# Quality Control Charts for Monitoring Process Mean Using Varied L Ranked Set Sampling

By

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بسم الله الرحمن الرحيم

*In The Name of Allah The Most Merciful and The Most Beneficent*

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*A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY IN STATISTICS*

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# Declaration

I "Muhammad Awais" hereby solemnly declare that this thesis entitled "Quality Control Charts for Monitoring Process Mean Using Varied L Ranked Set Sampling", submitted by me for the partial fulfillment of Master of Philosophy in Statistics, is the original work and has not been submitted concomitantly or latterly to this or any other university for any other Degree.

Dated: ________________ Signature: ________________

# Dedication

*I am feeling great honor and pleasure to dedicate this research work to*

**My Beloved Parents**

*Whose endless affection, prayers and wishes have been a great source of comfort for me during my whole education period*

# Acknowledgments

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# Abstract

In the statistical process control literature, there exist several improved quality control charts based on cost-effective sampling schemes, including the ranked set sampling (RSS) and median RSS (MRSS). A generalized cost-effective RSS scheme has been recently introduced for efficiently estimating the population mean, namely varied L RSS (VLRSS). In this thesis, we construct new quality control charts for monitoring the process mean using VLRSS scheme, including the cumulative sum (CUSUM), the exponentially weighted moving average (EWMA), the Shewhart-EWMA and Shewhart-CUSUM charts, with perfect and imperfect rankings. Extensive Monte Carlo simulations are used to compute the run length characteristics of the proposed control charts. These control charts are then compared with their existing counterparts based on simple random sampling, RSS and MRSS schemes. It is found that, with either perfect or imperfect rankings, the proposed control charts perform better than the existing control charts in detecting small to moderate shifts in the process mean. A real life dataset is also used to elucidate the implementation and working of the proposed control charts. An algorithm to generate a varied L ranked set sample is given in the Appendix.

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# Chapter 1

# Introduction

Quality can be defined in many ways. Some people have understanding of quality as relating to one or more desirable characteristics that a product or service should possess. However, quality has become an important consumer decision factor in selecting among competing products and services regardless of whether the consumer is an individual, an industrial organization, a retail store, a bank or financial institution, or a military defence program. Therefore, understanding and improving quality of the products are key factors that lead to success of a business, growth, and enhanced competitiveness.

In the present world of increasing global competition it is crucial for all manufacturing and service providing organizations/industries to improve the quality of their products. The quality of product depends on a number of factors which includes the organization itself, and control of the firm's employees, and speaking from technical point of view the quality of design and the production quality. Quality control describes the directed use of procedures to measure the achievement of a desired standard.

## 1.1 Statistical process control

The statistical process control (SPC) is a set of statistical tools that are used to monitor, control; and hence, improve the quality of the output of a production process. There are seven major tools in the SPC that are discussed below:

- **Histogram:** the commonly used graph for representing the frequency distributions, or to see how often each value in a set of data occurs.
- **Check sheet:** a structured, well-prepared form for collecting and analyzing the data: It is a general tool that can be adapted for various purposes.
- **Pareto chart:** a bar graph showing those factors that are more significant.
- **Cause and effect diagram:** also called fishbone diagram, it identifies several potential causes or problems and classify them into useful categories.

- **Defect concentration diagram:** a graphical tool which is useful in analyzing the causes of the product or parts that are defective.
- **Scatter diagram:** a graph in which the values of two variables are plotted to identify the relationship between the variables.
- **Control charts:** graphical tools that are used to see/study how a process changes over time.

Control charts are very effective tool of the SPC in distinguishing between two types of variations, namely natural- and special-cause variations. A process is considered to be in statistical control when it operating only in the presence of natural-cause variations, which are inherent part of a process (cf., Montgomery (2007)). However, a process is declared out-of-control when it is operating in the presence of special-cause variations. Quality control charts are often considered one of the more simplistic online SPC techniques, but can prove to be very powerful when used properly. The ultimate goal of any control chart is to detect the process variability and to improve the performance of a production process.

## 1.2 Control charts

The idea of control charts was first introduced by Walter A. Shewhart in the 1920s, for achieving statistical stability in a production process. Nowadays, control charts have wide applications in variety of different fields like quality control, fisheries, signal segmentation, navigation system monitoring, and hospital monitoring, to name a few, for monitoring, control and ultimately improving processes. Despite the fact that the initial aim of a control chart was to attain the statistical stability for a given process ("process control"), but since its introduction, many improvements have been suggested in the control charting schemes, like the cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts—more advanced process monitoring tools.

The control chart is a graphical representation of the quality statistic versus the sample number or time, in which samples are taken from a process at regular intervals of time, and the quality characteristics are measured. Some sample statistic, say the sample mean, is measured and is then plotted against the time or against the sample number and then compared to a pair of limits, called the control limits. If the sample statistic falls between the control limits, then it is assumed that the process is in-control. But, if the sample statistic falls outside the control limits, then it indicates that the process is working with the special-cause variations or the process is said to be out-of-control. Hence, investigation and corrective actions are taken to bring the process back into the control state.

The statistical control charting process is characterized into two phases, namely the Phase-I and Phase-II, with two different and distinct objectives. In Phase-I, a large historical sample of observations is evaluated in a retrospective analysis to determine the state of an in-control process over the period of time where the data were collected to estimate the unknown parameters.

Once the controlled state is achieved, the next step is to see if the reliable control limits can be constructed to monitor the future production. Phase-II begins after we have a clean set of process data collected under stable conditions and are representative of an in-control process performance. In Phase-II, generally it is assumed that the process is statistically controlled. Phase-II involves monitoring the process by comparing the sample statistic of each successive sample.

Like the statistical hypothesis, there are two types of errors in the control charting procedure. Type-I error is concluding the process out-of-control when actually it is in-control. Similarly, declaring the process in-control when actually it is out-of-control is Type-II error. Control charts are designed in such a way that both of these errors get minimized. However, this may not be the situation, i.e., an out-of-control may sometime be a false alarm. A false alarm is an out-of-control signal generated when actually the process is in-control. Hence, in order to prolong the reliability of a control chart, a good choice of control limits is required, which is a compromise between swiftly detecting a process change and avoiding a high false alarm rate. When the sample statistics that are being plotted are normal and independent then using three-sigma control limits (a common choice used in the control chart structure) for a control chart with the known parameters correspond to a false alarm once in every 370.4 samples on the average. Here, the term "sigma" refers to the standard deviation of the plotting statistic.

The control charts are characterized into two categories, namely the memory-less and memory-type charts. The well-known memory-less control charts includes the classical Shewhart-type charts, that were first introduced by Walter A. Shewhart in 1920s. These type of charts completely rely only on the current observations and do not take into account any past information. Because of this reason the Shewhart charts are more effective in detecting large shifts in a process. On the other hand, the memory-type charts do take into account the past information in order to update their plotting-statistics. This feature helps them in reacting swiftly against small to moderate shifts in the process parameter(s). The well-known memory-type charts are the CUSUM and the EWMA charts.

In this research the performances of the CUSUM, EWMA, and combined Shewhart chart with the EWMA and CUSUM charts are enhanced for monitoring the process mean with the help of a cost-effective sampling scheme, namely varied L ranked set sampling (VLRSS). These control charts are compared with the existing control charts in terms of run length characteristics.

In the Chapter 2, a brief review of the classical CUSUM chart along with its fast initial response (FIR) feature is presented. We propose a new CUSUM chart using VLRSS scheme for monitoring the process mean. We show that the proposed CUSUM chart with the VLRSS scheme is more sensitive than the CUSUM charts based on simple random sampling (SRS), RSS, and median RSS (MRSS) schemes when monitoring the process mean. The FIR feature is also discussed in the proposed chart. The performances of the proposed and existing charts are assessed in terms of the average run length (ARL) and the standard deviation of the run length (SDRL).

In the Chapter 3, we discuss the classical EWMA chart along with its FIR feature. A new

EWMA chart is proposed using the VLRSS scheme, and its run length performance is then compared with those of the existing EWMA charts based on SRS, RSS, and MRSS schemes when monitoring the process mean. The comparison is made in terms of ARL and SDRL. It is shown that the proposed EWMA chart perform well in detecting small to moderate shifts in the process mean than the EWMA charts based on SRS, RSS and MRSS schemes.

In the Chapter 4, a review of the classical Shewhart-EWMA and Shewhart-CUSUM charts is presented. Their FIR features are also discussed. New Shewhart-EWMA and Shewhart-CUSUM charts using the VLRSS scheme for monitoring the process mean are proposed. Their run length performances are compared with those of the existing Shewhart-EWMA and Shewhart-CUSUM charts, respectively, based on SRS, RSS and MRSS schemes. The proposed combined charts are found to be more effective than their existing counterparts.

Finally, in Chapter 5 we summarize the main findings and discuss the general conclusions. Further, some future recommendations are given which stem this work.

# Chapter 2

# A New Cumulative Sum Control Chart for Monitoring the Process Mean using Varied L Ranked Set Sampling

Recently, a varied L ranked set sampling (VLRSS) scheme has been introduced in the literature for estimating the population mean. The VLRSS scheme is a cost-efficient alternative to the classical ranked-set sampling (RSS) and median RSS (MRSS) schemes. In this chapter, we construct a new cumulative sum (CUSUM) control chart for efficiently monitoring the process mean using VLRSS, named the CUSUM-VLRSS chart. The CUSUM-VLRSS chart encompasses the existing CUSUM charts based on RSS and MRSS schemes, named the CUSUM-RSS and CUSUM-MRSS charts. We use extensive Monte Carlo simulations to compute the run length characteristics of the proposed CUSUM chart. It is shown that the CUSUM-VLRSS chart is able to perform uniformly better than the CUSUM-RSS and CUSUM-MRSS charts when detecting different kinds of shift in the process mean. A similar trend is observed when the proposed CUSUM chart is constructed under imperfect rankings. A real data set is also used to explain the implementation of the CUSUM-VLRSS chart.

## 2.1 Introduction

The statistical process control (SPC) includes the comparison of output of a service/process with the standard/target and then taking corrective actions when the discrepancy is present between the two. It also determines whether the process is in statistical control or not. In the SPC toolkit, the control charts are very important graphical tool that monitor the activity of an ongoing process. They are now frequently used in different applied fields, like the quality control, signal segmentation, fisheries, hospital monitoring, and navigation system monitoring, to name a very few, for monitoring the industrial processes and to give relevant feedback on the performance of a system or manufacturing process. Walter A. Shewhart was the first to introduce the concept of a control charts in the 1920s. Since then a variety of quality control charts have been developed in the SPC literature. The most commonly used control charts include the Shewhart, cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) charts. The main object of these control charts is to detect a shift in the process parameter(s) lest the defective items are produced. For more details, we refer to Oakland (2007), Montgomery (2007), Hawkins and Olwell (2012) and Qiu (2013).

The control charts are classified into two categories, memory-type and memory-less control charts. The Shewhart-type charts lie in the class of memory-less control chart. The reason being they are solely based on the current realizations of a process and do not consider the past relevant information. On the other hand, we have memory-type control charts. The most popular statistical process monitoring control charts include the CUSUM and EWMA charts. These charts are not only based on current information but also use the past information to update their plotting statistics. This feature helps memory-type control charts to swiftly respond against the small and moderate persistent shifts in the process parameters. The CUSUM and EWMA charts were first introduced by Page (1954) and Roberts (1959), respectively. These control charts are often used in chemical and process industries to detect small process disturbances where small shifts may impose serious financial penalties (cf., Montgomery (2007)). Lucas (1982) suggested using fast initial response (FIR) feature in the CUSUM chart to retrieve its performance for the start-up process shifts. Lucas (1982) attached the Shewhart chart with the CUSUM chart, named the Shewhart-CUSUM chart, to design a single control chart that is able to detect small as well as large shifts in the process mean. For some more relative works and interesting findings on the CUSUM and EWMA charts, we refer to Knoth (2005), Lucas and Saccucci (1990), Chiu (2009), Abbas et al. (2013), Haq (2013), Haq et al. (2014a) and the references cited therein.

The ranked-set sampling (RSS) scheme was first introduced by McIntyre (1952) for estimating mean pasture and forage yields. The RSS scheme is an efficient alternative to the simple random sampling (SRS) when it is possible to rank the values of the study variable visually or using ranks of an auxiliary variable—provided that it is highly correlated with the study variable. As an example, suppose that the quantification of the underlying quality characteristic(s) is difficult, expensive, time-consuming, and it may involve breaking the product (which is very expensive, and hard to construct, etc.), but it maybe possible to rank the quality characteristic according to its quality by using any cheap or non-destructive methods, e.g., testing the weight or shape of the product, or using expert's judgment, etc. The RSS scheme is able to draw a much more representative sample from the target population than that using the SRS scheme. There are situation where the error in ranking is inevitable, particularly when dealing with the large set sizes. Dell and Clutter (1972) showed that, despite the presence of ranking errors, the sampling not only remain an unbiased estimator of the population but it is more precise than the sample mean based on SRS. Stokes (1977) used a simple model for imperfect ranking and showed that the study variable could can be ranked using the ranks of a cheap auxiliary/concomitant variable. In some later interesting works, Samawi et al. (1996) and Muttlak (1997) introduced unbalanced RSS schemes, named extreme RSS (ERSS) and median RSS (MRSS), respectively, for estimating the population. The MRSS (ERSS) scheme is more efficient than the RSS (SRS) scheme when estimating the mean of a symmetric population. Muttlak (2003) investigated the use of quartile RSS (QRSS) for estimating the population mean. Al-Nasser (2007) suggested a generalized version of the RSS scheme, named the L RSS (LRSS), for estimating the population mean. The LRSS scheme is based on the idea of L moments, like the trimmed or winsorized means, and hence the name LRSS. Moreover, the LRSS scheme encompasses several existing RSS scheme, including RSS, QRSS, ERSS, and MRSS. Haq et al. (2015e) extended the work on LRSS scheme and further generalized the LRSS scheme, named the varied LRSS (VLRSS) scheme, for estimating the population mean. The VLRSS scheme is cost-effective alternative to the RSS, QRSS, ERSS and MRSS schemes when estimating the mean of a symmetric population (cf., Haq et al. (2015e)).

In the past decades, the RSS scheme has had popularity in constructing more sensitive quality control charts than those with the SRS scheme. As the mean estimator with the RSS scheme is more precise than the mean estimator with the SRS scheme, this fact has motivated the researchers to propose memoryless and memory-type control charts using the RSS protocol. Salazar and Sinha (1997) were the first ones to suggest the Shewhart control chart for monitoring the process mean using RSS. Later on, Muttlak and Al-Sabah (2003) explored the run length performances of several Shewhart mean control charts using RSS, median RSS (MRSS) and extreme RSS (ERSS) schemes under perfect and imperfect ranking settings. Furthermore, Abujiya and Muttlak (2004) and Al-Omari and Haq (2012) has constructed new Shewhart control charts using double RSS schemes for monitoring changes in the process mean. Al-Sabah (2010) proposed new CUSUM charts for monitoring the process mean using RSS and MRSS schemes. He showed that the CUSUM charts based on RSS schemes are more sensitive than that using the SRS scheme. Recently, Abujiya and Lee (2013) has used RSS with perfect and imperfect setups to construct new Shewhart, CUSUM and EWMA charts for monitoring the process mean. The literature on the quality control charts is growing at a good pace. For some fresh research works, we refer to Abujiya et al. (2013b,a, 2014), Mehmood et al. (2013, 2014), and other references cited therein.

As aforementioned that the VLRSS scheme not only encompasses several existing RSS scheme but it also provides a mean estimator which is more precise than that those of the RSS schemes. This fact has motivated us to suggest a new CUSUM chart using the VLRSS scheme for efficiently monitoring the process mean. Using extensive Monte Carlo simulation, the run length properties—the average run length (ARL) and the standard deviation of the run length (SDRL)—of the proposed CUSUM chart. The run length performances of the CUSUM chart with VLRSS is compared with that of the CUSUM charts with SRS, RSS, and MRSS schemes under both perfect and imperfect rankings. It turns out that the proposed CUSUM chart is able to perform uniformly better than the existing CUSUM charts.

The rest of the chapter is organized as follows: Section 2.2 contains a brief review of the CUSUM chart (with and without FIR features) using the SRS scheme. In Section 2.3, we explain the VLRSS scheme for estimating the population mean with perfect and imperfect ranking. In Section 2.4 we construct an CUSUM chart using VLRSS with and without FIR features. The run length performances of the proposed and existing charts are compared in Section 2.5. An illustrative example in give in Section 2.6. Finally, Section 2.7 summarizes the main findings.

## 2.2 The CUSUM control chart

In this section, we briefly review the classical CUSUM chart along with its FIR feature. The CUSUM chart can be designed by using either the individual observations or the mean of rational subgroups when monitoring changes in the process target. The CUSUM chart is very useful in detecting small as well as continual shifts in the process mean and/or dispersion. The CUSUM charts are of two types, i.e., the two-sided V-masked and two/one-sided tabular CUSUM. The latter one has been frequently used in the literature when monitoring the process parameter(s).

Let $Y$ denotes the quality characteristic under study and let $\{Y_t\}$ be the sequence of independent and identically distributed (IID) random variables, where $t = 1, 2, ....$ It is assumed that $Y_t$ is normally distributed with the in-control mean $\mu_Y$ and the in-control variance $\sigma_Y^2$, i.e., $Y_t \sim N(\mu_Y, \sigma_Y^2)$ for $t \geq 1$. Let $\{\overline{Y}_{\text{SRS},t}\}$ be a sequence of IID random variables, where $\overline{Y}_{\text{SRS},t} = (1/n)\sum_{i=1}^{n} Y_{i,t}$. Here, $Y_{i,t}$ is the $i$th observation in the $t$th sample, for $i = 1, 2, ..., n$. Note that $\overline{Y}_{\text{SRS},t}$ is also a normal random variable with the mean $\mu_Y$ and the variance $\sigma_Y^2/n$, i.e., $\overline{Y}_{\text{SRS},t} \sim N(\mu_Y, \sigma_Y^2/n)$. Using $\{\overline{Y}_{\text{SRS},t}\}$, the upper-side and the lower-side CUSUMs, say $C_t^+$ and $C_t^-$, respectively, of the CUSUM chart, are given by

$$C_t^+ = \text{Max}[0, +(\overline{Y}_{\text{SRS},t} - \mu_Y) - K + C_{t-1}^+], \tag{2.1}$$

$$C_t^- = \text{Max}[0, -(\overline{Y}_{\text{SRS},t} - \mu_Y) - K + C_{t-1}^-], \tag{2.2}$$

where Max[A,B] takes the maximum of A and B, and the initial values are usually set to zero, that is, $C_0^+ = C_0^- = 0$. Here, $K = k\sigma_Y/\sqrt{n}$ is the reference value of the CUSUM chart, where $k$ is usually taken as half of the magnitude of the shift $\delta$, measured in the standard deviation $\sigma_Y$ units, to be detected in the process mean $\mu_Y$ i.e., $k = \delta/2$, where $\delta = \sqrt{n}|\mu_{Y,1} - \mu_Y|/\sigma_Y$, and $\mu_{Y,1}$ is the out-of-control process mean. The process is said to be in-control when $\delta = 0$ and out-of-control when $\delta \neq 0$. The CUSUM chart begin by plotting $C_t^+$ and $C_t^-$ against the time $t$. The CUSUM charts declares the process in-control when both $C_t^+$ and $C_t^-$ are less than the decision interval $H = h\sigma_Y/\sqrt{n}$, where $h$ is selected to ensure that the in-control ARL of the CUSUM chart has reached to a desired level. However, if either $C_t^+$ or $C_t^-$ exceeds $H$, the CUSUM chart triggers an out-of-control signal to state that the process has gone out-of-control. For more details, we refer to Hawkins and Olwell (2012).

It is quite likely that the process gets away from the target initially or after recovering from an out-of-control state, and, if not stopped, the process may continue to produce defective items. To overcome such problems, it is customary to reset starting values to non-zero constants. This gives headstart to the CUSUM chart. The headstart feature helps in trigging out-of-control signals more frequently when the process suffers from startup problems. Lucas and Crosier (1982) introduced the headstart feature or fast initial response (FIR) with the CUSUM chart.

They recommended resetting the starting values of both plotting CUSUMs to $H/2$ for an 50% headstart, i.e., $C_0^+ = C_0^- = H/2$. For more details, see Lucas and Crosier (1982), Haq et al. (2014a), and the references cited therein.

## 2.3 The VLRSS scheme

In this section, we briefly review the VLRSS schemes and the mean estimators under this scheme using perfect and imperfect rankings.

Haq et al. (2015e) suggested the VLRSS scheme for precisely estimating the population mean. They have shown that the existing RSS schemes are special cases of the VLRSS scheme. One remarkable feature of the VLRSS scheme is that it not only improve the efficiency of the mean estimator (for a symmetric population) but it also helps in reducing the ranking cost when the existing RSS schemes cannot be applied, i.e., it requires fewer units than that required under the existing RSS schemes.

The main steps involved in selecting a varied L ranked set sample of size $n = mr$ are as follows:

Step 1: Select the value of the VLRSS coefficient, say $w = [al]$, where $0 \leq a < 0.5$. Here, $[\cdot]$ is the largest possible integer value.

Step 2: Select $2wl$ units from the target population, and then divide these units into $2w$ sets, with each set comprising $l$ units.

Step 3: Rank the units within each set by any cheap or inexpensive method with respect to the study variable or using ranks of an auxiliary variable.

Step 4: Select the $v$th and $(l - v + 1)$th smallest ranked units from the first and last $w$ sets, respectively, where $v = 1, 2, ..., [l/2]$.

Step 5: Identify $m(m - 2w)$ units from the target population, and then divide these units into $m - 2w$ sets, with each set comprising $m$ units.

Step 6: Select the $i$th smallest ranked unit from the $(i + w)$th set of $m$ units, for $i = w + 1, w + 2, ..., m - w$.

Step 7: This completes one cycle of a varied L ranked set sample of size $m$. The Steps 1-6 could be repeated, if necessary, $r$ number of times to get a total sample of size $n$ units.

Mathematically, let $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$, $i = 1, 2, ..., 2w$, denote $2w$ samples, each of size $l$, for the $j$th cycle, for $j = 1, 2, ..., r$. Let $Y_{i(v:l)j}$ be the $v$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = 1, 2, ..., w$, and let $Y_{i(l-v+1:l)j}$ denote the $(l - v + 1)$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = w + 1, w + 2, ..., 2w$. Let $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$, $i = w + 1, ..., m - w$ denote $m - 2w$ samples, each of size $m$, for the $j$th cycle. Let $Y_{i+w(i:m)j}$ be the $i$th order statistic of $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$, for $i = 1, 2, ..., m - w$.

The sample mean based on a varied L ranked set sample of size $mr$, denoted by $\overline{Y}_{\text{VLRSS}}$, and its variance are, respectively, given by

$$\overline{Y}_{\text{VLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right), \tag{2.3}$$

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y(v:l)} + \sigma^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w}\sigma^2_{Y(i:m)}\right), \tag{2.4}$$

where $\sigma^2_{Y(v:l)} = \text{Var}(Y_{i(v:l)j})$, $\sigma^2_{Y(l-v+1:l)} = \text{Var}(Y_{i(l-v+1:l)j})$, and $\sigma^2_{Y(i:m)} = \text{Var}(Y_{i(i:m)j})$. For more details regarding the computation of the variances of the order statistics, we refer to David and Nagaraja (2003).

Haq et al. (2015e) has shown that, when the underlying population is symmetric, $\overline{Y}_{\text{VLRSS}}$ is an unbiased estimator of the population mean $\mu_Y$. They also showed that with suitable choices of $v$, $l$, and $m$, the existing RSS schemes are special cases of the VLRSS scheme. For example, when $w = 0$, VLRSS reduces to RSS; when $w = [(m-1)/2]$, $l = m$ and $v = w + 1$, VLRSS reduces to MRSS, and so on. When selecting a varied L ranked set sample of size $n$, it requires identifying $nm - 2w(m-l)r$ units, and the classical RSS and MRSS schemes require $nm$ units. Note that, when $m > l$, it is possible to select a sample with VLRSS by identifying less $nm$ units, while it is not possible under the RSS and MRSS schemes as these two require exactly $nm$ units when selecting a sample of $n$ units. This shows that the VLRSS is more economical and more practical than the existing RSS schemes as it reduces the ranking costs. Besides, when the ranking costs are ignorable, with VLRSS scheme, it is possible to select a more representative sample than those with the RSS and MRSS scheme by identifying more than $nm$ units (cf., Haq et al. (2015e)).

### 2.3.1 The imperfect VLRSS scheme

Sometimes, for the experimenter, it is very difficult to rank the values of the study variable visually, or it is costly and/or time-consuming. This problem can be solved by ranking the study variable ($Y$) using the ranks of a highly correlated auxiliary variable, say $X$. A simple model for imperfect ranking has been suggested by Stokes (1977). The model works with some reasonable assumptions, which are given by:

(i) The regression of $Y$ on $X$ is linear.

(ii) The underlying distributions of the standardized study and auxiliary variables are the same.

These assumptions could be met easily when the underlying joint distribution of $(Y, X)$ is a bivariate normal distribution. The imperfect ranking model suggested by Stokes (1977) is:

$$Y_{i[i:u]j} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}\left(X_{i(i:u)j} - \mu_X\right) + \xi_{ij},\ i = 1, 2, ..., u,\ j = 1, 2, ..., r, \tag{2.5}$$

where $u = l, m$; $\mu_X$ and $\sigma_X$ are the population mean and standard deviation of $X$, respectively, and $\rho$ is the correlation between $Y$ and $X$. Here, $\xi_{ij}$ is the random error term (a normally distributed random variable and is independent of $X$) with the mean zero and the variance $\sigma_Y^2(1-\rho^2)$, i.e., $\xi_{ij} \sim N(0, \sigma_Y^2(1-\rho^2))$. Here, $Y_{i[i:u]j}$ is the $i$th concomitant (induced order statistic) corresponding to the $i$th order statistic $X_{i(i:u)j}$. The values of $X$ are perfectly ranked while the $Y$ values are ranked with error.

The sample mean under imperfect VLRSS (IVLRSS) scheme, using the above imperfect ranking model, say $\overline{Y}_{\text{IVLRSS}}$, and its variance are, respectively, given by:

$$\overline{Y}_{\text{IVLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w} Y_{i[v:l]j} + \sum_{i=w+1}^{2w} Y_{i[l-v+1:l]j} + \sum_{i=w+1}^{m-w} Y_{i+w[i:m]j}\right), \tag{2.6}$$

and

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y[v:l]} + \sigma^2_{Y[l-v+1:l]}) + \sum_{i=w+1}^{m-w}\sigma^2_{Y[i:m]}\right) \tag{2.7}$$

$$= \frac{1}{nm}\left\{m\sigma_Y^2(1-\rho^2) + \rho^2\frac{\sigma_Y^2}{\sigma_X^2}\left(2w\sigma^2_{X(v:l)} + \sum_{i=w+1}^{m-w}\sigma^2_{X(i:m)}\right)\right\}, \tag{2.8}$$

where $\sigma^2_{Y[v:l]} = \text{Var}(Y_{i[v:l]j})$, $\sigma^2_{Y[l-v+1:l]} = \text{Var}(Y_{i[l-v+1:l]j})$, and $\sigma^2_{Y[i:m]} = \text{Var}(Y_{i[i:m]j})$. For more details on the computation of these variances, we refer to see David and Nagaraja (2003).

## 2.4 The proposed CUSUM control chart

In this section, we propose new CUSUM charts using the VLRSS and IVLRSS schemes for efficiently monitoring the process mean.

For monitoring changes in the process mean $\mu_Y$, a sequence is generated by selecting a sample of size $mr$ under the S sampling scheme at each inspection point $t$, where S = VLRSS, IVLRSS. Let $\{\overline{Y}_{\text{S},t}\}$ be a sequence of IID random variables for $t = 1, 2, ....$ Using $\{\overline{Y}_{\text{S},t}\}$, it is possible to construct an CUSUM chart for monitoring $\mu_Y$. Similar to the CUSUM chart under SRS, the plotting-statistics (upper and lower CUSUMs) of the CUSUM chart using S sampling scheme are, respectively, defined by

$$C_t^+ = \text{Max}[0, +(\overline{Y}_{\text{S},t} - \mu_Y) - K + C_{t-1}^+], \tag{2.9}$$

$$C_t^- = \text{Max}[0, -(\overline{Y}_{\text{S},t} - \mu_Y) - K + C_{t-1}^-], \tag{2.10}$$

where $C_0^+ = C_0^- = 0$ and $\overline{Y}_{\text{S},t}$ is the mean of the sample obtained under S scheme. The references value $K$ and the decision interval $H$ of the CUSUM chart are

$$K = k\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}, \tag{2.11}$$

$$H = h\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}, \tag{2.12}$$

where the values of $k$ and $h$ are the same as explained in the previous section. The CUSUM chart begin by plotting the CUSUMs $C_t^+$ and $C_t^-$ versus the time $t$ and states that the process is in statistical control when both $C_t^+$ and $C_t^-$ are less than $H$. However, an out-of-signal is generated by the CUSUM chart when either $C_t^+$ or $C_t^-$ exceeds $H$. For the case when $C_t^+ > H$ ($C_t^- > H$), it is then stated that there is an upward (downward) shift in the process mean $\mu_Y$. The selection of $k$ and $h$ play an important role in optimizing the run length performance of the CUSUM chart. As aforementioned that the sensitivity of the CUSUM chart could be increased for earlier detection of startup problem by setting the starting values of $C_t^+$ and $C_t^-$ to non-zero constants, like $C_0^+ = C_0^- = H/2$ for an 50% headstart (cf., Lucas and Crosier (1982)). Here, we follow this approach to associate an FIR feature for the proposed CUSUM chart with 50% headstart.

The run length performance of a control chart is generally evaluated in terms of its run length characteristics, including the ARL and SDRL. For an in-control process, the in-control ARL should be large to avoid false alarms, while for an out-of-control process, it should be as small as possible to generate an out-of-control signal as soon as possible. Given the ARL, the smaller the SDRL the better will be the performance of a control chart. In the literature, there exist many approaches that are frequently used to approximate the run length of memory-type control chart, which include the Markov chain, integral equation, and Monte Carlo simulations. The last method is mostly used to explore the run length profiles of a control chart as it provide more efficient estimates of the run length characteristics. Thus, here we perform extensive Monte Carlo simulations from a standard normal distribution (for perfect rankings) and standard bivariate normal distribution (for imperfect rankings in next section) to compute the run length profiles of the proposed CUSUM chart. We consider different values of the shift $\delta$, i.e., $\delta = 0(0.25)4$. The in-control ARL is set to 500—a recommended choice of the SPC practitioners. For brevity of discussion, in Table 2.1, we report different values of $k$ and $h$ for different choices of $m$ and $l$ with $r = 1$. Moreover, in Tables 2.2–2.5, the run length profiles of the proposed CUSUM chart are presented using with and without FIR features for $n = 5$ with $m = 5$ and $r = 1$. Each simulation run comprises 50,000 iterations. Following Haq et al. (2015e), we consider the values of $w$, $k$ and $(l, v)$ for which the VLRSS mean estimator is more precise than that using other values.

From Tables 2.2–2.5, it is observed that having fixed $w$, $k$, $(l, v)$, as the values of $\delta$ increases the out-of-controls ARL tend to decrease and vice versa. The values of $k$ do affect the sensitivity of the CUSUM chart, i.e., the small values of $k$ are useful in detecting small shifts while the large values of $k$ help in earlier detection of large shifts. When $w = 2$, having fixed $\delta$ and $k$, as the value of $(l, v)$ increases, the sensitivity of the CUSUM also increases and vice versa. It is also observed that the CUSUM chart with $w = 2$ is better than the CUSUM chart with $w = 1$. The reason is with $w = 1$ the VLRSS scheme selects samples by identifying less than 25 units (less ranking cost is involved here) than that with $w = 2$ (more ranking cost is involved here). The trend observed here remains the same when the CUSUM charts are constructed with the FIR features.

### 2.4.1 When the process parameters are unknown

In case the process parameters are not known in advance, then it is customary to estimate them using a large historical data that have been recorded when the process was running in the control state. Suppose that $q$ subgroups, each of size $m$, are available under the S sampling scheme. For the perfect ranking case, we can estimate $\mu_Y$ and $\text{Var}(\overline{Y}_{\text{VLRSS}})$ by their respective unbiased estimators, given by

$$\overline{\overline{Y}}_{\text{VLRSS}} = \frac{1}{q}\sum_{j=1}^{q}\overline{Y}_{\text{VLRSS},j}, \tag{2.13}$$

$$\hat{\text{V}}\text{ar}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(S^2_{Y(v:l)} + S^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w} S^2_{Y(i:m)}\right), \tag{2.14}$$

where

$$\overline{Y}_{\text{VLRSS},j} = \frac{1}{m}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right),$$

$$S^2_{Y(i:u)} = \frac{1}{q-1}\sum_{j=1}^{q}\left(Y_{i'(i:u)j} - \frac{1}{q}\sum_{j=1}^{q} Y_{i'(i:u)j}\right)^2, \tag{2.15}$$

where $i$ and $i'$ may or may not be equal. In case of imperfect rankings, instead of selecting the order statistics, we are to select the induced order statistics or concomitants of the study variable $Y$ corresponding to the order statistics of the auxiliary variable $X$. Under imperfect, it is also possible to derive the unbiased estimators of the mean $\mu_Y$ and $\text{Var}(\overline{Y}_{\text{IVLRSS}})$. The unbiased estimator can be obtained from above expression but the parenthesis are not replace by the square brackets. For example, replace $Y_{i(v:l)j}$ by $Y_{i[v:l]j}$; $S^2_{Y(v:l)}$ by $S^2_{Y[v:l]}$, and similarly the others. To get better estimates of population parameters using the above formulae, it is recommended to keep number of subgroups $q$ as large as possible. Having the unknown parameters estimated using the large historical dataset, it is possible to construct the aforementioned CUSUM chart for monitoring the process mean.

## 2.5 Performance comparisons

To see how effective our proposed CUSUM chart is at detecting a wide range of shifts in the process mean, we compare its run length performance with some of the existing control charts that are designed for the similar objectives. The ARL and SDRL are used here as performance criterion. For a fair comparison of RSS schemes based control charts, we consider both perfect and imperfect rankings. For the perfect ranking case, the in-control ARLs are matched to 500. It is also possible to compare the run length performances of the control charts using imperfect rankings when matching their in-control ARLs to 500—which is not followed here. Here, for the comparisons under the imperfect rankings, we use same choices of $H$ to compute the run length profiles that were used in the perfect rankings for all of the control charts considered here. Without loss of generally, in all comparisons of the control charts, we consider $m = 5$.

### 2.5.1 The CUSUM charts with RSS and MRSS

The CUSUM charts using the RSS and MRSS schemes were designed by Al-Sabah (2010) and have had better run length performance than that of the CUSUM chart using SRS. Hence what follows, the VLRSS based CUSUM chart is compared with the CUSUM charts based on RSS and MRSS schemes using different choices of $k$ and $\delta$. The run length profiles of these CUSUM charts are presented in Table 2.6. It is observed that the proposed CUSUM chart performs uniformly and substantially better than the RSS and MRSS based CUSUM charts. The performance of the proposed CUSUM chart increases as the value of $(l, v)$ increases and vice versa.

### 2.5.2 The CUSUM charts with imperfect RSS and MRSS

On the lines of the perfect case comparison, the run length profiles of the proposed and existing CUSUM charts are compared when there are errors in ranking. Different values of $k$ and $\rho$ are considered here, including $k = 0.50, 1.00$ and $\rho = 0.25, 0.50, 0.75, 0.90$. The ARL profiles are presented in Table 2.7–2.8. It is observed that the in-control ARLs, though no matched to 500, remain close to 500. However, the out-of-control ARLs do get affected by the values of $\rho$, i.e., as the value of $\rho$ increases, the run length performance of the CUSUM chart increases and vice versa. It is worth mentioning that, despite the presence of small to large ranking errors, the proposed CUSUM chart is more sensitive than the existing CUSUM charts. The rest of the trends are the same as was observed in the perfect case.

### 2.5.3 The CUSUM chart with SRS

The proposed CUSUM chart is also compared with the SRS based CUSUM chart when ranking costs are high and it is not possible to select samples using RSS and MRSS schemes. But, with less identified units, the VLRSS scheme is able to draw representative samples from the underlying population. Here, we consider those choices of the VLRSS scheme in which the number of identified units are less than 25. For these choices the run length profiles of the proposed CUSUM chart under both perfect and imperfect rankings are compared with that of the SRS based CUSUM chart. The run length profiles of the proposed and existing CUSUMs are presented in Table 2.9–2.11. It is remarkable to see that the even with less number of identified units the proposed CUSUM chart surpasses the CUSUM chart using SRS. The performance of the CUSUM chart increases as the value of $\rho$ increases and vice versa.

## 2.6 An application

In this section, we apply the proposed and the existing CUSUM charts on the samples drawn from a real dataset under the MRSS and VLRSS schemes.

A hard-bake process is used in conjunction with photolithography in the semiconductor manufacturing. Our object is to establish statistical control of the flow width of the resist in this process using the CUSUM charts. For this purpose, forty-samples, each of size five wafers, have been taken from a process, which is assumed to be running in an in-control state. The complete data set is given in Montgomery (2007). All these samples are combined to generate a population that comprises 225 flow width measurements—measured in microns. Generate thirty samples, each of size five, from this population under the MRSS ($l = m, v = 3, w = 2$) and VLRSS ($l = 6, v = 3, w = 2$) schemes. Treat these data as Phase-I sample, and use these data to estimate the means and the variances of the mean estimator based on both RSS schemes. For both CUSUM charts the in-control ARLs are set to 500 with $k = 0.50$. For the CUSUM charts using MRSS and VLRSS schemes, we consider $h = 5.0758$ and $h = 5.0760$, respectively. Then, generate twenty samples, each of size 5, under both sampling schemes. In order to investigate the performances of these CUSUM charts, we add 0.02 to all the observations in the last twenty samples. The plotting-statistics of the CUSUM charts are computed for all fifty samples and are displayed in Figures 2.1 and 2.2.

From 2.1 and 2.2, it is clear that, for the first thirty samples, both the CUSUM charts are showing that the process is in statistical control. But, for the next twenty samples, both CUSUM charts are signaling out-of-control signals. It is interesting to see that the proposed CUSUM chart triggers the first out-of-control signal earlier than that of the CUSUM chart with MRSS, i.e., the CUSUM charts based on MRSS and VLRSS schemes are first signaling at the 39th and 35th samples, respectively.

## 2.7 Conclusion

In this chapter, we have proposed a new CUSUM control chart using the VLRSS scheme for efficiently monitoring the process mean. We have used extensive Monte Carlo simulations to estimate the run length characteristics of the proposed CUSUM chart. The CUSUM chart with the VLRSS scheme has been compared with the existing CUSUM charts based on SRS, RSS and MRSS schemes in terms of ARL and SDRL using perfect and imperfect rankings. It turned out that the proposed CUSUM chart performed uniformly and substantially better than its counterparts when detecting different kinds of shift in the process mean. Thus we recommend the use of the propose CUSUM chart with VLRSS, when possible, for efficiently monitoring the process mean.

Table 2.1: The values of $h$ with different choices of $(l, v)$ when the in-control ARL of the CUSUM-VLRSS chart is 500

| | $m = 2, w = 0$ | $m = 4, w = 1$ | | | | | |
|---|---|---|---|---|---|---|---|
| $k$ | $(0, 0)$ | $(2, 1)$ | $(3, 2)$ | $(4, 1)$ | $(4, 2)$ | $(5, 3)$ | $(6, 3)$ |
| 0.25 | 8.5800 | 8.5963 | 8.5955 | 8.5954 | 8.5955 | 8.5955 | 8.5953 |
| 0.50 | 5.0700 | 5.0741 | 5.0737 | 5.0749 | 5.0747 | 5.0750 | 5.0754 |
| 0.75 | 3.5600 | 3.5485 | 3.5476 | 3.5496 | 3.5469 | 3.5468 | 3.5469 |
| 1.00 | 2.6890 | 2.6785 | 2.6741 | 2.6836 | 2.6768 | 2.6751 | 2.6767 |

| | $m = 3, w = 1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $(2, 1)$ | $(3, 1)$ | $(3, 2)$ | $(4, 1)$ | $(4, 2)$ | $(5, 1)$ | $(5, 2)$ | $(5, 3)$ |
| 0.25 | 8.5900 | 8.6084 | 8.6022 | 8.5970 | 8.5927 | 8.5990 | 8.5982 | 8.5961 |
| 0.50 | 5.0800 | 5.0813 | 5.0750 | 5.0790 | 5.0715 | 5.0835 | 5.0788 | 5.0772 |
| 0.75 | 3.5530 | 3.5568 | 3.5459 | 3.5587 | 3.5463 | 3.5598 | 3.5501 | 3.5483 |
| 1.00 | 2.6778 | 2.6900 | 2.6740 | 2.6943 | 2.6773 | 2.7000 | 2.6840 | 2.6750 |

| | $m = 5$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $w = 1$ | | | | $w = 2$ | | | |
| | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| 0.25 | 8.5955 | 8.5954 | 8.5946 | 8.5890 | 8.5911 | 8.5911 | 8.5920 | 8.5909 |
| 0.50 | 5.0747 | 5.0753 | 5.0759 | 5.0766 | 5.0758 | 5.0760 | 5.0760 | 5.0759 |
| 0.75 | 3.5477 | 3.5467 | 3.5468 | 3.5528 | 3.5456 | 3.5454 | 3.5471 | 3.5469 |
| 1.00 | 2.6767 | 2.6747 | 2.6747 | 2.6827 | 2.6735 | 2.6731 | 2.6723 | 2.6725 |

Table 2.2: The run length profiles of the CUSUM-VLRSS chart for $w = 1$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 8.5955 | 8.5954 | 8.5946 | 8.5890 | 5.0747 | 5.0753 | 5.0759 | 5.0766 |
| 0.00 | ARL | 502.34 | 502.91 | 502.45 | 500.42 | 500.28 | 501.80 | 501.12 | 500.68 |
| | SDRL | 487.68 | 486.47 | 488.09 | 483.03 | 499.88 | 493.81 | 496.42 | 492.57 |
| 0.25 | ARL | 49.58 | 41.41 | 38.33 | 41.14 | 71.76 | 56.68 | 51.23 | 57.13 |
| | SDRL | 34.11 | 26.72 | 23.99 | 26.51 | 64.14 | 49.22 | 43.80 | 49.75 |
| 0.50 | ARL | 17.84 | 15.33 | 14.43 | 15.34 | 17.75 | 14.35 | 13.11 | 14.40 |
| | SDRL | 7.78 | 6.25 | 5.66 | 6.20 | 11.59 | 8.63 | 7.59 | 8.59 |
| 0.75 | ARL | 10.69 | 9.37 | 8.82 | 9.36 | 8.90 | 7.53 | 6.97 | 7.49 |
| | SDRL | 3.62 | 2.94 | 2.68 | 2.93 | 4.30 | 3.33 | 2.98 | 3.33 |
| 1.00 | ARL | 7.70 | 6.77 | 6.40 | 6.76 | 5.92 | 5.10 | 4.76 | 5.10 |
| | SDRL | 2.18 | 1.81 | 1.64 | 1.79 | 2.32 | 1.84 | 1.65 | 1.85 |
| 1.25 | ARL | 6.02 | 5.34 | 5.06 | 5.34 | 4.45 | 3.87 | 3.66 | 3.88 |
| | SDRL | 1.51 | 1.25 | 1.15 | 1.25 | 1.50 | 1.20 | 1.10 | 1.21 |
| 1.50 | ARL | 4.97 | 4.42 | 4.20 | 4.42 | 3.59 | 3.15 | 2.99 | 3.15 |
| | SDRL | 1.12 | 0.94 | 0.88 | 0.94 | 1.07 | 0.89 | 0.82 | 0.88 |
| 1.75 | ARL | 4.25 | 3.80 | 3.62 | 3.79 | 3.03 | 2.69 | 2.54 | 2.68 |
| | SDRL | 0.88 | 0.75 | 0.70 | 0.75 | 0.84 | 0.70 | 0.64 | 0.69 |
| 2.00 | ARL | 3.73 | 3.35 | 3.20 | 3.35 | 2.63 | 2.36 | 2.24 | 2.35 |
| | SDRL | 0.73 | 0.61 | 0.57 | 0.62 | 0.67 | 0.55 | 0.50 | 0.55 |
| 2.25 | ARL | 3.34 | 3.02 | 2.88 | 3.01 | 2.35 | 2.13 | 2.05 | 2.13 |
| | SDRL | 0.61 | 0.52 | 0.51 | 0.52 | 0.55 | 0.43 | 0.39 | 0.43 |
| 2.50 | ARL | 3.04 | 2.74 | 2.61 | 2.74 | 2.15 | 1.98 | 1.91 | 1.98 |
| | SDRL | 0.53 | 0.51 | 0.52 | 0.51 | 0.44 | 0.37 | 0.37 | 0.36 |
| 2.75 | ARL | 2.80 | 2.49 | 2.35 | 2.49 | 2.00 | 1.85 | 1.78 | 1.86 |
| | SDRL | 0.51 | 0.51 | 0.48 | 0.51 | 0.37 | 0.39 | 0.43 | 0.39 |
| 3.00 | ARL | 2.56 | 2.27 | 2.16 | 2.26 | 1.89 | 1.72 | 1.63 | 1.72 |
| | SDRL | 0.52 | 0.44 | 0.37 | 0.44 | 0.38 | 0.46 | 0.49 | 0.45 |
| 3.25 | ARL | 2.35 | 2.11 | 2.05 | 2.11 | 1.78 | 1.57 | 1.45 | 1.57 |
| | SDRL | 0.48 | 0.31 | 0.22 | 0.31 | 0.43 | 0.50 | 0.50 | 0.50 |
| 3.50 | ARL | 2.18 | 2.04 | 2.01 | 2.03 | 1.65 | 1.40 | 1.29 | 1.40 |
| | SDRL | 0.39 | 0.19 | 0.13 | 0.19 | 0.48 | 0.49 | 0.45 | 0.49 |
| 3.75 | ARL | 2.08 | 2.00 | 1.99 | 2.00 | 1.51 | 1.25 | 1.16 | 1.25 |
| | SDRL | 0.27 | 0.11 | 0.12 | 0.11 | 0.50 | 0.44 | 0.37 | 0.44 |
| 4.00 | ARL | 2.02 | 1.99 | 1.97 | 1.99 | 1.36 | 1.14 | 1.08 | 1.14 |
| | SDRL | 0.16 | 0.13 | 0.18 | 0.12 | 0.48 | 0.35 | 0.27 | 0.35 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 3.5477 | 3.5467 | 3.5468 | 3.5528 | 2.6767 | 2.6747 | 2.6747 | 2.6827 |
| 0.00 | ARL | 501.83 | 503.69 | 500.25 | 501.82 | 504.59 | 504.77 | 499.42 | 499.88 |
| | SDRL | 499.26 | 500.98 | 499.92 | 499.86 | 499.26 | 505.79 | 499.64 | 495.72 |
| 0.25 | ARL | 107.06 | 85.11 | 76.91 | 85.20 | 147.67 | 118.68 | 107.44 | 120.75 |
| | SDRL | 102.66 | 79.96 | 72.79 | 81.49 | 144.16 | 117.15 | 105.40 | 117.50 |
| 0.50 | ARL | 22.74 | 17.44 | 15.59 | 17.42 | 32.09 | 23.81 | 20.74 | 23.91 |
| | SDRL | 18.77 | 13.56 | 11.72 | 13.54 | 29.30 | 21.01 | 18.13 | 21.17 |
| 0.75 | ARL | 9.36 | 7.50 | 6.85 | 7.52 | 11.34 | 8.61 | 7.65 | 8.63 |
| | SDRL | 5.97 | 4.35 | 3.79 | 4.35 | 8.70 | 6.18 | 5.22 | 6.13 |
| 1.00 | ARL | 5.54 | 4.63 | 4.29 | 4.62 | 5.92 | 4.72 | 4.28 | 4.71 |
| | SDRL | 2.78 | 2.12 | 1.89 | 2.11 | 3.66 | 2.66 | 2.28 | 2.64 |
| 1.25 | ARL | 3.96 | 3.37 | 3.14 | 3.37 | 3.87 | 3.23 | 2.98 | 3.22 |
| | SDRL | 1.66 | 1.30 | 1.16 | 1.30 | 1.97 | 1.50 | 1.32 | 1.49 |
| 1.50 | ARL | 3.09 | 2.68 | 2.53 | 2.68 | 2.91 | 2.47 | 2.30 | 2.46 |
| | SDRL | 1.13 | 0.90 | 0.82 | 0.90 | 1.28 | 1.00 | 0.90 | 0.98 |
| 1.75 | ARL | 2.57 | 2.25 | 2.14 | 2.25 | 2.35 | 2.02 | 1.90 | 2.02 |
| | SDRL | 0.84 | 0.68 | 0.62 | 0.68 | 0.92 | 0.75 | 0.69 | 0.74 |
| 2.00 | ARL | 2.21 | 1.97 | 1.87 | 1.97 | 1.98 | 1.72 | 1.62 | 1.72 |
| | SDRL | 0.66 | 0.56 | 0.54 | 0.56 | 0.72 | 0.62 | 0.59 | 0.62 |
| 2.25 | ARL | 1.96 | 1.75 | 1.66 | 1.76 | 1.72 | 1.50 | 1.41 | 1.50 |
| | SDRL | 0.56 | 0.52 | 0.52 | 0.52 | 0.62 | 0.55 | 0.51 | 0.55 |
| 2.50 | ARL | 1.77 | 1.56 | 1.47 | 1.57 | 1.51 | 1.32 | 1.24 | 1.32 |
| | SDRL | 0.52 | 0.52 | 0.51 | 0.52 | 0.55 | 0.48 | 0.43 | 0.48 |
| 2.75 | ARL | 1.60 | 1.39 | 1.30 | 1.39 | 1.35 | 1.18 | 1.13 | 1.19 |
| | SDRL | 0.52 | 0.49 | 0.46 | 0.49 | 0.49 | 0.39 | 0.33 | 0.39 |
| 3.00 | ARL | 1.44 | 1.24 | 1.17 | 1.24 | 1.22 | 1.10 | 1.06 | 1.10 |
| | SDRL | 0.50 | 0.43 | 0.38 | 0.43 | 0.42 | 0.30 | 0.23 | 0.29 |
| 3.25 | ARL | 1.30 | 1.13 | 1.08 | 1.13 | 1.13 | 1.04 | 1.02 | 1.04 |
| | SDRL | 0.46 | 0.34 | 0.27 | 0.34 | 0.34 | 0.20 | 0.15 | 0.20 |
| 3.50 | ARL | 1.19 | 1.06 | 1.03 | 1.06 | 1.06 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.39 | 0.24 | 0.18 | 0.24 | 0.24 | 0.13 | 0.09 | 0.12 |
| 3.75 | ARL | 1.10 | 1.03 | 1.01 | 1.03 | 1.03 | 1.00 | 1.00 | 1.01 |
| | SDRL | 0.30 | 0.16 | 0.11 | 0.16 | 0.17 | 0.07 | 0.05 | 0.07 |
| 4.00 | ARL | 1.05 | 1.01 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.22 | 0.10 | 0.06 | 0.09 | 0.11 | 0.04 | 0.02 | 0.04 |

Table 2.3: The run length profiles of the CUSUM-VLRSS chart for $w = 2$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 8.5911 | 8.5911 | 8.5920 | 8.5909 | 5.0758 | 5.0760 | 5.0760 | 5.0759 |
| 0.00 | ARL | 501.77 | 502.16 | 502.16 | 503.08 | 502.20 | 503.39 | 502.37 | 501.02 |
| | SDRL | 484.64 | 484.37 | 485.51 | 487.30 | 498.61 | 495.14 | 495.00 | 494.45 |
| 0.25 | ARL | 34.64 | 31.56 | 28.80 | 26.96 | 44.85 | 39.36 | 35.12 | 31.95 |
| | SDRL | 20.65 | 18.21 | 15.89 | 14.49 | 37.64 | 32.44 | 28.14 | 25.09 |
| 0.50 | ARL | 13.25 | 12.31 | 11.43 | 10.84 | 11.77 | 10.68 | 9.73 | 9.07 |
| | SDRL | 4.99 | 4.47 | 3.98 | 3.68 | 6.52 | 5.69 | 4.90 | 4.41 |
| 0.75 | ARL | 8.21 | 7.66 | 7.18 | 6.84 | 6.41 | 5.89 | 5.45 | 5.14 |
| | SDRL | 2.42 | 2.17 | 1.96 | 1.83 | 2.61 | 2.30 | 2.05 | 1.86 |
| 1.00 | ARL | 5.96 | 5.61 | 5.27 | 5.03 | 4.42 | 4.10 | 3.82 | 3.64 |
| | SDRL | 1.48 | 1.35 | 1.22 | 1.14 | 1.47 | 1.32 | 1.19 | 1.10 |
| 1.25 | ARL | 4.73 | 4.45 | 4.19 | 4.02 | 3.40 | 3.18 | 2.99 | 2.85 |
| | SDRL | 1.04 | 0.95 | 0.87 | 0.81 | 0.99 | 0.89 | 0.81 | 0.76 |
| 1.50 | ARL | 3.95 | 3.72 | 3.52 | 3.38 | 2.79 | 2.63 | 2.47 | 2.37 |
| | SDRL | 0.80 | 0.73 | 0.67 | 0.62 | 0.74 | 0.67 | 0.61 | 0.56 |
| 1.75 | ARL | 3.40 | 3.23 | 3.06 | 2.95 | 2.40 | 2.27 | 2.16 | 2.08 |
| | SDRL | 0.64 | 0.58 | 0.53 | 0.52 | 0.57 | 0.51 | 0.45 | 0.41 |
| 2.00 | ARL | 3.02 | 2.86 | 2.71 | 2.60 | 2.13 | 2.04 | 1.96 | 1.91 |
| | SDRL | 0.53 | 0.51 | 0.51 | 0.52 | 0.43 | 0.38 | 0.37 | 0.37 |
| 2.25 | ARL | 2.72 | 2.56 | 2.40 | 2.28 | 1.97 | 1.89 | 1.81 | 1.74 |
| | SDRL | 0.51 | 0.51 | 0.49 | 0.45 | 0.37 | 0.38 | 0.41 | 0.45 |
| 2.50 | ARL | 2.44 | 2.28 | 2.16 | 2.09 | 1.83 | 1.74 | 1.62 | 1.53 |
| | SDRL | 0.50 | 0.45 | 0.36 | 0.29 | 0.40 | 0.45 | 0.49 | 0.50 |
| 2.75 | ARL | 2.20 | 2.10 | 2.04 | 2.01 | 1.67 | 1.55 | 1.42 | 1.32 |
| | SDRL | 0.40 | 0.30 | 0.20 | 0.14 | 0.47 | 0.50 | 0.49 | 0.46 |
| 3.00 | ARL | 2.07 | 2.02 | 2.00 | 1.99 | 1.49 | 1.36 | 1.23 | 1.16 |
| | SDRL | 0.25 | 0.16 | 0.11 | 0.12 | 0.50 | 0.48 | 0.42 | 0.36 |
| 3.25 | ARL | 2.01 | 2.00 | 1.98 | 1.96 | 1.31 | 1.19 | 1.10 | 1.06 |
| | SDRL | 0.13 | 0.11 | 0.15 | 0.20 | 0.46 | 0.39 | 0.30 | 0.23 |
| 3.50 | ARL | 1.99 | 1.97 | 1.93 | 1.87 | 1.17 | 1.09 | 1.04 | 1.02 |
| | SDRL | 0.12 | 0.16 | 0.26 | 0.33 | 0.37 | 0.28 | 0.19 | 0.13 |
| 3.75 | ARL | 1.97 | 1.92 | 1.83 | 1.72 | 1.08 | 1.03 | 1.01 | 1.00 |
| | SDRL | 0.18 | 0.27 | 0.38 | 0.45 | 0.26 | 0.18 | 0.10 | 0.06 |
| 4.00 | ARL | 1.92 | 1.82 | 1.67 | 1.52 | 1.03 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.27 | 0.39 | 0.47 | 0.50 | 0.17 | 0.10 | 0.05 | 0.02 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 3.5456 | 3.5454 | 3.5471 | 3.5469 | 2.6735 | 2.6731 | 2.6723 | 2.6725 |
| 0.00 | ARL | 500.45 | 500.65 | 503.77 | 500.36 | 502.12 | 501.54 | 501.99 | 502.86 |
| | SDRL | 495.73 | 496.84 | 496.35 | 497.14 | 501.75 | 497.85 | 496.91 | 499.68 |
| 0.25 | ARL | 66.97 | 58.55 | 51.30 | 46.47 | 94.43 | 83.80 | 73.08 | 66.33 |
| | SDRL | 63.03 | 54.01 | 46.96 | 42.32 | 91.85 | 80.89 | 70.17 | 64.08 |
| 0.50 | ARL | 13.52 | 11.79 | 10.41 | 9.60 | 17.55 | 15.08 | 13.04 | 11.58 |
| | SDRL | 9.74 | 8.14 | 6.92 | 6.18 | 14.77 | 12.40 | 10.28 | 9.09 |
| 0.75 | ARL | 6.11 | 5.53 | 5.01 | 4.70 | 6.64 | 5.88 | 5.19 | 4.80 |
| | SDRL | 3.23 | 2.75 | 2.38 | 2.18 | 4.30 | 3.63 | 3.07 | 2.72 |
| 1.00 | ARL | 3.91 | 3.60 | 3.33 | 3.14 | 3.83 | 3.46 | 3.15 | 2.95 |
| | SDRL | 1.64 | 1.43 | 1.27 | 1.16 | 1.93 | 1.68 | 1.45 | 1.29 |
| 1.25 | ARL | 2.90 | 2.70 | 2.52 | 2.40 | 2.71 | 2.49 | 2.29 | 2.17 |
| | SDRL | 1.02 | 0.91 | 0.82 | 0.75 | 1.15 | 1.01 | 0.89 | 0.82 |
| 1.50 | ARL | 2.35 | 2.21 | 2.08 | 1.99 | 2.12 | 1.97 | 1.83 | 1.74 |
| | SDRL | 0.73 | 0.66 | 0.60 | 0.57 | 0.79 | 0.72 | 0.66 | 0.63 |
| 1.75 | ARL | 2.01 | 1.89 | 1.78 | 1.70 | 1.76 | 1.63 | 1.53 | 1.45 |
| | SDRL | 0.57 | 0.54 | 0.52 | 0.52 | 0.63 | 0.59 | 0.56 | 0.53 |
| 2.00 | ARL | 1.76 | 1.65 | 1.54 | 1.46 | 1.50 | 1.39 | 1.30 | 1.24 |
| | SDRL | 0.52 | 0.52 | 0.51 | 0.51 | 0.55 | 0.51 | 0.47 | 0.43 |
| 2.25 | ARL | 1.55 | 1.44 | 1.33 | 1.26 | 1.31 | 1.22 | 1.14 | 1.10 |
| | SDRL | 0.51 | 0.50 | 0.47 | 0.44 | 0.47 | 0.41 | 0.35 | 0.31 |
| 2.50 | ARL | 1.36 | 1.25 | 1.17 | 1.12 | 1.16 | 1.10 | 1.05 | 1.04 |
| | SDRL | 0.48 | 0.44 | 0.37 | 0.32 | 0.37 | 0.30 | 0.23 | 0.18 |
| 2.75 | ARL | 1.20 | 1.12 | 1.07 | 1.04 | 1.07 | 1.04 | 1.02 | 1.01 |
| | SDRL | 0.40 | 0.33 | 0.25 | 0.20 | 0.26 | 0.19 | 0.13 | 0.09 |
| 3.00 | ARL | 1.10 | 1.05 | 1.02 | 1.01 | 1.03 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.30 | 0.22 | 0.15 | 0.10 | 0.16 | 0.11 | 0.07 | 0.04 |
| 3.25 | ARL | 1.04 | 1.02 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.19 | 0.12 | 0.07 | 0.05 | 0.09 | 0.05 | 0.03 | 0.02 |
| 3.50 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.06 | 0.03 | 0.02 | 0.05 | 0.02 | 0.01 | 0.01 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.03 | 0.01 | 0.00 | 0.02 | 0.01 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |

Table 2.4: The run length profiles of the CUSUM-VLRSS chart with the FIR feature for $w = 1$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 8.5955 | 8.5954 | 8.5946 | 8.5890 | 5.0747 | 5.0753 | 5.0759 | 5.0766 |
| 0.00 | ARL | 502.34 | 502.91 | 502.45 | 500.42 | 500.28 | 501.80 | 501.12 | 500.68 |
| | SDRL | 487.68 | 486.47 | 488.09 | 483.03 | 499.88 | 493.81 | 496.42 | 492.57 |
| 0.25 | ARL | 49.58 | 41.41 | 38.33 | 41.14 | 71.76 | 56.68 | 51.23 | 57.13 |
| | SDRL | 34.11 | 26.72 | 23.99 | 26.51 | 64.14 | 49.22 | 43.80 | 49.75 |
| 0.50 | ARL | 17.84 | 15.33 | 14.43 | 15.34 | 17.75 | 14.35 | 13.11 | 14.40 |
| | SDRL | 7.78 | 6.25 | 5.66 | 6.20 | 11.59 | 8.63 | 7.59 | 8.59 |
| 0.75 | ARL | 10.69 | 9.37 | 8.82 | 9.36 | 8.90 | 7.53 | 6.97 | 7.49 |
| | SDRL | 3.62 | 2.94 | 2.68 | 2.93 | 4.30 | 3.33 | 2.98 | 3.33 |
| 1.00 | ARL | 7.70 | 6.77 | 6.40 | 6.76 | 5.92 | 5.10 | 4.76 | 5.10 |
| | SDRL | 2.18 | 1.81 | 1.64 | 1.79 | 2.32 | 1.84 | 1.65 | 1.85 |
| 1.25 | ARL | 6.02 | 5.34 | 5.06 | 5.34 | 4.45 | 3.87 | 3.66 | 3.88 |
| | SDRL | 1.51 | 1.25 | 1.15 | 1.25 | 1.50 | 1.20 | 1.10 | 1.21 |
| 1.50 | ARL | 4.97 | 4.42 | 4.20 | 4.42 | 3.59 | 3.15 | 2.99 | 3.15 |
| | SDRL | 1.12 | 0.94 | 0.88 | 0.94 | 1.07 | 0.89 | 0.82 | 0.88 |
| 1.75 | ARL | 4.25 | 3.80 | 3.62 | 3.79 | 3.03 | 2.69 | 2.54 | 2.68 |
| | SDRL | 0.88 | 0.75 | 0.70 | 0.75 | 0.84 | 0.70 | 0.64 | 0.69 |
| 2.00 | ARL | 3.73 | 3.35 | 3.20 | 3.35 | 2.63 | 2.36 | 2.24 | 2.35 |
| | SDRL | 0.73 | 0.61 | 0.57 | 0.62 | 0.67 | 0.55 | 0.50 | 0.55 |
| 2.25 | ARL | 3.34 | 3.02 | 2.88 | 3.01 | 2.35 | 2.13 | 2.05 | 2.13 |
| | SDRL | 0.61 | 0.52 | 0.51 | 0.52 | 0.55 | 0.43 | 0.39 | 0.43 |
| 2.50 | ARL | 3.04 | 2.74 | 2.61 | 2.74 | 2.15 | 1.98 | 1.91 | 1.98 |
| | SDRL | 0.53 | 0.51 | 0.52 | 0.51 | 0.44 | 0.37 | 0.37 | 0.36 |
| 2.75 | ARL | 2.80 | 2.49 | 2.35 | 2.49 | 2.00 | 1.85 | 1.78 | 1.86 |
| | SDRL | 0.51 | 0.51 | 0.48 | 0.51 | 0.37 | 0.39 | 0.43 | 0.39 |
| 3.00 | ARL | 2.56 | 2.27 | 2.16 | 2.26 | 1.89 | 1.72 | 1.63 | 1.72 |
| | SDRL | 0.52 | 0.44 | 0.37 | 0.44 | 0.38 | 0.46 | 0.49 | 0.45 |
| 3.25 | ARL | 2.35 | 2.11 | 2.05 | 2.11 | 1.78 | 1.57 | 1.45 | 1.57 |
| | SDRL | 0.48 | 0.31 | 0.22 | 0.31 | 0.43 | 0.50 | 0.50 | 0.50 |
| 3.50 | ARL | 2.18 | 2.04 | 2.01 | 2.03 | 1.65 | 1.40 | 1.29 | 1.40 |
| | SDRL | 0.39 | 0.19 | 0.13 | 0.19 | 0.48 | 0.49 | 0.45 | 0.49 |
| 3.75 | ARL | 2.08 | 2.00 | 1.99 | 2.00 | 1.51 | 1.25 | 1.16 | 1.25 |
| | SDRL | 0.27 | 0.11 | 0.12 | 0.11 | 0.50 | 0.44 | 0.37 | 0.44 |
| 4.00 | ARL | 2.02 | 1.99 | 1.97 | 1.99 | 1.36 | 1.14 | 1.08 | 1.14 |
| | SDRL | 0.16 | 0.13 | 0.18 | 0.12 | 0.48 | 0.35 | 0.27 | 0.35 |
| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 3.5477 | 3.5467 | 3.5468 | 3.5528 | 2.6767 | 2.6747 | 2.6747 | 2.6827 |
| 0.00 | ARL | 501.83 | 503.69 | 500.25 | 501.82 | 504.59 | 504.77 | 499.42 | 499.88 |
| | SDRL | 499.26 | 500.98 | 499.92 | 499.86 | 499.26 | 505.79 | 499.64 | 495.72 |
| 0.25 | ARL | 107.06 | 85.11 | 76.91 | 85.20 | 147.67 | 118.68 | 107.44 | 120.75 |
| | SDRL | 102.66 | 79.96 | 72.79 | 81.49 | 144.16 | 117.15 | 105.40 | 117.50 |
| 0.50 | ARL | 22.74 | 17.44 | 15.59 | 17.42 | 32.09 | 23.81 | 20.74 | 23.91 |
| | SDRL | 18.77 | 13.56 | 11.72 | 13.54 | 29.30 | 21.01 | 18.13 | 21.17 |
| 0.75 | ARL | 9.36 | 7.50 | 6.85 | 7.52 | 11.34 | 8.61 | 7.65 | 8.63 |
| | SDRL | 5.97 | 4.35 | 3.79 | 4.35 | 8.70 | 6.18 | 5.22 | 6.13 |
| 1.00 | ARL | 5.54 | 4.63 | 4.29 | 4.62 | 5.92 | 4.72 | 4.28 | 4.71 |
| | SDRL | 2.78 | 2.12 | 1.89 | 2.11 | 3.66 | 2.66 | 2.28 | 2.64 |
| 1.25 | ARL | 3.96 | 3.37 | 3.14 | 3.37 | 3.87 | 3.23 | 2.98 | 3.22 |
| | SDRL | 1.66 | 1.30 | 1.16 | 1.30 | 1.97 | 1.50 | 1.32 | 1.49 |
| 1.50 | ARL | 3.09 | 2.68 | 2.53 | 2.68 | 2.91 | 2.47 | 2.30 | 2.46 |
| | SDRL | 1.13 | 0.90 | 0.82 | 0.90 | 1.28 | 1.00 | 0.90 | 0.98 |
| 1.75 | ARL | 2.57 | 2.25 | 2.14 | 2.25 | 2.35 | 2.02 | 1.90 | 2.02 |
| | SDRL | 0.84 | 0.68 | 0.62 | 0.68 | 0.92 | 0.75 | 0.69 | 0.74 |
| 2.00 | ARL | 2.21 | 1.97 | 1.87 | 1.97 | 1.98 | 1.72 | 1.62 | 1.72 |
| | SDRL | 0.66 | 0.56 | 0.54 | 0.56 | 0.72 | 0.62 | 0.59 | 0.62 |
| 2.25 | ARL | 1.96 | 1.75 | 1.66 | 1.76 | 1.72 | 1.50 | 1.41 | 1.50 |
| | SDRL | 0.56 | 0.52 | 0.52 | 0.52 | 0.62 | 0.55 | 0.51 | 0.55 |
| 2.50 | ARL | 1.77 | 1.56 | 1.47 | 1.57 | 1.51 | 1.32 | 1.24 | 1.32 |
| | SDRL | 0.52 | 0.52 | 0.51 | 0.52 | 0.55 | 0.48 | 0.43 | 0.48 |
| 2.75 | ARL | 1.60 | 1.39 | 1.30 | 1.39 | 1.35 | 1.18 | 1.13 | 1.19 |
| | SDRL | 0.52 | 0.49 | 0.46 | 0.49 | 0.49 | 0.39 | 0.33 | 0.39 |
| 3.00 | ARL | 1.44 | 1.24 | 1.17 | 1.24 | 1.22 | 1.10 | 1.06 | 1.10 |
| | SDRL | 0.50 | 0.43 | 0.38 | 0.43 | 0.42 | 0.30 | 0.23 | 0.29 |
| 3.25 | ARL | 1.30 | 1.13 | 1.08 | 1.13 | 1.13 | 1.04 | 1.02 | 1.04 |
| | SDRL | 0.46 | 0.34 | 0.27 | 0.34 | 0.34 | 0.20 | 0.15 | 0.20 |
| 3.50 | ARL | 1.19 | 1.06 | 1.03 | 1.06 | 1.06 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.39 | 0.24 | 0.18 | 0.24 | 0.24 | 0.13 | 0.09 | 0.12 |
| 3.75 | ARL | 1.10 | 1.03 | 1.01 | 1.03 | 1.03 | 1.00 | 1.00 | 1.01 |
| | SDRL | 0.30 | 0.16 | 0.11 | 0.16 | 0.17 | 0.07 | 0.05 | 0.07 |
| 4.00 | ARL | 1.05 | 1.01 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.22 | 0.10 | 0.06 | 0.09 | 0.11 | 0.04 | 0.02 | 0.04 |

Table 2.5: The run length profiles of the CUSUM-VLRSS chart with the FIR feature for $w = 2$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 8.8520 | 8.8450 | 8.8460 | 8.8455 | 5.1500 | 5.1495 | 5.1507 | 5.1505 |
| 0.00 | ARL | 502.84 | 502.59 | 502.78 | 504.24 | 499.68 | 501.73 | 501.89 | 503.37 |
| | SDRL | 551.31 | 550.05 | 551.61 | 552.91 | 530.20 | 534.18 | 530.84 | 531.94 |
| 0.25 | ARL | 22.24 | 19.92 | 18.00 | 16.72 | 35.55 | 30.30 | 26.59 | 23.81 |
| | SDRL | 18.85 | 16.30 | 14.22 | 12.87 | 37.63 | 31.68 | 27.60 | 24.68 |
| 0.50 | ARL | 7.62 | 7.06 | 6.51 | 6.17 | 7.42 | 6.60 | 5.94 | 5.53 |
| | SDRL | 3.98 | 3.55 | 3.12 | 2.85 | 5.75 | 4.89 | 4.18 | 3.78 |
| 0.75 | ARL | 4.66 | 4.34 | 4.08 | 3.89 | 3.82 | 3.48 | 3.23 | 3.05 |
| | SDRL | 1.83 | 1.64 | 1.50 | 1.39 | 2.12 | 1.82 | 1.63 | 1.47 |
| 1.00 | ARL | 3.40 | 3.21 | 3.02 | 2.90 | 2.63 | 2.43 | 2.28 | 2.17 |
| | SDRL | 1.12 | 1.02 | 0.93 | 0.86 | 1.15 | 1.03 | 0.93 | 0.87 |
| 1.25 | ARL | 2.74 | 2.59 | 2.45 | 2.35 | 2.04 | 1.92 | 1.80 | 1.73 |
| | SDRL | 0.79 | 0.72 | 0.65 | 0.61 | 0.78 | 0.72 | 0.67 | 0.64 |
| 1.50 | ARL | 2.32 | 2.20 | 2.11 | 2.04 | 1.69 | 1.58 | 1.50 | 1.43 |
| | SDRL | 0.59 | 0.53 | 0.49 | 0.46 | 0.63 | 0.58 | 0.56 | 0.53 |
| 1.75 | ARL | 2.05 | 1.96 | 1.88 | 1.82 | 1.44 | 1.35 | 1.28 | 1.22 |
| | SDRL | 0.46 | 0.44 | 0.44 | 0.44 | 0.53 | 0.50 | 0.46 | 0.42 |
| 2.00 | ARL | 1.86 | 1.77 | 1.68 | 1.61 | 1.26 | 1.19 | 1.13 | 1.09 |
| | SDRL | 0.44 | 0.46 | 0.48 | 0.49 | 0.44 | 0.39 | 0.34 | 0.29 |
| 2.25 | ARL | 1.69 | 1.58 | 1.48 | 1.39 | 1.13 | 1.08 | 1.05 | 1.03 |
| | SDRL | 0.48 | 0.50 | 0.50 | 0.49 | 0.34 | 0.28 | 0.22 | 0.17 |
| 2.50 | ARL | 1.51 | 1.39 | 1.28 | 1.21 | 1.06 | 1.03 | 1.02 | 1.01 |
| | SDRL | 0.50 | 0.49 | 0.45 | 0.40 | 0.23 | 0.17 | 0.12 | 0.09 |
| 2.75 | ARL | 1.32 | 1.21 | 1.13 | 1.09 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.47 | 0.41 | 0.34 | 0.28 | 0.14 | 0.10 | 0.06 | 0.04 |
| 3.00 | ARL | 1.18 | 1.10 | 1.05 | 1.03 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.38 | 0.30 | 0.22 | 0.16 | 0.08 | 0.04 | 0.03 | 0.02 |
| 3.25 | ARL | 1.08 | 1.04 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.27 | 0.19 | 0.12 | 0.08 | 0.04 | 0.02 | 0.01 | 0.00 |
| 3.50 | ARL | 1.03 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.18 | 0.11 | 0.06 | 0.04 | 0.01 | 0.01 | 0.00 | 0.00 |
| 3.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.10 | 0.06 | 0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.05 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 3.5749 | 3.5790 | 3.5797 | 3.5795 | 2.6891 | 2.6935 | 2.6927 | 2.6921 |
| 0.00 | ARL | 501.14 | 503.46 | 504.73 | 504.34 | 500.55 | 504.38 | 500.55 | 503.40 |
| | SDRL | 520.17 | 528.08 | 522.41 | 524.61 | 514.61 | 523.03 | 515.43 | 524.14 |
| 0.25 | ARL | 58.90 | 51.30 | 43.73 | 39.46 | 87.90 | 78.04 | 67.66 | 60.73 |
| | SDRL | 63.49 | 55.27 | 46.81 | 42.09 | 93.59 | 83.04 | 71.67 | 65.31 |
| 0.50 | ARL | 9.57 | 8.23 | 7.15 | 6.45 | 13.93 | 11.74 | 9.76 | 8.63 |
| | SDRL | 9.31 | 7.68 | 6.48 | 5.71 | 14.65 | 12.18 | 9.94 | 8.66 |
| 0.75 | ARL | 3.85 | 3.45 | 3.15 | 2.91 | 4.52 | 3.95 | 3.47 | 3.15 |
| | SDRL | 2.79 | 2.38 | 2.06 | 1.82 | 3.96 | 3.30 | 2.76 | 2.40 |
| 1.00 | ARL | 2.40 | 2.21 | 2.03 | 1.93 | 2.47 | 2.24 | 2.02 | 1.90 |
| | SDRL | 1.35 | 1.18 | 1.01 | 0.94 | 1.68 | 1.42 | 1.21 | 1.10 |
| 1.25 | ARL | 1.78 | 1.66 | 1.54 | 1.48 | 1.74 | 1.60 | 1.49 | 1.42 |
| | SDRL | 0.84 | 0.75 | 0.67 | 0.63 | 0.94 | 0.81 | 0.70 | 0.64 |
| 1.50 | ARL | 1.45 | 1.36 | 1.28 | 1.24 | 1.38 | 1.30 | 1.23 | 1.18 |
| | SDRL | 0.60 | 0.54 | 0.49 | 0.45 | 0.61 | 0.53 | 0.46 | 0.41 |
| 1.75 | ARL | 1.25 | 1.18 | 1.13 | 1.10 | 1.19 | 1.14 | 1.09 | 1.07 |
| | SDRL | 0.46 | 0.40 | 0.34 | 0.30 | 0.43 | 0.36 | 0.29 | 0.26 |
| 2.00 | ARL | 1.12 | 1.08 | 1.05 | 1.03 | 1.08 | 1.05 | 1.03 | 1.02 |
| | SDRL | 0.33 | 0.27 | 0.21 | 0.17 | 0.28 | 0.23 | 0.17 | 0.14 |
| 2.25 | ARL | 1.05 | 1.03 | 1.01 | 1.01 | 1.03 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.21 | 0.16 | 0.12 | 0.09 | 0.17 | 0.13 | 0.09 | 0.07 |
| 2.50 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.13 | 0.09 | 0.06 | 0.04 | 0.10 | 0.07 | 0.04 | 0.03 |
| 2.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.04 | 0.03 | 0.02 | 0.05 | 0.03 | 0.01 | 0.01 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.02 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.00 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 2.6: The run length comparison of the CUSUM-VLRSS chart with the CUSUM charts based on RSS and MRSS

| | | $k = 0.25$ | | | | | $k = 0.50$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.42 | 501.77 | 502.16 | 502.16 | 503.08 | 500.68 | 502.20 | 503.39 | 502.37 | 501.02 |
| | SDRL | 483.03 | 484.64 | 484.37 | 485.51 | 487.30 | 492.57 | 498.61 | 495.14 | 495.00 | 494.45 |
| 0.25 | ARL | 41.14 | 34.64 | 31.56 | 28.80 | 26.96 | 57.13 | 44.85 | 39.36 | 35.12 | 31.95 |
| | SDRL | 26.51 | 20.65 | 18.21 | 15.89 | 14.49 | 49.75 | 37.64 | 32.44 | 28.14 | 25.09 |
| 0.50 | ARL | 15.34 | 13.25 | 12.31 | 11.43 | 10.84 | 14.40 | 11.77 | 10.68 | 9.73 | 9.07 |
| | SDRL | 6.20 | 4.99 | 4.47 | 3.98 | 3.68 | 8.59 | 6.52 | 5.69 | 4.90 | 4.41 |
| 0.75 | ARL | 9.36 | 8.21 | 7.66 | 7.18 | 6.84 | 7.49 | 6.41 | 5.89 | 5.45 | 5.14 |
| | SDRL | 2.93 | 2.42 | 2.17 | 1.96 | 1.83 | 3.33 | 2.61 | 2.30 | 2.05 | 1.86 |
| 1.00 | ARL | 6.76 | 5.96 | 5.61 | 5.27 | 5.03 | 5.10 | 4.42 | 4.10 | 3.82 | 3.64 |
| | SDRL | 1.79 | 1.48 | 1.35 | 1.22 | 1.14 | 1.85 | 1.47 | 1.32 | 1.19 | 1.10 |
| 1.50 | ARL | 4.42 | 3.95 | 3.72 | 3.52 | 3.38 | 3.15 | 2.79 | 2.63 | 2.47 | 2.37 |
| | SDRL | 0.94 | 0.80 | 0.73 | 0.67 | 0.62 | 0.88 | 0.74 | 0.67 | 0.61 | 0.56 |
| 2.00 | ARL | 3.35 | 3.02 | 2.86 | 2.71 | 2.60 | 2.35 | 2.13 | 2.04 | 1.96 | 1.91 |
| | SDRL | 0.62 | 0.53 | 0.51 | 0.51 | 0.52 | 0.55 | 0.43 | 0.38 | 0.37 | 0.37 |
| 2.50 | ARL | 2.74 | 2.44 | 2.28 | 2.16 | 2.09 | 1.98 | 1.83 | 1.74 | 1.62 | 1.53 |
| | SDRL | 0.51 | 0.50 | 0.45 | 0.36 | 0.29 | 0.36 | 0.40 | 0.45 | 0.49 | 0.50 |
| 3.00 | ARL | 2.26 | 2.07 | 2.02 | 2.00 | 1.99 | 1.72 | 1.49 | 1.36 | 1.23 | 1.16 |
| | SDRL | 0.44 | 0.25 | 0.16 | 0.11 | 0.12 | 0.45 | 0.50 | 0.48 | 0.42 | 0.36 |
| 4.00 | ARL | 1.99 | 1.92 | 1.82 | 1.67 | 1.52 | 1.1 | 1.03 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.12 | 0.27 | 0.39 | 0.47 | 0.50 | 0.3 | 0.17 | 0.10 | 0.05 | 0.02 |

| | | $k = 0.75$ | | | | | $k = 1.00$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 501.82 | 500.45 | 500.65 | 503.77 | 500.36 | 499.88 | 502.12 | 501.54 | 501.99 | 502.86 |
| | SDRL | 499.86 | 495.73 | 496.84 | 496.35 | 497.14 | 495.72 | 501.75 | 497.85 | 496.91 | 499.68 |
| 0.25 | ARL | 85.20 | 66.97 | 58.55 | 51.30 | 46.47 | 120.75 | 94.43 | 83.80 | 73.08 | 66.33 |
| | SDRL | 81.49 | 63.03 | 54.01 | 46.96 | 42.32 | 117.50 | 91.85 | 80.89 | 70.17 | 64.08 |
| 0.50 | ARL | 17.42 | 13.52 | 11.79 | 10.41 | 9.60 | 23.91 | 17.55 | 15.08 | 13.04 | 11.58 |
| | SDRL | 13.54 | 9.74 | 8.14 | 6.92 | 6.18 | 21.17 | 14.77 | 12.40 | 10.28 | 9.09 |
| 0.75 | ARL | 7.52 | 6.11 | 5.53 | 5.01 | 4.70 | 8.63 | 6.64 | 5.88 | 5.19 | 4.80 |
| | SDRL | 4.35 | 3.23 | 2.75 | 2.38 | 2.18 | 6.13 | 4.30 | 3.63 | 3.07 | 2.72 |
| 1.00 | ARL | 4.62 | 3.91 | 3.60 | 3.33 | 3.14 | 4.71 | 3.83 | 3.46 | 3.15 | 2.95 |
| | SDRL | 2.11 | 1.64 | 1.43 | 1.27 | 1.16 | 2.64 | 1.93 | 1.68 | 1.45 | 1.29 |
| 1.50 | ARL | 2.68 | 2.35 | 2.21 | 2.08 | 1.99 | 2.46 | 2.12 | 1.97 | 1.83 | 1.74 |
| | SDRL | 0.90 | 0.73 | 0.66 | 0.60 | 0.57 | 0.98 | 0.79 | 0.72 | 0.66 | 0.63 |
| 2.00 | ARL | 1.97 | 1.76 | 1.65 | 1.54 | 1.46 | 1.72 | 1.50 | 1.39 | 1.30 | 1.24 |
| | SDRL | 0.56 | 0.52 | 0.52 | 0.51 | 0.51 | 0.62 | 0.55 | 0.51 | 0.47 | 0.43 |
| 2.50 | ARL | 1.57 | 1.36 | 1.25 | 1.17 | 1.12 | 1.32 | 1.16 | 1.10 | 1.05 | 1.04 |
| | SDRL | 0.52 | 0.48 | 0.44 | 0.37 | 0.32 | 0.48 | 0.37 | 0.30 | 0.23 | 0.18 |
| 3.00 | ARL | 1.24 | 1.10 | 1.05 | 1.02 | 1.04 | 1.10 | 1.03 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.43 | 0.30 | 0.22 | 0.15 | 0.20 | 0.29 | 0.16 | 0.11 | 0.07 | 0.09 |
| 4.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.09 | 0.03 | 0.01 | 0.01 | 0.00 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 |

Table 2.7: The run length comparison of the CUSUM-VLRSS chart with the CUSUM charts based on RSS and MRSS under imperfect ranking for $k = 0.50$

| | | $\rho = 0.25$ | | | | | $\rho = 0.50$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | $w = 2$ | | | | $w = 0$ | $w = 2$ | | | |
| | $(l, v)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 5.0766 | 5.0758 | 5.0760 | 5.0760 | 5.0759 | 5.0766 | 5.0758 | 5.0760 | 5.0760 | 5.0759 |
| 0.00 | ARL | 503.63 | 500.31 | 500.23 | 503.33 | 501.13 | 502.23 | 503.75 | 500.48 | 501.97 | 502.13 |
| | SDRL | 494.94 | 492.00 | 495.33 | 499.63 | 491.69 | 496.98 | 499.84 | 494.77 | 493.41 | 495.43 |
| 0.25 | ARL | 140.80 | 140.25 | 140.14 | 140.13 | 139.82 | 126.57 | 124.18 | 122.47 | 122.43 | 121.08 |
| | SDRL | 133.80 | 132.87 | 132.03 | 133.61 | 133.27 | 119.55 | 115.75 | 115.12 | 114.82 | 114.52 |
| 0.50 | ARL | 37.25 | 37.07 | 37.05 | 37.04 | 36.79 | 32.46 | 31.79 | 31.52 | 31.27 | 30.92 |
| | SDRL | 30.51 | 29.95 | 29.90 | 29.96 | 29.56 | 25.60 | 24.85 | 24.51 | 24.54 | 24.08 |
| 0.75 | ARL | 16.69 | 16.66 | 16.60 | 16.56 | 16.55 | 14.84 | 14.48 | 14.44 | 14.25 | 14.21 |
| | SDRL | 10.61 | 10.59 | 10.66 | 10.53 | 10.57 | 9.03 | 8.75 | 8.71 | 8.56 | 8.50 |
| 1.00 | ARL | 10.23 | 10.16 | 10.14 | 10.11 | 10.10 | 9.18 | 9.04 | 8.97 | 8.91 | 8.87 |
| | SDRL | 5.31 | 5.21 | 5.16 | 5.20 | 5.19 | 4.49 | 4.41 | 4.35 | 4.31 | 4.29 |
| 1.50 | ARL | 5.69 | 5.66 | 5.65 | 5.64 | 5.64 | 5.20 | 5.12 | 5.09 | 5.07 | 5.05 |
| | SDRL | 2.17 | 2.18 | 2.13 | 2.15 | 2.15 | 1.90 | 1.86 | 1.83 | 1.82 | 1.81 |
| 2.00 | ARL | 3.96 | 3.95 | 3.95 | 3.95 | 3.95 | 3.67 | 3.63 | 3.60 | 3.58 | 3.58 |
| | SDRL | 1.25 | 1.24 | 1.24 | 1.24 | 1.23 | 1.11 | 1.10 | 1.07 | 1.07 | 1.07 |
| 2.50 | ARL | 3.08 | 3.07 | 3.07 | 3.07 | 3.07 | 2.87 | 2.84 | 2.82 | 2.81 | 2.80 |
| | SDRL | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.77 | 0.75 | 0.75 | 0.74 | 0.74 |
| 3.00 | ARL | 2.55 | 2.54 | 2.54 | 2.54 | 2.53 | 2.39 | 2.36 | 2.36 | 2.34 | 2.34 |
| | SDRL | 0.64 | 0.64 | 0.64 | 0.63 | 0.64 | 0.57 | 0.55 | 0.56 | 0.54 | 0.55 |
| 4.00 | ARL | 2.00 | 2.00 | 2.00 | 2.00 | 2.00 | 1.92 | 1.91 | 1.90 | 1.89 | 1.89 |
| | SDRL | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.38 | 0.38 |

| | | $\rho = 0.75$ | | | | | $\rho = 0.90$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | $w = 2$ | | | | $w = 0$ | $w = 2$ | | | |
| | $(n, v)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 5.0766 | 5.0758 | 5.0760 | 5.0760 | 5.0759 | 5.0766 | 5.0758 | 5.0760 | 5.0760 | 5.0759 |
| 0.00 | ARL | 501.91 | 501.85 | 501.02 | 503.27 | 503.97 | 501.60 | 502.70 | 501.57 | 499.21 | 501.32 |
| | SDRL | 498.52 | 494.84 | 497.44 | 496.44 | 497.43 | 498.38 | 497.92 | 496.62 | 491.86 | 493.55 |
| 0.25 | ARL | 100.20 | 93.59 | 90.12 | 88.92 | 87.06 | 76.59 | 66.59 | 62.38 | 59.17 | 56.50 |
| | SDRL | 93.08 | 86.40 | 82.78 | 81.77 | 79.86 | 69.43 | 58.97 | 55.01 | 51.86 | 49.34 |
| 0.50 | ARL | 24.47 | 22.95 | 22.31 | 21.65 | 21.47 | 18.66 | 16.60 | 15.61 | 14.75 | 14.18 |
| | SDRL | 17.86 | 16.34 | 15.96 | 15.10 | 14.94 | 12.56 | 10.72 | 9.72 | 8.93 | 8.42 |
| 0.75 | ARL | 11.74 | 11.11 | 10.85 | 10.53 | 10.42 | 9.30 | 8.42 | 8.01 | 7.68 | 7.48 |
| | SDRL | 6.45 | 5.96 | 5.79 | 5.52 | 5.40 | 4.56 | 3.93 | 3.66 | 3.40 | 3.30 |
| 1.00 | ARL | 7.49 | 7.17 | 7.00 | 6.86 | 6.78 | 6.15 | 5.63 | 5.39 | 5.20 | 5.05 |
| | SDRL | 3.32 | 3.12 | 2.97 | 2.89 | 2.83 | 2.46 | 2.12 | 2.00 | 1.88 | 1.80 |
| 1.50 | ARL | 4.39 | 4.21 | 4.14 | 4.08 | 4.02 | 3.71 | 3.44 | 3.32 | 3.21 | 3.13 |
| | SDRL | 1.47 | 1.37 | 1.34 | 1.30 | 1.27 | 1.13 | 1.01 | 0.96 | 0.90 | 0.87 |
| 2.00 | ARL | 3.15 | 3.05 | 2.99 | 2.94 | 2.91 | 2.72 | 2.54 | 2.46 | 2.39 | 2.34 |
| | SDRL | 0.88 | 0.84 | 0.82 | 0.80 | 0.78 | 0.71 | 0.64 | 0.60 | 0.57 | 0.55 |
| 2.50 | ARL | 2.50 | 2.42 | 2.39 | 2.35 | 2.33 | 2.20 | 2.09 | 2.04 | 2.00 | 1.97 |
| | SDRL | 0.62 | 0.58 | 0.57 | 0.55 | 0.54 | 0.47 | 0.41 | 0.39 | 0.37 | 0.37 |
| 3.00 | ARL | 2.13 | 2.08 | 2.05 | 2.03 | 2.02 | 1.93 | 1.84 | 1.80 | 1.74 | 1.72 |
| | SDRL | 0.43 | 0.40 | 0.40 | 0.39 | 0.38 | 0.37 | 0.40 | 0.42 | 0.45 | 0.46 |
| 4.00 | ARL | 1.73 | 1.66 | 1.63 | 1.60 | 1.58 | 1.42 | 1.28 | 1.22 | 1.17 | 1.13 |
| | SDRL | 0.45 | 0.48 | 0.48 | 0.49 | 0.49 | 0.49 | 0.45 | 0.41 | 0.37 | 0.34 |

Table 2.8: The run length comparison of the CUSUM-VLRSS chart with the CUSUM charts based on RSS and MRSS under imperfect ranking for $k = 1.00$

| | | $\rho = 0.25$ | | | | | $\rho = 0.50$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | $w = 2$ | | | | $w = 0$ | $w = 2$ | | | |
| | $(l, v)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 2.6640 | 2.6639 | 2.6649 | 2.6651 | 2.6650 | 2.6641 | 2.6639 | 2.6652 | 2.6651 | 2.6652 |
| 0.00 | ARL | 500.05 | 502.90 | 500.14 | 499.56 | 501.19 | 500.33 | 499.42 | 502.08 | 502.29 | 500.73 |
| | SDRL | 498.03 | 498.52 | 495.80 | 494.28 | 498.89 | 498.66 | 494.50 | 499.75 | 500.45 | 493.84 |
| 0.25 | ARL | 244.31 | 242.89 | 242.48 | 241.88 | 241.54 | 226.18 | 221.44 | 220.29 | 220.06 | 218.84 |
| | SDRL | 242.23 | 238.36 | 238.75 | 241.18 | 240.93 | 223.87 | 217.39 | 218.18 | 217.61 | 218.20 |
| 0.50 | ARL | 77.90 | 77.22 | 77.18 | 77.16 | 77.11 | 67.72 | 65.26 | 64.93 | 64.18 | 64.17 |
| | SDRL | 74.98 | 74.51 | 74.74 | 74.58 | 74.50 | 64.67 | 62.59 | 62.15 | 61.41 | 61.89 |
| 0.75 | ARL | 29.31 | 29.01 | 28.98 | 28.64 | 28.03 | 24.52 | 23.83 | 23.65 | 23.53 | 23.27 |
| | SDRL | 26.63 | 26.26 | 25.96 | 26.23 | 26.41 | 21.81 | 21.12 | 20.86 | 20.82 | 20.49 |
| 1.00 | ARL | 13.91 | 13.86 | 13.85 | 13.81 | 13.76 | 11.71 | 11.49 | 11.23 | 11.23 | 11.17 |
| | SDRL | 11.27 | 11.20 | 11.09 | 11.08 | 11.05 | 9.17 | 8.85 | 8.59 | 8.66 | 8.59 |
| 1.50 | ARL | 5.52 | 5.49 | 5.47 | 5.46 | 5.45 | 4.83 | 4.74 | 4.71 | 4.68 | 4.64 |
| | SDRL | 3.35 | 3.28 | 3.30 | 3.28 | 3.29 | 2.74 | 2.69 | 2.67 | 2.64 | 2.59 |
| 2.00 | ARL | 3.31 | 3.30 | 3.28 | 3.28 | 3.28 | 2.99 | 2.93 | 2.91 | 2.90 | 2.89 |
| | SDRL | 1.56 | 1.54 | 1.55 | 1.53 | 1.55 | 1.33 | 1.29 | 1.28 | 1.28 | 1.26 |
| 2.50 | ARL | 2.39 | 2.38 | 2.38 | 2.37 | 2.37 | 2.18 | 2.15 | 2.14 | 2.13 | 2.12 |
| | SDRL | 0.95 | 0.95 | 0.94 | 0.94 | 0.94 | 0.83 | 0.81 | 0.81 | 0.80 | 0.80 |
| 3.00 | ARL | 1.89 | 1.89 | 1.88 | 1.88 | 1.88 | 1.75 | 1.73 | 1.72 | 1.71 | 1.70 |
| | SDRL | 0.69 | 0.69 | 0.68 | 0.68 | 0.68 | 0.63 | 0.62 | 0.62 | 0.62 | 0.61 |
| 4.00 | ARL | 1.34 | 1.34 | 1.34 | 1.34 | 1.34 | 1.25 | 1.23 | 1.22 | 1.22 | 1.21 |
| | SDRL | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 | 0.44 | 0.43 | 0.42 | 0.41 | 0.41 |

| | | $\rho = 0.75$ | | | | | $\rho = 0.90$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | $w = 2$ | | | | $w = 0$ | $w = 2$ | | | |
| | $(l, v)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 1)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| | | RSS | MRSS | VLRSS | VLRSS | VLRSS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 2.6641 | 2.6651 | 2.6652 | 2.6651 | 2.6652 | 2.6705 | 2.6651 | 2.6687 | 2.6685 | 2.6694 |
| 0.00 | ARL | 500.26 | 501.58 | 499.60 | 499.78 | 500.08 | 499.91 | 501.85 | 502.17 | 499.98 | 500.53 |
| | SDRL | 500.35 | 499.51 | 499.61 | 496.92 | 496.63 | 497.39 | 501.08 | 501.01 | 499.09 | 497.02 |
| 0.25 | ARL | 187.33 | 178.17 | 175.48 | 170.99 | 168.12 | 151.96 | 135.77 | 129.77 | 120.96 | 116.57 |
| | SDRL | 185.67 | 176.41 | 173.23 | 168.36 | 165.47 | 147.92 | 134.06 | 126.83 | 117.47 | 114.29 |
| 0.50 | ARL | 48.84 | 45.05 | 43.10 | 41.81 | 40.87 | 34.48 | 28.69 | 26.58 | 24.69 | 23.32 |
| | SDRL | 46.11 | 42.23 | 40.52 | 38.98 | 38.02 | 31.82 | 26.05 | 23.90 | 22.06 | 20.58 |
| 0.75 | ARL | 17.44 | 15.85 | 15.25 | 14.79 | 14.29 | 12.11 | 10.30 | 9.54 | 8.84 | 8.46 |
| | SDRL | 14.74 | 13.22 | 12.71 | 12.03 | 11.63 | 9.46 | 7.74 | 7.04 | 6.40 | 5.93 |
| 1.00 | ARL | 8.54 | 7.94 | 7.67 | 7.41 | 7.28 | 6.26 | 5.46 | 5.12 | 4.84 | 4.67 |
| | SDRL | 6.12 | 5.57 | 5.26 | 5.02 | 4.91 | 3.99 | 3.28 | 2.99 | 2.75 | 2.62 |
| 1.50 | ARL | 3.79 | 3.58 | 3.51 | 3.43 | 3.37 | 3.02 | 2.74 | 2.62 | 2.51 | 2.44 |
| | SDRL | 1.92 | 1.77 | 1.71 | 1.65 | 1.60 | 1.36 | 1.17 | 1.09 | 1.02 | 0.98 |
| 2.00 | ARL | 2.46 | 2.34 | 2.30 | 2.25 | 2.23 | 2.05 | 1.88 | 1.81 | 1.75 | 1.71 |
| | SDRL | 0.98 | 0.92 | 0.89 | 0.87 | 0.85 | 0.76 | 0.69 | 0.65 | 0.63 | 0.62 |
| 2.50 | ARL | 1.85 | 1.78 | 1.74 | 1.72 | 1.69 | 1.57 | 1.44 | 1.39 | 1.35 | 1.31 |
| | SDRL | 0.67 | 0.64 | 0.63 | 0.62 | 0.61 | 0.57 | 0.53 | 0.51 | 0.49 | 0.47 |
| 3.00 | ARL | 1.49 | 1.43 | 1.41 | 1.38 | 1.37 | 1.26 | 1.17 | 1.13 | 1.11 | 1.09 |
| | SDRL | 0.55 | 0.52 | 0.52 | 0.51 | 0.50 | 0.44 | 0.38 | 0.34 | 0.31 | 0.28 |
| 4.00 | ARL | 1.09 | 1.07 | 1.06 | 1.05 | 1.04 | 1.02 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.29 | 0.25 | 0.23 | 0.22 | 0.20 | 0.14 | 0.08 | 0.06 | 0.04 | 0.03 |

Table 2.9: The run length comparison of the CUSUM-VLRSS chart with the CUSUM chart based on SRS

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| $\delta$ | $h$ | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 499.87 | 502.34 | 502.91 | 502.45 | 500.28 | 500.28 | 501.80 | 501.12 |
| | SDRL | 481.07 | 487.68 | 486.47 | 488.09 | 495.53 | 499.88 | 493.81 | 496.42 |
| 0.25 | ARL | 94.83 | 49.58 | 41.41 | 38.33 | 145.87 | 71.76 | 56.68 | 51.23 |
| | SDRL | 77.68 | 34.11 | 26.72 | 23.99 | 138.48 | 64.14 | 49.22 | 43.80 |
| 0.50 | ARL | 31.21 | 17.84 | 15.33 | 14.43 | 38.83 | 17.75 | 14.35 | 13.11 |
| | SDRL | 17.73 | 7.78 | 6.25 | 5.66 | 31.71 | 11.59 | 8.63 | 7.59 |
| 0.75 | ARL | 17.56 | 10.69 | 9.37 | 8.82 | 17.36 | 8.90 | 7.53 | 6.97 |
| | SDRL | 7.60 | 3.62 | 2.94 | 2.68 | 11.28 | 4.30 | 3.33 | 2.98 |
| 1.00 | ARL | 12.13 | 7.70 | 6.77 | 6.40 | 10.51 | 5.92 | 5.10 | 4.76 |
| | SDRL | 4.35 | 2.18 | 1.81 | 1.64 | 5.51 | 2.32 | 1.84 | 1.65 |
| 1.50 | ARL | 7.58 | 4.97 | 4.42 | 4.20 | 5.82 | 3.59 | 3.15 | 2.99 |
| | SDRL | 2.14 | 1.12 | 0.94 | 0.88 | 2.25 | 1.07 | 0.89 | 0.82 |
| 2.00 | ARL | 5.54 | 3.73 | 3.35 | 3.20 | 4.07 | 2.63 | 2.36 | 2.24 |
| | SDRL | 1.32 | 0.73 | 0.61 | 0.57 | 1.30 | 0.67 | 0.55 | 0.50 |
| 2.50 | ARL | 4.41 | 3.04 | 2.74 | 2.61 | 3.15 | 2.15 | 1.98 | 1.91 |
| | SDRL | 0.94 | 0.53 | 0.51 | 0.52 | 0.88 | 0.44 | 0.37 | 0.37 |
| 3.00 | ARL | 3.69 | 2.56 | 2.27 | 2.16 | 2.60 | 1.89 | 1.72 | 1.63 |
| | SDRL | 0.72 | 0.52 | 0.44 | 0.37 | 0.66 | 0.38 | 0.46 | 0.49 |
| 4.00 | ARL | 2.84 | 2.02 | 1.99 | 1.97 | 2.03 | 1.36 | 1.14 | 1.08 |
| | SDRL | 0.51 | 0.16 | 0.13 | 0.18 | 0.38 | 0.48 | 0.35 | 0.27 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| $\delta$ | $h$ | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.11 | 501.83 | 503.69 | 500.25 | 500.80 | 504.59 | 504.77 | 499.42 |
| | SDRL | 496.76 | 499.26 | 500.98 | 499.92 | 497.95 | 499.26 | 505.79 | 499.64 |
| 0.25 | ARL | 200.07 | 107.06 | 85.11 | 76.91 | 249.75 | 147.67 | 118.68 | 107.44 |
| | SDRL | 195.33 | 102.66 | 79.96 | 72.79 | 245.44 | 144.16 | 117.15 | 105.40 |
| 0.50 | ARL | 57.01 | 22.74 | 17.44 | 15.59 | 81.50 | 32.09 | 23.81 | 20.74 |
| | SDRL | 52.84 | 18.77 | 13.56 | 11.72 | 79.31 | 29.30 | 21.01 | 18.13 |
| 0.75 | ARL | 22.07 | 9.36 | 7.50 | 6.85 | 30.88 | 11.34 | 8.61 | 7.65 |
| | SDRL | 18.10 | 5.97 | 4.35 | 3.79 | 27.88 | 8.70 | 6.18 | 5.22 |
| 1.00 | ARL | 11.61 | 5.54 | 4.63 | 4.29 | 14.65 | 5.92 | 4.72 | 4.28 |
| | SDRL | 7.97 | 2.78 | 2.12 | 1.89 | 11.95 | 3.66 | 2.66 | 2.28 |
| 1.50 | ARL | 5.44 | 3.09 | 2.68 | 2.53 | 5.76 | 2.91 | 2.47 | 2.30 |
| | SDRL | 2.72 | 1.13 | 0.90 | 0.82 | 3.53 | 1.28 | 1.00 | 0.90 |
| 2.00 | ARL | 3.55 | 2.21 | 1.97 | 1.87 | 3.40 | 1.98 | 1.72 | 1.62 |
| | SDRL | 1.40 | 0.66 | 0.56 | 0.54 | 1.63 | 0.72 | 0.62 | 0.59 |
| 2.50 | ARL | 2.67 | 1.77 | 1.56 | 1.47 | 2.45 | 1.51 | 1.32 | 1.24 |
| | SDRL | 0.89 | 0.52 | 0.52 | 0.51 | 0.99 | 0.55 | 0.48 | 0.43 |
| 3.00 | ARL | 2.18 | 1.44 | 1.24 | 1.17 | 1.95 | 1.22 | 1.10 | 1.06 |
| | SDRL | 0.64 | 0.50 | 0.43 | 0.38 | 0.71 | 0.42 | 0.30 | 0.23 |
| 4.00 | ARL | 1.63 | 1.05 | 1.01 | 1.00 | 1.38 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.52 | 0.22 | 0.10 | 0.06 | 0.50 | 0.11 | 0.04 | 0.02 |

Table 2.10: The run length comparison of the CUSUM-VLRSS chart with CUSUM chart based on SRS for $k = 0.50$ under imperfect ranking

| | | $\rho = 0$ | $\rho = 0.25$ | | | $\rho = 0.50$ | | |
|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 5.0710 | 5.0747 | 5.0753 | 5.0759 | 5.0747 | 5.0753 | 5.0759 |
| 0.00 | ARL | 500.28 | 501.79 | 503.77 | 502.22 | 501.88 | 501.67 | 501.99 |
| | SDRL | 495.53 | 495.00 | 500.14 | 498.36 | 496.63 | 497.16 | 492.92 |
| 0.25 | ARL | 145.87 | 142.88 | 141.33 | 140.75 | 128.88 | 127.19 | 126.21 |
| | SDRL | 138.48 | 135.55 | 133.79 | 133.50 | 121.10 | 119.29 | 118.52 |
| 0.50 | ARL | 38.83 | 37.42 | 37.30 | 37.23 | 33.36 | 32.57 | 32.08 |
| | SDRL | 31.71 | 30.39 | 30.17 | 30.07 | 26.44 | 25.62 | 25.08 |
| 0.75 | ARL | 17.36 | 16.79 | 16.70 | 16.68 | 15.13 | 14.77 | 14.66 |
| | SDRL | 11.28 | 10.72 | 10.72 | 10.61 | 9.28 | 8.99 | 8.93 |
| 1.00 | ARL | 10.51 | 10.24 | 10.20 | 10.18 | 9.42 | 9.16 | 9.07 |
| | SDRL | 5.51 | 5.29 | 5.27 | 5.24 | 4.66 | 4.48 | 4.44 |
| 1.50 | ARL | 5.82 | 5.70 | 5.67 | 5.65 | 5.29 | 5.19 | 5.17 |
| | SDRL | 2.25 | 2.18 | 2.14 | 2.16 | 1.95 | 1.89 | 1.87 |
| 2.00 | ARL | 4.07 | 3.98 | 3.97 | 3.96 | 3.73 | 3.66 | 3.63 |
| | SDRL | 1.30 | 1.25 | 1.25 | 1.25 | 1.14 | 1.11 | 1.09 |
| 2.50 | ARL | 3.15 | 3.10 | 3.08 | 3.08 | 2.91 | 2.87 | 2.85 |
| | SDRL | 0.88 | 0.86 | 0.85 | 0.85 | 0.78 | 0.76 | 0.76 |
| 3.00 | ARL | 2.60 | 2.56 | 2.55 | 2.55 | 2.42 | 2.39 | 2.38 |
| | SDRL | 0.66 | 0.64 | 0.64 | 0.64 | 0.59 | 0.57 | 0.56 |
| 4.00 | ARL | 2.03 | 2.01 | 2.00 | 2.00 | 1.93 | 1.92 | 1.91 |
| | SDRL | 0.38 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |

| | | $\rho = 0$ | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 5.0710 | 5.0747 | 5.0753 | 5.0759 | 5.0747 | 5.0753 | 5.0759 |
| 0.00 | ARL | 500.28 | 499.81 | 501.79 | 501.31 | 500.38 | 502.88 | 503.40 |
| | SDRL | 495.53 | 490.30 | 494.89 | 497.50 | 493.46 | 495.98 | 498.85 |
| 0.25 | ARL | 145.87 | 107.52 | 100.00 | 96.95 | 87.53 | 75.88 | 71.50 |
| | SDRL | 138.48 | 100.52 | 92.63 | 89.34 | 80.36 | 68.49 | 64.19 |
| 0.50 | ARL | 38.83 | 26.73 | 24.63 | 23.86 | 21.54 | 18.76 | 17.62 |
| | SDRL | 31.71 | 20.08 | 17.82 | 17.34 | 15.10 | 12.56 | 11.50 |
| 0.75 | ARL | 17.36 | 12.50 | 11.69 | 11.44 | 10.45 | 9.35 | 8.86 |
| | SDRL | 11.28 | 7.08 | 6.44 | 6.25 | 5.44 | 4.61 | 4.27 |
| 1.00 | ARL | 10.51 | 7.97 | 7.52 | 7.33 | 6.80 | 6.16 | 5.90 |
| | SDRL | 5.51 | 3.63 | 3.31 | 3.19 | 2.88 | 2.45 | 2.30 |
| 1.50 | ARL | 5.82 | 4.60 | 4.39 | 4.31 | 4.03 | 3.71 | 3.58 |
| | SDRL | 2.25 | 1.57 | 1.46 | 1.42 | 1.29 | 1.13 | 1.07 |
| 2.00 | ARL | 4.07 | 3.30 | 3.16 | 3.10 | 2.93 | 2.71 | 2.63 |
| | SDRL | 1.30 | 0.94 | 0.89 | 0.86 | 0.79 | 0.71 | 0.67 |
| 2.50 | ARL | 3.15 | 2.60 | 2.50 | 2.46 | 2.34 | 2.20 | 2.14 |
| | SDRL | 0.88 | 0.66 | 0.62 | 0.60 | 0.54 | 0.47 | 0.44 |
| 3.00 | ARL | 2.60 | 2.19 | 2.13 | 2.10 | 2.02 | 1.93 | 1.89 |
| | SDRL | 0.66 | 0.47 | 0.43 | 0.42 | 0.38 | 0.37 | 0.38 |
| 4.00 | ARL | 2.03 | 1.78 | 1.72 | 1.69 | 1.59 | 1.42 | 1.36 |
| | SDRL | 0.38 | 0.43 | 0.45 | 0.47 | 0.49 | 0.49 | 0.48 |

Table 2.11: The run length comparison of the CUSUM-VLRSS chart with CUSUM chart based on SRS for $k = 1.00$ under imperfect ranking

| | | $\rho = 0$ | $\rho = 0.25$ | | | $\rho = 0.50$ | | |
|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2,1)$ | $(3,2)$ | $(4,2)$ | $(2,1)$ | $(3,2)$ | $(4,2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 2.6655 | 2.6690 | 2.6670 | 2.6670 | 2.6690 | 2.6691 | 2.6690 |
| 0.00 | ARL | 500.80 | 502.18 | 501.70 | 500.22 | 500.82 | 499.93 | 501.59 |
| | SDRL | 497.95 | 502.82 | 499.32 | 493.99 | 500.97 | 497.68 | 502.69 |
| 0.25 | ARL | 249.75 | 246.71 | 244.87 | 243.31 | 228.77 | 226.49 | 225.70 |
| | SDRL | 245.44 | 243.13 | 241.88 | 241.77 | 225.13 | 223.04 | 224.48 |
| 0.50 | ARL | 81.50 | 78.80 | 78.59 | 77.93 | 69.19 | 68.17 | 66.95 |
| | SDRL | 79.31 | 75.95 | 76.23 | 75.24 | 66.95 | 65.31 | 64.14 |
| 0.75 | ARL | 30.88 | 29.71 | 29.41 | 29.22 | 25.53 | 24.58 | 24.39 |
| | SDRL | 27.88 | 27.04 | 26.51 | 26.52 | 22.75 | 21.76 | 21.72 |
| 1.00 | ARL | 14.65 | 14.06 | 13.93 | 13.85 | 12.18 | 11.89 | 11.57 |
| | SDRL | 11.95 | 11.41 | 11.26 | 11.13 | 9.56 | 9.22 | 9.05 |
| 1.50 | ARL | 5.76 | 5.55 | 5.52 | 5.52 | 4.96 | 4.85 | 4.81 |
| | SDRL | 3.53 | 3.36 | 3.35 | 3.34 | 2.87 | 2.77 | 2.73 |
| 2.00 | ARL | 3.40 | 3.32 | 3.30 | 3.30 | 3.04 | 2.99 | 2.96 |
| | SDRL | 1.63 | 1.56 | 1.55 | 1.56 | 1.37 | 1.33 | 1.31 |
| 2.50 | ARL | 2.45 | 2.40 | 2.39 | 2.39 | 2.22 | 2.19 | 2.18 |
| | SDRL | 0.99 | 0.95 | 0.95 | 0.95 | 0.86 | 0.84 | 0.82 |
| 3.00 | ARL | 1.95 | 1.91 | 1.89 | 1.89 | 1.77 | 1.76 | 1.74 |
| | SDRL | 0.71 | 0.69 | 0.69 | 0.69 | 0.64 | 0.63 | 0.63 |
| 4.00 | ARL | 1.38 | 1.35 | 1.35 | 1.34 | 1.26 | 1.24 | 1.24 |
| | SDRL | 0.50 | 0.49 | 0.49 | 0.49 | 0.45 | 0.43 | 0.43 |

| | | $\rho = 0$ | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2,1)$ | $(3,2)$ | $(4,2)$ | $(2,1)$ | $(3,2)$ | $(4,2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 2.6655 | 2.6696 | 2.6685 | 2.6691 | 2.6694 | 2.6685 | 2.6691 |
| 0.00 | ARL | 500.80 | 501.48 | 501.11 | 502.49 | 499.64 | 500.22 | 501.60 |
| | SDRL | 497.95 | 500.62 | 497.24 | 502.13 | 497.18 | 495.92 | 500.06 |
| 0.25 | ARL | 249.75 | 201.81 | 188.82 | 185.00 | 170.97 | 154.09 | 144.51 |
| | SDRL | 245.44 | 199.73 | 187.56 | 183.16 | 167.64 | 151.17 | 142.29 |
| 0.50 | ARL | 81.50 | 53.89 | 48.96 | 47.43 | 41.65 | 34.46 | 31.98 |
| | SDRL | 79.31 | 51.14 | 46.17 | 44.58 | 39.22 | 31.85 | 29.35 |
| 0.75 | ARL | 30.88 | 19.15 | 17.44 | 16.77 | 14.57 | 12.08 | 11.31 |
| | SDRL | 27.88 | 16.38 | 14.58 | 14.18 | 11.82 | 9.42 | 8.74 |
| 1.00 | ARL | 14.65 | 9.39 | 8.51 | 8.24 | 7.30 | 6.26 | 5.91 |
| | SDRL | 11.95 | 6.89 | 6.06 | 5.80 | 4.94 | 4.00 | 3.68 |
| 1.50 | ARL | 5.76 | 4.07 | 3.82 | 3.70 | 3.39 | 3.04 | 2.89 |
| | SDRL | 3.53 | 2.13 | 1.92 | 1.86 | 1.64 | 1.37 | 1.28 |
| 2.00 | ARL | 3.40 | 2.60 | 2.45 | 2.41 | 2.25 | 2.04 | 1.97 |
| | SDRL | 1.63 | 1.07 | 0.99 | 0.96 | 0.86 | 0.75 | 0.72 |
| 2.50 | ARL | 2.45 | 1.94 | 1.85 | 1.82 | 1.71 | 1.56 | 1.51 |
| | SDRL | 0.99 | 0.71 | 0.67 | 0.65 | 0.62 | 0.57 | 0.55 |
| 3.00 | ARL | 1.95 | 1.56 | 1.49 | 1.46 | 1.37 | 1.26 | 1.22 |
| | SDRL | 0.71 | 0.57 | 0.54 | 0.53 | 0.50 | 0.44 | 0.42 |
| 4.00 | ARL | 1.38 | 1.13 | 1.09 | 1.08 | 1.05 | 1.02 | 1.01 |
| | SDRL | 0.50 | 0.34 | 0.29 | 0.27 | 0.21 | 0.14 | 0.10 |

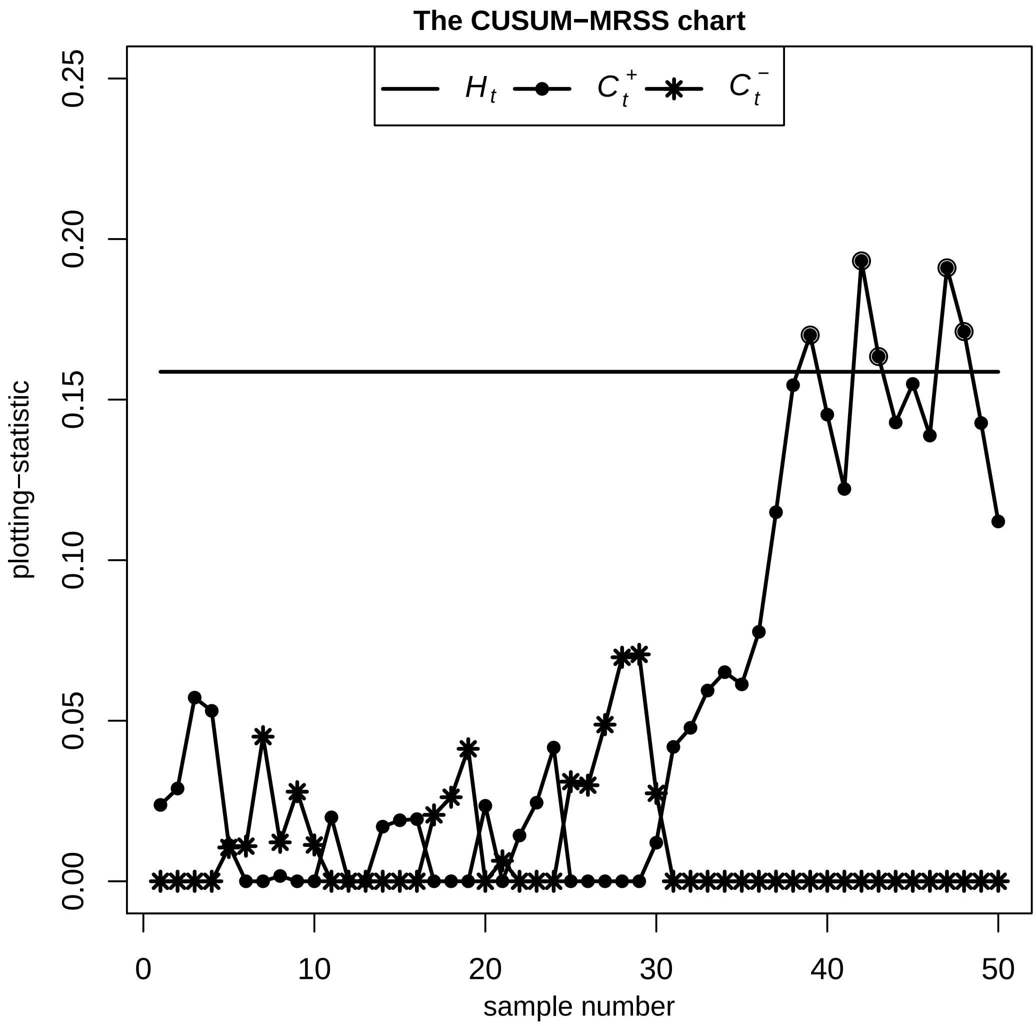


Figure 2.1: The CUSUM chart using MRSS scheme

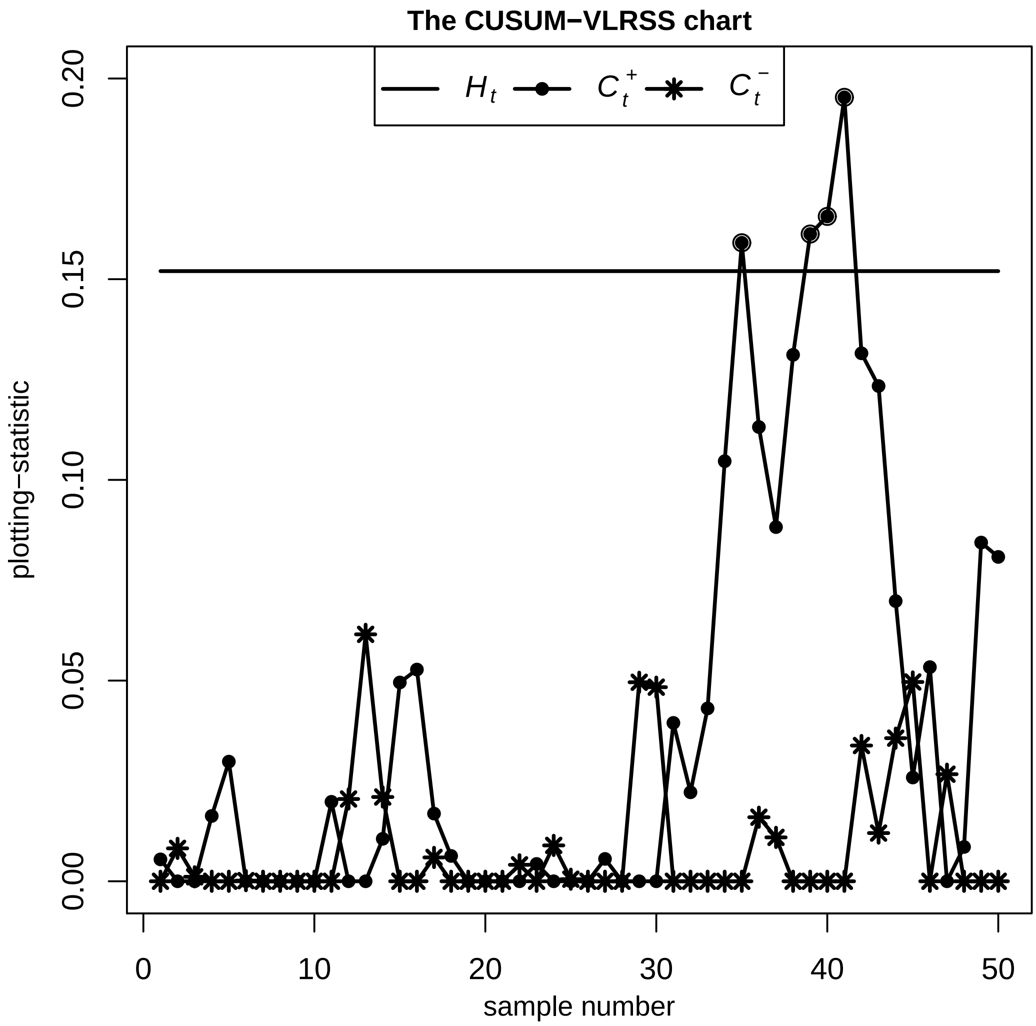


Figure 2.2: The CUSUM chart using VLRSS scheme

# Chapter 3

# An EWMA Chart for Monitoring Process Mean

In the statistical process control literature, there exist several improved quality control charts based on cost-effective sampling schemes, including the ranked set sampling (RSS) and median RSS (MRSS). A generalized cost-effective RSS scheme has been recently introduced for efficiently estimating the population mean, namely varied L RSS (VLRSS). In this chapter, we propose a new exponentially weighted moving average (EWMA) control chart for monitoring the process mean using a recently developed ranking scheme, named varied L ranked set sampling (VLRSS), named the EWMA-VLRSS chart, under both perfect and imperfect rankings. The EWMA-VLRSS chart encompasses the existing EWMA charts based on ranked set sampling (RSS) and median RSS (MRSS) (named the EWMA-RSS and EWMA-MRSS charts). We use extensive Monte Carlo simulations to compute the run length characteristics of the EWMA-VLRSS chart. The proposed chart is then compared with the existing EWMA charts. It is found that, with either perfect or imperfect rankings, the EWMA-VLRSS chart is more sensitive than the EWMA-RSS and EWMA-MRSS charts in detecting small to large shifts in the process mean. A real dataset is also used to explain the working of the EWMA-VLRSS chart.

## 3.1 Introduction

One of the main objectives of the statistical process control (SPC) is a swift detection of an out-of-control situation and to differentiate between the common and assignable causes—two distinct origins of variation in a production process. The common causes are due to inherent variability of a process while the assignable causes are due to the unknown factors that are not part of a process. A process operating with only common-cause variation is said to be in control state while the process with the special-cause variation is said to be out-of-control/unstable. Control charts are very important graphical tool in the SPC toolkit, and are effective in detecting the assignable cause or special-cause variation. In short, the control chart is a very productive tool in detecting and eliminating the infrequent process variations (cf., Montgomery (2007)).

A control chart is a graphical display of three horizontal lines, namely the upper control limit (UCL), the central limit (CL), and the lower control limit (LCL). A process is said to be in-control/stable when the plotting statistic falls within the control limits. Any point that falls outside the control limits is a sign of an out-of-control situation; and hence, necessary corrective actions have to be taken to bring the process back into the control state. The control charts are portrayed into two categories, the memory-less and memory-type charts. The well-known memory-less control charts are the Shewhart-type charts, first introduced by the Walter A. Shewhart in the 1920s. These charts completely rely on the present observations and does not consider the past data. This is the reason why the Shewhart charts are more effective in detecting large shifts. The other classification of the control charts is the memory-type charts, which takes into account both the current and the past data to overhaul their plotting statistics. This feature of the memory-type charts helps them to quickly react against the small and moderate shifts in the process parameter(s). The well-known memory-type control charts are the exponentially weighted moving average (EWMA) and the cumulative sum (CUSUM) charts. The CUSUM chart was first introduced by Page (1954) for monitoring the process dispersion while the EWMA chart was first introduced by Roberts (1959) for monitoring changes in the process mean. These control charts are frequently used in the chemical and process industries to detect small process disturbances where the small process shifts may inflict significant financial penalties (cf., Montgomery (2007)). Lucas and Saccucci (1990) suggested the fast initial response (FIR) feature to upgrade the performance of the EWMA chart for the start-up problems. They have also suggested a combination of two charting schemes that are able to control small as well as large shifts in the process mean. For more related works on the EWMA and CUSUM charts, we refer to Knoth (2005), Locas and Crosier (1982), Lucas and Saccucci (1990), Chiu (2009), Abbas et al. (2013), Haq (2013), Haq et al. (2014a), and therein cited references.

McIntyre (1952) first introduced the ranked-set sampling (RSS) for estimating the mean pasture and forage yields. He notices that the RSS scheme is highly beneficial and may be superior to the standard simple random sampling (SRS) when the values of the study variable are costly, time-consuming and/or destructive, but these values could be ranked visually/personal judgment or using the ranks of a cheap auxiliary variable. For example, if the quantification of the quality characteristic(s) is laborious, costly, time-consuming, and it may involve breaking the product (which is very expensive, and might be hard to construct, etc.), it might be possible to rank the quality characteristic according to its quality level by using some expert's opinion, etc. The RSS scheme draws a more representative sample form the target population than that using SRS scheme. In practice, while dealing with the large set sizes, there exist situations where the errors in the ranking are likely to arise. In such situations, Dell and Clutter (1972) showed that, in spite of the presence of ranking errors, the RSS based mean estimator not only remain unbiased but also it turns out to be more precise than that with SRS. A simple mathematical model was considered by Stokes (1977) for the imperfect ranking, whereby the study variable could be ranked using a cheap auxiliary/concomitant variable. Later on, extreme RSS (ERSS) and median RSS (MRSS) schemes were introduced by Samawi et al. (1996) and Muttlak (1997), respectively, for efficiently estimating the population mean. For a symmetric population, the mean estimator with MRSS is more precise than the ones with SRS and RSS. Quartile RSS (QRSS) was suggested by Muttlak (2003) for estimating the population mean. Al-Nasser (2007) used the idea of the L moments to propose a generalized RSS scheme, named L RSS (LRSS), for estimating the population mean. The LRSS scheme encompasses several existing RSS scheme, like the RSS, QRSS, ERSS, and MRSS. Recently, Haq et al. (2015e) further extended the work on LRSS scheme, and have suggested a more generalized form of the LRSS scheme for efficiently estimating the mean of a symmetric population, named varied LRSS (VLRSS). The VLRSS scheme is not only an alternative to the existing RSS schemes but also it encompasses them (cf., Haq et al. (2015e)).

As the mean estimator with the RSS scheme is more accurate than the mean estimator based on SRS scheme, this fact has inspired many researchers to propose either memory-less or memory-type charts for monitoring the process mean using the RSS scheme. The charts with RSS are more sensitive than those with SRS. Under the RSS protocol, the Shewhart chart was first suggested by Salazar and Sinha (1997) for monitoring the process mean. Their work, later, extended by Muttlak and Al-Sabah (2003), who suggested several improved Shewhart-type mean charts using the RSS, MRSS, and ERSS schemes for both perfect and imperfect setups. The RSS schemes' based Shewhart charts outperformed their counterparts based on SRS. Abujiya and Muttlak (2004) and Al-Omari and Haq (2012) utilized double RSS schemes to propose new Shewhart charts for the process mean. Recently, Abujiya and Lee (2013) used RSS with both perfect and imperfect rankings to construct new Shewhart, EWMA and CUSUM charts for monitoring the process mean. Haq (2014) used the sample mean deviation to monitor the process dispersion using the EWMA chart with the RSS scheme. Haq et al. (2015b) used a mixed RSS scheme, that mixes both SRS and RSS schemes, to monitor the process mean. Haq et al. (2015a) used the ordered RSS based mean and location estimators for simultaneously monitoring the process mean and variability with the help of an EWMA chart. Haq et al. (2015c) used ordered double RSS scheme to construct a new maximum EWMA chart for monitoring both the process mean and dispersion.

As aforementioned that the VLRSS scheme incorporates several existing RSS schemes, and it provides a precise mean estimator than those with the existing RSS schemes when sampling from a symmetric population. This fact has motivated us to suggest an improved EWMA chart using the VLRSS for monitoring the process mean, the EWMA-VLRSS chart. Here, Monte Carlo simulations are used in computing the run length properties of the EWMA-VLRSS chart, including the average run length (ARL) and the standard deviation of the run length (SDRL). The run length performance of the EWMA-VLRSS is compared with that of the existing EWMA charts based on SRS, RSS and MRSS schemes under both perfect and imperfect rankings, named the EWMA-SRS, EWMA-RSS, EWMA-MRSS charts. It turns out that the EWMA-VLRSS chart is better than the existing EWMA charts.

The rest of the chapter is in the following order: Section 3.2 contains a review of the EWMA chart with and without FIR features under SRS. In Section 3.3, the VLRSS scheme is reviewed for estimating the population mean. The EWMA-VLRSS chart with and without FIR features is proposed in Section 3.4. In Section 3.5, the performances comparisons of the proposed and existing EWMA charts are made. An illustrative example is given in Section 3.6. Section 3.7 concludes the chapter.

## 3.2 The EWMA chart

In this section, we briefly review the classical EWMA chart along with its FIR feature. The EWMA chart is exceptionally useful in detecting small to moderate shifts in the process parameter(s) more rapidly than the classical Shewhart chart. The reason is that the EWMA chart suitably assigns weights to both the current and the past information—memory-type chart.

Let $Y$ be our study variable and let $\{Y_t\}$, for $t = 1, 2, ...$, be a sequence of independent and identically distributed (IID) random variables. Here, we assume that $Y_t$ is a normal random variable with the mean $\mu_Y$ and the variance $\sigma_Y^2$, i.e., $Y_t \sim N(\mu_Y, \sigma_Y^2)$ for $t \geq 1$. Let $\{\overline{Y}_{\text{SRS},t}\}$ be a sequence of IID random variables, where $\overline{Y}_{\text{SRS},t} = (1/n)\sum_{i=1}^{n} Y_{i,t}$. Here, $Y_{i,t}$ is the $i$th observation in the $t$th simple random sample of size $n$, for $i = 1, 2, ..., n$. It is easy to show that $\overline{Y}_{\text{SRS},t}$ is also a normal random variable with the mean $\mu_Y$ and the variance $\sigma_Y^2/n$, i.e., $\overline{Y}_{\text{SRS},t} \sim N(\mu_Y, \sigma_Y^2/n)$. Using $\overline{Y}_{\text{SRS},t} = (1/n)\sum_{i=1}^{n} Y_{i,t}$, the EWMA statistic is given by

$$Z_t = \lambda\overline{Y}_{\text{SRS},t} + (1-\lambda)Z_{t-1}, \tag{3.1}$$

where $\lambda \in (0, 1]$ is a smoothing constant. The starting value of $Z_t$ is set equal to the in-control process mean $\mu_Y$, i.e., $Z_0 = \mu_Y$. The variance of $Z_t$ is

$$\text{Var}(Z_t) = \frac{\sigma_Y^2}{n} \cdot \frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]. \tag{3.2}$$

The control limits—UCL, CL, LCL—of the EWMA chart at the time $t$ are

$$\text{UCL} = \mu_Y + L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}, \tag{3.3}$$

$$\text{CL} = \mu_Y, \tag{3.4}$$

$$\text{LCL} = \mu_Y - L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2t}]}, \tag{3.5}$$

where $L$ is the design parameter of the EWMA chart, and its value depends on the choice of the smoothing constant $\lambda$ and the desired in-control ARL. If the time $t$ gets large, the term $[1-(1-\lambda)^{2t}]$ approaches to unity. It is interesting to note that the convergence rate depends on the value of $\lambda$, i.e., for very small values of $\lambda$ the control limits converge slowly. For small values of $t$, the variable control limits (given in (3.3) and (3.5)) are recommended for better performance of the EWMA chart in detecting initial/startup problem (cf., Montgomery (2007)). The EWMA chart works by plotting $Z_t$ with the control limits against the time $t$. The EWMA chart initiates an out-of-control signal whenever $Z_t > \text{UCL}$ or $Z_t < \text{LCL}$.

In some cases, the process may makes tracks in a different direction from the process target initially or after the process is recouped from an out-of-control state; and, if not ceased, the process may keep on producing the defective items. To overcome such problems, Lucas and Saccucci (1990) associated an FIR feature—resetting the initial value of the plotting-statistics to some constants—with the EWMA chart to enhance its performance for the start-up problems. They used two one-sided EWMA charts each with a head-start. Rhoads et al. (1996) extended the work of Lucas and Saccucci (1990), and has suggested using two one-sided EWMA charts with time-varying control limits with the head-start. Steiner (1999) further reduces the time-varying control limits of the EWMA chart for the first few sample points using an FIR-adjustment. To narrow the control limits, he utilized an exponentially decreasing adjustment factor. The new control limits of the EWMA chart with the FIR-adjustment factor are:

$$\text{UCL} = \mu_Y + L\frac{\sigma_Y}{\sqrt{n}}\left(1-(1-f)^{1+a(t-1)}\right)\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}, \quad (3.6)$$

$$\text{LCL} = \mu_Y - L\frac{\sigma_Y}{\sqrt{n}}\left(1-(1-f)^{1+a(t-1)}\right)\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}, \quad (3.7)$$

where $f$ and $a$ are known constants. Steiner (1999) suggested that the choice of $a$ for which the FIR-adjustment feature has little effect after the 20th observations, i.e., $a = -(1/19)(2/\log(1-f)+1)$. For example, for $f = 0.5$, we get $a = 0.3$. For more details on the FIR features with the EWMA chart, see Rhoads et al. (1996), Steiner (1999), Knoth (2005), and Haq et al. (2014a), to name a few.

## 3.3 The VLRSS scheme

In this section, we briefly review the VLRSS schemes' based mean estimators under both perfect and imperfect rankings.

The VLRSS scheme is a productive plan for efficiently selecting the samples when sampling from a symmetric population. The VLRSS encompasses several existing RSS schemes. An interesting characteristic of the VLRSS scheme is that, with some reasonable assumptions, it requires fewer units in selecting the sample than that using the RSS schemes. Thus, it is useful and helps in reducing the ranking costs when the other RSS schemes could not be employed with full confidence (cf., Haq et al. (2015e)).

The steps involved in selecting a varied L ranked set sample of size $n = mr$ are as follows:

Step 1: Select the value of the VLRSS coefficient, say $w = [al]$ for $0 \leq a < 0.5$, where $[\cdot]$ is the largest possible integer value.

Step 2: Select $2wl$ units from the target population and then divide them into $2w$ sets of $l$ units each.

Step 3: Rank the units within each set with respect to the study variable by using any cheap method or by using the ranks of an auxiliary variable.

Step 4: Select the $v$th and $(l-v+1)$th smallest ranked units from the first and the last $w$ sets, respectively, for $v = 1, 2, ..., [l/2]$.

Step 5: Identify $m(m-2w)$ units form the target population and then divide them into $m-2w$ sets of $m$ units each.

Step 6: Select the $i$th smallest ranked unit from the $(i+w)$th set of $m$ units, for $i = w+1, w+2, ..., m-w$.

Step 7: This completes the one cycle of a varied L ranked set sample of size $m$. The Steps 1-6 could be repeated, if necessary, $r$ number of times to get a total sample size of $n$ units.

Let $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$, $i = 1, 2, ..., 2w$, denote $2w$ samples, each of size $l$, for the $j$th cycle, where $j = 1, 2, ..., r$. Let $Y_{i(v:l)j}$ denote the $v$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = 1, 2, ..., w$, and let $Y_{i(l-v+1:l)j}$ be the $(l-v+1)$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = w+1, w+2, ..., 2w$. Let $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$, $i = w+1, 2, ..., m-w$, denote $m-2w$ samples, each of size $m$, for the $j$th cycle. Let $Y_{i+w(i:m)j}$ denote the $i$th order statistic of $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$ for $i = 1, 2, ..., m-w$.

The sample mean based on a varied L ranked set sample of size $n$, denoted by $\overline{Y}_{\text{VLRSS}}$, and its variance are, respectively, given by

$$\overline{Y}_{\text{VLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right), \tag{3.8}$$

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y(v:l)} + \sigma^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w} \sigma^2_{Y(i:m)}\right), \tag{3.9}$$

where $\sigma^2_{Y(v:l)} = \text{Var}(Y_{i(v:l)j})$, $\sigma^2_{Y(l-v+1:l)} = \text{Var}(Y_{i(l-v+1:l)j})$, and $\sigma^2_{Y(i:m)} = \text{Var}(Y_{i(i:m)j})$. For further details on the computation of the variance of the order statistics, we refer to David and Nagaraja (2003).

It has been shown by Haq et al. (2015e) that, for a symmetric population, $\overline{Y}_{\text{VLRSS}}$ is an unbiased estimator of $\mu_Y$. They showed that with the appropriate choices of $v$, $l$, and $m$, the existing RSS schemes are special cases of VLRSS. For example, when $w = 0$, VLRSS becomes RSS; when $w = [(m-1)/2]$, $l = m$, and $v = w+1$, VLRSS becomes MRSS, etc. For selecting a varied L ranked set sample of size $n$, the experimenter needs to identify $nm - 2w(m-l)r$ units whereas the traditional RSS and MRSS schemes require $nm$ units. Thus, with the less number of identified units ($w \geq 1, l \leq m$), the VLRSS scheme is more economical and practical than the existing RSS schemes as it reduces the ranking costs. Furthermore, when the ranking costs are negligible, it is possible to select more representative samples with the VLRSS scheme than those with the RSS and MRSS scheme by identifying more than $m^2r$ units (cf., Haq et al. (2015e)).

### 3.3.1 The Imperfect VLRSS scheme

There exist the situations where it is difficult for the experimenter to rank visually, or it is costly and/or time-consuming. This issue is resolvable by ranking the study variable ($Y$) using the ranks of a highly correlated auxiliary variable, say $X$—provided that it is readily available. Stokes (1977) recommended a simple model for imperfect rankings, given by

$$Y_{i[i:u]j} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}\left(X_{i(i:u)j} - \mu_X\right) + E_{ij},\ i = 1, 2, ..., u,\ j = 1, 2, ..., r, \tag{3.10}$$

where $u = l, m$; $\mu_X$ and $\sigma_X$ are the population mean and standard deviation of $X$, respectively, and $\rho$ is the correlation between $Y$ and $X$. Here, $E_{ij}$ is the normal random error term with mean zero and variance $\sigma_Y^2(1-\rho^2)$, i.e., $E_{ij} \sim N(0, \sigma_Y^2(1-\rho^2))$, and $E$ is independent of $X$. Here, $Y_{i[i:u]j}$ is the $i$th concomitant variable corresponding to the $i$th order statistic $X_{i(i:u)j}$ for $i = 1, 2, ..., u$. The $X$ values are ranked perfectly here but the $Y$ values are ranked with error. Using the above imperfect ranking model, the sample mean under imperfect VLRSS (IVLRSS) scheme, say $\overline{Y}_{\text{IVLRSS}}$, and its variance are, respectively, given by (cf., Haq et al. (2015e)):

$$\overline{Y}_{\text{IVLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w} Y_{i[v:l]j} + \sum_{i=w+1}^{2w} Y_{i[l-v+1:l]j} + \sum_{i=w+1}^{m-w} Y_{i+w[i:m]j}\right), \tag{3.11}$$

and

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y[v:l]} + \sigma^2_{Y[l-v+1:l]}) + \sum_{i=w+1}^{m-w}\sigma^2_{Y[i:m]}\right) \tag{3.12}$$

$$= \frac{1}{nm}\left\{m\sigma_Y^2(1-\rho^2) + \rho^2\frac{\sigma_Y^2}{\sigma_X^2}\left(2w\sigma^2_{X(v:l)} + \sum_{i=w+1}^{m-w}\sigma^2_{X(i:m)}\right)\right\}, \tag{3.13}$$

where $\sigma^2_{Y[v:l]} = \text{Var}(Y_{i[v:l]j})$, $\sigma^2_{Y[l-v+1:l]} = \text{Var}(Y_{i[l-v+1:l]j})$, and $\sigma^2_{Y[i:m]} = \text{Var}(Y_{i[i:m]j})$. For more details on the computation of these variances, see David and Nagaraja (2003).

## 3.4 The proposed EWMA chart

In this section, we propose new EWMA charts using the VLRSS and IVLRSS schemes for efficiently monitoring $\mu_Y$.

Suppose that a sample of size $n$ is selected with the S scheme at each time point $t (t \geq 1)$, where S = VLRSS and IVLRSS. Let $\{\overline{Y}_{\text{S},t}\}$ be a sequence of IID random variables for $t = 1, 2, ...$. Using $\{\overline{Y}_{\text{S},t}\}$, it is possible to construct an EWMA chart for monitoring $\mu_Y$. The plotting-statistic of the EWMA chart with the S scheme is given by:

$$M_t = \lambda\overline{Y}_{\text{S},t} + (1-\lambda)M_{t-1},\ M_0 = \mu_Y, \tag{3.14}$$

where $\lambda$ is a smoothing constant as aforementioned. The variance of $M_t$ is

$$\text{Var}(M_t) = \text{Var}(\overline{Y}_{\text{S},t}) \cdot \frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2t}\right], \tag{3.15}$$

where $\text{Var}(\overline{Y}_{\text{S},t})$ is the variance of $\{\overline{Y}_{\text{S},t}\}$ at the time $t$. The control limits of the EWMA chart with the S scheme are

$$\text{UCL} = \mu_Y + L\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}, \tag{3.16}$$

$$\text{CL} = \mu_Y, \tag{3.17}$$

$$\text{LCL} = \mu_Y - L\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]}, \tag{3.18}$$

where $L$ is a positive control charting multiplier that is selected to ensure that the in-control ARL of the EWMA chart has reached to a certain level. The EWMA chart triggers an out-of-control signal whenever $M_t$ goes outside the interval [LCL,UCL]. As mentioned earlier that the sensitivity of the EWMA chart can be enhanced by giving a head-start to the EWMA chart by using the FIR-adjustment (cf., Steiner (1999)). In what follows, we use the same approach to associate an FIR feature with the proposed EWMA chart by setting $f = 0.5$ and $a = 0.3$—recommended by Steiner (1999).

Generally, the run length performance of a control chart is assessed in terms of its run length characteristics, including the ARL and SDRL. The ARL is defined as the average number of samples that are required by a control chart before initiating an out-of-control signal. In the SPC literature, there are some methods that could be used to compute the run length characteristics of a control chart, including the integral equation, Markov chain, and the Monte Carlo simulation. The Monte Carlo simulation approach is broadly used to investigate the run length characteristics of a control chart, the reason being the accuracy of run length characteristics' estimates. In what follows, we perform extensive Monte Carlo simulations from standard normal and bivariate normal distributions for generating samples under VLRSS and IVLRSS schemes, respectively. In order to assess the run length performance of the proposed EWMA chart, different values of the shift $\delta = \sqrt{n}|\mu_{Y,1} - \mu_Y|/\sigma_Y$ are considered, where $\mu_{Y,1}$ is the out-of-control process mean, i.e., $\delta = 0(0.25)4$. The in-control ARL of the proposed EWMA chart is set to 500—a common choice set by the SPC practitioners. Each simulation run comprises 50,000 iterations. In Table 3.1, for different choices of $m, l$ and $v$ with $r = 1$ , we report the values of $L$ with different choices of $\lambda$ for which the in-control ARLs are matched to 500. Tables 3.2-3.5 present the ARLs and SDRLs of the proposed EWMA chart with and without FIR features for $n = 5$ with $m = 5$ and $r = 1$. Tables 3.2 and 3.3 present the ARL values for different choices of $w$ and $(l, v)$ against different choices of $\delta$. It is observed that the out-of-control ARLs tend to decrease as the value of $\delta$ increases for the proposed EWMA chart with and without the FIR feature. It is also observed that the EWMA chart with the FIR feature is more sensitive than that without the FIR. It is to be noted here that those values of $w$ and $(l, v)$ are considered for which the VLRSS/IVLRSS mean estimator is most precise for a given sample size $n$ (cf., Haq et al. (2015e)).

### 3.4.1 The process parameters are unknown—phase-I monitoring

There may exist a situation when the underlying process parameters are not known in advance. In such a situation, it is conventional to estimate the unknown parameters using a large historical data obtained from an in-control process. Assume that $q$ subgroups each of size $m$ are available from an in-control process, selected using the S scheme. In case of perfect rankings, $\mu_Y$ and $\text{Var}(\overline{Y}_{\text{VLRSS}})$ could be estimated by using their respective unbiased estimators, say $\overline{\overline{Y}}_{\text{VLRSS}}$ and $\hat{\text{V}}\text{ar}(\overline{Y}_{\text{VLRSS}})$, respectively, given by:

$$\overline{\overline{Y}}_{\text{VLRSS}} = \frac{1}{q}\sum_{j=1}^{q}\overline{Y}_{\text{VLRSS},j}, \tag{3.19}$$

$$\hat{\text{V}}\text{ar}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(S^2_{Y(v:l)} + S^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w} S^2_{Y(i:m)}\right), \tag{3.20}$$

where

$$\overline{Y}_{\text{VLRSS},j} = \frac{1}{m}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right),$$

$$S^2_{Y(i:u)} = \frac{1}{q-1}\sum_{j=1}^{q}\left(Y_{i'(i:u)j} - \frac{1}{q}\sum_{j=1}^{q} Y_{i'(i:u)j}\right)^2, \tag{3.21}$$

where $i$ and $i'$ may or may not be equal. In imperfect rankings, the concomitants of $Y$ corresponding to the order statistics of $X$, are used to estimate the aforementioned parameters. From the above expressions, it is possible to obtain the unbiased estimators given that the parenthesis are replaced by the square brackets, i.e., we replace the order statistics by their corresponding concomitants. As an example, replace $Y_{i(v:l)j}$ by $Y_{i[v:l]j}$ and $S^2_{Y(v:l)}$ by $S^2_{Y[v:l]}$, and similarly the others steps. In order to get the precise estimates of the unknown parameters, the large number of subgroups are required. Having estimated the unknown parameters, it is then possible to construct the proposed EWMA chart for monitoring the process mean—phase-II monitoring.

## 3.5 Performance comparison

In this section, we compare the proposed EWMA chart with its competitors—EWMA-SRS, EWMA-RSS, EWMA-MRSS— in terms of the ARL and SDRL. For an in-control process, the ARL should be large enough to avoid frequent false alarms while for an out-of-control process the ARL should be the other way around. The comparisons among the considered EWMA charts are made for both perfect and imperfect rankings. The in-control ARLs are matched to 500 in all run length comparisons with $n = 5$.

### 3.5.1 EWMA charts with SRS, RSS and MRSS—perfect ranking

The ARLs of the EWMA-VLRSS chart is compared with those of the EWMA charts using different choices of $\lambda$ and $\delta$, i.e., we consider $\lambda = 0.05$, 0.10, 0.25, 0.50 and $\delta = 0$, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 4. The run length profiles of these EWMA charts are reported in Table 3.6. From Table 3.6, it is interesting to observe that the EWMA-VLRSS chart performs better than the other considered EWMA charts. The performance of the EWMA chart increases as the value of pair $(l, v)$ increases and vice versa.

### 3.5.2 EWMA charts with SRS, RSS and MRSS—imperfect ranking

On similar steps, the run length profiles of the EWMA-VLRSS chart are compared with the considered EWMA charts but when there are errors in ranking. The aforementioned choices of $\lambda$ and $\rho$ are considered here. The ARL profiles of the EWMA charts are presented in Table 3.7. It is observed that, despite the presence of ranking errors, the EWMA-VLRSS chart is able to perform better than the EWMA charts—considered here. It is worth noting that the sensitivity of an EWMA chart is increasing with $\rho$, i.e., the larger the $\rho$ the better the run length performance of an EWMA chart.

### 3.5.3 EWMA chart with SRS

There may exist a situation when the ranking costs are high, and it may not be possible to select a sample using RSS or MRSS. But, it is possible with VLRSS to select a sample with less number of identified units than those with the RSS and MRSS schemes. Hence, we compare the EWMA-VLRSS chart with the EWMA-SRS chart for those choices $(l, v)$ for which the ranking cost is less than that using the RSS and MRSS. Here, we consider those choices of the VLRSS scheme with $n = 5$ for which the total number of identified units are less than 25. Both perfect and imperfect rankings are used to construct the EWMA chart for these choices. The run length profiles for these cases are presented in Tables 3.8 and 3.9. It is worth noting that the VLRSS based EWMA chart perform uniformly better than the EWMA-SRS chart. A similar trend is observed when studying the impact of imperfect rankings on the performance of the EWMA-VLRSS and EWMA-SRS charts. As expected, as the value of $\rho$ increase so does the sensitivity of the EWMA-VLRSS chart and vice versa.

## 3.6 An illustrative example

In this section, a real dataset is used to explain the working of the proposed EWMA chart, and to compare its detection ability with that of the EWMA-MRSS chart.

In semiconductor manufacturing, a hard-bake process is used in conjunction with the photolithography. Our objective is to establish statistical control for the flow width of the resist in this process using the proposed and existing EWMA charts. For this purpose, forty-five samples, each of size five wafers, are drawn from an in-control process—the complete data is reported in Montgomery (2007). As the data are not available under the MRSS and VLRSS scheme; here, we generate datasets under both sampling schemes by combining whole data to generate a population. Then, the population comprises 225 flow width measurements—measured in microns. Thirty samples, each of size five under the MRSS ($l = m, v = 3, w = 2$) and the VLRSS ($l = 6, v = 3, w = 2$) scheme are generated, and we treat them as phase-I samples. Here, under each RSS scheme, we consider sampling with replacement. Using these data, we estimate the means and the variances of the mean estimators with the MRSS and VLRSS. For the EWMA charts, the in-control ARLs are set to 500 with $\lambda = 0.25$. The values of $L$ for the EWMA-MRSS and EWMA-VLRSS charts are set to 3.0020 and 3.0035, respectively. Then the control limits of both the EWMA charts are estimated. For phase-II monitoring, generate twenty samples, each of size five, under the MRSS and VLRSS from the same population. Then, add a shift of size 0.03, i.e., $\delta = 0.03$, in all of the observations in the last generated twenty samples—phase-II samples. The plotting statistics of the EWMA charts are computed for all fifty samples and are presented in the Figures 3.1 and 3.2.

From Figures 3.1 and 3.2, it is clear that, for the first thirty samples, both the EWMA charts are showing that the process is in statistical control; while for the last twenty samples, both EWMA charts are issuing out-control signals. It is interesting to see that the EWMA-VLRSS chart triggers the first out-of-control signal at the 42nd sample while the EWMA-MRSS chart triggers its first out-of-control signal at the 45th sample. This earlier detection shows that the EWMA-VLRSS chart is more sensitive than the EWMA-MRSS chart.

## 3.7 Concluding remarks

In this chapter, we have proposed an EWMA-VLRSS chart—with perfect and imperfect rankings—for monitoring the changes in the process mean. Monte Carlo simulations have been used to compute the run length profiles of the proposed EWMA chart. The EWMA-VLRSS chart has been compared with the EWMA-SRS and EWMA-MRSS charts in terms of ARL and SDRL under both perfect and imperfect rankings. The proposed EWMA chart has found to be uniformly better than the EWMA charts considered here. Thus, when possible; we recommend using the EWMA-VLRSS chart for improved process mean monitoring.

Table 3.1: The values of $L$ with different choices of $(l, v)$ when the in-control ARL of the EWMA-VLRSS chart is 500

| | $m=2$ | $m=4$ | | | | | |
|---|---|---|---|---|---|---|---|
| $\lambda$ | $(0,0)$ | $(2,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,3)$ | $(6,3)$ |
| 0.05 | 2.2616 | 2.6400 | 2.6410 | 2.6410 | 2.6410 | 2.6435 | 2.6444 |
| 0.10 | 2.4197 | 2.8271 | 2.8257 | 2.8272 | 2.8261 | 2.8273 | 2.8274 |
| 0.25 | 2.5700 | 3.0055 | 3.0055 | 3.0080 | 3.0031 | 3.0048 | 3.0048 |
| 0.50 | 2.6310 | 3.0818 | 3.0769 | 3.0865 | 3.0786 | 3.0778 | 3.0790 |
| 0.75 | 2.6450 | 3.1008 | 3.0979 | 3.1099 | 3.0958 | 3.0950 | 3.0935 |
| 1.00 | 2.6468 | 3.1055 | 3.0996 | 3.1155 | 3.0994 | 3.0988 | 3.0988 |

| | $m=3, w=1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $(2,1)$ | $(3,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,1)$ | $(5,2)$ | $(5,3)$ |
| 0.05 | 2.6390 | 2.6400 | 2.6400 | 2.6420 | 2.6407 | 2.6410 | 2.6410 | 2.6400 |
| 0.10 | 2.8261 | 2.8270 | 2.8259 | 2.8270 | 2.8257 | 2.8273 | 2.8250 | 2.8255 |
| 0.25 | 3.0050 | 3.0070 | 3.0055 | 3.0105 | 3.0048 | 3.0110 | 3.0050 | 3.0049 |
| 0.50 | 3.0832 | 3.0907 | 3.0790 | 3.0938 | 3.0781 | 3.0951 | 3.0805 | 3.0800 |
| 0.75 | 3.1019 | 3.1125 | 3.0967 | 3.1178 | 3.0967 | 3.1220 | 3.1008 | 3.0980 |
| 1.00 | 3.1055 | 3.1186 | 3.1017 | 3.1277 | 3.1017 | 3.1295 | 3.1055 | 3.0994 |

| | $m=5$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $w=1$ | | | | $w=2$ | | | |
| | $(2,1)$ | $(3,2)$ | $(4,2)$ | $(5,1)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| 0.05 | 2.6425 | 2.6408 | 2.6430 | 2.6418 | 2.6390 | 2.6400 | 2.6420 | 2.6410 |
| 0.10 | 2.8245 | 2.8258 | 2.8232 | 2.8263 | 2.8236 | 2.8260 | 2.8235 | 2.8236 |
| 0.25 | 3.0048 | 3.0050 | 3.0044 | 3.0068 | 3.0020 | 3.0035 | 3.0050 | 3.0042 |
| 0.50 | 3.0778 | 3.0772 | 3.0777 | 3.0868 | 3.0750 | 3.0771 | 3.0755 | 3.0762 |
| 0.75 | 3.0975 | 3.0963 | 3.0948 | 3.1107 | 3.0925 | 3.0948 | 3.0929 | 3.0931 |
| 1.00 | 3.1010 | 3.0990 | 3.0998 | 3.1150 | 3.0965 | 3.0970 | 3.0965 | 3.0965 |

Table 3.2: The run length profiles of the EWMA-VLRSS chart for $w = 1$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 2.6425 | 2.6408 | 2.6430 | 2.6418 | 2.8245 | 2.8258 | 2.8232 | 2.8263 |
| 0.00 | ARL | 500.96 | 501.54 | 500.03 | 500.87 | 501.56 | 501.17 | 502.06 | 502.33 |
| | SDRL | 520.72 | 517.41 | 517.15 | 516.20 | 504.59 | 509.28 | 500.37 | 507.48 |
| 0.25 | ARL | 39.88 | 32.74 | 29.89 | 32.76 | 51.16 | 41.07 | 37.16 | 40.85 |
| | SDRL | 32.47 | 25.72 | 23.50 | 26.03 | 45.04 | 34.96 | 31.15 | 34.58 |
| 0.50 | ARL | 12.11 | 10.02 | 9.12 | 9.89 | 13.90 | 11.30 | 10.32 | 11.35 |
| | SDRL | 8.33 | 6.76 | 6.04 | 6.65 | 9.74 | 7.60 | 6.82 | 7.58 |
| 0.75 | ARL | 6.16 | 5.07 | 4.68 | 5.08 | 6.84 | 5.65 | 5.17 | 5.67 |
| | SDRL | 3.88 | 3.13 | 2.84 | 3.12 | 4.23 | 3.38 | 3.05 | 3.39 |
| 1.00 | ARL | 3.84 | 3.22 | 2.97 | 3.21 | 4.24 | 3.53 | 3.26 | 3.54 |
| | SDRL | 2.27 | 1.82 | 1.65 | 1.81 | 2.39 | 1.93 | 1.77 | 1.96 |
| 1.25 | ARL | 2.72 | 2.30 | 2.14 | 2.30 | 2.99 | 2.50 | 2.32 | 2.51 |
| | SDRL | 1.48 | 1.20 | 1.09 | 1.20 | 1.59 | 1.29 | 1.16 | 1.27 |
| 1.50 | ARL | 2.09 | 1.78 | 1.67 | 1.78 | 2.27 | 1.92 | 1.79 | 1.92 |
| | SDRL | 1.06 | 0.86 | 0.78 | 0.85 | 1.13 | 0.91 | 0.83 | 0.92 |
| 1.75 | ARL | 1.69 | 1.46 | 1.38 | 1.46 | 1.83 | 1.57 | 1.47 | 1.57 |
| | SDRL | 0.80 | 0.64 | 0.58 | 0.64 | 0.86 | 0.69 | 0.63 | 0.69 |
| 2.00 | ARL | 1.43 | 1.26 | 1.21 | 1.27 | 1.53 | 1.34 | 1.26 | 1.34 |
| | SDRL | 0.62 | 0.48 | 0.43 | 0.48 | 0.67 | 0.53 | 0.47 | 0.53 |
| 2.25 | ARL | 1.26 | 1.14 | 1.10 | 1.14 | 1.33 | 1.19 | 1.13 | 1.18 |
| | SDRL | 0.48 | 0.35 | 0.30 | 0.36 | 0.53 | 0.40 | 0.35 | 0.40 |
| 2.50 | ARL | 1.15 | 1.07 | 1.04 | 1.07 | 1.20 | 1.09 | 1.06 | 1.09 |
| | SDRL | 0.37 | 0.25 | 0.20 | 0.25 | 0.42 | 0.30 | 0.24 | 0.30 |
| 2.75 | ARL | 1.08 | 1.03 | 1.01 | 1.03 | 1.11 | 1.04 | 1.02 | 1.04 |
| | SDRL | 0.27 | 0.16 | 0.12 | 0.16 | 0.31 | 0.20 | 0.15 | 0.20 |
| 3.00 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.05 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.18 | 0.10 | 0.07 | 0.10 | 0.22 | 0.13 | 0.08 | 0.12 |
| 3.25 | ARL | 1.02 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.01 |
| | SDRL | 0.12 | 0.05 | 0.04 | 0.05 | 0.15 | 0.07 | 0.05 | 0.07 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.03 | 0.01 | 0.03 | 0.10 | 0.04 | 0.02 | 0.04 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.01 | 0.01 | 0.01 | 0.06 | 0.02 | 0.01 | 0.02 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.01 | 0.03 | 0.01 | 0.01 | 0.01 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.75$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 3.0048 | 3.0050 | 3.0044 | 3.0068 | 3.0778 | 3.0772 | 3.0777 | 3.0868 |
| 0.00 | ARL | 501.55 | 502.62 | 502.67 | 499.77 | 499.25 | 501.43 | 499.51 | 502.27 |
| | SDRL | 503.67 | 506.70 | 502.71 | 499.01 | 501.43 | 500.68 | 498.12 | 500.92 |
| 0.25 | ARL | 87.39 | 70.37 | 62.53 | 70.02 | 152.46 | 126.43 | 114.71 | 128.41 |
| | SDRL | 83.63 | 67.11 | 59.05 | 66.77 | 150.60 | 124.52 | 113.59 | 127.01 |
| 0.50 | ARL | 19.84 | 15.54 | 13.87 | 15.49 | 36.78 | 27.88 | 24.41 | 28.36 |
| | SDRL | 16.50 | 12.35 | 10.71 | 12.33 | 35.16 | 25.83 | 22.49 | 26.61 |
| 0.75 | ARL | 8.44 | 6.73 | 6.12 | 6.74 | 13.17 | 9.78 | 8.63 | 9.96 |
| | SDRL | 5.85 | 4.38 | 3.88 | 4.38 | 11.40 | 8.05 | 6.92 | 8.15 |
| 1.00 | ARL | 4.90 | 3.99 | 3.64 | 4.00 | 6.49 | 4.98 | 4.47 | 5.04 |
| | SDRL | 2.93 | 2.26 | 2.00 | 2.26 | 4.84 | 3.44 | 3.01 | 3.56 |
| 1.25 | ARL | 3.34 | 2.77 | 2.56 | 2.77 | 3.97 | 3.15 | 2.87 | 3.17 |
| | SDRL | 1.80 | 1.42 | 1.28 | 1.41 | 2.57 | 1.87 | 1.66 | 1.90 |
| 1.50 | ARL | 2.50 | 2.10 | 1.95 | 2.10 | 2.78 | 2.27 | 2.08 | 2.29 |
| | SDRL | 1.25 | 0.99 | 0.91 | 0.99 | 1.58 | 1.20 | 1.06 | 1.20 |
| 1.75 | ARL | 1.98 | 1.69 | 1.57 | 1.69 | 2.14 | 1.78 | 1.65 | 1.78 |
| | SDRL | 0.93 | 0.75 | 0.68 | 0.75 | 1.10 | 0.85 | 0.76 | 0.85 |
| 2.00 | ARL | 1.65 | 1.41 | 1.34 | 1.42 | 1.73 | 1.47 | 1.38 | 1.48 |
| | SDRL | 0.73 | 0.58 | 0.53 | 0.58 | 0.81 | 0.63 | 0.57 | 0.64 |
| 2.25 | ARL | 1.41 | 1.24 | 1.18 | 1.24 | 1.46 | 1.27 | 1.20 | 1.27 |
| | SDRL | 0.58 | 0.46 | 0.40 | 0.45 | 0.63 | 0.49 | 0.42 | 0.49 |
| 2.50 | ARL | 1.26 | 1.13 | 1.08 | 1.13 | 1.28 | 1.14 | 1.10 | 1.14 |
| | SDRL | 0.47 | 0.34 | 0.28 | 0.34 | 0.50 | 0.36 | 0.30 | 0.36 |
| 2.75 | ARL | 1.15 | 1.06 | 1.03 | 1.06 | 1.16 | 1.07 | 1.04 | 1.07 |
| | SDRL | 0.36 | 0.23 | 0.18 | 0.24 | 0.38 | 0.25 | 0.20 | 0.25 |
| 3.00 | ARL | 1.08 | 1.02 | 1.01 | 1.02 | 1.09 | 1.03 | 1.02 | 1.03 |
| | SDRL | 0.27 | 0.15 | 0.11 | 0.15 | 0.28 | 0.17 | 0.12 | 0.17 |
| 3.25 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.04 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.18 | 0.09 | 0.06 | 0.09 | 0.20 | 0.10 | 0.06 | 0.10 |
| 3.50 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.12 | 0.05 | 0.03 | 0.05 | 0.13 | 0.06 | 0.03 | 0.06 |
| 3.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.03 | 0.01 | 0.02 | 0.08 | 0.03 | 0.02 | 0.03 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.05 | 0.01 | 0.00 | 0.01 | 0.05 | 0.01 | 0.01 | 0.01 |

Table 3.3: The run length profiles of the EWMA-VLRSS chart for $w = 2$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 2.6390 | 2.6400 | 2.6420 | 2.6410 | 2.8236 | 2.8260 | 2.8235 | 2.8236 |
| 0.00 | ARL | 500.07 | 499.91 | 502.42 | 501.98 | 499.74 | 499.87 | 501.25 | 499.89 |
| | SDRL | 519.38 | 512.62 | 518.08 | 519.27 | 504.06 | 505.47 | 507.38 | 507.63 |
| 0.25 | ARL | 26.75 | 24.15 | 21.90 | 20.20 | 32.76 | 29.61 | 26.08 | 24.17 |
| | SDRL | 20.64 | 18.34 | 16.30 | 14.89 | 26.93 | 23.87 | 20.62 | 18.73 |
| 0.50 | ARL | 8.20 | 7.39 | 6.72 | 6.27 | 9.26 | 8.39 | 7.53 | 6.96 |
| | SDRL | 5.42 | 4.81 | 4.30 | 3.97 | 5.98 | 5.33 | 4.71 | 4.30 |
| 0.75 | ARL | 4.20 | 3.81 | 3.49 | 3.27 | 4.67 | 4.23 | 3.83 | 3.60 |
| | SDRL | 2.52 | 2.23 | 2.01 | 1.86 | 2.70 | 2.39 | 2.15 | 1.98 |
| 1.00 | ARL | 2.69 | 2.46 | 2.25 | 2.12 | 2.95 | 2.70 | 2.45 | 2.31 |
| | SDRL | 1.47 | 1.31 | 1.17 | 1.08 | 1.57 | 1.40 | 1.25 | 1.16 |
| 1.25 | ARL | 1.94 | 1.79 | 1.66 | 1.57 | 2.12 | 1.94 | 1.79 | 1.69 |
| | SDRL | 0.97 | 0.87 | 0.78 | 0.72 | 1.04 | 0.92 | 0.84 | 0.77 |
| 1.50 | ARL | 1.53 | 1.43 | 1.34 | 1.28 | 1.65 | 1.53 | 1.42 | 1.35 |
| | SDRL | 0.69 | 0.61 | 0.54 | 0.49 | 0.75 | 0.66 | 0.59 | 0.54 |
| 1.75 | ARL | 1.29 | 1.21 | 1.15 | 1.12 | 1.36 | 1.28 | 1.20 | 1.16 |
| | SDRL | 0.50 | 0.44 | 0.37 | 0.33 | 0.55 | 0.49 | 0.42 | 0.37 |
| 2.00 | ARL | 1.14 | 1.09 | 1.06 | 1.04 | 1.19 | 1.13 | 1.08 | 1.06 |
| | SDRL | 0.36 | 0.30 | 0.24 | 0.20 | 0.41 | 0.34 | 0.28 | 0.24 |
| 2.25 | ARL | 1.06 | 1.03 | 1.02 | 1.01 | 1.09 | 1.05 | 1.03 | 1.02 |
| | SDRL | 0.24 | 0.18 | 0.13 | 0.10 | 0.28 | 0.22 | 0.17 | 0.13 |
| 2.50 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.03 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.14 | 0.10 | 0.07 | 0.05 | 0.18 | 0.13 | 0.09 | 0.06 |
| 2.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.08 | 0.05 | 0.03 | 0.02 | 0.10 | 0.07 | 0.04 | 0.02 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.02 | 0.02 | 0.01 | 0.05 | 0.03 | 0.01 | 0.01 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.01 | 0.00 | 0.02 | 0.01 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 3.0020 | 3.0035 | 3.0050 | 3.0042 | 3.0750 | 3.0771 | 3.0755 | 3.0762 |
| 0.00 | ARL | 500.39 | 501.09 | 503.18 | 501.75 | 500.83 | 499.88 | 502.47 | 501.58 |
| | SDRL | 500.30 | 500.69 | 502.64 | 500.80 | 501.81 | 498.60 | 502.22 | 502.54 |
| 0.25 | ARL | 54.86 | 48.49 | 42.71 | 42.39 | 101.50 | 91.24 | 80.26 | 80.11 |
| | SDRL | 51.36 | 45.10 | 39.20 | 38.84 | 99.95 | 89.25 | 78.33 | 78.44 |
| 0.50 | ARL | 12.07 | 10.63 | 9.38 | 9.42 | 20.62 | 17.68 | 15.20 | 15.18 |
| | SDRL | 9.12 | 7.78 | 6.68 | 6.71 | 18.81 | 15.77 | 13.33 | 13.33 |
| 0.75 | ARL | 5.40 | 4.87 | 4.38 | 4.37 | 7.36 | 6.42 | 5.60 | 5.59 |
| | SDRL | 3.33 | 2.91 | 2.54 | 2.51 | 5.68 | 4.80 | 4.00 | 4.02 |
| 1.00 | ARL | 3.28 | 2.98 | 2.72 | 2.72 | 3.90 | 3.46 | 3.07 | 3.08 |
| | SDRL | 1.76 | 1.56 | 1.39 | 1.38 | 2.50 | 2.14 | 1.81 | 1.81 |
| 1.25 | ARL | 2.31 | 2.11 | 1.94 | 1.95 | 2.55 | 2.30 | 2.08 | 2.09 |
| | SDRL | 1.13 | 1.01 | 0.90 | 0.91 | 1.41 | 1.22 | 1.06 | 1.06 |
| 1.50 | ARL | 1.77 | 1.64 | 1.51 | 1.51 | 1.88 | 1.72 | 1.58 | 1.58 |
| | SDRL | 0.80 | 0.72 | 0.64 | 0.65 | 0.92 | 0.81 | 0.71 | 0.71 |
| 1.75 | ARL | 1.45 | 1.35 | 1.26 | 1.26 | 1.51 | 1.39 | 1.29 | 1.29 |
| | SDRL | 0.60 | 0.54 | 0.47 | 0.47 | 0.66 | 0.58 | 0.50 | 0.51 |
| 2.00 | ARL | 1.24 | 1.17 | 1.12 | 1.12 | 1.27 | 1.20 | 1.13 | 1.13 |
| | SDRL | 0.45 | 0.39 | 0.33 | 0.33 | 0.49 | 0.42 | 0.34 | 0.35 |
| 2.25 | ARL | 1.12 | 1.07 | 1.04 | 1.04 | 1.13 | 1.08 | 1.05 | 1.05 |
| | SDRL | 0.33 | 0.26 | 0.20 | 0.20 | 0.35 | 0.28 | 0.22 | 0.22 |
| 2.50 | ARL | 1.05 | 1.03 | 1.01 | 1.01 | 1.06 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.22 | 0.16 | 0.11 | 0.11 | 0.23 | 0.17 | 0.12 | 0.12 |
| 2.75 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.13 | 0.08 | 0.05 | 0.05 | 0.14 | 0.09 | 0.06 | 0.06 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.04 | 0.02 | 0.02 | 0.08 | 0.05 | 0.02 | 0.02 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.02 | 0.01 | 0.01 | 0.04 | 0.02 | 0.01 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.01 | 0.02 | 0.01 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 3.4: The run length profiles of the EWMA-VLRSS chart with the FIR feature for $w = 1$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 2.7490 | 2.7480 | 2.7485 | 2.7474 | 2.9140 | 2.9148 | 2.9139 | 2.9180 |
| 0.00 | ARL | 501.96 | 502.82 | 502.33 | 501.04 | 499.78 | 501.24 | 501.13 | 502.55 |
| | SDRL | 660.01 | 657.93 | 659.27 | 659.57 | 626.82 | 628.07 | 628.57 | 630.93 |
| 0.25 | ARL | 31.55 | 25.41 | 22.76 | 25.13 | 41.70 | 32.34 | 28.63 | 32.75 |
| | SDRL | 36.23 | 28.55 | 25.61 | 28.34 | 50.26 | 38.13 | 33.89 | 38.66 |
| 0.50 | ARL | 8.10 | 6.39 | 5.78 | 6.35 | 9.11 | 7.10 | 6.32 | 7.16 |
| | SDRL | 8.90 | 6.98 | 6.36 | 7.03 | 10.10 | 7.84 | 6.90 | 7.83 |
| 0.75 | ARL | 3.56 | 2.85 | 2.60 | 2.86 | 3.83 | 3.05 | 2.79 | 3.05 |
| | SDRL | 3.64 | 2.78 | 2.45 | 2.79 | 3.90 | 2.93 | 2.64 | 2.94 |
| 1.00 | ARL | 2.13 | 1.75 | 1.65 | 1.76 | 2.24 | 1.86 | 1.73 | 1.86 |
| | SDRL | 1.82 | 1.32 | 1.16 | 1.32 | 1.91 | 1.41 | 1.25 | 1.42 |
| 1.25 | ARL | 1.53 | 1.34 | 1.28 | 1.34 | 1.60 | 1.39 | 1.32 | 1.39 |
| | SDRL | 1.01 | 0.73 | 0.64 | 0.73 | 1.07 | 0.78 | 0.68 | 0.78 |
| 1.50 | ARL | 1.26 | 1.16 | 1.12 | 1.15 | 1.30 | 1.18 | 1.14 | 1.18 |
| | SDRL | 0.60 | 0.44 | 0.38 | 0.44 | 0.66 | 0.48 | 0.41 | 0.47 |
| 1.75 | ARL | 1.13 | 1.07 | 1.05 | 1.07 | 1.15 | 1.08 | 1.06 | 1.08 |
| | SDRL | 0.39 | 0.27 | 0.23 | 0.27 | 0.42 | 0.30 | 0.25 | 0.29 |
| 2.00 | ARL | 1.06 | 1.03 | 1.02 | 1.03 | 1.07 | 1.03 | 1.02 | 1.03 |
| | SDRL | 0.25 | 0.17 | 0.13 | 0.16 | 0.27 | 0.18 | 0.15 | 0.18 |
| 2.25 | ARL | 1.03 | 1.01 | 1.01 | 1.01 | 1.03 | 1.01 | 1.01 | 1.01 |
| | SDRL | 0.16 | 0.10 | 0.07 | 0.09 | 0.18 | 0.11 | 0.08 | 0.11 |
| 2.50 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.10 | 0.05 | 0.03 | 0.05 | 0.11 | 0.06 | 0.04 | 0.06 |
| 2.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.03 | 0.02 | 0.03 | 0.07 | 0.03 | 0.02 | 0.03 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.01 | 0.01 | 0.04 | 0.02 | 0.01 | 0.02 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.00 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 3.0815 | 3.0789 | 3.0781 | 3.0845 | 3.1537 | 3.1514 | 3.1530 | 3.1593 |
| 0.00 | ARL | 500.09 | 499.58 | 500.17 | 500.14 | 501.98 | 501.78 | 502.70 | 499.85 |
| | SDRL | 618.48 | 626.43 | 623.66 | 629.27 | 628.05 | 628.00 | 631.27 | 620.79 |
| 0.25 | ARL | 73.16 | 55.89 | 48.99 | 57.02 | 128.94 | 102.35 | 93.74 | 104.30 |
| | SDRL | 95.32 | 73.58 | 64.29 | 74.85 | 175.27 | 141.36 | 130.65 | 144.67 |
| 0.50 | ARL | 12.23 | 8.90 | 7.91 | 9.04 | 21.82 | 14.70 | 12.31 | 14.99 |
| | SDRL | 16.01 | 11.39 | 10.01 | 11.56 | 34.41 | 23.73 | 19.84 | 24.20 |
| 0.75 | ARL | 4.27 | 3.32 | 2.99 | 3.33 | 5.64 | 3.93 | 3.42 | 3.95 |
| | SDRL | 4.79 | 3.42 | 2.94 | 3.43 | 8.43 | 5.26 | 4.31 | 5.29 |
| 1.00 | ARL | 2.37 | 1.95 | 1.80 | 1.95 | 2.57 | 2.05 | 1.87 | 2.06 |
| | SDRL | 2.09 | 1.51 | 1.31 | 1.52 | 2.71 | 1.80 | 1.53 | 1.84 |
| 1.25 | ARL | 1.66 | 1.43 | 1.36 | 1.44 | 1.72 | 1.47 | 1.38 | 1.47 |
| | SDRL | 1.13 | 0.83 | 0.73 | 0.84 | 1.28 | 0.90 | 0.77 | 0.89 |
| 1.50 | ARL | 1.34 | 1.21 | 1.16 | 1.21 | 1.36 | 1.22 | 1.17 | 1.22 |
| | SDRL | 0.70 | 0.51 | 0.44 | 0.50 | 0.74 | 0.53 | 0.45 | 0.53 |
| 1.75 | ARL | 1.17 | 1.09 | 1.07 | 1.09 | 1.18 | 1.10 | 1.07 | 1.10 |
| | SDRL | 0.45 | 0.32 | 0.27 | 0.32 | 0.48 | 0.34 | 0.28 | 0.33 |
| 2.00 | ARL | 1.08 | 1.04 | 1.03 | 1.04 | 1.09 | 1.04 | 1.03 | 1.04 |
| | SDRL | 0.30 | 0.20 | 0.16 | 0.20 | 0.31 | 0.21 | 0.16 | 0.21 |
| 2.25 | ARL | 1.04 | 1.01 | 1.01 | 1.01 | 1.04 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.20 | 0.12 | 0.09 | 0.12 | 0.20 | 0.13 | 0.10 | 0.13 |
| 2.50 | ARL | 1.02 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.01 |
| | SDRL | 0.13 | 0.07 | 0.05 | 0.07 | 0.13 | 0.07 | 0.05 | 0.07 |
| 2.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.03 | 0.02 | 0.04 | 0.08 | 0.04 | 0.03 | 0.03 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.02 | 0.01 | 0.02 | 0.05 | 0.02 | 0.01 | 0.02 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.01 | 0.02 | 0.00 | 0.00 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |

Table 3.5: The run length profiles of the EWMA-VLRSS chart with the FIR feature for $w = 2$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 2.7480 | 2.7479 | 2.7479 | 2.7480 | 2.9136 | 2.9125 | 2.9130 | 2.9127 |
| 0.00 | ARL | 501.53 | 500.55 | 499.24 | 500.75 | 501.81 | 501.24 | 501.11 | 501.16 |
| | SDRL | 655.40 | 660.59 | 657.77 | 664.54 | 632.48 | 630.66 | 630.84 | 634.18 |
| 0.25 | ARL | 20.27 | 18.10 | 15.97 | 14.68 | 25.34 | 22.01 | 19.12 | 17.60 |
| | SDRL | 22.49 | 20.01 | 17.71 | 16.13 | 29.74 | 25.43 | 22.15 | 20.16 |
| 0.50 | ARL | 5.02 | 4.43 | 3.96 | 3.62 | 5.55 | 4.86 | 4.30 | 3.95 |
| | SDRL | 5.45 | 4.74 | 4.15 | 3.73 | 5.96 | 5.17 | 4.47 | 4.07 |
| 0.75 | ARL | 2.30 | 2.09 | 1.90 | 1.80 | 2.49 | 2.21 | 2.01 | 1.89 |
| | SDRL | 2.07 | 1.77 | 1.52 | 1.38 | 2.23 | 1.88 | 1.62 | 1.47 |
| 1.00 | ARL | 1.52 | 1.41 | 1.33 | 1.28 | 1.58 | 1.47 | 1.38 | 1.31 |
| | SDRL | 0.99 | 0.82 | 0.70 | 0.64 | 1.05 | 0.90 | 0.76 | 0.67 |
| 1.25 | ARL | 1.21 | 1.16 | 1.12 | 1.09 | 1.25 | 1.19 | 1.14 | 1.11 |
| | SDRL | 0.53 | 0.45 | 0.37 | 0.33 | 0.57 | 0.49 | 0.40 | 0.36 |
| 1.50 | ARL | 1.08 | 1.06 | 1.04 | 1.03 | 1.10 | 1.07 | 1.05 | 1.03 |
| | SDRL | 0.30 | 0.25 | 0.20 | 0.17 | 0.33 | 0.28 | 0.22 | 0.19 |
| 1.75 | ARL | 1.03 | 1.02 | 1.01 | 1.01 | 1.04 | 1.02 | 1.01 | 1.01 |
| | SDRL | 0.18 | 0.14 | 0.10 | 0.08 | 0.19 | 0.15 | 0.11 | 0.10 |
| 2.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.10 | 0.07 | 0.04 | 0.04 | 0.11 | 0.08 | 0.05 | 0.05 |
| 2.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.05 | 0.03 | 0.02 | 0.01 | 0.05 | 0.04 | 0.03 | 0.02 |
| 2.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.01 | 0.00 | 0.03 | 0.02 | 0.01 | 0.01 |
| 2.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 3.0812 | 3.0810 | 3.0796 | 3.0797 | 3.1525 | 3.1517 | 3.1508 | 3.1515 |
| 0.00 | ARL | 502.23 | 500.27 | 501.63 | 502.10 | 500.66 | 500.38 | 501.64 | 502.46 |
| | SDRL | 618.63 | 620.63 | 620.23 | 619.57 | 626.89 | 630.80 | 629.78 | 630.29 |
| 0.25 | ARL | 41.83 | 36.18 | 31.09 | 27.70 | 79.86 | 69.06 | 59.03 | 52.74 |
| | SDRL | 55.73 | 48.57 | 41.73 | 36.95 | 112.82 | 98.58 | 85.70 | 77.65 |
| 0.50 | ARL | 6.63 | 5.65 | 4.90 | 4.42 | 9.98 | 8.15 | 6.64 | 5.80 |
| | SDRL | 8.21 | 6.82 | 5.67 | 4.99 | 16.04 | 12.80 | 10.32 | 8.79 |
| 0.75 | ARL | 2.64 | 2.36 | 2.12 | 1.97 | 2.90 | 2.54 | 2.27 | 2.08 |
| | SDRL | 2.44 | 2.08 | 1.75 | 1.55 | 3.30 | 2.67 | 2.21 | 1.85 |
| 1.00 | ARL | 1.65 | 1.52 | 1.42 | 1.35 | 1.70 | 1.56 | 1.45 | 1.38 |
| | SDRL | 1.11 | 0.95 | 0.80 | 0.71 | 1.22 | 1.03 | 0.86 | 0.76 |
| 1.25 | ARL | 1.28 | 1.22 | 1.16 | 1.13 | 1.30 | 1.23 | 1.17 | 1.14 |
| | SDRL | 0.61 | 0.52 | 0.44 | 0.38 | 0.65 | 0.54 | 0.46 | 0.40 |
| 1.50 | ARL | 1.12 | 1.08 | 1.05 | 1.04 | 1.13 | 1.09 | 1.06 | 1.05 |
| | SDRL | 0.37 | 0.30 | 0.24 | 0.21 | 0.38 | 0.31 | 0.25 | 0.22 |
| 1.75 | ARL | 1.04 | 1.03 | 1.02 | 1.01 | 1.05 | 1.03 | 1.02 | 1.01 |
| | SDRL | 0.21 | 0.17 | 0.13 | 0.10 | 0.22 | 0.17 | 0.13 | 0.11 |
| 2.00 | ARL | 1.01 | 1.01 | 1.00 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.12 | 0.09 | 0.06 | 0.05 | 0.12 | 0.09 | 0.07 | 0.05 |
| 2.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.04 | 0.02 | 0.02 | 0.07 | 0.04 | 0.03 | 0.02 |
| 2.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.02 | 0.01 | 0.00 | 0.03 | 0.02 | 0.01 | 0.00 |
| 2.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 3.6: The run length comparison of the EWMA-VLRSS chart with the EWMA-SRS, EWMA-RSS, and EWMA-MRSS charts

| | | $\lambda = 0.05$ | | | | | | $\lambda = 0.10$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.54 | 500.87 | 500.07 | 499.91 | 502.42 | 501.98 | 500.76 | 502.33 | 499.74 | 499.87 | 501.25 | 499.89 |
| | SDRL | 515.09 | 516.20 | 519.38 | 512.62 | 518.08 | 519.27 | 514.08 | 507.48 | 504.06 | 505.47 | 507.38 | 507.63 |
| 0.25 | ARL | 78.30 | 32.76 | 26.75 | 24.15 | 21.90 | 20.20 | 103.42 | 40.85 | 32.76 | 29.61 | 26.08 | 24.17 |
| | SDRL | 70.16 | 26.03 | 20.64 | 18.34 | 16.30 | 14.89 | 98.37 | 34.58 | 26.93 | 23.87 | 20.62 | 18.73 |
| 0.50 | ARL | 23.79 | 9.89 | 8.20 | 7.39 | 6.72 | 6.27 | 28.95 | 11.35 | 9.26 | 8.39 | 7.53 | 6.96 |
| | SDRL | 17.98 | 6.65 | 5.42 | 4.81 | 4.30 | 3.97 | 23.32 | 7.58 | 5.98 | 5.33 | 4.71 | 4.30 |
| 0.75 | ARL | 11.83 | 5.08 | 4.20 | 3.81 | 3.49 | 3.27 | 13.59 | 5.67 | 4.67 | 4.23 | 3.83 | 3.60 |
| | SDRL | 8.10 | 3.12 | 2.52 | 2.23 | 2.01 | 1.86 | 9.48 | 3.39 | 2.70 | 2.39 | 2.15 | 1.98 |
| 1.00 | ARL | 7.32 | 3.21 | 2.69 | 2.46 | 2.25 | 2.12 | 8.19 | 3.54 | 2.95 | 2.70 | 2.45 | 2.31 |
| | SDRL | 4.76 | 1.81 | 1.47 | 1.31 | 1.17 | 1.08 | 5.19 | 1.96 | 1.57 | 1.40 | 1.25 | 1.16 |
| 1.50 | ARL | 3.76 | 1.78 | 1.53 | 1.43 | 1.34 | 1.28 | 4.17 | 1.92 | 1.65 | 1.53 | 1.42 | 1.35 |
| | SDRL | 2.19 | 0.85 | 0.69 | 0.61 | 0.54 | 0.49 | 2.37 | 0.92 | 0.75 | 0.66 | 0.59 | 0.54 |
| 2.00 | ARL | 2.42 | 1.27 | 1.14 | 1.09 | 1.06 | 1.04 | 2.66 | 1.34 | 1.19 | 1.13 | 1.08 | 1.06 |
| | SDRL | 1.28 | 0.48 | 0.36 | 0.30 | 0.24 | 0.20 | 1.38 | 0.53 | 0.41 | 0.34 | 0.28 | 0.24 |
| 2.50 | ARL | 1.78 | 1.07 | 1.02 | 1.01 | 1.00 | 1.00 | 1.92 | 1.09 | 1.03 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.86 | 0.25 | 0.14 | 0.10 | 0.07 | 0.05 | 0.92 | 0.30 | 0.18 | 0.13 | 0.09 | 0.06 |
| 3.00 | ARL | 1.41 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.52 | 1.02 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.60 | 0.10 | 0.04 | 0.02 | 0.02 | 0.01 | 0.65 | 0.12 | 0.05 | 0.03 | 0.01 | 0.01 |
| 4.00 | ARL | 1.09 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.28 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | | | $\lambda = 0.50$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 499.99 | 499.77 | 500.39 | 501.09 | 503.18 | 501.75 | 500.53 | 502.27 | 500.83 | 499.88 | 502.47 | 501.58 |
| | SDRL | 497.14 | 499.01 | 500.30 | 500.69 | 502.64 | 500.80 | 499.16 | 500.92 | 501.81 | 498.60 | 502.22 | 502.54 |
| 0.25 | ARL | 168.27 | 70.02 | 54.86 | 48.49 | 42.71 | 42.39 | 255.04 | 128.41 | 101.50 | 91.24 | 80.26 | 80.11 |
| | SDRL | 165.39 | 66.77 | 51.36 | 45.10 | 39.20 | 38.84 | 253.92 | 127.01 | 99.95 | 89.25 | 78.33 | 78.44 |
| 0.50 | ARL | 47.34 | 15.49 | 12.07 | 10.63 | 9.38 | 9.42 | 88.15 | 28.36 | 20.62 | 17.68 | 15.20 | 15.18 |
| | SDRL | 44.01 | 12.33 | 9.12 | 7.78 | 6.68 | 6.71 | 86.16 | 26.61 | 18.81 | 15.77 | 13.33 | 13.33 |
| 0.75 | ARL | 19.41 | 6.74 | 5.40 | 4.87 | 4.38 | 4.37 | 35.62 | 9.96 | 7.36 | 6.42 | 5.60 | 5.59 |
| | SDRL | 16.04 | 4.38 | 3.33 | 2.91 | 2.54 | 2.51 | 33.68 | 8.15 | 5.68 | 4.80 | 4.00 | 4.02 |
| 1.00 | ARL | 10.39 | 4.00 | 3.28 | 2.98 | 2.72 | 2.72 | 17.11 | 5.04 | 3.90 | 3.46 | 3.07 | 3.08 |
| | SDRL | 7.59 | 2.26 | 1.76 | 1.56 | 1.39 | 1.38 | 15.16 | 3.56 | 2.50 | 2.14 | 1.81 | 1.81 |
| 1.50 | ARL | 4.77 | 2.10 | 1.77 | 1.64 | 1.51 | 1.51 | 6.27 | 2.29 | 1.88 | 1.72 | 1.58 | 1.58 |
| | SDRL | 2.83 | 0.99 | 0.80 | 0.72 | 0.64 | 0.65 | 4.65 | 1.20 | 0.92 | 0.81 | 0.71 | 0.71 |
| 2.00 | ARL | 2.96 | 1.42 | 1.24 | 1.17 | 1.12 | 1.12 | 3.38 | 1.48 | 1.27 | 1.20 | 1.13 | 1.13 |
| | SDRL | 1.55 | 0.58 | 0.45 | 0.39 | 0.33 | 0.33 | 2.07 | 0.64 | 0.49 | 0.42 | 0.34 | 0.35 |
| 2.50 | ARL | 2.09 | 1.13 | 1.05 | 1.03 | 1.01 | 1.01 | 2.26 | 1.14 | 1.06 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.99 | 0.34 | 0.22 | 0.16 | 0.11 | 0.11 | 1.19 | 0.36 | 0.23 | 0.17 | 0.12 | 0.12 |
| 3.00 | ARL | 1.62 | 1.02 | 1.00 | 1.00 | 1.00 | 1.00 | 1.69 | 1.03 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.72 | 0.15 | 0.07 | 0.04 | 0.02 | 0.02 | 0.79 | 0.17 | 0.08 | 0.05 | 0.02 | 0.02 |
| 4.00 | ARL | 1.16 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.19 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.38 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 3.7: The run length comparison of the EWMA-VLRSS chart with the EWMA-SRS, EWMA-RSS, and EWMA-MRSS charts under imperfect ranking for $\lambda = 0.25$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) |
| | | SRS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS |
| $\delta$ | $L$ | 3.0007 | 3.0062 | 3.0020 | 3.0037 | 3.0027 | 3.0020 | 3.0026 | 3.0013 | 3.0020 | 3.0019 | 3.0059 | 3.0013 | 3.0027 |
| 0.00 | ARL | 499.99 | 502.45 | 502.02 | 502.22 | 501.97 | 501.56 | 502.14 | 499.51 | 501.08 | 501.88 | 501.71 | 499.80 | 501.86 |
| | SDRL | 497.14 | 502.67 | 503.30 | 504.12 | 502.48 | 505.50 | 505.00 | 499.36 | 498.61 | 498.49 | 502.17 | 501.17 | 501.90 |
| 0.25 | ARL | 168.27 | 166.39 | 164.61 | 163.64 | 149.14 | 146.60 | 145.01 | 119.37 | 112.58 | 106.84 | 92.84 | 81.37 | 72.60 |
| | SDRL | 165.39 | 163.77 | 161.84 | 161.17 | 146.59 | 143.58 | 142.29 | 116.03 | 109.10 | 104.33 | 89.86 | 77.63 | 69.46 |
| 0.50 | ARL | 47.34 | 45.87 | 45.82 | 44.37 | 39.22 | 38.49 | 37.37 | 29.07 | 26.95 | 25.24 | 21.30 | 18.28 | 15.84 |
| | SDRL | 44.01 | 42.32 | 41.57 | 41.20 | 35.59 | 35.14 | 34.17 | 25.64 | 23.64 | 21.85 | 18.00 | 15.15 | 12.64 |
| 0.75 | ARL | 19.41 | 18.50 | 18.44 | 18.32 | 15.91 | 15.56 | 15.28 | 11.99 | 11.13 | 10.38 | 8.96 | 7.86 | 6.94 |
| | SDRL | 16.04 | 15.22 | 15.10 | 15.18 | 12.80 | 12.34 | 12.22 | 9.12 | 8.24 | 7.52 | 6.29 | 5.32 | 4.55 |
| 1.00 | ARL | 10.39 | 10.04 | 9.93 | 9.92 | 8.77 | 8.51 | 8.41 | 6.71 | 6.28 | 5.92 | 5.17 | 4.56 | 4.09 |
| | SDRL | 7.59 | 7.27 | 7.18 | 7.15 | 6.14 | 5.88 | 5.81 | 4.37 | 4.01 | 3.71 | 3.15 | 2.67 | 2.34 |
| 1.50 | ARL | 4.77 | 4.61 | 4.60 | 4.58 | 4.10 | 4.03 | 3.97 | 3.27 | 3.09 | 2.95 | 2.61 | 2.35 | 2.14 |
| | SDRL | 2.83 | 2.70 | 2.70 | 2.69 | 2.34 | 2.29 | 2.26 | 1.75 | 1.63 | 1.53 | 1.32 | 1.16 | 1.03 |
| 2.00 | ARL | 2.96 | 2.86 | 2.82 | 2.81 | 2.56 | 2.52 | 2.48 | 2.09 | 1.99 | 1.91 | 1.72 | 1.56 | 1.44 |
| | SDRL | 1.55 | 1.47 | 1.46 | 1.46 | 1.28 | 1.27 | 1.24 | 0.99 | 0.94 | 0.88 | 0.77 | 0.68 | 0.60 |
| 2.50 | ARL | 2.09 | 2.03 | 2.01 | 2.00 | 1.84 | 1.82 | 1.80 | 1.53 | 1.47 | 1.42 | 1.29 | 1.21 | 1.14 |
| | SDRL | 0.99 | 0.96 | 0.95 | 0.95 | 0.85 | 0.83 | 0.82 | 0.66 | 0.62 | 0.59 | 0.50 | 0.43 | 0.36 |
| 3.00 | ARL | 1.62 | 1.58 | 1.56 | 1.55 | 1.45 | 1.42 | 1.41 | 1.24 | 1.20 | 1.17 | 1.10 | 1.05 | 1.03 |
| | SDRL | 0.72 | 0.68 | 0.67 | 0.68 | 0.60 | 0.58 | 0.58 | 0.45 | 0.42 | 0.38 | 0.30 | 0.22 | 0.17 |
| 4.00 | ARL | 1.16 | 1.14 | 1.14 | 1.14 | 1.09 | 1.08 | 1.07 | 1.02 | 1.02 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.38 | 0.36 | 0.35 | 0.35 | 0.29 | 0.28 | 0.26 | 0.15 | 0.12 | 0.10 | 0.05 | 0.03 | 0.01 |

Table 3.8: The run length comparison of the EWMA-VLRSS chart with the EWMA-SRS chart

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (2,1) | (3,2) | (4,2) | | (2,1) | (3,2) | (4,2) |
| $\delta$ | | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.54 | 500.96 | 501.54 | 500.03 | 500.76 | 501.56 | 501.17 | 502.06 |
| | SDRL | 515.09 | 520.72 | 517.41 | 517.15 | 514.08 | 504.59 | 509.28 | 500.37 |
| 0.25 | ARL | 78.30 | 39.88 | 32.74 | 29.89 | 103.42 | 51.16 | 41.07 | 37.16 |
| | SDRL | 70.16 | 32.47 | 25.72 | 23.50 | 98.37 | 45.04 | 34.96 | 31.15 |
| 0.50 | ARL | 23.79 | 12.11 | 10.02 | 9.12 | 28.95 | 13.90 | 11.30 | 10.32 |
| | SDRL | 17.98 | 8.33 | 6.76 | 6.04 | 23.32 | 9.74 | 7.60 | 6.82 |
| 0.75 | ARL | 11.83 | 6.16 | 5.07 | 4.68 | 13.59 | 6.84 | 5.65 | 5.17 |
| | SDRL | 8.10 | 3.88 | 3.13 | 2.84 | 9.48 | 4.23 | 3.38 | 3.05 |
| 1.00 | ARL | 7.32 | 3.84 | 3.22 | 2.97 | 8.19 | 4.24 | 3.53 | 3.26 |
| | SDRL | 4.76 | 2.27 | 1.82 | 1.65 | 5.19 | 2.39 | 1.93 | 1.77 |
| 1.50 | ARL | 3.76 | 2.09 | 1.78 | 1.67 | 4.17 | 2.27 | 1.92 | 1.79 |
| | SDRL | 2.19 | 1.06 | 0.86 | 0.78 | 2.37 | 1.13 | 0.91 | 0.83 |
| 2.00 | ARL | 2.42 | 1.43 | 1.26 | 1.21 | 2.66 | 1.53 | 1.34 | 1.26 |
| | SDRL | 1.28 | 0.62 | 0.48 | 0.43 | 1.38 | 0.67 | 0.53 | 0.47 |
| 2.50 | ARL | 1.78 | 1.15 | 1.07 | 1.04 | 1.92 | 1.20 | 1.09 | 1.06 |
| | SDRL | 0.86 | 0.37 | 0.25 | 0.20 | 0.92 | 0.42 | 0.30 | 0.24 |
| 3.00 | ARL | 1.41 | 1.03 | 1.01 | 1.00 | 1.52 | 1.05 | 1.02 | 1.01 |
| | SDRL | 0.60 | 0.18 | 0.10 | 0.07 | 0.65 | 0.22 | 0.13 | 0.08 |
| 4.00 | ARL | 1.09 | 1.00 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.28 | 0.02 | 0.01 | 0.00 | 0.33 | 0.03 | 0.01 | 0.01 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (2,1) | (3,2) | (4,2) | | (2,1) | (3,2) | (4,2) |
| $\delta$ | | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 499.99 | 501.55 | 502.62 | 502.67 | 500.53 | 499.25 | 501.43 | 499.51 |
| | SDRL | 497.14 | 503.67 | 506.70 | 502.71 | 499.16 | 501.43 | 500.68 | 498.12 |
| 0.25 | ARL | 168.27 | 87.39 | 70.37 | 62.53 | 255.04 | 152.46 | 126.43 | 114.71 |
| | SDRL | 165.39 | 83.63 | 67.11 | 59.05 | 253.92 | 150.60 | 124.52 | 113.59 |
| 0.50 | ARL | 47.34 | 19.84 | 15.54 | 13.87 | 88.15 | 36.78 | 27.88 | 24.41 |
| | SDRL | 44.01 | 16.50 | 12.35 | 10.71 | 86.16 | 35.16 | 25.83 | 22.49 |
| 0.75 | ARL | 19.41 | 8.44 | 6.73 | 6.12 | 35.62 | 13.17 | 9.78 | 8.63 |
| | SDRL | 16.04 | 5.85 | 4.38 | 3.88 | 33.68 | 11.40 | 8.05 | 6.92 |
| 1.00 | ARL | 10.39 | 4.90 | 3.99 | 3.64 | 17.11 | 6.49 | 4.98 | 4.47 |
| | SDRL | 7.59 | 2.93 | 2.26 | 2.00 | 15.16 | 4.84 | 3.44 | 3.01 |
| 1.50 | ARL | 4.77 | 2.50 | 2.10 | 1.95 | 6.27 | 2.78 | 2.27 | 2.08 |
| | SDRL | 2.83 | 1.25 | 0.99 | 0.91 | 4.65 | 1.58 | 1.20 | 1.06 |
| 2.00 | ARL | 2.96 | 1.65 | 1.41 | 1.34 | 3.38 | 1.73 | 1.47 | 1.38 |
| | SDRL | 1.55 | 0.73 | 0.58 | 0.53 | 2.07 | 0.81 | 0.63 | 0.57 |
| 2.50 | ARL | 2.09 | 1.26 | 1.13 | 1.08 | 2.26 | 1.28 | 1.14 | 1.10 |
| | SDRL | 0.99 | 0.47 | 0.34 | 0.28 | 1.19 | 0.50 | 0.36 | 0.30 |
| 3.00 | ARL | 1.62 | 1.08 | 1.02 | 1.01 | 1.69 | 1.09 | 1.03 | 1.02 |
| | SDRL | 0.72 | 0.27 | 0.15 | 0.11 | 0.79 | 0.28 | 0.17 | 0.12 |
| 4.00 | ARL | 1.16 | 1.00 | 1.00 | 1.00 | 1.19 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.38 | 0.05 | 0.01 | 0.00 | 0.41 | 0.05 | 0.01 | 0.01 |

Table 3.9: The run length comparison of the EWMA-VLRSS chart with the EWMA-SRS chart under imperfect ranking for $\lambda = 0.25$

| | | $\rho = 0$ | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (0,0) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $L$ | 3.0007 | 3.0024 | 3.0021 | 3.0017 | 3.0007 | 3.0050 | 3.0044 | 3.0007 | 3.0050 | 3.0044 | 3.0048 | 3.0050 | 3.0044 |
| 0.00 | ARL | 499.99 | 501.97 | 499.73 | 500.92 | 501.74 | 502.33 | 501.52 | 501.18 | 502.09 | 500.04 | 502.06 | 501.74 | 501.60 |
| | SDRL | 497.14 | 502.01 | 494.78 | 505.57 | 499.31 | 503.62 | 502.62 | 501.14 | 504.89 | 499.98 | 505.50 | 503.24 | 504.14 |
| 0.25 | ARL | 168.27 | 165.96 | 164.82 | 164.09 | 151.60 | 150.44 | 148.82 | 128.14 | 120.25 | 116.02 | 106.42 | 93.30 | 88.47 |
| | SDRL | 165.39 | 163.01 | 163.27 | 161.99 | 148.40 | 147.72 | 146.31 | 124.73 | 116.80 | 112.12 | 104.01 | 90.00 | 85.10 |
| 0.50 | ARL | 47.34 | 46.05 | 45.72 | 45.32 | 40.08 | 39.50 | 38.74 | 31.65 | 29.25 | 28.32 | 25.15 | 21.38 | 19.91 |
| | SDRL | 44.01 | 42.78 | 42.34 | 41.83 | 36.71 | 36.07 | 35.03 | 28.03 | 25.99 | 24.81 | 21.69 | 18.04 | 16.68 |
| 0.75 | ARL | 19.41 | 18.83 | 18.49 | 18.50 | 16.50 | 15.99 | 15.91 | 12.98 | 11.97 | 11.48 | 10.38 | 8.94 | 8.37 |
| | SDRL | 16.04 | 15.71 | 15.18 | 15.29 | 13.27 | 12.81 | 12.79 | 9.95 | 9.01 | 8.61 | 7.59 | 6.27 | 5.74 |
| 1.00 | ARL | 10.39 | 10.08 | 9.97 | 9.98 | 8.95 | 8.79 | 8.70 | 7.19 | 6.67 | 6.53 | 5.90 | 5.15 | 4.88 |
| | SDRL | 7.59 | 7.25 | 7.14 | 7.26 | 6.33 | 6.13 | 6.08 | 4.78 | 4.33 | 4.20 | 3.71 | 3.10 | 2.93 |
| 1.50 | ARL | 4.77 | 4.63 | 4.62 | 4.59 | 4.20 | 4.11 | 4.08 | 3.49 | 3.26 | 3.19 | 2.93 | 2.62 | 2.49 |
| | SDRL | 2.83 | 2.71 | 2.72 | 2.69 | 2.42 | 2.34 | 2.32 | 1.91 | 1.76 | 1.70 | 1.53 | 1.31 | 1.24 |
| 2.00 | ARL | 2.96 | 2.86 | 2.85 | 2.84 | 2.62 | 2.56 | 2.54 | 2.22 | 2.09 | 2.04 | 1.90 | 1.71 | 1.65 |
| | SDRL | 1.55 | 1.48 | 1.47 | 1.47 | 1.33 | 1.29 | 1.27 | 1.07 | 0.99 | 0.97 | 0.88 | 0.77 | 0.73 |
| 2.50 | ARL | 2.09 | 2.04 | 2.03 | 2.03 | 1.88 | 1.85 | 1.83 | 1.62 | 1.54 | 1.50 | 1.41 | 1.29 | 1.25 |
| | SDRL | 0.99 | 0.96 | 0.96 | 0.95 | 0.87 | 0.84 | 0.84 | 0.71 | 0.65 | 0.64 | 0.58 | 0.49 | 0.46 |
| 3.00 | ARL | 1.62 | 1.58 | 1.57 | 1.57 | 1.47 | 1.44 | 1.44 | 1.29 | 1.24 | 1.22 | 1.16 | 1.10 | 1.07 |
| | SDRL | 0.72 | 0.68 | 0.68 | 0.68 | 0.62 | 0.60 | 0.60 | 0.49 | 0.45 | 0.44 | 0.38 | 0.30 | 0.26 |
| 4.00 | ARL | 1.16 | 1.14 | 1.14 | 1.14 | 1.10 | 1.09 | 1.08 | 1.04 | 1.02 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.38 | 0.36 | 0.36 | 0.36 | 0.30 | 0.29 | 0.28 | 0.19 | 0.15 | 0.14 | 0.10 | 0.06 | 0.04 |

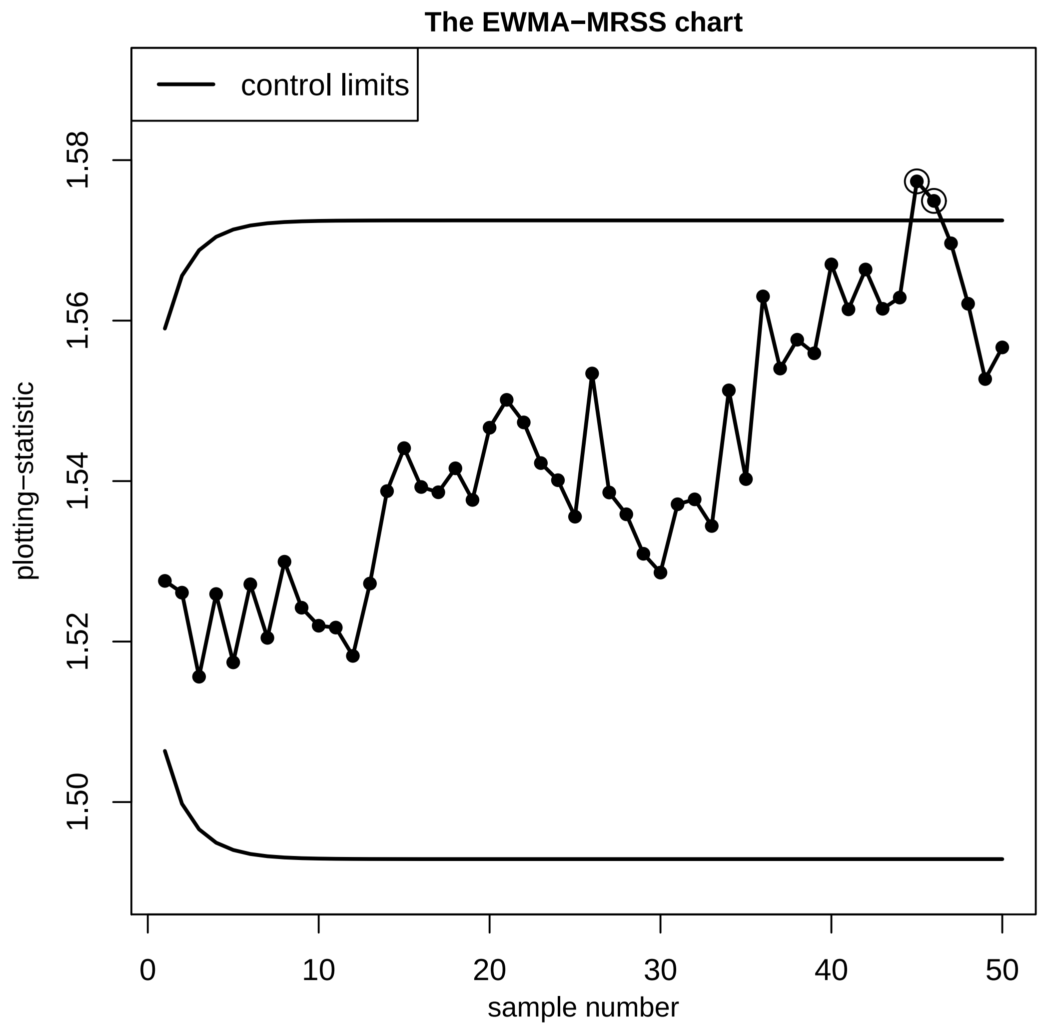


Figure 3.1: The EWMA chart using MRSS scheme

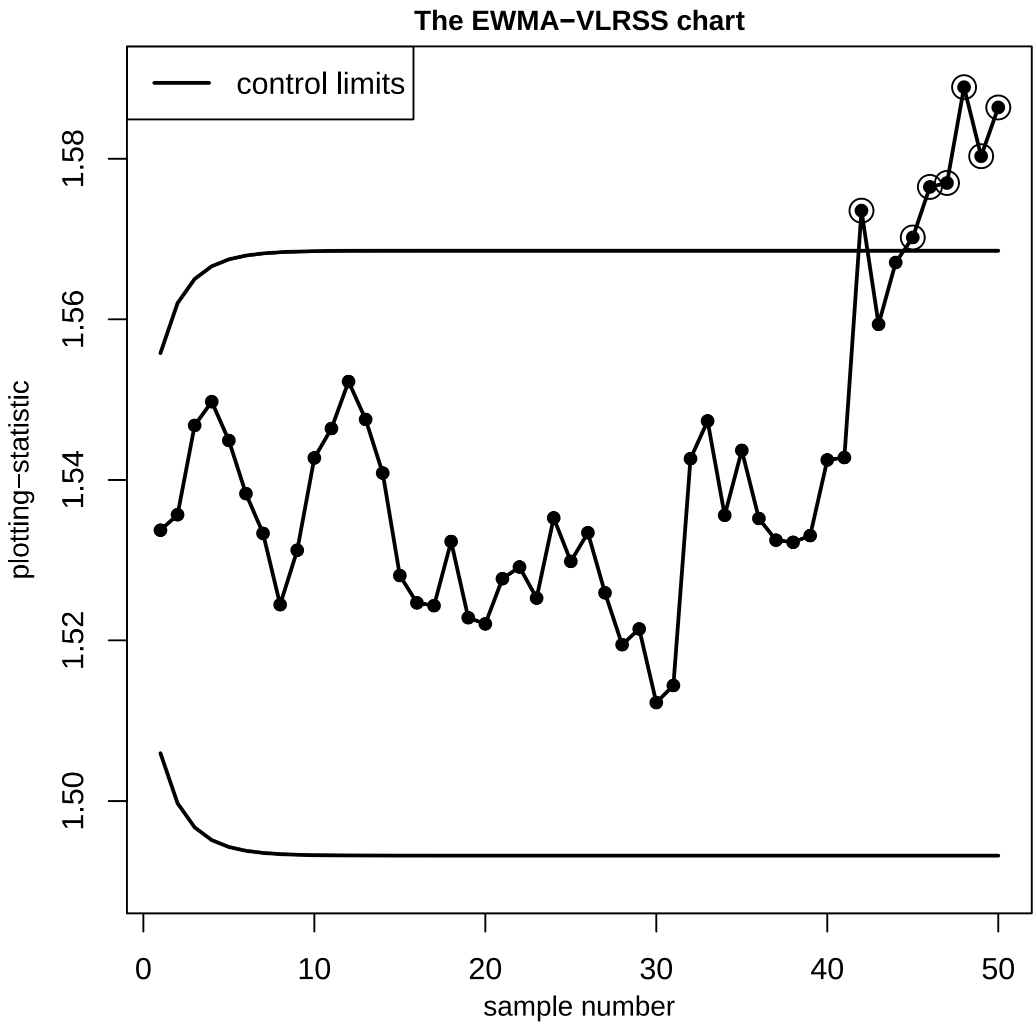


Figure 3.2: The EWMA chart using VLRSS scheme

# Chapter 4

# New Shewhart-EWMA and Shewhart-CUSUM Control Charts for Monitoring Process Mean

In this chapter, we propose new Shewhart-EWMA (SEWMA) and Shewhart-CUSUM (SCUSUM) control charts using the varied L ranked set sampling (VLRSS) for monitoring the process mean, namely the SEWMA-VLRSS and SCUSUM-VLRSS charts. The run length characteristics of the proposed charts are computed using extensive Monte Carlo simulations. The proposed charts are compared with their existing counterparts in terms of the average and standard deviation of run lengths. It is found that, with perfect and imperfect rankings, the SEWMA-VLRSS and SCUSUM-VLRSS charts are more sensitive than their analogous charts based on simple random sampling, ranked set sampling (RSS) and median RSS schemes. A real dataset is also used to explain the implementation of the proposed control charts.

## 4.1 Introduction

The Statistical Process Control (SPC) is a collection of tools that help in distinguishing between two types of variation, namely the natural- and special-cause variations. A process is said to be in statistical control when only natural-cause variations are present while the process with the special-cause variations is said to be out-of-control. There are seven major tools in the SPC, including the histogram, check sheet, Pareto chart, cause and effect diagram, defect concentration diagram, scatter diagram, and control charts. Statistical quality control charts are very effective SPC tool that are frequently used to monitor special-cause variations in a production/manufacturing process.

The control charts are divided into two categories, memory-less and memory-type control charts. The Shewhart-type charts fall in the memory-less category because they completely rely on the present information. A major limitation of the Shewhart control chart is that it is less sensitive against small and moderate shifts in the process parameter(s). On the other hand, the exponentially weighted moving average (EWMA) and the cumulative sum (CUSUM) control charts fall in the memory-type category. The reason being that both of these control charts take into account both past and current information to maintain their plotting-statistics. This feature of the memory-type control charts helps them to swiftly react against small to moderate shifts in the process parameter(s).

The CUSUM chart was first developed by Page (1954). Lucas and Crosier (1982) associated a fast initial response (FIR) feature with the CUSUM chart to further enhance its sensitivity by giving head-starts to the plotting CUSUMs at the beginning of a process. Lucas (1982) used both the Shewhart and CUSUM charts simultaneously for monitoring small as well as large shifts in a process, named the Shewhart-CUSUM (SCUSUM) chart. For monitoring changes in the process mean, Roberts (1959) was the first to introduce the EWMA chart. Lucas and Saccucci (1990) attached an FIR feature with the EWMA chart for increasing its sensitivity for the start-up/initial problems. Moreover, they coupled the Shewhart chart with the EWMA chart, named the Shewhart-EWMA (SEWMA) chart, for detecting small and large shifts simultaneously. There are many new advancements and improvements in the control charting structure of the EWMA and CUSUM charts. For some related works on these control charts, we refer to Knoth (2005), Lucas and Crosier (1982), Chiu (2009), Abbas et al. (2013), Haq (2013), Haq et al. (2014a), and the references cited therein.

Ranked-set sampling (RSS) was first introduced by McIntyre (1952) for estimating mean pasture and forage yields. RSS is a cost-effective alternative to the simple random sampling (SRS) in situations where the units to be sampled could be ranked relative to each other prior to the formal measurements. Ranking may be done visually, on personal judgment or using the ranks of an auxiliary variable, provided that it is highly correlated with the study variable. For example, if the quantification of the underlying quality characteristic(s) is laborious, costly, time-consuming, may involve breaking the product—which is expensive and might be hard to construct, etc.—but using some experts' knowledge, it might be possible to rank the quality characteristic according to its quality level or using any less expensive method. There also exist the situations where the error while ranking the units is inevitable, particularly when ranking the units in large set sizes. Dell and Clutter (1972) has shown that, despite the presence of ranking errors, the mean estimator with RSS is not only unbiased but also is more precise than the mean estimator with SRS. A simple imperfect ranking model was designed by Stokes (1977) whereby the study variable could be ranked using the ranks of an auxiliary variable. For precisely estimating the mean of a symmetric population, extreme RSS (ERSS) and median RSS (MRSS) schemes were suggested by Samawi et al. (1996) and Muttlak (1997), respectively. The mean estimator with MRSS is more precise than those with the SRS and RSS, when sampling from a symmetric population. Muttlak (2003) suggested using the quartile RSS (QRSS), as a better alternative to SRS, RSS and MRSS schemes, for estimating the population mean when sampling from an asymmetric population. Utilizing the idea of L moments, Al-Nasser (2007) proposed a generalized sampling scheme, named L RSS (LRSS) for estimating the population mean. The LRSS scheme encompasses existing RSS schemes, like the RSS, QRSS, ERSS, and MRSS. Haq et al. (2015e) further extended the work of Al-Nasser (2007) and further generalized the LRSS scheme for efficiently estimating the population mean, named the varied LRSS (VLRSS) scheme.

For a symmetric population, the VLRSS scheme—with both perfect and imperfect rankings—is not only a cost-effective alternative to the exiting ranking schemes but it also encompasses them, i.e., the mean estimator with the VLRSS scheme is better than the mean estimator based on SRS, RSS, ERSS, MRSS, and QRSS. For more details, we refer to Haq et al. (2015e).

As the mean estimators with the RSS schemes are more precise than the mean estimator based on SRS scheme, this fact has led many researchers to construct more sensitive quality control charts. Salazar and Sinha (1997) were the first to propose a Shewhart chart using RSS for monitoring the process mean. Their work, later on, extended by Muttlak and Al-Sabah (2003), who suggested several Shewhart-type mean charts using the RSS, ERSS and MRSS schemes under both perfect and imperfect rankings. Abujiya and Muttlak (2004) and Al-Omari and Haq (2012) used the double RSS schemes to construct the Shewhart charts for monitoring the process mean. Al-Sabah (2010) suggested new CUSUM mean charts using RSS and MRSS. He showed that the CUSUM charts with RSS and MRSS schemes are more sensitive than that using SRS. Recently, Abujiya and Lee (2013) constructed new Shewhart, EWMA and CUSUM mean charts using RSS with both perfect and imperfect rankings. Abujiya et al. (2013a,b) proposed SEWMA and SCUSUM mean charts using RSS and MRSS schemes. For more related works on the RSS based control charts, we refer to Haq (2014), Mehmood et al. (2013, 2014), Haq et al. (2014b, 2015d,b,c), Abbasi and Riaz (2016), Abid et al. (2016b,a), and the references cited therein.

As the VLRSS mean estimator is better than the mean estimator based on SRS, RSS, MRSS schemes when sampling from a symmetric population, we believe that the control charts with the VLRSS would be more sensitive than those based on SRS, RSS, MRSS schemes. In this chapter, we propose new SEWMA and SCUSUM mean chart using VLRSS, named the SEWMA-VLRSS and SCUSUM-VLRSS charts, respectively. Monte Carlo simulations are used in computing the run length characteristics of the proposed control charts, including the average run length (ARL) and the standard deviation of the run length (SDRL). The proposed charts are compared with their counterparts based on SRS, RSS and MRSS schemes. It turns out that the proposed charts are more sensitive than the existing charts.

The rest of the chapter is in following order: Section 4.2 and 4.3 briefly review the the SEWMA and SCUSUM charts with SRS, respectively. In Section 4.4, the VLRSS scheme is discussed with both perfect and imperfect rankings. The proposed charts are presented in Section 4.5. A comparative study is conducted in Section 4.6. An illustrative example is presented in Section 4.7, and Section 4.8 summarizes the main findings.

## 4.2 The Shewhart-EWMA chart

The classical SEWMA chart is a mixture of two control charts. The Shewhart and EWMA charts provide protection against the large and small-to-moderate shifts in the process mean (cf., Lucas and Saccucci (1990)).

Let $Y$ denotes the study variable and let $\{Y_t\}$, for $t = 1, 2, ...$, be a sequence of independent and identically distributed (IID) random variables. Here, it is assumed that $Y_t$ is a normally distributed random variable with the in-control mean $\mu_Y$ and the in-control variance $\sigma_Y^2$, i.e., $Y_t \sim N(\mu_Y, \sigma_Y^2)$ for $t \geq 1$. Let $\{\overline{Y}_{\text{SRS},t}\}$ be a sequence of IID random variables with SRS, where $\overline{Y}_{\text{SRS},t} = (1/n)\sum_{i=1}^{n} Y_{i,t}$. Here, $Y_{i,t}$ is the $i$th observation in the $t$th simple random sample of size $n$, for $i = 1, 2, ..., n$. Note that, $\overline{Y}_{\text{SRS},t}$, is also a normal random variable with the mean $\mu_Y$ and the variance $\sigma_Y^2/n$, i.e., $\overline{Y}_{\text{SRS},t} \sim N(\mu_Y, \sigma_Y^2/n)$. Let $\delta = \sqrt{n}|\mu_Y - \mu_{Y,1}|/\sigma_Y$ be the amount of standardized shift to be detected in the in-control process mean $\mu_Y$, where $\mu_{Y,1}$ is the out-of-control process mean.

Using $\overline{Y}_{\text{SRS},t}$, an EWMA statistic, say $Z_t$, is given by

$$Z_t = \lambda \overline{Y}_{\text{SRS},t} + (1-\lambda) Z_{t-1}, \tag{4.1}$$

where $Z_t$ and $Z_{t-1}$ are the current and past information, respectively, and $0 < \lambda \leq 1$ is a smoothing constant. The starting value of $Z_t$ is set equal to the in-control process mean $\mu_Y$, i.e., $Z_0 = \mu_Y$. The variance of $Z_t$ is

$$\text{Var}(Z_t) = \frac{\sigma_Y^2}{n} \cdot \frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2t}\right]. \tag{4.2}$$

Here, if the time $t$ gets large, the term $[1 - (1-\lambda)^{2t}]$ approaches to unity. The asymptotic variance of the EWMA statistic $Z_t$ is given by

$$\text{Var}(Z_t) = \frac{\sigma_Y^2}{n} \cdot \frac{\lambda}{(2-\lambda)}. \tag{4.3}$$

The upper control limit (UCL) and the lower control limit (LCL) of the EWMA chart based on the asymptotic variance of $Z_t$ are given by

$$\text{UCL} = \mu_Y + L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}} \quad \text{and} \quad \text{LCL} = \mu_Y - L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}}. \tag{4.4}$$

Similarly, the UCL and LCL of the Shewhart chart based on $\overline{Y}_{\text{SRS},t}$ are given by

$$\text{UCL} = \mu_Y + d_1\frac{\sigma_Y}{\sqrt{n}} \quad \text{and} \quad \text{LCL} = \mu_Y - d_1\frac{\sigma_Y}{\sqrt{n}}. \tag{4.5}$$

The central limits (CLs) of both the EWMA and Shewhart charts are set equal to the in-control process mean, i.e., CL $= \mu_Y$. Here, $L$ and $d_1$ are the design parameters of the SEWMA chart, respectively, and their values depend on the choices of $\lambda$ and the desired in-control ARL. The SEWMA chart triggers an out-of-control signal when either $Z_t$ falls outside the EWMA control limits or $\overline{Y}_{\text{SRS},t}$ falls outside the Shewhart control limits. When working with the SEWMA chart, the range of $d_1$ should be $3.0 < d_1 < 4.5$ (cf., Lucas and Saccucci (1990)). Here, the value of $d_1$ is set equal to 3.31.

As aforementioned, the EWMA chart is very effective in detecting small to moderate process shifts. However, there might exist a situation where the process—initially or in the startup—may make tracks in a different direction from the process target or after the process is recouped from an out-of-control state. In such situations, giving a head-start to the EWMA chart may help in earlier detection of shifts in the process target. The FIR feature in the EWMA charting structure was first suggested by Lucas and Saccucci (1990) to overcome such situations. They suggested using two one-sided EWMA charts, each with a head-start. Their work was further extended by Rhoads et al. (1996) who used two one-sided EWMA charts with head-starts and the time-varying control limits. To further reduce the time-varying control limits of the EWMA chart for the first few samples, say ten or twenty, Steiner (1999) used an exponentially decreasing adjustment factor. The new control limits of the EWMA chart with the FIR-adjustment factor are:

$$\text{UCL} = \mu_Y + L\frac{\sigma_Y}{\sqrt{n}}\left(1-(1-f)^{1+a(t-1)}\right)\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{4.6}$$

$$\text{LCL} = \mu_Y - L\frac{\sigma_Y}{\sqrt{n}}\left(1-(1-f)^{1+a(t-1)}\right)\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{4.7}$$

where $f$ and $a$ are known constants. The choice of $a$, suggested by Steiner (1999), for which the FIR-adjustment has little effect after the 20th observation is $a = -(1/19)(2/\log(1-f)+1)$. For instance, with $f = 0.5$, we get $a = 0.3$. For more details regarding the FIR feature with the EWMA chart, we refer to Rhoads et al. (1996), Steiner (1999), Knoth (2005), and Haq et al. (2014a).

## 4.3 The Shewhart-CUSUM chart

The SCUSUM chart was first suggested by Lucas (1982), which integrates the Shewhart chart with the CUSUM chart, and it useful to detect small and large shifts in the process target simultaneously. In the SCUSUM charting structure, the CUSUM chart quickly detects small shifts whereas the Shewhart chart swiftly detects large shifts in the process target.

The SCUSUM chart works in a similar way like the classical CUSUM chart. The CUSUM chart works with the two CUSUMs, upward and downward, say $C_t^+$ and $C_t^-$, respectively, given by

$$C_t^+ = \text{Max}[0, +(\overline{Y}_{\text{SRS},t} - \mu_Y) - K + C_{t-1}^+], \tag{4.8}$$

$$C_t^- = \text{Max}[0, -(\overline{Y}_{\text{SRS},t} - \mu_Y) - K + C_{t-1}^-], \tag{4.9}$$

where $K = k\sigma_Y/\sqrt{n}$ is reference of slack value of the CUSUM chart. Here, $k$ is usually taken as half of the magnitude of the shift $\delta$ to be detected in $\mu_Y$, i.e., $k = \delta/2$. The CUSUM chart declares the process out-of-control if either $C_t^+$ or $C_t^-$ exceeds the predetermined decision value $H = h\sigma_Y$, where $h$ is selected to get the desired in-control ARL of the CUSUM chart. In the SCUSUM chart, the above CUSUM is integrated with the Shewhart chart, explained in the previous section. The SCUSUM chart triggers an out-of-control signal when $C_t^+$ or $C_t^-$ exceeds $H$ or if $Y_{\text{SRS},t}$ exceeds the control limits given in Eq. (4.5) in either direction. It is customary to take $d_1 = 3.5$ (cf., Lucas (1982)).

The FIR feature with the CUSUM chart was first suggested by Lucas and Crosier (1982) which enables the CUSUM chart to react quickly against the start-up/initial problems. The FIR feature in the CUSUM chart works by resetting the starting values of both plotting-CUSUMs to some non-zero constant. They recommended using $H/2$ for a 50% head-start, i.e. by setting $C_0^+ = C_0^- = H/2$. For more details, see Lucas and Crosier (1982) and Haq et al. (2014a).

## 4.4 The VLRSS scheme

In this section, we briefly review the mean estimator using VLRSS under both perfect and imperfect rankings. The VLRSS scheme is a cost-effective alternative to the SRS and RSS schemes. This scheme not only provides unbiased and precise mean estimator when sampling from a symmetric population, but it also provides plenty of options to the experimenter in selecting different representative samples with the less number of identified units than that using the RSS scheme, i.e., the ranking costs with VLRSS could be more or less than that with the RSS. It is worth mentioning that the VLRSS scheme provides more efficient mean estimator than the mean estimators based on RSS and MRSS schemes when ranking costs are negligible. But, when ranking costs are high, it is still beneficial to use VLRSS scheme with the less ranking cost than that with the RSS schemes (cf., Haq et al. (2015e)).

The main steps involved in selecting a varied L ranked set sample of size $n$ are as follows:

Step 1: Select the value of the VLRSS coefficient, say $w = [al]$, where $0 \leq a < 0.5$. Here, $[\cdot]$ is the largest possible integer value.

Step 2: Select $2wl$ units from the target population. Divide these units into $2w$ sets, with each set consisting of $l$ units.

Step 3: Rank the units within each set by any cheap or inexpensive method with respect to the study variable or using ranks of an auxiliary variable.

Step 4: Select the $v$th and $(l - v + 1)$th smallest ranked units from the first and last $w$ sets, respectively, where $v = 1, 2, ..., [l/2]$.

Step 5: Identify $m(m - 2w)$ units from the target population, and then divide these units into $m - 2w$ sets, with each set comprising $m$ units.

Step 6: Select the $i$th smallest ranked unit from the $(i + w)$th set of $m$ units, for $i = w + 1, w + 2, ..., m - w$.

Step 7: This completes one cycle of a varied L ranked set sample of size $m$. The Steps 1-6 could be repeated, if necessary, $r$ number of times to get a total sample of size $n = mr$ units.

Symbolically, let $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$, $i = 1, 2, ..., 2w$, be $2w$ samples, each of size $l$, for the $j$th cycle, where $j = 1, 2, ..., r$. Let $Y_{i(v:l)j}$ denote the $v$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = 1, 2, ..., w$, and let $Y_{i(l-v+1:l)j}$ be the $(l - v + 1)$th order statistic of $(Y_{i1j}, Y_{i2j}, ..., Y_{ilj})$ for $i = w + 1, w + 2, ..., 2w$. Let $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$, $i = w + 1, 2, ..., m - w$, denote $m - 2w$ samples, each of size $m$, for the $j$th cycle. Let $Y_{i+w(i:m)j}$ denote the $i$th order statistic of $(Y_{(i+w)1j}, Y_{(i+w)2j}, ..., Y_{(i+w)mj})$ for $i = 1, 2, ..., m - w$.

The sample mean based on a varied L ranked set sample of size $n$, denoted by $\overline{Y}_{\text{VLRSS}}$, and its variance, respectively, are given by

$$\overline{Y}_{\text{VLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right), \tag{4.10}$$

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y(v:l)} + \sigma^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w} \sigma^2_{Y(i:m)}\right), \tag{4.11}$$

where $\sigma^2_{Y(v:l)} = \text{Var}(Y_{i(v:l)j})$, $\sigma^2_{Y(l-v+1:l)} = \text{Var}(Y_{i(l-v+1:l)j})$, and $\sigma^2_{Y(i:m)} = \text{Var}(Y_{i(i:m)j})$. For more details regarding the computation of the variances of order statistics, we refer to David and Nagaraja (2003).

For a symmetric population, Haq et al. (2015e) have shown that $\overline{Y}_{\text{VLRSS}}$ is an unbiased estimator of $\mu_Y$. They have also shown that, with some suitable choices of $v, l$, and $w$, the existing RSS schemes are special cases of VLRSS. For instance, for $w = 0$, VLRSS becomes RSS; for $w = [(m - 1)/2]$, $l = m$, and $v = w + 1$, VLRSS becomes MRSS, etc. While selecting a varied L ranked set sample of size $n$, the experimenter needs to identify $nm - 2w(m - l)r$ units, while the classical RSS and MRSS require identifying $nm$ units when selecting a sample of size $n$. It is to be noted that when $m > l$, VLRSS requires less number of identified units than that using the RSS or MRSS (cf., Haq et al. (2015e)).

### 4.4.1 The imperfect VLRSS scheme

There may exists a situation where it is not possible to rank the study variable visually, or it is costly and time-consuming. This issue can be solved by ranking the study variable ($Y$) using the ranks of a highly correlated variable, say $X$, given that it is readily available. Stokes (1977) suggested a simple model for the imperfect rankings, given by

$$Y_{i[i:u]j} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}\left(X_{i(i:u)j} - \mu_X\right) + \xi_{ij},\ i = 1, 2, ..., u,\ j = 1, 2, ..., r, \tag{4.12}$$

where $u = l, m$; $\mu_X$ and $\sigma_X$ are the population mean and standard deviation of $X$, respectively, and $\rho$ is the correlation between $Y$ and $X$. Here, $\xi_{ij} \sim N(0, \sigma^2_Y(1 - \rho^2))$, and $X_{i(i:u)j}$ and $\xi_{ij}$ are mutually independent. $Y_{i[i:u]j}$ is the $i$th concomitant or induced order statistic corresponding to the $i$th order statistic $X_{i(i:u)j}$, $i = 1, 2, ..., u$. The values of $X$ are perfectly ranked but the values of $Y$ are ranked with error. On the lines of Stokes (1977), the sample mean under imperfect

VLRSS (IVLRSS), say $\overline{Y}_{\text{IVLRSS}}$, and its variance, are, respectively, given by

$$\overline{Y}_{\text{IVLRSS}} = \frac{1}{n}\sum_{j=1}^{r}\left(\sum_{i=1}^{w}Y_{i[v:l]j} + \sum_{i=w+1}^{2w}Y_{i[l-v+1:l]j} + \sum_{i=w+1}^{m-w}Y_{i+w[i:m]j}\right), \tag{4.13}$$

and

$$\text{Var}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(\sigma^2_{Y[v:l]} + \sigma^2_{Y[l-v+1:l]}) + \sum_{i=w+1}^{m-w}\sigma^2_{Y[i:m]}\right) \tag{4.14}$$

$$= \frac{1}{nm}\left\{m\sigma^2_Y(1-\rho^2) + \rho^2\frac{\sigma^2_Y}{\sigma^2_X}\left(2w\sigma^2_{X(v:l)} + \sum_{i=w+1}^{m-w}\sigma^2_{X(i:m)}\right)\right\}, \tag{4.15}$$

where $\sigma^2_{Y[v:l]} = \text{Var}(Y_{i[v:l]j})$, $\sigma^2_{Y[l-v+1:l]} = \text{Var}(Y_{i[l-v+1:l]j})$, and $\sigma^2_{Y[i:m]} = \text{Var}(Y_{i[i:m]j})$. For more details on the computation of these variances, we refer to see David and Nagaraja (2003). When sampling from a symmetric bivariate population, $\overline{Y}_{\text{IVLRSS}}$ is not only unbiased, and with reasonable assumptions, it is more precise than the mean estimators based on imperfect RSS and MRSS schemes (cf., Haq et al. (2015e)).

## 4.5 The proposed control charts

In this section, we propose new SEWMA and SCUSUM control charts for efficiently monitoring the process mean $\mu_Y$ under both perfect and imperfect VLRSS schemes. The run length characteristic of these control charts are also computed and explored using Monte Carlo simulations.

### 4.5.1 The SEWMA chart

Suppose that a sample of size $n$ is selected from the target population with the S sample scheme at each time point $t(\geq 1)$, where S = VLRSS and IVLRSS. Let $\{\overline{Y}_{\text{S},t}\}$ be a sequence of IID random variables for $t = 1, 2, ....$ Using $\{\overline{Y}_{\text{S},t}\}$, it is possible to construct an SEWMA chart for monitoring $\mu_Y$. The plotting-statistic of the SEWMA chart with the S scheme is given by:

$$Q_t = \lambda\overline{Y}_{\text{S},t} + (1-\lambda)Q_{t-1}, \tag{4.16}$$

where $\lambda$ is a smoothing constant. The asymptotic variance of $Q_t$ is

$$\text{Var}(Q_t) = \text{Var}(\overline{Y}_{\text{S},t})\cdot\frac{\lambda}{(2-\lambda)}, \tag{4.17}$$

where $\text{Var}(\overline{Y}_{\text{S},t})$ denotes the variance of $\{\overline{Y}_{\text{S},t}\}$ at time $t$. The control limits of the SEWMA chart with the S scheme are

$$\text{UCL} = \mu_Y + L\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}\sqrt{\frac{\lambda}{(2-\lambda)}} \quad \text{and} \quad \text{LCL} = \mu_Y - L\sqrt{\text{Var}(\overline{Y}_{\text{S},t})}\sqrt{\frac{\lambda}{(2-\lambda)}}, \tag{4.18}$$

Similarly, the control limits of the Shewhart chart, based on $\{\overline{Y}_{\mathrm{S},t}\}$, are given by

$$\mathrm{UCL} = \mu_Y + d_1\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})} \qquad \text{and} \qquad \mathrm{LCL} = \mu_Y - d_1\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})}, \tag{4.19}$$

where $L$ and $d_1$ are positive control charting multipliers that are selected to ensure that the in-control ARL of the SEWMA chart has reached to a certain level. The SEWMA chart triggers an out-of-control signal whenever $Q_t$ or $\{\overline{Y}_{\mathrm{S},t}\}$ falls outside their respective control limits' interval, i.e., [LCL,UCL]. As aforementioned that the sensitivity of the SEWMA chart can be enhanced by giving a head-start to the SEWMA chart with the FIR-adjustment (cf., Steiner (1999)); on the similar lines, we associate an FIR feature with the proposed SEWMA chart by setting $f = 0.5$ and $a = 0.3$ as recommended by Steiner (1999).

### 4.5.2 The SCUSUM chart

To construct an SCUSUM chart for monitoring $\mu_Y$, consider the sequence $\{\overline{Y}_{\mathrm{S},t}\}$ for $t = 1, 2, ....$ The plotting-statistics (upper and lower CUSUMs) of the proposed SCUSUM chart using S sampling scheme are, respectively, defined by

$$C_t^+ = \mathrm{Max}[0, +(\overline{Y}_{\mathrm{S},t} - \mu_Y) - K + C_{t-1}^+], \tag{4.20}$$
$$C_t^- = \mathrm{Max}[0, -(\overline{Y}_{\mathrm{S},t} - \mu_Y) - K + C_{t-1}^-], \tag{4.21}$$

where $C_0^+ = C_0^- = 0$. The reference value $K$ and the decision interval $H$ of the SCUSUM chart are

$$K = k\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})}, \tag{4.22}$$
$$H = h\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})}, \tag{4.23}$$

where the values of $k$ and $h$ are the same as explained in the previous section. Similarly, the control limits of the Shewhart chart based on $\{\overline{Y}_{\mathrm{S},t}\}$ are given by

$$\mathrm{UCL} = \mu_Y + d_1\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})} \qquad \text{and} \qquad \mathrm{LCL} = \mu_Y - d_1\sqrt{\mathrm{Var}(\overline{Y}_{\mathrm{S},t})}. \tag{4.24}$$

The SCUSUM triggers an out-of-control signal if $C_t^+$ or $C_t^-$ exceeds $H$ or if $\overline{Y}_{\mathrm{S},t}$ is less than LCL or greater than UCL of the Shewhart chart. The sensitivity of the CUSUM chart for start-up problems, as suggested by Lucas and Crosier (1982), could be increased using a head-start feature. They recommended resetting the starting values of $C_t^+$ and $C_t^-$ to non-zero constants, like $C_0^+ = C_0^- = H/2$ for an 50% head-start. (cf., Lucas and Crosier (1982)). On the same lines, we attach an FIR feature with the SCUSUM chart with 50% head-start.

### 4.5.3 Run length evaluation

Generally, the run length performance of a control chart is evaluated in terms of its run length characteristics including the ARL and the SDRL. For an in-control process, the in-control ARL should be large enough to avoid false alarms, while for an out-of-control process, it should be as small as possible to swiftly trigger an out-of-control signal. In the literature, there exist some methods that could be used to compute the run length characteristics of a control chart, which include the integral equations, Markov chain, and the Monte Carlo simulations. The Monte Carlo simulation method is broadly used in computing the run length characteristics as it provide more accurate results.

In order to evaluate the run length performances of the proposed control charts, we generate samples under VLRSS from the standard normal distribution. The in-control ARL is set to 500— a choice recommended by the SPC practitioners. Here, each simulation run comprises 50,000 iterations. In Tables 4.1 and 4.2, we report the values of $(\lambda, L)$ and $(k, h)$ for the SEWMA and SCUSUM charts, respectively, with different possible values of $(m, l, v)$ with $r = 1$ when the in-control ARL for each case is set to 500. These constants could be used when using the proposed charts with different choices of $(m, l, v)$ when the in-control ARL is fixed to 500.

For brevity of discussion, without loss of generality, with $n = 5$ and $r = 1$, using different pairs of $(l, v)$, we compute the ARLs and SDRLs of the proposed control charts in Tables 4.3–4.10 (with and without FIR features). It is to be noted that, for a given sample size $n$, we consider those choices of $w$ and $(l, v)$ with the VLRSS scheme for which the mean estimator is precise (cf., Haq et al. (2015e)). Different values of $\delta$ are considered, i.e., $\delta = 0(0.25)4$. For both the SEWMA and SCUSUM charts, we consider different values of $\lambda$ and $k$. Moreover, The results are computed when sampling from a standard normal distribution. Here, under each simulations run, 50,000 replications are considered. It is observed that the out-of-control ARLs tend to decrease as the value of $\delta$ increases and vice versa. A similar trend is observed when a control chart is constructed with the FIR feature.

### 4.5.4 When the process parameters are unknown

If the underlying process parameters are not known in advance—phase-I monitoring, then it is customary to estimate them using a large historical dataset provided that it has been obtained from an in-control process. Suppose that, from an in-control process, $q$ subgroups each of size $m$ are available under the S scheme. In the perfect ranking case, $\mu_Y$ and $\text{Var}(\overline{Y}_{\text{VLRSS}})$ could be estimated by using their respective unbiased estimators, say $\overline{\overline{Y}}_{\text{VLRSS}}$ and $\hat{\text{V}}\text{ar}(\overline{Y}_{\text{VLRSS}})$, respectively, defined by

$$\overline{\overline{Y}}_{\text{VLRSS}} = \frac{1}{q}\sum_{j=1}^{q}\overline{Y}_{\text{VLRSS},j}, \tag{4.25}$$

$$\hat{\text{V}}\text{ar}(\overline{Y}_{\text{VLRSS}}) = \frac{1}{nm}\left(w(S^2_{Y(v:l)} + S^2_{Y(l-v+1:l)}) + \sum_{i=w+1}^{m-w} S^2_{Y(i:m)}\right), \tag{4.26}$$

where

$$\overline{Y}_{\text{VLRSS},j} = \frac{1}{m}\left(\sum_{i=1}^{w} Y_{i(v:l)j} + \sum_{i=w+1}^{2w} Y_{i(l-v+1:l)j} + \sum_{i=w+1}^{m-w} Y_{i+w(i:m)j}\right),$$

$$S^2_{Y(i:u)} = \frac{1}{q-1}\sum_{j=1}^{q}\left(Y_{i'(i:u)j} - \frac{1}{q}\sum_{j=1}^{q} Y_{i'(i:u)j}\right)^2, \tag{4.27}$$

where $i$ and $i'$ may or may not be equal. Under imperfect ranking, the concomitants of the study variable $Y$ corresponding to the order statistics of the auxiliary variable $X$, are used to estimate the aforementioned parameters. From the above expressions, it is possible to obtain the unbiased estimators given that the parenthesis are replaced by the square brackets, i.e., replace the order statistics by their corresponding concomitants. For example, replace $Y_{i(v:l)j}$ by $Y_{i[v:l]j}$ and $S^2_{Y(v:l)}$ by $S^2_{Y[v:l]}$, and similarly the others. For a precise estimation of the unknown parameters using the above formulae, large number of subgroups $q$ have to be used. Once the unknown parameters get estimated, it is possible to construct the aforementioned proposed control charts for monitoring the process mean.

## 4.6 Performance comparison

In this section, we compare the ARL and SDRL of the proposed charts with that of their respective counterparts, i.e., SEWMA-VLRSS chart is compared with SEWMA-SRS, SEWMA-RSS and SEWMA-MRSS (SEWMA charts based on SRS, RSS, MRSS), and the SCUSUM-VLRSS chart is compared with SCUSUM-SRS, SCUSUM-RSS and SCUSUM-MRSS (SCUSUM charts based on SRS, RSS, MRSS) in terms of ARL and SDRL. The same implies when these control charts are compared when there are errors in ranking. For the brevity of discussion, the ARL characteristics are compared using both perfect and imperfect rankings. Note that for the imperfect ranking case, samples are drawn from a standard bivariate normal distribution with different choices of $\rho$. The in-control ARLs are matched to 500 in all comparisons with $n = 5$ and $r = 1$.

### 4.6.1 SEWMA and SCUSUM charts with perfect rankings

The ARLs and SDRLs of the SEWMA-VLRSS and SCUSUM-VLRSS charts are compared with those of the SEWMA-SRS, SEWMA-RSS, SEWMA-MRSS (SEWMA based on SRS, RSS, MRSS) and SCUSUM-SRS, SCUSUM-RSS, SCUSUM-MRSS (SCUSUM based on SRS, RSS, MRSS) charts, respectively, with different choices of $\delta$, $\lambda$ and $k$ in Tables 4.11 and 4.12. It is observed that, as expected, the SEWMA-VLRSS (SCUSUM-VLRSS) chart performs uniformly better than the SEWMA (SCUSUM) charts based on SRS, RSS and MRSS schemes. The sensitivity of the SEWMA-VLRSS/SCUSUM-VLRSS chart increases as the value of pair $(l, v)$ increases and vice versa.

### 4.6.2 SEWMA and SCUSUM charts with imperfect rankings

On similar lines, the SEWMA and SCUSUM charts are compared with those based on SRS and imperfect RSS schemes. The run length profiles of the considered SEWMA and SCUSUM charts are reported in Tables 4.13, 4.14 and 4.15, 4.16, respectively. This time, for brevity, we consider $\lambda = 0.25, 0.50$ and $k = 0.50, 1.00$ for the SEWMA and SCUSUM charts, respectively. It is observed that, despite the presence of ranking errors, the SEWMA-VLRSS (SCUSUM-VLRSS) chart turns out to be more sensitive than its existing counterparts. As expected, the sensitivity of a control chart increases as the values of $\rho$ increases and vice versa.

### 4.6.3 SEWMA and SCUSUM charts with SRS

In the situation where the ranking costs are high, then it may not be possible to select a sample using RSS and MRSS schemes. However, it is possible to select a sample using VLRSS scheme with less number of identified units than those of the RSS and MRSS schemes. Hence, we compare SEWMA-VLRSS and SCUSUM-VLRSS charts with the SEWMA-SRS and SCUSUM-SRS charts, respectively, for those choices of $(l, v)$ for which the ranking cost is less than that using RSS and MRSS schemes, i.e., we consider those choices of VLRSS scheme with $n = 5$ for which the number of identified units are less than 25. Both perfect and imperfect rankings are used to construct SEWMA and SCUSUM charts. Under perfect rankings, the ARL and SDRL profiles of the SEWMA-SRS and SEWMA-VLRSS charts are given in Table 4.17 and those of the SCUSUM-SRS and SCUSUM-VLRSS charts are given in Table 4.20. From Tables 4.17 and 4.20, it is observed that the SEWMA- and SCUSUM-VLRSS charts are better than the SEWMA- and SCUSUM-SRS charts. Similarly the run length profiles of the proposed (VLRSS) and existing (SRS) SEWMA and SCUSUM charts with the imperfect ranking are reported in Tables 4.18, 4.19 and Tables 4.21-4.21, respectively. It turns out that the proposed charts outperform their counterparts based on SRS. As expected, with the increase in the value of $\rho$, the sensitivity of the SEWMA and SCUSUM charts increases and vice versa.

## 4.7 Illustrative example

In this section, an illustrative example is presented to explain the working of the proposed and existing SEWMA and SCUSUM charts based on MRSS and VLRSS schemes.

A hard-bake process in conjunction with the photolithography is used in a semiconductor manufacturing process. Our objective is to establish statistical control of the flow width of the resist for this process using the proposed control charts. For this purpose, forty-five samples, each of size five wafers (measured in microns), are drawn from an in-control process. The complete dataset is given in Montgomery (2007). As the samples are selected with SRS, in order to generate samples using MRSS and VLRSS, the whole dataset is combined to generate a population that comprises 225 observations. Then, thirty samples, each of size five, are generated using MRSS $(l = m, v = 3, w = 2)$ and VLRSS $(l = 6, v = 3, w = 2)$. We treat these thirty samples as phase-I samples. Here, under both MRSS and VLRSS, the samples are drawn using with replacement selection. These data are used to estimate the means and variances of the mean estimators with MRSS and VLRSS. For the SEWMA and SCUSUM charts, the in-control ARLs are set equal to 500 with $\lambda = 0.25$ and $k = 0.50$, respectively. The values of $(L, d_1)$ for the SEWMA-MRSS and SEWMA-VLRSS charts are set to (3.1570,3.31) and (3.1579,3.31), respectively. Similarly, the values of $(k, d_2)$ for the SCUSUM-MRSS and SCUSUM-VLRSS charts are set to to (5.2930,3.5) and (5.2935,3.5). Then, the control limits of both SEWMA and SCUSUM charts are estimated using thirty phase-I samples. Now we generate the data for the phase-II monitoring. For this purpose, from the same population; generate twenty samples, each of size five, under MRSS and VLRSS. Then, add $\delta = 0.02$ in the last twenty samples' observations. The plotting-statistics of the SEWMA and SCUSUM charts are computed for all fifty samples (obtained using MRSS and VLRSS), and are displayed in Figures 4.1–4.4.

From Figures 4.1–4.4 it can be seen that the process remains in the control state for the first thirty samples while in the last twenty samples the control charts are issuing out-of-control signals to indicate an upward shift in the underlying process mean. It is observed that the SEWMA-MRSS and SEWMA-VLRSS charts detect an upward shift in the process mean at the 45th and 42nd samples, respectively, while the SCUSUM-MRSS and SCUSUM-VLRSS charts detect the same shift at the 45th and 42nd samples, respectively. Note that the median and varied L ranked set samples are independently drawn; thus, their charts have different behavior. But, both schemes' samples (last twenty) are contaminated with the same value of $\delta$. This the reason why the Shewhart sub-charts with the SEWMA- and SCUSUM-VLRSS charts are issuing out-of-control signals while their counterparts do not.

## 4.8 Conclusion

In this chapter, we proposed new SEWMA and SCUSUM charts using VLRSS and IVLRSS schemes for monitoring the process mean. We computed the run length characteristics of the proposed charts using Monte Carlo simulations. The run length performances of the proposed charts were compared with those of their existing counterparts based on SRS, RSS and MRSS. It was found that the proposed charts have uniform improvement over their analogous charts. Thus we recommend using the proposed chart, when possible, for efficiently monitoring changes in the process mean.

Table 4.1: The values of $L$ with different choices of $(l, v)$ when the in-control ARL of the SEWMA-VLRSS chart is 500

| | $m=2$ | $m=4$ | | | | | |
|---|---|---|---|---|---|---|---|
| $\lambda$ | $(0,0)$ | $(2,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,3)$ | $(6,3)$ |
| 0.05 | 2.2582 | 2.8510 | 2.8523 | 2.8800 | 2.8490 | 2.8492 | 2.8485 |
| 0.10 | 2.4260 | 3.0270 | 3.0164 | 3.0463 | 3.0257 | 3.0225 | 3.0230 |
| 0.25 | 2.5789 | 3.1700 | 3.1666 | 3.1856 | 3.1650 | 3.1659 | 3.1648 |
| 0.50 | 2.6350 | 3.1755 | 3.1700 | 3.1889 | 3.1747 | 3.1695 | 3.1755 |
| 0.75 | 2.6470 | 3.1230 | 3.1186 | 3.1390 | 3.1190 | 3.1155 | 3.1161 |
| 1.00 | 2.6465 | 3.1050 | 3.1000 | 3.1158 | 3.0990 | 3.1000 | 3.1000 |

| | $m=3, w=1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $(2,1)$ | $(3,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,1)$ | $(5,2)$ | $(5,3)$ |
| 0.05 | 2.8600 | 2.8800 | 2.8519 | 2.8890 | 2.8510 | 2.8990 | 2.8585 | 2.8522 |
| 0.10 | 3.0307 | 3.0469 | 3.0285 | 3.0599 | 3.0285 | 3.0690 | 3.0308 | 3.0290 |
| 0.25 | 3.1730 | 3.1860 | 3.1710 | 3.2004 | 3.1710 | 3.2100 | 3.1683 | 3.1660 |
| 0.50 | 3.1763 | 3.1921 | 3.1758 | 3.2056 | 3.1759 | 3.2105 | 3.1800 | 3.1759 |
| 0.75 | 3.1240 | 3.1377 | 3.1200 | 3.1499 | 3.1207 | 3.1545 | 3.1230 | 3.1203 |
| 1.00 | 3.1070 | 3.1157 | 3.1025 | 3.1260 | 3.1028 | 3.1313 | 3.1070 | 3.1013 |

| | $m=5$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $w=1$ | | | | $w=2$ | | | |
| $\lambda$ | $(2,1)$ | $(3,2)$ | $(4,2)$ | $(5,1)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| 0.05 | 2.8494 | 2.8490 | 2.8480 | 2.8740 | 2.8450 | 2.8460 | 2.8444 | 2.8448 |
| 0.10 | 3.0270 | 3.0190 | 3.0250 | 3.0450 | 3.0170 | 3.0179 | 3.0164 | 3.0164 |
| 0.25 | 3.1695 | 3.1654 | 3.1645 | 3.1850 | 3.1570 | 3.1579 | 3.1584 | 3.1590 |
| 0.50 | 3.1747 | 3.1690 | 3.1755 | 3.1920 | 3.1660 | 3.1669 | 3.1670 | 3.1673 |
| 0.75 | 3.1195 | 3.1150 | 3.1150 | 3.1370 | 3.1150 | 3.1150 | 3.1134 | 3.1155 |
| 1.00 | 3.1000 | 3.1000 | 3.1000 | 3.1147 | 3.0980 | 3.0968 | 3.0945 | 3.0970 |

Table 4.2: The values of $h$ with different choices of $(l, v)$ when the in-control ARL of the SCUSUM-VLRSS chart is 500

| | $m = 2$ | $m = 4, w = 1$ | | | | | |
|---|---|---|---|---|---|---|---|
| $k$ | $(0,0)$ | $(2,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,3)$ | $(6,3)$ |
| 0.25 | 9.1100 | 9.0950 | 9.0740 | 9.1100 | 9.0970 | 9.0760 | 9.0875 |
| 0.50 | 5.3280 | 5.3166 | 5.3105 | 5.3360 | 5.3050 | 5.3050 | 5.3077 |
| 0.75 | 3.6980 | 3.6785 | 3.6765 | 3.6950 | 3.6753 | 3.6758 | 3.6760 |
| 1.00 | 2.7450 | 2.7282 | 2.7255 | 2.7440 | 2.7245 | 2.7240 | 2.7252 |

| | $m = 3, w = 1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $k$ | $(2,1)$ | $(3,1)$ | $(3,2)$ | $(4,1)$ | $(4,2)$ | $(5,1)$ | $(5,2)$ | $(5,3)$ |
| 0.25 | 9.0953 | 9.1109 | 9.0700 | 9.1230 | 9.0950 | 9.1560 | 9.0950 | 9.0747 |
| 0.50 | 5.3164 | 5.3340 | 5.3133 | 5.3430 | 5.3085 | 5.3780 | 5.3143 | 5.3060 |
| 0.75 | 3.6791 | 3.6980 | 3.6767 | 3.7122 | 3.6790 | 3.7181 | 3.6800 | 3.6767 |
| 1.00 | 2.7320 | 2.7450 | 2.7280 | 2.7547 | 2.7284 | 2.7670 | 2.7290 | 2.7287 |

| | $m = 5$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $w = 1$ | | | | $w = 2$ | | | |
| $k$ | $(2,1)$ | $(3,2)$ | $(4,2)$ | $(5,1)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| 0.25 | 9.0959 | 9.0739 | 9.0755 | 9.1110 | 9.0490 | 9.0499 | 9.0508 | 9.0510 |
| 0.50 | 5.3109 | 5.3080 | 5.3080 | 5.3270 | 5.2930 | 5.2935 | 5.2938 | 5.2938 |
| 0.75 | 3.6760 | 3.6755 | 3.6746 | 3.6960 | 3.6670 | 3.6682 | 3.6666 | 3.6682 |
| 1.00 | 2.7250 | 2.7245 | 2.7260 | 2.7460 | 2.7170 | 2.7193 | 2.7185 | 2.7186 |

Table 4.3: The run length profiles of the SEWMA-VLRSS chart for $w = 1$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 2.8494 | 2.8490 | 2.8480 | 2.8740 | 3.0270 | 3.0190 | 3.0250 | 3.0450 |
| 0.00 | ARL | 499.32 | 500.62 | 499.16 | 500.35 | 500.76 | 500.68 | 500.31 | 500.85 |
| | SDRL | 486.43 | 492.10 | 491.01 | 492.00 | 496.14 | 495.93 | 494.79 | 496.51 |
| 0.25 | ARL | 52.99 | 43.71 | 40.14 | 44.57 | 66.29 | 52.12 | 47.97 | 53.65 |
| | SDRL | 37.77 | 29.17 | 25.84 | 30.00 | 55.93 | 42.36 | 38.18 | 43.56 |
| 0.50 | ARL | 18.20 | 15.53 | 15.52 | 14.73 | 18.09 | 14.95 | 13.76 | 15.10 |
| | SDRL | 8.69 | 6.87 | 6.28 | 7.00 | 10.73 | 8.28 | 7.33 | 8.30 |
| 0.75 | ARL | 10.64 | 9.14 | 8.58 | 9.25 | 9.59 | 8.07 | 7.54 | 8.20 |
| | SDRL | 4.20 | 3.50 | 3.27 | 3.56 | 4.40 | 3.53 | 3.20 | 3.60 |
| 1.00 | ARL | 7.34 | 6.30 | 5.86 | 6.32 | 6.39 | 5.44 | 5.08 | 5.48 |
| | SDRL | 2.82 | 2.48 | 2.36 | 2.51 | 2.62 | 2.23 | 2.08 | 2.23 |
| 1.25 | ARL | 5.42 | 4.56 | 4.23 | 4.60 | 4.72 | 3.97 | 3.70 | 4.02 |
| | SDRL | 2.26 | 2.06 | 1.97 | 2.07 | 1.93 | 1.70 | 1.62 | 1.73 |
| 1.50 | ARL | 4.11 | 3.38 | 3.07 | 3.39 | 3.61 | 3.01 | 2.77 | 3.03 |
| | SDRL | 1.94 | 1.75 | 1.65 | 1.76 | 1.60 | 1.42 | 1.35 | 1.43 |
| 1.75 | ARL | 3.12 | 2.50 | 2.24 | 2.49 | 2.82 | 2.30 | 2.10 | 2.32 |
| | SDRL | 1.67 | 1.43 | 1.31 | 1.44 | 1.37 | 1.19 | 1.10 | 1.20 |
| 2.00 | ARL | 2.39 | 1.88 | 1.69 | 1.88 | 2.24 | 1.80 | 1.63 | 1.80 |
| | SDRL | 1.39 | 1.10 | 0.97 | 1.11 | 1.16 | 0.95 | 0.85 | 0.96 |
| 2.25 | ARL | 1.87 | 1.48 | 1.35 | 1.48 | 1.79 | 1.46 | 1.34 | 1.46 |
| | SDRL | 1.10 | 0.79 | 0.67 | 0.80 | 0.95 | 0.72 | 0.62 | 0.72 |
| 2.50 | ARL | 1.51 | 1.25 | 1.16 | 1.24 | 1.48 | 1.24 | 1.16 | 1.24 |
| | SDRL | 0.83 | 0.54 | 0.44 | 0.54 | 0.74 | 0.52 | 0.42 | 0.52 |
| 2.75 | ARL | 1.28 | 1.11 | 1.07 | 1.12 | 1.27 | 1.11 | 1.07 | 1.11 |
| | SDRL | 0.59 | 0.36 | 0.27 | 0.36 | 0.56 | 0.35 | 0.27 | 0.35 |
| 3.00 | ARL | 1.14 | 1.05 | 1.03 | 1.05 | 1.14 | 1.05 | 1.03 | 1.05 |
| | SDRL | 0.40 | 0.22 | 0.17 | 0.22 | 0.39 | 0.22 | 0.16 | 0.22 |
| 3.25 | ARL | 1.07 | 1.02 | 1.01 | 1.02 | 1.07 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.28 | 0.14 | 0.10 | 0.14 | 0.27 | 0.14 | 0.09 | 0.13 |
| 3.50 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.03 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.18 | 0.08 | 0.05 | 0.08 | 0.18 | 0.08 | 0.05 | 0.08 |
| 3.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.04 | 0.02 | 0.04 | 0.11 | 0.04 | 0.02 | 0.04 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.02 | 0.01 | 0.02 | 0.06 | 0.02 | 0.01 | 0.02 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 3.1695 | 3.1654 | 3.1645 | 3.1850 | 3.1747 | 3.1690 | 3.1755 | 3.1920 |
| 0.00 | ARL | 500.97 | 501.83 | 502.89 | 502.46 | 500.45 | 500.41 | 500.25 | 500.08 |
| | SDRL | 496.30 | 497.12 | 501.76 | 502.52 | 501.39 | 500.87 | 503.79 | 502.07 |
| 0.25 | ARL | 112.27 | 89.38 | 80.32 | 92.18 | 175.36 | 145.23 | 136.20 | 150.44 |
| | SDRL | 108.33 | 85.21 | 75.99 | 87.60 | 172.28 | 143.50 | 134.42 | 148.76 |
| 0.50 | ARL | 24.45 | 18.90 | 16.81 | 19.35 | 43.67 | 32.37 | 28.74 | 33.73 |
| | SDRL | 20.19 | 14.74 | 12.75 | 15.20 | 41.58 | 30.11 | 26.79 | 31.40 |
| 0.75 | ARL | 10.12 | 8.05 | 7.32 | 8.16 | 15.30 | 11.19 | 9.88 | 11.43 |
| | SDRL | 6.57 | 4.78 | 4.19 | 4.96 | 13.18 | 9.17 | 7.91 | 9.39 |
| 1.00 | ARL | 5.92 | 4.89 | 4.50 | 4.93 | 7.32 | 5.55 | 4.98 | 5.62 |
| | SDRL | 3.14 | 2.44 | 2.17 | 2.45 | 5.46 | 3.81 | 3.29 | 3.84 |
| 1.25 | ARL | 4.12 | 3.45 | 3.18 | 3.47 | 4.39 | 3.49 | 3.17 | 3.52 |
| | SDRL | 1.94 | 1.56 | 1.44 | 1.58 | 2.76 | 2.01 | 1.77 | 2.03 |
| 1.50 | ARL | 3.10 | 2.61 | 2.42 | 2.62 | 3.07 | 2.49 | 2.30 | 2.51 |
| | SDRL | 1.40 | 1.17 | 1.10 | 1.19 | 1.67 | 1.26 | 1.13 | 1.27 |
| 1.75 | ARL | 2.47 | 2.06 | 1.90 | 2.07 | 2.35 | 1.94 | 1.80 | 1.96 |
| | SDRL | 1.11 | 0.95 | 0.88 | 0.96 | 1.16 | 0.90 | 0.81 | 0.92 |
| 2.00 | ARL | 2.00 | 1.67 | 1.53 | 1.67 | 1.89 | 1.60 | 1.49 | 1.60 |
| | SDRL | 0.93 | 0.77 | 0.70 | 0.78 | 0.87 | 0.69 | 0.62 | 0.69 |
| 2.25 | ARL | 1.66 | 1.40 | 1.30 | 1.39 | 1.59 | 1.36 | 1.28 | 1.36 |
| | SDRL | 0.77 | 0.60 | 0.52 | 0.60 | 0.69 | 0.54 | 0.48 | 0.55 |
| 2.50 | ARL | 1.41 | 1.21 | 1.15 | 1.22 | 1.38 | 1.20 | 1.14 | 1.20 |
| | SDRL | 0.61 | 0.45 | 0.38 | 0.45 | 0.56 | 0.42 | 0.36 | 0.42 |
| 2.75 | ARL | 1.24 | 1.11 | 1.07 | 1.11 | 1.23 | 1.10 | 1.07 | 1.10 |
| | SDRL | 0.47 | 0.32 | 0.25 | 0.32 | 0.44 | 0.31 | 0.25 | 0.31 |
| 3.00 | ARL | 1.13 | 1.05 | 1.03 | 1.05 | 1.13 | 1.05 | 1.03 | 1.05 |
| | SDRL | 0.36 | 0.21 | 0.16 | 0.22 | 0.34 | 0.21 | 0.16 | 0.21 |
| 3.25 | ARL | 1.07 | 1.02 | 1.01 | 1.02 | 1.07 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.25 | 0.14 | 0.09 | 0.13 | 0.25 | 0.13 | 0.09 | 0.13 |
| 3.50 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.03 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.17 | 0.08 | 0.05 | 0.07 | 0.17 | 0.08 | 0.05 | 0.08 |
| 3.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.04 | 0.02 | 0.04 | 0.11 | 0.05 | 0.02 | 0.04 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.02 | 0.01 | 0.02 | 0.07 | 0.03 | 0.01 | 0.02 |

Table 4.4: The run length profiles of the SEWMA-VLRSS chart for $w = 2$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| $\delta$ | $L$ | 2.8450 | 2.8460 | 2.8444 | 2.8448 | 3.0170 | 3.0179 | 3.0164 | 3.0164 |
| 0.00 | ARL | 500.25 | 499.43 | 500.87 | 499.53 | 502.58 | 502.48 | 500.45 | 502.76 |
| | SDRL | 494.42 | 490.67 | 491.50 | 490.78 | 494.28 | 500.68 | 500.09 | 498.02 |
| 0.25 | ARL | 36.42 | 32.97 | 30.09 | 28.20 | 42.10 | 37.61 | 33.53 | 30.84 |
| | SDRL | 22.74 | 19.85 | 17.63 | 15.99 | 32.78 | 28.23 | 24.51 | 22.16 |
| 0.50 | ARL | 13.33 | 12.29 | 11.37 | 10.77 | 12.37 | 11.34 | 10.34 | 9.73 |
| | SDRL | 5.62 | 5.08 | 4.58 | 4.28 | 6.33 | 5.57 | 4.93 | 4.53 |
| 0.75 | ARL | 7.89 | 7.29 | 6.72 | 6.35 | 6.89 | 6.34 | 5.84 | 5.53 |
| | SDRL | 3.01 | 2.80 | 2.62 | 2.52 | 2.89 | 2.61 | 2.40 | 2.26 |
| 1.00 | ARL | 5.35 | 4.93 | 4.48 | 4.19 | 4.64 | 4.27 | 3.90 | 3.67 |
| | SDRL | 2.24 | 2.13 | 2.03 | 1.96 | 1.90 | 1.79 | 1.68 | 1.61 |
| 1.25 | ARL | 3.81 | 3.41 | 3.04 | 2.81 | 3.34 | 3.04 | 2.77 | 2.56 |
| | SDRL | 1.86 | 1.76 | 1.64 | 1.56 | 1.52 | 1.43 | 1.34 | 1.28 |
| 1.50 | ARL | 2.69 | 2.39 | 2.10 | 1.92 | 2.47 | 2.21 | 1.99 | 1.83 |
| | SDRL | 1.52 | 1.38 | 1.23 | 1.13 | 1.25 | 1.15 | 1.05 | 0.97 |
| 1.75 | ARL | 1.95 | 1.72 | 1.53 | 1.41 | 1.87 | 1.66 | 1.50 | 1.40 |
| | SDRL | 1.15 | 1.00 | 0.84 | 0.73 | 0.99 | 0.87 | 0.76 | 0.67 |
| 2.00 | ARL | 1.49 | 1.34 | 1.23 | 1.16 | 1.46 | 1.33 | 1.22 | 1.16 |
| | SDRL | 0.80 | 0.65 | 0.52 | 0.44 | 0.72 | 0.61 | 0.49 | 0.42 |
| 2.25 | ARL | 1.23 | 1.14 | 1.08 | 1.06 | 1.22 | 1.14 | 1.08 | 1.05 |
| | SDRL | 0.52 | 0.40 | 0.30 | 0.24 | 0.50 | 0.39 | 0.30 | 0.24 |
| 2.50 | ARL | 1.10 | 1.05 | 1.03 | 1.01 | 1.10 | 1.05 | 1.02 | 1.01 |
| | SDRL | 0.33 | 0.24 | 0.16 | 0.12 | 0.32 | 0.24 | 0.16 | 0.12 |
| 2.75 | ARL | 1.03 | 1.02 | 1.01 | 1.00 | 1.04 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.19 | 0.13 | 0.08 | 0.05 | 0.20 | 0.13 | 0.08 | 0.06 |
| 3.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.10 | 0.07 | 0.04 | 0.02 | 0.11 | 0.07 | 0.03 | 0.03 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.05 | 0.03 | 0.01 | 0.00 | 0.06 | 0.03 | 0.01 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.00 | 0.02 | 0.01 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| $\delta$ | $L$ | 3.1570 | 3.1579 | 3.1584 | 3.1590 | 3.1660 | 3.1669 | 3.1670 | 3.1673 |
| 0.00 | ARL | 500.20 | 500.80 | 502.48 | 500.74 | 501.10 | 500.45 | 501.94 | 500.08 |
| | SDRL | 500.00 | 497.87 | 501.34 | 498.89 | 508.07 | 504.69 | 500.30 | 502.16 |
| 0.25 | ARL | 69.36 | 61.38 | 53.18 | 48.84 | 118.70 | 106.57 | 93.83 | 85.87 |
| | SDRL | 64.36 | 57.35 | 48.42 | 44.58 | 117.02 | 103.89 | 91.67 | 83.88 |
| 0.50 | ARL | 14.56 | 12.78 | 11.35 | 10.30 | 23.68 | 20.41 | 17.45 | 15.45 |
| | SDRL | 10.63 | 9.03 | 7.69 | 6.74 | 21.49 | 18.15 | 15.24 | 13.37 |
| 0.75 | ARL | 6.52 | 5.88 | 5.30 | 4.94 | 8.30 | 7.19 | 6.24 | 5.67 |
| | SDRL | 3.59 | 3.12 | 2.74 | 2.47 | 6.38 | 5.31 | 4.39 | 3.89 |
| 1.00 | ARL | 4.04 | 3.68 | 3.37 | 3.16 | 4.30 | 3.79 | 3.40 | 3.13 |
| | SDRL | 1.89 | 1.69 | 1.53 | 1.43 | 2.70 | 2.26 | 1.95 | 1.73 |
| 1.25 | ARL | 2.88 | 2.63 | 2.40 | 2.25 | 2.82 | 2.52 | 2.28 | 2.14 |
| | SDRL | 1.29 | 1.19 | 1.09 | 1.03 | 1.49 | 1.29 | 1.12 | 1.02 |
| 1.50 | ARL | 2.18 | 1.99 | 1.80 | 1.69 | 2.06 | 1.88 | 1.72 | 1.62 |
| | SDRL | 1.00 | 0.92 | 0.84 | 0.79 | 0.98 | 0.87 | 0.76 | 0.70 |
| 1.75 | ARL | 1.71 | 1.55 | 1.43 | 1.34 | 1.64 | 1.50 | 1.39 | 1.31 |
| | SDRL | 0.79 | 0.71 | 0.63 | 0.56 | 0.71 | 0.63 | 0.56 | 0.51 |
| 2.00 | ARL | 1.39 | 1.29 | 1.20 | 1.15 | 1.37 | 1.27 | 1.19 | 1.14 |
| | SDRL | 0.60 | 0.51 | 0.43 | 0.37 | 0.54 | 0.47 | 0.41 | 0.35 |
| 2.25 | ARL | 1.20 | 1.13 | 1.08 | 1.05 | 1.19 | 1.13 | 1.08 | 1.05 |
| | SDRL | 0.43 | 0.35 | 0.28 | 0.23 | 0.41 | 0.34 | 0.27 | 0.22 |
| 2.50 | ARL | 1.09 | 1.05 | 1.03 | 1.01 | 1.09 | 1.05 | 1.03 | 1.01 |
| | SDRL | 0.29 | 0.22 | 0.16 | 0.12 | 0.29 | 0.22 | 0.16 | 0.12 |
| 2.75 | ARL | 1.03 | 1.02 | 1.01 | 1.00 | 1.03 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.18 | 0.12 | 0.08 | 0.05 | 0.18 | 0.13 | 0.08 | 0.05 |
| 3.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.06 | 0.04 | 0.02 | 0.10 | 0.07 | 0.04 | 0.02 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.03 | 0.01 | 0.01 | 0.05 | 0.03 | 0.01 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.01 | 0.01 | 0.03 | 0.01 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4.5: The run length profiles of the SCUSUM-VLRSS chart for $w = 1$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $h$ | 9.0959 | 9.0739 | 9.0755 | 9.1110 | 5.3109 | 5.3080 | 5.3080 | 5.3270 |
| 0.00 | ARL | 500.18 | 501.73 | 500.67 | 500.73 | 500.20 | 500.71 | 500.14 | 501.50 |
| | SDRL | 492.28 | 484.81 | 488.69 | 488.51 | 498.31 | 503.18 | 499.89 | 492.83 |
| 0.25 | ARL | 52.45 | 43.30 | 39.93 | 43.60 | 76.63 | 60.27 | 54.50 | 60.85 |
| | SDRL | 36.17 | 28.12 | 24.94 | 28.10 | 69.39 | 52.27 | 46.70 | 53.31 |
| 0.50 | ARL | 18.53 | 15.86 | 14.86 | 15.93 | 18.42 | 14.86 | 13.58 | 14.89 |
| | SDRL | 8.21 | 6.62 | 6.01 | 6.63 | 12.04 | 8.94 | 7.88 | 8.92 |
| 0.75 | ARL | 10.99 | 9.55 | 8.96 | 9.55 | 9.18 | 7.68 | 7.13 | 7.74 |
| | SDRL | 4.00 | 3.36 | 3.13 | 3.36 | 4.48 | 3.49 | 3.15 | 3.50 |
| 1.00 | ARL | 7.72 | 6.67 | 6.22 | 6.69 | 6.02 | 5.11 | 4.77 | 5.13 |
| | SDRL | 2.67 | 2.35 | 2.23 | 2.35 | 2.50 | 2.03 | 1.85 | 2.03 |
| 1.25 | ARL | 5.79 | 4.94 | 4.56 | 4.94 | 4.44 | 3.79 | 3.54 | 3.80 |
| | SDRL | 2.13 | 1.97 | 1.89 | 1.97 | 1.72 | 1.47 | 1.38 | 1.47 |
| 1.50 | ARL | 4.47 | 3.71 | 3.40 | 3.73 | 3.46 | 2.94 | 2.74 | 2.95 |
| | SDRL | 1.88 | 1.74 | 1.66 | 1.73 | 1.35 | 1.20 | 1.14 | 1.21 |
| 1.75 | ARL | 3.48 | 2.80 | 2.51 | 2.79 | 2.78 | 2.35 | 2.17 | 2.35 |
| | SDRL | 1.68 | 1.50 | 1.39 | 1.50 | 1.16 | 1.03 | 0.99 | 1.04 |
| 2.00 | ARL | 2.69 | 2.11 | 1.89 | 2.10 | 2.27 | 1.88 | 1.73 | 1.89 |
| | SDRL | 1.46 | 1.21 | 1.09 | 1.21 | 1.02 | 0.89 | 0.83 | 0.89 |
| 2.25 | ARL | 2.10 | 1.64 | 1.48 | 1.64 | 1.87 | 1.54 | 1.42 | 1.54 |
| | SDRL | 1.21 | 0.92 | 0.78 | 0.92 | 0.89 | 0.73 | 0.65 | 0.73 |
| 2.50 | ARL | 1.67 | 1.34 | 1.23 | 1.34 | 1.57 | 1.30 | 1.22 | 1.30 |
| | SDRL | 0.94 | 0.65 | 0.53 | 0.65 | 0.75 | 0.55 | 0.47 | 0.56 |
| 2.75 | ARL | 1.38 | 1.16 | 1.10 | 1.16 | 1.34 | 1.16 | 1.10 | 1.15 |
| | SDRL | 0.69 | 0.43 | 0.34 | 0.42 | 0.59 | 0.40 | 0.31 | 0.39 |
| 3.00 | ARL | 1.21 | 1.07 | 1.04 | 1.07 | 1.19 | 1.07 | 1.04 | 1.07 |
| | SDRL | 0.49 | 0.28 | 0.21 | 0.28 | 0.44 | 0.27 | 0.20 | 0.26 |
| 3.25 | ARL | 1.10 | 1.03 | 1.01 | 1.03 | 1.10 | 1.03 | 1.01 | 1.03 |
| | SDRL | 0.33 | 0.18 | 0.12 | 0.17 | 0.32 | 0.17 | 0.12 | 0.17 |
| 3.50 | ARL | 1.05 | 1.01 | 1.00 | 1.01 | 1.05 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.22 | 0.10 | 0.07 | 0.10 | 0.22 | 0.10 | 0.06 | 0.10 |
| 3.75 | ARL | 1.02 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.15 | 0.06 | 0.03 | 0.06 | 0.14 | 0.05 | 0.03 | 0.06 |
| 4.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.09 | 0.03 | 0.01 | 0.03 | 0.09 | 0.03 | 0.01 | 0.03 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $h$ | 3.6760 | 3.6755 | 3.6746 | 3.6960 | 2.7250 | 2.7245 | 2.7260 | 2.7460 |
| 0.00 | ARL | 502.50 | 502.97 | 500.93 | 501.24 | 499.56 | 500.75 | 502.78 | 500.15 |
| | SDRL | 499.33 | 500.89 | 501.22 | 500.01 | 498.18 | 497.22 | 502.07 | 498.37 |
| 0.25 | ARL | 113.71 | 90.64 | 80.81 | 91.33 | 149.44 | 121.97 | 111.38 | 124.39 |
| | SDRL | 109.91 | 87.06 | 75.61 | 87.58 | 147.02 | 118.80 | 109.08 | 123.01 |
| 0.50 | ARL | 23.70 | 18.04 | 16.04 | 18.18 | 32.93 | 24.29 | 21.36 | 24.45 |
| | SDRL | 19.66 | 13.98 | 12.11 | 14.05 | 30.11 | 21.43 | 18.65 | 21.59 |
| 0.75 | ARL | 9.64 | 7.66 | 6.98 | 7.73 | 11.45 | 8.71 | 7.70 | 8.78 |
| | SDRL | 6.18 | 4.45 | 3.94 | 4.44 | 8.80 | 6.24 | 5.29 | 6.25 |
| 1.00 | ARL | 5.67 | 4.69 | 4.33 | 4.71 | 5.95 | 4.75 | 4.31 | 4.78 |
| | SDRL | 2.89 | 2.23 | 1.99 | 2.22 | 3.74 | 2.70 | 2.36 | 2.73 |
| 1.25 | ARL | 3.97 | 3.35 | 3.10 | 3.35 | 3.91 | 3.22 | 2.96 | 3.22 |
| | SDRL | 1.78 | 1.42 | 1.30 | 1.43 | 2.04 | 1.53 | 1.37 | 1.53 |
| 1.50 | ARL | 3.04 | 2.59 | 2.40 | 2.59 | 2.89 | 2.44 | 2.27 | 2.46 |
| | SDRL | 1.26 | 1.07 | 0.99 | 1.06 | 1.31 | 1.04 | 0.95 | 1.05 |
| 1.75 | ARL | 2.45 | 2.07 | 1.93 | 2.08 | 2.31 | 1.97 | 1.84 | 1.98 |
| | SDRL | 1.01 | 0.86 | 0.80 | 0.87 | 0.96 | 0.79 | 0.73 | 0.79 |
| 2.00 | ARL | 2.02 | 1.72 | 1.59 | 1.71 | 1.92 | 1.66 | 1.55 | 1.66 |
| | SDRL | 0.84 | 0.71 | 0.66 | 0.72 | 0.76 | 0.65 | 0.60 | 0.65 |
| 2.25 | ARL | 1.70 | 1.45 | 1.36 | 1.45 | 1.65 | 1.43 | 1.34 | 1.43 |
| | SDRL | 0.71 | 0.58 | 0.53 | 0.59 | 0.64 | 0.54 | 0.50 | 0.55 |
| 2.50 | ARL | 1.47 | 1.27 | 1.19 | 1.27 | 1.45 | 1.26 | 1.19 | 1.26 |
| | SDRL | 0.59 | 0.47 | 0.41 | 0.47 | 0.55 | 0.45 | 0.40 | 0.45 |
| 2.75 | ARL | 1.29 | 1.14 | 1.10 | 1.15 | 1.29 | 1.15 | 1.09 | 1.14 |
| | SDRL | 0.49 | 0.36 | 0.30 | 0.36 | 0.47 | 0.35 | 0.29 | 0.35 |
| 3.00 | ARL | 1.18 | 1.07 | 1.04 | 1.07 | 1.17 | 1.07 | 1.04 | 1.07 |
| | SDRL | 0.39 | 0.25 | 0.19 | 0.25 | 0.38 | 0.25 | 0.19 | 0.25 |
| 3.25 | ARL | 1.10 | 1.03 | 1.01 | 1.03 | 1.09 | 1.03 | 1.01 | 1.03 |
| | SDRL | 0.30 | 0.17 | 0.12 | 0.17 | 0.29 | 0.17 | 0.12 | 0.17 |
| 3.50 | ARL | 1.04 | 1.01 | 1.00 | 1.01 | 1.05 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.21 | 0.10 | 0.07 | 0.10 | 0.21 | 0.10 | 0.06 | 0.10 |
| 3.75 | ARL | 1.02 | 1.00 | 1.00 | 1.00 | 1.02 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.14 | 0.06 | 0.03 | 0.05 | 0.14 | 0.06 | 0.03 | 0.06 |
| 4.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.09 | 0.03 | 0.01 | 0.03 | 0.08 | 0.03 | 0.02 | 0.03 |

Table 4.6: The run length profiles of the SCUSUM-VLRSS chart for $w = 2$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| $\delta$ | $h$ | 9.0490 | 9.0499 | 9.0508 | 9.0510 | 5.2930 | 5.2935 | 5.2938 | 5.2938 |
| 0.00 | ARL | 500.80 | 500.62 | 500.72 | 500.26 | 502.65 | 500.08 | 501.11 | 499.16 |
| | SDRL | 486.65 | 490.59 | 492.92 | 487.82 | 497.32 | 493.26 | 494.90 | 491.04 |
| 0.25 | ARL | 36.00 | 32.87 | 30.02 | 28.32 | 47.16 | 41.55 | 36.91 | 33.68 |
| | SDRL | 21.62 | 18.97 | 16.75 | 15.31 | 39.74 | 34.25 | 29.62 | 26.48 |
| 0.50 | ARL | 13.64 | 12.66 | 11.76 | 11.17 | 12.17 | 11.00 | 9.96 | 9.32 |
| | SDRL | 5.36 | 4.82 | 4.36 | 4.09 | 6.73 | 5.83 | 5.06 | 4.60 |
| 0.75 | ARL | 8.25 | 7.65 | 7.11 | 6.73 | 6.48 | 5.95 | 5.50 | 5.17 |
| | SDRL | 2.88 | 2.66 | 2.49 | 2.38 | 2.76 | 2.44 | 2.22 | 2.06 |
| 1.00 | ARL | 5.71 | 5.27 | 4.85 | 4.54 | 4.37 | 4.05 | 3.73 | 3.52 |
| | SDRL | 2.11 | 2.02 | 1.94 | 1.88 | 1.71 | 1.56 | 1.44 | 1.37 |
| 1.25 | ARL | 4.15 | 3.76 | 3.38 | 3.12 | 3.23 | 2.97 | 2.72 | 2.55 |
| | SDRL | 1.81 | 1.73 | 1.65 | 1.58 | 1.28 | 1.21 | 1.14 | 1.09 |
| 1.50 | ARL | 3.02 | 2.69 | 2.37 | 2.16 | 2.48 | 2.26 | 2.05 | 1.91 |
| | SDRL | 1.56 | 1.45 | 1.33 | 1.24 | 1.08 | 1.01 | 0.95 | 0.90 |
| 1.75 | ARL | 2.18 | 1.92 | 1.69 | 1.55 | 1.94 | 1.75 | 1.58 | 1.48 |
| | SDRL | 1.25 | 1.11 | 0.96 | 0.85 | 0.91 | 0.84 | 0.75 | 0.69 |
| 2.00 | ARL | 1.65 | 1.46 | 1.31 | 1.23 | 1.54 | 1.40 | 1.28 | 1.21 |
| | SDRL | 0.92 | 0.77 | 0.62 | 0.51 | 0.73 | 0.64 | 0.53 | 0.46 |
| 2.25 | ARL | 1.31 | 1.20 | 1.12 | 1.08 | 1.28 | 1.19 | 1.12 | 1.08 |
| | SDRL | 0.62 | 0.49 | 0.36 | 0.30 | 0.54 | 0.43 | 0.34 | 0.28 |
| 2.50 | ARL | 1.14 | 1.08 | 1.04 | 1.02 | 1.13 | 1.08 | 1.04 | 1.02 |
| | SDRL | 0.39 | 0.29 | 0.20 | 0.16 | 0.36 | 0.28 | 0.20 | 0.15 |
| 2.75 | ARL | 1.05 | 1.03 | 1.01 | 1.01 | 1.05 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.24 | 0.16 | 0.11 | 0.07 | 0.23 | 0.16 | 0.11 | 0.08 |
| 3.00 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.13 | 0.09 | 0.04 | 0.04 | 0.13 | 0.09 | 0.05 | 0.04 |
| 3.25 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.04 | 0.02 | 0.01 | 0.07 | 0.04 | 0.02 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.02 | 0.01 | 0.00 | 0.04 | 0.02 | 0.01 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ | $(5, 3)$ | $(6, 3)$ | $(7, 4)$ | $(8, 4)$ |
| $\delta$ | $h$ | 3.6670 | 3.6682 | 3.6666 | 3.6682 | 2.7170 | 2.7193 | 2.7185 | 2.7186 |
| 0.00 | ARL | 502.85 | 502.68 | 502.63 | 500.05 | 500.36 | 499.33 | 501.55 | 500.13 |
| | SDRL | 499.40 | 497.36 | 501.27 | 495.78 | 497.91 | 496.82 | 500.44 | 498.09 |
| 0.25 | ARL | 70.37 | 61.78 | 53.89 | 48.85 | 96.77 | 86.37 | 75.50 | 68.85 |
| | SDRL | 66.21 | 57.04 | 49.40 | 44.55 | 94.02 | 84.06 | 72.87 | 66.29 |
| 0.50 | ARL | 13.86 | 12.16 | 10.70 | 9.81 | 17.85 | 15.30 | 13.16 | 11.72 |
| | SDRL | 10.04 | 8.45 | 7.10 | 6.29 | 15.22 | 12.52 | 10.48 | 9.08 |
| 0.75 | ARL | 6.23 | 5.60 | 5.11 | 4.73 | 6.69 | 5.91 | 5.26 | 4.81 |
| | SDRL | 3.32 | 2.85 | 2.51 | 2.25 | 4.378 | 3.68 | 3.13 | 2.76 |
| 1.00 | ARL | 3.92 | 3.59 | 3.28 | 3.09 | 3.84 | 3.48 | 3.16 | 2.96 |
| | SDRL | 1.75 | 1.55 | 1.38 | 1.30 | 1.99 | 1.71 | 1.49 | 1.36 |
| 1.25 | ARL | 2.83 | 2.61 | 2.40 | 2.25 | 2.68 | 2.46 | 2.27 | 2.13 |
| | SDRL | 1.17 | 1.07 | 0.99 | 0.93 | 1.18 | 1.05 | 0.95 | 0.87 |
| 1.50 | ARL | 2.19 | 2.01 | 1.84 | 1.74 | 2.08 | 1.92 | 1.77 | 1.67 |
| | SDRL | 0.90 | 0.84 | 0.77 | 0.73 | 0.84 | 0.76 | 0.69 | 0.65 |
| 1.75 | ARL | 1.76 | 1.61 | 1.48 | 1.40 | 1.69 | 1.57 | 1.46 | 1.38 |
| | SDRL | 0.73 | 0.67 | 0.60 | 0.56 | 0.66 | 0.61 | 0.56 | 0.52 |
| 2.00 | ARL | 1.45 | 1.34 | 1.25 | 1.19 | 1.44 | 1.33 | 1.24 | 1.19 |
| | SDRL | 0.58 | 0.52 | 0.46 | 0.40 | 0.55 | 0.49 | 0.44 | 0.40 |
| 2.25 | ARL | 1.25 | 1.17 | 1.11 | 1.07 | 1.24 | 1.17 | 1.11 | 1.07 |
| | SDRL | 0.45 | 0.39 | 0.32 | 0.26 | 0.44 | 0.38 | 0.31 | 0.26 |
| 2.50 | ARL | 1.12 | 1.07 | 1.04 | 1.02 | 1.12 | 1.07 | 1.04 | 1.02 |
| | SDRL | 0.33 | 0.26 | 0.20 | 0.15 | 0.33 | 0.26 | 0.20 | 0.15 |
| 2.75 | ARL | 1.05 | 1.02 | 1.01 | 1.01 | 1.05 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.22 | 0.15 | 0.10 | 0.08 | 0.22 | 0.16 | 0.10 | 0.07 |
| 3.00 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.13 | 0.08 | 0.05 | 0.03 | 0.13 | 0.09 | 0.04 | 0.03 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.04 | 0.02 | 0.01 | 0.08 | 0.04 | 0.02 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.02 | 0.00 | 0.00 | 0.04 | 0.01 | 0.01 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |

Table 4.7: The run length profiles of the SEWMA-VLRSS chart with the FIR feature for $w = 1$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 2.8527 | 2.8489 | 2.8499 | 2.8760 | 3.0329 | 3.0308 | 3.0295 | 3.0560 |
| 0.00 | ARL | 500.68 | 501.79 | 502.80 | 502.32 | 499.00 | 500.60 | 501.11 | 502.99 |
| | SDRL | 494.46 | 494.91 | 493.45 | 495.72 | 499.77 | 509.28 | 502.23 | 507.14 |
| 0.25 | ARL | 52.10 | 42.94 | 39.40 | 43.79 | 63.48 | 49.96 | 45.05 | 51.37 |
| | SDRL | 37.46 | 29.26 | 26.24 | 30.05 | 56.68 | 43.18 | 38.41 | 44.66 |
| 0.50 | ARL | 17.15 | 14.39 | 13.31 | 14.59 | 15.76 | 12.76 | 11.59 | 12.95 |
| | SDRL | 9.08 | 7.30 | 6.71 | 7.37 | 11.01 | 8.61 | 7.61 | 8.64 |
| 0.75 | ARL | 9.30 | 7.81 | 7.21 | 7.91 | 7.52 | 6.21 | 5.71 | 6.30 |
| | SDRL | 4.44 | 3.67 | 3.35 | 3.69 | 4.54 | 3.57 | 3.21 | 3.60 |
| 1.00 | ARL | 6.00 | 5.03 | 4.65 | 5.10 | 4.66 | 3.89 | 3.63 | 3.94 |
| | SDRL | 2.74 | 2.26 | 2.10 | 2.30 | 2.48 | 1.95 | 1.77 | 1.98 |
| 1.25 | ARL | 4.27 | 3.58 | 3.29 | 3.59 | 3.32 | 2.83 | 2.62 | 2.84 |
| | SDRL | 1.91 | 1.61 | 1.48 | 1.61 | 1.58 | 1.26 | 1.15 | 1.28 |
| 1.50 | ARL | 3.21 | 2.69 | 2.47 | 2.69 | 2.58 | 2.21 | 2.06 | 2.22 |
| | SDRL | 1.45 | 1.23 | 1.15 | 1.24 | 1.12 | 0.91 | 0.84 | 0.92 |
| 1.75 | ARL | 2.53 | 2.09 | 1.92 | 2.11 | 2.10 | 1.81 | 1.70 | 1.82 |
| | SDRL | 1.17 | 0.99 | 0.92 | 1.01 | 0.85 | 0.71 | 0.66 | 0.72 |
| 2.00 | ARL | 2.03 | 1.69 | 1.55 | 1.69 | 1.78 | 1.54 | 1.45 | 1.55 |
| | SDRL | 0.97 | 0.80 | 0.72 | 0.80 | 0.70 | 0.60 | 0.55 | 0.60 |
| 2.25 | ARL | 1.67 | 1.40 | 1.30 | 1.40 | 1.54 | 1.35 | 1.27 | 1.34 |
| | SDRL | 0.79 | 0.62 | 0.54 | 0.62 | 0.59 | 0.50 | 0.46 | 0.50 |
| 2.50 | ARL | 1.42 | 1.21 | 1.15 | 1.21 | 1.36 | 1.20 | 1.14 | 1.20 |
| | SDRL | 0.63 | 0.45 | 0.38 | 0.45 | 0.51 | 0.41 | 0.35 | 0.41 |
| 2.75 | ARL | 1.25 | 1.11 | 1.07 | 1.11 | 1.22 | 1.10 | 1.07 | 1.10 |
| | SDRL | 0.49 | 0.32 | 0.25 | 0.32 | 0.43 | 0.30 | 0.25 | 0.30 |
| 3.00 | ARL | 1.13 | 1.05 | 1.03 | 1.04 | 1.13 | 1.05 | 1.03 | 1.04 |
| | SDRL | 0.36 | 0.21 | 0.16 | 0.21 | 0.34 | 0.21 | 0.16 | 0.21 |
| 3.25 | ARL | 1.07 | 1.02 | 1.01 | 1.02 | 1.06 | 1.02 | 1.01 | 1.02 |
| | SDRL | 0.26 | 0.14 | 0.09 | 0.13 | 0.25 | 0.14 | 0.09 | 0.13 |
| 3.50 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.03 | 1.01 | 1.00 | 1.01 |
| | SDRL | 0.17 | 0.08 | 0.05 | 0.08 | 0.17 | 0.08 | 0.05 | 0.08 |
| 3.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.05 | 0.02 | 0.04 | 0.11 | 0.04 | 0.02 | 0.05 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.02 | 0.00 | 0.02 | 0.07 | 0.02 | 0.01 | 0.02 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(5, 1)$ |
| $\delta$ | $L$ | 3.2088 | 3.2044 | 3.1990 | 3.2251 | 3.2544 | 3.2527 | 3.2520 | 3.2750 |
| 0.00 | ARL | 502.21 | 500.21 | 501.34 | 501.42 | 500.26 | 500.35 | 501.83 | 500.83 |
| | SDRL | 534.12 | 537.79 | 538.93 | 535.06 | 578.28 | 583.88 | 579.04 | 577.25 |
| 0.25 | ARL | 102.79 | 80.20 | 71.16 | 83.36 | 160.29 | 132.19 | 118.86 | 137.48 |
| | SDRL | 113.49 | 89.24 | 79.90 | 93.41 | 200.06 | 168.10 | 153.20 | 173.38 |
| 0.50 | ARL | 18.15 | 13.19 | 11.26 | 13.46 | 29.09 | 19.79 | 16.46 | 20.67 |
| | SDRL | 20.29 | 14.31 | 11.89 | 14.67 | 42.72 | 30.05 | 25.13 | 31.18 |
| 0.75 | ARL | 6.33 | 4.82 | 4.34 | 4.92 | 7.13 | 4.94 | 4.22 | 5.05 |
| | SDRL | 5.85 | 4.10 | 3.54 | 4.17 | 10.38 | 6.63 | 5.33 | 6.79 |
| 1.00 | ARL | 3.44 | 2.82 | 2.58 | 2.84 | 3.06 | 2.39 | 2.17 | 2.43 |
| | SDRL | 2.50 | 1.85 | 1.61 | 1.87 | 3.17 | 2.09 | 1.76 | 2.15 |
| 1.25 | ARL | 2.38 | 2.01 | 1.88 | 2.03 | 1.97 | 1.65 | 1.55 | 1.66 |
| | SDRL | 1.41 | 1.08 | 0.97 | 1.09 | 1.45 | 1.03 | 0.88 | 1.02 |
| 1.50 | ARL | 1.84 | 1.60 | 1.50 | 1.61 | 1.52 | 1.34 | 1.27 | 1.34 |
| | SDRL | 0.93 | 0.75 | 0.67 | 0.75 | 0.86 | 0.63 | 0.54 | 0.64 |
| 1.75 | ARL | 1.53 | 1.35 | 1.28 | 1.35 | 1.28 | 1.17 | 1.13 | 1.17 |
| | SDRL | 0.69 | 0.55 | 0.49 | 0.55 | 0.57 | 0.42 | 0.36 | 0.42 |
| 2.00 | ARL | 1.32 | 1.19 | 1.14 | 1.19 | 1.15 | 1.07 | 1.05 | 1.08 |
| | SDRL | 0.53 | 0.42 | 0.36 | 0.42 | 0.40 | 0.27 | 0.23 | 0.28 |
| 2.25 | ARL | 1.19 | 1.09 | 1.06 | 1.10 | 1.08 | 1.03 | 1.02 | 1.03 |
| | SDRL | 0.41 | 0.30 | 0.25 | 0.30 | 0.28 | 0.17 | 0.14 | 0.18 |
| 2.50 | ARL | 1.10 | 1.04 | 1.03 | 1.04 | 1.03 | 1.01 | 1.01 | 1.01 |
| | SDRL | 0.31 | 0.20 | 0.16 | 0.20 | 0.18 | 0.10 | 0.08 | 0.11 |
| 2.75 | ARL | 1.05 | 1.02 | 1.01 | 1.02 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.22 | 0.13 | 0.09 | 0.13 | 0.11 | 0.06 | 0.04 | 0.07 |
| 3.00 | ARL | 1.02 | 1.01 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.15 | 0.07 | 0.05 | 0.08 | 0.07 | 0.03 | 0.02 | 0.03 |
| 3.25 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.09 | 0.04 | 0.02 | 0.04 | 0.04 | 0.02 | 0.01 | 0.02 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.05 | 0.02 | 0.01 | 0.02 | 0.02 | 0.00 | 0.00 | 0.01 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4.8: The run length profiles of the SEWMA-VLRSS chart with the FIR feature for $w = 2$ when the in-control ARL is 500

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 2.8479 | 2.8479 | 2.8475 | 2.8472 | 3.0297 | 3.0294 | 3.0294 | 3.0283 |
| 0.00 | ARL | 499.65 | 501.08 | 502.93 | 500.20 | 501.52 | 501.09 | 502.12 | 500.81 |
| | SDRL | 493.15 | 496.13 | 494.98 | 492.83 | 503.53 | 510.07 | 504.01 | 507.82 |
| 0.25 | ARL | 35.57 | 32.18 | 29.27 | 27.25 | 39.47 | 34.99 | 30.98 | 28.24 |
| | SDRL | 23.10 | 20.41 | 17.88 | 16.22 | 33.06 | 28.62 | 24.90 | 22.39 |
| 0.50 | ARL | 12.07 | 11.06 | 10.06 | 9.46 | 10.28 | 9.22 | 8.29 | 7.70 |
| | SDRL | 5.93 | 5.39 | 4.87 | 4.51 | 6.59 | 5.79 | 5.08 | 4.64 |
| 0.75 | ARL | 6.53 | 5.96 | 5.46 | 5.10 | 5.12 | 4.64 | 4.21 | 3.96 |
| | SDRL | 3.02 | 2.72 | 2.46 | 2.32 | 2.80 | 2.45 | 2.19 | 2.00 |
| 1.00 | ARL | 4.20 | 3.83 | 3.50 | 3.28 | 3.27 | 3.01 | 2.76 | 2.60 |
| | SDRL | 1.89 | 1.72 | 1.57 | 1.48 | 1.54 | 1.37 | 1.22 | 1.13 |
| 1.25 | ARL | 2.98 | 2.71 | 2.46 | 2.30 | 2.40 | 2.22 | 2.06 | 1.95 |
| | SDRL | 1.35 | 1.24 | 1.14 | 1.08 | 1.02 | 0.92 | 0.83 | 0.78 |
| 1.50 | ARL | 2.23 | 2.01 | 1.83 | 1.70 | 1.90 | 1.76 | 1.64 | 1.56 |
| | SDRL | 1.05 | 0.96 | 0.87 | 0.80 | 0.76 | 0.69 | 0.63 | 0.60 |
| 1.75 | ARL | 1.73 | 1.57 | 1.43 | 1.35 | 1.58 | 1.47 | 1.37 | 1.30 |
| | SDRL | 0.82 | 0.72 | 0.64 | 0.57 | 0.61 | 0.56 | 0.52 | 0.48 |
| 2.00 | ARL | 1.40 | 1.29 | 1.20 | 1.15 | 1.35 | 1.26 | 1.19 | 1.14 |
| | SDRL | 0.62 | 0.53 | 0.44 | 0.38 | 0.50 | 0.45 | 0.39 | 0.35 |
| 2.25 | ARL | 1.20 | 1.13 | 1.08 | 1.05 | 1.19 | 1.13 | 1.08 | 1.05 |
| | SDRL | 0.44 | 0.36 | 0.28 | 0.23 | 0.40 | 0.33 | 0.27 | 0.22 |
| 2.50 | ARL | 1.09 | 1.05 | 1.03 | 1.01 | 1.09 | 1.05 | 1.03 | 1.01 |
| | SDRL | 0.30 | 0.22 | 0.16 | 0.12 | 0.28 | 0.22 | 0.16 | 0.12 |
| 2.75 | ARL | 1.04 | 1.02 | 1.01 | 1.00 | 1.03 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.19 | 0.12 | 0.08 | 0.06 | 0.18 | 0.13 | 0.08 | 0.06 |
| 3.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.10 | 0.07 | 0.04 | 0.02 | 0.11 | 0.06 | 0.04 | 0.02 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.03 | 0.01 | 0.01 | 0.05 | 0.03 | 0.02 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.01 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ | $(5,3)$ | $(6,3)$ | $(7,4)$ | $(8,4)$ |
| $\delta$ | $L$ | 3.2014 | 3.1999 | 3.1990 | 3.1995 | 3.2490 | 3.2490 | 3.2486 | 3.2480 |
| 0.00 | ARL | 500.07 | 500.87 | 501.93 | 502.28 | 499.56 | 499.95 | 500.56 | 501.07 |
| | SDRL | 535.91 | 537.36 | 535.19 | 534.75 | 579.34 | 580.42 | 584.53 | 583.79 |
| 0.25 | ARL | 61.29 | 52.22 | 45.27 | 40.63 | 103.34 | 89.54 | 76.76 | 69.54 |
| | SDRL | 69.10 | 59.21 | 51.49 | 45.93 | 134.54 | 117.61 | 102.91 | 94.93 |
| 0.50 | ARL | 9.50 | 8.22 | 7.08 | 6.38 | 13.13 | 10.53 | 8.48 | 7.42 |
| | SDRL | 9.74 | 8.15 | 6.77 | 5.94 | 20.25 | 15.89 | 12.67 | 10.89 |
| 0.75 | ARL | 3.83 | 3.42 | 3.06 | 2.84 | 3.50 | 3.02 | 2.66 | 2.41 |
| | SDRL | 2.91 | 2.47 | 2.11 | 1.87 | 3.97 | 3.19 | 2.52 | 2.12 |
| 1.00 | ARL | 2.34 | 2.14 | 1.98 | 1.86 | 1.93 | 1.77 | 1.63 | 1.54 |
| | SDRL | 1.38 | 1.18 | 1.06 | 0.95 | 1.40 | 1.17 | 0.99 | 0.88 |
| 1.25 | ARL | 1.73 | 1.60 | 1.50 | 1.42 | 1.43 | 1.34 | 1.27 | 1.22 |
| | SDRL | 0.85 | 0.75 | 0.67 | 0.61 | 0.74 | 0.64 | 0.55 | 0.49 |
| 1.50 | ARL | 1.40 | 1.32 | 1.24 | 1.20 | 1.20 | 1.15 | 1.10 | 1.08 |
| | SDRL | 0.59 | 0.52 | 0.46 | 0.42 | 0.46 | 0.39 | 0.33 | 0.28 |
| 1.75 | ARL | 1.21 | 1.15 | 1.11 | 1.08 | 1.09 | 1.06 | 1.04 | 1.02 |
| | SDRL | 0.43 | 0.37 | 0.31 | 0.27 | 0.30 | 0.24 | 0.19 | 0.15 |
| 2.00 | ARL | 1.10 | 1.06 | 1.04 | 1.02 | 1.03 | 1.02 | 1.01 | 1.01 |
| | SDRL | 0.30 | 0.24 | 0.19 | 0.15 | 0.18 | 0.14 | 0.10 | 0.08 |
| 2.25 | ARL | 1.04 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.19 | 0.14 | 0.10 | 0.08 | 0.10 | 0.07 | 0.05 | 0.03 |
| 2.50 | ARL | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.08 | 0.05 | 0.03 | 0.05 | 0.04 | 0.02 | 0.01 |
| 2.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.06 | 0.03 | 0.02 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.03 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4.9: The run length profiles of the SCUSUM-VLRSS chart with the FIR feature for $w = 1$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 9.3436 | 9.3438 | 9.3440 | 9.4160 | 5.3798 | 5.3799 | 5.3743 | 5.4160 |
| 0.00 | ARL | 499.46 | 499.84 | 500.39 | 500.07 | 500.37 | 499.83 | 499.62 | 500.67 |
| | SDRL | 536.97 | 543.13 | 539.01 | 541.17 | 528.85 | 520.28 | 522.45 | 530.21 |
| 0.25 | ARL | 35.96 | 28.72 | 26.35 | 29.07 | 64.98 | 49.85 | 43.78 | 50.35 |
| | SDRL | 34.08 | 25.63 | 23.09 | 26.22 | 69.51 | 53.49 | 46.69 | 54.21 |
| 0.50 | ARL | 10.85 | 9.21 | 8.61 | 9.32 | 12.29 | 9.65 | 8.79 | 9.68 |
| | SDRL | 6.61 | 5.14 | 4.68 | 5.17 | 11.09 | 8.04 | 7.11 | 8.14 |
| 0.75 | ARL | 6.33 | 5.53 | 5.19 | 5.54 | 5.59 | 4.63 | 4.28 | 4.68 |
| | SDRL | 2.91 | 2.37 | 2.16 | 2.36 | 3.69 | 2.77 | 2.44 | 2.80 |
| 1.00 | ARL | 4.51 | 3.95 | 3.73 | 3.98 | 3.62 | 3.10 | 2.92 | 3.13 |
| | SDRL | 1.77 | 1.48 | 1.36 | 1.49 | 1.87 | 1.46 | 1.32 | 1.48 |
| 1.25 | ARL | 3.51 | 3.07 | 2.90 | 3.10 | 2.71 | 2.39 | 2.25 | 2.40 |
| | SDRL | 1.28 | 1.10 | 1.05 | 1.11 | 1.18 | 0.95 | 0.88 | 0.96 |
| 1.50 | ARL | 2.84 | 2.49 | 2.34 | 2.50 | 2.21 | 1.96 | 1.86 | 1.97 |
| | SDRL | 1.03 | 0.93 | 0.89 | 0.93 | 0.86 | 0.72 | 0.68 | 0.73 |
| 1.75 | ARL | 2.37 | 2.04 | 1.92 | 2.06 | 1.88 | 1.68 | 1.59 | 1.69 |
| | SDRL | 0.89 | 0.81 | 0.77 | 0.82 | 0.68 | 0.61 | 0.57 | 0.61 |
| 2.00 | ARL | 2.00 | 1.70 | 1.59 | 1.70 | 1.65 | 1.46 | 1.38 | 1.47 |
| | SDRL | 0.80 | 0.70 | 0.65 | 0.71 | 0.59 | 0.54 | 0.51 | 0.54 |
| 2.25 | ARL | 1.69 | 1.45 | 1.35 | 1.45 | 1.46 | 1.29 | 1.23 | 1.30 |
| | SDRL | 0.69 | 0.58 | 0.53 | 0.58 | 0.53 | 0.46 | 0.42 | 0.47 |
| 2.50 | ARL | 1.47 | 1.26 | 1.19 | 1.26 | 1.30 | 1.17 | 1.12 | 1.17 |
| | SDRL | 0.59 | 0.47 | 0.41 | 0.47 | 0.47 | 0.37 | 0.32 | 0.38 |
| 2.75 | ARL | 1.30 | 1.14 | 1.09 | 1.14 | 1.19 | 1.08 | 1.05 | 1.09 |
| | SDRL | 0.49 | 0.36 | 0.29 | 0.36 | 0.39 | 0.28 | 0.23 | 0.28 |
| 3.00 | ARL | 1.17 | 1.07 | 1.04 | 1.07 | 1.10 | 1.04 | 1.02 | 1.04 |
| | SDRL | 0.39 | 0.25 | 0.20 | 0.25 | 0.31 | 0.18 | 0.14 | 0.19 |
| 3.25 | ARL | 1.09 | 1.03 | 1.01 | 1.03 | 1.05 | 1.01 | 1.01 | 1.01 |
| | SDRL | 0.29 | 0.17 | 0.12 | 0.17 | 0.22 | 0.12 | 0.08 | 0.12 |
| 3.50 | ARL | 1.04 | 1.01 | 1.00 | 1.01 | 1.02 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.20 | 0.10 | 0.06 | 0.10 | 0.15 | 0.07 | 0.04 | 0.07 |
| 3.75 | ARL | 1.02 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.14 | 0.06 | 0.03 | 0.06 | 0.09 | 0.03 | 0.02 | 0.03 |
| 4.00 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.08 | 0.03 | 0.02 | 0.03 | 0.06 | 0.02 | 0.01 | 0.02 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (2,1) | (3,2) | (4,2) | (5,1) | (2,1) | (3,2) | (4,2) | (5,1) |
| $\delta$ | $h$ | 3.7099 | 3.7070 | 3.7070 | 3.7360 | 2.7495 | 2.7483 | 2.7435 | 2.7720 |
| 0.00 | ARL | 502.76 | 499.88 | 500.07 | 501.10 | 500.15 | 500.89 | 499.93 | 500.64 |
| | SDRL | 523.36 | 517.07 | 518.10 | 518.08 | 516.09 | 523.01 | 513.04 | 522.59 |
| 0.25 | ARL | 104.48 | 81.58 | 72.94 | 84.08 | 144.39 | 117.87 | 104.37 | 119.05 |
| | SDRL | 111.86 | 86.48 | 78.70 | 90.30 | 153.46 | 123.51 | 110.40 | 126.45 |
| 0.50 | ARL | 18.40 | 13.34 | 11.79 | 13.48 | 28.45 | 20.10 | 17.33 | 20.60 |
| | SDRL | 19.23 | 13.50 | 11.73 | 13.66 | 30.71 | 21.38 | 18.52 | 21.92 |
| 0.75 | ARL | 6.40 | 5.00 | 4.50 | 5.04 | 8.54 | 6.22 | 5.41 | 6.27 |
| | SDRL | 5.51 | 3.96 | 3.42 | 3.97 | 8.53 | 5.87 | 4.90 | 5.95 |
| 1.00 | ARL | 3.55 | 2.93 | 2.71 | 2.95 | 4.04 | 3.13 | 2.82 | 3.15 |
| | SDRL | 2.40 | 1.80 | 1.57 | 1.80 | 3.37 | 2.37 | 2.00 | 2.37 |
| 1.25 | ARL | 2.49 | 2.11 | 1.98 | 2.12 | 2.53 | 2.08 | 1.92 | 2.10 |
| | SDRL | 1.38 | 1.07 | 0.96 | 1.05 | 1.71 | 1.25 | 1.09 | 1.27 |
| 1.50 | ARL | 1.95 | 1.68 | 1.59 | 1.70 | 1.88 | 1.60 | 1.50 | 1.61 |
| | SDRL | 0.94 | 0.75 | 0.69 | 0.76 | 1.06 | 0.80 | 0.71 | 0.81 |
| 1.75 | ARL | 1.61 | 1.42 | 1.35 | 1.43 | 1.53 | 1.34 | 1.27 | 1.35 |
| | SDRL | 0.70 | 0.58 | 0.53 | 0.58 | 0.74 | 0.57 | 0.50 | 0.57 |
| 2.00 | ARL | 1.39 | 1.24 | 1.19 | 1.25 | 1.31 | 1.18 | 1.14 | 1.18 |
| | SDRL | 0.56 | 0.45 | 0.41 | 0.45 | 0.54 | 0.41 | 0.36 | 0.41 |
| 2.25 | ARL | 1.24 | 1.13 | 1.09 | 1.13 | 1.17 | 1.09 | 1.06 | 1.09 |
| | SDRL | 0.44 | 0.34 | 0.29 | 0.34 | 0.40 | 0.29 | 0.24 | 0.29 |
| 2.50 | ARL | 1.13 | 1.06 | 1.04 | 1.06 | 1.09 | 1.04 | 1.02 | 1.04 |
| | SDRL | 0.35 | 0.24 | 0.19 | 0.24 | 0.30 | 0.19 | 0.15 | 0.20 |
| 2.75 | ARL | 1.07 | 1.02 | 1.01 | 1.03 | 1.05 | 1.01 | 1.01 | 1.02 |
| | SDRL | 0.26 | 0.15 | 0.12 | 0.16 | 0.21 | 0.12 | 0.08 | 0.12 |
| 3.00 | ARL | 1.03 | 1.01 | 1.00 | 1.01 | 1.02 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.18 | 0.10 | 0.07 | 0.09 | 0.14 | 0.07 | 0.04 | 0.07 |
| 3.25 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.11 | 0.05 | 0.03 | 0.06 | 0.09 | 0.03 | 0.02 | 0.04 |
| 3.50 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.03 | 0.02 | 0.03 | 0.05 | 0.02 | 0.01 | 0.02 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.01 | 0.01 | 0.01 | 0.03 | 0.01 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.01 | 0.02 | 0.00 | 0.00 | 0.00 |

Table 4.10: The run length profiles of the SCUSUM-VLRSS chart with the FIR feature for $w = 2$ when the in-control ARL is 500

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 9.3440 | 9.3443 | 9.3435 | 9.3432 | 5.3740 | 5.3757 | 5.3737 | 5.3747 |
| 0.00 | ARL | 502.80 | 500.96 | 500.58 | 500.21 | 500.91 | 499.34 | 500.56 | 501.70 |
| | SDRL | 545.00 | 542.88 | 541.06 | 546.99 | 527.12 | 528.30 | 526.67 | 525.66 |
| 0.25 | ARL | 23.38 | 20.78 | 18.75 | 17.50 | 37.59 | 32.45 | 28.15 | 25.20 |
| | SDRL | 19.59 | 16.69 | 14.59 | 13.14 | 39.69 | 34.13 | 29.05 | 25.74 |
| 0.50 | ARL | 7.92 | 7.32 | 6.79 | 6.43 | 7.64 | 6.83 | 6.12 | 5.69 |
| | SDRL | 4.08 | 3.61 | 3.24 | 2.99 | 5.83 | 4.99 | 4.27 | 3.79 |
| 0.75 | ARL | 4.80 | 4.48 | 4.20 | 3.99 | 3.89 | 3.60 | 3.33 | 3.14 |
| | SDRL | 1.93 | 1.76 | 1.59 | 1.51 | 2.11 | 1.87 | 1.63 | 1.49 |
| 1.00 | ARL | 3.46 | 3.24 | 3.04 | 2.89 | 2.69 | 2.52 | 2.35 | 2.25 |
| | SDRL | 1.25 | 1.16 | 1.09 | 1.04 | 1.17 | 1.04 | 0.93 | 0.87 |
| 1.25 | ARL | 2.70 | 2.51 | 2.33 | 2.22 | 2.11 | 1.98 | 1.85 | 1.77 |
| | SDRL | 0.98 | 0.93 | 0.89 | 0.85 | 0.79 | 0.72 | 0.68 | 0.64 |
| 1.50 | ARL | 2.16 | 1.99 | 1.83 | 1.72 | 1.74 | 1.64 | 1.55 | 1.48 |
| | SDRL | 0.85 | 0.79 | 0.75 | 0.71 | 0.63 | 0.59 | 0.56 | 0.54 |
| 1.75 | ARL | 1.75 | 1.61 | 1.48 | 1.40 | 1.49 | 1.40 | 1.31 | 1.26 |
| | SDRL | 0.72 | 0.66 | 0.60 | 0.56 | 0.55 | 0.51 | 0.47 | 0.44 |
| 2.00 | ARL | 1.46 | 1.34 | 1.25 | 1.19 | 1.30 | 1.22 | 1.15 | 1.11 |
| | SDRL | 0.59 | 0.52 | 0.45 | 0.40 | 0.46 | 0.42 | 0.36 | 0.32 |
| 2.25 | ARL | 1.26 | 1.17 | 1.11 | 1.08 | 1.16 | 1.10 | 1.06 | 1.04 |
| | SDRL | 0.46 | 0.39 | 0.31 | 0.27 | 0.36 | 0.30 | 0.24 | 0.19 |
| 2.50 | ARL | 1.12 | 1.07 | 1.04 | 1.02 | 1.07 | 1.04 | 1.02 | 1.01 |
| | SDRL | 0.33 | 0.26 | 0.20 | 0.15 | 0.25 | 0.19 | 0.14 | 0.10 |
| 2.75 | ARL | 1.05 | 1.02 | 1.01 | 1.01 | 1.03 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.22 | 0.15 | 0.10 | 0.07 | 0.16 | 0.11 | 0.06 | 0.05 |
| 3.00 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.13 | 0.09 | 0.05 | 0.03 | 0.09 | 0.05 | 0.03 | 0.02 |
| 3.25 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.05 | 0.02 | 0.01 | 0.04 | 0.03 | 0.01 | 0.01 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.01 | 0.01 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (5,3) | (6,3) | (7,4) | (8,4) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | $h$ | 3.7070 | 3.7070 | 3.7067 | 3.7063 | 2.7430 | 2.7435 | 2.7437 | 2.7423 |
| 0.00 | ARL | 501.74 | 502.89 | 502.93 | 500.90 | 500.36 | 500.86 | 501.63 | 500.88 |
| | SDRL | 519.76 | 519.43 | 522.94 | 519.50 | 512.85 | 518.16 | 518.54 | 518.06 |
| 0.25 | ARL | 62.48 | 54.44 | 46.76 | 41.78 | 92.28 | 80.17 | 69.60 | 62.42 |
| | SDRL | 67.54 | 58.46 | 49.93 | 45.09 | 97.53 | 85.55 | 74.02 | 66.23 |
| 0.50 | ARL | 9.93 | 8.44 | 7.31 | 6.58 | 14.24 | 11.90 | 10.02 | 8.83 |
| | SDRL | 9.67 | 7.87 | 6.59 | 5.74 | 14.97 | 12.34 | 10.21 | 8.87 |
| 0.75 | ARL | 3.92 | 3.55 | 3.20 | 2.98 | 4.63 | 4.00 | 3.50 | 3.19 |
| | SDRL | 2.78 | 2.40 | 2.04 | 1.84 | 4.05 | 3.35 | 2.77 | 2.42 |
| 1.00 | ARL | 2.45 | 2.26 | 2.08 | 1.97 | 2.52 | 2.26 | 2.04 | 1.92 |
| | SDRL | 1.35 | 1.20 | 1.04 | 0.95 | 1.70 | 1.43 | 1.20 | 1.09 |
| 1.25 | ARL | 1.83 | 1.70 | 1.59 | 1.51 | 1.75 | 1.62 | 1.50 | 1.43 |
| | SDRL | 0.85 | 0.76 | 0.68 | 0.64 | 0.94 | 0.82 | 0.71 | 0.65 |
| 1.50 | ARL | 1.48 | 1.39 | 1.31 | 1.25 | 1.40 | 1.31 | 1.23 | 1.19 |
| | SDRL | 0.62 | 0.55 | 0.50 | 0.46 | 0.62 | 0.54 | 0.46 | 0.42 |
| 1.75 | ARL | 1.27 | 1.20 | 1.14 | 1.11 | 1.20 | 1.14 | 1.10 | 1.07 |
| | SDRL | 0.47 | 0.41 | 0.36 | 0.31 | 0.43 | 0.36 | 0.30 | 0.26 |
| 2.00 | ARL | 1.13 | 1.09 | 1.05 | 1.04 | 1.09 | 1.06 | 1.03 | 1.02 |
| | SDRL | 0.34 | 0.28 | 0.23 | 0.19 | 0.29 | 0.23 | 0.18 | 0.15 |
| 2.25 | ARL | 1.06 | 1.03 | 1.02 | 1.01 | 1.03 | 1.02 | 1.01 | 1.01 |
| | SDRL | 0.23 | 0.18 | 0.13 | 0.10 | 0.18 | 0.14 | 0.10 | 0.07 |
| 2.50 | ARL | 1.02 | 1.01 | 1.00 | 1.00 | 1.01 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.14 | 0.10 | 0.06 | 0.04 | 0.10 | 0.07 | 0.04 | 0.03 |
| 2.75 | ARL | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.07 | 0.05 | 0.03 | 0.02 | 0.05 | 0.03 | 0.02 | 0.01 |
| 3.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.04 | 0.02 | 0.01 | 0.01 | 0.02 | 0.02 | 0.01 | 0.00 |
| 3.25 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.02 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.50 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |
| 3.75 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 4.00 | ARL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4.11: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS, SEWMA-RSS, and SEWMA-MRSS charts

| | | $\lambda = 0.05$ | | | | | | $\lambda = 0.10$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 499.66 | 500.35 | 500.25 | 499.43 | 500.87 | 499.53 | 500.37 | 500.85 | 502.58 | 502.48 | 500.45 | 502.76 |
| | SDRL | 494.39 | 492.00 | 494.42 | 490.67 | 491.50 | 490.78 | 494.51 | 496.51 | 494.28 | 500.68 | 500.09 | 498.02 |
| 0.25 | ARL | 100.51 | 44.57 | 36.42 | 32.97 | 30.09 | 28.20 | 131.40 | 53.65 | 42.10 | 37.61 | 33.53 | 30.84 |
| | SDRL | 82.86 | 30.00 | 22.74 | 19.85 | 17.63 | 15.99 | 121.68 | 43.56 | 32.78 | 28.23 | 24.51 | 22.16 |
| 0.50 | ARL | 32.60 | 15.73 | 13.33 | 12.29 | 11.37 | 10.77 | 36.68 | 15.10 | 12.37 | 11.34 | 10.34 | 9.73 |
| | SDRL | 19.58 | 7.00 | 5.62 | 5.08 | 4.58 | 4.28 | 27.23 | 8.30 | 6.33 | 5.57 | 4.93 | 4.53 |
| 0.75 | ARL | 17.80 | 9.25 | 7.89 | 7.29 | 6.72 | 6.35 | 17.66 | 8.20 | 6.89 | 6.34 | 5.84 | 5.53 |
| | SDRL | 8.38 | 3.56 | 3.01 | 2.80 | 2.62 | 2.52 | 10.37 | 3.60 | 2.89 | 2.61 | 2.40 | 2.26 |
| 1.00 | ARL | 12.12 | 6.32 | 5.35 | 4.93 | 4.48 | 4.19 | 11.19 | 5.48 | 4.64 | 4.27 | 3.90 | 3.67 |
| | SDRL | 4.94 | 2.51 | 2.24 | 2.13 | 2.03 | 1.96 | 5.46 | 2.23 | 1.90 | 1.79 | 1.68 | 1.61 |
| 1.50 | ARL | 7.17 | 3.39 | 2.69 | 2.39 | 2.10 | 1.92 | 6.27 | 3.03 | 2.47 | 2.21 | 1.99 | 1.83 |
| | SDRL | 2.77 | 1.76 | 1.52 | 1.38 | 1.23 | 1.13 | 2.58 | 1.43 | 1.25 | 1.15 | 1.05 | 0.97 |
| 2.00 | ARL | 4.83 | 1.88 | 1.49 | 1.34 | 1.23 | 1.16 | 4.21 | 1.80 | 1.46 | 1.33 | 1.22 | 1.16 |
| | SDRL | 2.12 | 1.11 | 0.80 | 0.65 | 0.52 | 0.44 | 1.77 | 0.96 | 0.72 | 0.61 | 0.49 | 0.42 |
| 2.50 | ARL | 3.37 | 1.24 | 1.10 | 1.05 | 1.03 | 1.01 | 3.00 | 1.24 | 1.10 | 1.05 | 1.02 | 1.01 |
| | SDRL | 1.73 | 0.54 | 0.33 | 0.24 | 0.16 | 0.12 | 1.42 | 0.52 | 0.32 | 0.24 | 0.16 | 0.12 |
| 3.00 | ARL | 2.34 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 | 2.19 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 1.36 | 0.22 | 0.10 | 0.07 | 0.04 | 0.02 | 1.14 | 0.22 | 0.11 | 0.07 | 0.03 | 0.03 |
| 4.00 | ARL | 1.32 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.31 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.63 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |

| | | $\lambda = 0.25$ | | | | | | $\lambda = 0.50$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.72 | 502.46 | 500.20 | 500.80 | 502.48 | 500.74 | 500.21 | 500.08 | 501.10 | 500.45 | 501.94 | 500.08 |
| | SDRL | 500.86 | 502.52 | 500.00 | 497.87 | 501.34 | 498.89 | 500.15 | 502.07 | 508.07 | 504.69 | 500.30 | 502.16 |
| 0.25 | ARL | 206.35 | 92.18 | 69.36 | 61.38 | 53.18 | 48.84 | 280.77 | 150.44 | 118.70 | 106.57 | 93.83 | 85.87 |
| | SDRL | 200.96 | 87.60 | 64.36 | 57.35 | 48.42 | 44.58 | 279.12 | 148.76 | 117.02 | 103.89 | 91.67 | 83.88 |
| 0.50 | ARL | 59.56 | 19.35 | 14.56 | 12.78 | 11.35 | 10.30 | 104.08 | 33.73 | 23.68 | 20.41 | 17.45 | 15.45 |
| | SDRL | 54.98 | 15.20 | 10.63 | 9.03 | 7.69 | 6.74 | 102.02 | 31.40 | 21.49 | 18.15 | 15.24 | 13.37 |
| 0.75 | ARL | 23.67 | 8.16 | 6.52 | 5.88 | 5.30 | 4.94 | 41.73 | 11.43 | 8.30 | 7.19 | 6.24 | 5.67 |
| | SDRL | 19.38 | 4.96 | 3.59 | 3.12 | 2.74 | 2.47 | 39.42 | 9.39 | 6.38 | 5.31 | 4.39 | 3.89 |
| 1.00 | ARL | 12.43 | 4.93 | 4.04 | 3.68 | 3.37 | 3.16 | 19.72 | 5.62 | 4.30 | 3.79 | 3.40 | 3.13 |
| | SDRL | 8.70 | 2.45 | 1.89 | 1.69 | 1.53 | 1.43 | 17.58 | 3.84 | 2.70 | 2.26 | 1.95 | 1.73 |
| 1.50 | ARL | 5.77 | 2.62 | 2.18 | 1.99 | 1.80 | 1.69 | 7.02 | 2.51 | 2.06 | 1.88 | 1.72 | 1.62 |
| | SDRL | 3.03 | 1.19 | 1.00 | 0.92 | 0.84 | 0.79 | 5.13 | 1.27 | 0.98 | 0.87 | 0.76 | 0.70 |
| 2.00 | ARL | 3.65 | 1.67 | 1.39 | 1.29 | 1.20 | 1.15 | 3.75 | 1.60 | 1.37 | 1.27 | 1.19 | 1.14 |
| | SDRL | 1.67 | 0.78 | 0.60 | 0.51 | 0.43 | 0.37 | 2.22 | 0.69 | 0.54 | 0.47 | 0.41 | 0.35 |
| 2.50 | ARL | 2.60 | 1.22 | 1.09 | 1.05 | 1.03 | 1.01 | 2.49 | 1.20 | 1.09 | 1.05 | 1.03 | 1.01 |
| | SDRL | 1.17 | 0.45 | 0.29 | 0.22 | 0.16 | 0.12 | 1.25 | 0.42 | 0.29 | 0.22 | 0.16 | 0.12 |
| 3.00 | ARL | 1.96 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 | 1.86 | 1.05 | 1.01 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.90 | 0.22 | 0.11 | 0.06 | 0.04 | 0.02 | 0.85 | 0.21 | 0.10 | 0.07 | 0.04 | 0.02 |
| 4.00 | ARL | 1.28 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.26 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.50 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |

Table 4.12: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS, SCUSUM-RSS, and SCUSUM-MRSS charts

| | | $k = 0.25$ | | | | | | $k = 0.50$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.86 | 500.73 | 500.80 | 500.62 | 500.72 | 500.26 | 500.43 | 501.50 | 502.65 | 500.08 | 501.11 | 499.16 |
| | SDRL | 490.40 | 488.51 | 486.65 | 490.59 | 492.92 | 487.82 | 500.13 | 492.83 | 497.32 | 493.26 | 494.90 | 491.04 |
| 0.25 | ARL | 99.60 | 43.60 | 36.00 | 32.87 | 30.02 | 28.32 | 154.81 | 60.85 | 47.16 | 41.55 | 36.91 | 33.68 |
| | SDRL | 81.21 | 28.10 | 21.62 | 18.97 | 16.75 | 15.31 | 147.97 | 53.31 | 39.74 | 34.25 | 29.62 | 26.48 |
| 0.50 | ARL | 32.44 | 15.93 | 13.64 | 12.66 | 11.76 | 11.17 | 40.86 | 14.89 | 12.17 | 11.00 | 9.96 | 9.32 |
| | SDRL | 18.57 | 6.63 | 5.36 | 4.82 | 4.36 | 4.09 | 33.67 | 8.92 | 6.73 | 5.83 | 5.06 | 4.60 |
| 0.75 | ARL | 18.17 | 9.55 | 8.25 | 7.65 | 7.11 | 6.73 | 17.99 | 7.74 | 6.48 | 5.95 | 5.50 | 5.17 |
| | SDRL | 8.01 | 3.36 | 2.88 | 2.66 | 2.49 | 2.38 | 11.83 | 3.50 | 2.76 | 2.44 | 2.22 | 2.06 |
| 1.00 | ARL | 12.49 | 6.69 | 5.71 | 5.27 | 4.85 | 4.54 | 10.81 | 5.13 | 4.37 | 4.05 | 3.73 | 3.52 |
| | SDRL | 4.77 | 2.35 | 2.11 | 2.02 | 1.94 | 1.88 | 5.71 | 2.03 | 1.71 | 1.56 | 1.44 | 1.37 |
| 1.50 | ARL | 7.56 | 3.73 | 3.02 | 2.69 | 2.37 | 2.16 | 5.89 | 2.95 | 2.48 | 2.26 | 2.05 | 1.91 |
| | SDRL | 2.61 | 1.73 | 1.56 | 1.45 | 1.33 | 1.24 | 2.42 | 1.21 | 1.08 | 1.01 | 0.95 | 0.90 |
| 2.00 | ARL | 5.21 | 2.10 | 1.65 | 1.46 | 1.31 | 1.23 | 3.98 | 1.89 | 1.54 | 1.40 | 1.28 | 1.21 |
| | SDRL | 2.00 | 1.21 | 0.92 | 0.77 | 0.62 | 0.51 | 1.53 | 0.89 | 0.73 | 0.64 | 0.53 | 0.46 |
| 2.50 | ARL | 3.70 | 1.34 | 1.14 | 1.08 | 1.04 | 1.02 | 2.93 | 1.30 | 1.13 | 1.08 | 1.04 | 1.02 |
| | SDRL | 1.73 | 0.65 | 0.39 | 0.29 | 0.20 | 0.16 | 1.20 | 0.56 | 0.36 | 0.28 | 0.20 | 0.15 |
| 3.00 | ARL | 2.62 | 1.07 | 1.02 | 1.01 | 1.00 | 1.00 | 2.23 | 1.07 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 1.43 | 0.28 | 0.13 | 0.09 | 0.04 | 0.04 | 1.01 | 0.26 | 0.13 | 0.09 | 0.05 | 0.04 |
| 4.00 | ARL | 1.43 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.38 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.74 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.62 | 0.03 | 0.00 | 0.01 | 0.00 | 0.00 |

| | | $k = 0.75$ | | | | | | $k = 1.00$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) | | (5,1) | (5,3) | (6,3) | (7,4) | (8,4) |
| $\delta$ | | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS | SRS | RSS | MRSS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 501.84 | 501.24 | 502.85 | 502.68 | 502.63 | 500.05 | 499.85 | 500.15 | 500.36 | 499.33 | 501.55 | 500.13 |
| | SDRL | 498.78 | 500.01 | 499.40 | 497.36 | 501.27 | 495.78 | 500.34 | 498.37 | 497.91 | 496.82 | 500.44 | 498.09 |
| 0.25 | ARL | 210.92 | 91.33 | 70.37 | 61.78 | 53.89 | 48.85 | 255.35 | 124.39 | 96.77 | 86.37 | 75.50 | 68.85 |
| | SDRL | 207.75 | 87.58 | 66.21 | 57.04 | 49.40 | 44.55 | 252.81 | 123.01 | 94.02 | 84.06 | 72.87 | 66.29 |
| 0.50 | ARL | 60.35 | 18.18 | 13.86 | 12.16 | 10.70 | 9.81 | 83.74 | 24.45 | 17.85 | 15.30 | 13.16 | 11.72 |
| | SDRL | 56.15 | 14.05 | 10.04 | 8.45 | 7.10 | 6.29 | 81.36 | 21.59 | 15.22 | 12.52 | 10.48 | 9.08 |
| 0.75 | ARL | 22.98 | 7.73 | 6.23 | 5.60 | 5.11 | 4.73 | 31.82 | 8.78 | 6.69 | 5.91 | 5.26 | 4.81 |
| | SDRL | 18.98 | 4.44 | 3.32 | 2.85 | 2.51 | 2.25 | 29.24 | 6.25 | 4.38 | 3.68 | 3.13 | 2.76 |
| 1.00 | ARL | 11.94 | 4.71 | 3.92 | 3.59 | 3.28 | 3.09 | 15.02 | 4.78 | 3.84 | 3.48 | 3.16 | 2.96 |
| | SDRL | 8.30 | 2.22 | 1.75 | 1.55 | 1.38 | 1.30 | 12.34 | 2.73 | 1.99 | 1.71 | 1.49 | 1.36 |
| 1.50 | ARL | 5.55 | 2.59 | 2.19 | 2.01 | 1.84 | 1.74 | 5.80 | 2.46 | 2.08 | 1.92 | 1.77 | 1.67 |
| | SDRL | 2.83 | 1.06 | 0.90 | 0.84 | 0.77 | 0.73 | 3.55 | 1.05 | 0.84 | 0.76 | 0.69 | 0.65 |
| 2.00 | ARL | 3.55 | 1.71 | 1.45 | 1.34 | 1.25 | 1.19 | 3.42 | 1.66 | 1.44 | 1.33 | 1.24 | 1.19 |
| | SDRL | 1.53 | 0.72 | 0.58 | 0.52 | 0.46 | 0.40 | 1.68 | 0.65 | 0.55 | 0.49 | 0.44 | 0.40 |
| 2.50 | ARL | 2.57 | 1.27 | 1.12 | 1.07 | 1.04 | 1.02 | 2.43 | 1.26 | 1.12 | 1.07 | 1.04 | 1.02 |
| | SDRL | 1.07 | 0.47 | 0.33 | 0.26 | 0.20 | 0.15 | 1.04 | 0.45 | 0.33 | 0.26 | 0.20 | 0.15 |
| 3.00 | ARL | 1.99 | 1.07 | 1.02 | 1.01 | 1.00 | 1.00 | 1.89 | 1.07 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.82 | 0.25 | 0.13 | 0.08 | 0.05 | 0.03 | 0.75 | 0.25 | 0.13 | 0.09 | 0.04 | 0.03 |
| 4.00 | ARL | 1.33 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.32 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.51 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.03 | 0.01 | 0.00 | 0.00 | 0.00 |

Table 4.13: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS, SEWMA-RSS, and SEWMA-MRSS charts under imperfect ranking for $\lambda = 0.25$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) |
| | | SRS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS |
| $\delta$ | $L$ | 3.1560 | 3.1508 | 3.1515 | 3.1518 | 3.1508 | 3.1517 | 3.1524 | 3.1518 | 3.1519 | 3.1522 | 3.1572 | 3.1523 | 3.1523 |
| 0.00 | ARL | 500.72 | 501.58 | 499.93 | 499.64 | 500.07 | 499.47 | 502.31 | 500.49 | 499.89 | 500.76 | 499.44 | 500.21 | 500.96 |
| | SDRL | 500.86 | 498.78 | 496.30 | 500.16 | 501.82 | 493.61 | 499.13 | 498.04 | 496.68 | 499.31 | 495.83 | 500.79 | 495.48 |
| 0.25 | ARL | 206.35 | 200.30 | 200.03 | 199.38 | 180.19 | 180.16 | 176.13 | 147.35 | 139.91 | 132.65 | 115.52 | 102.44 | 90.41 |
| | SDRL | 200.96 | 197.01 | 194.92 | 195.98 | 176.22 | 175.74 | 172.66 | 143.83 | 136.35 | 127.58 | 110.53 | 98.66 | 85.64 |
| 0.50 | ARL | 59.56 | 56.90 | 56.63 | 56.12 | 48.61 | 47.74 | 46.07 | 35.61 | 33.16 | 31.06 | 25.95 | 22.02 | 19.29 |
| | SDRL | 54.98 | 52.50 | 51.94 | 51.41 | 44.01 | 43.35 | 41.59 | 31.23 | 28.70 | 26.74 | 21.74 | 17.75 | 15.13 |
| 0.75 | ARL | 23.67 | 22.45 | 22.32 | 22.26 | 19.43 | 18.69 | 18.49 | 14.38 | 13.38 | 12.47 | 10.70 | 9.33 | 8.27 |
| | SDRL | 19.38 | 18.26 | 18.20 | 18.18 | 15.28 | 14.53 | 14.23 | 10.46 | 9.58 | 8.70 | 7.10 | 5.91 | 5.02 |
| 1.00 | ARL | 12.43 | 11.91 | 11.83 | 11.64 | 10.35 | 10.22 | 9.96 | 7.99 | 7.54 | 7.12 | 6.21 | 5.53 | 4.97 |
| | SDRL | 8.70 | 8.19 | 8.18 | 8.19 | 6.82 | 6.69 | 6.47 | 4.80 | 4.43 | 4.09 | 3.36 | 2.88 | 2.46 |
| 1.50 | ARL | 5.77 | 5.55 | 5.54 | 5.54 | 4.98 | 4.90 | 4.80 | 4.01 | 3.81 | 3.64 | 3.24 | 2.92 | 2.66 |
| | SDRL | 3.03 | 2.90 | 2.88 | 2.88 | 2.49 | 2.43 | 2.36 | 1.87 | 1.76 | 1.67 | 1.46 | 1.31 | 1.20 |
| 2.00 | ARL | 3.65 | 3.53 | 3.51 | 3.50 | 3.19 | 3.14 | 3.09 | 2.59 | 2.46 | 2.35 | 2.10 | 1.87 | 1.71 |
| | SDRL | 1.67 | 1.61 | 1.59 | 1.60 | 1.43 | 1.41 | 1.38 | 1.16 | 1.12 | 1.07 | 0.97 | 0.87 | 0.79 |
| 2.50 | ARL | 2.60 | 2.51 | 2.51 | 2.50 | 2.27 | 2.23 | 2.20 | 1.84 | 1.74 | 1.66 | 1.47 | 1.34 | 1.24 |
| | SDRL | 1.17 | 1.14 | 1.13 | 1.13 | 1.04 | 1.02 | 1.01 | 0.86 | 0.81 | 0.76 | 0.66 | 0.56 | 0.47 |
| 3.00 | ARL | 1.96 | 1.90 | 1.88 | 1.88 | 1.71 | 1.68 | 1.65 | 1.39 | 1.32 | 1.28 | 1.17 | 1.10 | 1.06 |
| | SDRL | 0.90 | 0.88 | 0.88 | 0.87 | 0.79 | 0.78 | 0.76 | 0.60 | 0.55 | 0.51 | 0.40 | 0.30 | 0.23 |
| 4.00 | ARL | 1.28 | 1.24 | 1.24 | 1.23 | 1.16 | 1.14 | 1.13 | 1.05 | 1.03 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.50 | 0.47 | 0.47 | 0.47 | 0.38 | 0.37 | 0.35 | 0.21 | 0.18 | 0.15 | 0.08 | 0.05 | 0.02 |

Table 4.14: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS, SEWMA-RSS, and SEWMA-MRSS charts under imperfect ranking for $\lambda = 0.50$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) | (5,1) | (5,3) | (7,4) |
| | | SRS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS |
| $\delta$ | $L$ | 3.1630 | 3.1610 | 3.1616 | 3.1605 | 3.1612 | 3.1618 | 3.1605 | 3.1618 | 3.1618 | 3.1614 | 3.1698 | 3.1620 | 3.1616 |
| 0.00 | ARL | 500.21 | 499.54 | 501.09 | 501.43 | 501.20 | 501.61 | 500.30 | 499.81 | 501.07 | 500.26 | 501.87 | 499.66 | 499.90 |
| | SDRL | 500.15 | 498.66 | 501.07 | 500.30 | 499.34 | 500.00 | 499.62 | 499.83 | 501.63 | 199.35 | 503.45 | 500.42 | 497.46 |
| 0.25 | ARL | 280.77 | 275.75 | 275.01 | 274.52 | 256.99 | 255.46 | 249.31 | 218.94 | 212.17 | 202.75 | 183.85 | 163.41 | 148.17 |
| | SDRL | 279.12 | 271.65 | 273.12 | 273.64 | 256.16 | 252.17 | 248.06 | 217.01 | 209.68 | 52.86 | 181.19 | 161.92 | 145.99 |
| 0.50 | ARL | 104.08 | 99.15 | 98.10 | 97.87 | 86.00 | 84.34 | 82.64 | 64.23 | 59.50 | 55.22 | 46.54 | 38.57 | 33.40 |
| | SDRL | 102.02 | 97.44 | 97.78 | 96.02 | 83.93 | 82.14 | 81.03 | 61.75 | 57.03 | 17.71 | 44.52 | 36.11 | 31.10 |
| 0.75 | ARL | 41.73 | 39.16 | 39.10 | 39.04 | 33.18 | 32.48 | 31.75 | 23.54 | 21.41 | 19.91 | 16.21 | 13.54 | 11.57 |
| | SDRL | 39.42 | 36.50 | 37.17 | 36.98 | 30.94 | 30.26 | 29.45 | 21.37 | 19.29 | 7.43 | 14.05 | 11.46 | 9.55 |
| 1.00 | ARL | 19.72 | 18.73 | 18.58 | 18.40 | 15.76 | 15.20 | 14.79 | 11.02 | 10.09 | 9.39 | 7.73 | 6.56 | 5.70 |
| | SDRL | 17.58 | 16.80 | 16.48 | 16.33 | 13.58 | 13.09 | 12.73 | 8.98 | 8.11 | 3.77 | 5.84 | 4.72 | 3.94 |
| 1.50 | ARL | 7.02 | 6.68 | 6.64 | 6.56 | 5.75 | 5.58 | 5.47 | 4.29 | 3.98 | 3.75 | 3.23 | 2.85 | 2.57 |
| | SDRL | 5.13 | 4.82 | 4.84 | 4.75 | 3.98 | 3.81 | 3.72 | 2.70 | 2.43 | 1.49 | 1.81 | 1.51 | 1.31 |
| 2.00 | ARL | 3.75 | 3.59 | 3.57 | 3.56 | 3.17 | 3.10 | 3.04 | 2.49 | 2.35 | 2.24 | 1.97 | 1.78 | 1.63 |
| | SDRL | 2.22 | 2.10 | 2.07 | 2.07 | 1.76 | 1.70 | 1.68 | 1.25 | 1.16 | 0.84 | 0.92 | 0.80 | 0.71 |
| 2.50 | ARL | 2.49 | 2.41 | 2.39 | 2.39 | 2.15 | 2.11 | 2.08 | 1.75 | 1.67 | 1.60 | 1.43 | 1.32 | 1.22 |
| | SDRL | 1.25 | 1.21 | 1.18 | 1.19 | 1.03 | 1.01 | 0.98 | 0.78 | 0.73 | 0.57 | 0.59 | 0.51 | 0.44 |
| 3.00 | ARL | 1.86 | 1.80 | 1.79 | 1.79 | 1.64 | 1.61 | 1.59 | 1.36 | 1.30 | 1.26 | 1.16 | 1.10 | 1.05 |
| | SDRL | 0.85 | 0.81 | 0.80 | 0.80 | 0.72 | 0.70 | 0.68 | 0.54 | 0.50 | 0.37 | 0.38 | 0.30 | 0.23 |
| 4.00 | ARL | 1.26 | 1.23 | 1.23 | 1.22 | 1.15 | 1.14 | 1.13 | 1.04 | 1.03 | 1.02 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.47 | 0.45 | 0.44 | 0.44 | 0.36 | 0.35 | 0.34 | 0.21 | 0.18 | 0.15 | 0.09 | 0.05 | 0.03 |

Table 4.15: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS, SCUSUM-RSS, and SCUSUM-MRSS charts under imperfect ranking for $k = 0.50$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ |
| | | SRS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS |
| $\delta$ | $h$ | 5.2910 | 5.2915 | 5.2925 | 5.2935 | 5.2927 | 5.2925 | 5.2937 | 5.2919 | 5.2924 | 5.2931 | 5.2918 | 5.2914 | 5.2926 |
| 0.00 | ARL | 500.43 | 501.99 | 502.68 | 501.69 | 500.63 | 501.39 | 501.80 | 501.31 | 501.93 | 500.09 | 500.95 | 501.66 | 501.16 |
| | SDRL | 500.13 | 502.84 | 494.43 | 500.18 | 499.63 | 501.20 | 497.50 | 497.70 | 500.64 | 497.11 | 495.87 | 494.82 | 495.17 |
| 0.25 | ARL | 154.81 | 150.10 | 150.00 | 149.46 | 134.96 | 131.88 | 129.58 | 105.57 | 99.76 | 94.26 | 80.67 | 70.77 | 62.33 |
| | SDRL | 147.97 | 143.65 | 142.35 | 143.08 | 127.39 | 124.68 | 122.40 | 98.02 | 92.67 | 86.62 | 73.03 | 63.15 | 54.64 |
| 0.50 | ARL | 40.86 | 39.40 | 38.89 | 38.63 | 34.10 | 33.48 | 32.64 | 25.67 | 24.03 | 22.59 | 19.34 | 17.18 | 15.23 |
| | SDRL | 33.67 | 31.94 | 31.54 | 31.34 | 26.78 | 26.29 | 25.43 | 18.80 | 17.36 | 15.89 | 12.87 | 10.96 | 9.26 |
| 0.75 | ARL | 17.99 | 17.33 | 17.17 | 17.10 | 15.27 | 14.94 | 14.80 | 12.02 | 11.41 | 10.86 | 9.60 | 8.64 | 7.86 |
| | SDRL | 11.83 | 11.15 | 10.84 | 10.94 | 9.31 | 9.04 | 8.90 | 6.61 | 6.18 | 5.73 | 4.83 | 4.15 | 3.61 |
| 1.00 | ARL | 10.81 | 10.48 | 10.41 | 10.31 | 9.40 | 9.23 | 9.15 | 7.70 | 7.31 | 6.97 | 6.22 | 5.70 | 5.21 |
| | SDRL | 5.71 | 5.46 | 5.41 | 5.43 | 4.67 | 4.55 | 4.45 | 3.51 | 3.27 | 3.06 | 2.62 | 2.31 | 2.08 |
| 1.50 | ARL | 5.89 | 5.73 | 5.73 | 5.70 | 5.22 | 5.16 | 5.10 | 4.36 | 4.16 | 4.01 | 3.59 | 3.29 | 3.01 |
| | SDRL | 2.42 | 2.34 | 2.34 | 2.32 | 2.08 | 2.05 | 2.01 | 1.68 | 1.61 | 1.54 | 1.40 | 1.31 | 1.22 |
| 2.00 | ARL | 3.98 | 3.89 | 3.88 | 3.86 | 3.55 | 3.49 | 3.45 | 2.94 | 2.81 | 2.68 | 2.38 | 2.14 | 1.93 |
| | SDRL | 1.53 | 1.50 | 1.48 | 1.50 | 1.38 | 1.36 | 1.36 | 1.20 | 1.16 | 1.13 | 1.05 | 0.98 | 0.91 |
| 2.50 | ARL | 2.93 | 2.85 | 2.85 | 2.83 | 2.59 | 2.54 | 2.51 | 2.09 | 1.97 | 1.88 | 1.64 | 1.47 | 1.33 |
| | SDRL | 1.20 | 1.17 | 1.17 | 1.17 | 1.10 | 1.08 | 1.08 | 0.96 | 0.92 | 0.89 | 0.78 | 0.69 | 0.58 |
| 3.00 | ARL | 2.23 | 2.17 | 2.15 | 2.14 | 1.93 | 1.91 | 1.87 | 1.54 | 1.45 | 1.39 | 1.24 | 1.14 | 1.08 |
| | SDRL | 1.01 | 0.99 | 0.98 | 0.98 | 0.91 | 0.90 | 0.89 | 0.72 | 0.67 | 0.63 | 0.49 | 0.38 | 0.29 |
| 4.00 | ARL | 1.38 | 1.34 | 1.34 | 1.33 | 1.22 | 1.20 | 1.19 | 1.07 | 1.05 | 1.04 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.62 | 0.59 | 0.58 | 0.58 | 0.47 | 0.45 | 0.44 | 0.26 | 0.22 | 0.19 | 0.11 | 0.06 | 0.03 |

Table 4.16: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS, SCUSUM-RSS, and SCUSUM-MRSS charts under imperfect ranking for $k = 1.00$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ | $(5, 1)$ | $(5, 3)$ | $(7, 4)$ |
| | | SRS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS | RSS | MRSS | VLRSS |
| $\delta$ | $h$ | 2.7150 | 2.7170 | 2.7164 | 2.7180 | 2.7170 | 2.7167 | 2.7179 | 2.7170 | 2.7165 | 2.7175 | 2.7164 | 2.7161 | 2.7171 |
| 0.00 | ARL | 499.85 | 502.53 | 501.04 | 500.31 | 500.10 | 500.47 | 502.48 | 501.94 | 501.22 | 501.83 | 499.87 | 500.30 | 500.98 |
| | SDRL | 500.34 | 502.33 | 496.84 | 501.88 | 500.49 | 504.26 | 503.32 | 496.91 | 502.42 | 496.23 | 497.76 | 496.09 | 495.03 |
| 0.25 | ARL | 255.35 | 250.12 | 250.02 | 249.97 | 232.82 | 228.11 | 225.23 | 195.12 | 184.27 | 177.46 | 156.55 | 139.32 | 124.66 |
| | SDRL | 252.81 | 248.42 | 248.70 | 247.65 | 232.71 | 225.29 | 222.44 | 193.43 | 181.48 | 173.93 | 153.93 | 136.86 | 121.59 |
| 0.50 | ARL | 83.74 | 80.89 | 79.92 | 79.80 | 69.41 | 67.76 | 66.35 | 50.33 | 46.56 | 43.25 | 35.22 | 29.81 | 25.07 |
| | SDRL | 81.36 | 78.17 | 77.17 | 77.05 | 66.91 | 65.15 | 63.78 | 47.51 | 43.79 | 40.48 | 32.44 | 26.91 | 22.40 |
| 0.75 | ARL | 31.82 | 30.18 | 29.96 | 29.84 | 25.14 | 24.50 | 24.19 | 17.75 | 16.14 | 14.96 | 12.27 | 10.46 | 9.04 |
| | SDRL | 29.24 | 27.45 | 27.20 | 27.21 | 22.49 | 21.76 | 21.28 | 15.06 | 13.50 | 12.26 | 9.65 | 7.88 | 6.54 |
| 1.00 | ARL | 15.02 | 14.19 | 14.13 | 14.05 | 11.99 | 11.67 | 11.46 | 8.62 | 8.02 | 7.49 | 6.32 | 5.49 | 4.89 |
| | SDRL | 12.34 | 11.49 | 11.49 | 11.40 | 9.32 | 9.01 | 8.88 | 6.17 | 5.57 | 5.16 | 4.04 | 3.36 | 2.79 |
| 1.50 | ARL | 5.80 | 5.60 | 5.58 | 5.51 | 4.90 | 4.80 | 4.70 | 3.81 | 3.61 | 3.44 | 3.03 | 2.74 | 2.50 |
| | SDRL | 3.55 | 3.38 | 3.37 | 3.37 | 2.84 | 2.74 | 2.64 | 1.96 | 1.82 | 1.69 | 1.40 | 1.22 | 1.08 |
| 2.00 | ARL | 3.42 | 3.30 | 3.30 | 3.30 | 2.98 | 2.93 | 2.89 | 2.44 | 2.32 | 2.23 | 2.00 | 1.83 | 1.69 |
| | SDRL | 1.68 | 1.59 | 1.60 | 1.59 | 1.38 | 1.34 | 1.32 | 1.04 | 0.97 | 0.92 | 0.80 | 0.72 | 0.66 |
| 2.50 | ARL | 2.43 | 2.36 | 2.35 | 2.35 | 2.15 | 2.11 | 2.09 | 1.80 | 1.72 | 1.65 | 1.50 | 1.38 | 1.28 |
| | SDRL | 1.04 | 1.00 | 0.99 | 0.98 | 0.88 | 0.86 | 0.85 | 0.70 | 0.67 | 0.64 | 0.57 | 0.52 | 0.46 |
| 3.00 | ARL | 1.89 | 1.84 | 1.83 | 1.83 | 1.69 | 1.67 | 1.65 | 1.43 | 1.37 | 1.32 | 1.21 | 1.13 | 1.08 |
| | SDRL | 0.75 | 0.72 | 0.72 | 0.72 | 0.66 | 0.65 | 0.64 | 0.54 | 0.52 | 0.49 | 0.41 | 0.34 | 0.27 |
| 4.00 | ARL | 1.32 | 1.29 | 1.28 | 1.28 | 1.19 | 1.18 | 1.17 | 1.07 | 1.05 | 1.03 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.48 | 0.47 | 0.47 | 0.46 | 0.40 | 0.39 | 0.38 | 0.25 | 0.21 | 0.18 | 0.11 | 0.06 | 0.04 |

Table 4.17: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS chart

| | | $\lambda = 0.05$ | | | | $\lambda = 0.10$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | (2,1) | (3,2) | (4,2) | | (2,1) | (3,2) | (4,2) |
| $\delta$ | | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 499.66 | 499.32 | 500.62 | 499.16 | 500.37 | 500.76 | 500.68 | 500.31 |
| | SDRL | 494.39 | 486.43 | 492.10 | 491.01 | 494.51 | 496.14 | 495.93 | 494.79 |
| 0.25 | ARL | 100.51 | 52.99 | 43.71 | 40.14 | 131.40 | 66.29 | 52.12 | 47.97 |
| | SDRL | 82.86 | 37.77 | 29.17 | 25.84 | 121.68 | 55.93 | 42.36 | 38.18 |
| 0.50 | ARL | 32.60 | 18.20 | 15.53 | 14.52 | 36.68 | 18.09 | 14.95 | 13.76 |
| | SDRL | 19.58 | 8.69 | 6.87 | 6.28 | 27.23 | 10.73 | 8.28 | 7.33 |
| 0.75 | ARL | 17.80 | 10.64 | 9.14 | 8.58 | 17.66 | 9.59 | 8.07 | 7.54 |
| | SDRL | 8.38 | 4.20 | 3.50 | 3.27 | 10.37 | 4.40 | 3.53 | 3.20 |
| 1.00 | ARL | 12.12 | 7.34 | 6.30 | 5.86 | 11.19 | 6.39 | 5.44 | 5.08 |
| | SDRL | 4.94 | 2.82 | 2.48 | 2.36 | 5.46 | 2.62 | 2.23 | 2.08 |
| 1.50 | ARL | 7.17 | 4.11 | 3.38 | 3.07 | 6.27 | 3.61 | 3.01 | 2.77 |
| | SDRL | 2.77 | 1.94 | 1.75 | 1.65 | 2.58 | 1.60 | 1.42 | 1.35 |
| 2.00 | ARL | 4.83 | 2.39 | 1.88 | 1.69 | 4.21 | 2.24 | 1.80 | 1.63 |
| | SDRL | 2.12 | 1.39 | 1.10 | 0.97 | 1.77 | 1.16 | 0.95 | 0.85 |
| 2.50 | ARL | 3.37 | 1.51 | 1.25 | 1.16 | 3.00 | 1.48 | 1.24 | 1.16 |
| | SDRL | 1.73 | 0.83 | 0.54 | 0.44 | 1.42 | 0.74 | 0.52 | 0.42 |
| 3.00 | ARL | 2.34 | 1.14 | 1.05 | 1.03 | 2.19 | 1.14 | 1.05 | 1.03 |
| | SDRL | 1.36 | 0.40 | 0.22 | 0.17 | 1.14 | 0.39 | 0.22 | 0.16 |
| 4.00 | ARL | 1.32 | 1.00 | 1.00 | 1.00 | 1.31 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.63 | 0.06 | 0.02 | 0.01 | 0.59 | 0.06 | 0.02 | 0.01 |
| | | $\lambda = 0.25$ | | | | $\lambda = 0.50$ | | | |
| | | $(l, v)$ | (2,1) | (3,2) | (4,2) | | (2,1) | (3,2) | (4,2) |
| $\delta$ | | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.72 | 500.97 | 501.83 | 502.89 | 500.21 | 500.45 | 500.41 | 500.25 |
| | SDRL | 500.86 | 496.30 | 497.12 | 501.76 | 500.15 | 501.39 | 500.87 | 503.79 |
| 0.25 | ARL | 206.35 | 112.27 | 89.38 | 80.32 | 280.77 | 175.36 | 145.23 | 136.20 |
| | SDRL | 200.96 | 108.33 | 85.21 | 75.99 | 279.12 | 172.28 | 143.50 | 134.42 |
| 0.50 | ARL | 59.56 | 24.45 | 18.90 | 16.81 | 104.08 | 43.67 | 32.37 | 28.74 |
| | SDRL | 54.98 | 20.19 | 14.74 | 12.75 | 102.02 | 41.58 | 30.11 | 26.79 |
| 0.75 | ARL | 23.67 | 10.12 | 8.05 | 7.32 | 41.73 | 15.30 | 11.19 | 9.88 |
| | SDRL | 19.38 | 6.57 | 4.78 | 4.19 | 39.42 | 13.18 | 9.17 | 7.91 |
| 1.00 | ARL | 12.43 | 5.92 | 4.89 | 4.50 | 19.72 | 7.32 | 5.55 | 4.98 |
| | SDRL | 8.70 | 3.14 | 2.44 | 2.17 | 17.58 | 5.46 | 3.81 | 3.29 |
| 1.50 | ARL | 5.77 | 3.10 | 2.61 | 2.42 | 7.02 | 3.07 | 2.49 | 2.30 |
| | SDRL | 3.03 | 1.40 | 1.17 | 1.10 | 5.13 | 1.67 | 1.26 | 1.13 |
| 2.00 | ARL | 3.65 | 2.00 | 1.67 | 1.53 | 3.75 | 1.89 | 1.60 | 1.49 |
| | SDRL | 1.67 | 0.93 | 0.77 | 0.70 | 2.22 | 0.87 | 0.69 | 0.62 |
| 2.50 | ARL | 2.60 | 1.41 | 1.21 | 1.15 | 2.49 | 1.38 | 1.20 | 1.14 |
| | SDRL | 1.17 | 0.61 | 0.45 | 0.38 | 1.25 | 0.56 | 0.42 | 0.36 |
| 3.00 | ARL | 1.96 | 1.13 | 1.05 | 1.03 | 1.86 | 1.13 | 1.05 | 1.03 |
| | SDRL | 0.90 | 0.36 | 0.21 | 0.16 | 0.85 | 0.34 | 0.21 | 0.16 |
| 4.00 | ARL | 1.28 | 1.00 | 1.00 | 1.00 | 1.26 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.50 | 0.07 | 0.02 | 0.01 | 0.47 | 0.07 | 0.03 | 0.01 |

Table 4.18: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS chart under imperfect ranking for $\lambda = 0.25$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (0,0) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $L$ | 3.1560 | 3.1505 | 3.1505 | 3.1508 | 3.1515 | 3.1511 | 3.1510 | 3.1514 | 3.1512 | 3.1512 | 3.1519 | 3.1519 | 3.1520 |
| 0.00 | ARL | 500.72 | 501.16 | 499.86 | 500.46 | 500.09 | 499.78 | 499.80 | 499.73 | 500.05 | 501.67 | 499.77 | 501.17 | 500.47 |
| | SDRL | 500.86 | 496.30 | 496.88 | 496.95 | 492.92 | 495.39 | 493.58 | 495.24 | 492.21 | 494.59 | 494.96 | 499.63 | 497.19 |
| 0.25 | ARL | 206.35 | 201.12 | 199.02 | 198.13 | 185.43 | 180.99 | 179.38 | 156.63 | 148.55 | 142.21 | 131.05 | 115.14 | 109.16 |
| | SDRL | 200.96 | 197.13 | 195.72 | 195.03 | 181.49 | 176.87 | 175.81 | 153.83 | 143.06 | 137.80 | 126.76 | 110.68 | 104.09 |
| 0.50 | ARL | 59.56 | 57.06 | 56.75 | 56.59 | 50.30 | 49.06 | 48.14 | 39.36 | 35.89 | 34.39 | 30.74 | 25.76 | 24.13 |
| | SDRL | 54.98 | 52.78 | 52.19 | 51.57 | 45.81 | 44.80 | 43.36 | 34.63 | 31.40 | 29.79 | 26.08 | 21.35 | 19.87 |
| 0.75 | ARL | 23.67 | 22.54 | 22.34 | 22.27 | 20.00 | 19.22 | 19.01 | 15.66 | 14.28 | 13.79 | 12.34 | 10.63 | 9.99 |
| | SDRL | 19.38 | 18.25 | 18.15 | 18.00 | 15.74 | 15.05 | 15.02 | 11.59 | 10.44 | 9.90 | 8.59 | 7.01 | 6.46 |
| 1.00 | ARL | 12.43 | 12.04 | 11.95 | 11.88 | 10.73 | 10.41 | 10.32 | 8.56 | 8.01 | 7.77 | 7.05 | 6.21 | 5.88 |
| | SDRL | 8.70 | 8.30 | 8.23 | 8.16 | 7.13 | 6.86 | 6.74 | 5.25 | 4.78 | 4.62 | 4.03 | 3.34 | 3.12 |
| 1.50 | ARL | 5.77 | 5.60 | 5.57 | 5.53 | 5.10 | 4.98 | 4.92 | 4.27 | 4.01 | 3.92 | 3.61 | 3.24 | 3.10 |
| | SDRL | 3.03 | 2.93 | 2.92 | 2.88 | 2.56 | 2.50 | 2.43 | 2.03 | 1.87 | 1.81 | 1.64 | 1.46 | 1.40 |
| 2.00 | ARL | 3.65 | 3.55 | 3.53 | 3.52 | 3.26 | 3.19 | 3.17 | 2.75 | 2.60 | 2.53 | 2.34 | 2.09 | 1.99 |
| | SDRL | 1.67 | 1.62 | 1.61 | 1.61 | 1.47 | 1.43 | 1.43 | 1.23 | 1.17 | 1.14 | 1.07 | 0.97 | 0.92 |
| 2.50 | ARL | 2.60 | 2.53 | 2.51 | 2.50 | 2.32 | 2.27 | 2.26 | 1.95 | 1.84 | 1.78 | 1.64 | 1.48 | 1.41 |
| | SDRL | 1.17 | 1.14 | 1.14 | 1.13 | 1.06 | 1.04 | 1.03 | 0.91 | 0.85 | 0.82 | 0.76 | 0.66 | 0.61 |
| 3.00 | ARL | 1.96 | 1.90 | 1.89 | 1.89 | 1.75 | 1.70 | 1.69 | 1.47 | 1.39 | 1.36 | 1.27 | 1.16 | 1.13 |
| | SDRL | 0.90 | 0.88 | 0.88 | 0.88 | 0.81 | 0.79 | 0.78 | 0.66 | 0.60 | 0.57 | 0.50 | 0.39 | 0.35 |
| 4.00 | ARL | 1.28 | 1.24 | 1.24 | 1.24 | 1.17 | 1.15 | 1.14 | 1.07 | 1.05 | 1.04 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.50 | 0.48 | 0.47 | 0.47 | 0.40 | 0.38 | 0.37 | 0.26 | 0.21 | 0.20 | 0.14 | 0.08 | 0.06 |

Table 4.19: The run length comparison of the SEWMA-VLRSS chart with the SEWMA-SRS chart under imperfect ranking for $\lambda = 0.50$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $(l, v)$ | (0,0) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) | (2,1) | (3,2) | (4,2) |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $L$ | 3.1630 | 3.1608 | 3.1608 | 3.1610 | 3.1612 | 3.1610 | 3.1609 | 3.1611 | 3.1613 | 3.1609 | 3.1613 | 3.1615 | 3.1614 |
| 0.00 | ARL | 500.21 | 499.54 | 501.40 | 501.38 | 501.75 | 501.73 | 502.13 | 501.38 | 499.49 | 501.38 | 500.44 | 499.44 | 499.65 |
| | SDRL | 500.15 | 497.48 | 500.64 | 500.82 | 500.77 | 500.91 | 501.47 | 502.60 | 498.68 | 500.10 | 498.68 | 497.10 | 497.37 |
| 0.25 | ARL | 280.77 | 275.61 | 273.90 | 273.46 | 261.06 | 256.73 | 255.03 | 230.60 | 219.25 | 215.06 | 200.01 | 180.57 | 172.63 |
| | SDRL | 279.12 | 273.71 | 272.77 | 270.02 | 258.84 | 255.75 | 252.10 | 227.76 | 217.89 | 213.85 | 198.81 | 178.68 | 170.22 |
| 0.50 | ARL | 104.08 | 99.31 | 99.01 | 98.86 | 89.20 | 86.63 | 85.52 | 70.25 | 64.40 | 62.32 | 54.40 | 45.98 | 42.84 |
| | SDRL | 102.02 | 96.68 | 97.06 | 96.73 | 86.82 | 84.81 | 83.84 | 67.88 | 62.72 | 60.25 | 52.32 | 43.81 | 40.70 |
| 0.75 | ARL | 41.73 | 39.78 | 39.42 | 39.18 | 34.45 | 33.39 | 32.95 | 26.13 | 23.58 | 22.37 | 19.49 | 16.15 | 14.81 |
| | SDRL | 39.42 | 37.17 | 37.76 | 36.92 | 32.28 | 31.17 | 31.07 | 24.10 | 21.39 | 20.22 | 17.40 | 13.91 | 12.70 |
| 1.00 | ARL | 19.72 | 18.87 | 18.71 | 18.63 | 16.15 | 15.79 | 15.45 | 12.21 | 11.03 | 10.60 | 9.24 | 7.76 | 7.18 |
| | SDRL | 17.58 | 16.81 | 16.64 | 16.54 | 14.11 | 13.71 | 13.37 | 10.15 | 9.02 | 8.60 | 7.30 | 5.89 | 5.29 |
| 1.50 | ARL | 7.02 | 6.70 | 6.66 | 6.63 | 5.93 | 5.73 | 5.66 | 4.62 | 4.27 | 4.14 | 3.70 | 3.24 | 3.06 |
| | SDRL | 5.13 | 4.83 | 4.79 | 4.81 | 4.15 | 3.96 | 3.89 | 2.95 | 2.66 | 2.52 | 2.19 | 1.82 | 1.66 |
| 2.00 | ARL | 3.75 | 3.61 | 3.58 | 3.56 | 3.25 | 3.16 | 3.13 | 2.66 | 2.48 | 2.42 | 2.22 | 1.98 | 1.89 |
| | SDRL | 2.22 | 2.11 | 2.08 | 2.08 | 1.83 | 1.76 | 1.73 | 1.39 | 1.25 | 1.22 | 1.07 | 0.92 | 0.86 |
| 2.50 | ARL | 2.49 | 2.41 | 2.41 | 2.40 | 2.20 | 2.15 | 2.14 | 1.85 | 1.75 | 1.70 | 1.58 | 1.44 | 1.38 |
| | SDRL | 1.25 | 1.21 | 1.20 | 1.21 | 1.05 | 1.03 | 1.02 | 0.85 | 0.78 | 0.75 | 0.68 | 0.59 | 0.55 |
| 3.00 | ARL | 1.86 | 1.80 | 1.80 | 1.79 | 1.66 | 1.63 | 1.62 | 1.43 | 1.36 | 1.33 | 1.26 | 1.16 | 1.13 |
| | SDRL | 0.85 | 0.81 | 0.81 | 0.81 | 0.73 | 0.71 | 0.70 | 0.59 | 0.54 | 0.52 | 0.46 | 0.38 | 0.34 |
| 4.00 | ARL | 1.26 | 1.23 | 1.23 | 1.23 | 1.17 | 1.15 | 1.14 | 1.07 | 1.05 | 1.04 | 1.02 | 1.01 | 1.00 |
| | SDRL | 0.47 | 0.44 | 0.44 | 0.44 | 0.38 | 0.36 | 0.35 | 0.26 | 0.21 | 0.20 | 0.14 | 0.08 | 0.07 |

Table 4.20: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS chart

| | | $k = 0.25$ | | | | $k = 0.50$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| $\delta$ | $h$ | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 500.86 | 500.18 | 501.73 | 500.67 | 500.43 | 500.20 | 500.71 | 500.14 |
| | SDRL | 490.40 | 492.28 | 484.81 | 488.69 | 500.13 | 498.31 | 503.18 | 499.89 |
| 0.25 | ARL | 99.60 | 52.45 | 43.30 | 39.93 | 154.81 | 76.63 | 60.27 | 54.50 |
| | SDRL | 81.21 | 36.17 | 28.12 | 24.94 | 147.97 | 69.39 | 52.27 | 46.70 |
| 0.50 | ARL | 32.44 | 18.53 | 15.86 | 14.86 | 40.86 | 18.42 | 14.86 | 13.58 |
| | SDRL | 18.57 | 8.21 | 6.62 | 6.01 | 33.67 | 12.04 | 8.94 | 7.88 |
| 0.75 | ARL | 18.17 | 10.99 | 9.55 | 8.96 | 17.99 | 9.18 | 7.68 | 7.13 |
| | SDRL | 8.01 | 4.00 | 3.36 | 3.13 | 11.83 | 4.48 | 3.49 | 3.15 |
| 1.00 | ARL | 12.49 | 7.72 | 6.67 | 6.22 | 10.81 | 6.02 | 5.11 | 4.77 |
| | SDRL | 4.77 | 2.67 | 2.35 | 2.23 | 5.71 | 2.50 | 2.03 | 1.85 |
| 1.50 | ARL | 7.56 | 4.47 | 3.71 | 3.40 | 5.89 | 3.46 | 2.94 | 2.74 |
| | SDRL | 2.61 | 1.88 | 1.74 | 1.66 | 2.42 | 1.35 | 1.20 | 1.14 |
| 2.00 | ARL | 5.21 | 2.69 | 2.11 | 1.89 | 3.98 | 2.27 | 1.88 | 1.73 |
| | SDRL | 2.00 | 1.46 | 1.21 | 1.09 | 1.53 | 1.02 | 0.89 | 0.83 |
| 2.50 | ARL | 3.70 | 1.67 | 1.34 | 1.23 | 2.93 | 1.57 | 1.30 | 1.22 |
| | SDRL | 1.73 | 0.94 | 0.65 | 0.53 | 1.20 | 0.75 | 0.55 | 0.47 |
| 3.00 | ARL | 2.62 | 1.21 | 1.07 | 1.04 | 2.23 | 1.19 | 1.07 | 1.04 |
| | SDRL | 1.43 | 0.49 | 0.28 | 0.21 | 1.01 | 0.44 | 0.27 | 0.20 |
| 4.00 | ARL | 1.43 | 1.01 | 1.00 | 1.00 | 1.38 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.74 | 0.09 | 0.03 | 0.01 | 0.62 | 0.09 | 0.03 | 0.01 |

| | | $k = 0.75$ | | | | $k = 1.00$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| $\delta$ | $h$ | SRS | VLRSS | VLRSS | VLRSS | SRS | VLRSS | VLRSS | VLRSS |
| 0.00 | ARL | 501.84 | 502.50 | 502.97 | 500.93 | 499.85 | 499.56 | 500.75 | 502.78 |
| | SDRL | 498.78 | 499.33 | 500.89 | 501.22 | 500.34 | 498.18 | 497.22 | 502.07 |
| 0.25 | ARL | 210.92 | 113.71 | 90.64 | 80.81 | 255.35 | 149.44 | 121.97 | 111.38 |
| | SDRL | 207.75 | 109.91 | 87.06 | 75.61 | 252.81 | 147.02 | 118.80 | 109.08 |
| 0.50 | ARL | 60.35 | 23.70 | 18.04 | 16.04 | 83.74 | 32.93 | 24.29 | 21.36 |
| | SDRL | 56.15 | 19.66 | 13.98 | 12.11 | 81.36 | 30.11 | 21.43 | 18.65 |
| 0.75 | ARL | 22.98 | 9.64 | 7.66 | 6.98 | 31.82 | 11.45 | 8.71 | 7.70 |
| | SDRL | 18.98 | 6.18 | 4.45 | 3.94 | 29.24 | 8.80 | 6.24 | 5.29 |
| 1.00 | ARL | 11.94 | 5.67 | 4.69 | 4.33 | 15.02 | 5.95 | 4.75 | 4.31 |
| | SDRL | 8.30 | 2.89 | 2.23 | 1.99 | 12.34 | 3.74 | 2.70 | 2.36 |
| 1.50 | ARL | 5.55 | 3.04 | 2.59 | 2.40 | 5.80 | 2.89 | 2.44 | 2.27 |
| | SDRL | 2.83 | 1.26 | 1.07 | 0.99 | 3.55 | 1.31 | 1.04 | 0.95 |
| 2.00 | ARL | 3.55 | 2.02 | 1.72 | 1.59 | 3.42 | 1.92 | 1.66 | 1.55 |
| | SDRL | 1.53 | 0.84 | 0.71 | 0.66 | 1.68 | 0.76 | 0.65 | 0.60 |
| 2.50 | ARL | 2.57 | 1.47 | 1.27 | 1.19 | 2.43 | 1.45 | 1.26 | 1.19 |
| | SDRL | 1.07 | 0.59 | 0.47 | 0.41 | 1.04 | 0.55 | 0.45 | 0.40 |
| 3.00 | ARL | 1.99 | 1.18 | 1.07 | 1.04 | 1.89 | 1.17 | 1.07 | 1.04 |
| | SDRL | 0.82 | 0.39 | 0.25 | 0.19 | 0.75 | 0.38 | 0.25 | 0.19 |
| 4.00 | ARL | 1.33 | 1.01 | 1.00 | 1.00 | 1.32 | 1.01 | 1.00 | 1.00 |
| | SDRL | 0.51 | 0.09 | 0.03 | 0.01 | 0.48 | 0.08 | 0.03 | 0.02 |

Table 4.21: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS chart under imperfect ranking for $k = 0.50$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 5.2910 | 5.2911 | 5.2916 | 5.2918 | 5.2919 | 5.2913 | 5.2914 | 5.2915 | 5.2905 | 5.2911 | 5.2908 | 5.2914 | 5.2916 |
| 0.00 | ARL | 500.43 | 501.66 | 501.96 | 500.13 | 501.84 | 500.72 | 501.11 | 500.76 | 501.56 | 500.89 | 500.00 | 501.15 | 499.63 |
| | SDRL | 500.13 | 494.04 | 495.18 | 496.14 | 500.20 | 497.93 | 500.57 | 497.06 | 500.39 | 501.72 | 491.27 | 493.49 | 495.51 |
| 0.25 | ARL | 154.81 | 151.13 | 149.58 | 149.27 | 136.73 | 133.23 | 132.38 | 113.21 | 106.37 | 101.97 | 92.35 | 81.37 | 76.09 |
| | SDRL | 147.97 | 144.07 | 142.20 | 143.31 | 130.63 | 125.65 | 125.29 | 105.38 | 99.58 | 94.52 | 84.91 | 73.47 | 67.76 |
| 0.50 | ARL | 40.86 | 39.52 | 39.18 | 38.73 | 34.76 | 34.14 | 33.74 | 27.71 | 25.64 | 24.80 | 22.35 | 19.57 | 18.29 |
| | SDRL | 33.67 | 32.23 | 31.90 | 31.51 | 27.35 | 27.02 | 26.59 | 20.57 | 18.79 | 18.09 | 15.70 | 13.14 | 12.05 |
| 0.75 | ARL | 17.99 | 17.35 | 17.32 | 17.30 | 15.62 | 15.27 | 15.15 | 12.88 | 12.03 | 11.75 | 10.70 | 9.60 | 9.10 |
| | SDRL | 11.83 | 11.11 | 11.15 | 11.15 | 9.61 | 9.40 | 9.24 | 7.32 | 6.62 | 6.40 | 5.61 | 4.79 | 4.45 |
| 1.00 | ARL | 10.81 | 10.52 | 10.48 | 10.44 | 9.63 | 9.42 | 9.32 | 8.11 | 7.67 | 7.47 | 6.93 | 6.24 | 6.00 |
| | SDRL | 5.71 | 5.51 | 5.48 | 5.47 | 4.86 | 4.69 | 4.59 | 3.77 | 3.51 | 3.36 | 3.01 | 2.61 | 2.48 |
| 1.50 | ARL | 5.89 | 5.76 | 5.76 | 5.72 | 5.33 | 5.25 | 5.18 | 4.59 | 4.37 | 4.25 | 3.97 | 3.59 | 3.44 |
| | SDRL | 2.42 | 2.36 | 2.36 | 2.34 | 2.13 | 2.09 | 2.07 | 1.80 | 1.69 | 1.65 | 1.53 | 1.40 | 1.35 |
| 2.00 | ARL | 3.98 | 3.90 | 3.89 | 3.86 | 3.62 | 3.55 | 3.52 | 3.10 | 2.94 | 2.87 | 2.66 | 2.38 | 2.27 |
| | SDRL | 1.53 | 1.51 | 1.49 | 1.49 | 1.40 | 1.39 | 1.37 | 1.25 | 1.20 | 1.18 | 1.12 | 1.05 | 1.02 |
| 2.50 | ARL | 2.93 | 2.86 | 2.86 | 2.85 | 2.62 | 2.58 | 2.57 | 2.24 | 2.10 | 2.04 | 1.86 | 1.64 | 1.57 |
| | SDRL | 1.20 | 1.17 | 1.17 | 1.18 | 1.12 | 1.10 | 1.10 | 1.00 | 0.97 | 0.95 | 0.88 | 0.78 | 0.74 |
| 3.00 | ARL | 2.23 | 2.17 | 2.16 | 2.16 | 1.98 | 1.93 | 1.93 | 1.64 | 1.54 | 1.50 | 1.38 | 1.24 | 1.19 |
| | SDRL | 1.01 | 0.98 | 0.99 | 0.98 | 0.93 | 0.91 | 0.90 | 0.79 | 0.73 | 0.70 | 0.61 | 0.49 | 0.44 |
| 4.00 | ARL | 1.38 | 1.35 | 1.34 | 1.34 | 1.24 | 1.22 | 1.21 | 1.10 | 1.07 | 1.06 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.62 | 0.59 | 0.59 | 0.59 | 0.50 | 0.47 | 0.47 | 0.32 | 0.26 | 0.24 | 0.18 | 0.11 | 0.09 |

Table 4.22: The run length comparison of the SCUSUM-VLRSS chart with the SCUSUM-SRS chart under imperfect ranking for $k = 1.00$

| | | | $\rho = 0.25$ | | | $\rho = 0.50$ | | | $\rho = 0.75$ | | | $\rho = 0.90$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | | $w = 0$ | $w = 2$ | |
| | | $(l, v)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ | $(2, 1)$ | $(3, 2)$ | $(4, 2)$ |
| | | SRS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS | VLRSS |
| $\delta$ | $h$ | 2.7150 | 2.7154 | 2.7158 | 2.7162 | 2.7166 | 2.7160 | 2.7169 | 2.7163 | 2.7155 | 2.7169 | 2.7150 | 2.7159 | 2.7161 |
| 0.00 | ARL | 499.85 | 501.46 | 499.97 | 501.19 | 501.62 | 502.54 | 501.85 | 502.93 | 501.76 | 501.09 | 501.17 | 500.28 | 500.08 |
| | SDRL | 500.34 | 500.51 | 494.31 | 499.77 | 501.33 | 500.99 | 500.82 | 503.07 | 500.31 | 502.34 | 489.54 | 499.82 | 498.26 |
| 0.25 | ARL | 255.35 | 250.90 | 249.01 | 247.75 | 236.66 | 232.07 | 230.74 | 204.30 | 194.32 | 188.69 | 174.01 | 156.48 | 148.18 |
| | SDRL | 252.81 | 249.15 | 248.09 | 244.18 | 234.52 | 229.64 | 229.62 | 203.10 | 192.35 | 184.96 | 171.42 | 152.64 | 144.56 |
| 0.50 | ARL | 83.74 | 80.30 | 80.22 | 80.04 | 71.41 | 69.64 | 68.25 | 55.42 | 50.06 | 48.52 | 42.51 | 35.20 | 32.62 |
| | SDRL | 81.36 | 77.39 | 77.28 | 77.95 | 69.03 | 66.87 | 65.35 | 52.75 | 47.47 | 45.90 | 39.62 | 32.49 | 29.79 |
| 0.75 | ARL | 31.82 | 30.43 | 30.19 | 29.94 | 26.20 | 25.32 | 24.98 | 19.73 | 17.72 | 17.01 | 14.76 | 12.27 | 11.37 |
| | SDRL | 29.24 | 27.68 | 27.46 | 27.29 | 23.55 | 22.62 | 22.28 | 17.03 | 15.03 | 14.30 | 11.97 | 9.65 | 8.82 |
| 1.00 | ARL | 15.02 | 14.26 | 14.26 | 14.17 | 12.44 | 11.97 | 11.83 | 9.47 | 8.66 | 8.39 | 7.41 | 6.32 | 5.94 |
| | SDRL | 12.34 | 11.54 | 11.53 | 11.59 | 9.79 | 9.37 | 9.23 | 6.99 | 6.20 | 5.97 | 5.01 | 4.06 | 3.72 |
| 1.50 | ARL | 5.80 | 5.62 | 5.59 | 5.57 | 5.03 | 4.90 | 4.84 | 4.11 | 3.81 | 3.73 | 3.39 | 3.02 | 2.88 |
| | SDRL | 3.55 | 3.42 | 3.43 | 3.38 | 2.95 | 2.80 | 2.76 | 2.18 | 1.97 | 1.90 | 1.66 | 1.40 | 1.30 |
| 2.00 | ARL | 3.42 | 3.34 | 3.31 | 3.29 | 3.06 | 2.98 | 2.96 | 2.58 | 2.44 | 2.38 | 2.22 | 2.00 | 1.92 |
| | SDRL | 1.68 | 1.62 | 1.60 | 1.59 | 1.42 | 1.38 | 1.36 | 1.12 | 1.04 | 1.00 | 0.92 | 0.81 | 0.77 |
| 2.50 | ARL | 2.43 | 2.38 | 2.37 | 2.36 | 2.20 | 2.16 | 2.14 | 1.89 | 1.79 | 1.76 | 1.64 | 1.50 | 1.44 |
| | SDRL | 1.04 | 1.00 | 1.00 | 1.00 | 0.90 | 0.88 | 0.87 | 0.75 | 0.71 | 0.69 | 0.64 | 0.58 | 0.55 |
| 3.00 | ARL | 1.89 | 1.86 | 1.85 | 1.84 | 1.72 | 1.69 | 1.68 | 1.50 | 1.43 | 1.40 | 1.31 | 1.21 | 1.17 |
| | SDRL | 0.75 | 0.73 | 0.73 | 0.72 | 0.67 | 0.66 | 0.65 | 0.57 | 0.54 | 0.53 | 0.48 | 0.41 | 0.38 |
| 4.00 | ARL | 1.32 | 1.29 | 1.29 | 1.29 | 1.21 | 1.20 | 1.19 | 1.10 | 1.07 | 1.06 | 1.03 | 1.01 | 1.01 |
| | SDRL | 0.48 | 0.47 | 0.47 | 0.47 | 0.42 | 0.40 | 0.40 | 0.30 | 0.25 | 0.23 | 0.18 | 0.11 | 0.09 |

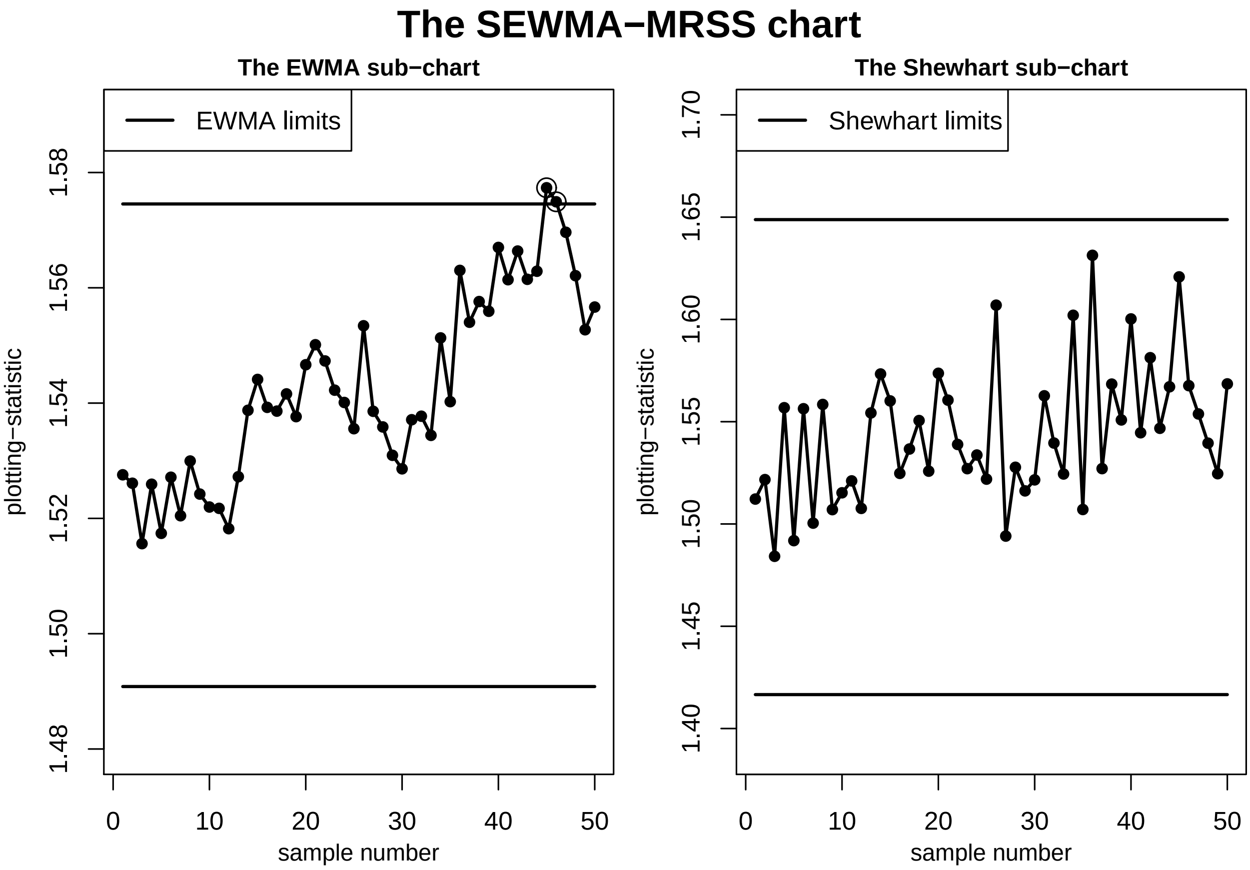


Figure 4.1: The SEWMA-MRSS chart

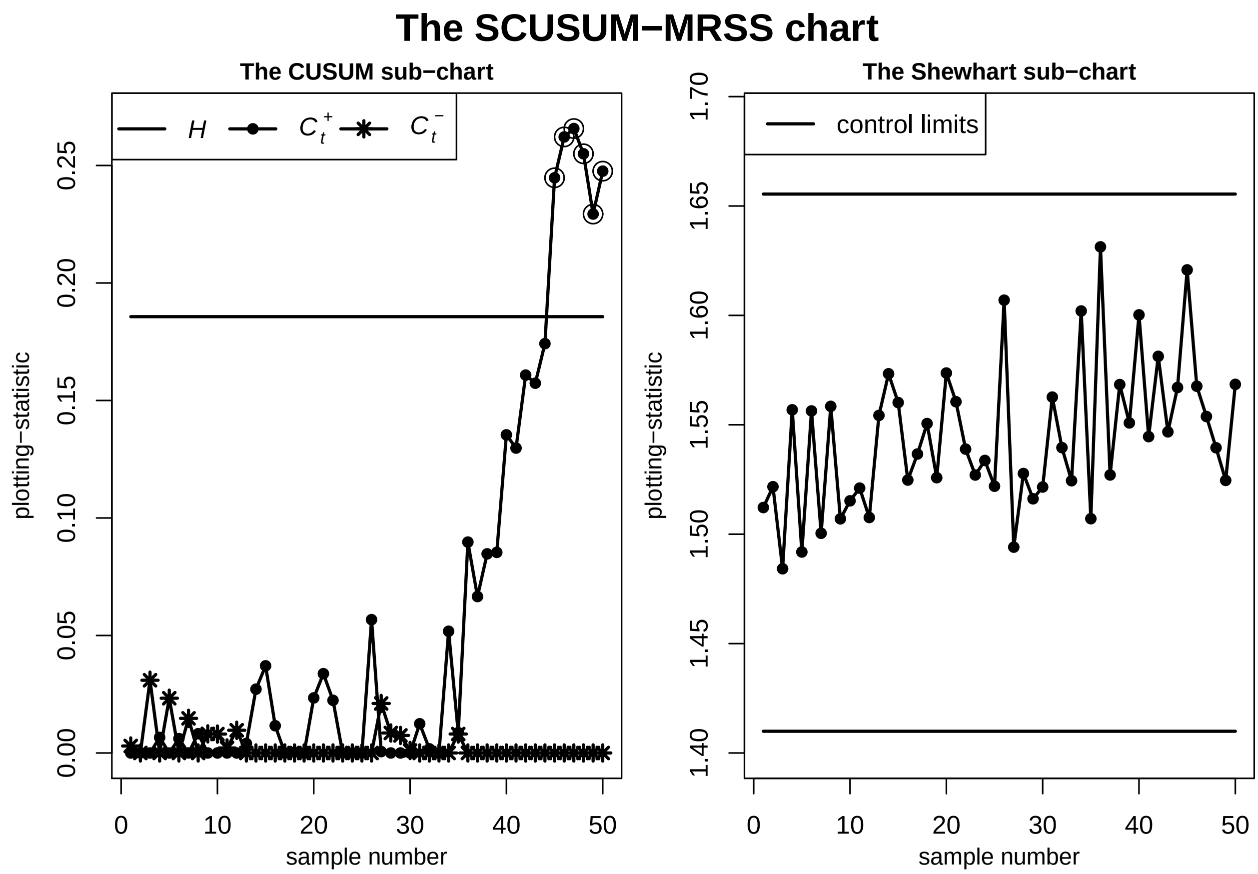


Figure 4.2: The SCUSUM-MRSS chart

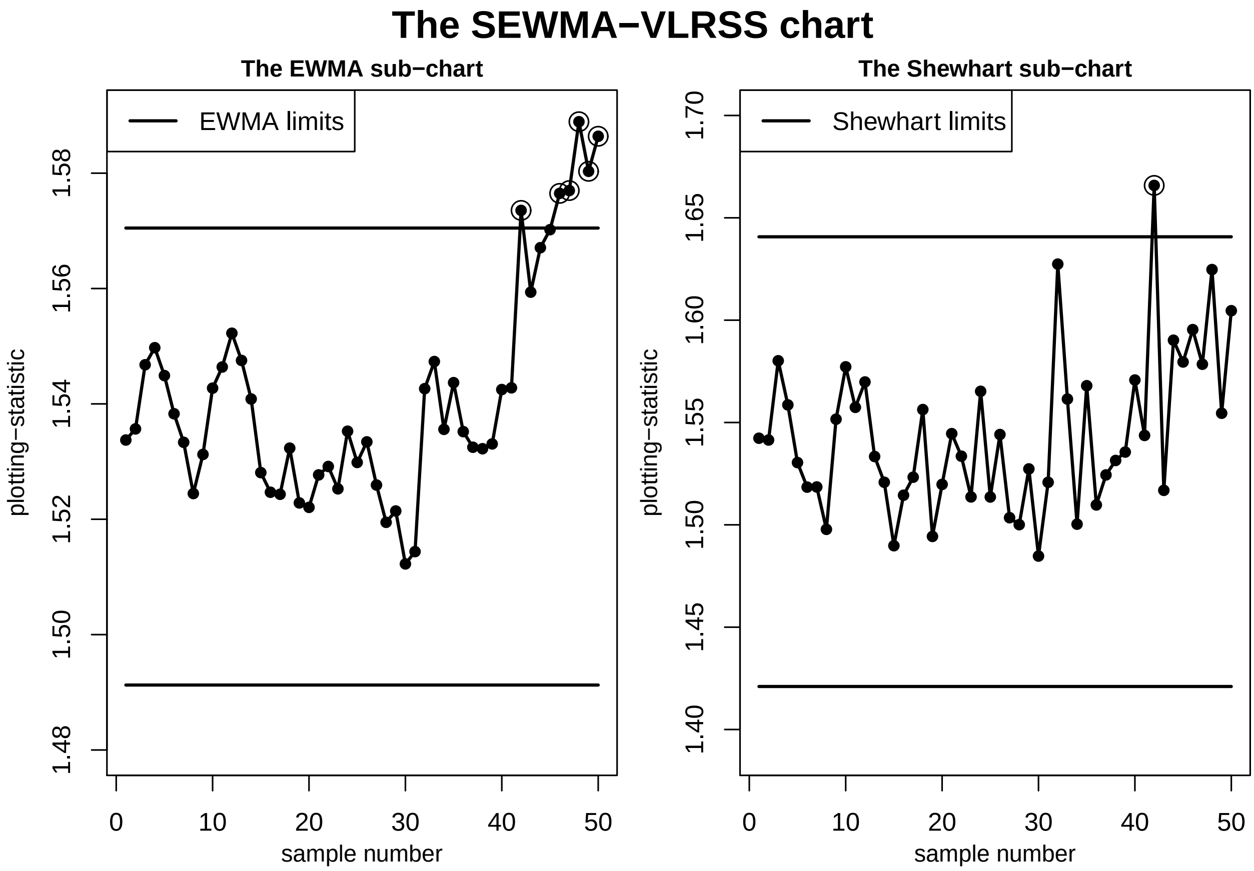


Figure 4.3: The SEWMA-VLRSS chart

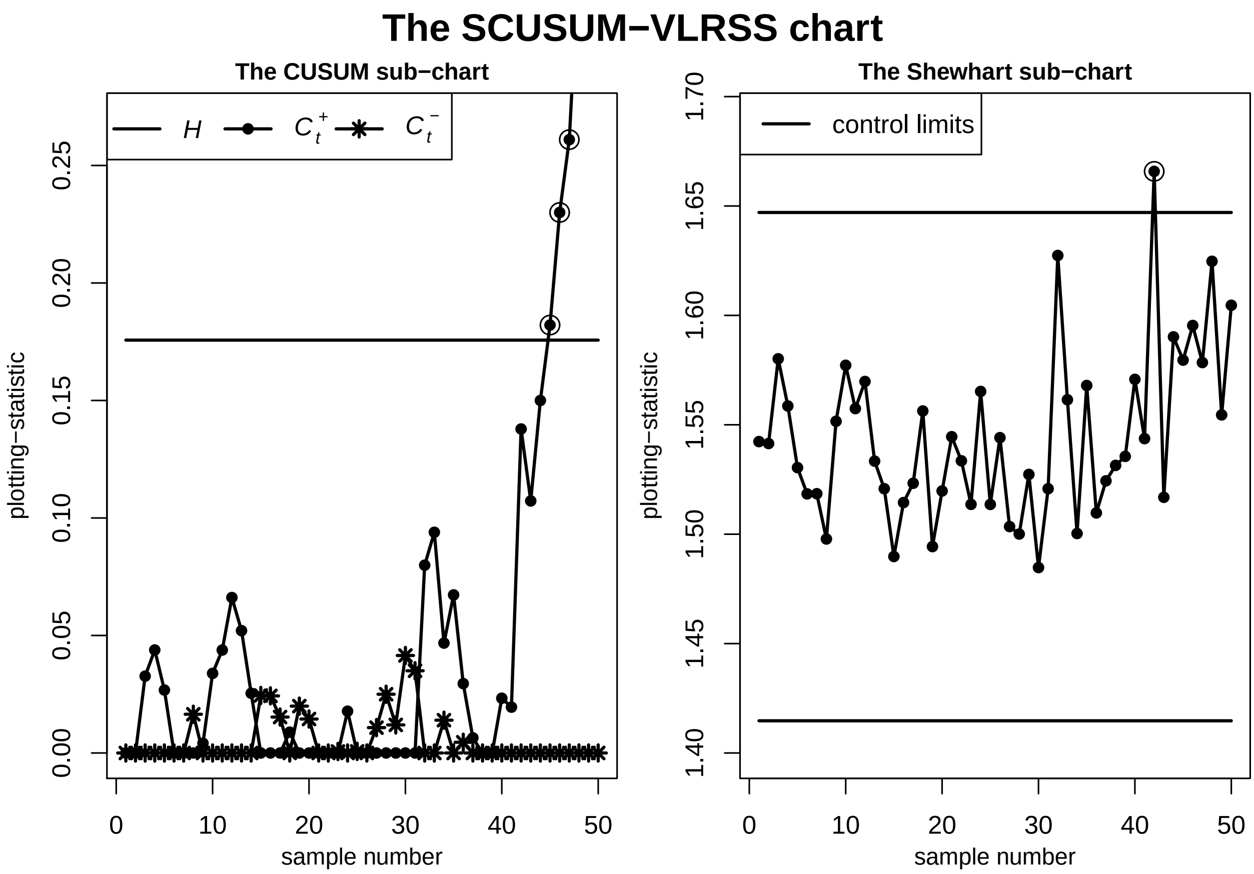


Figure 4.4: The SCUSUM-VLRSS chart

# Chapter 5

# Conclusion and Future Work

## 5.1 Conclusion

One of the main reasons for using control charts in the statistical process control is to detect an out-of-control situation of a production process as quick as possible. The control charts based on the SRS scheme were in use for a long time but since the introduction of RSS scheme, a variety of control charts based on RSS scheme have been produced in the literature if one can bear the ranking cost. However, there may exist a situation when one can not do so. To overcome such situations, we used a newly developed cost-effective ranking scheme to construct new quality control charts, namely VLRSS scheme, for efficiently monitoring the process mean. Extensive Monte Carlo simulations were used to estimate the run length characteristics of the proposed control charts. The run length performances of the proposed control charts were evaluated in terms of the ARL and SDRL with perfect and imperfect rankings.

In Chapter 2, we proposed the CUSUM chart with VLRSS for monitoring the process mean, which was then compared with the existing CUSUM charts based on SRS, RSS, and MRSS schemes. It has been found that the proposed CUSUM chart is uniformly efficient in detecting small to moderate shifts in the process mean than its competitors.

In Chapter 3, we proposed an EWMA chart based on the VLRSS scheme and compared its run length performances with those of the EWMA charts based on SRS, RSS, and MRSS schemes. The results showed that if there were small to moderate shifts in the process mean, the proposed EWMA chart performed uniformly better than the EWMA chart with SRS, RSS, and MRSS schemes.

In Chapter 4, we considered the situation when the interest lies in simultaneously detecting both small and large shifts in the process mean. In order to detect these shifts, using VLRSS scheme, we combined Shewhart chart with the EWMA and the CUSUM charts, named the SEWMA and SCUSUM charts, respectively. From the results it has been seen that the SEWMA and SCUSUM charts with the VLRSS scheme performed uniformly better than the existing combined SEWMA and SCUSUM charts based on the SRS, RSS, and MRSS schemes, respectively.

## 5.2 Future Work

In this study our focus was on improving the run length performances of the quality control charts for monitoring the process mean using VLRSS. The performances of the proposed control charts were evaluated and then compared with the existing control charts. From the results presented in Chapters 2-4, it has been seen that the proposed control charts performed uniformly better than the existing control charts in detecting different shifts in the process mean. Thus, for efficiently monitoring the process mean, the proposed control charts are recommended to use.

The scope of this study can be further extended to other control charting structures. For example, the adaptive CUSUM charts, in which by suitably selecting the reference parameter $k$, these control charts give better performance to monitor changes in a range of shifts than the classical CUSUM charts. Thus, on the lines of the CUSUM chart proposed in this research, an adaptive CUSUM chart can be constructed for efficiently detecting changes in a range of process mean shifts.

Similarly, the proposed EWMA chart can be extended to adaptive EWMA chart in which weights to the past observations of the monitored process are given as a function of the current "error". The resulting control chart scheme can be seen as a smooth function of two control charting schemes, i.e., a combination of the Shewhart and EWMA charts. Moreover, the proposed study can be further extended to the dual CUSUM chart in which two CUSUM charts are used to detect changes in a range, particularly small to large shifts in the process mean.

# Appendix

**Algorithm 1** To generate a Varied L Ranked Set Sample of size $n = mr$

**Require:** A matrix $M$ of $m$ rows and $r$ columns
**Ensure:** Required parameters $(m, l, w, v)$—all integers

**for** $j$ in $1 : r$ **do**
    **for** $i$ in $1 : m$ **do**
        **if** $i \leq w$ **then**
            Generate a random sample of size $l$
            Rank these units $\rightarrow x$
            Select $v$th ranked unit from $x \rightarrow$ M$[i, j]$
        **end if**
        **if** $i > w$ and $i \leq (m - w)$ **then**
            Generate a random sample of size $m$
            Rank these units $\rightarrow y$
            Select $i$th ranked unit from $y \rightarrow$ M$[i, j]$
        **end if**
        **if** $i > (m - w)$ and $i \leq m$ **then**
            Generate a random sample of size $l$
            Rank these units $\rightarrow z$
            Select $(l - v + 1)$th ranked unit from $z \rightarrow$ M$[i, j]$
        **end if**
    **end for**
**end for**

The R codes used in computing the run length characteristics of the proposed control charts are taken from Abbas (2012, pp. 84-85) and then modified.

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