prandtl number). Also we define the velocity slip, thermal slip and microorganism slip factor as,

$$\begin{aligned} \delta_\nu &= (N_1)_0,\ \delta_T = (D_1)_0\frac{1}{\sqrt{\nu}}, \\ \delta_n &= (E_1)_0\frac{1}{\sqrt{\nu}}. \end{aligned} \tag{2.26}$$

Note N_1, D_1,and E_1 are proportional to $\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$. In above equation (2.25) λ is the constant of stretching with $\lambda < 0$ for shrinking, $\lambda = 0$, for stationary and $\lambda > 0$, for stretching plate, K_1 is the constant and having value $K_1 = 1$ with the pressure gradient, and $K_1 = 0$ without pressure gradient.

2.2.1 Physical Quantities

Physical quantities are very important from an engineering perspective and tells us the behavior of flow, rate of transfer heat and microorganism rate. The local skin friction, local Nusselt number as well as the motile microorganism flux are defined as,

$$C_{fx} = \frac{\tau_w}{\rho u_e^2}, \qquad Nu_x = \frac{xp_w}{T_w - T_\infty}, \qquad Q_{nx} = \frac{xp_n}{D_m n_w},$$

$$\tau_w = 2\left[(\kappa+\mu)\left.\frac{\partial u}{\partial y}\right|_{y=0} + \kappa(N)_{y=0}\right],\ p_w = -\kappa\left.\frac{\partial T}{\partial y}\right|_{y=0}, \tag{2.27}$$

$$p_n = -D_B\left.\frac{\partial n}{\partial y}\right|_{y=0}.$$

Where τ_w, denotes the skin fraction, p_w is surface heat flux, and p_n is motile microorganism flux. Using (2.18), (2.19) in (2.27) we get,

$$\mathrm{Re}_x^{1/2}\, C_{fx} = 2(1+\Delta)f''(0), \tag{2.28}$$

$$\mathrm{Re}_x^{-1/2}\, Nu_x = -\theta'(0), \tag{2.29}$$

$$\mathrm{Re}_x^{-1/2}\, Q_{nx} = -\chi'(0). \tag{2.30}$$

Here $Re_x = u_e\sqrt{\frac{\nu(1-\alpha_0 t)}{a}}$ is denotes the Local Reynold number.

2.3 Homotopy Analysis Method

To find the solution of non-linear differential equations we used homotopy analysis method. The linear operators and initial guesses for above flow problem are chosen as,

$$\begin{aligned} L_f(f) &= \frac{d^3 f}{d\eta^3} + \frac{d^2 f}{d\eta^2},\ L_h(h) = \frac{d^2 h}{d\eta^2} - h,\ L_\theta(\theta) = \frac{d^2\theta}{d\eta^2} - \theta, \\ L_\phi(\phi) &= \frac{d^2\phi}{d\eta^2} - \phi, L_\chi(\chi) = \frac{d^2\chi}{d\eta^2} - \chi. \end{aligned} \tag{2.31}$$

$$f_0(\eta) = \frac{\lambda - K_1}{\delta_\nu + 1} + K_1\eta + \left(\frac{K_1 - \lambda_0}{\delta_\nu + 1}\right) Exp(-\eta), \tag{2.32}$$

$$h_0(\eta) = 0, \tag{2.33}$$

$$\theta_0(\eta) = \left(\frac{1}{\delta_T + 1}\right) Exp(-\eta), \tag{2.34}$$

$$\phi_0(\eta) = -\left(\frac{Nt}{Nb}\right)\left(\frac{1}{\delta_T + 1}\right) Exp(-\eta), \tag{2.35}$$

$$\chi_0(\eta) = \left(\frac{1}{\delta_n + 1}\right) Exp(-\eta). \tag{2.36}$$

Where $\mathcal{L}_f\,(f)\,, \mathcal{L}_h\,(h)\,, \mathcal{L}_\theta\,(\theta)\,, \mathcal{L}_\phi\,(\phi)\,,$ and $\mathcal{L}_\chi\,(\chi)$ defines the linear operators, besides $f_0(\eta)$, $h_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$, and $\chi_0(\eta)$ is the respective initial guesses of f, h, θ, ϕ, and χ.

with the properties

$$\begin{aligned}
&\mathcal{L}_f\left[C_1e^{-\eta} + C_2e^{\eta} + C_3\right] = 0,\\
&\mathcal{L}_h\left[C_4e^{-\eta} + C_5e^{\eta}\right] = 0,\\
&\mathcal{L}_\theta\left[C_6e^{-\eta} + C_7e^{\eta}\right] = 0,\\
&\mathcal{L}_\phi\left[C_8e^{-\eta} + C_9e^{\eta}\right] = 0,\\
&\mathcal{L}_\chi\left[C_{10}e^{-\eta} + C_{11}e^{\eta}\right] = 0.
\end{aligned} \tag{2.37}$$

here the arbitrary constants $C_i\,(i = 1 - 11)$.

2.3.1 Zeroth order problem

$$\begin{aligned}
(1-q)\,\mathcal{L}_f\left[f(\eta;q) - f_0(\eta)\right] &= q\hbar_f\mathcal{N}_f\left[f(\eta;q)\right],\\
(1-q)\,\mathcal{L}_h\left[h(\eta;q) - h_0(\eta)\right] &= q\hbar_h\mathcal{N}_h\left[h(\eta;q)\right],\\
(1-q)\,\mathcal{L}_\theta\left[\theta(\eta;q) - \theta_0(\eta)\right] &= q\hbar_\theta\mathcal{N}_\theta\left[\theta(\eta;q)\right],\\
(1-q)\,\mathcal{L}_\phi\left[\phi(\eta;q) - \phi_0(\eta)\right] &= q\hbar_\phi\mathcal{N}_\phi\left[\phi(\eta;q)\right],\\
(1-q)\,\mathcal{L}_\chi\left[\chi(\eta;q) - \chi_0(\eta)\right] &= q\hbar_\chi\mathcal{N}_\chi\left[\chi(\eta;q)\right],
\end{aligned} \tag{2.38}$$

$$\begin{aligned}
&f(\eta;q) = 0,\ \tfrac{\partial f(\eta;q)}{\partial\eta} = \lambda + \delta_\nu\tfrac{\partial^2 f(\eta;q)}{\partial\eta^2},\ h(\eta;q) = 0,\ \theta(\eta;q) = 1 + \delta_T\tfrac{\partial\theta(\eta;q)}{\partial\eta},\\
&Nb\tfrac{\partial\phi(\eta;q)}{\partial\eta} + Nt\tfrac{\partial\theta(\eta;q)}{\partial\eta} = 0,\ \chi(0;q) = 1 + \delta_n\tfrac{\partial\chi(\eta;q)}{\partial\eta},\ as\ \eta = 0.\\
&\tfrac{\partial f(\eta;q)}{\partial\eta} = K_1,\ h(\eta;q) = \theta(\eta;q) = \phi(\eta;q) = \chi(\eta;q) = 0,\ as\ \eta\to\infty.
\end{aligned} \tag{2.39}$$

Where non-zero auxiliary parameters are $\hbar_f$ $\hbar_h$ $\hbar_\theta$ $\hbar_\phi$ and $\hbar_\chi$, $q \in [0,1]$ is the enclose parameter and $\mathcal{N}_f$, $\mathcal{N}_h$, $\mathcal{N}_\theta$, $\mathcal{N}_\phi$, and $\mathcal{N}_\chi$ are the non-linear operators.

$$\begin{aligned}
&\mathcal{N}_f\left[f(\eta,q)\right]=(1+\Delta)\tfrac{\partial^3 f}{\partial\eta^3}-\left(\tfrac{\partial f}{\partial\eta}\right)^2+f\tfrac{\partial^2 f}{\partial\eta^2}-\tfrac{A}{2}\left(2\tfrac{\partial f}{\partial\eta}+\eta\tfrac{\partial^2 f}{\partial\eta^2}\right)+\Delta\tfrac{\partial h}{\partial\eta}+K_1\left(1+A\right),\\
&\mathcal{N}_h\left[h(\eta,q)\right]=\lambda_0\tfrac{\partial^2 h}{\partial\eta^2}+\tfrac{\partial h}{\partial\eta}f-h\tfrac{\partial f}{\partial\eta}-\tfrac{A}{2}\left(3h+\eta\tfrac{\partial h}{\partial\eta}\right)-\Delta B(2h+\tfrac{\partial^2 f}{\partial\eta^2}),\\
&\mathcal{N}_\theta\left[\theta(\eta,q)\right]=\tfrac{1}{\Pr}\tfrac{\partial^2\theta}{\partial\eta^2}+\tfrac{\partial\theta}{\partial\eta}f-\theta\tfrac{\partial f}{\partial\eta}-\tfrac{A}{2}\left(4\theta+\eta\tfrac{\partial\theta}{\partial\eta}\right)+Nb\tfrac{\partial\theta}{\partial\eta}\tfrac{\partial\phi}{\partial\eta}+Nt\left(\tfrac{\partial^2\theta}{\partial\eta^2}\right)^2,\\
&\mathcal{N}_\phi\left[\phi(\eta,q)\right]=\tfrac{1}{Sc}\tfrac{\partial^2\phi}{\partial\eta^2}+\tfrac{\partial\phi}{\partial\eta}f-\phi\tfrac{\partial f}{\partial\eta}-\tfrac{A}{2}\left(4\phi+\eta\tfrac{\partial\phi}{\partial\eta}\right)+\tfrac{Nt}{Nb}\tfrac{1}{Sc}\tfrac{\partial^2\theta}{\partial\eta^2},\\
&\mathcal{N}_\chi\left[\chi(\eta,q)\right]=\tfrac{1}{Sb}\tfrac{\partial^2\chi}{\partial\eta^2}+\tfrac{\partial\chi}{\partial\eta}f-\chi\tfrac{\partial f}{\partial\eta}-\tfrac{A}{2}\left(4\chi+\eta\tfrac{\partial\chi}{\partial\eta}\right)-\tfrac{Pe}{Sb}\left[\chi\tfrac{\partial^2\phi}{\partial\eta^2}+\tfrac{\partial\phi}{\partial\eta}\tfrac{\partial\chi}{\partial\eta}\right],
\end{aligned}\tag{2.40}$$

$$\begin{aligned}
&f_0(\eta)=f(\eta,0),\ h_0(\eta)=h(\eta,0),\ \theta_0(\eta)=\theta(\eta,0),\ \phi_0(\eta)=\phi(\eta,0),\ \chi_0(\eta)=\chi(\eta,0),\\
&f(\eta,1)=f(\eta),\ h(\eta,1)=h(\eta),\ \theta(\eta,1)=\theta(\eta),\ \phi(\eta,1)=\phi(\eta),\ \chi(\eta,1)=\chi(\eta).
\end{aligned}\tag{2.41}$$

kth-order deformation problems

Zeroth-order deformation problem (2.23) is differentiating k-times w.r.t q and dividing by $k!$ also setting $q=0$, then kth-order deformation problem is given by

$$\begin{aligned}
\mathcal{L}_f\left[f_k(\eta)-\psi_k f_{k-1}(\eta)\right]&=\hbar_f\mathcal{R}_{f,k}(\eta),\\
\mathcal{L}_h\left[h_k(\eta)-\psi_k h_{k-1}(\eta)\right]&=\hbar_h\mathcal{R}_{h,k}(\eta),\\
\mathcal{L}_\theta\left[\theta_k(\eta)-\psi_k\theta_{k-1}(\eta)\right]&=\hbar_\theta\mathcal{R}_{\theta,k}(\eta),\\
\mathcal{L}_\phi\left[\phi_k(\eta)-\psi_k\phi_{k-1}(\eta)\right]&=\hbar_\phi\mathcal{R}_{\phi,k}(\eta),\\
\mathcal{L}_\chi\left[f_k(\eta)-\psi_k\chi_{k-1}(\eta)\right]&=\hbar_\chi\mathcal{R}_{\chi,k}(\eta).
\end{aligned}\tag{2.42}$$

$$\begin{aligned}
&f_k(0)=0,\ f_k'(0)=1+\delta_\nu\ f_k''(0),\ h_k(0)=0,\ \theta_k(0)=1+\delta_T\ \theta_k'(0),\\
&Nb\phi_k'(0)+Nt\ \theta_k'(0)=0,\ \chi_k(0)=1+\delta_n\ \chi_k'(0),\\
&f_k'(\eta)=K_1,\ h(\eta)=0\ \theta(\eta)=0\ \phi(\eta)=0\ \chi(\eta)=0,\quad \text{as } \eta\to\infty.
\end{aligned}\tag{2.43}$$

With the results stated below,

$$\begin{aligned}
\mathcal{R}_k^f(\eta) \;=\;& (1+\Delta)\frac{\partial^3 f_{k-1}}{\partial\eta^3}+\sum_{i=0}^{k-1}f_i\frac{\partial^2 f_{k-i-1}}{\partial\eta^2}-\sum_{i=0}^{k-1}\frac{\partial f_i}{\partial\eta}\frac{\partial f_{k-i-1}}{\partial\eta}-\frac{A}{2}\left[2\frac{\partial f_{k-1}}{\partial\eta}+\eta\frac{\partial^2 f_{k-1}}{\partial\eta^2}\right]\\
&+\Delta\frac{\partial h_{k-1}}{\partial\eta}+K_1(1+A),
\end{aligned}\tag{2.44}$$

$$\mathcal{R}_k^h(\eta) = \lambda_0 \frac{\partial^2 h_{k-1}}{\partial \eta^2} + \sum_{i=0}^{k-1} f_i \frac{\partial h_{k-i-1}}{\partial \eta} - \sum_{i=0}^{k-1} \frac{\partial f_i}{\partial \eta} h_{k-i-1} - \frac{A}{2}\left[3h_{k-1} + \eta \frac{\partial h_{k-1}}{\partial \eta}\right]$$
$$-\Delta B(2h_{k-1} + \frac{\partial^2 f_{k-1}}{\partial \eta^2}), \tag{2.45}$$

$$\mathcal{R}_k^\theta(\eta) = \frac{1}{\Pr} \frac{\partial^2 \theta_{k-1}}{\partial \eta^2} + \sum_{i=0}^{k-1} f_i \frac{\partial \theta_{k-i-1}}{\partial \eta} - \sum_{i=0}^{k-1} \frac{\partial f_i}{\partial \eta} \theta_{k-i-1} - \frac{A}{2}\left[4\theta_{k-1} + \eta \frac{\partial \theta_{k-1}}{\partial \eta}\right]$$
$$+Nb \sum_{i=0}^{k-1} \frac{\partial \phi_i}{\partial \eta} \frac{\partial \theta_{k-i-1}}{\partial \eta} + Nt \sum_{i=0}^{k-1} \frac{\partial \theta_i^2}{\partial \eta^2} \frac{\partial \theta_{k-i-1}^2}{\partial \eta^2}, \tag{2.46}$$

$$\mathcal{R}_k^\phi(\eta) = \frac{1}{Sc} \frac{\partial^2 \phi_{k-1}}{\partial \eta^2} + \sum_{i=0}^{k-1} f_i \frac{\partial \phi_{k-i-1}}{\partial \eta} - \sum_{i=0}^{k-1} \frac{\partial f_i}{\partial \eta} \phi_{k-i-1} - \frac{A}{2}\left[4\phi_{k-1} + \eta \frac{\partial \phi_{k-1}}{\partial \eta}\right]$$
$$+\frac{Nb}{Nt} \frac{1}{Sc} \frac{\partial^2 \theta_{k-1}}{\partial \eta^2}, \tag{2.47}$$

$$\mathcal{R}_k^\chi(\eta) = \frac{1}{Sb} \frac{\partial^2 \chi_{k-1}}{\partial \eta^2} + \sum_{i=0}^{k-1} f_i \frac{\partial \chi_{k-i-1}}{\partial \eta} - \sum_{i=0}^{k-1} \frac{\partial f_i}{\partial \eta} \chi_{k-i-1} - \frac{A}{2}\left[4\chi_{k-1} + \eta \frac{\partial \chi_{k-1}}{\partial \eta}\right]$$
$$-\frac{Pe}{Sb}\left[\sum_{i=0}^{k-1} \chi_i \frac{\partial \phi_{k-i-1}^2}{\partial \eta^2} + \frac{\partial \phi_{k-1}}{\partial \eta} \frac{\partial \chi_{k-i-1}}{\partial \eta}\right]. \tag{2.48}$$

where

$$\psi_k = \begin{cases} 0, & k \leq 1 \\ 1, & k > 1 \end{cases}. \tag{2.49}$$

the symbolic computation software MATHEMATICA when $k = 1, 2, 3, ...$

When q changes from 0 to 1 then $f(\eta, q)$, $h(\eta, q)$, $\theta(\eta, q)$, $\phi(\eta, q)$, and $\chi(\eta, q)$ change from the initial guesses $f_0(\eta)$, $h_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$, and $\chi_0(\eta)$ to the final solutions $f(\eta)$, $h(\eta)$, $\theta(\eta)$, $\phi(\eta)$, and $\chi(\eta)$ respectively. Expanding $f(\eta, q)$, $h(\eta, q)$, $\theta(\eta, q)$, $\phi(\eta, q)$, and $\chi(\eta, q)$

in Taylor's series w.r.t the enclose parameter q one can write

$$\begin{aligned}
f(\eta,q) &= f_0(\eta) + \textstyle\sum_{k=1}^{\infty} f_k(\eta)\, q^k, & f_k(\eta) &= \frac{1}{k!}\frac{\partial^k f(\eta,q)}{\partial q^k}\Big|_{q=0}, \\
h(\eta,q) &= h_0(\eta) + \textstyle\sum_{k=1}^{\infty} h_k(\eta)\, q^k, & h_k(\eta) &= \frac{1}{k!}\frac{\partial^k h(\eta,q)}{\partial q^k}\Big|_{q=0}, \\
\theta(\eta,q) &= \theta_0(\eta) + \textstyle\sum_{k=1}^{\infty} \theta_k(\eta)\, q^k, & \theta_k(\eta) &= \frac{1}{k!}\frac{\partial^k \theta(\eta,q)}{\partial q^k}\Big|_{q=0}, \\
\phi(\eta,q) &= \phi_0(\eta) + \textstyle\sum_{k=1}^{\infty} \phi_k(\eta)\, q^k, & \phi_k(\eta) &= \frac{1}{k!}\frac{\partial^k \phi(\eta,q)}{\partial q^k}\Big|_{q=0}, \\
\chi(\eta,q) &= \chi_0(\eta) + \textstyle\sum_{k=1}^{\infty} \chi_k(\eta)\, q^k, & \chi_k(\eta) &= \frac{1}{k!}\frac{\partial^k \chi(\eta,q)}{\partial q^k}\Big|_{q=0}.
\end{aligned} \tag{2.50}$$

The auxiliary parameters $\hbar_f$, $\hbar_h$, $\hbar_\theta$, $\hbar_\phi$, and $\hbar_\chi$ are chosen as that the series is converge for $q = 1$, we have

$$f(\eta) = f_0(\eta) + \sum_{k=1}^{\infty} f_k(\eta), \tag{2.51}$$

$$h(\eta) = h_0(\eta) + \sum_{k=1}^{\infty} h_k(\eta), \tag{2.52}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{k=1}^{\infty} \theta_k(\eta), \tag{2.53}$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{k=1}^{\infty} \phi_k(\eta), \tag{2.54}$$

$$\chi(\eta) = \chi_0(\eta) + \sum_{k=1}^{\infty} \chi_k(\eta). \tag{2.55}$$

2.4 HAM solution

2.4.1 Covergence analysis

Here the series solution of (2.50) involve the auxiliary parameters ($\hbar_f$, $\hbar_h$, $\hbar_\theta$, $\hbar_\phi$, $\hbar_\chi$). Surely the auxiliary variables ($\hbar_f$, $\hbar_h$, $\hbar_\theta$, $\hbar_\phi$, $\hbar_\chi$) in the series solution accelerate the convergence. To choose the appropriate values of ($\hbar_f$, $\hbar_h$, $\hbar_\theta$, $\hbar_\phi$, $\hbar_\chi$) the h-curves have been displayed at 25^{th} order of approximations. It is noticed from Figs. $(1-5)$ that the acceptable range of $\hbar_f$, $\hbar_h$, $\hbar_\theta$, $\hbar_\phi$, and $\hbar_\chi$ are $-1.5 \leq h_f \leq -0.5$, $-1.7 \leq h_h \leq -0.5$, $-1.0 \leq h_\theta \leq -0.6$, $-1.5 \leq h_\phi \leq -0.1$, $-1.5 \leq h_\chi \leq -0.5$. Table (2.1) notify that the 6^{th} order of deformation is enough for the convergent solution of velocity and micro rotation and 24^{th} order of deformation is enough for temperature, nanoparticles concentration and microorganism respectively.

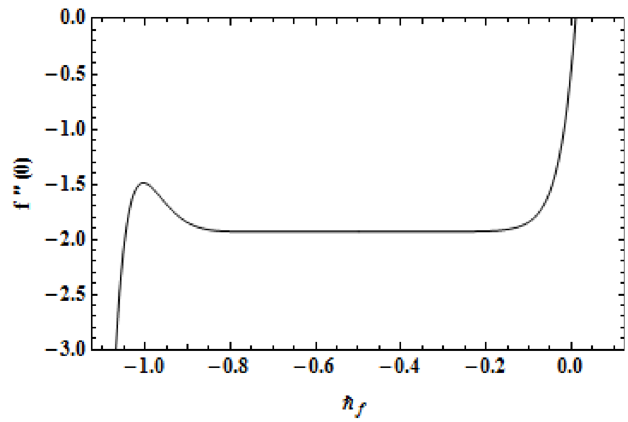


Fig. 2.2; h-curve for the function $f(\eta)$.

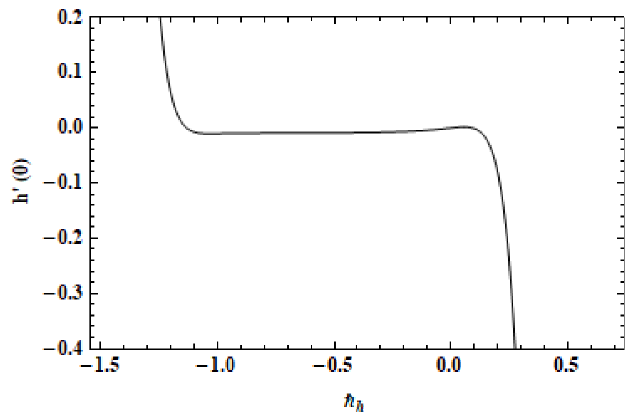


Fig. 2.3; h-curve for the function $h(\eta)$.

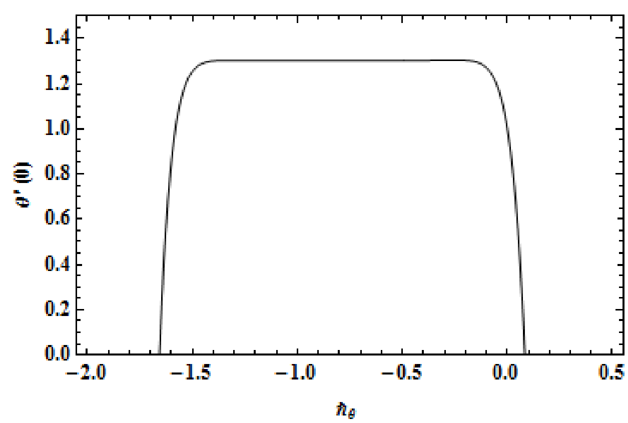


Fig. 2.4; h-curve for the function $\theta(\eta)$.

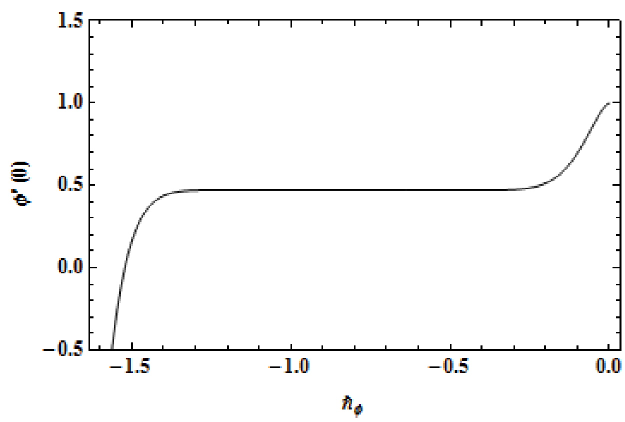


Fig. 2.5; h-curve for the function $\phi(\eta)$.

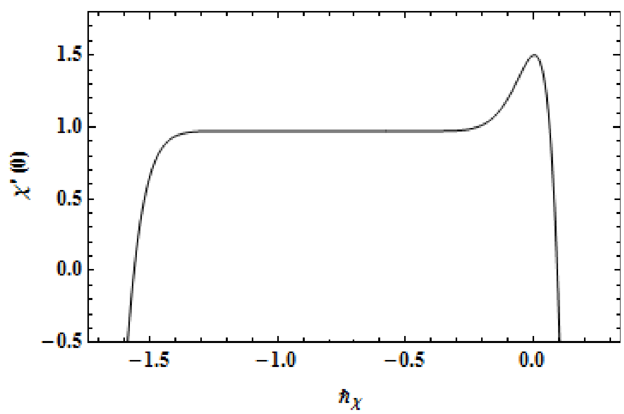


Fig. 2.6; h-curve for the function $\chi(\eta)$.

Table (2.1) convergence of homotopy solution when $\delta_\nu = \delta_T = \delta_n = \Delta = B = 0.1$, $\lambda_0 = 0.3$, $\Pr = 6.8$, $Nb = Nt = 0.01$, $A = K = Sc = Sb = Pe = 1$, and $\lambda = 0.5$.

Table (2.1); Convergence of homotopy solution.

Order of approximation	$-f''(0)$	$-h'(0)$	$\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
1	1.10556	1.52788	0.57334	0.50011	0.96250
2	1.11538	1.52125	0.57940	0.54874	0.35960
3	1.11548	1.52280	0.56212	0.50871	0.44361
6	1.11502	1.52251	0.55041	0.48740	0.47235
10	1.11502	1.52251	0.54745	0.48044	0.47225
15	1.11502	1.52251	0.54698	0.47343	0.47214
18	1.11502	1.52251	0.54691	0.47913	0.47213
21	1.11502	1.52251	0.54689	0.47898	0.47208
22	1.11502	1.52251	0.54688	0.47897	0.47202
24	1.11502	1.52251	0.54687	0.47896	0.47201
25	1.11502	1.52251	0.54687	0.47896	0.47201
26	1.11502	1.52251	0.54687	0.47896	0.47201

2.5 Results and discussion

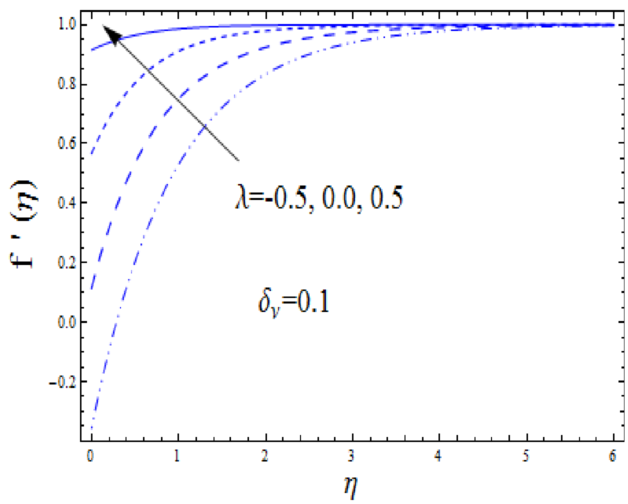
The mathematical model and computational simulations can be used to determine the dominant features of the flow. The dimensionless quantities such as velocity, temperature, nanoparticles volume fraction, micro-rotation, and microorganisms are dealed with analytically, same as the local skin friction, rate of transfer heat and microorganism are observed. In this analysis, we elaborate the influence of δ_ν, δ_T, δ_n Δ, and λ on the behavior of flow, heat transfer and microorganism. The fixed value of parameters are $\delta_\nu = \delta_T = \delta_n = \Delta = B = 0.1$, $\lambda_0 = 0.3$, $\Pr = 6.8$, $Nb = Nt = 0.01$, $A = K = Sc = Sb = Pe = 1$, $\lambda = 0.5$.

In this section analyzed the impact of parameter δ_ν. Occurrence of slip velocity due to shrinking/stretching surface plate, by this we investigate the characteristics of the velocity, nanoparticle volume fraction, temperature, micro-rotation, and microorganism profile. Fig. (2.7) represents the effects velocity slip and shrinking/stretching sheet on the dimensionless velocity profile. Physically by rising the value of velocity slip parameter, the fluid slip at the surface of plate, so by the increment in the velocity slip parameter then the velocity profile is increases for both shrinking/stretching and stationary sheet. No velocity slip effect, when the zero value of dimensionless velocity at the surface of the plate. It is noted that for different value of λ the velocity of fluid is greater to the plate velocity by the slip effect ($\delta_\nu = 0.1$). Fig. (2.8) reported that the micro-rotation mechanism is increased by the increasing the magnitude of velocity slip. The velocity slip parameter is increased, the reduction in dimensionless temperature profile occur for both shrinking/stretching sheet is observed in Fig. (2.9). It is depicted that the hot plate lesser energy transmit to the fluid, when the fluid penetrates through the boundary layer and thereby decreases the temperature profile. Fig. (2.10) shows that with increment in the slip velocity parameter enhance the nanoparticle volume fraction profile. Fig. (2.11) designates the reduction is occurs in the dimensionless micro-organism density profile when velocity slip parameter is rises.

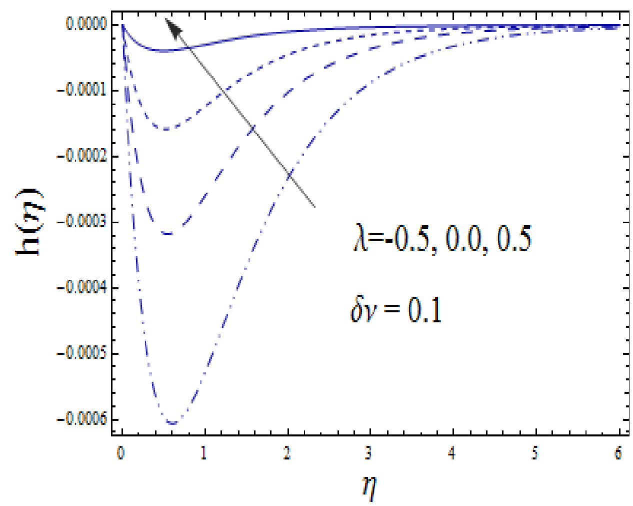
Due to thermal slip parameter the change in the temperature, nanoparticle volume fraction and micro-organisms mechanism with shrinking/stretching sheet at the boundary layer are observed in the figures (2.12−2.14). When the thermal slip parameter is rises than at the surface the nanoparticle volume fraction enhances whereas the reduction is occurs in the temperature and microorganisms. Fig. (2.12) shows that the surface temperature decline to minimum with greater thermal slip and become maximum with no thermal slip effects. This produces, the less heat transferred from the plate to body of nanofluid, because the boundary layers being unresponsive to the impacts of heating at the surface of plate. Fig. (2.13) display the effect of thermal slip parameter, increment in δ_T, the nanoparticle volume fraction profile at the wall is rises whereas away from the wall is decreases. Fig. (2.14) illustrates the impact δ_T on the microorganism profile. It is found that the reduction is occurs in the microorganism profile when the thermal slip factor is increases. Also it has been observed that due to thermal slip parameter the change in velocity and micro-rotation is not mentioned.

In Fig. (2.15), it is illustrated that the density of motile microorganism is rises when microorganism slip parameter is increases. At the surface the density of microorganism decline minimum with combined effects of microorganism non slip and contrary attain maximum with joined effects of microorganism slip for both stretching/shrinking and stationary sheet. It has been depicted that change in velocity, temperature, micro - rotation, and nanoparticle volume fraction with change in microorganism slip factor are not significant.

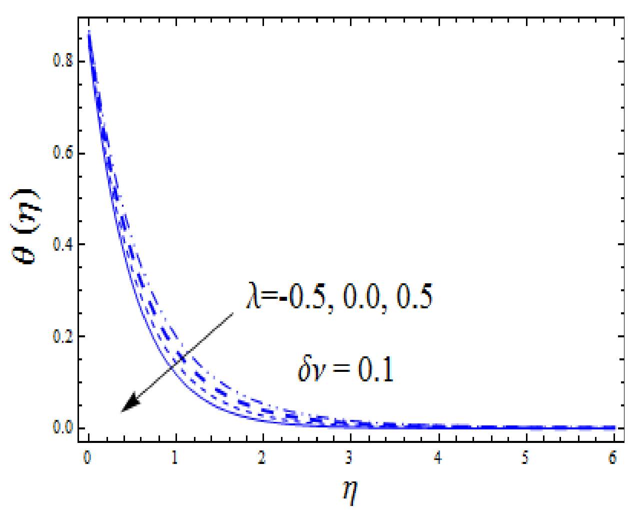
In the Figures. $(2.16 - 2.18)$, it is observed that the effects of the different parameters on the skin friction, Nusselt and microorganism rate. The theoretical conclusions shows that, the micro-rotation of the nanoparticles in the suspension increases, therefore skin friction is reduce, while microorganism transfer rate and transfer of heat increases. With no slip velocity the skin friction has greater value of the joined effect of lower micro - rotation factor and shrinking sheet. In the case of stretching sheet, for the no slip condition the skin friction shows lowest value and micro-rotation factor shows highest value. Fig. (2.16) designates that the value of skin friction is decline as the slip velocity parameter and micro-rotation factor is increases. In Fig. (2.17) depicts that increment in the velocity slip parameter and micro-rotation factor, the increment is occur in the Nusselt number. The rate of heat transfer for the stretching sheet is greater then the shrinking sheet. The negative value shows that the heat flow from higher to lower temperature from shrinking/stretching sheet. The change in the heat transfer rate of velocity slip and microorganism slip parameters are not observed. Fig. (2.18) shows that increased the micro-rotation factor and micro-organism slip parameter then microorganism transfer rate is enhanced. These conclusions are adopting to improving the performance of microbial fuel cells.


f ' (η)
η
λ=-0.5, 0.0, 0.5
δv=0.1

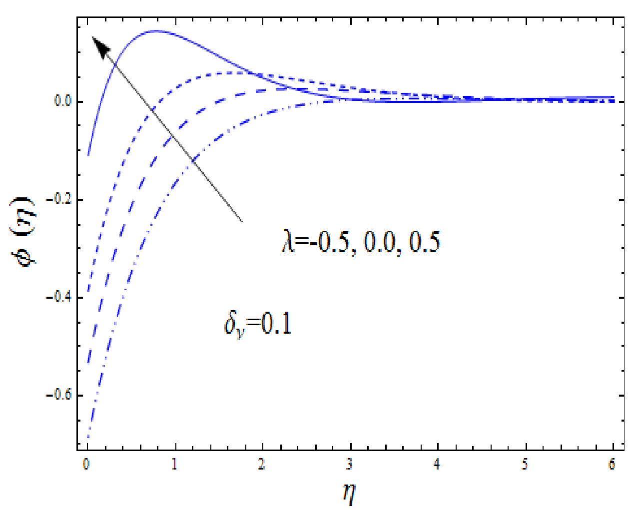
(2.7)


h(η)
η
λ=-0.5, 0.0, 0.5
δν = 0.1

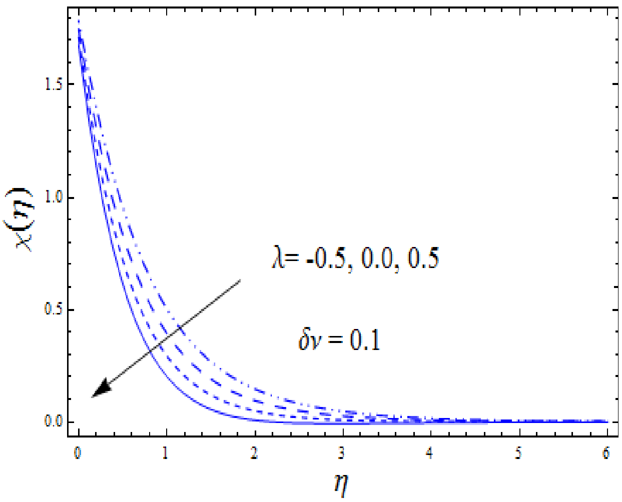
2.8


θ (η)
η
λ=-0.5, 0.0, 0.5
δν = 0.1

(2.9)

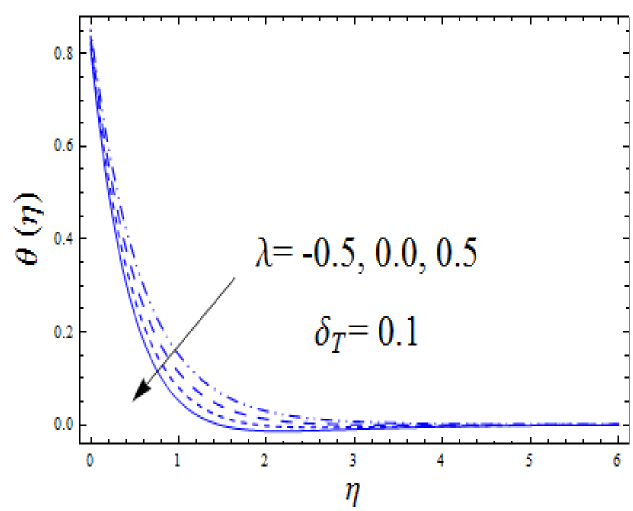

φ (η)
η
λ=-0.5, 0.0, 0.5
δv=0.1

(2.10)

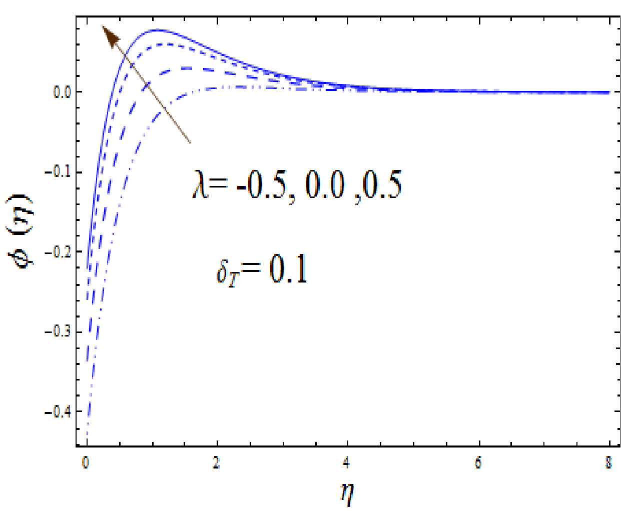


(2.11)

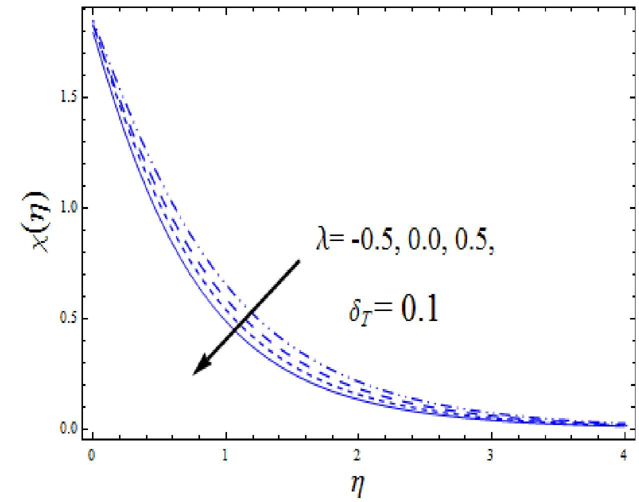
Figure (**2.7 − 2.11**) : Impacts of velocity slip parameter over a stretching/shrinking sheet on (2.7) velocity, (2.8) angular velocity,(2.9) temperature,(2.10) nanoparticle volume fraction and (2.11) micro-organism profiles.



(2.12)

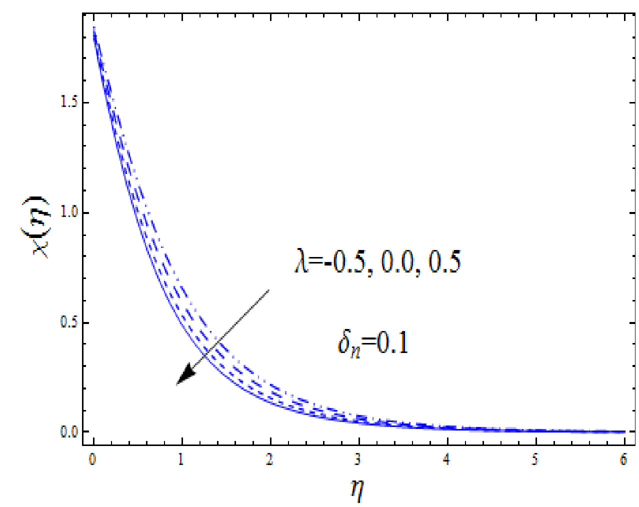


(2.13)



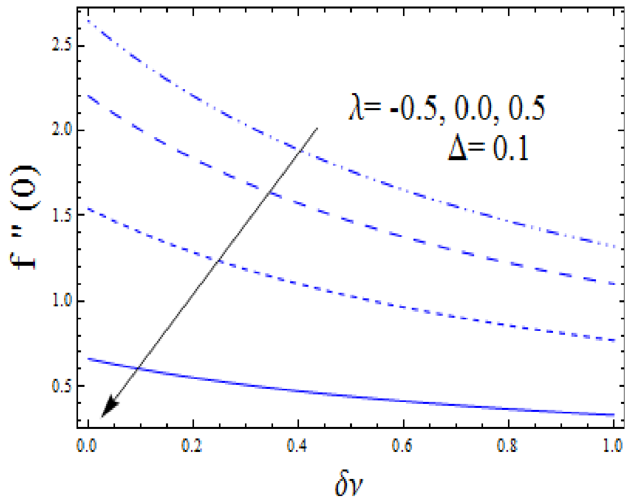
(2.14)

Figure (**2.12** − **2.14**) : Effects of thermal slip parameter over a stretching/shrinking sheet on (2.12) temperature, (2.13) nanoparticle volume fraction, and (2.14) micro-organism density profiles.

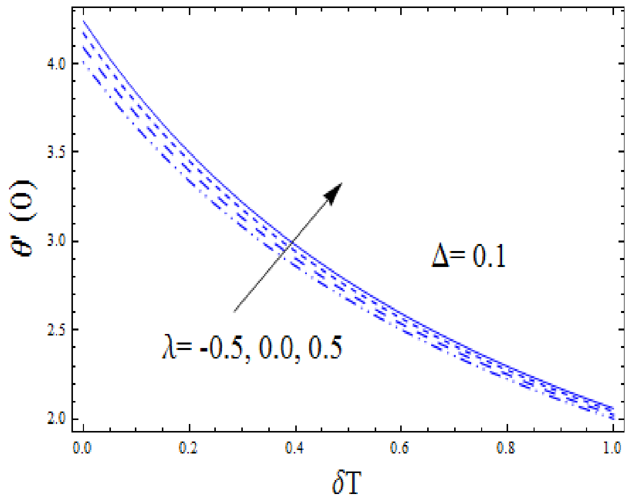


(2.15)

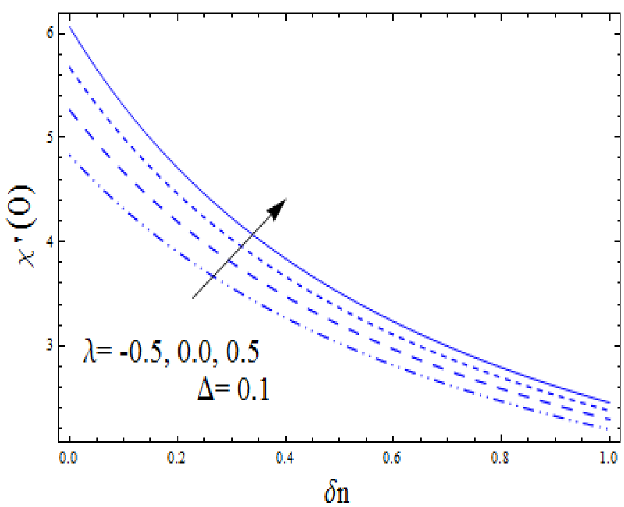
Figure **2.15** : Impacts of microorganism slip parameter over a stretching/shrinking sheet on microorganism profile.



(2.16)



(2.17)



(2.18)

Figure (**2.16 − 2.18**) : Conclusions of (2.16) skin friction, (2.17) Nusselt and (2.18) density of motile microorganisms for various values of velocity slip, thermal slip and microorganism slip parameters with stretching/shrinking sheet respectively.

2.6 Concluding Remarks

Here we have concluded the analytical results for an unsteady two dimensional, viscous, and incompressible flow of a micropolar nanofluid containing microorganisms past a perpendicular and non porous stretching/shrinking sheet. Converted mathematical model in the set of ordinary equations by used of suitable transformation and solve these equations by homotopy analysis method. The assumed conclusions are granted graphically to clarify the effect of different parameters on velocity, temperature, nanoparticle volume fraction, micro - rotation, and motile microorganism density function, furthermore the local skin fricition, Nusselt number, and microorganism number.

The important points of the flow problem are highlighted as,

▶When the velocity slip parameter is increases than the reduction is occur in the local skin friction.

▶ Due to increment of thermal slip parameter, the Nusselt number profile is decreases.

▶Increases the microorganism slip parameter, the local microorganism number is ehanced.

▶ Increment in the micro-rotation factor, the reduction is occur in the value of Nusselt number and skin friction.

▶ When the velocity slip, thermal slip and microorganism slip parameters are increases respectively than the microorganism number profile rises.

Chapter 3

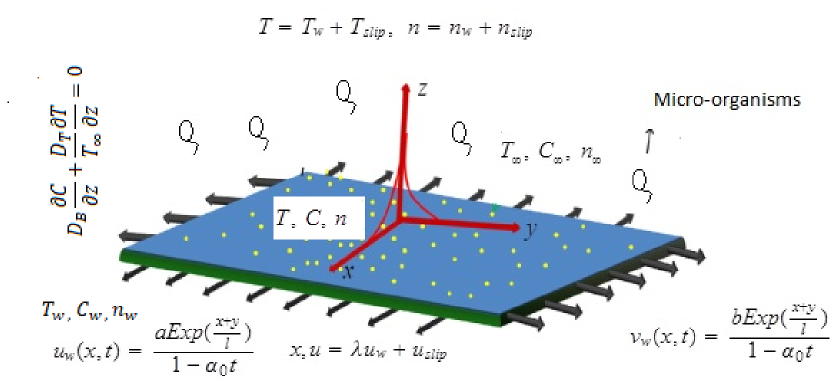
Theoretical investigation of unsteady forced bio-convective slip flow of exponentially stretching sheet

3.1 Introduction

In this chapter, we have described the three dimensional unsteady forced bio-convection viscous flow. An incompressible flow of a micropolar nanofluid encloses micro-organisms past an exponentially stretching sheet in the presence of magnetic field is analyzed. By using convenient transformation the governing partial differential equations are converted into the set of coupled non-linear ordinary differential equations. These equations are solved numerically by using shooting/bvp4c method. The influence of the determining parameters on the dimensionless velocity, micro-rotation, temperature, nanoparticle volume fraction, microorganism are incorporated. The wall shear stress, rate of transfer heat, and the rate of microorganism are analyzed graphically. The results depicts that the value of the Nusselt number and skin friction coefficient are declined while an enhancement take place in the microorganism number. The slip parameters increases the velocity, thermal energy, and microorganism number consequentially. The present investigation is important in improving achievement of microbial fuel cells.

3.2 Mathematical model

The numerical study of an unsteady incompressible micropolar nanofluid containing the microorganisms past an exponentially stretching sheet is examined. The geometry of problem is shown in Fig. (3.1). The velocity field in x, y and z directions are u, v and w respectively. n, T, C represent the density of motile micro-organisms, the temperature, and the nanoparticle volume fraction inside the boundary layer respectively, whereas T_w, C_w, n_w, represents the fluid temperature, the nanoparticle volume friction and the density of motile micro-organisms at the wall respectively, while away from the wall they are denoted by T_∞, C_∞, n_∞ respectively. The surface of plate having multiple slip and zero mass flux conditions. By considering above supposition the required equations of the motion, energy, nanoparticle and motile microorganism are described as,



Figure(3.1) = *Physical* mod *el and coordinate system*

$$\boldsymbol{\nabla} \cdot \mathbf{V} = 0, \tag{3.1}$$

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div} \boldsymbol{\tau} + \mathbf{J} \times \mathbf{B}, \tag{3.2}$$

$$\boldsymbol{\tau} = -p\mathbf{I} + \mu \mathbf{A}_1^*. \tag{3.3}$$

In above, equation (3.3) $\mathbf{I}$ denotes the identity tensor, p represents the pressure, dynamic viscosity is μ, $\mathbf{A}_1^*$ is the first Rivlin-Ericksen tensor. Mathematically $\mathbf{A}_1^*$ is expressed as

$$\mathbf{A}_1^* = \boldsymbol{\nabla} \cdot \mathbf{V} + (\boldsymbol{\nabla} \cdot \mathbf{V})^T. \tag{3.4}$$

Therefore after simplification, we get the required momentum equation in the form of the partial differentional equation along with micropolar as,

$$\rho \frac{D\mathbf{V}}{Dt} = -\boldsymbol{\nabla}\mathbf{p} + (\mu + \kappa)\nabla^2\mathbf{V} + \kappa(\boldsymbol{\nabla} \times \mathbf{N}) + \mathbf{J} \times \mathbf{B}, \tag{3.5}$$

$$\rho j \frac{D\mathbf{N}}{Dt} = \gamma\nabla^2\mathbf{N} - \kappa(2\mathbf{N} - \boldsymbol{\nabla} \times \mathbf{V}). \tag{3.6}$$

The energy equation for the present problem is,

$$\frac{DT}{Dt} = \alpha\boldsymbol{\nabla}^2 T + (\tau)\left[D_B \boldsymbol{\nabla}T \cdot \boldsymbol{\nabla}C + (\frac{D_T}{T_\infty})\boldsymbol{\nabla}T \cdot \boldsymbol{\nabla}T\right]. \tag{3.7}$$

The concentration equation for the present problem is,

$$\frac{DC}{Dt} = D_B \boldsymbol{\nabla}^2 C + (\frac{D_T}{T_\infty})\boldsymbol{\nabla}^2 T. \tag{3.8}$$

The microorganism equation for the present problem is,

$$\frac{Dn}{Dt} + \frac{\widetilde{b}W_c}{\nabla C}\left[\boldsymbol{\nabla}n \cdot \boldsymbol{\nabla}C\right] = D_m(\boldsymbol{\nabla}^2 n). \tag{3.9}$$

In this model at the wall the fluid velocity, temperature, and nanoparticle volume fraction all are varying with respect to the coordinates x, y, and t are given below,

$$u_w(x, y, t) = \frac{aExp(\frac{x+y}{l})}{1 - \alpha_0 t}, \qquad T_w = T_\infty + \frac{bExp(\frac{x+y}{2l})}{(1 - \alpha_0 t)^2},$$

$$C_w = C_\infty + \frac{b_1 Exp(\frac{x+y}{2l})}{(1 - \alpha_0 t)^2}, \text{ and } n_w = n_\infty + \frac{b_2 Exp(\frac{x+y}{2l})}{(1 - \alpha_0 t)^2}.$$

Here both α_0 and a are positive constant with dimension per unit time, b_2, b_1, and b are the constants having the dimensions microorganisms, nanoparticle volume fraction and temperature respectively.

The velocity field for the current flow problem is,

$$\mathbf{V} = \left[u\left(x,y,t\right),\ v\left(x,y,t\right),\ w(x,y,t)\right], \tag{3.10}$$

Using these assumption and imposing the boundary layer approximation the components form of above equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{3.11}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\kappa}{\rho}\frac{\partial N_2}{\partial y} - \frac{\sigma}{\rho}B^{*2}u, \tag{3.12}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial z^2} + \frac{\kappa}{\rho}\frac{\partial N_1}{\partial y} - \frac{\sigma}{\rho}B^{*2}v, \tag{3.13}$$

$$\rho j\left(\frac{\partial N_1}{\partial t} + u\frac{\partial N_1}{\partial x} + v\frac{\partial N_1}{\partial y} + w\frac{\partial N_1}{\partial z}\right) = \gamma\frac{\partial^2 N_1}{\partial z^2} - k\left(2N_1 + \frac{\partial v}{\partial z}\right), \tag{3.14}$$

$$\rho j\left(\frac{\partial N_2}{\partial t} + u\frac{\partial N_2}{\partial x} + v\frac{\partial N_2}{\partial y} + w\frac{\partial N_2}{\partial z}\right) = \gamma\frac{\partial^2 N_2}{\partial z^2} - k\left(2N_2 - \frac{\partial u}{\partial z}\right), \tag{3.15}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} + \tau\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2 + \tau D_B\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}, \tag{3.16}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial z^2}, \tag{3.17}$$

$$\frac{\partial n}{\partial t} + u\frac{\partial n}{\partial x} + v\frac{\partial n}{\partial y} + w\frac{\partial n}{\partial z} = D_m\frac{\partial^2 n}{\partial z^2} - \frac{\widetilde{b}W_c}{C_w - C_\infty}\left[\frac{\partial}{\partial z}\left(n\frac{\partial C}{\partial z}\right)\right]. \tag{3.18}$$

Here, $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$, is the material derivative with respect to time, ρ is the density of nanofluid, $\mathbf{N}$ is the micro-rotation vector, the velocity vector is $\mathbf{V}$, α is the diffusivity constant of thermal fluid, σ is the electric conductivity, γ is spin gradient viscosity, κ is the coefficient of micro - rotation viscosity, j is the density of micro - inertia, τ is the ratio of heat capacity of nanoparticle to the fluid, $\widetilde{b}$ denotes the chemotaxis constant, W_c represents the maximum cell speed D_B, D_T, and D_m represents the Brownian, thermophoretic, and microorganism diffusion coefficient respectively.

The concerned boundary conditions of the problem are assumed to be of the form,

$$\begin{aligned} &u = \lambda u_w(x,t) + \nu N_1(x,y,t)\left(\tfrac{\partial u}{\partial z}\right),\ v = \lambda v_w(y,t) + \nu N_2(x,y,t)\left(\tfrac{\partial v}{\partial z}\right), \\ &N_1 = n\tfrac{\partial v}{\partial z},\ N_2 = -n\tfrac{\partial u}{\partial z},\ \ T = T_w(x,y,t) + D_1(x,y,t)\left(\tfrac{\partial T}{\partial z}\right), \\ &D_B\tfrac{\partial C}{\partial z} + \tfrac{D_T}{T_\infty}\tfrac{\partial T}{\partial z} = 0,\ n = n_w(x,y,t) + E_1(x,y,t)\left(\tfrac{\partial n}{\partial z}\right),\quad at\ z \longrightarrow 0. \end{aligned} \tag{3.19}$$

$$u = 0,\ v = 0,\ N \longrightarrow 0,\ T \longrightarrow T_\infty,\ C \longrightarrow C_\infty,\ n \longrightarrow n_\infty,\quad at\ z \longrightarrow \infty. \tag{3.20}$$

Here, we define velocity, thermal and microorganism factor by,
$N_1(x,y,t) = (N_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, $D_1(x,y,t) = (D_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, and $E_1(x,y,t) = (E_1)_0\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$, respectively. In our study we have assumed mass flux is zero as, $\frac{D_T}{T_\infty}\frac{\partial T}{\partial z} + D_B\frac{\partial C}{\partial z} = 0$. Suggested by Nield and Kuznetsov [37] simultaneously slip boundary conditions are taken by saturated fluid wall system [38] and surface irregularity [39]. Now we introduce specified dimensionless variables of the following form,

$$\begin{aligned} \eta &= z\sqrt{\frac{a}{\nu(1-\alpha_0 t)}}Exp(\frac{x+y}{2l}), \\ N_1 &= F(\eta)\sqrt{\frac{a^3}{\nu(1-\alpha_0 t)^3}}Exp(\frac{3(x+y)}{2l}), \\ N_2 &= G(\eta)\sqrt{\frac{a^3}{\nu(1-\alpha_0 t)^3}}Exp(\frac{3(x+y)}{2l}), \\ T &= T_\infty + \frac{bExp(\frac{x+y}{2l})}{(1-\alpha_0 t)^2}\theta(\eta), \\ C &= C_\infty + \frac{b_1Exp(\frac{x+y}{2l})}{(1-\alpha_0 t)^2}\phi(\eta),\ n = n_\infty + \frac{b_2Exp(\frac{x+y}{2l})}{(1-\alpha_0 t)^2}\chi(\eta), \\ \theta &= \frac{T-T_\infty}{T_w-T_\infty},\ \phi = \frac{C-C_\infty}{C_w-C_\infty},\ \chi = \frac{n-n_\infty}{n_w-n_\infty}. \end{aligned} \tag{3.21}$$

Here η is the variable of similarity and $f(\eta)$, $h(\eta)$, $\theta(\eta)$, $\phi(\eta)$ and $\chi(\eta)$ are the dimensionless variables of the linear velocity, micro-rotation, temperature, nanoparticle volume fraction and microorganisms respectively. The components of velocity are defined as,

$$\begin{aligned} u &= \frac{af'(\eta)Exp(\frac{x+y}{l})}{(1-\alpha_0 t)},\ v = \frac{ag'(\eta)Exp(\frac{x+y}{l})}{(1-\alpha_0 t)}, \\ w &= -(f + \eta f' + g + \eta g')\sqrt{\frac{a\nu}{2l(1-\alpha_0 t)}}Exp(\frac{x+y}{2l}). \end{aligned} \tag{3.22}$$

In above equation (3.22) u, v, and w denotes the velocity components in x, y, and z direction respectively, prime denotes the differentiation with respect to η. Using similarity variables transformation above system of equations take the form,

$$(1+\delta)f''' + (g+f)\,f'' - 2(g'+f')f' - A(2f'+\eta f'') - Mf' - \delta G' = 0, \tag{3.23}$$

$$(1+\delta)g''' + (f+g)g'' - 2(g'+f')g' - A(2g'+\eta g'') - Mg' + \delta F' = 0, \tag{3.24}$$

$$\lambda_0 F'' + (g+f)\,F' - 3(g'+f')F - A(3F+\eta F^{'}) - \delta B(2g''+4F) = 0, \tag{3.25}$$

$$\lambda_0 G'' + (g+f)\,G' - 3(g'+f')G - A(3G+\eta G') + \delta B(2f''+4G) = 0, \tag{3.26}$$

$$\frac{1}{\Pr}\theta'' + (g+f)\theta' - (g'+f')\theta - A(4\theta+\eta\theta') + Nb\theta'\phi' + Nt\theta'^2 = 0, \tag{3.27}$$

$$\frac{1}{Sc}\phi'' + (g+f)\,\phi' - (g'+f')\phi - A(4\phi+\eta\phi') + \frac{Nt}{Nb}\frac{1}{Sc}\theta'' = 0, \tag{3.28}$$

$$\frac{1}{Sb}\chi'' + (g+f)\,\chi' - (g'+f')\chi - A(4\chi+\eta\chi') - \frac{Pe}{Sb}(\chi\phi''+\chi'\phi') = 0. \tag{3.29}$$

The relevant boundary conditions expressed as,

$$\begin{aligned}
f(\eta) &= 0,\ f'(\eta) = \lambda + \delta_\nu f''(\eta),\ f(\eta) = 0, g(\eta) = 0,\ g'(\eta) = \lambda c + \delta_\nu g''(\eta),\\
F(\eta) &= n g''(\eta),\ G(\eta) = -n f''(\eta), \qquad (3.30)\\
\theta(\eta) &= 1 + \delta_T\theta'(\eta),\ Nt\theta'(\eta) + Nb\phi'(\eta) = 0,\ \chi(\eta) = 1 + \delta_n\chi'(\eta),\ \text{when}\ \ \eta \longrightarrow 0.\\
f'(\eta) &= 0,\ g'(\eta) = 0,\ \ F(\eta) = 0,\ \ G(\eta) = 0,\ \theta(\eta) = 0,\\
\phi(\eta) &= 0,\ \chi(\eta) = 0 \text{ when } \eta \to \infty. \qquad (3.31)
\end{aligned}$$

In the above system of non-linear ordinary differential equations, the parameters are defined as, micropolar parameter is $\delta = \frac{\kappa}{\mu}$, $A = \frac{\alpha_0}{a}$ is the unsteadiness parameter, $c = \frac{b}{a}$ is stretching parameter, $M = \frac{\sigma l B^{*2}}{\rho u_w}$ is magnetic field parameter, $\lambda_0 = \frac{\gamma}{j\mu}$ (spin gradient viscosity parameter), $B = \frac{\nu}{j u_w}$ (micro-inertia density parameter), n is micro-gyration parameter, $Nb = \frac{\tau D_B \Delta C}{\nu}$ (Brownian motion parameter), $Nt = \frac{\tau D_T \Delta T}{\nu T_\infty}$ (thermosphoresis parameter), $Sc = \frac{\nu}{D_B}$ (Schmidt number), $Sb = \frac{\nu}{D_m}$ (bio-convection Schmidt number), $Pe = \frac{b W_c D_m}{\nu^2}$ (bio-convection Peclet number) and $\Pr = \frac{\upsilon}{\alpha}$ (Prandtl number). Also we define the velocity slip, thermal slip and microorganism slip factor as,

$$\begin{aligned}\delta_\nu &= (N_1)_0,\ \delta_T = (D_1)_0\frac{1}{\sqrt{\nu}},\\ \delta_n &= (E_1)_0\frac{1}{\sqrt{\nu}}.\end{aligned}\tag{3.32}$$

Note N_1, D_1, and E_1 are proportional to $\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$. In above equations c and λ, is the constant of stretching, with $\lambda < 0$ for shrinking, $\lambda = 0$ for stationary and $\lambda > 0$ for stretching plate.

3.2.1 Physical Quantities

Physical quantities are very important from an engineering point of view and tells us the characteristics of flow, rate of heat transfer and microorganism transfer rate. The skin friction coefficient are Cf_x and Cf_y in the direction of x and y respectively as well as the local Nusselt number and the local motile microorganism flux are defined as,

$$\begin{aligned}C_{fx} &= \frac{\tau_{wx}}{\rho u_w^2},\ \ C_{fy} = \frac{\tau_{wy}}{\rho u_w^2},\ \ Nu_x = \frac{xq_w}{T_w - T_\infty},\ \ \ Q_{nx} = \frac{xj_w}{D_m n_w}\\ \tau_{wx} &= 2\left[(\mu+\kappa)\left.\frac{\partial u}{\partial z}\right|_{z=0} + \kappa(N_2)_{z=0}\right],\ \ \tau_{wy} = 2\left[(\mu+\kappa)\left.\frac{\partial v}{\partial z}\right|_{z=0} + \kappa(N_1)_{z=0}\right],\\ q_w &= -\kappa\left.\frac{\partial T}{\partial z}\right|_{z=0},\ \ j_w = -D_B\left.\frac{\partial n}{\partial z}\right|_{z=0}.\end{aligned}\tag{3.33}$$

In above equations τ_{wx} and τ_{wy} are the shear stresses in the direction of x and y respectively. Also the surface heat flux q_w and motile microorganism flux is j_w. Using above similarity variables transformation we get,

$$\mathrm{Re}_x^{1/2}\, C_{fx} = 2(1+\delta(1-n))f''(0),\tag{3.34}$$

$$\mathrm{Re}_x^{1/2}\, C_{fy} = 2(1+\delta(1+n))g''(0),\tag{3.35}$$

$$\mathrm{Re}_x^{-1/2}\, Nu_x = -\theta'(0), \quad (3.36)$$

$$\mathrm{Re}_x^{-1/2}\, Q_{nx} = -\chi' 0). \quad (3.37)$$

Where $Re_x = u_w\sqrt{\frac{(1-\alpha_0 t)}{\nu a}}$ is denotes the Local Reynold number.

3.3 Numerical solution

We converted the partial differential equations into the set of coupled non-linear ordinary differential equations. These equations are solved numerically by using shooting/bvp4c method. To apply these method into the set of non-linear ordinary differential equations we converted these equations into the system of 1^{st} order equations as,

$$\begin{aligned} f &= y_1,\ f' = y_2,\ f'' = y_3,\ g = y_4,\ g' = y_5,\ g'' = y_6,\ F = y_7,\ F' = y_8, \\ G &= y_9,\ G' = y_{10},\ \theta = y_{11},\ \theta' = y_{12},\ \phi = y_{13},\ \phi' = y_{14},\ \chi = y_{15},\ \chi' = y_{16}. \end{aligned} \quad (3.38)$$

Also relevant boundary conditions are converted as,

$$\begin{aligned} y_1(0) &= 0,\ y_2(0) = \lambda + \delta_\nu y_3(0),\ y_4(0) = 0,\ y_5(0) = \lambda c + \delta_\nu y_6(0),\ y_7(0) = n y_6(0), \\ y_9(0) &= -n y_3(0),\ y_{11}(0) = 1 + \delta_T y_{12}(0),\ Nb y_{14}(0) + Nt y_{12}(0) = 0, \\ y_{15}(0) &= 1 + \delta_n y_{16}(0), \\ y_2(\infty) &= 0,\ y_5(\infty) = 0,\ y_7(\infty) = 0,\ y_9(\infty) = 0,\ y_{11}(\infty) = 0,\ y_{13}(\infty) = 0, \\ y_{15}(\infty) &= 0. \end{aligned} \quad (3.39)$$

3.4 Results and discussion

The solution of above equations are found numerically by using shooting/bvp4c method. The dominant efficiencies of the flow behavior can be determined with the mathematical model and computational simulations. An analysis is performed by using different parameters. The conclusions are conducted in the form of dimensionless velocity, temperature, nanoparticles volume fraction, micro-rotation, and microorganisms. Further, local skin friction, rate of heat transfer, and rate of microorganism are studied. In this analysis we investigated the influence of different parameters on the behavior of flow and also the behavior of microorganism and heat transfer.

In Fig. (3.2), it is noted that the increment in the value of stretching parameter λ, the fluid velocity at the wall is increases. The parameter λ is stretching constant and have different value for stretching plate $\lambda < 0$, for shrinking $\lambda = 0$, and for stationary plate its value is $\lambda > 0$. In Fig. (3.3) it can be noted that increment in the value of unsteadiness parameter A, the velocity field rises. The unsteadiness parameter is directly proportional to the stretching velocity rate, as the stretching velocity rate increases then the unsteadiness parameter is also increases and vice versa. Fig. (3.4) illustrates the impact on the velocity field due to material parameter δ. It is found that reduction is occur in the velocity distribution when material parameter rises. The material parameter have inverse relation with dynamic viscosity, increment in the dynamic viscosity the reduction is occur in the material parameter. Fig. (3.5) is sketched to see the impact of magnetic field on velocity. The velocity field decreases due to increment in the magnetic field parameter. It is depicted in Fig. (3.6) that the velocity slip parameter δ_ν increases, the velocity profile reduce. The impact of spin gradient viscosity parameter λ_0 on micro - rotation profile is depicted in Fig. (3.7) and (3.8). In Fig. (3.7) the micro - rotation profile decreases when λ_0 is increased while the micro - rotation profile increases when λ_0 is increased in Fig. (3.8). The effects of micro - inertia density parameter B on micro - rotation profile is presented in Fig. (3.9). From this figure an enhance in micro - rotation profile is observed with an increment in B. In Fig. (3.10), the temperature profile is shown for various value of thermal slip parameter δ_T. The temperature profile found to be decreases with increasing thermal slip parameter. Fig. (3.11) designates the influence of thermophoresis parameter Nt on the temperature. It is illustrates that increases the value of Nt, the temperature is rises as well as thermal boundary layer becomes thicker. Fig. (3.12) illustrates the impact of Prandtl number Pr on the temperature profile. It is exhibited that, both temperature and thermal boundary layer thickness are reduces by increase the value of Pr. It designates that the larger value of Pr cause the reduction in the thermal diffusivity. Fig. (3.13) display the influence of Brownian motion parameter Nb on concentration. A reduction in concentration profile is observed by the increment in Brownian motion parameter Nb. Fig. (3.14) depicts the concentration field for changing Schmidt number Sc. Since Schmidt number expressed the ratio of momentum diffusivity (viscous diffusion rate) to species diffusivity (molecular diffusion rate). It has been noted that increment in Sc leads to decay in nanoparticle concentration distribution. The behavior of microorganism slip parameter δ_n on microorganism is reported in Fig. (3.15). The density of motile microorganism is rises, when microorganism slip factor growing. Fig. (3.16) and (3.17) shows the effects of Pe and Sb on microorganism profile. Microorganism is found to be increased with increasing bio-convection Peclet number. Since Pe is directly proportional to maximum cell swimming motion and chemotaxis constant and inversely to diffusivity of micro-organisms. Therefore for larger value of Pe the micro-organisms profile will be increased while reduction occur in the motile microorganism density and boundary layer thickness with enhancing the value of bio - convection Schmidt number Sb.

The numerical values of skin frictions for different parameter M, δ, A, and B shown in Table (3.1). it can be seen that for increment the value of magnetic parameter M, material parameter δ and unsteadiness parameter A the numerical value of skin friction enhances, whereas the value of skin friction reduces due to increment in the value of micro-inertia parameter B The effect of stretching parameter on Nusselt number and microorganism number are observed in (3.18) and (3.19). Fig. (3.18) shows that enhance the stretching parameter, the increment in the rate of heat transfer is occur. The heat transfer rate for the shrinking sheet is less than the stretching sheet. The negative sign shows that the heat flow from higher to lower temperature from shrinking/stretching sheet. The change in the heat transfer rate of velocity slip and microorganism slip parameters are not observed. Fig. (3.19) shows that the stretching parameter is increases, then the microorganism transfer rate is enhanced. These conclusions are adopting to developing the performance of microbial fuel cells. The microbial fuel cells leading to the change in the efficiency of fuel cell, specially at the fuel cell wall.

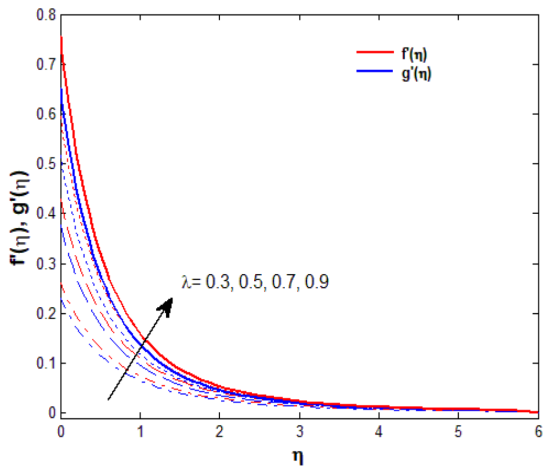


Fig. (3.2); Impact of λ on velocity profile.

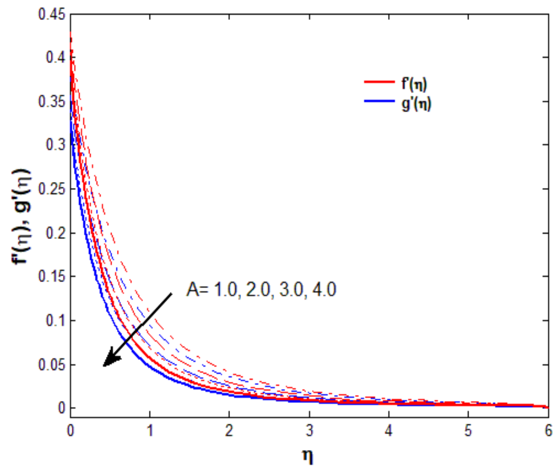


Fig. (3.3); Impact of A on velocity profile.

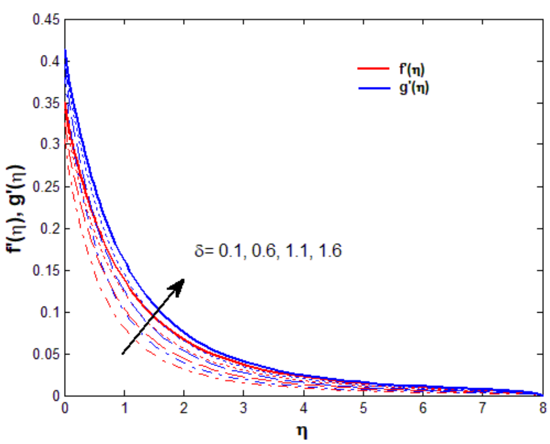


Fig. (3.4); Impact of δ on velocity profile.

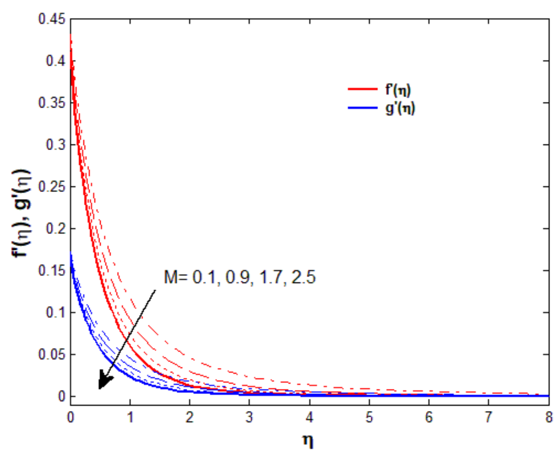


Fig. (3.5); Impact of M on velocity profile.

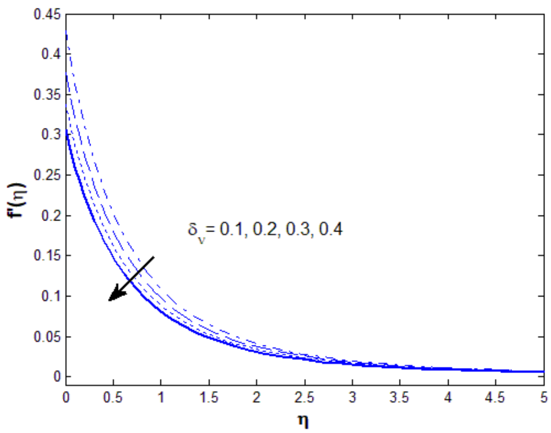


Fig. (3.6); Impact of velocity slip δ_ν on velocity profile.

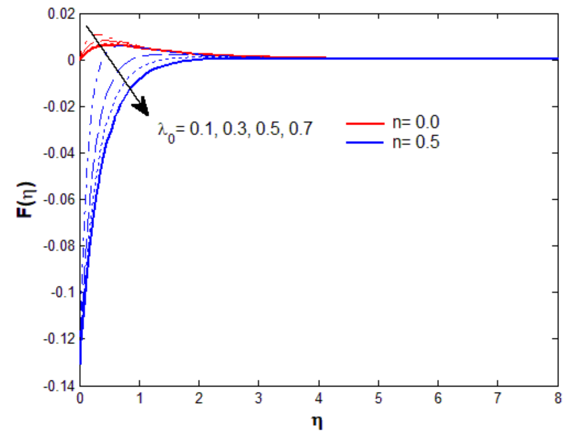


Fig. (3.7); Impact of λ_0 on angular velocity field $F(\eta)$.

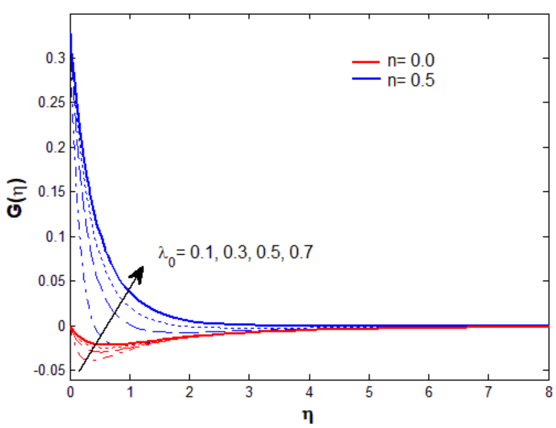


Fig. (3.8); Impact of λ_0 on angular velocity field $G(\eta)$.

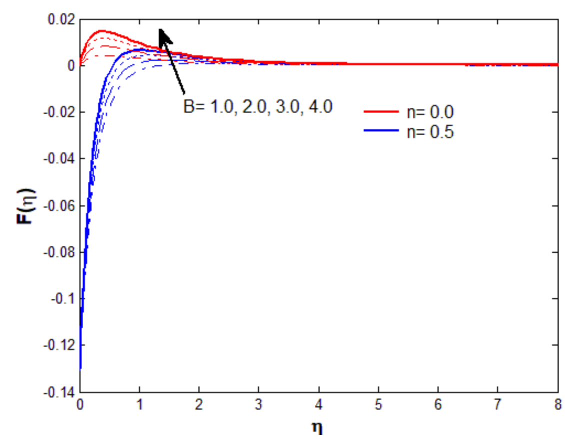


Fig. (3.9); Impact of B on angular velocity field $F(\eta)$.

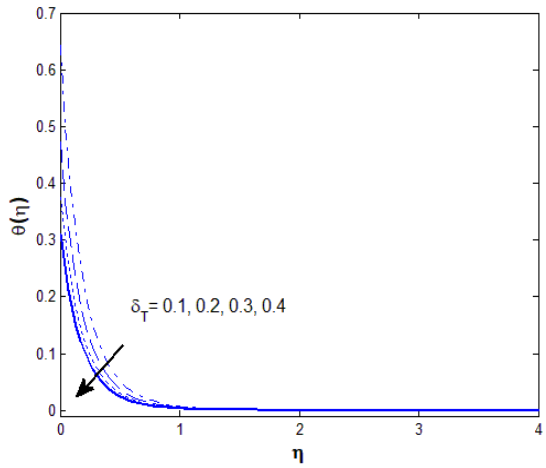


Fig. (3.10); Impact of thermal slip δ_T on temperature profile.

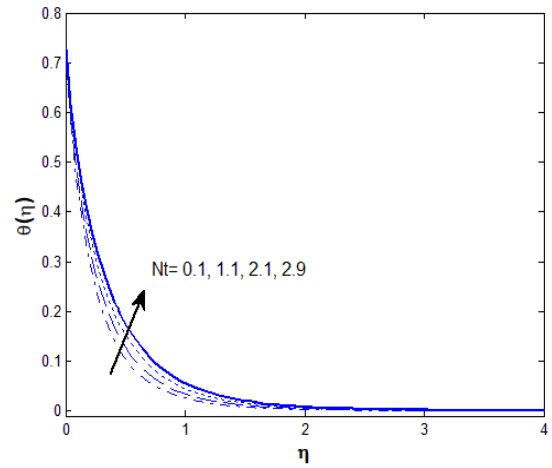


Fig. (3.11); Impact of Nt on temperature profile.

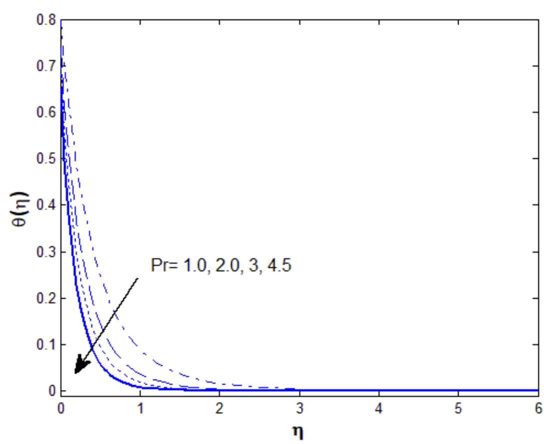


Fig. (3.12); Impact of Prandtl number Pr on temperature profile.

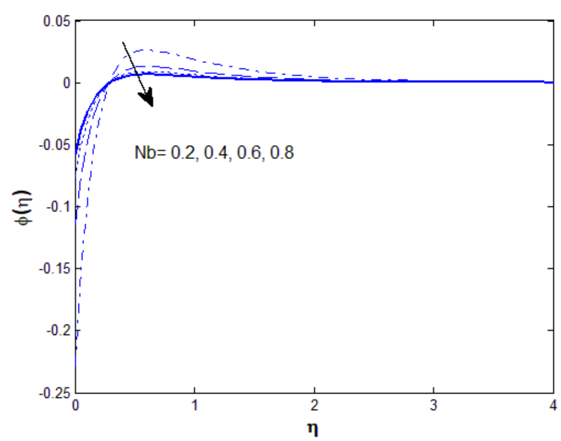


Fig. (3.13); Impact of Nb on concentration profile.

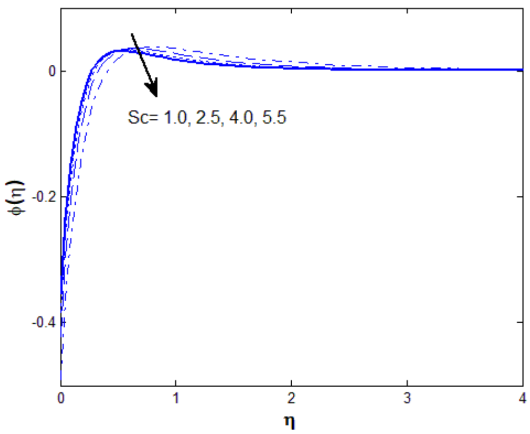


Fig. (3.14); Impact of Sc on concentration profile.

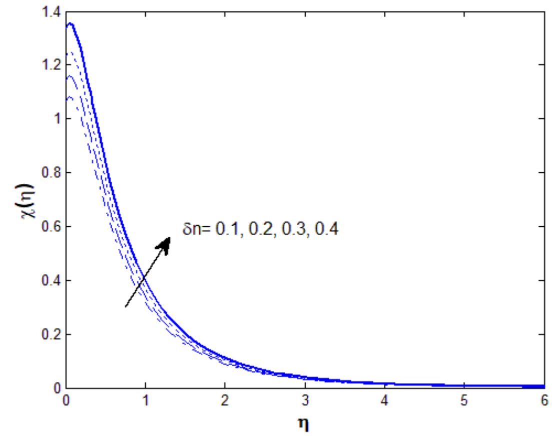


Fig. (3.15); Impact of δ_n on microorganism profile.

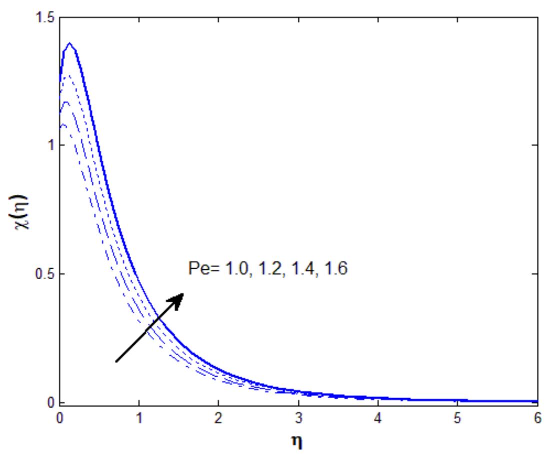


Fig. (3.16); Impact of Pe on microorganism profile.

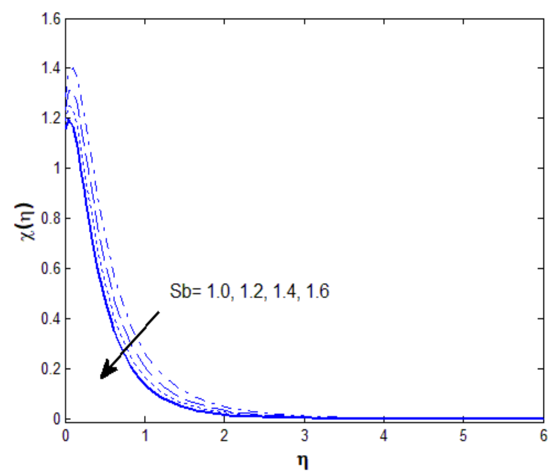


Fig. (3.17); Impact of Sb on microorganism profile.

3.5 Physical quantities

Table (3.1); Numerical value $f''(0)$, and $g''(0)$ for M, δ, A, B .

M	δ	A	B	$-\mathrm{Re}_x^{\frac{1}{2}}\, C_{fx}$	$-\mathrm{Re}_x^{\frac{1}{2}}\, C_{fy}$
0.1	0.1	0.1	0.2	1.21662	1.22813
0.2				1.24812	1.25560
0.3				1.27808	1.28311
	0.2			1.34265	1.34912
	0.3			1.40845	1.41124
	0.4			1.46333	1.47254
		0.2		1.32115	1.32219
		0.3		1.34979	1.34983
		0.4		1.36201	1.36225
		0.1	0.3	1.36167	1.36188
			0.5	1.36121	1.36153
			0.7	1.36072	1.36122

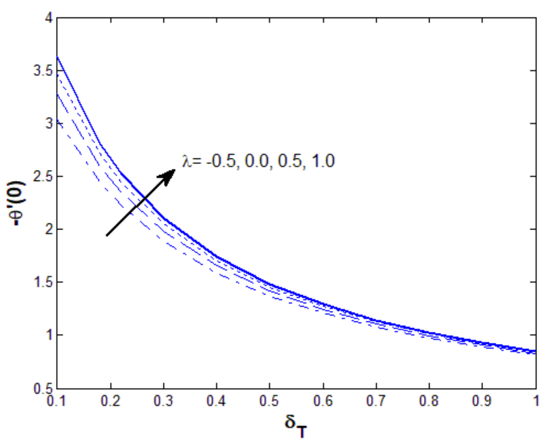


Fig. (3.18); Depict the behavior of Nusselt number for λ.

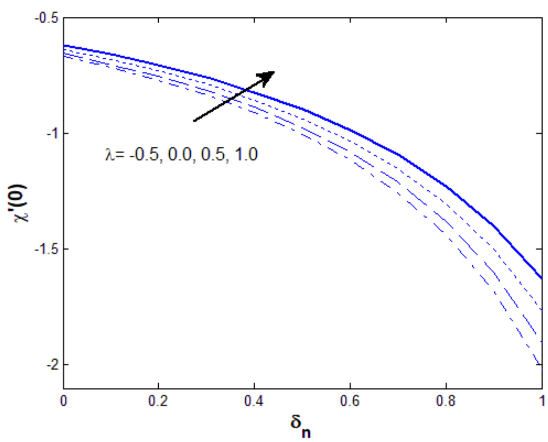


Fig. (3.19); Depict the behavior of microorganism number for λ.

3.6 Concluding remarks

We observed numerically an unsteady three dimensional viscous incompressible flow containing micropolar nanofluid over a stretching sheet. Converted mathematical model into the set of coupled ordinary differential equations by applying suitable transformation. These equations are dealed with numerically by using shooting method. The assumed conclusions are granted graphically to illustrate the effects of different parameters on the velocity, temperature,nanoparticle volume fraction, micro - rotation, and motile microorganism profile. Furthermore the local skin friction, the heat transfer rate and the microorganism rate are observed graphically. The important points of the flow problem are highlighted as,

▶As increases the value of parameters λ and δ, the velocity profile rises. Also for the parameters A, δ_ν and M the velocity profile reduces by increment in the values of A, δ_ν and M.

▶When the value of spin gradient viscosity parameter increases, the micro-rotation profile for $F(\eta)$ is decreases while for $G(\eta)$ it increases. Also for micro-inertia parameter, the the micro - rotation profile $F(\eta)$ increases by increment in the value of B.

▶The temperature profile is decreases for the Prandtl number and increases for the thermophoresis parameter by the increment of $\Pr$ and Nt.

▶ Increment in the value of Schmidt number and Brownian motion parameter, the concentration profile is decline for stretching sheet.

▶ Microorganism profile is rises for the Peclet number and microorganism slip parameter while it decreases for the bio - convection Schmidt number by increment in the value Pe, δ_T, and Sb.

▶ The local skin friction reduces for the various values of micropolar parameter.

▶ When rises the value of stretching parameter λ, the local Nusselt number and local microorganism number is increases.

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