RADIATION FROM A LINE SOURCE BURIED UNDER UNIAXIAL DIELECTRIC NON-INTEGER DIMENSIONAL PLANAR INTERFACE

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by

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CERTIFICATE

It is to be certified that Ms. Tayyaba Naz carried out the work contained in this dissertation, under my supervision.

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Tayyaba Naz

Dedicated to Ammi, Abbu
Brothers and Sisters

Abstract

Radiation from a two dimensional canonical source carrying time harmonic magnetic current and buried below free space-non-integer dimensional (NID) uniaxial dielectric planar interface has been investigated. Half space carrying current source is NID in direction normal to the planar interface whereas other half space is free space. Far-zone radiated field is obtained in the free space and behavior is studied with respect to NID parameter and permittivity of the uniaxial medium.

Contents

CHAPTER 1

Introduction

1.1. Green's function in electromagnetics

One of the most fundamental problem in electromagnetics is to obtain solution to the Maxwell equations or Helmholtz's equation when source is present. This is because such situations lead to find the solutions to the inhomogeneous partial differential equation. Green's function is considered as a powerful and efficient approach to obtain solution in these situations. Potential due to a point charge which satisfies certain boundary conditions is the most simple example of Green's function. The concept of Green's function in electromagnetics has been introduced by George Green [1]. His work provides the use of mathematical analysis to the theory of electromagnetism. Later, Poincare [2] briefly discussed the theory about Green's functions. Mackie [3] soughted out how Green's functions and intergral transformation approaches can be applied to the boundary value problems. Neumann [4] developed and extended this concept and gave the idea of two-dimensional Green's function by solving two dimensional potential equation. Meutzner continued Neumann's work and proposed Green's function for the regions that are enclosed by an ellipse and a circle [5]. Sommerfeld [6] established a technique using integration to develop the method of images to various three dimensional geometries. Later, Waldmann [7] used this technique for finding electrostatic solutions of an electron lens. It is important to mention that, Green's function for various geometries are available in scientific literature.

1.2. Green's function for unbounded isotropic dielectric medium

Radiation of electromagnetic waves from buried object has applications in various disciplines including remote sensing and military. The problems dealing with buried object are considered more difficult than when object is placed in unbounded medium. Difficulty arises due to electromagnetic interactions between buried object and interfaces surrounding the object. Interaction between buried object and its surrounding interfaces yields complex current distribution on the buried object. Complexity of buried object problems depends on the shape of the object and type of the medium in which the object is buried. Green's function technique have been widely used to treat such problems. Hohmann [8] obtained scattered fields outside the cylindrical inhomogeneity by using the Green's function approach. His work was further extended by Howard [9]. Couple integral equations had been solved by Parry and Ward [10]. Engheta and Papas [11] derived the far-zone radiated fields from a line source placed at the planar interface of two dielectric media. Chalmers et. al [12] solved integral equation numerically to obtain the induced currents on the buried cylinder. Using these currents, they calculated the far-zone scattered field. Lakhtakia et al. [13] obtained ordinary and extraordinary fields for canonical sources by using electric and magnetic Green's function. Chen [14] determined radiation of sources in anisotropic medium with the help of Fourier integrals and dyadic Green's function.

In order to give more insight, two dimensional Green's function for free-space geometry is discussed below. A magnetic line source is located at (x', y') bearing time harmonic magnetic current. The space is divided in two regions so that solution of homogeneous Helmholtz's equation can be used. Above the line source is Region I and below the line source is Region II. Expressions for unknown radiated field in both regions are given below

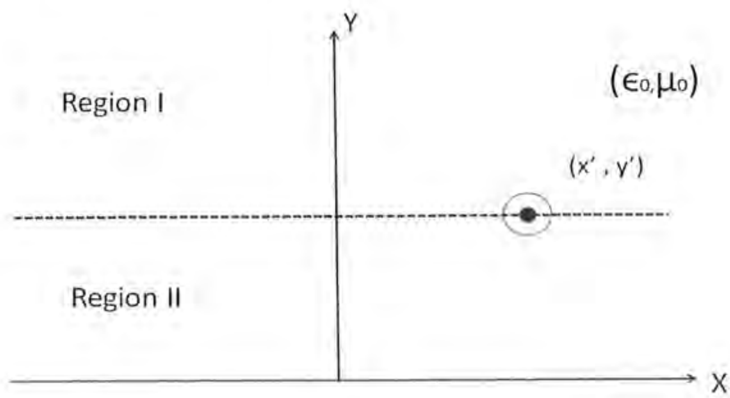

Figure 1: *Radiation from a magnetic line source in unbounded isotropic dielectric medium*

$$H_{1z}(x,y) = \int_{-\infty}^{\infty} A_1(k_x)\exp(ik_y y + ik_x x)dk_x, \qquad y > y' \tag{1}$$

$$H_{2z}(x,y) = \int_{-\infty}^{\infty} A_2(k_x)\exp(ik_x x - ik_y y)dk_x, \qquad y < y' \tag{2}$$

where $k_y = \sqrt{k_0^2 - k_x^2}$ and $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ using corresponding boundary condition, unknown coefficients are obtained. After substitution of unknowns, field in each region is given below

$$H_{1z}(x,y) = \frac{-\omega\epsilon_0 I_0}{4\pi}\int_{-\infty}^{\infty}\frac{1}{k_y}\exp\{ik_x(x-x') + ik_y(y-y')\}dk_x \tag{3a}$$

$$H_{2z}(x,y) = \frac{-\omega\epsilon_0 I_0}{4\pi}\int_{-\infty}^{\infty}\frac{1}{k_y}\exp\{ik_x(x-x') - ik_y(y-y')\}dk_x \tag{3b}$$

1.3. Non-integer dimensional space in electromagnetics

In 1977, the idea of non-integer dimensional (NID) space was first proposed by Stillinger [15] and suggested the Laplacian operator for (NID) space. Later Palmer and Stavrinou [16] extended his work and derived equations of motion for NID space. After these contributions the concept of NID space has been used succesfully in various fields of science and enginering [17]. Electromagnetic differential equations in NID space were solved by Zubair et. al [18]. Sadallah et. al [19] presented the solution of Euler-Langrange equation in NID space. Engheta [20] proposed the fractional curl operator for NID space. The differential operator ∇_D and Laplacian operator ∇_D^2 for NID space are given in [21-22].

$$\nabla_D = \left(\frac{\partial}{\partial x} + \frac{\alpha_1 - 1}{2x}\right)\hat{x} + \left(\frac{\partial}{\partial y} + \frac{\alpha_2 - 1}{2y}\right)\hat{y} + \left(\frac{\partial}{\partial z} + \frac{\alpha_3 - 1}{2z}\right)\hat{z}$$

$$\nabla_D^2 = \frac{\partial^2}{\partial x^2} + \frac{\alpha_1 - 1}{x}\frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} + \frac{\alpha_2 - 1}{y}\frac{\partial}{\partial y} + \frac{\partial^2}{\partial z^2} + \frac{\alpha_3 - 1}{z}\frac{\partial}{\partial z}$$

where parameters $0 < \alpha_1 \leq 1$, $0 < \alpha_2 \leq 1$ and $0 < \alpha_3 \leq 1$ are measured distributions on coordinates x, y and z. $D = \alpha_1 + \alpha_2 + \alpha_3$ which is the entire dimension of space.

In NID space curl operator of cartesian coordinates are given below

$$\begin{aligned}\nabla_D \times \mathbf{F} = &\left\{\left(\frac{\partial}{\partial y}F_z + \frac{\alpha_2 - 1}{2y}F_z\right) - \left(\frac{\partial}{\partial z}F_y + \frac{\alpha_3 - 1}{2z}F_y\right)\right\}\hat{x} \\ &+ \left\{\left(\frac{\partial}{\partial z}F_x + \frac{\alpha_3 - 1}{2z}F_x\right) - \left(\frac{\partial}{\partial x}F_z + \frac{\alpha_1 - 1}{2x}F_z\right)\right\}\hat{y} \\ &+ \left\{\left(\frac{\partial}{\partial x}F_y + \frac{\alpha_1 - 1}{2x}F_y\right) - \left(\frac{\partial}{\partial y}F_x + \frac{\alpha_2 - 1}{2y}F_x\right)\right\}\hat{z}\end{aligned}$$

Homogeneous Helmholtz's equation for NID space is

$$\nabla_D^2 E(x,y) + k^2 E(x,y) = 0$$

Assuming that space is NID along y-axis and yields solution

$$E(x,y) = \exp\left(k_x x\right)\left(k_y y\right)^n H_n^{(i)}\left(k_y y\right), \qquad i = 1, 2$$

where $n = (3 - D)/2, 1 < D \leq 2$

1.4. Fractal/NID space

The idea of fractal was first introduced by mandelbrot [23] for complex structures by using small number of parameters. Study of fractal media depends on model of continuity [24-25]. The concept of NID space is useful to reveal the properties of fractal media [26]. Due to this reason, fractal media has been considered as continuum model with NID space [27]. Zubair et. al derived the expressions for spherical [28] and cylindrical [29] waves in NID space. Antenna radiation problems in fractional space were discussed in [30]. Analysis of acoustic waves in isotropic fractal materials were discussed in [31]. The different ways to explain fractal media were divided into following approaches by Tarasov [32]. i) Analysis on fractal (ii) Continum model for fractional-differentials (iii) Continuum model for fractional integrals (iv) Fractional space approach (v) NID space approach. Fractal electrodynamics depends on continum model of fields [33-37]. For fractional space Green's function has been derived in [38]. Green's function was derived for half space geometry by taking two different NID spaces above and below the planar interface [39]. Ampere's law and faraday's were derived for NID space by Martin [40]. Vector differential operators and vector Laplacian for NID space were presented in [41].

1.5. Uniaxial-anisotropic media

During the recent years, the interaction between electromagnetic waves and uniaxial anisotropic medium is topic of research in growing number of scientific publications. This is due to the presence of anisotropy in many natural and artificial materials. Variety of applications may be proposed in many scientific and engineering purposes. Anisotropic media can be defined as media having different physical properties along different directions. Uniaxial medium is the medium with different physical properties restricted along single direction. The permittivity of the uniaxial medium can be written in tensor form

$$\underline{\underline{\epsilon}} = \epsilon_0 \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

where ϵ_o is the permittivity of vacuum, ϵ_x is permittivity along x-axis and ϵ is permittivity along y-axis and z-axis. Mukherjee and Mann [42] derived the far-zone radiated field expressions and also obtained the pattern of electromagnetic radiations for line sources placed in a uniaxial medium. Jokob et al. [43] obtained a spectrum of plane waves from an isolated current source radiated in a uniaxial anisotropic medium. Electromagnetic fields produced by an electric dipole immersed in anisotropic medium has been calculated by Martin [44]. The behavior of plane waves in a uniaxial medium has been presented by McDonald [45]. Kojima et al. [46] determined the radiated fields and also pattern of radiation for magnetic line source coated with a uniaxial isotropic moving sheath. Fleck et al. [47] solved the praxial wave equations to determine the generation of beams in uniaxial anisotropic media.

1.6. Thesis plan

Objective of the thesis is to determine the radiation from a magnetic line source buried in planar half space geometry. Half space containing line source is uniaxial NID whereas other half space is free space. Before addressing this geometry, first magnetic line source in unbounded isotropic dielectric, isotropic dielectric half space, unbounded uniaxial dielectric and uniaxial dielectric halfspace geometries are addressed.

This thesis contains four chapters. First chapter includes introduction and literature review about Green's function. Two dimensional Green's function for unbounded dielectric medium has been presented. NID space in electromagnetics and uniaxial-anisotropic media has also been discussed.

Second chapter deals with radiation from a magnetic line source buried beneath planar isotropic dielectric interface. Far-zone radiated field has been obtained by using asymptotic technique (saddle-point method of integration). Behavior of far-zone radiated magnetic field has been presented.

In the third chapter the objective is to address main problem of M.Phil. research. That is, to determine the radiation from a magnetic line source buried under uniaxial dielectric NID planar interface. Initially, radiated magnetic field expression has been derived for magnetic line source placed in unbounded uniaxial dielectric medium. The discussion is extended by treating magnetic line source buried under uniaxial dielectric planar interface. Behavior of radiated field from magnetic line source buried in uniaxially anisotropic medium has been studied by taking different values of permittivity of uniaxial medium. Finally radiation from a magnetic line source buried under uniaxial dielectric NID planar interface has been investigated. Behavior of the far-zone radiated magnetic field has been studied with respect to different values of NID parameter. Chapter four contains conclusions drawn in the research work.

CHAPTER 2

Line source buried under isotropic dielectric planar interface

In this chapter, the goal is to determine the radiation from a magnetic line source buried beneath a planar isotropic dielectric interface. Expressions for radiated fields are derived in terms of spectrum of plane waves. Far-zone radiated field has been obtained by using the saddle point technique of integration.

2.1. Formulation

Consider a line source having time harmonic magnetic current is located beneath planar dielectric interface. Half space y>b is free space whereas other half space is isotropic dielectric medium. (ϵ_0, μ_0) are the constitutive parameters of free space and (ϵ_1, μ_0) are the parameters of dielectric media. It has been assumed that magnetic line source is located at (x', y'). Geometry is divided into three regions. Region I is above the planar interface. Region II is between the line source and planar interface and region III is below the line source. Expressions for radiated magnetic field in each region are given below

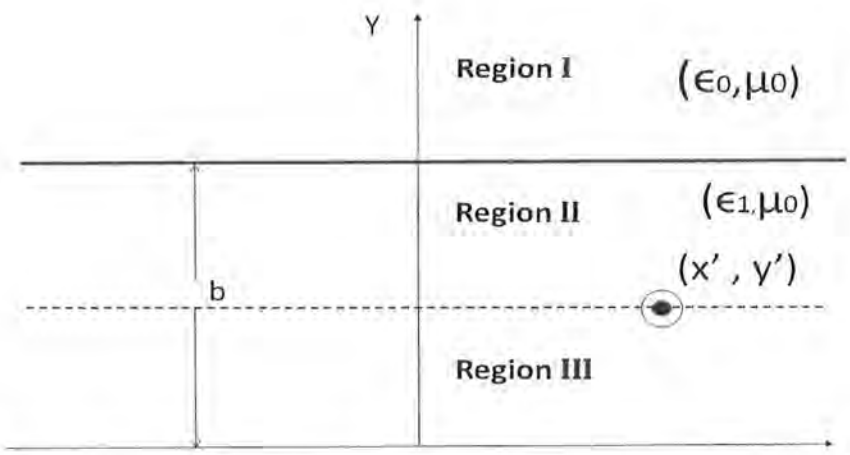

Figure 2.1: *magnetic line source buried under planar isotropic dielectric interface.*

$$H_{1z}(x,y) = \int_{-\infty}^{\infty} A_1(k_{1x}) \exp(ik_{1x}x + ik_{1y}y)dk_{1x} \qquad y > b \tag{4a}$$

$$H_{2z}(x,y) = \int_{-\infty}^{\infty} A_2(k_{2x}) \exp(ik_{2x}x + ik_{2y}y)dk_{2x}+$$

$$\int_{-\infty}^{\infty} A_3(k_{2x}) \exp(ik_{2x}x - ik_{2y}y)dk_{2x}) \qquad y' < y < b \tag{4b}$$

$$H_{3z}(x,y) = \int_{-\infty}^{\infty} A_4(k_{2x}) \exp(ik_{2x}x - ik_{2y}y)dk_{2x} \qquad y < y' \tag{4c}$$

where k_{1y}=$\sqrt{k_0^2 - k_{1x}^2}$ and k_{2y}=$\sqrt{k_1^2 - k_{1x}^2}$. Boundary conditions are used to determine the unknown coefficients A_1, A_2, A_3 and A_4.

Continuity of tangential components of magnetic field at y=b,

$$H_{1z} = H_{2z} \tag{5}$$

Application of above yields

$$A_1(u)\exp(i\gamma_1 b) = A_2(u)\exp(i\gamma_2 b) + A_3(u)\exp(-i\gamma_2 b) \tag{6}$$

where γ_1=$\sqrt{k_0^2 - u^2}$, γ_2=$\sqrt{k_1^2 - u^2}$ and u is dummy variable.

Continuity of tangential component of electric fields at y=b

$$\frac{1}{\epsilon_0}\frac{\partial H_{1z}}{\partial y} = \frac{1}{\epsilon_1}\frac{\partial H_{2z}}{\partial y} \tag{7}$$

$$\frac{1}{\epsilon_0}\frac{\partial H_{1z}}{\partial y} = \frac{1}{\epsilon_0 \epsilon}\frac{\partial H_{2z}}{\partial y} \tag{8}$$

where $\epsilon_1 = \epsilon_0 \epsilon$

$$\gamma_1 \epsilon A_1(u)\exp(i\gamma_1 b) = \gamma_2 A_2(u)\exp(i\gamma_2 b) - \gamma_2 A_3(u)\exp(-i\gamma_2 b) \tag{9}$$

At $y = y'$, tangential components of magnetic field must be continuos

$$H_{2z} = H_{3z} \tag{10}$$

yields

$$A_2(u)\exp(i\gamma_2 y') + A_3(u)\exp(-i\gamma_2 y') = A_4(u)\exp(-i\gamma_2 y') \tag{11}$$

At $y = y'$, tangential component of electric field must be discontinuos

$$\frac{\partial H_{3z}}{\partial y} - \frac{\partial H_{2z}}{\partial y} = i\omega\epsilon_0\epsilon I_0 \exp(-iux') \tag{12}$$

Following is obtained

$$A_2(u)\exp(i\gamma_2 y') - A_3(u)\exp(-i\gamma_2 y') + A_4(u)\exp(-i\gamma_2 y') = \frac{1}{2\pi}\frac{\omega\epsilon_0\epsilon I_0}{\gamma_2}\exp(-iux') \tag{13}$$

Solving above equations yields

$$A_1(u) = -\frac{1}{2\pi}\frac{\omega\epsilon_0\epsilon I_0}{\gamma_2 + \epsilon\gamma_1}\exp(-i\gamma_1 b + i\gamma_2 b - iux' - i\gamma_2 y') \tag{14a}$$

$$A_2(u) = -\frac{\omega\epsilon_0\epsilon I_0}{4\pi\gamma_2}\exp(-iux' - i\gamma_2 y') \tag{14b}$$

$$A_3(u) = -\frac{\omega\epsilon_0\epsilon}{4\pi\gamma_2}\frac{\gamma_2 - \epsilon\gamma_1}{\gamma_2 + \epsilon\gamma_1}\exp(2i\gamma_2 b - iux' - i\gamma_2 y') \tag{14c}$$

$$A_4(u) = A_3(u) - \frac{\omega\epsilon_0\epsilon I_0}{4\pi\gamma_2}\exp(-iux' + i\gamma_2 y') \tag{14d}$$

Radiated field in region I is

$$\begin{aligned} H_{1z}(x,y) = \frac{-\omega\epsilon_0\epsilon I_0}{2\pi}\int_{-\infty}^{\infty} & \frac{1}{\sqrt{k_1^2 - k_{1x}^2} + \epsilon\sqrt{k_0^2 - k_{1x}^2}} \\ & \times \exp\left\{-i\sqrt{k_1^2 - k_{1x}^2}(y' - b)\right\} \\ & \times \exp\{ik_{1x}(x - x') + ik_{1y}(y - b)\}dk_{1x} \end{aligned} \tag{15}$$

2.2 Asymptotic analysis

Setting

$$k_{1x} = k_0 \cos\theta, \qquad k_{1y} = k_0 \sin\theta$$

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi$$

$$x^{'} = \rho'\cos\phi^{'} \qquad y^{'} = \rho^{'}\sin\phi^{'}$$

in equation (15) yields

$$\begin{aligned} H_{1z}(\rho,\phi) = \frac{-\omega\epsilon_0\epsilon I_0 k_0}{2\pi}\int_0^{\pi}\left\{\frac{\epsilon k_0\sin^2\theta - \sin\theta\sqrt{k_1^2 - k_0^2\cos^2\theta}}{k_1^2 - k_0^2(\cos^2\theta + \epsilon^2\sin^2\theta)}\right\} \\ \times \exp\{-ik_0\rho'\cos\theta\cos\phi' - i\rho'\sqrt{k_1^2 - k_0^2\cos^2\theta}\sin\phi'\} \\ \times \exp\{ib\sqrt{k_1^2 - k_0^2\cos^2\theta} - ik_0 b\sin\theta\} \\ \times \exp\{ik_0\rho\cos(\theta - \phi)\}d\theta \end{aligned} \tag{16}$$

Taking $k_0\rho >> 1$, saddle point method of integration can be applied to find the far-zone radiated field. Saddle point is located at $\theta = \phi$. Dominant contribution is given below

$$\begin{aligned} H_{1z} \simeq \frac{-\omega\epsilon_0\epsilon I_0 k_0}{\sqrt{2\pi}}\left\{\frac{\epsilon k_0\sin^2\phi - \sin\phi\sqrt{k_1^2 - k_0^2\cos^2\phi}}{k_1^2 - k_0^2(\cos^2\phi + \epsilon^2\sin^2\phi)}\right\} \\ \times \exp\{-ik_0\rho'\cos\phi\cos\phi' - i\rho'\sqrt{k_1^2 - k_0^2\cos^2\phi}\sin\phi'\} \\ \times \exp\{-ik_0 b\sin\phi + ib\sqrt{k_1^2 - k_0^2\cos^2\phi}\} \\ \times \frac{\exp(ik_0\rho - i\pi/4)}{\sqrt{k_0\rho}}, \quad k_0\rho >> 1 \end{aligned} \tag{17}$$

Behavior of far zone radiated magnetic field of a line source buried beneath isotropic dielectric planar interface with respect to following parameters ϵ=4 and b=0.4 $\rho' = 0.2$ is given in Figure 2.2.

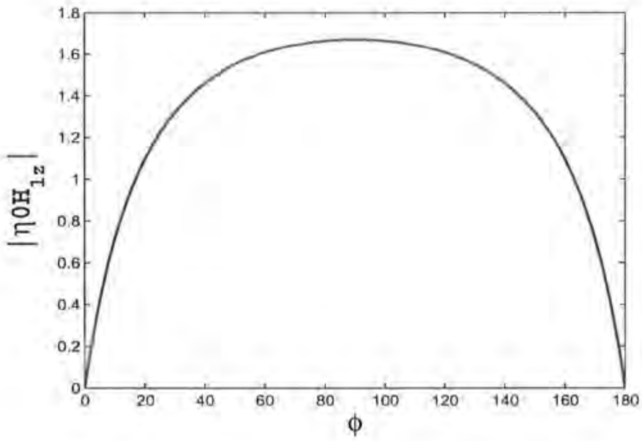

Figure 2.2: *Behavior of the far-zone radiated magnetic field for isotropic half space geometry*

CHAPTER 3

Radiation from a line source buried under uniaxial dielectric non-integer dimensional planar interface

First, expression for radiated field from a magnetic line source placed in unbounded uniaxial dielectric medium has been derived. The discussion is extended by treating a magnetic line source buried under planar uniaxial dielectric interface. Radiation from a magnetic line source buried in planar NID uniaxial dielectric interface has been investigated at the end. Behavior of the far-zone radiated magnetic field has been studied with respect to different values of NID parameter.

3.1. Line source in unbounded uniaxial dielectric medium

In this section, field radiated from a magnetic line source, placed in unbounded uniaxially anisotropic dielectric medium, is obtained in terms of plane waves. The permittivity of the mediums are different while the permeability are assumed to be same. The line source is placed at (x', y') bearing harmonic magnetic current. It may be noted that radiated fields contain components (H_z, E_x, E_y). Uniaxially anisotropic dielectric medium may be described through following expressions for the constitutive parameters

$$\underline{\underline{\epsilon}} = \epsilon_0 \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

$$\mu = \mu_0$$

where ϵ_0 are the permittivity and the μ_0 are the permeability of the free space. ϵ_x and ϵ are relative permittivities. For this situation, Ampere's law

$$\nabla \times \mathbf{H} = -i\omega\underline{\underline{\epsilon}}.\mathbf{E} = -i\omega\epsilon_0\epsilon_x E_x\hat{x} - i\omega\epsilon_0\epsilon E_y\,\hat{y}$$

yields following relations

$$E_x = -\frac{1}{i\omega\epsilon_0\epsilon_x}\frac{\partial H_z}{\partial y}$$
$$E_y = \frac{1}{i\omega\epsilon_0\epsilon}\frac{\partial H_z}{\partial x}$$

As the line source carries current in z-direction, therefore Faraday's Maxwell equation

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H} - \mathbf{M}$$

simplifies as written below

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega\mu_0 H_z - M_z$$

Substituting relations derived from Ampere's law in above equation leads to following Helmholtz's equation

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\epsilon}{\epsilon_x}\frac{\partial^2 H_z}{\partial y^2} = -\omega^2\mu_0\epsilon_0\epsilon H_z - i\omega\epsilon_0\epsilon M_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\epsilon}{\epsilon_x}\frac{\partial^2 H_z}{\partial y^2} + k_0^2\epsilon H_z = -i\omega\epsilon_0\epsilon M_z$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. As canonical source is situated at (x', y'), therefore the Helmholtz equation becomes

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\epsilon}{\epsilon_x}\frac{\partial^2 H_z}{\partial y^2} + k_0^2\epsilon H_z = -i\omega\epsilon_0\epsilon I_o\delta(x - x')\delta(y - y')$$

Solution of above equation in form of Fourier transform are given below

$$H_z(x, y) = \int_{-\infty}^{\infty} \tilde{H}_z(k_x, y)\exp(ik_x x)dk_x$$

Using Fourier transform representation in Helmholtz's equation gives

$$\frac{\partial^2 \tilde{H}_z}{\partial y^2} + k_0^2\frac{\epsilon_x}{\epsilon}\left(\epsilon - k_x^2/k_0^2\right)\tilde{H}_z = -i\omega\epsilon_0\epsilon_x I_o\exp(ik_x x')\delta(y - y')$$

or

$$\frac{d^2\tilde{H}_z}{dy^2} + q^2\tilde{H}_z = -i\omega\epsilon_0\epsilon_x I_o \exp(-ik_x x')\delta(y - y')$$

where $q = k_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - k_x^2/k_0^2)}$. Therefore the spectrum representation in uniaxially anisotropic medium are following

$$H_z(x,y) = \int_{-\infty}^{\infty} A(k_x)\exp(ik_x x + iqy)dk_x$$

Now assume that space in y-direction is (NID), that is

$$\frac{d^2\tilde{H}_z}{dy^2} + \frac{\alpha - 1}{y}\frac{d\tilde{H}_z}{dy} + q^2\tilde{H}_z = -i\omega\epsilon_0\epsilon_x I_o \exp(ik_x x')\delta(y - y')$$

where $D = \alpha + 1$ is dimension of the NID space. Solution in NID uniaxial dielectric medium is

$$H_z(x,y) = \int_{-\infty}^{\infty} A(k_x)(qy)^n H_n^{(1)}(qy)\exp(ik_x x)dk_x, \qquad n = \frac{3-D}{2}$$

3.2: Free space-uniaxial dielectric interface

Consider free space-uniaxial dielectric interface as shown in Figure 1. Half space $y > b$ is filled with free space whereas half space $y < b$ is occupied by uniaxially anisotropic dielectric medium. Constitutive parameters for free space are (ϵ_0, μ_0). Consider a line source carrying time harmonic magnetic current is situated at (x', y') in a uniaxially anisotropic dielectric medium. The geometry is splitted into three regions. Region I is above the planar interface. Region II is between the line source and planar interface whereas region III is below the line source. Helmholt'z equation can be used.

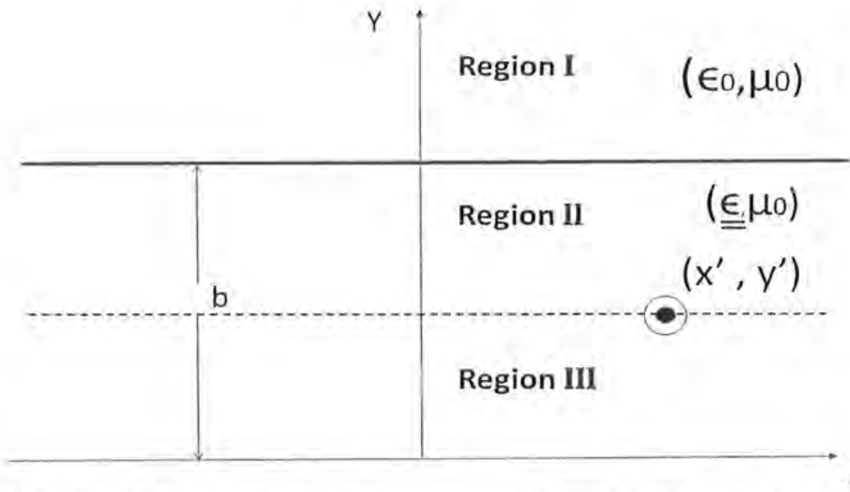

Figure 3.1: *Radiation from a magnetic line source buried under uniaxial dielectric interface.*

Expression for radiated magnetic field in each region is written below

$$H_{1z}(x,y) = \int_{-\infty}^{\infty} A_1(k_{1x}) \exp(ik_{1y}y + ik_{1x}x)dk_{1x}, \qquad y > b \qquad (18a)$$

$$H_{2z}(x,y) = \int_{-\infty}^{\infty} A_2(k_{2x}) \exp(iq_2y + ik_{2x}x)dk_{2x} + \int_{-\infty}^{\infty} A_3(k_{2x}) \exp(-iq_2y + ik_{2x}x)dk_{2x}, \qquad y' < y < b \quad (18b)$$

$$H_{3z}(x,y) = \int_{-\infty}^{\infty} A_4(k_{2x}) \exp(-iq_2y + ik_{2x}x)dk_{2x}, \qquad y < y' \qquad (18c)$$

where $k_{1y} = \sqrt{k_0^2 - k_{1x}^2}$ and $q_2 = k_0\sqrt{\frac{\epsilon_x}{\epsilon}\left(\epsilon - k_{2x}^2/k_0^2\right)}$. Boundary conditions are applied to find the unknown co-effiecients A_1, A_2, A_3 and A_4.

Continuity of tangential components magnetic fields at $y = b$,

$$H_{1z} = H_{2z}, \qquad \text{at} \qquad y = b \qquad (19)$$

gives

$$A_1(u)\exp(i\gamma_1 b) = A_2(u)\exp(i\gamma_2 b) + A_3(u)\exp(-i\gamma_2 b) \tag{20}$$

where $\gamma_1 = \sqrt{k_1^2 - u^2}$, $\gamma_2 = k_0\sqrt{\frac{\epsilon_x}{\epsilon}\left(\epsilon - u^2/k_0^2\right)}$ and u is dummy variable.

Continuity of tangential component of electric fields at $y = b$

$$\frac{1}{\epsilon_0}\frac{\partial H_{1z}}{\partial y} = \frac{1}{\epsilon_0\epsilon_x}\frac{\partial H_{2z}}{\partial y}, \qquad \text{at} \qquad y = b \tag{21}$$

yields

$$\gamma_1\epsilon_x A_1(u)\exp(i\gamma_1 b) = \gamma_2 A_2(u)\exp(i\gamma_2 b) - \gamma_2 A_3(u)\exp(-i\gamma_2 b) \tag{22}$$

At $y = y'$, continuity of tangential component of magnetic field.

$$H_{2z} = H_{3z}, \qquad \text{at} \qquad y = y' \tag{23}$$

Therefore

$$A_2(u)\exp(i\gamma_2 y') + A_3(u)\exp(-i\gamma_2 y') = A_4(u)\exp(-i\gamma_2 y') \tag{24}$$

At $y = y'$, tangential component of electric field must be discontinuous

$$\frac{\partial H_{3z}}{\partial y} - \frac{\partial H_{2z}}{\partial y} = i\omega\epsilon_0\epsilon_x I_o\exp(-iux') \qquad \text{at} \qquad y = y' \tag{25}$$

Therefore

$$\begin{aligned} A_2(u)\exp(i\gamma_2 y') - A_3(u)\exp(-i\gamma_2 y') + A_4(u)\exp(-i\gamma_2 y') \\ = -\frac{1}{2\pi}\frac{\omega\epsilon_0\epsilon_x I_o}{\gamma_2}\exp(-iux') \end{aligned} \tag{26}$$

Solving algebraic equations simultaneously yields

$$A_1(u) = -\frac{1}{2\pi}\frac{\omega\epsilon_0\epsilon_x I_0}{\gamma_2 + \epsilon_x\gamma_1}\exp(-i\gamma_1 b + i\gamma_2 b - iux' - i\gamma_2 y') \tag{27a}$$

$$A_2(u) = -\frac{\omega\epsilon_0\epsilon_x I_0}{4\pi\gamma_2}\exp(-iux' - i\gamma_2 y') \tag{27b}$$

$$A_3(u) = -\frac{\omega\epsilon_0\epsilon_x}{4\pi\gamma_2}\frac{\gamma_2 - \epsilon_x\gamma_1}{\gamma_2 + \epsilon_x\gamma_1}\exp(2i\gamma_2 b - iux' - i\gamma_2 y') \tag{27c}$$

$$A_4(u) = A_3(u) - \frac{\omega\epsilon_0\epsilon_x I_0}{4\pi\gamma_2}\exp(-iux' + i\gamma_2 y') \tag{27d}$$

Radiated field in region I

$$H_{1z}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{-\omega\epsilon_0\epsilon_x I_0}{k_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - k_{1x}^2/k_0^2)} + \epsilon_x k_{1y}}$$

$$\times \exp\left\{-ik_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - k_{1x}^2/k_0^2)}(y'-b)\right\}\exp\{ik_{1x}(x-x') + ik_{1y}(y-b)\}dk_{1x} \quad (28)$$

3.3: Free space-NID uniaxial dielectric interface

It is suppose $y < b$, is lower half space which is NID normal to the planar interface. Now field in each region is given as

$$H_{1z}(x,y) = \int_{-\infty}^{\infty} A_1(k_{1x})\exp(ik_{1y}y + ik_{1x}x)dk_{1x}, \qquad y > b \qquad (29a)$$

$$H_{2z}(x,y) = \int_{-\infty}^{\infty} A_2(k_{2x})\exp(ik_{2x}x)(q_2y)^n H_n^{(1)}(q_2y)dk_{2x}$$
$$+ \int_{-\infty}^{\infty} A_3(k_{2x})\exp(ik_{2x}x)(q_2y)^n H_n^{(2)}(q_2y)dk_{2x}, \qquad y' < y < b \quad (29b)$$

$$H_{3z}(x,y) = \int_{-\infty}^{\infty} A_4(k_{2x})\exp(ik_{2x}x)(q_2y)^n H_n^{(2)}(q_2y)dk_{2x}, \qquad y < y' \qquad (29c)$$

Continuity of tangential component of magnetic fields at $y = b$

$$H_{1z} = H_{2z}, \qquad \text{at} \qquad y = b \qquad (30)$$

gives

$$A_1(u)\exp(i\gamma_1 b) = A_2(u)(\gamma_2 b)^n H_n^{(1)}(\gamma_2 b) + A_3(u)(\gamma_2 b)^n H_n^{(2)}(\gamma_2 b) \qquad (31)$$

where $\gamma_1 = \sqrt{k_1^2 - u^2}$, $\gamma_2 = k_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - u^2/k_0^2)}$ and u is dummy variable.

Continuity of tangential component of electric fields

$$\frac{1}{\epsilon_0}\frac{\partial H_{1z}}{\partial y} = \frac{1}{\epsilon_0\epsilon_x}\left[\frac{\partial H_{2z}}{\partial y} + \frac{1}{2}\frac{D-2}{y}H_{2z}\right], \qquad \text{at} \qquad y = b \qquad (32)$$

yields

$$i\gamma_1\epsilon_x A_1(u)\exp(i\gamma_1 b) = \gamma_2 A_2(u)(\gamma_2 b)^n H_{n_h}^{(1)}(\gamma_2 b) + \gamma_2 A_3(u)(\gamma_2 b)^n H_{n_h}^{(2)}(\gamma_2 b) \tag{33}$$

At $y = y'$, continuity of tangential component of magnetic field

$$H_{2z} = H_{3z}, \qquad \text{at} \qquad y = y' \tag{34}$$

Therefore

$$A_2(u)(\gamma_2 y')^n H_n^{(1)}(\gamma_2 y') + A_3(u)(\gamma_2 y')^n H_n^{(2)}(\gamma_2 y') = A_4(u)(\gamma_2 y')^n H_n^{(2)}(\gamma_2 y') \tag{35}$$

At $y = y'$, tangential component of electric field must be discontinuous

$$\frac{\partial H_{3z}}{\partial y} - \frac{\partial H_{2z}}{\partial y} = \frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\exp(-iux') \qquad \text{at} \qquad y = y' \tag{36}$$

Therefore

$$\begin{aligned} A_2(u)(\gamma_2 y')^n H_{n_h}^{(1)}(\gamma_2 y') + A_3(u)(\gamma_2 y')^n H_{n_h}^{(2)}(\gamma_2 y') - A_4(u)(\gamma_2 y')^n H_{n_h}^{(2)}(\gamma_2 y') \\ = -i\frac{\omega\epsilon_0\epsilon_x I_o}{2\pi\gamma_2}\exp(-iux') \end{aligned} \tag{37}$$

Solving algebraic equations simultaneously yields spectrum function as

$$\begin{aligned} A_1(u) = {} & \frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\frac{(\gamma_2 b)^n}{(\gamma_2 y')^n}\frac{H_n^{(2)}(\gamma_2 y')}{H_n^{(1)}(\gamma_2 y')H_{n_h}^{(2)}(\gamma_2 y') - H_{n_h}^{(1)}(\gamma_2 y')H_n^{(2)}(\gamma_2 y')} \\ & \times \frac{H_n^{(1)}(\gamma_2 b)H_{n_h}^{(2)}(\gamma_2 b) - H_n^{(2)}(\gamma_2 b)H_{n_h}^{(1)}(\gamma_2 b)}{\gamma_2 H_{n_h}^{(2)}(\gamma_2 b) - i\gamma_1\epsilon_x H_n^{(2)}(\gamma_2 b)}\exp(-iux' - i\gamma_1 b) \end{aligned} \tag{38a}$$

$$A_2(u) = \frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\frac{\exp(-iux')}{(\gamma_2 y')^n\gamma_2}\frac{H_n^{(2)}(\gamma_2 y')}{H_n^{(1)}(\gamma_2 y')H_{n_h}^{(2)}(\gamma_2 y') - H_{n_h}^{(1)}(\gamma_2 y')H_n^{(2)}(\gamma_2 y')} \tag{38b}$$

$$\begin{aligned} A_3(u) = {} & -\frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\frac{\exp(-iux')}{(\gamma_2 y')^n\gamma_2}\frac{H_n^{(2)}(\gamma_2 y')}{H_n^{(1)}(\gamma_2 y')H_{n_h}^{(2)}(\gamma_2 y') - H_{n_h}^{(1)}(\gamma_2 y')H_n^{(2)}(\gamma_2 y')} \\ & \times \left[\frac{i\gamma_1\epsilon_x H_n^{(1)}(\gamma_2 b) - \gamma_2 H_{n_h}^{(1)}(\gamma_2 b)}{i\gamma_1\epsilon_x H_n^{(2)}(\gamma_2 b) - \gamma_2 H_{n_h}^{(2)}(\gamma_2 b)}\right] \end{aligned} \tag{38c}$$

$$A_4(u) = A_3(u) + \frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\frac{\exp(-iux')}{(\gamma_2 y')^n\gamma_2}\frac{H_n^{(1)}(\gamma_2 y')}{H_n^{(1)}(\gamma_2 y')H_{n_h}^{(2)}(\gamma_2 y') - H_{n_h}^{(1)}(\gamma_2 y')H_n^{(2)}(\gamma_2 y')} \tag{38d}$$

Radiated magnetic field in upper half space is

$$H_{1z}(x,y) = \frac{i\omega\epsilon_0\epsilon_x I_o}{2\pi}\int_{-\infty}^{\infty} \frac{(\Gamma_2 b)^n}{(\Gamma_2 y')^n}\frac{H_n^{(2)}(\Gamma_2 y')}{H_n^{(1)}(\Gamma_2 y')H_{n_h}^{(2)}(\Gamma_2 y') - H_{n_h}^{(1)}(\Gamma_2 y')H_n^{(2)}(\Gamma_2 y')}$$
$$\times \frac{H_n^{(1)}(\Gamma_2 b)H_{n_h}^{(2)}(\Gamma_2 b) - H_n^{(2)}(\Gamma_2 b)H_{n_h}^{(1)}(\Gamma_2 b)}{\Gamma_2 H_{n_h}^{(2)}(\Gamma_2 b) - ik_{1y}\epsilon_x H_n^{(2)}(\Gamma_2 b)}$$
$$\times \exp(-ik_{1x}x' - ik_{1y}b)\exp(ik_{1y}y + ik_{1x}x)dk_{1x} \qquad (39)$$

where $\Gamma_2(k_{1x}) = k_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - k_{1x}^2/k_0^2)}$. Setting

$$k_{1x} = k_0\cos\theta, \qquad k_{1y} = k_0\sin\theta$$

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi$$

$$x' = \rho'\cos\phi', \qquad y' = \rho'\sin\phi'$$

in above equation yields

$$H_{1z}(\rho,\phi) = \frac{i\omega\epsilon_0\epsilon_x I_0 k_0}{2\pi}\int_0^{\pi} \frac{(\Gamma_2 b)^n}{(\Gamma_2\rho'\sin\phi')^n}$$
$$\times \frac{\sin\theta H_n^{(2)}(\Gamma_2 y')}{H_n^{(1)}(\Gamma_2 y')H_{n_h}^{(2)}(\Gamma_2 y') - H_{n_h}^{(1)}(\Gamma_2 y')H_n^{(2)}(\Gamma_2 y')}$$
$$\times \frac{H_n^{(1)}(\Gamma_2 b)H_{n_h}^{(2)}(\Gamma_2 b) - H_n^{(2)}(\Gamma_2 b)H_{n_h}^{(1)}(\Gamma_2 b)}{\Gamma_2 H_{n_h}^{(2)}(\Gamma_2 b) - ik_0\epsilon_x\sin\theta H_n^{(2)}(\Gamma_2 b)}$$
$$\times \exp(-ik_0\rho'\cos\theta\cos\phi' - ik_0 b\sin\theta)$$
$$\times \exp\{ik_0\rho\cos(\theta - \phi)\}d\theta \qquad (40)$$

where $\Gamma_2(\theta) = k_0\sqrt{\frac{\epsilon_x}{\epsilon}(\epsilon - \cos^2\theta)}$

Applying saddle point technique of integration to find out the far-zone radiated field by taking $k_0\rho \gg 1$,. It is obvious that saddle point is located at $\theta = \phi$.

$$H_{1z}(\rho,\phi) = \frac{i\omega\epsilon_0\epsilon_x I_o k_0}{\sqrt{2\pi}}\frac{(\Gamma_2 b)^n}{(\Gamma_2\rho'\sin\phi')^n}$$
$$\times \frac{\sin\phi H_n^{(2)}(\Gamma_2\rho'\sin\phi')}{H_n^{(1)}(\Gamma_2\rho'\sin\phi')H_{n_h}^{(2)}(\Gamma_2\rho'\sin\phi') - H_{n_h}^{(1)}(\Gamma_2\rho'\sin\phi')H_n^{(2)}(\Gamma_2\rho'\sin\phi')}$$
$$\times \frac{H_n^{(1)}(\Gamma_2 b)H_{n_h}^{(2)}(\Gamma_2 b) - H_n^{(2)}(\Gamma_2 b)H_{n_h}^{(1)}(\Gamma_2 b)}{\Gamma_2 H_{n_h}^{(2)}(\Gamma_2 b) - ik_0\epsilon_x\sin\phi H_n^{(2)}(\Gamma_2 b)}$$
$$\times \exp(-ik_0\rho'\cos\phi\cos\phi' - ik_0 b\sin\phi)$$
$$\times \frac{\exp(ik_0\rho - i\pi/4)}{\sqrt{k_0\rho}} \qquad (41)$$

where $\Gamma_2(\phi) = k_0\sqrt{\frac{\epsilon_x}{\epsilon}\left(\epsilon - \cos^2\phi\right)}$. It is obvious that for $\epsilon_x = \epsilon$ yields

$$\Gamma_2(\phi) = k_0\sqrt{\epsilon - \cos^2\phi}.$$

3.4. Numerical results:

In this section impact of material and NID parameters on the far-zone radiated field has been presented and discussed. In all plots, behavior of $|\eta_o H_{1z}|$ is presented where η_o is impedance of the free space. Factor $\frac{\exp(ik_0\rho - i\pi/4)}{\sqrt{k_0\rho}}$ has not been taken into account in the code. Values of different parameters are taken as b=$0.4\lambda_o$, $\rho^{'} = 0.2\lambda_o$ and I_o=1 volt. Where λ_o is wavelength in free space and its value is unity in this paper. Figure 2 and Figure 3 depict the impact of material parameter whereas Figure 4 depicts the impact of NID parameter on radiated field. It is noted that impact of variation of value of ϵ_x is stronger than ϵ. It is also noticed from Figure 2 and Figure 3 that, for all observation angles, the value of $|\eta_o H_{1z}|$ decreases as ϵ_x decreases whereas variation of ϵ has negligible effects. It is noted that, for all observation angles, $|\eta_o H_{1z}|$ decreases as value of NID parameter increases.

Behavior of far-zone radiated field with respect to NID parameter is shown in Figure 5 to Figure 7. Figure 5 contains three plots, each for a specific observation angle. At each specific angle and over entire range of NID parameter, $|\eta_o H_{1z}|$ decreases as value of the NID parameter increases. Impact of variation of observation angle on field is slightly more effective near the small of NID parameter. Plots in Figure 6 and Figure 7 are obtained for specific values of material parameters. Insignificant change in behavior of the radiated field, over entire range of NID parameter, is noted when ϵ varies whereas change in behavior has been noted when ϵ_x varies. Impact of variation of ϵ_x near small values of NID parameter ($D \rightarrow 1$) is more prominent than near obtained when $D \rightarrow 2$. In Figure 8 and Figure 9 far-zone radiated magnetic field with respect to components of the tensor permittivity is examined taking specific values of

NID parameter. Over a wide range of values of permittivity, same observation has been obtained as derived from plots given in previous Figures. Over the considered range of components of permittivity, the impact of NID parameter is slightly more effective near large of ϵ_x and small values of ϵ.

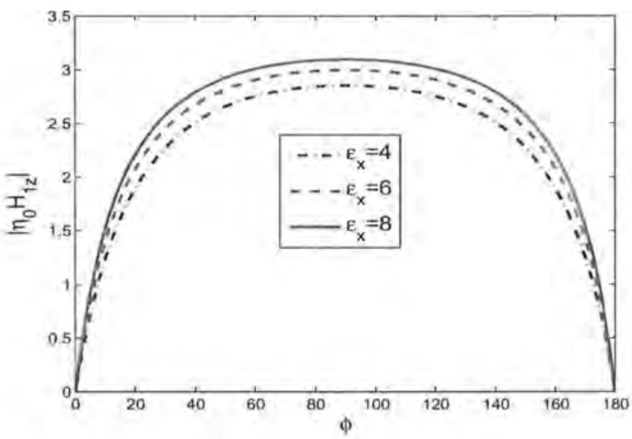

Figure 3.2: *Behavior of the far-zone radiated magnetic field with respect to observation angle for different values of ϵ_x. $\epsilon = 4$ and D=1.3.*

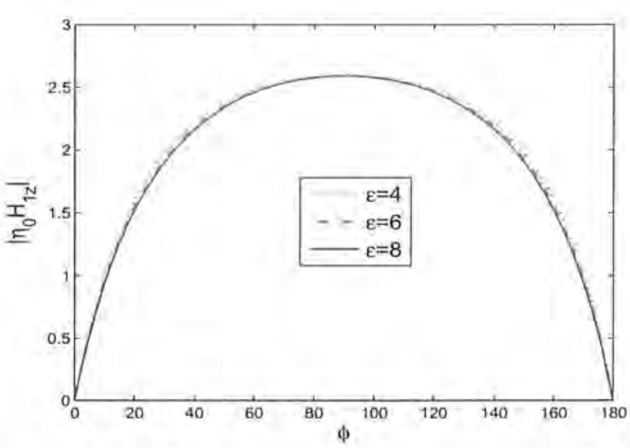

Figure 3.3: *Behavior of the far-zone radiated magnetic field with respect to observation angle for different values of ϵ. $\epsilon_x = 2$ and D=1.3.*

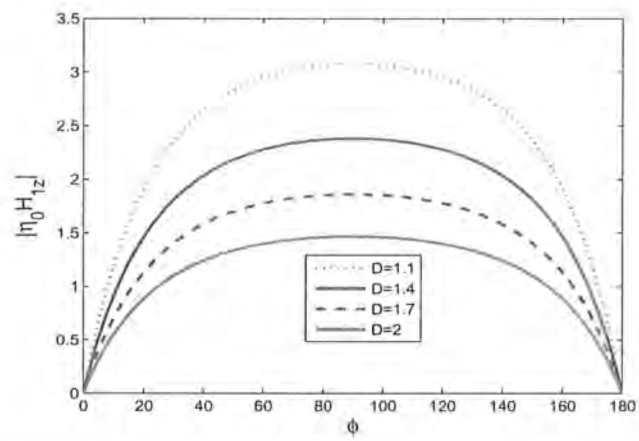

Figure 3.4: *Behavior of the far-zone radiated magnetic field with respect to observation angle for different values of NID parameter.* $\epsilon = 4$ $\epsilon_x = 2$.

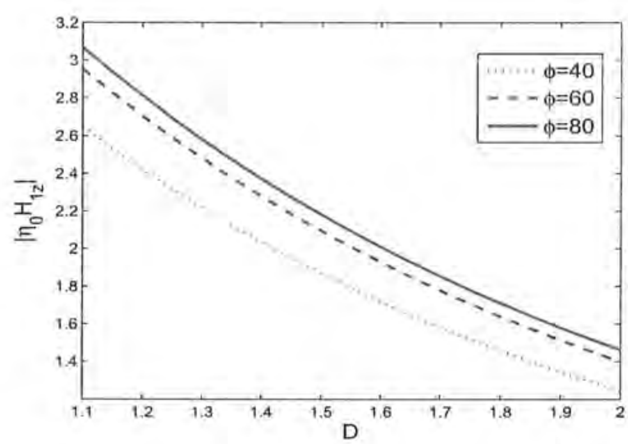

Figure 3.5: *Behavior of the far-zone radiated magnetic field with respect to NID parameter for different values of* ϕ. $\epsilon = 4$ and $\epsilon_x = 2$.

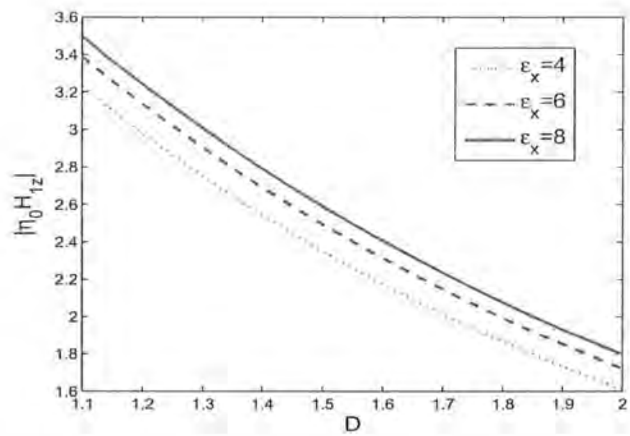

Figure 3.6: *Behavior of the far-zone radiated magnetic field with respect to NID parameter for different values of ϵ_x. $\epsilon = 4$ and $\phi = 60°$.*

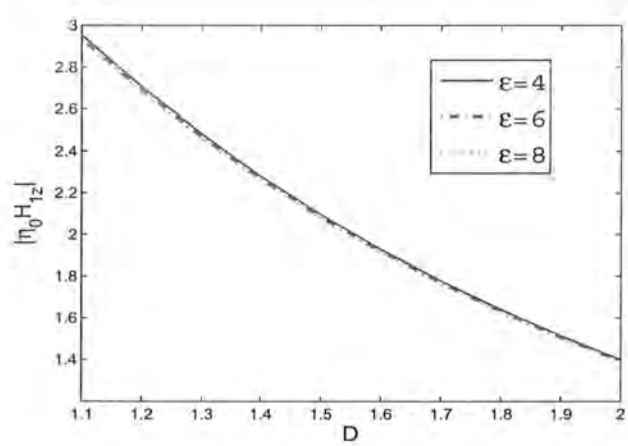

Figure 3.7: *Behavior of the far-zone radiated magnetic field with respect to NID parameter for different values of ϵ. $\epsilon_x = 2$ and $\phi = 60°$.*

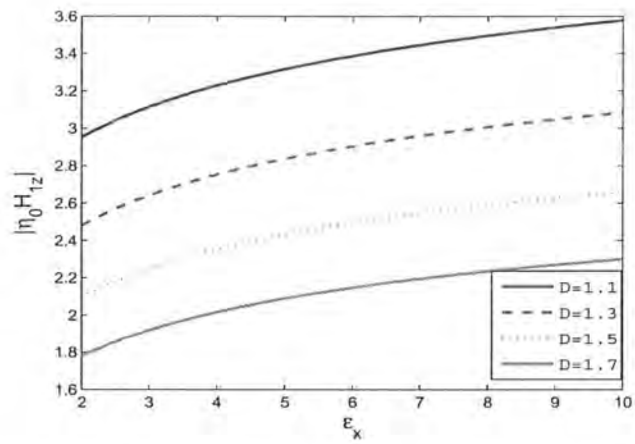

Figure 3.8: *Behavior of the far-zone radiated magnetic field with respect to ϵ_x for different values of NID parameter. $\phi = 60°$ and ϵ=4.*

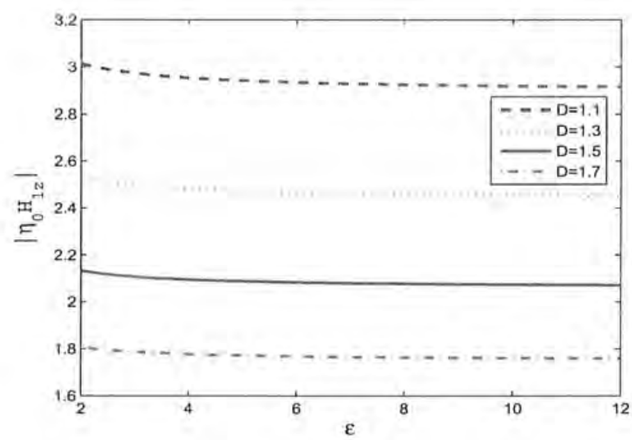

Figure 3.9: *Behavior of the far-zone radiated magnetic field with respect to ϵ for different values of NID parameter. $\phi = 60°$ and ϵ_x=2.*

CHAPTER 4

Conclusions

Conclusions and brief summary of research work are given below.

Radiation from a line source in unbounded isotropic dielectric medium has been addressed at the beginning. Secondly radiation from a line source buried beneath planar isotropic dielectric interface has been studied by using Green's function. Expressions for radiated fields have been derived by applying boundary conditions. Far-zone radiated field has been find out by applying saddle point technique of integration. Radiated field has been obtained and has maximum value at 90° and has minimum value at 0° and 180° .

Mathematical formulation of radiated field for magnetic line source buried under uniaxial anisotropic medium has been derived. Numerical results has been reported for different parameters. Radiated field for anisotropic half space has also been obtained by taking different values of ϵ_x which is permittivity of the uniaxial medium. Value of field has been increased by increasing the value of ϵ_x. Behavior of the field has also been studied by taking different values of $\epsilon_y = \epsilon_z = \epsilon$. Variation of ϵ_x is stronger than variation of ϵ when radiated magnetic field is observed with respect to observation angle and also with respect to NID parameter.

Radiation from a line source buried under uniaxial dielectric NID planar interface has been investigated at the end that was the main objective of the research work. Far-zone radiated field expressions has been derived asymptotically for buried magnetic line source by using saddle-point technique of integration. Radiated field has been studied for different values of permittivity of the uniaxial medium. Radiated field has also been studied for different values of NID parameter. Strong impact of NID parameter has been noted on the far-zone radiated magnetic field. By increasing the values of NID parameter, the value of radiated field decreases. It has also been noted behavior of radiated field with respect to NID parameter for specific values of material parameters and behavior of radiated field with respect to material parameters for specific values of NID parameter are consistent. Only a shift in values of the curve is noted.

Appendix A

Curl operator for Cartesian coordinates in NID space is given below,

$$\begin{aligned}\nabla_D \times F(x,y,z) = &\left[\left(\frac{\partial}{\partial y}F_z + \frac{1}{2}\frac{\alpha_2 - 1}{y}F_z\right) - \left(\frac{\partial}{\partial z}F_y + \frac{1}{2}\frac{\alpha_3 - 1}{z}F_y\right)\right]\hat{x} \\ &+ \left[\left(\frac{\partial}{\partial z}F_x + \frac{1}{2}\frac{\alpha_3 - 1}{z}F_x\right) - \left(\frac{\partial}{\partial x}F_z + \frac{1}{2}\frac{\alpha_1 - 1}{x}F_z\right)\right]\hat{y} \\ &+ \left[\left(\frac{\partial}{\partial x}F_y + \frac{1}{2}\frac{\alpha_1 - 1}{x}F_y\right) - \left(\frac{\partial}{\partial y}F_x + \frac{1}{2}\frac{\alpha_2 - 1}{y}F_x\right)\right]\hat{z}\end{aligned} \tag{1.A}$$

For partial derivatives in NID space following identities are used.

$$\begin{aligned}\frac{\partial}{\partial y}\left[(k_{1y}y)^n H_n^{(1)}(k_{1y}y)\right] &= k_{1y}(k_{1y}y)^n H_{n-1}^{(1)}(k_{1y}y) \\ \frac{\partial}{\partial y}\left[(k_{1y}y)^n H_n^{(2)}(k_{1y}y)\right] &= k_{1y}(k_{1y}y)^n H_{n-1}^{(2)}(k_{1y}y)\end{aligned}$$

and

$$\begin{aligned}H_{n_h}^{(1)}(k_{1y}y) &= H_{n-1}^{(1)}(k_{1y}y) + \frac{D-2}{2(k_{1y}y)}H_n^{(1)}(k_{1y}y) \\ H_{n_h}^{(2)}(k_{1y}y) &= H_{n-1}^{(2)}(k_{1y}y) + \frac{D-2}{2(k_{1y}y)}H_n^{(2)}(k_{1y}y)\end{aligned}$$

These identities are used to find the expressions for NID space.

$$H_{-n}^{(1)} = \exp(jn\pi)H_n^{(1)}(k_{1y}y) \tag{2.A}$$

$$H_{-n}^{(2)} = \exp(-jn\pi)H_n^{(2)}(k_{1y}y) \tag{3.A}$$

$$H_{\frac{1}{2}}^{(1)}(k_{1y}y) = \sqrt{\frac{2}{\pi}}\frac{\exp\{j(k_{1y}y - \frac{\pi}{2})\}}{\sqrt{k_{1y}y}} \tag{4.A}$$

$$H_{-\frac{1}{2}}^{(1)}(k_{1y}y) = j\sqrt{\frac{2}{\pi}}\frac{\exp\{j(k_{1y}y - \frac{\pi}{2})\}}{\sqrt{k_{1y}y}} \tag{5.A}$$

$$H_{\frac{1}{2}}^{(2)}(k_{1y}y) = \sqrt{\frac{2}{\pi}}\frac{\exp\{-j(k_{1y}y - \frac{\pi}{2})\}}{\sqrt{k_{1y}y}} \tag{6.A}$$

$$H_{-\frac{1}{2}}^{(2)}(k_{1y}y) = -j\sqrt{\frac{2}{\pi}}\frac{\exp\{-j(k_{1y}y - \frac{\pi}{2})\}}{\sqrt{k_{1y}y}} \tag{7.A}$$

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