RENORMALIZATION OF ELECTROWEAK THEORY AT FINITE TEMPERATURE

By

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Certificate

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Abstract

The renormalizability of the standard electroweak theory at finite temperatures and densities much below the electroweak scale at the one loop level in the background of hot and dense leptons have been checked. Moreover, we calculate self-mass and charge renormalization in such a background. Some of the applications of these results are also mentioned in this work.

Introduction

Human nature is so enthusiastic that even the astonishing discoveries, sometimes, can open new venues of research in the particular area. Human curiosity always demands the improvement of knowledge. Therefore, the scientific discoveries lead to new directions of research.

The intellectual mind is always exuberent to find the origin of the universe through the discovery of new laws of nature. Therefore the beginning of the universe has been studied since long. In this regard the standard big bang model of cosmology is quite well accepted. This model also depicts the thermal history of the universe. The information about the prevalant thermodynamic conditions is very important because it helps to understand the existence of our galaxy, solar system and the earth itself.

Present knowledge of the early history of the universe [1] starts from the Plancks epoch (i.e., $t \sim 10^{-43}$ sec and $T = 10^{19}$ GeV), the point at which quantum corrections to general relativity should render it invalid. At the earliest times the universe was a plasma of relativistic particles, including quarks, leptons, gauge bosons, and the Higgs bosons. If current ideas are correct, a number of spontaneous symmetry breaking (SSB) phase transitions should take place during the course of the early evolution of the universe. They include the grand unified theories (GUTs) and phase transition at a temperature of 10^{14} to 10^{16} GeV, and the electroweak SSB phase transitions at temperatures of around 300 GeV. During these SSB phase transitions some of the gauge bosons aquire mass via the Higgs mechanism and the entire symmetry of the theory is broken to a lower symmetry.

The standard cosmology of the universe and the microwave background give a clear evidence of the electroweak processes in the early universe. There is an evidence for leptonic scatterings at $T < 2$ MeV i.e., in the highly dense medium and then β-decays before nucleosynthesis led to the formation of He at $T \sim m_e$. All these calculations, even in the vacuum theories of particle physics, do not give the desired information and the techniques have been developed to improve these results incorporating different type of corrections. For any physically acceptable quantum field theory (QFT) the question of renormalization is one of the basic requirements. A most important step in this direction would be the evaluation of the renormalization constants in the relevant background. In this regard the effects of the thermal background are also being incorporated through the calculation of the radiative corrections at the 1-loop level [2, 3, 4, 5] as well as at the 2-loop level [6, 7, 8] in hot and dense medium.

Different renormalization schemes for the electroweak theory have been proposed in literature [9]. The most successful among these is the one by t'Hooft [10], Sirlin [11] and some others [12, 13]. Since we are dealing with the first order corrections at the moment and have used the on-shell scheme [14], following Hollik [15], it is worthwhile to study the radiative corrections to electroweak processes and to check the renormalizability of the standard electroweak theory.

The renormalizability of electroweak theory in vacuum is quite well understood now. As a next step, we want to renormalize the theory in hot and dense medium so that the relevant background effects can be correspondingly incorporated. In this connection, following Hollik [15], we try to evaluate the renormalization constants of electroweak theory at finite temperature and density (FTD) below the threshold for creation of electroweak bosons in the background upto the 1-loop level. We use the real-time formalism for this purpose and see to what extent the thermal background can affect the physical results.

Chapter 1

Gauge Theories and the Standard Model

1.1 Gauge Theories: An Introduction

The present understanding of various forces of nature relies on three basic principles of physics which can be summarised as:

1. Invariance of the lagrangian under local gauge symmetries in Yang-Mills theories.
2. Nambu-Goldstone realization of a symmetry or non-invariance of vacuum under the symmetry transformation.
3. Higgs mechanism that combines (1) and (2) to eliminate the massless unphysical scalar fields from the theory and produce mass terms for the gauge bosons so that the weak interactions could be short ranged.

1.1.1 Abelian Gauge Theories

The most well tested theory of the particle interaction, i.e., Quantum Electrodynamics (QED), is based on the principles of local gauge invariance. To see this, consider a fermion carrying an electric charge 'e' described by the field ψ. Its electromagnetic (e.m) interaction can be derived by demanding that the lagrangian for this field should be invariant under the local symmetry transformation:

$$\psi(x) \rightarrow \exp\{i\alpha(x)\}\psi(x). \tag{1.1}$$

The kinetic energy term involving $\partial_\mu \psi(x)$ can be made invariant under this transformation only if ∂ is replaced by a covariant derivative , i.e.,

$$\partial_\mu\psi(x) \rightarrow D_\mu\psi(x) = (\partial_\mu - ieA_\mu(x)). \tag{1.2}$$

where A_μ transforms as

$$A_\mu(x) \rightarrow A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x). \tag{1.3}$$

Eqs.(1.1) to (1.3) are collectively called gauge group transformations. It can be shown that the invariant kinetic energy term for ψ is $\bar{\psi}\gamma_\mu D^\mu\psi$ and that for A_μ, is $F_{\mu\nu}F^{\mu\nu}$, where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1.4}$$

is the field strength tensor. This leads to the lagrangian density,

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma_\mu D^\mu\psi. \tag{1.5}$$

The form of the electromagnetic interaction lagrangian dictated by the local symmetry is

$$\mathcal{L}^{\text{e.m.}}_{\text{int.}} = ie\bar{\psi}\gamma_\mu\psi A^\mu. \tag{1.6}$$

The transformation in eq.(1.1) generates the U(1) gauge group.

1.1.2 Non-Abelian Gauge Theories

In 1954, Yang and Mills generalized the abelian U(1) transformation to the case where the local symmetry is associated with the non-abelian group. The idea is that, in such a situation, one could generate interactions that take one type of particle to another, e.g., photon plays a role of mediator in the e.m interaction.

To study the formalism of non-abelian gauge theories consider a simple non-abelian group G. Let ψ be a multiplet of fermions transforming as an irreducible representation under G as follows

$$\psi(x) \xrightarrow{G} \psi'(x) = U(x)\psi(x). \tag{1.7}$$

where U is a unitary representation of G in the space ψ. To construct a G-invariant kinetic energy in ψ, note that

$$\partial_\mu \Psi \rightarrow U(x)\partial_\mu \Psi + (\partial_\mu U)\Psi. \tag{1.8}$$

In analogy with the abelian case, if we replace

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu, \tag{1.9}$$

then

$$\bar{\Psi}(\partial_\mu - A_\mu)\Psi \rightarrow \bar{\Psi}\partial_\mu \Psi + \bar{\Psi}U^\dagger \partial_\mu U \Psi - \Psi U^\dagger A_\mu U \Psi. \tag{1.10}$$

For this to be invariant, obviously one must have

$$A_\mu = UA_\mu U^\dagger + (\partial_\mu U)U^\dagger. \tag{1.11}$$

Eqs.(1.8) and (1.11), therefore, constitute the generalized non-abelian gauge transformations for ψ and A_μ. We now have to construct the counterpart of $F_{\mu\nu}$ for the non-abelian case. Using eq.(1.3), one finds by simple algebra that.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu], \tag{1.12}$$

and it transforms under G as

$$F_{\mu\nu} = UF_{\mu\nu}U^\dagger. \tag{1.13}$$

One can now write the following gauge invariant lagrangian for the whole system as

$$\mathcal{L} = -\bar{\psi}\gamma^\mu D_\mu \psi + \frac{1}{4g^2}\text{Tr } F_{\mu\nu}F^{\mu\nu}. \tag{1.14}$$

Here 'g' is some dimensionless scale parameter. We notice that in order to preserve the gauge invariance there should be as many gauge fields as the number of generators of the gauge group. The gauge potential is known to be the adjoint representation of the gauge group. We can write it as,

$$A_\mu = ig\theta_\alpha A_{\alpha\mu}, \tag{1.15}$$

where the sum over the repeated index is known and θ_α are the generators of the gauge group, and they satisfy the following commutation relation:

$$[\theta_a, \theta_b] = if_{abc}\theta_c, \tag{1.16}$$

f_{abc} are the structure constants of the corresponding group, e.g., for SU(2), $f_{abc} = \varepsilon_{abc}$ the totally anti-symmetric tensor. It then follows that

$$F_{\mu\nu} = ig\theta_\alpha f_{\mu\nu,\alpha}, \tag{1.17}$$

where

$$f_{\mu\nu,\alpha} = \partial_\mu A_{\nu,\alpha} - \partial_\nu A_{\mu,\alpha} + gf_{\alpha\beta\gamma}A_{\mu,\beta}A_{\nu,\gamma}. \tag{1.18}$$

Here g is the universal gauge coupling constant that couples the gauge fields among themselves with the same strength. This is known as the so called universality of the gauge coupling. For example, the β-decay and μ-decay have the same weak coupling $\frac{G_F}{2}$ despite their widely different lifetimes.

Let us see how the non-abelian gauge theories describe weak interactions. Take a model [16] based on an SU(2) gauge group with proton and neutron transforming as an SU(2) doublet (ignoring helicities); neutrino and electron transforming as another doublet, i.e.,

$$\psi = \begin{bmatrix} p \\ n \end{bmatrix}, \quad \psi = \begin{bmatrix} \nu_e \\ e \end{bmatrix}. \tag{1.19}$$

Since SU(2) has three generators, i.e., the Pauli matrices ($\frac{\tau_\alpha}{2}$, $\alpha = 1, 2, 3$), we can write

$$\begin{aligned} A_\mu &= \frac{i}{2}\tau_\alpha W_{\alpha,\mu} \\ &= \frac{i}{2}\cdot\begin{bmatrix} W_{3\mu} & (W_1 - iW_2)_\mu \\ (W_1 + iW_2)_\mu & -W_{3\mu} \end{bmatrix} \\ &= \frac{i}{2}\cdot\begin{bmatrix} W_{3\mu} & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} \end{bmatrix}. \end{aligned} \tag{1.20}$$

The interaction lagrangian generated by local SU(2) symmetry follows from the first term in eq.(1.14) to be

$$
\begin{aligned}
\mathcal{L}_W &= -\frac{ig}{2}(\bar{p} \quad \bar{n})\gamma^\mu \begin{bmatrix} W_{3\mu} & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} \end{bmatrix} \begin{bmatrix} p \\ n \end{bmatrix} \\
&\quad -i\frac{g}{2}(\bar{\nu}_e \quad \bar{e})\gamma^\mu \begin{bmatrix} W_{3\mu} & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} \end{bmatrix} \begin{bmatrix} \nu_e \\ e \end{bmatrix} + \text{h.c} \\
&= -i\frac{g}{\sqrt{2}}W_\mu^+(\bar{P}\gamma^\mu n + \bar{\nu}\gamma^\mu e) \\
&\quad -i\frac{g}{\sqrt{2}}W_{3\mu}(\bar{P}\gamma^\mu P + \bar{\nu}\gamma^\mu \nu - \bar{n}\gamma^\mu n - \bar{e}\gamma^\mu e) + \text{h.c.} \qquad (1.21)
\end{aligned}
$$

The first term in eq.(1.21) leads via the second order perturbation to the β-decay process

$$n \to p + e^- + \bar{\nu}_e.$$

Note that the fermions, of course, could have mass since $\bar{\psi}\psi$ is invariant under the transformation (1.8) in this model. One can immediately see from eqs.(1.8) and (1.11) that the gauge transformations prevent the W-bosons from acquiring mass, because the mass term $W_\mu W^\mu$ is not invariant under eq.(1.11). Therefore, this theory predicts that weak interactions should be long ranged, in contradiction with the observation.

One might think, why we do not simply add a mass term $\frac{1}{2}(m_W^2 W_\mu W^\mu)$ to the lagrangian and ignore its invariance under the local symmetry. Such an approach was first attemped by Veltman [17]. However, it leads to divergences beyond the one loop level [18] in perturbation theory, rendering it not too useful for making predictions. This approach, therefore, had to be abandoned and an alternative solution of the problem had to be searched. Of course the solution was found under the idea of SSB. We consider this idea in the next section and observe how SSB of the local symmetry produces mass terms for the mediating gauge bosons.

1.2 Spontaneous Breaking of Local Symmetry

In quantum field theories with symmetries, two conditions determine the way in which a symmetry manifests itself. First, the lagrangian must be invariant under the symmetry transformation. Secondly, the vacuum state must be invariant under it. However, Nambu and Goldstone [19] discovered, around 1960, that a lagrangian may be invariant under a symmetry transformation, but the vacuum state may not be so. If the symmetry is global, the spectrum of the theory contains massless particles i.e., the Goldstone bosons [19]. It was realized [20] afterwards that if a gauge symmetry is spontaneously broken, then ultimately we have no such particle. But the gauge bosons corresponding to the broken generators acquire masses. This is known as the Higgs mechanism. The $SU(2)_L\otimes U(1)_Y$ model of electroweak interactions uses this mechanism in order to get masses for the gauge bosons and, in fact, for all fermions.

We take here a simple abelian example of Higgs mechanism. Consider a U(1) local symmetry with a scalar field ϕ transforming under U(1) as

$$\phi \to e^{ie\lambda(x)}\phi. \tag{1.22}$$

If we demand that the spin 1 gauge field transforms under the local U(1) as

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \lambda(x), \tag{1.23}$$

then the gauge invariant lagrangian for the system can be written as

$$\mathcal{L} = \mathcal{L}_o - \mathrm{V}(x), \tag{1.24}$$

where

$$\mathcal{L}_o = (D_\mu\phi)^\dagger(D^\mu\phi) - \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}. \tag{1.25}$$

The potential $\mathrm{V}(\phi)$ must be gauge invariant. The most general form of the potential is

$$\mathrm{V}(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \tag{1.26}$$

λ must be positive in order that the potential has a lower bound. There is no restriction on the sign of μ^2. If μ^2 is positive, then the shape of the potential $V(\phi)$ appears as in Fig.(1). Minimization of the potential yields

$$\phi(x) = \phi_o = \sqrt{\frac{\mu^2}{2\lambda}} e^{i\theta}. \tag{1.27}$$

The lagrangian is invariant under the $e^{\iota\theta}$ transformation, therefore, θ can be chosen as zero for convenience, so that

$$\phi_o = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{V}{\sqrt{2}}, \quad V = \sqrt{\frac{\mu^2}{\lambda}} > 0. \tag{1.28}$$

Let us redefine

$$\phi(x) = \frac{1}{\sqrt{2}}[\sigma(x) + V], \tag{1.29}$$

with

$$\sigma(x) = \eta(x) + i\xi(x), \tag{1.30}$$

where V is the minimum value of ϕ (x) and σ (x) (a complex field) is a small fluctuation around this minimum, so that σ(x) can be expanded in terms of creation and annihilation operators. Substituting eqs.(1.4), (1.7) and (1.8) into eq.(1.5), one can see that the kinetic energy term for ϕ leads to a term $\frac{1}{2}(e^2 v^2 A_\mu A^\mu)$ in the lagrangian, which is a mass term for A_μ. Thus, the process of SSB gives mass to gauge bosons. In fact, the particle spectrum of the theory also appears to have a massless Goldstone boson ξ and a massive scalar η. But these are not the physical particles, because by changing the field variables one can find a particular gauge, such that, the massless boson is eliminated from the lagrangian. Thus one is left with only two interacting massive particles, a vector gauge boson A_μ and a massive scalar, say H which is called the Higgs particle. This phenomenon is known as the 'Higgs mechanism'.

We have discussed SSB of a U(1) gauge symmetry for illustration. It can easily be extended to the case of SU(2) local gauge symmetry, in which one introduces a Higgs weak isospin doublet coupled to the fermions and gauge bosons.

The three basic ingredients needed for building a gauge model can be summarized as:

- Choice of the gauge group G,
- Assignment of the fermions to a suitable representation of the gauge group,
- Choice of the Higgs bosons and their expectation values to break the gauge symmetry. The choice must be good enough to reproduce the quark and lepton masses in a phenomenologically acceptable way.

1.3 The Standard Model

According to the standard model of particle physics, all the matter in the universe is made of two kinds of particles, the 'quarks' and the 'leptons'. Quarks make up 'hadrons' and can participate in all type of interactions whereas, the leptons are affected by electromagnetic and weak forces only. It should be mentioned here that the neutral leptons i.e., the neutrinos, interact only weakly.

As already stated, the hadrons are made up of quarks that come in six flavors, labelled as u(up), d(down), s(strange), c(charm), b(bottom), and t(top). The 'baryons' are assumed to be composites of three quarks qqq and 'mesons' as $q\bar{q}$ (quark-antiquark) composites. Considerable hadronic spectra can be gained using potential picture binding 3q's and $q\bar{q}$. It follows that quarks have the baryon number $B = \frac{1}{3}$ and also to reconcile with statistics it becomes necessary to give an additional SU(3) quantum number to each quark flavor, called 'color'. Each quark flavor comes in three colors, Red, Green, and Blue. All the six quark flavors are now experimently verified. Various kinds of forces (including gravitation) are supposed to arise from the exchange of spin 1 bosons whose couplings are dictated by the underlying local symmetry. Before proceeding further, we notice a very fundamental symmetry [21] between quarks and leptons, for each type (flavor) of quark there exists a corresponding lepton and weak interactions respect quark-lepton symmetry, such that, all the lepton flavors have corresponding quark flavors in different colors.

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \qquad 1$$

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L \qquad 2$$

$$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L \qquad 3.$$

The six flavors are grouped into three sets (called generations) of two each with $i =$ R, G, and B, standing for Red, Green, and Blue respectively. All known forms of weak interactions are assumed to operate only inside each generation. Observed cross-generation transitions such as $\Delta S = 1$, i.e., the strangeness changing weak decays are explained in terms of a mixing angle, the Cabbibo angle [22], and its extended version, the CKM (Cabibbo, Kobayashi, and Maskawa) matrix [23].

In the rest of this chapter we will work only with the first generation, which can be simply generalized to higher generations by similar type of analysis.

1.4 $SU(2)_L \otimes U(1)_Y$ Electroweak Model

Historically, the $SU(2)_L \otimes U(1)_Y$ model modifying weak and electromagnetic interactions was first suggested by Glashow in 1961 [24] with the W-boson masses inserted by hand. This was subsequently discussed by Salam and Ward [25] also. The model in this form with Higgs mechanism for the generation of the W-bosons was represented for the case of leptons by Salam and Weinberg [26]. The incorporation of hadrons into the model was done following the suggestion of Glashow, Illiopous and Maiani (known as the GIM mechanism [27]), which incorporated the six flavors of quarks (not then discovered). In the present section we will give an outline of this model.

The structure of the model is completely determined by assignment of quarks and leptons to the representation of the group. For simplicity we consider only the first generation, i.e., $(\nu_e, e; u, d)$. We denote their $SU(2)_L \otimes U(1)_Y$ contents by (I_W, Y), where I_W stands for the weak isospin and Y for the weak hypercharge.

$$\psi_L = \begin{bmatrix} \nu_e \\ e \end{bmatrix}_L \qquad (\tfrac{1}{2}, -1),$$

$$e_R \qquad (0, -2) \tag{1.31}$$

$$Q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L \qquad (\tfrac{1}{2}, \tfrac{1}{3}),$$

$$u_R \qquad (0, \tfrac{4}{3})$$

$$d_R \qquad (0, \tfrac{-2}{3}). \tag{1.32}$$

We choose the Higgs doublet denoted by

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^- \end{bmatrix}, \tag{1.33}$$

transforming like $(\frac{1}{2}, +1)$. The electric charge in this model is given by

$$Q = I_{3W} + \frac{Y}{2}, \tag{1.34}$$

that is

$$eJ_\mu^{em} = e(j_\mu^3 + \frac{j_\mu^Y}{2}), \tag{1.35}$$

j_μ represents the corresponding four currents. Y generates the symmetry group $U(1)_Y$ just as Q generates the group $U(1)_{em}$. In this way the electromagnetic and weak interactions are incorporated in a single framework, the symmetry group $SU(2)_L \otimes U(1)_Y$.

The basic electroweak interaction can be written as

$$\mathcal{L}_{ew} = -ig(j_i^\mu)W_{i\mu} - i\acute{g}(j_Y^\mu)\frac{B_\mu}{2}, \tag{1.36}$$

showing the isotriplet of vector fields $W_{\imath\mu}$ coupled to the weak isospin current $j_\imath^\mu$ with strength g, and the single vector field B_μ coupled to the weak hypercharge current j_Y^μ with strength conventionally taken to be $\frac{\acute{g}}{2}$ with $\imath$ is the SU(2) index. The fields

$$W_\mu^\mp = \frac{1}{\sqrt{2}}(W_{1\mu} \pm \imath W_{2\mu}), \tag{1.37}$$

describe massive charged bosons $W^\mp$, whereas $W_{3\mu}$ and B_μ are neutral fields. The electromagnetic interaction is embeded in eq.(1.6) through the eq.(1.4) and (1.5).

The charged current ($C.C.$) weak lagrangian can now be written as

$$\mathcal{L}_W^{C.C.} = \frac{\imath g}{2\sqrt{2}} W_\mu \left[\bar{\nu}\gamma^\mu(1-\gamma_5)e + \bar{u}\gamma^\mu(1-\gamma_5)d\right], \tag{1.38}$$

which leads to the expression for the fermion coupling:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}. \tag{1.39}$$

The two neutral fields $W_{3\mu}$ and B_μ must mix in such a way that the massive fields are expressed as a linear combination of the gauge fields.

$$A_\mu = B_\mu \cos\theta_W + W_{3\mu}\sin\theta_W \text{ (massless)}, \tag{1.40}$$

$$Z_\mu = -B_\mu \sin\theta_W + W_{3\mu}\cos\theta_W \text{ (massive)}, \tag{1.41}$$

θ_W is the weak mixing angle.

One can write from eq.(1.6) the expression for neutral current ($N.C.$) electroweak interaction as

$$\begin{aligned}\mathcal{L}_{\text{ew}}^{N.C.} &= -gj_3^\mu W_{3\mu} - \imath\acute{g}j_Y^\mu B_\mu \\ &= -\imath\left[g\sin\theta_W j_3^\mu + \frac{\acute{g}}{2}\cos\theta_W j_Y^\mu\right]A_\mu \\ &\quad -\imath\left[g\cos\theta_W j_3^\mu - \frac{\acute{g}}{2}\sin\theta_W j_Y^\mu\right]Z_\mu.\end{aligned} \tag{1.42}$$

By using eqs.(1.10) and (1.11). The first term in the last equation gives the electromagnetic interaction. Comparing it with eq.(1.5), we have

$$g\sin\theta_W = \acute{g}\cos\theta_W = e, \tag{1.43}$$

which yields

$$\tan\theta_W = \frac{\acute{g}}{g}. \tag{1.44}$$

To extract further physical contents of the model we write the gauge invariant lagrangian involving the fermion fields, the gauge fields and the Higgs fields such that

$$\mathcal{L} = \mathcal{L}_G = \mathcal{L}_Y - V(\phi), \tag{1.45}$$

where $\mathcal{L}_G$ is the part of the lagrangian involving the gauge invariant derivatives for all fields, $\mathcal{L}_Y$ is the Yuakawa coupling of fermions and Higgs bosons, while $V(\phi)$ is the Higgs potential. The explicit form of $\mathcal{L}_G$ is

$$\begin{aligned}\mathcal{L}_G &= -\bar{\psi}_L\gamma^\mu D_\mu\psi_L - \bar{e}_R\gamma^\mu D_\mu e_R - \bar{Q}_L\gamma^\mu D_\mu Q_L \\ &\quad -\bar{u}_R\gamma^\mu D_\mu u_R - \bar{d}_R\gamma^\mu D_\mu d_R - (D^\star_\mu\phi)(D^\mu\phi) \\ &\quad -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},\end{aligned} \tag{1.46}$$

whereas, the covariant derivatives D_μ corresponding to fermion fields are

$$\begin{aligned} D_\mu\psi_L &= (\partial_\mu - i\frac{g}{2}\tau_i W_{i\mu} + \iota\frac{\acute{g}}{2}B_\mu)\psi_L, \\ D_\mu e_R &= (\partial_\mu + i\acute{g}B_\mu)e_R, \\ D_\mu Q_L &= (\partial_\mu - \frac{i}{g}\tau_i W_{i\mu} - i\frac{\acute{g}}{6}B_\mu)Q_L, \\ D_\mu u_R &= (\partial_\mu - i\frac{\acute{g}}{3}B_\mu)u_R, \\ D_\mu d_R &= (\partial_\mu + i\frac{\acute{g}}{3}B_\mu)d_R, \end{aligned}$$

and the boson field are,

$$D_\mu\phi = (\partial_\mu - i\frac{g}{2}\tau_i.W_{i\mu} - i\acute{g}B_\mu)\phi,$$

with the tensor fields,

$$\begin{aligned} f_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu + gW_\mu \times W_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{1.47}$$

and

$$\mathcal{L}_Y = h_e\bar{\psi}_L\tilde{\phi}e_R + h_u\bar{Q}_L\phi u_R + h_d\bar{Q}_L\phi d_R, \tag{1.48}$$

$\tilde{\phi} = \iota\tau_2\phi^*$ is the charge conjugate representation of ϕ, τ_ι's are the weak isospin generators, and

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2. \tag{1.49}$$

It is now clear from eq.(1.29) that the minimum energy solution of the Hamiltonian corresponds to

$$\phi_o = \frac{\nu}{\sqrt{2}}, \text{ with } \quad \nu = \sqrt{\frac{\mu^2}{\lambda}}. \tag{1.50}$$

To get masses of the fermion and the gauge bosons we use eq.(1.50) which leads to

$$M_W^2 = \frac{g^2\nu^2}{4}. \tag{1.51}$$

$$m_f = h_f\frac{\nu}{\sqrt{2}}, \tag{1.52}$$

due to the choice of only one helicity state for the neutrino field, $m_\nu = 0$ in this model.

Once again we consider the ($W_{3\mu}$ - B_μ) sector. One obtains the following mass mixing matrix:

$$\begin{array}{c} \\ W_{3\mu} \\ B_\mu \end{array} \begin{array}{c} \begin{array}{cc} W_{3\mu} & B_\mu \end{array} \\ \left[\begin{array}{cc} \frac{g^2\nu^2}{4} & \frac{-g\acute{g}\nu^2}{4} \\ \frac{-g\acute{g}\nu^2}{4} & \frac{\acute{g}^2\nu^2}{4} \end{array}\right] \end{array} \tag{1.53}$$

To identify the physical gauge bosons, we have to diagonalize the above mass matrix. The eigenvectors are those given by eqs.(1.40) and (1.41) with the masses

$$M_Z^2 = \frac{(g^2 + \acute{g}^2)\nu^2}{4} = \frac{g\nu \sec\theta_W}{4}, \tag{1.54}$$

and

$$m_A^2 = 0. \tag{1.55}$$

From eqs.(1.21) and (1.22), one can see that M_W and M_Z are related by

$$M_Z \cos\theta_W = M_W. \tag{1.56}$$

Now eqs.(1.9) and (1.13) imply

$$M_W = \sqrt{\frac{\sqrt{2}e^2}{8G_F \sin^2\theta_W}} = \frac{37.35}{\sin\theta_W}. \tag{1.57}$$

Thus we are left with only one parameter, the weak mixing angle θ_W, which is measured experimentally from the neutral current phenomena.

The standard SU(2) $\otimes$ U(1) model of electroweak unification has been described briefly in this chapter. This model works very well in explaining the low energy electroweak interactions. Most of the predictions of the model have been confirmed experimentally. The Higgs sector of the model is still unexplored. Also the CP violation problem is a great challenge to the model. In the next chapter we shall discuss the problem of renormalization of electroweak processes in the context of standard model.

Chapter 2

Renormalizability of Gauge Theories

2.1 Renormalization

Renormalizibility means that the amplitudes of different processes associated with an interaction should be finite i.e., non-divergent at higher energies and in higher orders of perturbation. The prototype field theory i.e., Quantum Electrodynamics (QED), does in fact contain divergent terms associated with integrals over intermediate states, but it is found that these divergences can always be thrown away in a redefinition of the 'bare' lepton charges and masses, which are in any case arbitrary, as being equal to the physically measured values. Thus we can say that a theory is renormalizable, if at the cost of introducing a finite number of arbitrary parameters (to be determined experimentally) the predicted amplitudes for physical processes remain finite at all energies and to all orders in the coupling constant. QED is an example of such a theory and for many years, was the only one.

The true nature of QED resides in the concept of renormalization. Tomonaga called this concept by a name, in Japanese which phrases 'compounding interest' in bank accounts i.e., to say, putting the interest back into the account to earn more interest.

To explain the idea behind the theory by S. Tomonaga and many others [28], it seems appropriate to quote again the phrase 'principle of renunciation' used by Tomonaga. The meaning of this phrase is, one should give up the hope that the theory is perfect and that everything can be calculated from it, and instead, one should make a definite distinction between things that can be calculated and those that cannot. However, the theorists after spending some 20 years in the dark, finally, got the idea that the addition of some new elements to the renormalization theory would pave the way for a solution. However, early theories of weak interactions though well behaved at low energy and to first order, involved divergences at higher orders. These could be calculated only at the cost of introducing an indefinitely large number of arbitrary constants, thus loosing essentially their predictive power, if any, as discussed above. A good high energy behaviour and cancellation of divergent terms at higher orders are thus sensible demands for any physical theory.

In order to analyse the different renormalization schemes, consider the tree level lagrangian of the minimal SU(2)$_W\otimes$ U(1)$_Y$ model

$$\mathcal{L}_c = \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_F, \tag{2.1}$$

with the gauge field part

$$\mathcal{L}_G = -\frac{1}{4}W^{\alpha}_{\mu\nu}W^{\mu\nu,\alpha} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{2.2}$$

and the Higgs part

$$\mathcal{L}_H = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi), \tag{2.3}$$

with the covariant derivative,

$$D_\mu = \partial_\mu - ig_2 J_\alpha W^\alpha_\mu + i\frac{g_1}{2}B_\mu,$$

and the corresponding Higgs field self-interaction (Higgs potential),

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2,$$

whereas the induced fermion gauge field interaction via the minimal substitution rule

$$\mathcal{L}_F = \sum_j \bar{\psi}_j^L i\gamma^\mu D_\mu \psi_j^L + \sum_{j,\sigma} \bar{\psi}_j {\sigma^R} i\gamma^\mu D_\mu \psi_j {\sigma^R}, \tag{2.4}$$

involves a number of free parameters which are not fixed by the theory. The definition of these parameters and their relation to measureable quantities is the main content of a renormalization scheme. The parameters (or appropriate combinations) can be determined from specific experiments with the help of theoretical calculations of the physical processes. After this procedure of defining the physical input, other observables can be predicted allowing verification of the theory by comparison with the corresponding experimental results. In higher order perturbation theory the relation between the formal parameters and measurable quantities are different from the tree level relations in general. Moreover, the procedure is obscured by the appearance of divergences from the loop integrations. For a mathematically consistent treatment one has to regularize the theory, e.g., by dimensional regularization (performing the calculations in D-dimensions). However, the relation between physical quantities and the parameters then become cut off dependent. Hence the parameters of the basic lagrangian, the 'bare' parameters, have no physical meaning. On the other hand, relations between measureable physical quantities, where the parameters drop out, are finite and independent of the cut off. It is therefore, possible to perform tests of the theory in terms of such relations by eliminating the bare parameters [29, 30]. The minimal $SU(2)_W \otimes U(1)_Y$ model, involves free parameters such as:

$$e, M_W, M_Z, M_H, m_{f,i},$$

which have to be determined experimentally. These are chosen such that they comprise the physical meaning of different parameters of the theory, means that these are related to experimental quantities. This direct relation is also destroyed through higher order corrections. Also the parameters of the original lagrangian called the bare parameters, differ from the corresponding physical quantities by UV-divergent contributions (but appear in the higher values of momentum).

These divergences are cancelled in relations between the physical quantities, allowing for meaningful predictions in the renormalizable theories. The possibility to evaluate predictions of a renormalizable model is the following:

- calculate physical quantities in terms of the bare parameters.
- use as many of the resulting relations as bare parameters present, to express these in terms of physical observations.
- insert the resulting expressions into the remaining relations.

In this way we can predict physical observables in terms of other physical quantities, which have to be determined from experiments. In these predictions all UV-divergences cancel in any order of perturbation theory. The predictions obtained from different input parameters differ in finite orders of perturbation theory, in higher order contributions. This electroweak on-shell scheme is the straight forward extension of the familiar QED, first proposed by Ross and Taylor [14] and used in many practical applications [11, 31, 32, 33, 34, 35].

For stable particles, the masses are well defined quantities and can be measured with high accuracy. The masses of W and Z bosons are related to the resonance peaks in cross sections where they are produced and hence can also be accurately determined. The mass of the Higgs boson, as long as it is experimently unknown, is treated as a free input parameter. Before we can make predictions from the theory, a set of independent parameters has to be determined from experiments. This can either be done for the bare quantities or for renormalized parameters which have a simple physical interpretation. In a more restrictive sense, a renormalization scheme characterizes a specific choice of experimental data points to be used as input, defining the basic parameters of the lagrangian in terms of which the perturbative calculation of physical amplitudes is performed. Predictions for the relations between physical quantities do not depend on the choice of a specific renormalization scheme, if we perform the calculation to all orders in the perturbative expansion. Practical calculations, however, are obtained by truncating perturbation series, making the predictions dependent on the chosen set of basic parameters and thereby leading to scheme dependence.

Parameterizations or 'renormalization schemes' frequently used in the electroweak calculations are:

1. The on-shell(OS) scheme with free parameters $\alpha, M_W, M_Z, m_f, M_H$
2. The G$_\mu$ scheme with the basic parameters $\alpha, G_\mu, M_Z, m_f, M_H$
3. The low energy scheme with the mixing angle as a basic parameter defined in neutrino electron scattering $\alpha, G_\mu, Sin^2\theta_{\nu_e}, m_f, M_H$.
4. Dimensional Regularization: in which a divergent multiple integral may be made convergent by reducing the number of multiple integrals. In dimensional regularization we keep the space-time dimension-D lower than four dimensional integral by a convergent D-dimensional one. Explicit momentum integrations give an analytical expression in the dimension D. The original divergence will appear as a pole at D $=4$ [40] upon integration on D after the analytical continuation. Now we will discuss the 1-loop contributions to the on-shell parameters and their renormalization. Since the boson masses appear in the particle propagators, we have to investigate the effects of the W and Z self-energies.
5. The $\bar{\text{M}}$S-scheme. The modified minimal substraction scheme ($\bar{\text{M}}$S-scheme) [36, 37, 38] is one of the simplest way to obtain finite 1-loop expressions by performing the substitution,
$$\frac{2}{\epsilon} - \gamma + \ln 4\pi + \ln \mu^2 \rightarrow \ln \mu^2_{\bar{M}S},$$
in the divergent part of loop integral [39].
6. The star $\star$ scheme where the bare parameters e_o, G^o_μ, S^2_o are eliminated and replaced in terms of dressed running (K^2-dependent) parameters [40], $e^2(K^2)$, $G_\mu(K^2)$, $S^2(K^2)$, m_f, M$_H$.

We restrict our discussion to the transverse parts Π_T. In the electroweak theory, different from QED, the longitudinal components Π_L. of the vector boson propagators do not vanish in physical matrix elements. But for light external fermions, the contributions are suppressed by $(\frac{m_f}{M_Z})^2$ and we can safely neglect them. Writing the self-energies as [41].

$$\sum_{\mu\nu}^{W,Z} = g_{\mu\nu} \sum^{W,Z} + \dots., \tag{2.5}$$

with scalar functions $\sum^{W,Z}(q^2)$, we have for the 1-loop propagators (V = W, Z)

$$\frac{-ig^{\mu\sigma}}{q^2 - M_{V^2}} \left(-i \sum_{\rho}^{V} \right) \frac{-ig^{\rho\nu}}{q^2 - M_V^2} = \frac{-ig^{\mu\nu}}{q^2 - M_V^2} \left(\frac{-\sum^V(q^2)}{q^2 - M_V^2} \right), \tag{2.6}$$

Besides the fermion loop contributions in the electroweak theory there are also the non-Abelian gauge boson loops and loops involving the Higgs boson. In the graphical representation, the self energies for the vector bosons denote the sum of all the diagrams with virtual fermions, vector bosons, Higgs and ghost loops. Resumming all self energy insertions yields a geometrical series for the dressed propagators:

$$\frac{-ig_{\mu\nu}}{q^2 - M_V^2} \left[1 + (\frac{\sum^V}{q^2 - M_V^2}) + (\frac{\sum^V}{q^2 - M_V^2})^2 + . \right] = \frac{-ig_{\mu\nu}}{q^2 - M_V^2 + \sum^V(q^2)}. \tag{2.7}$$

The self energies have the following properties:

- Im $\sum^V M_V^2 \neq 0$ for both W and Z. This is because W and Z are unstable particles and can decay into pairs of light fermions. The imaginary parts correspond to the total decay widths of W, Z and remove the poles from the real axis.

- $Re \sum^V M_V^2 \neq 0$ for both W and Z as they are UV-divergent.

The second feature shows that the location of the poles in the propagators is shifted by the loop contributions. Consequently, the important steps in mass renormalization consist of a re-interpretation of the parameters. The masses in the lagrangian cannot be the physical masses of W and Z but are the 'bare-masses' related to the physical masses M_W, M_Z by :

$$M_W^2 = M_W^2 + \delta M_W^2, \tag{2.8}$$

$$M_Z^{2} = M_Z^{2} + \delta M_Z^{2}, \tag{2.9}$$

with counterterms of 1-loop order. The 'correct' propagators according to this prescription are given by:

$$\frac{-ig_{\mu\nu}}{q^2 - M_V^0 - \delta M_V^2 + \sum^V(q^2)} = \frac{-ig_{\mu\nu}}{q^2 - M_V^2 + \sum^V(q^2)}, \tag{2.10}$$

instead of eq.(2.3). The renormalization conditions which ensure that $M_{W,Z}$ are the physical masses fix the mass counterterms to be

$$\delta M_W^2 = Re \sum^{W}(M_W^2), \tag{2.11}$$

$$\delta M_Z^2 = Re \sum^{Z}(M_Z^2). \tag{2.12}$$

In this way, two of these input parameters and their counterterms have been defined. Another parameter is the electromagnetic charge e. The electroweak charge renormalization is very similar to that in pure QED. As in QED, we want to retain the definition of 'e' as the classical charge in the Thomson cross-section,

$$\sigma_{Th} = \frac{e^4}{6\pi m_e^2}.$$

Accordingly, the lagrangian carries the bare charge $e_o = e + \delta e$ with the charge counterterm δe being absorbed in the electroweak loop contributions to the $ee\gamma$ vertex in the Thomson limit. This charge renormalization condition is simplified by the validity of a generalization of the QED Ward identity [42] which implies that the corrections related to the external particles cancel each other. Thus for δe only two universal contributions are left:

$$\frac{\delta e}{e} = \frac{1}{2}\Pi^{\gamma}(0) - \frac{\sin\theta_W \sum^{\gamma Z}}{\cos\theta_W M_z^2}. \tag{2.13}$$

The first one in analogy with QED, is given by the vacuum polarization of the photon. But now, besides the fermion loops, it also contains bosonic loop diagrams from W^+W^- virtual states and the corresponding ghosts. The second term contains the mixing between photon and Z, in general described as a mixing propagator with $\sum^{\gamma Z}$ normalized as

$$\triangle^{\gamma Z} = \frac{-ig_{\mu\nu}}{q^2}\left(\frac{-\sum^{\gamma Z}(q^2)}{q^2 - M_Z^2}\right).$$

The fermion loop contribution to $\sum^{\gamma Z}$ vanishes at $q^2 = 0$; only the non-abelian bosonic loop yields $\sum^{\gamma Z}(0) \neq 0$. To be more precise, the charge renormalization as discussed above, is a condition for the vector coupling constant of the photon only. The axial coupling vanishes for on-shell photons as a consequence of the Ward identity. From the diagonal photon self energy

$$\sum^{\gamma}(q^2) = q^2\Pi^{\gamma}(q^2),$$

no mass term arises for the photon. Besides the fermion loops, the boson loops behave like

$$\sum_{bos}^{\gamma}(q^2) \simeq q^2\Pi^{\gamma}_{bos}(0) \rightarrow 0,$$

for $q^2 \rightarrow 0$ leaving the pole at $q^2 = 0$ in the propagator. The absence of mass terms for photon at all orders is a consequence of the unbroken electromagnetic gauge invariance.

Concluding this discussion we summarize the principal structure of electroweak calculations.

- The classical lagrangian $\mathcal{L}$ $(e, M_W, M_Z, ...)$ is sufficient for lowest order calculations and the parameters can be identified with the physical parameters.

- For higher order calculations, $\mathcal{L}$ has to be considered as the 'bare' lagrangian of the theory $\mathcal{L}(e_o, M_W^o, M_Z^o, ...)$ with 'bare' parameters which are related to the physical ones by

$$e_o = e + \delta e;\, M_W^{o2} = M_W^2 + \delta M_W^2,$$

$$M_Z^{o2} = M_Z^2 + \delta M_Z^2,$$

The counter terms are fixed in terms of a certain subset of 1-loop diagrams by specifying the definition of the physical parameters.

- For any 4-fermion process we can write down the 1-loop matrix element with the bare parameters and the relevant loop diagrams. Together with the counter terms, the matrix element is finite when expressed in terms of the physical parameters, i.e; all UV-singularities are removed.

It is a well known fact that in QED, the Feynman diagrams for the self-energy at the vertex which contribute to the radiative corrections to the decay processes can be divergent. These divergences are cancelled by adding mass counterterms to the lagrangian. The method by which these infinities are removed defines the renormalization procedure in the theory. The singularities are separated in the form of the renormalization of fermion mass, wavefunction and charge, usually expressed in the form of corresponding renormalization constants.

The techniques for the renormalization of the standard model in vacuum have been extended to include thermal background effects. But before going to these finite temperature (FT) effects we will give the basic formalisms used to incorporate the statistical background effects. In quantum field theory (QFT), the temperature effects are incorporated in the Euclidean space through imaginary time formalism [43]. This formalism was originally developed by Matsubara which was later expressed in terms of functional path integrals. In this framework, the diagrammatic methods are essentially equivalent to the Feynman-Dyson perturbation theory of zero temperature QFT. The only exception is that the imaginary time domain is finite and periodic. The imaginary time formalism is particularly suited to the direct computation of static quantities and to the use of high temperature expansions. However, low temperature expansions and the properties of explicitly time dependent quantities are much less accessible. These, in practice, involve analytical continuation to formulate the finite-temperature field theories in terms of the real-time or Minkowski space variables, which are obtained by Wick's rotation ($x_o \rightarrow x_o + i\beta$) from Euclidean space. As mentioned above, in the Euclidean space the energies are discrete given as,

$$w_n = \frac{2n\pi}{-i\beta}, \tag{2.14}$$

in boson propagator, and

$$w_n = \frac{(2n+1)\pi}{-i\beta}, \tag{2.15}$$

in the fermion propagator. These discrete energies are summed over infinite values of n which yield a divergent series even for fixed temperatures breaking the Lorentz covariance which is in turn restored in the form of manifest covariance. But a very crucial problem associated with the real time formalism [44] is that of higher order graphs. The product of delta functions of the propagators give rise to $\delta(0)$ type singularities, an unusual singularity which is difficult to handle. At this stage it is useful to enumerate the advantages of the real-time formalism over the imaginary-time formalism, which are:

1. As discussed above, it is not diffcult to see that summations over infinite energies in propagators are avoided in this formalism. In this way a possible divergent summation can be replaced by the distribution function in the theory.

2. The removal of the discrete energy summations leads to the restoration of covariance in the real time formalism. This is done by introducing the manifest covariance and employing the unidirectional time like four velocity of the heat bath, u_μ, defined as:

$$u_\mu = (1,0,0,0).$$

   Whence the exponentials in the propagator can be written as:

$$e^{\beta|E|} = e^{\beta|p.u|} = e^{\beta|p_\mu u^\mu|}, \tag{2.16}$$

   with the particle propagators expressed in the covariant form such that the fermion and boson distribution functions are:

$$n_F(p) = \frac{1}{e^{\beta|p.u|}+1}, \tag{2.17}$$

and

$$n_B(k) = \frac{1}{e^{\beta|k.u|} - 1}, \tag{2.18}$$

respectively. One can see that this introduces u_μ covariance in this formalism. Due to this manifest covariance almost all the problems can be studied involving quantum statistical background effects in quantum field theory.

3. Another advantage of the real time formalism is that it immediately splits the calculations into a zero temperature and a temperature dependent part by virtue of such a splitting in the particle propagators. Thus it is always convenient to calculate the finite temperature part separately and add its contribution to T = 0 part. Thereby, the ratio of the finite temperature corrections to the uncorrected (T = 0) results for various temperature regions can be relatively easily evaluated and compared with the results of any available physical process.

4. In the process of analytical continuation for the real-time formalism from the imaginary-time one, the imposed periodic boundary conditions on the temperature $|0, -i\beta|$ are removed and the formalism becomes valid for all temperatures.

5. An extension of the real-time formalism to higher order graphs which enables to cancel $\delta(0)$ type singularities has been given in the thermofield dynamics (TFD). However, we shall not go beyond one loop approximation in the perturbative expansion, therefore, TFD method will not be needed in our analysis.

6. Finally, in contrast to the imaginary time formalism the real time formalism has a well defined zero temperature limit and systematic low temperature expansions become accessible. It is, of course, possible to compute time dependent quantities, such as the linear response functions, directly without using potentially very complicated analytical continuations.

7. In this formalism the fermion propagator is:

$$S_\beta(p) = \frac{i(\not p + m)}{\not p - m + i\epsilon} - 2\pi\delta(p^2 - m^2)(\not p + m)n_F(p),$$

whereas the boson propagator in the Feynman gauge is [45].

$$D_\beta^{\mu\nu}(p) = -g^{\mu\nu}\left[\frac{i}{p^2 - m^2 + \iota\epsilon} + 2\pi\delta(p^2 - m^2)n_B(p)\right],$$

where $n_F(p)$ and $n_B(p)$ are the Fermi-Dirac and Bose-Einstein distribution functions given in eqs.(2.10) and (2.11) respectively. Feynman diagrams are calculated by replacing the vacuum propagators by the above propagators. The fermion distribution functions act as a regularization parameter thereby providing an ultraviolet cutoff. However, the divergence in the infrared region appears in an enhanced form i.e.,

$$I_A \sim \int_0^\infty \frac{dk}{k} n_B(k),$$

which can be eliminated in the physical processes [46, 47, 48, 49]. We therefore, propose to preferably use the real-time formalism for the renormalization. This is necessary because the renormalization prescriptions developed at zero temperatures cannot be directly applied to the finite temperature theory, because of the absence of Lorentz invariance, which is an essential ingredient of the zero temperature theory.

We have already discussed the zero temperature renormalization methods in this chapter. Now we present the finite temperature renormalization procedure in QED developed by Donoghue, Holstein [3] and Robinett [50].

2.2 Renormalization of QED at Finite Temperature

It is already discussed that in QED, the Feynman diagrams for the self-energy and vertex graphs, which contribute to the radiative corrections to the QED processes, contain divergences. These divergences are cancelled by adding mass counter terms to the lagrangian. The method by which these infinities are removed defines the renormalization procedure in the theory. The singularities are separated in the form of the renormalization constants Z_1, Z_2, and Z_3 which renormalize the fermion mass, wavefunction, and charge respectively.

The techniques for the renormalization in zero temperature field theory have been extended to include finite temperature effects [3, 51, 52]. The electron self-energy up to the order α in Fig.(2) at finite temperature is:

$$\begin{aligned}\Sigma_\beta(p) &= \Sigma_{T=0}(p) + \frac{e^2}{4\pi^3}\int d^4q(2m - \not{p} + \not{k}) \times \\ &\left[\frac{n_F(p-k)\delta(p-k)^2 - m^2}{k^2 + \imath\epsilon} - \frac{n_B(k)\delta(k^2)}{(p-k)^2 - m^2 + \imath\epsilon}\right], \end{aligned} \tag{2.19}$$

giving,

$$\begin{aligned} m_{phys}^2 &= m^2\left[1 - \frac{6\alpha}{\pi}b(m\beta)\right] + \frac{4\alpha}{\pi}mTa(m\beta) \\ &+\frac{2}{3}\alpha\pi T^2\left[1 - \frac{6}{\pi^2}c(m\beta)\right], \end{aligned} \tag{2.20}$$

The temperature dependent radiative corrections to the electron mass upto the first order in α, is obtained from

$$m_{phys} = m + \delta m. \tag{2.21}$$

Squaring (2.21) and neglecting the $(\delta m)^2$ term , the correction is

$$\begin{aligned}\frac{\delta m}{m} &\simeq \frac{1}{2m^2}(m_{phys}^2 - m^2) \\ &\simeq \frac{\alpha\pi T^2}{3m^2}\left[1 - \frac{6}{\pi^2}c(m\beta)\right] + \frac{2\alpha}{\pi}\frac{T}{m}a(m\beta) - \frac{3\alpha}{\pi}b(m\beta), \end{aligned} \tag{2.22}$$

with

$$\begin{aligned} a(m\beta) &= \ln(1 + e^{-m\beta}), \\ b(m\beta) &= \sum_{n=1}^{\infty}(-1)^n e^{\mp n\beta} E\iota(-nm\beta), \end{aligned}$$

$$c(m\beta) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-nm\beta}, . \tag{2.23}$$

$$\tag{2.24}$$

At low temperature, the functions $a(m\beta)$, $b(m\beta)$, and $c(m\beta)$ fall off in powers of $e^{-m\beta}$ in comparison with $\frac{T^2}{m^2}$ and can be neglected so that

$$\frac{\delta m}{m} \overset{T \ll m_e}{\longrightarrow} \frac{\alpha\pi T^2}{3m^2}. \tag{2.25}$$

Moreover, in the high temperature limit, $a(m\beta)$ and $b(m\beta)$ are negligibly small whereas $c(m\beta) \to -\frac{\pi^2}{12}$. When T becomes very large compared to m_e, the term with $(\frac{T}{m})^2$ dominates giving:

$$\frac{\delta m}{m} \overset{T \gg m_e}{\longrightarrow} \frac{\alpha\pi T^2}{2m^2}, \tag{2.26}$$

Therefore, eq.(2.16) is valid for all temperature in QED including $T \sim m_e$. This range of temperature is particularly important from the point view of cosmology. It has been found that some parameters in the early universe such as the energy density ρ_T and the helium abundance parameter Y become slowly varying functions of temperature [53] whereas they are constant at $T \gg m_e$ and $T \ll m_e$ [54, 55].

The temperature dependent wavefunction renormalization constant has been obtained as [5].

$$\begin{aligned} Z_2^{-1}(\beta) &= Z_2^{-1}(T=0) - \frac{2\alpha}{\pi} \int_0^{\infty} \frac{dk}{k} n_B(k) - \frac{5\alpha}{\pi} b(m\beta) \\ &+ \frac{\alpha T^2}{\pi E^2 V} \ln \frac{1+v}{1-v} [\frac{\pi^2}{6} - mc(m\beta) + m\beta a(m\beta)], \end{aligned} \tag{2.27}$$

The charge renormalization constant at finite temperature has been calculated from the vaccum polarization of a photon in Fig.(3) by writing

$$\begin{aligned} \Pi_\beta^{\mu\nu}(k) &= \Pi_{T=0}^{\mu\nu}(q) + \frac{e^2}{(2\pi)^3} \int d^4p Tr \{\gamma_\mu(\not p + \not q + m)\gamma_\nu(\not p + m)\} \times \\ & \left\{ \frac{\delta[(p+k)^2 - m^2] n_F(P+k)}{p^2 - m^2 + i\epsilon} + \frac{\delta(p^2 - m^2) n_F(p)}{(p+k)^2 - m^2 + i\epsilon} \right\}, \end{aligned} \tag{2.28}$$

where

$$\mathrm{k}^2 = \omega^2 - k^2,$$

with

$$\omega = k_\alpha u^\alpha.$$

Eq.(2.27) is now solved to obtain the longitudinal and transverse components of $\Pi_{\mu\nu}$, using

$$\Pi_{\mu\nu}(k) = \Pi_T(\omega, k) P_{\mu\nu} + \Pi_L(\omega, k) Q_{\mu\nu}, \tag{2.29}$$

where,

$$\Pi_L(\omega, k) = -\frac{\mathrm{k}^2}{k^2} u^\mu u^\nu \Pi_{\mu\nu}(k),$$

$$\Pi_T(\omega, k) = -\frac{1}{2}\Pi_L(\omega, k) + \frac{1}{2} g^{\mu\nu}\Pi_{\mu\nu}(k),$$

and,

$$\begin{aligned} P_{\mu\nu} &\equiv \tilde{g}_{\mu\nu} + \frac{\tilde{k}_\mu \tilde{k}_\nu}{k^2}, \\ Q_{\mu\nu} &\equiv \frac{-1}{K^2 k^2}(k^2 u_\mu + \omega \tilde{k}_\mu)(k^2 u_\nu + \omega \tilde{k}_\nu), \end{aligned} \tag{2.30}$$

with

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu,$$

$$\tilde{k}_\mu = k_\mu - \omega u_\mu,$$

Thus, the finite temperature correction to the longitudinal and transverse components of the photon self-energy upto the first order in α become

$$\begin{aligned} \Pi_L \simeq\ & \frac{4e^2}{\pi^2}[1 - \frac{\omega^2}{k^2}]\Bigg[\Bigg\{(1 - \frac{\omega}{2k}\ln\frac{\omega + k}{\omega - k})(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2})\Bigg\} \\ & + \frac{1}{4}\Bigg\{2m^2 - \omega^2 + \frac{11k^2 + 37\omega^2}{72} b(m\beta)\Bigg\}\Bigg], \end{aligned} \tag{2.31}$$

$$\begin{aligned} \Pi_T \simeq\ & \frac{2e^2}{\pi^2}\Bigg[\Bigg\{\frac{\omega^2}{k^2} + (1 - \frac{\omega^2}{k^2})\ln\frac{\omega + k}{\omega - k}\Bigg\}\Bigg\{\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2}\Bigg\} \\ & + \frac{1}{8}\Bigg\{\Bigg[2m^2 - \omega^2 + \frac{107\omega^2 + 131k^2}{72}\Bigg\} b(m\beta)\Bigg\}\Bigg], \end{aligned} \tag{2.32}$$

respectively, using

$$\acute{D}_{F_{\mu\nu}} = \frac{P_{\mu\nu}}{k^2 - \Pi_T}. \tag{2.33}$$

With $k^2 \to 0$, the charge renormalization has been determined, giving

$$Z_3^{-1} = 1 - \frac{4e^2}{m^2\pi^2}\left\{\frac{c(m\beta)}{\beta^2} - \frac{ma(m\beta)}{\beta} - \frac{1}{4}(m^2 - \frac{\omega^2}{3})b(m\beta)\right\}. \tag{2.34}$$

In the next section, we give the generalization of these results after including the density effects.

2.3 Renormalization of QED at Finite Temperature and Density

The study of hot and dense systems like the quark-gluon plasma requires the incorporation of the finite density effects alongwith those of temperature. A systematic development of the FTD dynamics has been done and the question of renormalization examined in detail [56]. The massless bosons do not exhibit the chemical potential(μ). The fermion distribution function, however, has to be changed to include the density effects as [57].

$$n_F(k) = \frac{1}{e^{\beta|k|} + 1}, \tag{2.35}$$

in the fermion propagator giving

$$\begin{aligned} S_F(p) &= \frac{i(\not p + m)}{\not p - m + i\epsilon} - 2\pi(\not p + m)\delta(p^2 - m^2) \\ & [\theta(E_p)n_F(p+\mu) + \theta(-E_p)n_F(p-\mu)], \end{aligned} \tag{2.36}$$

The sign of chemical potential corresponds to the charge of the fermion. The $a(m\beta), b(m\beta)$ and $c(m\beta)$ functions in eq.(2.23) are replaced by $a(m\beta, \pm\mu), b(m\beta, \pm\mu)$ and $c(m\beta, \pm\mu)$ given by

$$\begin{aligned} a(m\beta, \pm\mu) &= \ln(1 + e^{-(m\pm\mu)\beta}), \\ b(m\beta, \pm\mu) &= \sum_{n=1}^{\infty}(-1)^n e^{\mp n\beta\mu} Ei(-nm\beta), \end{aligned}$$

$$c(m\beta, \pm\mu) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-n\beta(m\pm\mu)}, \tag{2.37}$$

where $\pm$ correspond to particles and antiparticles in the background. If there are particles and antiparticles with the same chemical potential μ in the medium we can simply have the following functions.

$$\tilde{a}(m\beta, \mu) = \frac{1}{2} \ln \left\{ [1 + e^{-(m-\mu)\beta}][1 + e^{-(m+\mu)\beta}] \right\},$$

$$\tilde{b}(m\beta, \mu) = \sum_{n=1}^{\infty} (-1)^n \cosh(n\beta\mu) E_i(-nm\beta),$$

$$\tilde{c}(m\beta, \mu) = \sum_{n=1}^{\infty} (-1)^n \cosh(n\beta\mu) \frac{e^{-nm\beta}}{n^2}.$$

The FT as well as FTD corrections have been used to evaluate the change in the decay rate of the scalar Higgs bosons [3, 51, 56]. We will calculate similar type of corrections to the electroweak decays in the next chapter and evaluate the renormalization constants of electroweak theory at temperatures sufficiently below the electroweak scale such that $T << 100$ GeV, i.e., low temperatures on the electroweak scale.

Chapter 3

Renormalization of Electroweak Processes in Hot and Dense Medium

The renormalization [56] of QED at FTD involves the replacement of the cold propagators of temperature independent theories by the hot and dense propagators [57], mentioned in the last chapter. The self energy corrections to the electron mass at FTD upto the first order is calculated in ref.[56], from which the wavefunction renormalization can also be directly obtained. The electron mass shift at FTD has important physical implications. Such calculations have also been done upto 2-loop level [6, 7, 8] in QED. These renormalized mass, wavefunction and charge of the particles at FTD in QED give some corrections to the parameters in cosmology and astrophysics. So the most important applications of these electroweak and QED processes at FTD are expected in cosmology and astrophysics and the universe can be considered as the best test laboratory for FTD theories.

3.1 Calculations of the Renormalization Constants

The calculation of the renormalization constants of the electroweak theory at FTD upto the one loop level are evaluated in the background of hot and dense leptons. We work at the temperature and the chemical potential sufficiently below the electroweak scale where we do not have any hot and dense electroweak gauge boson or Higgs particles in the background. Therefore, the statistical corrections due to the background of such particles need not be evaluated. We only mention those diagrams which acquire the background corrections whereas all the other diagrams have the same vacuum contributions. Here we use the frame work of real-time formalism where the background corrections appear as the additive corrections to the vacuum results. We prefer to evaluate the background contributions to these renormalization constants in parallel to the vacuum calculations [15]

3.1.1 Mass Renormalization

Fermion Self-mass

i. Leptons:
The self mass of charged leptons at low temperature has been calculated from the matrix amplitude of Fig.(2) which can be written as:

$$\begin{aligned} ie_o^2\sum(p) &= \frac{ie_o^2}{(2\pi)^4}\int d^4k\mathrm{Tr}\left\{-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)\right. \\ &\left.\frac{i(\not{p}-\not{k})+m^2}{(p+k)-m_l^2}\times\frac{i}{k^2-m_W^2}\times\frac{-ig\gamma^\nu(1-\gamma_5)}{2\sqrt{2}}\right\}, \end{aligned} \quad (3.1)$$

This diagram gives a zero contribution at low temperature. However, in case of neutral leptons, Figs.(8a) and (8b) contribute to the self-mass of neutrino if it is considered to be a massive particle [58, 59].

ii. Quarks:
In the case of quarks, their masses have no effect at this temperature and remain just the same as in vacuum.

Self mass of Higgs

Since we are working with temperatures sufficiently below the Higgs, W, and Z masses, the self mass of Higgs does not get any significant corrections from the background of hot and dense particles except leptons. Considering Fig.(4), the result is obtained as

$$\tilde{m}^2 = (m + \delta m)^2, \tag{3.2}$$

with

$$\tilde{m_H}^2 = m_H^2 + 2m\frac{8k(ie)^2}{(2\pi)^4 m_H^2}\left[\frac{ma(m\beta, \pm\mu)}{\beta} + \frac{c(m\beta, \pm\mu)}{\beta^2}\right\}, \tag{3.3}$$

Gauge Bosons

The diagrams contributing to the self-energies of the photon, W, Z and γZ transition contain fermion, vector boson, Higgs and ghost loops. Only the fermion loops at such low temperatures need to be cosidered in more detail. In this section we are dealing with the self-mass effects so we discuss 1-loop contributions to the on-shell parameters and their renormalization. Since the boson masses are a part of the propagators so we have to investigate only the effects of the W and Z self-energies here.

Following Hollik [15], all self-energy insertions yield a geometrical series for the dressed propagators in Fig.(5). eq.(2.7) shows that the poles exist at:

$$\tilde{k}^2 - M_W^2 + \sum^{W}(k^2) = 0,$$

which then gives

$$\tilde{k}^2 - \tilde{m}^2 = 0,$$

with ref.[A.4]

$$\tilde{m}^2 = (m + \delta m)^2, \tag{3.4}$$

$$\tilde{m}^2 = M_W^2 + \sum^{W}(k^2),$$

$$\tilde{m}^2 = M_W^2 + (\frac{-ig_W}{2\sqrt{2}})^2 \times \frac{2\pi i}{(2\pi)^4}\mathbf{k}^2 \left[\left\{+\frac{10k_o}{m_W^2} - \frac{2m}{m_W^2} - \frac{2}{m}\right\}\frac{a(m\beta,\pm\mu)}{\beta} + \left\{-\frac{2m^2}{m_W^2}\right\} b(m\beta,\pm\mu) + \left\{\frac{20k_o}{mm_W^2} + \frac{2}{m_W^2} - \frac{2}{m^2}\right\}\frac{c(m\beta,\pm\mu)}{\beta^2} + \left\{\frac{20k_o}{m^2m_W^2}\right\}\frac{d(m\beta,\pm\mu)}{\beta^3}\right] \quad (3.5)$$

Similarly the self-energy of Z in, Fig.(6). Again using eq.(2.7), we get

$$\tilde{k}^2 - M_Z^2 + \sum^Z(k^2) = 0,$$

which then gives

$$\tilde{k}^2 - \tilde{m}^2 = 0,$$

with ref.[A.8]

$$\tilde{m}^2 = (m + \delta m)^2. \quad (3.6)$$

We have an expression for $\sum^\gamma(k^2) = \mathbf{k}^2\Pi^\gamma$, for Fig.(3) with

$$\begin{aligned}\tilde{m}^2 &= M_Z^2 + \sum^Z(k^2)\\ &= M_Z^2 + \frac{(ig_Z)^2(-2i)(\pi^2)}{4(2\pi)^2}\mathbf{k}^2\left[\left\{\frac{-7m}{m_Z^2}\right\}\frac{a(m\beta,\pm\mu)}{\beta}\left\{\frac{-3m^2}{m_Z^2} - 1\right\}\right.\\ &\quad b(m\beta,\pm\mu) + \left\{\frac{-11}{m_Z^2}\right\}\frac{c(m\beta,\pm\mu)}{\beta^2} + \left\{\frac{-2}{mm_Z^2}\right\}\frac{d(m\beta,\pm\mu)}{\beta^3}\\ &\quad \left. + \left\{\frac{-2}{m_Z^2}\right\}\frac{f(m\beta,\pm\mu)}{\beta^4}\right], \end{aligned} \quad (3.7)$$

3.1.2 Coupling Constants Renormalization

Charge Renormalization

We have another input parameter i.e., electromagnetic charge 'e'. Charge renormalization is connected to the gauge invariance. For temperature field theory it is not simple to establish the gauge invariance at high temperature. Donoghue, Holstein and Robinett [50] established the gauge invariance explicitly at low temperature. Here we will discuss the charge renormalization constants obtained which is the same as eq.(3.10). The other renormalization constants are as

$$
\begin{aligned}
\delta Z_2^Z &= -\Pi^\gamma(0) - 2\frac{\cos^2\theta_W - \sin^2\theta_W}{\sin\theta_W\cos\theta_W}\frac{\sum^{\gamma Z}(0)}{M_Z^2} + \\
&\quad \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right), \qquad (3.13)
\end{aligned}
$$

Using eq.(3.12) in eq.(3.13), we get

$$
\delta Z_2^Z = -\Pi^\gamma(0) + \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right), \qquad (3.14)
$$

Again make use of eq.(3.10) in eq.(3.14),

$$
\begin{aligned}
\delta Z_2^Z &= -\frac{4e^2}{m^2\pi^2}\left[\left\{\frac{k_o^2}{|k^2|} + \left(\frac{k_o}{2|k|} - \frac{k_o^3}{2|k^3|}\right)\ln\frac{k_o+|k|}{k_o-|k|}\right\}\frac{ma(m\beta,\pm\mu)}{\beta}\right. \\
&\quad + \frac{1}{4}\left\{m^2\left(1+\frac{2k_o^2}{|k^2|}\right) + k_o^2\left(1+\frac{k_o^2}{|k^2|}\right) + \frac{107k_o^2}{72} - \frac{37k_o^4}{72|k^2|} - \frac{131|k^2|}{72}\right. \\
&\quad \left. -\frac{11k_o^2}{72}\right\} b(m\beta,\pm\mu) + \left\{\frac{k_o^3}{2|k^3|}\ln\left(\frac{k_o+|k|}{k_o-|k|}\right) - \frac{k_o^2}{|k^2|}\right. \\
&\quad \left.\left. +\frac{k_o}{2|k|}\ln\left(\frac{k_o+|k|}{k_o-|k|}\right)\right\}\frac{c(m\beta,\pm\mu)}{\beta^2}\right] + \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W} \\
&\quad \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right), \qquad (3.15)
\end{aligned}
$$

and for

$$
\begin{aligned}
\delta Z_1^Z &= -\Pi^\gamma(0) - \frac{(3\cos^2\theta_W - 2\sin^2\theta_W)\sum^{\gamma z}(0)}{\cos\theta_W\sin\theta_W M_z^2} \\
&\quad + \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right). \qquad (3.16)
\end{aligned}
$$

Using eqs.(3.10) and (3.12), in eq.(3.15), we get

$$
\begin{aligned}
\delta Z_1^Z &= -\frac{4e^2}{m^2\pi^2}\left[\left\{\frac{k_o^2}{|k^2|} + \left(\frac{k_o}{2|k|} - \frac{k_o^3}{2|k^3|}\right)\ln\frac{k_o+|k|}{k_o-|k|}\right\}\frac{ma(m\beta,\pm\mu)}{\beta} + \frac{1}{4}\right. \\
&\quad \left\{m^2\left(1+\frac{2k_o^2}{|k^2|}\right) + k_o^2\left(1+\frac{k_o^2}{|k^2|}\right) + \frac{107k_o^2}{72} - \frac{37k_o^4}{72|k^2|} - \frac{131|k^2|}{72}\right. \\
&\quad \left. -\frac{11k_o^2}{72}\right\} b(m\beta,\pm\mu) + \left\{\frac{k_o^3}{2|k^3|}\ln\left(\frac{k_o+|k|}{k_o-|k|}\right) - \frac{k_o^2}{|k^2|} + \frac{k_o}{2|k|}\right.
\end{aligned}
$$

$$\ln(\frac{k_o + |k|}{k_o - |k|})\Big\} \frac{c(m\beta, \pm\mu)}{\beta^2}\Big] + \frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}$$
$$(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}). \quad (3.17)$$

Now in the case of W self-energy

$$\delta Z_2^W = -\Pi^\gamma(0) - 2\frac{\cos\theta_W \sum^{\gamma Z}}{\sin\theta_W M_Z^2} + \frac{\cos^2\theta_W}{\sin^2\theta_W}(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}). \quad (3.18)$$

Using eqs.(3.10) and (3.12), we have

$$\delta Z_2^W = -\frac{4e^2}{m^2\pi^2}\Big[\Big\{\frac{k_o^2}{|k^2|} + (\frac{k_o}{2|k|} - \frac{k_o^3}{2|k^3|})\ln\frac{k_o + |k|}{k_o - |k|}\Big\}\frac{ma(m\beta, \pm\mu)}{\beta}$$
$$+\frac{1}{4}\Big\{m^2(1 + \frac{2k_o^2}{|k^2|}) + k_o^2(1 + \frac{k_o^2}{|k^2|}) + \frac{107k_o^2}{72} - \frac{37k_o^4}{72|k^2|}$$
$$-\frac{131|k^2|}{72} - \frac{11k_o^2}{72}\Big\} b(m\beta, \pm\mu) + \Big\{\frac{k_o^3}{2|k^3|}\ln(\frac{k_o + |k|}{k_o - |k|}) - \frac{k_o^2}{|k^2|}$$
$$+\frac{k_o}{2|k|}\ln(\frac{k_o + |k|}{k_o - |k|})\Big\} \frac{c(m\beta, \pm\mu)}{\beta^2}\Big] + \frac{\cos^2\theta_W}{\sin^2\theta_W}(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}) \quad (3.19)$$

The results obtained in this section will be discussed in some more detail in the following section.

3.2 Results and Discussions

It can be seen from the calculations of the renormalization constants in the last section that the lepton background can affect the renormalization constants of the electroweak theory. These results are obviously different from QED because the hot photon background does not affect the physical processes whereas the massive neutrino background can have significant effects. This point is clear from eqs.(3.5) and (3.7) because the $\tilde{a}, \tilde{b}, \tilde{c}$ functions involving the expressions of the renormalization constants are always functions of mβ and $\mu\beta$. The contribution of these functions can only be significant when we deal with the massive leptons and temperature is either of the order of the lepton mass or greater. If the neutrino is considered as a massless particle, the neutrino background does not contribute at least upto the 1-loop level. However, even for a very tiny mass of neutrino the tilde functions give significant corrections at $T < m_e$ because of the exponential dependence of these functions on the parameter $m_\nu\beta$. Therefore, in the standard electroweak model the hot and dense charged lepton background contributions start when temperature goes upto the order of the lepton mass. In this regime the fermion self-mass is simply vanishingly small because of the W or Z loop suppression. Whereas, in the standard model with massive neutrinos, the neutrino mass can get some corrections to form the hot charged lepton background which is supressed by $\frac{1}{m_W^2}$ [58, 59]. Similarly the Higgs self-mass corrections appear to be $0(\frac{m_l^2}{m_H^2})$, hence ignorable. The self-mass of the gauge vector bosons, however, get significant contribution from the background and is given in eqs.(3.5) and (3.7) for $T \sim m_l$. When $T >> m_l$, these equations attain a simple form. For eq.(3.5) we have

$$m_W^2 \sim m_W^{o2} + (\frac{g_W^2 \mathrm{T}^2}{384}),$$

and eq.(3.7) gives

$$m_Z^2 \sim m_Z^{o2} + O(\frac{\mathrm{T}^2}{m_Z^2}).$$

Similarly the charge renormalization gives. First we consider eq.(3.10)

$$\delta Z_2^\gamma \sim -\frac{4e^2}{m^2\pi^2}\left\{\frac{k_o^3}{2|k^3|}\ln(\frac{k_o+|k|}{k_o-|k|}) - \frac{k_o^2}{|k^2|} + \frac{k_o}{2|k|}\ln(\frac{k_o+|k|}{k_o-|k|})\right\}\frac{\pi^2\mathrm{T}^2}{12},$$

Now eq.(3.14) becomes

$$\delta Z_2^Z \sim -\frac{4e^2}{m^2\pi^2}\left\{\frac{k_o^3}{2|k^3|}\ln(\frac{k_o+|k|}{k_o-|k|}) - \frac{k_o^2}{|k^2|} + \frac{k_o}{2|k|}\ln(\frac{k_o+|k|}{k_o-|k|})\right\}\frac{\pi^2\mathrm{T}^2}{12}$$
$$+\frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}),$$

Now eq.(3.16) gets the form

$$\delta Z_1^Z \sim -\frac{4e^2}{m^2\pi^2}\left\{+\frac{k_o^3}{2|k^3|}\ln(\frac{k_o+|k|}{k_o-|k|}) - \frac{k_o^2}{|k^2|} + \frac{k_o}{2|k|}\ln(\frac{k_o+|k|}{k_o-|k|})\right\}\frac{\pi^2 T^2}{12}$$

$$+\frac{\cos^2\theta_W - \sin^2\theta_W}{\sin^2\theta_W}(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}).$$

And eq.(3.18) becomes

$$\delta Z_1^W \sim -\frac{4e^2}{\pi^2}\left\{+\frac{k_o^3}{2|k^3|}\ln(\frac{k_o+|k|}{k_o-|k|}) - \frac{k_o^2}{|k^2|} + \frac{k_o}{2|k|}\ln(\frac{k_o+|k|}{k_o-|k|})\right\}\frac{\pi^2 T^2}{12}$$

$$+\frac{\cos^2\theta_W}{\sin^2\theta_W}(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}),$$

It is also worth mentioning that $\sum^{\gamma z} = 0$ for the fermion loops, even in the statistical background.

It is therefore clear that there is no problem with the renormalizability of electroweak theory in hot and dense lepton background. The self-masses and charges of the particles are corrected only when they propagate in a medium. These effective masses and effective charges are expected to be relevant in the calculations of the physical processes taking place in astrophysics and cosmology.

3.3 Implications

The renormalized mass, wavefunction and charge of the electroweak particles at FTD in QED and also at low temperatures on the electroweak scale, give some corrections to the parameters in cosmology and astrophysics. Some of the physically measureable parameters such as charge, wavefunction and mass may also change due to these corrections. The most important of all these applications can be found in cosmology and astrophysics for which the universe can be considered as the best test laboratory.

3.3.1 Cosmological

It is now evident from the standard big bang model that the temperatures in the early universe were very high. The existing matter dominated era has lasted throughout most of the history of the universe. But if we go back to the early epochs of the radiation era, the existing energies started converting into matter which was obviously the relativistic matter at extremely high temperatures. The ordinary vacuum quantum field theories are not enough to obtain the correct information, so we have to evaluate the background corrections also. We want to turn our attention back to an earlier period when the radiation and relativistic particles were more important than ordinary matter. The thermal history of the universe can be seen in Fig.(9). It is clear from this Fig.(9) that at T of the order of a few seconds after the creation of the universe, the electroweak lepton scatterings were taking place and the background temperature was high enough to give non-ignorable effects. In this case, for the precise calculation of these processes the effective charges and masses have to be incorporated. The check of renormalizability of the theory is also important in such a background.

3.3.2 Astrophysical

The background effects are not only important in the early universe but they are worth incorporating among the stellar objects. It is expected that the stellar cores have very high temperatures. It is seen through detailed calculations that in the stellar objects such as sun, T $\sim 10^7$ K which does not give significant thermal corrections. Even in the dense objects like neutron stars, the density corrections are not significant. However, in the superdense collapsing stars like supernovae, the statistical corrections are non-ignorable. Therefore, the thermal as well as the density corrections are worth studying, though the theoretical models describing such systems are not so well understood. However, it is expected that the tremendous amount of energy emitted from SN1987A can only be explained if the corrections of the electroweak processes taking place in the core of these stars are also studied in the superdense background, i.e., $\mu >$ T $> m_l$; where

l stands for the corresponding lepton flavour. The existing limits on T and μ in supernovae [60] is T $\sim$ 30-70 MeV and $\mu \sim$ 250-300 MeV where the leptonic scatterings take place and the background contributions to these processes have to be carefully calculated from the renormalization constants of the electroweak theory in the relevant background.

3.3.3 Heavy ion collisions and quark-gluon plasma

In the heavy ion collisions and quark-gluon plasma, if it exists, the above mentioned calculations may be important because of very high temperatures and densities. In hadronic physics where densities are somewhat higher than temperature, weak hadronic processes can also take place. Therefore, in the perturbative study of strangelet production, the above mentioned calculations are important for the processes, e.g.,

$$u + d \leftrightarrow u + s.$$

3.4 Future Perspectives

The application of the above results to astrophysics, cosmology and heavy-ion collisions can be in the systems studied where the hot and dense media are expected to exist and the background effects are supposed to be non-ignorable.

The above calculations are done at low temperatures where chemical potentials are even lower than the temperatures. A similar type of calculations can be done at lepton densities greater than the lepton temperatures, which is a more relevant regime for superdense stars. For higher temperatures and densities, which are of the order of electroweak scales, more graphs involving the hot gauge bosons and Higgs have to be incorporated at the 1-loop level. These will be more important regarding electroweak phase transition which are thought to lead the baryogenesis in the early universe. In addition to this, the higher loop calculations are worth-while to check the validity of perturbative expansion.

The calculational techniques developed here can be helpful in determining such parameters and processes in other theories as well.

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Appendix A

A.1 Appendix A

A.2 Calculation of Self energy of W

$$\begin{aligned}\Pi^W_{\mu\nu}(k) &= (\frac{-ig_W}{2\sqrt{2}})^2\frac{1}{(2\pi)^4}\int d^4p\mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)(\not p+m_l)\right.\\ &\times\left\{\frac{1}{(p^2-m_l^2)}+2\pi i\delta(p^2-m_l^2)n_F(p)\right\}\gamma_\nu(1-\gamma_5)(\not p+\not k+m_\nu)\\ &\times\left.\left\{\frac{1}{(p+k)^2-m_\nu^2}+2\pi i\delta[(p^2+k)^2-m_\nu^2]n_F(p)\right\}\right],\end{aligned} \quad (A.1)$$

which then goes to

$$\begin{aligned}\Pi^W &= (\frac{-ig_W}{2\sqrt{2}})^2\frac{2\pi i}{(2\pi)^4}\left[\int\frac{p^3dpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\right.\\ &\times\left\{\ln|m_W^2+2p(1+\frac{m^2}{2p^2})k_o-(2pk)\cos\theta|\right\}_{-1}^{+1}-\frac{1}{2}\int pdpn_F(p)(1-\frac{m^2}{2p^2})\\ &\times\left\{\cos\theta+\frac{m_W^2+2p(1+\frac{m^2}{2p^2})k_o}{-2pk}\ln|m_W^2+2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right)\Bigg\}_{-1}^{+1}\\ &-\int\frac{p^2dp[p(1+\frac{m^2}{2p^2})+k_o]n_F(p)}{-2pk}\left\{\ln|m_W^2+2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}\\ &\times+2m^2\int\frac{pdpn_F(p)}{(-2pk)}\left\{\ln|m_W^2+2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}\end{aligned}$$

$$+\int\frac{p^3dpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\left\{\ln|m_W^2-2|p|(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}$$

$$-\frac{1}{2}\int pdpn_F(p)(1-\frac{m^2}{2p^2})\left\{\cos\theta+\frac{m_W^2-2p(1+\frac{m^2}{2p^2})k_o}{-2pk}\left(\ln|m_W^2\right.\right.$$

$$\left.\left.-2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right)\right\}_{-1}^{+1}-\int\frac{p^2dp[p(1+\frac{m^2}{2p^2})-k_o]n_F(p)}{-2pk}$$

$$\times\left\{\ln|m_W^2-2|p|(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}+2m^2\int\frac{pdpn_F(p)}{(-2pk)}$$

$$\left.(1-\frac{m^2}{2p^2})\left\{\ln|m_W^2-2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}\right], \tag{A.2}$$

Expanding the logarithmic function in Taylor series and neglecting the square and higher terms, $\theta(\frac{m_l^2}{m_W^2})$ we have,

$$\Pi^W = (\frac{-ig_W}{2\sqrt{2}})^2\times\frac{2\pi i}{(2\pi)^4}\left[\left\{\frac{10k_o}{m_W^2}-\frac{2m}{m_W^2}-\frac{2}{m}\right\}\frac{a(m\beta,\pm\mu)}{\beta}+\left\{+\frac{2m^2}{m_W^2}\right\}b(m\beta,\pm\mu)\right.$$

$$\left.+\left\{\frac{20k_o}{mm_W^2}-\frac{2}{m_W^2}-\frac{2}{m^2}\right\}\frac{c(m\beta,\pm\mu)}{\beta^2}+\left\{\frac{20k_o}{m^2m_W^2}\right\}\frac{d(m\beta,\pm\mu)}{\beta^3}\right], \tag{A.3}$$

The a, b, c, d, f, and g, functions are defined as:

$$\begin{aligned}
a(m\beta,\pm\mu) &= \ln(1+e^{-(m\pm\mu)\beta}),\\
b(m\beta,\pm\mu) &= \sum_{n=1}^{\infty}(-1)^n e^{\mp n\beta\mu}E\iota(-nm\beta),\\
c(m\beta,\pm\mu) &= \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-n\beta(m\pm\mu)},\\
d(m\beta,\pm\mu) &= \sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}e^{-n\beta(m\pm\mu)},\\
f(m\beta,\pm\mu) &= \sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}e^{-n\beta(m\pm\mu)},\\
g(m\beta,\pm\mu) &= \sum_{n=1}^{\infty}(-1)[\beta nE\iota(-nm\beta)+n\beta\frac{e^{-nm\beta}}{m}]e^{\mp n\beta\mu}.
\end{aligned} \tag{A.4}$$

A.3 Calculation of Self energy of Z

$$\begin{aligned}\Pi^Z_{\mu\nu} &= \frac{(ig_Z)^2}{2(2\pi)^4}\int d^4p\mathrm{Tr}\Big[\gamma_\mu(c_V^f - c_A^f\gamma_5)(\not p + m_l)\Big\{\frac{1}{p^2-m_l^2}\\ &+2\pi i\delta(p^2-m_l^2)n_F(p)\Big\}\gamma_\nu(c_V^f - c_A^f\gamma_5)\times(\not p + \not k + m_l)\times\\ &\Big\{\frac{1}{(p+k)^2-m_l^2} + 2\pi i\delta[(p+k)^2-m_l^2]\Big\}n_F(p+k)\Big],\end{aligned} \quad (A.5)$$

after simplifying, we get

$$\begin{aligned}\Pi^Z &= \frac{(ig_Z)^2(-2i)\pi^2}{4(2\pi)^4}\Big[\Big[\int\frac{p^3dpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\Big\{\ln|m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o\\ &-2pk\cos\theta|\}_{-1}^{+1} - \int p^2dp\frac{p(1+\frac{m^2}{2p^2})+k_o}{-2pk}n_F(p)\{\ln|m_Z^2+2p(1\\ &+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} + 2m^2\int\frac{pdpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\{\ln|m_z^2\\ &+2p(1+\frac{m^2}{2p})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} + \int\frac{p^3dpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\\ &\Big\{\ln|m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} - 2\int pdpn_F(p)\\ &\times(1-\frac{m^2}{2p^2}) + \frac{1}{2}\int\frac{pdpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})[m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o]\\ &\times\Big\{\ln|m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} - \int\frac{p^3dpn_F(p)(1+\frac{m^2}{2p^2})}{(-2pk)}\\ &\times\Big\{\ln|m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} + k_o\int\frac{p^2dpn_F(p)}{(-2pk)}\\ &\times\Big\{\ln|m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1} + 2m^2\int\frac{pn_F(p)dp}{(-2pk)}\\ &\times(1+\frac{m^2}{2p^2})\Big\{\ln|m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1}\\ &+\frac{1}{2}\int\frac{pdpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})[m_Z^2 - 2p(1+\frac{m^2}{2p^2})k_o]\{\ln|m_Z^2 - 2p\\ &\times(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\Big\}_{-1}^{+1}\Big],\end{aligned} \quad (A.6)$$

neglecting the square and higher terms, and ignoring $\theta(\frac{m_l^2}{m_Z^2})$, we have

$$\Pi^Z = \frac{(ig_Z)^2(-2i)(\pi^2)}{4(2\pi)^2}\left[\left\{\frac{-7m}{m_Z^2}\right\}\frac{a(m\beta,\pm\mu)}{\beta} - \left\{\frac{-3m^2}{m_Z^2} - 1\right\} b(m\beta,\pm\mu) + \left\{\frac{-11}{m_Z^2}\right\}\frac{c(m\beta,\pm\mu)}{\beta^2} + \left\{\frac{-2}{mm_Z^2}\right\}\frac{d(m\beta,\pm\mu)}{\beta^3} + \left\{\frac{-2}{m^2m_Z^2}\right\}\frac{f(m\beta,\pm\mu)}{\beta^4}\right], \quad (A.7)$$

Dropping the terms containing $\frac{1}{m_Z^2}$, since m_Z very large than m_l we get the result in eqn.(3.7).

A.4 Calculation of Self energy of γZ

$$\Pi_{\mu\nu}^{\gamma Z} = \frac{(ie)^2}{(2\pi)^4}\int d^4p \mathrm{Tr}\left[\gamma_\mu(\not p + \not k + m)\left\{\frac{1}{(p+k)^2 - m^2} + 2\pi i\delta(p+k)^2 - m^2 n_F(p+k)\right\}\gamma_\nu(1-\gamma_5)(\not p + m)\left\{\frac{1}{p^2-m^2} + 2\pi i\delta(p^2 - m^2)n_F(p)\right\}\right], \quad (A.8)$$

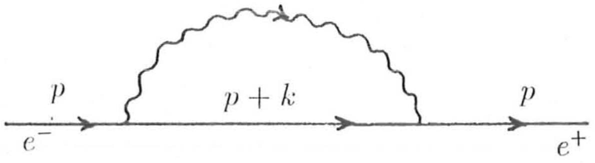
After simpifying, we get

$$\Pi^{\gamma Z} = \frac{16i(ie\pi)^2}{(2\pi)^4}\left[\int \frac{p^3dpn_F(p)}{(-2pk)}(1 - \frac{m^2}{2p^2})\left\{\ln|m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\right\}_{-1}^{+1} - \frac{1}{2}\int pdpn_F(p)(1-\frac{m^2}{2p^2})\left\{\cos\theta - \frac{[m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o]}{-2pk}\left(\ln|m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\right)\right\}_{-1}^{+1} - \int \frac{p^3dpn_F(p)}{-2pk}(1+\frac{m^2}{2p^2})\left\{\ln|m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\right\}_{-1}^{+1} + k_o\int \frac{p^2dpn_F(p)}{-2pk}\left\{\ln|m_Z^2 + 2p(1+\frac{m^2}{2p^2})k_o - 2pk\cos\theta|\right\}_{-1}^{+1} + 2m^2\right.$$

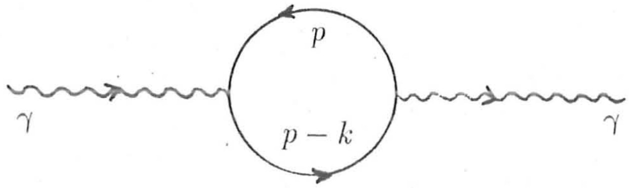
$$\times \int \frac{pdpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\left\{\ln|m_Z^2+2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}$$
$$+\int \frac{p^3dpn_F(p)}{-2pk}(1+\frac{m^2}{2p^2})\left\{\ln|m_Z^2-2p(1+\frac{m^2}{2p^2}k_o-2pk\cos\theta|\right\}_{-1}^{+1}$$
$$-\frac{1}{2}\int pdpn_F(p)(1-\frac{m^2}{2p^2})\left\{\cos\theta-\frac{[m_Z^2+2p(1+\frac{m^2}{2p^2})k_o]}{-2pk}(\ln|m_Z^2\right.$$
$$\left.+2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|)\right\}_{-1}^{+1}-\int\frac{p^2dpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})\{\ln|m_Z^2$$
$$-2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\Big\}_{-1}^{+1}+k_o\int\frac{p^2dpn_F(p)}{-2pk}\{\ln|m_Z^2-2p$$
$$\times(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\Big\}_{-1}^{+1}+2m^2\int\frac{pdpn_F(p)}{(-2pk)}(1-\frac{m^2}{2p^2})$$
$$\left.\left\{\ln|m_Z^2-2p(1+\frac{m^2}{2p^2})k_o-2pk\cos\theta|\right\}_{-1}^{+1}\right], \tag{A.9}$$

neglecting the higher terms, since m_Z is large as compared to lepton mass, we then have,

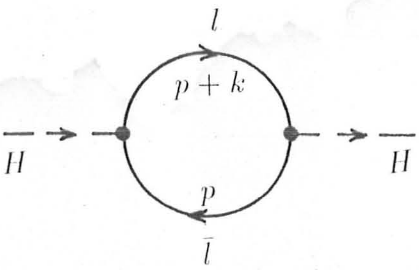
$$\Pi^{\gamma Z} = \frac{16i(ie\pi)^2}{(2\pi)^4}\left[\left\{\frac{2k}{m_Z^2}+\frac{2k_o}{m_Z^2}-\frac{4m}{m_Z^2}+\frac{2}{m}\right\}\frac{a(m\beta,\pm\mu)}{\beta}-\left\{\frac{-3m^2}{m_Z^2}\right\}\right.$$
$$b(m\beta,\pm\mu)+\left\{\frac{8k}{mm_Z^2}+\frac{4k}{mm_Z^2}-\frac{4}{m_Z^2}+\frac{2}{m^2}\right\}\frac{c(m\beta,\pm\mu)}{\beta^2}+\left\{\frac{8k}{m^2m_Z^2}\right.$$
$$\left.\left.+\frac{4k_o}{m^2m_Z^2}\right\}\frac{d(m\beta,\pm\mu)}{\beta^3}\right]. \tag{A.10}$$



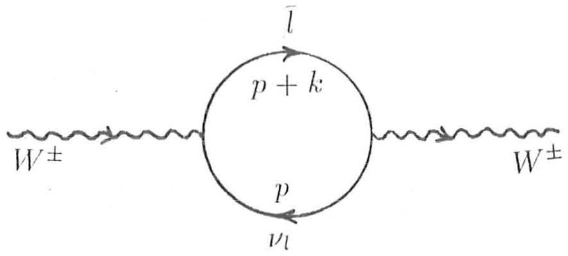
• **Fig.(2)**



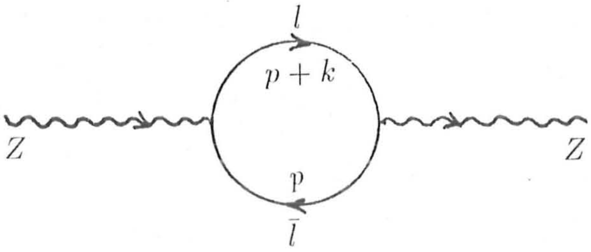
• **Fig.(3)**



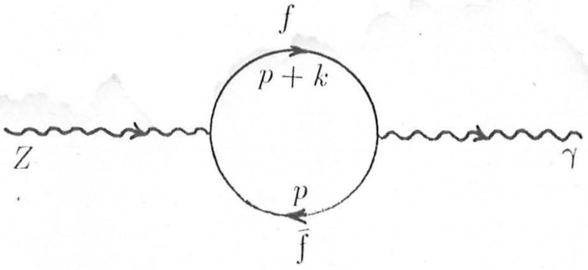
• **Fig.(4)**



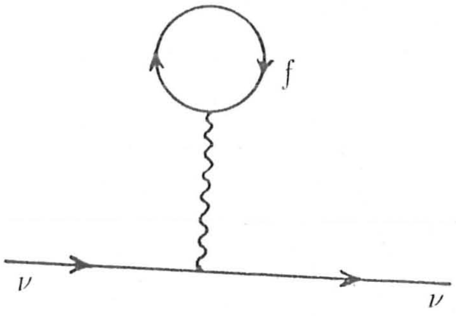
• **Fig.(5)**



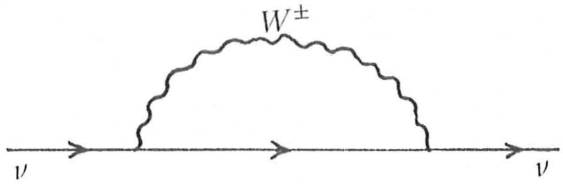
• **Fig.(6)**



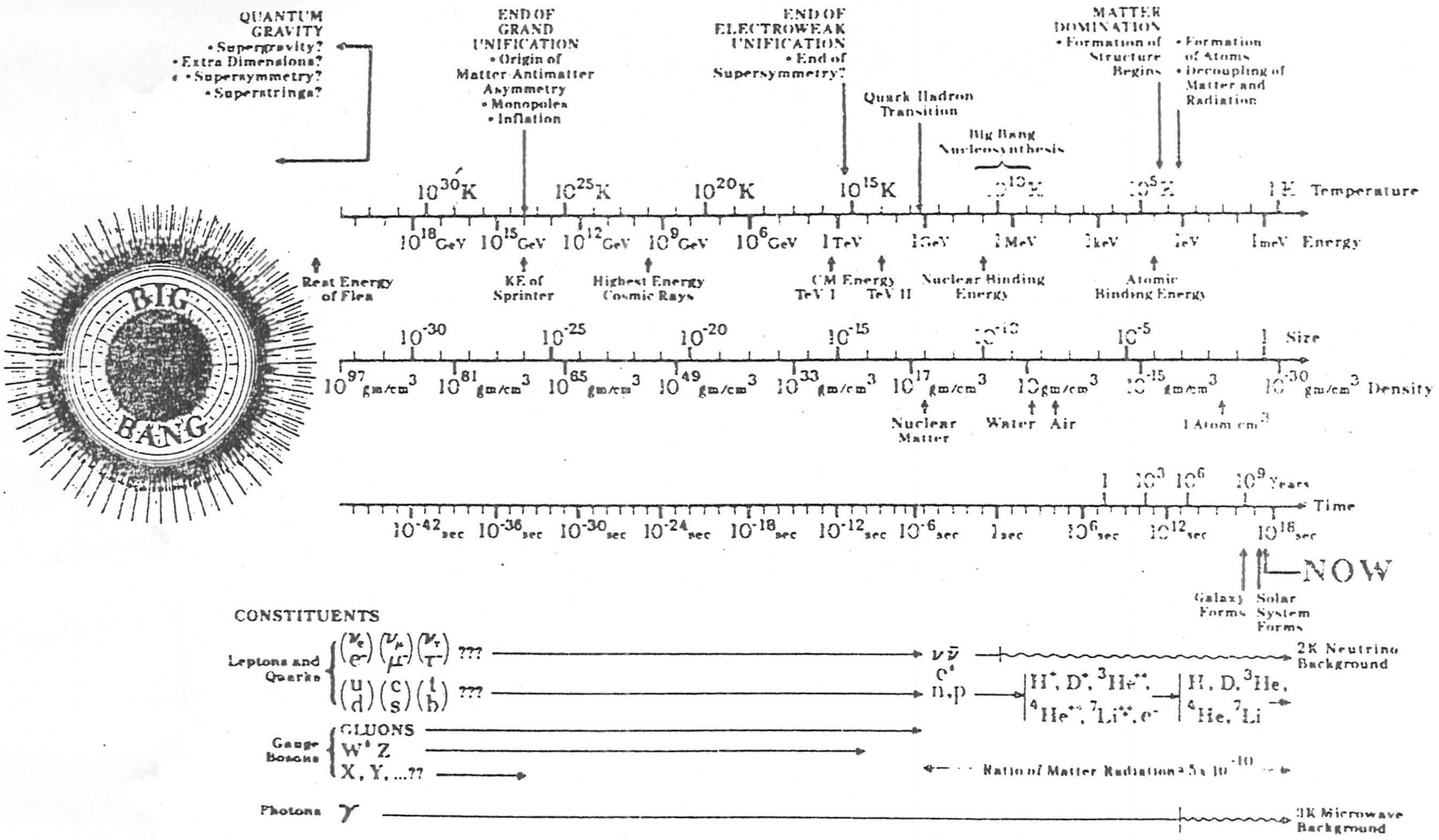
• **Fig.(7)**



• Fig.(8a)



• Fig.(8b)


QUANTUM GRAVITY
• Supergravity?
• Extra Dimensions?
• Supersymmetry?
• Superstrings?
END OF GRAND UNIFICATION
• Origin of Matter-Antimatter Asymmetry
• Monopoles
• Inflation
END OF ELECTROWEAK UNIFICATION
• End of Supersymmetry?
Quark Hadron Transition
Big Bang Nucleosynthesis
MATTER DOMINATION
• Formation of Structure Begins
• Formation of Atoms
• Decoupling of Matter and Radiation
Temperature
Energy
Rest Energy of Flea
KE of Sprinter
Highest Energy Cosmic Rays
CM Energy
TeV I
TeV II
Nuclear Binding Energy
Atomic Binding Energy
BIG BANG
Size
Density
Nuclear Matter
Water
Air
Time
NOW
Galaxy Forms
Solar System Forms
CONSTITUENTS
Leptons and Quarks
Gauge Bosons
GLUONS
Photons
2K Neutrino Background
3K Microwave Background

• Fig.(9)