Engineering Entanglement in Cavity Quantum Electrodynamical Systems

By

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The work presented in this thesis is dedicated

To

- *My uncle, Raja M. Fayyaz who has persistently encouraged me to pursue higher studies*

&

To the memories of

- *My late father, Raja Mulazim Hussain who unconsciously inculcated an intellectual longing for the books into the family*
- *My late Director, Dr. Sarfraz A. Bhatti who was more of a friend than a boss*

This work is submitted as a thesis in partial fulfillment for the award of the degree of the Doctor of Philosophy in Electronics, Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan.

Certificate

It is certified that the work contained in this thesis is carried out by **Mr. Rameez-ul-Islam** under my supervision.

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"That complementarity evoked in my mind a very classical Taoist idea about using language as an instrument to capture meaning. The metaphor may be misleading, but the notion is that using language to understand meaning is like using a net to catch fish. Often people who are not trained are confused, identifying the net with the fish, confusing language with meaning. The instrument that you use to catch the fish defines your conception of what the fish is, and it becomes instrumentalized in the wrong way. But there is no way of catching the fish other than with the net. This is the only instrument that we have, and therefore the instrument becomes a constitutive part. Language becomes a constitutive part of the meaning we try to capture. No matter how effectively we try to use the language, our meaning is being conditioned and shaped by this particular procedure."

The new physics and cosmology: Dialogues with the Dalai Lama.

"For surely, to explain something is to reduce it to what is already known. But it may turn out that we will never be able to reduce the quantum universe to our customary ways of thinking. Perhaps we will have to adjust our ways of thinking to it. Perhaps, years from now, people will think in new and unfamiliar ways, ways in which the quantum universe is no longer a challenge, but rather simple everyday reality."

The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics.

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The corporate world, we inhabit, is also holistic in a sense somewhat different from quantum theory. All that one achieves owes much to the community of friends and foes around him, with each category contributing in one way or the other. Anyhow, it is better not to name the foes because, like measurement in quantum mechanics, it can potentially bring them face to face; not a pleasant situation after all. However, like Zeno effect, frequent mentioning and thanksgiving keeps friendship intact and prosperous.

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Abstract

Phenomenon of entanglement was justly referred by Erwin Schrödinger as the characteristic trait of the quantum theory. On one hand, the phenomenon helped a lot to clarify the conceptual foundations of the theory. On the other hand, the very same nonlocal, counterintuitive correlations that bind entangled entities are now being employed as a backbone resource for most of the quantum informatics tasks including cryptography, teleportation, entanglement swapping, quantum computation and many others. Therefore controlled engineering of entangled states becomes vitally important.

Present work deals with four theoretical proposals for the cavity QED based generation of a variety of entangled atomic and cavity field states including Bell, W, NOON, cluster and graph states. In first two proposals, atom interferometry in Bragg regime has been utilized for the engineering of Bell, W and NOON cavity field states. In this respect, basic constituents of Mach-Zehnder-Bragg (MZB) interferometers i.e. atomic de Broglie wave mirrors and beam splitters have been explored in detail. It is further demonstrated that by manipulating split atomic de Broglie wavepackets in a MZB interferometer, the required states can be engineered in an experimentally feasible manner while utilizing time-tested standard cavity QED tools. This work therefore opens up a new vista for quantum state engineering based on atom interferometry.

Remaining two proposals aim at the generation of atomic and cavity field cluster and graph states and employ dispersive as well as resonant atom-field interactions as architectural components of the phase gate. First scheme in this section utilizes the concept of collective eraser whereas the second proposal, the most resource economical one, is based on the simultaneous resonant and dispersive interactions of two two-level atoms with an initially vacuum state high-Q cavity. Here the phase gate operation and hence the state engineering is accomplished when cavity is detected again into vacuum state after culmination of the interactions. Various parameters affecting success probability and fidelity of the proposed protocol have also been elucidated briefly. The parametric dependence of success probability and fidelity on most crucial factors of imprecision in the interaction times have also been plotted for the sake of quantitative assessment. This section is also concluded by providing a comprehensive note on the experimental feasibility of the presented work.

List of Publications

1. * Engineering maximally entangled *N*-photon NOON field states using an atom interferometer based on Bragg regime cavity QED.
   Rameez-ul-Islam, Manzoor Ikram and Farhan Saif,
   J. Phys. B: At. Mol. Opt. Phys. **40,** 1359 (2007).
2. Generation of Maximally Entangled States of Two Cavity Modes.
   F. Saif, M. Abdel-Aty, M. Javed, and **R. Ul-Islam**
   Applied Mathematics & Information Sciences **1,** 3 (2007).
3. Engineering Quantum Universal Logic Gates in Electromagnetic Field Modes.
   Farhan Saif, **Rameez-ul-Islam,** and Mazhar Javed
   Journal of Russian Laser Research **28,** 401 (2007).
4. * Generation of Bell, NOON and W states via atom interferometry.
   Rameez-ul-Islam, Ashfaq H. Khosa and Farhan Saif,
   J. Phys. B: At. Mol. Opt. Phys. **41,** 035505 (2008).
5. Remote Field and Atomic state preparation.
   Rameez-ul-Islam, Manzoor Ikram, Ashfaq H. Khosa and Farhan Saif
   International Journal of Quantum Information **6(2),** 393 (2008).
6. * Generation of field cluster states through collective operation of cavity QED disentanglement eraser.
   R. ul-Islam, A. H. Khosa, Hai-Woong Lee and Farhan Saif
   Eur. Phys. J. D **48,** 271 (2008).
7. Quantum teleportation of a high-dimensional entangled state.
   Qurat-ul-Ain Gulfam, **Rameez-ul-Islam** and Manzoor Ikram.
   J. Phys. B: At. Mol. Opt. Phys. **41,** 165502 (2008).
8. * Atomic cluster and graph states: An engineering proposal
   Rameez-ul-Islam, Ashfaq H. Khosa and Farhan Saif
   Submitted: J. Phys. B: At. Mol. Opt. Phys.
9. Generation of atomic momentum cluster and graph states via cavity QED
   Rameez-ul-Islam, Ashfaq H. Khosa, Farhan Saif and J. Bergou
   Submitted: Phys. Rev. A

10. Atomic State Teleportation: From internal to external degrees of freedom
Rameez-ul-Islam, Manzoor Ikram, Rizwan Ahmed, Ashfaq H. Khosa and Farhan Saif
J. Mod. Opt. **56**, 875 (2009).
11. Remote preparation of atomic and field cluster states from a pair of tri-partite GHZ states.
Rameez-ul-Islam, Ashfaq H. Khosa, Farhan Saif and Hai-Woong Lee
Submitted: J. Phys. B: At. Mol. Opt. Phys.
12. Generation of two-field mode NOON state in spatially separated high-Q cavities using Bragg diffraction of atoms.
Farhan Saif, **Rameez-ul-Islam** and Ashfaq H. Khosa
Submitted: Phys. Lett. A
13. Engineering entangled momenta states of identical atoms through projective measurements in Bragg regime cavity QED.
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14. Generation of cavity field cluster states through cascaded Mach-Zenhder atom interferometry in Bragg Regime.
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(To be submitted.)
15. Entangling atoms during flight
Rameez-ul-Islam, Tasawar Abbas, Ashfaq H. Khosa and Farhan Saif
(To be submitted.)

Conference Presentations:

1. Generation of field entangled state through quantum eraser.
Rameez-ul-Islam, Ashfaq H. Khosa and Farhan Saif
1st International Symposium on Quantum Optics, 9-11 January 2007,
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2. Engineering entangled states through Bragg regime cavity QED.
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ISLAMABAD.

3. Teleportation via Bragg regime cavity QED.
   Rizwan Ahmed and **Rameez-ul-Islam**
   3rd International Symposium on Quantum Optics, (5-7) August 2008,
   Centre for Quantum Physics, COMSATS Institute of Information Technology,
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[* Work on which present thesis is based.]

Contents

Chapter 1

Entanglement: Historical review and applications

1.1 First quantum revolution: The birth in an arena

The dawn of quantum theory presents a classic example of paradigm shift in science [1].The theory was initially proposed as a model based on arbitrary assumptions deemed fit to explain the profound dilemmas posed by experimental physics in the beginning of 20^{th}century. This maze of experimental data came from microscopic studies of a multitude of phenomena including black body radiations, atomic structure and photoelectric effect [2]. Historical developments of quantum mechanics can easily be divided into two eras: first generation quantum theory, from 1900 to 1925, and second generation quantum mechanics from 1925 to 1928 [3]. In the first era, it was realized that problems faced in quantitative analysis of black body radiations and atomic structural studies were intractable through the sole tools of classical physics and electrodynamics. Thus, this was the era of imposed quantization with working quantum postulate being the discretization of atomic energy levels along with the photon or light quanta hypothesis and marks the breakdown of classical concepts at microscopic level [4]. However, the emerging ideas were equally difficult to assimilate even for the founding fathers of the theory. For example, Max Plank, who introduced the concept of light quanta was never at home with it completely in his mind [5]. With setbacks and successes, nature kept on indicating hidden laws in old quantum model. For example, hydrogen spectrum was explained successfully but in case of helium spectrum, the model was a hopeless failure. Final blow to the old theory came through the negative result of BKS (Bohr-Kramers-Slater) proposal. The proposal envisioned that the conservation of momentum and energy hold only statistically in, for example, Compton scattering. However, from 1925 onward, a coherent and logically consistent picture of the theory began to emerge. Heisenberg, Born, Pauli and Jordan developed the theory in terms of observable quantities. They were able to solve the nontrivial problem of harmonic oscillator. The resulting theory was named as matrix quantum mechanics. In 1927, Heisenberg, in another epic making work [6], introduced the concept of uncertainty and noncommuting complementary variables. This work attracted attention of not only the physicists but also that of philosophers due to its underpinning concerning inherent indeterminism at microscopic level. Similarly, in 1926, Zurich-based physicist Erwin Schrödinger extended de Broglie's suggestion of wave-corpuscle duality to material particles satisfying momentum-frequency relations and the result was what is now known as Schrödinger's wave equation. In this differential equation, momentum and energy were replaced with their corresponding differential operators and quantization naturally came out of the physical requirement that wave function describing particle's dynamics should be single-valued. Schrödinger's approach was widely appreciated in the community compared to Heisenberg's matrix formalism because working physicists of the time were well acquainted with the mathematical tools of differential calculus. However, soon in 1926, Schrödinger himself proved the conceptual equivalence of the both formalisms. The same was done, not much later, by Carl Eckart and Paul Dirac. Dirac, who introduced the theme that physical quantities should be represented by operators also coined the idea that quantum states are vectors in Hilbert space [7]. He further showed that commutating operators can be connected with the classical Poisson brackets. His major contribution to the field was, however, later development of relativistic quantum mechanics and subsequent prediction of anti-particles. Max Born, at about the same time, argued that wave function Ψ of Schrödinger's equation does not describe a real wave attached with the particle, and rather $|\Psi(x,t)|^2$ gives probability of finding the particle at position x at a time t. These hectic developments finally culminated in the shape of a coherent, mathematically consistent and logically complete structure known as modern quantum mechanics with the publication of John Von Neumann's classic book, Mathematical foundations of quantum mechanics, first published in 1932 [8]. In the same book he additionally introduced projection postulate in place of wavefunction collapse to describe the non-unitary measurement process in quantum theory. However, from the very beginning people like Einstein and Schrödinger were utterly dissatisfied with the philosophical implications and interpretational issues raised by the theory. For them, the main task of physics was to unveil the mechanisms underlying a phenomena and to provide an ontology i.e. to explain things as they really are. Whereas the orthodox quantum theory insists that physics does not aim to describe mechanisms but rather relationships [9]. The theory is inherently indeterministic and does not adhere to a pre-existing reality beyond the phenomenon. Rather, in a crude sense, it says that we can talk of reality only after a measurement is made and results recorded. Thus according to orthodox Copenhagen interpretation, quantum theory deals with a world of potentialities and a quantity becomes real only when it has been measured [10]. Einstein, being a realist at heart, was much intrigued by the ongoing state of affairs and repeatedly tried to find out flaws in the theory through thought experiments presented at various Solvay conferences held in between 1927 to 1933. His arrogance is also reflected in his remarks, " Is the moon not there when nobody looks at it" [11]. On the other hand, in experimental arena, the situation was altogether different and spectacularly amazing. The theory had proved itself to be a tremendous success in describing and predicting microscopic phenomena. It successfully explained atomic structure and molecular properties and by 1930s, Linus Pauling and others have explained chemical bonding using quantum mechanics [12]. Rudolf Pierls, Flex Bloch and Alan Wilson applied it to describe various properties of metals and developed band theory of solids. Semi-conductor physics based on this band theory paved way the path towards micro-electronics revolution that we now witness in form of computers, internet and communication industry [13]. Similarly Heisenberg, Fermi, Gamou and others demonstrated the validity of the theory at nuclear dimensions. The theory, in a sense, also predicted the possibility of an atomic bomb and nuclear reactors. By now the theory has also proved to be a marvellous success at the level of the least i.e. quarks, the fundamental particles making protons and neutrons. Therefore, under the shadow of operational excellence, mainstream working physicist paid little attention to the vitally important but in essence philosophical battle going on among Schrödinger, Einstein and Bohr about the foundational issues of the quantum theory. Most of the researchers were indeed deeply engaged in understanding more and more subtle phenomena and in inventing new devices. Thus the prevailing situation of the time was that of " Shut up and Calculate" and there was no time for "Sit down and Contemplate" [9]. Hence by the early thirties most of the interpretational/methodological opposition to the theory had died down with the sole exception of Einstein and Schrödinger. Einstein was, however, no way content. On 4^{th} December 1926, he wrote to Max Born;

> "Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing"[14].

Schrodinger, while exploring the hidden meanings of the quantum mechanics, came up with the concept of entangled state in 1935 [15]. In Schrödinger's words;

> "When two systems, of which we know the states by their respective representation, enter into a temporary interaction due to known forces between them and when after a time of mutual influence the systems separate again, then they no longer be described as before, viz, by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics".

Mathematically speaking, if two independent quantum systems, say A and B at a time t_1, before interaction, possess state vectors $|\Psi_A(t_1)\rangle$ and $|\Psi_B(t_1)\rangle$, then their interaction for a time t_2 leads to,

$$|\Psi_A(t_1)\rangle \otimes |\Psi_B(t_1)\rangle \overset{Interaction}{\Rightarrow} |\Psi_{AB}(t \geq t_2)\rangle . \tag{1.1}$$

We say that systems are entangled iff

$$|\Psi_{AB}(t)\rangle \neq |\Psi_A(t)\rangle \otimes |\Psi_B(t)\rangle . \tag{1.2}$$

Thus their collective state vector is no more factorizable into original components or subsystems. Physically it implies that the systems A and B have now lost their individual identities and are no more separable in their behaviour irrespective of the mutual spatial separation that may extend to even light years. Existence of such strong correlations between entangled entities is the main crossroad where quantum theory split paths with the classical physics and local realism. These mysterious and counterintutive correlations, on the other hand, has prompted Einstein to coin the phrase "spooky action at a distance" and were later on elaborated in now famous and most cited Einstein-Podolosky-Rosen (EPR) paper [16]. This work highlights the conflict between quantum theory and local realism in most profound manner. Historically, the EPR idea was initially conceived somewhere in 1933 and Einstein discussed it privately with Rosenfeld during 1933 Solvay conference. According to Rosenfeld, Einstein asked;

> "What would you say of the following situation?. Suppose two particles are set in motion towards each other with the same, very large momentum, and that they interact with each other for a very short time when they pass at known positions. Consider now an observer who gets hold of one of the particles, far away from the region of interaction, and measures its momentum, then, from the conditions of the experiment, he will obviously be able to deduce the momentum of the other particle. If, however, he chooses to measure the position of the first particle, he will be able to tell where the other particle is. This is perfectly correct and straight forward deduction from the principles of quantum mechanics; but is it not very paradoxical ? How can the final state of the second particle be influenced by a measurement performed on the first, after all physical interactions has ceased between them" [5].

However, in 1935 the idea was presented in mature form carrying precisely defined philosophical concepts in the form of a paper, mentioned above. In EPR scenario, Einstein et al.[16] assumed a similar situation where two particles get entangled in position-momentum space after a brief interaction generated out of common emission. In order to analyze post-entanglement situation, they put forward a precise criteria of physical existence or reality based on scientific realism and locality or Einstein separability. It says;

> " If, without in anyway disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then their exists an element of physical reality corresponding to this physical quantity".

There are the two fundamental assumptions implicit in this definition; a). existence prelude measurement and; b). Two non-interacting, spatially well-separated objects can not influence each other globally. It is worth mentioning here that EPR do not claim mathematical inconsistence or falsehood of quantum theory rather they asserted that the theory is incomplete and hence incapable of describing the physical reality. In their assumed situation, two particle in composite state has initial momentum $P_1 + P_2 = 0$. Then they fall apart keeping same net conserved momentum all along the way. Now, after they are well separated, we are free to measure position or momentum of any one or both of them. Since particles are no more interacting so momentum measurement for particle-2 i.e. P_2 does not have any effect on momentum or position measurement performed on particle-1. From measurement $P_1 = P$, we infer $P_2 = -P$ as $P_1 + P_2 = 0$. But if we opt to measure the position of first particle, then uncertainty principle forbids to measure P_1. Now since P_1 is no more measurable so $P_1 + P_2 = 0$ implies that P_2 is also not measurable under the premises of quantum theory. This violates the basic assumption-that P_2 measurement is independent of what we opt to measure (position or momentum) on well separated particle-1. Since we can yet measure P_2 which is not predicted by quantum theory therefore EPR criterion i.e. "every element of physical reality must have a counterpart in the physical theory", is not satisfied and hence quantum theory must be treated as incomplete [17]. For Copenhagen school along with its mentor Niels Bohr, EPR publication was like bolt from the blue which left them shocked, confused and angry. Erwing Schrödinger, originator of the theory however told Einstein, "You have publicly caught dogmatic quantum mechanics by its throat" [5]. Although the issues raised by EPR like, does interference between two particles well separated in space is possible? Do the entangled states exist? Are they manipulatable experimentally?, were very important but they were sidestepped, relegated or even ridiculed by most of the working physicist of the age. These were termed as interpretational questions and were treated as speculations or mere metaphysical gossip. In short, studying the foundations of quantum mechanics was not a fashionable thing to do. Wolfgang Pauli's letter to Max Born (1954) explicitly shows the prevailing hostile attitude;

> " As O. Stern said recently, one should no more rack one's brain about the problem of whether something one can not know anything about exists all the same, than about the ancient question of how many angels are able to sit on point of a needle. But it seems to me that Einstein's questions are ultimately always of this kind" [18].

Therefore, along with founding fathers like Einstein, Schrödinger and Bohr, we find only few names including Eugene Wigner, David Bohm, John Wheeler, Hugh Everett III , John Bell and Abner Shimony who were genuinely worried about philosophical foundations and nonlocal character of the quantum theory. David Bohm, accepting the incompleteness suggested by EPR, proposed Hidden-variable theory [19]. However his most important work in present context is that he transformed EPR logic from position-momentum basis to electron spin basis, now called EPRB thought experiment, thus bringing it on conceptually clear footing bearing envisionable affinity to real experiments. In EPRB, two spin entangled particles, say electrons, are considered. Furthermore, instead of using two continuous variables like position and momentum, as was the case with original EPR proposal, Bohm considered just one property i.e. electron spin, a single discrete variable. Once the spin of one of the electron is measured and is found to be, say down denoted by " $\downarrow$ ", the spin of the other automatically turns up " $\uparrow$ ", irrespective of the distance between them. This is true for all possible measurement directions or basis [20]. Such a spin entangled state of the two electrons may be mathematically represented by

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow_1, \downarrow_2\rangle - |\downarrow_1, \uparrow_2\rangle \right), \tag{1.3}$$

John Stewart Bell, a frontline particle physicist working in CERN near Geneva, was also passionately but rather secretly involved in exploring the perplexities and philosophical issues raised by the quantum theory. Bell, like Einstein, was one of the few, who from very beginning realized the strange nonlocal consequences of the quantum theory. As a quantum field theorist, he was an expert of the instrumental side of the theory but was, at the same time deeply disturbed by its philosophical implications. Once he remarked that;

" I am a quantum engineer, but on Sundays I have principles" [21].

In 1964, Bell showed that no local hidden variable theory can reproduce the predictions of quantum mechanics. In a paper, published in an obscure journal that folded shortly afterwards [22], Bell analyzed the EPR argument keeping in view the locality and reality assumptions. He came up with a statistical inequality that would always be satisfied classically but might be violated by quantum theory provided entangled states actually bind particles via nonlocal and strong correlations. This is now famously known as Bell's inequality and it has lifted EPR gedenken experiment from a mere interpretational issue to an experimentally verifiable question of whether nature follows quantum mechanics with ingrained nonlocality or casual realism as envisaged by Einstein [9]. Bell's work, at the time of its publication, received almost negligible attention, not to mention appreciation but within next few years it emerged out of the debris of consciously ignored ideas. In 1968, J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt (CHSH) generalized Bell's inequality in a form that was easily testable using correlated pairs of polarization entangled photons rather than Bell's originally suggested electrons [23]. This was a positive step forward because precision experiments based on photon's polarization (equivalent to spin in case of an electron) are much easier on executional level as compared with the ones based on electrons or any other half integral spin particles [12]. Here we reproduce the CHSH argument in a brief manner. Suppose two parties, conventionally called Alice and Bob, are mutually sharing a large number of identically prepared entangled quantum systems. Such an entangled state may be expressed as follows

$$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}} \left(|\updownarrow_A, \longleftrightarrow_B\rangle - |\longleftrightarrow_A, \updownarrow_B\rangle \right). \tag{1.4}$$

Here $\updownarrow_i$ ($\longleftrightarrow_i$) stands for vertical (horizontal) polarization of a photon and $i = A, B$. Spatially separated parties Alice and Bob then decide a polarization measurement basis. Alice has measurement choice along polarization direction a and a' whereas Bob has to comply with the measurement axis b and b'. In the language of operator algebra, these local measurements are represented by set of Pauli's matrices $\left(\sigma_a^A, \sigma_{a'}^A\right)$ and $\left(\sigma_b^B, \sigma_{b'}^B\right)$ for Alice and Bob respectively. As stated earlier, this is a statistical experiment performed over a large ensemble of identically prepared entangled states such that in each run Alice is free to measure polarization of her component in either a or a' direction. Similarly Bob can choose any one i.e. either b or b' polarization orientation for measuring polarization of his qubit. Their measurement results denoted by $(\delta_a, \delta_{a'})$ and $(\delta_b, \delta_{b'})$ respectively are random with a value ± 1. Now locality-reality criteria put forward by EPR suggest that these measurements, performed at two different locations, are completely independent of each other with no foreseeable mutual influence or interaction. If accepted, then in all cases, each of these random set of measured values, must satisfy the following simple expression

$$(\delta_a - \delta_{a'})\,\delta_b + (\delta_a + \delta_{a'})\,\delta_{b'} = \pm 2. \tag{1.5}$$

Above equation is quite easy to comprehend in classical terms through an analogy suggested by S. Haroche and J.-M. Raimond [24]. The expression can be verified by tossing again and again two nickels and two dimes and recording $+1$ (*heads*) and -1 (*tails*) in each case. Furthermore, it is simple to note that either of the terms $(\delta_a \pm \delta_{a'})$ is zero whereas the other one is always equal to ± 2. After sufficient runs, data accumulated can be averaged out in form of four terms $\langle \sigma_a^A, \sigma_b^B \rangle$, $\langle \sigma_{a'}^A, \sigma_b^B \rangle$, $\langle \sigma_a^A, \sigma_{b'}^B \rangle$ and $\langle \sigma_{a'}^A, \sigma_{b'}^B \rangle$ of above expression. From these quantum mechanical expectation values, we can straightforwardly formulate the algebraic sum

$$S_A = \langle \sigma_a^A, \sigma_b^B \rangle - \langle \sigma_{a'}^A, \sigma_b^B \rangle + \langle \sigma_a^A, \sigma_{b'}^B \rangle + \langle \sigma_{a'}^A, \sigma_{b'}^B \rangle . \tag{1.6}$$

This stands for the average of a quantity that has only values ± 2 and therefore it must be bounded by the same limits. Hence based on local realism criteria, we get the following inequality, known as CHSH-Bell inequality

$$-2 \leq S_A \leq +2. \tag{1.7}$$

Thus Bell was first to show that Einstein's questions were indeed not synonym to " how many angles can sit on the point of a needle", as conjectured by the grand master Pauli. Hence Bell's inequality, described by noble prize winning physicist Brian Josephson as the most important recent advance in physics [25], in a single masterstroke subjected the conflict between local realistic world view of Einstein and that of nonlocal traits of the quantum theory into an experimental scenario. A situation, where actual experiments can be employed to distinguish between any hidden variable theory including local realistic model and the quantum theory [26]. This amazing result paved the way for what Abner Shimony later described as "experimental metaphysics" [27]. Once the importance of the Bell's work was recognized, many frontline experimental groups became fully interested in pursuing the goal. In pioneering experiments using two-photon transition from calcium atomic cascade, Stuart Freedman and J. S. Clauser tested the inequality [28, 29] and provided the first hint of confirmation that quantum mechanics is intrinsically nonlocal bearing correlations that are impossible to explain classically through any local realistic model. At Texas A&M University, Ed. S. Fry and R. C. Thompson [30] carried out similar experiments using ^{200}Hg Mercury isotope but this time excited with lasers . This enhanced their signal by several order of magnitude. In 1976, M. Lamehi-Rachti and W. Mittig at Saclay Nuclear Research Center showed results favouring quantum mechanics. They used controlled pairs of protons in the singlet state [31]. However the most profound, timely, decisive and ground breaking series of experiments are those performed by Aspect et al. [32, 33]. They also employed two-photon transition of ^{40}Ca atomic cascade to generate counterpropagating pairs of entangled photons. The results violated Bell's inequality by good enough margin and were generally accepted by the working community of the physicists. Aspect and co-workers also pointed out various possible loopholes and to some extent blocked locality loophole. The research on the topic continued onward. Leonard Mandel was the first quantum optical experimentalist that employed optical spontaneous down conversion (SPDC) method to explore the entangled states [34] and Yanhua Shih, University of Maryland, demonstrated violation of Bell's inequality by hundred standard deviations using entangled photons generated through SPDC [35]. Later on, Weihs et al. [36] permanently closed the locality loophole by rapid switching of the analyzers placed 400 meters apart. Quantum nonlocality was also verified over a distance of about 10 Km using Bell's inequality [37] whereas M. A. Rowe et al. [38] demonstrated the closure of detection loophole using ion entangled states in an ion trap. However, since the entangled ions were close enough, the locality loophole stayed open in this case. Nonlocality of multi-partite states, like GHZ state [39], has also been verified experimentally using triplets of entangled photons. The good feature of such higher dimensional states is that Bell's inequality is not violated statistically but rather only one state suffice to settle the case [40]. For recent reviews of Bell's inequality tests, please see the references [41, 42, 43]. Anyhow, by now the nonlocality and counterintutive nature of entangled states have been proved beyond any doubt. For Bell, these results only deepens the mystery. He, in an interview with J. Bernstein said;

> " The discomfort that I feel is associated with the fact that the observed perfect quantum correlations seems to demand something like the *Genetic Hypothesis* (identical twins carrying with them identical genes). For me it is reasonable to assume

that the photons in those experiments carry with them programs, which have been correlated in advance, telling them how to behave. This is so rational that I think that when Einstein saw that, and others refused to see it, he was the rational man. The other people, although history has justified them, were burying their heads in sand. I feel that Einstein's intellectual superiority over Bohr, in this instance, was enormous; a vast gulf between the man who saw clearly what was needed, and the obscurantist. So for me, it is pity that Einstein's idea does not work. The reasonable thing just does not work" [12].

1.2 Second quantum revolution: From metaphysics to technology

Nonlocality of entangled quantum states on one hand has affects on philosophical foundations of quantum mechanics as mentioned above but on the other hand it also opened a new vista of positive consequences. These fruitful novel off-shoots of proven nonlocality that stem out of the Bell's reanalysis of the EPR argument has, it seems, much wider technological impact. The era has been justly termed as "The Second Quantum Revolution" by A. Aspect [44]. This second revolution was based on two facts.

i). Bell's theorem and inequality.

ii). Availability of technical maturity required for engineering, exploration and manipulation of entangled states generated between single microscopic quantum objects like ions, atoms, electrons and photons.

Thus once the nonlocality of quantum entangled states was proved beyond any reasonable doubt, scientists also started pondering how these novel features of the theory can be employed to harness practical applications. First of the such applicative examples of quantum weirdness is what we now call Quantum Cryptography that root back to work of Stephen Wiesner done in early 70s but published much later in 1983 [45]. Cryptography is probably one of the most ancient technical field dating back to B.C. era and aims at communicating data/information among the legitimate parties leaving almost negligible chance of eavesdropping. From Cesar cipher to Vernam cipher and then RSA, a lot of classical cryptosystems were employed from time to time with each having its own problems. Wiesner, however, in his ground breaking research combined quantum physics with cryptography and demonstrated that how, using quantum states, confidentiality level can be raised beyond the accessibility of classical physics. In brief, using these ideas he showed that a) Bank notes can be designed that are impossible to counterfeit, and b) It is possible to send two classical messages through quantum data transmission such that receiver can retrieve either one or the other message but not the both simultaneously. Charles Bennett and Gilles Brassard, aware of yet unpublished work of Wiesner, extended the idea to a practically feasible quantum cryptography proposal. In essence, it is a Quantum Key Distribution (QKD) protocol christened as BB84 protocol, designated so through the initial of their names and its year of publications [46]. The protocol is based on the fact that, in contrast to classical physics, quantum states are inherently fragile and any attempt at state measurement, irrespective of its carefulness and subtlety, is guaranteed to either destroy or at least change the state in a detectable manner. This feature of unavoidable disturbance caused by an intruder into the quantum channel ensure the security of the protocol. On practical side, BB84 is based on randomly sending and measuring photons by two parties, conventionally called Alice and Bob, in mutually orthogonal basis i.e. ($\rightarrow$ or $\uparrow$) and ($\nearrow$ or $\searrow$). The same authors along with F. Bessette, L. Salvail and J. Smolin practically demonstrated the effectiveness of their protocol as well as feasibility of quantum cryptography in general by sending and receiving quantum encrypted signals across a lab. bench with sender and receiver only separated by 32 cm and a feeble data transfer rate of only ten bits per second. This simple experiment, however proved to be a successful leap into quantum informatics age [47]. QKD based research progressed onward in pace with the advancement of photonics technologies and in 2002 the encryptic key was communicated over a distance of 20 Km [48]. Moreover just after one year, in 2003, Quantum Information Group at Cambridge University headed by Andrew Shields demonstrated a prototype QKD system capable of transmitting encrypted data at a rate of 2 kb/s over a distance of 122 Km through an optical fiber [49]. From 2004 onward such quantum cryptography systems are commercially available and are being used in defence and commerce sectors. As an example, we cite the commercial manufacturer of quantum cryptographic systems like MagicQ (www.MagicQtech.com) and ID Quantique (www.idQuantique.com) companies. However, first entanglement-based quantum cryptography proposal was presented by Artur Ekert in 1991, then a Ph.D. student at the University of Oxford [50]. Here entangled pairs of photons are assumed to be shared by the two secretly communicating parties. With the absence of eavesdropper, their spatially separated, local measurement results on entangled pairs are deemed to be strongly correlated as envisioned by the Bell's theorem. The improved versions of entanglement-based cryptographic techniques were soon published by Bennett et al. [51], and by Ekert et al. [52]. These proposals were also experimentally implemented, initially over a distance of 10 Km in 1998 [37, 53, 54] and later on up to 50 Km through fiber optic by the same group [55, 56]. Another group headed by Anton Zeilinger at University of Vienna has demonstrated Internet Bank transfer over a distance of 1.45 Km using a prototype photon polarization entanglement-based quantum cryptographic system [57]. Similarly free-space optical links with entangled photons have also been demonstrated successfully over a distance of about 7.8 Km [58]. An extension of the work to launch quantum communication in space using satellites is right now underway with financial support from European Space Agency (ESA) and the feasibility for such a communication has been recently verified [59]. In short, quantum cryptography, from its very inception, has gained a fully involved attention of the working scientific community and the subject has been thoroughly explored both theoretically and experimentally as suggested by the bulk of research papers published in recent past [60].

At about the same time, when quantum cryptography was being invented, famous theoretical physicist and major inventor of Quantum Electrodynamics Richard P. Feynman asked the question whether a classical computer can simulate a quantum phenomenon. He conjectured that it is impossible without going through an exponential computational slowdown [61]. However the idea of a Quantum Computer, in its physical essence, was proposed by Paul Benioff [62] and was later on developed by David Deutsch in 1985. He laid down the foundations of quantum computations by showing that, in principle, a quantum computer is perfectly capable of simulating any phenomena [63]. Quantum computation, along with most of other quantum informatics ideas, thoroughly relies on the counter intuitive traits of the theory like superposition and entanglement. Classical computers operate on digitized binaries called bits. A bit is represented by 0 or 1 that usually corresponds to observable levels of either voltage or a current. Peculiar thing with the quantum theory is that a quantum system may exist, in general, in superposition of these measurable quantities. Such a two-dimensional superposition is termed as quantum bit or simply qubit, and can be expressed, using Dirac notation, as follows

$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,. \tag{1.8}$$

Here α and β are called probability amplitudes and normalized probability conservation demands that $|\alpha|^2 + |\beta|^2 = 1$. In this case, Hilbert space vectors $|0\rangle$ and $|1\rangle$ form sole computational basis for two-dimensional system. Similarly, for an $n-$dimensional system, we may write the superposition as

$$|\Psi\rangle = \sum_{j=0}^{n-1} C_j\,|j\rangle\,, \tag{1.9}$$

with $\sum |C_j|^2 = 1$ and $\langle i \mid j\rangle = \delta_{ij}$. However, the main feature of the computation in this domain comes from what is usually termed as *"Quantum Parallelism"*. This stems from the fact that each and all components of a n-dimensional superposition can be manipulated simultaneously on parallel grounds. For further details, please consult the frequently cited reviews [64, 65, 66] and excellent books like Nielson and Chuang [67] and Mermin [68]. Peter Shor in 1994 really broadened the spectrum of quantum computing to practical horizons by showing that factorization of large numbers, needed for example to break RSA code, can be carried out much more efficiently through a quantum computer as compared to exponential computational complexity faced in using classical machines. Here, the number of steps in Shor's algorithm increases only polynomialy with increasing input and this gives tremendous boost in speed making practically intractable problems to be solvable in physically acceptable times [69]. Similarly D. Simon in 1997, suggested an algorithm that aimed at finding the periodicity of a 2-1 binary function incorporating a periodic element [70]. The proposed algorithm again demonstrate an exponential speed up over classical counterparts. Another breakthrough came at about the same time when Lov Grover showed that for sorting a N item database, quantum computer needs to perform only $O(\sqrt{N})$ queries whereas classical computer has to go through N queries for the same task [71]. Involvement of entanglement into quantum computation is vitally important. Jozsa et al. [72] have shown that participation of entangled states should always be ensured at one step or the other in order to have exponential speed up in quantum computers. Link between exponential speed up and entanglement have also been highlighted by Linden et al. [73] and by Harrow et al. [74]. Furthermore, information needs to be encoded in the entangled correlations, if a quantum computer is to work in noisy environment, a criterion much closer to actual lab. scenario [75]. Role of entanglement has been experimentally elucidated during execution of Shor's algorithm with the help of photonic techniques [76]. More recently, an altogether new model, termed as one-way quantum computing model has been proposed [77, 78, 79] that employ newly discovered entangled cluster states as resource [80]. Information is processed through local qubit measurement. One-way computational model has also been verified experimentally through the manipulation of four qubit polarization cluster state [81, 82]. Half of the present work is concerned with the engineering of cluster states. Quantum computation has been implemented using NMR techniques and prime factorization of the number 15 was carried out through a seven-qubit Shor's algorithm [83]. Similarly using ion traps, Cirac-Zoller gate [84], a geometric bi-ionic phase gate [85], the Deutsch-Jozsa algorithm [86] and a semi-classical quantum-Fourier transforms [87] have been demonstrated successfully. Moreover, using photonic qubits, a compiled version of Shor's algorithm have been implemented [88] and Grover's search algorithm have also been demonstrated using two-photon four-qubit cluster state under one-way computing model [89]. A cavity QED based tunable phase gate in microwave regime has also been experimentally implemented [90]. In summary, experimental quantum computing is being explored through utilization of diverse technologies like Cavity QED systems, NMR, Ion traps, Semiconductor quantum dots, Superconductor devices and Photonics techniques. For a comprehensive review, one may consult the Ref. [91].

Another landmark in the history of second quantum revolution is the discovery of quantum teleportation in 1993 by Bennett et al. [92]. Teleportation, in present context, means quantum state delivery from one location to the other with the help of preexisting entangled link and aided by classical communication. It is interesting to note that during quantum teleportation, apart from information nothing physical is imparted from one place to the other. The procedure is quite simple to follow. We assume that Alice and Bob are sharing a maximally entangled state, say $\left|\Psi_{AB}^{(+)}\right\rangle = \left(|1_A, 0_B\rangle + |0_A, 1_B\rangle\right)/\sqrt{2}$. Now Alice is provided a quantum system in unknown state $|\xi_T\rangle = \alpha\,|0_T\rangle + \beta\,|1_T\rangle$, that is to be teleported to Bob. Alice then perform a joint measurement, called Bell Basis measurement, on her part of entangled state and the unknown state that is to be teleported i.e. $|\xi_T\rangle$. The initial product state may be expressed as

$$|\Psi\rangle = \left|\Psi_{AB}^{(+)}\right\rangle \otimes |\xi_T\rangle = (|1_A, 0_B\rangle + |0_A, 1_B\rangle) \otimes (\alpha\,|0_T\rangle + \beta\,|1_T\rangle)\,/\sqrt{2}. \tag{1.10}$$

Introducing Bell-basis for Alice i.e.

$$\left|\Psi_{AT}^{(\pm)}\right\rangle = (|1_A, 0_T\rangle \pm |0_A, 1_T\rangle)\,/\sqrt{2} \qquad \text{and} \qquad \left|\phi_{AT}^{(\pm)}\right\rangle = (|0_A, 0_T\rangle \pm |1_A, 1_T\rangle)\,/\sqrt{2},$$

we may rearrange above expression as

$$\begin{aligned} |\Psi\rangle = &-\frac{1}{2}\left|\Psi_{AT}^{(-)}\right\rangle \otimes (\alpha\,|0_B\rangle + \beta\,|1_B\rangle) + \frac{1}{2}\left|\phi_{AT}^{(-)}\right\rangle \otimes (\alpha\,|1_B\rangle + \beta\,|0_B\rangle) \\ &+ \frac{1}{2}\left|\phi_{AT}^{(+)}\right\rangle \otimes (\alpha\,|1_B\rangle - \beta\,|0_B\rangle) - \frac{1}{2}\left|\Psi_{AT}^{(+)}\right\rangle \otimes (\alpha\,|0_B\rangle - \beta\,|1_B\rangle). \end{aligned} \tag{1.11}$$

Now this indistinguishable Bell-Basis measurement leaves four equally probable detection patterns corresponding to $\left|\Psi_{AB}^{(+)}\right\rangle$, $\left|\phi_{AT}^{(+)}\right\rangle$, $\left|\Psi_{AT}^{(-)}\right\rangle$ and $\left|\Psi_{AT}^{(+)}\right\rangle$. Hence if, after Bell measurement, Alice finds her parts in $\left|\Psi_{AT}^{(-)}\right\rangle$ then the unknown state is faithfully teleported to Bob. For other options, Bob upon reception of two bit classical message from Alice, can locally transform it back to the original state through unitary transforms at his end [67]. Publication of Bennett et al.'s pioneering paper was followed by a hectic research activity in the area and many theoretical proposals for teleporting atomic or field states were presented. A few are being cited here [93, 94, 95, 96, 97, 98, 99]. Since present thesis is concerned about atom-field interactions so in this short review emphasis will be mostly limited to the concerned disciplines in order to avoid undue expansion. After a wholehearted theoretical activity, the experimentalists joined in to demonstrate the feasibility of the phenomenon. The real difficulty here, especially in photonics case, was the operational procedure of Bell-State analyzer that should meet the stringent criteria of two-qubit indistinguishability during measurement. Anton Zeilinger's group at the University of Innsbruck, Austria demonstrated quantum teleportation of an unknown photonic qubit in 1997 [100]. They employed polarization entangled photon pairs produced through SPDC process but the condition of simultaneous interaction of independent qubits (i.e. photons) at the beam splitter restricted success probability of their Bell-state measurement to only 25%. At about the same time Boschi et al. [101] demonstrated teleportation with unit success probability. They, however, employed a variant of standard protocol that did not requires, apart from shared entangled photons, any extra photonic qubit in unknown state. NMR technology was also tested successfully for the task [102]. Jeff Kimble at the Caltech's Quantum Optics group successfully reconstructed an object by successive teleportation of photons with the help of squeezed state beam entanglement [103]. These all experiments were limited within the space offered by an optical Lab. bench but, by now, operational potentiality of the protocol has been shown to be independent of the distances involved [104]. In 2004, two independent teams reported atomic state teleportation [105, 106, 107]. This success served as a tremendous boost to the field because standard protocol was followed in both cases and teleportation of atoms was fully deterministic up to the "Push Button" level. Teleportation of comparatively larger objects seems accomplishable in near future because the major prerequisite i.e. mesoscopic spin entanglement between two trillion-atom cesium clouds has already been demonstrated [108]. Research on quantum teleportation is yet being pursued rigorously both on theoretical and experimental grounds [109, 110, 111, 112, 113, 114] including state-independent teleportation and squeezed state storage and teleportation [115, 116]. This interest mainly owes to applications of teleportation in quantum computation. This is specifically significant for transmission of data encoded in quantum states to remote sites within a quantum computer [117, 118]. For a comprehensive and technology based review of optical systems utilized for quantum information processing, please see T. C. Ralph's recent work [119]. Another good review covering practically implementable quantum information protocols based mostly on weak coherent light is to be found in Wang et al. [120].

Now, from previous discussions, it is amply evident that from quantum cryptography to teleportation, entangled states serve most of the purposes as an essential resource. So it is indeed fair to say that entanglement is the foundational stone upon which the edifice of quantum informatics has been solidly erected. Along with cryptography, quantum computation and teleportation, entanglement is also involved in Secret Sharing [121, 122], Quantum Error Correction [123], Entanglement Purification [124], Dense Coding [125], Competitive Quantum Games [126] and Distributed Computation [127]. Apart from quantum informatics tasks mentioned above, entangled states has also been found vital in, for example, frequency standards [128, 129] and lithography [130]. Further, it has also been shown that entanglement may cause specific phase transition in materials near absolute zero [131, 132]. The effect was theoretically predicted by V. Vedral and co-workers few years back [133]. Similarly Benni Reznik of Tel Aviv University have conjectured that quantum vacuum is filled with pairs of particles that are entangled [134]. Moreover, Brukner et al. [135] have suggested that temporal events can also get entangled. It has also been further speculated that almost all quantum interactions result in entanglement, with no restriction what so ever-thus entanglement is everywhere [136]. From above discussion, entanglement emerges as not just the characteristic trait of quantum theory [15] but rather also as a phenomenon underlying most of the microphysics with added advantage of being a potential resource in emerging science and technology of quantum information.

Since present work is concerned with the generation of entangled states using cavity QED tools, therefore in next chapter we give a brief review of entanglement engineering based on Atom-field interactions in fully controlled cavity QED environment.

Chapter 2

Cavity QED based entanglement generation: Review, preliminaries and mathematical tools

2.1 Entanglement generation in cavity QED: A brief review

Cavity QED has served as a pioneer tool in handling various quantum information tasks including entanglement generation and manipulation [137]. High-Q cavities with a lifetime ranging up to seconds in microwave regime and that of millisecond in optical regime have been reported [138, 139] whereas miniaturization of the technology in micrometer cavity dimensions has also been achieved [140]. Therefore, we can safely say that, by now, the discipline has gained the status of a mature technology. This has also been manifested through recent fascinating experiments on single atoms trapped in high-Q cavities [141, 142, 143, 144, 145]. Keeping in view the evident importance of entangled states, related both to foundational issues as well as to quantum information, hundreds of research proposals for generation of such states using cavity QED tools have been published in last 10-15 years [146, 147]. Cavity QED techniques have been suggested to generate atom-atom, atom-field and field-field entanglements. We will, however, limit our description to only the most important or the most relevant ones. Broadly speaking, these all proposals can be roughly categorized into two major types. The first methodology relies on the fully controlled unitary evolution of coherent atom-field dynamics in High-Q cavities. The desired state in this case generates through precise selection of Rabi oscillations. The first proposal of this type was given by Davidovich et al. [96] which aimed at teleporting an unknown field state between the two entangled cavities. A similar scheme was the first one to be implemented experimentally by Hagely et al. [148] where they successfully generated Einstein-Podolosky-Rosen pairs of atoms. The same group later on engineered a Bell state between two field modes in a single microwave cavity using precisely controlled coherent interactions of a Rydberg atom [149]. Following the same methodology, a tripartite GHZ state among two-atoms and a cavity has also been demonstrated in 2000 [150]. B. B. Blinov and co-workers demonstrated entanglement between a trapped atom and a flying qubit i.e. photon [151]. Such demonstrations later on paved the way for the concepts like quantum repeaters and Tangled Memories [152]. Now to begin with, Cirac and Zoller showed that a maximally entangled Bell state of two-level atoms can be engineered if atoms, one in ground but other in excited state, interact with an initially vacuum state cavity for a well calculated time, such that, at the termination of interaction, cavity is again left into vacuum state [153]. Bogar et al. presented a scheme for generation of entangled pairs of atoms using two micromaser cavities [154]. However, keeping the technical difficulties involved in first approach like precise control of atom-field interactions in view, an alternative procedure was also sorted out for entanglement generation in cavity QED scenario. This second approach is fundamentally a measurement based approach. The technique do not require a tight control over various parameters of interacting entities and, in most cases, entanglement is generated through indistinguishable detection of photons emitted either from the cavities or from the atoms. The first proposal of this type was presented by Cabrillo et al. [155]. The scheme is based on the interactions of two atoms with a weak laser pulse. The state is engineered with a subsequent detection of a single spontaneous emission event. Plenio and co-workers employed cavity-loss for generation of such states [156]. This theme was refined and made more experimentally accessible by many researchers [157, 158, 159, 160, 161]. Such schemes, though seems easier to implement experimentally, suffer through some inherent drawbacks like significantly lower success probability (in most cases 50%) and inefficient photo-detection. The trend, however,continues with further suggestions of improvements [162, 163, 164, 165, 166]. Schemes based on atom-cavity systems for the generation of entangled photons on demand have also been suggested [167, 168, 169, 170]. Zubairy and co-workers have recently introduced some novel ideas concerning entanglement generation through atom-field interactions. These include coherence-induced entanglement [171], correlated spontaneous emission laser based entanglement generation and amplification [172, 173] and utilization of single atom as a macroscopic entanglement source [174]. In a recent proposal by Pielawa et al. [175], entangled radiations were produced through a cavity QED based atomic reservoir comprised of a beam of atoms with random arrival times. Another emerging trend points to the macroscopic entanglements [176] between, for example atom-light-mirrors [177], flux qubits/ superconducting devices [178, 179] and Bose-Einstein Condensates [180, 181]. A brief review on entanglement in many-body systems is given by Amico et al. [182]. Apart from atomic or field entangled states, Bell state engineering between position-momentum degrees of freedom of distantly separated trapped atoms is also taken into consideration [183] whereas yet in another proposal, two atoms were entangled in their external degrees of freedom i.e. transverse momentum along cavity axis, through Bragg scattering [184]. Similarly many schemes for generation of multipartite entangled states like GHZ state [39] and W state [185] have been forwarded in cavity QED background. Present thesis also includes a scheme for four-partite cavity field W state engineered through atomic interferometry in Bragg regime cavity QED [186]. For these so called W states, it has been shown that such states are highly robust against the loss of one or two qubits compared to their counterpart GHZ states [185]. Multiqubit W states are also shown to exhibit stronger nonclassicality than GHZ states [187]. These unique characteristics make W states an ideal resource for communication based on multinodal networks [188], teleportation and dense coding [189, 190, 191, 192] and optimal universal quantum cloning [193]. Keeping such vital applications in view, various schemes based on cavity QED [166, 194, 195, 196, 197, 198, 199] and photonic [200, 201, 202, 203, 204, 205] systems have been proposed with cavity QED based schemes having evident edge of generating the states with good success probabilities. Similarly numerous schemes for GHZ state engineering with the aid of atom-field interactions in high-Q resonators have been proposed over the years but those are irrelevant concerning present work and hence are being omitted here for the sake of brevity. We, rather want to discuss bipartite $N-$photon entangled NOON state [130, 206] i.e. $\left(|N_A, 0_B\rangle \pm |0_A, N_B\rangle\right)/\sqrt{2}$. Such states serve as an important resource for Heisenberg-limited metrology and quantum lithography [130, 207, 208]. These states also help to cope with detection efficiency loopholes. It is worth noting that most of the earlier work for production of such states depended heavily on photonic techniques [209, 210] utilizing optical parametric down conversion as their fundamental tool [211, 212, 213]. Thus they were not able to demonstrate the basic character of such states, that is, an interference pattern with a resolution of λ/N, in a clear cut manner. Here N stands for the number of entangled photons. Furthermore, success probabilities of photonics based techniques are comparatively small due to both post-selection procedure as well as problems related to low count rates. Post selection makes these photonics based techniques inherently probabilistic and state engineering schematics beyond $N = 2$ become quite inefficient and cumbersome [214]. Furthermore, states engineered through optical techniques yield entangled photons acting solely as flying qubits that can not be employed for information processing based on fully controlled atom-field interactions. However, up to our best knowledge, no cavity QED based scheme has been proposed for generation of such states, at least for large enough N. We, in present work, have suggested two such schemes [186, 215]. The schemes are deterministic and are based on Bragg diffraction of atoms in interferometric setups. Here, in principle, we can go for sufficiently large N. Next, we give a brief review of recently introduced so called cluster states [80]. This general class of entangled state exhibit some novel characteristics. As discussed earlier, the states are found to be more resistant to decoherence as compared to their counterpart GHZ states [216]. Furthermore, the cluster states exhibit a rich nonlocality structure different from the one possessed by the GHZ states. For example, one can construct new types of Bell inequalities that are maximally violated by the four-qubit cluster states but not violated at all by the four qubit GHZ states [217, 218]. The emerging interest in cluster states is linked with the newly proposed one-way quantum computing model whose operatibility depends upon such states [77, 78, 79]. The model performs universal quantum computing through local single-qubit measurements of the cluster states. This has been experimentally verified by optical exploration of a four photon cluster state [81, 82]. Thus, owing to their evident importance, many schemes for the engineering of the cluster states have been suggested using diverse technologies based on photonic [196, 219, 220, 221, 222, 223], atomic [113, 199, 224, 225, 226, 227, 228, 229] and solid-state [188, 230, 231] systems. However, due to their relative easiness concerning implementation, the schemes employing linear optical techniques have already been experimentally demonstrated [81, 82, 89]. Photonics techniques, however, suffer from the fact that they are inherently probabilistic. Whereas schemes based on cavity QED technologies [199, 224, 225, 226, 227] have an ideal success probability of unity, although in reality such schemes are subjected to experimental imperfections such as cavity photon loss, atomic spontaneous emission, and violation of the Lamb-Dicke condition. Cluster states are subset of more general states called graph states [232]. Second part of the present work deals with generation of cluster and graph states. In this respect, we have forwarded two proposals for field as well as atomic cluster and graph state engineering [233, 234]. The schemes are based on resonant as well as dispersive atom-field interactions in cavity QED environment.

2.2 Preliminaries and mathematical tools

The work presented in the thesis relies on the quantized atom-field interaction in cavity quantum electrodynamics (CQED). This includes resonant and dispersive atomic interactions with cavity fields as well as Bragg diffraction of matter waves. We, therefore, provide a short mathematical overview of these techniques and details can be found in graduate level books on the subject [235, 236, 237, 238]. For entirely cavity QED specific expositions, books by Haroche and Raimond [24] or Dutra [239] can be consulted, in addition to recent review of the cavity quantum electrodynamics [240].

2.2.1 Cavity field quantization

Analogy of a field mode with the simple harmonic oscillator (SHO) is the comprehensive way to understand field quantization in a cavity. Each cavity mode may be considered as a SHO having a generalized amplitude q and hence the total energy of the radiation field will be summation over energies of the individual oscillators. Classically the amplitude q_j of the j^{th} oscillator is treated as a continuous variable but quantization of electromagnetic field restricts mode amplitude q_j to certain regimes depending upon the field state [237]. The classical Hamiltonian for the j^{th} SHO with unit mass may be expressed as

$$H_j = \frac{1}{2}\dot{q}_j + \frac{1}{2}\Omega_j q_j^2. \tag{2.1}$$

Here $\dot{q_j} = \partial q_j/\partial t$ and $\Omega_j = c|k_j|$ with $\vec{k_j}$ being the wave vector determined by the boundary conditions. In above expression, the conjugate variables q_j and $p_j \equiv \dot{q_j}$ obey Hamilton's equations,

$$\dot{q_j} = \frac{\partial H_j}{\partial p_j} = p_j, \quad \dot{p_j} = -\frac{\partial H_j}{\partial q_j} = -\Omega_j^2 q_j, \tag{2.2}$$

and lead to the equation of motion

$$\ddot{q_j} + \Omega_j^2 q_j = 0. \tag{2.3}$$

Now by analogy, magnetic field is proportional to q_j and the electric field is proportional to $\dot{q_j} = p_j$ and hence both the fields behave as the two conjugate variables i.e. position and momentum of a mechanical harmonic oscillator. We here introduce, for the sake of quantization, the complex amplitudes

$$a_j = \frac{1}{\sqrt{2\hbar\Omega_j}}\left(\Omega_j q_j + ip_j\right), \qquad a_j^* = \frac{1}{\sqrt{2\hbar\Omega_j}}\left(\Omega_j q_j - ip_j\right). \tag{2.4}$$

The complex amplitudes a_j and a_j^* will be later on recognized as field annihilation and creation operators. With these definitions at hand, we may write conjugate variables q_j and p_j as follows

$$q_j = \sqrt{\frac{\hbar}{2\Omega_j}}\left(a_j + a_j^*\right), \qquad p_j = \frac{1}{i}\sqrt{\frac{\hbar\Omega_j}{2}}\left(a_j - a_j^*\right). \tag{2.5}$$

Substituting the values of q_j and p_j, the above Hamiltonian simplifies to

$$H_j = \frac{1}{2}\hbar\Omega_j\left(a_j^* a_j + a_j a_j^*\right). \tag{2.6}$$

Here we are in position to impose the quantization through the commutation relation $\left[q_j, p_{j'}\right] = i\hbar\delta_{j,j'}$. Using the values of q_j and p_j, this commutation can be easily transformed to

$$\left[a_j, a_{j'}^\dagger\right] = \delta_{j,j'}. \tag{2.7}$$

Here we have used $\left[a_j, a_{j'}\right] = \left[a_j^\dagger, a_{j'}^\dagger\right] = 0$. Thus with the help of commutation relation, Hamiltonian H_j simplifies to

$$H_j = \frac{1}{2}\hbar\Omega_j\left(a_j^\dagger a_j + a_j a_j^\dagger\right) = \hbar\Omega_j\left(a_j^\dagger a_j + \frac{1}{2}\right). \tag{2.8}$$

This gives the final expression of Hamiltonian for a quantized electromagnetic field inside a cavity. The operators a_j and $a_j^\dagger$ are called annihilation and creation field operators for j^{th} cavity mode and possess the following properties

$$a\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle, \quad a^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle, \tag{2.9}$$

here $\left|n\right\rangle$ stands for a field Fock state containing precisely n photons. Clearly a and $a^\dagger$ themselves are not Hermitian operators but the number operator $n = a^\dagger a$ is Hermitian with $a^\dagger a\left|n\right\rangle = n\left|n\right\rangle$.

The multimode field linearly polarized along x-axis in the cavity may be expressed as

$$E_x(z,t) = \sum_j A_j q_j(t)\sin(k_j z). \tag{2.10}$$

Here, as stated earlier, q_j has the dimension of length and stands for normal mode amplitude. Different modes are represented by $k_j = j\pi/L$ with $j = 1, 2, ...$and $A_j = 2\Omega_j^2/V\epsilon_o$. The frequency of j^{th}mode inside a cavity of length L is denoted by $\Omega_j = j\pi c/L$ and V is the resonator volume. Substitution of $q_j(t)$ and little mathematical simplification gives following quantum mechanical expression for multimode electric field in a cavity of volume V

$$\mathbf{E} = \sum_j \epsilon_j \varepsilon_j\left(a_j + a_j^\dagger\right), \tag{2.11}$$

with $\varepsilon_j = (\hbar\Omega_j/2\epsilon_o V)^2$ [235].

2.2.2 Quantized atom-field interactions

An n-level atom interacting with a multimode Electric field $\mathbf{E}$ may be expressed by the Hamiltonian

$$H_T = H_A + H_F + H_{Int}, \tag{2.12}$$

here H_A and H_F are the independent energies of the atom and field respectively and $H_{Int.}$ stands for the interaction energy of the Atom-field system. We have already evaluated $H_F = \hbar\sum_k \upsilon_k \left(a_k^\dagger a_k + \frac{1}{2}\right)$. Atomic Hamiltonian H_A, is simply obtained by considering completeness of atomic energy eigen states (or levels) i.e. $\sum_i |i\rangle\langle i| = 1$ and eigenvalue equation $H_A|i\rangle = E_i|i\rangle$. Thus

$$H_A = \sum_i E_i |i\rangle\langle i| = \sum_i E_i \sigma_{ii}. \tag{2.13}$$

Now using dipole approximation (i.e. field is taken uniform over whole of the atom), Atom-field interaction H_{Int} may be written as

$$H_{Int} = -\mathbf{er}.\mathbf{E}. \tag{2.14}$$

Here e is the charge on an electron and $\mathbf{r}$ is its position vector. Again with the help of atomic eigenvalue completeness, we may write

$$\mathbf{er} = \sum_{i,j} e|i\rangle\langle i|\mathbf{r}|j\rangle\langle j| = \sum_{i,j} \wp_{ij}\sigma_{ij}. \tag{2.15}$$

In above expression $\wp_{ij} = e\langle i|\mathbf{r}|j\rangle$ and is called electric-dipole transition matrix element. Now, in the language of field creation and annihilation operators, the multimode electric field $\mathbf{E}$ may be expressed quantum mechanically as follows

$$\mathbf{E} = \sum_k \epsilon_k \varepsilon_k \left(a_k + a_k^\dagger\right), \tag{2.16}$$

with $\varepsilon_k = (\hbar\upsilon_k/2\epsilon_o V)^2$. Substituting all these expressions into H_T and ignoring the zero-energy terms, we get,

$$H_T = \sum_k \hbar\upsilon_k a_k^\dagger a_k + \sum_i E_i\sigma_{ii} + \hbar\sum_{i,j}\sum_k \mu_k^{ij}\sigma_{ij}\left(a_k + a_k^\dagger\right), \tag{2.17}$$

where $\mu_k^{ij} = -\wp_{ij}.\epsilon_k\varepsilon_k/\hbar$ is known as atom-field coupling constant. Now for a two-level atom with lower and upper level being denoted by $|g\rangle$ and $|e\rangle$ respectively, we have $\wp_{eg} = \wp_{ge}$ and hence we may take $\mu_k = \mu_k^{eg} = \mu_k^{ge}$. Thus above expression for H_T reduces to

$$H_T = \sum_k \hbar \upsilon_k a_k^\dagger a_k + (E_e \sigma_{ee} + E_g \sigma_{gg}) + \hbar \sum_k \mu_k (\sigma_{eg} + \sigma_{ge}) \left(a_k + a_k^\dagger \right). \tag{2.18}$$

Now $(E_e \sigma_{ee} + E_g \sigma_{gg}) = \frac{1}{2} \hbar \omega (\sigma_{ee} - \sigma_{gg}) + \frac{1}{2} (E_e + E_g)$. Here we have taken $\hbar \omega = (E_e - E_g)$ and $(\sigma_{ee} + \sigma_{gg}) = 1$ as it represents the completeness of a two-level system. Constant energy term i.e. $(E_e + E_g)/2$ will again be ignored. Further, we will adopt following notations of Pauli's matrices

$$\sigma_z = \sigma_{ee} - \sigma_{gg} = |e\rangle \langle e| - |g\rangle \langle g|, \; \sigma_+ = \sigma_{eg} = |e\rangle \langle g|, \; \sigma_- = \sigma_{ge} = |g\rangle \langle e|. \tag{2.19}$$

Under this notation, above Hamiltonian transforms to

$$H_T = \sum_k \hbar \upsilon_k a_k^\dagger a_k + \frac{1}{2} \hbar \omega \sigma_z + \hbar \sum_k \mu_k (\sigma_+ + \sigma_-) \left(a_k + a_k^\dagger \right). \tag{2.20}$$

In above equation, the terms $\sigma_+ a^\dagger$ and $\sigma_- a$ violate energy conservation and are being dropped here under rotating wave approximation. Thus final expression of the Hamiltonian for a two-level atom interacting with n-mode field becomes

$$H_T = \sum_k \hbar \upsilon_k a_k^\dagger a_k + \frac{1}{2} \hbar \omega \sigma_z + \hbar \sum_k \mu_k \left(\sigma_+ a_k + \sigma_- a_k^\dagger \right). \tag{2.21}$$

For single mode, above expression becomes

$$H_T = \hbar \upsilon a^\dagger a + \frac{1}{2} \hbar \omega \sigma_z + \hbar \mu \left(\sigma_+ a + \sigma_- a^\dagger \right) = H_0 + H_I, \tag{2.22}$$

with $H_0 = \hbar \upsilon a^\dagger a + \frac{1}{2} \hbar \omega \sigma_z$ and $H_I = \hbar \mu \left(\sigma_+ a + \sigma_- a^\dagger \right)$. For description of atom-field dynamics, interaction picture Hamiltonian is usually found more convenient and is given by, $\nu = e^{\frac{iH_0 t}{\hbar}} H_I e^{\frac{-iH_0 t}{\hbar}}$. Substituting values and simplifying with the help of identity,

$$e^{\alpha A} B e^{-\alpha A} = B + \alpha [A, B] + \frac{\alpha^2}{2!} [A, [A, B]] + ..., \tag{2.23}$$

the interaction picture Hamiltonian comes to be,

$$V = \hbar\mu\left(\sigma_+ a e^{i\Delta t} + \sigma_- a^\dagger e^{-i\Delta t}\right). \tag{2.24}$$

Here Δ is called Atom-field frequency detuning and numerically $\Delta = \omega - \upsilon$. For resonant case, $\Delta = 0$ and the Hamiltonian becomes

$$V_R = \hbar\mu\left(\sigma_+ a + \sigma_- a^\dagger\right). \tag{2.25}$$

Whereas for sufficiently large detuning i.e $\Delta >> \mu$, the Hamiltonian becomes dispersive and induces only the virtual atomic transitions accompanied by the corresponding phase modifications. The expression thus comes to be [241],

$$V_D = \frac{\hbar\mu^2}{\Delta}\left(aa^\dagger \left|e\right\rangle \left\langle e\right| - a^\dagger a \left|g\right\rangle \left\langle g\right|\right) = \frac{\hbar\mu^2}{\Delta}\left(aa^\dagger \sigma_{ee} - a^\dagger a \sigma_{gg}\right). \tag{2.26}$$

2.2.3 Cavity QED based Bragg diffraction of atoms

Mechanical action of light on material particles is a well explored physical phenomenon [242]. Kapitza and Dirac [243] showed that electrons could be diffracted from a standing light wave. In 1970, Ashkin [244] theoretically investigated that interaction with laser beam is capable to deflect the atoms by appreciable angles. Bernhardt and co-workers [245, 246] laid the fundamentals concerning momentum exchange between atoms and the field during interaction. This momentum exchange and subsequent light induced forces on atoms can be attributed to two major factors. a) Spontaneous Emission Force: During spontaneous emission, a recoil is imparted to the atom in random direction and hence the atomic momentum component along standing wave axis is modified in range from $-\hbar k$ to $\hbar k$ [247]. Here k is the wave vector of the emitted photon. b) Dipole Force: The dipole force on the atoms results from absorption and stimulated emission of field quanta [242]. Thus atomic momentum along cavity direction is altered in the quantized multiples of $2\hbar k$. Therefore, for a sufficiently large atomic life-time, the probability for spontaneous emission becomes negligibly small and the dipole force takes predominant part in the process of momentum exchange between atoms and the cavity field.

This scattering or deflection of atoms by the cavity field is divided into two interaction regimes. The short interaction time regime is called Raman-Nath regime [248] and here the recoil energy is much less than the energy associated with the Rabi frequency [249]. However, in large interaction time regime, known as Bragg regime [184, 250, 251, 252, 253, 254, 255], the recoil energy is much larger than the energy associated with Rabi frequency [245, 246, 256]. We are here concerned with the later case only i.e. Bragg regime, and so its mathematical treatment will be explored a bit further. Bragg's diffraction [257] is a interference induced process that coherently transforms a quantum mechanical system from one state to another equal-energy state by a periodic interaction. It take place only when quantum system has a wavelength closer to the periodicity of interaction. Initially, W. H. Bragg and his son W. L. Bragg considered it as the constructive interference of the x-rays reflected from a crystal lattice. Bragg found out that strong reflection occurs when the reflection angle θ satisfy the condition,

$$2d\sin\theta = n\lambda. \tag{2.27}$$

Here d is the separation between lattice planes and λ is the wavelength of x-rays. In atom optics, the role of light and matter are reversed for the Bragg diffraction of atoms. Theory of atomic Bragg diffraction from photonic crystal i.e. electromagnetic standing wave was developed by Bernhardt and co-workers [245, 246] and was verified in various experiments [250, 258, 259, 260, 261]. These experiments, which have successfully demonstrated up to 8^{th} order Bragg diffraction, were all done with classical lasers. Later on, diffraction of atoms from quantized cavity fields was also carried out [262, 263]. Martin et al. [258], in 1988, demonstrated the Bragg diffraction of sodium atomic beam and observed that ratio of diffracted and transmitted parts of the beam depends on laser intensity of the standing light field. It exhibits an oscillatory behaviour known as Pendellosung oscillations. This controllable oscillatory behaviour ensures that every splitting ratio between 0 and 1 can be achieved. Since the splitting is coherent, atomic mirrors, beam splitters and even atomic interferometers can be constructed in Bragg regime [250, 264, 265, 266, 267]. Bragg Scattering is a multiphoton energy conserving Raman process. This implies that atom exits from the interaction region with the same initial kinetic energy that it have at the time of entrance. Large uncertainty in interaction time Δt corresponds to a minute uncertainty in energy i.e. $\Delta E \cong \hbar/\Delta t$, such that atom with initial momentum $l\hbar k$ get effectively constrained into two adjacent equal-energy momentum states $-l\hbar k$ and $l\hbar k$. This forbids population from moving to other momentum states with a different energy. Since the energy difference with the neighboring states is of the order of $\hbar\omega_r$, so the Bragg regime is characterized by the condition $\hbar\omega_r > 1$. Here $\omega_r = \hbar k^2/2M$ is called photon recoil frequency. Thus we see that energy and momentum conservation imply the condition $p_\perp = n\hbar k$ for scattering. This can also be understood in another, more explicit way. In Bragg diffraction, the condition for constructive interference of atomic de Broglie waves requires that the angle of incidence to the standing wave plane must be the one of the n^{th} order scattering angles θ_n that satisfies Bragg's relation i.e. $\lambda \sin\theta_n = n\lambda_{dB}$, where λ is the wavelength of standing wave field and λ_{dB} is the de Broglie wavelength of the atomic wave. The momentum transfer result only in the discrete initial values of atomic momentum along k vector of the field. It is important to note that the interaction only reverses the direction (and hence changes atomic momentum from $l\hbar k$ to $-l\hbar k$) but does not alter the magnitude of the momentum. Therefore energy and momentum both are conserved in atomic Bragg diffraction. If $\mathbf{p}_{in} = l_0\hbar k/2$ denotes initial atomic momentum, then conservation of momentum implies that

$$\mathbf{p}_{out} = \mathbf{p}_{in} + l\hbar k. \tag{2.28}$$

Here l_0 is an even integer and represent the order of Bragg diffraction [184, 253, 254, 255] and $\mathbf{p}_{out}$ denotes the atomic momentum after l interactions with the cavity field. Now energy conservation requires that

$$\frac{|\mathbf{p}_{in}|^2}{2M} = \frac{|\mathbf{p}_{out}|^2}{2M}. \tag{2.29}$$

Here M denotes the mass of the atom. Substituting initial and final momentum into above expression, we get resonance condition for Bragg regime, that is

$$\frac{l\,(l+l_0)}{2M}\hbar^2 k^2 = 0. \tag{2.30}$$

This equation has only two solutions. a) $l = 0$, this corresponds to undeflected atomic beam and b) $l = -l_0$, which corresponds to deflected beam. It is further to be noted that transverse atomic momentum (i.e. along cavity axis) is always taken quantized with $\Delta p < \hbar k$ whereas longitudinal momentum, usually very large, is always treated classically. In summary, we can say that Bragg scattering is an energy conserving multiphoton process through which atomic de Broglie wavepacket is coherently split into two counterpropagating momenta states due to its interaction with standing wave field. Thus an atom with a transverse initial momentum of $P_{in} = l_0\hbar k$, k being the wave number of the field, exits the interaction zone in superposition of initial momentum state $P_{in} = l_0\hbar k$ and the state with momentum $P_{out} = -l_0\hbar k$. As stated earlier, the integer l_0 is called order of the Bragg diffraction and implies an evident exchange of $2l$ photons during the atom-field interaction. Now assume that a two-level atom, initially in ground state $|g\rangle$ with a transverse momentum $|P_0\rangle$ interacts with a cavity field in Fock state $|n_A\rangle$. The state vector of the atom interacting with cavity field at any arbitrary time t may be expressed as [253, 255],

$$|\Psi_{AF}(t)\rangle = e^{-i\left(\frac{P_0^2}{2M\hbar}-\frac{\Delta_A}{2}\right)t} \sum_{l=-\infty}^{\infty} C^{P_l}_{n_A,g}(t)\,|n_A, g, P_l\rangle + C^{P_l}_{(n-1)_A,e}(t)\,|(n-1)_A, e, P_l\rangle . \quad (2.31)$$

Here $C^{P_l}_{n_A,g}(t)[C^{P_l}_{(n-1)_A,e}(t)]$ is the probability amplitude for finding the atom in ground state $|g\rangle$ [excited state $|e\rangle$] with transverse atomic momentum $P_l = P_0 + l\hbar k$ after l interactions. Evidently l is an even number, because during one Rabi cycle the momentum imparted to the atom by its interaction with cavity field is either zero or $2\hbar k$. The summation over l signifies the accumulative nature of transverse atomic momenta during interaction. Corresponding interaction picture Hamiltonian, describing interaction of a two-level atom having quantized center-of-mass motion with cavity field under dipole and rotating wave approximations, can be written as

$$H_I = \frac{P_x^2}{2M} + \frac{\hbar\Delta_A}{2}\sigma_z + \hbar\mu\cos(kx)\left[\sigma_+ a + a^+\sigma_-\right], \quad (2.32)$$

where M is mass of the atom, μ atom-field coupling constant, $a(a^\dagger)$ are the annihilation(creation) field operators and Δ is detuning of atomic frequency with the cavity field. Similarly $\sigma_+(\sigma_-)$ and σ_z are the atomic raising (lowering) and inversion operators, respectively. The P_x and x are the momentum and position operators for the center-of-mass motion of the atom along x-axis, respectively. Since Bragg diffraction is mathematically less treated therefore here we give a rather detailed exposition. This will help to understand the models being presented in next chapter. Now Schrödinger's equation i.e. $i\hbar\frac{\partial}{\partial t}\left|\Psi_{AF}(t)\right\rangle = H_I\left|\Psi_{AF}(t)\right\rangle$ implies

$$\begin{aligned} &i\hbar\left[\sum_{l=-\infty}^{\infty}\left\{\frac{\partial}{\partial t}C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+\frac{\partial}{\partial t}C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right. \\ &\left.-i\left(\frac{P_0^2}{2M\hbar}-\frac{\Delta_A}{2}\right)\sum_{l=-\infty}^{\infty}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right] \\ &=\left[\frac{\hbar\Delta_A}{2}\sigma_z+\frac{P_x^2}{2M}+\hbar\mu\cos(kx)\left[\sigma_+a+a^+\sigma_-\right]\right] \\ &\times\sum_{l=-\infty}^{\infty}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\} \end{aligned} \tag{2.33}$$

Further simplification on the right hand side of the above equation. yields

$$\begin{aligned} &\sum_{l=-\infty}^{\infty}\left[i\hbar\left\{\frac{\partial}{\partial t}C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+\frac{\partial}{\partial t}C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right. \\ &\left.-i\left(\frac{P_0^2}{2M\hbar}-\frac{\Delta_A}{2}\right)\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right] \\ &=\sum_{l=-\infty}^{\infty}\left[\frac{\hbar\Delta}{2}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle-C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right. \\ &+\frac{\hbar^2k^2}{2M}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\} \\ &\left.+\hbar\mu\cos(kx)\left\{C^{P_l}_{n_A,g}(t)\sqrt{n_A}\left|(n-1)_A,e,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\sqrt{n_A}\left|n_A,g,P_l\right\rangle\right\}\right]. \end{aligned} \tag{2.34}$$

Operating $\cos(kx)$ and dividing both sides by $i\hbar$, above expression becomes

$$\begin{aligned} &\sum_{l=-\infty}^{\infty}\left[\left\{\frac{\partial}{\partial t}C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+\frac{\partial}{\partial t}C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right. \\ &\left.-i\left(\frac{P_0^2}{2M\hbar}-\frac{\Delta_A}{2}\right)\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right] \\ &=-i\sum_{l=-\infty}^{\infty}\left[\frac{\Delta}{2}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle-C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right. \\ &+\frac{\hbar k^2}{2M}\left\{C^{P_l}_{n_A,g}(t)\left|n_A,g,P_l\right\rangle+C^{P_l}_{(n-1)_A,e}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}+\frac{\mu}{2}\left\{C^{P_{l+1}}_{n_A,g}(t)\sqrt{n_A}\left|(n-1)_A,e,P_{l+1}\right\rangle\right. \\ &\left.\left.+C^{P_{l-1}}_{n_A,g}(t)\sqrt{n_A}\left|(n-1)_A,e,P_{l-1}\right\rangle+C^{P_{l+1}}_{(n-1)_A,e}(t)\sqrt{n_A}\left|n_A,g,P_{l+1}\right\rangle+C^{P_{l-1}}_{(n-1)_A,e}(t)\sqrt{n_A}\left|n_A,g,P_{l-1}\right\rangle\right\}\right] \end{aligned} \tag{2.35}$$

Rearrangement and simplification gives,

$$\sum_{l=-\infty}^{\infty}\left\{\frac{\partial}{\partial t}C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle+\frac{\partial}{\partial t}C_{(n-1)_A,e}^{P_l}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}$$
$$=-i\sum_{l=-\infty}^{\infty}\left[\Delta_A C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle+\left(\frac{\hbar k^2}{2M}-\frac{\hbar k_0^2}{2M}\right)\left\{C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle+C_{(n-1)_A,e}^{P_l}(t)\right.\right.$$
$$\left.\left|(n-1)_A,e,P_l\right\rangle\right\}+\frac{\mu\sqrt{n_A}}{2}\left\{C_{n_A,g}^{P_{l+1}}(t)\left|(n-1)_A,e,P_{l+1}\right\rangle+C_{n_A,g}^{P_{l-1}}(t)\left|(n-1)_A,e,P_{l-1}\right\rangle\right.$$
$$\left.\left.+C_{(n-1)_A,e}^{P_{l+1}}(t)\left|n_A,g,P_{l+1}\right\rangle+C_{(n-1)_A,e}^{P_{l-1}}(t)\left|n_A,g,P_{l-1}\right\rangle\right\}\right]. \tag{2.36}$$

This implies that

$$\sum_{l=-\infty}^{\infty}\left\{\frac{\partial}{\partial t}C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle+\frac{\partial}{\partial t}C_{(n-1)_A,e}^{P_l}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}$$
$$=-i\sum_{l=-\infty}^{\infty}\left[\left(\frac{\hbar k^2}{2M}-\frac{\hbar k_0^2}{2M}\right)\left\{C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle+C_{(n-1)_A,e}^{P_l}(t)\left|(n-1)_A,e,P_l\right\rangle\right\}\right.$$
$$\left.+\Delta_A C_{n_A,g}^{P_l}(t)\left|n_A,g,P_l\right\rangle\right]-i\frac{\mu\sqrt{n_A}}{2}\sum_{l=-\infty}^{\infty}\left\{C_{n_A,g}^{P_{l+1}}(t)\left|(n-1)_A,e,P_{l+1}\right\rangle+C_{n_A,g}^{P_{l-1}}(t)\right.$$
$$\left.\left|(n-1)_A,e,P_{l-1}\right\rangle+C_{(n-1)_A,e}^{P_{l+1}}(t)\left|n_A,g,P_{l+1}\right\rangle+C_{(n-1)_A,e}^{P_{l-1}}(t)\left|n_A,g,P_{l-1}\right\rangle\right\}. \tag{2.37}$$

Taking projections over $\left|n_A,g,P_l\right\rangle$ and $\left|(n-1)_A,e,P_l\right\rangle$ while keeping in view the accumulative nature of transverse atomic momenta, we get the following expression for the rate of change of probability amplitudes,

$$\frac{\partial}{\partial t}C_{n_A,g}^{P_l}(t)=-i\left[\left(\frac{l\,(l_0+l)\,\hbar k^2}{2M}\right)C_{n_A,g}^{P_l}(t)+\frac{\mu\sqrt{n_A}}{2}\left(C_{(n-1)_A,e}^{P_{l+1}}(t)+C_{(n-1)_A,e}^{P_{l-1}}(t)\right)\right], \tag{2.38}$$

$$\frac{\partial}{\partial t}C_{(n-1)_A,e}^{P_l}(t)=-i\left[\left(\frac{l\,(l_0+l)\,\hbar k^2}{2M}+\Delta_A\right)C_{(n-1)_A,e}^{P_l}(t)+\frac{\mu\sqrt{n_A}}{2}\left(C_{n_A,g}^{P_{l+1}}(t)+C_{n_A,g}^{P_{l-1}}(t)\right)\right]. \tag{2.39}$$

These two expressions represent an infinite set of coupled differential equations valid both for resonant and off-resonant interactions. In resonant case, atom goes through real excitation and deexcitation Rabi cycles. This leaves an appreciable chance for spontaneous emission that can potentially destroy the coherence of the diffraction process. Therefore, in our calculations, we have opted for the off-resonant Bragg diffraction of the atom from the cavity field. Now the interactions will be off-resonant when detuning Δ_A is much larger than recoil frequency $\omega_r = \hbar k^2/2M$ [184, 253, 255]. Thus under the condition $\Delta_A >> \omega_r$, above expressions simplify to,

$$i\frac{\partial}{\partial t}C^{P_l}_{n_A,g}(t) = \left[\left(\frac{l\,(l_0+l)\,\hbar k^2}{2M}\right)C^{P_l}_{n_A,g}(t) + \frac{\mu\sqrt{n_A}}{2}\left(C^{P_{l+1}}_{(n-1)_A,e}(t) + C^{P_{l-1}}_{(n-1)_A,e}(t)\right)\right], \quad (2.40)$$

$$i\frac{\partial}{\partial t}C^{P_l}_{(n-1)_A,e}(t) = \left[\Delta_A C^{P_l}_{(n-1)_A,e}(t) + \frac{\mu\sqrt{n_A}}{2}\left(C^{P_{l+1}}_{n_A,g}(t) + C^{P_{l-1}}_{n_A,g}(t)\right)\right]. \quad (2.41)$$

Above infinite coupled set of equations can describe off-resonant atomic Bragg diffraction of any arbitrary order l_0, with l_0 being any even integer. For first order Bragg diffraction $l_0 = 2$, and therefore above infinite set reduces to only five significant expressions for which l varies from 1 to -3. Since atom is initially taken in ground state therefore expression (2.40) describe the even and (2.41) relate to the odd number of interactions [245, 246]. Thus we get,

$$\text{For } l=1, \quad i\frac{\partial}{\partial t}C^{P_1}_{(n-1)_A,e}(t) = \left[\Delta_A C^{P_1}_{(n-1)_A,e}(t) + \frac{\mu\sqrt{n_A}}{2}\left(C^{P_2}_{n_A,g}(t) + C^{P_0}_{n_A,g}(t)\right)\right], \quad (2.42)$$

$$\text{For } l=0, \quad i\frac{\partial}{\partial t}C^{P_0}_{n_A,g}(t) = \left[\frac{\mu\sqrt{n_A}}{2}\left(C^{P_1}_{(n-1)_A,e}(t) + C^{P_{-1}}_{(n-1)_A,e}(t)\right)\right], \quad (2.43)$$

$$\text{For } l=-1, \quad i\frac{\partial}{\partial t}C^{P_{-1}}_{(n-1)_A,e}(t) = \left[\Delta_A C^{P_{-1}}_{(n-1)_A,e}(t) + \frac{\mu\sqrt{n_A}}{2}\left(C^{P_0}_{n_A,g}(t) + C^{P_{-2}}_{n_A,g}(t)\right)\right], \quad (2.44)$$

$$\text{For } l=-2, \quad i\frac{\partial}{\partial t}C^{P_{-2}}_{n_A,g}(t) = \left[\frac{\mu\sqrt{n_A}}{2}\left(C^{P_{-1}}_{(n-1)_A,e}(t) + C^{P_{-3}}_{(n-1)_A,e}(t)\right)\right], \quad (2.45)$$

$$\text{For } l=-3, \quad i\frac{\partial}{\partial t}C^{P_{-3}}_{(n-1)_A,e}(t) = \left[\Delta_A C^{P_{-3}}_{(n-1)_A,e}(t) + \frac{\mu\sqrt{n_A}}{2}\left(C^{P_{-2}}_{n_A,g}(t) + C^{P_{-4}}_{n_A,g}(t)\right)\right]. \quad (2.46)$$

Now adiabatic approximation for off-resonant Bragg diffraction holds only when the inequality $\Delta_A > \omega_r > \mu^2 n_A/2$ is satisfied and implies that $\frac{\partial}{\partial t}C^{P_1}_{(n-1)_A,e}(t) = \frac{\partial}{\partial t}C^{P_{-1}}_{(n-1)_A,e}(t) = \frac{\partial}{\partial t}C^{P_{-3}}_{(n-1)_A,e}(t) = 0$. Further, under the same approximation, we can also ignore the probability amplitudes $C^{P_2}_{n_A,g}(t)$ and $C^{P_{-4}}_{n_A,g}(t)$ [253, 256, 260]. Using these values and simplifying algebracially, we get the following two coupled equations,

$$i\frac{\partial}{\partial t}C^{P_0}_{n_A,g}(t) = -\frac{\mu^2 n_A}{2\Delta_A}C^{P_0}_{n_A,g}(t) - \frac{\mu^2 n_A}{4\Delta_A}C^{P_{-2}}_{n_A,g}(t), \quad (2.47)$$

$$i\frac{\partial}{\partial t}C^{P_{-2}}_{n_A,g}(t) = -\frac{\mu^2 n_A}{2\Delta_A}C^{P_{-2}}_{n_A,g}(t) - \frac{\mu^2 n_A}{4\Delta_A}C^{P_0}_{n_A,g}(t). \quad (2.48)$$

We can easily solve these coupled expressions using Laplace transforms, which yields,

$$C^{P_0}_{n_A,g}(t) = e^{2i\beta_{NR}n_At}\left\{C^{P_0}_{n_A,g}(t=0)\cos(\beta_{NR}n_At) + iC^{P_{-2}}_{n_A,g}(t=0)\sin(\beta_{NR}n_At)\right\}, \quad (2.49)$$

$$C^{P_{-2}}_{n_A,g}(t) = e^{2i\beta_{NR}n_At}\left\{C^{P_{-2}}_{n_A,g}(t=0)\cos(\beta_{NR}n_At) + iC^{P_0}_{n_A,g}(t=0)\sin(\beta_{NR}n_At)\right\}. \quad (2.50)$$

Where we have taken $\beta_{NR} = \mu^2/4\Delta_A$. Since initially, at $t = 0$, the atom is taken in ground state $|g\rangle$ with momentum $|P_0\rangle$, therefore $C^{P_0}_{n_A,g}(t=0) = 1$ and $C^{P_{-2}}_{n_A,g}(t=0) = 0$. Thus above expressions reduce to,

$$C^{P_0}_{n_A,g}(t) = e^{2i\beta_{NR}n_At}\cos(\beta_{NR}n_At), \quad (2.51)$$

$$C^{P_{-2}}_{n_A,g}(t) = ie^{2i\beta_{NR}n_At}\sin(\beta_{NR}n_At). \quad (2.52)$$

These very simple expressions explicitly show that off-resonant Bragg diffraction is capable of dealing with all aspects of atom optics. For example, interaction for $t = \pi/4\beta_{NR}n_A$ yields a symmetric atomic de Broglie wave beam splitter. Similarly, interaction for $t = \pi/2\beta_{NR}n_A$ corresponds to an atomic mirror under first order off-resonant Bragg diffraction.

Chapter 3

Cavity field entangled state engineering through atomic interferometry in Bragg regime

We demonstrate two techniques [186, 215] for the generation of cavity field entangled states including Bell, NOON and W state in this chapter. Both the schemes are based on atom interferometry in Bragg regime cavity QED. We have utilized off-resonant Bragg diffraction of atomic de Broglie wave packets under single loop as well as cascaded Mach-Zehnder-Bragg (MZB) interferometric geometries. Experimental feasibility of the proposals has also been pointed out briefly in the closing section.

3.1 Engineering maximally entangled N-photon NOON field states

Almost all the entanglement generation techniques mentioned in previous chapter deal either with the generation of Bell, GHZ, Werner or cluster states and usually become cumbersome and less efficient for the production of the field-entangled states of the form,

$$|\Psi_{DE}\rangle = \frac{1}{\sqrt{2}}\left(|n_D, 0_E\rangle \pm |0_D, n_E\rangle\right). \tag{3.1}$$

Here subscripts D and E stand for the two spatially separated high-Q cavities containing either zero or n photons. Such states are commonly termed as NOON states [206] and their vital importance have already been elaborated at length previously. However, through present scheme [215], we are firstly proposing a feasible method to engineer such high-NOON states inside high-Q cavities with sufficiently high efficiency approaching unity under ideal experimental conditions. The proposed scheme utilizes atomic analog of Mach-Zehnder optical interferometer. The interferometer is based on successive diffraction of atoms in Bragg regime cavity QED and result into the generation of required NOON field state in two separate high-Q cavities, say D and E, respectively. In past, cavity QED systems operating in Bragg regime have been comparatively less explored in conjunction with quantum informational aspects[184, 253, 254, 255, 268, 269]. However, recent advances both in atom interferometry as well as in Cavity QED render it more feasible for generation of entangled field states of the type mentioned above [140, 270]. In our proposed model (Fig. 3-1.), a stream of identical two-level atoms, initially in their ground state $|g_i\rangle$ with transverse momenta $\left|P_0^{(i)}\right\rangle$, passes through a high-Q cavity A. This cavity contains symmetric field superposition of zero and one photon. The atomic beam density is kept low enough to ensure that only one atom passes through the cavity at a time. These atoms, one at a time, interact with the cavity in first order off-resonant Bragg diffraction for a specific time and emerge out of the cavity with their external momenta states i.e., $\left|P_0^{(i)}\right\rangle$ and $\left|P_{-2}^{(i)}\right\rangle$ entangled with the cavity field. Cavity A thus acts as first atomic beam splitter of the Mach-Zehnder-Bragg (MZB) interferometer. These split de-Broglie wavepacket components after travelling a sufficiently large distance are reflected back over each other through two similar cavities B and C, each containing Fock state $|m_{B(C)}\rangle$. Cavities B and C thus act as two symmetric atomic mirrors of MZB interferometer. After reflection, both arms pass through a $\pi-$pulse Ramsey field (in vertical direction, perpendicular to the plane of the paper as shown in Fig. 3-1.). This interaction with Ramsey field flips the internal state of split atomic wavepacket from ground to excited level in a coherent manner. Excited split wavepackets then traverse through two independent, high-Q cavities D and E, initially prepared in vacuum, fitted adjacent and aligned parallel to the external Ramsey fields. The excited wavepackets after interaction with the cavity fields D or E, contribute the photon in either of the cavities by precisely controlled interaction through applied Stark fields across cavities D and E. Such an interaction is, however described

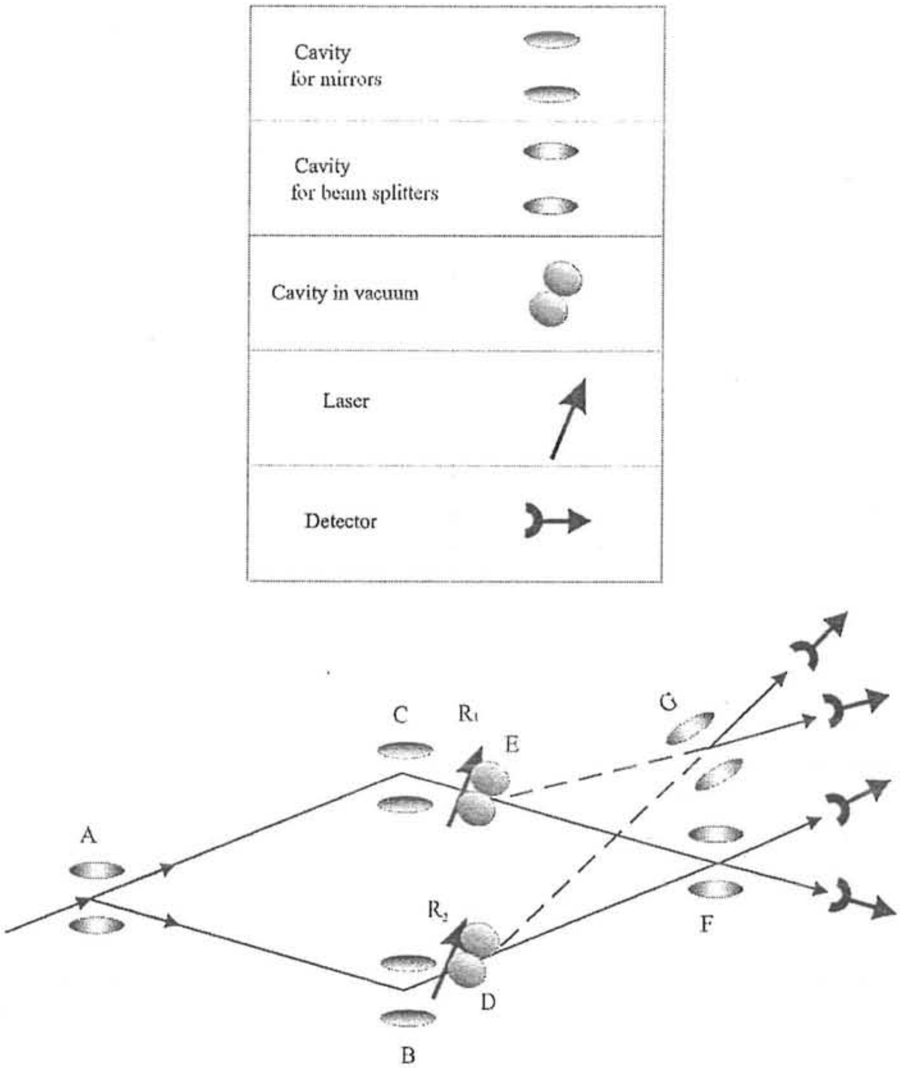

Figure 3-1: Schematics of the proposed interferometric setup for generation of higher entangled states. Cavity A, containing superposition of zero and one photon, acts as the atomic beam splitter whereas the cavities B and C having identical Fock field serve the purpose of atomic mirrors. Reflected two-level atomic de-Broglie waves are then raised to excited level through Ramsey zones R_1 and R_2. The excited atoms then deposit their excitations either into Cavity D or E and finally the which-path information carried by the travelling atomic de-Broglie wavepackets is erased through the atom-field interactions in cavities F and G that act as second beam splitter of the interferometer. Atomic mirrors B and C and exit beam splitters F and G which are shown here as high-Q cavities may equally be replaced with counter propagating laser fields forming standing wave pattern.

by fully quantized atom-field model. The split atomic wavepacket, now in ground internal state, travel toward each other and finally overlap in either cavity F or G. Both these cavities, with G stacked above F, operate in first order off-resonant Bragg regime. The atom-field interaction times for these cavities are selected such that wavepacket components entering into them emerge out in symmetric momenta superpositions while erasing completely the which-path information carried by the components prior to their entrance into the cavities. Same procedure continues for successive atoms as they pass through all the cavities in a similar manner. At the end, when sufficient number of atoms have undergone through similar interactions, resulting into generation of higher order entangled states of the form $\left[|0_A, 0_D, n_E\rangle \pm |1_A, n_D, 0_E\rangle\right]/\sqrt{2}$, the field in cavity A is disentangled from the system by transferring its quantum information to a resonant two-level atom that passes through it. This atom then interacts with a Ramsey field and finally detected in ground or excited state. The process consequently yields desired NOON field state in spatially separated high-Q cavities D and E. Production of maximally entangled n photon entangled state relies on successive passage of an equal number of atoms through the setup and is of course limited mainly by the coherence and life-time of the cavity A, i.e. first atomic beam splitter of MZB apparatus.

3.1.1 Schematics of the state engineering

In high-Q cavity A, atoms go through first order off-resonant Bragg diffraction. Bragg scattering, as stated earlier, is an energy conserving multiphoton process through which atomic de Broglie wavepacket is coherently split into two counterpropagating momenta states due to its interaction with standing wave field. Thus an atom with a transverse initial momentum of $P_{in} = N\hbar k$, k being the wave number of the field, exits the interaction zone in superposition of initial momentum state $P_{in} = N\hbar k$ and the state with momentum $P_{out} = -N\hbar k$. The integer N is called order of the Bragg diffraction and implies an evident exchange of $2N$ photons during the atom-field interaction. On the other hand longitudinal momentum component being large enough, is always treated classically. The state vector for atom-1 interacting with cavity mode

A at any arbitrary time t_1 may be expressed as [255]

$$\left|\Psi_{a-P}^{A}(t_1)\right\rangle_1 = e^{-i\left(\frac{P_{0_{(1)}}^2}{2M\hbar}-\frac{\Delta_A}{2}\right)t_1} \sum_{l=-\infty}^{\infty} \left\{ C_{0_A,g_1}^{P_l^{(1)}}(t_1)\left|0_A, g_1, P_l^{(1)}\right\rangle \right.$$

$$\left. + C_{1_A,g_1}^{P_l^{(1)}}(t_1)\left|1_A, g_1, P_l^{(1)}\right\rangle + C_{0_A,e_1}^{P_l^{(1)}}(t_1)\left|0_A, e_1, P_l^{(1)}\right\rangle \right\}. \tag{3.2}$$

Here l stands for the number of interactions of atom-1 with the cavity field, an even number, because during one Rabi cycle the momentum imparted to the atom by its interaction with cavity field is either zero or $2\hbar k$. $C_{0_A,g_1}^{P_l^{(1)}}(t_1)$ and $C_{1_A,g_1}^{P_l^{(1)}}(t_1)$ denote the probability amplitudes for finding the atom-1 in ground state with transverse momenta$\left|P_l^{(1)}\right\rangle$ when corresponding cavity field carries either zero or one photon respectively. Similarly $C_{0_A,e_1}^{P_l^{(1)}}(t_1)$ represents the probability amplitude for the atom to be in excited state with momenta $\left|P_l^{(1)}\right\rangle$ when cavity field is in vacuum state. As mentioned earlier, summation over l represents the accumulative nature of transverse atomic momenta incurred during the interaction. Hence atom-field interactions lasting for $N/2$ Rabi cycles may exhibit a coherent spectrum of transverse atomic momenta $\left|P_l^{(1)}\right\rangle$ with a general pattern $(\left|-N\hbar k\right\rangle, ..., \left|-4\hbar k\right\rangle, \left|-2\hbar k\right\rangle, \left|0\hbar k\right\rangle, \left|2\hbar k\right\rangle, \left|4\hbar k\right\rangle, ..., \left|N\hbar k\right\rangle)$. However, as will be shown next, imposition of Bragg diffraction criteria for a specific order limits it to only two oppositely directed transverse momenta states for the outgoing atom. Global phase factor has been introduced for the sake of simplicity and $P_{0_{(1)}}$ is the initial momentum of the atom-1 [255]. Corresponding off-resonant interaction picture Hamiltonian, describing interaction of a two-level atoms with cavity field under dipole and rotating wave approximations, can be written as

$$H_{I_i} = \frac{P_{x_i}^2}{2M} + \frac{\hbar\Delta_{(j)}}{2}\sigma_z^{(i)} + \hbar\mu_{(j)}\cos(kx_i)\left[\sigma_+^{(i)}a + a^+\sigma_-^{(i)}\right]. \tag{3.3}$$

Here M is mass of atom, $\mu_{(j)}$ atom-field coupling constants for various cavities (j=A,B,C,D,E,F,G), $a(a^\dagger)$ are the annihilation(creation) field operators and $\Delta_{(j)}$ is detuning of atomic frequency with the cavity field. Similarly $\sigma_+^{(i)}(\sigma_-^{(i)})$ and $\sigma_z^{(i)}$ are the atomic raising (lowering) and inversion operators, respectively, for the i^{th} atom. The P_{x_i} and x_i are the momentum and position operators for the center-of-mass motion of the i^{th} atom along cavity axis (i.e. x-axis), respectively. Solution of the Schrödinger's equation yields the following expressions for rate of change of probability amplitudes for atom-1

$$i\frac{\partial}{\partial t}C_{0_A,g_1}^{P_l^{(1)}}(t_1)=\left[\frac{l\,(l_0+l)\,\hbar k^2}{2M}\right]C_{0_A,g_1}^{P_l^{(1)}}(t_1), \tag{3.4}$$

$$i\frac{\partial}{\partial t}C_{0_A,e_1}^{P_l^{(1)}}(t_1)=\left[\frac{l\,(l_0+l)\,\hbar k^2}{2M}+\Delta_A\right]C_{0_A,e_1}^{P_l^{(1)}}(t_1)+\frac{\mu_A}{2}\left[C_{1_A,g_1}^{P_{l+1}^{(1)}}(t_1)+C_{1_A,g_1}^{P_{l-1}^{(1)}}(t_1)\right], \tag{3.5}$$

$$i\frac{\partial}{\partial t}C_{1_A,g_1}^{P_l^{(1)}}(t_1)=\left[\frac{l\,(l_0+l)\,\hbar k^2}{2M}\right]C_{1_A,g_1}^{P_l^{(1)}}(t_1)+\frac{\mu_A}{2}\left[C_{0_A,e_1}^{P_{l+1}^{(1)}}(t_1)+C_{0_A,e_1}^{P_{l-1}^{(1)}}(t_1)\right]. \tag{3.6}$$

Expression (3.4) is a simple first order differential equation whereas expressions (3.5) and (3.6) represent a set of coupled first order differential equations. If atom-1 is initially in ground state then expression (3.5) describe the odd and (3.6) relate to the even number of interactions [245, 246]. In the above equations, l_0 and l denote the order of Bragg diffraction and the number of Rabi's interactions completed therein by atom-1, respectively. For the first order Bragg diffraction $l_0 = 2$, then out of the above infinite set only significantly relevant equations remains those for which l varies from 1 to -3 [253]. Furthermore, for off-resonant Bragg diffraction, we assume that detuning Δ_A is very large compared to photon recoil frequency $\omega_{r_1} = \hbar k^2/2M$ such that ω_{r_1}can be safely ignored in comparison with detuning. Assumption of large detuning further enable adiabatic removal of fast oscillating terms and prevents decoherence which might occur due to spontaneous emission [184, 253, 255]. We also assume that at $t_1 = 0$, the atom initially prepared in its ground state $|g_1\rangle$, enters with initial momentum $\left|P_0^{(1)}\right\rangle = |2\hbar k\rangle$ into the cavity A. This cavity is initially prepared in superposition of zero and one photon. Therefore, initial conditions will be, $C_{0_A,g_1}^{P_0^{(1)}}(0) = C_{1_A,g_1}^{P_0^{(1)}}(0) = 1/\sqrt{2}$ and $C_{0_A,e_1}^{P_0^{(1)}}(0) = C_{0_A,g_1}^{P_{-2}^{(1)}}(0) = C_{1_A,g_1}^{P_{-2}^{(1)}}(0) = 0$. Thus, under these conditions, equations (3.4-3.6) yield the following wavefunction for an interaction time $t_1 = 2\pi\Delta_{(A)}/\mu_{(A)}^2$

$$\left|\Psi_{a-P}^{A}\right\rangle_1=\frac{1}{\sqrt{2}}\left[\left|0_A,g_1,P_0^{(1)}\right\rangle-i\left|1_A,g_1,P_{-2}^{(1)}\right\rangle\right]. \tag{3.7}$$

Here we see that after completion of the interaction, counterpropagating atomic momenta components get entangled with the field in cavity A. These split atomic momenta components then pass through high-Q cavities B and C. Each of these cavities is assumed to possess Fock state $\left|m_{B(C)}\right\rangle$. Momentum component $P_{-2}^{(1)}$ interacts with cavity B whereas the component $P_0^{(1)}$ interacts with cavity C respectively. Interaction times of atoms with cavities B and

C are controlled in such a way that these cavities act as atomic mirrors and enact NOT-gate transformation to momenta states passing through them. Hence after first order Bragg diffraction for an interaction time equal to $t_{B(C)} = 2\pi\Delta'/\mu'^2$, the state moulds to

$$|\Psi_{a-P}^{ABC}\rangle_1 = \frac{1}{\sqrt{2}} e^{i\frac{3\pi}{2}} \left[\left|0_A, m, g_1, P_{-2}^{(1)}\right\rangle - i\left|1_A, m, g_1, P_0^{(1)}\right\rangle\right]. \tag{3.8}$$

Where we have taken $\mu_C = \mu_B = \mu'$, $\Delta_C = \Delta_B = \Delta'$ and $|m_C\rangle = |m_B\rangle = |m\rangle$. This is because we may use a single cavity instead of two independent but identical cavities. Furthermore, atomic mirrors and exit beam splitter which are taken here as high-Q cavities containing Fock fields for mathematical consistency may equally be replaced with two counterpropagating laser fields, effectively forming a standing wave pattern [265, 271, 272]. However, in that case, dipole transition operator $\mathbf{d}$ incorporated into Hamiltonian (3.3) will have to be replaced with $-\wp \mathrm{e}(\sigma_+^{(i)} + \sigma_-^{(i)})$. Here $\wp$ denotes reduced matrix element of the dipole transition. Tracing over the field state $|m\rangle$, we may write the above state in simple form

$$|\Psi_{a-P}^{A}\rangle_1 = \frac{1}{\sqrt{2}} e^{i\frac{3\pi}{2}} \left[\left|0_A, g_1, P_{-2}^{(1)}\right\rangle - i\left|1_A, g_1, P_0^{(1)}\right\rangle\right]. \tag{3.9}$$

The split de-Broglie wavepacket then passes through a π-pulse of two co-propagating Ramsey fields intersecting the atomic paths in perpendicular direction. This interaction is based on semi-classical treatment of the atom-field system expressed through the Hamiltonian $H^{(sc)} = \frac{\hbar\Omega_R}{2}\left(\sigma_+^{(1)} + \sigma_-^{(1)}\right)$ and leads to atomic internal state inversion i.e. from ground to excited state, for a time equal to π/Ω_R. Here $\Omega_R = |\wp_{ge}|\,\varepsilon/\hbar$ is Rabi frequency with ε representing classical field amplitude and $\wp_{ge}$ stands for the transition dipole matrix element [235]. So after interaction with Ramsey fields the state becomes

$$|\Psi_{a-P-f}^{A}\rangle_1 = \frac{1}{\sqrt{2}} e^{i\frac{3\pi}{2}} \left[\left|0_A, e_1, P_{-2}^{(1)} + \hbar k_\perp\right\rangle - i\left|1_A, e_1, P_0^{(1)} + \hbar k_\perp\right\rangle\right], \tag{3.10}$$

where $\hbar k_\perp$ is the vertical momentum kick imparted by the field to the atom due to absorption of one photon. After this interaction, split atomic wavepacket traverses through cavities D and E, both initially in vacuum state i.e. $|0_D, 0_E\rangle$, placed adjacent and aligned in parallel with the respective Ramsey zones. At this stage, internal dynamics of atom under near-resonant interaction with such cavities may be described by fully quantized interaction picture Hamiltonian written under dipole and rotating wave approximations as

$$H_I = \hbar\mu_{D(E)}\left[\sigma_+^{(j)} a e^{i\Delta_{D(E)}^{(j)}t} + a\dagger\sigma_-^{(j)} e^{-i\Delta_{D(E)}^{(j)}t}\right], \tag{3.11}$$

with j denoting the number of specific atom interacting with cavities $D\,(E)$ [235]. All the atoms in the beam are assumed to be resonant with the cavity mode. However, a large detuning $\Delta_{D(E)}^{(j)}$ can be induced after completion of resonant atom-field interactions culminating into deposition of atomic excitation in either of the cavities D or E using Stark field whenever necessary. Such a large detuning, when switched on, transform near-resonant Hamiltonian H_I into a dispersive one with negligible chance for real photon exchange between atom and the field. However, for the first atom, Stark field is kept off so $\Delta_{D(E)}^{(1)} = 0$, implying throughout resonant interactions. Then unitary time evolution for an interaction time $t_{D(E)} = \pi/2\mu_{D(E)}$, leaves the atom into ground state while imparting one photon to either cavity D or E. The state vector thus becomes

$$\begin{aligned}|\Psi_{a-P}^{ADE}\rangle_1 = \frac{e^{i\pi}}{2}\Big[&|0_A,0_D,1_E\rangle\left|g_1,P_{-2}^{(1)}\right\rangle - i\,|1_A,1_D,0_E\rangle\left|g_1,P_0^{(1)}\right\rangle \\ &+|0_A,0_D,1_E\rangle\left|g_1,P_{-2}^{(1)}+2\hbar k_\perp\right\rangle - i\,|1_A,1_D,0_E\rangle\left|g_1,P_0^{(1)}+2\hbar k_\perp\right\rangle\Big].\end{aligned} \tag{3.12}$$

This is because $\left|e_1,P_0^{(1)}+\hbar k_\perp\right\rangle$ either changes to $\left|g_1,P_0^{(1)}\right\rangle$ or $\left|g_1,P_0^{(1)}+2\hbar k_\perp\right\rangle$ owing to the atom-field interaction process (in cavities D or E) that can yield photon coherently in either upward or downward direction. Finally the split atomic de Broglie wave passes through the cavities F and G containing identical Fock state, say $|n_{F(G)}\rangle$. These cavities are placed over one another in two different planes and aligned to ensure the first order off-resonant Bragg diffraction for the wavepacket components $\left(P_0^{(1)},P_{-2}^{(1)}\right)$ and $\left(P_0^{(1)}+2\hbar k_\perp, P_{-2}^{(1)}+2\hbar k_\perp\right)$, respectively. Again if the interaction time is chosen such that $t_{F(G)} = \pi\Delta_{F(G)}/\mu_{F(G)}^2 n_{F(G)}$ then cavity field acts as Hadamard operator for discrete momenta states leading to the transformations; $|P_0\rangle \to i\left(|P_{0'}\rangle + i\,|P_{-2'}\rangle\right)/\sqrt{2}$, $|P_0+2\hbar k_\perp\rangle \to i\left(|P_{0'}+2\hbar k_\perp\rangle + i\,|P_{-2'}+2\hbar k_\perp\rangle\right)/\sqrt{2}$, $|P_{-2}\rangle \to i\left(i\,|P_{0'}\rangle + |P_{-2'}\rangle\right)/\sqrt{2}$ and $|P_{-2}+2\hbar k_\perp\rangle \to i\left(i\,|P_{0'}+2\hbar k_\perp\rangle + |P_{-2'}+2\hbar k_\perp\rangle\right)/\sqrt{2}$. Dashed variables used here indicate atomic momenta states emerging out of the cavities F and G. Thus if the atom is detected either with momentum $P_{0'}$ or $P_{0'}+2\hbar k_\perp$ (i.e., out of the cavity

F and G respectively), then we get the state

$$\left|\Psi_{a-P_0}^{ADE}\right\rangle_1 = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, 1_E\rangle - |1_A, 1_D, 0_E\rangle\right]. \tag{3.13}$$

However, if the atom is detected with momentum $P_{-2'}$ or $P_{-2'} + 2\hbar k_{\perp}$, then the corresponding state will be

$$\left|\Psi_{a-P_{-2}}^{ADE}\right\rangle_1 = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, 1_E\rangle + |1_A, 1_D, 0_E\rangle\right]. \tag{3.14}$$

Here we have ignored the unphysical global phases. It should be noted that since all operations are carried out symmetrically in both arms of the interferometer therefore resulting phases becomes global and thus can be ignored on physical grounds.

Now the initial state vector for the second atom entering cavity A in its ground state $|g_2\rangle$ at $t_2 = 0$ with initial momentum $\left|P_0^{(2)}\right\rangle$, may be expressed as

$$\left|\Psi_{a-P}^{ADE}(t_2 = 0)\right\rangle_2 = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, 1_E\rangle \pm |1_A, 1_D, 0_E\rangle\right] \otimes \left|g_2, P_0^{(2)}\right\rangle. \tag{3.15}$$

This state vector for atom-2 at any arbitrary time $t_2 > t_1$, after the completion of the interaction of the first one, may be written as

$$\begin{aligned}\left|\Psi_{a-P}^{ADE}(t_2)\right\rangle_2 = e^{-i\left(\frac{P_{0(2)}^2}{2M\hbar} - \frac{\Delta_A}{2}\right)t_2} \sum_{\xi=-\infty}^{\infty} \Big\{ & C_{0_A,0_D,1_E,g_2}^{P_\xi^{(2)}}(t_2)\left|0_A, 0_D, 1_E, g_2, P_\xi^{(2)}\right\rangle \\ & + C_{1_A,1_D,0_E,g_2,}^{P_\xi^{(2)}}(t_2)\left|1_A, 1_D, 0_E, g_2, P_\xi^{(2)}\right\rangle + C_{0_A,1_D,0_E,e_2,}^{P_\xi^{(2)}}(t_2)\left|0_A, 1_D, 0_E, e_2, P_\xi^{(2)}\right\rangle \Big\}.\end{aligned} \tag{3.16}$$

Again for atom-2, we consider first order off-resonant Bragg interaction with cavity A. Therefore, $\xi_0 = 2$ and significant transverse momenta contributions only come from values of ξ in-between $\xi = 1$ to $\xi = -3$. The system's evolution thus essentially goes through the same procedure except that now interaction of the atom-2 with the cavities D and E is modified by the presence of one photon a priori which reduces the effective interaction times. This is because the Rabi frequency in a single photon field is $\sqrt{2}$ larger than corresponding vacuum field [273]. Since fixed geometrical design of the interferometer allows the usage of only monochromatic atomic beams of specific velocity therefore interaction parameters are controlled by switching on the external stark field across cavities D and E just after deposition of one photon by the excited atom undergoing a $\pi-$pulse into the respective cavities. The resulting large Stark shift effectively decouples the subsystems and prohibits further exchange of field quanta between cavity-atom system. Resonant interaction spans for successive atoms, prior to Stark field trigger causing large dispersive detuning, will be simply $t_{D(E)} = \pi/2\sqrt{(n+1)}\mu_{D(E)}$ with $n = 0, 1, 2, 3, ...$ for consecutive interactions of a stream of atoms with cavities $D(E)$. Effectiveness of this mechanism has both been theoretically proposed and experimentally implemented [149, 274]. Hence the state vector of the system after emergence of atom-2 from cavity $D(E)$ comes out to be

$$
\begin{aligned}
|\Psi_{a-P}^{ADE}\rangle_2 = \frac{1}{2}\Big\{ & |0_A, 0_D, 2_E\rangle \left|g_2, P_{-2}^{(2)}\right\rangle \pm e^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle \left|g_2, P_0^{(2)}\right\rangle \\
& + |0_A, 0_D, 2_E\rangle \left|g_2, P_{-2}^{(2)} + 2\hbar k_\perp\right\rangle \pm e^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle] \left|g_2, P_0^{(2)} + 2\hbar k_\perp\right\rangle \Big\}. \qquad (3.17)
\end{aligned}
$$

This state then passes through cavities $F(G)$ yielding Hadamard momentum transformations for an interaction time equal to $\pi\Delta_{F(G)}/\mu_{F(G)}^2 n_{F(G)}$ in first order off-resonant Bragg interaction regime. The state vector thus finally transforms to

$$
\begin{aligned}
|\Psi_{a-P}^{ADEFG}\rangle_2 = \frac{i}{2\sqrt{2}}\Big[& \left(i|0_A, 0_D, 2_E\rangle \pm e^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle\right) \left|g_2, m_F, P_{0'}^{(2)}\right\rangle \\
& + \left(|0_A, 0_D, 2_E\rangle \pm ie^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle\right) \left|g_2, m_F, P_{-2'}^{(2)}\right\rangle \\
& + \left(i|0_A, 0_D, 2_E\rangle \pm e^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle\right) \left|g_2, m_G, P_{0'}^{(2)} + 2\hbar k_\perp\right\rangle \\
& + \left(|0_A, 0_D, 2_E\rangle \pm ie^{i\frac{3\pi}{2}} |1_A, 2_D, 0_E\rangle\right) \left|g_2, m_G, P_{-2'}^{(2)} + 2\hbar k_\perp\right\rangle \Big]. \qquad (3.18)
\end{aligned}
$$

We now see that if, for example, in previous step atom-1 was detected either in $\left|P_{0'}^{(1)}\right\rangle$ or $\left|P_{0'}^{(1)} + 2\hbar k_\perp\right\rangle$ then second atom detected in either $\left|P_{-2'}^{(2)}\right\rangle$ or $\left|P_{-2'}^{(2)} + 2\hbar k_\perp\right\rangle$ yields the state

$$
|\Psi_{a-P}^{ADE}\rangle_2 = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, 2_E\rangle - |1_A, 2_D, 0_E\rangle\right]. \qquad (3.19)
$$

Otherwise it changes to

$$\left|\Psi_{a-P}^{ADE}\right\rangle_2 = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, 2_E\rangle + |1_A, 2_D, 0_E\rangle\right], \tag{3.20}$$

if atom comes out with momentum states $\left|P_{0'}^{(2)}\right\rangle$ or $\left|P_{0'}^{(2)} + 2\hbar k_\perp\right\rangle$ after interaction with cavities fields F and G respectively. Same is true for the other alternatives. Now if we keep up the procedure of sending n identical atoms successively, all initially in their ground states with external transverse momenta $\left|P_0^{(i)}\right\rangle$, then we end up in generation of the either of the states

$$\left|\Psi_{a-P}^{ADE}\right\rangle_n = \frac{1}{\sqrt{2}}\left[|0_A, 0_D, n_E\rangle \pm |1_A, n_D, 0_E\rangle\right], \tag{3.21}$$

with equal probability of 1/2 for each case. At the end, in order to disentangle the field state in cavity A from the field in cavities D and E, we perform disentangling eraser [241] following the procedure described in [184]. We send a probe atom resonant with the cavity A for an interaction corresponding to half a Rabi cycle and hence transform the above state into,

$$\left|\Psi_a^{ADE}\right\rangle_n = \frac{1}{\sqrt{2}}\left[|g_{probe}, 0_D, n_E\rangle \pm |e_{probe}, n_D, 0_E\rangle\right], \tag{3.22}$$

while leaving cavity A into vacuum state $|0_A\rangle$ which is traced out of the above expression. Next the probe atom passes through a $\pi/2$ Ramsey pulse that performs the transformations; $|g_{probe}\rangle \rightarrow \left(|g_{probe}\rangle + |e_{probe}\rangle\right)/\sqrt{2}$, $|e_{probe}\rangle \rightarrow \left(|g_{probe}\rangle - |e_{probe}\rangle\right)/\sqrt{2}$ and consequently converts above state to

$$\left|\Psi_a^{DE}\right\rangle_n = \frac{1}{2}\left[\left(|0_D, n_E\rangle \pm |n_D, 0_E\rangle\right) \otimes |g_{probe}\rangle \pm \left(|0_D, n_E\rangle \mp |n_D, 0_E\rangle\right) \otimes |e_{probe}\rangle\right]. \tag{3.23}$$

Thus if the probe atom is found in its ground state then we get the state,

$$\left|\Psi^{DE}\right\rangle_n = \frac{1}{\sqrt{2}}\left[|0_D, n_E\rangle \pm |n_D, 0_E\rangle\right]. \tag{3.24}$$

and alternatively if found in excited state then the entangled state generated will be,

$$\left|\Psi^{DE}\right\rangle_n = \frac{1}{\sqrt{2}}\left[|0_D, n_E\rangle \mp |n_D, 0_E\rangle\right]. \tag{3.25}$$

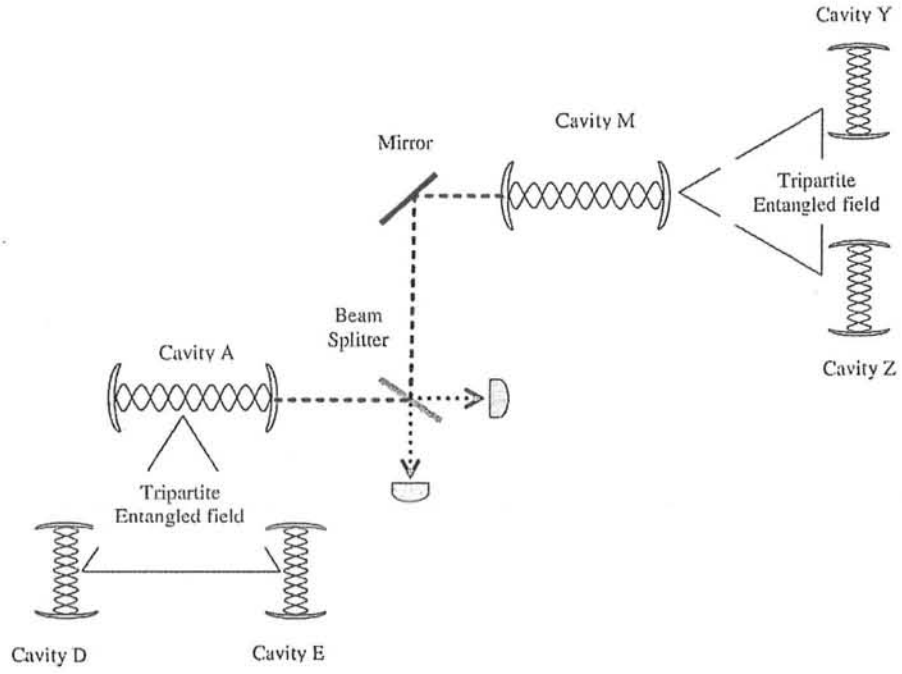

Figure 3-2: Cavities A and M, initially containing similar superpositions of zero and one identical photons get entangled with cavities D, E and Y, Z, respectively, in higher photon states through a procedure described in the text. The field in cavities A and M is then projected over one another through an ideal 50-50 beam splitter. The procedure yields entangled field state (3.28) among cavities D, E and Y, Z.

Another interesting possibility appears if we have two states of the form,

$$\left|\Psi^{ADE}\right\rangle_n = \frac{1}{\sqrt{2}}\left[\left|0_A, 0_D, n_E\right\rangle - \left|1_A, n_D, 0_E\right\rangle\right], \tag{3.26}$$

$$\left|\Psi^{MYZ}\right\rangle_n = \frac{1}{\sqrt{2}}\left[\left|0_M, 0_Y, n_Z\right\rangle - \left|1_M, n_Y, 0_Z\right\rangle\right], \tag{3.27}$$

engineered using the same procedure as described above then a projective measurement over the cavities A and M, both initially in the superposition of zero and one identical photons, through a beam splitter (Fig. 3-2.) produces the four-partite field entangled state of the form,

$$\left|\Psi^{DEYZ}\right\rangle_n = \frac{1}{\sqrt{2}}\left[\left|0_D, n_E, n_Y, 0_Z\right\rangle - \left|n_D, 0_E, 0_Y, n_Z\right\rangle\right], \tag{3.28}$$

among the cavities D, E, Y and Z on detection of single photon after passage through beam splitter with a 50% success probability.

3.2 Generation of Bell, NOON and W states via atom interferometry

This proposal [186] is concerned with the cavity field entangled state engineering using MZB in single loop as well as in cascaded, multiport geometries. Here we discuss the generation of Bell, NOON and W field entangled states. In previous scheme [215], NOON states have been generated with unit probability but the method was a bit fragile because the setup incorporates sensitive superposition of field Fock states, $|0\rangle$ and $|1\rangle$. However, in present proposal, we engineer entangled field states in spatially separated high-Q cavities using comparatively more robust technique based on off-resonant Bragg diffraction of a single atom from Fock field that can be replaced with counter propagating laser beams.

As mentioned before, Bragg scattering is a coherent multiphoton Raman process where an atom with an initial transverse momentum $p_{\xi_0} = \xi_0 \hbar k$ exits in a superposition of initial and inverted momentum state, viz. $p_{-\xi_0} = -\xi_0 \hbar k$, where ξ_0 is an integer that designates the order of Bragg diffraction and k denotes the wave number of field. Under off-resonance condition only virtual transitions take place leading to very low probability of decoherence due to atomic decay [184, 253, 255]. Hence the absence of spontaneous emission and the availability of only two discrete atomic paths make Bragg scattering an ideal candidate for lossless, efficient Mach-Zehnder Bragg Interferometer (MZBI) [272, 275]. Geometrical symmetry of MZBI provides an additional benefit in the form of cancellation of diffraction and other such phases incurred during atom-field interactions inside the interferometer.

In present work, we employ atom interferometric setups with second order off-resonant Bragg diffraction of atoms from high-Q cavities. Second order diffraction is selected to yield a comparatively large separation between split wavepackets after the interaction. By choosing specific conditions we can set interferometric cavities to act as atomic beam splitters and mirrors. Briefly, the setups consist of cascaded MZBI loops that additionally incorporate Ramsey zones and initially vacuum state cavities, distributed evenly in both the arms of the interferometer. The operational procedure of our proposed schemes is quite simple to follow. We use beam splitters of MZBI to coherently split atomic de Broglie wavepacket in two equal weighted, symmetric components in the momentum space. These well separated wavepacket momenta components are then deflected through atomic mirrors of the MZBI. Later we pass the wavepackets through travelling wave classical fields that cause the transformation both in the atomic momenta as well as in atomic internal state. Excited atoms are then allowed to coherently deposit their respective excitations in the form of photons into the adjacent initially vacuum state cavities. Finally the which-path information carried out by the atomic momenta components is erased by passing them through exit beam splitter(s), prior to detection. The procedure consequently ends up in generation of entangled field states between initially vacuum state cavities.

3.2.1 Atom optics through second order off-resonant Bragg diffraction

Here we develop the expressions for Atomic beam splitter and mirrors using off-resonant second order Bragg diffraction of a two-level atom. Treatment here is kept very brief because the same has already been described in previous chapter for the case of first order Bragg diffraction and the mathematical formalism is almost similar for both the cases. We consider a two-level atom initially prepared in ground state $|g\rangle$ with momentum $|p_0\rangle$. Excited state of such an atom is denoted by $|e\rangle$. Atomic interaction with Fock field $|m\rangle$ for an arbitrary time t may be expressed by the state vector

$$|\Psi(t)\rangle = e^{-i\left(\frac{p_0^2}{2M\hbar}-\frac{\Delta}{2}\right)t}\sum_{\xi=-\infty}^{\infty}\left[C_{g,m}^{p_\xi}(t)\,|g,m,p_\xi\rangle + C_{e,m-1}^{p_\xi}(t)\,|e,m-1,p_\xi\rangle\right], \tag{3.29}$$

where, ξ is the number of interactions between atom and field in the cavity and summation suggests the accumulative nature of momenta so acquired. Probability amplitude $C_{g,m}^{p_\xi}(t)$ $\left(C_{e,m-1}^{p_\xi}(t)\right)$ dictates the situation where the atom is in ground(excited) state with momentum state $|p_\xi\rangle$ and the cavity field is in Fock state $|m\rangle$ ($|m-1\rangle$). The interaction picture Hamiltonian under dipole and rotating wave approximations is written as

$$H = \frac{\hat{P}_x^2}{2M} + \frac{\hbar\Delta}{2}\sigma_z + \hbar\mu\cos(k\hat{x})\left[\sigma_+\hat{a} + \hat{a}^\dagger\sigma_-\right], \tag{3.30}$$

where $\sigma_+ = |e\rangle\langle g|$ ($\sigma_- = |g\rangle\langle e|$) is the raising(lowering) operator, $\hat{a}^\dagger(\hat{a})$ denotes field creation(annihilation) ladder operator, μ is vacuum Rabi frequency and Δ measures the atom-field detuning [184, 255]. Moreover, in Bragg regime [253, 255], the longitudinal component of atomic momenta (along y-axis) is treated classically while transverse momentum component along x-axis with negligible spread i.e. $\Delta p \ll \hbar k$, is treated quantum mechanically. We further assume that atomic momenta along z-axis is also quantized. The Schrödinger equation yields following set of coupled differential equations for probability amplitudes:

$$i\frac{\partial}{\partial t}C_{e,m-1}^{p_\xi}(t) = \left[\xi(\xi_0+\xi)\omega_r + \Delta\right]C_{e,m-1}^{p_\xi}(t) + \frac{\mu\sqrt{m}}{2}\left[C_{g,m}^{p_{\xi+1}}(t) + C_{g,m}^{p_{\xi-1}}(t)\right], \tag{3.31}$$

$$i\frac{\partial}{\partial t}C_{g,m}^{p_\xi}(t) = \left[\xi(\xi_0+\xi)\omega_r\right]C_{g,m}^{p_\xi}(t) + \frac{\mu\sqrt{m}}{2}[C_{e,m-1}^{p_{\xi+1}}(t) + C_{e,m-1}^{p_{\xi-1}}(t)], \tag{3.32}$$

where $\omega_r = \hbar k^2/2M$, is recoil frequency [184]. Equation (3.31) corresponds to odd number of interactions whereas equation (3.32) expresses the system's behavior for even number of interactions. Here, we apply adiabatic approximation which, as seen from above expressions, clearly demands that $\omega_r + \Delta \gg \mu\sqrt{m}/2$, a condition that must be followed in off-resonant Bragg diffraction [253, 255]. Furthermore, under the assumption of large detuning, i.e. $\Delta \gg \hbar k^2/2M$, above expressions reduce to

$$i\frac{\partial}{\partial t}C^{p_\xi}_{e,m-1}(t)=\Delta C^{p_\xi}_{e,m-1}(t)+\frac{\mu\sqrt{m}}{2}\left[C^{p_{\xi+1}}_{g,m}(t)+C^{p_{\xi-1}}_{g,m}(t)\right], \tag{3.33}$$

$$i\frac{\partial}{\partial t}C^{p_\xi}_{g,m}(t)=\left[\xi\left(\xi_0+\xi\right)\omega_r\right]C^{p_\xi}_{g,m}(t)+\frac{\mu\sqrt{m}}{2}\left[C^{p_{\xi+1}}_{e,m-1}(t)+C^{p_{\xi-1}}_{e,m-1}(t)\right]. \tag{3.34}$$

Equations (3.33) and (3.34) collectively represent a set of infinite coupled equations. This infinite set, however, simplifies to only nine significant expressions under second order Bragg diffraction for which $\xi_0=4$ and ξ varies from 3 to -5. Under adiabatic approximation, we ignore the probability amplitudes $C^{p_4}_{g,m}(t)$ and $C^{p_{-6}}_{g,m}(t)$ and take $\frac{\partial}{\partial t}C^{p_j}_{e,m-1}(t)=0$ for $j=$ odd number of interactions [253, 255]. After few mathematical steps (for details, please see chapter 2.), we obtain

$$C^{p_0}_{g,m}(t)=e^{-i\alpha_m t}\left[C^{p_0}_{g,m}(0)\cos\left(\frac{\beta_m}{2}t\right)+iC^{p_{-4}}_{g,m}(0)\sin\left(\frac{\beta_m}{2}t\right)\right], \tag{3.35}$$

$$C^{p_{-4}}_{g,m}(t)=e^{-i\alpha_m t}\left[C^{p_{-4}}_{g,m}(0)\cos\left(\frac{\beta_m}{2}t\right)+iC^{p_0}_{g,m}(0)\sin\left(\frac{\beta_m}{2}t\right)\right], \tag{3.36}$$

where $\alpha_m=\mu^4m^2/4\Delta^2\omega_r$ and $\beta_m=\alpha_m/8=\mu^4m^2/32\Delta^2\omega_r$ are field dependent quantities [184]. Discarding the global phase factor and substituting initial conditions, the state of the atom-field system at any arbitrary time becomes

$$|\Psi(t)\rangle=\cos\left(\frac{\beta_m}{2}t\right)|g,m,p_0\rangle+i\sin\left(\frac{\beta_m}{2}t)\right)|g,m,p_{-4}\rangle. \tag{3.37}$$

Since under off-resonant atomic Bragg diffraction, the atomic internal state and the cavity field states do not change, so we introduce here the reduced state vector

$$|\Phi(t)\rangle=\langle g,m\,|\Psi(t)\rangle=\cos\left(\frac{\beta_m}{2}t\right)|p_0\rangle+i\sin\left(\frac{\beta_m}{2}t\right)|p_{-4}\rangle. \tag{3.38}$$

Equation (3.38) indicates that the beam splitter operation is achieved when the interaction time $t=\pi/2\beta_m$, and leads to momentum state transformations such that

$$|p_0\rangle\rightarrow\left(|p_0\rangle+i\,|p_{-4}\rangle\right)/\sqrt{2},\quad\text{and}\quad|p_{-4}\rangle\rightarrow\left(i\,|p_0\rangle+|p_{-4}\rangle\right)/\sqrt{2}$$

and and thus acts like Hadamard gate. Whereas atomic mirror operation, corresponding to a NOT logic gate, is achieved for time $t = \pi/\beta_m$ and yields

$$|p_0\rangle \rightarrow i\,|p_{-4}\rangle\,, \quad \text{and} \quad |p_{-4}\rangle \rightarrow i\,|p_0\rangle\,.$$

Now once we have the atomic beam splitter and mirror transformations then construction of MZB interferometer becomes trivial in second order off-resonant Bragg regime.

3.2.2 Generation of maximal entangled cavity field Bell states

We consider a two-level atom, initially in its ground state $|g\rangle$ with transverse momentum $|p_0\rangle$. The atom performs off-resonant interaction in second order Bragg regime with a high-Q cavity (BS_1) as shown in Fig. 3-3. The atom-field interaction time is controlled such that the atom has equal probabilities for the momentum states $|p_0\rangle$ and $|p_{-4}\rangle$. The deflected atom in momentum states $|p_0\rangle$ or $|p_{-4}\rangle$, after free evolution, interacts with two other cavities in Fock field state $|n\rangle$. These cavities act as atomic mirrors (M_1 and M_2) for incoming atomic wave packet (corresponding to interaction time $t_2 = \pi/\beta_m$). Deflected atomic wavepackets are then excited through interaction with Ramsey zones $R_1^{p_0}$ and $R_1^{p_{-4}}$. These Ramsey fields are directed perpendicular to the basic plane (i.e. xy-plane) of the interferometer. Two other cavities A and B (initially in vacuum state), where the entangled state is to be prepared, are placed parallel to the Ramsey fields. At the end, the two possible planes of the interferometer contain two more beam splitters BS_2 and BS_3 in order to ensure the eraser of which-path information.

Under dipole and rotating wave approximations the atom-field interaction in Ramsey fields is described using semi-classical interaction Hamiltonian

$$H^{(sc)} = \frac{\hbar\Omega_R e^{i\phi}}{2}[|e\rangle\langle g| + |g\rangle\langle e|]e^{-ik\hat{z}}, \tag{3.39}$$

where $\Omega_R = |\wp_{ge}|\,\varepsilon/\hbar$ is Rabi frequency with ε representing the classical field amplitude and $\wp_{ge} = e\langle g|\,r\,|e\rangle$ stands for transition dipole matrix element. The term $e^{-ik\hat{z}}$ is included to incorporate the running wave contribution of the laser field and ϕ denotes the phase between atomic dipole moment and the laser field. In our calculations, we take $\phi = 0$ for convenience. The interaction of a ground state atom with the field for the time $t = \pi/\Omega_R$, flips atomic

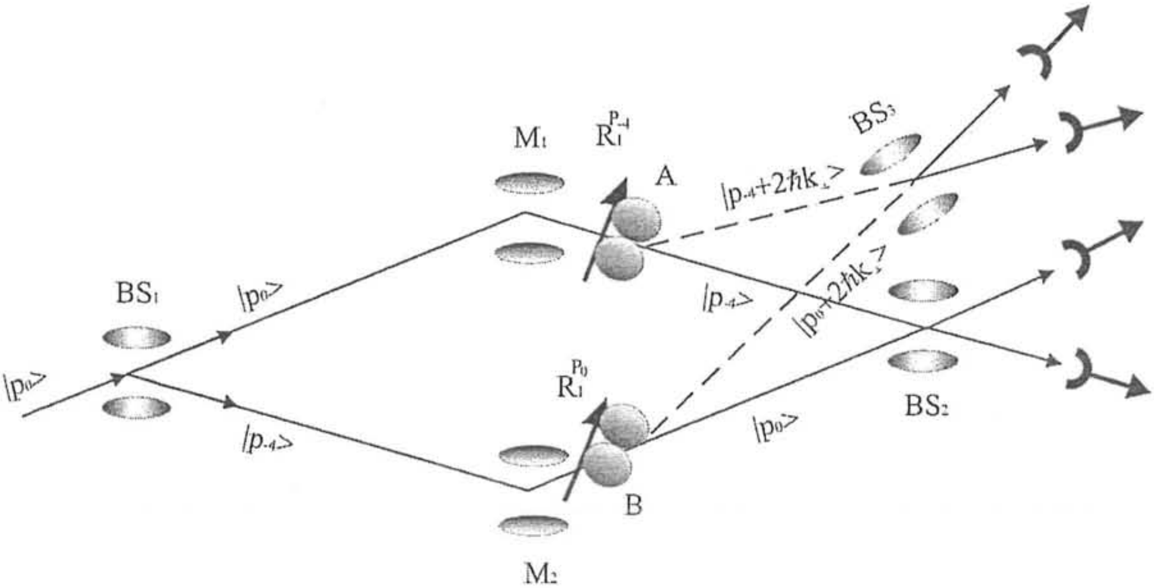

Figure 3-3: Schematics for generation of Bell states. Here BS_j, for $j = 1, 2, 3$, represent atomic de Broglie wave beam splitters, each consisting of a high-Q cavity containing Fock field $|m\rangle$. On the other hand, M_k, for $k = 1, 2$, are also high-Q cavities with the Fock field $|n\rangle$ but act as mirrors of the interferometer. Each mirror cavity is accompanied by an adjacent Ramsey excitational zone, namely $R_1^{p_0}$ and $R_1^{p_{-4}}$, aligned perpendicular to the mirror cavities (shown upward in the figure). These zones are then followed by two initially empty high-Q cavities A and B, each fitted parallel to the Ramsey fields. Finally, the deflected and undeflected momenta components going through the interferometric arms are mixed at either the second or the third beam splitter. The notations are the same as defined in Fig. 3-1.

internal state [235] as well as changes atomic external momentum equal to $\hbar k_\perp$ in the z-axis. Following, we give the detailed calculations concerning engineering aspects of cavity field Bell state.

The initial state of the system at $t = 0$ is expressed as

$$|\Psi(0)\rangle = |g, p_0\rangle \left(\prod_{k=A,B} |0_k\rangle\right) \left(\prod_{j=1}^{2} |n\rangle_{M_j}\right) \left(\prod_{l=1}^{3} |m\rangle_{BS_l}\right). \tag{3.40}$$

Here subscript k describes the cavities in their initial vacuum state whereas j and l denotes the cavities that act like mirrors and beam splitters respectively. The off-resonant atom-field interactions with first cavity BS_1, for a time $t_1 = \pi/2\beta_m$, leaves the atom into a symmetric superposition of atomic momenta states $|p_0\rangle$ and $|p_{-4}\rangle$ as stated earlier. This, however does not change in internal state of the atom. Therefore, the interaction being off-resonant, leaves the atom in its ground state $|g\rangle$. The step-wise interaction of atom with the field in cavities M_1 and M_2 (for an interaction time $t_2 = \pi/\beta_m$) and with the Ramsey zones $R_1^{p_0}$ and $R_1^{p_{-4}}$ (that transforms internal atomic state from $|g\rangle$ to $|e\rangle$ while imparting a momentum kick of $\hbar k_\perp$ in a direction perpendicular to the xy plane) leads to the state vector

$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}} |e\rangle \left(i\, |p_{-4} + \hbar k_\perp\rangle - |p_0 + \hbar k_\perp\rangle\right) \left(\prod_{k=A,B} |0_k\rangle\right) \left(\prod_{j=1}^{2} |n\rangle_{M_j}\right) \left(\prod_{l=1}^{3} |m\rangle_{BS_l}\right). \tag{3.41}$$

After interaction with the Ramsey field the excited atom deposits the excitation coherently into either cavity A or B through resonant interaction. We find that the atom exits cavity A or B with either an unaltered transverse momenta in the plane of interferometer or with an additive momenta $2\hbar k_\perp$ in upward direction in a plane inclined to the basic plane of the interferometer. The angle of the new plane is proportional to the change in the momentum of the atom in the perpendicular direction. The resonant interaction with cavities A and B is now governed by the Janynes-Cummings-Paul Hamiltonian $H^{(q)} = \hbar\mu_r \cos(k\hat{z})[\sigma_+ \hat{b} + \sigma_- \hat{b}^\dagger]$, where $\hat{b}(\hat{b}^\dagger)$ stands for field lowering(raising) ladder operator, μ_r is the atom-field coupling constant for resonant interaction and $\cos(k\hat{z})$ describes the spectral distribution of the field [235]. As the atomic transition from excited to ground state within these cavities occurs at $t = \pi/2\mu_r$, we may express the state of the system after atomic wavepacket exits the cavities as

$$|\Psi(t_4)\rangle = \frac{i}{2}|g\rangle \{|p_{-4}\rangle |1_A, 0_B\rangle + i|p_0\rangle |0_A, 1_B\rangle + |p_{-4} + 2\hbar k_{\perp}\rangle |1_A, 0_B\rangle \\ + i|p_0 + 2\hbar k_{\perp}\rangle |0_A, 1_B\rangle\} \left(\prod_{j=1}^{2} |n\rangle_{M_j}\right) \left(\prod_{l=1}^{3} |m\rangle_{BS_l}\right). \tag{3.42}$$

Finally, wavepacket momenta component $|p_0\rangle$ ($|p_{-4}\rangle$) and $|p_0 + 2\hbar k_{\perp}\rangle$ ($|p_{-4} + 2\hbar k_{\perp}\rangle$) passes through beam splitter cavities BS_2 and BS_3 respectively, that erase the prior which-path information carried by split atomic de Broglie wavepackets. It is to be noted that the cavity acting as BS_3 is stacked above BS_2 and aligned with the respective incoming momenta states. Hence we find the simplified expression of the system in its final state as

$$|\Psi(t_5)\rangle = \frac{1}{2\sqrt{2}}|g\rangle [(-|1_A, 0_B\rangle - |0_A, 1_B\rangle)|p_0\rangle + i(|1_A, 0_B\rangle - |0_A, 1_B\rangle)|p_{-4}\rangle - (|1_A, 0_B\rangle \\ + |0_A, 1_B\rangle)|p_0 + 2\hbar k_{\perp}\rangle + i(|1_A, 0_B\rangle - |0_A, 1_B\rangle)|p_{-4} + 2\hbar k_{\perp}\rangle] \otimes \left(\prod_{j=1}^{2} |n\rangle_{M_j}\right) \left(\prod_{l=1}^{3} |m\rangle_{BS_l}\right). \tag{3.43}$$

From equation (3.43) we can trace out the interferometric fields. Thus as the atom is detected either in momentum state $|p_0\rangle$ or $|p_0 + 2\hbar k_{\perp}\rangle$, with an occurrence probability of 50%, after BS_2 or BS_3 respectively, we find the entangled state

$$\left|\psi_{A,B}^{(+)}\right\rangle = \frac{1}{\sqrt{2}}(|1_A, 0_B\rangle + |0_A, 1_B\rangle), \tag{3.44}$$

however possibility to find the atom is state $|p_{-4}\rangle$ or $|p_{-4} + 2\hbar k_{\perp}\rangle$ yields the antisymmetric Bell state

$$\left|\psi_{A,B}^{(-)}\right\rangle = \frac{1}{\sqrt{2}}(|1_A, 0_B\rangle - |0_A, 1_B\rangle), \tag{3.45}$$

with an equivalent success probability of 50%. On obtaining any of these states, we can transform it to alternative Bell states $\left|\psi_{A,B}^{(\pm)}\right\rangle$ or $\left|\phi_{A,B}^{(\pm)}\right\rangle$, if needed, by local operations [67]. Our proposed method generates maximally entangled Bell states in deterministic way with unit success probability while employing only a single two-level atom.

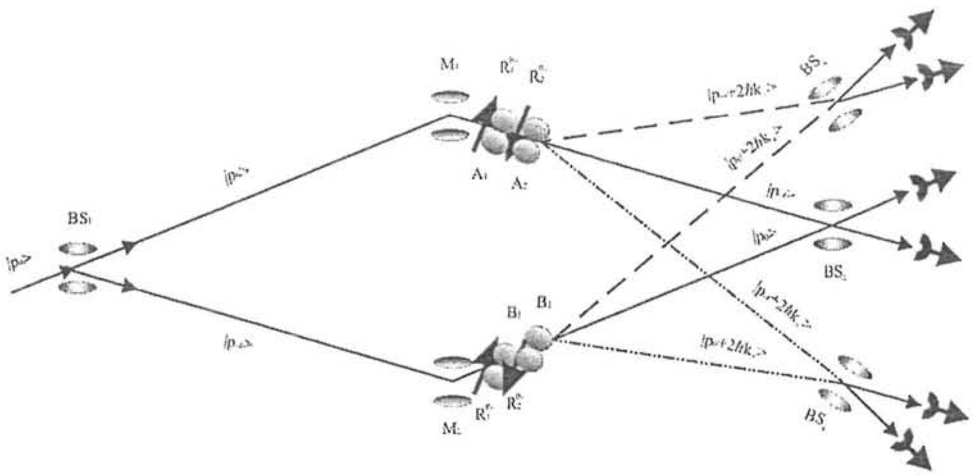

Figure 3-4: Setup for the generation of $N = 2$ cavity field NOON state.

3.2.3 Generation of NOON state

We present an extended version of MZBI to engineer entangled NOON states. Here we incorporate a series of consecutive counterpropagating Ramsey zones R_i^{p0}, R_i^{p-4} accompanied with cavities A_j and B_j initially in vacuum state, where $i, j = 1, 2, ..., n$. Following we explain procedural steps of our setup that generate two and four partite NOON states. The technique can be extended to generate any NOON state with arbitrary number of entangled photons.

Two-partite NOON state

For the generation of two-partite NOON state we suggest an experimental setup as presented Fig. 3-4. In this setup, three final transverse momentum states are possible. Therefore we need two additional beam splitters to erase the which-path information along with two beam splitters BS_1 and BS_2, which are part of the main frame MZBI. Initial state of such an atom-field system may be expressed as follows

$$|\Psi(0)\rangle = |g, p_0\rangle \left(\prod_{i=1}^{2} |0_{A_j}\rangle\right) \left(\prod_{j=1}^{2} |0_{B_k}\rangle\right) \left(\prod_{l=1}^{2} |n\rangle_{M_l}\right) \left(\prod_{q=1}^{4} |m\rangle_{BS_q}\right). \tag{3.46}$$

The state given in equation (3.46) after the passage of atom through BS_1, M_1, M_2, $R_1^{p_{-4}}$, $R_1^{p_0}$, A_1 and B_1 becomes

$$|\Psi(t_4)\rangle = \frac{1}{2}\{(-i\,|1_{A_1}, 0_{B_1}\rangle\,|p_{-4}\rangle + |0_{A_1}, 1_{B_1}\rangle\,|p_0\rangle) + (-i\,|1_{A_1}, 0_{B_1}\rangle\,|p_{-4} + 2\hbar k_\perp\rangle + |0_{A_1}, 1_{B_1}\rangle\,|p_0 + 2\hbar k_\perp\rangle)\}\,|g\rangle\,|0_{A_2}, 0_{B_2}\rangle\left(\prod_{l=1}^{2}|n\rangle_{M_l}\right)\left(\prod_{q=1}^{4}|m\rangle_{BS_q}\right). \tag{3.47}$$

The atom-field interaction with cavities A_1 and B_1 develops an entanglement of external degrees of freedom with cavity field. After emergence from cavities A_1 and B_1, the atom is again excited by Ramsey zones $R_2^{p_{-4}}$, $R_2^{p_0}$. This excitation is then transferred to initially vacuum state cavities A_2 and B_2. In order to erase the which path information we pass the atom through the final beam splitters BS_2, BS_3 and BS_4 and get the final state of the system as

$$\begin{aligned}|\Psi(t_7)\rangle = \frac{1}{2}\Bigg\{&\frac{-1}{\sqrt{2}}\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle + |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)|p_0\rangle + \frac{i}{\sqrt{2}}\{|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle \\ &- |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\}\,|p_{-4}\rangle - \frac{1}{2}\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle - |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)|p_0 + 2\hbar k_\perp\rangle \\ &+ \frac{i}{2}\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle + i\,|0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)|p_{-4} + 2\hbar k_\perp\rangle - \frac{1}{2}\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle\right. \\ &\left.- |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)|p_0 + 2\hbar k_\top\rangle + \frac{i}{2}\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle + i\,|0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right) \\ &|p_{-4} + 2\hbar k_\top\rangle\Bigg\} \otimes |g\rangle\left(\prod_{l=1}^{2}|n\rangle_{M_l}\right)\left(\prod_{q=1}^{4}|m\rangle_{BS_q}\right).\end{aligned} \tag{3.48}$$

Here $k_\perp$ and $k_\top$ indicate wave number corresponding to two planes where the atom is incident after momentum change. Detection of the atom in the momentum state $|p_0\rangle$, $|p_0 + 2\hbar k_\perp\rangle$ and $|p_0 + 2\hbar k_\top\rangle$ generates the state $\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle + |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)/\sqrt{2}$ with a success probability of 50%. Similarly the other alternative detection patterns yield equally probable state $\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle - |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)/\sqrt{2}$.

The states $\left(|1_{A_1}, 1_{A_2}, 0_{B_1}, 0_{B_2}\rangle \pm |0_{A_1}, 0_{A_2}, 1_{B_1}, 1_{B_2}\rangle\right)/\sqrt{2}$ are tomographically similar to Bell states extended over four dimensional Hilbert space, a phenomena being introduced first time. These, however can be converted to an explicit NOON state by employing two more two-level atoms which pick up field photons from a cavity and deposit in the other. Atom-1 initially in its ground state interacts resonantly with the cavity A_1 and its evolution is controlled by the interaction picture Hamiltonian $\hat{H} = \hbar\mu_r[\sigma_+\hat{b} + \sigma_-\hat{b}^\dagger]$. If the cavity has a photon then interacting atom after an interaction time $t_{A_1} = \pi/2\mu_r$ leaves cavity A_1 into vacuum state and transforms field-field entanglement to atom-field entanglement. The states thus become $(-i\left|e_1, 1_{A_2}, 0_{B_1}, 0_{B_2}\right\rangle \pm \left|g_1, 0_{A_2}, 1_{B_1}, 1_{B_2}\right\rangle) \otimes \left|0_{A_1}\right\rangle/\sqrt{2}$. Latter the atom interacts with cavity A_2, again resonantly for a time $t_{A_2} = \pi/2\sqrt{2}\mu_r$, and leads to the state $(-\left|2_{A_2}, 0_{B_1}, 0_{B_2}\right\rangle \pm \left|0_{A_2}, 1_{B_1}, 1_{B_2}\right\rangle) \otimes \left|g_1, 0_{A_1}\right\rangle/\sqrt{2}$. At this stage, cavity A_1 and atom-1 can be safely traced out of the system. Now atom-2, initially in its ground state $|g_2\rangle$, interacts successively with the cavities B_1 and B_2 under the same interaction parameters employed for atom-1 and yield the following final state vector

$$\left|\Psi_F^{\pm}\right\rangle = \frac{e^{i\pi}}{\sqrt{2}}\left(\left|2_{A_2}, 0_{B_2}\right\rangle \pm \left|0_{A_2}, 2_{B_2}\right\rangle\right) \otimes \left|g_2, 0_{B_1}\right\rangle.$$

Tracing out cavity field B_1 and atom-2 and eliminating the global phase factor of $e^{i\pi}$, we are left with

$$\left|\Psi_F^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|2_{A_2}, 0_{B_2}\right\rangle \pm \left|0_{A_2}, 2_{B_2}\right\rangle\right). \tag{3.49}$$

This expression corresponds to the deterministic generation of two-photon NOON state between the cavities A_2 and B_2.

Four-partite NOON state

The setup utilized for the generation of four-partite NOON state is a simple extension of the one used for two-partite NOON case and is shown in Fig. 3-5. Here, after two mirror cavities, we have four counterpropagating Ramsey zones along with four initially vacuum state high-Q cavities in each arm of the MZBI (i.e., we take $i, j = 1, 2, 3$ and 4 only). In this setup, five final transverse momentum states are possible. Thus we need four additional beam splitters to erase the which-path information. Out of the four, two beam splitters, BS_3 and BS_5 are stacked above and remaining two BS_4 and BS_6 are placed below BS_2 (only BS_3 and BS_4 are shown in Fig. 3-5.). Initial state of such an atom-field system may be expressed as follows

$$|\Psi(0)\rangle = |g, p_0\rangle \left(\prod_{i=1}^{4}\left|0_{A_i}\right\rangle\right)\left(\prod_{j=1}^{4}\left|0_{B_k}\right\rangle\right)\left(\prod_{l=1}^{2}|n\rangle_{M_l}\right)\left(\prod_{q=1}^{6}|m\rangle_{BS_q}\right). \tag{3.50}$$

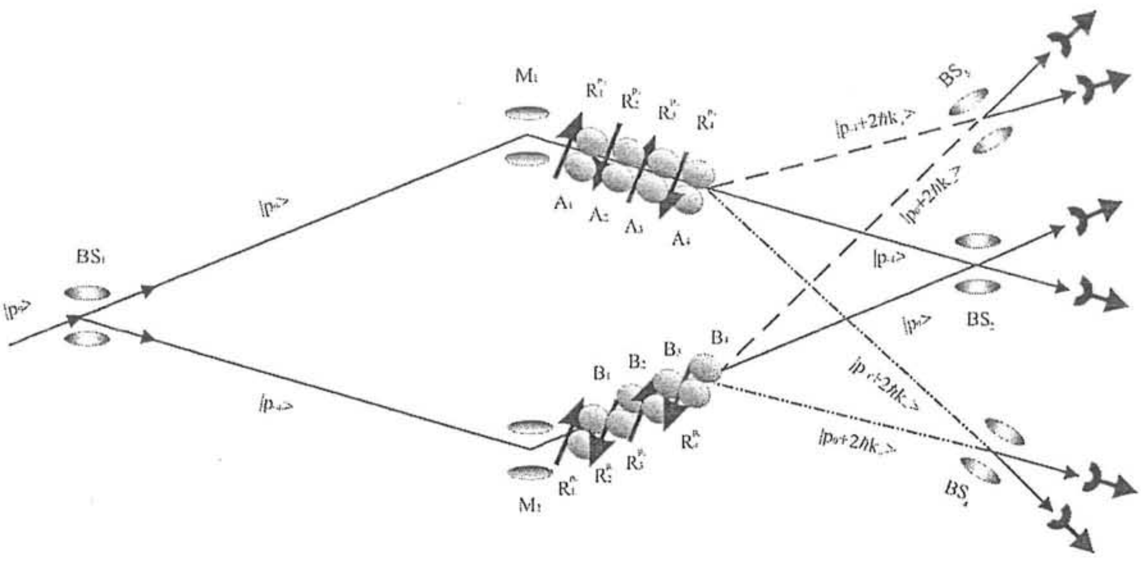

Figure 3-5: Setup for the generation of $N = 4$ cavity field NOON state. Here we show three planes out of five.

The state (3.50) after passing through BS_1, M_1, M_2, $R_1^{p_{-4}}$, $R_1^{p_0}$, A_1 and B_1 becomes

$$|\Psi(t_4)\rangle = \frac{1}{2}\{(-i\,|1_{A_1}, 0_{B_1}\rangle\,|p_{-4}\rangle + |0_{A_1}, 1_{B_1}\rangle\,|p_0\rangle) + (-i\,|1_{A_1}, 0_{B_1}\rangle\,|p_{-4} + 2\hbar k_{\perp}\rangle$$
$$+ |0_{A_1}, 1_{B_1}\rangle\,|p_0 + 2\hbar k_{\perp}\rangle)\}\,|g\rangle \left(\prod_{i=2}^{4} |0_{A_j}\rangle\right) \left(\prod_{j=2}^{4} |0_{B_k}\rangle\right) \left(\prod_{l=1}^{2} |n\rangle_{M_l}\right) \left(\prod_{q=1}^{6} |m\rangle_{BS_q}\right). \quad (3.51)$$

After emergence from cavities A_1 and B_1, split wavepackets are again excited by Ramsey zones $R_2^{p_{-4}}$, $R_2^{p_0}$. This excitation is then transferred to initially vacuum state cavities A_2 and B_2 through resonant π Rabi cycle. Same procedure is then repeated for Ramsey zones $R_3^{p_{-4}}$, $R_3^{p_0}$ and initially vacuum state cavities A_3 and B_3. The state after passage of split wavepackets through cavities A_4 and B_4 comes to be

$$|\Psi(t_{10})\rangle = \left\{ \frac{\sqrt{3}}{4} \left(i \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) |p_{-4}\rangle - \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) |p_0\rangle \right) \right.$$
$$+ \frac{1}{2\sqrt{2}} \left(i \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) |p_{-4} + 2\hbar k_\perp\rangle - \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) |p_0 + 2\hbar k_\perp\rangle \right)$$
$$+ \frac{1}{2\sqrt{2}} \left(i \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) |p_{-4} + 2\hbar k_\top\rangle - \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) |p_0 + 2\hbar k_\top\rangle \right)$$
$$+ \frac{1}{4\sqrt{2}} \left(i \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) |p_{-4} + 4\hbar k_\perp\rangle - \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) |p_0 + 4\hbar k_\perp\rangle \right)$$
$$\left. + \frac{1}{4\sqrt{2}} \left(i \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) |p_{-4} + 4\hbar k_\top\rangle - \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) |p_0 + 4\hbar k_\top\rangle \right) \right\}$$
$$\otimes |g\rangle \left(\prod_{l=1}^{2} |n\rangle_{M_l} \right) \left(\prod_{q=1}^{6} |m\rangle_{BS_q} \right). \tag{3.52}$$

This state, after interaction with the final symmetric atomic beam splitters BS_i, $i = 2, ..., 6$, transforms to

$$|\Psi(t_{11})\rangle = \left\{ \frac{1}{4\sqrt{2}} \left(-\left|\Psi^{(+)}_{A_j,B_k}\right\rangle |p_0 + 4\hbar k_\perp\rangle + i \left|\Psi^{(-)}_{A_j,B_k}\right\rangle |p_{-4} + (4\hbar k)_\perp\rangle \right) \right.$$
$$+ \frac{1}{2\sqrt{2}} \left(-\left|\Psi^{(+)}_{A_j,B_k}\right\rangle |p_0 + 2\hbar k_\perp\rangle + i \left|\Psi^{(-)}_{A_j,B_k}\right\rangle |p_{-4} + 2\hbar k_\perp\rangle \right) - \frac{\sqrt{3}}{4} \left(\left|\Psi^{(+)}_{A_j,B_k}\right\rangle |p_0\rangle \right.$$
$$\left. - i \left|\Psi^{(-)}_{A_j,B_k}\right\rangle |p_{-4}\rangle \right) - \frac{1}{2\sqrt{2}} \left(\left|\Psi^{(+)}_{A_j,B_k}\right\rangle |p_0 + 2\hbar k_\top\rangle - i \left|\Psi^{(-)}_{A_j,B_k}\right\rangle |p_{-4} + 2\hbar k_\top\rangle \right)$$
$$\left. - \frac{1}{4\sqrt{2}} \left(\left|\Psi^{(+)}_{A_j,B_k}\right\rangle |p_0 + 4\hbar k_\top\rangle - i \left|\Psi^{(-)}_{A_j,B_k}\right\rangle |p_{-4} + 4\hbar k_\top\rangle \right) \right\} \otimes |g\rangle \left(\prod_{l=1}^{2} |n\rangle_{M_l} \right) \prod_{q=1}^{6} |m\rangle_{BS_q}, \tag{3.53}$$

where we have taken

$$\left|\Psi^{(\pm)}_{A_j,B_k}\right\rangle = \frac{1}{\sqrt{2}} \left\{ \left(\prod_{j=1}^{4} |1_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |0_{B_k}\rangle \right) \pm \left(\prod_{j=1}^{4} |0_{A_j}\rangle \right) \left(\prod_{k=1}^{4} |1_{B_k}\rangle \right) \right\}. \tag{3.54}$$

We note that, during this process, atomic transverse momenta components execute a quantum random-walk forming binomial distribution around initial state $|p_0\rangle$ ($|p_{-4}\rangle$), a phenomenon quite interesting in itself . Now if we resolute to remain only in the main frame of the interferometer by employing just one output beam splitter i.e. BS_2, even then we have a net success probability of 37.5% of obtaining equally probable states $\left|\Psi^{(+)}_{A_j,B_k}\right\rangle$ and $\left|\Psi^{(-)}_{A_j,B_k}\right\rangle$. The collective success probability, however reaches to 87.5% with the inclusion of beam splitters BS_3 and BS_4 stacked above and below the main frame output beam splitter BS_2 and aligned at inclinations adjusted to meet $2\hbar k_\perp$ and $2\hbar k_\top$ momenta components respectively. Unit success probability is, however, achievable only when protocol is executed with the inclusion of all the exit beam splitters. The states $\left|\Psi^{(\pm)}_{A_j,B_k}\right\rangle$ can be converted into

$$\left|\Psi^{(\pm)}_{A_j,B_k}\right\rangle = \frac{1}{\sqrt{2}}\left(|4,0\rangle \pm |0,4\rangle\right), \tag{3.55}$$

by either using state transfer based on atom-field resonant interaction as discussed in previous section or by employing fiber optical coupling.

Generation of W state

Complete setup employed to generate four-partite field W state is depicted in Fig. 3-6. Here only main plane of the multiport interferometric configuration is shown. However, the dotted area is duplicated above with an inclination matching $2\hbar k_\perp$ momentum kick in upward direction. The initial state of the atom-field system may be expressed as follows

$$|\Psi(0)\rangle = |g,p_0\rangle \left(\prod_{j=A}^{D}|0_j\rangle\right)\left(\prod_{k=1}^{13}|m\rangle_{BS_k}\right)\left(\prod_{l=1}^{6}|n\rangle_{M_l}\right). \tag{3.56}$$

After interaction with BS_1, BS_2, BS_3 , M_1 and M_2, the state takes the form

$$\begin{aligned}|\Psi(t_3)\rangle &= \frac{1}{2}\{i\,|g,p_{-4}\rangle_6 - |g,p_0\rangle_5 + i\,|g,p_{-4}\rangle_4 - |g,p_0\rangle_{10}\}\\ &\otimes \left(\prod_{j=A}^{D}|0_j\rangle\right)\left(\prod_{k=1}^{13}|m\rangle_{BS_k}\right)\left(\prod_{l=1}^{6}|n\rangle_{M_l}\right),\end{aligned} \tag{3.57}$$

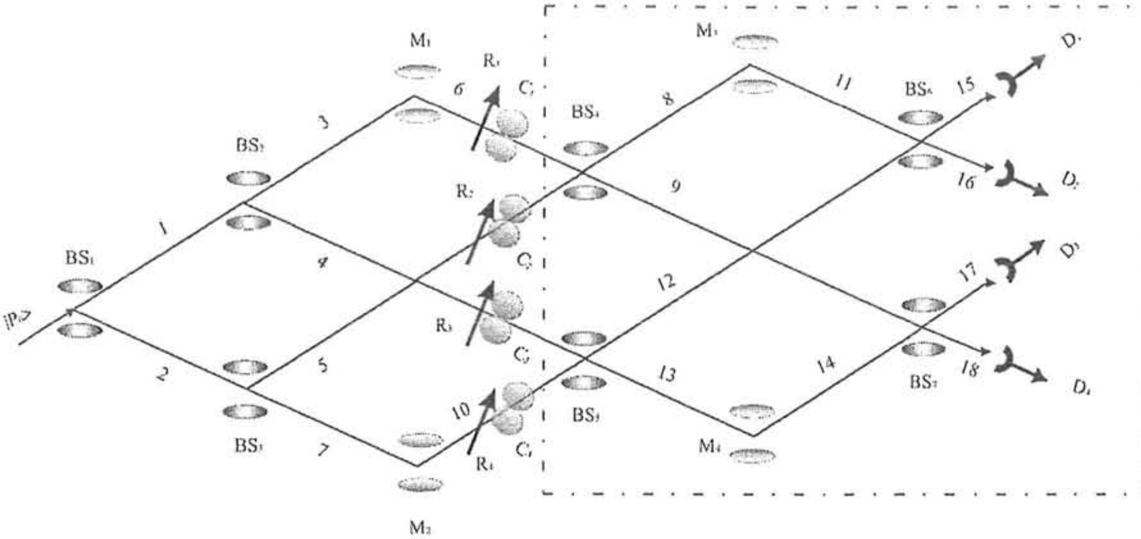

Figure 3-6: Sketch of the proposed setup for W state engineering. The dotted portion is also duplicated in upper plane.

where the subscripts below each ket denote the various paths being followed as indicated in Fig. 3-6. Next these paths are intercepted by excitational Ramsey zones labelled as R_l, where $l = 1, 2, 3$ and 4, causing the change in atomic internal state and external transverse momenta as $|g, p_{-4}\rangle \rightarrow -i\,|e, p_{-4} + \hbar k_{\perp}\rangle$. This excitation is consequently deposited into any one of the cavities $j = A$ to D which are aligned and placed after their respective Ramsey zones and are initially all taken to be in vacuum state. Therefore, atom-field entangled state after passing through these four resonant cavities becomes

$$
\begin{aligned}
|\Psi(t_5)\rangle = \frac{1}{\sqrt{2}}\{&\frac{1}{2}[-i\,|1_A, 0_B, 0_C, 0_D\rangle\,|p_{-4}\rangle_6 + |0_A, 1_B, 0_C, 0_D\rangle\,|p_0\rangle_5 - i\,|0_A, 0_B, 1_C, 0_D\rangle\,|p_{-4}\rangle_4 \\
&+ |0_A, 0_B, 0_C, 1_D\rangle\,|p_0\rangle_{10}] + \frac{1}{2}[-i\,|1_A, 0_B, 0_C, 0_D\rangle\,|p_{-4} + 2\hbar k_{\perp}\rangle_{6\uparrow} \\
&+ |0_A, 1_B, 0_C, 0_D\rangle\,|p_0 + 2\hbar k_{\perp}\rangle_{5\uparrow} - i\,|0_A, 0_B, 1_C, 0_D\rangle\,|p_{-4} + 2\hbar k_{\perp}\rangle_{4\uparrow} \\
&+ |0_A, 0_B, 0_C, 1_D\rangle\,|p_0 + 2\hbar k_{\perp}\rangle_{10\uparrow}]\} \otimes |g\rangle \left(\prod_{k=1}^{13} |m\rangle_{BS_k}\right)\left(\prod_{l=1}^{6} |n\rangle_{M_l}\right).
\end{aligned}
\tag{3.58}
$$

The paths subscripted with arrowheads lies in the upper plane which dose not appears in Fig. 3-6. It is clear from above expression that atom has 50% probability to remain in the interferometric fundamental plane and 50% probability to jump to upper inclined plane defined by upwardly acquired transverse momenta component $2\hbar k_{\perp}$. Thus we will get W field state with unit probability only when both the planes are considered. However since mathematical description is identical for both planes, we describe only the case when the atom is detected in the fundamental plane of the setup. This partial state after passing through the atomic beam splitters BS_4, BS_5 and atomic mirrors M_3, M_4 becomes

$$\begin{aligned} |\Psi(t_7)\rangle = &\frac{1}{2}\{[\frac{i}{\sqrt{2}}|1_A,0_B,0_C,0_D\rangle|p_{-4}\rangle_{11} - \frac{i}{\sqrt{2}}|1_A,0_B,0_C,0_D\rangle|p_{-4}\rangle_9] \\ &+[\frac{i}{\sqrt{2}}|0_A,1_B,0_C,0_D\rangle|p_{-4}\rangle_{11} + \frac{i}{\sqrt{2}}|0_A,1_B,0_C,0_D\rangle|p_{-4}\rangle_9] \\ &+[\frac{1}{\sqrt{2}}|0_A,0_B,1_C,0_D\rangle|p_0\rangle_{12} + \frac{1}{\sqrt{2}}|0_A,0_B,1_C,0_D\rangle|p_0\rangle_{14}] \\ &+[\frac{1}{\sqrt{2}}|0_A,0_B,0_C,1_D\rangle|p_0\rangle_{12} - \frac{1}{\sqrt{2}}|0_A,0_B,0_C,1_D\rangle|p_0\rangle_{14}]\} \\ &\otimes|g\rangle\left(\prod_{k=1}^{13}|m\rangle_{BS_k}\right)\left(\prod_{l=1}^{6}|n\rangle_{M_l}\right). \end{aligned} \tag{3.59}$$

Finally the momenta components traversing paths labelled as 11 and 12 pass through beam splitters BS_6 whereas components coming simultaneously through paths 9 and 14 pass through beam splitters BS_7. Thus the final state may be expressed as follows

$$\begin{aligned} |\Psi(t_8)\rangle = &\frac{1}{4}[\{-|1_A,0_B,0_C,0_D\rangle - |0_A,1_B,0_C,0_D\rangle + |0_A,0_B,1_C,0_D\rangle + |0_A,0_B,0_C,1_D\rangle\}|p_0\rangle_{D_1} \\ &+i\{|1_A,0_B,0_C,0_D\rangle + |0_A,1_B,0_C,0_D\rangle + |0_A,0_B,1_C,0_B\rangle + |0_A,0_B,0_C,1_D\rangle\}|p_{-4}\rangle_{D_2} \\ &\{-|1_A,0_B,0_C,0_D\rangle - |0_A,1_B,0_C,0_D\rangle + |0_A,0_B,1_C,0_D\rangle - |0_A,0_B,0_C,1_D\rangle\}|p_0\rangle_{D_3} \\ &i\{|1_A,0_B,0_C,0_D\rangle + |0_A,1_B,0_C,0_D\rangle + |0_A,0_B,1_C,0_D\rangle - |0_A,0_B,0_C,1_D\rangle\}|p_{-4}\rangle_{D_4}] \\ &\otimes|g\rangle\left(\prod_{k=1}^{13}|m\rangle_{BS_k}\right)\left(\prod_{l=1}^{6}|n\rangle_{M_l}\right). \end{aligned} \tag{3.60}$$

Here $|p_0\rangle_{D_{1(3)}}$ and $|p_{-4}\rangle_{D_{2(4)}}$ denotes the momenta components leading to the detectors $D_{1(3)}$ and $D_{2(4)}$, respectively. Multiport field components along with atomic internal state can be safely traced out as they are in outer product configuration. Thus we get maximally entangled W state whose local phases depends upon the pattern in which the atom is finally detected.

3.3 Experimental aspects of the proposed schemes

The schemes presented in the chapter bear promising executational aspects when viewed in the background of prevailing cavity QED experimental research scenario. Cavity quality factors of the order of 10^{12} in microwave regime have been achieved leading to the lifetimes as large as in the range of seconds [138, 139]. With the availability of these sufficiently long lifetimes, Khosa et al. [255] have shown the possibility of first order Bragg diffraction of about $15 - 20$ helium atoms successively through a quantized cavity field before decoherence takes place. Therefore, in our case, atom-field interaction times do not pose any stringent constraint because in all the proposed schemes, atom only once interacts with each cavity. Furthermore, almost all the techniques invoked here have already been experimentally implemented in one context or the other. Atomic beam splitters and even MZB interferometry [276] as a whole is well explored in the theoretical as well as experimental domain, and with both classical and quantized fields [258, 260, 261, 264, 263, 265]. Recently such an apparatus has produced good signal-to-noise ratios and a fringe contrast of $(74 - 84)$ % has been achieved with a thermal atomic beam source [267] whereas a fringe contrast of almost 100% has been attained through a MBZI utilizing rubidium Bose-Einstein condensate [265]. The most relevant setup here is the one reported by Koolen et al. [266] where they have achieved a coherent superposition of about $2mm$ spatially separated split atomic de Broglie wavepackets of helium during a travel time of only about $17\mu s$ with the total distance covered being $4.2mm$ under first order Bragg diffraction. However, for second order Bragg diffraction, this spatial separation between split component goes up to $2.88mm$ for the same longitudinal distance of $4.2mm$. Since coherence can be retained up to the order of few meters in long arm atomic interferometers [277, 278] therefore even larger spatial separation between split atomic de Broglie wavepackets can be achieved by extending the MZB interferometer's arm lengths. Thus with the availability of sufficient space, small volume high-Q microcavities [140] can be incorporated into the MZB arms as conjectured by A. E. A. Koolen and co-workers [266].

Cascaded MZB interferometric loops proposed here for the generation of four-partite cavity field W state are also no way beyond the access of present technology because the operation of multiple beam atomic interferometers with good results have already been demonstrated in some other context [272, 275]. Most importantly, interferometric mirrors and beam splitters that are described here, for the sake of mathematical brevity, as high-Q cavities containing Fock field can be safely replaced with counterpropagating laser beams forming a standing wave pattern [265, 271, 272]. To be more specific, we suggest using cold ^{85}Rb atoms that interact with a $\lambda = 780nm$ optical field [252, 271, 279]. Here the recoil frequency, detuning and the coupling constant μ of the atom field system are respectively $24.25KHz$, $16.75MHz$ and $703.72KHz$. Thus the required inequality i.e. detuning >> recoil frequency >> effective Rabi frequency holds. Same task can be done at room temperature using ^{23}Na atoms, where we can observe the 2^{nd} order off-resonant Bragg diffraction with $\mu = 703KHz$ and $\Delta = 1.68GHz$. Here the inequality comes to be (detuning $= 1.68MHz$) >> (recoil frequency $= 0.16MHz$) >> (effective Rabi frequency $= 34.7KHz$). For these calculations, we have considered the diffraction of ^{23}Na atoms from an optical field of wavelength $\lambda = 589nm$ and the interaction time comes out to be $t = 17.5\mu s$ [255].

Chapter 4

Cluster and graph state engineering

This chapter deals with the proposals for generation of cavity field as well as atomic cluster and graph states [233, 234]. The schemes presented here relies on cavity QED based atom-field interactions. Both resonant and dispersive interactions are utilized for the engineering of the desired entangled states. As described in chapter 2, these recently introduced so called cluster states [80] exhibit some novel characteristics. For example, these states are resistant to decoherence [216] and possess rich nonlocality features compared to the GHZ states [217, 218]. However, mainly the interest in cluster states is linked with the newly proposed one-way quantum computing model whose operatibility depends upon such states [77, 78, 79]. The model performs universal quantum computing through local single-qubit measurements of the cluster states. The experimental feasibility of such one-way computing model has also been verified through the operational exploration of the four-photon cluster state [81, 82]. Thus, owing to their evident importance, many schemes for generating the cluster states through multidisciplinary technologies have been proposed which are based on photonics [196, 219, 220, 221, 222, 223], atomic interaction in cavity QED [113, 199, 224, 225, 226, 227, 228, 229] and solid-state systems [188, 230, 231]. Due to their relative simplicity, the schemes utilizing linear optics techniques have already been experimentally demonstrated [81, 82, 89]. Photonics techniques, however, suffer from the fact that they are inherently probabilistic. Whereas schemes based on cavity QED technologies [199, 224, 225, 226, 227] have an ideal success probability of unity, although in reality such schemes are subjected to experimental imperfections such as cavity photon loss, atomic spontaneous emission, and violation of the Lamb-Dicke condition. We also conclude this chapter with a brief discussion covering experimental feasibility of our proposed schemes.

4.1 Electromagnetic field cluster and graph states through collective cavity QED disentanglement eraser.

Since here we intend to utilize the concept of disentanglement eraser for the generation of cavity field cluster and graph states therefore it seems appropriate to first mention the idea of quantum eraser briefly. Quantum eraser, introduced by M. O. Scully and K. Drühl in 1982 [280, 281], has played its due part in this quest for an in-depth understanding of what happens at the microscopic level, especially concerning the relationship of time and quantum dynamics as well as interconnections among information eraser, complementarity and entanglement [272, 282, 283]. In this respect, various types of quantum erasers have been proposed and experimentally demonstrated both with photons as well as with atom-field systems [282, 284, 285, 286, 287]. All of these are basically concerned with the restoration or revival of initial coherence that gets destroyed due to coupling of some tag to a coherent superposition state. Therefore, an optimal quantum eraser may be taken as comprised of coupling of a tag qubit via its external, internal or intrinsic degrees of freedom to any given superposition state, which binds the tag and the superposition state under non-local, entangled correlations through interactions of the labelling system i.e. the tag with either one or both components of the original superposition. This consequently enlarges the Hilbert space of the new non-factorizable system. The mutual orthogonality of the tag qubit and the initial superposition effectively destroys the interference pattern characteristic of the untagged state. Thus this interference loss is attributed to the possible distinguishability of the final tag states. The measurement of the tag qubit in its original basis, say $|\alpha_T\rangle$ and $|\beta_T\rangle$, collapses the state, washing out interference fringes completely. The fringe revival, however, can be achieved if the tag qubit is measured in a rotated basis, i.e., $\frac{1}{\sqrt{2}}(|\alpha_T\rangle + |\beta_T\rangle)$ [285]. Consider, for example, any arbitrary two-level quantum system initially in superposition state $\frac{1}{\sqrt{2}}(|0_s\rangle + |1_s\rangle)$. The system then interacts with a tag qubit taken initially to be in ground state $|\alpha\rangle$. The unitary evolution can then be described symbolically by;

$$\frac{1}{\sqrt{2}}(|0_s\rangle + |1_s\rangle) \otimes |\alpha_T\rangle \overset{Interaction}{\rightarrow} \frac{1}{\sqrt{2}}(|0_s, \alpha_T\rangle + |1_s, \beta_T\rangle). \tag{4.1}$$

Now it is quite easy to see that coherence of the initial superposition $(|0_s\rangle + |1_s\rangle)/\sqrt{2}$ is lost due to its coupling with the tagging qubit. This coherence, is however retrievable if we measure the tag in transformed basis i.e. $|\alpha_T\rangle \rightarrow (|\alpha_T\rangle + |\beta_T\rangle)/\sqrt{2}$ and $|\beta_T\rangle \rightarrow (|\alpha_T\rangle - |\beta_T\rangle)/\sqrt{2}$. Above expression under this transformation becomes

$$\begin{aligned}\frac{1}{\sqrt{2}}(|0_s, \alpha_T\rangle + |1_s, \beta_T\rangle) &\rightarrow \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[|0_s\rangle \otimes (|\alpha_T\rangle + |\beta_T\rangle) + |1_s\rangle \otimes (|\alpha_T\rangle - |\beta_T\rangle)\right] \\ &= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0_s\rangle + |1_s\rangle) \otimes |\alpha_T\rangle + \frac{1}{\sqrt{2}}(|0_s\rangle - |1_s\rangle) \otimes |\beta_T\rangle\right].\end{aligned} \tag{4.2}$$

Here we specifically want to mention the class of quantum erasers introduced by Garisto and Hardy [288] that not just deal with the restoration of interference but also suggests a method for the recovery of coherence of an initially two-partite entangled state by removing the unwanted tag qubit. Such a type of eraser is usually termed as disentanglement eraser and has been experimentally verified in NMR based systems [289, 290].

Suppose we initially have an entangled Bell state

$$\left|\Phi^{(\pm)}_{A,B}\right\rangle = \frac{1}{\sqrt{2}}\left(|0_A, 0_B\rangle \pm |1_A, 1_B\rangle\right). \tag{4.3}$$

An extra tag qubit can get entangled with the system either through its interaction with the component A or B or both. The state will therefore comes to be

$$|\Psi_{A,B,T}\rangle = \frac{1}{\sqrt{2}}\left(\left|0_A, 0_B, p^{(T)}\right\rangle \pm \left|1_A, 1_B, q^{(T)}\right\rangle\right), \tag{4.4}$$

with $p, q = 0, 1$ and $p \neq q$. This tagged information poses a potential threat to coherence of the initial state $\left|\Phi^{(\pm)}_{A,B}\right\rangle$ unless it is removed or erased in a satisfactory manner. As stated earlier, the usual method to erase such tagged information is to measure the ancilla qubit in an obliquely rotated basis, i.e., $\left|p^{(T)}\right\rangle \pm \left|q^{(T)}\right\rangle$. Implementation of such disentanglement eraser for cavity QED based atom-field systems has already been proposed with experimentally feasible schematics for coupling and removal of a tag atomic qubit imposed over an initially entangled field Bell state [241].

In present section we extend the procedure proposed in [241] and show that operation of a collective eraser over two tagged states of the type expressed in equation (4.4) not only safeguards coherence of the states but also lead to a four-partite field cluster state. Collective eraser is performed through consecutive resonant and dispersive interactions of the atoms with the cavity field, followed by their passage through Ramsey zone, prior to detection. Subsequent detection of atoms in excited or ground state culminates into the generation of cluster state.

The scheme we consider here is also a cavity QED based scheme and generates the four-qubit field cluster state in four spatially separated cavities [233]. First, we briefly describe the tagging procedure in section 4.1.1. The collective operation of a disentanglement eraser on the tagged system, leading to the generation of the four-qubit cluster state, is then described in next section. A schematic view of our scheme for tagging and erasing information is presented in Fig. 4-1., which will be described in detail in Secs. 4.1.1. and 4.1.2. Finally, in Sec. 4.1.3.,the scheme is generalized to cover the engineering of any arbitrary field graph state.

4.1.1 Tagging procedure

Suppose we are provided with two independent but identical field Bell states

$$\left|\Phi_{j,k}^{(+)}\right\rangle = \frac{1}{\sqrt{2}}\left(|0_j, 0_k\rangle + |1_j, 1_k\rangle\right), \tag{4.5}$$

where $j, k = A, B(C, D)$, and $|0\rangle$ and $|1\rangle$ refer to the vacuum and one-photon state, respectively. Such a state can be prepared by sending successively an excited two-level atom through two initially vacuum state cavities. Atom interacts resonantly with first cavity for $\pi/2$ Rabi pulse. After leaving the cavity, atom is passed through a Ramsey classical field for a time corresponding to π pulse. The atom finally interacts resonantly with the second cavity, again for a π pulse. The detection of atom in ground state, after emerging from second cavity ensures the generation of desired state. This can be done, for example, using a set up similar to the one employed in Ref. [96].

In order to place a tag over each pair of the field Bell states $\left|\Phi_{j,k}^{(+)}\right\rangle$, we take two identical

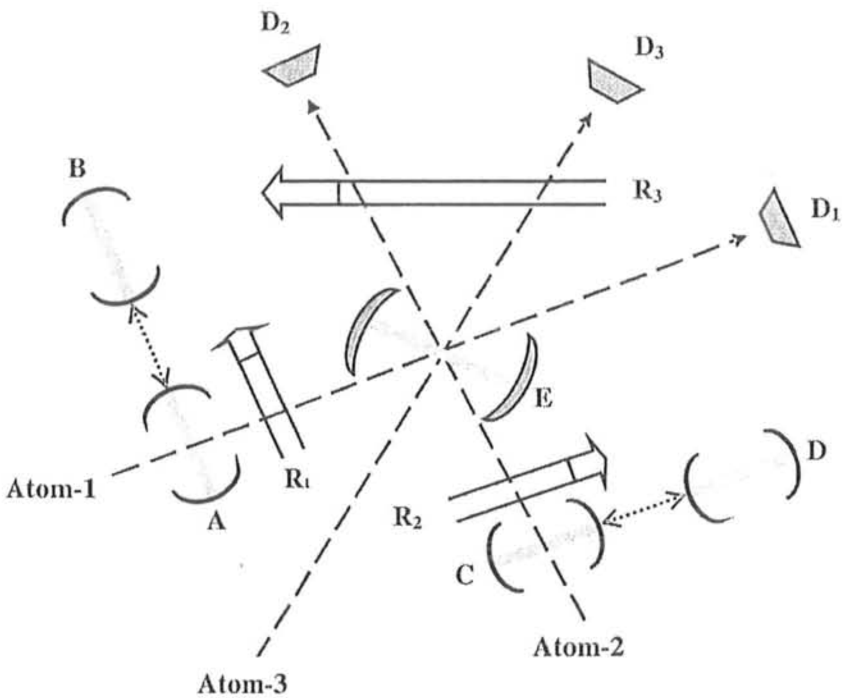

Figure 4-1: Schematic representation of the proposed setup for tagging and operating disentanglement eraser.

three-level cascade atoms, initially prepared into a symmetric superposition of the two lower levels $|l_1\rangle$ and $|l_2\rangle$, and pass them separately through cavities A and C, respectively (see Fig. 4-1.). The atoms are selected such that the lower atomic transition $|l_1\rangle \rightarrow |l_2\rangle$ is far detuned and hence effectively decoupled from the cavity field contained in cavities A and C, whereas the upper transition $|l_2\rangle \rightarrow |l_3\rangle$ only yields dispersive interactions through an atom-field detuning of δ. The effective Hamiltonian for such a set of system will be

$$H_{eff}^{(T_q)} = \frac{\hbar\mu^2}{\delta}\left(aa^{\dagger}\left|l_3^{(T_q)}\right\rangle\left\langle l_3^{(T_q)}\right| - a^{\dagger}a\left|l_2^{(T_q)}\right\rangle\left\langle l_2^{(T_q)}\right|\right). \tag{4.6}$$

Here superscript T_q ($q = 1$ or 2) denotes the tag atom (1 or 2), $a^{\dagger}a$ stands for the Fock field number operator, with $a^{\dagger}a\,|n\rangle = n\,|n\rangle$, and μ is the atom-field coupling constant. Corresponding state vector for an interaction time τ_q may be written as

$$\begin{aligned}|\Psi_{j,k,Tq}(\tau_q)\rangle &= C^{l_1^{(T_q)}}_{0_j,0_k}(\tau_q)\left|0_j,0_k,l_1^{(T_q)}\right\rangle + C^{l_2^{(T_q)}}_{0_j,0_k}(\tau_q)\left|0_j,0_k,l_2^{(T_q)}\right\rangle + C^{l_1^{(T_q)}}_{1_j,0_k}(\tau_q)\left|1_j,0_k,l_1^{(T_q)}\right\rangle \\ &+ C^{l_1^{(T_q)}}_{1_j,1_k}(\tau_q)\left|1_j,1_k,l_1^{(T_q)}\right\rangle + C^{l_2^{(T_q)}}_{0_j,1_k}(\tau_q)\left|0_j,1_k,l_2^{(T_q)}\right\rangle + C^{l_2^{(T_q)}}_{1_j,1_k}(\tau_q)\left|1_j,1_k,l_2^{(T_q)}\right\rangle \\ &+ C^{l_1^{(T_q)}}_{2_j,1_k}(\tau_q)\left|2_j,1_k,l_1^{(T_q)}\right\rangle. \end{aligned} \tag{4.7}$$

We note that atom-1 interacts with cavity A and atom-2 interacts with cavity C. Schrödinger's equation under initial conditions $C^{l_1^{(T_q)}}_{0_j,0_k}(0) = C^{l_2^{(T_q)}}_{0_j,0_k}(0) = C^{l_1^{(T_q)}}_{1_j,1_k}(0) = C^{l_2^{(T_q)}}_{1_j,1_k}(0) = 1/2$ and $C^{l_1^{(T_q)}}_{1_j,0_k}(0) = C^{l_2^{(T_q)}}_{0_j,1_k}(0) = C^{l_1^{(T_q)}}_{2_j,1_k}(0) = 0$ gives the following state vector for an interaction time τ_q,

$$|\Psi_{j,k,Tq}(\tau_q)\rangle = \frac{1}{\sqrt{2}}\left[\left|l_1^{(T_q)}\right\rangle \otimes \frac{1}{\sqrt{2}}\left(|0_j,0_k\rangle + |1_j,1_k\rangle\right) + \left|l_2^{(T_q)}\right\rangle \otimes \frac{1}{\sqrt{2}}(|0_j,0_k\rangle + e^{i\frac{\mu^2\tau_q}{\delta}}\,|1_j,1_k\rangle)\right], \tag{4.8}$$

where $j,k,q = A,B,1(C,D,2)$. We select the interaction times of the atoms in cavities A and C such that $\mu^2\tau_q/\delta = \pi$ for both $q = 1$ and 2. The atoms after completing dispersive interaction with their respective cavities then pass through Ramsey zone R_1 and R_2 as shown in Fig. 4-1., where their internal states transform to

$$\left|l_1^{(T_q)}\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|l_1^{(T_q)}\right\rangle + \left|l_2^{(T_q)}\right\rangle\right), \qquad \left|l_2^{(T_q)}\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\left|l_1^{(T_q)}\right\rangle - \left|l_2^{(T_q)}\right\rangle\right), \tag{4.9}$$

and the state listed in (4.8) consequently becomes

$$|\Psi_{j,k,T_q}\rangle = \frac{1}{\sqrt{2}}\left(\left|0_j,0_k,l_1^{(T_q)}\right\rangle + \left|1_j,1_k,l_2^{(T_q)}\right\rangle\right). \tag{4.10}$$

This completes the tagging procedure as atom-1 and atom-2 are now tagged with the initial entangled states $\left|\Phi^{(+)}_{A,B}\right\rangle$ and $\left|\Phi^{(+)}_{C,D}\right\rangle$, respectively, through their internal degrees of freedom forming tri-partite GHZ states. The product of these two GHZ states may be expressed as

$$\begin{aligned}\left|\Psi^{C,D,T_2}_{A,B,T_1}\right\rangle &= |\Psi_{A,B,T_1}\rangle \otimes |\Psi_{C,D,T_2}\rangle \\ &= \frac{1}{2}(|X_1\rangle\left|l_1^{(T_1)},l_1^{(T_2)}\right\rangle + |X_2\rangle\left|l_1^{(T_1)},l_2^{(T_2)}\right\rangle + |X_3\rangle\left|l_2^{(T_1)},l_1^{(T_2)}\right\rangle + |X_4\rangle\left|l_2^{(T_1)},l_2^{(T_2)}\right\rangle).\end{aligned} \tag{4.11}$$

Here we have taken $|X_1\rangle = |0_A, 0_B, 0_C, 0_D\rangle$, $|X_2\rangle = |0_A, 0_B, 1_C, 1_D\rangle$, $|X_3\rangle = |1_A, 1_B, 0_C, 0_D\rangle$ and $|X_4\rangle = |1_A, 1_B, 1_C, 1_D\rangle$.

4.1.2 Generation of cluster states through operation of disentanglement eraser

In this section, we discuss the evolution of the tagged systems under application of a collective disentanglement eraser. This is done by passing atom-1 through an initially empty cavity E where it goes through a resonant π Rabi cycle and hence exits out in the ground state with its state effectively transferred to the cavity. The state of the system thus comes to be

$$\left|\Psi_{A,B,C,T_1}^{D,E,T_2}\right\rangle = \frac{1}{2}[|X_1\rangle \otimes \left|0_E, l_1^{(T_2)}\right\rangle + |X_2\rangle \otimes \left|0_E, l_2^{(T_2)}\right\rangle \\ - i\,|X_3\rangle \otimes \left|1_E, l_1^{(T_2)}\right\rangle - i\,|X_4\rangle \otimes \left|1_E, l_2^{(T_2)}\right\rangle] \otimes \left|l_1^{(T_1)}\right\rangle . \tag{4.12}$$

Atom-1, detected in the ground state, is consequently traced out of the system. Atom-2, then traverses cavity E, such that the transition $|l_1\rangle \to |l_2\rangle$ now couples dispersively with the cavity field. This can be done, for example, by applying an external Stark field that induces a detuning Δ between field and atomic transition frequencies, and/or by moving the cavity mirrors if necessary. Such dispersive interactions are again described by the Hamiltonian $H_{eff}^{(T_2)} = \frac{\hbar\lambda^2}{\Delta}\left(aa^\dagger \left|l_2^{(T_2)}\right\rangle \left\langle l_2^{(T_2)}\right| - a^\dagger a \left|l_1^{(T_2)}\right\rangle \left\langle l_1^{(T_2)}\right|\right)$, where λ denotes the atom-field coupling constant in this case. The corresponding state vector of the system after an arbitrary interaction time t, may be expressed as

$$|\Psi(t)\rangle = C_{X_1,0_E}^{l_1^{(T_2)}}(t)\left|X_1, 0_E, l_1^{(T_2)}\right\rangle + C_{X_2,0_E}^{l_2^{(T_2)}}(t)\left|X_2, 0_E, l_2^{(T_2)}\right\rangle + C_{X_2,1_E}^{l_1^{(T_2)}}(t)\left|X_2, 1_E, l_1^{(T_2)}\right\rangle \\ + C_{X_3,1_E}^{l_1^{(T_2)}}(t)\left|X_3, 1_E, l_1^{(T_2)}\right\rangle + C_{X_3,0_E}^{l_2^{(T_2)}}(t)\left|X_3, 0_E, l_2^{(T_2)}\right\rangle + C_{X_4,1_E}^{l_2^{(T_2)}}(t)\left|X_4, 1_E, l_2^{(T_2)}\right\rangle \\ + C_{X_4,2_E}^{l_1^{(T_2)}}(t)\left|X_4, 2_E, l_1^{(T_2)}\right\rangle . \tag{4.13}$$

Schrödinger's equation under initial conditions $C_{X_1,0_E}^{l_1^{(T_2)}}(0) = C_{X_2,0_E}^{l_2^{(T_2)}}(0) = 1/2$, $C_{X_3,1_E}^{l_1^{(T_2)}}(0) = C_{X_4,1_E}^{l_2^{(T_2)}}(0) = -i/2$ and $C_{X_2,1_E}^{l_1^{(T_2)}}(0) = C_{X_3,0_E}^{l_2^{(T_2)}}(0) = C_{X_4,2_E}^{l_1^{(T_2)}}(0) = 0$ yields the following expression for the state vector

$$|\Psi(t)\rangle = \frac{1}{2}\left[\left|X_1, 0_E, l_1^{(T_2)}\right\rangle + e^{-\frac{i\lambda^2}{\Delta}t}\left|X_2, 0_E, l_2^{(T_2)}\right\rangle - ie^{\frac{i\lambda^2}{\Delta}t}\left|X_3, 1_E, l_1^{(T_2)}\right\rangle - ie^{-2\frac{i\lambda^2}{\Delta}t}\left|X_4, 1_E, l_2^{(T_2)}\right\rangle\right]. \tag{4.14}$$

Atom-2 is then passed through a Ramsey zone R_3 which produces internal state transformations similar to (4.9). If the interaction time of the second atom with cavity E is selected such that $\lambda^2 t/\Delta = \pi/2$, then the state of the system after atom's passage through R_3 becomes

$$\begin{aligned}|\Psi\rangle = \frac{1}{\sqrt{2}}&\left\{\frac{1}{2}\left[|X_1, 0_E\rangle - i|X_2, 0_E\rangle + |X_3, 1_E\rangle + i|X_4, 1_E\rangle\right] \otimes \left|l_1^{(T_2)}\right\rangle\right.\\ &\left.+\frac{1}{2}\left[|X_1, 0_E\rangle + i|X_2, 0_E\rangle + |X_3, 1_E\rangle - i|X_4, 1_E\rangle\right] \otimes \left|l_2^{(T_2)}\right\rangle\right\}.\end{aligned} \tag{4.15}$$

Thus, when atom-2 is detected in either $\left|l_1^{(T_2)}\right\rangle$ or $\left|l_2^{(T_2)}\right\rangle$, we get the corresponding equiprobable entangled field state. However, since our goal is to perform a collective eraser, therefore to complete the procedure duly, we pass another atom, say atom-3, initially in its ground state, through cavity E, where it goes through a resonant interaction with the cavity mode for a time needed to complete a π Rabi cycle. This operation effectively sweeps the field information to the atom while leaving cavity E in vacuum state. This atom, after emerging out of the cavity E, passes through R_3 and is finally detected at its respective state-sensitive detector D_3 as depicted in Fig. 4-1. The final expression of the state vector therefore comes to be

$$\begin{aligned}|\Psi_F\rangle = \frac{1}{2}&\left\{\frac{1}{2}\left[|X_1\rangle - i|X_2\rangle - i|X_3\rangle + |X_4\rangle\right] \otimes \left|l_1^{(T_2)}, l_1^{(3)}\right\rangle\right.\\ &+\frac{1}{2}\left[|X_1\rangle - i|X_2\rangle + i|X_3\rangle - |X_4\rangle\right] \otimes \left|l_1^{(T_2)}, l_2^{(3)}\right\rangle\\ &+\frac{1}{2}\left[|X_1\rangle + i|X_2\rangle - i|X_3\rangle - |X_4\rangle\right] \otimes \left|l_2^{(T_2)}, l_1^{(3)}\right\rangle\\ &\left.+\frac{1}{2}\left[|X_1\rangle + i|X_2\rangle + i|X_3\rangle + |X_4\rangle\right] \otimes \left|l_2^{(T_2)}, l_2^{(3)}\right\rangle\right\} \otimes |0_E\rangle.\end{aligned} \tag{4.16}$$

Thus, upon detection of atoms through state sensitive detectors D_2 and D_3 in combination of the internal states of either $\left|l_1^{(T_2)}, l_1^{(3)}\right\rangle$, $\left|l_1^{(T_2)}, l_2^{(3)}\right\rangle$, $\left|l_2^{(T_2)}, l_1^{(3)}\right\rangle$,or $\left|l_2^{(T_2)}, l_2^{(3)}\right\rangle$, we get any one of the corresponding equally probable linear field cluster states that can subsequently be converted into the standard form through local operations, if needed [80, 232]. One sees that the two independent pairs of two-field Bell states have now been merged to form a four-field cluster state. Thus, the collective operation of a disentanglement eraser can generate four-qubit coherence out of the initial coherence of a pair of two entangled qubits.

The proposal presented here may be generalized in a straightforward way to generate (2n)-qubit field cluster states of the form,

$$|\Psi_F\rangle = \frac{1}{2}\left(\left|X_1^{(2n)}\right\rangle + \left|X_2^{(2n)}\right\rangle + \left|X_3^{(2n)}\right\rangle - \left|X_4^{(2n)}\right\rangle\right), \tag{4.17}$$

where

$$\begin{aligned}\left|X_1^{(2n)}\right\rangle &= |0_{A_1}, 0_{A_2}, \ldots, 0_{A_n}, 0_{C_1}, 0_{C_2}, \ldots, 0_{C_n}\rangle, \quad \left|X_2^{(2n)}\right\rangle = |0_{A_1}, 0_{A_2}, \ldots, 0_{A_n}, 1_{C_1}, 1_{C_2}, \ldots, 1_{C_n}\rangle, \\ \left|X_3^{(2n)}\right\rangle &= |1_{A_1}, 1_{A_2}, \ldots, 1_{A_n}, 0_{C_1}, 0_{C_2}, \ldots, 0_{C_n}\rangle, \quad \left|X_4^{(2n)}\right\rangle = |1_{A_1}, 1_{A_2}, \ldots, 1_{A_n}, 1_{C_1}, 1_{C_2}, \ldots, 1_{C_n}\rangle,\end{aligned} \tag{4.18}$$

provided we are in possession of a pair of tagged states of the type

$$\left|\Psi_{j_1, j_2, \ldots, j_n, T_q}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0_{j_1}, 0_{j_2}, \ldots, 0_{j_n}, l_1^{(T_q)}\right\rangle + \left|1_{j_1}, 1_{j_2}, \ldots, 1_{j_n}, l_2^{(T_q)}\right\rangle\right), \tag{4.19}$$

where $j, q = A, 1(C, 2)$, and $A_1, A_2, \ldots, A_n$ $(C_1, C_2, \ldots, C_n)$ represent n cavities initially entangled with one another and tagged with atom-1 (atom-2). Through collective operation of a disentanglement eraser, $(2n)$ entangled qubits can be generated out of a pair of (n) entangled qubits.

4.1.3 Generation of cavity field graph states

In the scheme described above, collective operation of a disentanglement eraser on two independent cavity fields and the resulting generation of coherence between them are accomplished by tagging the two fields with an atom each and letting the two tag atoms interact one after the other with a common radiation field. Alternatively, it can be achieved by tagging one cavity field with an atom and utilizing interaction between this atom and the field of the other cavity. Such a tagged field-state that serves as the basic building block for the generation of field graph states can be engineered by passing an excited two-level atom through an initially vacuum state cavity for a time corresponding to $\pi/2$ Rabi pulse. The state is engineered when, atom after emergence from the cavity, is passed through a Ramsey zone for a π Rabi oscillation. In particular, the controlled-Z operation between two cavity fields can be accomplished by letting the atom tagged with one cavity field go through a dispersive interaction with the field of the other cavity and pass through a Ramsey zone. Consider, for example, cavity-1 tagged with atom-1 in state $\frac{1}{\sqrt{2}}\left(\left|0_1, l_1^{(1)}\right\rangle + \left|1_1, l_2^{(1)}\right\rangle\right)$ and cavity-2 prepared in a symmetric superposition $\frac{1}{\sqrt{2}}\left(|0_2\rangle + |1_2\rangle\right)$. After a dispersive interaction of atom-1 with the field of cavity-2, the state of the system becomes

$$\left|\Psi^{(2)}(\tau^{(d)})\right\rangle = \frac{1}{2}\left(\left|0_1, 0_2, l_1^{(1)}\right\rangle + e^{\frac{i\lambda^2}{\Delta}\tau^{(d)}}\left|0_1, 1_2, l_1^{(1)}\right\rangle \right.$$
$$\left. + e^{\frac{-i\lambda^2}{\Delta}\tau^{(d)}}\left|1_1, 0_2, l_2^{(1)}\right\rangle + e^{\frac{-2i\lambda^2}{\Delta}\tau^{(d)}}\left|1_1, 1_2, l_2^{(1)}\right\rangle\right). \quad (4.20)$$

Here $\tau^{(d)}$ stands for the time of dispersive interaction of the atom with the second cavity. We then let atom-1 pass through a Ramsey zone. The state of the system then becomes

$$\left|\Psi^{(2)}\right\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{2}\left(|0_1, 0_2\rangle + e^{\frac{i\lambda^2}{\Delta}\tau^{(d)}}|0_1, 1_2\rangle + e^{\frac{-i\lambda^2}{\Delta}\tau^{(d)}}|1_1, 0_2\rangle + e^{\frac{-2i\lambda^2}{\Delta}\tau^{(d)}}|1_1, 1_2\rangle\right) \otimes \left|l_1^{(1)}\right\rangle \right.$$
$$\left. + \frac{1}{2}\left(|0_1, 0_2\rangle + e^{\frac{i\lambda^2}{\Delta}\tau^{(d)}}|0_1, 1_2\rangle - e^{\frac{-i\lambda^2}{\Delta}\tau^{(d)}}|1_1, 0_2\rangle - e^{\frac{-2i\lambda^2}{\Delta}\tau^{(d)}}|1_1, 1_2\rangle\right) \otimes \left|l_2^{(1)}\right\rangle\right]. \quad (4.21)$$

Thus the detection of the atom in either of the state i.e. $\left|l_1^{(1)}\right\rangle$ or $\left|l_2^{(1)}\right\rangle$, we get the corresponding linear two partite field cluster state. With appropriate selection of the interaction time aided with subsequent local operations, we can convert the state, if needed, into standard form $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle - |1_1, 1_2\rangle)$. In general, the initial state for the generation of n-partite linear field cluster state may be expressed as follows

$$\left|\Psi^{(n)}(0)\right\rangle = \frac{1}{2^{\frac{n}{2}}}\prod_{k=1}^{n-1}\left(\left|0_k, l_1^{(k)}\right\rangle + \left|1_k, l_2^{(k)}\right\rangle\right) \otimes (|0_n\rangle + |1_n\rangle). \quad (4.22)$$

The state engineering procedure is quite easy to follow. The k^{th} atom, tagged with the k^{th} cavity field, interacts dispersively with the $(k+1)^{th}$ cavity field for a time $\tau_k^{(d)}$. After the completion of dispersive interactions, this k^{th} atom passes through Ramsey zone prior to its detection. The procedure continues until $(n-1)^{th}$ atom completes its dispersive interactions with the

n^{th} cavity field and then detected after its traversal through the Ramsey zone. Culmination of the process subsequently yields any one of the 2^{n-1} equiprobable n-partite linear cluster states corresponding to the specific pattern of the recorded atoms out of the 2^{n-1} possible permutations. In order to elaborate it a little bit further, we present the case of four partite linear field cluster state. Here, firstly atom-1, tagged with cavity-1, interacts dispersively with the field in cavity-2 for a time $\tau_1^{(d)}$ and passes through the Ramsey zone. Next, atom-2, tagged with cavity-2, performs dispersive interactions with cavity-3 for a time $\tau_2^{(d)}$ and then goes through the Ramsey zone after which it is duly recorded through a state selective detector. Finally, atom-3 which was initially tagged with the cavity-3 interacts dispersively with the field in cavity-4 and then it also passes through the Ramsey zone. It should be noted that being dispersive in nature, the temporal ordering of the interactions is neither specific nor important. Thus the atoms may interact with the fields simultaneously or in any arbitrary sequence deemed experimentally feasible. Since three atoms are involved in the generation of four-partite field cluster state, so $2^{4-1} = 8$ equiprobable states are possible corresponding to the recording of any one out of the eight atomic internal state detection possibilities $\left|l_i^{(1)}, l_j^{(2)}, l_k^{(3)}\right\rangle$, for i, j, $k = 1, 2$. The four-partite linear field cluster state generated when atoms are detected in, for example $\left|l_1^{(1)}, l_2^{(2)}, l_1^{(3)}\right\rangle$ pattern is given by

$$\begin{aligned}
\left|\Psi^{(4)}\right\rangle = &\frac{1}{4}(|0_1,0_2,0_3,0_4\rangle + e^{\frac{i\lambda^2}{\Delta}\tau_3^{(d)}}|0_1,0_2,0_3,1_4\rangle \\
&+ e^{\frac{i\lambda^2}{\Delta}\left(\tau_2^{(d)}-\tau_3^{(d)}\right)}|0_1,0_2,1_3,0_4\rangle + e^{\frac{i\lambda^2}{\Delta}\left(\tau_2^{(d)}-2\tau_3^{(d)}\right)}|0_1,0_2,1_3,1_4\rangle \\
&- e^{\frac{i\lambda^2}{\Delta}\left(\tau_1^{(d)}-\tau_2^{(d)}\right)}|0_1,1_2,0_3,0_4\rangle - e^{\frac{i\lambda^2}{\Delta}\left(\tau_1^{(d)}-\tau_2^{(d)}+\tau_3^{(d)}\right)}|0_1,1_2,0_3,1_4\rangle \\
&- e^{\frac{i\lambda^2}{\Delta}\left(\tau_1^{(d)}-2\tau_2^{(d)}-\tau_3^{(d)}\right)}|0_1,1_2,1_3,0_4\rangle - e^{\frac{i\lambda^2}{\Delta}\left(\tau_1^{(d)}-2\tau_2^{(d)}-2\tau_3^{(d)}\right)}|0_1,1_2,1_3,1_4\rangle \\
&+ e^{\frac{-i\lambda^2}{\Delta}\tau_1^{(d)}}|1_1,0_2,0_3,0_4\rangle + e^{\frac{-i\lambda^2}{\Delta}\left(\tau_1^{(d)}-\tau_3^{(d)}\right)}|1_1,0_2,0_3,1_4\rangle \\
&+ e^{\frac{-i\lambda^2}{\Delta}\left(\tau_1^{(d)}-\tau_2^{(d)}+\tau_3^{(d)}\right)}|1_1,0_2,1_3,0_4\rangle + e^{\frac{-i\lambda^2}{\Delta}\left(\tau_1^{(d)}-\tau_2^{(d)}+2\tau_3^{(d)}\right)}|1_1,0_2,1_3,1_4\rangle \\
&- e^{\frac{-i\lambda^2}{\Delta}\left(2\tau_1^{(d)}+\tau_2^{(d)}\right)}|1_1,1_2,0_3,0_4\rangle - e^{\frac{-i\lambda^2}{\Delta}\left(2\tau_1^{(d)}+\tau_2^{(d)}-\tau_3^{(d)}\right)}|1_1,1_2,0_3,1_4\rangle \\
&- e^{\frac{-2i\lambda^2}{\Delta}\left(\tau_1^{(d)}+\tau_2^{(d)}+\frac{\tau_3^{(d)}}{2}\right)}|1_1,1_2,1_3,0_4\rangle - e^{\frac{-2i\lambda^2}{\Delta}\left(\tau_1^{(d)}+\tau_2^{(d)}+\tau_3^{(d)}\right)}|1_1,1_2,1_3,1_4\rangle). \quad (4.23)
\end{aligned}$$

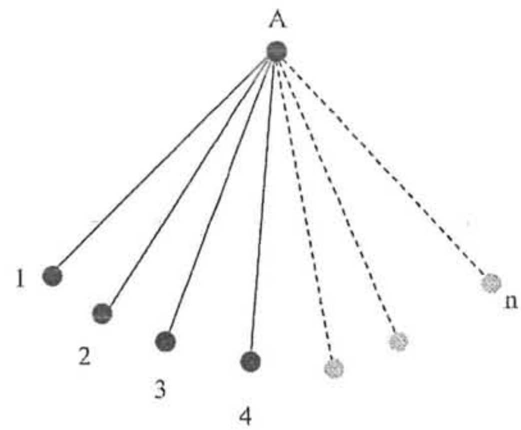

Figure 4-2: A n-partite cavity field star graph state. Dots denote vertices 1, 2, . . . , n and A which are high-Q cavities containing Fock field superpositions. Vertex A forms the sole edge of this graph.

Since in dispersive interactions we are free to select arbitrary values for the interaction times $\tau_1^{(d)}, \tau_2^{(d)}$ and $\tau_3^{(d)}$, so we may engineer weighted entangled states along with standard cluster states. The above $n = 4$ case corresponds to the four-qubit cluster state we considered in the previous section.

Now, with the availability of Ising interactions based on controlled-Z gate as described above, we are in a position to engineer any arbitrary graph state [232, 291]. Here, for the sake of demonstration, we consider the case of n-qubit star graph states (e.g., No.3(n=4), No.5(n=5), No.9(n=6) of Fig. 4 of ref. [232]. These states (Fig. 4-2.) are local unitarily equivalent to their counterpart GHZ states. For the generation of such states, we prepare cavity-1 into superposition of zero and one photon whereas the remaining $(n-1)$ cavity fields are taken to be tagged with respective atoms. Therefore the initial state for such a system will be

$$\left|\Psi^{(n)}(0)\right\rangle = \frac{1}{2^{\frac{n}{2}}} \prod_{k=2}^{n} \left(\left|0_k, l_1^{(k)}\right\rangle + \left|1_k, l_2^{(k)}\right\rangle\right) \otimes \left(|0_1\rangle + |1_1\rangle\right). \tag{4.24}$$

The procedure for the generation of desired star graph state is now quite simple. All the tagged atoms interact dispersively in a successive manner with the fields in cavity-1 and then pass through Ramsey zones, prior to their detection. The procedure finally culminates into the generation of required graph states whose weights can be chosen appropriately by proper selection of dispersive atom-field interaction times. Similarly for the generation of n-qubit ring-type graph state (e.g., No.8(n=5), No.18(n=6) of Fig. 4 of ref. [232]), we propose to prepare all n cavities in the tagged state $\frac{1}{\sqrt{2}}\left(\left|0_k, l_1^{(k)}\right\rangle + \left|1_k, l_2^{(k)}\right\rangle\right)$ where $k = 1, 2, \ldots, n$. Now k^{th} tag atom interact dispersively with the $(k-1)^{th}$-cavity field of and then traverses through a Ramsey zone. Finally atom-1 interacts dispersively with the n^{th} cavity field of and pass through a Ramsey zone. This generates the tomographically desired state by effectively closing the ring. It is now clear that an appropriate application of the collective disentanglement eraser allows one to generate any arbitrary field graph state one desires. Furthermore, the scheme can be used to generate weighted graph states [291] that result when particles and fields interact for different interaction times. This is because in our scheme dispersive atom-field interaction times can be controlled precisely and fixed to any arbitrary value we desire.

4.2 Atomic cluster and graph states in internal degrees of freedom

This section is concerned with the engineering of entangled atomic cluster states as well as the most generalized group, so called the graph states [232]. The scheme presented here is essentially a proposal utilizing cavity QED tools that employs minimum atomic and cavity resources and generates entangled cluster and graph states in atomic internal degrees of freedom. Initially we take two two-level atoms prepared independently in superposition of states $(|g_1\rangle + |e_1\rangle)/\sqrt{2}$ and $(|g_2\rangle + |e_2\rangle)/\sqrt{2}$. These atoms interact with a vacuum state cavity C in such a way that atom-1 has a resonant interaction whereas atom-2 goes through the dispersive interaction. The dispersive interaction can follows naturally if the atoms are of different kind but for identical atoms it may equally be achieved by applying localized Stark field to half of the cavity. The other alternative way is to use L shaped cavity [292] with one wing placed under the Stark field. We control interaction times corresponding to resonant and dispersive interactions such that when both the atoms exit the cavity, it is left again into vacuum state. This guarantees the generation of bipartite atomic cluster state. The same task can be done remotely by coupling two independent cavities through fiber [293, 294] but here we do not intend to invoke such

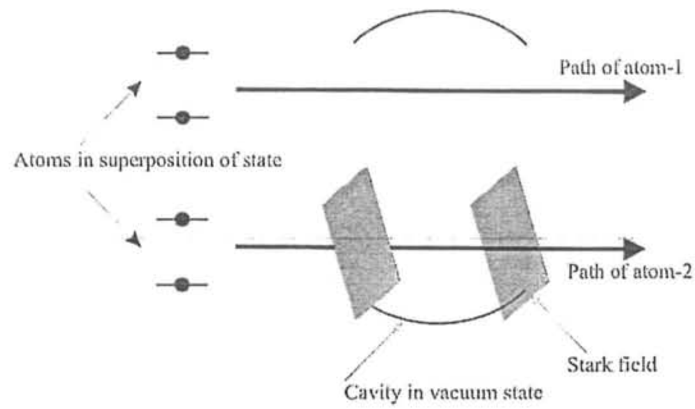

Figure 4-3: Schematic representation of the proposed atom-cavity field system. Dispersive interactions are induced in lower portion of the cavity by application of local Stark field. Whereas the two two-level atoms are prepared in internal supperposition states.

complications.

The work presented here is organized as follows. Section 4.2.1. introduces basic theme along with details of the steps needed for the generation of bipartite linear cluster state. The generation of four-partite linear cluster atomic state as a straight forward extension of two-partite case is also discussed in this section. In section 4.2.2., we generalize the schematics and propose methodologies for engineering of any arbitrary atomic graph state. Finally, in section 4.2.3., we provide an assessment of our work through success probability and fidelity of the engineered state keeping in view the present status of experimental cavity QED based research.

4.2.1 Generation of bi- and four-partite atomic cluster state

We start with two atoms initially prepared in superposition of internal state $|g_i\rangle \rightarrow (|g_i\rangle + |e_i\rangle)/\sqrt{2}$ and a single mode vacuum state cavity $|0\rangle$. The interaction scheme is as follows. Atom-1 enters in a region where the Stark field is not present (say the upper part of the cavity C), see Fig. 4-3., and interacts resonantly for a time t_1. The interaction in this case can be described by the Jaynes Cummings Hamiltonian under dipole and rotating wave approximations [235]. It is easy to find the state vector of the atom-field system after a time t_1

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}\left(|g_1,0\rangle + \cos(\mu t_1)|e_1,0\rangle - i\sin(\mu t_1)|g_1,1\rangle\right), \tag{4.25}$$

where μ is the vacuum Rabi frequency. Now atom-2 enters the region where the Stark field is present (lower part of the cavity) and goes through dispersive interactions for a time t_2 while atom-1 is still engaged in resonant interaction. Such dispersive interactions induced by the local Stark field is a well explored cavity QED technique [149, 235, 241]. Further, we assume that the atoms are far apart and dipole-dipole interaction is not present. Thus the initial state vector of the system may be expressed as

$$\begin{aligned}|\Psi(t_1, t_2 = 0)\rangle &= |\Psi(t_1)\rangle \otimes \frac{1}{\sqrt{2}}\left(|g_2\rangle + |e_2\rangle\right) \\ &= \frac{1}{2}(|g_1, g_2, 0\rangle + \cos\left(\mu t_1\right)|e_1, g_2, 0\rangle - i\sin\left(\mu t_1\right)|g_1, g_2, 1\rangle \\ &+ |g_1, e_2, 0\rangle + \cos\left(\mu t_1\right)|e_1, e_2, 0\rangle - i\sin\left(\mu t_1\right)|g_1, e_2, 1\rangle),\end{aligned} \tag{4.26}$$

Now the evolution of the system is governed by the Hamiltonian

$$H = \hbar\mu(|e_1\rangle\langle g_1|\, a + |g_1\rangle\langle e_1|\, a^\dagger) + \hbar\lambda(aa^\dagger|e_2\rangle\langle e_2| - a^\dagger a\,|g_2\rangle\langle g_2|), \tag{4.27}$$

First part of the above expression describes the resonant interactions of the atom-1 and second part represents the dispersive interaction of the atom-2. Here $\lambda = \mu^2/\Delta$ stands for the effective Rabi frequency, $a^\dagger a$ is photon number operator and Δ is atom-field detuning for the atom-2. The 2^{nd} part of above equation is written under large detuning limit. The simultaneous interactions for a time t_2 under initial conditions $C^0_{g_1,g_2}(t_1, t_2 = 0) = C^0_{g_1,e_2}(t_1, t_2 = 0) = 1/2$, $C^0_{e_1,g_2}(t_1, t_2 = 0) = C^0_{e_1,e_2}(t_1, t_2 = 0) = \cos\left(\mu t_1\right)/2$ and $C^1_{g_1,g_2}(t_1, t_2 = 0) = C^1_{g_1,e_2}(t_1, t_2 = 0) = -i\sin\left(\mu t_1\right)/2$ therefore lead to the following expressions for the probability amplitudes

$$C^0_{g_1,g_2}(t_1, t_2) = \frac{1}{2}, \tag{4.28}$$

$$C^0_{g_1,e_2}(t_1, t_2) = \frac{1}{2}e^{-i\lambda t_2}, \tag{4.29}$$

$$C^0_{e_1,g_2}(t_1, t_2) = \frac{1}{4\alpha}e^{i\frac{\lambda}{2}t_2}\left[2\alpha\cos(\mu t_1)\cos(\alpha t_2) - i\lambda\cos(\mu t_1)\sin(\alpha t_2) - 2\mu\sin(\mu t_1)\sin(\alpha t_2)\right], \tag{4.30}$$

$$C^0_{e_1,e_2}(t_1, t_2) = \frac{1}{4\alpha}e^{-i\frac{3\lambda}{2}t_2}\left[2\alpha\cos(\mu t_1)\cos\left(\alpha t_2\right) + i\lambda\cos(\mu t_1)\sin\left(\alpha t_2\right) - 2\mu\sin(\mu t_1)\sin\left(\alpha t_2\right)\right], \tag{4.31}$$

$$C^1_{g_1,g_2}(t_1,t_2) = \frac{1}{4\alpha} e^{i\frac{\lambda}{2}t_2} \left[-i2\alpha\sin(\mu t_1)\cos(\alpha t_2) + \lambda\sin(\mu t_1)\sin(\alpha t_2) - i2\mu\cos(\mu t_1)\sin(\alpha t_2)\right], \tag{4.32}$$

$$C^1_{g_1,e_2}(t_1,t_2) = \frac{1}{4\alpha} e^{-i\frac{3\lambda}{2}t_2} \left[-i2\alpha\sin(\mu t_1)\cos(\alpha t_2) - \lambda\sin(\mu t_1)\sin(\alpha t_2) + i2\mu\cos(\mu t_1)\sin(\alpha t_2)\right], \tag{4.33}$$

where $\alpha = \sqrt{\lambda^2 + 4\mu^2}/2$. It is clear from the above set of equations that except for the case of both atoms in ground state, along with the vacuum state in the cavity, all other probability amplitudes are accompanied by different phase factors that are dependent over the interaction time t_2. Thus in general the state vector cannot be expressed as product of separable states of the subsystem. This shows that procedure being considered here is potentially capable of generating entangled states for different choices of λ and t_2. Thus the condition for the generation of bi-partite linear atomic cluster state is to select the times t_1 and t_2 in such a way that after completion of interactions the cavity C should be left again in vacuum state $|0\rangle$. This implies that the probabilities corresponding to the amplitudes $C^1_{g_1,e_2}(t_1,t_2)$ and $C^1_{g_1,g_2}(t_1,t_2)$ should both be minimized near the ideal value of zero . Here various solutions can be envisioned.

When atom-1 is ahead of atom-2 with $t_1 = \pi/2\mu$. In this case $\sin(\mu t_1)\,[\cos(\mu t_1)] = 1\,[0]$ and the equations (4.30)-(4.33) become

$$C^0_{e_1,g_2}(t_1,t_2) = \frac{1}{4\alpha} e^{i\frac{\lambda}{2}t_2} \left[-2\mu\sin(\alpha t_2)\right], \tag{4.34}$$

$$C^0_{e_1,e_2}(t_1,t_2) = \frac{1}{4\alpha} e^{-i\frac{3\lambda}{2}t_2} \left[-2\mu\sin(\alpha t_2)\right], \tag{4.35}$$

$$C^1_{g_1,g_2}(t_1,t_2) = \frac{1}{4\alpha} e^{i\frac{\lambda}{2}t_2} \left[-i2\alpha\cos(\alpha t_2) + \lambda\sin(\alpha t_2)\right], \tag{4.36}$$

$$C^1_{g_1,e_2}(t_1,t_2) = \frac{1}{4\alpha} e^{-i\frac{3\lambda}{2}t_2} \left[-i2\alpha\cos(\alpha t_2) - \lambda\sin(\alpha t_2)\right], \tag{4.37}$$

We plot the probabilities corresponding to these amplitudes in Fig. 4-4 which show that of the numerical probability values for $t_2 = n\pi/2\alpha$ are 0.248781, 0.248781, 0.001219 and 0.001219, respectively. This restricts the maximum achievable success probability to about 99.7% for the present case. Here we have used the values of coupling constant $\mu = 2\pi \times 1.12 \times 10^5$ and detuning $\Delta = 2\pi \times 0.8 \times 10^6$. Further the state generated is also not ideally maximally entangled.

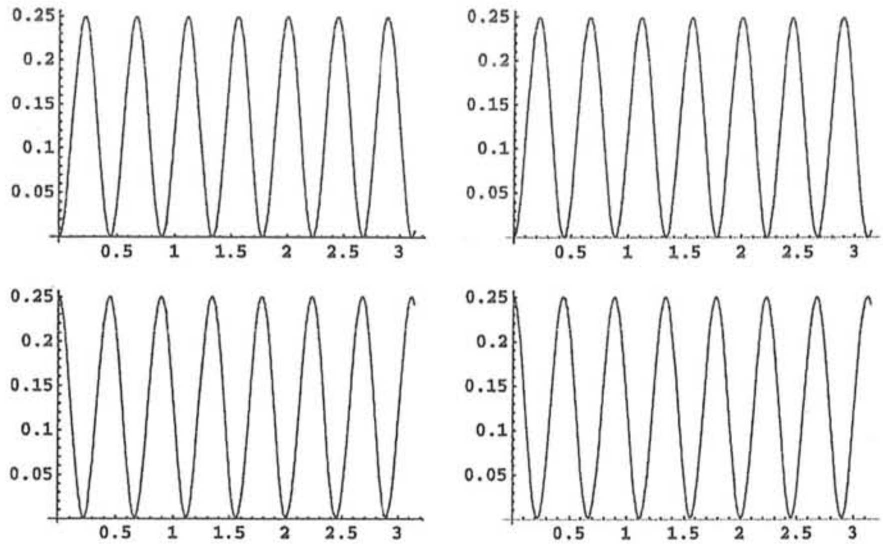

Figure 4-4: Probability plots for amplitudes $C^{0_c}_{g_1,e_2}(t_1,t_2)$, $C^{0_c}_{e_1,e_2}(t_1,t_2)$, $C^{1_c}_{g_1,g_2}(t_1,t_2)$ and $C^{1_c}_{g_1,e_2}(t_1,t_2)$ in (a), (b), (c) and (d) versus interaction time t_2. Here we fix the interaction time of atom-1 $t_1 = \pi/2\mu$ before the entrance of atom-2 into the dispersive region.

When atom-1 and atom-2 are launched simultaneously $t_1 = 0$ In this case $\sin(\mu t_1)$ $[\cos(\mu t_1)] = 0\,[1]$ and therefore equations (4.30)-(4.33) simplify to

$$C^{0}_{e_1,g_2}(t_1,t_2) = \frac{1}{4\alpha}e^{i\frac{\lambda}{2}t_2}\left[2\alpha\cos(\alpha t_2) - i\lambda\sin(\alpha t_2)\right], \tag{4.38}$$

$$C^{0}_{e_1,e_2}(t_1,t_2) = \frac{1}{4\alpha}e^{-i\frac{3\lambda}{2}t_2}\left[2\alpha\cos(\alpha t_2) + i\lambda\sin(\alpha t_2)\right], \tag{4.39}$$

$$C^{1}_{g_1,g_2}(t_1,t_2) = \frac{1}{4\alpha}e^{i\frac{\lambda}{2}t_2}\left[-i2\mu\sin(\alpha t_2)\right], \tag{4.40}$$

$$C^{1}_{g_1,e_2}(t_1,t_2) = \frac{1}{4\alpha}e^{-i\frac{3\lambda}{2}t_2}\left[i2\mu\sin(\alpha t_2)\right], \tag{4.41}$$

It is clear from this set of equations that when both atoms are injected into the cavity simultaneously then unit success probability is achievable. Because the probability of $C^{1}_{g_1,e_2}(t_1,t_2)$ and $C^{1}_{g_1,g_2}(t_1,t_2)$ becomes zero at $t_2 = n\pi/2\alpha$ for $n = 2, 4, 6, ...$

Now instead of discussing the particular cases we plot the probabilities corresponding to Eqs. (4.30)-(4.33) in Fig. 4-5. (a) (b) (c) and (d), respectively, verses interaction times t_1 and t_2 varying from zero to $\pi/2$ using the experimental parameters employed by Rempe and co-workers for quantum optical exploration of ^{85}Rb atoms [252, 255, 271, 272, 279]. In Fig. 4-5. (a) and (b),

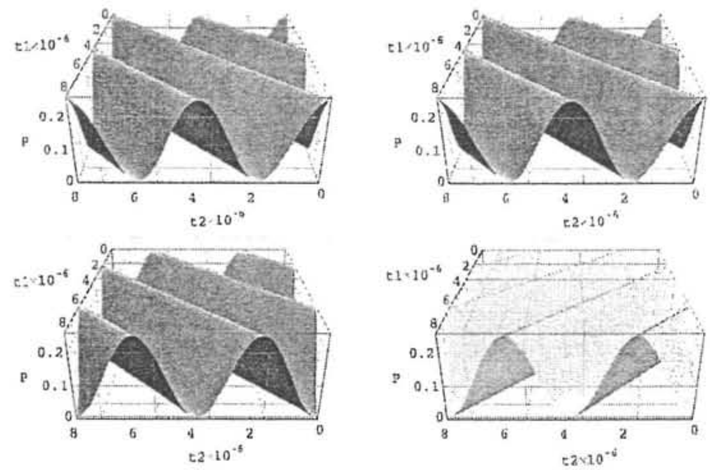

Figure 4-5: Probability plots for amplitudes $C^{0_c}_{g1,e2}(t_1,t_2)$, $C^{0_c}_{e1,e2}(t_1,t_2)$, $C^{1_c}_{g1,g2}(t_1,t_2)$ and $C^{1_c}_{g1,e2}(t_1,t_2)$ in (a), (b), (c) and (d) versus interaction time t_1 and t_2 showing that the (a) and (b) bear same trend whereas (c) and (d) exhibit opposite tendency. This also illustrates the probability accumulation for the above stated amplitudes with variations in the interaction time t_1

the probabilities $\left|C^{0}_{e1,g2}(t_1,t_2)\right|^2$ and $\left|C^{0}_{e1,e2}(t_1,t_2)\right|^2$ bears the same trend and achieve maxima at $t_2 = n\pi/2\alpha$ where as the probabilities $\left|C^{1}_{g1,g2}(t_1,t_2)\right|^2$ and $\left|C^{1}_{g1,e2}(t_1,t_2)\right|^2$ plotted in (c) and (d) exhibit the opposite trend and have minima at said points. So when the probabilities corresponding to equations (4.32) and (4.33) go to zero the probabilities corresponding to equations (4.30) and (4.31) reach the highest value of 0.25.

Here we consider the case where $t_1 = 0$ and $t_2 = n\pi/2\alpha$. For $n = 2$ the remaining probability amplitudes simplify to yield following expression for the state vector

$$|\Psi_{12}\rangle = \frac{1}{2}(\Big(|g_1,g_2\rangle - e^{i\pi\xi}|e_1,g_2\rangle + e^{-2i\pi\xi}|g_1,e_2\rangle - e^{-3i\pi\xi}|e_1,e_2\rangle\Big), \tag{4.42}$$

where $\xi = \lambda/2\alpha$. In equation (4.42) we have traced out the cavity field $|0\rangle$. This expression represent maximally entangled bi-partite atomic cluster state. Entangled nature of above state is evident because $\left|C^{0}_{g1,g2}C^{0}_{e1,e2} - C^{0}_{e1,g2}C^{0}_{g1,e2}\right| \neq 0$ for any arbitrary values of μ and Δ.

The technique presented here for the generation of bi-partite atomic cluster state may equally be employed as the most simple and resource economical method to engineer Bell states by applying local Hadamard transform on any one of the two atomic qubits in equation

(4.42) forming the bi-partite cluster state.

We may extend the same method, using bipartite atomic cluster state (i.e., expression (4.42)) as the basic building block, to produce a four-partite atomic cluster state as follows. We generate another bipartite cluster state between atom-3 and atom-4 following the procedure described above such that atom-3 interacts resonantly whereas atom-4 interacts dispersively with an initially vacuum state cavity C. The expression for the engineered state is

$$|\Psi_{34}\rangle = \frac{1}{2}(|g_3, g_4\rangle - e^{i\pi\xi}|e_3, g_4\rangle + e^{-2i\pi\xi}|g_3, e_4\rangle - e^{-3i\pi\xi}|e_3, e_4\rangle . \quad (4.43)$$

The combined state of the four atoms at this stage is a product state of the two entangled state, i.e., $|\Psi_{1234}(0)\rangle = |\Psi_{12}\rangle \otimes |\Psi_{34}\rangle$. We use this initial product state to generate a four-partite cluster state by allowing atom-2 and atom-3 to interact simultaneously with the initially vacuum state cavity C_3, in such a way that atom-2 interacts resonantly while atom-3 interacts dispersively. For the interaction time $t_{23} = n\pi/2\alpha$ with $n = 2$ the four-partite linear atomic cluster state obtained is

$$\begin{aligned}|\Psi_{1234}\rangle = \frac{1}{4}\{&\left|L^{g_3,g_4}_{g_1,g_2}\right\rangle - e^{-i\pi\xi}\left|L^{e_3,g_4}_{g_1,g_2}\right\rangle + e^{-2i\pi\xi}\left|L^{g_3,e_4}_{g_1,g_2}\right\rangle - e^{-5i\pi\xi}\left|L^{e_3,e_4}_{g_1,g_2}\right\rangle \\ &- e^{i\pi\xi}\left|L^{g_3,g_4}_{e_1,g_2}\right\rangle + \left|L^{e_3,g_4}_{e_1,g_2}\right\rangle - e^{-i\pi\xi}\left|L^{g_3,e_4}_{e_1,g_2}\right\rangle + e^{-4i\pi\xi}\left|L^{e_3,e_4}_{e_1,g_2}\right\rangle \\ &- e^{-i\pi\xi}\left|L^{g_3,g_4}_{g_1,e_2}\right\rangle + e^{-4i\pi\xi}\left|L^{e_3,g_4}_{g_1,e_2}\right\rangle - e^{-3i\pi\xi}\left|L^{g_3,e_4}_{g_1,e_2}\right\rangle + e^{-8i\pi\xi}\left|L^{e_3,e_4}_{g_1,e_2}\right\rangle \\ &+ e^{-2i\pi\xi}\left|L^{g_3,g_4}_{e_1,e_2}\right\rangle - e^{-5i\pi\xi}\left|L^{e_3,g_4}_{e_1,e_2}\right\rangle + e^{-4i\pi\xi}\left|L^{g_3,e_4}_{e_1,e_2}\right\rangle - e^{-9i\pi\xi}\left|L^{e_3,e_4}_{e_1,e_2}\right\rangle\}. \quad (4.44)\end{aligned}$$

The notation$\left|L^{l,m}_{j,k}\right\rangle = |j, k, l, m\rangle$ with $j, k, l, m = g_1\,(e_1)\,, g_2\,(e_2)\,, g_3(e_3), g_4\,(e_4)$ is adapted here for the sake of mathematical brevity. The very same method mentioned above may be extended to generate any n-partite linear atomic cluster state within allowed span of decoherence time. Further interesting point to note is that theoretically there is no constraint on the resonant/dispersive interactional sequence of atoms with the vacuum state cavities. Thus the interactions may be initiated in any sequence deemed experimentally feasible.

4.2.2 Generation of atomic graph states

The technique employed to yield controlled phase operation for engineering of cluster states can be straightforwardly generalized to cover the generation of any arbitrary graph state [232]. This is because, now we have, at our disposal the Ising interactions comprised of controlled phase operation defined by; $|g,g\rangle \rightarrow |g,g\rangle$, $|g,e\rangle \rightarrow e^{-2i\pi\xi}|g,e\rangle$, $|e,g\rangle \rightarrow -e^{i\pi\xi}|e,g\rangle$ and $|e,e\rangle \rightarrow -e^{-3i\pi\xi}|e,e\rangle$. Here, as mentioned earlier, $\xi = \lambda/2\alpha$.

Now in order to generate n-partite atomic graph state, we take n atomic qubits of two-level atoms in independent superpositions. The initial state needed for the generation of n-partite graph state may be expressed as

$$\left|\Psi^{(n)}(0)\right\rangle = \frac{1}{2^{\frac{n}{2}}}\prod_{j=1}^{n}\left(|g_j\rangle + |e_j\rangle\right). \tag{4.45}$$

Here, as an example, we present the engineering schematics of only two relatively simple atomic graph states. a). Five-partite ring graph state (Fig.1, sketch-1 of Ref [232]) and, b) Four-partite star graph state.

Five-partite ring graph state

For five-partite atomic ring graph state, we take the initial state to be

$$\left|\Psi^{(5)}(0)\right\rangle = \frac{1}{2^{\frac{5}{2}}}\prod_{j=1}^{5}\left(|g_j\rangle + |e_j\rangle\right)\otimes|0_c\rangle. \tag{4.46}$$

Atom-1 and atom-2 are then simultaneously sent into an initially vacuum state cavity i.e. $|0_c\rangle$, such that atom-1 interacts resonantly whereas atom-2 goes through dispersive interactions. These interactions, lasting for a time $t_{12} = \pi/\alpha$ leads to the state vector

$$\left|\Psi^{(5)}(t_{12})\right\rangle = \frac{1}{2^{\frac{5}{2}}}\left(|g_1,g_2\rangle - e^{i\pi\xi}|e_1,g_2\rangle + e^{-2i\pi\xi}|g_1,e_2\rangle - e^{-3i\pi\xi}|e_1,e_2\rangle\right)\otimes\prod_{j=3}^{5}\left(|g_j\rangle + |e_j\rangle\right). \tag{4.47}$$

After completion of the interaction, both the atoms exit the cavity, leaving it into vacuum state $|0_c\rangle$ that we have traced out of the above expression. Further since at the termination of each interaction the cavity field comes to be in vacuum, i.e. $|0_c\rangle$, factorizable from the state expression and henceforth it will not be mentioned in subsequent calculations and discussions. In next step, atom-2 and atom-3 interact with the cavity such that atom-2 now performs resonant interaction but atom-3 at the same time interact dispersively. Again after an interaction time $t_{23} = \pi/\alpha$, both the atoms exit the cavity (leaving it in vacuum state) and the state vector becomes

$$\begin{aligned}
\left|\Psi^{(5)}(t_{12}, t_{23})\right\rangle &= \frac{1}{2^{\frac{5}{2}}}(|g_1, g_2, g_3\rangle - e^{i\pi\xi}|e_1, g_2, g_3\rangle - e^{-i\pi\xi}|g_1, e_2, g_3\rangle + e^{-2i\pi\xi}|e_1, e_{2,}, g_3\rangle \\
&+ e^{-2i\pi\xi}|g_1, g_2, e_3\rangle - e^{-i\pi\xi}|e_1, g_2, e_3\rangle - e^{-5i\pi\xi}|g_1, e_2, e_3\rangle + e^{-6i\pi\xi}|e_1, e_2, e_3\rangle \\
&\otimes \prod_{j=4}^{5}(|g_j\rangle + |e_j\rangle). \qquad (4.48)
\end{aligned}$$

Next, the same procedure is repeated for atom-3 and atom-4 such that atom-3 interacts resonantly and atom-4 interact dispersively. As soon as atom-3 and atom-4 exit the cavity leaving it again into vacuum, the interaction of atom-4 and atom-5 is initiated that thoroughly follows the pattern described earlier. Thus simultaneous interactions of atom-4 (resonant) and atom-5 (dispersive) for a time again equal to π/α leads to the following state vector

$$\begin{aligned}
\left|\Psi^{(5)}(t_{\gamma\theta})\right\rangle &= \frac{1}{2^{\frac{5}{2}}}(\left|L^{g_4,g_5}_{g_1,g_2,g_3}\right\rangle - e^{i\pi\xi}\left|L^{g_4,g_5}_{e_1,g_2,g_3}\right\rangle - e^{-i\pi\xi}\left|L^{g_4,g_5}_{g_1,e_2,g_3}\right\rangle + e^{-2i\pi\xi}\left|L^{g_4,g_5}_{e_1,e_2,,g_3}\right\rangle \\
&- e^{-i\pi\xi}\left|L^{g_4,g_5}_{g_1,g_2,e_3}\right\rangle + \left|L^{g_4,g_5}_{e_1,g_2,e_3}\right\rangle + e^{-4i\pi\xi}\left|L^{g_4,g_5}_{g_1,e_2,e_3}\right\rangle - e^{-5i\pi\xi}\left|L^{g_4,g_5}_{e_1,e_2,e_3}\right\rangle \\
&- e^{-i\pi\xi}\left|L^{e_4,g_5}_{g_1,g_2,g_3}\right\rangle + \left|L^{e_4,g_5}_{e_1,g_2,g_3}\right\rangle + e^{-2i\pi\xi}\left|L^{e_4,g_5}_{g_1,e_2,g_3}\right\rangle - e^{-3i\pi\xi}\left|L^{e_4,g_5}_{e_1,e_2,g_3}\right\rangle \\
&+ e^{-4i\pi\xi}\left|L^{e_4,g_5}_{g_1,g_2,e_3}\right\rangle - e^{-3i\pi\xi}\left|L^{e_4,g_5}_{e_1,g_2,e_3}\right\rangle - e^{-7i\pi\xi}\left|L^{e_4,g_5}_{g_1,e_2,e_3}\right\rangle + e^{-8i\pi\xi}\left|L^{e_4,g_5}_{e_1,e_2,e_3}\right\rangle \\
&+ e^{-2i\pi\xi}\left|L^{g_4,e_5}_{g_1,g_2,g_3}\right\rangle - e^{-i\pi\xi}\left|L^{g_4,e_5}_{e_1,g_2,g_3}\right\rangle - e^{-3i\pi\xi}\left|L^{g_4,e_5}_{g_1,e_2,g_3}\right\rangle + e^{-4i\pi\xi}\left|L^{g_4,e_5}_{e_1,e_2,g_3}\right\rangle \\
&- e^{-3i\pi\xi}\left|L^{g_4,e_5}_{g_1,g_2,e_3}\right\rangle + e^{-2i\pi\xi}\left|L^{g_4,e_5}_{e_1,g_2,e_3}\right\rangle + e^{-6i\pi\xi}\left|L^{g_4,e_5}_{g_1,e_2,e_3}\right\rangle - e^{-7i\pi\xi}\left|L^{g_4,e_5}_{e_1,e_2,e_3}\right\rangle \\
&- e^{-5i\pi\xi}\left|L^{e_4,e_5}_{g_1,g_2,g_3}\right\rangle + e^{-4i\pi\xi}\left|L^{e_4,e_5}_{e_1,g_2,g_3}\right\rangle + e^{-6i\pi\xi}\left|L^{e_4,e_5}_{g_1,e_2,g_3}\right\rangle - e^{-7i\pi\xi}\left|L^{e_4,e_5}_{e_1,e_2,g_3}\right\rangle \\
&+ e^{-8i\pi\xi}\left|L^{e_4,e_5}_{g_1,g_2,e_3}\right\rangle - e^{-7i\pi\xi}\left|L^{e_4,e_5}_{e_1,g_2,e_3}\right\rangle - e^{-11i\pi\xi}\left|L^{e_4,e_5}_{g_1,e_2,e_3}\right\rangle + e^{-12i\pi\xi}\left|L^{e_4,e_5}_{e_1,e_2,e_3}\right\rangle), \qquad (4.49)
\end{aligned}$$

where $(\gamma, \theta) = (1,2), (2,3), (3,4), (4,5)$ stand for the interactional sequence of atoms and $\left|L^{m,n}_{j,k,l}\right\rangle = |j, k, l, m, n\rangle$ abbreviate Hilbert space vectors with j, k, l, m, n again representing the atomic internal state indices for first, second, third, fourth and fifth atoms respectively.

Now finally in order to close the loop leading to the generation of desired five-partite ring atomic graph state, we pass atom-5 and atom-1 through a similar initially vacuum state cavity. This is necessary because we have to contact vertex-5 and vertex-1 of the atomic graph so that it converges to a ring morphism. At this final stage, atom-5 partake in resonant interaction whereas atom-1 at the same time goes through dispersive interactions. When interaction time is taken to be $t_{51} = \pi/\alpha$, then we get the desired ring graph state with, in principle, unit success probability. Thus the final state comes to be

$$\begin{aligned}\left|\Psi^{(5)}(t_{\gamma\theta})\right\rangle = \frac{1}{2^{\frac{5}{2}}}(&\left|L^{g_4,g_5}_{g_1,g_2,g_3}\right\rangle - e^{-i\pi\xi}\left|L^{g_4,g_5}_{e_1,g_2,g_3}\right\rangle - e^{-i\pi\xi}\left|L^{g_4,g_5}_{g_1,e_2,g_3}\right\rangle + e^{-4i\pi\xi}\left|L^{g_4,g_5}_{e_1,e_2,,g_3}\right\rangle \\ &- e^{-i\pi\xi}\left|L^{g_4,g_5}_{g_1,g_2,e_3}\right\rangle + e^{-2i\pi\xi}\left|L^{g_4,g_5}_{e_1,g_2,e_3}\right\rangle + e^{-4i\pi\xi}\left|L^{g_4,g_5}_{g_1,e_2,e_3}\right\rangle - e^{-7i\pi\xi}\left|L^{g_4,g_5}_{e_1,e_2,e_3}\right\rangle \\ &- e^{-i\pi\xi}\left|L^{e_4,g_5}_{g_1,g_2,g_3}\right\rangle + e^{-2i\pi\xi}\left|L^{e_4,g_5}_{e_1,g_2,g_3}\right\rangle + e^{-2i\pi\xi}\left|L^{e_4,g_5}_{g_1,e_2,g_3}\right\rangle - e^{-5i\pi\xi}\left|L^{e_4,g_5}_{e_1,e_2,g_3}\right\rangle \\ &+ e^{-4i\pi\xi}\left|L^{e_4,g_5}_{g_1,g_2,e_3}\right\rangle - e^{-5i\pi\xi}\left|L^{e_4,g_5}_{e_1,g_2,e_3}\right\rangle - e^{-7i\pi\xi}\left|L^{e_4,g_5}_{g_1,e_2,e_3}\right\rangle + e^{-10i\pi\xi}\left|L^{e_4,g_5}_{e_1,e_2,e_3}\right\rangle \\ &- e^{-i\pi\xi}\left|L^{g_4,e_5}_{g_1,g_2,g_3}\right\rangle + e^{-4i\pi\xi}\left|L^{g_4,e_5}_{e_1,g_2,g_3}\right\rangle + e^{-2i\pi\xi}\left|L^{g_4,e_5}_{g_1,e_2,g_3}\right\rangle - e^{-7i\pi\xi}\left|L^{g_4,e_5}_{e_1,e_2,g_3}\right\rangle \\ &+ e^{-2i\pi\xi}\left|L^{g_4,e_5}_{g_1,g_2,e_3}\right\rangle - e^{-5i\pi\xi}\left|L^{g_4,e_5}_{e_1,g_2,e_3}\right\rangle - e^{-5i\pi\xi}\left|L^{g_4,e_5}_{g_1,e_2,e_3}\right\rangle + e^{-10i\pi\xi}\left|L^{g_4,e_5}_{e_1,e_2,e_3}\right\rangle \\ &+ e^{-4i\pi\xi}\left|L^{e_4,e_5}_{g_1,g_2,g_3}\right\rangle - e^{-7i\pi\xi}\left|L^{e_4,e_5}_{e_1,g_2,g_3}\right\rangle - e^{-5i\pi\xi}\left|L^{e_4,e_5}_{g_1,e_2,g_3}\right\rangle + e^{-10i\pi\xi}\left|L^{e_4,e_5}_{e_1,e_2,g_3}\right\rangle \\ &- e^{-7i\pi\xi}\left|L^{e_4,e_5}_{g_1,g_2,e_3}\right\rangle + e^{-10i\pi\xi}\left|L^{e_4,e_5}_{e_1,g_2,e_3}\right\rangle + e^{-10i\pi\xi}\left|L^{e_4,e_5}_{g_1,e_2,e_3}\right\rangle - e^{-15i\pi\xi}\left|L^{e_4,e_5}_{e_1,e_2,e_3}\right\rangle).\end{aligned} \tag{4.50}$$

where $(\gamma, \theta) = (1,2), (2,3), (3,4), (4,5)$ and $(5,1)$.

Four-partite star graph state

The star states bear specific relevance to quantum informatics because such states are LU-equivalent to GHZ states. The initial state for the generation of four-partite star graph state may be expressed as

$$\left|\Psi^{(4)}(0)\right\rangle = \frac{1}{4}\prod_{j=1}^{4}(|g_j\rangle + |e_j\rangle) \otimes |0_c\rangle. \tag{4.51}$$

Here, for the sake of brevity, we produce only the state engineering schematics along with the final expression of the state. In this case the qubit defined by atom-1 will serve as the sole vertex of the atomic star graph whereas all other qubits for $j = 2,3,4$ will form the distinct edges, all connected independently to first qubit (i.e. atom-1). In order to achieve this, we first simultaneously send atom-1 and atom-2 into the initially vacuum state cavity. In the cavity, atoms collectively engage themselves into the interactions that are resonant for atom-1 and dispersive for atom-2, for a time $t_{12} = \pi/\alpha$. However after the lapse of this time, only atom-2 exit while leaving atom-1 still in the cavity. Then atom-3 enters to perform dispersive interaction. Thus the next interaction, again lasting for $t_{13} = \pi/\alpha$, follows the same pattern with atom-1 going through resonant interactions but atom-3 interacting dispersively. Finally, when the atom-3 exit the cavity after duly completing its dispersive interactions, atom-4 enters into the cavity. Please note that during this whole process, atom-1 remains continuously engaged in resonant interactions inside the cavity. At the end, after an interaction time $t_{14} = \pi/\alpha$, both the atoms emerge out of the cavity, leaving it into vacuum. This generates the desired four-partite atomic ring graph state. The final expression for such a state is given by

$$\begin{aligned}\left|\Psi^{(4)}(t_{1,\beta})\right\rangle = \frac{1}{4}(&\left|L^{g_3,g_4}_{g_1,g_2}\right\rangle - e^{3i\pi\xi}\left|L^{g_3,g_4}_{e_1,g_2}\right\rangle + e^{-2i\pi\xi}\left|L^{g_3,g_4}_{g_1,e_2}\right\rangle - e^{-i\pi\xi}\left|L^{g_3,g_4}_{e_1,e_2}\right\rangle \\ &+ e^{-2i\pi\xi}\left|L^{e_3,g_4}_{g_1,g_2}\right\rangle - e^{-i\pi\xi}\left|L^{e_3,g_4}_{e_1,g_2}\right\rangle + e^{-4i\pi\xi}\left|L^{e_3,g_4}_{g_1,e_2}\right\rangle - e^{-5i\pi\xi}\left|L^{e_3,g_4}_{e_1,e_2}\right\rangle \\ &+ e^{-2i\pi\xi}\left|L^{g_3,e_4}_{g_1,g_2}\right\rangle - e^{-i\pi\xi}\left|L^{g_3,e_4}_{e_1,g_2}\right\rangle + e^{-4i\pi\xi}\left|L^{g_3,e_4}_{g_1,e_2}\right\rangle - e^{-5i\pi\xi}\left|L^{g_3,e_4}_{e_1,e_2}\right\rangle \\ &+ e^{-4i\pi\xi}\left|L^{e_3,e_4}_{g_1,g_2}\right\rangle - e^{-5i\pi\xi}\left|L^{e_3,e_4}_{e_1,g_2}\right\rangle + e^{-6i\pi\xi}\left|L^{e_3,e_4}_{g_1,e_2}\right\rangle - e^{-9i\pi\xi}\left|L^{e_3,e_4}_{e_1,e_2}\right\rangle). \qquad (4.52)\end{aligned}$$

Here $\beta = 2, 3, 4$ and stands for the dispersive interactions of atom-2, atom-3 and atom-4 respectively.

4.2.3 Success probability and fidelity

Although in principle the proposed scheme is deterministic with unit success probability but practically it may be affected by various parameters including uncontrollable atomic velocity spread, coupling dispersion, operational imprecisions in the interaction times of the atoms, cavity decay and delays in simultaneous injection of atoms into the cavity. However, the most crucial ones in the present situation are delays in the simultaneous injection of atoms δt_d into the cavity and imprecision in atom-cavity interaction time δt_{int} because most of the factors cited above may be effectively incorporated into δt_d and δt_{int}. Success probability P_s of the basic unit i.e. two-partite atomic cluster state versus delay δt_d in simultaneous injection of

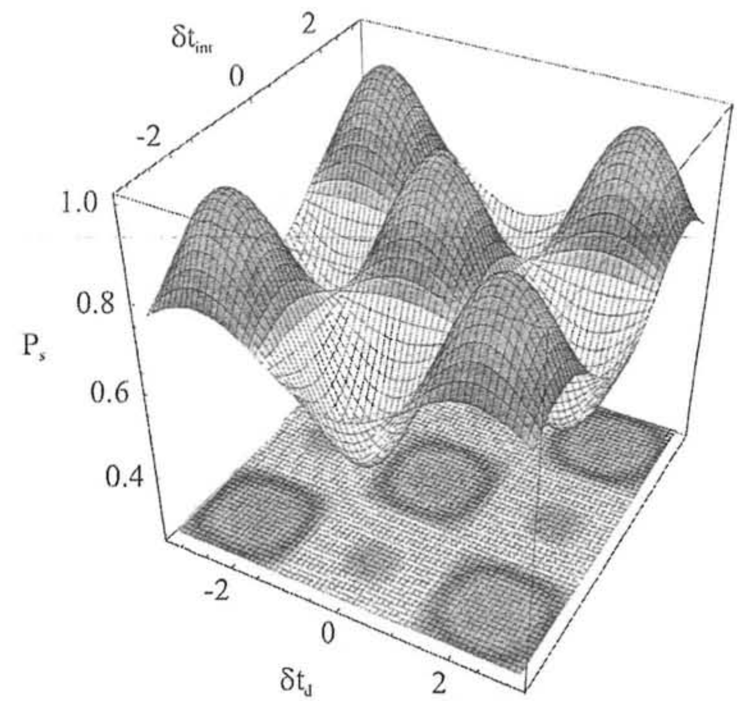

Figure 4-6: Success probability P_s for generation of bipartite atomic cluster state versus delay in simultaneous atomic injection $10^{-6} \times \delta t_d$ and imprecision in interaction time $10^{-6} \times \delta t_{int}$.

atoms and imprecision in quantized atom-field interaction time δt_{int} is given by

$$P_s = \frac{3(4\Delta^2 + \mu^2) + \cos(2\mu\delta t_d)(\mu^2 + 4\Delta^2 \cos(2\alpha\delta t_{int})}{4(4\Delta^2 + \mu^2)} \tag{4.53}$$

In Fig. 4-6., we show success probability P_s versus δt_d and δt_{int} by employing the experimental parameters cited in [252, 255, 271, 272, 279]. Whereas in Fig. 4-7., we plot the normalized fidelity using the same parameters as in the case of success probability. We note that main factor contributing to these uncontrollable imperfections basically owes to the non-monochromatic nature of atomic beams. In this respect A. Rauschenbeutel et al. [149] have employed an oven-based atomic beam technology that produced atoms with a velocity spread of $\pm 2m/s$ when an average atomic velocity was taken to be $503m/s$. However, even better control over atomic motion is now available through state-of-the art magneto-optical traps [295, 296].

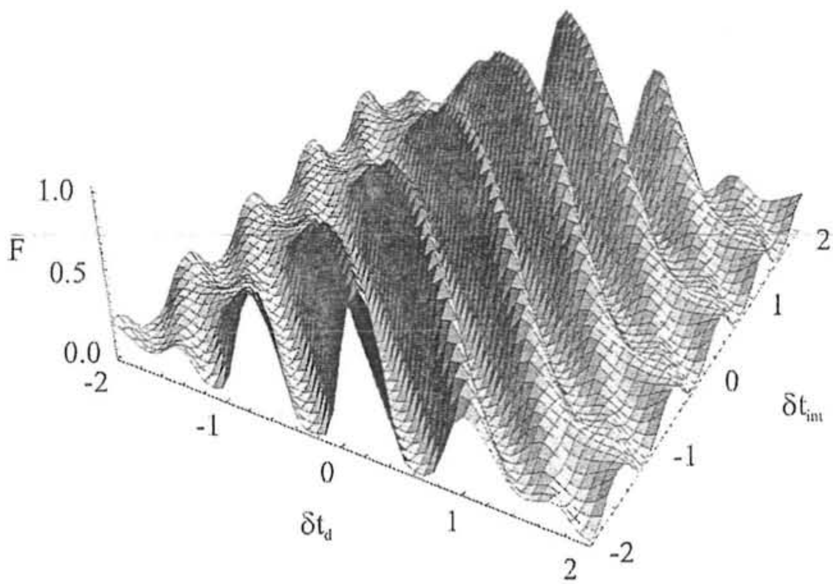

Figure 4-7: Fidelity of bipartite atomic cluster state versus delay in simultaneous atomic injection $10^{-5} \times \delta t_d$ and imprecision in interaction time $10^{-5} \times \delta t_{int}$.

4.3 Experimental feasibility and general discussions

In this chapter we have proposed two theoretical schemes for preparation of field as well as atomic cluster states. Both the schemes were generalized to cover the engineering aspects of any arbitrary respective graph state and are based on standard cavity QED tools along with Ramsey interferometry. Another common feature of the proposals is the employment of resonant as well as dispersive atom-field interactions in a fully controllable cavity QED environment. First scheme utilizes the concept of collective eraser to generate cavity field cluster and graph states . Here we have demonstrated that operation of a disentanglement eraser not only restores coherence of the original untagged state but can also bind coherent subsystems bearing independent tags into an extended single coherent entangled system occupying effectively doubled Hilbert space, if performed collectively over a set of mutually factorizable states. A new feature in our scheme originates from an addition of one more pair of entangled cavity fields to the scheme of Zubairy et al.[241]. By tagging each pair of the cavity fields with an atom and exploiting interaction of the two tagged atoms with a common radiation field, two independent pairs of entangled fields have been transformed to four fields all entangled with one another in the form of the four-field cluster state. Similarly in second scheme that aims to engineer atomic cluster and graph states, the method proposed for Ising interactions to join graph vertices is based on controlled phase operation which is resource economical and can be implemented using presently available cavity QED resources. The method utilizes simultaneous resonant and dispersive interactions of two two-level atoms with an initially vacuum state high-Q cavity for a predetermined time. At the end, the cavity is again left into vacuum state while atoms get duly entangled into the desired state. Thus the cavity is automatically recycled and is readily available for further interactions, if needed. Its economical nature also make it the most easy technique to generate Bell states from two-partite cluster states that are local-unitarily equivalent to the Bell states. One of the important feature is the interaction pattern of atoms invoked in the present scheme. In most cases, each atom successively goes through dispersive interactions followed by the resonant one. This effectively refresh the quantum memory of the atom and therefore helps to sustain the state coherence for comparatively longer duration.

Present cavity QED research scenario [24, 240] exhibit promising features for experimental execution of the proposed schemes. Microwave cavities with quality factor as high as 10^{12} has been reported that straightforwardly leads to cavity life-times in the range of seconds [138, 297]. Such large cavity life-times are many order of magnitude higher than usual atom-field interaction times. All the techniques invoked in present proposals have already been perfected experimentally [96, 146]. Ramsey interferometry used to produce atomic superpositions is also one of the standard tool extensively employed to supplement cavity QED based experimental research [146]. Dispersive interactions initiated with the aid of Stark field has also proved its effectiveness beyond doubt [24, 146, 149] and along with usual applications, the success list also includes the beautiful demonstration of quantum phase gate based on dispersive atom-field interactions [90]. Furthermore, in case of the first scheme, Zubairy et al. [241] have already proved the feasibility of tag addition and removal over individual Bell states. These atomic tags, however, can also be implemented through resonant interactions on atomic transition $\left|l_2^{(T_q)}\right\rangle \to \left|l_3^{(T_q)}\right\rangle$ for a time corresponding to a 2π Rabi rotation. Since a resonant 2π transition requires a comparatively smaller interaction time than a dispersive interaction, such an alteration in the scheme will make it more resistant to decoherence and experimentally more feasible [241]. This

2π-phase flip has already been experimentally demonstrated [298]. Although in first proposal, we have mentioned five cavities but only three of them take part in the atom-field interactions. Thus, if we take two-mode entangled state in a single cavity [149, 292], then the same task of four-field cluster state generation can be accomplished by using only three cavities. Indeed cavity QED based proposals bearing equivalent or even higher complexity have already been published much earlier [299]. Second proposal, on the other hand, require bare minimum cavity resources and in most cases a single high-Q cavity suffices to complete the job. Furthermore, sufficiently precise control over synchronized atomic dynamics has been achieved in recent past [142, 295, 300, 301]. Last but no way least, cavity QED technology has also proved its operational smartness at the lowest quantum level [142, 302, 303]. In summary, the overall cited situation manifestly justifies our optimism about the envisioned experimental feasibility of the proposals.

Chapter 5

Summary and Conclusions

The present thesis is concerned with the generation of entangled states in quantum optics. In first chapter we have given a brief historical review about the advent of quantum theory and subsequent emergence of the concept of entanglement. Keeping the vitality of the subject in view, Bohr-Einstein debate has been given due elucidation. Main turning point in this context i.e. the publication of the Bell's theorem, its philosophical ramifications and experimental violation of the envisioned inequality have also been discussed at length here. Second part of the chapter deals solely with the applicative side of the phenomenon of entanglement. Here we have reviewed Quantum Cryptography, Quantum Computation and Teleportation as prototype examples to mark the power and resourcefulness of entangled correlations for the emerging field of Quantum Informatics. Both theoretical as well as experimental perspective have been covered here to make the discussion comprehensive and meaningful. Recent trends in entanglement based research like, for example, dependence of phase transitions in condensed matter on entanglement, have also been mentioned here.

First part of the second chapter is then devoted to a comprehensive review of entanglement engineering in cavity QED based systems. Various strategies adapted so far for the production of entangled states with controlled atom-field interactions in cavity QED background have been mentioned with citation of the relevant literature. The review being presented here encapsulates almost all types of entangled states including Bell, NOON, W, Cluster and Graph states. This chapter, being introductory in essence, also embarks upon a quick review of mathematical tools that were later on employed for the state engineering. Here along with brief description of atom-field interactions in cavity QED, we have also given a detailed exposition of the Bragg scattering of atomic de Broglie wavepackets. In this respect, off-resonant interactions were opted for state engineering in order to keep the decoherence linked with the spontaneous emission at the minimum level. Various working approximations have been duly mentioned and incorporated into the flow of mathematical treatment of the subject.

Engineering of cavity field entangled states including Bell, NOON and W states relie on the off-resonant Bragg diffraction of atomic de Broglie wavepackets in Mach-Zenhder-Bragg (MZB) interferometric geometries as discussed in chapter 3. We employ both single as well as multiple loop atomic MZB interferometers for the generation of desired states in high-Q cavities placed in the arms of the interferometers. Here we have elaborated at length on the modelling of the architectural components like atomic mirrors and beam splitters of the interferometers. In most cases, a single split atomic de Broglie wavepacket suffice for the purpose of state engineering. Upto our best knowledge, this the first work that envisions atomic interferometry as a potential tool for entangled state generation. In essence, the proposed schematics are geometrically similar to the Bell multiport designs employed in optical systems for the production of photon's flying qubit entangled states [304, 305]. However, there are few profound differences that need to be pointed out: First and for most, the multiport MZB loops employed here are, in principle, deterministic when viewed from the perspective of state engineering whereas their counterparts in photonics are inherently probabilistic because state engineering procedure thereof depends mostly on post-selective mechanisms. Second, in contrast with the less efficient photodetection, the atomic detection is almost ideal. This chapter ends with a mention of experimental parameters needed for the practical implementation of the proposed schemes, using presently available laboratory setups. During present study, it was however noted that Bragg diffraction of neutral atoms can serve as a good pathway for state engineering related to both atomic momenta states as well as cavity field states. Its relevance becomes even more evident when viewed in the background of decoherence, a general threat for all quantum informatics tasks. This is because, as mentioned earlier, matter waves (i.e. neutral atomic de Broglie wavepackets) can retain their coherence up to distances of many meters in clean and noise free environment [277, 278]. Therefore, work with atom interferometry in Bragg domain is worth pursuing and it may serve as an alternative strategy against decoherence for the subject of applied quantum information. Apart from present thesis, we have also done some further work in this regard [306, 307, 308, 309, 310, 311] but the field needs to be explored with even more vigor and earnestness.

Keeping in view the emerging importance of cluster states along with their most general set of so called graph states, we present two independent proposals for the engineering of these states in chapter 4. First scheme addresses the generation of cavity field cluster and graph states through the collective operation of disentanglement eraser. The scheme explicitly explores the link between information eraser and quantum coherence. Here we have shown that the collective eraser operation over a pair of individually tagged bipartite field states not only safeguards the initial coherence but also extends it to a four-partite horse-shoe cavity field cluster state. The phase gate procedure employed here was then generalized to demonstrate that collective operation of disentanglement eraser is potentially capable of generating any arbitrary field graph state. A cavity field superposition tagged with an atom was taken as the basic building block for the construction of various graph morphologies. The second scheme included in this chapter is concerned with the engineering of cluster and graph states in atomic internal degrees of freedom. Here simultaneous resonant and dispersive interactions of two two-level atoms, both initially prepared in their respective internal level superpositions, with a vacuum state cavity are utilized for the sake of Ising interactions. At the culmination of the interactions, the cavity is again left into vacuum and hence gets automatically recycled for further use. Thus the phase gate employed here is most resource economical as it entangles the two atoms with the help of a high-Q cavity that acts more or less like a catalyst. Moreover the method is flexible enough to efficiently generate any arbitrary atomic graph state. Further novel feature of the scheme is successive interaction pattern of the atoms with the vacuum state high-Q cavity. In most of the state engineering cases discussed here, an atom after having dispersive interactions goes through a cycle of resonant interactions. This consequently refreshes the quantum memory of the atom and helps to sustain the coherence for comparatively longer times bearing a close affinity with the phenomenon of quantum repeaters [312]. Success probability and fidelity of the engineered state are also calculated and sketched out for the sake of practical assessment. It was thus demonstrated that the scheme is expected to yield good overall results under prevailing cavity QED experimental research scenario. At the end of the chapter, we have further added a general discussion of the experimental feasibility of both the proposals included in this chapter. It is shown that the schemes rely on time-tested standard cavity QED tools and are easily implementable with currently available resources. Although favourable experimental features of the presented schemes are given independently in previous chapters but a recent advance is worth mentioning that further support our optimism concerning experimental execution of various proposals. Haroche group has recently demonstrated, in principle, the passage of thousands of atoms from a high-Q cavity before the decoherence effectively takes place [313].

In summary, the present contribution extends the research horizons on quantum entanglement. The applicability of entangled states engineered in present work cover almost all active research areas of Quantum Informatics. For example Bell states, along with other applications, are poineer in handling the foundational issues through Bell's inequality violation. Similarly W states are helpful in data distribution via quantum communication networks, teleportation, dense coding and optimal universal quantum cloning [188, 189, 190, 191, 192, 193]. NOON states, apart from ultrahigh resolution interferometry, can equally be employed in Ramsey-type interferometry with enhanced resultant phase shift. Another very important technical advantage of NOON state is that it can be utilized to efficiently cope photodetection problems/loopholes. One-way quantum computing is an emerging computational alternative and employes entagled cluster states as a resource [77, 78, 79]. More recently, we are witnessing the novel emergence of mesoscopic, multipartite entanglements utilizing the phenomenon of Bose-Einstein Condensation [314, 315, 316]. Further interesting point to note is that entanglement mechanisms and even Bose-Einstein Interferometry based on Bragg interactions have already been investigated out [269, 317]. Therefore, we are optimistic that work presented in the thesis may equally be extended in the regime of mesoscopic entangled state engineering. Such macroscopic entangled states, on the one hand, are expected to shed ample light on the outstanding controversies and philosophical implications of the quantum theory [318, 319]. Whereas, on the other hand they can serve better as the main impetus behind the newly born field of Quantum Informatics. In general, with the experimental demonstrations of the basic ingredients of Quantum Informatics, such as, quantum logic gates, quantum teleportation and quantum cryptography, it is evident that the future research on the phenomenon of entanglement especially macroscopic entanglement will turn the subject into an even more fascinating era.

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