DESIGN OF ELECTRONIC CIRCUITRY FOR SHORT TIME MEASUREMENTS OF THERMAL PROPERTIES OF SOLIDS USING THE TRANSIENT HOT STRIP METHOD

by

M U H A M M A D   A S I F

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Certified that the work in this dissertation was carried out and completed under my supervision.

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T O

T H E   T O I L I N G

W O R K E R S   A N D   P E A S A N T S

CONTENTS

A B S T R A C T

A non steady state 'TRANSIENT HOT STRIP'method has been used for measuring the thermal conductivity and thermal diffusivity of non conducting solids. The technique uses a metal film deposited on a substrate sample. The dimensions of strip aresuch as to realize an infinitely long continuous plane heat source of negligible thickness and finite width. Power supplied to the strip is electrical the heat conduction equation for the strip being solved in terms of the voltage developed across it. This voltage is time varying because resistance of strip increases as temperature increases with time. From the voltage variation we can get the thermal properties of substrate. Whereas the THS method has been used before[3] ,in this work the technique was improvised for milli-second time range. Short time measurements required fast switching with rather small voltage variation. This necessiated use of electronic instrumentation. Various sophisticated circuits were designed and tested. Finally a Bridge Circuit was made which was simple and accurate. Short time THS method was used for measurements on pure fused quartz, an isotropic material . It is possible to use the THS for anisotropic samples; a solution of the heat conduction equation for such a situation has been attempted. The results for fused quartz are consistent with those obtained by others.[3,4]

CHAPTER ONE

INTRODUCTION AND GENERAL THEORY

1.1 CONDUCTION OF HEAT

Heat flows in a body from higher temperature part to a lower temperature part. Transfer of heat takes place in three different ways. These are conduction, convection and radiation.

In solids, conduction is dominant while convection is absent and radiation can usually be neglected. In liquids and gases, convection and radiation dominate. We are concerned here with conduction of heat in solids only, so we will consider conduction of heat only.

1.2 THERMAL CONDUCTIVITY

Consider a plate of some solid bounded by planes of very large dimensions such that the points at the bounderies may be considered at infinite distances w.r.t. points in the centre of the planes (Fig.1.1). The planes are kept at different temperatures of some 10 degrees. After a sufficient time, a steady state is achieved, and the points away from the planes and lying on planes parallel to the bounding planes in the plate, will be at the same temperature.

Consider an imaginary cylinder of cross-section S with axis normal to the surface of the plate bounding the part of a solid. The cylinder is supposed so that no flow of heat takes place across its generating lines.

Now, let the temperature of the right-hand surface be T_o ^oC and that of left-hand T_1 ^oC; T_o being higher temperature than T_1. Let the thickness of the plate be d cm. Then the quantity of heat which flows from lower temperature to higher temperature surface in time t seconds over the surface S is given by:

$$Q = \frac{K(T - T)S\ t}{d} \quad \ldots \quad (1.1)$$

where K is a constant called "Thermal Conductivity" of the substance. The relation for thermal conductivity from equation (1) becomes:

$$K = \frac{Q\ d}{(T_o - T_1)St} \quad \ldots \quad (1.2)$$

This constant depends upon the material. This result is suggested by the experiment.

The thermal conductivity K is a function of temperature and is not constant for the same substance. The dependence of K on temperature may be approximated by making K to be a linear function of T like

$$K = K_o\ (1 + \beta T)$$

where β is small. This result, perhaps, proves good around room temperature only, with β negative for most of the substances.

But as we go nearer to the absolute zero degree, the results are rather strange. As shown in the Fig.1.2, the temperature dependence of K becomes of higher degree and after reaching a maximum, the curve falls to lower ones.

1.3 FLUX OF HEAT ACROSS ANY SURFACE

The rate at which heat is transferred across any surface S at a point per unit area per unit time is called the flux of heat at that point across that surface.

It can be shown that the continuity of flux does not depend on the continuity of thermal properties of the media.

If the values of flux 'f' are given for three mutually perpendicular planes meeting at a point, its value for any other plane through the point may be written down. This can be shown that if the three fluxes f_x, f_y, and f_z at a point P across planes parallel to the coordinate planes are known, the flux across any other plane through P can be determined from the following equation:

$$f = \lambda f_x + \mu f_y + \nu f_z \qquad \ldots . \qquad (1.3)$$

where λ , μ , ν , are the direction cosines of normal to plane through P.

A "flux vector" $\underline{f}$ at every point P of the solid is defined where components are f_x, f_y and f_z with magnitude

$$f_m = \sqrt{f_x^2 + f_y^2 + f_z^2}$$

and lying along the line with direction cosines f_x/f_m, f_y/f_m and f_z/f_m

The flux at P across a plane whose normal makes an angle θ with the line of direction of flux is $f_m \cos\theta$

1.4 ISOTHERMAL SURFACES

In a solid with a temperature distribution which is a function of position and time, the points with equal temperature (say T) will constitute a surface. This surface is called the isothermal surface for temperature T. Since no point or part of a body can have two temperatures, so no two isothermal surfaces cut each other.

1.5 HEAT CONDUCTION IN ISOTROPIC SOLIDS

A solid is said to be isotropic if all the directions for heat conduction are equally favourable, i.e. when a point within a solid is heated, the heat spreads out equally well in all directions. On the other hand, there are crystalline and anisotropic solids in whch certain directions are more favourable for the conduction of heat. There are also hetrogeneous solids in which the conduction of heat vary from point to point as well as in direction at each point.

In the experiment described for the thermal conductivity, the isothermal surfaces are planes parallel to the faces of the plates. The isothermals for temperatures T and $T+\delta T$ are at a distance of δx. Then the rate of flow of heat per unit time per unit area in the direction of x is: $-K\frac{\delta T}{\delta x}$ and as $\delta x \to 0$, we have:

$$f_x = -K\frac{\partial T}{\partial x}$$

We may generalize it by saying that the rate at which heat crosses from inside to the outside of an isothermal surface per unit area per unit time at a point is equal to $-K\,\partial T/\partial x$; where $\partial T/\partial x$ denotes differentiation along the outward normal drawn to the surface.

Generally, the flux of heat at a point across any surface is $-K\,\partial T/\partial h$, where $\partial/\partial h$ denotes differentiation in the direction of the outward normal. When planes are parallel to the coordinate axes, the fluxes are given by:

$$f_x = -K\frac{\partial T}{\partial x} \qquad \ldots \qquad (1.4a)$$

$$f_y = -K\frac{\partial T}{\partial x} \qquad \ldots \qquad (1.4b)$$

$$f_z = -K\frac{\partial T}{\partial x} \qquad \ldots \qquad (1.4c)$$

that is $\underline{f} = -K \underline{\nabla} T$ ... (1.5)

where $\underline{f}$ is a flux vector.

1.6 THE DIFFERENTIAL EQUATION OF CONDUCTION OF HEAT IN AN ISOTROPIC SOLID.

Considering the case of solid within which no heat is being generated but is flowing through it. The temperature and flux at point P(x,y,z) will be continuous functions of space and time coordinates.

A rectangular parallelopiped is considered in the solid with point P(x,y,z) at its centre and edges being parall to the coordinate axes; the lengths of the coordinates being 2dx, 2dy, and 2dz. The faces ABCD and ABCD be in the planes x-dx and x+dx respectively. Let the flux across the plane at P parallel to A B C D is f_x then the flux through the plane A B C D will be

$$f_x - \frac{\partial f_x}{\partial x} . dx$$

So the rate of flow of heat into the parallelpiped over the face A B C D will be

$$4(f_x - \frac{\partial f_x}{\partial x} . dx) \; dy \; dz \qquad \ldots \qquad (1.6)$$

Since the area of the face A B C D is 4 dy. dz. Similarly rate of flow of heat over the face A'B'C'D' which is at x+dx is given by

$$4(f_x + \frac{\partial f_x}{\partial x} . dx) dy dz \quad \ldots \quad (1.7)$$

Subtracting (1.6) from (1.7) the rate of gain of heat is given by

$$- 8 \frac{\partial f_x}{\partial x} . dxdydz$$

Similarly the rate of gain of heat from the flow across the planes parallel to z-x plane is

$$- 8 \frac{\partial f_y}{\partial y} . dxdydz$$

and that for the planes parallel to y-x plane is

$$- 8 \frac{\partial f_z}{\partial z} . dxdydz$$

So the total rate of gain of heat of the parallelopiped from the flow across its faces is found to be

$$-8(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial x}).dxdydz = -8 \; dxdydz.\underline{\nabla} \, \underline{f} \quad \ldots(1.8)$$

$\underline{f}$ being the flux vector.

The rate of gain of heat is also given by

$$8 \rho c \frac{\partial T}{\partial t} dxdydz \quad \ldots \quad (1.9)$$

where ρ and c are density and specific heat of the solid.

From equation (8) and (9), we get

$$\rho c \frac{\partial T}{\partial t} + \left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}\right) = 0 \qquad \ldots \qquad (1.10)$$

This equation corresponds to the equation of continuity in hydrodynamics. This equation holds for every point of the solid except the point where heat is being supplied. Also it is not necessary that the solid should be homogeneous or isotropic.

From equation (4), we have

$$f_x = - K \frac{\partial T}{\partial x}$$

$$f_y = - K \frac{\partial T}{\partial y}$$

$$f_z = - K \frac{\partial T}{\partial z}$$

The equations are true for the homogeneous isotropic solid whose thermal conductivity is independent of temperature.

So we have

$$\frac{\partial f_x}{\partial x} = - K \frac{\partial^2 T}{\partial x^2} \qquad \ldots \qquad (1.11a)$$

$$\frac{\partial f_y}{\partial y} = - K \frac{\partial^2 T}{\partial y^2} \qquad \ldots \qquad (1.11b)$$

$$\frac{\partial f_z}{\partial z} = - K \frac{\partial^2 T}{\partial z^2} \qquad \ldots \qquad (1.11c)$$

Substituting equation(1.11) in equation(1.10) we get

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa} \cdot \frac{\partial T}{\partial t} = 0 \qquad \ldots \qquad (1.12)$$

where $\kappa = K/\rho c$

Kelvin called this "the diffusivity of the substance" and Clerk Maxwell called it "the thermometric conductivity of the substance".

Equation (12) is known as the equation of conduction of heat.

1.7 INITIAL AND BOUNDARY CONDITIONS:

The temperature satisfies some boundary and initial conditions. Temperature T is considered as a continuous function of space co-ordinates and time. Also that it is true for the first differential coefficient w.r.t. time and upto second differential coefficient w.r.t. x, y, and z.

(i): INITIAL CONDITIONS

We suppose that at time t=0, the temperature is given by some arbitrary function, i.e.

$$T = f(x,y,z)$$

So the solution of the equation of conduction of heat

$$\kappa \nabla^2 T = \frac{\partial T}{\partial t}$$

tends to value of T at t = 0 as t tends to 0 i.e.

$$\lim_{t \to 0}(T) = f(x,y,z)$$

at all points of solid.

(ii): BOUNDARY CONDITIONS

The boundary or surface conditions which usually arise are the following:

(a): PRESCRIBED SURFACE TEMPERATURE

The prescribed temperature may be a constant or function of space or time or of both space and time. It is often difficult to prescribe surface temperature. A better condition may be given like the radiation boundary condition -- described later.

(b): NO FLUX ACROSS THE SURFACE

This condition suggests that the differentiation of temperature in the direction of outward normal to the surface is zero at all points of surface i.e.

$$\frac{\partial T}{\partial x} = 0$$

(c): PRESCRIBED FLUX ACROSS THE SURFACE

Like prescribed temperature, this flux also may be constant or function of space coordinates or position or both.

(d): LINEAR HEAT TRANSFER AT THE BOUNDARY; THE RADIATION BOUNDARY CONDITION

The boundary condition is given by

$$K \frac{\partial T}{\partial x} + H(T - T_o) = 0 \qquad \ldots \qquad (1.13)$$

Here $(T - T_o)$ is the difference of temperature between surface and surrounding media, T being the temperature of the medium and H is a constant. The second term is the flux across the surface which is proportional to $(T - T_o)$. Equation (1.13) can be written as

$$\frac{\partial T}{\partial x} + h(T - T_o) = 0 \qquad \ldots \qquad (1.14)$$

where $h = H/K$

This condition tends to the condition "no flux across the surface" as h tends to zero, and tends to the condition "prescribed surface temperature" as h tends to ∞ .

H is called the "Surface conductance" or "the coefficient of surface heat transfer" and 1/H is called the "Surface thermal resistance per unit area".

Also if a flux F is prescribed into the surface, equation(1.13)will become

$$K \frac{\partial T}{\partial x} + H(T - T_o) + F = 0 \quad \ldots \quad (1.15)$$

or

$$\frac{\partial T}{\partial x} + h(T - T_o - F/H) = 0 \quad \ldots \quad (1.16)$$

this condition is called the "Radiation boundary condition"; the reason being is that the heat transfer by radiation, which actually is proportional to the fourth power of the absolute temperatures can be approximated to first order of the absolute temperature, provided the temperature difference is small.

(e) NON-LINEAR HEAT TRANSFER

When temperature difference between the surface and surrounding medium is small, the flux dependence on temperature difference is approximately linear. But in many cases, this is not a linear function of temperature difference. For example the rate of loss of heat from a body at absolute temperature T surrounded by a black body at temperature T is given by

$$\sigma \, \varepsilon \, (T^4 - T_o^4) \qquad \ldots \qquad (1.17)$$

where σ is the Stefan-Boltzman's constant and ε is the emissivity of the surface, which is the ratio of the heat emitted by it to that emitted by a black body at same temperature.

If $T - T_o$ is small, this may be approximated as

$$4 \, \sigma \, \varepsilon \, T_o^3 \, (T - T_o) \qquad \ldots \qquad (1.18)$$

and if T_o is considered as a constant, the flux is directly proportional to $(T - T_o)$.

The second example for non-linear heat transfer can be natural convection, i.e. where a body is surrounded by fluid, the heat is transferred by convection. It is found that this heat transfer is nearly proportional to the 5/4th power of the temperature difference.

(f): CONTACT WITH A WELL-STIRRED FLUID OR PERFECT CONDUCTOR

In this case, the surface of solid is in contact with fluid which is well stirred so that its temperature may be taken as constant throughout. Now let a well stirred fluid of specific heat C' be in contact with a solid surface of area S, surface temperature T, and conductivity K. The temperature of fluid is supposed to be T' and mass M. We suppose that the fluid of mass M receives heat from external source at rate Q per unit time, and loses heat at the rate H_1 (T' - T_o) by radiation into a medium at temperature T. If rise in temperature of this fluid in time δt is $\delta T'$ then we have

$$Q\delta t - H_1(T' - T_o)\delta t - K\delta t \iint \frac{\partial T}{\partial x}\, dS = MC'\delta T' \quad \ldots(1.19)$$

$$K \iint \frac{\partial T}{\partial x}\, dS + MC' \frac{\partial T'}{\partial dt} + H_1(T'-T_o)-Q = 0 \quad \ldots(1.20)$$

If we also assume that

$$T = T' \qquad \text{for} \quad t > 0$$

where T and T' arc thc temperatures of surface and of fluid respectively for t > 0. And if heat transfer is taking place at a rate proportional to the difference of temperatures, then we have

$$K \frac{\partial T}{\partial x} + H(T - T') = 0$$

If mass m of fluid is withdrawn per unit time and replaced by the same amount of fluid at temperature T, we have

$$MC' \frac{dT}{dt} + K\iint \frac{\partial T}{\partial x}\, dS + mC'(T'-T_o)=0 \qquad \ldots \quad (1.21)$$

The above condition holds as well if instead of fluid, a perfectly conducting solid is in contact with the solid surface. A metallic conductor may be treated as a perfect conductor when it is in contact with non-metal.

(g): THE SURFACE OF SEPARATION OF TWO MEDIA OF DIFFERENT CONDUCTIVITIES

The fact that the flux is continuous over the surface of separation suggests this condition. Let K_1 and K_2 be the conductivities and T_1 and T_2 be the temperature of two media, then from flux continuity condition, we have

$$K_1 \frac{\partial T_1}{\partial x} = K_2 \frac{\partial T_2}{\partial x} \qquad \ldots \qquad (1.22)$$

$\frac{\partial}{\partial x}$ being the differentiation along the normal to the surface of separation.

Another condition that may be supposed to be valid for very intimate contact, such as soldered joint is that the two surfaces have equal temperature, i.e.

$$T_1 = T_2 \qquad \ldots \qquad (1.23)$$

For the surfaces not in contact like this (not soldered) heat transfer takes place so that flux across the surfaces is proportional to the difference of temperature, i.e.

$$-K_1 \frac{\partial T_1}{\partial x} = H(T_1 - T_2) \qquad \ldots \qquad (1.24)$$

(h): CONTACT WITH A THIN SKIN OF MUCH BETTER CONDUCTOR

Example of this boundary condition is a thin metal sheet or wire in contact with a relatively poor conductor, such as soil, food-stuff, etc. The skin is assumed so thin that the temperature throughout across its thickness may be considered as constant. Let T' be the temperature of the skin and K_1 and κ are the conductivity and diffusivity. Then the equation of conduction of heat may be written as

$$\frac{\partial^2 T'}{\partial \xi^2} + \frac{\partial^2 T'}{\partial \eta^2} - \frac{1}{K_1} - \frac{\kappa}{K_1} \cdot \frac{\partial T}{\partial n} = 0 \qquad \ldots \qquad (1.25)$$

where $\frac{\partial}{\partial n}$ is differentiation along the over ward normal direction and $\frac{\partial}{\partial \xi}$ and $\frac{\partial}{\partial \eta}$ along two perpendicular directions.

If we assume that the temperature of skin is equal to that of the solid, then another boundary condition arises i.e.

$$T' = T$$

1.8 SOURCES

1.8 (a): THE POINT SOURCE

When a finite quantity of heat is instataneously liberated at a point, at given time in an infinite solid, the source is called "Instataneous Point Source". In the theory of conduction of heat this point source has proved most useful.

The solution of this source may be taken as fundamental. A solution of continuous point source may be obtained by integrating it w.r.t. time. The continuous point source is that which release heat at a given point at a prescribed rate per unit time.

The solution of point sources may be integrated w.r.t. appropriate space variables to obtain solutions for instantaneous and continuous line, plane. spherical surface, and cylindrical surface sources.

(b): THE INSTANTANEOUS POINT SOURCE

The equation of conduction of heat is given by (1.12) to be

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa} \frac{\partial T}{\partial t}$$

The solution of this equation is

$$T = \frac{Q}{8(\pi\kappa t)^{3/2}} \exp\left[-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4\kappa t\right] \quad \ldots (1.26)$$

As t tends to zero; T tends to zero for all points except (x' , y' , z')

The total quantity of heat in infinite region is

$$\frac{Q\rho c}{8(\pi\kappa t)^{3/2}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \exp\left[-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4\kappa t\right] dx\,dy\,dz$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho c T \, dx\,dy\,dz$$

$$= Q\rho c \quad \ldots \quad (1.27)$$

Therefore the equation(1.26) may be thought as the temperature in an infinite solid due to a quantity of heat instantaneously generated at t=0 at point (x', y', z'). So the equation(1.26) is called the temperature due to an instantaneous point source of strength Q at point (x',y',z') at time t=0.

(c): THE CONTINUOUS POINT SOURCE

The temperature at point (x,y,z) at time t, due to a source at point (x', y', z') liberating heat at the rate $\phi(t)\rho c$ per unit time from t=0 to t=t is by integrating equation (1.26).

$$\frac{1}{8(\pi\kappa)^{3/2}}\int_{0}^{t}\frac{\phi(t')}{(t-t')^{3/2}}\exp[-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4\kappa(t-t')].dt' \quad \ldots.\ (1.28)$$

or if we put

$$r^2 = (x-x)^2+(y-y)^2+(z-z)^2$$

then (28) becomes

$$\frac{1}{8(\pi\kappa)^{3/2}}\int_{0}^{t}\frac{\phi(t')}{(t-t')^{3/2}}\exp[-r^2/4\kappa(t-t')].dt' \quad \ldots\ (1.29)$$

This distribution of temperature is said to be a continuous point source of strength from t=0 onward.

Now putting

$$\tau = (t - t')^{-1/2}$$

we get

$$d\tau = \tfrac{1}{2}(t - t')^{-3/2}.dt'$$

or $$2d\tau = dt'/(t - t')^{3/2}$$

and for lower limit i.e. for t = 0

$$= 1/\sqrt{t}$$

and for upper limit $t = t'$

$$\tau = 1/0 = \infty$$

also if $\phi(t)$=q (a constant), then from(1.29) we have

$$T = \frac{q}{4(\pi\kappa)^{3/2}} \int_{1/\sqrt{t}}^{\infty} \exp[-r^2\tau^2/4\kappa].d\tau$$

$$= \frac{q}{4(\pi\kappa r)} \text{erfc} \frac{r}{\sqrt{4\kappa t}} \qquad \ldots \qquad (1.30)$$

As $t \to \infty$ this reduces to $T=q/(4\pi\kappa r)$ a steady temperature distribution in which a constant supply of heat is continuously introduced at (x', y', z') and spreads outwards in the infinite solid.

(d): INSTANTANEOUS LINE SOURCE

The line source is considered parallel to the z-axis passing through point (x', y') with strength Q at time t = 0.

Considering instaneous point sources of strength Q.dz at z distributed along the line; the temperature due to this distribution can be obtained by integrating the solution of instaneous point source. So

$$T = \frac{Q}{8(\pi\kappa t)^{3/2}} \int_{\infty}^{\infty} dz'.\exp\left[-\{(x-x')^2+(y-y')^2+(z-z')^2\}/4\kappa t\right]$$

$$= \frac{Q}{4\pi\kappa t}.\exp\left[-\{(x-x')^2+(y-y')^2\}/4\kappa t\right] \quad \ldots \quad (1.31)$$

is the quantity of heat liberated by unit length of the line.

(e): INSTANTANEOUS PLANE SOURCE

The source is considered to be parallel to the y-z plane and passing through the point (x, 0, 0) with strength Q at t = 0.

The temperature due to this source can be obtained by integrating the solution for the instantaneous line source i.e.

$$T = \frac{Q}{4\pi\kappa t} \int_{-\infty}^{+\infty} \exp\left[-\{(x-x')^2+(y-y')^2\}/4\kappa t\right].dy'$$

$$= \frac{Q}{2\sqrt{\pi\kappa t}} \exp\left[\{-(x-x')^2\}/4\kappa t\right] \quad \ldots \quad (1.32)$$

and the quantity of heat liberated per unit area of the plain is

$Q\rho c$

(f): THE CONTINUOUS LINE SOURCE

Consider a solid at zero temperature at time t=0, when the supply heat starts. Temperature due to line source parallel to z-axis through the point (x', y'), liberating heat at the rate $\phi(t)\rho c$ per unit length per unit time, at time t is determined by integrating equation (1.26) i.e

$$T = \frac{1}{4\pi\kappa}\int_0^t \frac{\phi(t')}{t-t'} \exp\left[-\{(x-x')^2+(y-y')^2\}/4\kappa t\right].dt'$$

$$= \frac{1}{4\pi\kappa}\int_0^t \frac{\phi(t')}{t-t'} \exp\{-r^2/4\kappa(t-t')\}.dt'$$

where $r^2 = (x-x')^2 + (y-y')^2$

Now if $\phi(t) = q$ a constant

Putting

$$u = \frac{r^2}{4\kappa(t-t')}$$

then $$\frac{du}{u} = \frac{dt}{(t-t')}$$

and the lower limit (t=0) becomes

$$u = \frac{r^2}{4\kappa t}$$

and upper limit (t=t') becomes

$$u = \infty$$

also $\phi(t)=q$ a constant

we get

$$T = \frac{q}{4\pi\kappa} \int_{r^2/4\kappa t}^{\infty} \frac{\exp(-u)}{u}.du$$

$$= - \frac{q}{4\pi\kappa} \text{Ei} \quad -r^2/(4\kappa t)$$

where $$-\text{Ei}(-x) = \int_{x}^{\infty} \frac{\exp(-u)}{u}.du$$

for small values of x

$$\text{Ei}(-x) = \gamma + \ln x - x + x^2/4 + 0(x^3)$$

Where γ is Euler's constant having numerical value 0.5772...
Thus for large values of t we have

$$T = \frac{q}{4\pi k} \{ \ln (\frac{4kt}{r^2}) - r \} \quad \ldots \quad (1.33)$$

The term $\frac{q}{2\pi k} \ln(1/r)$ in Eq.(1.33) is the temperature due to a steady supply of heat. The rate of heat supply being $q \rho c$ heat units per unit length per unit time.

Also the solution for T in Eq.(1.33) gives the temperature in an infinite solid which is heated along a line say a wire carrying electric current.

(g): THE CONTINUOUS PLANE SOURCE

Let heat be liberated at the rate $\rho\ c\,\phi\,(t)$ per unit area per unit time in a plane x' starting at time t=0.

The temperature at time 't' is obtained by integrating Eq.(1.32)with respect to time. The temperature being then gives by

$$T = \frac{1}{2(\pi k)^{\frac{1}{2}}} \int_{0}^{t} \exp\ \{ -(x-x')^{2}/4k\ (t-t')\}\phi(t')/(t-t')^{\frac{1}{2}}.dt' \qquad \ldots.\ (1.34)$$

It $\phi\,(t) = q$, constant this becomes

$$T = q\left(\frac{t}{\pi k}\right)^{\frac{1}{2}} \exp\ \{ -(x-x')^{2}/4kt\} -q(|x-x'|)/2k.\mathrm{erfc}(|x-x'|)/2\sqrt{kt} \qquad \ldots\ (1.35)$$

C H A P T E R T W O

EXPERIMENTAL METHODS FOR THERMAL PROPERTIES

2.1 Linear Heat Flow

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2.7 Radial Heat Flow

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2.10 Dynamic Electrical Heating Method

EXPERIMENTAL METHODS

Some general and many special methods are in use for experimental determination of thermal conductivity of substances. The choice of a particular approach depends upon the following aspects.

i) Type of specimen

ii) Gradient heater

iii) Differential thermometer

iv) Calibration or reference

v) Thermal isolation etc.

GENERAL METHODS

2.1 LINEAR HEAT FLOW

In this method usually a rod of uniform cross-section is used and it is assumed that heat flows along the rod in one direction only. The arrangement is shown in Fig. 2.1.

S is a heat source at higher temperature and 0 is heat sink. The specimen is in the rod form of cross-section A.

(The form factor ℓ/d of rod is selected to give accurate results for the particular metal). T_1 and T_2 are the temperature sensors on the specimen a distance ℓ apart. It is assumed that there are no loses or heat generation along the length of the specimen.

Measuring the temperature difference $\Delta T = T_2 - T_1$ and rate of heat flow Q; the mean thermal conductivity is derived from the equation

$$Q = \frac{K(T)\ A\,\Delta T}{\ell} \qquad \ldots \qquad (2.1)$$

2.2 DIFFUSIVITY PLATE AND DIFFUSIVITY ROD

Steady state methods have been popular in thermal conductivity measurements; due to their simplicity in experimental implementation. But now non-steady state methods are comming up. Example of a non-steady method is given below.

In this method, pulses of heat are applied to one end of the specimen and the temperature along the specimen is measured as a function of time. The specimen is made in the form of plate or rod shape.

Knowing the temperature variations as a function of time, the thermal conductivity can be calculated from the thermal diffusivity equation if the specific heat is known.

2.3 SELF HEATING

In this method the energy is supplied to the specimen by passing a current direct through the specimen. Measuring the electrical conductivity, and the temperature at various points on the specimen, thermal conductivity may then be indirectly be calculated.

It is difficult to apply this method accurately if either the thermal or electrical conductivities vary significantly with temperature.

2.4 FORBES METHOD

Like linear heat flow method, the specimen used in this method is in rod form. But radial heat flow is allowed along the specimen, and corrections and control experiments are applied to simulate the ideal axial heat flow conditions.

This method has been used mostly in the temperature range above 300 K. With this method conductivity measurements are often made by three sets of readings; an "isothermal" an "unmatched guards" and then a "matched guard".

2.5 COMPARATIVE METHOD

This method is simple so far as the experimental setup and operation is concerned, but this at the expense of accuracy in the measurements. The power input in this method is determined by using a material with known thermal conductivity. This specimen is placed next to or in series with the standard. The comparative thermal conductivity values can be calculated by determining the different temperature gradients. The conductivity mismatch between standard and the specimen must not be too large, and also one should take care of inter facial resistance between the specimen and the standard, since it effects the temperature distribution.

2.6 GUARDED FLAT PLATE OR SPHERE

This method is very useful in measurement of poor heat conductors, like plastics and soldified gases. The potential heat loses are of great importance since the thermal conductivity of some of the materials is pressure dependent. So the apparatus must allow for any possible change in loading factor caused by thermal expansion or change of gas pressure. The method is principally same as the axial heat flow method except the form factor 1/A is very much smaller and auxiliary heat losses effects very prominent.

2.7 RADIAL HEAT FLOW

The method is very useful in high temperature range and is rarely used in low temperature range. The comparative or absolute methods can be used as in the axial heat flow methods.

In this method the discs stacked in the form of a cylindrical specimen having axial holes through them to provide thermocouple probe entry. The thermocouple probe is moved along the entire length of the stacked discs to measure the axial temperature gradients. A disadvtnage of the technique is the necessary waiting time for the system to reach equilibrium after each change in the probe position. This problem can be solved by installing many thermocouples at various radial and axial positions. The electrical resistivity and thermal conductivity measurements can be made simultancously with this apparatus. The radiative loses for high emissivity, low conductivity specimens at high temperature can be much reduced by using this method.

2.8 HOT WIRE METHOD

The method is used for determining the thermal conductivity of molten salts. Cylindrical liquid films of variable thickness were used (steady state method) by Lucks and Deam (1950).[2] Sand which had a conductivity similar to those of salts, was used as a calibration material. This avoided convection, but its purity and packing are often difficult to reproduce. Radial heat leaks and the radiation loses were large. To avoid convection is difficult with such thick films, and most of the errors had been reduced by extrapolating to zero film thickness, but the overall accuracy was still only ± 25%.

The transient hot wire method is found suitable instead of steady-state method. This method uses a thin wire in the liquid, heated by a constant current. The rate of rise in temperature of wire is measured, and the thermal conductivity of liquid can be calculated from the rate of production of heat and the temperature time relation.

Simplicity of the probe, auxiliary apparatus, speed, ease of operation, and avoidence of radiation and convection errors are the advantages of transient methods over steady-state method.

2.8 a THEORY OF HOT WIRE

Considering an ideal case of an infinitely long wire with radius r_o and infinite thermal conductivity. is immersed in a liquid having infinite extensions in space with ρ , c and k as its density, specific heat ahd thermal conductivity respectively. It is assumed that there is no thermal resistance at the surface of the wire to the liquid. The wire is heated at a constant rate q per unit length. The exact solution of the Fourier differential equation for these conditions is given by equation (1.33) which is

$$T = \frac{q}{4\pi k} \{ \ln \frac{4kt}{r^2} - r \}$$

Which gives for thermal conductivity

$$K = \frac{q}{4\pi T} \{ \ln 4\tau - 0.577 \}$$

where $\tau = Kt/\rho c r_o^2$

Now in given experiment K, ρ , c and r_o are constant. Conductivity K can be evaluated from the linear relation of T and ln t. For large times the approximation becomes better.

The following considerations must be made since the actual wire differs from the ideal case.

i) Small radius of wire is used to approximate to a line source, and reduce radiation.

ii) The longest possible length of the wire is used to reduce end loses. This length is limited by isothermal

length of the thermostat and amount of specimen available.

iii) The material of the wire must be corrosion resistance and has a high resistive temperature coefficient, which is also reproducable.

iv) The resistivity of the wire should be low compared with that of salt, but high enough to enable accurate temperature measurements from resistance change.

v) The temperature rise (i.e. heating current) should be small to reduce the change in heating rate, delay the onset of convection, avoid convection currents in the salt, and reduce the change in K itself due to change in temperature.

2.8. b EXPERIMENTAL ARRANGEMENT

Bridge circuit shown in Fig. (2.2) is used. The probe is a platinium wire, welded to short thick platinum leads, with a fine potential taping. Thick coppor leads of equal resistance are used to connect the probes to the bridge circuit. Thermo-electric effects are balanced out automatically since all the three copper-platinium junctions are at the same temperature.

In the bridge circuit the resistance of leads and wire end effects are balanced out since these are in adjacent bridge arms. Very small current (causing negligible heating) is passed to determine the resistance of the wire. A dummy resistor having resistance approximately equal to the resistance of the bridge is used to stablize the heating current. The current is known by passing it through a standard resistance and measuring the potential drop across it. The unbalanced voltage which is proportional to the temperature rise of the wire, is recorded by a short periodic time galvenometer or fast potentiometric recorder. The resistive temperature coefficient of wire is found by measuring resistance at the ice, steam, nephthaleane and sulfur points.

2.9 HOT WIRE SHORT TIME METHOD

The method is principally same as described in the previous article i.e. the thermal conductivity of the medium may be deduced from the temperature variations of the wire, immersed in an electrically insulating medium and is heated by a known constant power. Here it is shown that the specific heat capacity or the thermal diffusivity can be determined by the further analysis of temperature variation.

Starting from equation (1.33)

$$T = \frac{q}{4\pi k} \{ \ln \frac{4kt}{r^2} - 0.5772 \} \quad \ldots \quad (2.2)$$

This equation is valid for $kt/r^2 >> 1$ or for large times. Thermal conductivity K may be determined from T versus ln t plot (T being linear in ln t for small temperature rise) since $q/r\pi k$ is the coefficient of the slope.

Rearranging equation (2.2) the thermal diffusivity is given by the equation

$$k = (0.4453\ r^2/t) \exp \{ 4\pi K(T-T_o)/q \} \quad \ldots \quad (2.3)$$

Now all the quantities on the right are observable except possibly the initial temperature. This can be determined by measuring the initial resistance using a very small current, as explained in the previously described method. Another method which gives more reproducible results is of small but finite time measurements of resistance and hence temperature. The extrapolation then to zero and time gives the required value. This requires some rough knowledge of the expression $T - T_o$ in small time limit. This approximation is given in the form of the following equation

$$T-T_o = \frac{q\,\alpha}{2\,\pi\,\lambda}\{T - 0.7523\ \alpha\ \tau^{1.5} + 0\ \tau^2\}\ \ldots\ldots \quad (2.4)$$

where $\tau = \alpha^t/r^2$

2.10 DYNAMIC ELECTRICAL - HEATING METHOD

This method is used by S. R. Chin and W. K. Zwicker to determine the thermal conductivity of Neodynium Pentaphosphate (NdP O, abbreviated as NPP), used in miniature efficient lasers.

Two parallel strips of metal are grown on the surface of the sample. One strip is used as a heater and other as a sensor. An alternating current is passed through heater. The resistance variation (due to temperature variation) is measured as a function of heating current frequency. Knowing the electrical parameters for the heater and sensor, and the temperature coefficient of sensor resistance, alongwith measured data, the thermal conductivity and specific heat can be calculated.

C H A P T E R   T H R E E

THEORY FOR THE TRANSIENT HOT STRIP METHOD

3.1 FIRST ORDER APPROXIMATION OF TRANSIENT HOT STRIP METHOD.

The solution for the differential equation of conduction of heat, for a point source is given by Eq.(1.26)

$$T = \frac{Q}{8(\pi \kappa t)^{3/2}} \cdot \exp\left[-\{(x-x')^2 + (y-y')^2 + (z-z')^2\} / 4\kappa t \right]$$

If $\phi(t)\rho c$ is the heat liberated per unit time then the solution for a continuous infinitely long strip of width ' 2d ' lying in the y-z plane is obtained by integrating the above equation with respect to y' , z' and t'. (It is assumed that the thickness of strip is negligible).

$$\Delta T = \frac{1}{8(\pi\kappa)^{3/2}} \int_0^t \frac{dt'}{(t-t')^{3/2}} \cdot \exp\{-x^2/4\kappa(t-t')\}$$

$$\int_{-d}^{d} dy'\phi(t',y') \exp\{-(y-y')^2/4\kappa(t-t')\}$$

$$\int_{-\infty}^{+\infty} dz' \exp\{-(z-z')^2/4\kappa(t-t')\} \quad \ldots \quad (3.1)$$

The 'z' dependent part gives us a constant $2\sqrt{\pi\kappa(t-t')}$ Equation 3.1 becomes

$$\Delta T(y,t) = \frac{1}{4\pi\kappa} \int_0^t \frac{dt'}{(t-t')} \cdot \exp\{-x^2/4\kappa(t-t')\}$$

$$\int_{-d}^{d} dy'\phi(t',y')\exp.\{-(y-y')/4\kappa(t-t')\} \quad \ldots \quad (3.2)$$

Considering the strip to be in $x = 0$ plane we will get

$$\Delta T = \frac{1}{4\pi\kappa} \int_0^t \frac{dt'}{(t-t')} \int_{-d}^{d} dy'\phi(t',y')\exp\{-(y-y')^2/4\kappa(t-t')\}$$

$$\ldots \quad (3.3)$$

Make a change of variable

Let $\sigma^2 = \frac{\kappa(t-t')}{d^2}$

$\Longrightarrow$ $dt' = -\frac{2d^2}{\kappa}\, d\sigma$

and when $t' = 0$, $\sigma = \sqrt{\kappa t/d}$

$t' = t$, $\sigma = 0$

with these Eq. 3.3 changes to

$$\Delta T(y,t) = \frac{1}{4\pi\kappa} \int_{\sqrt{\kappa t}/d}^{0} \frac{-\,d\sigma}{\sigma} \int_{-d}^{d} dy'\,\phi(t - \frac{d^2\sigma^2}{\kappa}, y')$$

$$\exp\{-(y-y')^2/4\,\sigma^2 d^2\}$$

$$\Rightarrow \qquad \Delta T(y,t) = \frac{1}{4\pi\kappa} \int_{0}^{\sqrt{\kappa t}/d} \frac{d\sigma}{\sigma} \int_{-d}^{d} dy'\,\phi(t - \frac{d^2\sigma^2}{\kappa}, y')$$

$$\exp\{-(y-y')^2/4\sigma^2 d^2\} \qquad \ldots \qquad (3.4)$$

Let $\quad y - y' = 2\,\eta\,\sigma\,d$

$\Rightarrow \quad dy' = (2\,\sigma\,d)\;d\,\eta$

and for $\quad y' = d \quad , \quad \eta = (y - d)/2\sigma d$

$\quad y' = -d \quad , \quad \eta = (y + d)/2\sigma d$

Then Eq. (4) becomes

$$\Delta T(y,t) = \frac{1}{2\pi\kappa} \int_{0}^{\sqrt{\kappa t}/d} \frac{d\sigma}{\sigma} \int_{(y-d)/2\sigma d}^{(y+d)/2\sigma d} 2\sigma d.\phi(t - \frac{d^2\sigma^2}{\kappa}, y-2\sigma\eta d) \times$$

$$\exp(-\,\eta^2).\;d\eta$$

or

$$\Delta T(y,t) = \frac{d}{\pi \kappa} \int_{0}^{\sqrt{\kappa t}/d} d\sigma \int_{(y-d)/2\sigma d}^{(y+d)/2\sigma d} d\eta \, \phi(t - \frac{d^2\sigma^2}{\kappa}, y - 2\sigma\eta d) \times$$

$$\exp(-\eta^2) \quad \ldots \quad (3.5)$$

Making another change of variables

$$\tau = \sqrt{\kappa}t/d \quad , \quad t_c = d^2/\kappa \quad , \quad \xi = y/d$$

Eq. (5) becomes

$$\Delta T(\xi, \tau) = \frac{d}{\pi \kappa} \int_{0}^{\tau} d\sigma \int_{(\xi-1)/2\sigma}^{(\xi+1)/2\sigma} d\eta \, \phi(t_c(\tau^2 - \sigma^2,$$

$$d(\xi - 2\sigma\eta)) \exp(-\eta^2) \quad \ldots \quad (3.6)$$

We know that $\phi(y', t')$ is the output of energy per unit area per unit time.

Consider a strip of unit breadth along the length of the main strip. Let the output of power for this unit breadth strip be $P(y', t')$

$$\Rightarrow \qquad \phi(y',t') = \frac{1}{\rho c} \frac{P(y',t')}{2h}$$

Where $\frac{P(y',t')}{2h}$ is the output of power per unit area of the strip and '2h' is total length of strip.

Let the voltage across the strip be U(t'), the resistance of the strip be $\Delta R(y',t')$ and thermal conductivity of the strip be Ω.

We can then write $\phi(y',t')$ as

$$\phi(y',t') = \frac{\kappa}{2h\Omega} \cdot \frac{U^2(t')}{\Delta R(y',t')} \quad \ldots \qquad (3.7)$$

Rewriting (7) in terms of electrical conductivity σ_E which is inverse of electrical resistivity.

$$\phi(y',t') = \frac{\kappa}{2h\Omega} \; U^2(t') \; \Delta\sigma_E \, (y',t') \quad \ldots \qquad (3.8)$$

If thickness of the strip is '2ν' and ρ(T) is the time dependent resistivity of the strip then electrical conductivity is given by

$$\Delta\sigma_E(y',t') = \frac{1}{2h} \cdot \frac{2\nu}{\rho(T)}$$

Where $\rho(T) = \rho_o \; \{ 1 + \alpha \, \Delta \, T(y',t')\}$

' α ' is the temperature coefficient of resistance. So we can write $\Delta\sigma_E$ as

$$\Delta\sigma_E = \frac{\nu}{h\ \rho_o\ \{1+\alpha\Delta T(y',t')\}}$$

At initial temperature, before any change in temperature occurs $\Delta T = 0$ so we can write for electrical conductivity at initial temperature

$$\sigma_{Eo} = \frac{2d.2\nu}{2h\rho_o}$$

$$\Rightarrow \qquad \Delta\sigma_E = \frac{\sigma_o}{2d\ \{1+\alpha\ \Delta T(y',t')\}}$$

Substituting this in Eq. (8) we get

$$(y',t') = \frac{\kappa\ \sigma_{Eo}}{(4dh\ \Omega)} \cdot \frac{U^2(t')}{\{1+\alpha\Delta T(y',t')\}} \qquad \ldots \qquad (3.9)$$

Now

$$\sigma_{Eo} = \frac{J_o}{U_o} = \frac{I_o U_o}{U_o^2} = \frac{P_o}{U_o^2}$$

Let $\quad \dfrac{U(t')}{U_o} = v(t')$

So Eq. (9) can be rewritten as

$$\phi(y',t') = \frac{\kappa P_o}{4dh\Omega} \cdot \frac{v^2(t')}{1+\alpha\Delta T(y',t')} \qquad \ldots \qquad (3.10)$$

Combining Eqs. 3.6 and 3.10 we get

$$\Delta T = \frac{P_o}{4\pi\kappa\Omega} \int_0^{\tau} d\sigma \int_{(\xi-1)/2}^{(\xi+1)/2} d\eta \frac{V^2(t_c(\tau^2-\sigma^2))}{\{1+\alpha\Delta T(d(\xi-2\sigma\eta),t_c(\tau^2-\sigma^2))\} \cdot \exp(-\eta^2)} \quad \ldots \quad (3.11)$$

We had $\Delta\sigma_E = \dfrac{\nu \cdot dy}{h\,\rho_o\{1+\alpha\Delta T(y,t)\}}$

$$\Longrightarrow \quad \sigma_E = \frac{\nu}{h\rho_o} \int_{-d}^{d} \frac{dy}{\{1+\alpha\Delta T(y,t)\}}$$

$$= \frac{I_o}{U}$$

where I_o is the constant current input

or

$$\sigma_E = \frac{I_o}{2dU_o} \int_{-d}^{d} \frac{dy}{\{1+\alpha\Delta T(y,t)\}}$$

or using $\dfrac{U(t)}{U_o} = v(t)$ we get

$$\frac{U_o}{U} = \frac{1}{v(t)} = \frac{1}{2d} \int_{-d}^{d} \frac{dy}{\{1+\alpha\Delta T(y,t)\}} \quad \ldots \quad (3.12)$$

and with $\xi = y/d$ we get

$$\frac{1}{v(t)} = \frac{1}{2}\int_{-1}^{+1} \frac{d}{1+\alpha\Delta T(\xi,\ t)} \qquad \ldots \qquad (3.13)$$

Typically α is of the order 10^{-3} and the rise in temperature 'ΔT' is kept below 1^{o}. So to a zero order approximation $\alpha . \Delta T(y,t)$ is neglected in Eq. (13). This gives $v(t) \simeq 1$ and Eq. (11) becomes

$$\Delta T(\xi,\tau) = \frac{P_o}{4\pi\kappa\Omega}\int_{o}^{\tau} d\sigma \int_{(\xi-1)/2\sigma}^{(\xi+1)/2\sigma} d\eta \;.\; \exp\;(\;-\;\eta^2) \;\ldots \qquad (3.14)$$

$$\Delta T(\xi,\tau) = \frac{P_o}{8\;\kappa\Omega\sqrt{\pi}}\int_{o}^{\alpha} d\sigma .\; \frac{2}{\sqrt{\pi}}\left[\int_{(\xi-1)/2\sigma}^{\infty} d\eta\ \exp(-\eta^2) - \int_{(\xi+1)/2\sigma}^{\infty} d\eta . \exp(-\eta^2)\right]$$

$$\ldots \; (3.15)$$

Using the definition $\frac{2}{\sqrt{\pi}}\int_{x}^{\infty} e^{-m^2}\, dm = \text{erfc}\ x = 1 - \text{erf} x$

Eq. (15) is written as

$$\Delta T(\xi,\tau) = \frac{P_o}{8\kappa\Omega\sqrt{\pi}}\int_{o}^{\tau} d\sigma\ \{\text{erfc}\ (\tfrac{\xi-1}{2\sigma}) - \text{erfc}\ (\ \tfrac{\xi+1}{2\sigma}\ )\} \quad \ldots \qquad (3.16)$$

Let $\qquad x = \frac{\xi-1}{2\sigma} \qquad \Rightarrow \qquad dx = -\ \frac{d\sigma}{\sigma^2}\ .\ (\ \xi - 1)$

This means that the integral dependent on 'σ' becomes

$$\int_0^{\tau} d\sigma.\ \mathrm{erfc}\ (\frac{\xi-1}{\sigma}) = \frac{\xi-1}{2} \int_{(\xi-1)/2\tau}^{\infty} dx.\ \frac{1}{x^2}\ .\ \mathrm{erfc}\ x \qquad \ldots \qquad (3.16a)$$

Integration by parts the R.H.S. gives

$$\frac{\xi-1}{2} \int_{(\xi-1)/2\tau}^{\infty} dx.\ \frac{1}{x}2.\ \mathrm{erfc}\ x = (\frac{\xi-1}{2})\ \frac{2\tau}{\xi-1}\ .\ \mathrm{erfc}\ (\frac{\xi-1}{2\tau}) - \frac{2}{\sqrt{\pi}} \int_{(\xi-1)/2\tau}^{\infty} \frac{dx}{x}\ .\ \exp\ (-x)$$

More simplification can be obtained by putting

$$x^2 = y \qquad \Rightarrow \qquad 2x\ dx = dy$$

We get from Eq. (16a)

$$\frac{\xi-1}{2} \int_{(\xi-1)/2\tau}^{\infty} \frac{dx}{x^2}\ .\ \mathrm{erfc}\ x = \tau.\ \mathrm{erfc}\ (\frac{\xi-1}{2})+(\frac{\xi-1}{2\tau})\ \frac{}{2\sqrt{\pi}} \times (-(\frac{\xi-1}{2\tau})^2) \qquad \ldots \qquad (3.16b)$$

where $\quad -E_i(-x) = \int_x^{\infty} \frac{1}{U}\ .\ \exp(-U).du$

Using Eq.(316a) and(3.16b) in Eq.(3.16) we get

$$\Delta T(\xi, \tau) = \frac{P_o}{8h\,\Omega\sqrt{\pi}} \{ \tau \operatorname{erfc} (\frac{\xi -1}{2\tau}) - \tau \operatorname{erfc} (\frac{\xi+1}{2\tau})$$

$$+ \frac{\xi-1}{2\sqrt{\pi}} E_i(-(\frac{\xi-1}{2\tau})^2) - \frac{\xi+1}{2\sqrt{\pi}} E_i(-(\frac{\xi+1}{2\tau})^2) \} \quad \ldots \quad (3.17)$$

For first order approximation we consider Eq.(3.13) which is

$$\frac{1}{v(t)} = \frac{1}{2} \int_{-1}^{+1} \frac{d\xi}{1 + \alpha\,\Delta T(\xi,t)}$$

Since $\alpha\,\Delta T(\xi,t) << 1$ we can write

$$\frac{1}{v(t)} = \frac{1}{2} \int_{-1}^{+1} \{ 1 - \alpha\,\Delta T(\xi,t) \} \, d\xi$$

$$= 1 - \frac{\alpha}{2} \int_{-1}^{+1} d\xi\,\Delta T(\xi,t)$$

Or

$$v(t) = 1 + \frac{\alpha}{2} \int_{-1}^{+1} d\xi\,\Delta T(\xi,t) \quad \ldots \quad (3.18)$$

Consider the integral $\int_{-1}^{+1} d\xi\,\Delta T(\xi,t)$, in it we can substitute for $\Delta T(\xi,t)$ from Eq.(3.17) so that

$$\int_{-1}^{+1} d\xi \Delta T(\xi,\tau) = \frac{P_o}{8h\Omega\sqrt{\pi}} \int_{-1}^{+1} d\xi \{\tau\, \mathrm{erfc}(\frac{\xi-1}{2\tau}) - \tau\, \mathrm{erfc}\, (\frac{\xi+1}{2\tau})$$

$$+ (\frac{\xi-1}{2\sqrt{\pi}})\, E_i(-(\frac{\xi-1}{2\sqrt{\pi}})^2) - (\frac{\xi+1}{2\sqrt{\pi}})\, E_i(-(\frac{\xi+1}{2\tau})^2)\, \} \quad \ldots \quad (3.19)$$

Consider the integral $\tau \int_{-1}^{+1} d\xi \;\; \mathrm{erfc}\; (\frac{\xi-1}{2\tau})$

Let $\frac{\xi-1}{2\tau} = x \quad \Rightarrow \quad d\xi = 2\,\tau\, dx$

$$\Rightarrow \quad \tau \int_{-1}^{+1} d\xi\; \mathrm{erfc}\; (\frac{\xi-1}{2\tau}) = 2\tau^2 \int_{-1/}^{0} \mathrm{erfc}\; x.\; dx$$

$$= 2\tau^2 \left|\, x.\mathrm{erfc}\; x - \frac{1}{\sqrt{\pi}} \exp\; (-x^2) \,\right|_{1/\tau}^{0}$$

$$\text{or} \quad \tau \int_{-1}^{+1} d\xi\; \mathrm{erfc}(\frac{\xi-1}{2\tau}) = 2\,\tau\,(2 - \mathrm{erfc}\; 1/\tau) - \frac{2\tau^2}{\sqrt{\pi}}(1-\exp(-1/\tau^2) \quad \ldots \quad (3.20)$$

Similarly

$$\tau \int_{-1}^{+1} d\xi\; \mathrm{erfc}(\frac{\xi+1}{2\tau}) = 2\tau.\mathrm{erfc}\; 1/\tau + \frac{2\tau^2}{\sqrt{\pi}}(1-\exp(-1/\tau^2) \quad \ldots \quad (3.21)$$

Let $(\frac{\xi-1}{2\tau})^2 = x \quad \Rightarrow \quad (\xi-1)\; d\xi = 2\tau^2\; dx$

So that

$$\frac{1}{2\sqrt{\pi}} \int_{-1}^{+1} d\xi(\xi-1)\, E_i\{-(\frac{\xi-1}{2\tau})^2\} = \frac{\tau^2}{\sqrt{\pi}} \int_{1/\tau^2}^{0} E_i(-x)dx$$

$$= \frac{\tau^2}{\sqrt{\pi}} \Big| x.E_i(-x) + \exp(-x) \Big|_{1/\ ^2}^{0}$$

$$\Rightarrow \quad \frac{1}{2\sqrt{\pi}} \int_{-1}^{+1} d\xi\ (\xi-1)\ E_i\{-(\frac{-1}{2})^2\}$$

$$= \frac{\tau^2}{\sqrt{\pi}} \{1-\exp(-1/\tau^2)\} - \frac{1}{\sqrt{\pi}} E_i(-1/\tau^2) \quad \ldots \quad (3.22)$$

Similarly

$$\frac{1}{2\sqrt{\pi}} \int_{-1}^{+1} d\xi\ (\xi+1)E_i\ \{-(\frac{\xi+1}{2\tau})^2 = \frac{1}{\sqrt{\pi}} E_i(-1/\tau^2) - \frac{\tau^2}{\sqrt{\pi}}$$

$$\{1 - \exp\ (-1/\tau^2)\} \quad \ldots \quad (3.23)$$

Combining (19), (20), (21), (22) and (23) we get

$$\int_{-1}^{+1} d\xi\Delta T(\xi,\tau) = \frac{P_o}{2h\Omega\sqrt{\pi}} \Big[\tau \mathrm{erfc}(1/\tau) - \frac{\tau^2}{2\sqrt{\pi}} \{1 - \exp(-1/\tau^2)\}$$

$$- \frac{1}{2\sqrt{\pi}} E_i(-1/\tau^2) \Big] \quad \ldots \quad (3.24a)$$

Denoting the terms in brackets on R.H.S. of3.24a by $f(\tau)$ we rewrite

$$\int_{-1}^{+1} d\xi \ \Delta T(\xi,\tau) = \frac{P_o}{2h\Omega\sqrt{\pi}} . \ f(\tau) \qquad \ldots \qquad (3.24b)$$

where $f(\tau) = \text{erfc}(1/\tau) - \frac{1}{2\sqrt{\pi}} \tau^2 \{1-\exp(-1/\tau^2)\} - \frac{1}{2\sqrt{\pi}} E_i(-1/\tau^2)$

$$\ldots \qquad (3.25)$$

$$v(\tau) = 1 + \frac{\alpha P_o}{4h\Omega\sqrt{\pi}} f(\tau)$$

and substituting G for $\frac{P_o}{4h\Omega\sqrt{\pi}}$ we can write

$$v(\tau) = 1 + \alpha G \ f(\tau) \qquad \ldots \qquad (3.26)$$

$$\int_{-1}^{+1} d\xi \ \Delta T(\xi,\tau) = 2 \ G. \ f(\tau) \qquad \ldots \qquad (3.27)$$

$$\Delta T(\xi,\tau) = \frac{1}{2} G \ \Psi(\xi,\tau) \qquad \ldots \qquad (3.28)$$

where $\Psi(\xi,\tau) = \tau \ \text{erfc} \ (\frac{\xi-1}{2\tau}) - \tau \text{erfc}(\frac{\xi+1}{2\tau})$

$$+ \frac{\xi-1}{2\sqrt{\pi}} E_i\{-(\frac{\xi-1}{2\tau})^2\} - \frac{\xi+1}{2\sqrt{\pi}} E_i\{-(\frac{\xi+1}{2\tau})^2\} \qquad \ldots \qquad (3.29)$$

Equation (3.25) above is the first order approximation. A second order[3] can be obtained by plugging (3.25) back into Eq. (3.11) and including terms of second order.

From the above we know that

$$v(t) = 1 + \alpha G f(\tau)$$

Now $v(t) = u(t)/U_o$

$$\Rightarrow \quad u(t) = U_o + U_o \alpha G . f(\tau) \quad \ldots \quad (3.30)$$

It is possible to plot u(t) against $f(\tau)$. This would give us a straight line. The intercept of this line gives U_o and the slope gives the thermal conductivity as Ω^{-1}. The function $f(\tau)$ being numerically evaluated.

Generally it would be convenient to approximate the complicated $f(\tau)$ function by some simple expression

The $f(\tau)$ function is given by Eq. (3.25) as

$$f(\tau) = \text{erfc}(1/\tau) - \frac{1}{2\sqrt{\pi}} . \tau^2 \{1 - \exp(-1/\tau^2)\}$$

$$- \frac{1}{2\sqrt{\pi}} E_i(-1/\tau^2) .$$

For small values of τ, $f(\tau)$ can be expanded by Taylor's expansion around $\tau \approx 0$.

$$f(\tau)=f(o) + \tau f'(o)+ \frac{\tau^2}{2.1} .f''(o)+\frac{\tau^3}{3.2.1}.f'''(o) + \ldots$$

Now

$$f(o) = 0$$

$$f'(\tau) = \text{erfc}(1/\tau) - \frac{1}{\sqrt{\pi}} . \tau \{1 - \exp(-1/\tau^2)\}$$

$\Rightarrow$ $$f'(o) = 1$$

$$f''(\tau) = - \frac{1}{\sqrt{\pi}} \{1 - \exp(-1/\tau^2)\}$$

$\Rightarrow$ $$f''(o) = - \frac{1}{\sqrt{\pi}}$$

$$f'''(\tau) = \frac{1}{\sqrt{\pi}} \frac{1}{\tau^3} . \exp(-1/\tau^2)$$

$\Rightarrow$ $$f'''(o) = 0$$

$\vdots$

etc.

The third and higher order derivatives may be written in the form

$$\tau^{-m} \exp(-1/\tau^2)$$

where $m \geq 3$

Now as $\tau \to 0$, $e^{-m} \exp(-1/\tau^2) \to 0$

and so sum of all terms above the second order derivative will be zero. That is, the contribution is from the first two derivatives only in the Taylors Expansion of $f(\tau)$ around τ x 0, so that we keep τ^2 terms only. The function $f(\tau)$ is then approximated as

$$f(\tau) = \tau - \frac{1}{\sqrt{4\pi}} \tau^2 \qquad \ldots \qquad (3.31)$$

The voltage variation as a function of time can be written as

$$u(t) = a_1 + a_2\, t^{\frac{1}{2}} + a_3 t \qquad \ldots \qquad (3.32)$$

where the coeffecients a_1, a_2 and a_3 would give the thermal properties to be determined.

We had Eq. (3.26) as

$$v(t) = 1 + \alpha\, G\, f(\tau)$$

where now $f(\tau) = \tau - \frac{1}{\sqrt{4\pi}}\, \tau^2$

and $G = \frac{P_o}{4\, h\, \Omega \sqrt{\pi}}$

Comparing v(t) with u(t) above we get the thermal conductivity Ω for a metal strip inside a semi-infinite medium.

$$\Omega = \frac{\alpha\, I_o}{2h} \left(- \frac{a_1^2\, a_3}{a_2^2} \right) \qquad \ldots \qquad (3.33)$$

For a thin metallic film evaporated on the surface of a semi-infinite medium we get for the thermal conductivity

$$\Omega = \frac{\alpha I_o}{h} \left(- \frac{a_1^2\, a_3}{a_2^2} \right) \qquad \ldots \qquad (3.34)$$

For the thermal diffusivity κ in both cases of a strip inside a semi-infinite medium and a strip on the surface of a semi-infinite sample we have

$$\kappa = 4\pi d^2 \ (a_3/a_2)^2 \qquad \ldots \qquad (3.35)$$

3.2 CONDUCTIVITY TENSOR

Solution of the heat conduction equation for an instantaneous point source in an an-isotropic medium, with principle conductivities Ω_1, Ω_2 and Ω_3 along x, y and z axes is given by[3]

$$\frac{Q(\rho C)^{3/2}}{8(\pi \Omega_1 \Omega_2 \Omega_3)^{\frac{1}{2}}(t-t')^{3/2}} \cdot \exp\left\{- \frac{\rho C}{4(t-t')}\left(\frac{(x-x')^2}{\Omega_1} + \frac{(y-y')^2}{\Omega_2} + \frac{(z-z')^2}{\Omega_3}\right)\right\}$$

The temperature increase at time 't' for an infinitely long strip along z and of width 2d along y and lying in the plane x = 0 is

$$\Delta T = \frac{1}{8(\pi^3 \kappa_1 \kappa_2 \kappa_3)^{\frac{1}{2}}} \int_o^t \frac{dt'}{(t-t')^{3/2}} \int_{-d}^{d} dy' \phi(t',y') \exp\left\{- \frac{(y-y')^2}{4\kappa_2(t-t')}\right\} \times \int_{-\infty}^{+\infty} dz' \cdot \exp\left\{\frac{-(z-z')^2}{4\kappa_3(t-t')}\right\} \qquad \ldots \qquad (3.36)$$

where we have put $\kappa_i = \Omega_i/\rho C$. The part gives us $2\sqrt{\kappa_3(t-t')}$ so that

$$\Delta T = \frac{1}{4\pi(\kappa_1\kappa_2)^{\frac{1}{2}}} \int_o^t \frac{dt'}{(t-t')} \int_{-d}^{d} dy' \cdot \phi(t',y') \exp\left\{- \frac{(y-y')^2}{4\kappa_2(t-t')}\right\} \qquad \ldots \qquad (3.37)$$

From Eq. 3.10 we have for $\phi(t',y')$

$$\phi(t',y') = \frac{\kappa P_o}{4dh\Omega} \cdot \frac{v^2(t')}{1+\alpha\cdot\Delta T(t',y')}$$

Substituting this in Eq. (3.37) we get

$$\Delta T = \frac{1}{16\pi dh\rho C\sqrt{\kappa_1\kappa_2}} \int_o^t dt' \frac{v^2(t')}{(t-t')} \cdot \int_{-d}^{d} \frac{dy'}{1+\alpha\cdot\Delta T(y',t')} \cdot \exp\left\{\frac{-(y-y')}{4\kappa_2(t-t')}\right\} \qquad (3.38)$$

Now α and ΔT is small and a first order approximation can be used as done before to get

$$v(\tau_2) = 1 + \frac{P_o\ \alpha}{4h\ \sqrt{\pi\Omega_1\Omega_2}} \cdot f(\tau_2) \qquad (3.39)$$

where $\tau_2 = (t/\theta_2)^{\frac{1}{2}}$ and $\theta_2 = d^2/\kappa_2$. θ is called the characteristic time and $f(\tau)$ is

$$f(\tau) = \tau\ \mathrm{erfc}(1/\tau) - \frac{\tau^2}{2\sqrt{\pi}}\{1-\exp(-1/\tau^2)\} - \frac{1}{2\sqrt{\pi}} E_i(-1/\tau^2)$$

From Eq. (3.38) we see that if the strip is in y-z phase then the conductivities along x and y directions and the thermal diffusivity along y-axis can be determined.

In Figure (3.1) the three axes are designated as 1,2 and

3. Three independent measurements can be made with the three orientations of the strip, position A, position B and position C.

According to the above analysis about the direction dependent conductivities and the position of the strip the following information can be obtained from the three positions A, B and C of the strip.

$$A : \Omega_1 \cdot \Omega_2 = a_A \text{ and } \kappa_1 = b_A \quad \ldots. \quad (3.40a)$$

$$B : \Omega_2 \cdot \Omega_3 = a_B \text{ and } \kappa_2 = b_B \quad \ldots. \quad (3.40b)$$

$$C : \Omega_3 \cdot \Omega_1 = a_C \text{ and } \kappa_3 = b_C \quad \ldots. \quad (3.40c)$$

where we know that $\Omega_i = \kappa_i \rho C$; $i = 1,2,3$.

We can re-write Eqn.(3.40) as

$$\Omega_1 = \left(\frac{a_A \cdot a_C}{a_B}\right)^{\frac{1}{2}} \quad \ldots. \quad (3.41a)$$

$$\Omega_2 = \left(\frac{a_B \cdot a_A}{a_C}\right)^{\frac{1}{2}} \quad \ldots. \quad (3.41b)$$

$$\Omega_3 = \left(\frac{a_C \cdot a_B}{a_A}\right)^{\frac{1}{2}} \quad \ldots. \quad (3.41c)$$

and

$$\rho C = \left(\frac{a_A \; a_C}{a_B}\right)^{\frac{1}{2}} \cdot b_A^{-1} = \left(\frac{a_B \; a_A}{a_C}\right)^{\frac{1}{2}} \cdot b_B^{-1} = \left(\frac{a_C \; a_B}{a_A}\right)^{\frac{1}{2}} \cdot b_C^{-1} \quad \ldots. \quad (3.42)$$

Thus with three strips along three orientations the conductivities along the three principal directions can be found in a single crystal.

3.3 SOME FACTORS AFFECTING THE EXPERIMENT

In the mathematical model developed for the T.H.S. some factors like radiation losses, thickness of heater and end contacts which might affect the experiment were not considered. If the experimental conditions are such that the contribution due to the above factors is small then their effects may safely be neglected.

We now show that the contribution from radiation, thickness of heater and the end contacts is negligible.

(a) Radiation from the strip

The loss of heat per unit area into a black enclosure due to radiation is

$$Q_r = \bar{\sigma}(\varepsilon T_1^4 - \alpha_{12} T_2^4) \quad .... \quad (3.43)$$

where T_1 is temperature of radiating body, T_2 is temperature of the black enclosure, $\bar{\sigma}$ is the Stefan-Boltzman constant, ε is the emissivity co-efficient and α_{12} is the absorption co-efficient.

For a silver film $\varepsilon \approx \alpha_{12} \approx 0.02$ and $T_1 \approx T_2$ so that

$$Q_r \approx 0.08\ \bar{\sigma}\ T_o^3\ \overline{\Delta T} \qquad \ldots. \qquad (3.44)$$

where $\overline{\Delta T}$ is the mean rise in temperature and T_o is initial we can write

$$u = U_o\ (1 + \alpha\ \overline{\Delta T})$$

$$\Rightarrow \qquad \overline{\Delta T} = \frac{u-U_o}{U_o \alpha}$$

Eq.(3.42) gives

$$v(\tau_2) = 1 + \frac{P_o \alpha}{4h\ \sqrt{\pi \Omega_1 \Omega_2}}\ f(\tau_2)$$

where $v(\tau) = \dfrac{u-U_o}{U_o}$

So far $\overline{\Delta T}$ we get for a film in a semi-infinite medium

$$\Delta T(\tau_2) = \frac{P_o}{4\ \sqrt{\pi \Omega_1 \Omega_2}}\ f(\tau_2)$$

and for our case of a film on a sample i.e., a film on the surface of a semi-infinite medium we have

$$\Delta T(\tau_2) = \frac{P_o}{2h\sqrt{\pi \Omega_1 \Omega_2}}\ f(\tau_2) \qquad \ldots. \qquad (3.45)$$

The extensions of the strip are 2h x 2d where 'h' is the half length and 'd' is the half width. So the total power loss from the strip due to radiation is

$$P_r = 0.08\ \bar{\sigma}\ T_o^3\ \overline{\Delta T}\ 2h\cdot 2d$$

$$\Rightarrow \qquad P_r = \frac{0.32\ \bar{\sigma}\ T_o^3\ h\cdot d\cdot P_o}{2h\sqrt{\pi\Omega_1\Omega_2}}\cdot f(\tau_2)$$

The relative power loss is thus

$$P_r/P_o = \frac{0.16\ \bar{\sigma}\ T_o^3\ d}{\sqrt{\pi\Omega_1\Omega_2}}\cdot f(\tau_2) \qquad \ldots\ldots \qquad (3.46)$$

For a typical experiment $\tau = 0.7$

$$\text{Now } f(\tau) \approx \tau - \frac{1}{\sqrt{4\pi}}\tau^2$$

and for $\tau = 0.7$ we get $f(\tau) = 0.56$

Keeping a usually acheivable width of strip $2d = 2 \times 10^{-3}$ m

with $\bar{\sigma} = 5.7 \times 10^{-8}$ watts/m^2K^4

$(\Omega_1\Omega_2)^{\frac{1}{2}} = 1.42$ watts/mK

and $T_o = 300^oK$

We get for the relative loss of power

$$P_f/P_o \approx 10^{-4}$$

So loss of power due to radiation can be neglected.

(b) Thickness of the Heater

The strip has a length 2h, width 2d and thickness 2ν. If the density and specific heat of the strip are ρ_f and C_f then the power needed to heat the strip is

$$P_f = \rho_f \, C_f \, \frac{d(\overline{\Delta T})}{dt} \cdot 2d \cdot 2h \cdot 2\nu \qquad \ldots . \qquad (3.47)$$

$$\Rightarrow \qquad P_f = 8\,\rho_f C_f \, \frac{d(\overline{\Delta T})}{dt} \cdot d \cdot h \cdot \nu$$

The mean rise in temperature is

$$\overline{\Delta T} = \frac{P_o}{2h\sqrt{\pi \Omega_1 \Omega_2}} \cdot f(\tau_2)$$

Initially high power is needed to heat the strip, but after a very short time the power required becomes negligible. So that $f(\tau_2) \approx \tau_2$

Now $\tau_2 = \sqrt{t/\theta_2}$

and $\frac{d}{dt} f(\tau_2) = \frac{1}{\sqrt{4t\theta_2}}$

So the relative power loss P_f/P_o is given as

$$\frac{P_f}{P_o} = \frac{8\,\rho_f C_f \; d \cdot h \cdot \nu}{2h\sqrt{\pi \Omega_1 \Omega_2}} \cdot \frac{1}{2\sqrt{t\theta_2}} \qquad \ldots . \qquad (3.48)$$

Putting $\tau_o \theta_2 = \sqrt{t\theta_2}$, $\kappa_2 = d^2/\theta_2$, and $\Omega_2 = \kappa_2/\rho C$

So for relative power loss we have

$$\frac{P_f}{P_o} = \frac{2\rho_f C_f}{\sqrt{\pi\kappa_1\kappa_2}} \cdot \frac{\kappa_2}{\rho C d\tau_o}$$

$$\frac{P_f}{P_o} = \frac{2}{\sqrt{\pi}} \left(\frac{\rho_f C_f}{\rho C}\right) \sqrt{\kappa_2/\kappa_1} \cdot \frac{\nu}{d\tau_o}$$

With $\nu = 1000\,{}^{o}A$, $d = 0.2$ mm

$$\frac{\rho_f C_f}{\rho C} \approx 1, \; \frac{\kappa_2}{\kappa_1} \approx 1$$

and $\tau_o = 16$ we get the relative power loss to be $\approx 10^{-3}$ which is negligible.

(c) End Contacts

The relative temperature decrease due to end contacts for an isotropic solid is given by

$$R = \frac{1}{2h} \sqrt{\kappa t/\pi} \qquad \ldots. \qquad (3.49)$$

with the usual $\theta = d^2/\kappa$ and $\tau = \sqrt{t/\theta}$

and $\tau_{max} \approx 0.7$ and $t_{max} \approx 0.5$ we get

$$R = \frac{d}{2h} \sqrt{0.5/\pi}$$

If $d \leq \frac{1}{40} h$ then

$$R \leq 5 \times 10^{-3}$$

Thus if width of strip is less than $\frac{1}{40}$th of the strip length then the effect from the end contacts can be neglected.

C H A P T E R   F O U R

ELECTRONIC CIRCUITRY

4.1 Circuit Requirments For The THS Method

4.2 Brief Theory of Circuit Design

4.3 First Model

4.4 Second Model

4.5 Final Model - The Bridge Arrangement

4.1 CIRCUIT REQUIRMENTS FOR THE 'THS' EXPERIMENT

In the T.H.S. method a constant current is allowed to flow through the strip. Due to this input of constant current the temperature of the strip increases and its resistance increases too. This results in a time varying voltage u(t) across the strip. The whole experiment depends upon the accurate monitoring of u(t) against time. We know from Chapter Three that

$$u(t) = U_o + U_o\, C\, f(\tau)$$

where U_o is the voltage across the strip when the current is switched on, C is a constant and $f(\tau)$ function has been defined in Chapter 3. in equation (3.25) .

The trace of U(t) against t is as shown in Figure (4.1).

Usually U_o is of the order of 1 volt and the increase in voltage above U_o is from a few micro-volts to a few millivolts at most. The increase in voltage $\Delta U(t)$ above U_o is the interesting part because that is due to the change in resistivity of the strip.

To detect the increase in voltage $\Delta U(t)$ above U_o

requires amplification of the increase. If the whole signal i.e. U_o included is amplified a lot of information about the change $\Delta U(t)$ above U_o would be lost. It would be best for the reading instruments and for the sake of accuracy to somehow offset the initial voltage U_o.

In short to make a better experiment it is required that:

1. There should be extremely fast switching on of the current through the strip, so that voltage U_o is achieved virtuilly at $t = 0$.

2. There is some arrangement to offset U_o.

3. Sufficient amplification is provided to read the increase above U_o on a convenient setting using a storage oscilloscope.

The different circuitry attempted all revolved around maximization of the above three points. The initial set ups were too complicated, but finally a neat and simple arrangement developed which was highly accurate also.

Before describing the different set ups which were tried it would be appropriate to talk briefly about circuit design.

4.2 BRIEF THEORY OF CIRCUIT DESIGN

(1) Constant Currnt Source

A constant current source is one which supplies a direct current, the magnitude of which is independent of the load into which the current flows. Also where such condition require the current should remain constant with respect to supply voltage and temperature.

Theoretically an ideal constant current source would be one having an infinite supply voltage. However in practise we cannot have an infinite supply of voltage in the first place, secondly the actual supply voltage is limited by sort of usage for which it is required. So it turns out that there is a limit on the load carrying the current, beyond which the constant current source will not supply constant current.

If a constant current source has a maximum supply voltage V_m and the current is I then the maximum load R_L for a constant current is such that

$$R_L < \frac{V_m}{I}$$

The simplest arrangement for a constant current source is to have a voltage source V_o and a high resistance R (as compared to R_L) in series with the load R_L (Fig. 4.2).

If the voltage across load is much smaller than the voltage V_1 across R_1 we would have a constant current source. Slight changes in load would not influence the current I_L in load, which is given by

$$I_L = \frac{V_o - V_L}{R_1} \qquad \ldots\ldots \qquad (4.1)$$

Alternatively

$$I_L = \frac{V_1}{R_1 + R_2}$$

since $\qquad R_1 >> R_L$

$$I_L \approx \frac{V_o}{R_1} \qquad \ldots\ldots \qquad (4.2)$$

The percentage departure from the constant current is $V_L/V_1 \times 100\%$. This means if current is to be held within a few percent and V_L is several volts then V_1 must be a few hundred volts. Such a high value of V_1 might not be convenient in many situations.

If the voltages in a circuit are not to be very big we can rig up a constant current source with the help of the magic genie 'the transistor'. Consider the circuit given in Fig. (4.3)

$$I_E = \frac{V_{BB} - V_{BE}}{R_E} \qquad \ldots\ldots \qquad (4.3)$$

$$I_L = I_C = \text{collector current}$$

$$= \alpha I_E + I_{CBO}$$

where I_{CBO} is the leakage current

$$\Rightarrow \quad I_L = \alpha \frac{V_{BB} - V_{BE}}{R_E} + I_{CBO} \qquad \ldots. \quad (4.4)$$

The parameters α, I_{CBO} and V_{BE} for a particular transistor(at a particular temperature) are constant.

I_L will be independent of load since V_{BB} and R_E are fixed.

A practical circuit used as a constant current includes a zener diode as shown in Fig. (4.4). The zener supplies a constant voltage V_{BB}, which can be fixed to get a current in the load from 0 to 200 mA.

(2) Transistor Switch

For the sake of high speed and bounce free switching a transistor is used. Ideally in the 'ON' condition the transistor should offer zero resistance and in the 'OFF' condition the resistance offered should be infinite.

A circuit for such a transistor switch is shown in Fig. (4.5). The collector to emitter voltage is given by

$$V_{CE} = V_{CC} - I_C R_L \qquad \ldots. \quad (4.5)$$

where V_{CC} is the supply voltage, I_C is collector current and R_L is the load resistance.

Now if I_B is the base current and 'β' is the common-emitter amplification factor, than

$$I_C = \beta I_B \qquad \ldots. \qquad (4.6)$$

If I_B is made zero then I_C is zero and we will get $V_{CE} = V_{CC}$, i.e. the whole voltage is across the transistor and no current is flowing through R_L. (Actually a small current does flow which is the leakage current.) When $V_{CE} = V_{CC}$, we have the 'OFF' condition of the switch.

In the 'ON' condition there should be zero voltage across the transistor which means that $V_{CE} = 0$. Using Eq.(4.5) we get

$$V_{CC} - I_C R_L = 0$$

$$\Rightarrow \quad V_{CC} = I_C R_L$$

So that when the whole voltage is across the load we have the 'ON' condition of the switch. In actuality there is a small voltage drop across the transistor because of reasons like internal resistances of the transistor.

In switching, the 'ON' and 'OFF' time of a transistor are

of great importance. These times would depend upon the characteristics of transistor especially base-emitter and base collector capacitances. Also the external circuit plays an important part.

The design of a switch requires special transistors having very small rise and fall times. In the external circuitry the inductive and capacitive component of the load is minimized. A capacitor across the base resistance R_B greatly helps in increasing the switching speed.

(3) Operational Amplifier

Historically operational amplifiers (op.amp) have been used for mathematical operations such as addition, substraction, function generation etc., therefore called operational amplifiers. This amplifier has a very big gain, very high input and low output impedence etc.

Ideally this amplifier is characterised as having infinite gain, infinite input impedence, infinite band width, zero output impedence and zero voltage and current offset.

Equivalent circuit for an ideal op.amp is given in Fig.(4.6).

Here R_i, $A_{V_o} \longrightarrow \infty$, $R_o \longrightarrow 0$ and band width is infinite. Since $R_i = \infty$, the current I_1 is zero into the terminal, which is defined as the summing point.

Since A_{V_o} is infinite, except zero, for any value of V_1,V_2 will be infinite. Practically V_2 has saturating value V^+ or V^- limited by supply voltages. The configuration in which no feedback is applied is called 'open loop' configuration. A very stable operation of the amplifier can be obtained by the introduction of feedback. The gain, then becomes almost independent of the gain of the amplifier, and depends only on the external circuit components.

The circuit with feedback loop is shown in Fig. (4.7). The circuit in which feedback is applied is called 'closed loop' circuit. Since $R_{in} = \infty$; $I_3 = 0$, also $V_i = 0$ since $V_2 = A_{V_o} V_i$ and for V_2 within limits $V_i \simeq 0$ (A_{V_o} tends to infinity). So point A is at virtual ground and we can write

$$I_1 = \frac{V_1 - V_i}{R_1} \simeq \frac{V_1}{R_1} \qquad \ldots\ldots \qquad (4.9)$$

$$I_2 = \frac{V_2 - V_i}{R_F} \simeq \frac{V_2}{R_F} \qquad \ldots\ldots \qquad (4.10)$$

Also

$$I_1 + I_2 = I_3 \simeq 0$$

or

$$I_1 = -I_2$$

So

$$\frac{V_1}{R_1} = - \frac{V_2}{R_2}$$

or

$$\frac{V_2}{V_1} = - \frac{R_F}{R_1} = A_V \qquad \ldots. \qquad (4.11)$$

A_V is gain of the circuit independent of A_{V_o}. In fact A_{V_o} need not be infinite in the real sense of the word, but it is only necessary that A_{V_o} be large for a reasonably good approximation.

Since point A appears to be at ground potential so the load for the input source is only R_1. Therefore, R_1 isthe input resistance of the amplifier. So both the input resistance and gain can be set with R_1 and R_F only. This configuration is called the inverting amplifier, since the output is always out of phase with input.

In Fig. (4.8), a non-inverting amplifier is shown. R_F is feedback resistance and input is applied at the non-inverting input of the amplifier. Since $V_i \approx 0$; V_1 can be considered as a voltage across R_1, so we can write

$$V_2 = I_1 R_F + I_2 R_1$$

$$= I_1 R_F + V_1$$

$$= \frac{V_1}{R_1} R_F + V_1 \qquad \ldots. \qquad (4.12)$$

or $$\frac{V_2}{V_1} = 1 + \frac{R_F}{R_1} = A_V \ , \qquad \ldots. \qquad (4.13)$$

A_V is gain of this amplifier and here input impedance is the input resistance (R_{in}) of the amplifier, which is infinite ideally.

A useful version of a non-inverting amplifier is 'voltage follower' which can be considered as a special case of a non-inverting amplifier by letting $R_F = 0$. The voltage gain is one and input impedance is infinite. This can be used as a buffer or decoupling stage between load and driving source. The circuit is shown in Fig. (4.9).

(4) Differential Amplifier

A differential amplifier is also called a substractor or difference amplifier and is shown in Fig. (4.10). Here we can easily apply superposition theorem to analyze the circuit. So first shortening V_2, we get inverting amplifier, giving

$$V_{01} = - \frac{R_F}{R_1} V_1 \qquad \ldots. \qquad (4.14)$$

Now shortening V_1 a non-inverting amplifier, but with R_2 at its non-inverting input, is formed. Voltage V_2 is divided by R_1 and R_2 and produced voltage V_2' at the non-inverting input. So we get

$$V_2' = \frac{R_2}{R_1 + R_2} V_2 \qquad \ldots\ldots \qquad (4.15)$$

and

$$V_{02} = \left(1 + \frac{R_F}{R_1}\right) V_2'$$

$$= \left(1 + \frac{R_F}{R_1}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_2 , \qquad \ldots\ldots \qquad (4.16)$$

if we let $R_2 = R_F$, then we get

$$V_{02} = \frac{R_F}{R_1} V_2 . \qquad \ldots\ldots \qquad (4.17)$$

Therefore the total output is given by

$$V_0 = V_{01} + V_{02}$$

$$= - \frac{R_F}{R_1} V_1 + \frac{R_F}{R_1} V_2$$

$$= \frac{R_F}{R_1} (V_2 - V_1) \qquad \ldots\ldots \qquad (4.18)$$

So the output voltage is proportional to the difference of the voltages at the inputs.

(5) Actual Operational Amplifier

Although very good approximations can be made, yet following departures from the ideal can be realized.

Obviously in actual amplifiers, infinite gain, infinite

input impedance, zero output resistance and infinite bandwidth are impossible to achieve. Typically the open loop gain is of the order of 10^5, the input impedance in the range of several M.ohm output impedance around 50 ohms and the bandwidth a few M. Hertz. Alongwith these the following differences are also observed.

(a) Offset: When both the inputs of an op.amp are at ground, ideally there should be zero output, see Fig. (4.11). But in a practical op.amp an undesired non-zero output appears. V_{00} with input grounded is called 'output offset voltage'. This voltage can be related to the input voltage (in built) as $V_{00}/Av_o = V_{i0}$. V_{i0} is called the 'input offset voltage'. 'Input offset currents' are also present in a practical op.amp. Consider Fig. (4.12), the current flowing in or out of the input terminals. The currents I_{B_1} and I_{B_2} are usually not equal, and the difference $(I_{B_1} - I_{B_2}) = I_{i0}$ is called the 'input offset current'. V_0 in this case is due to the input offset voltage and not due to I_{i0}. To see the effect of I_{i0} consider the circuit in Fig.(4.13). Current I_{B_1} develops a voltage $I_{B_1} R_1$ accross resistance R_1. This voltage appears as an input voltage, the amplifier yielding output voltage.

Now V_0 is not only due to V_{i0} but is also due to I_{i0}. The effect due to I_{B_1} can be nullified by inserting a resistance in the other input terminal, see Fig. (4.14).One thing must be noted here, that even if $R_1 = R_2$, V_0 will not be equal to zero if $I_{B1} \neq I_{B2}$.

A pair of terminals is provided in most of the amplifiers for offset adjustment. A potentiometer is usually connected accross these terminals and the adjusting arm connected to V_{CC} or V_{EE}.

(b) Drift: The input offset current and voltage drifts with temperature and to some extent with time. Thermal drift is usually the most significant. Offset voltage and current drifts are specified in units of $\mu V/^{o}C$ and $\mu A/^{o}C$ respectively.

(c) Common Mode Error: Ideally one expects that the output voltage is gain times the difference of the voltages at the inputs, i.e.

$$V_0 = A_\alpha(V_1 - V_2)$$

where V_1 and V_2 are the voltages at the non-inverting and inverting points. But in practical amplifiers output also has some dependence on the average value of the two inputs. For practical op.amp we have

$$V_0 = A_\alpha(V_1 - V_0) + A_{cm}\left(\frac{V_1 + V_2}{2}\right) \quad \ldots\ldots \quad (4.19)$$

where A_{cm} is called the common mode gain. A term 'common mode rejection ratio' (CMRR) is defined as

$$CMRR = A_\alpha / A_{cm} . \quad \ldots\ldots \quad (4.20)$$

Eq.(4.19) can be written as

$$V_0 = A_\alpha(V_1 - V_2) + \frac{A_\alpha}{CMRR}\frac{(V_1 + V_2)}{2}, \quad \ldots. \ (4.21)$$

CMR ratios of 20,000:1 to 200,000:1 are typical in practical amplifiers.

(d) Slew Rate: Slew rate is a measure of how fast the output voltage can be varied with respect to time, and hence it is generally specified as so many volts per μ sec, as measured in a particular feedback configuration with some particular load.

If an op.amp is operated open loop, and the input signal is sinusoidal (amplitude is small enough not to saturate the amplifier), the output will also be sinusoidal of the same frequency.

If this amplifier is used in closed loop configuration and the magnitude of the input is increased (still not to saturate the amplifier in this configuration) the output will be distorted. This is due to the various internal and external capacitive loads. The output voltage cannot instantaneously follow the input due to charging time of the capacitor. So the high frequency response of an op.amp is different for small signal and large signal.

For a large step signol input the output rises at

a fixed rate, see Fig. (4.15). The rate limit is the slew rate and determines the speed with which the amplifier can respond to large signals.

The maximum frequency of operation (with sinusoidal signals) of the op.amp is limited by the slew rate. If output is given by

$$V = V_m \sin(\omega t)$$

time rate of change of voltage is given by

$$\frac{dV}{dt} = V_m \, \omega \cos(\omega t)$$

and the maximum value of this rate of change is

$$\left.\frac{dV}{dt}\right|_{max} = V_m \, \omega_{max}$$

When $\frac{dV}{dt}$ is slew rate of an op.amp the maximum operating frequency due to slew rate limit is

$$\omega_{max} = \frac{1}{V_m} \frac{dV}{dt} \quad \text{....} \quad (4.22)$$

It can be seen from Eq.(4.22) that the maximum operating frequency decreases with increasing amplitudes.

(e) Non-Linearities: The input, output and gain characteristics all show some non-linearity in the region of operation. These non-linearities are negligible for most purposes, but would have to be taken into account when do ing sensitive measurements.

There are limits to the current and voltage when using an op.amp. because there is both input and output saturation. The input must not exceed a certain peak to peak value and so must the output be restricted too. These saturation limits are imposed by the maximum rated power supply voltage which drives the op.amp .

In this saturation bussiness the load impedence plays an important role. If the load impedence is below a certain value (which is specified) then the output current saturates before the voltage saturation occurs. Hence the load impedence of an op. amp. should be greater than that specified for a particular op. amp.

Now that we have described thedifferent aspect of circuit design which would be useful in the experiment on the 'transient hot strip' we go to describe the different models attempted. These attempts finally ended in a 'bridge circuit' which we beleive did the best job.

4.3 FIRST MODEL

Circuit diagram of the first attempted model is shown in Figure (4.16).

R_L is the current limiting resistance and S. R. is a standard resistance of 1 ohm in series with the THS. The other and of THS being grounded.

Voltage across the S.R., which would give us the current through the THS, is amplified by a differential amplifier DA1 and fed to channel A of the oscilloscope.

The potential difference U(t) across the THS is amplified by a variable gain amplifier A1. The gain is adjusted so that at point 1 the amplified U_o is equal to the zener voltage. e.g. if zener voltage is $10U_o$ the gain of A1 is made ten. The signal at point 1 is thus fed to a fixed gain amplifier with gain 10, from where it goes to channel B of the oscilloscope.

The disadvantages associated with this model were:

1. If current through THS was made to change then the gain of A1 had to be changed also to bring its output equal to the zener voltage. This practically had to be done every time a run was made.

2. Also since each run required an adjustment of A1 its gain could not be find. To find this gain each time an external signal had to be applied to A1 and its gain studied on an oscilloscope.

3. Because the gain of A1 had to be changed as required each time its off setting had to be adjusted accordingly.

However, inspite of the disadvantages the attempted circuit gave us the proper insight into the requirements of the experiment and led to better changes later on.

4.4 SECOND MODEL

The second attempted model is based on the four probe method. First a block working is explained and thus follows a detailed description.

a) BLOCK DIAGRAM

Block diagram of the circuit is given in Fig. (4.17). A constant current is passed through the strip. Standard resistance (S.R.) is also in series with the strip to measure the current. The constant current develops voltage across SR and strip which are proportional to their resistances. The voltage across the SR remains constant throughout the experiment, due to its constant resistance, but the voltage across the strip is a function of time. The resistance of strip being a function of the temperature.

The voltage across the inner ends of strip is applied to the inputs of a differential amplifier, DA2 with gain 1. DA2 amplifies only the difference of the voltages at its inputs, so only the voltage across the inner ends of the strip will be amplified irrespective of any voltage present at the strip w.r.t. ground.

The output of DA2 is applied to the inverting input of a reference or difference amplifier DA3. A reference voltage is applied at its non-inverting input. Output of DA3 will be the reference voltage minus input voltage at the inverting terminal of DA3. The reference voltage is set equal to the initial voltage U_o developed across the strip. So at the output of DA3 we are left with only the voltage variation above U_o across the strip but inverted.

This voltage variation is again given to the input of an inverting amplifier A1 with fixed gain 10. Output of A1 is amplified and inverted so we get the change above U_o with original polarity. This voltage is

given to the storage oscilloscope (channel A) where it can be photographed for a permanent record.

Initial voltage step is measured by measuring the reference voltage at DA3. A voltage follower V.F. stage is used between the digital voltmeter and DA3 stage to decouple these from each other.

A differential amplifier DA1 is used to measure the voltage across S.R. which actually gives the current through THS. The gain of this amplifier is one. The output of this amplifier is applied to channel B of the storage oscilloscope, from which it can be recorded at same time as the $\Delta U(t)$.

b) DETAILED DESCRIPTION OF SECOND MODEL

Block diagram of this model is given in Fig. (4.17). In Fig. (4. 18) the detailed diagram is shown. Transistor T_{r1} is used as a constant current source. The transistor switch T_{r2}, standard resistance (S.R.) and the strip are the load for this constant current source (being in the collector circuit). The necessary constant base voltage supply V_{BB} is made available using a zener diode DZ_1. This voltage can be fixed at any value from zero to V_Z by adjusting a potentiometer, across D_{Z1}. V_Z is the zener voltage of the diode.

Transistor T_{r2} is used as a switch, drivem by a single shot multivibrator. A buffer stage using T_{r3} as an emitter follower is used between switch and single shot. The "ON" duration of single shot is determined by the R_1, C_1. R_1 is a variable resistance, and by adjusting it ON duration can be fixed for any value between 0 and 200 milli-seconds.

Operation amplifier Op_1 is half 747 and is used as a differential amplifier, with gain one. A variable resistance is connected in the feedback loop which is used to caliberate the gain of amplifier. A 10 K potentiometer is used for offset adjustments. This amplifier is across S.R. to measure the current. The Output of it is connected to channel A of the storage oscilloscope.

Op-amp-2 (½747) is also used as a differential amplifier across strip. The circuit configuration of this amplifier is same as that of Op-amp. 1.

Op-amp-3 is also difference amplifier. It is used to cut the initial step of voltage across the strip. Its inverting input is connected to the output of Op-amp-2. The non-inverting terminal is kept at a voltage, which is approximately equal to the initial step U_o. Transistor T_{r3} is used to provide this voltage. T_{r3} provides a constant voltage determined by zener diode D_{Z2}. This voltage can be fixed at any value from 0 to V_Z = 5V with the aid of a potentiometer in the emitter leg of T_{r3}.

Op-amp-4 (½747) is a simple inverting amplifier with gain 10. The gain of this amplifier can be calibrated by a variable resistance in its feed back circuit. The input of this amplifier is output of Op-amp-3, which is only the voltage rise $\Delta U(t)$ across the strip. The output of Op-amp-4 is connected to channel B of storage oscilloscope.

Op-amp-5 is a voltage follower stage which is used as a buffer stage between digital volt-meter (DVM) and the rest of the circuit. Digital voltmeter measures the voltage present at the output of Op-amp-3, which is the reference voltage (approximately equal to U_o) at its non-inverting input.

The traces of $\Delta U(t)$ recorded on the storage oscilloscope were as expected but accuracy was lacking. Reproduction of results was very difficult the reasons being basically the complex nature of the circuit and also lack of precise value of the U_o leading to bad precision in setting of the references voltage. A slight mismatch between U_o and the reference voltage made the image on oscilloscope go out of the viewing window.

The reasons for the models short coming and for the search of a better one were:

1. Each time gain calibration of the amplifiers had to be done.

2. Each time the offset of every operational amplifier had to be adjusted.

3. The current and reference voltages had to be adjusted so as to make them compatible.

Clearly the biggest problem was to select a certain current to get a reasonable U_o, and adjust the reference voltage so that U_o could be offset. The best choice would be to have an arrangement whereby U_o could be balanced out easily. This pointed the way towards the use of a Bridge Circuit on lines of the famous Risistance Bridge.

4.5 FINAL MODEL — THE BRIDGE ARRANGEMENT

i) THE BRIDGE:

It is a well known circuit having four arms as shown in Fig. (4.19). When the bridge is balanced the voltages at C and D are equal. The balance condition in terms of arm resistances is given by

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad R_1R_4 = R_2R_3 \qquad \text{....} \quad (4.23)$$

The second class of bridge is an unbalanced bridge. When the bridge is imbalanced, a little voltage difference arises between C and D, which is a function of the imbalance.

From Fig. (4. 20) we have

$$V = I_1R_1 - I_2R_2$$

$$= I_1R_1 - U(t) \qquad \text{....} \quad (4.24)$$

$$I_1 = \frac{I(R_2 + R_1)}{2R_1+R_2+R_3}$$

$$= \frac{(I_1+I_2)(R_2+R_3)}{2R_1+R_2+R_3}$$

or

$$I_1(1 - \frac{R_2 + R_3}{2R_1+R_2+R_3}) = I_2 \frac{(R_2 + R_3)}{2R_1+R_2+R_3}$$

or

$$I_1(\frac{2R_1}{2R_1+R_2+R_3}) = I_2 \frac{(R_2 + R_3)}{2R_1+R_2+R_3}$$

or

$$I_1 = I_2 \frac{(R_2 + R_3)}{2R_1} \qquad \text{....} \quad (4.25)$$

From Equations (4.24) and (4.25) we get

$$V = I_2 \frac{(R_2 + R_3)}{2R_1} - R_1 - U(t)$$

$$= \frac{I_2R_2}{2} + \frac{I_2R_3}{2} - U(t)$$

Now I_2R_2 is U_o, the initial voltage across the strip and I_2R_3 is $U(t)$, so

$$V(t) = \frac{U_o}{2} + \frac{U(t)}{2} - U(t)$$

or

$$2V(t) = U_o - U(t) \quad \ldots\ldots \quad (4.26)$$

ii) DETAILED CIRCUIT OF BRIDGE ARRANGEMENT:

The circuit is shown in Fig. (4.21). This is a method which uses three probes across the THS for current and voltage isolated measurement.

Initially the bridge is balanced by passing a very little current through the circuit. Resistances R_1 and R_2 are equal. R_3 is varied to balance the bridge. Section 'a' of the strip see Fig. (4.21 b) is in the R_3 arm of the bridge. Section 'a' being in the lower arm balances the section 'b' of the strip (it being assumed that the strip is symmetric). The effective length of the strip then becomes h. Standard resistance (S.R.) is in the strip arm but it does not contribute to the trace, since it remains constant for current values lower then the current specified on standard resistance.

The initial voltage U_o is automatically balanced out when the bridge gets balanced using R_a. The heating current increases the temperature of the strip and so imbalances the bridge. This imbalance and hence the voltage at the input of differential amplifier DA1 (which is a Tektronics AM502) is proportional to the voltage increase across the strip, Eqn.(4.26). This voltage is amplified 200 times by DA1. The output of the amplifier is connected to a storage oscilloscope, an HP 1741A.

A potentiometer acting as an offset alongwith a differential amplifier DA2 (same made as DA1) is used to measure the current. The DA2 amplifies only the difference of the voltage across SR and the potentiometer. The potentiometer voltage is set approximately equal to the voltage across S.R. Votlage of the potentiometer is measured by a digital voltmeter.an HP instrument No. 3466A. Difference voltage, after amplification is given to storage oscilloscope. Detailed discription of switch 'S' is given below.

Figure 4.[illegible] : Circuit for Electronic Switch

iii) ELECTRONIC SWITCH

Normally it is good idea to keep the battery loaded, which means that current is always flowing through the limiting resistor. Only when an experimental run is made the current is switched on to the bridge. So normally two points B and C are closed and the two contacts B and A close only when required (at this time B and C are open).

To perform such an operation we use two single pole single throw reed relays RL_1 and RL_2 are used because they have low inertia, can be driven by low current and low voltages. Reed relays are also bounce free. Transistors are used to switch ON and switch OFF the relays.

Referring to Fig. (4. 22). Initially T_{r1} is "ON" because it has positive voltage at its base. At the same time T_{r2} is OFF since its base is grounded through resistance R_1. This leads to the RL_1 to be closed, and so contacts B and C to be closed. This would be the normal position.

When we want contacts A and B to close the manual switch 'S_1' is closed giving a positive voltage to the base of T_{r2} making it ON. This actuates RL_2 closing contacts A and B. At the same time T_{r1} goes to 'OFF' state, opening RL_1 and so the contacts B and C also.

The switch is extremely fast with switching time of about 10 micro-seconds and so suitable for the milli-second range of work.

CHAPTER FIVE

EXPERIMENTAL ARRANGEMENT, DATA AND RESULTS

5.1 Substrate and The Deposited Film

5.2 Temperature Range

5.3 Electrical Arrangement

5.4 Measurement of Temperature Dependent Co-effecient of Resistivity (TCR)

5.5 Analysis of Photographs

5.6 Tables

5.7 Errors

EXPERIMENTAL REQUIRMENTS

In the Transient Hot Strip (THS) method a thin film of metal is deposited on a substrate, and a constant current is passed through the film. Essentially what is done is to read the voltage across the film as a function of time. The things to get in the experiment are, one the Temperature Coefficient of Resistance (TCR), 'α' and the other is to get a visual trace giving the voltage time relation.

5.1 SUBSTRATE AND THE DEPOSITED FILM:

The substrate sample selected was fused quartz chosen because of its well known thermal properties. The sample was a rectangular parallelpiped of extinsions 35x15x10 mm^3. The sample was optically polished on two opposite sides. The known thermal propcrties of fused quartz being

$$\text{density} = \rho = 2.25\ gm/cm^3$$

$$\text{Thermal Diffusivity} = \kappa = 8.333\times10^{-3}\ cm^2/sec$$

$$\text{Thermal Conductivity} = \Omega = 1.4\times10^{-2}\ watts/cm\text{-}^{\circ}C$$

The metal film was deposited by evaporation under vaccum.

The metal used for the hot strip was copper. The strip was 0.45mm wide and 27.3mm long. The pads for current and voltage along with the film were made in one deposition. The thickness of the film was about 700 Å.

The substrate and the film size are shown in Fig.(5.1)

5.2 TEMPERATURE RANGE:

The measurements were made over a temperature range of approximately $40^{o}C$, from room temperature of $20^{o}C$ to $60^{o}C$.

The sample alongwith the probe leads is suspended in a Dewar flask. The sample is surrounded by a thick mild steel sylinder, which acts as a heat reservoir, next to which is the wall of the Dewar. Near the sample is placed a thermometer, reading to a minimum interval of $0.2^{o}C$. The heating was done by using a hot air blower and then waiting a sufficient time for the temperature to stabalize.

5.3 ELECTRICAL ARRANGEMENT:

A short explaination is given below. For detailed review refer to section dealing with the Bridge Arrangement in Chapter Four.

The electrical set up is shown in Fig. (4.21). Voltage source is a heavy duty battery of 12V with a series current limiting resistance $R_s \approx 80$ ohms. This voltage is given across the bridge circuit. As explained in Chapter-4 the voltage across S.R.(current through the strip) and voltage trace are simultaneously displayed on the oscilloscope. These traces on the oscilloscope are photographed for further analysis. A sample of trace is shown in Fig. (5. 2).

The current used for THS was about 80mA.

5.4 MEASUREMENT OF TEMPERATURE DEPENDENT COEFFICIENT OF RESISTIVITY (TCR).

TCR 'α' was measured in two different ways, one was to do a completely separate experiment for determination of α, and second was to combine the determination of α with experiment to determine the thermal properties of fused quartz.

i) EXPERIMENT TO DETERMINE 'α' ALONE:

The sample in the Dewar flask is heated using a hot blower for some time. Then blowing is stopped and a wait ensues till the temperature stabalizes.

At this stage a number of steps given below are undertaken to determine α.

DETERMINATION OF TCR α

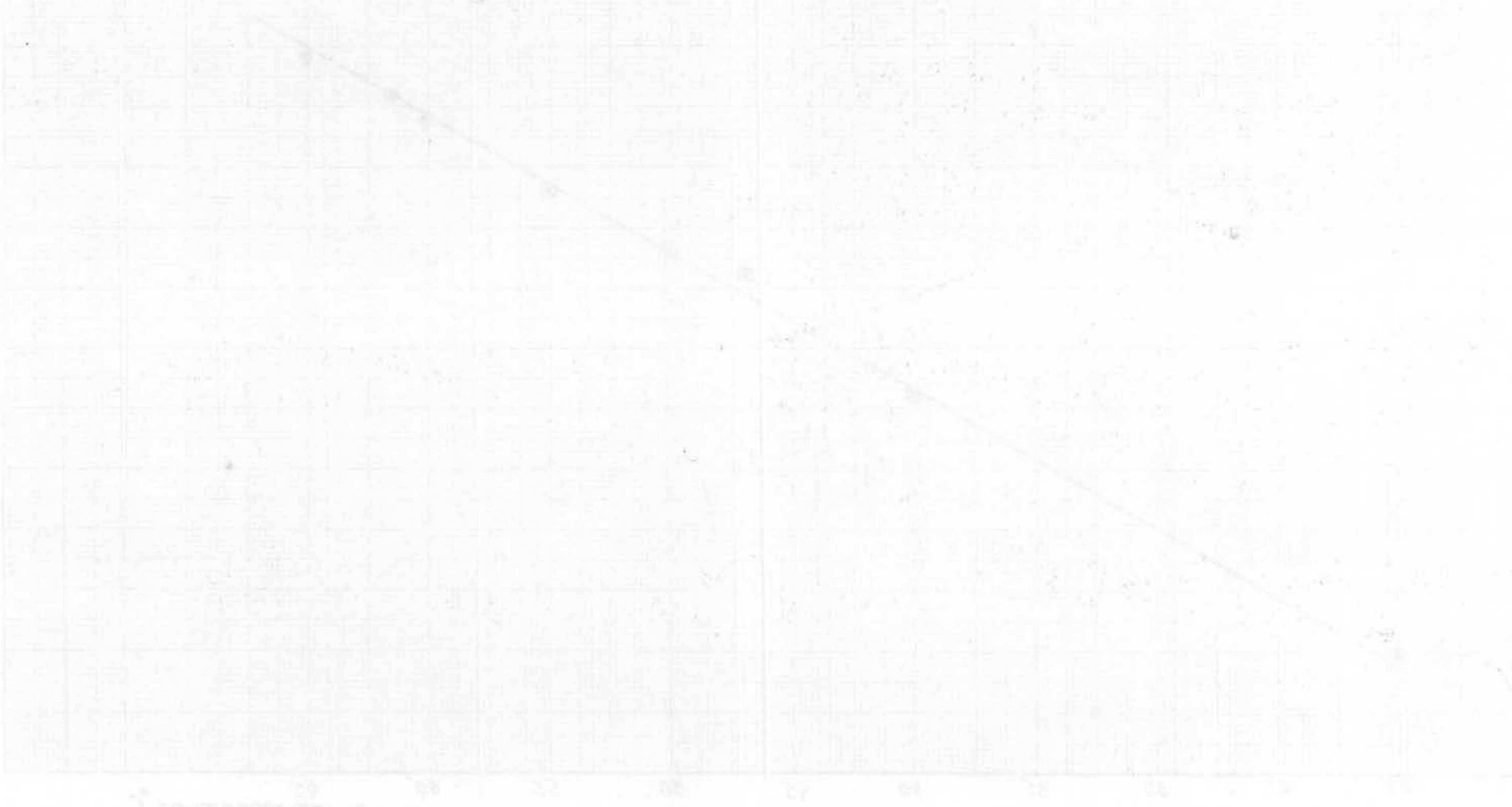

Figure 5.2

1. Note the temperature.

2. Balance the Bridge with adjustable resistance R_a.

3. Then resistance of THS at that temperature is $R = (R_a - 1)$ ohms.
   Repeat step, 1,2,3, for different temperature.

4. Plot R(T) against T and from the slope we get 'α'. Such a plot for our experiment is given in Fig. (5. 3).

ii) SECOND WAY FOR DETERMINATION OF α.

The same procedure as in the first method was followed except that at the same time the traces of $\Delta U(t)$ against t were taken on the storage Oscilloscope at different temperatures. So that making a plot of $R(T) = R_a(T) - 1$ against T would give us 'α' ; and also we would be getting the thermal properties from analysis of each photograph taken at some temperature.

5.5 ANALYSIS OF PHOTOGRAPHS:

For all traces taken on the storage oscilloscope, the time base of the oscilloscope was 5 m. sec/div. and the vertical voltage scale was 5 v/div.

The voltage before being fed to the oscilloscope for the voltage versus time trace had been amplified 200 times, i.e. the trace on oscilloscope (and the photograph) was 200 times the original voltage developed across the THS.

The photograph of the trace was magnified about 2 times (Actually the magnification of the original photograph does not change the data, but it only help in making reading more easy).

The magnified photograph is placed under a travelling microscope, its horizontal movement is related to time t and the vertical to the voltage. Where the voltage scale is read as (5/200) volts/div.

The voltage at different data pertaining to

$$\Delta U(t) = U(t) - U_{ref.}$$

$$U_{ref.} = (\text{current through THS}) \times R_a$$

where current through the THS is obtained by noting the voltage across the SR.

At this stage we have the data $U(t_i)$ for different t_i.

From the theory of THS we know that

$$U(t) = a_1 + a_2\sqrt{t} + a_3 t$$

We utilize services of the faithful friend 'the desk top computer' in the Department of Physics to fit this equation $U(t) = a_1 + a_2\sqrt{t} + a_3 t$ to our data points $(U(t_i), t_i)$ and give us the coefficient a_1, a_2, and a_3 for the best fit.

Knowing a_1, a_2, a_3, half width of strip d, half length of strip h, TCR 'α' and the current through the strip I_o, we can find thermal diffusivity, the thermal conductivity.

Thermal diffusivity of sample = $\kappa = 4\pi d^2 \left(\frac{a_3}{a^2} \right)^2$

Thermal conductivity of sample = $\Omega = \frac{I_o}{h}\left(- \frac{a_1^2 \, a_3}{a_2^2} \right)$

knowing κ we get characteristic time $= \theta_c = \frac{d^2}{\kappa}$

These quantities determined for the sample are given in TABLE 5

5.6 DATA AND RESULTS

Experimental data and subsequent results obtained are given in form of TABLES from page 100 to page 104.

TABLE -1

Time (Sec.)	Experimental $U(t)-U_{ref.}$ (Volts)	$U-U_{ref.}$ After fit (Volts)	Difference
0.005	0.0064	0.0064246	-2.00644E-05
0.01	0.009802	0.00976843	3.35698E-05
0.015	0.01211	0.012114	-4.69297E-06
0.02	0.01397	0.0139443	2.57004E-05
0.025	0.015428	0.0154437	-1.57345E-05
0.03	0.01664	0.0167086	-4.46382E-05
0.035	0.017792	0.0177958	-3.82560E-06
0.04	0.018772	0.0187423	2.96853E-05

I_o = 72.29 mA $U_{ref.}$ = 5.50V

TABLE - 2

Time (Sec.)	Experimental $U(t)-U_{ref.}$ (Volts)	$U-U_{ref.}$ After fit (Volts)	Difference
0.005	0.004064	0.004082	-1.86385E-05
0.01	0.006624	0.006584	3.97336E-05
0.015	0.008326	0.008344	-1.82873E-05
0.02	0.009718	0.009720	-2.42408E-06
0.025	0.010880	0.018512	2.87622E-05
0.03	0.011784	0.011807	-2.37909E-05
0.035	0.012608	0.012632	-2.42979E-05
0.04	0.01335	0.013352	-2.26374E-06
0.045	0.014008	0.013986	2.12066E-05

$I_o = 65.57$ mA $\quad U_{ref.} = 4.983$V

TABLE - 3

Time (Sec.)	Experimental $U(t)-U_{ref.}$ (Volts)	$U-U_{ref.}$ After fit (Volts)	Difference
0.005	0.0059140	0.0059279	-1.39114E-05
0.01	0.0084900	0.0084614	2.85748E-05
0.015	0.0102380	0.0102357	2.23187E-06
0.02	0.0116060	0.0116171	-1.11354E-05
0.25	0.0127360	0.0127473	-1.13694E-05
0.03	0.0136880	0.0136992	-1.12149E-05
0.035	0.0145220	0.0145158	6.11115E-06
0.04	0.0152460	0.0152255	2.04552E-05
0.045	0.0158380	0.0158477	-9.74118E-06

I_o = 65.55mA $U_{ref.}$ = 4.982V.

TABLE - 4

Time (Sec.)	Experimental $U(t)-U_{ref.}$ (Volts)	$U-U_{ref.}$ After fit (Volts)	Difference
0.005	0.0029740	0.0029803	-6.34690E-06
0.01	0.0040880	0.0040809	7.09917E-06
0.015	0.0048620	0.0048544	7.59121E-06
0.02	0.0054580	0.0054586	-6.25010E-07
0.025	0.0059560	0.0059546	1.34474E-06
0.03	0.0063580	0.0063738	-1.58369E-05
0.035	0.0067340	0.0067347	-7.86768E-07
0.04	0.0070520	0.0070496	2.36568E-06
0.045	0.0073320	0.0073268	5.19513E-06

I_o = 49.32 mA $U_{ref.}$ = 3.748V

TABLE - 5

	$U_{ref.}$	I_o (mA)	U_o(V)	θ(Sec.)	$\Omega = \frac{\alpha}{h} \frac{I_o}{U_5}$ (W/mk)	$\kappa = d^2/\theta$
1.	5.50	72.29	5.497	0.0477	1.35	0.992
2.	4.983	65.57	4.980	0.0478	1.32	0.989
3	4.982	65.55	4.981	0.501	1.34	0.944
4.	3.748	49.32	3.747	0.0492	1.29	0.962
				$\bar{\theta} = 0.0487$	$\bar{\Omega} = 1.33$	

$\alpha = 0.96 \times 10^{-3}$ Ohm/degree, $2h = 0.01365$m $2d = 0.435$ mm.

$U_5 = \frac{a_2^2}{U_o^2 a_3}$, $\theta_c = (\frac{a_2}{a_3})^2/4$, are computed with Computer Program.

5.7 ERRORS

While the errors dependent upon the design of the experiment have been dealt in Section 3.3 of Chapter (3) an estimate of the errors in the results due to errors in measurements and observation is required.

The different measurements donefor the determination of the TCR and the thermal properties and the errors in them are taken up turn by turn.

i) Voltage across standard resistor S.R of 1 ohm.

The error in measuring this voltage is 0.17%

The voltage across S.R gives us the current I_o through the hot strip

Error in I_o is 0.17 %

ii) The adjustable resistance R_a in the Bridge (Section 4.5 , Chapter 4).

The error in R_a is 0.05 %

Now $U_{ref} = I_o R_a$

Using the standard error analysis methods we get

Error in U_{ref} is 0.22 %

iii) In the determination of α we read R_a with changing temperature T .

The error in R_a is 0.05 %

The error in reading T is 0.1 %

Now $$\alpha = \frac{R - R_o}{R_o (T_1 - T_2)}$$

where $T_2 > T_1$

$\Longrightarrow$ Error in α is 0.15 %

iv) In the estimation of the thermal diffusivity and thermal conductivity we need information about the width ' 2d ' and the length '2h ' of the hot strip.

The error in h is 0.04 %

The error in d is 1 %

Now thermal Diffusivity $\kappa = \frac{d^2}{\theta}$

where ' θ ' has been evaluated by fitting the data about u(t) to

$$u(t) = a_1 + a_2\sqrt{t} + a_3 t$$

with the help of the computer.(This gives us the co-effecients of t).

$\Longrightarrow$ Error in κ is 2 %

Now thermal conductivity is given by

$$\Omega = \frac{\alpha I_o}{h U_5}$$

where $U_5 = \dfrac{a_2}{U_o a_3}$

and $U_o = I_o \times$ Resistance of the THS at a given temperature.

Knowing the errors in α , h and U_o we get

Error in Ω is 0.6 %

To sum up the error in the estimation of the thermal properties is not greater than 2 % .