Radiative Leptonic B Decays Beyond the Standard Model

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TO MY LOVING AMMEE *AND* ABBU

Certificate

It is certified that the work contained in this dissertation is carried out and completed under my supervision at National Centre for Physics, Quaid-i-Azam University Campus, Islamabad, Pakistan.

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Ishtiaq Ahmed

Abstract

Among the various rare B-meson decays, the semileptonic $B_s \to \gamma l^+ l^-$ ($l = e$, μ, τ) decays are specially intersting due to their relative cleanliness and sensitivity to new physics. This channel also provides us with a very large number of possible observables, such as the Forward Backword (FB) asymmetry, etc. Of special interest is the zero which the FB asymmetry has in this decay mode. In this work we have studied this zero in the most general model independent framework.

Contents

List of Figures

Chapter 1

Introduction

High-energy physics is the search for elementary particles and basic laws of nature. What are the smallest building blocks out of which protons, neutrons, atoms and all matter are made? Do such elementary particles exist? And if so, what are they? This search to unveil the elementary constituents of matter, along with the forces that link them, involves distances thousands of times smaller than nuclear sizes, about one ten trillionth of a centimeter, or 10^{-13}cm. Accelerators must have very large energies to probe nature at such smaller distances. The ultimate goal of this quest is to find out the underlying first principles that govern our entire physical universe.

In recent years, we have realized a strong and growing synergism between the physics of short distances and large- scale structure of the universe. This development reflects the unity of science as explored on both the high-energy and particle astrophysics frontiers. With this connection, we are now addressing some of the most basic questions one can ask: How did our physical universe begin? How did it evolve to its present state? What will be its final fate?

Physicists currently believe that the four interactions (gravitational, weak, electromagnetic and strong) are different expressions of a single force, the same force that was at work when the universe first came into being. At the time of the Big Bang, the particle world could be described with perfectly symmetrical laws. When the universe cooled down, it is believed that a number of rifts occurred in this symmetry and the four forces became differentiated. Since their effects on particles became very different at short and long distances, their masses and messengers also became very different.

Particle physicists are working on the development of a single theoretical framework to describe this major unification. The standard model has already unified weak and electromagnetic forces by a single electroweak force. The messengers of the weak force ($W_{+}, W_{-}$and Z_{o}) have acquired a mass by interacting with the Higgs field whilst the photon has acquired none.

Due to complex theoretical reasons, most physicists agree that the unification of forces involves a new basic symmetry which has never before been observed: super-symmetry, which connects fermions and bosons. This type of theory predicts the existence of numerous new particles, which are super-symmetrical partners of standard particles. The LHC would be the ideal tool for discovering these new particles and understanding their interactions.

Despite the spectacular success of the Standard Model, it is generally accepted that the Standard Model can not be the ultimate, fundamental theory of nature. As the theory stands, it accommodates but does not explain the masses of fundamental particles, and while it explains the strengths of many of their interactions, the strengths of other interactions are unexplained. Moreover, the standard model of particle physics is currently incomplete because it does not contain a quantum theory of gravity. In our attempt to develop a more complete theory of nature, particle theorists attempted to embed the Standard Model in grander theories and then study the testable predictions of these models. The predictions of the Standard Model and theories which attempt to go beyond the Standard Model are tested at particle accelerators.

Our best window towards identifying the more fundamental theory of nature comes to us from particle accelerators. Theories which attempt to go beyond the standard model predict new particles which are not part of the standard model. Searching for these hypothetical new particles is one of the central missions of these accelerators. High energy physicists attempt to produce new particles through the high-energy collisions of other particles. Today's high-energy particle accelerators collide various particles: Collisions of protons with anti-protons (the anti-matter version of the proton) are made nearby at Fermilab's Tevatron. Future upgrades of this collider will continue to collide protons and anti-protons into the next comming years. In the near future, protons will be collided with themselves at tremendously high energies at CERN's LHC, in Europe. Collisions of electrons with positrons (the anti-particle of the electron) have been studied in Europe at CERN's LEP and the SLAC, a linear accelerator in Stanford, California. In the future a 'next generation' electron-positron collider may be constructed in the USA, Japan, or Europe.

Roughly speaking, the heavier a particle is, the harder (with more energy) we need to smash lighter particles together to create it. Almost all theories which attempt to embed the Standard Model in a more complete theory predict new, heavy particles with masses several hundred times heavier than the proton. These heavy particles require so much energy to create that they could not have been produced by previous collider experiments. The new experiments upcoming and underway discussed above will change this situation in the next decade.

One very interesting and far reaching idea in theoretical physics is grand unification. We know of four fundamental forces in nature; (1) electricity and magnetism, (2) the weak nuclear force, (3) the strong nuclear force, and (4) gravity. As mentioned above, the Standard Model is a quantum theory of the first three of these forces. Each of these forces, more appropriately referred to as interactions, can be characterized by its strength. When measured at low energies, the strengths of these interactions are very different. These strengths, however, depend on the energy at which they are measured. In some theories, if we extrapolate the measured strengths of these three interactions from low energies to high energies, all three forces appear to unify into a single force at very high energies. This unification is predicted by grand unified theories. If grand unification proves to be correct, all three of these forces can be understood as different manifestations of a single force. More ambitious still are string theories, which attempt to unify these forces with gravitation at and even higher scale.

As disscussed above that the standard model is not a fundamental theory still incomplete (because it does not contain a quantum theory of gravity) and not give answers to many questions. In order to have a complete picture of what is happerning at the very fundamental leve, we have to go beyond the standard model and test its predictions.

Within this dissertation, we work in radiative dileptonic B-meson decay. The prospect of physics beyond the Standard Model (SM) is not just observing, but quantitatively measuring at the B-factories (BELLE, BaBar, LHC etc.) in a matter of years, if not months, has elicited much excitement in the high-energy phenomenology community. The primary reason for this is that data from these factories is alredy streaming in, whilst other projects capable of probing the frontiers of known physics may not even commenc for several years [1]. With such a valuable resource at hand it is incumbent upon us to propose the most experimentally viable tests of

e = 0.51 MeV
μ = 106 MeV
τ = 1780 MeV

any possible observables of new physics effects.

Note that the reason for the flavour-changing neutral current(FCNC) transitions of $b \to s(d)$ (which occur only through loops in the SM) having been extensively studied is that these FCNC decays provide an extremely sensitive test of the gauge structure of the SM at loop level whilst simultaneously constituting a very suitable tool for probing new physics beyond the SM [2].

Among the rare B-meson decays, the radiative lptonic $B_s \to \gamma l\bar{l}$ ($l = e, \mu, \tau$) decays are especially intersting due to their relative cleanliness (No QCD effect) and sensitivity to new physics. $B_s \to \gamma l\bar{l}$ induced by $B \to l\bar{l}$ one, which can be in principle serve as a useful process to determine the fundamental parameters of the SM since the only non-perturbative quantity in its theoretical calculation is the decay constant, which is reliably known. However, in the SM, matrix element of $B \to l\bar{l}$ decay is proportional to the lepton mass and therefore corresponding branching ratio will be helicity suppressed. Although $l = \tau$ channel is free from this suppression, its experimental observation is quite difficult due to low efficiency. In this connection, it has been pointed out [3] that the radiative leptonic $B^+ \to \gamma l^+ \nu_l$ ($l = e, \mu$) decays have larger branching ratios than purely leptonic modes. It has been shown that similar enhancements take place also in the radiative decay $B_s \to \gamma l\bar{l}$, in which the photon emitted from any of the charged lines in addition to the lepton pair makes it possible to overcome the helicity suppression. For that reason, the investigation of the $B_s \to \gamma l\bar{l}$ decays becomes intersting.

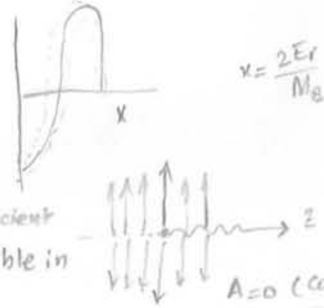
In this work, we will investigate the new physics effects in the forward-backward (FB) asymmetries in the $B_s \to \gamma l\bar{l}$ decay. Since this asymmetry contains different Wilson coefficients and hence provides independent information they are thought to play important role in further investigations of the SM and its possible extensions. As for the new physics effects, in rare B meson decays they can appear in two different ways: one way is through new contributions to the Wilson coefficients that is already present in the SM and the other is through the new operators in the effective Hamiltonian which is absent in the SM. In this dissertation we use a most general model independent effective Hamiltonian that combines both these approaches and contains the scalar and tensor type interactions as well as the vector types.

We have organized the subsequent pages as follows: In chapter 2 we mentioned the present status of particle physics, a breif overview on gauge symmetry and a breif review of the Standard Model.

Wilson Coefficients

A = C . Q
(HQ effect as const.)
C_{LR}, $C_{..}$ are determined through

SM
V, V−A currents
⇓
O_9, O_{10}, O_7
C_9, C_{10}, C_7
Wilson Coefficient
LR not possible in SM.
But C_{LR}, C_{RL} incorporate in A, zero shifts.



$x = \frac{2E_\gamma}{M_B}$
A = 0 (common sense)
A ≠ 0 asymmetries

In chapter 3 we begin with a breif survey of the physics of heavy quark systems. This discussion motivates the introduction of the Heavy Quark Effective theory (HQET) which captures a great deal of the intuition developed. A derivation of the HQET from QCD is presented as well as an analysis of its special properties such as operator product expension (OPE) and Factorization. The heavy quark flavour- and spin- symmetry of the effective lagrangian is an offspring of this.

In chapter 4 we study the radiative leptonic decay $B \to \gamma l\bar{l}$. In first section (review work) we take the effective hamiltonian within the SM and calculate the forward-backward asymmetry by using the Large Energy Effective form factors and the dispersion form factors describe in Eq. (4.38). variation in the behaviour of asymmetry with scaled energy of photon (x) is observed. In the second section the general, model independent hamiltonian is used to analyze the FB asymmetry and results shown in graphs.

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Chapter 2

STANDARD MODEL

2.1 INTRODUCTION

The world is full of diverse physical objects. It is a natural curiosity of man to ask, are "all the bewildering variety of objects that we see in nature made up of a small number of elementary (fundamental) particles?" And "how the fundamental particles cohere to make all the objects of the universe?". These questions are underlying the subject of elementary particle physics. The past two decades have seen a remarkable progress in the physics of elementary particlees and our greatest endeavour in basic science, undoubtedly, been the study of the matter and its constituent particles. Now strengethening the arena of basic scientific research, the greatest synthesis of all time, which describes the basic interactions of all the particles, has been achieved in the form of the Standard Model of Particle Physics.

The Standard Model (SM) is a gauge theory, based on the symmetry group, which describes strong, weak and electromagnetic interactions, via the exchange of the corresponding spin-1 gauge fields: 8 massless gluons and 1 massless photon for the strong and electromagnetic intractions respectively and 3 massive bosons, $W^{\pm}$ and Z, for the weak interaction. The SM constitutes one of the most successful achievements in modern physics. It provides a very elegant theoretical frame work which is able to describe the known experimental facts in particle physics with high precision. In this chapter, we give a short introduction into this beautiful theory [1].

2.2 The Picture of Elementary Particle Physics

Elementary particle physics began when humans first started wondering what we and every thing around us are made of and what is happening to hold matter together. Certainly throughout recorded history man has searched for the basic building blocks of matter and their interactions.

The picture of fundamental constitutes of matter and the interactions among them that has emerged recent years is one of great beauty and simplicity. All matter seemed to be composed of quarks and leptons, which are supposedly point-like (structureless), spin 1/2 particles. The quarks bound together by the strong force and made hadrons. Hadrons are further classified into Baryons and Mesons. Assuming that mesons are $M \equiv q\overline{q}$ states, while baryons have three quark constituents $B \equiv qqq$ one can nicely classify the existence of a new quantum number, colour, such that each species of quark may have $N_c = 3$ different colours: $q^\alpha, \alpha = 1, 2, 3$ (red, green, blue).

Leaving aside gravitation, which is a negligible perturbation at the energy scales usually considered, all the three interacions namely weak, electromagnetic and strong are described by gauge theories and are mediated by spin one gauge quanta. A general feature of quantum field theory is that each particle has its own antiparticle with opposite charge and magnetic moment but with same mass and spin [2]. Accordingly we have positron (e^+) and the up and down quark also have the anti-up $\overline{u}$ and the anti-down quark $\overline{d}$. Antiparticles may also be found within hadrons. For example, the positively charged pi-meson (or simply pion) consists of an up quark and an anti-down quark ($\overline{\pi}^+ = u\overline{d}$).

2.2.1 Present Status: Fundamental Constituents of matter

Fermions are divided into, as indicated above, Leptons and Quarks. They are further divided into three families or generation (by their historical backgrounds) .

In the Lepton generations, electron was the first of the leptons and much lighter ($m_e = 0.51 Mev/c^2$) than the muon ($m_\mu = 105.7 Mev/c^2$) or taon ($m_\tau = 1777.1 Mev/c^2$). Each of these leptons is associated with a neutral partner called a neutrino. And three generations of 3 quarks, each generation has two flavours.

All the six flavours u (up), d (down), c (charm), b (bottom) and t (top) quarks have been observed. Each flavour quark comes in three colours. Colour is just a quantum number like the charge and bears no similarity with the visual colour.

2.2.2 Mediators of Force

According to quantum field theory, the forces between the fundamental constituents of matter arise due to the exchange of gauge bosons. The mediators of the fundamental forces are: γ, W^+, W^-, Z and 8 coloured gluons. They are all spin 1 objects.

2.2.3 Higgs

Finally, there are Higgs particles (scalor spin 0 objects) which are supposed to be responsible for giving mass to the quarks, lepons and the intermediate vector bosons W^+, W^- and Z. They have not been seen so far.

Leptons

Particle	Mass (Mev)	Electriccharge	Lifetime
Electron	.511	-1	$> 10^{24} Yr$
Electron neutrino	$< .000003$	0	$> 300 s/ev$
Muon	106	-1	2.2×10^{-6s}
Mu neutrino	$< .19$	0	$> 15.4 s/ev$
Tau	1780	-1	$290 \times 10^{-15} s$
Tau neutrino	< 18.2	0	??

local Gauge

$\mathcal{L}_{QED}$ → Symmetry → Gauge field

$\mathcal{L}_{ew}$ → Symmetry → Gauge9 field (corresponding mass term)
⇓
SSB (vacuum) Not by hand
⇓
Higgs

Quarks

Particle	Mass(Mev)	Electriccharge
Upquark	$1 - 5$	$2/3$
Downquark	$3 - 9$	$-1/3$
Strangequark	$75 - 170$	$-1/3$
Charmquark	$1150 - 1350$	$2/3$
Bottomquark	$40000 - 4400$	$-1/3$
Topquark	174000	$2/3$

2.2.4 Fundamental Interactions

All known processes in nature from micrscopic to macroscopic (i.e. from sub-nuclear to extra galactic) can be understood as a manifestation of one or, more of the four fundamental interactions: (a) gravitation, (b) electromagnetic, (c) weak and (d) strong. These interactions have different strengths and obey different coservation laws.

Electromagnetic interactions are associated with the fermion electric charges, while the quark flavous (up, down, strange, charm, bottom, top) are related to electroweak phenomena. The strong forces are flavour conserving and flavour independent. On the other side, the carriers of the electroweak interaction (γ, Z, $W^{\pm}$) do not couple to the quark colour. Thus, it seems natural to take colour as the charge associated with the strong forces only and try to build a quntum field theory based on it. A great success was made with the help of the principle of gauge invariance by the work of S. L. Glashow, S. Weinberg and A. Salam where they synthesise the electromagnetic and weak forces into a unified electroweak force. As a result, gauge symmetry assumed its role as the guiding principle for particle interactions [1].

2.3 Gauge Symmetry as the Guiding Principle for Particle Interaction

The past 15 years has seen the gradual emergence of the idea that the fundamental physical interactions are determined by gauge symmetry or, more precisely, by hidden (spontaneously broken) gauge symmetry. The importance of gauge symmetry is that it reduces considerably the possible forms of interaction, gives the interactions a geometrical meaning, and introduces a certain degree of unification to the different known interactions (gravitational, weak, etc.). Gauge invariance is a powerful tool to determine the dynamics of the electroweak and strong forces. It is now believed that all fundamental interactions are described by some form of gauge theory. We also say that Gauge invariance is a powerful symmetry that tames uncontrollable infinities in quantum amplitudes and encodes the rich symmetry structure in elementary particle physics.

A gauge theory involves two kinds of particles, those which carry charge and those which mediate interactions between currents by coupling directly to charge. In the former class are the fundamental fermions and non-abelian gauge bosons, whereas the latter consists solely of gauge bosons, both abelian and non-abelian.

Today, three of the observed forces in Nature have been successfully described as theories of gauge symmetry, and it turns out that these three forces can be described in terms of unitary groups of different dimensions. Physicists write this combination of gauge groups as $SU(3) \times SU(2) \times U(1)$. In the gauge theory described by the groups $SU(N)$, there are $(N^2 - 1)$ gauge bosons. The group $SU(3)$ is the gauge group of the theory of the strong interactions known as QCD. The massless gauge field of this theory is known as the gluon. The group $SU(3)$ has eight generators, and this turns out to mean that there are eight types of gluons predicted by the theory.

The $SU(2) \times U(1)$ part that remains is a bit more complicated, the $U(1)$ has associated with it a, B single gauge boson, known to everyone as the photon. The $SU(2)$ refer to the weak interaction, has three generators of gauge symmetry, and that would give three massless gauge bosons W^+, W^- and W^0 to mediate the weak nuclear force. Now B and W^0 can mix, one linear combination is identical with photon, the after with another neutral boson Z^0, which mediate neutral weak interaction.

A Lagrangian density (L) is used to describe the dynamics of interating fermions. The Standard Model Lagrangian L_{SM} embodies our knowledge of the strong and electroweak intractions, as disscussed above. So by describing these gauge groups we can say that in the standard model, each Lagrangian density is generated by requiring local gauge invariance. Physically this means that transformations of the form

$$\Psi(\overline{x}, t) \rightarrow e^{iH(\overline{x},t)}\Psi(\overline{x}, t)$$

will not alter the physically observable effects. The quantity $H(x, t)$ is referred to as the gauge and may be any $n \otimes n$ Hermitian matrix.

The prototype gauge theory is quantum electrodynamics (QED), an abelian $U(1)$ gauge theory. It is instructive to show that the theory can actually be derived by requiring the Dirac free electron theory to be gauge invariant and renormalizable [3].

Consider the Lagrangian for a free-electron field $\Psi(x)$

$$L_{\circ} = \overline{\Psi}(x)(i\gamma^{\mu}\partial_{\mu} - m)\Psi(x).$$

The requirement of invariance for this Lagrangian under above gauge transformation we need to form a gauge-covariant derivative D_{μ} , to replace ∂_{μ} and $D_{\mu}\Psi(x)$ will have the transformation

$$D_{\mu}\Psi(x) \rightarrow [D_{\mu}\Psi(x)]' = e^{iH(\overline{x},t)}D_{\mu}\Psi(x)$$

so

$$D_{\mu}\Psi = (\partial_{\mu} + ieA_{\mu})\Psi$$

here A_{μ} is the gauge field (photon) and it should be transformed as

$$A_{\mu}(x) \rightarrow A'_{\mu}(x) = A_{\mu}(x) + \frac{1}{e}\partial_{\mu}H(x)$$

Our final QED lagrangian

$$L_{QED} = \overline{\Psi}(x)i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\Psi - m\Psi\overline{\Psi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

By this Lagrangian it should be noted that photon is massless because $A_{\mu}A^{\mu}$ term is not gauge invariant [4]. The minimal coupling of photon to the electron is $-(e\overline{\Psi}\gamma^{\mu}\Psi)$.

The Lagrangian does not have a gauge-field self-coupling because the photon does not carry a charge.

The property that ditinguishes quarks from leptons is color, so it is natural to attempt to construct a theory of the strong intractions among quarks based upon a local color gauge symmetry.

As in the QED case, we now require the Lagrangian to be also invariant under local $SU(3)_c$ transformations. To satisfy this requirement, we need to change the quark derivatives by covariant derivatives. Since we have now eight independent gauge parameters, eight different gauge bosons the so-called gluons are needed.

The gauge transformation of the gluon fields is more complicated than the one obtained in QED for the photon. The non-commutativity of the $SU(3)_c$ matrices gives rise to an additional term involving the gluon fields themselves. Taking the proper normalization for the gluon kinetic term, we finally have the $SU(3)_c$ invariant Lagrangian of Quantum Chromodynamics (QCD).

$$L_{QCD} \equiv -\frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a + \sum_f \overline{q}_f(i\gamma^{\mu}D_{\mu} - m_f)q_f$$

The color interaction between quarks and gluons is

$$g_s G_a^{\mu} \sum_f \overline{q}_f^{\alpha}\gamma_{\mu}(\frac{\lambda^a}{2})_{\alpha\beta}q_f^{\beta}$$

it involves the $SU(3)_c$ matrices λ_a. Finally, owing to the non-abelian character of the colour group, the $G_a^{\mu\nu}G_{\mu\nu}^a$ term generates the cubic and quartic gluon self-interactions, the strength of these interactions is given by the same coupling constant g_s which appears in the fermionic piece of Lagrangian.

In spite of the rich physics contained in it, the Lagrangian looks very simple because of its color symmetry properties. All interactions are given in terms of a single universal coupling g_s which is called the strong cupling constant. The existence of self-interactions among the gauge fields is a new feature that was not present in QED. It seems reasonable to expect that these gauge self-interactions could explain properties like asymptotic freedom (strong interactions become weaker at short distances) and confinement (the strong forces increase at large distances) which do not appear in QED.

Without any detailed calculation, one can already extract qualitative physical consequences from L_{QCD}. Quarks can emit gluons. At lowest order in g_s, the dominant process will be emission of a single gauge boson.

The electromagnetic and the weak interactions have been integrated into a single gauge theory so called electroweak theory. Using gauge invariance, we have been able to determine the right QED and QCD Lagrangians. To describe weak interactions, we need a more elaborated structure, with several fermionic flavours and different properties for left- and right-handed fields. Moreover, the left-handed fermions should appear in doublets and we would like to have massive gauge bosons $W^{\pm}$ and Z in addition to the photon [5]. The simplest group with doublet representations is $SU(2)$. We want to include also the electromagnetic interactions; thus we need an additional $U(1)$ group. The obvious symmetry group to consider is then

$$G \equiv SU(2)_L \otimes U(1)_Y$$

where L refers to left-handed fields and Y is the hypercharge.

In the electroweak theory, the interaction Lagrangian for the first family or 'generation' of fermion is [6]

$$\begin{aligned} L_{EW} &= \sum_{f=l,q} g_1 \overline{f}\gamma^{\mu} f A^{\mu} + \frac{g_2}{\cos\theta_w} \sum_{f=l,q} [\overline{f}_L \gamma^{\mu} f_L (T_f^3 - Q_f \sin^2\theta_w) + \overline{f}_R \gamma^{\mu} f_R (-Q_f \sin^2\theta_w)] Z_{\mu} \\ &\quad + \frac{g_2}{\sqrt{2}} [\overline{u}_L \gamma^{\mu} d_L + \overline{v}_{eL} \gamma^{\mu} e_L) W_{\mu}^{+} + h.c.] \end{aligned}$$

where f and $\overline{f}$ are the fields of fermions. A_{μ} is the field of the photon and Z and W are the fields of the two weak gauge bosons; g_1 is the electric coupling strength (or electric charge), g_2 is the weak coupling strength and T_f^3 is the third component of the weak isospin of the interacting fermions. The subscripts L and R denote the chirality or handedness of the fermions. For massless fermions the chirality is equal to the helicity, which is positive (negative) if the fermion spin is directed towards (away from) its direction of motion. The Weinberg or weak mixing angle, θ_w is a measure of a relative strength of the electromagnetic coupling and weak coupling strength. In electroweak interactions the neutral currents involve left and right-handed of the left-handed charged current is parity violation, the hallmark of the weak interaction.

In the end the weak nuclear force is a short range force, behaving as if the gauge bosons are very heavy. In order to make a gauge invariant theory work for the weak nuclear force, theorists had to come up with a way to make heavy gauge bosons that wouldn't destroy the consistency of the quantum theory. The method they came up with is called Spontaneous Symmetry Breaking, where massless gauge bosons acquire mass by interacting with a scalar field called the Higgs field. The resulting theory has massive gauge bosons but still retains the nice properties of a fully gauge invariant theory where the gauge bosons would normally be massless.

	Forces and Symmetries		
Force	Gauge bosons	Gauge group	Details
Electromagnetism	1 Photon	The unbroken $U(1)$ combination of $SU(2)_L \times U(1)_Y$ symmetry	Photon is massless and neutral, couples to electric charge, force is inf inite range, theory is called Quantum Electrodynamics, or QED for short
Weak nuclear Force	W^+,W^- ,Z	The broken combination of $SU(2)_L \times U(1)_Y$ symmetry	Gauge symmetry is hidden by interaction with scalar particle called Higgs, W and Z are massive, have weak and electric charge, interaction is short range
Strong nuclear Force	8 Gluons	$SU(3)$	Gluon is massless but self-interacting. Charge is called quark color, theory is called Quantum chro mod ynamics, or QCD for short.

2.3.1 The Higgs-Kibble Phenomenon

Gauge symmetry also guarantees that we have a well-defined renormalizable Lagrangian. However, this Lagrangian has very little to do with reality. Our gauge bosons are massless particles; while this is fine for the photon field, the physical $W^{\pm}$ and Z bosons should be quite heavy objects.

In order to generate masses, we need to break the gauge symmetry in some way; however, we also need a fully symmetric Lagrangian to preserve renormalizability. A proposed solution to this dilemma, is based on the fact that it is possible to get non-symmetric results from an invariant Lagrangian. It means the gauge symmetry must be broken somehow. If we introduce explicit breaking terms in the form of arbitrary gauge boson masses we alter the high-energy behaviour of the theory in such a way that the renomalizability of the theory is lost. We may contemplate the possibility of the spontaneous breaking of the symmetry. The symmetry breaking mechanism must not only generate the fermion and boson masses, but also lead to a renormalizable theory. In the Salam Weinberg model, this is accomplished by introducing a scalar isospin doublet of complex Higgs fields [7]

$$\phi(x) \equiv \begin{pmatrix} \phi^{+}(x) \\ \phi^{\circ}(x) \end{pmatrix}$$

expanding the Higgs fields around an asymmetrical ground state i.e. with non vanishing vacuum expectation value and demanding local gauge invariance. Therefore, we have found a clever way of giving masses to the intermediate carriers of the weak force. Three of the four scalars degree of freedom of the Higgs field give masses to the W and Z bosons. The remaining one manifests itself in a massive neutral spin zero boson, the physical Higgs boson. It is the only particle of the Standard Model which lacks direct experimental detection.

Fermion masses in Salam-Weinberg model are generated via a Yukawa interaction $\overline{\psi}(x)\phi(x)\psi(x)$ with the Higgs field. The terms representing the fermion-Higgs interaction in the Lagrangian are not necessarily diagonal in fermion generations. Since fermion-Higgs interaciton must be expressed in terms of mass eigenstates, the weak eigenstates giving currents diagonal in generations are not the same as the mass eigenstates. Hence, intergenrational mixing between fermions can occur.

To express the fermion-Higgs interaction in terms of mass eigenstates, the mass matrix is diagonalized using a pair of unitary transformations (one for each quark charge) relating the physical and weak quark bases. The product of unitary matrices that accomplish this task and appears in the charge current interaction Lagrangian, is known as the mixing matrix. For neutral currents the mass matrix stays diagonal and mixing does not occur.

The mixing matrix is unitary by construction, and therefore contains n^2 parameters. An overall phase can be chosen to render one of these operations ineffective, so we can remove a total of $2n-1$ phases. Of the n^2-2n+1 parameters, it can be shown that $\frac{1}{2}(n-1)(n-2)$ are imaginary parameters.

For two generations ($n=2$), the matrix contains one real paramter: the Cabibbo angle, θ_c. The resulting charge current (CC) part of the Lagrangian is:

$$L_{cc} = W_+^\mu(\overline{u}_L\overline{c_L})\gamma_\mu \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d_L \\ sL \end{pmatrix} + h.c.$$

where all coupling constants are real. The well-known GIM mechanism uses the notion of Cabibbo-rotated quark states to explain the suppression of flavor-changing neutral currents and justify the existence of the charm quark. By this rotating or mixing matrix between flavours, alter the coupling strength by an extra factor of cosθ_c, and sinθ_c as shown in Fig. (1).

For three generations ($n=3$), the resulting charge current part of the Lagrangian is:

$$L_{cc} = W_+^\mu(\overline{u}_L\overline{c}_L\overline{t}_L)\gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

Like fermion masses, the matrix elements of above mixing matrix for three generations are fundamental input parameters and must be determined experimentally. For three generations, the matrix contains four independent quantities: three real parameters (or angles) and one imaginary parameter (a complex phase) as discussed below.

2.4 Flavor Mixing (CKM) Matrix

The weak interaction is the only one in which a quark can change into another type (flavour) of quark or a lepton into another type of lepton. In this transformation, a quark is allowed to change charge by unit amount e. Because quarks can change flavour by weak interactions, only the lightest quarks and leptons are included in the stable matter of the world around us - all heavier ones decay to one or another of the lighter ones. If we look at all the ways in which one quark can change into another quark with a charge change of e, i.e. just all quarks with charge $+\frac{2}{3}e$ (u, c *or* t) paired with quarks with charge $-\frac{1}{3}e$ (d, s or b). i.e. nine possible pairings. Each of these pairings has its own weak charge associated with it, which is related to a physical constant which we called a 'coupling constant' that contains real and imaginary parts and is *complex*. The set of coupling constants can be represented by a 3×3 matrix.

$$\begin{array}{ccc|c} d & s & b & \\ \hline V_{ud} & V_{us} & V_{ub} & u \\ V_{cd} & V_{cs} & V_{cb} & c \\ V_{td} & V_{ts} & V_{tb} & t \end{array}$$

2.5 B-mesons and their Decays

We will deal with the bottom quark system, which is an ideal laboratory for studying flavour physics. By defination, flavour physics deals with that part of the SM that distinguishes between the three generations of fundamental fermions. Flavour physics can be regarded as the least tested part of the SM. The history of B physics started in 1977 with the observation of a dimuon resonance at 9.5 Gev in 400 Gev proton-nucleon collisions at Fermilab.

What is B meson? Due to confinement quarks appear in nature not separately, but have to be bound into colourless hadrons. Considering constituent quarks only, the simplest possible of such objects consists of a quark and an antiquark, called a meson. The bound states with a bquark and a $\overline{d}$ or $\overline{u}$ antiquark only and is called a B-meson. The bound states with a b quark and $\overline{d}$ or $\overline{u}$ antiquark are referred to as the $\overline{B}^0$ and B^- mesons, respectively. B-mesons containing an s or c quark are denoted by B_s and B_c, respectively.

The B-meson is a relatively heavy particle having a mass of $5.28Gev/c^2$, which is more than five times the mass of a proton. This is because the b-quark it conatins is almost that massive.

2.6 B-Meson factories

The experimental situation concerning flavour physics is drastically changing. Several B physics experiments are successfully running at the moment and, in the upcoming years, new facilities will start to explore B physics with inceasing sensitivity and within various experimental settings: apart from the CLEO experiment (cornell, USA), located at the Cornell Electron-Positron Storage Ring (CESR), two B factories, operating at the $\Upsilon(4S)$ resonance in an asymmetric mode, are successfully obtaining data: the BABAR experiment at SLAC (Stanford, USA) and the BELLE experiment at KEK (Tsukuba, Japan). Besides the hadronic B physics program at FERMILAB (Batavia, USA) there are B physics program at hadronic colliders [8]. Within the LHC project at CERN at Geneva all three experiments have strong B physics programs. Also at FERMILAB an independent B physics experiment, BTeV, is planned. The main motivation for a B physics program at hadron colliders is the huge b quark production cross section with respect to the one at e^+e^- machines.

While the time of electroweak precision physics focusing on the gauge sector of the SM, draws to a close with the completion of the LEP experiments at CERN and the SLAC experiment in Stanford, the era of precision flavour physics, focusing on the scalar sector of the SM, just begun with the start of the B factories.

2.7 Standard Model and the main goal of B-physics

In spite of its impressive successes, the Sandard Model is believed to be not complete. For a really final theory it is too arbitrary, especially considering the large number of, sometimes even unnatural parameters in the Lagrangian. Examples for such parameters, that are largely differnt from what one naively expects them to be, are the weak scale compared with the Planck scale or the small value of the strong CP-violation parameter θ_{QCD}. Questions like: "Why are there three particle generations?" "Why is the gauge structure with the assignment of charges as it is?" or "What is the origin of the mass spectrum?" demand an answer by a really fundamental theory but the Standard Model gives no replies. Furthermore, the union of gravity with quantum theory yields a nonrenomalizable quantum field theory, indicating that New Physics should show up at very high energies.

The ideas of grand unification, extra demensions, or supersymmetry were put forward to find a more complete theory. But applying these ideas has not yet led to theories that are substantially simpler or less arbitrary than the Standard Model. To date, string theory, the relativistic quantum theory of one-dimensional objects, is a promising and so far the only candidate for such a Theory of Everything.

The main goal of B-physics, which is a precision study of the flavour sector with its phenomenon of CP violation to pass the buck of being the experimentally least constrained part of the Standard Model. This is not only to pin down the parameters of the Standard Model but in particular to reveal New Physics effects via deviations of measured observables from the Standard Model expectation. Such an indirect search for New Physics and the direct search at particle accelerators invite both experimenters and theoreticians to work with precision. We need accurate and reliable measurements and calculations.

B-decays show an extremely rich phenomenology and theoretical techniques using an expansion in the heavy mass allow for model-independent predictions. The rich phenomenology is based on the large available phase space, on the one hand, allowing for a plethora of possible final states and on the other hand on the possiblity for large CP-violating asymmetries in B decays. The latter feature is in contrast to the Standard Model expectations for decays of K and D mesons. In D decays only the comparably light d, s, and c quarks can enter internal loops which leads to a strong GIM suppression of CP-violating phenomena.

Rare decays induced by flavor changing neutral currents can also be used as tests of the Standard Model (SM) and are sensitive to new physics. It is expected that at future B factories and fixed target machines, other rare decay channels of the bottom quark will be discovered in addition to the observed $b \to s\gamma$ transition. These processes also offer useful information for extracting the fundamental parameters of the SM, such as $\mid V_{ub} \mid$. Rare decays can also serve as alternative channels to measure some elementary hadronic parameters. For instance, the decay constants f_{B_q}, $q = s, d$ can be extracted from $B_q \to \gamma\nu\overline{\nu}$.

In the end, the B meson system offers an excellent laboratory to quantitatively test the CP-violating sector of the Standard Model, determine fundamental parameters, study the interplay of strong and electroweak interactions, or search for New Physics.

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Chapter 3

Theoretical background for B-Physics

3.1 Introduction

Quantum chromo dynamics (QCD) as the theory of strong interactions has been with us for over twenty years. It has been remarkably successful in describing high energy physics. The discovery of asymptotic freedom has allowed for many perturbative calculations of physical quantities within QCD when combined with the parton model.

However, during the last few years there has been a resurging interest in heavy quark physics within the context of QCD. The reason for this is that one has been able to extract general principles from particular models of heavy flavour transitions. Model independent implication, namely, that for infinitely heavy quarks, velocity is the only important parameter.

Further,

(1) there is a heavy quark flavour symmetry,

(2) there is a two-fold spin degeneracy (because the spin coupling is $\propto \frac{1}{m_Q}$ which tends to zero), and

(3) at the zero recoil point, or equivalently at maximum momentum transfer,the elastic transition is absolutely normalized [1].

The new symmetries in heavy quark theory give rise to numerous predictions for free!

The heavy quark effective theory HQET, as it has come to be known, captures the physics of the heavy quark systems which brings to light these new symmetries. HQET, as we will see, is an expansion of the QCD action in inverse powers of the heavy quark mass. Heavy quarks Q, for present purposes will be (c, b, t) with masses $m_Q \approx (1.5, 5, 175)\ GeV$. The scale is set by Λ_{QCD}/m_Q.

3.2 Physics of a Heavy Hadron

The classical picture of a heavy hadron that one has is in many ways similar to that of the Hydrogen atom. The typical momentum carried by the light degrees of freedom (light quarks plus glue) inside a hadron Λ_{QCD} is of the order of the proton mass divided by three $\sim 330MeV$. This is a measurement of how far the light quarks are off shell or, rather more correctly, this is about what their constituent masses are. Typically, the light quarks and this gluonic cloud are carrying momentum Λ_{QCD}. In heavy hadrons the mass of the heavy quark m_Q is much greater than the typical scale, $m_Q \gg \Lambda_{QCD}$ and the heavy quark is carrying most of the momentum of the heavy hadron. The interactions of the heavy quark with the light degrees of freedom will also only change the momentum of the heavy quark by the order of Λ_{QCD}, so that the heavy quark is then almost on mass shell. Indeed one expects $M_Q \approx m_Q + O(\Lambda_{QCD})$.

While the change in momentum of the heavy quark is of the order Λ_{QCD}, its change in velocity, $\Lambda_{QCD}/m_Q \ll 1$, is negligible as the mass of the heavy quark goes to infinity. The picture one has then is of the heavy quark moving with constant velocity and this velocity is that of the heavy hadron. It is important to emphasis that it is not momentum that is being equated but, rather, velocity. In the rest frame of the heavy hadron the heavy quark is almost at rest; it is only slightly recoiling from the emission and absorption of soft gluons. This picture does not depend on the actual value of m_Q but just that it satisfies $m_Q \gg \Lambda_{QCD}$. As the mass of the heavy quark is taken to be bigger and bigger the recoil is less and less until ultimately, in the limit $m_Q \to \infty$, the heavy quark does not recoil at all from the emission and absorption of soft gluons. In this limit the binding is independent of the flavor and hence the difference between the mass of the heavy hadron and the heavy quark, $\overline{\Lambda} = M_Q - m_Q$, is a universal, flavor independent constant.

In many ways we have also just described the Hydrogen atom. Take the proton to be a fundamental particle. The typical momentum imparted to the proton by the electron and the photon cloud is very much smaller than the mass of the proton (m_p). The proton can be taken to be a static photon source which binds the electron to form the Hydrogen atom. The fact that the Schrodinger equation for the electron in a $1/r$ potential describes quantitatively the Hydrogen atom so well is indicative of the success of this picture. One of the things that is missed by this non-relativistic analysis is the hyperfine splitting of energy levels. But such corrections are rather small compared to the energy level, $\Delta E/E \ll 1$. One can derive the Shrodinger equation from the fully relativistic and interacting Dirac theory for the Hydrogen atom and systematically incorporate corrections such as the Thomas term for the spin-orbit coupling.

Likewise for the heavy hadron, it is immaterial, in a first approximation, what the spin state of the heavy quark is and in analogy to the above discussion one can give a systematic derivation of corrections to this picture. The corrections will be of the form of a series in $\frac{1}{m_Q}$ with the spin coupling coming in at next to leading order [2].

3.3 Physics of Flavour Changing Transitions

We will be interested in the flavour changing electroweak transitions of one heavy hadron into another. The transitions of prime interest will be where the heavy quark in the first interacts with the electro-weak particle to flavuor change into the heavy quark of the second. The light quarks will essentially be spectators. The reason for this is that in the heavy mass limit the heavy quark, as we have seen, is essentially on-shell and acts as a colour source for the light degrees of freedom. In particular the spin of the heavy quark decouples from the dynamics. Thus the dynamics of the weak transition of a heavy hadron are essentially determined by the point like interaction of the weak current with the heavy quark. The picture which emerges of the transition is as follows.

In the infinite mass limit in the rest frame of the heavy hadron, the heavy quark Q is at rest and is surrounded by the light cloud with no spin interaction between them. In the transition the heavy quark Q emits a W meson and becomes another heavy quark $\overline{Q}$ moving with some velocity (the velocity of the final heavy hadron). The light cloud has thus to adjust its velocity to keep up with the heavy quark $\overline{Q}$ in forming the new hadron. It is this adjustment, or overlap, of the light degrees of freedom which gives rise to the form factors. We immediately see from this simple, atomic physics, picture the physical reason why the unique form factor in heavy meson decays, for example, is normalized to one at zero recoil (or q^2_{max}). At this kinamtical point, the daughter heavy quark is produced at rest in the initial rest frame. It is clear that nothing has changed as far as the light degrees of freedom are concerned because there is no flavor dependence in the static colour source due to the heavy quark and the light degrees of freedom do not feel the effect of the change in the heavy quark mass (both are infinite) unless it moves. Thus there is a complete overlap of the light wavefunctions before and after the transition at this kinematical point and hence the form factor is one. Put bluntly there is no dynamics in the transition at this point [2].

3.4 Effective Field Theories

Effective field theories are an important tool in theoretical physics. The reason is simple: For the understanding of a physical process it is usually counterproductive to consider it in the context of a "theory of everything" (even if this existed). It is better to use a level of description that is most adequate to the problem at hand. In other words, one takes into account those aspects of the "full theory" which are important, and ignores others which are irrelevant. So far, Newton's laws, Maxwell's equations, and the laws of thermodynamics sufficient to account for most of the phenomena of our everyday life, whereas the more refined descriptions of quantum mechanics and relativity are necessary to understand the physics at smaller distances and larger energies. For the energies presently accessible in particle accelerators, the language of local quantum field theories, in form of the standard model of the strong and electroweak interactions, has proved to provide a most adequate level of description. We are well aware that none of these concepts truly is the "theory of everything". But nevertheless, in their range of applicability they can all be used to make calculations of sometimes incredible accuracy.

In particle physics, it is often the case that the effects of a very heavy particle become irrelevant at low energies. It is then useful to construct a low energy effective theory in which this heavy particle no longer appears. Eventually, this effective theory will be easier to deal with than the full theory. A familiar example is Fermi's theory of the weak interactions. For the description of weak decays of hadrons one can safely approximate the weak interactions by point-like four-fermion interactions governed by a dimensionful coupling constant G_F . Only at energies much larger than the masses of hadrons can one resolve the structure of the intermediate vector bosons $W^{\pm}$. and Z. This example is also instructive in that it shows that it is usually the low energy effective theory which is known first. Only as one proceeds to higher energies its limitations become apparent. In fortunate circumstances, this leads to the discovery of a new effective theory (in this case the standard model of electroweak interactions), which provides an adequate level of description for higher energies.

The heavy quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts with light degrees of freedom by the exchange of soft gluons. Clearly, m_Q is the high energy scale in this case and Λ_{QCD} is the scale of the hadronic physics one is interested in. However, a subtlety arises since one wants to describe the properties and decays of hadrons which contain a heavy quark. Hence it is not possible, however, is to integrate out the "small components" in the full heavy quark spinor, which describe fluctuations around the mass shell.

3.5 Basic Formalism

Heavy quark effective theory is an effective field theory designed to systematically exploit the simplifications of QCD interactions in the heavy-quark limit for the case of hadrons containing a single heavy quark. The HQET Lagrangian can be derives as follows. QCD Lagrangian for a heavy-quark field ψ with mass m

$$L = \overline{\psi} i \not{D} \psi - m \overline{\psi \psi} \tag{3.1}$$

with the covariant derivative

$$D_\mu = \partial_\mu - igT^a A_\mu^a \tag{3.2}$$

The heavy-quark momentum can be decomposed as

$$p = mv + k \tag{3.3}$$

where v is the 4-velocity of the heavy hadron. Once mv, the large kinematical part of the momentum is singled out, the remaining component k is determined by soft QCD bound state interactions, and thus $k = O(\Lambda_{QCD}) \ll m$. We next decompose the quark field ψ into

$$h_v(x) \equiv e^{imv.x} \frac{1+ \not{v}}{2} \psi(x) \tag{3.4}$$

$$H_v(x) \equiv e^{imv.x} \frac{1- \not{v}}{2} \psi(x) \tag{3.5}$$

which implies

$$\psi(x) = e^{-imv.x}(h_v(x) + H_v(x)) \tag{3.6}$$

The expressions $P_{\pm} = (1\pm \not{v})/2$ are projection operators. Their action represents the covariant generalization of decomposing $\psi(x)$ into upper and lower components. Using the standard representation for $\gamma-$matrices, this is evident in the rest frame where $\not{v} = \gamma^{\circ}$. Note also that the equation of motion with respect to the large momentum components, $m(\not{v} - 1)h_v = 0$, is manifest for h_v.

The exponential factor $exp(imv.x)$ in Eqs. (3.4) and (3.5) removes the large frequency part of the x-dependence in $\psi(x)$ resulting from the large momentum mv. Consequently, the x-dependence of h_v and H_v is only governed by the small residual momentum and derivatives acting on h_v and H_v count as $O(\Lambda_{QCD})$.

By the above equation we can easily get

$$iv \cdot Dh_v = -i \not{D}_{\perp} H_v \tag{3.7}$$

$$(iv \cdot D + 2m)H_v = i \not{D}_{\perp} h_v \tag{3.8}$$

Eqs. (3.7 and 3.8) represent the equation of motion in terms of h_v and H_v. The Eq. (3.8) implies that $H_v = O(\Lambda_{QCD})h_v$ by power counting. Hence H_v is suppressed with respect to h_v in the heavy quark limit. In other words, h_v contains the large components, H_v the small components of ψ.

where,

$$\not{D}_\perp \equiv D^\mu - v^\mu v.D$$

and

$$H_v = \frac{1}{(iv \cdot D + 2m_Q - i\varepsilon)} i \not{D}_\perp h_v \tag{3.9}$$

The HQET lagrangian is obtained starting from Eq. (3.1), expressing ψ in terms of h_v, H_v and eliminating H_v using Eq. (3.8). We find

$$L = \overline{h}_v iv \cdot Dh_v + \overline{h}_v i \not{D}_\perp \frac{1}{iv \cdot D + 2m} i \not{D}_\perp h_v \tag{3.10}$$

The second term in above Eq. (3.10) contains the nonlocal operator $(iv \cdot D + 2m)^{-1}$. It can be expended in powers of Λ_{QCD}/m tp yield a series of local operators. Keeping only the leading-power correction we can simply replace $(iv \cdot D + 2m)^{-1}$ by $(2m)^{-1}$

$$(iv \cdot D + 2m)^{-1} = \left[1 - \frac{iv.D}{2m} + \cdots\right] \tag{3.11}$$

Keeping only the leading-power correction and using the following identity

$$\gamma \cdot D\gamma \cdot D = D^2 - \frac{i}{2}\sigma^{\mu\nu}G_{\mu\nu} \tag{3.12}$$

we get

$$L = \overline{h}_v iv \cdot Dh_v + \frac{1}{2m}\overline{h}_v(i \not{D}_\perp)^2 h_v + \frac{g}{4m}\overline{h}_v \sigma^{\mu\nu}G_{\mu\nu}h_v \tag{3.13}$$

Let us discuss some important aspects of this result.

The first term on the r.h.s of Eq. (3.13) is the basic, lowest-order Lagrangian of HQET. It describes the "residual" QCD dynamics of the heavy quark once the kinematic dependence on

m is separated out. Since there is no longer any reference to the mass m, the only parameter to distinguish quark flavors, this term is flavor symmetric: The dynamics is the same for b and c quarks in the static limit. Since the operator $v \cdot D$ contains no $\gamma-$matrices, which would act on the spin degrees of freedom, the leading HQET Lagrangian also exhibits a spin symmetry. This corresponds to the decoupling of the heavy-quark spin in the $m \to \infty$ limit. Together we have the famous spin-flavor symmetries of HQET.

From the Lagrangian $\overline{h}_v iv \cdot Dh_v$ the Feynman rules for HQET can be read off. The propagator is

$$\frac{i}{v.k}\frac{1+\not{v}}{2}$$

The interaction of the heavy-quark field h_v with gluons is given by the vertex

$$igv^{\mu}T^a$$

These Feynman rules enter in the computation of QCD quantum corrections. The remaining terms are the leading power corrections [3].

3.5.1 Effective weak Hamiltonians

The task of computing weak decays of hadrons represents a complicated problem in quantum field theory. Two typical cases, the first-order nonleptonic process $\overline{B}^{\circ} \to \pi^+\pi^-$ and the loop-induced, second-order weak transition $B^- \to K^-\nu\overline{\nu}$ are illustrated in Fig. (3-1)

The dynamics of the decays is determined by a nontrivial interplay of strong and electroweak forces which is characterized by several energy scales of very different magnitude, the W mass, the various quark masses and the QCD scale: m_t, $M_W \gg m_b$, $m_c \gg \Lambda_{QCD} \gg m_u$, m_d, (m_s). While it is usually sufficient to treat electroweak interactions to lowest nonvanishing order in perturbation theory, it is necessary to consider all orders in QCD. Asymptotic freedom still allows us to compute the effect of strong interactions at short distances perturbatively. However, since the participating hadrons are bound states with light quarks, confined inside the hadron by long-distance dynamics, it is clear that also nonperturbative QCD interactions enter the decay process in an essential way.

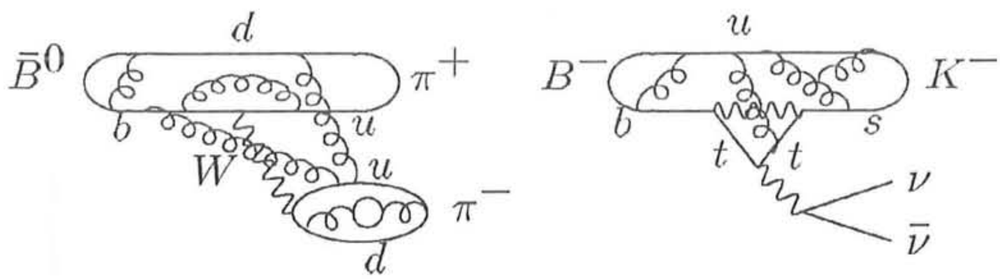

Figure 3-1: QCD effects in weak decays.

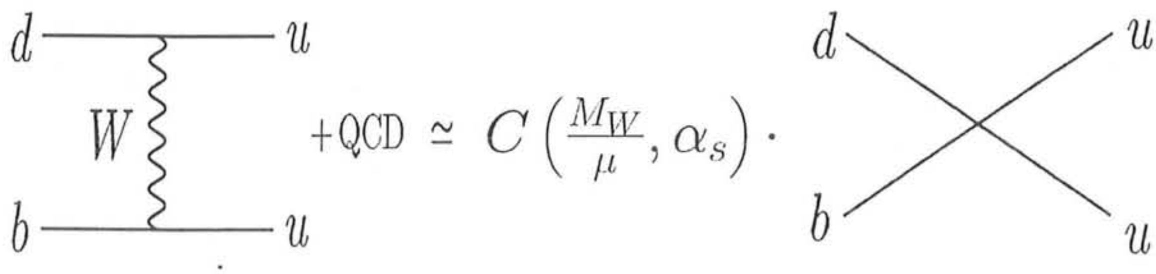

Figure 3-2: OPE for weak decays

To deal with this situation, we need a method to disentangle long- and short-distance contributions to the decay amplitude in a systematic fashion. A basic tool for this purpose is provided by the operator product expansion (OPE).

3.5.2 Operator product expansion

The OPE is of crucial importance for the theory of weak decay processes, not only in the case of B mesons, but also for kaons, mesons with charm, light or heavy baryons and weakly decaying hadrons in general. Consider, for instance, the basic W-boson exchange process shown on the left-hand side of Fig. (3-2)

This diagram mediates the decay of a b quark and triggers the nonleptonic decay of a B meson. Here, a key feature is provided by the fact that the W mass M_W is very much heavier than the other momentum scales p in the problem (m_b, Λ_{QCD}, m_u, m_d, m_s). We can therefore expand the full amplitude A, schematically, as follows.

$$A = C\left(\frac{M_W}{\mu}, \alpha_s\right) \cdot \langle Q \rangle + O\left(\frac{p^2}{M_W^2}\right) \tag{3.14}$$

Which is sketched in Fig. (3-2). Up to negligible power corrections of $O\left(\frac{p^2}{M_W^2}\right)$, the full amplitude on the left-hand side is written as the matrix element of a local four-quark operator Q, multiplied by a Wilson coefficient C. This expansion in $1/M_W$ is called a (short-distance)operator product expansion because the nonlocal product of two bilinear quark - current operators $(\overline{d}u)$ and $(\overline{u}b)$ that interact via W exchange, is expanded into a series of local operators. Physically, the expansion in Fig(3-3). means that the exchange of the very heavy W boson can be approximated by a point-like four-quark interaction. With this picture the formal terminology of the OPE can be expressed in a more intuitive and the Wilson coefficient as the corresponding coupling constant. Together they define an effective Hamiltonian $H_{eff} = C \cdot Q$, describing weak interactions of light quarks at low energies. Ignoring QCD the OPE reads explicitly (in momentum space)

$$\begin{aligned} A &= \frac{g_W^2}{8} V_{ud}^* V_{ub} \frac{i}{k^2 - M_W^2} (\overline{d}u)_{V-A} (\overline{u}b)_{V-A} \\ &= -i \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} C \cdot \langle Q \rangle + o\left(\frac{k^2}{M_W^2}\right) \end{aligned} \tag{3.15}$$

with $C = 1$, $Q = (\overline{d}u)_{V-A}(\overline{u}b)_{V-A}$ and

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{ub} (\overline{d}u)_{V-A} (\overline{u}b)_{V-A} \tag{3.16}$$

3.5.3 Factorization

The most important property of the OPE in Eq. (3.14) is the factorization of long-and short-distance contributions: All effects of QCD interactions above some factorization scale μ (short distances) are contained in the Wilson coefficient C. All the low-energy contributions below μ (long distances are collected into the matrix elements of local operators $\langle Q \rangle$. In this way the short-distance part of the amplitude can be systematically extracted and calculated in

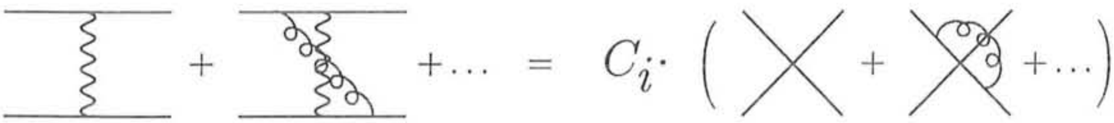

Figure 3-3: Calculation of Wilson coefficients of the OPE.

perturbation theory [3]. The problem to evaluate the matrix elements of local operators between hadron states remains.

The short-distance OPE that we have describes, the resulting effective Hamiltonian,and the factorization property are fundamental for the theory of B decays. In fact, the idea of factorization, in various forms and generalizations, is the key to essentially all applications of perturbative QCD. The reason is the same in all cases: perturbative QCD is a theory of quarks and gluons, but those never appear in isolation and are always bound inside hadrons. Nonperturbative dynamics is therefore always relevant to some extent in hadronic reactions, even if these occur at very high energy or with a large intrinsic mass scale. Thus, before perturbation theory can be applied, nonperturbative input has to be isolated in a systematic way, and this is achieved by establishing the property of factorization. It turns out that the weak effective Hamiltonian for nonleptonic B decays provides a nice example to demonstrate the general idea of factorization in simple and explicit terms.

A diagrammatic representation for the OPE is shown in Fig. (3-3)

The key to calculating the coefficients C_i is again the property of factorization. Since factorization implies the separation of all long-distance sensitive features of the amplitude into the matrix elements of $\langle Q_i \rangle$, the short-distance quantities C_i are, in particular, independent of the external states. This means that the C_i are always the same, no matter whether we consider the actual physical amplitude where the quarks are bound inside mesons, or any other, unphysical amplitude with on-shell or even off-shell external quark lines. The effective Hamiltonian necessary for the calculations to follow [4]

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1 + C_2 Q_2 + \sum_{i=3,\ldots 8} C_i Q_i \right] \tag{3.17}$$

The operators that appear follow from the actual calculations. Without QCD corrections there is only one operator of dimension 6

$$Q_1 = (\overline{d}_i u_i)_{V-A} (\overline{u}_j b_j)_{V-A}$$

where i and j are color indices have been made explicit. To $O(\alpha_s)$ QCD generates another operator

$$Q_2 = (\overline{d}_i u_j)_{V-A} (\overline{u}_j b_i)_{V-A}$$

which has the same Dirac and flavour structure, but a different colour form.

The other operators which describe penguin and dipole processes are given, these operators originate from the diagrams in Fig. (3-4)

$$\begin{aligned} Q_3 &= (\overline{s}b)_{V-A} \sum_q (\overline{q}q)_{V-A} \\ Q_4 &= (\overline{s}_i b_j)_{V-A} \sum_q (\overline{q}_j q_i)_{V-A} \\ Q_5 &= (\overline{s}b)_{V-A} \sum_q (\overline{q}q)_{V+A} \\ Q_6 &= (\overline{s}_i b_j)_{V-A} \sum_q (\overline{q}_j q_i)_{V+A} \\ Q_{7\gamma} &= \frac{e}{8\pi^2} m_b \overline{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu} \\ Q_8 &= \frac{g_s}{8\pi^2} m_b \overline{s}_i \sigma^{\mu\nu} (1+\gamma_5) T^a_{ij} b_j G^a_{\mu\nu} \end{aligned}$$

with e and g_s the coupling constants of electromagnetic and strong interaction and $F_{\mu\nu}$ and $G_{\mu\nu}$ the photonic and gluonic field strength tensors, respectively. The i, j are colour indices. If no colour index is given the two operators are assumed to be in a colour singlet state. The operator basis consists of all possible gauge invariant operators with energy dimension six. Diagramatic way of these operators are in Fig. (3-4)

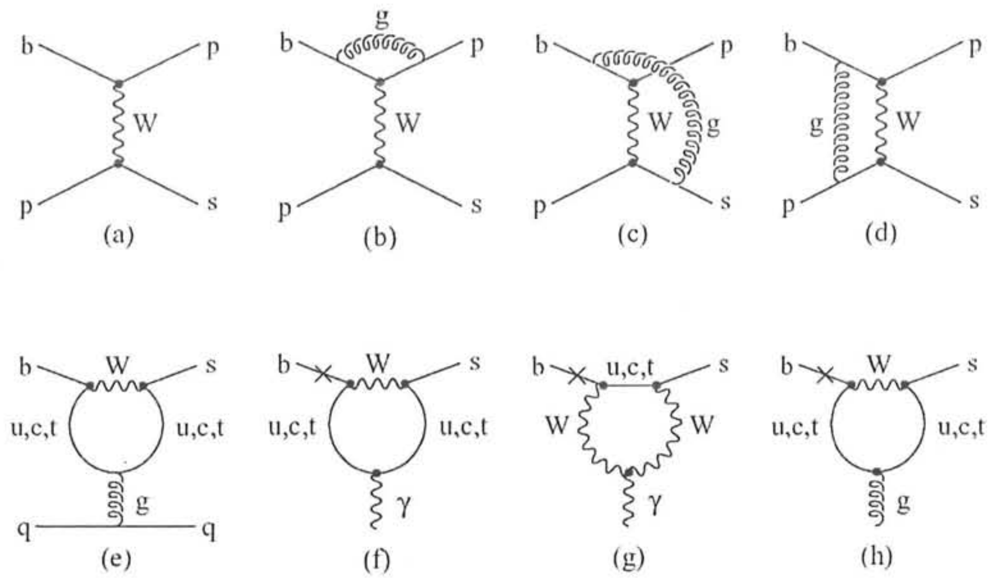

Figure 3-4: The diagrams where the operator basis are: Current-Current diagram (a) with QCD corrections (b), (c), (d); gluon penguin diagram (e), magnetic photon penguin diagrams (f), (g), and magnetic gluon penguin diagram (h)

3.5.4 Spin Symmetry of Heavy Quark

In the limit $m \to \infty$, the Lagrangian L given in Eq. (4.12)

$$L = \overline{h}_v iv \cdot Dh_v + \frac{1}{2m}\overline{h}_v(i \not{D}_\perp)^2 h_v + \frac{g}{4m}\overline{h}_v \sigma^{\mu\nu} G_{\mu\nu} h_v$$

has additional symmetries not present in the full QCD Lagrangian. One such symmetry namely the spin symmetry of heavy quark is reflected in the fact that in the limit $m \to \infty$, Lagrangian becomes

$$L = \overline{h}_v iv \cdot Dh_v \tag{3.18}$$

which makes no reference to the Dirac structure at all which can couple to the spin degrees of h_{+v}.

More explicitly define the spin:

$$S^i = -S_i = -\gamma_5 \not{v}\gamma e_i \tag{3.19}$$

where

$$\begin{aligned} e_i.v_i &= v_\mu e_i^\mu = 0 \\ e_{j_\mu} e_k^\mu &= -\delta_{jk} \end{aligned} \tag{3.20}$$

In the rest frame of h_v

$$\begin{pmatrix} v \to 0 \\ v_\circ = 1 \end{pmatrix}, e_i^\mu = \delta_i^\mu$$

Thus in the rest frame

$$\begin{aligned} S_i &= \gamma_5 \left(\gamma^\circ v_\circ\right) \gamma_\mu \delta_i^\mu \\ &= \gamma^\circ v_\circ \gamma_5 \gamma_i \\ &= \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma_i \end{pmatrix} = -S^i \end{aligned} \tag{3.21}$$

i.e we get the usual definition of the spin. We note that the Lagrangian L given in Eq. (3.13) is invariant under the infinitesimal transformation

$$\begin{aligned} \delta h_{+v} &= i\theta.sh_{+v} \\ \delta\overline{h}_{+v} &= -i\theta.s\overline{h}_{+v} \end{aligned} \tag{3.22}$$

Now the Noether current is given by

$$J^{\mu} = -i\frac{\partial L}{\partial(D_{\mu}h_{+v})}sh_{+v}$$
$$= \overline{h}_{+v}v^{\mu}sh_{+v} \quad (3.23)$$

Hence the spin operator is given by

$$S = \int J^{\circ}(x,t)d^3x$$
$$= v_{\circ}\int \overline{h}_{+v}sh_{+v}d^3x \quad (3.24)$$

We note that

$$[S^i, h_{+v}] = -s_i h_{+v} \quad (3.25)$$

We conclude that the Lagrangian L in Eq. (3.13) is invariant under $SU(2)$ of heavy quark spin symmetry [5].

3.6 Summary

We would finally like to summarize the basic ideas and virtues of HQET, and to re-emphasize the salient points.

(1) HQET describes the static approximation for a heavy quark, covariantly formulates as an effective field theory and allowing for a systematic inclusion of power corrections.

(2) A cruicial and general idea for dealing with QCD effects is the *factorization* of short distance and long-distance dynamics.

(3) The OPE to construct the effective weak Hamiltonians (H_{eff}) factorizes the short-distance scales of order M_W, m_t from the scales of order m_b.

The heavy-quark scale m treated as short-distance scale can be factorized further from the intrinsic long-deistance scale of QCD, Λ_{QCD}. This leads to a systamatic expansion of observables simultaneously in $1/m$ and $\alpha_s(m)$ with often very important simplifications.

With these tools at hand we are in a good position to make full use of the rich experimmental results in the physics of heavy flavours (particularly, focussing on the important case of B physics). We can determine fundamental parameters of the flavour sector and probe electroweak dynamics at the quantum level through $b \to s\gamma$. This will enable us to throughly test the standard model and to learn about new structures and phenomena that are yet to be discoverd.

3.7 Forward-backward asymmetry

In the next chapter we analyze the $B_s \to \gamma l^+ l^-$ due to its relative cleanliness and sensitivity to new physics among the semileptonic rare B-meson decays. Various kinematical distributions of the $B_s \to \gamma l^+ l^-$ decays have been studied in many earlier works. The analiysis in the frame work of the SM can be found in [6] . The new physics has been studied in some models [7] and it has been shown that different observables, like branching ratio, lepton and photon polarization asymmetries, etc., are very sensitive to the physics beyond the SM. In addition to these observables, it is possible to study the lepton pair forward backward asymmetry in the $B_s \to \gamma l^+ l^-$ decay. In addition in a recent work [8] the effect of new physics on the zero of the forward backward asymmetry in the $B_s \to K^* l^+ l^-$ decay has been considered and it is shown that its spectrum is sensitive to the new physics effects. In principle new Wilson coefficients can be determined using measurements of the zero of the the Forward-backward (FB) asymmetry. Due to sensitivity of the zero of the Forward-backward (FB) asymmetry, we incorporate it to $B_s \to \gamma l^+ l^-$ decay.

The forward-backward asymmetry is determined using the

$$A_{FB}(x) = \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{dx d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx d\cos\theta}}{\int_0^1 d\cos\theta \frac{d\Gamma}{dx d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx d\cos\theta}}$$

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Chapter 4

Forward Backword Asymmetry In $B_S \to l^+l^-\gamma$ decay

Among the rare B-meson decays, the semileptonic $B \to l^+l^-\gamma$ $(l = e, \mu, \tau)$ decays are especially intersting due to their relative cleanliness and sensitivity to new physics. $B \to l^+l^-\gamma$ decay induced by $B \to l^+l$ can be in principle serve as a useful process to determine the fundamental parameters of the SM since the only nonperturbative quantity in its theoretical calculation is the decay constant f_{B_S}, which is reliably known. However, in the SM, matrix element of $B \to l^+l^-$decay is proportional to the lepton mass and therefore corresponding branching ratio will be helicity supressed (disscussed in section 1.1). Although $l = \tau$ channel is free from this suppression, its experimental observation is quite difficult due to low efficiency.

The radiative dilepton decay receives various contributions. The main contribution in the case of light leptons comes from the so-called structure-dependent (SD) part, where the photon is emitted from the external quark line as shown in Fig. (4-1 c). Contributions coming from photons attached to charged internal lines are suppressed by $\frac{m_b^2}{M_w^2}$ [1]. The bremsstrahlung contribution (4-1 a and b) due to emission of the photon from the external leptons is suppressed by the mass of the light leptons $l \equiv e, \mu$ and affects the photon energy spectrum only in the low E region [2].

Neglecting the bramsstrahlung contributions, the decay is then governed by the effective Hamiltonian describing the $b \to sl^+l^-$ decay, together with the form factors parameterizing the

$B \to \gamma$ transition.

4.1 Helicity Suppression:

To understand heliciy suppression effect, consider the decay $B^- \to l\bar{l}$ in the B rest frame Fig. (4-2). Since the B has zero spin, the two leptons which are emitted in opposite directions, must have the same helicity so that their spins add to zero, by angular momentum conservation. Weak interactions, however, couple only to the left-handed chiral component $(1-\gamma_5)\overline{u}_l$, i.e. $(V-A)$ coupling, the leptons produced are preferentially left-handed, so that this decay would be completely forbidden in the limit $m_l = 0$. Since, $ml \neq 0$, both positive and negative helicity states are mixed by an amount proportional to the mass, $(1-\gamma_5)\overline{u}_l$ contains a small part of positive helicity, resulting in non-zero decay rates but the amplitude for this process is suppressed by a factor $\frac{m_l^2}{M_B^2}$. The dependence of helicity suppression on the lepton mass is given by

$$\begin{aligned}\text{Helicity Suppression} &\sim 1-\beta_l \\ &= \frac{2m_l^2}{m_B^2+m_l^2}\end{aligned}$$

where β_l is the velocity of the lepton. For $l = \mu$, e the same argument holds. The helicity suppression factor for τ, μ and e is approximately $1/5$, $1/1000$ and $1/50,000,00$ respectively. The experimental confirmation of helicity suppression in $\pi^- \to l^-\overline{\nu}_l$ decays [?] is one of the greatest achievements of the Standard Model.

4.2 Effective Hamiltonian

The most important contribution to $B_s \to l^+l^-\gamma$ stemes from the effective Hamiltonian which induces the pure leptonic process $B_s \to l^+l^-$. The short distance contributions to $b \to sl^+l^-$ decay, comes from Magnetic penguins operators [?]

$$Q_{7\gamma} = \frac{e}{8\pi^2}m_b\overline{s}_i\sigma^{\mu\nu}(1+\gamma_5)b_iF_{\mu\nu} \tag{4.1}$$

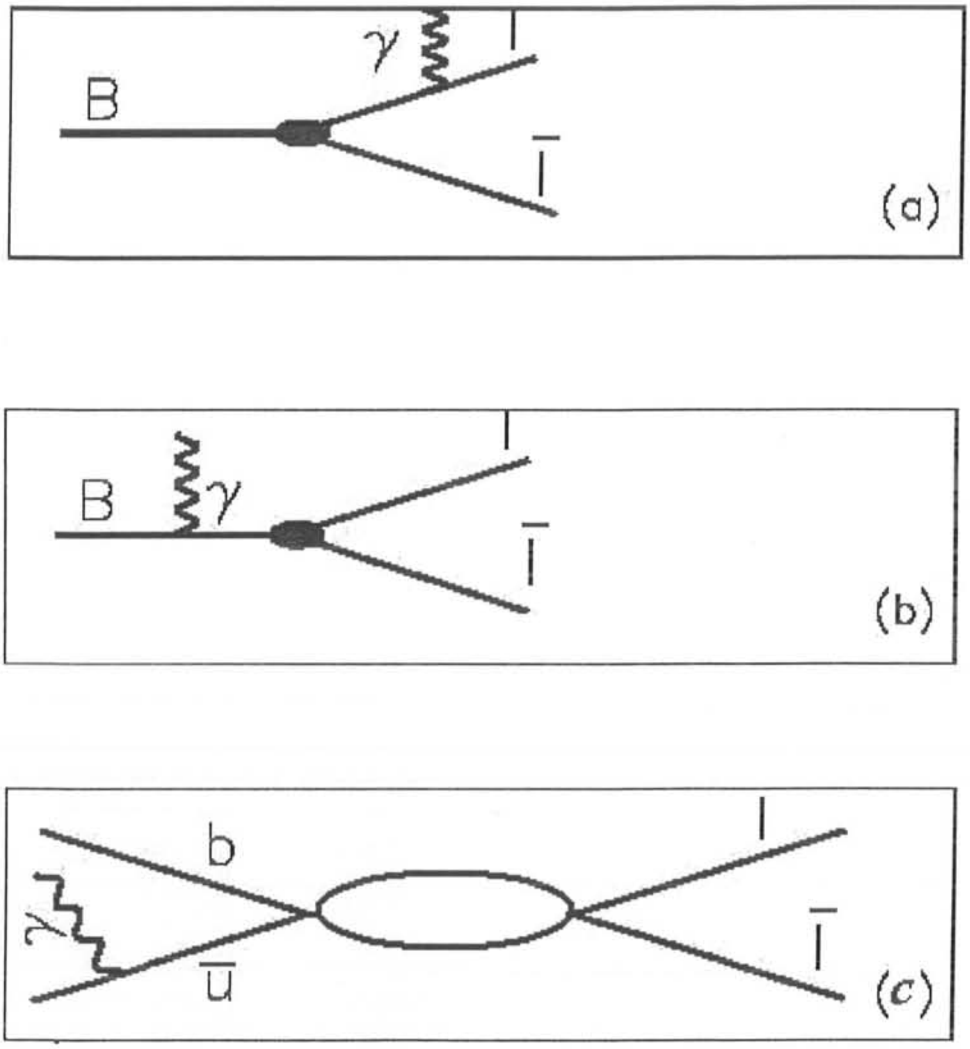

Figure 4-1: (a) and (b) show the Internal Bremsstrahlung contribution , (c) shows the Structure dependent part (SD).

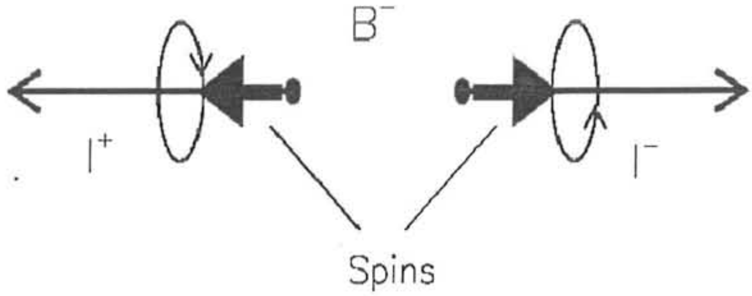

Figure 4-2: Lepton helicity states in $B \to l^+ l^-$

and Semileptonic operators

$$Q_9 = \overline{s}_i \gamma^\mu (1-\gamma_5) b_i (\bar{l}\gamma^\mu l)$$

$$Q_{10} = \overline{s}_i \gamma^\mu (1-\gamma_5) b_i (\bar{l}\gamma^\mu \gamma_5 l) \tag{4.2}$$

Figs. 4-3 and 4-4 correspond to these operators.

The QCD corrected quark level effective hamiltonian in the SM [3] can be written as ($m_s = 0$)

$$\begin{aligned} H_{SM} &= \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts}^* V_{tb} [(C_9^{eff} - C_{10}) \overline{s}_L \gamma^\mu b_L \bar{l}_L \gamma^\mu l_L \\ &+ (C_9^{eff} + C_{10}) \overline{s}_L \gamma^\mu b_L \bar{l}_R \gamma^\mu l_R \\ &- 2C_7^{eff} \overline{s} i \sigma^{\mu\nu} \frac{q^\nu}{q^2} (m_b R) b \bar{l} \gamma^\mu l], \end{aligned} \tag{4.3}$$

where $q = p - k$ and $L, R = (1 \mp \gamma_5)$.

where $L = \frac{(1-\gamma_5)}{2}$ and $R = \frac{(1-\gamma_5)}{2}$ are the chiral projection operators. Thus the amplitude

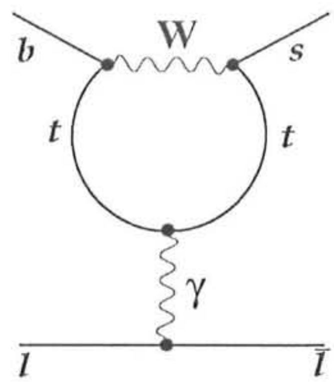

Figure 4-3: This diagram correspond to the semileptonic operators.

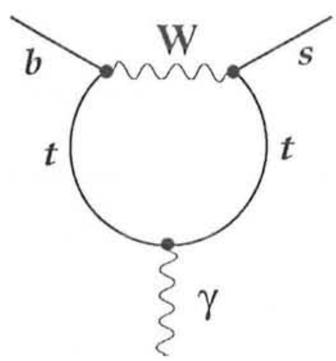

Figure 4-4: This diagram correspond to the magnetic penguins operators.

is

$$M_{SM} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{ts}^*V_{tb}[(C_9^{eff}\bar{l}\gamma^\mu l + C_{10}\bar{l}\gamma^\mu\gamma_5 l) \times \langle\gamma(k) \mid \bar{s}\gamma^\mu(1-\gamma_5)b \mid B(P_B)\rangle$$
$$-\frac{2C_7^{eff}m_b}{q^2}\langle\gamma(k) \mid \bar{s}i\sigma_{\mu\nu}q^\nu Rb \mid B(P_B)\rangle\bar{l}_L\gamma^\mu l_L], \quad (4.4)$$

4.3 Decay kinematics and the matrix element

For the study of the decay $B \to l^+l^-\gamma$, we introduce here the decay kinamatics. $p = (E, \mathbf{P})$, $k = (E_\gamma, \mathbf{k})$, $p_l = (E_l, \mathbf{p}_l)$, $p_{\bar{l}} = (E_{\bar{l}}, \mathbf{p}_{\bar{l}})$, $\epsilon = (0, \sin\theta\cos\phi, \sin\theta\cos\phi, 0)$ are the four-momentua of the charged B-meson, photon, charged lepton, anti lepton and the polarization of photon respectively. The equations of energy-momentum conservation is read as

$$E_B = E_\gamma + E_l + E_{\bar{l}} \quad (4.5)$$

and

$$\mathbf{p} = \mathbf{k} + \mathbf{p}_l + \mathbf{p}_{\bar{l}} \quad (4.6)$$

The scaled energy variable is defined as

$$x \equiv \frac{2p\cdot k}{M_B^2} = \frac{2E_\gamma}{M_B} \quad (4.7)$$

Using the energy-momentum conservation law, in the B-meson rest frame, it is useful to express the scalar products as follows:

$$p\cdot p = p^2 = M_B^2 \quad (4.8)$$
$$k\cdot k = k^2 = 0 \quad (4.9)$$
$$p_l\cdot k = EE_\gamma - \mid \mathbf{p} \mid E_\gamma\cos\theta \quad (4.10)$$
$$p_{\bar{l}}\cdot k = EE_\gamma + \mid \mathbf{p} \mid E_\gamma\cos\theta \quad (4.11)$$
$$p\cdot p_l = p.p_{\bar{l}} = M_B E \quad (4.12)$$
$$p\cdot k = M_B E_\gamma \quad (4.13)$$

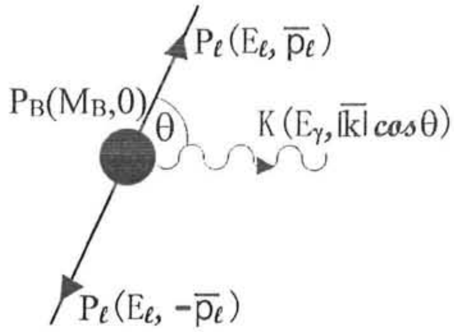

Figure 4-5: This diagram shows the decay kinamatics in B-meson rest frame.

where θ is the angle between the three-momentum vectors of l and the photon in the dilepton centre of mass, as shown in Fig. (4-5).

For the transition to a real photon, the matrix elements are given in [4],

$$\langle\gamma(k) \mid \overline{s}\gamma^{\mu}\gamma_5 b \mid B(P_B)\rangle = ie\epsilon^{*\alpha}(k)[g_{\mu\alpha}(p.k) - p_{\alpha}k^{\mu}]\frac{F_A}{M_{B_s}}, \tag{4.14}$$

$$\langle\gamma(k) \mid \overline{s}\gamma^{\mu}b \mid B(P_B)\rangle = e\epsilon^{*\alpha}(k)\varepsilon_{\mu\alpha\rho\sigma}p^{\rho}k^{\sigma}]\frac{F_V}{M_{B_s}}, \tag{4.15}$$

$$\langle\gamma(k) \mid \overline{s}\sigma_{\mu\nu}\gamma_5 b \mid B(P_B)\rangle(p-k)^{\nu} = e\epsilon^{*\alpha}(k)[g_{\mu\alpha}(p.k) - p_{\alpha}k_{\mu}]F_{TA}, \tag{4.16}$$

$$\langle\gamma(k) \mid \overline{s}\sigma_{\mu\nu}b \mid B(P_B)\rangle(p-k)^{\nu} = ie\epsilon^{*\alpha}(k)\varepsilon_{\mu\alpha\rho\sigma}p^{\rho}k^{\sigma}F_{TV}, \tag{4.17}$$

4.4 Differential Decay Rate

The next task is the calculation of the differential decay rate of $B_s \to l^+l^-\gamma$ decay as a function of dimensionless parameter $x = \frac{2E_\gamma}{M_B}$, where E_γ is the photon energy. In the center of mass (CM) frame of the dileptons l^+l^-, we take $z = \cos\theta$ where θ is the angle between the momentum of the B_s-meson and that of l^-. The decay width is found to be

$$d\Gamma = \frac{1}{2M_B} \mid M \mid^2 d_{LIPS} \tag{4.18}$$

with

$$d_{LIPS} = (2\pi)^4 \delta^4(p_B - p_1 - p_2 - k) \frac{d^3 p_l}{(2\pi)^3 2E_1} \frac{d^3 p_{\bar{l}}}{(2\pi)^3 2E_2} \frac{d^3 k}{(2\pi)^3 2E_\gamma} \tag{4.19}$$

Now solving $\mid M \mid^2$by using matrix element and kinamatics for $l = \mu$,

$$\mid M \mid^2 = \mid MM^\dagger \mid = \frac{G_F^2 \alpha^3 M_B^3}{2^2 \pi} \mid V_{tb} V_{ts}^* \mid^2 x^2 [B_0(x) + B_1(x) \cos\theta + B_{21}(x) \cos^2\theta]. \tag{4.20}$$

Here, we have summed over the spins of the particles in the final state, and have introduced the auxiliary functions

$$B_0(x) = (1 - x + 4\widehat{m}_\mu^2)(F_1 + F_2) - 8\widehat{m}_\mu^2 \mid C_{10} \mid^2 (F_V^2 + F_A^2), \tag{4.21}$$

$$\begin{aligned} B_1(x) &= 8\left(1 - \frac{4\widehat{m}_\mu^2}{1-x}\right)^{1/2} \mathrm{Re}\{C_{10}[C_9^{eff*}(1-x) F_V F_A \\ &\quad + C_7^{eff} \widehat{m}_b (F_V F_{TA} + F_{TA} F_V)]\}, \end{aligned} \tag{4.22}$$

where we expressF_1 and F_2 in the form of F_A and F_V.

$$\begin{aligned} F_1 &= (\mid C_9^{eff} \mid^2 + \mid C_{10} \mid^2) F_V^2 + \frac{4 \mid C_7^{eff} \mid^2 \widehat{m}_b^2}{(1-x)^2} F_{TV}^2 \\ &\quad + \frac{4\,\mathrm{Re}(C_7^{eff} C_9^{eff*}) \widehat{m}_b}{1-x} F_V F_{TV} \end{aligned} \tag{4.23}$$

$$F_2 = (\mid C_9^{eff} \mid^2 + \mid C_{10} \mid^2) F_A^2 + \frac{4 \mid C_7^{eff} \mid^2 \widehat{m}_b^2}{(1-x)^2} F_{TA}^2$$

$$+\frac{4\,\mathrm{Re}(C_7^{eff}C_9^{eff*})\widehat{m}_b}{1-x}F_AF_{TA} \tag{4.24}$$

where $\widehat{m}_i \equiv m_i/M_{B_s}$ and

$$d_{LIPS} = \frac{M_B^2}{2^8\pi^3}\int\int_0^\pi \sqrt{1-\frac{4\widehat{m}_\mu^2}{(1-\frac{E_\gamma}{M_B})^2}}\,x d(\cos\theta)dx \tag{4.25}$$

here

$$E_\gamma = \frac{M_B x}{2} \tag{4.26}$$

and

$$E_l = \frac{M_B(2-x)}{4} \tag{4.27}$$

$$\mid p_l \mid = \frac{M_B}{2}\sqrt{(1-\frac{E_\gamma}{M_B})^2-4\widehat{m}_\mu^2} \tag{4.28}$$

Finally we get the double differential decay rate

$$\frac{d\Gamma}{dx d\cos\theta} = \frac{G_F^2\alpha^3 M_B^5}{2^{11}\pi^4}\mid V_{tb}V_{ts}^* \mid^2 x^3 \times \left(1-\frac{4\widehat{m}_\mu^2}{1-x}\right)^{1/2}[B_0(x)+B_1(x)\cos\theta+B_{21}(x)\cos^2\theta]. \tag{4.29}$$

4.5 Numerical Parameters

We first give the input parameters used in our numerical analysis:

$$\begin{aligned} \mid & V_{tb}V_{ts}^* \mid^2 = 0.045,\ \alpha^{-1}=137,\ G_F = 1.17\times 10^{-5}\ Gev^{-2} \\ M_B &= 5.28\ Gev,\ m_b = 4.8\ Gev,\ m_\mu = 0.105\ Gev, \end{aligned}$$

Table(1) : Values of the SM Wilson coefficients at $\mu \sim m_b$ scale.

C_1	C_2	C_3	C_4	C_5	C_6	C_7^{eff}	C_9	C_{10}
-0.248	$+1.107$	$+0.011$	-0.026	$+0.007$	-0.031	-0.313	$+4.344$	-4.624

The values of the individual Wilson coefficients that appear in the SM are listed in Table (1).

It should be noted here that the value of the Wilson coefficient C_9 in Table (1) corresponds only to the short-distance contributions. C_9 also receives long-distance contributions due to conversion of the real $\bar{c}c$ into lepton pair $\bar{l}l$ and they are usually absorbed into a redefinition of the short-distance Wilson coefficients:

$$C_9^{eff}(\mu) = C_9(\mu) + Y(\mu), \tag{4.30}$$

where

$$\begin{aligned} Y(\mu) &= Y_{reson} + h(\widehat{m}_c, \widehat{s})[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)] \\ &\quad -\frac{1}{2}h(\widehat{m}_b, \widehat{s})(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ &\quad -\frac{1}{2}h(\widehat{m}_s, \widehat{s})(C_3(\mu) + 3C_4(\mu)) \\ &\quad +\frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)), \end{aligned} \tag{4.31}$$

with

$$\begin{aligned} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) &= 0.359, \\ (4C_3 + 4C_4 + 3C_5 + C_6) &= -6.749 \times 10^{-2} \\ (3C_3 + C_4 + 3C_5 + C_6) &= -1.558 \times 10^{-3} \\ (C_3 + 3C_4) &= -6.594 \times 10^{-2} \end{aligned} \tag{4.32}$$

where we have introduced the notation $\widehat{s} = q^2/m_b^2, \widehat{m}_i = m_i/m_b$, while $h(\widehat{m}_i, \widehat{s})$ arises from the one-loop contributions of the four-quark operators O_1 and O_2 and is given by [5]

$$
\begin{aligned}
h(\widehat{m}_i,\widehat{s}) &= -\frac{8}{9}\ln(\widehat{m}_i)+\frac{8}{27}+\frac{4}{9}y_i-\frac{2}{9}(2+y_i)\sqrt{|1-y_i|} \\
&\times\left\{\Theta(1-y_i)(\ln\left(\frac{1+\sqrt{1-y_i}}{1-\sqrt{1-y_i}}\right)-i\pi)+\Theta(y_i-1)2\arctan\frac{1}{\sqrt{y_i-1}}\right\}
\end{aligned}
\tag{4.33}
$$

where $y_i = 4\widehat{m}_i^2/\widehat{s}$.

4.5.1 Long-Distance Contributions

In addition to the short-distance interaction defined by Eq. (4.33) it is possible to take into account long-distance effects, associated with real $c\bar{c}$ resonances in the intermediate states, i.e. with the reaction chain $B \to X_s + V(c\bar{c}) \to X_s l\bar{l}$. This can be accomplished in an approximate manner through the Breit-Wigner substitution [6].

$$
Y_{reson} = -\frac{3\pi}{\alpha^2}\sum_{V=j/\psi,\psi',\ldots}\frac{\widehat{m}_V Br(V\to l\bar{l})\widehat{\Gamma}^V_{total}}{\widehat{s}-\widehat{m}_V^2+i\widehat{m}_V\widehat{\Gamma}^V_{total}} \tag{4.34}
$$

where the properties of the vector mesons are summarized in Table (2). There are six

TABLE(2) : Charmonium ($c\bar{c}$) masses and widths [7]

$Meson$	$Mass(Gev)$	$Br(V \to l\bar{l})$	Γ_{tot} (Mev)
$J/\psi(1S)$	3.097	6.0×10^{-2}	0.088
$\psi(2S)$	3.686	8.3×10^{-3}	0.277
$\psi(3770)$	3.770	1.1×10^{-5}	23.6
$\psi(4040)$	4.040	1.4×10^{-5}	52
$\psi(4160)$	4.159	1.0×10^{-5}	78
$\psi(4415)$	4.415	1.1×10^{-5}	43

known resonances in the $c\bar{c}$ system that can contribute to the decay modes $B \to X_s e^+e^-$ and $B \to X_s\mu^+\mu^-$. Recall that the Wilson coefficient C_9^{eff} depands on x via $q^2 = M_B^2(1-x)$.

4.6 Forward-Backward Asmmetry within the SM

The term odd in cosθ in Eq. (4.29) produces a Forward-Backward asymmetry, defined as

$$A_{FB}(x) = \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{dx d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx d\cos\theta}}{\int_0^1 d\cos\theta \frac{d\Gamma}{dx d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d\Gamma}{dx d\cos\theta}} \tag{4.35}$$

which is given by

$$A_{FB}(x) = 3\left(1 - \frac{4\widehat{m}_\mu^2}{1-x}\right)^{1/2} \times \frac{\mathrm{Re}\{C_{10}[C_9^{*eff}(1-x)F_V F_A + C_7^{eff}\widehat{m}_b(F_V F_{TA} + F_A F_{TV})]\}}{[(F_1+F_2)(1-x+2\widehat{m}_\mu^2) - 6\widehat{m}_\mu^2 \mid C_{10}\mid^2 (F_V^2 + F_A^2)]} \tag{4.36}$$

Utilizing Large Energy Effective Theory (LEET) form factors which are given in [8],

$$\begin{aligned} F_{TA} &= F_{TV} = 0.115. \\ F_A &= 0.09 \text{ and } F_V = 0.105. \end{aligned} \tag{4.37}$$

We plot the FB symmetry as a function of the scaled photon energy x, by using above form factors in Figs. (4-6) and (??).

The $1/M_B$ and $1/E$ corrections to rhe LEET form factors are well parametrized by a particularly simple formula:

$$F_i(E_\gamma) = \beta\frac{f_B M_B}{\Delta + E_\gamma},\ i = A,\ V,\ TA,\ TV. \tag{4.38}$$

The numerical parameters are listed in Table (3) [4].

Table(3) : parameters of the $B \to \gamma$ form factors as defined in Eq. (4.38)

$Parameter$	F_V	F_{TV}	F_A	F_{TA}
$\beta(Gev)^{-1}$	0.28	0.30	0.26	0.33
$\Delta(Gev)$	0.04	0.04	0.30	0.30

Finally, in our numerical analysis we have considered only the final state lepoton as being the muon (μ) and plotted the FB symmetry as a function of the scaled photon energy x, by using form factors of Eq. (4.38) in Fig. (4-7).

4.7 Calculation beyond the SM

The general (model independent) effective hamiltonian is the combination of the SM contribution and the contribution from the local four-Fermi interactions,

$$H_{eff} = H_{SM} + H_{NEW} \tag{4.39}$$

where H_{SM} is the SM part and is given by

$$\begin{aligned} H_{SM} &= \frac{G_F\alpha}{\sqrt{2}\pi}V_{ts}^*V_{tb}[(C_9^{eff} - C_{10})\overline{s}_L\gamma^\mu b_L\bar{l}_L\gamma^\mu l_L \\ &+(C_9^{eff} + C_{10})\overline{s}_L\gamma^\mu b_L\bar{l}_R\gamma^\mu l_R \\ &-2C_7^{eff}\overline{s}i\sigma^{\mu\nu}\frac{q^\nu}{q^2}(m_sL + m_bR)b\bar{l}\gamma^\mu l], \end{aligned} \tag{4.40}$$

There are ten independent local four-Fermi interactions which may contribute to the process. H_{NEW} is a function of the coefficients of local four-Fermi interactions and is defined as

$$\begin{aligned} H_{NEW} &= \frac{G_F\alpha}{\sqrt{2}\pi}V_{ts}^*V_{tb}[C_{LL}\overline{s}_L\gamma^\mu b_L\bar{l}_L\gamma^\mu l_L \\ &+C_{LR}\overline{s}_L\gamma^\mu b_L\bar{l}_R\gamma^\mu l_R \\ &+C_{RL}\overline{s}_R\gamma^\mu b_R\bar{l}_L\gamma^\mu l_L \\ &+C_{RR}\overline{s}_R\gamma^\mu b_R\bar{l}_R\gamma^\mu l_R \\ &+C_{LRLR}\overline{s}_L b_R\bar{l}_L l_R \\ &+C_{RLLR}\overline{s}_R b_L\bar{l}_L l_R \\ &+C_{LRRL}\overline{s}_L b_R\bar{l}_R l_L \\ &+C_{RLRL}\overline{s}_R b_L\bar{l}_R l_L \end{aligned}$$

$$+C_T\bar{s}\sigma_{\mu\nu}b\bar{l}\sigma^{\mu\nu}l$$
$$+iC_{TE}\bar{s}\sigma_{\mu\nu}b\bar{l}\sigma_{\alpha\beta}l\epsilon^{\mu\nu\alpha\beta}] \tag{4.41}$$

where $L = \frac{(1-\gamma_5)}{2}$ and $R = \frac{(1-\gamma_5)}{2}$ are the chiral projection operators. In Eq. (4.41), C_X are the coefficients of the four-Fermi interactions with $X = LL, LR, RL, RR$ describing vector, $X = LRLR, RLLR, LRRL, RLRL$ scalar and $X = T, TE$ tensor type interactions [9].

Having established the general form of the effective Hamiltonian, the next step is to calculate the matrix element of the $B_s \to \gamma l^+ l^-$ decay which can be written using the Eqs. (4.42 to 4.47) [10]

$$\langle\gamma(k) \mid \bar{s}\gamma^\mu(1 \mp \gamma_5)b \mid B(P_B)\rangle = \frac{e}{M_B^2}\{\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}q^\lambda k^\sigma g(q^2)$$
$$\pm i[\varepsilon^{*\mu}(kq) - (\varepsilon^* q)k^\mu]f(q^2)\}, \tag{4.42}$$

$$\langle\gamma(k) \mid \bar{s}\sigma^{\mu\nu}b \mid B(P_B)\rangle = \frac{e}{m^2}\epsilon_{\mu\nu\lambda\sigma}[G\varepsilon^{*\lambda}k^\sigma + H\varepsilon^{*\lambda}q^\sigma + L(\varepsilon^* q)q^\lambda k^\sigma], \tag{4.43}$$

here, ε^* and k are the four vector polarization and momentum of the photon, respectively, $q = p_B - k$ is the momentum transfer, p_B is the momentum of the B meson and $g(q^2)$, $f(q^2)$, $G(q^2)$, $H(q^2)$, $L(q^2)$ are the $B_s \to \gamma$ transition form factors. The matrix element $\langle\gamma(k) \mid \bar{s}\sigma^{\mu\nu}\gamma_5 b \mid B(P_B)\rangle$ can be obtained from Eq. (4.43) using the identity

$$\sigma_{\mu\nu} = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5. \tag{4.44}$$

The matrix elements $\langle\gamma(k) \mid \bar{s}(1 \mp \gamma_5)b \mid B(P_B)\rangle$ and $\langle\gamma(k) \mid \bar{s}\sigma_{\mu\nu}q^\nu b \mid B(P_B)\rangle$ can be obtained from Eqs. (4.42) and (4.43) by multiplying them with q^ν and q^μ respectively, as a result of which we get

$$\langle\gamma(k) \mid \bar{s}(1 \mp \gamma_5)b \mid B(P_B)\rangle = 0 \tag{4.45}$$

$$\langle\gamma(k) \mid \bar{s}\sigma^{\mu\nu}q^\nu b \mid B(P_B)\rangle = \frac{e}{m^2}i\epsilon_{\mu\nu\alpha\beta}q^\nu\varepsilon^{*\alpha}k^\beta G \tag{4.46}$$

The matrix element $\langle \gamma(k) \mid \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \mid B(P_B)\rangle$ can be written in terms of the two form factors $f_1(q^2)$ and $g_1(q^2)$.

$$\langle \gamma(k) \mid \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b \mid B(P_B)\rangle = \langle \gamma(k) \mid \bar{s}\sigma^{\mu\nu}b \mid B(P_B)\rangle g_1(q^2) + i[\varepsilon^*_\mu(kq) - (\varepsilon^* q)k_\mu]f_1(q^2)\}. \tag{4.47}$$

These above equations allow us to express G, H and L in terms of f_1 and g_1.

The amplitude can be written as

$$M = \frac{G_F\alpha}{4\sqrt{2}\pi}V_{ts}^*V_{tb}\frac{e}{m^2}\left\{\begin{array}{c} \bar{l}\gamma^\mu(1-\gamma_5)l\left[A_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + iA_2\left(\varepsilon^*_\mu(kq) - (\varepsilon^* q)k_\mu\right)\right] \\ +\bar{l}\gamma^\mu(1+\gamma_5)l\left[B_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + iB_2\left(\varepsilon^*_\mu(kq) - (\varepsilon^* q)k_\mu\right)\right] \\ +i\epsilon_{\mu\nu\alpha\beta}\bar{l}\sigma^{\mu\nu}l\left[G\varepsilon^{*\alpha}k^\beta + H\varepsilon^{*\alpha}q^\beta + L(\varepsilon^* q)q^\alpha k^\beta\right] \\ +i\bar{l}\sigma_{\mu\nu}l\left[\begin{array}{c} G_1(\varepsilon^{*\mu}k^\nu - \varepsilon^{*\nu}k^\mu) + H_1(\varepsilon^{*\mu}q^\nu - \varepsilon^{*\nu}q^\mu) \\ +L_1(\varepsilon^* q)(q^\mu k^\nu - q^\nu k^\mu) \end{array}\right] \end{array}\right\} \tag{4.48}$$

where

$$\begin{aligned}
A_1 &= \frac{1}{q^2}(C_{BR}+C_{SL})g_1 + (C_{LL}^{tot}+C_{RL})g, \\
A_2 &= \frac{1}{q^2}(C_{BR}-C_{SL})f_1 + (C_{LL}^{tot}-C_{RL})f, \\
B_1 &= \frac{1}{q^2}(C_{BR}+C_{SL})g_1 + (C_{LR}^{tot}+C_{RR})g, \\
B_2 &= \frac{1}{q^2}(C_{BR}-C_{SL})f_1 + (C_{LR}^{tot}-C_{RR})f, \\
G &= 4C_T g_1, \\
L &= -4C_T\frac{1}{q^2}(f_1+g_1), \\
H &= N(qk), \\
G_1 &= -8C_{TE}g_1, \\
N_1 &= 8C_{TE}\frac{1}{q^2}(f_1+g_1), \\
H_1 &= N_1(qk),
\end{aligned}$$

where C_{BR} and C_{SL} correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, while C_{LL} and C_{LR} are expressed in the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$,respectively. Therefore, writing

$$C_{LL}^{tot} = C_9^{eff} - C_{10} + C_{LL},$$

$$C_{LR}^{tot} = C_9^{eff} + C_{10} + C_{LR},$$

we observe that C_{LL}^{tot} and C_{LR}^{tot} cantain the contributions from the SM and also from the new physics.

For double differential decay the amplitude square is found to be $(m_s = 0)$,

$$\begin{aligned}
\mid M \mid^2 = \mid MM^\dagger \mid = \frac{G_F^2 \alpha^3}{2^6 \pi M_B^3} \mid V_{tb} V_{ts}^* \mid^2 \{ & x^2[-16r^2 A_1^2 + 3x^2 A_1^2 - 8x A_1^2 - B' \cos(2\theta) A_1^2 \\
& +6A_1^2 + 8A_2 A(x-1)\cos(\theta) A_1 - 32 A_2^2 r^2 + 4A_2^2 x) M_B^5 + x^2[-16\widehat{m}_\mu^2 B_1^2 + 3x^2 B_1^2 \\
& +8x B_1^2 - B' \cos(2\theta) B_1^2 - 8B_2 A(x-1)\cos(\theta) B_1 - 6B_2^2 + 32 B_2^2 \widehat{m}_\mu^2 + 4B_2^2 x] M_B^5 \\
& +64(A_1 B_1 + 2A_2 B_2) m_\mu^2 x^2 M_B^3 + 32(G-H) m_\mu x^2 [A_1(x-1) + \frac{3}{2} A_2 A \cos(\theta)] M_B^3 \\
& +32(G-H) m_\mu x^2 [B_1(x-1) - \frac{3}{2} B_2 A \cos(\theta)] M_B^3 2(4(48 m_\mu^2 + M_B^2) B')(x-1) H^2 \\
& -4G(48 m_\mu^2 + M_B^2 B') x H + L M_B^2 (16 m_\mu^2 + M_B^2 B')(2G - L M_B^2 (x-1)) x^2 \\
& +2M_B^2 B' (L^2 (x-1) M_B^4 - 2G^2 - 2H^2 + G(4H - 2L M_B^2)) x^2 \cos^2(\theta) \\
& +M_B^2 (x-2)^2 [4(x^2 - 3x + 3) H^2 - 4Gx(2x-3) H + (L^2 (x-1) M_B^4 - 2GL M_B^2 \\
& +4G^2) x^2)] M_B + 8(-(G_1 - H_1)^2 B' x^2 \cos^2(\theta) M_B^3 \\
& +(x-2)^2 (3H_1^2 + 3(G_1 - H_1) x H_1 + (G_1 - H_1)^2 x^2 M_B^3 \\
& +4H_1 (12 m_\mu^2 + \frac{1}{4} M_B^2 B')(-x H_1 + H_1 + G_1 x) M_B)) \}
\end{aligned} \tag{4.49}$$

where

$$\begin{aligned}
A &= \sqrt{((x-2)^2 - 16\widehat{m}_\mu^2)} \\
B' &= (16\widehat{m}_\mu^2 - (x-2)^2)
\end{aligned}$$

The dalitz density using above Eq. (4.18),

$$d\Gamma = \frac{1}{2M_B} \mid M \mid^2 d_{LIPS}$$

here

$$d_{LIPS} = \frac{M_B}{2^9\pi^3}\int\int_0^{\pi}\frac{A}{\sqrt{(x-2)^2}}xd(\cos\theta)dx \tag{4.50}$$

we get

$$\begin{aligned}
\frac{d\Gamma}{dxd\cos\theta} &= \frac{G_F^2\alpha^3}{2^{15}\pi^4M_B^2}\frac{A}{\sqrt{(x-2)^2}}x \mid V_{tb}V_{ts}^* \mid^2 \{x^2[-16r^2A_1^2+3x^2A_1^2-8xA_1^2-B^{'}\cos(2\theta)A_1^2 \\
&+6A_1^2+8A_2A(x-1)\cos(\theta)A_1-32A_2^2r^2+4A_2^2x)M_B^5+x^2[-16r^2B_1^2+3x^2B_1^2 \\
&+8xB_1^2-B^{'}\cos(2\theta)B_1^2-8B_2A(x-1)\cos(\theta)B_1-6B_2^2+32B_2^2r^2+4B_2^2x]M_B^5 \\
&+64(A_1B_1+2A_2B_2)m_\mu^2x^2M_B^3+32(G-H)m_\mu x^2[A_1(x-1)+\frac{3}{2}A_2A\cos(\theta)]M_B^3 \\
&+32(G-H)m_\mu x^2[B_1(x-1)-\frac{3}{2}B_2A\cos(\theta)]M_B^3 2(4(48m_\mu^2+M_B^2)B^{'})(x-1)H^2 \\
&-4G(48m_\mu^2+M_B^2B^{'})xH+LM^2(16m_\mu^2+M_B^2B^{'})(2G-LM_B^2(x-1))x^2 \\
&+2M_B^2B^{'}(L^2(x-1)M_B^4-2G^2-2H^2+G(4H-2LM_B^2))x^2\cos^2(\theta) \\
&+M_B^2(x-2)^2[4(x^2-3x+3)H^2-4Gx(2x-3)H+(L^2(x-1)M_B^4-2GLM_B^2 \\
&+4G^2)x^2)]M_B+8(-(G_1-H_1)^2B^{'}x^2\cos^2(\theta)M_B^3 \\
&+(x-2)^2(3H_1^2+3(G_1-H_1)xH_1+(G_1-H_1)^2x^2M_B^3 \\
&+4H_1(12m_\mu^2+\frac{1}{4}M_B^2B^{'})(-xH_1+H_1+G_1x)M_B))\}
\end{aligned} \tag{4.51}$$

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4.8 Forward-Backward Asymmetry - Prob to New physics

Using the farmula of forward-backward asymmetry which is given in Eq. (4.35),

$$A_{FB}(x) = \frac{\int_0^1 d\cos\theta\frac{d\Gamma}{dxd\cos\theta} - \int_{-1}^0 d\cos\theta\frac{d\Gamma}{dxd\cos\theta}}{\int_0^1 d\cos\theta\frac{d\Gamma}{dxd\cos\theta} + \int_{-1}^0 d\cos\theta\frac{d\Gamma}{dxd\cos\theta}}$$

we get

$$A_{FB}(x) = 6(x-2)\left(1-\frac{16\widehat{m}_\mu^2}{(x-2)^2}\right)^{1/2} M_B^2 x^2$$
$$\times \mathrm{Re}[\frac{A_1 A_2 A' - B_1 B_2 A' + 6(A_2 - B_2)(G-H)m_\mu}{8H[72(x-1)\widehat{m}_\mu^2 + M_B^2(C+D)] - 16x^2(3A_1 + B_1)m_\mu(x-1)M_B^2 + J}] \quad (4.52)$$

where

$$\begin{aligned} J &= GH[36m_\mu^2 + M_B^2 E] + 8M_B^6 L^2 x^2[x^2(x-5) - 4(1-2x) + 10\widehat{m}_\mu^2(1-x)] \\ &+ M_B^4 x^2[(A_1^2 + B_1^2)(7 - 16\widehat{m}_\mu^2 + 4x^2 - 10x) - (A_2^2 + B_2^2)(9 - 48\widehat{m}_\mu^2 + 6x)] \\ &+ 48\widehat{m}_\mu^2[L^2(1-x) + 2(A_1 B_1 + 2A_2 B_2)] + M_B^2 x^2[8(G^2 + G_1^2)F + 16GLI \\ &+ 48Gm_\mu(x-1)(A_1 + B_1)] - 16G_1 H_1 x(M_B^2 E - 36\widehat{m}_\mu^2) + 8H_1^2[72m_\mu(x-1) \\ &- M_B^2(C+D)]. \end{aligned}$$

and

$$\begin{aligned} A' &= M_B^2(x-1) \\ C &= 8(x^2 + 3x - 3)\widehat{m}_\mu^2 \\ E &= 4(2x+3)\widehat{m}_\mu^2 + (x-3)(x-2)^2 \\ F &= 8\widehat{m}_\mu^2 + (x-2)^2 \\ I &= 6m_\mu^2 + M_B^2(10\widehat{m}_\mu^2 - (x-2)^2). \end{aligned}$$

To make some numerical predictions, we also need the explicit forms of the form factors g, f, g_1 and f_1. The form factors F_V, F_A, F_{TV} and F_{TA} are dimensionless, and related to g, f, g_1 and f_1 by

$$F_V = \frac{g}{M_B}, F_A = \frac{f}{M_B}$$
$$F_{TV} = \frac{-g_1}{M_B^2}, F_{TA} = \frac{-f_1}{M_B^2} \quad (4.53)$$

and the numerical values of these dimensionless form factors are given in Eq. (4.37)

As for the values of the new Wilson coefficients, they are the free parameters in this work, but it is possible to establish ranges out of experimentally measured branching ratios of the semileptonic and also purely leptonic rare B-meson decays

$$BR(B \rightarrow Kl^+l^-) = 0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6},$$
$$BR(B \rightarrow K\mu^+\mu^-) = 0.9^{+1.3}_{-0.9} \pm 0.1) \times 10^{-6},$$

reported by Belle and Babar collaborations [12]. It is now also available an upper bound of pure leptonic rare B-decays in the $B^o \rightarrow \mu^+\mu^-$ mode [13].

$$BR(B^o \rightarrow \mu^+\mu^-) \leq 2.0 \times 10^{-7}.$$

Being in accordance with this upper limit and also the above mentioned measurments of the branching ratios for the semileptonic rare B-decays, we take in this work all new Wilson coefficients as real and varying in the region -4 $\leq C_X \leq$ 4 [10]. In Figs. (4-9), (4-10) and (4-11), we have plotted the FB asymmetry for the values of the New Wilsons coefficients as $C_{LL} = C_{LR} = C_{RL} = C_{RR} = 3$ and $C_T = -0.3$, $C_{TE} = -0.1$ without $c\bar{c}$ resonances. For $c\bar{c}$ resonanceswe have plotted in Figs. (4-12, 4-13 and 4-14).

4.9 Comparison and Discussion

We have calculated the Forward-Backward asymmetry $A_{FB}(x)$ and have plotted it as a function of scaled photon energy x in Figs. (4-6) and (4-8) (without and with resonances respectively) using the LEET form factors as defined in Eq. (4.37). In the Fig. (4-7), we have used $1/M_B$

and $1/E$ corrections to the LEET form factors. From Fig. (4-8) one infers an intersting feature of $A_{FB}(x)$ in the SM i.e. for a given photon energy $x = x_o$ and far from the $c\bar{c}$ resonances, the Forward -Backward asymmetry vanishes. As can be seen from Figs. (4-6 and 4-7), $1/M_B$ corrections to the form factors, shift the location of the zero by only a few percent, but do not change the qualitative picture of the asymmetry.

In the second part of the calculation, new physics contributions were included and using the explicit expression of the FB asymmetry given in the previous section, the dependence of the zero of the FB asymmetry on the various new Wilson coefficients was observed. Our SM value of the zero of the FB asymmetry can be seen from Figs. (4-9, 4-10 and 4-11) and (4-12, 4-13 and 4-14) without resonances and with resonances respectively, substantially changed for different choices of the new Wilson coefficients.

From our analysis we have demonstrated that the zero of the FB asymmetry will not only serve as a valuable test of the SM, but will be a useful probe of any possible new physics. One can also pick the values of new Wilsons coefficients, via the experimental data of the zero of the FB asymmetry that are expected by new facilities to explore B physics in near future, like the LHC-B experiment at CERN and BTev at FERMILAB.

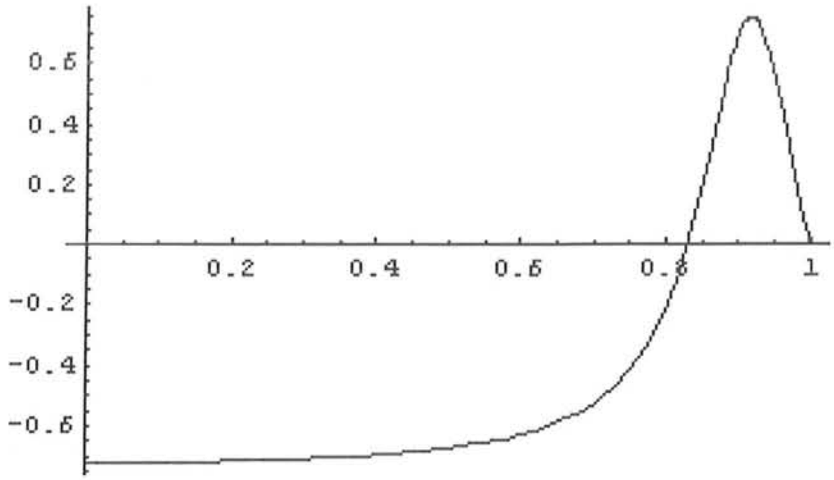

Figure 4-6: The SM prediction for the FB asymmetry of μ^- in the decay $B_s \to \gamma\mu^-\mu^+$ without resonances as a function of x scaled photon energy, utilizing the leading order LEET form factors.

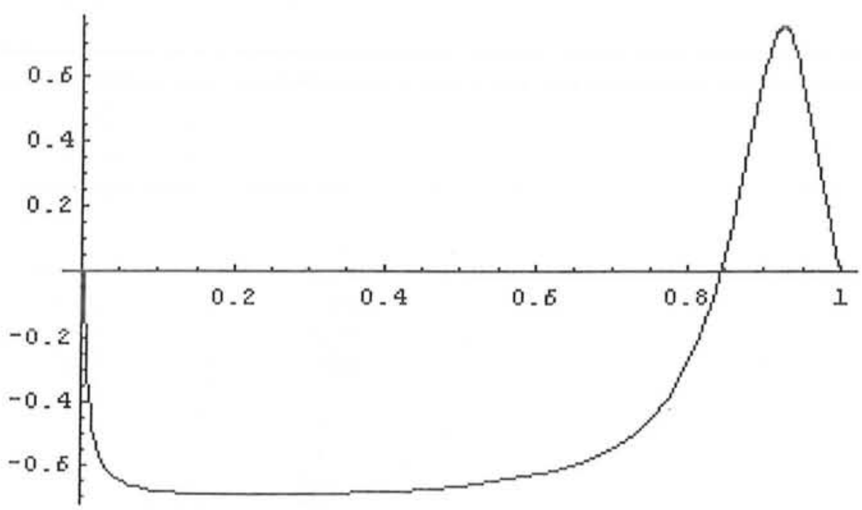

Figure 4-7: SM prediction for the FB asymmetry of μ^- in the decay $B_s \to \gamma\mu^-\mu^+$ as a function of x scaled photon energy, utilizing the form factors of Eq4.38.

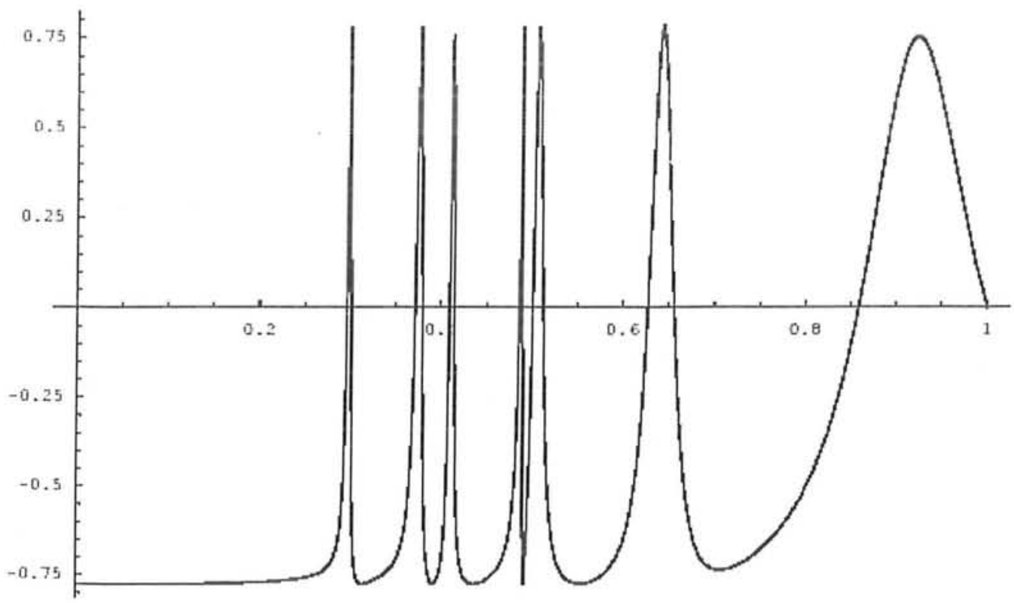

Figure 4-8: FB asymmetry in the decay $B_s \to \gamma\mu^-\mu^+$ as a function of x scaled photon energy with $c\bar{c}$ resonances.

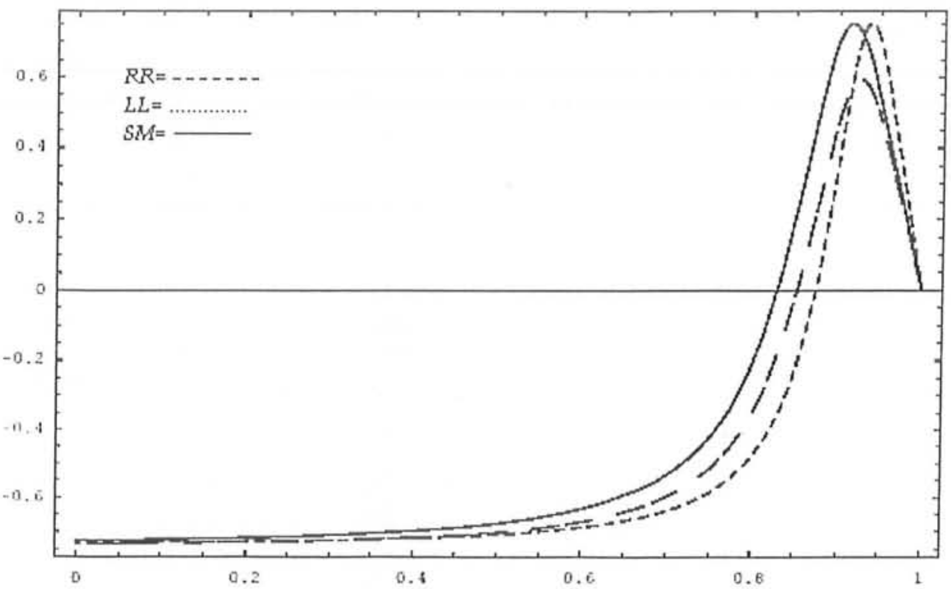

Figure 4-9: The FB asymmetry in the decay $B_s \to \gamma\mu^-\mu^+$ using $CLL = CRR = 3$ without resonances.

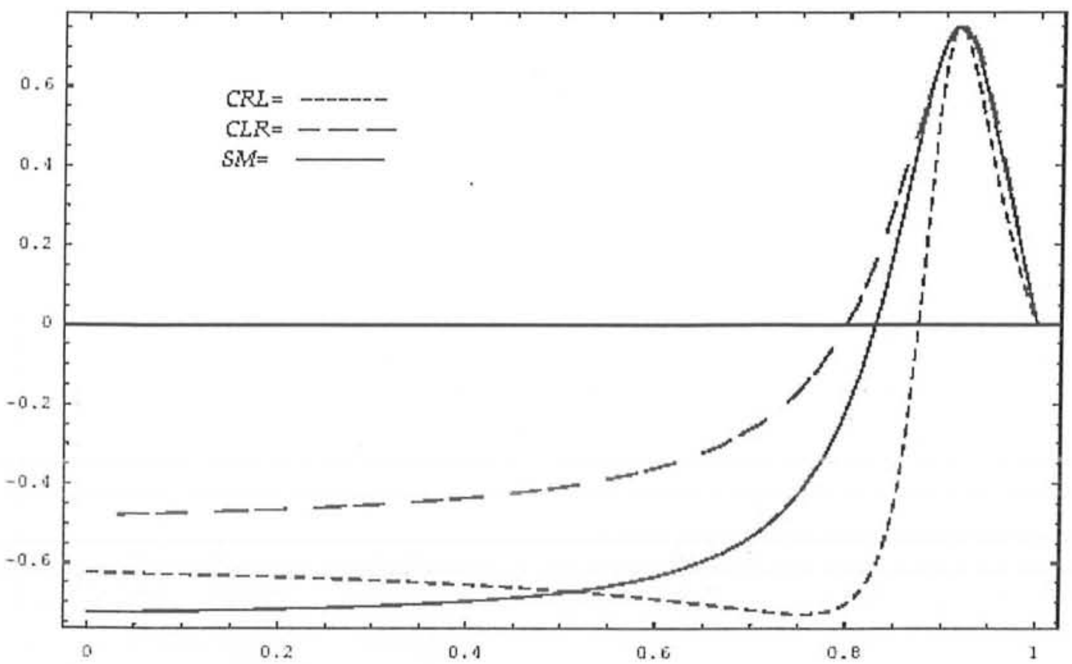

Figure 4-10: The FB asymmetry in the decay $B_s \to \gamma\mu^-\mu^+$ using $CLR = CRL = 3$ without resonances.

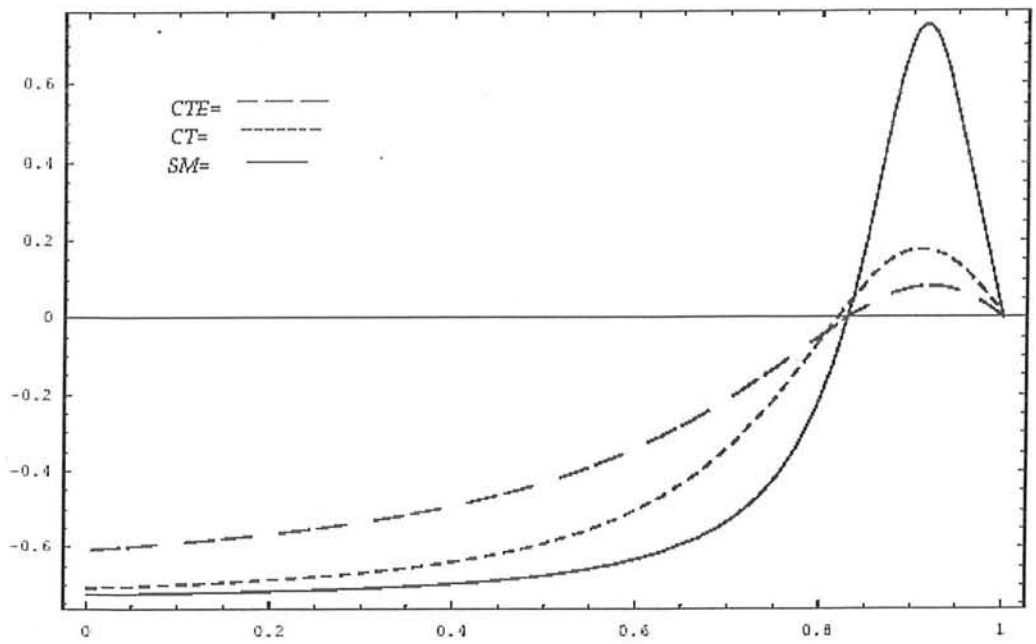

Figure 4-11: The FB asymmetry in the decay $B_s \to \gamma\mu^-\mu^+$ using $CT = -0.1$ and $CTE = -0.3$ without resonances.

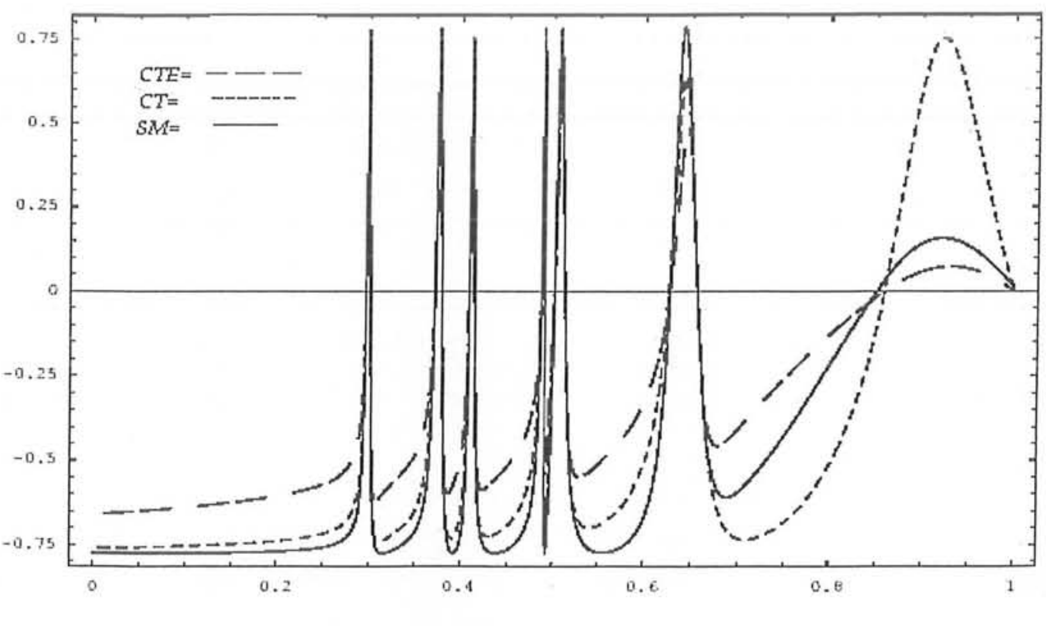

Figure 4-12: The FB asymmetry in the decay $B_s \to \gamma\mu^-\mu^+$ using $CT = -0.1$ and $CTE = -0.3$ with resonances.