RESONANCES IN CHARM AND BOTTOM BARYONS

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By

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RESONANCES IN CHARM AND BOTTOM BARYONS

This work is submitted as a dissertation in partial fulfillment
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DECLARATION

I, Ms. Anam Nawaz Roll No: 02181613007, student of Mphil, in the subject of Physics session 2016-2018, hereby declare that the matter printed in the thesis titled **Resonances In Charm and Bottom Baryons** is my review work and has not been printed, published or submitted as research work, thesis or publication in any form in any University and Research Institution etc, in Pakistan.

Dated: February 25, 2019

This work is dedicated to the sole treasure of my life, my parents, my brothers and sister.

RESEARCH COMPLETION CERTIFICATE

This is to certify that **Anam Nawaz** (Roll No:02181613007) has carried out the theoretical work contained in this dissertation under my supervision and is accepted by the Department of Physics, Quaid-i-Azam University Islamabad as fulfilling the dissertation for the degree of Master of Philosophy in Physics.

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Abstract

This dissertation presents the study of resonances of charmed and bottom baryons using spin-dependent potential. This potential model includes tensor interaction term, hyper-fine splitting and spin orbit terms responsible for fine structure. In the recent report, LHCb announced the discovery of five narrow excited Ω_c baryons which decays into Ξ_c^+ K^-.The Ω_c^0 charmed baryon is a bound state of css quarks, which can be treated as a P-wave (ss) diquark in our case and a heavy (c) quark. There are five possible combinations of orbital angular momentum and spin for such system. The narrowness of the states signifies that it is very hard to break the two s quarks in a diquark. The mass spectrum for the Ω_c(css) states has been measured by the LHCb, confirmed recently by Belle, and the masses of the states is consistently increasing with their total spin. LHCb also determined their widths through $\Omega_c^\circ \rightarrow \Xi_c^+$ K^-decay channels, which didn't exceeds a few MeV. Five states with negative parity have been predicted, two states decay in S-wave, and three states decay in D-wave. The D-wave states might be narrower than the S-wave states. We expect the similar pattern in excited Ω_b states with negative parity. In this work, we have evaluated eigenstates of the newly observed excited states of Ω_c baryons and mass shifts (ΔM) using these states. In the end, we have calculated the numerical values of parameters $(a_1, a_2, b, c, M_\circ)$ in MeV. by using effective Hamiltonian for Ω_c baryons in the quark - diquark description.

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Chapter 1

Introduction

The aim at understanding the universe and what it is made of has always driven the mankind curiosity. Ancient Greek philosophers, Democritus and Leuippus, introduced the idea that matter is composed of unbreakable atoms, surrounded by blank space. But this idea is very different from today's understanding of nature which is similar to the modern explanation of the atomic structure of matter.

1.1 Standard Model

The comprehensive model which outlined three out of four elementary forces of nature (electromagnetic, strong and weak interactions) and arranging all elementary particles is known as Standard Model in particle physics.The " Standard Model " is the consequence of an immense experimental and theoretical effort, spanning over fifty years. It was more than a hundred years ago, when the electron was discovered by J. J Thomson in 1897, which is still thought as structure less i.e., point like particle and he was awarded with Nobel prize in 1906 for this discovery. Discovery of electron was a consequence of cathodes rays that were observed by William Crooks in the discharge tube experiment.Actually J. J Thomson concluded that these rays consist of particles having negative charge. Later on Stoney named them " electrons ". Atom was considered neutral, so it was obvious that there exist some positively charged particles in an atom.Finally, in 1886, Eugene discovered positively charged rays in discharge tube experiment and later on in 1914 Rutherford observed in his experiment that these are particles.In 1920 Rutherford assigned name to these particles " protons " and neutron was discovered by Chadwick in 1932.Fermions includes the proton, neutron and electron having angular momentum which is half integral multiple $(\frac{\hbar}{2})$ and spin.At first proton and neutron were considered as elementary particles, but later on it was experimentally confirmed that these particles are not point like, but have a complex structure.After these facts, some questions were raised that what are the ultimate constituents of matter? How can we categorize them? In what way do they interact? What should be the mathematical formalism for elementary particles? Physicists tried to answer these questions and Standard Model (SM) is a result of such efforts.The SM describes that all the matter of universe is composed of fundamental fermions that interact through fields.There are some particles which are associated with field interaction called gauge bosons because their interaction can be described by gauge theory.Standard Model is composed of twelve fundamental particles that are the constituents of all the matter around us.In current time the known elementary particles are fermions as well as the fundamental bosons which are commonly " force carriers " (gauge bosons and the Higgs boson).

1.2 Elementary Particle Physics

Elementary particle physics is related with the fundamental constituents of matter and their interactions. Particle Physics or high energy physics deals with the elementary constituents of matter, interactions of elementary particles and their properties which is disclosed in experiments using particle accelerators. Fundamental constituents of matter interact with each other through fundamental forces and particles physics covers the interaction and classification of these elementary particles.

Elementary Particle physics begins in 1897 with J. J Thomson's discovery of cathode rays composed of negatively charged particles called " electrons " using glass discharge tube.In 1911, Ernest Rutherford's scattering experiment revealed that the most of the mass the positive charge, was concentrated at the center of atom called 'nucleus' as displayed in Fig.(1.1). He demonstrated his famous experiment by firing alpha particles beam onto a thin sheet of gold foil. Neil's Bohr proposed an atomic model, which describe that nucleus consists of protons inside it and electrons are orbiting in certain circular orbits which have fixed angular momentum around a nucleus. Later on, in 1932 James Chadwick discovered neutrons by bombarding beam of α-particles from a polonium source on a beryllium target at the Cavendish Laboratory and it was the phenomenal achievement of that time.Experimental results proved that even the atom's nucleus is not fundamental, but comprised of protons and neutrons instead. In the beginning, only four particles are considered to be fundamental particles, the neutron (n^0), the electron (e^-), the proton (p^+) , and the neutrino (v). Two particles, proton and electron are positively and negatively charged respectively, while the other two neutron and neutrino are electrically neutral. Atomic nuclei is formed by the proton and neutron whereas, an atom is formed together with the nucleus. The neutrino is achieved in the process of neutron decay when it decomposes into an electron, a proton and an anti-neutrino.

1.3 Classification Of Particles

On the basis of spin, Standard model categorized particles in three distinct types as shown in Fig.(1.2).

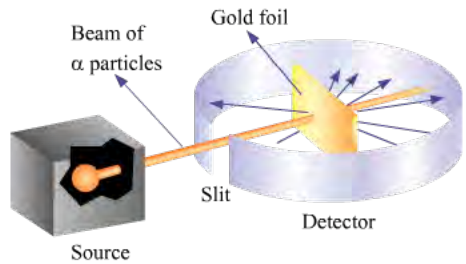


Figure 1.1: Rutherford Gold Foil Experiment.

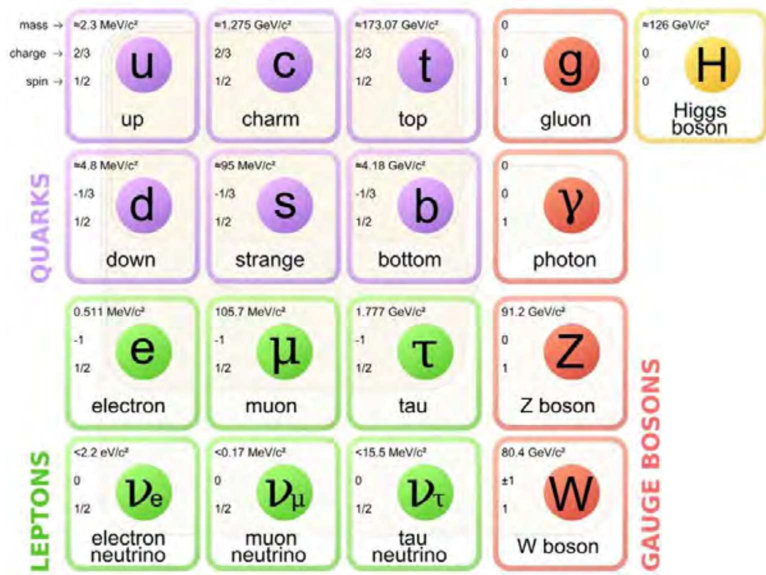


Figure 1.2: Standard Model of Elementary Particles.

- Fermions are the particles with half integer spin and obey Fermi - Dirac statistics.
- Leptons carry 1/2 integral spin and have six flavors which are arranged in three generations.
- Quarks carry 1/2 integral spin and always exist in bound state.
- While particles having integer spin are called guage Bosons, and are force carrier in Standard Model.In addition there is another boson, which explains the origin of mass in SM called Higgs Boson. In the absence of Higgs all the particles of SM should have zero mass and Bosons obeys Bose - Einstein statistics.

The wave-function of fermions must be anti-symmetric on exchange of two identical particles whereas the wave-function which describes a collection of bosons must be symmetric.

1.3.1 Fermions

According to the Standard Model, matter is constructed from small number of elementary spin $\frac{1}{2}$ particles or fermions: six quarks and six leptons as represented in Fig.(1.3).Each fermion have its corresponding anti-particle

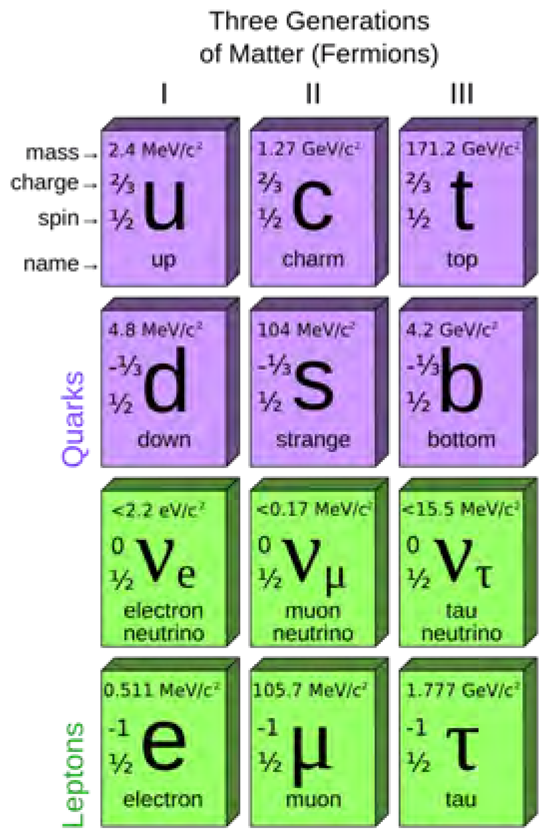


Figure 1.3: Fundamental Matter Particles.

with opposite electric charge but same quantum numbers.With the exchange of identical particles, the wave function of fermions must be anti symmetric and follow the Pauli's Exclusion Principle as shown in Fig.(1.4) i.e. "Two fermions cannot exists in the same quantum mechanical state simultaneously" [1].Each fermion have its corresponding anti-particle with opposite electric charge but same quantum numbers.

1.3.1.1 Leptons

The origin of word lepton is a Greek word " leptos" having meanings of fine, small or thin.There are six known leptons whose list is shown in Table (1.1). Leptons are the building blocks of matter with the half - integer spin that can be up or down [2] and may be charged or neutral.Integral electric charge is carried by the leptons as represented in Table (1.2). The leptons which carry charge are e, μ and τ, whereas the leptons which are electrically neutral are called neutrinos represented by symbol v. Leptons are the fundamental particles and do not interact through the strong nuclear force. They can only experience electromagnetic and weak interactions.The charged leptons can interact through electromagnetic as well as weak interactions, the neutral ones only interact weakly.There are six flavors of leptons which are arranged in three generations, each generation comprises of 2 leptons and differs from the other on the basis of their mass. The first generation also known as electronic leptons are the lightest leptons includes the electron (e) and electron neutrino (v_{e}). The second generation called the muonic leptons includes muon (μ) and its muon neutrino (v_μ) and the third generation also called tauonic leptons is composed of the tau (τ) and its neutrino (v_τ). Electron, muon and tau carry –1 as an electric charge. Lepton masses increases from left to right and μ,τ are heavy versions of electron as represented in Table (1.3). An electron is emitted together with an electron neutrino (v_e) in nuclear β-decay whereas the charged tauon and muon, both are unstable and exhibit spontaneous decay to neutrinos, electrons and other particles. The mean life time of tauon and muon are 2.9×10^{-13}s and 2.2×10^{-6}s, respectively. The instability of leptons increases with their masses and they decay very quickly. Dirac equation of charged massive particle (i.e.fermion) predicts the existence of an antiparticle having opposite charge, same mass, magnetic moment to the direction specifying the spin. Similarly, it also predicts the existence of anti-neutrino corresponding to each neutrino. From these three generations of leptons, only electron and its antiparticle termed as positron are stable while other leptons i.e. muon, tau and their antiparticles differ from electron e^- and positron e^+ in masses and lifetimes. It is experimentally well established fact that all leptons including an electron, tau and muon have the own associated neutrinos and anti-neutrinos. In 1930, Pauli postulated about the existence of neutrinos (electron neutrino) to understand and explain the experimental beta decay results

$$(\mathrm{Z}, \mathrm{A}) \rightarrow (\mathrm{Z}–1, \mathrm{A}) + \mathrm{e}^+ + \nu_{\mathrm{e}} \ ,$$

$$(\mathrm{Z}', \mathrm{A}') \rightarrow (\mathrm{Z}+1, \mathrm{A}) + \mathrm{e}^- + \overline{\nu}_{\mathrm{e}}.$$

In these reactions Z indicates the atomic number and A is mass number of nucleon. First reaction is decay of bound proton to neutron whereas in second reaction bound neutron decays to proton via basic processes.

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^+ + \nu_e,$$

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_e.$$

In free space only neutron decay can occur because mass of neutron is larger in magnitude than the combined mass of an electron and proton.

$$\mathrm{m}_n > (\mathrm{m}_p + \mathrm{m}_e).$$

1^{st} Generation		2^{nd} Generation		3^{rd} Generation	
Electron		**Muon**		**Tau**	
Symbol	e^-	Symbol	μ^-	Symbol	τ^-
Mass	0.51 MeV.	Mass	105.7 MeV.	Mass	1.77 GeV.
Charge	-1	Charge	-1	Charge	-1
Electron neutrino		**Muon neutrino**		**Tau neutrino**	
Symbol	ν_e	Symbol	ν_μ	Symbol	ν_τ
Mass	2.2 MeV.	Mass	0.17 MeV.	Mass	15.5 MeV.
Charge	0	Charge	0	Charge	0

Table 1.1: Generation of Leptons.

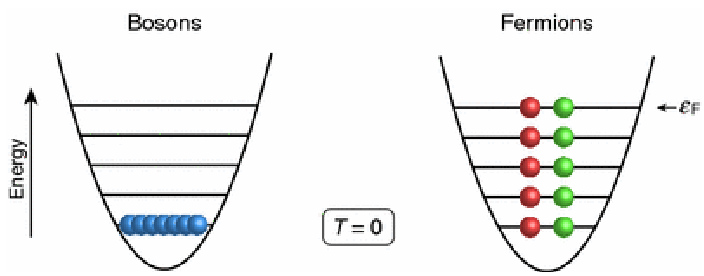


Figure 1.4: Bosons Vs Fermions.

Due to apparent violation of energy in bound state, proton decay is possible which can be balanced with change of binding energy. Initially ν_e and $\overline{\nu}_e$ were not observed experimentally, they were inferred from conservation laws. Muon is a highly penetrating particle having mass 105.7 MeV. and it was identified in 1936 by Naddermeyer and Anderson in experiments of cosmic rays. Muons have an associated magnetic moment,

$$\mu = \frac{e}{m_\mu} S.$$

Similar to that of spin half point like particles and have similar electromagnetic properties as that of electrons taking into account the mass difference.Tauons are even heavier (1.77 GeV.) discovered in 1975 in e^+e^- annihilation experiment. The properties of tau leptons were not studied in much details as that of muon, but the studied properties were enough to list it in point like particles category whose electromagnetic properties are similar to that of electron and muon.Tauon and muon both are unstable having life time 0.2 pico seconds and 2.2 micro seconds, respectively and both decay to electron via weak interaction. Therefore, what we observe in an interaction is that a lepton changing to only same type of another lepton along with a pair of lepton and anti-lepton of same family. This is experimentally established fact and called as Lepton Number Conservation Law

$$\mu^- \to \nu_\mu + e^- + \overline{\nu}_e.$$

Particle	Flavor			Q / |e|
Leptons	e	μ	τ	-1
	v_e	v_μ	v_τ	0
Quarks	u	c	t	$+\frac{2}{3}$
	d	s	b	$-\frac{1}{3}$

Table 1.2: The Fundamental Fermions.

1.3.1.2 Quarks

If we look at past, initially it was thought that atom is indivisible later on when electron, proton and neutron were discovered it was proposed that there exist sub-atomic particles. The progress did not stop and physicists proposed that neutrons and protons are not fundamental particles but are made up of fundamental particles called quarks. The existence of quarks was verified by a series of experiments (during electron-nucleon inelastic scattering) performed at Stanford Linear Accelerator Center between 1967 and 1973.Unlike other particles, quarks have a striking property called as quark 'confinement' [31]. According to this property quarks always exists in the bound state are not found as independent entities or free particle. This bound state is composed of either quark-anti quark pair or three quarks anti quarks.The fractional charges$+\frac{2}{3} \mid e \mid$ and $-\frac{1}{3} \mid e \mid$ are carried by quarks, like leptons, quarks are classified into pairs with difference by one unit of electric charge. The up, charm and top i.e. up-type quarks carry $+\frac{2}{3}$ e as an electric charge and down, bottom and strange quarks (i.e., down-type quarks) have a charge of $-\frac{1}{3}$e. Like leptons, quarks also have six different flavors in the standard model which are placed in three generations. The quark type or 'flavor' is indicated by a symbol: u for 'up', d for 'down', c for 'charmed', s for 'strange', b for 'bottom' and t for 'top'. For each of the different fundamental constituents, the ratio of its electric charge Q to the elementary charge e of the electron and its symbol are given in Table (1.2) [32]. Like leptons shown in Table (1.3) quark masses increases from left to right i.e., up (u) quark is the lightest in mass, whereas top (t) quark is the heaviest of all quarks as given in Table (1.4).

1.4 The Fundamental Forces

Physicists know about four different forces, Gravity is most familiar one, but is too weak for the particles considered in the high energy physics. Other three forces are strong force, electromagnetic force and weak force. All the three forces relevant to particle physics are described by Quantum Field Theory by the exchange of mediating particle having spin-1 known as gauge bosons.Most familiar spin-1 particle " the photon ", is a gauge boson of Quantum Electrodynamics.

Flavor	Charged Lepton Mass	Neutral Lepton Mass
e	m_e = 0.511 Me V	$m_{v_e} \leq$ 10 Me V
μ	m_μ = 105.66 Me V	$m_{v_\mu} \leq$ 0.16 Me V
τ	m_τ = 1777 Me V	$m_{v_\tau} \leq$ 18 Me V

Table 1.3: Leptons masses in energy units mc^2.

Flavor	Quantum Number	Rest mass GeV./c^2
up or down	—	$m_u \simeq m_d \simeq 0.31$
strange	S = –1	$m_s \simeq 0.50$
charm	C = +1	$m_c \simeq 1.6$
bottom	B = –1	$m_b \simeq 4.6$
top	T = +1	$m_t \simeq 180$

Table 1.4: Constituent Quark Masses.

1.4.1 Classification Of Interactions

The particles interaction takes place through the four fundamental forces [33] as displayed in Fig.(1.5) as,

- Gravity
- Electromagnetism
- The strong force
- The weak force

The gravitational force between the two particles can be omitted in the study of particle interactions because its magnitude is very small. Depending on the types of interactions they experience, the properties of twelve elementary fermions are classified as summarized in Table (1.5). All twelve elementary particles experience weak force and suffer weak interactions, excluding electrically neutral neutrinos, the other nine particles which are charged electrically participate in electromagnetic interactions of QED. Color charge is a characteristic of gluons and quarks which is

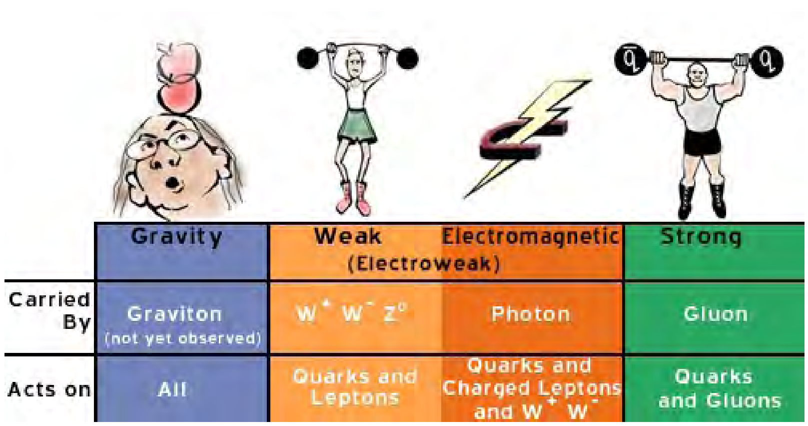


Figure 1.5: Types of Interactions.

Particle	Types of particles	Flavor			Strong	Electromagnetic	Weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged-neutrinos	e^-	μ^-	τ^-	✓		✓
	neutral-neutrinos	v_e	v_μ	v_τ			✓

Table 1.5: Forces experienced by fermions.

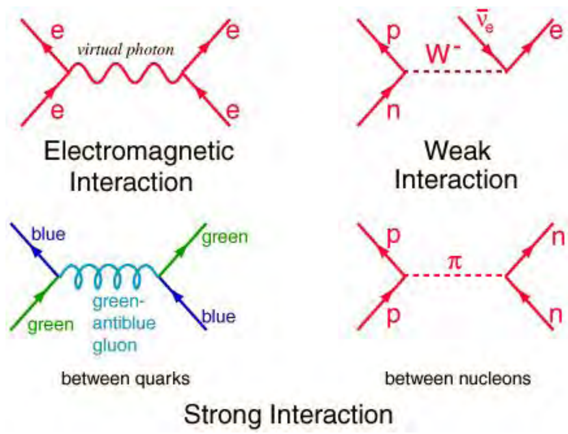


Figure 1.6: Fundamental Interactions of The Standard Model.

related to particles strong interactions in the theory of Quantum Chromodynamics (QCD) and as a result only the quarks undergo the strong force. Properties of quarks are distinct from those of leptons because they feel the strong force, and for every quark, there exists its anti-particle called anti-quark. Quarks are not observed as the free particles and are always confined in a bound state, called Hadrons like proton (uud) and neutron (udd).

1.4.2 Bosons

Bosons are the force carrying particles and are responsible for mediating fundamental interactions between fermions and between bosons itself. They have integral spin of 1 or 0 and therefore obeys the Bose-Einstein statistics [35] which means that by exchanging the position of any two particles, wave-function of a system of identical bosons remain symmetric [7, 8]. There exist two kinds of bosons, gauge bosons and higgs bosons, the Gauge bosons carry integral spin of 1.

1.4.2.1 The Interactions

Gauge bosons are the force mediators in the standard model and these boson mediators are recorded in Table (1.6). There are four kind of fundamental field or interactions which are given as follows:

- Strong interactions are responsible for binding neutrons and protons within nuclei and quarks in the proton and neutron. The massless and chargeless particle, called gluon (g) mediates the inter quark force and there are total eight gluons and they all carry color charge [9].

- Weak interactions are represented by the slow procedure of nuclear β-decay, which involves the release of a radioactive nucleus of neutrino and an electron. Mediators of weak interactions are Z^0(neutral)and $W^\pm$ (charged), W^+ and W^- contain the masses of 80.39 GeV./c^2 and Z^0 has a mass of 91.188 GeV./c^2. In particular, W^+ and W^- bosons are responsible for mediating the charged weak interactions, and Z^0 bosons for mediating neutral weak interactions as shown in Fig.(1.6).

- Electromagnetic interactions are responsible for the inter molecular forces in liquids and solids and for the bound state of electrons with nuclei (molecules and atoms). Photon (γ) is the mediator of electromagnetic interaction and has zero mass which implies that electromagnetic force has infinite range.

- Gravitational interactions proceed between all types of particles, and the strength of gravitational force is very weak between the individual particles so, we safely ignore it in the experimental high energy particle experiment.It is mediated by exchange of boson of spin -2 called the graviton. Experiments to discover graviton are currently in progress.

1.4.2.2 Photon And Gluon Exchange

In Classical electrodynamics we describe the force between charged particles using scalar potential. But this classical explanation of forces is not satisfactory at many stages. For example, when we study the scattering of an electron and a proton, momentum is transferred without any apparently seen mediating particle.In this regard of action-at-a-distance, Newton famously wrote " It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact ". Explanation of force on the basis of potential is convenient as expressed in classical electromagnetism but it does not inform about the origin of interaction.

Forces are described by Quantum Field Theory in modern particle physics. Quantum Electrodynamics explains the electromagnetic interaction between two charged particles.According to QED, this interaction is mediated by exchanging the virtual photons and mass of the mediating particle is zero.The range of this interaction is infinite which is due to the fact that the rest mass of the mediator is zero.Considering this mediating particle theory description, there remains no mysterious theory of action at a distance.Fig.(1.7), shows an example of photon exchange between two electrons.First diagram shows that a photon is emitted by upper electron and absorbed by lower one which causes transfer of momentum between electrons.Other possibility of time-ordering where the upper electron absorbs the photon which is emitted by the lower electron as shown in second diagram.

In 1920 when it was proposed by Rutherford that the nucleus of an atom consists of a proton and a neutron, it was realized that due to proton-proton repulsion, it should have been burst.To solve this problem, Eugene Wigner

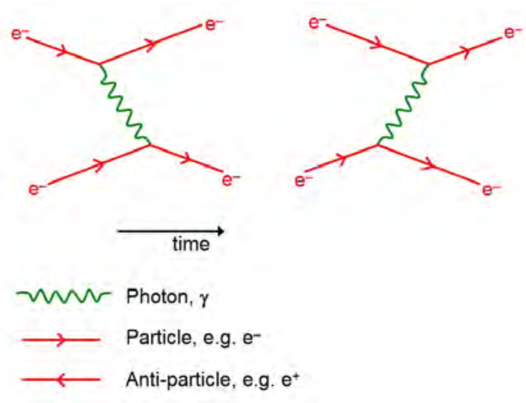


Figure 1.7: Photon (γ) exchange between two electrons.

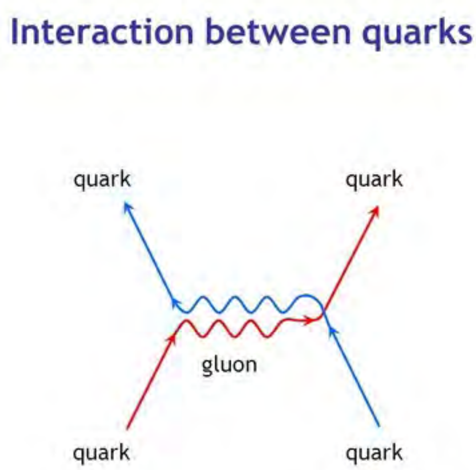


Figure 1.8: Gluon exchange between two quarks.

proposed that there should be another force called strong force between nucleons which should be responsible for the stability of nucleus and overcomes electromagnetic repulsion between protons. Since protons (uud) and neutrons (udd) both consist of quarks, it means quarks are combined through strong interaction / force to form proton and neutrons. In case of strong interaction, mediating particle is gluon, which has a color charge and the theory of strong interaction is called Quantum Chromodynamics (QCD). Force is strongest for this interaction amongst the other interactions but its range is very short i.e., in the order of Fermi (1fermi $= 10^{-15}$ m).Gluon exchange is shown in Fig.(1.8).

The study of beta decay reveals that weak charged current is responsible for β - decay. Beta decay is mediated by W bosons where neutron decays to proton as shown in Fig.(1.9). Similarly, we can see exchange of Z boson in elastic scattering of electron and muon neutrino, as demonstrated in Fig.(1.10) and these interactions are consequences of

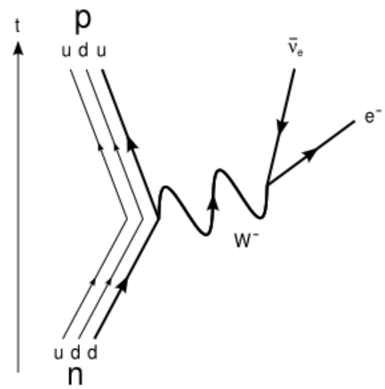


Figure 1.9: β- decay.

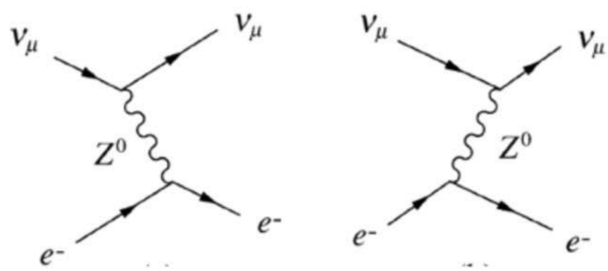


Figure 1.10: Elastic scattering of electron and muon neutrino.

weak interaction.The range of this interaction is very short, less than that of the strong interaction. Force carriers in this interaction are $W^{\pm}$ and Z bosons. Unlike to photon and gluons, Z and $W^{\pm}$ bosons are massive particles. $W^{\pm}$ bosons carry weak charge while Z boson is responsible for the weak neutral current. Relative strength of forces, mediators and masses of mediators is represented in Table (1.6).

1.4.2.3 Higgs Boson

In 1960 the British theoretical physicist, Peter Ware Higgs, gave the idea that there is some energy field permeated throughout the universe which was called Higgs field. One of the successful prediction of the standard model is the discovery of Higgs Boson. It is the remarkable discovery of standard model called as Brout-Englert-Higgs (BEH) boson and it was discovered by the ATLAS and CMS experiments at the Large Hadron Collider (LHC), CERN [34, 11] in 2012. The quanta or excitation of this field is called Higgs particle. The mass of a particle is due to its interaction with Higgs field via the process of electroweak symmetry breaking. If interaction of a particle with the Higgs field is high, then its mass will be greater.It is the only observed and detected boson which has zero spin hence being called scalar particle. Its mass range is up to 125 GeV./c^2, so it quickly decays to other particles due to its heavy mass and it is unique due to its spin with zero value, among all the other particles of standard model with

Interactions	Mediator (Boson)	Spin / Parity	Charge	Strength	Lifetime	Mass / GeV.
Strong	(8 Gluons) g	1^-	0	1	∞	0
Electromagnetic	Photon γ	1^-	0	10^{-3}	∞	0
Weak	$W^{\pm}$(charged) Z^0(neutral)	$1^-, 1^+$	±1 0	10^{-8}	3.11×10^{-25}	80.4 91.2
Gravity	Graviton G	2^+		10^{-37}	2.64×10^{-25}	0

Table 1.6: The Boson Mediators (Spin -1).

either spin-half or spin-1. All those particles that interact with the higgs field acquire masses and cannot propagate at the speed of light. Particles which do not interact with higgs field i.e., gluons and photon remain massless. Unlike the fundamental fermions (spin $= \frac{1}{2}$) and gauge bosons (spin $= -1$) particles, the only fundamental spin - 0 scalar particle is the Higgs boson which is discovered so far as developed in the Standard Model.

Chapter 2

The Quark Model

2.1 Quarks

Even though only three quarks were suggested in the beginning, six are known to exist now. Similar to leptons, the six various types or flavors, appear in generations or pairs represented by,

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}, \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}, \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}.$$

Each pair / generation includes a quark with electric charge +2/3 (u, c , t) together with a quark of charge −1/3 (d, s, b), in units of the magnitude of the electron charge e [12]. They are known as the charmed (c),strange (s),up (u),down (d), top (t) and bottom (b) quarks. Table (2.1) shows the summary of quarks. There exists an anti quark corresponding to each quark as,

$$\begin{pmatrix} \bar{\mathrm{d}} \\ \bar{\mathrm{u}} \end{pmatrix}, \begin{pmatrix} \bar{\mathrm{s}} \\ \bar{\mathrm{c}} \end{pmatrix}, \begin{pmatrix} \bar{\mathrm{b}} \\ \bar{\mathrm{t}} \end{pmatrix}.$$

The anti-quarks carry the charges +1/3 ($\bar{\mathrm{d}}$, $\bar{\mathrm{s}}$, $\bar{\mathrm{b}}$) and −2/3 ($\bar{\mathrm{u}}$, $\bar{\mathrm{c}}$, $\bar{\mathrm{t}}$). There is a good experimental verification for the existence of all six flavors (u , d, c, s, t, b).Approximate values of masses of quarks in the units of GeV./c^2, their electric charge Q in the units of e, values of quantum numbers such as baryon number B, strangeness S, charm C, bottomness$\tilde{\mathrm{B}}$ and topness T are summarized in Table (2.2). All these quarks have antiparticles having the same mass and total angular momentum J, but opposite electric charge, I_z(Isospin z-component), the values of flavor quantum numbers and flavor (e.g., anti-charm $\bar{\mathrm{c}}$, anti-strange $\bar{\mathrm{s}}$, etc.).

In 1964 Green-berg suggested that quarks must have another characteristic in addition to spin and space degrees

1^{st} Generation	2^{nd} Generation	3^{rd} Generation
up	**charm**	**top**
Symbol u	Symbol c	Symbol t
Mass 2.4 MeV.	Mass 1.3 MeV.	Mass 173 GeV.
Charge 2/3	Charge 2/3	Charge 2/3
Down	**Strange**	**bottom**
Symbol d	Symbol s	Symbol b
Mass 4.8 MeV.	Mass 104 M eV.	Mass 4.2 M eV.
Charge −1/3	Charge −1/3	Charge −1/3

Table 2.1: Generation of quarks

Particle name	Symbol	Mass	Q-electric charge	J	B	S	C	$\tilde{B}$	T	Isospin	I_z
Down	d	$m_d \approx 0.33$	−1/3	1/2	+1/3	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
Up	u	$m_u \approx m_d$	2/3	1/2	+1/3	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
Strange	s	$m_s \approx 0.5$	−1/3	1/2	+1/3	−1	0	0	0	0	0
Charmed	c	$m_c \approx 1.5$	2/3	1/2	+1/3	0	1	0	0	0	0
Bottom	b	$m_b \approx 4.5$	−1/3	1/2	+1/3	0	0	−1	0	0	0
Top	t	$m_t \approx 173$	2/3	1/2	+1/3	0	0	0	1	0	0

Table 2.2: Properties of Quarks

of freedom and called it as color. The fundamental assumption in color theory implies that a quark can exist in three different colors (r, b, g) i.e., red, blue and green.

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Three colors of quarks

2.1.1 Quantum Numbers of The Quarks

By convention, quarks have positive parity and are strongly interacting fermions having spin 1/2, whereas anti quarks have negative parity. Quarks have 1/3 baryon number and anti quarks have −1/3. Table (2.2), shows the other additive quantum numbers (flavors) for the three generations of quarks and are related to the charge Q (in units of elementary charge e) through the generalized formula proposed by Gell -Mann-Nishijima

$$Q = I_z + \frac{B + S + C + \tilde{B} + T}{2}.$$

where B is the baryon number, flavor of a quark (I_z, S, C, $\tilde{B}$ or T) has the same sign as its charge Q by the convention. Any flavor carried by a charged meson has the same sign as its charge, with this convention, e.g., the

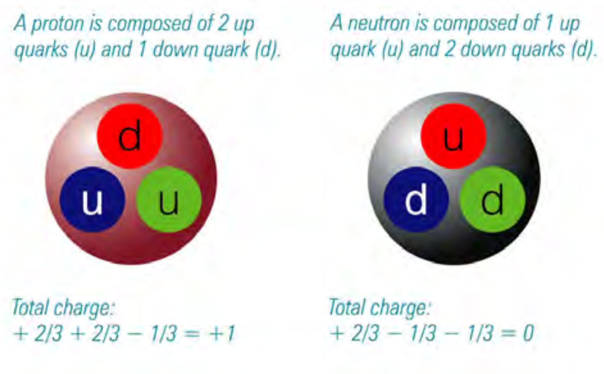


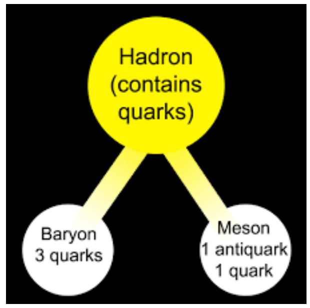
Figure 2.1: Building Blocks of Proton and Neutron.

bottomness of B^+ is +1, the strangeness of the K^+ is +1 and the strangeness and charm of the D_s^- are each −1. The flavor signs for antiquarks are opposite.

2.1.2 Classification of Hadrons (The Bound State of Quarks)

Quarks participate in strong interaction therefore they have quite different properties from leptons. Due to QCD interactions, quarks do not exist freely, but are always found in bound states i.e., hadrons, for example neutron and proton. In high energy atomic collision, one can split an atom into its constituents - atomic nucleus and electrons. In high energy nucleus - nucleus, nucleus can be splitted into its constituents i.e., protons and neutrons. But in high energy hadron-hadron collisions, hadron do not breaks into its constituents i.e., free quarks, it results into other hadrons and this is the hypothesis of quark confinement. Quarks are confined in a hadron by the strong fundamental force and the strength of this force is indicated by a dimensionless coupling constant $\alpha_s = \frac{g^2_s}{4\Pi}$, which is energy dependent. Classification of hadrons was necessary for new experimental techniques and knowing that hadrons are not elementary. In particle physics, hadron is any type of particle which is composed of quarks which are held together by the strong nuclear force. Gell - Mann developed a model for their classification, depending upon the number of quarks and spins, hadrons are subdivided into two classes named Baryons and Mesons. The mesons are arranged in octet and singlet representation whereas, the baryons with spin $\frac{1}{2}$ into an octet, and baryons with spin$\frac{3}{2}$ forming a decuplet. However, a particle which had not been discovered until that was also predicted which should be in decuplet. Gell-Mann called this particle Ω^- which was observed latter on in 1964 [13].

- Baryons are made up of three quarks or anti quarks (includes proton and neutron). Baryons are fermions, carry half integer spin and are massive particles. Proton and neutrons are the most eminent examples of baryons. Proton is composed of two up and one down quark (uud) whereas neutron is made up of two down and one up quark (ddu)as shown in Fig. (2.1). Other examples of baryons are sigma, omega, lambda etc, as shown in Fig.(2.2).



Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Figure 2.2: Examples of Hadrons.

- Mesons are hadronic subatomic particles, which are formed from a quark and an anti quark pair. Mesons are bosons and have an integral spin. Kaon ($s\bar{u}$) and pions ($u\bar{d}$) are the examples of mesons as shown in Fig. (2.2).

- Anti-baryons, which are bound state of 3 anti quarks .and considered as anti-particles of baryons, as shown in Fig.(2.3).

The Hadrons were organized into SU(3) representation multiplets, the baryons and mesons. The mesons are arranged in octet and singlet representation whereas,the baryons with spin $\frac{1}{2}$ into an octet and baryons with spin $\frac{3}{2}$ forming a decuplet.

2.1.3 Quark Model

The quark model is the grouping strategy for hadrons in terms of valence quarks (quarks and anti- quarks), in particle physics which give rise to the quantum number of the hadrons. It gives natural explanation of strangeness

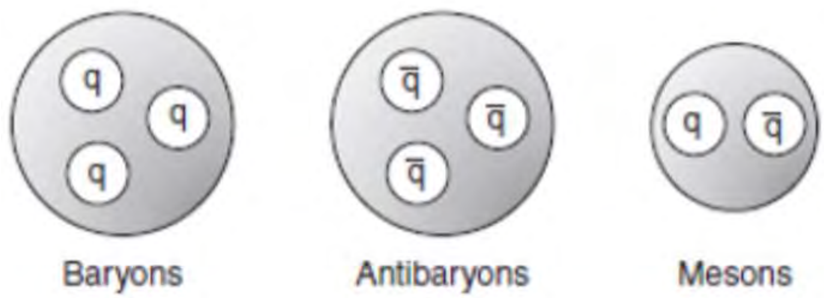


Figure 2.3: Composition of baryons, anti-baryons, mesons.

Quark	Q	Spin	I	I_3	B	S	Y
up (u)	2/3	1/2	1/2	1/2	1/3	0	1/3
down (d)	–1/3	1/2	1/2	–1/2	1/3	0	1/3
strange (s)	–1/3	1/2	0	0	1/3	–1	–2/3

Table 2.3: Quantum Numbers Of The Quarks

and isospin. According to the quark model, all hadrons are not fundamental particles as they are made up of small variety of more basic entities called quarks bound together (in different manner) by the strong force. They follow fundamental representation of SU(3) multiplet from which we can form all others. The original model carried a triplets of quarks

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}.$$

with the quantum numbers as shown in Table (2.3),where Q,Y, I , B, S and I_3 are charge, hyper charge, isospin, baryon number, strangeness and third component of isospin respectively. Each quark is awarded baryon number 1/3 and spin1/2. An additive quantum number, which is termed as " hyper charge " (Y) [14] which distinguish quark and anti quark multiplets. These quantum number of quarks satisfy the Gell-Mann following relation proposed for hadronic states.

$$Y \equiv B + S,$$

$$Q = I_3 + \frac{Y}{2}.$$

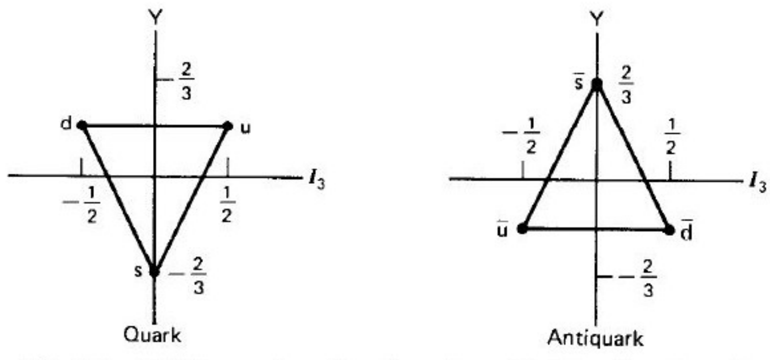


Figure 2.4: SU(3) quark and anti quark multiplets, Y= B + S.

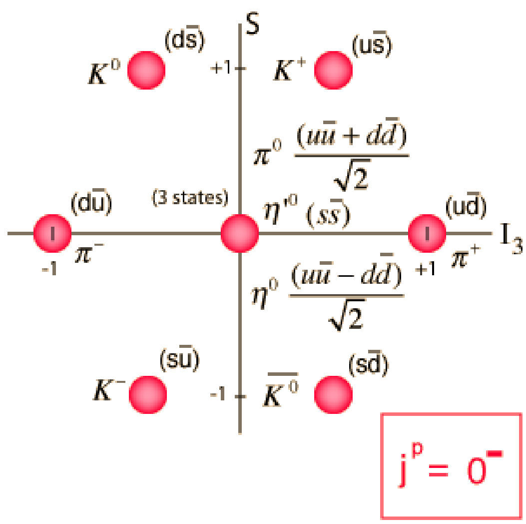


Figure 2.5: Weight diagram for pseudo scalar meson octet.

It is suitable to plot each state on a I-Y plot of the triplet representation as shown in Fig.(2.4). On horizontal axis third component of isospin is shown. Isospin is mathematical structure which explains similarity of neutron and proton.Therefore, Y is equal to $\frac{1}{3}$ for the up and down quarks,$-\frac{2}{3}$ for the strange quark and 0 for all other quarks.

2.2 Mesons (Bound $q\bar{q}$ states)

Mesons are bosons made up of a quark q (u, d, s) and an anti quark $\bar{q}$ $(\bar{u}, \bar{d}, \bar{s})$. The quarks and anti quarks are spin $\frac{1}{2}$ particles and mesons are the composites of $(q\bar{q})_{L=0}$ in the ground state. Now q and $\bar{q}$ spins are merged to form a total spin S, which results 0 and 1.If the orbital angular momentum of the $q\bar{q}$ state is L, then the parity of the $q\bar{q}$ system is P $(q\bar{q})= P_q P_{\bar{q}}(-1)^L = (+1)(-1)(-1)^L = -1$ as intrinsic parity of quark (q) and an anti quark ($\bar{q}$) are opposite.

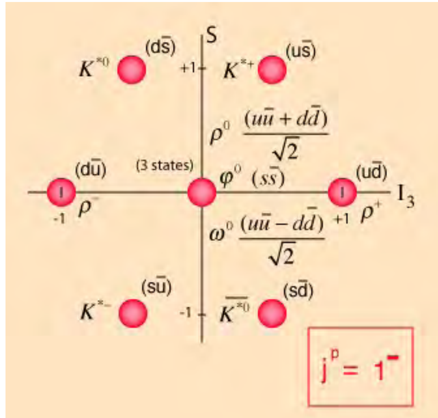


Figure 2.6: Weight diagram for vector meson octet.

Quark Content	0^-state	1^-state	Q	I_3	I	S	Y = B + S
$u\bar{s}$	$K^+(494)$	$K^{*+}(892)$	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1
$d\bar{s}$	$K^0(498)$	$K^{*0}(896)$	0	$-\frac{1}{2}$	$\frac{1}{2}$	1	1
$u\bar{d}$	$\prod^+(140)$	$\rho^+(786)$	1	1	1	0	0
$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	$\prod^0(135)$	$\rho^0(768)$	0	0	1	0	0
$d\bar{u}$	$\prod^-(140)$	$\rho^-(768)$	–1	–1	1	0	0
$s\bar{d}$	$\overline{K}^0(498)$	$\overline{K}^{*0}(896)$	0	$\frac{1}{2}$	$\frac{1}{2}$	–1	–1
$s\bar{u}$	$K^-(494)$	$K^{*-}(892)$	–1	$-\frac{1}{2}$	$\frac{1}{2}$	–1	–1
$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$\eta^0(549)$	$\omega^0(782)$	0	0	0	0	0
$s\bar{s}$	$\eta^{0\prime}(958)$	$\varphi^0(1019)$	0	0	0	0	0

Table 2.4: The states of the light L = 0 meson nonet.

The lightest mesons experimentally detected are a family of nine particles called the pseudo scalar meson nonet with spin - parity J^P=0^- and a family of nine particles with spin - parity J^P= 1^-known as the vector meson nonet [15] as shown in Table (2.4). The nine feasible $q\bar{q}$ combinations carrying the u, d, s quarks are classified into a singlet and an octet octet of light quark mesons:

$$3 \otimes \bar{3} = 1 \otimes 8.$$

Mesons have net baryon number B $= (+\frac{1}{3}) + (-\frac{1}{3}) = 0$. The eight lightest mesons form a hexagonal pattern making the pseudo scalar meson octet. The weight diagram for the ground state pseudo scalar (J^P= 0^-) and vector (J^P =1^-) mesons is shown in Fig. (2.5) and Fig. (2.6), respectively.

2.2.0.1 Baryons (Bound qqq states)

Baryons are subatomic particles, made up of three quarks (qqq) with baryon number B = 1 and are fermions. Three spin half quarks give a total spin of either $\frac{1}{2}$ or $\frac{3}{2}$ as the spin of baryons. The two known baryons are proton

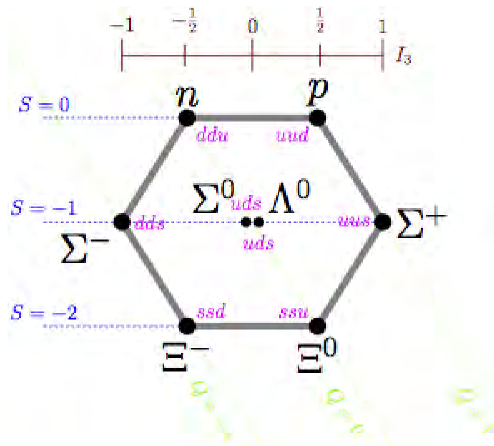


Figure 2.7: The baryon octet.

uud(made up of two up and one down quark) and neutron ddu (made up of two down and one up quark) as summarized in Table (2.5). The baryon is composed of three quarks states containing the light quarks u, d, s with zero orbital angular momentum, therefore the parity is $\mathrm{P} = \mathrm{P_q P_q P_q} = (+1)(+1)(+1)(-1)^{\mathrm{L}} = 1$. The baryon made up of three quarks belongs to the multiplets of :

$$3 \otimes 3 \otimes 3 = 10_{\mathrm{S}} \oplus 8_{\mathrm{M,S}} \oplus 8_{\mathrm{M,A}} \oplus 1_{\mathrm{A}}.$$

The baryon supermultiplets can have 10, 8,1 members and are known as decuplets, octets and singlets, respectively as shown in Table (2.6). The lightest baryon supermultiplets detected experimentally are an octet of $\mathrm{J^P} = \frac{1}{2}^+$ particles and a decuplet of $\mathrm{J^P} = \frac{3}{2}^+$ particles. The hyper charge for baryons is Y = S + 1 and the diagram for the baryon decuplet and octet representation is depicted in Fig.(2.7) and Fig.(2.8), respectively.

Mesons	Baryons
$(\mathrm{q\bar{q}})_{\mathrm{L=0}}$	$(\mathrm{qqq})_{\mathrm{L=0}}$
$\mathrm{S} = 0, 1$	$\mathrm{S} = \frac{1}{2}, \frac{3}{2}$
$\mathrm{J^P} = 0^-, 1^-$	$\mathrm{J^P} = \frac{1}{2}^+, \frac{3}{2}^+$

2.3 Wave Functions For Mesons

The mesons are made up of quark and an anti quark $\mathrm{q\bar{q}}$. The lowest mesons have

$$(\mathrm{q\bar{q}})_{\mathrm{L=0}} \text{ and Parity } \mathrm{P} = (-1)(-1)^{\mathrm{L}} = -1.$$

The spin wave functions are written as,

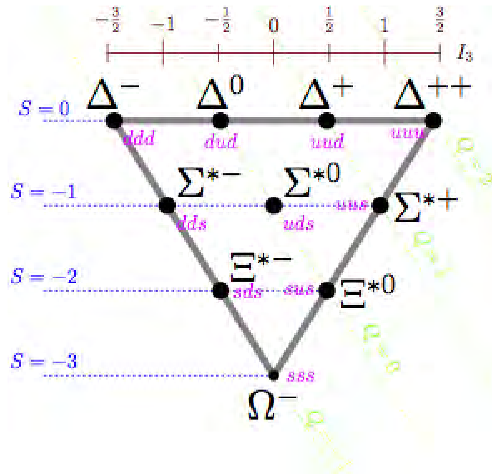


Figure 2.8: The baryon decuplet.

Quark composition	Observed state	Q	I	I_3	S	Y= B + S
uud	p(*938*)	1	$\frac{1}{2}$	$\frac{1}{2}$	0	1
udd	n (940)	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	1
uds	Λ^0 (1116)$^{(*)}$	0	0	0	−1	0
uus	Σ^+(1189)	1	1	1	−1	0
uds	Σ^0(1193)	0	1	0	−1	0
dds	Σ^-(1197)	−1	1	−1	−1	0
uss	Ξ^0(1315)	0	$\frac{1}{2}$	$\frac{1}{2}$	−2	−1
dss	Ξ^-(1321)	−1	$\frac{1}{2}$	$-\frac{1}{2}$	−2	−1

Table 2.5: The states of L = 0, $J^P = \frac{1}{2}^+$ octet of light baryons.

Quark composition	Observed state	Q	I	I_3	S	Y= B + S
uuu	$\Delta^{++}(1232)^{(*)}$	2	$\frac{3}{2}$	$\frac{3}{2}$	0	1
uud	$\Delta^+(1232)^{(*)}$	1	$\frac{3}{2}$	$\frac{1}{2}$	0	1
udd	$\Delta^0(1232)^{(*)}$	0	$\frac{3}{2}$	$-\frac{1}{2}$	0	1
ddd	$\Delta^-(1232)^{(*)}$	−1	$\frac{3}{2}$	$-\frac{3}{2}$	0	1
uus	Σ^+(1383)	+1	1	1	−1	0
uds	Σ^0(1384)	0	1	0	−1	0
dds	Σ^-(1387)	−1	1	−1	−1	0
uss	Ξ^0(1532)	0	$\frac{1}{2}$	$\frac{1}{2}$	−2	−1
dss	Ξ^-(1535)	−1	$\frac{1}{2}$	$-\frac{1}{2}$	−2	−1
sss	Ω^-(1672)	−1	0	0	−3	−2

Table 2.6: The states of L = 0, $J^P = \frac{3}{2}^+$ decuplet of light baryons.

Spin singlet and triplet state,

$$\chi_{\mathrm{A}} = \frac{1}{\sqrt{2}} \left|\uparrow\downarrow - \downarrow\uparrow\right\rangle, \tag{2.1}$$

$$\chi_{\mathrm{S}}^{1,0,-1} = \left|\uparrow\uparrow\right\rangle, \frac{1}{\sqrt{2}} \left|\uparrow\downarrow + \downarrow\uparrow\right\rangle, \left|\downarrow\downarrow\right\rangle \tag{2.2}$$

The spin singlet state is appeared to be anti symmetric and it has $\mathrm{J^P} = 0^-$, on the other hand spin triplet states appeared to be symmetric and gives $\mathrm{J^P} = 1^-$ therefore, SU(6) states can be written as

$$\mathrm{J^P} = 0^- \left({}^1\mathrm{S}_0\right),$$

$$(\mathrm{q_i \bar{q}_j})_{\mathrm{L=0}}\ \chi_{\mathrm{A}} = (\mathrm{q_i \bar{q}_j})\ \frac{1}{\sqrt{2}} \left|\uparrow(\mathrm{i}) \downarrow(\mathrm{j}) - \downarrow(\mathrm{i}) \uparrow(\mathrm{j})\right\rangle.$$

.For example,

$$\prod^{+} = (\mathrm{u}^{\uparrow}\overline{\mathrm{d}}^{\downarrow} - \mathrm{u}^{\downarrow}\overline{\mathrm{d}}^{\uparrow}),$$

$$\prod^{0} = \frac{1}{2}\ (\mathrm{u}^{\uparrow}\overline{\mathrm{u}}^{\downarrow} - \mathrm{u}^{\downarrow}\overline{\mathrm{u}}^{\uparrow} - \mathrm{d}^{\uparrow}\overline{\mathrm{d}}^{\downarrow} + \mathrm{d}^{\downarrow}\overline{\mathrm{d}}^{\uparrow}).$$

In the same way, we can write SU(6) states for other members of the nonets. For example $\mathrm{J^P} = 1^-({}^3\mathrm{S}_1)$, SU(6) states are summarized as follows:

For the P-wave i.e., L = 1 mesons

$$(\mathrm{q\bar{q}})_{\mathrm{L=1}} \text{ and Parity } \mathrm{P} = (-1)(-1)^1 = +1.$$

We have the following SU(6) wave functions, for L = 1 mesons

$$(\mathrm{q\bar{q}})_{\mathrm{L=0}}\ \chi_{\mathrm{A}}^{0} : ({}^1\mathrm{P}_1) \text{ state } 1^+,$$

$$(\mathrm{q\bar{q}})_{\mathrm{L=1}}\ \chi_{\mathrm{s}}^{1} : ({}^3\mathrm{P}_0) \text{ state } 0^+,$$

$$(\mathrm{q\bar{q}})_{\mathrm{L=1}}\ \chi_{\mathrm{s}}^{0} : ({}^3\mathrm{P}_1) \text{ state } 1^+,$$

	$J_z = 1$	$J_z = 0$	$J_z = -1$
	$(q_i \bar{q_j})_{L=0}\ \chi_s^1$	$(q_i \bar{q_j})_{L=0}\ \chi_s^0$	$(q_i \bar{q_j})_{L=0}\ \chi_s^{-1}$
	$(q_i \bar{q_j})(\uparrow(i) \downarrow(j))$	$(q_i \bar{q_j})\ \frac{1}{\sqrt{2}} \vert\uparrow(i) \downarrow(j) + \downarrow(i) \uparrow(j)\rangle$	$(q_i \bar{q_j})(\downarrow(i) \downarrow(j))$
ρ^+ :	$(u^\uparrow \overline{d}^\uparrow)$	$\frac{1}{\sqrt{2}}\ (u^\uparrow \overline{d}^\downarrow + u^\downarrow \overline{d}^\uparrow)$	$(u^\downarrow \overline{d}^\downarrow)$
ω^0 :	$\frac{1}{\sqrt{2}}(u^\uparrow \overline{d}^\uparrow + u^\uparrow \overline{d}^\uparrow)$	$\frac{1}{2}\ (u^\uparrow \overline{u}^\downarrow + u^\downarrow \overline{u}^\uparrow + d^\uparrow \overline{d}^\downarrow + d^\downarrow \overline{d}^\uparrow)$	$\frac{1}{\sqrt{2}}(u^\downarrow \overline{d}^\downarrow + u^\downarrow \overline{d}^\downarrow)$

$$(q\bar{q})_{L=1}\ \chi_s^{-1} : (^3P_2) \text{ state } 2^+.$$

The C parity is composite of same flavor quark and anti-quark for the mesons and it is given by $C = (-1)^{L+S}$ and G parity $G = C(-1)^I$. Hence for

$$L = 0\ S = 0\ C = +1\ G = 1^-\ \Pi,$$

$$L = 0\ S = 0\ C = +1\ G = 1^+\ \eta, \eta',$$

$$L = 0\ S = 1\ C = -1\ G = 1^+\ \rho,$$

$$L = 0\ S = 1\ C = -1\ G = 1^-\ \omega, \phi.$$

For

$$^1P_1 : 1^{+-} : L = 1\ S = 0\ C = -1,$$

$$^3P_0 : 0^{++} : L = 1\ S = 1\ C = 1,$$

$$^3P_1 : 1^{++} : L = 1\ S = 1\ C = 1,$$

$$^3P_2 : 2^{++} : L = 1\ S = 1\ C = 1.$$

Therefore we have nonent of such states in the quark model. These states can be recognized in the Particle book data [16].

$S_z = 3/2$	$S_z=1/2$	$S_z = -1/2$	$S_z = -3/2$
$\chi_s^{3/2}$ Symmetric: $\|\uparrow\uparrow\uparrow\rangle$,	$\frac{1}{\sqrt{3}}\|\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow\rangle$	$\frac{1}{\sqrt{3}}\|\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow\rangle$	$\|\downarrow\downarrow\downarrow\rangle$
$\chi_{MS}^{1/2}$ (Mixed symmetry) :	$\frac{1}{\sqrt{3}}\left\|-\frac{(\uparrow\downarrow+\downarrow\uparrow)\uparrow}{\sqrt{2}} + \sqrt{2}\uparrow\uparrow\downarrow\right\rangle$	$\frac{1}{\sqrt{3}}\left\|\frac{(\uparrow\downarrow+\downarrow\uparrow)\downarrow}{\sqrt{2}} - \sqrt{2}\downarrow\downarrow\uparrow\right\rangle$	

Table 2.7: S = $\frac{1}{2}$ and S= $\frac{3}{2}$ Spin wave functions which results by combining the spins (S =1/2 and S = 1).

2.3.1 Wave Function For Baryons

Baryons are composed of three quarks and lowest lying: $(qqq)_{L=0}$, Parity P = $(1)^3(-1)^0$ = 1. According to the group theory, the direct product of three basic (two dimensional) representation of SU(2) decomposes in the following manner (into the direct sum of a four dimensional and two dimensional representation) [17].

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2.$$

In above decomposition, 4 represents completely symmetric spin $\frac{3}{2}$, 2 represents mixed symmetry spin $\frac{1}{2}$. In case of baryon three spin $1/2'$s are combined. It is easy when we combine spin as S= $\frac{1}{2}\pm\frac{1}{2}$, and therefore we get S = 0 which has a wave function χ_A as given in Eq (2.1) and S = 1 which have wave function χ_s as given in Eq (2.2). Now we combine the remaining spin 1/2 with S = 0 and we get a total spin S = 1/2 and we write wave function of these states as:

$$\chi_{MA}^{1/2} = \frac{1}{\sqrt{2}} \mid \uparrow\downarrow - \downarrow\uparrow > \uparrow > \qquad \chi_{MA}^{-1/2} = \frac{1}{\sqrt{2}} \mid \uparrow\downarrow - \downarrow\uparrow > \downarrow > \tag{2.3}$$

Now combine remaining spin 1/2 with spin 1 and in this way, we get spin3/2 and spin 1/2 and for this case the spin wave functions are represented in Table (2.7). The numerical coefficients are the Clebsch Gordon coefficients which results by combining the spins as $(1 \pm \frac{1}{2})= \frac{1}{2}, \frac{3}{2}$, as shown in the Table (2.7).

2.4 Magnetic Moments

The magnetic dipole moments of the particles of the $1/2^+$octet of baryons can be evaluated as an application of baryon spin / flavor wave functions. The net magnetic moment of the baryon is the vector sum of the moments of three quarks as its constituents, in the absence of orbital motion.

$$\mu = \mu_1 + \mu_2 + \mu_3.$$

It depends on the spin configuration and on the quark flavors(because different magnetic moments are carried by three flavors). The magnetic dipole moment of point particle of charge q, mass m and having spin - $\frac{1}{2}$ is given by,

$$\mu = \frac{\mathrm{q}}{\mathrm{mc}} \quad \mathrm{S}\mu = \frac{\mathrm{q}\hbar}{2\mathrm{mc}}. \quad \text{(magnitude of magnetic dipole moment)}$$

$S_z = \frac{\hbar}{2}$ for the spin-up state,

For the quarks,

$$\mu_{\mathrm{u}} = \frac{2\mathrm{e}\hbar}{3.2\mathrm{m_u c}},\ \mu_{\mathrm{d}} = -\frac{1e\hbar}{3.2\mathrm{m_d c}}, \mu_s = -\frac{1\mathrm{e}\hbar}{3.2\mathrm{m_s c}}.$$

The baryon B has the magnetic moment,

$$\mu_{\mathrm{B}} = \langle \mathrm{B}\uparrow |(\mu_1 + \mu_2 + \mu_3)|\ \mathrm{B}\uparrow\rangle\,,$$

$$\mu_{\mathrm{B}} = \frac{2}{\hbar}\sum_{\mathrm{i}=1}^{3} \langle \mathrm{B}\ \uparrow |(\mu_{\mathrm{i}}\mathrm{S}_{iz})|\ \ \mathrm{B}\uparrow\rangle\,. \tag{2.4}$$

2.4.0.1 Baryon Magnetic Moment

The wave function for a proton with spin up is given by,

$$|\ \mathrm{p} : \frac{1}{2}\frac{1}{2} >= \frac{2}{3\sqrt{2}}\,(\mathrm{u}(\uparrow)\mathrm{u}(\uparrow)\mathrm{d}(\downarrow)) - \frac{1}{3\sqrt{2}}\,(\mathrm{u}(\uparrow)\mathrm{u}(\downarrow)\mathrm{d}(\uparrow)) - \frac{1}{3\sqrt{2}}\,(\mathrm{u}(\downarrow)\mathrm{u}(\uparrow)\mathrm{d}(\uparrow)) + \text{ permutations.} \tag{2.5}$$

Consider the first term i.e., $\frac{2}{3\sqrt{2}}$ (u(↑)u(↑)d(↓)) and from Eq (2.4),

$$(\mu_1 \mathrm{S}_{1z} + \mu_2\mathrm{S}_{2z} + \mathrm{S}_{3z}\mu_3)\,|(\mathrm{u}(\uparrow)\mathrm{u}(\uparrow)\mathrm{d}(\downarrow))\rangle$$

$$= \left[\mu_{\mathrm{u}}\frac{\hbar}{2} + \mu_{\mathrm{u}}\frac{\hbar}{2} + \mu_{\mathrm{d}}\left(\frac{-\hbar}{2}\right)\right]\,|(\mathrm{u}(\uparrow)\mathrm{u}(\uparrow)\mathrm{d}(\downarrow))\rangle\,,$$

Therefore, this term makes a contribution as,

$$\begin{aligned}\mu_{\mathrm{p}} &= \frac{2}{\hbar}\sum_{\mathrm{i}=1}^{3} \langle \mathrm{B}\uparrow |(\mu_{\mathrm{i}} S_{\mathrm{iz}})|\ \mathrm{B}\uparrow\rangle\,, \\ &= \frac{2}{3\sqrt{2}}.\frac{2}{3\sqrt{2}}.\frac{2}{\hbar}\left[\mu_{\mathrm{u}}\frac{\hbar}{2} + \mu_{\mathrm{u}}\frac{\hbar}{2} - \mu_{\mathrm{d}}\left(\frac{\hbar}{2}\right)\right] = \frac{2}{9}\,(2\mu_{\mathrm{u}} - \mu_{\mathrm{d}})\,.\end{aligned}$$

In the same way, the second and third term in Eq (2.5) contributes an amount $\frac{1}{18}\mu_{\mathrm{d}}$.The result is given by,

Baryon	Moment	Prediction	Experiment
p	$\left(\frac{4}{3}\right)\mu_u - \left(\frac{1}{3}\right)\mu_d$	2.79	2.793
n	$\left(\frac{4}{3}\right)\mu_d - \left(\frac{1}{3}\right)\mu_u$	–1.86	–1.913
Λ	μ_s	–0.58	–0.613
Σ^+	$\left(\frac{4}{3}\right)\mu_u - \left(\frac{1}{3}\right)\mu_s$	2.68	2.458
Σ^-	$\left(\frac{4}{3}\right)\mu_d - \left(\frac{1}{3}\right)\mu_s$	–1.05	–1.160
Ξ^0	$\left(\frac{4}{3}\right)\mu_s - \left(\frac{1}{3}\right)\mu_u$	–1.40	–1.250
Ξ^-	$\left(\frac{4}{3}\right)\mu_s - \left(\frac{1}{3}\right)\mu_d$	–0.47	–0.651

Table 2.8: Magnetic Dipole moments of Octet Baryons.

$$\mu_p = 3\left[\frac{2}{9}\left(2\mu_u - \mu_d\right) + \frac{1}{18}\mu_d + \frac{1}{18}\mu_d\right] = \frac{1}{3}\left(4\mu_u - \mu_d\right).$$

In the similar way, we can calculate all the magnetic moments in the form of μ_u, μ_d, μ_s, as displayed in Table (2.8).

2.5 Spin - Spin Interaction

A charged particle having charge eQ_i with spin 1/2 has a magnetic momentum

$$\mu_i = \frac{e\ Q_i}{2m_i}\ \sigma_i.$$

The energy splitting between the states with zero orbital angular momentum (L= 0) say S-states, according to quantum mechanics, can be written by Fermi contact term (two particle operator),

$$H^M_{ij} = \frac{1}{4\Pi}\left[-\frac{8\Pi}{3}\mu_i \cdot \mu_j \delta^3(r_i - r_j)\right] = -\frac{8\Pi}{3}\left(\frac{e^2}{4\Pi}\right)\frac{Q_i Q_j}{2m_i m_j}\sigma_i \cdot \sigma_j \delta^3(r) = -\frac{8\Pi}{3}\alpha\frac{Q_i Q_j}{2m_i m_j}\sigma_i \cdot \sigma_j \delta^3(r), \tag{2.6}$$

Since $\alpha = \left(\frac{e^2}{4\Pi}\right)$ likewise in QCD, where eight color-magnetic moments are present.

$$\mu^i_A = \frac{g_s}{2m_i}\left(\frac{\Lambda_A}{2}\right)\sigma \qquad \text{with } A = 1, 2, 3, ..., 8. \tag{2.7}$$

The similar interaction for two-particles in QCD is written as,

$$\begin{aligned} H^M_{ij} &= \frac{1}{4\Pi}\left[-\frac{8\Pi}{3}\mu^i_{(A)} \cdot \mu^j_{(A)}\delta^3(r_i - r_j)\right], \\ &= -\frac{8\Pi}{3}\frac{g_s^2}{4\Pi}\left(\frac{\Lambda_A^2}{4}\right)\frac{\sigma_i \cdot \sigma_j}{4m_i m_j}\ \delta^3(r), \end{aligned} \tag{2.8}$$

$$H_{ij}^{M} = -\frac{8\Pi}{3}\,\alpha_s k_s \frac{\sigma_i \cdot \sigma_j}{4m_i m_j}\,\delta^3(r). \tag{2.9}$$

Since $\alpha_s = \frac{g_s^2}{4\Pi}$ and $\alpha \rightarrow k_s\alpha_s$ for a color singlet system. Eq (2.9), implies that m $\left({}^3S_1\right) >$ m $\left({}^1S_0\right)$ for example, $(m_\rho > m_\Pi)$. This strengthen the fact that the gluons are particles with spin S = 1.

2.5.1 Mass Spectrum

When we sum over all quark indices that are possible in V_{ij}^G in a multi-quark system like baryons (qqq) and mesons $(q\bar{q})$, the one gluon exchange potential is established. Therefore,

$$\begin{aligned} V^G &= \frac{1}{2}\sum_{i\neq j} V_{ij}^G, \\ &= \frac{1}{2}\left[\sum_{i>j} V_{ij}^G + \sum_{i>j} V_{ij}^G\right], \\ &= \frac{1}{2}\left[\sum_{i>j}\left(V_{ij}^G + V_{ji}^G\right)\right], \end{aligned}$$

$$V^G = \sum_{i>j} V_{ij}^G. \tag{2.10}$$

In non-relativistic limit the potential for S-states, V_G and we keep the terms up to momentum square is given by,

$$V_G = k_s\alpha_s \sum_{i>j}\left[\frac{1}{r} - \frac{1}{2m_i m_j}\left(\frac{p_i \cdot p_j}{r} + \frac{r\,(r\,\cdot\,p_i)\,\cdot\,p_j}{r^3}\right) - \frac{\Pi}{2}\delta^3(r)\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\,S_i\,\cdot\,S_j}{3m_i m_j}\right)\right]. \tag{2.11}$$

We will ignore the second term in the bracket because we know that the one gluon exchange potential is independent of velocity.On the right hand side the first term is the potential in extreme non-relativistic limit $\left(\frac{v}{c} \simeq 0\right)$. The term dependent on spin is because of color magnet moments interaction.

For S-states,

$$\langle\psi_s\left|\delta^3(r)\right|\psi_s\rangle = \int \psi_s^*(r)\delta^3(r)\psi_s^*(r)d^3r = \mid \psi_s(0)\mid^2 .$$

Now the Hamiltonian, having the rest masses of quarks is given by,

$$H\,(r) = \sum_i m_i + \sum_i \frac{\hat{p}_i^2}{m_i} + V_C\;(r) + V_G\;(r), \tag{2.12}$$

where

$$\hat{p}_{\mathrm{i}}^2 = -\frac{\hbar^2}{2\mathrm{m_i}}\nabla_{\mathrm{i}}^2$$

Here $V_C(r)$ is defined as the confining potential, V_G (r)is termed as the one gluon exchange potential which is given in Eq (2.12), and for hadron, i is the index of quark flavor,(i = up,down,strange). Here let's assume mass of up quark is equal to the mass of down quark ($m_u = m_d$).We are interested in discussion of the mass spectrum of hadrons so we will consider the expectation values of Hamiltonian H(r) with respect to $\psi_s(r)$ hadron wave function.The hadron wave function is actually consists of three parts (space, spin and unitary spin).For S-wave, The space function can be written as $\psi_s(r)$, for S-wave.The expectation value of the Hamiltonian H(r) gives us,

$$\mathrm{m} \equiv \langle \psi_s\,(r)\,|H|\,\psi_s\,(r)\rangle\,,$$

$$\mathrm{m} = \sum_i \mathrm{m_i} + \sum_i \frac{1}{\mathrm{m_i}} \langle \psi_s\,(r)\,\left|\hat{p}_i{}^2\right|\,\psi_s\,(r)\rangle + \langle \psi_s\,(r)\,|V_C\,(r)|\,\psi_s\,(r)\rangle + k_s\alpha_s \frac{1}{r}\,|\psi_s\,(r)\rangle - k_s\alpha_s \frac{\Pi}{2}\left(\frac{1}{\mathrm{m_i^2}} + \frac{1}{\mathrm{m_j^2}} + \frac{16\ \mathrm{S_i} \cdot \mathrm{S_j}}{3\mathrm{m_i}\mathrm{m_j}}\right) |\,\psi_s(0)\,|^2\,.$$

We have noted that in the spin space and in the unitary spin, the mass operator is still an operator.

2.5.1.1 Baryon Mass Spectrum

If we want to study the mass spectrum for baryons, it is useful to first work out the matrix elements for the spin operator between spin states.

$$\Omega_{ss} = \sum_{i>j} \frac{1}{\mathrm{m_i}\mathrm{m}_{\,j}} \mathrm{S_i} \cdot \mathrm{S_j}\,, \tag{2.13}$$

The eigen values for $S_i \cdot S_j$ are $-3/4$ for spin the singlet and $1/4$ for spin the triplet states, as we have calculated before. So,

$$\mathrm{S_i} \cdot \mathrm{S_j}\,|\uparrow\uparrow\rangle = -\frac{3}{4}\,|\uparrow\uparrow\rangle\,,$$

$$\mathrm{S_i} \cdot \mathrm{S_j}\left[\frac{1}{\sqrt{2}}\,|\uparrow\downarrow + \downarrow\uparrow\rangle\right] = \frac{1}{4\sqrt{2}}\,|\uparrow\downarrow\ + \downarrow\uparrow\rangle\,,$$

$$\mathrm{S_i} \cdot \mathrm{S_j}\,|\downarrow\downarrow\rangle = \frac{1}{4}\,|\downarrow\downarrow\rangle \tag{2.14}$$

$$S_i \cdot S_j \left[\frac{1}{\sqrt{2}}\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle\right] = -\frac{3}{4\sqrt{2}}\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle, \tag{2.15}$$

From Eq (2.14) and Eq (2.15), we get

$$S_i \cdot S_j \left|i^{\uparrow}j^{\downarrow}\right\rangle = -\frac{1}{4}\left|i^{\uparrow}j^{\downarrow}\right\rangle + \frac{1}{2}\left|i^{\downarrow}j^{\uparrow}\right\rangle, \tag{2.16}$$

$$S_i \cdot S_j \left|i^{\downarrow}j^{\uparrow}\right\rangle = -\frac{1}{4}\left|i^{\downarrow}j^{\uparrow}\right\rangle + \frac{1}{2}\left|i^{\uparrow}j^{\downarrow}\right\rangle. \tag{2.17}$$

The baryons spin wave functions are represented in Table (2.7) and Eq (2.3).We now use these wave functions, and also using the results we found in Eq (2.14) , Eq (2.15), Eq (2.16) and Eq (2.17), we get for $\frac{1}{2}^{+}$ baryons having $S_z = \frac{1}{2}$,

$$\Omega_{ss}\left|p\right\rangle = \left|uud\right\rangle \frac{1}{m_u^2}(-\frac{3}{4})\frac{1}{\sqrt{6}}\left|(\uparrow\downarrow + \downarrow\uparrow)-2\uparrow\uparrow\downarrow\right\rangle = -\frac{3}{4m_u^2}\left|p\right\rangle. \tag{2.18}$$

In the same manner by using,

$$\left|\Lambda\right\rangle = -\left|uds\right\rangle \chi_{MS}^{1/2}, \tag{2.19}$$

$$\left|\Sigma^0\right\rangle = \left|uds\right\rangle \chi_{MS}^{1/2}, \tag{2.20}$$

$$\left|\Xi^0\right\rangle = \left|ssu\right\rangle \chi_{MS}^{1/2}, \tag{2.21}$$

we get,

$$\Omega_{ss}\left|\Lambda\right\rangle = -\frac{3}{4m_u^2}\left|\Lambda\right\rangle, \tag{2.22}$$

$$\Omega_{ss}\left|\Sigma^0\right\rangle = \frac{1}{4}\left(\frac{1}{m_u^2} - \frac{4}{m_u m_s}\right)\left|\Sigma^0\right\rangle, \tag{2.23}$$

$$\Omega_{ss}\left|\Xi^0\right\rangle = \frac{1}{4}\left(\frac{1}{m_u^2} - \frac{4}{m_u m_s}\right)\left|\Xi^0\right\rangle, \tag{2.24}$$

We take $S_z = \frac{3}{2}$ for $\frac{3}{2}^{+}$baryons, now we will calculate the matrix elements for Ω_{ss}.

$$
\begin{aligned}
\Omega_{ss}\left|\Delta^{++}\right\rangle &= \sum_{i>j}\frac{1}{m_i m_j}S_i \cdot S_j\,|uuu\rangle\,|\uparrow\uparrow\uparrow\rangle = \frac{3}{4m_u^2}\left|\Delta^{++}\right\rangle, \\
\left|\Sigma^{*+}\right\rangle &= |uus\rangle\,|\uparrow\uparrow\uparrow\rangle, \\
\left|\Xi^{*+}\right\rangle &= |ssu\rangle\,|\uparrow\uparrow\uparrow\rangle, \\
\left|\Omega^{-}\right\rangle &= |sss\rangle\,|\uparrow\uparrow\uparrow\rangle,
\end{aligned}
$$

we get,

$$
\begin{aligned}
\Omega_{ss}\left|\Sigma^{*+}\right\rangle &= \frac{1}{4}\left(\frac{2}{m_s m_u}+\frac{1}{m_u^2}\right)\left|\Sigma^{*+}\right\rangle, \\
\Omega_{ss}\left|\Xi^{*+}\right\rangle &= \frac{1}{4}\left(\frac{2}{m_s m_u}+\frac{1}{m_u^2}\right)\left|\Xi^{*+}\right\rangle, \\
\Omega_{ss}\left|\Omega^{-}\right\rangle &= \frac{3}{4}\left(\frac{1}{m_u^2}\right)\left|\Omega^{-}\right\rangle,
\end{aligned}
$$

The spin-spin interaction term from gluon exchange potential,

$$
\frac{16\pi}{9}\alpha_s \mid \psi_s(0)\mid^2 \Omega_{ss}.
$$

we concluded from Eq (2.22) and Eq (2.24),

$$
m\left(J=\frac{3}{2}\right) > m\left(J=\frac{1}{2}\right).
$$

This agrees with experimental values for gluons having color, $k_s = -\frac{2}{3}$. If gluons do not have color then the values of k_s would be 1 instead of −2/3, this would be a contradiction with the experimental observations. And this tells the fact that vector gluons carry the color. Now we discuss the masses of baryon with same spin, and for this purpose we use the following equation,

$$
m = m_0 + m_1 + m_2 + a\left(\frac{1}{m_1}+\frac{1}{m_2}\right) + \bar{d}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}+\frac{16\,S_1 \cdot S_2}{3\,m_1\,m_2}\right), \tag{2.25}
$$

where

$$\begin{aligned}
m_0 &= A_0 + k_s\alpha_s b, \\
a &= \langle \psi_s \left| \hat{p}_i^2 \right| \psi_s \rangle , \\
A_0 &= \langle \psi_s \mid |V_C(r)| \; \psi_s \rangle , \\
b &= \langle \psi_s \left| \frac{1}{r} \right| \psi_s \rangle , \\
\overline{d} &= -\frac{4}{3}\alpha_s \frac{\pi}{2} \langle \psi_s \left| \delta^3(r) \right| \psi_s \rangle .
\end{aligned}$$

Using Eq (2.19) and Eq (2.25), the baryon mass formula can be written as,

$$m = (m_1 + m_2 + m_3) \left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} \right) \langle \psi_s \left| \hat{p}_i^2 \right| \psi_s \rangle + \langle \psi_s \left| V_C - \frac{2}{3}\alpha_s \frac{1}{r} \right| \psi_s \rangle + \sum_{i>j} \left(\frac{1}{m_i^2} + \frac{1}{m_i^2} \right) \frac{\Pi}{3}\alpha_s \mid \psi_s(0) \mid^2$$

$$+ \frac{16\pi}{3}\alpha_s \mid \psi_s(0) \mid^2 \langle B \left| \Omega_{ss} \right| B \rangle .$$

We concluded that using the non-relativistic quantum mechanics, mass spectra for baryons can be explained for strange, up and down quarks. Results are fine, despite the fact that this approximation is not appropriate for these quarks, because the masses of these quarks are smaller than $\frac{1}{2}$ G eV., but still the results are good [18].

2.5.1.2 Baryon Masses

All the octet baryons would have same weight if flavor SU(3) were a perfect symmetry. But they do not weigh the same due to the fact that s quark is more heavier than u and d. Clearly, there is remarkable spin - spin (hyper fine) contribution,that is proportional to the dot product of the spins and it is inversely proportional to the product of the masses.

$$M(\text{baryons}) = m_1 + m_2 + m_3 + A' \left[\frac{S_1 \cdot S_2}{m_1 m_2} + \frac{S_1 \cdot S_3}{m_1 m_3} + \frac{S_2 \cdot S_3}{m_2 m_3} \right],$$

where A' is a constant. When three quarks have the mass, the spin products are easy to evaluate,

$$J^2 = (S_1 + S_2 + S_3)^2 = (S_1)^2 + (S_2)^2 + (S_3)^2 + 2\,(S_1 \cdot S_2 + S_1 \cdot S_3 + S_2 \cdot S_3)\,,$$

Therefore,

$$(\mathrm{S}_1 \cdot \mathrm{S}_2 + \mathrm{S}_1 \cdot \mathrm{S}_3 + \mathrm{S}_2 \cdot \mathrm{S}_3) = \left[\frac{\hbar^2}{2}\mathrm{j}(\mathrm{j}+1) - \frac{9}{4}\right],$$

$$= \left\{\frac{3\hbar^2}{4} \text{for j} = \frac{3}{2}(\text{decuplet})\right\} \text{ and } \left\{\frac{-3\hbar^2}{4} \text{for j} = \frac{1}{2}(\text{octet})\right\}. \tag{2.26}$$

Hence,The nucleon (proton or neutron) mass is,

$$\mathrm{M_N} = 3\mathrm{m_u} - \frac{3}{4}\frac{\hbar^2}{\mathrm{m_u^2}}\mathrm{A}',$$

The mass of Δ is,

$$\mathrm{M}_\Delta = 3\mathrm{m_u} + \frac{3}{4}\frac{\hbar^2}{\mathrm{m_u^2}}\mathrm{A}',$$

The mass of Ω^- is,

$$\mathrm{M}_{\Omega^-} = 3\mathrm{m_s} + \frac{3}{4}\frac{\hbar^2}{\mathrm{m_s^2}}\mathrm{A}'.$$

All the spins are parallel in the case of decuplet (each pair combines to form spin 1), so

$$(\mathrm{S}_1 + \mathrm{S}_2)^2 = (\mathrm{S}_1)^2 + (\mathrm{S}_2)^2 + 2\mathrm{S}_1 \cdot \mathrm{S}_2 = 2\hbar^2,$$

Therefore, for the decuplet

$$(\mathrm{S}_1 \cdot \mathrm{S}_2) = (\mathrm{S}_1 \cdot \mathrm{S}_3) = (\mathrm{S}_2 \cdot \mathrm{S}_3) = \frac{\hbar^2}{4},$$

.,Hence,

$$\mathrm{M}_{\Sigma^*} = 2\mathrm{m_u} + \mathrm{m_s} + \frac{\hbar^2}{4}\mathrm{A}'\left(\frac{1}{\mathrm{m_u^2}} + \frac{2}{\mathrm{m_u m_s}}\right),$$

while,

$$\mathrm{M}_{\Xi^*} = 2\mathrm{m_s} + \mathrm{m_u} + \frac{\hbar^2}{4}\mathrm{A}'\left(\frac{1}{\mathrm{m_s^2}} + \frac{2}{\mathrm{m_u m_s}}\right).$$

The up and down quarks combines to form isospin 0 and 1 respectively, the spins must therefore combine to a total of 0 and1, in order for the spin / flavor wave function to be symmetric, under the exchange of u and d quarks. For the Σ ′ s then we have,

Baryon	Calculated	Observed
N	939	m
Λ	1114	1116
Σ	1179	1193
Ξ	1327	1318
Δ	1239	1232
Σ^*	1381	1385
Ξ^*	1529	1533
Ω	1682	1672

Table 2.9: Baryon decuplet and octet masses (MeV./ c^2).

$$(S_u + S_d)^2 = (S_u)^2 + (S_d)^2 + 2S_u \cdot S_d = 2\hbar^2 \quad \text{So,} \quad S_u \cdot S_d = \frac{\hbar^2}{4},$$

whereas for the Λ, we have

$$(S_u + S_d)^2 = 0 \qquad \text{So,} \quad S_u \cdot S_d \; = \frac{-3\hbar^2}{4},$$

Using this result along with Eq (2.26), we get

$$\begin{aligned} M_\Sigma &= 2m_u + m_s + A' \left(\frac{S_u \cdot S_d}{m_u m_d} + \frac{(S_1 \cdot S_2 + S_1 \cdot S_3 + S_2 \cdot S_3 - S_u \cdot S_d)}{m_u m_s} \right), \\ M_\Sigma &= 2m_u + m_s + \frac{\hbar^2}{4} A' \left(\frac{1}{m_u^2} - \frac{4}{m_u m_s} \right), \end{aligned}$$

And

$$M_\Lambda = 2m_u + m_s - \frac{3\hbar^2}{4} \frac{A'}{m_u^2},$$

And mass of Ξ 's is

$$M_\Xi = 2m_s + m_u + \frac{\hbar^2}{4} A' \left(\frac{1}{m_s^2} - \frac{4}{m_u m_s} \right).$$

Considering $A' = \left(\frac{2m_u}{\hbar^2}\right) 50$ MeV./c^2 and using the masses of the up, down and strange quark, $m_u = m_d = 363.2$ MeV./c^2, $m_s = 538.1$ Me V/$c^2$and we obtain the results which fits to the experimental data shown in Table (2.9).

2.6 Quarkonium

Quarkonium is the name of sub-atomic system, which is composed of a heavy quark Q and an anti-quark $\overline{\mathrm{Q}}$ and bound by strong interaction, $(\mathrm{Q}\overline{\mathrm{Q}}\ , \mathrm{Q} = \mathrm{c}, \mathrm{b})$ for example bottomonium $\mathrm{b}\overline{\mathrm{b}}$ and charmonium $\mathrm{c}\overline{\mathrm{c}}$. Quarks bound state can be written as $\left(\mathrm{Q}\overline{\mathrm{Q}}\right)_{\mathrm{L,S}}$, since quarks have Spin 1/2 as that of fermion. Now spin S can have two values, 0 and 1 having anti symmetric and symmetric spin wave functions, respectively. Consider Q $\overline{\mathrm{Q}}$ as identical fermions, and their charges are different, then generalized Pauli Exclusion principle can be stated as: With exchange of particles (heavy quark Q and an anti-quark $\overline{\mathrm{Q}}$), the wave function is anti symmetric. Under the exchange of the particle, we get the exchange of space coordinates by a factor $(-1)^{\mathrm{L}}$, the exchange of spin coordinates by a factor $(-1)^{\mathrm{S}+1}$and the exchange of charge by a factor C (where C is called C - parity) [19].Therefore, the Pauli principle gives,

$$(-1)^{\mathrm{L}+\mathrm{S}+1}\mathrm{C} = -1,$$

,Hence,

$$\mathrm{C} = (-1)^{\mathrm{L}+\mathrm{S}}.$$

we get the following expression,

$$\mathrm{C} = \begin{cases} -1 & \mathrm{L}+\mathrm{S} \quad \text{odd} \\ +1 & \mathrm{L}+\mathrm{S} \quad \text{even} \end{cases}.$$

For the $\left(\mathrm{Q}\overline{\mathrm{Q}}\right)$ system, we have the parity as,

$$\mathrm{P} = (-1)(-1)^{\mathrm{L}} = (-1)^{\mathrm{L}+1}.$$

Let 's introduce the spectroscopic notation,

$$L = \begin{matrix} 0, & 1, & 2, & 3, \\ \mathrm{S}, & \mathrm{P}, & \mathrm{D}, & \mathrm{E}, \end{matrix}$$

A state is completely described as,

$$\mathrm{n}\ {}^{2\mathrm{S}+1}L_{\mathrm{J}}.$$

where n describes principal quantum number and J is the total angular momentum. Therefore,for L = 0, we have the following states,

n	${}^{1}S_{0}$	C = +1	n = 1, 2, ...	0^{-+}
n	${}^{3}S_{1}$	C = −1	n = 1, 2, ...	1^{--}

Therefore, the ground state is a hyper fine doublet $1^{1}S_{0}$ (0^{-+}) and $1^{3}S_{1}$(1^{--}). For or L = 1, we have the following states,

n	${}^{1}P_{J}$	J = +1	C = −1	n = 1, 2, ...	1^{+-}
n	${}^{3}P_{J}$	J = 0, 1, 2	C = 1	n = 1, 2, ...	$0^{++}, 1^{++}, 2^{++}$

For L =2, we have the following states,

n	${}^{1}D_{J}$	J = 2	C = +1	n = 1, 2, ...	2^{-+}
n	${}^{3}D_{J}$	J = 1, 2, 3	C = −1	n = 1, 2, ...	$1^{--}, 2^{--}, 3^{--}$

2.7 The bottom quark

2.7.1 Discovery

The first evidence for the existence of the bottom quark [20] was provided in1977, by the Fermi lab experiment directed by Leon Lederman,when fixed target experiment (i.e.400 GeV. proton collides with nuclear target) produced bottom quark. A particle discovered by the experiment, known as the upsilon (Υ) which a mass of 9.512 GeV. It consists of a new type of quark, called bottom quark (b) and its anti-quark known as anti-bottom ($\bar{b}$).

2.7.2 Properties

The contribution of the bottom quark is significant in both the electroweak and strong interactions. It is the member of the third generation of Standard Model. It was considered the most massive particle ever discovered, at the time of discovery. Mass of the bottom quark is 4.20 Ge V/c^2 [21] and spin 1/2. It has a charge of –1/3 e, isospin 0, even parity and bottomness number –1. It holds several unique properties and it has a mass a bit more than 4 times the mass of a proton and many orders of magnitude higher than common light-quarks.

Chapter 3

Construction of States

3.1 S-S Coupling

A Ω_c baryon can be written as c(ss), where c is a charm quark and [ss] is a diquark and the spin of diquark is $S_{[ss]}$ $= 1$, $M_{S_{[ss]}} = -1, 0, 1$. The spin of heavy (charm) quark $S_Q = \frac{1}{2}$ and $M_{S_Q} = -\frac{1}{2}, \frac{1}{2}$

$$\begin{aligned} S = & \quad S_{[ss]} \pm S_Q = & \frac{1}{2}, \frac{3}{2}, \\ M = & \quad M_{S_{[ss]}} + M_{S_Q} = & -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}. \end{aligned}$$

Let's define $S_{[ss]} = S_1 = 1, S_Q = \frac{1}{2} = S_2 = \frac{1}{2}, M_{S_{[ss]}} = M_{S_1}$ and $M_{S_Q} = M_{S_2}$, here M_{S_1} and M_{S_2} is the third component of S_1 and S_2, respectively.

$$|S, M_S\rangle = \sum_{m_{s_1}, m_{s_2}} C^{S,S_1,S_2}_{M,M_{S_1},M_{S_2}} \, |S_1, m_{s_1}\rangle \otimes |S_2, m_{s_2}\rangle \,, \tag{3.1}$$

For $S = \frac{3}{2}$ possible values of $M_S = \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}$ because $-S \leq M \geq +S$, similarly for M_{S_1},there are three possible value as $-1, 0, 1$ and M_{S_2} have possible value as $-\frac{1}{2}, \frac{1}{2}$, similarly for spin $= \frac{3}{2}$ we have an eigenstates as follows,

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = C^{\frac{3}{2},1,\frac{1}{2}}_{\frac{3}{2},1,\frac{1}{2}} \, |1, 1\rangle \otimes \left|\frac{1}{2}, \frac{1}{2}\right\rangle,$$

Using Clebsch Gordan coefficients, we substitute the value of C ′s,we have

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = |1,1\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle,$$
$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \mathrm{C}^{\frac{3}{2},1,\frac{1}{2}}_{\frac{1}{2},1,-\frac{1}{2}} |1,1\rangle \otimes \left|\frac{1}{2},\frac{-1}{2}\right\rangle + \mathrm{C}^{\frac{3}{2},1,\frac{1}{2}}_{\frac{1}{2},0,\frac{1}{2}} |1,0\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle,$$

Similarly, for the other states we have,

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|1,1\rangle \otimes \left|\frac{1}{2},\frac{-1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1,0\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle, \tag{3.2}$$
$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \mathrm{C}^{\frac{3}{2},1,\frac{1}{2}}_{-\frac{1}{2},-1,\frac{1}{2}} |1,-1\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle + \mathrm{C}^{\frac{3}{2},1,\frac{1}{2}}_{-\frac{1}{2},0,-\frac{1}{2}} |1,0\rangle \otimes \left|\frac{1}{2},\frac{-1}{2}\right\rangle, \tag{3.3}$$

$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|1,-1\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1,0\rangle \otimes \left|\frac{1}{2},\frac{-1}{2}\right\rangle, \tag{3.4}$$
$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = 1\,|1,-1\rangle \otimes \left|1,-\frac{1}{2}\right\rangle. \tag{3.5}$$

For S $= \frac{1}{2}$, possible values of $\mathrm{M_S} = -\frac{1}{2}, \frac{1}{2}$ because $-\mathrm{S} \leq \mathrm{M} \geq +\mathrm{S}$, and similarly for this value of spin we have an eigenstates as follows,

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{\frac{1}{2},0,\frac{1}{2}} |1,0\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle + \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{\frac{1}{2},1,-\frac{1}{2}} |1,1\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$

Using Clebsch Gordan coefficients, we substitute the value of C ′s

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{3}}|1,0\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1,1\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}|1,1\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}|1,0\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle, \tag{3.6}$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{\frac{1}{2},0,-\frac{1}{2}} |1,0\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle + \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{-\frac{1}{2},-1,\frac{1}{2}} |1,-1\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle, \tag{3.7}$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{\frac{1}{2},0,-\frac{1}{2}} |1,0\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle + \mathrm{C}^{\frac{1}{2},1,\frac{1}{2}}_{-\frac{1}{2},-1,\frac{1}{2}} |1,-1\rangle \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle. \tag{3.8}$$

Then upon using the value of Clebsch-Gordan coefficients give

$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}|1,0\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}|1,-1\rangle \otimes \left|1,\frac{1}{2}\right\rangle. \tag{3.9}$$

3.2 L - S Coupling

We have $S_{[ss]} = S_1 = 1$ and $S_c = S_2 = \frac{1}{2}$, so total spin will be $S = S_1 \pm S_2 = 1 \pm \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$. Now first we will calculate P − wave states for $S = \frac{1}{2}$, here L =1 and for J we have,

$$J = L \pm S = 1 \pm \frac{1}{2} = \frac{1}{2}, \frac{3}{2},$$

As value of orbital angular momentum(L =1), possible values of $J = \frac{1}{2}$,$\frac{3}{2}$,from Eq (3.1), we have

$$|J, J_3\rangle = \sum_{L_3,S_3} C^{J,L,S}_{J,L_3,S_3} |L, L_3\rangle \otimes |S, S_3\rangle ,$$

$$\left|\frac{3}{2}, J_3\right\rangle = C^{\frac{3}{2},1,\frac{1}{2}}_{J_3,L_3,S_3} |L, L_3\rangle \otimes |S, S_3\rangle ,$$

Now substituting the third component of angular momentum

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = C^{\frac{3}{2},1,\frac{1}{2}}_{\frac{3}{2},1,\frac{1}{2}} |1, 1\rangle \otimes \left|\frac{1}{2}, \frac{1}{2}\right\rangle ,$$

By substituting the value of coefficient we get,

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = |1, 1\rangle \otimes \left|\frac{1}{2}, \frac{1}{2}\right\rangle ,$$

Now, substituting Eq (3.6) in place of spin wave function, we get the following result

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = |1, 1\rangle \otimes \left[\sqrt{\frac{2}{3}} |1, 1\rangle \otimes \left|\frac{1}{2}, -\frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}} |1, 0\rangle \otimes \left|\frac{1}{2}, \frac{1}{2}\right\rangle\right] ,$$

Lets write above states as $|L_3 , m_{s_1}, m_{s_2}\rangle$=$|L, L_3\rangle \otimes |S_1, m_{s_1}\rangle \otimes |S, m_{s_2}\rangle$, we have

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = \sqrt{\frac{2}{3}} \left|1, 1, -\frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}} \left|1, 0, \frac{1}{2}\right\rangle ,$$

And in simplified notation, $\left|S_{d_3}, S_{Q_3}, L_3\right\rangle$ it will take the form

$$\left|\frac{3}{2}, \frac{3}{2}\right\rangle = \sqrt{\frac{2}{3}} \left|1, -\frac{1}{2}, 1\right\rangle - \frac{1}{\sqrt{3}} \left|0, \frac{1}{2}, 1\right\rangle ,$$

States are well described by term symbol $({}^{2S+1}L_J)$, where 2S+1 is the spin multiplicity, S is the total spin quantum number, L is the total orbital momentum quantum number and J is the total angular momentum quantum number.

Here in above case $J = \frac{3}{2}, L = 1$ and $S = \frac{1}{2}$, the term symbol becomes, ${}^{2S+1}L_J = {}^{2(\frac{1}{2})+1}P_{\frac{3}{2}} = {}^{2}P_{\frac{3}{2}}$. So we can write the baryon state as

$$\left|{}^2\mathrm{P}_{\frac{3}{2}},\ \mathrm{J}_3=\frac{3}{2}\right\rangle=\sqrt{\frac{2}{3}}\left|1,-\frac{1}{2},1\right\rangle-\frac{1}{\sqrt{3}}\left|0,\frac{1}{2},1\right\rangle. \tag{3.10}$$

Likewise, we can calculate the following remaining four states,

$$\left|{}^2\mathrm{P}_{\frac{1}{2}},\mathrm{J}_3=\frac{1}{2}\right\rangle=\frac{\sqrt{2}}{3}\left|1,-\frac{1}{2},0\right\rangle-\frac{\sqrt{2}}{3}\left|0,\frac{-1}{2},1\right\rangle+\frac{2}{3}\left|-1,\frac{1}{2},1\right\rangle-\frac{1}{3}\left|0,\frac{1}{2},0\right\rangle, \tag{3.11}$$

$$\left|{}^4\mathrm{P}_{\frac{1}{2}},\mathrm{J}_3=\frac{1}{2}\right\rangle=\frac{1}{\sqrt{2}}\left|1,\frac{1}{2},-1\right\rangle-\frac{1}{3}\left|1,-\frac{1}{2},0\right\rangle-\frac{\sqrt{2}}{3}\left|0,\frac{1}{2}0\right\rangle+\frac{1}{3}\left|0,-\frac{1}{2},1\right\rangle+\frac{1}{3\sqrt{2}}\left|-1,\frac{1}{2},1\right\rangle, \tag{3.12}$$

$$\left|{}^4\mathrm{P}_{\frac{3}{2}},\mathrm{J}_3=\frac{3}{2}\right\rangle=\sqrt{\frac{3}{5}}\left|1,\frac{1}{2},0\right\rangle-\sqrt{\frac{2}{15}}\left|1,\frac{-1}{2},1\right\rangle-\frac{2}{\sqrt{15}}\left|0,\frac{1}{2},1\right\rangle, \tag{3.13}$$

$$\left|{}^4\mathrm{P}_{\frac{5}{2}},\ \mathrm{J}_3=\frac{5}{2}\right\rangle=\left|1,\frac{1}{2},1\right\rangle. \tag{3.14}$$

3.2.1 Formula For LS Coupling

Let's introduce an orbital angular momentum vector L defined by three components $\mathrm{L_x}, \mathrm{L_y}$ and $\mathrm{L_z}$ and satisfy the following commutation relations,

$$[\mathrm{L_x},\mathrm{L_y}]=i\hbar\,\mathrm{L_z}\quad[\mathrm{L_y},\mathrm{L_z}]=i\hbar\,\mathrm{L_x}\quad[\mathrm{L_z},\mathrm{L_x}]=i\hbar\,\mathrm{L_y}.$$

The operators of angular momentum are defined as L_+ and L_-, raising and lowering operator respectively, such that, $\mathrm{L}_\pm=\mathrm{L_x}\pm i\mathrm{L_y}$, where i is the imaginary unit. The spin is also represented by a vector operator S whose components are defined as $\mathrm{S_x},\mathrm{S_y},\mathrm{S_z}$, and obey the following commutation relations,

$$[\mathrm{S_x}\ ,\mathrm{S_y}]=i\hbar\,\mathrm{S_z}\qquad[\mathrm{S_y}\ ,\mathrm{S_z}]=i\hbar\,\mathrm{S_x}\qquad[\mathrm{S_z}\ ,\mathrm{S_x}]=i\hbar\,\mathrm{S_y}\ .$$

Lets introduce raising and lowering spin operators S_+ and S_- respectively, such that,

$$\mathrm{S}_\pm=\mathrm{S_x}\ \pm\ i\ \mathrm{S_y},$$

This leads to,

$$\mathrm{L}_+\mathrm{S}_-=(\mathrm{L_x}+i\mathrm{L_y})\left(\mathrm{S}_{\mathrm{x_i}}-i\mathrm{S}_{\mathrm{y_i}}\right)=\mathrm{L_x}\ \mathrm{S}_{\mathrm{x_i}}-i\ \mathrm{L_x}\ \mathrm{S}_{\mathrm{y_i}}+\ \mathrm{i}\ \mathrm{L_y}\ \mathrm{S}_{\mathrm{x_i}}+\mathrm{L_y}\ \mathrm{S}_{\mathrm{y_i}},$$

Using the property $[\mathrm{L_x}\ ,\mathrm{S_y}]=0$, the above equation becomes

$$L_+S_- = L_x\ S_{x_i} + L_y\ S_{y_i}, \tag{3.15}$$

In the similar way,

$$L_-S_+ = \ (L_x - iL_y)\left(S_{x_i} + iS_{y_i}\right) = L_x\ S_{x_i} + i\ L_x\ S_{y_i} -\ i\ L_y\ S_{x_i} + L_y\ S_{y_i},$$

Again using the property of commutators $[L_x\ , S_y] = 0$, the above equation becomes,

$$L_-S_+ = L_x\ S_{x_i} + L_y\ S_{y_i}, \tag{3.16}$$

Adding Eq (3.15) and Eq (3.16), we get the following result

$$\begin{aligned} L_+S_- +\ L_-S_+ &= 2\left(L_x\ S_{x_i} + L_y\ S_{y_i}\right), \\ L_x\ S_{x_i} + L_y\ S_{y_i} &= \frac{1}{2}\ (L_+S_- + L_-S_+), \end{aligned}$$

An operator $L\ \cdot S_i$, where $i = d, Q$ may be expressed as $L \cdot S_i = L_x\ S_{x_i} + L_y\ S_{y_i} + L_z\ S_{z_i}$.Hence,

$$L\,.\,S_i \ = \ \frac{1}{2}\,(L_+S_{i-} + L_-S_{i+}) + L_3\ S_{i3}. \tag{3.17}$$

3.2.2 Determination of matrix elements of $\mathbf{L \cdot S_i (i = d, Q)}$ in the basis $[^2\mathbf{P_J}, ^4\mathbf{P_J}]$

We will first calculate:

$$L\,.\,S_{[ss]} = \frac{1}{2}\left(L_+\ S_{[ss]-} +\ L_-\ S_{[ss]+}\right) + L_3\ S_{[ss]3}.$$

To find matrix elements of $|\ L \cdot S_d >_{J=\frac{1}{2}}$, consider Eq (3.11),

$$\left|^2P_{\frac{1}{2}}, J_3 = \frac{1}{2}\right\rangle = \frac{\sqrt{2}}{3}\left|1, -\frac{1}{2}, 0\right\rangle - \frac{1}{3}\left|0, \frac{1}{2}, 0\right\rangle - \frac{\sqrt{2}}{3}\left|0, -\frac{1}{2}, 1\right\rangle + \frac{2}{3}\left|-1, \frac{1}{2}, 1\right\rangle,$$

Lets label the above equation as

$$\left|{}^2P_{\frac{1}{2}}, J_3 = \frac{1}{2}\right\rangle \equiv |1'\rangle\,;\left|1,-\frac{1}{2},0\right\rangle = \ \left|S_{d_3}, S_{Q_3}, L_3\right\rangle \equiv |1\rangle\,,$$
$$\left|0,\frac{1}{2},0\right\rangle = \left|S_{d_3},\ S_{Q_3}, L_3\right\rangle \equiv |2\rangle\,;\left|0,-\frac{1}{2},1\right\rangle = \left|S_{d_3}, S_{Q_3}, L_3\right\rangle \equiv |3\rangle\,,$$
$$\left|-1,\frac{1}{2},\ 1\right\rangle = \left|S_{d_3}, S_{Q_3}, L_3\right\rangle \equiv |4\rangle\,,$$

Eq (3.11) takes the following form,

$$\left|{}^2P_{\frac{1}{2}}, J_3 = \frac{1}{2}\right\rangle = |1'\rangle = \frac{\sqrt{2}}{3}\,|1\rangle - \frac{1}{3}\,|2\rangle - \frac{\sqrt{2}}{3}\,|3\rangle + \frac{2}{3}\,|4\rangle\,. \tag{3.18}$$

$$\begin{aligned} L_3 \cdot S_{d_3}\,|1\,\rangle &= 0 \ \ L_3 \cdot S_{d_3}\,|2\rangle = 0 \ \ L_3 \cdot S_{d_3}\,|3\rangle = 0, \\ L_3 \cdot S_{d_3}\,|4\,\rangle &= (-1)(1)\frac{2}{3}\,|4\,\rangle = -\frac{2}{3}\,|4\,\rangle\,. \end{aligned}$$

Therefore,

$$L_3 \cdot S_{d_3}\,|1'\rangle = -\frac{2}{3}\,|4\,\rangle\,. \tag{3.19}$$

Now let's evaluate $L_+ \ S_{i_-=}L_+ \ S_{d_-}|1'>$. Using

$$\begin{aligned} L_\pm\,|l, m_l\rangle &= \sqrt{l\ (l+1) - m_l\ (m_l \pm 1)}\,|l, m_l\ \pm\ 1\rangle\,, \\ S_\pm\,|S, m_s\rangle &= \sqrt{s\ (s+1) - m_s\ (m_s \pm 1)}\,|S, m_s\ \pm 1\rangle\,, \\ L_+\ S_{d_-}\,|1\rangle &= \frac{\sqrt{2}}{3} L_+\ S_{d_-}\left|1,-\frac{1}{2},0\right\rangle, \\ &= \frac{\sqrt{2}}{3}\ \sqrt{1(1+1)-0(0+1)}\sqrt{1(1+1)-1(1-1)}\left|0,-\frac{1}{2},1\right\rangle, \\ &= 2\,\frac{\sqrt{2}}{3}\left|0,-\frac{1}{2},1\right\rangle. \end{aligned}$$

$$\begin{aligned} L_+\ S_{d_-}\,|2\rangle &= -\frac{1}{3}\ L_+\ S_{d_-}\left|0,\frac{1}{2},0\right\rangle, \\ &= -\frac{1}{3}\ \sqrt{1(1+1)-0\ (0+1)}\sqrt{1(1+1)-0\ (0-1)}\left|-1,\frac{1}{2},1\right\rangle, \\ L_+\ S_{d_-}\,|2\rangle &= -\frac{2}{3}\left|-1,\frac{1}{2},1\right\rangle. \end{aligned}$$

$$\begin{aligned}
\mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\ |3\rangle &= -\frac{\sqrt{2}}{3}\ \mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\left|0,-\frac{1}{2},1\right\rangle, \\
&= -\frac{\sqrt{2}}{3}\sqrt{1(1+1)\ -1\ (1+1)}\sqrt{1(1+1)-0\ (0\ -1)}\left|-1,-\frac{1}{2},2\right\rangle, \\
\mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\ |3\rangle &= -\frac{\sqrt{2}}{3}\sqrt{2}\ \sqrt{2-2}\left|-1,-\frac{1}{2},2\right\rangle = 0.
\end{aligned}$$

Now for the fourth state we have

$$\begin{aligned}
\mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\ |4\rangle &= \frac{2}{3}\ \mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\left|-1,\frac{1}{2},1\right\rangle, \\
&= \frac{2}{3}\sqrt{1(1+1)\ -1(1+1)}\sqrt{1(1+1)-(-1)(-1-1)}\left|-2,\frac{1}{2},2\right\rangle, \\
\mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\ |4\rangle &= \frac{2}{3}\ \sqrt{2-2}\sqrt{2-2}\left|-2,\frac{1}{2},2\right\rangle = 0.
\end{aligned}$$

Therefore we get the following result,

$$\mathrm{L}_{+}\ \mathrm{S}_{\mathrm{d}_-}\ |1'\rangle = 2\frac{\sqrt{2}}{3}\left|0,-\frac{1}{2},1\right\rangle - \frac{2}{3}\left|-1,\frac{1}{2},1\right\rangle. \tag{3.20}$$

Now let' s evaluate $\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{i}_+ = }\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\ |1'\rangle$. Using

$$\begin{aligned}
\mathrm{L}_{\pm}\,|l,\mathrm{m}_l\rangle &= \sqrt{l\ (l+1)-\mathrm{m}_l\ (\mathrm{m}_l \pm 1)}\,|l,\mathrm{m}_l\ \pm\ 1\rangle, \\
\mathrm{S}_{\pm}\,|\mathrm{S},\mathrm{m}_s\rangle &= \sqrt{\mathrm{s}\ (\mathrm{s}+1)\ -\mathrm{m}_s\ (\mathrm{m}_s \pm 1)}\,|S,\mathrm{m}_s\ \pm 1\rangle, \\
\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\ |1\rangle &= \frac{\sqrt{2}}{3}\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\left|1,-\frac{1}{2},\ 0\right\rangle, \\
&= \frac{\sqrt{2}}{3}\ \sqrt{1(1+1)-0(0-1)}\sqrt{1(1+1)\ -1\ (1+1)}\left|2,-\frac{1}{2},-1\right\rangle, \\
\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\ |1\rangle &= \frac{\sqrt{2}}{3}\sqrt{2}\sqrt{2-2}\left|2,-\frac{1}{2},-1\right\rangle = 0.
\end{aligned}$$

Now for the second state

$$\begin{aligned}
\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\ |2\rangle &= -\frac{1}{3}\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\left|0,\frac{1}{2},0\right\rangle, \\
&= -\frac{1}{3}\sqrt{1(1+1)-0(0-1)}\sqrt{1(1+1)-0(0+1)}\left|1,\frac{1}{2},-1\right\rangle, \\
\mathrm{L}_{-}\ \mathrm{S}_{\mathrm{d}_+}\ |2\rangle &= -\frac{2}{3}\left|1,\frac{1}{2},-1\right\rangle.
\end{aligned}$$

Similarly, for the third and fourth state, one can see

$$\begin{aligned}
\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|3\right\rangle &= -\frac{\sqrt{2}}{3}\ \mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|\ 0,-\frac{1}{2},1\right\rangle, \\
&= -\frac{\sqrt{2}}{3}\sqrt{1(1+1)-1(1-1)}\sqrt{1(1+1)-0(0+1)}\left|1,-\frac{1}{2},0\right\rangle, \\
\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|3\right\rangle &= -\frac{2\sqrt{2}}{3}\left|1,-\frac{1}{2},0\right\rangle,
\end{aligned}$$

"

$$\begin{aligned}
\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|4\right\rangle &= \frac{2}{3}\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|-1,\frac{1}{2},1\right\rangle, \\
&= \frac{2}{3}\ \sqrt{1(1+1)-1(1-1)}\sqrt{1(1+1)-(-1)(-1+1)}\left|0,\frac{1}{2},0\right\rangle, \\
\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|4\right\rangle &= \frac{4}{3}\left|0,\frac{1}{2},0\right\rangle.
\end{aligned}$$

Now for the state $|1'\rangle$, we have

$$\mathrm{L}_-\ \mathrm{S}_{\mathrm{d}_+}\left|1'\right\rangle = -\frac{2}{3}\left|1,\frac{1}{2},-1\right\rangle - \frac{2\sqrt{2}}{3}\left|1,-\frac{1}{2},0\right\rangle + \frac{4}{3}\left|0,\frac{1}{2},0\right\rangle. \tag{3.21}$$

Therefore, from Eq (3.18) and Eq (3.19), we get the following matrix element,

$$\left\langle 1'\left|\mathrm{L}_3\cdot\mathrm{S}_{\mathrm{d}_3}\right|1'\right\rangle_{J=\frac{1}{2}} = -\frac{4}{9}\left\langle -1,\frac{1}{2},1\ \middle|\ -1,\frac{1}{2},1\right\rangle = -\frac{4}{9},$$

$$\left\langle 1'\left|\mathrm{L}_3\cdot\mathrm{S}_{\mathrm{d}_3}\right|1'\right\rangle_{J=\frac{1}{2}} = -\frac{4}{9}. \tag{3.22}$$

Now from Eq (3.18) and Eq (3.20), we get the following result,

$$\left\langle 1'\left|\mathrm{L}_+\mathrm{S}_{\mathrm{d}_-}\right|1'\right\rangle = -2\frac{\sqrt{2}}{3}\frac{\sqrt{2}}{3} - \frac{2}{3} - \frac{2}{3} = -\frac{8}{9}. \tag{3.23}$$

From Eq (3.18) and Eq (3.21) we get the following result ,

$$\left\langle 1'\left|\mathrm{L}_-\mathrm{S}_{\mathrm{d}_+}\right|1'\right\rangle_{\mathrm{J}=\frac{1}{2}} = -\frac{4}{9}-\frac{4}{9} = -\frac{8}{9}. \tag{3.24}$$

Substituting the values from Eq (3.22), Eq (3.23) and Eq (3.24) in Eq (3.17), we obtain

$$\langle \mathrm{L} \cdot \mathrm{S_d}\rangle_{1'} = \frac{1}{2}\left(\mathrm{L_+ S_{d-}} + \mathrm{L_- S_{d+}}\right) + \mathrm{L_3 S_{d_3}} = \frac{1}{2}\left[-\frac{8}{9} - \frac{8}{9}\right] - \frac{4}{9},$$

Therefore,

$$\langle L \cdot S_d\rangle_{1'} = -\frac{4}{3}. \tag{3.25}$$

Now consider Eq (3.12),

$$\left| {}^4\mathrm{P}_{\frac{1}{2}}, \mathrm{J_3} = \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|1, \frac{1}{2}, -1\right\rangle - \frac{1}{3}\left|1, -\frac{1}{2}, 0\right\rangle - \frac{\sqrt{2}}{3}\left|0, \frac{1}{2}, 0\right\rangle + \frac{1}{3}\left|0, -\frac{1}{2}, 1\right\rangle + \frac{1}{3\sqrt{2}}\left|-1, \frac{1}{2}, 1\right\rangle,$$

Lets label the above equation as,

$$\left|{}^4\mathrm{P}_{\frac{1}{2}}, \mathrm{J_3} = \frac{1}{2}\right\rangle = |2'\rangle \quad \left|1, \tfrac{1}{2}, -1\right\rangle = |1''\rangle \quad \left|1, -\frac{1}{2}, 0\right\rangle = |2''\rangle,$$
$$\left|0, \frac{1}{2}, 0\right\rangle = |3''\rangle \quad \left|0, -\tfrac{1}{2}, 1\right\rangle = |4''\rangle \quad \left|-1, \frac{1}{2}, 1\right\rangle = |5''\rangle,$$

Eq (3.12) takes the following form,

$$\left|{}^4\mathrm{P}_{\frac{1}{2}}, \mathrm{J_3} = \frac{1}{2}\right\rangle = |2'\rangle = \frac{1}{\sqrt{2}}|1''\rangle - \frac{1}{3}|2''\rangle - \frac{\sqrt{2}}{3}|3''\rangle + \frac{1}{3}|4''\rangle + \frac{1}{3\sqrt{2}}|5''\rangle, \tag{3.26}$$

Therefore,

$$\mathrm{L_3} \cdot \mathrm{S_{d_3}}|2'\rangle = -\frac{1}{\sqrt{2}}|1''\rangle - \frac{1}{3\sqrt{2}}|5''\rangle. \tag{3.27}$$

Now let′s evaluate $\mathrm{L_+\ S_{i_-}{=}L_+\ S_{d_-}}\ |2'\rangle$. Using

$$\begin{aligned}
\mathrm{L_\pm}\ |l, \mathrm{m}_l\rangle &= \sqrt{l\ (l+1) - \mathrm{m}_l\ (\mathrm{m}_l \pm 1)}\,|l, \mathrm{m}_l\ \pm\ 1\rangle, \\
\mathrm{S_\pm}\,|\mathrm{S}, \mathrm{m_s}\rangle &= \sqrt{\mathrm{s}\ (\mathrm{s}+1) - \mathrm{m}_s\ (\mathrm{m}_s \pm 1)}\,|\mathrm{S}, \mathrm{m}_s\ \pm 1\rangle,
\end{aligned}$$

$$\mathrm{L_+}\ \mathrm{S}_{d_-}\ |2'\rangle = \frac{2}{\sqrt{2}}\left|0, \frac{1}{2}, 0\right\rangle - \frac{2}{3}\left|0, -\frac{1}{2}, 1\right\rangle - 2\frac{\sqrt{2}}{3}\left|-1, \frac{1}{2}, 1\right\rangle. \tag{3.28}$$

Now let′s evaluate $\mathrm{L_-\ S}_{i_+}{=}\mathrm{L_-\ S_{d_+}}\ |2'\rangle$.

$$\mathrm{L_-\ S_{d_+}}\,|2'\rangle = -\frac{2\sqrt{2}}{3}\left|1,\frac{1}{2},-1\right\rangle + \frac{2}{3}\left|1,-\frac{1}{2},0\right\rangle + \frac{\sqrt{2}}{3}\left|0,\frac{1}{2},0\right\rangle. \tag{3.29}$$

From Eq (3.26) and Eq (3.27) we get the following result,

$$\langle 2'\,|\mathrm{L_3} \cdot \mathrm{S_{d_3}}|\ 2'\rangle = -\frac{5}{9}. \tag{3.30}$$

Similarly, from Eq (3.26) and Eq (3.28) we get,

$$\langle 2'\,\left|\mathrm{L_+\ S_{d_-}}\right|\, 2'\rangle_{\,\mathrm{J}=\frac{1}{2}} = -\frac{2}{3} - \frac{2}{9} - \frac{2}{9} = -\frac{10}{9}. \tag{3.31}$$

Also Eq (3.26) & Eq (3.29)gives,

$$\langle 2'\,\left|\mathrm{L_-\ S_{d_+}}\right|\, 2'\rangle_{\,\mathrm{J}=\frac{1}{2}} = -\frac{2}{3} - \frac{2}{9} - \frac{2}{9} = -\frac{10}{9}. \tag{3.32}$$

Substituting values from Eq (3.30), Eq (3.31) and Eq (3.32) in Eq (3.17), we obtain

$$\begin{aligned}
\langle \mathrm{L} \cdot \mathrm{S_d}\rangle_{2'} &= \langle 2'\,|\mathrm{L_3} \cdot \mathrm{S_{d_3}}|\, 2'\rangle_{\mathrm{J}=\frac{1}{2}} + \frac{1}{2}\left[\langle 2'\,\left|\mathrm{L_+\ S_{d_-}}\right|\, 2'\rangle_{\,\mathrm{J}=\frac{1}{2}} + \langle 2'\,\left|\mathrm{L_-\ S_{d_+}}\right|\, 2'\rangle_{\,\mathrm{J}=\frac{1}{2}}\right], \\
\langle \mathrm{L} \cdot \mathrm{S_d}\rangle_{2'} &= -\frac{5}{9} + \frac{1}{2}\left[-\frac{10}{9} - \frac{10}{9}\right] = -\frac{5}{9} - \frac{10}{9} = -\frac{15}{9},
\end{aligned}$$

$$\langle \mathrm{L} \cdot \mathrm{S_d}\rangle = -\frac{5}{3}. \tag{3.33}$$

From Eq (3.19) and Eq (3.26) we obtain,

$$\langle 2'\,|\mathrm{L_3} \cdot \mathrm{S_{d_3}}|\ 1'\rangle_{\mathrm{J}=\frac{1}{2}} = -\frac{\sqrt{2}}{9}. \tag{3.34}$$

From Eq (3.20) and Eq (3.26) we get,

$$\langle 2'\,\left|\mathrm{L_+\ S_{d_-}}\right|\, 1'\rangle_{\,\mathrm{J}=\frac{1}{2}} = \frac{\sqrt{2}}{9}. \tag{3.35}$$

From Eq (3.21) and Eq (3.26) we obtain,

$$\langle 2'\,\left|\mathrm{L_-\ S_{d_+}}\right|\, 1'\rangle_{\,\mathrm{J}=\frac{1}{2}} = -\frac{2}{3} \times \frac{1}{\sqrt{2}}\left\langle 1,\frac{1}{2},-1\,\middle|\,1,\frac{1}{2},-1\right\rangle,$$

which gives

$$\left\langle 2' \left| \mathrm{L}_- \ \mathrm{S}_{\mathrm{d}_+} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} = -\frac{5\sqrt{2}}{9}, \tag{3.36}$$

Therefore, substituting the values from Eq (3.34), Eq (3.35) and Eq (3.36) in Eq (3.17), we obtain

$$\begin{aligned}
\left\langle 2' \left| \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} &= \left\langle 2' \left| \mathrm{L}_3 \cdot \ \mathrm{S}_{\mathrm{d}_3} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} + \frac{1}{2}\left[\left\langle 2' \left| \mathrm{L}_+ \ \mathrm{S}_{\mathrm{d}_-} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} + \left\langle 2' \left| \mathrm{L}_- \ \mathrm{S}_{\mathrm{d}_+} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} \right], \\
\left\langle 2' \left| \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} &= -\frac{\sqrt{2}}{9} + \frac{1}{2}\left[\frac{\sqrt{2}}{9} - \frac{5\sqrt{2}}{9} \right] = -\frac{\sqrt{2}}{9} + \frac{1}{2}\left(-\frac{4\sqrt{2}}{9} \right),
\end{aligned}$$

$$\left\langle 2' \left| \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right| 1' \right\rangle_{\mathrm{J}=\frac{1}{2}} = -\frac{\sqrt{2}}{3}. \tag{3.37}$$

Similarly,

$$\left\langle 1' \left| \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right| 2' \right\rangle_{\mathrm{J}=\frac{1}{2}} = \left\langle 2' \left| \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right| 1' \right\rangle^{\dagger}_{\mathrm{J}=\frac{1}{2}} = -\frac{\sqrt{2}}{3}. \tag{3.38}$$

Substituting the matrix elements from Eq (3.25), Eq (3.33) , Eq (3.37) and Eq (3.38),we obtain the Matrix Form,

For $\mathrm{J} = \frac{1}{2}$ states ${}^2\mathrm{P}_{\frac{1}{2}}$ and ${}^4\mathrm{P}_{\frac{1}{2}}$ we have,

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right\rangle_{\mathrm{J}=\frac{1}{2}} = \begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix}. \tag{3.39}$$

Likewise, we can calculate the following remaining five matrices,

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{Q}} \right\rangle_{\mathrm{J}=\frac{1}{2}} = \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix}. \tag{3.40}$$

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right\rangle_{\mathrm{J}=\frac{3}{2}} = \begin{bmatrix} \frac{2}{3} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{2}{3} \end{bmatrix}. \tag{3.41}$$

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{Q}} \right\rangle_{\mathrm{J}=\frac{3}{2}} = \begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3} \end{bmatrix}. \tag{3.42}$$

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{d}} \right\rangle_{\mathrm{J}=\frac{5}{2}} = 1. \tag{3.43}$$

$$\left\langle \mathrm{L} \cdot \mathrm{S}_{\mathrm{Q}} \right\rangle_{\mathrm{J}=\frac{3}{2}} = \frac{1}{2}. \tag{3.44}$$

Chapter 4

Narrow Resonances In Charmed Baryons

4.1 Narrow Resonance

In particle physics, the peak which is located around a specific energy and found in differential cross section of the scattering experiments is called Resonance. These peaks are sometimes associated to elementary particles, which includes quarks, hadrons and a variety of bosons and their excitation. Commonly, resonance describes particles with very short lifetimes, usually high energy hadrons exist for 10^{-23} seconds or even less than it. The width of the resonance (Γ) is described in terms of mean life time (τ) of the particle or its excited state and is written in the following form as $\Gamma = \frac{\hbar}{\tau}$.

Therefore, the lifetime of a particle is the inversely proportional to the decay width of the particle. For example, it is observed that the charged pion has the second largest lifetime among all mesons i.e. 2.6×10^{-8}s and therefore it has a very small resonance width of 2.5×10^{-8}eV. So, charged pions are considered as " resonances ". On the other hand, the charged ρ meson has the short lifetime and it has a very large resonance width, which is nearly equal to the one-fifth of the rest mass of the particle.

4.1.1 Narrow Excited Ω_c Baryons

In the recent report, LHCb announced the discovery of five narrow excited Ω_c baryons which decays into Ξ_c^+ K^-. The Ω_c^0 charmed baryon is a combination of css quarks, which can be treated as a light quark (ss) in our case and a heavy(c) quark. Symmetry rules don't allow a spin-0 diquark so the ground state of Ω_c^0 is considered as a spin -1 diquark with the charm quark. These baryons are clarified as bound states of c quark and P-wave ss diquark. For such system, there are five possible combinations of orbital angular momentum and spin. The narrowness of the states signifies that it is very hard to break the two s quarks in a diquark. We predict all states with negative parity which includes two of spin 1/2, two of spin3/2, and one of spin 5/2. Out of the five states, two decay in S-wave, and three decay in D-wave, those with $J^P = \frac{1}{2}^-$decay to Ξ_c^+ K^-in S-wave, while those with $J^P = \frac{3}{2}^-, \frac{5}{2}^-$ decay to Ξ_c^+ K^- in a D-wave. D-wave states might be narrower than the S-wave states. We expect the similar pattern in excited Ω_b states with negative parity.

These states has been discussed in number of models and much work has been done on these states, such as Diquark model which uses net effective Hamiltonian containing tensor, spin - orbit, spin - spin interactions and compared the charm P-wave of tetra quarks with the P-wave system of these newly discovered narrow excited baryons [22]. Then another model proposed by M. Karliner and J. L.Rosner discussed these states using the spin - dependent potential between a heavy quark Q and the (ss) spin -1 diquark and calculated linear relation among the mass shifts [23]. These states are also discussed in the reanalysis of newly observed Ω_c^0 baryons with spin $\frac{1}{2}$ and $\frac{3}{2}$, decaying into Ξ_c^+ K^-using the strong coupling constants of baryons under the framework of QCD rules. Decay widths of these narrow excited baryons were calculated which were different from the experimental results [24].

This chapter includes the calculation of eigenstates for these five narrow excited Ω_c baryons and a linear relation among mass shifts using these states as well as the determination of the values of parameters $(a_1, a_2, b, c, M_\circ)$ using effective Hamiltonian for the Ω_c baryons.

4.2 P - WAVE c (ss) SYSTEM

We consider that the (ss) is a S-wave color $\bar{3}c$ diquark in c(ss) baryon then it must have spin $S_{[ss]} = 1$, this spin of diquark then combines with the spin 1/2 of the charm quark c and we have a total spin S =1/2, 3/2 for our baryon. By considering all the states have orbital angular momentum L= 1 between the spin-1 diquark and the charm quark, we combine angular momentum L= 1 with total spin $S = \frac{1}{2}$,we evaluate states with total spin $J = 1 \pm \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$,and combining L= 1 with total spin $S = \frac{3}{2}$, $J = 1 \pm \frac{3}{2} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. All five states are predicted with negative parity P (with the assumption that the diquark remains in its ground state). Let′ s introduce the notation $^{2S+1}P_J$ to describe these states as$^2P_{\frac{1}{2}}, {}^2P_{\frac{3}{2}}, {}^4P_{\frac{1}{2}}, {}^4P_{\frac{3}{2}}, {}^4P_{\frac{5}{2}}$, respectively.

4.2.1 Interpretation of Newly Observed Ω_c^0 Resonances

The hadron spectroscopy is the sub field of particle physics that plays a vital role in understanding basic theory of strong interactions i.e., QCD (Quantum Chromodynamics) which predicts that the mesons are the bound states of quark - anti quark pair while the baryons are made up of three quarks. In a quark - diquark model, Wei Wang and Rui -Lin Zhu studied the charmed and bottomed with two strange quarks. The two strange quarks are supposed to lie in S -wave and hence, their total spin is 1. The mass spectra of the S and P wave orbitally excited states was calculated within the heavy - quark -light-quark structure and it was observed that Ω_c (2695)°and Ω_c (2770)°are

State	Mass (MeV.)	Width(MeV.)	Proposed J^P
$\Omega_c(3000)^\circ$	$3000.4 \pm 0.2 \pm 0.1$	$4.5 \pm 0.6 \pm 0.3$	$\frac{1}{2}^-$
$\Omega_c(3050)^\circ$	$3050.2 \pm 0.1 \pm 0.1$	$0.8 \pm 0.2 \pm 0.1$	$\frac{1}{2}^-$
$\Omega_c(3066)^\circ$	$3065.6 \pm 0.1 \pm 0.3$	$3.5 \pm 0.4 \pm 0.2$	$\frac{3}{2}^-$
$\Omega_c(3090)^\circ$	$3090.2 \pm 0.3 \pm 0.5$	$8.7 \pm 1.0 \pm 0.8$	$\frac{3}{2}^-$
$\Omega_c(3119)^\circ$	$3119.1 \pm 0.3 \pm 0.9$	$1.1 \pm 0.8 \pm 0.4$	$\frac{5}{2}^-$

Table 4.1: LHCb Collaboration reported masses and widths (MeV.) of Ω_c = css candidates. Important quantum numbers (J^P), namely spin (J) and parity (P) are mentioned in the Table.

interpreted as the S wave states of charmed doubly strange baryons. The five recently discovered Ω_c^0 resonances by LHCb Collaboration i.e., Ω_c $(3000)^\circ$,$\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$, $\Omega_c(3090)^\circ$ and $\Omega_c(3090)^0$ fit well as the P-wave orbitally excited states [25] of charmed baryon with two strange quarks. In heavy quark effective (HQET), their decays are examined into Ξ_cK and Ξ_c'K.

4.2.2 Nature of Newly Discovered Ω_c States

LHCb Collaboration announced the discovery of five new narrow excited states Ω_c decaying into Ξ_c^+ K^-. The mass spectrum of Ξ_c^+ $K^-$is studied with a sample of pp collision data at center of mass energies $7, 8, 13$ TeV. to an integrated luminosity of 3.3 fb^{-1}. The masses of Ω_c states were measured which is equal to (in MeV.) M = $\Omega_c(3000)^\circ$, $\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$, $\Omega_c(3090)^\circ$and $\Omega_c(3119)^\circ$. LHCb also determined their widths through $\Omega_c^\circ \to \Xi_c^+$ K^-decay channels, which didn't exceeds a few MeV. In heavy quark effective theory, the decays of the five P- wave Ω_c states are analyzed into Ξ_c^+ $K^-$and $\Xi_c'^-$ $K^-$and are suppressed by either heavy quark symmetry or phase space. We can understand the narrowness of the newly five observed Ω_c states by using heavy quark symmetry [26].

Quantum numbers of these five Ω_c resonances are predicted using lattice quantum chromodynamics (lattice QCD), that is Ω_c $(3000)^\circ$and $\Omega_c(3050)^\circ$ have spin parity $J^P = \frac{1}{2}^-$, the states $\Omega_c(3066)^\circ$, $\Omega_c(3090)^\circ$have $J^P = \frac{3}{2}^-$,whereas $\Omega_c(3119)^\circ$is possibly a $\frac{5}{2}^-$. For extracting these energy level and in identifying their spins an elaborate and well-established lattice method is followed. The results are collected in Table (4.1).

4.3 Study of Ω_c And Ω_c^{*0} Baryons At Belle

The experimental analysis of charmed baryons which includes determination of their decay modes, masses and widths is a major test of several theoretical models that present predictions for the properties of heavy hadrons. A complete experimental analysis of charmed doubly strange baryons Ω_c^0 is delayed unlike the Λ_c^+,$\Xi_c^{++,+,0}$, $\Xi_c^{+,0}$. The Ω_c^0 $\left[J^P = (\frac{1}{2})^+\right]$ decays weakly and is the most heaviest singly charmed hadron known and its quark content is c(ss), where the ss pair has a symmetric state. The mass of Ω_c^0 is predicted by several theoretical models. Results are reported at Belle under the study of charmed double strange baryons Ω_c^0 and Ω_c^{*0}, the Ω_c^0 is restored by using

State	Mass (MeV.)	Width (MeV.)	Decay mode
$\Omega_c(2700)^\circ$	2695.2 ± 1.7		Weak
$\Omega_c(2770)^\circ$	2765.9 ± 2.0		$\Omega_c^0\gamma$
$\Omega_c(3000)^\circ$	$3000.4 \pm 0.2 \pm 0.1$	$4.5 \pm 0.6 \pm 0.3$	$\Xi_c K$
$\Omega_c(3050)^\circ$	$3050.2 \pm 0.1 \pm 0.1$	$0.8 \pm 0.2 \pm 0.1$	$\Xi_c K$
$\Omega_c(3066)^\circ$	$3065.6 \pm 0.1 \pm 0.3$	$3.5 \pm 0.4 \pm 0.2$	$\Xi_c K$, $\Xi_c' K$
$\Omega_c(3090)^\circ$	$3090.2 \pm 0.3 \pm 0.5$	$8.7 \pm 1.0 \pm 0.8$	$\Xi_c K$, $\Xi_c' K$
$\Omega_c(3119)^\circ$	$3119.1 \pm 0.3 \pm 0.9$	$1.1 \pm 0.8 \pm 0.4$	$\Xi_c K$, $\Xi_c' K$
$\Omega_c(3188)^\circ$	$3188 \pm 5 \pm 13$	$60 \pm 15 \pm 11$	$\Xi_c K$

Table 4.2: The experimental data including masses, widths and decay modes is presented for newly observed Ω_c states at LHCb.

$\Omega_c^0 \longrightarrow \Omega^- \Pi^+$decay mode and its mass is measured to be $M_{\Omega_c^0} = [2693.6 \pm 0.3(\text{stat.})]^{+1.8}_{-1.5}$ MeV./c^2. The Ω_c^{*0} is resorted in $\Omega_c\gamma$ mode and singly charmed, baryon Ω_c^0 (css) in the radiate decay $\Omega_c^0\gamma$ has recently been observed by BaBar. The mass difference is evaluated and it comes to be $[70.7 \pm 0.9(\text{stat.})]^{+0.1}_{-0.9}$ Me V/c^2 [27].

4.4 New Baryons Ω_c^0 Discovered As Member of 1P And 2S Charmed Baryons

The new baryons Ω_c^0states announced by LHCb let us to find the way how to categorize all of them to Ω_c^0 charmed family and makes it abundant. Observation of these five states stimulated our interest due to broad structure in the $\Xi_c^+ K^-$invariant mass spectrum, and this broad structure at about 3188 MeV. may be due to a single resonance or due to superposition of several states, and this broad structure is represented by $\Omega_c(3188)^\circ$. The properties of newly detected Ω_c^0 states at LHCb are being studied by executing the analysis of mass spectrum and behavior of their decay. Working on these states show that the five narrow states, i.e., $\Omega_c(3000)^\circ$, $\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$,$\Omega_c(3090)^\circ$and $\Omega_c(3119)^\circ$states can be grouped in the P-wave charmed baryon family with negative parity and css configuration.

Out of these states, $\Omega_c(3000)^\circ$and $\Omega_c(3090)^\circ$states can have spin parity $J^P = \frac{1}{2}^-$ with the mixture of $\mid 0, \frac{1}{2}^- >$ and $\mid 1, \frac{1}{2}^- >$, whereas the states $\Omega_c(3050)^\circ$,$\Omega_c(3119)^\circ$can be the $J^P = \frac{3}{2}^-$ candidates with the mixture of $\mid 1, \frac{3}{2}^- >$ and $\mid 2, \frac{3}{2}^- >$ $\Omega_c(3066)^\circ$can be suggested as $J^P = \frac{5}{2}^-$state. In addition to this, 2S Ω_c^0 state with $J^P = \frac{1}{2}^+ and$ $J^P = \frac{3}{2}^+$as the possible broad structure of $\Omega_c(3188)^\circ$ is discussed whereas $\Omega_c(3119)^\circ$cannot be a 2S Ω_c^0 state due to its very narrow decay width [28]. Some important properties have not been measured yet such as spin - parity quantum numbers, the electromagnetic EM transitions and the hadronic decay channels $\Omega_c^{(*)}\pi$. The experimental data including masses, widths and decay modes is presented in Table (4.2) for newly observed Ω_c^0states at LHCb with css configuration.

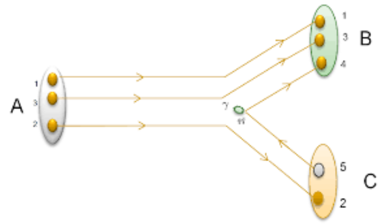


Figure 4.1: Baryon decay process of A $\rightarrow$ B+C in the $^3P_0$ model.

4.5 Hadronic Decay Properties of Newly Observed Ω_c Baryons

$^3P_0$model also known as Quark Pair Creation (QPC) model, assumes that a pair of quark $q\bar{q}$ is created from vacuum having J^{PC} $=0^{++}(^{2S+1}L_J{=}^3P_0)$ as $u\bar{u}, d\bar{d}, s\bar{s}$ are created in vacuum. The quarks created regroup with the quarks from hadron A initially present to form two daughter hadrons B and C. The baryon decay process is shown in Fig.(4.1).

The hadronic decay widths of newly observed charmed strange baryons Ω_c $(3000)^\circ$,$\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$, Ω_c $(3090)^\circ$ and $\Omega_c(3119)^\circ$have been calculated in $^3P_0$ model. There are two decay channels of Ω_c $(3000)^\circ$, $\Omega_c(3050)^\circ$,$\Omega_c(3066)^0$, when the masses of Ω_c baryons and threshold mass of the final particles are taken into account, the channels $\Xi_c'^{+}K^-$ and $\Xi_c'^{0}K$ are allowed for $\Omega_c(3090)^\circ$and $\Omega_c(3119)^\circ$. All newly observed charmed strange baryons are observed in $\Xi_c^+K^-$channel having very narrow decay widths. In assignments of P-wave and D-wave Ω_c, possible decay modes and corresponding hadronic decay widths of these baryons have been computed and result shows that Ω_c^0 can be P- wave or D-wave Ω_c^0 baryon. The results indicates that Ω_c $(3000)^\circ$ is 1D-wave with possible assignment of Ω_{c1} $(\frac{1}{2}^+)$ or Ω_{c1} $(\frac{3}{2}^+)$, $\Omega_c(3050)^\circ$is 1D-wave with possible assignment of Ω_{c3} $(\frac{5}{2}^+)$ or Ω_{c1} $(\frac{7}{2}^+)$ and four D-wave with assignments Ω_{c3} $(\frac{5}{2}^+)$, Ω_{c3} $(\frac{7}{2}^+)$, Ω_{c3} $(\frac{5}{2}^+)$ and Ω_{c3} $(\frac{7}{2}^+)$ are possible for $\Omega_c(3066)^0$and $\Omega_c(3090)^\circ$and $\Omega_c(3119)^\circ$ are interpreted as 1P-wave Ω_{c2} $(\frac{3}{2}^-)$ or Ω_{c2} $(\frac{5}{2}^-)$ [29]. The prediction of total decay widths are in agreement with the experimental results. Only $\Omega_c{\rightarrow}\Xi_c^+K^-$is observed experimentally which do not provide sufficient information for their identification. Moreover resonance with the same J^Pnumbers and similar masses interfere with each other, which make it difficult to differentiate them. The channels $\Xi_c'^{+}K^-$ and $\Xi_c'^{0}K^0 is$ hard to observe for 1D-wave $\Omega_c$baryons for the small decay widths. The narrowness of the states signifies that is separate two s quarks in a diquark. One s quark has to share its spin into K^- and the other in to Ξ_c^+.

4.6 Strong And Radiative Decays of Low Lying S -Wave And P-Wave Singly Heavy Baryons

Great development on the heavy baryon spectra has been made in experimental work during the past few years, and provides an interesting environment to study spectroscopy of heavy baryons. Besides the ground state, in the single charmed baryons with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ (1S wave) , many P-wave states $\Lambda_c(2593)$ $\left[J^P = \frac{1}{2}^-\right]$,$\Lambda_c(2625)$ $\left[J^P = \frac{3}{2}^-\right]$, $\Xi_c(2790)$ $\left[J^P = \frac{1}{2}^-\right]$ and $\Xi_c(2815)$ $\left[J^P = \frac{3}{2}^-\right]$ have been constructed. Higher charmed baryons such as $\Lambda_c(2880)$, $\Sigma_c(2800)$, $\Lambda_c(2880)$, $\Sigma_c(2800)$, $\Xi_c(2930)$, $\Xi_c(2970, 3080)$, $\Xi_c(3055, 3123)$ and $\Lambda_c(2860)$ have been reported by experimental observation. Recently, five narrow baryon $\Omega_c(X)$ states , $\Omega_c(3000)^\circ$, $\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$, Ω_c $(3090)^\circ$ and $\Omega_c(3119)^\circ$ have been observed in the $\Xi_c^+ K^-$channel at LHCb. The strong and radiate decays of low - lying S - and P - wave baryons $\Lambda_{c(b)}$, $\Sigma_{c(b)}$,$\Xi_{c(b)}$,$\Xi'_{c(b)}$ and $\Omega_{c(b)}$ are studied in quark model. It has been noted that the radiative decay mode $\Lambda_b^0\gamma$ is very useful to establish the missing neutral states Σ_b^0 and Σ_b^{*0} [30]. Calculation predicts that most of those missing λ-mode P-wave singly heavy baryons have a comparatively narrow decay width of less than 30 MeV. The $\Sigma_c(2800)$ resonance may be allot to $\left|\Sigma_c\ {}^2P_\lambda \frac{3}{2}^-\right\rangle$ with $J^P = \frac{3}{2}^-$ or $\left|\Sigma_c\ {}^4P_\lambda \frac{5}{2}^-\right\rangle$ with $J^P = \frac{5}{2}^-$. Generally, the ex-citations of $\left|{}^2P_\lambda \frac{3}{2}^-\right\rangle$ and $\left|{}^4P_\lambda \frac{5}{2}^-\right\rangle$ of the 6_F multiplet have identical strong decay properties. In order to recognize them, angular distributions of their decays in either strong decay modes or radiative transitions are required.

4.7 Effective Hamiltonian For the Ω_c Baryons

In the diquark - quark interpretation, the Hamiltonian [31] for the Ω_c states is written in the following form as follows,

$$H_{eff} = m_c + 2\ m_s\ + 2\ \mathscr{K}_{ss} S_{s_1}\ .\ S_{s_2} + B\frac{L(L+1)}{2} + V_{SD}, \tag{4.1}$$

where,

$$V_{SD} = a_1\ L . S_d + a_2\ L . S_Q + b\ \frac{S_{12}}{4} + c\ S_d\ .\ S_Q.$$

In Eq (4.1),where the first two terms m_c and m_s describes the masses of c and s quarks, respectively,$\mathscr{K}_{ss}$ is the spin-spin coupling of the quarks in the diquark, and L is the orbital angular momentum of the diquark-quark system.The spin operators of strange and charm quarks are given by operators S_d and S_Q, respectively, the fourth term is the pure orbital interactions. The coefficients a_1 and a_2 represents the strengths of the spin-orbit terms involving the spin of the diquark $S_{[ss]}(S_d)$ and the charm- quark spin$S_c(S_Q)$, respectively and c is the strength of the spin-spin interaction between the diqurak and the charm quark, whereas $\frac{S_{12}}{4}$ represents the tensor interaction term.

4.7.1 Spin Dependent Mass Shift Calculation Using Potential Model

The spin dependent potential between a heavy quark Q and the (ss) spin -1 diquark may be written as,

$$V_{SD} = a_1\ L . S_d + a_2\ L . S_Q + b\left[-S_d . S_Q + 3(S_d . r)(S_Q . r)/r^2\right] + c\ S_d\ .\ S_Q. \tag{4.2}$$

where the first two terms describes spin - orbit forces, the third is a tensor force term, and the last term describes hyperfine splitting. L represents the orbital angular momentum of diquark-quark system system, the coefficients a_1 and a_2 are the strengths spin-orbit terms involving the spin of the diquark $S_{[ss]}$ and the charm quark spin $S_c$respectively, c is the strength of the spin - spin interaction between the diquark and the charm quark and coefficient of b represents the tensor interaction term [32].

If $a_1 = a_2$, the spin orbit force becomes proportional to $L.(S_d+S_Q) = L.S$, where S is the total spin so states may be classified as ${}^{2S+1}P_J = {}^2P_{\frac{1}{2}}, {}^2P_{\frac{3}{2}}, {}^4P_{\frac{1}{2}}, {}^4P_{\frac{3}{2}}$ and ${}^4P_{\frac{5}{2}}$. When $a_1 \neq a_2$, the states with same J but different S mix with one another and are eigenstates of 2×2 matrices.The Matrix elements of $L . S_d$ and $L . S_Q$ may be evaluated by construction of states with a given J_3 as linear combinations of states $| S_{d_3}, S_{Q_3}, L_3 >$, where$S_{d_3} + S_{Q_3} + L_3 = J_3$. It is sufficient to use a single J_3 for each matrix element ,by angular momentum in-variance.

4.7.2 Calculating Mass Shift for $J = \frac{1}{2}$ States

The linearized approximation for the mass shift is derived using potential model between a heavy Quark Q and the (ss) spin -1 diquark. We can write from Eq (4.2),

$$V_{SD} = a_1\, L . S_d + a_2\, L . S_Q + b\left[-S_d . S_Q + 3(S_d . r)(S_Q . r)/r^2\right] + cS_d . S_Q$$

Substituting tensor interaction term in terms of LS coupling we get,

$$V_{SD} = a_1\, L . S_d + a_2\, L . S_Q + b\frac{S_{12}}{4} + c\, S_d . S_Q.$$

4.7.2.1 Heavy Quark Effective Theory

Heavy quark effective theory (HQET) is an effective field theory which describes the physics of heavy (i.e.,of mass far greater than QCD scale) quarks and used in studying the properties of hadron which contain a single charm or bottom quark in quantum in quantum chromodynamics(QCD).

The tensor force term in Eq (4.1) is defined as,

$$\frac{S_{12}}{4} = \left[-S_d . S_Q + 3(S_d . r)(S_Q . r)/r^2\right] = \frac{3}{r^2}\left[(S_d . r)(S_Q . r)\right] - S_d . S_Q.$$

where S_{12} represents the tensor interaction term. This is a kind of spin dependent force having the same form as the interaction between magnetic dipoles between nucleons. Tensor interaction term is calculated by using the following identity present in Landau- Lifshitz,

$$
\begin{aligned}
< n_i n_j > -\frac{1}{3}\delta_{ij} &= a\left[L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij} L(L+1)\right], \\
a &= -\frac{1}{(2L-1)(2L+3)},
\end{aligned}
$$

To determine the matrix elements of the tensor operator between the states with the same fixed value L=1, one can use the identity from Landau and Lifshitz [33]. In the present case we suppose L= 1 then $a = -\frac{1}{5}$. Therefore, we have

$$
\langle n_i n_j \rangle - \frac{1}{3}\delta_{ij} = -\frac{1}{5}\left[L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij}(2)\right],
$$

$$
\langle n_i n_j \rangle = -\frac{1}{5}\left[L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij}(2)\right] + \frac{1}{3}\delta_{ij}. \tag{4.3}
$$

Average value of tensor term can be written as

$$
\begin{aligned}
\left\langle \frac{S_{12}}{4} \right\rangle &= \frac{3}{r^2} < \left[(S_d \cdot r)(S_Q \cdot r)\right] > - \langle S_d \cdot S_Q \rangle, \\
&= \frac{3}{r^2}\left\langle \left[S_{d_i} r_i S_{Q_j} r_j\right] \right\rangle - \left\langle S_{d_i} S_{Q_j} \right\rangle, \\
&= \frac{3}{r^2} r^2 \left\langle S_{d_i} S_{Q_j} \right\rangle \langle n_i n_j \rangle - \left\langle S_{d_i} S_{Q_j} \right\rangle, \\
&= 3\left\langle S_{d_i} S_{Q_j} \right\rangle \langle n_i n_j \rangle - \left\langle S_{d_i} S_{Q_j} \right\rangle,
\end{aligned}
$$

Now substitute the identity given in Eq (4.3) in above equation and let′s denote $\frac{S_{12}}{4} = \hat{B}_J$

$$
\begin{aligned}
\hat{B}_J &= 3\left\langle S_{d_i} S_{Q_j} \right\rangle \left(\frac{1}{3}\delta_{ij} - \frac{1}{5}\left[L_i L_j + L_j L_i - \frac{4}{3}\delta_{ij}\right]\right) 1 - \left\langle S_{d_i} S_{Q_j} \right\rangle, \\
&= \left\langle S_{d_i} S_{Q_j} \right\rangle \delta_{ij} - \frac{3}{5}\left\langle S_{d_i} S_{Q_j} \right\rangle [L_i L_j + L_j L_i] + \frac{4}{5}\left\langle S_{d_i} S_{Q_j} \right\rangle \delta_{ij} - \left\langle S_{d_i} S_{Q_j} \right\rangle, \\
&= \left(1 + \frac{4}{5} - 1\right) S_{d_i} S_{Q_j} - \frac{3}{5}[L_i L_j + L_j L_i]\ S_{d_i} S_{Q_j}, \\
\hat{B}_J &= \frac{4}{5}(S_{d_i} S_{Q_j}) - \frac{3}{5}\left[L_i L_j S_{d_i} S_{Q_j} + L_j L_i S_{d_i} S_{Q_j}\right],
\end{aligned}
$$

Hence the tensor interaction term can be written in terms of LS coupling,

$$
\hat{B}_J = -\frac{3}{5}\left([(L \cdot S_d)(L \cdot S_Q) + (L \cdot S_Q)(L \cdot S_d)] - \frac{4}{3} S_d \cdot S_Q\right). \tag{4.4}
$$

We want to calculate matrix elements of $\hat{B}_J$ between the states of $J = 1/2$, $J = 3/2$ and $J = 5/2$. This can be evaluated in terms of known matrix elements of three other operators, $\hat{A}^1 \equiv L \cdot S_d$, $\hat{A}^2 \equiv L \cdot S_Q$ and $\hat{C} \equiv S_d \cdot S_Q$.Then,

$$\hat{B}_J = -\frac{3}{5}\left(\hat{A}_J^1 \cdot \hat{A_J}^2 + \hat{A_J}^2 \cdot \hat{A_J}^1 - \frac{4}{3}\hat{C}_J\right). \tag{4.5}$$

4.8 Hyper fine Interactions Between Quarks

The structure of spin-dependent potential contains the spin operators of diquark(S_d) and quark (S_Q) respectively.

$$\langle S_d \cdot S_Q\rangle = \frac{1}{2}\left(S^2 - S_d^2 - S_Q^2\right) = \frac{1}{2}\left[S(S+1) - S_d(S_d+1) - S_Q(S_Q+1)\right],$$

$$\text{For } J = \frac{1}{2} \text{ states } {}^2P_{\frac{1}{2}},\ S = \frac{1}{2},\ S_d = 1, S_Q = \frac{1}{2},$$

$$\langle S_d \cdot S_Q\rangle = \frac{1}{2}\left[\frac{1}{2}(\frac{1}{2}+1) - 1\,(1+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = -1,$$

$$\text{For } J = \frac{1}{2} \text{ states } {}^4P_{\frac{1}{2}},\ S = \frac{3}{2},\ S_d = 1,\ S_Q = \frac{1}{2},$$

$$\langle S_d \cdot S_Q\rangle = \frac{1}{2}\left[\frac{3}{2}(\frac{3}{2}+1) - 1\,(1+1) - \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\right] = \frac{1}{2}.$$

Therefore, The spin- spin interaction term is evaluated as follows,

$$\langle S_d \cdot S_Q\rangle_{J=\frac{1}{2}} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \tag{4.6}$$

From Eq (4.5), we have ,$\frac{S_{12}}{4} = -\frac{3}{5}\left(\hat{A}_J^1 \cdot \hat{A_J}^2 + \hat{A_J}^2 \cdot \hat{A_J}^1 - \frac{4}{3}\hat{C}_J\right)$. For $J = \frac{1}{2}$, using Eq (3.39) , Eq (3.40) and Eq (4.6) we get,

$$\hat{B}_{\frac{1}{2}} = -\frac{3}{5}\left(\begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix}\begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix}\begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix} - \frac{4}{3}\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\right),$$

$$\hat{B}_{\frac{1}{2}} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix}. \tag{4.7}$$

In the similar way we obtain,

$$J = \frac{1}{2}: \quad \frac{S_{12}}{4} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix}.$$

$$J = \frac{3}{2}: \quad \frac{S_{12}}{4} = \begin{bmatrix} 0 & -\frac{\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{10} & \frac{4}{5} \end{bmatrix}.$$

$$J = \frac{5}{2}: \quad \frac{S_{12}}{4} = -\frac{1}{5}.$$

The linearized approximation for the mass shift is derived using Eq (4.5) (by applying potential model between a heavy Quark Q and the (ss) spin -1 diquark). Now mass shift for state $J = \frac{1}{2}$ is given by,

$$\triangle M_{1/2} = a_1 \begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix} + a_2 \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix} + b \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix},$$

$$= \begin{bmatrix} \frac{1}{3}a_2 - \frac{4}{3}a_1 & \frac{\sqrt{2}}{3}(a_2 - a_1) \\ \frac{\sqrt{2}}{3}(a_2 - a_1) & -\frac{5}{3}a_1 - \frac{5}{6}a_2 \end{bmatrix} + b \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$$

Similarly mass splitting for the state $J = \frac{3}{2}$ and $\frac{5}{2}$ is given by,

$$\triangle M_{3/2} = a_1 \begin{bmatrix} \frac{2}{3} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{2}{3} \end{bmatrix} + a_2 \begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3} \end{bmatrix} + b \begin{bmatrix} 0 & \frac{-\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{10} & \frac{4}{5} \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix},$$

$$= \begin{bmatrix} \frac{2}{3}a_2 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) \\ \frac{\sqrt{5}}{3}(a_2 - a_1) & -\frac{2}{3}a_1 - \frac{1}{3}a_2 \end{bmatrix} + b \begin{bmatrix} 0 & -\frac{\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{10} & \frac{4}{5} \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}.$$

$$\triangle M_{5/2} = a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c.$$

4.9 j-j Coupling

When m_Q is much larger than the mass of diquark (m_d), the terms a_2, b and c in Eq (4.2), all act as 1/ m_Q and their expectation values are restrained in comparison with that ofa_1 term. Therefore, it is possible to expand a_1 term in a basis where $L \cdot S_d$ is diagonal, and taking the other terms in V_{SD} as perturbations [34]. In a heavy quark limit, it is obvious to couple orbital angular momentum $L = 1$ with spin of diquark S_d. Let 's define $j = L + S_d$

($S_{[ss]} = 1$) and squaring it we can find,

$$j^2 = L^2 + S_d^2 + 2\, L \cdot S_d.$$

Eigenvalues of $L \cdot S_d$ can be written as,

$$\begin{aligned} \langle L \cdot S_d \rangle &= \frac{1}{2}\left[j\ (j+1) - L\ (L+1)\ - S_d\ (S_d+1)\right], \\ &= \frac{1}{2}[j\ (j+1) - L\ (L+1)\ - S_d\ (1+1)] = (-2,-1,1).\ \text{for } j = (0,1,2) \end{aligned}$$

States of definite J and j can be expressed in states with definite J and S in a linear combinations and therefore, for $J = 1/2$,

$$|J = 1/2, j = 0\rangle = \frac{1}{\sqrt{3}} \left| {}^2P_{1/2} \right\rangle + \sqrt{\frac{2}{3}} \left| {}^4P_{1/2} \right\rangle, \tag{4.8}$$

$$|J = 1/2, j = 1\rangle = \sqrt{\frac{2}{3}} \left| {}^2P_{1/2} \right\rangle - \frac{1}{\sqrt{3}} \left| {}^4P_{1/2} \right\rangle. \tag{4.9}$$

For $J = 3/2$,

$$|J = 3/2, j = 1\rangle = \frac{1}{\sqrt{6}} \left| {}^2P_{3/2} \right\rangle + \sqrt{\frac{5}{6}} \left| {}^4P_{3/2} \right\rangle, \tag{4.10}$$

$$|J = 3/2, j = 1\rangle = \sqrt{\frac{5}{6}} \left| {}^2P_{3/2} \right\rangle - \frac{1}{\sqrt{6}} \left| {}^4P_{3/2} \right\rangle. \tag{4.11}$$

For $J = 5/2$,

$$|J = 5/2, j = 2\rangle = \left| {}^4P_{5/2} \right\rangle. \tag{4.12}$$

Let 's calculate mass shift $\triangle M\ (J = \frac{1}{2}, j = 0)$ using Eq (3.39) & Eq (4.8),

$$\begin{aligned} a_1 \langle L \cdot S_d \rangle_{J=\frac{1}{2}} &= a_1 \left\langle J = \frac{1}{2}, j = 0 \,|L \cdot S_d|\, J = \frac{1}{2}, j = 0 \right\rangle, \\ &= a_1 \left[\frac{1}{3}(-\frac{4}{3}) + \frac{\sqrt{2}}{3}\ (-\ \frac{\sqrt{2}}{3}) + \frac{\sqrt{2}}{3}\ (-\ \frac{\sqrt{2}}{3}) + \frac{2}{3}\left(-\frac{5}{3}\right)\right] = a_1\ \left[-\frac{4}{9} - \frac{2}{9} - \frac{2}{9} - \frac{10}{9}\right] = -2a_1. \end{aligned}$$

Using Eq (3.40) & Eq (4.8) we have,

$$a_2 \langle L \cdot S_Q \rangle_{J=\frac{1}{2}} == a_2 \left\langle J = \frac{1}{2}, j = 0 \left| L \cdot S_Q \right| J = \frac{1}{2}, j = 0 \right\rangle$$
$$= a_2 \left[\frac{1}{3}(\frac{1}{3}) + \frac{\sqrt{2}}{3}(\frac{\sqrt{2}}{3}) + \frac{\sqrt{2}}{3}(\frac{\sqrt{2}}{3}) + \frac{2}{3}\left(-\frac{5}{6}\right)\right] = a_2 \left[\frac{1}{9} + \frac{2}{9} + \frac{2}{9} - \frac{10}{18}\right] = 0$$

Using Eq (4.7) & Eq (4.8) we have,

$$b < \frac{S_{12}}{4} >_{J=\frac{1}{2}} = b \left\langle J = \frac{1}{2}, j = 0 \left| \frac{S_{12}}{4} \right| J = \frac{1}{2}, j = 0 \right\rangle,$$
$$= b \left[\frac{1}{3}(0) + \frac{\sqrt{2}}{3}(\frac{1}{\sqrt{2}}) + \frac{\sqrt{2}}{3}(\frac{1}{\sqrt{2}}) + \frac{2}{3}(-1)\right] = \frac{2b - 2b}{3} = 0.$$

Using Eq (4.6) & Eq (4.8) we have,

$$c \langle S_d \cdot S_Q \rangle = \left\langle J = \frac{1}{2}, j = 0 \left| S_d \cdot S_Q \right| J = \frac{1}{2}, j = 0 \right\rangle,$$
$$= c \left[\frac{1}{3}(-1) + \frac{\sqrt{2}}{3}(0) + \frac{\sqrt{2}}{3}(0) + \frac{2}{3}\left(\frac{1}{2}\right)\right] = -\frac{1}{3}c + \frac{1}{3}c = 0.$$

Therefore,

$$\triangle M \left(J = \frac{1}{2}, j = 0\right) = -2a_1.$$

Similarly, we can evaluate the expression for the five mass shifts in terms of four parameters (a_1, a_2, b, c).

$$\triangle M \left(J = \frac{1}{2}, j = 1\right) = -a_1 - \frac{1}{2}a_2 - b - \frac{1}{2}c.$$

$$\triangle M \left(J = \frac{3}{2}, j = 1\right) = -a_1 + \frac{1}{4}a_2 + \frac{1}{2}b + \frac{1}{4}c.$$

$$\triangle M \left(J = \frac{3}{2}, j = 2\right) = a_1 - \frac{3}{4}a_2 + \frac{3}{10}b - \frac{3}{4}c.$$

$$\triangle M \left(J = \frac{5}{2}, j = 2\right) = -a_1 + \frac{1}{2}a_2 - \frac{1}{5}b + \frac{1}{2}c.$$

States	Mass (MeV.)	Proposed J^P
$M_1 \equiv \Omega_c\ (3000)^\circ$	$3000.4 \pm 0.2 \pm 0.1$	$\frac{1}{2}^-$
$M_2 \equiv \Omega_c(3050)^\circ$	$3050.2 \pm 0.1 \pm 0.1$	$\frac{1}{2}^-$
$M_3 \equiv \Omega_c\ (3066)^\circ$	$3065.6 \pm 0.1 \pm 0.3$	$\frac{3}{2}^-$
$M_4 \equiv \Omega_c(3090)^\circ$	$3090.2 \pm 0.3 \pm 0.5$	$\frac{3}{2}^-$
$M_5 \equiv \Omega_c\ (3119)^\circ$	$3119.1 \pm 0.3 \pm 0.9$	$\frac{5}{2}^-$

a_1	a_2	b	c	M_0
26.95	25.75	13.52	4.07	3079.94

Table 4.3: Values of the parameters a_1, a_2, b, c and M_0 (in MeV.) using potential model.

In diquark - quark description, the Hamiltonian for the Ω_c states is written in the following form as follows,

$$H_{eff} = m_c + 2\ m_s\ + 2\ \mathscr{K}_{ss} S_{s_1} \cdot S_{s_2} + B\ \frac{L(L+1)}{2} + a_1\ L \cdot S_d + a_2\ L \cdot S_Q + b\ \frac{S_{12}}{4} + c\ S_d \cdot S_Q\ .$$

Diagonlizing the matrices for ${}^2P_{\frac{1}{2}}, {}^2P_{\frac{3}{2}}, {}^4P_{\frac{1}{2}}, {}^4P_{\frac{3}{2}}, {}^4P_{\frac{5}{2}}$ states, we evaluate the mass corrections appearing from the Hamiltonian as in Eq (4.1). In all the five states, there is common mass term M_0

$$\begin{aligned}
M_0 &\equiv m_c + 2m_s + \frac{1}{2}\mathscr{K}_{ss} + B, \\
M_1 &= M_0 + \left(-2a_1 - a_2 + b\ \left(\frac{-1\ -\ \sqrt{3}}{2} \right) \right) + \frac{c}{2}. \\
M_2 &= M_0 + \left(-a_1\ + \frac{a_2}{2} + b\ \left(\frac{-1\ +\ \sqrt{3}}{2} \right) \right) - c. \\
M_3 &= M_0 + \left(-a_1 - a_2 + b\ \left(\frac{4\ +\ \sqrt{21}}{10} \right) \right) + \frac{c}{2}. \\
M_4 &= M_0 + \left(+a_1\ + \frac{a_2}{2} + b\ \left(\frac{4\ -\ \sqrt{21}}{10} \right) \right) - c\ . \\
M_5 &= M_0 + \left(+a_1\ + \frac{a_2}{2} - \frac{b}{5} \right) + \frac{c}{2}.
\end{aligned}$$

By solving above equations on mathematica, values of the parameters a_1, a_2, b, c and M_0 (in MeV.) are evaluated using the masses of Ω_c baryons and is summarized in the Table (4.3). The parameters which reproduce the masses in Table (4.3) are:

M_0 (css) = 3079.94 MeV., a_1(css) = 26.95 MeV., a_2 (css) = 25.75MeV. , b(css) = 13.52 MeV., c (css) = 4.07 MeV.

- The strength of the spin - spin interaction between the charm quark and the diquark (hyper fine splitting parameter c) should be small as it rely on a P- wave function near the origin.

- The value of strength of spin - orbit term a_2(css) should be close to the estimated value of $\Lambda_c$system (a_2= 23.9 MeV.) as it refers to the matrix elements of a term $L \cdot S_Q$.
- The co-efficient of $L \cdot S_d$ a_1 should be positive and less than 55.1 MeV. as estimated for Σ_c and Σ_b. For the Ω_c system, the ratio of diquark masses gives $a_1 = \frac{m_{(uu)}}{M_{(ss)}} \times 55.1 = \frac{783}{1095} \times (55.1) = 39.4$ MeV. For (ss) diquark, using parameters from Table of [35] , we can evaluate the mass of the (ss) diquark to be,

$$M_{(ss)} = 2m_s^b + \frac{a}{(m_s^b)^2},$$

where,

$$\frac{a}{\left(m_q^b\right)^2} = 49.3\ MeV. \Rightarrow a = 49.3 MeV. \left(m_q^b\right)^2,$$

$$m_s^b = 536.3\ MeV., m_u^b = m_d^b = m_q^b = 363.7\,\mathrm{MeV}.$$

$$M_{(ss)}{=}2(536.3){+}49.3\left(\frac{363.7}{536.3}\right)^2 = 1095\ \mathrm{MeV}.$$

The (uu) diquark, is evaluated in [34] which is given by,

$$m_{(uu)} = m_{(ud)} = m_{(dd)} = \frac{m_{\Sigma+2m_{\Sigma^*}}}{3} - m_s = \left[\frac{1193+2(1385)}{3} - 538\right]\ \mathrm{MeV.} = 783\ \mathrm{MeV.}$$

4.10 Predictions For $\Omega_b = b(ss)$ States

The discovery of five narrow excited baryons Ω_c states allows us to postulate the properties of a identical system consisting of a b quark and a spin-1 (ss) diquark. The large mass of bottom quark implies that the linear approximation to the masses is effective and can be used with the following inputs.

- The strength of the spin - spin interaction between the charm quark and the diquark (hyper fine splitting parameter c) is set to zero.
- The co-efficient of $L \cdot S_{ss}$ a_1 for the (css) is the same for the b(ss) system,

$$a_1\,[b\,(ss)] = a_1\,[c\,(ss)] = 26.95\ \mathrm{MeV}.$$

- We can relatea_2 [c (ss)] to the term a_2 describing the coefficient of $L \cdot S_Q$ in b(ss) system,

$$a_2\,[b\,(ss)] = \frac{m_c^b}{m_b^b} a_2\,[c\,(ss)] = \left(\frac{1708.8}{5041.8}\right) 25.74\ \mathrm{MeV.} = 8.72\ \mathrm{MeV.}$$

Masses of the bottom and charm quark as given in [34] are, where

$$\mathrm{m_c = m_{\mathit{\Lambda}_c} - m_{\mathit{\Lambda}} + m_s = 2286.5\ MeV.–1115.7\,MeV. + 538\ MeV. = 1708.8\ MeV.}$$

$$\mathrm{m_b = m_{\mathit{\Lambda}_b} - m_{\mathit{\Lambda}} + m_s = 5619.5\ MeV.–1115.7\ MeV. + 538\ MeV. = 5041.8\ MeV.}$$

- The coefficient of tensor interaction term b is taken to have a range of ±20 MeV. around zero .
- The reduced mass of b(ss) system is roughly of the order of 300 MeV.

4.11 Conclusion

In this dissertation, we have reviewed the resonances in the charmed and bottom baryons based on the potential model. This potential model includes tensor interaction term, hyper fine splitting and spin orbit terms responsible for fine structure.We have examined the eigenstates for recently reported five narrow excited Ω_c baryons and evaluated the coefficients of the spin-orbit forces, tensor force and hyper fine splitting in the spin-dependent potential using effective Hamiltonian for the Ω_c baryons in the diquark-quark description. The spectroscopist are delighted on the discovery of new excited Ω_c states reported at LHCb Collaboration decaying into Ξ_c^+ K^-, due to their narrow widths and great importance. We have explained these five states in terms of five states assumed when (ss) spin - 1 diquark is excited with respected to charm quark with relative orbital angular momentum L = 1. As seen from the plot in Fig.(4.2), there are the five resonances observed experimentally if all the five states are P-wave excitations of (ss) diquark with respect to charm quark. The mass spectrum for the Ω_c (css) states has been measured by the LHCb, confirmed recently by Belle, and the masses of the states is consistently increasing with their total spin. The masses of Ω_c states were measured which is equal to (in MeV.) M $=\Omega_c$ $(3000)^\circ$, $\Omega_c(3050)^\circ$, $\Omega_c(3066)^\circ$, $\Omega_c(3090)^\circ$ and $\Omega_c(3119)^\circ$. LHCb also determined their widths through $\Omega_c^\circ \to \Xi_c^+$ K^-decay channels, which didn't exceeds a few MeV. The presence of broad structure is also indicated around 3188 MeV. in the data which is fitted as a single resonance but can also be produced by the superposition of several states. The $\Omega_c(3050)^\circ$ and $\Omega_c(3119)^\circ$ resonances have very small widths. The three lower states are P-wave ex-citations of the (ss) diquark with respect to the charmed quark, on the basis of their narrow widths having $J^P = \frac{3}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ and two upper states are 2S ex-citations with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. The prediction of two states with $J^P = \frac{1}{2}^-$ are still remain to locate, one around 2904 MeV. decaying to $\Omega_c\gamma$ or $\Omega_c\Pi^0$ and the other around 2978 MeV. decaying to Ξ_c^+ K^- in S -wave. The discovery of five narrow excited baryon states allow us to postulate the properties of an identical (bss) system which consists of bottom quark and spin-1 (ss) diquark.

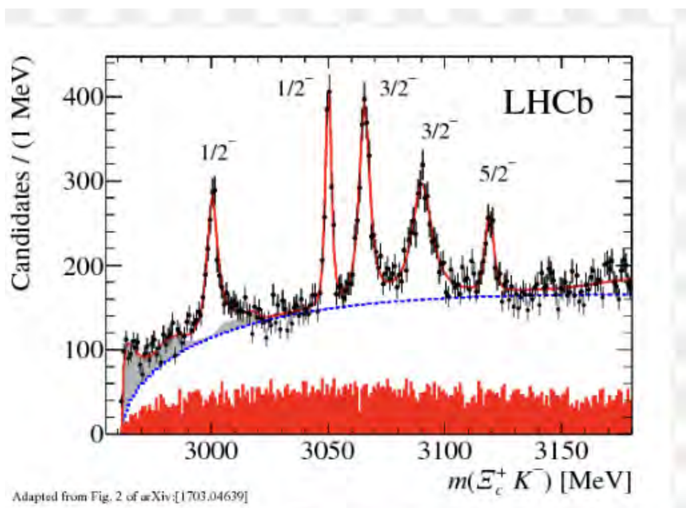


Figure 4.2: Proposed assignment of spins and parities of excited Ω_c= c (ss) states observed by the LHCb Collaboration.

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