# Standard Model Higgs Inflation

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# Certificate

It is certified that the work contained in this dissertation was carried out by **Ms. Noor-ul-Ain** for his M.Phil degree under my supervision.

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*“Do not the unbelievers see that the heavens and the earth was a closed-up mass then WE clove them asunder?"*

***(Al-Anbya, 21:30, Quarn)***

*“And WE have built the heavens with OUR own hands; and, verify, it is WE who are steadily expanding it."*

***(Adh-Dhariyat, 51:47, Quarn)***

❃ ❃ ❃ ❃ ❃ ❃ ❃ ❃

*To*

*the Lord of the worlds*

**THE SUPREME CREATOR**

*my source of inspiration*

**Prof. Riazuddin**

**Prof. Fayyazuddin**

*and*

**My Loving Parents**

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Glory is to GOD, the keeper of wisdom, WHO created this beautiful universe in discipline with symmetry, homogeneity and isotropy, yet it is so mysterious that the generations of inquiring minds have failed to unveil its realities. I thank HIM for providing me with an opportunity to probe into the secrets of the beginning.

May GOD bestow HIS countless blessings upon the ***Holy Prophet (Peace Be Upon Him)***, the Seal of the Prophethood, mercy for all worlds.

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*Noor-ul-Ain*

**Abstract**

In this work, the possibility of the Standard Model Higgs inflation is considered. Similar to the other models of inflation, if the minimal coupling of Higgs with gravity is considered, it is necessary to assume an extremely small self-coupling of Higgs (of the order of $10^{-13}$), which leads to the required mass limit of the Higgs particle below the acceptable range. In order to solve the small self-coupling problem of the Higgs particle, it is required to add a non-minimal coupling term, $\xi\phi^2\mathscr{R}/2$, with a very large coupling constant, $\xi = 10^4$. Calculations are made in the Einstein frame using a conformal transformation. The classical results for inflationary parameters are also revised which come out to be independent of the Higgs mass and are in good agreement with the cosmological data. However, the quantum analysis is not only strongly affected by the various parameters of the Standard Model but also the energy scale of inflation becomes larger than the cutoff scale, which is a challenge in this analysis. To resolve this challenge, the Higgs field is perturbed around a non-zero background field value. Consequently, the energy scale of inflation comes within the cutoff scale range.

# Contents

# List of Figures

# Chapter 1

# Introduction

Cosmology is one of the oldest sciences known to mankind. Man has always been curious to know more about his position and neighbours in space. In the 20th century advances were made in the field of cosmology and a theory of Big Bang was developed which gave a satisfactory explanation of the beginning and time evolution of the universe. Despite of all its accuracy the standard model of cosmology was unable to account for certain observations like the large scale homogeneity, flatness of the universe, absence of magnetic monopoles etc. In order to account for all of these observed facts, a set of initial conditions was required which was to be put in by hands.

To counter the problem of initial conditions, an American physicist, Alan Guth gave the idea of Cosmic Inflation in 1981. Cosmic inflation is a rapid exponential expansion of the early universe, driven by vacuum energy density having a negative pressure . It appeared about $10^{-36}$ seconds after the event of Big Bang and lasted for some time after that. During this period, universe expanded by a huge amount doubling in size after every $10^{-34}$ seconds, which is the value of the time constant of the exponential expansion. After the inflationary period, the universe continued to expand at a slower rate.

The idea of cosmic inflation is a modification of the standard Big Bang theory. Inflationary theory explains why the temperatures and curvatures of different regions in the universe are so nearly equal, and it predicts that the total curvature of space at constant global time is zero. Most strikingly, inflation allows us to calculate the minute differences in temperature of different regions, from quantum fluctuations during the inflationary era and these predictions have also been confirmed by experimental observations. Nowadays, no theory of universe is complete without a reference to inflation. Inflation achieved this success not only because it resolves many puzzles about the nature of the Universe, but also because it did so using the grand unified theories (GUTs) along with quantum field theory developed by particle physicists. Thus, inflation is sort of a marriage of particle physics and cosmology and this marriage seems to provide an explanation of how the Universe began and evolved. Inflation is thus regarded as the most important development in cosmology since the discovery that the Universe began in a Big Bang, which came from the observation that it is expanding.

An important quest is to find from the known particles, the field that drove inflation, called the inflaton field. The properties of the inflaton field are obtained by the observations of the Cosmic Microwave Background (CMB) Radiations. Observations tell that the inflaton has to be a scalar field. Many approaches are being followed to find this scalar field. One of the approaches, the one which I am working on, is to consider a particle from the Standard Model (SM) of the Particle Physics. Higgs is the only scalar particle present in SM. In order to have successful inflation with Higgs field, it is required to consider a non-minimal coupling between the Higgs field and gravity through the term $\xi\phi^2\mathcal{R}/2$. The non-minimal coupling constant represented by $\xi$ is required to be very large, of the order of $\sim 10^4$. With this non-minimal coupling the calculations can no longer be done in the Einstein frame under general relativity which works on the minimal coupling scheme and so the Lagrangian is written in the Jordan frame, however the classical calculations can be transformed into the Einstein frame, through a conformal transformation, where the coupling is minimal.

Quantum corrections arise due to non-minimal coupling and so they are treated in the Jordan frame. The mass of the Higgs field required to have observable values of the inflationary parameters comes out to be $m_H \geqslant 230 GeV$.In order to reduce it we include the renormalization group improvement which gives the following range for the Higgs mass:

$$135.62 GeV \leq m_H \leq 184.49 GeV$$

which is acceptable. Hence the quantum corrections bring the results in harmony with the observations.

Observations of the **Planck** satellite are expected to come soon. They together with the results of the Higgs boson search at the LHC will decide the final status of this inflationary model. For the time being, with uncertainty in the Higgs mass and the values of the inflationary parameters, the model seems promising.

# Chapter 2

# The Standard Model of Cosmology

## 2.1 The Big Bang Theory

In the 20th century, cosmology began with the work of Einstein. His theory of General Relativity (GR) describes gravity as the distortion of the geometry of space and time and is consistent with the ideas of special relativity. Within a year after the publication of general relativity, Einstein applied it to the universe as a whole. Einstein realized that it was impossible to build a static model of the universe consistent with general relativity. Therefore he modified his equations by including a term called a "cosmological term", a kind of universal repulsion which prevents the distribution of matter from collapsing under the force of gravity. This term fits neatly into the equations.

In the late 1960s and early 1970s, three British astrophysicists, Steven Hawking, George Ellis, and Roger Penrose started working on the theory of relativity and its implications regarding our understanding of time. In 1968 and 1970, they published papers in which they extended Einstein's theory of general relativity to include measurements of time and space. According to their calculations, time and space had a finite beginning that corresponded to the origin of matter and energy. The singularity didn't appear in space; rather, space began inside the singularity. Nothing existed, before the singularity. This was the beginning of the universe. This theory is a successful description of the evolution of the universe.

The hot big bang cosmology is a remarkable achievement. It gives a reliable account of the universe from about $10^{-2}$ seconds to the present. The Big Bang model of cosmology has been tested with precision and the results have proved to be correct. The precision of these tests provides confidence that the picture of the expansion is basically correct.

### 2.1.1 Evidence Supporting the Big Bang Theory

First evidence in favour of the Big Bang theory is the observation of the Cosmic Microwave Background (CMB) radiations. The early universe was at a very high temperature and so it would have been permeated by the glow of light emitted by the hot matter, as the universe expanded, the wavelengths of these photons also expanded and so this light would have redshifted. The universe still is bathed by the radiation which have now redshifted into the microwave region of the spectrum.

The second evidence in support of the Big Bang theory is associated with the big bang nucleosynthesis. The early universe was so hot that even nuclei were not stable. At about 2 minutes after the Big Bang there were virtually no nuclei at all. The universe was filled with hot gas of photon and neutrinos, with a much smaller density of protons, neutrons and electrons. As the universe cooled, the protons and neutrons began to coalesce to form nuclei. From the reaction rates, one can calculate the expected abundance of the different types of nuclei that would have formed. The observations made today support the results from calculations. If there was never a Big Bang, there would be no reason whatsoever to expect that Helium-4 would be $10^8$ times as abundant as Lithium-7, it might just as well have been the other way round but when calculated in the context of the Big Bang theory, the ratio works out just right.

The Big Bang model of cosmology is based on the FRW (Friedmann Robertson Walker) metric.

## 2.2 FRW Cosmology

The homogeneous and isotopic universe is represented by the Friedmann Robertson Walker metric [1]

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi \right) \tag{2.1}$$

where, $a(t)$ characterizes the relative size of the universe at different times and $\kappa$ is the curvature parameter, which is

$$k = \begin{cases} +1 & \text{for} \quad closed\ universe \\ 0 & \text{for} \quad flat\ universe \\ -1 & \text{for} \quad open\ universe \end{cases}$$

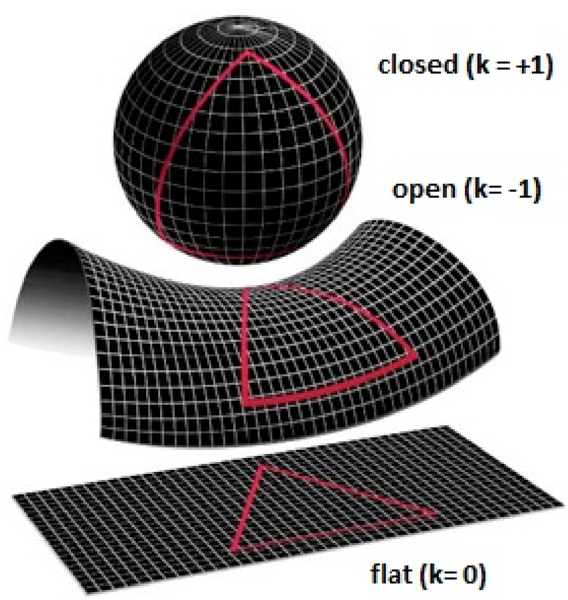



Figure 2-1: **Geometery of the universe** *with respect to different values of curvature parameter* $k$ *.*

### 2.2.1 Hubble Parameter

The expansion rate of the universe is given by the quantity known as the Hubble parameter, defined as

$$H = \frac{\dot{a}}{a} \tag{2.2}$$

where a is the scale factor of the universe.

$H$ has the units of inverse time and its value is negative for shrinking universe and positive for the expanding one.

### 2.2.2 Einstein Equations

The dynamics of the universe in FRW cosmology are determined by the Einstein Equation [2]

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \tag{2.3}$$

where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is the energy momentum tensor.

### 2.2.3 Friedmann Equations

After simplification and making assumptions of homogeneity and isotropy, the Einstein Equations take the form of two ordinary differential equations called the Friedmann Equations.

The first and the second Friedmann equation is obtained by calculating the $G_{00}$ and the $G_{ij,}$ component of the Einstein equation respectively [3].

The first Friedmann equation is,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{\kappa}{a^2} \tag{2.4}$$

where $\rho$ is the energy density of the universe. The other equation is,

$$\dot{H} + H^2 \equiv \frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho + 3p\right) \tag{2.5}$$

in units of $\hbar = c = 8\pi G = m_p = 1$, where $p$ is isotropic pressure.

## 2.3 Shortcomings of the Standard Big Bang Picture

The CMB radiation and the big bang nucleosynthesis calculations probe the history of the universe at different periods of time. The CMB samples the conditions in the universe about $10^5$ years after the bang when the universe became cool enough for the plasma of free nuclei and electrons to condense into neutral atoms. The plasma that filled the universe at earlier times was opaque to photons. With formation of neutral atoms, the universe became highly transparent. On the other hand nucleosynthesis calculations probe the history at much earlier times. It considers the abundance of nuclei that occurred at times ranging from about 1 second to 4 minutes after the Bang. Nonetheless, the standard Big Bang model has serious shortcomings.

In the Standard Big Bang theory, initial homogeneity and isotropy of the universe is assumed and this gives rise to a number of fundamental problems, such as:

1. Homogeneity Problem or the Horizon Problem

2. The Flatness Problem

3. Monopole Density Problem

### 2.3.1 The Horizon Problem

The extreme homogeneity at the cosmological scales seems to be violating the casualty principle. Observations of the CMB show that the inhomogeneities were smaller at earlier times and the observed small scale inhomogeneities were formed later by gravitational clumping. For the universe to attain a uniform temperature, it should be left undisturbed for long enough time that the distant points come in casual contact with each other. Thus the uniformity of the universe requires a connection between the causally disconnected regions but in Standard Big Bang theory, the universe evolves so quickly that it is impossible for any physical process to create uniformity.

Using the two Friedmann equations, the equation of state can be derived. It is given as:

$$\frac{d\rho}{dt} = -3H\rho\left(1 + \frac{p}{\rho}\right) \tag{2.6}$$

This can also be written in a slightly different form as

$$\frac{d\ln\rho}{d\ln a} = -3\left(1 + \omega\right) \tag{2.7}$$

where $\omega \equiv p/\rho$ . Solution to the above equation is

$$\rho \propto a^{-3(1+\omega)} \tag{2.8}$$

Plugging this in the first Friedmann equation, we get

$$\dot{a} \propto a^{-\frac{1}{2}(1+3\omega)} \tag{2.9}$$

From the definitions of Hubble parameter

$$\dot{a} = Ha$$

and the quantity $(aH)^{-1}$ is termed as the comoving Hubble radius. Combining this with the above equation, the comoving Hubble radius can be written as:

$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3\omega)} \tag{2.10}$$

This shows that the behaviour of Hubble radius depends upon the value of $\omega$.

The comoving particle horizon, which is the maximum distance a light signal can travel from the initial time $t_i$ to some given time $t$,is given by

$$\tau = \int_{t_i}^{t} \frac{dt}{a(t)} \tag{2.11}$$

Using

$$H = \frac{1}{a}\frac{da}{dt}$$

we get

$$\frac{dt}{da} = \frac{1}{Ha}$$

and thus for $t_i = 0$ and $t = a$, we have

$$\tau = \int_{0}^{a} \frac{da}{a^2 H} = \int_{0}^{a} d\ln a \left(\frac{1}{aH}\right) \tag{2.12}$$

Substituting the expression for $(aH)^{-1}$ from equation (2.10) in the above expression for $\tau$, we get

$$\tau \propto a^{\frac{1}{2}(1+3\omega)} \tag{2.13}$$

This shows that the comoving horizon grows with time which implies that scales entering the horizon now, have been far outside the horizon at photon decoupling. Why then we find high level of uniformity in the CMB ? This is known as the Horizon problem.

### 2.3.2 Homogeneity Problem

The homogeneity and isotropy of the universe was an assumption, known as the **Cosmological Principle.** The universe that we observe is very inhomogeneous, stars and galaxies form a lumpy distribution. Cosmologically, all of this structure in the universe is on a very small scale. If one averages over large scale of about 300 million light years then the universe appears to be very homogeneous. This large-scale homogeneity is most evident in the CMB for which temperature fluctuations are observed to be of the order $dT/T \sim 10^{-5}$.

Inhomogeneities are gravitationally unstable so they grow with time and therefore if there were initial inhomogeneities they should have grown to massive scales today but it is not so, on the contrary we observe homogeneous universe. Therefore in the Standard Cosmology we are required to assume uniformity as an initial condition.

### 2.3.3 The Flatness Problem

The flatness problem is related to the mass density of the universe. The mass density is measured relative to the "critical density", which is defined in terms of the expansion of the universe. If the mass density exceeds the critical density, then the gravitational pull will be strong enough to stop the expansion and the universe would eventually collapse. On the other hand, if the mass density is less than the critical density, the universe will expand forever.

This problem concerns the spatial flatness of the present-day universe. The observations indicate that the curvature term of the Friedmann equation is consistent with zero. In general relativity, the space-time curves in response to matter in universe but still we approximate universe by flat Euclidean space. In order to understand the severity of the problem, we consider the Friedmann equation, (2.4), written as

$$1 - \Omega = \frac{-\kappa}{(aH)^2} \tag{2.14}$$

where

$$\Omega \equiv \frac{\rho}{\rho_{crit}} \tag{2.15}$$

is called *cosmological density parameter* and

$$\rho_{crit} = 3H^2 \tag{2.16}$$

is the energy density for the flat universe ($\kappa = 0$).

Therefore, from equation (2.14) , if $\Omega$ is ever exactly equal to 1, it will remain equal to 1 forever. In Standard Cosmology, the comoving Hubble radius, $(aH)^{-1}$, grows with time and the quantity $|\Omega - 1|$ must thus diverge with time. Hence the near-flatness observed today ($\Omega \sim 1$) requires an extreme fine-tuning of $\Omega$ close to 1 in the infant universe. The deviation from $\Omega = 1$ (flatness) during the Big Bang Nucleosynthesis (BBN) at GUT and the Planck scale is respectively given by

$$\begin{aligned}
|\Omega\left(a_{BBN}\right) - 1| &\leq O\left(10^{-16}\right) \\
|\Omega\left(a_{GUT}\right) - 1| &\leq O\left(10^{-55}\right) \\
|\Omega\left(a_{Plan}\right) - 1| &\leq O\left(10^{-61}\right)
\end{aligned}$$

The Flatness and the Horizon problems are severe shortcomings in the predictive power of the Big Bang theory because flatness and large-scale homogeneity of the universe has to be assumed.

### 2.3.4 Magnetic Monopole Problem

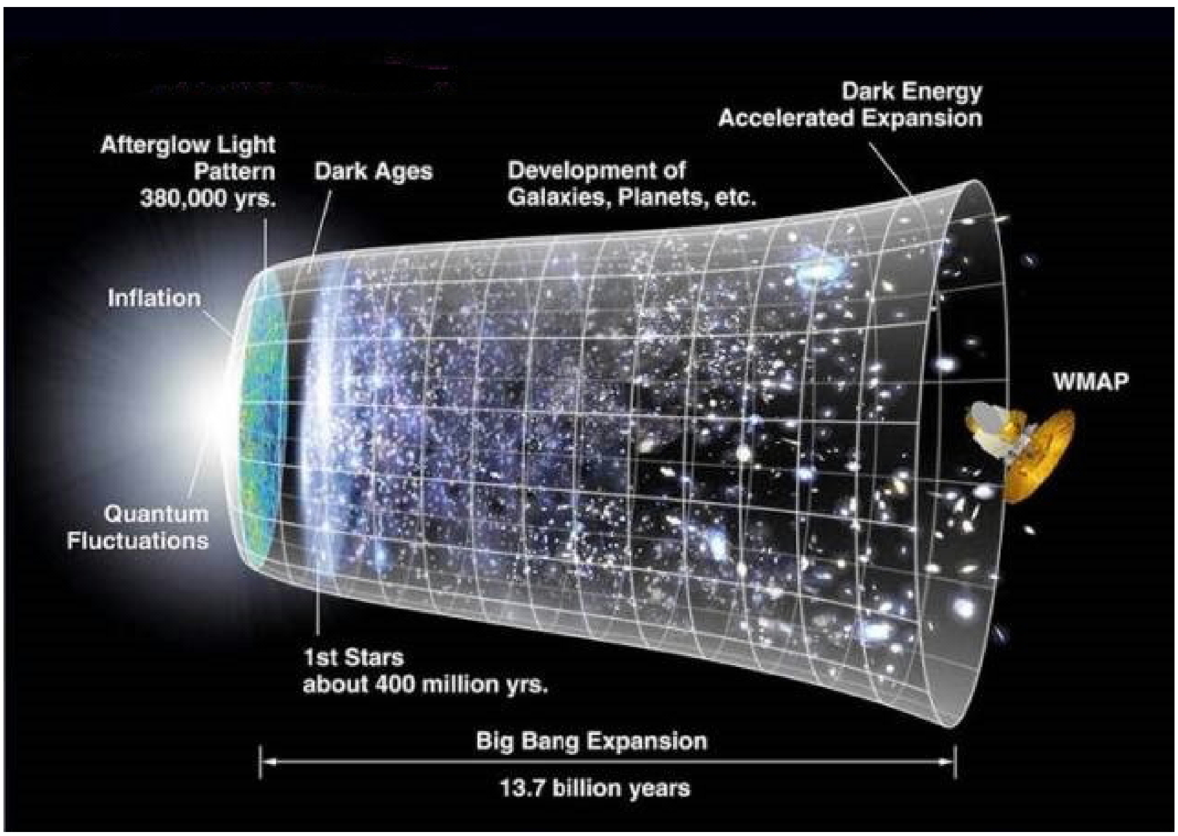
All Grand Unified Theories (GUTs) predict the presence of magnetic monopoles which are massive particles carrying a net magnetic charge. By combining GUTs with classical cosmology without inflation, Preskill in 1979 found that in the conventional Big Bang theory, magnetic monopoles would be produced in such high amounts that they would outweigh everything else in the universe by a factor of about $10^{12}$. But this case is grossly at odds with observations. The Big Bang theory is unable to give a satisfactory explanation of the very low density of magnetic monopoles in the universe today.

### 2.3.5 Ratio of the Number of Photons to the Number of Nucleons

Photons are found mainly in the CMB and the nucleons (protons and neutrons) form the atomic nuclei of matter. The observed universe contains about $10^{10}$ photons for about every proton or neutron. The Standard Big Bang theory does not explain this ratio, but this has to be assumed as initial conditions.

# Chapter 3

# Cosmic Inflation




**A representation of the evolution of the universe** *over 13.77 billion years. The far left depicts the earliest moment we can now probe, when a period of "inflation" produced a burst of exponential growth in the universe. (Courtesy NASA/WMAP Science Team.)*

## 3.1 The General Idea of Inflation

The idea of Cosmic Inflation was presented as a solution to the problems of the initial conditions of the Standard Big Bang theory. The inflationary scenario was proposed by Alan Guth in 1981 [4]. The original model had a problem. A variation that avoided that flaw was later proposed by Andre Linde [5].

The basic idea of inflation is that at early times the universe underwent a period of exponential expansion defined as a period when

$$\ddot{a} > 0 \tag{3.1}$$

where $a$ is the scale factor of the universe.

The effect of this acceleration was to quickly expand a small region of space to a huge size, diminishing spatial curvature in the process and making the universe extremely close to a flat one. In addition the horizon size, given by $(aH)^{-1}$, was greatly increased so that distant points in the universe were in causal contact and unwanted excitations were diluted. Moreover, quantum fluctuations prevented inflation to smooth out the universe perfectly and hence the universe contained the seeds for large scale structures.

## 3.2 Conditions for Inflation

The three conditions required for inflation are [6]:

**1**. Comoving Hubble radius must decrease

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \tag{3.2}$$

**2**. Accelerated expansion

$$\frac{d^2a}{d^2t} > 0 \tag{3.3}$$

**3**. Negative pressure i.e. violation of the strong energy condition .

$$p < -\frac{1}{3}\rho \tag{3.4}$$

where $p$ is the pressure and $\rho$ is the mass density. During the inflationary era, $\omega = -1$.

### 3.2.1 Decreasing Comoving Hubble Radius

The concept of comoving Hubble radius is important in the idea of inflation. According to relation (2.10)

$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3\omega)} \tag{3.5}$$

This shows that the behaviour of Hubble radius depends upon the sign of the exponent, hence as $\omega = -1$ during inflation, the Hubble radius decreases.

### 3.2.2 Accelerated Expansion

Using the definition of $H$, we have

$$(Ha)^{-1} = \dot{a}^{-1}$$

which gives

$$\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{(\dot{a})^2} = -\frac{\ddot{a}}{(aH)^2} \tag{3.6}$$

so if $\left[d\,(aH)^{-1}/dt\right] < 0$ then $\ddot{a} > 0$. It means that the shrinking comoving Hubble radius implies accelerated expansion

$$\frac{d^2a}{dt^2} > 0 \tag{3.7}$$

This explains why inflation is known as a period of exponential expansion.

### 3.2.3 Negative Pressure

We have one of the Friedmann equations as

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P) \tag{3.8}$$

which implies that the accelerated expansion ($\ddot{a} > 0$) requires the pressure to be negative ($P < -\rho/3$).

The inflationary mechanism produces an entire universe starting from essentially nothing. The energy of the universe actually came from the *gravitational field* [7]. The universe did not begin with energy stored in the gravitational field, but the gravitational field can supply the energy because **its energy can become negative without bound** [8]. As the positive energy materialized in the form of an ever-growing region filled with a high energy scalar field, negative energy materialized in the form of an expanding region filled with a gravitational field. The total energy remained constant at some very small value or even at zero. *There is nothing known that places any limit on the amount of inflation that can occur while the total energy remains exactly zero.*

## 3.3 Evidence for Inflation

There is now a great deal of evidence confirming the existence of a very hot and extremely dense early stage of the universe. Major portion of this data comes from a detailed study of the CMB radiation measured by NASA's WMAP satellite. Irrespective of the form of inflation which is considered, the evidence that our universe underwent a period of inflation is more or less the same. Few observations supporting the theory of inflation are as under.

### 3.3.1 The Vast Universe

It is a general observation that the universe is incredibly large. The visible part of the universe contains about $10^{90}$ particles [9]. In standard FRW cosmology, without inflation, one has to postulates the presence of $10^{90}$ from the beginning. A valid theory has to give a satisfactory explanation for the existence of such a large number of particles. The easiest way to obtain a large number, in result with a small numbers as input, is to calculate using an exponential. The exponential expansion of the universe, during inflation, reduces the problem of explaining $10^{90}$ particles to the problem of explaining 50 to 60 e-foldings [8].

Therefore, the theory of inflation suggests that although the universe is huge today, it may have begun from a small patch.

### 3.3.2 The Hubble Expansion

The Hubble expansion is known from earliest readings in cosmology. In standard FRW cosmology, the Hubble expansion is present in the list of postulates that define the initial conditions. On the other hand, inflationary theory explains the reasons for Hubble expansion. According to the theory, "the repulsive gravity associated with the false vacuum started the Hubble expansion" [7]. It provides the kind of force needed to drive the universe into a form of motion such that each pair of particles is moving apart with a velocity $v$ that is proportional to the distance $d$ of their separation, i.e.

$$v = Hd \tag{3.9}$$

where $H$ is the Hubble parameter.

## 3.4 Solution to the Problems of Standard Cosmology

### 3.4.1 Solution of the Horizon Problem

Inflation resolves the causal connection problem existing in the Standard Big Bang theory, by explaining how the apparently causally disconnected regions were in contact in the early universe. We saw that during inflation the comoving Hubble radius decreases in contrast to its behavior in the Standard scenario.

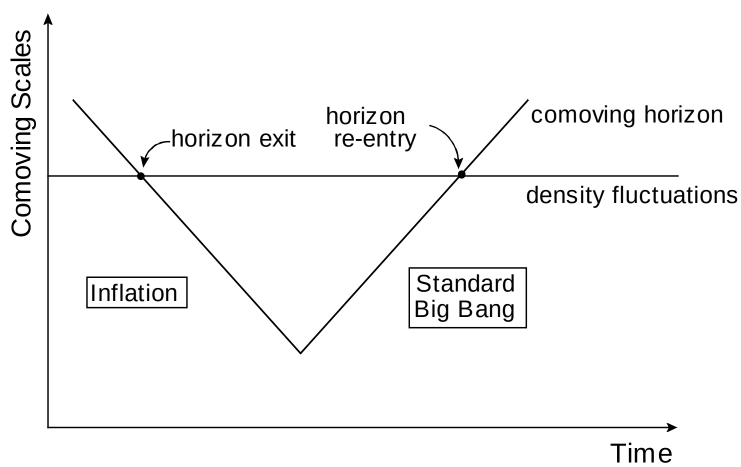



Figure 3-1: **Evolution of comoving Hubble radius** ***during inflation*****.**

Thus all comoving scales were smaller than the Hubble radius during inflation, hence all regions were within the horizon and were causally connected. After inflation Hubble radius began to increase again but at a smaller rate and so those scales seem to be coming inside the horizon today.

### 3.4.2 Homogeneity and Isotropy Explained

The CMB was released about 300,000 years after the Big Bang [7], when the universe cooled enough so that the opaque plasma neutralized into a transparent gas and the photons decoupled from the rest of matter-radiation soup [10]. Therefore, the observed uniformity of the radiations implies that the observed universe had become uniform in temperature by that time. Theory of inflation explains that the uniformity was actually created on microscopic scales, by normal thermal-equilibrium processes, and then during inflation the regions of uniformity were stretched to such large size that they encompassed the observed universe.

### 3.4.3 Solution of the Flatness Problem

The theory of inflation naturally predicts the extraordinary flatness of the early universe. As mentioned in the previous chapter, the Friedmann Equation can be put in the form ( with $\kappa = 1$) as:

$$|1 - \Omega| = \frac{1}{(aH)^2} \tag{3.10}$$

According to inflation, the decreasing comoving Hubble radius drives the universe toward flatness, by defining $\Omega = 1$ as an "attractor" during inflation. So, the universe may have begun with any value for $\Omega$ but during inflation it was exponentially driven towards $\Omega = 1$ rather than away from it.

Thus, as long as there is a long enough period of inflation, universe can start at any value for $\Omega$, and it will be driven to unity by the exponential expansion [2].

### 3.4.4 Solution of the Monopole Problem

Inflation is certainly the simplest known mechanism to eliminate monopoles from the visible universe, although they are still in the spectrum of possible particles. In inflationary theory monopoles are eliminated by arranging the parameters such that inflation took place during or after the monopole production, and so during inflation the monopole density was diluted to a completely negligible level [3].

### 3.4.5 CMB Anisotropy Explained

Despite of the fact that the intensity of the CMB radiations is 99.999% isotropic, it still contains small temperature anisotropies. Inflationary theory has the explanation. The universe was smoothed out by the process of rapid expansion, but the density perturbations were generated due to the quantum fluctuations of the inflaton field. This resulted in the formation of these anisotropies. New observations are arriving, but so far the data is in an excellent agreement with the predictions of the inflationary models.

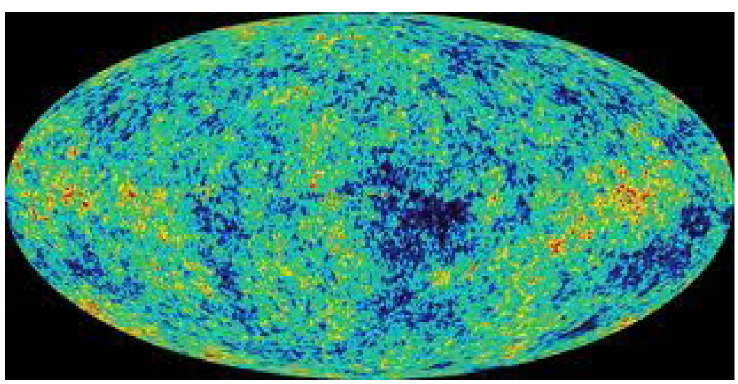


Figure 3-2: **WMAP picture of the CMB anisotropies.**

## 3.5 New and Old Inflation

The field studied in the original model of inflation was **false vacuum,** because it behaves as if it were the state of lowest possible energy density. Classically this state would be stable, as there would be no state of lower energy available for the field to decay. However, this false vacuum can decay by quantum tunneling. In the initial days, it was hoped that the process of tunneling could end inflation, but it was found that the randomness of decay can produce inhomogeneities.

In Guth's 1981 paper [4], where he introduced the idea of inflation, it was mentioned that inflation is not easy to end. The metastable false vacuum decays through bubble formation. Bubbles expand at speeds approaching the speed of light, but the false vacuum regions that separate them expand even faster. Thus, the bubbles (regions where inflation ends) never fill the entire space and inflation never ends. This is the graceful exit problem of the old inflationary scenario [11].

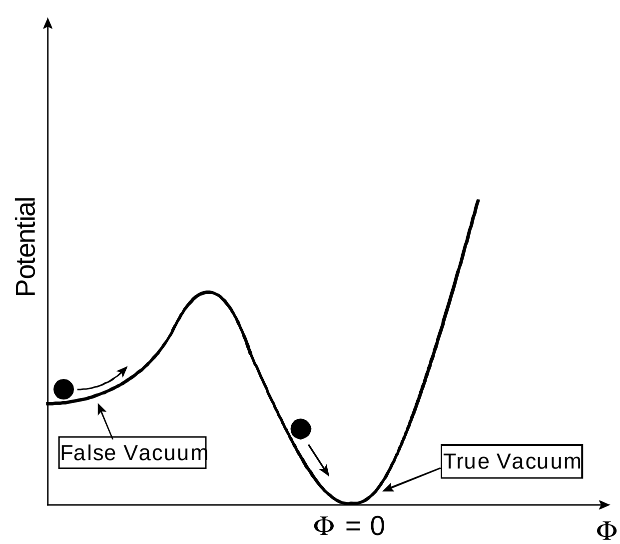



Figure 3-3: **Old infation potential**. *The scalar field makes transition from false to true vacuum by tunneling process.*

This "graceful exit" problem was solved by the invention of the new inflationary universe model by Linde [12] and by Albrecht and Steinhardt [13]. New inflation achieved enormous success. In this theory, inflation is driven by a *scalar field,* called the **inflaton**, perched on a plateau of the potential energy diagram [14]. If the plateau is flat enough, such a state can be stable enough for successful inflation. Later, Linde showed that the inflaton potential need not have either a local minimum or a gentle plateau and he named this model of inflation as chaotic inflation.

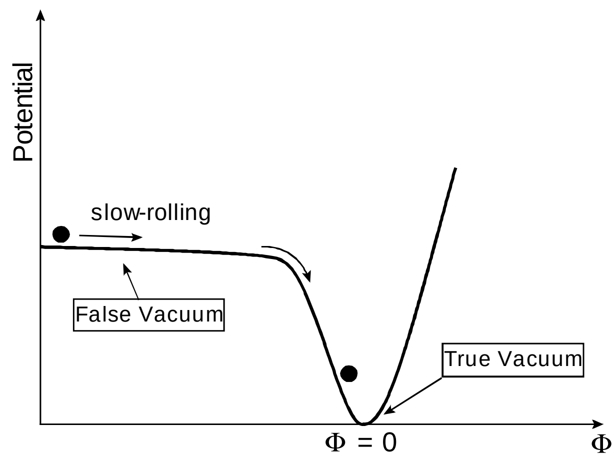



Figure 3-4: **New inflation potential**. *In here, transition from false to true vacuum is made by the scalar filed through quantum fluctuations.* $\phi = 0$ *corresponds to value of field at which potential has a global minimum.*

## 3.6 How Inflation Works

As mentioned earlier, inflation provides a brief prequel to the theory of Standard Big Bang cosmology. The exponential expansion of the universe started about $10^{-34}$ second after the 'bang', when initially the universe was in a phase of dense hot plasma, termed as the radiation dominated era, in which the energy density ($\rho$) decreased as $a^{-1/4}$ [7]. During inflation, the scale factor was given by $a \sim \exp(Ht)$ and the energy density of the universe was independent of the scale factor ( $\rho \sim a^0$), which is an equivalent statement to the violation of the strong energy condition i.e. $P = -\rho$ or $\omega = -1$. The process of inflation lasted for another fraction of a second and ended with a reheating phase. We don't have any direct observation of the processes that took place during inflation but we expect that the incidences which occurred then followed the physical laws justified by the theories that we have today, although the universe was in a state of very high temperature with enormously high density.

### 3.6.1 False Vacuum

Theory of inflation can explain the outward propulsion of the infant universe on the basis of gravitational repulsion, which is a property of matter having negative pressure [8]. The combination of modern particle physics and general relativity predicts that at very high energies, forms of matter that can create gravitational repulsion may exist. Inflation assumes that a patch of such repulsive gravity material existed in the early universe which is sort of a correct assumption because the universe then was in a state of its highest energy ever. If inflation took place at the GUT energy scale ($\sim 10^{16}$ $GeV$) then the patch needs to be only as large as $10^{-28}$ $cm$ [8]. Since any such patch is enlarged by inflation, the initial density of the patch need not be very high. According to inflation, the gravitational repulsion drove the universe into the exponential expansion, doubling in size after every $10^{-37}$ second or so. The patch expanded exponentially by a factor of at least $10^{28}$ which corresponds to about 65 time constants which, in the terms of inflationary theory, are called "number of e-folds". Inflation lasted for about $10^{-36}$ seconds and at the end, the universe was of the size of a marble. The repulsive gravity material is unstable and it decayed, calling an end to inflation. The decay released energy which produced ordinary particles. The density of the repulsive gravity material was not lowered as it expanded. Although more and more energy appeared as the repulsive gravity material expanded but the total energy remained conserved. It is due to the fact that the energy of a gravitational field is negative.

### 3.6.2 GUTs, Higgs Particle and Theory of Inflation

According to the Standard Big Bang theory, the expansion of the universe started from a state of enormously high density and at a temperature which was much higher than the critical temperature of a phase transition. So the symmetry between the strong and electroweak interactions in the Grand Unified Theories (GUTs) was restored in very early stages of the evolution of universe.

According to the general definition, a spontaneously broken symmetry is the one which is present in the theory describing a system, but is hidden when the system is in equilibrium state. For GUTs, the symmetry relates the behavior of one type of particle to the behavior of another. The symmetry of GUTs implies that the three interactions of the Standard Model of particle physics i.e. $U(1), SU(2)$ and $SU(3)$, are really a single one, and hence indistinguishable. The symmetry also implies that the individual particles, which are normally distinguished from each other by how they participate in these interactions, will necessarily lose their identity. In particular, GUTs propose that the underlying laws of physics make no distinction between an electron, a neutrino or a quark. In GUTs, a set of fields is added for the specific purpose of spontaneously breaking the symmetry. These fields are called as Higgs fields, after Peter W. Higgs of the University of Edinburgh, and the spontaneous symmetry breaking mechanism, which occurs in a variety of particle physics theories, is known as the Higgs mechanism.

The Higgs fields are on equal footing with the other fundamental fields, such as the electromagnetic field. It is postulated that these fields exist and that they evolve according to a specified set of equations. While the electromagnetic field gives rise to photons, the Higgs fields give rise to Higgs particles. The Higgs particles associated with the breaking of the grand unified symmetry are expected to have masses corresponding to energies in the vicinity of $10^{14}\ GeV$, which means that they are far too massive to be produced in the foreseeable future. Higgs is a scalar field and it existed at the time when we expect inflation to have taken place, we may consider it to have acted as an inflaton.

### 3.6.3 Spontaneous Symmetry Breaking and Phase Transitions in a Hot Universe

In the GUT sector of studies, spontaneous symmetry breaking is accomplished by developing the theory in such a way that the Higgs fields have non-zero values in vacuum. The other particles in the theory interact with the Higgs fields, producing the apparent distinction between the $U(1), SU(2)$ and $SU(3)$ interactions and also between electrons, neutrinos and quarks. The distinct properties that we observe for electron, neutrinos and quark are not fundamental, instead they represent the different ways in which that particles can interact with the Higgs field.

According to GUTs, a phase transition occurs at a temperature of the order of $10^{27}$ Kelvin. At temperatures higher than this value, the Higgs field enters a different phase. The Higgs field would oscillate wildly under thermal agitation, but the mean value of each field would be zero, so grand unified symmetry would be restored. In this phase, the $U(1)$, $SU(2)$ and $SU(3)$ interactions would all merge into a single interaction. This phase transition is closely linked to the spontaneous symmetry breaking i.e. at temperature greater than $10^{27}$ Kelvin, there is only one type of interaction and at temperatures below that value, the grand unification symmetry is broken and the $U(1)$, $SU(2)$ and the $SU(3)$ interactions acquire their separate identities.

When the universe cooled down to the temperature of the phase transition, there are two possibilities that might have happened. The phase transition might have occurred instantaneously or have been delayed, occurring after a large amount of supercooling. If the correct GUT and the values of its parameters were known, there would be no ambiguity about the nature of the phase transition. In the absence of this knowledge, however, either of the two possibilities are plausible. If the phase transition occurred immediately, then its cosmological consequences would be very problematical. In that case a large number of magnetic monopoles would be produced. For the most of the GUTs, these monopoles would survive to the present day, leading to predictions which are at odds with observations.

The inflationary scenario is based on the possibility that the phase transition was delayed and the universe underwent extreme supercooling. When the gas filling the universe supercooled to temperatures below the temperature of the phase transition, a false vacuum state would have been approached. The energy density required to produce this false vacuum is about 60 orders of magnitude larger than the density of the atomic nucleus [8]. Due to the gravitational repulsion, this false vacuum state underwent inflation at a rate which was larger than the expansion rate given by the Standard Cosmology. As a result, the universe doubled in size in about $10^{-34}$ seconds and continued to double in size during each successive interval of $10^{-34}$ seconds until the universe remained in false vacuum state. Eventually the phase transition occurred and the energy density of the false vacuum was released, which was the latent heat of phase transition. This energy produced particles which reheated the universe back to the temperature of phase transition, about $10^{27}$ Kelvin. After this phase transition, the universe was filled with a hot gas of particles, exactly as postulated in the initial conditions of the Standard Big Bang theory and from here the two models agree in explaining the evolution of the universe.

## 3.7 Quantum Fluctuations During Inflation

Basic quantum mechanics follows the uncertainty principle

$$\Delta E \Delta t \geqslant \frac{\hbar}{2}$$

During inflation, the universe expanded over large regions driving itself into homogeneity but still we have local concentration of matter announcing the local inhomogeneity. It is due to the fact that the field that drived inflation, like all quantum fields, underwent quantum fluctuations in accordance with the Heisenberg uncertainty principle. During inflation, these quantum fluctuations were stretched proportional to the scale factor, $a(t)$, and so they grew rapidly to macroscopic scales. As a result, we have a set of almost scale-invariant perturbations having a huge range of wavelengths [8]. The spectrum of primordial perturbations are parameterize by a spectral index, $n_s$. A spectrum that is scale-invariant would have $n_s = 1$. Inflationary models generically predict $n_s = 1$ to within 10%. The latest measurements of these perturbations by WMAP reveal $n_s = 0.977 + 0.039(8) - 0.025$.

### 3.7.1 Creation and Evolution of Fluctuations

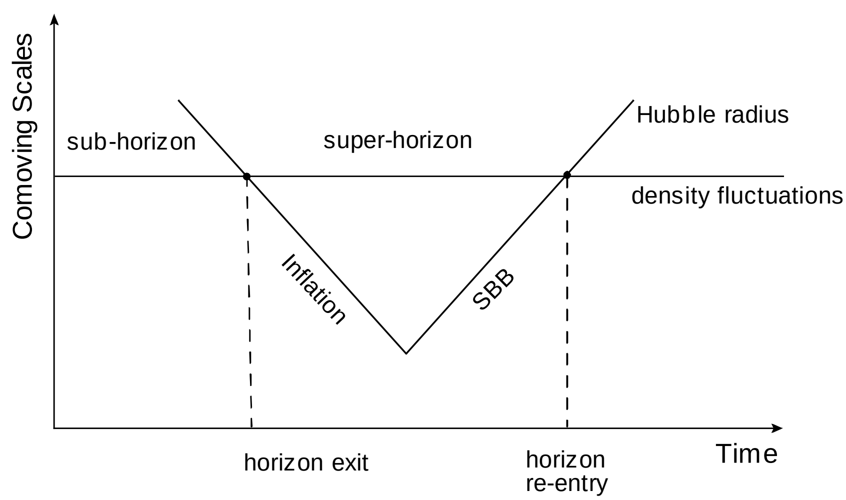



Figure 3-5: **Creation and evolution of perturbation during inflation and afterwards**. *Quatum perturbations are generated at sub-horizon scales. During inflation, when comoving Hubble radius decreases and comoving scales remain fixed, the fluctuations come outside the horizon. The perturbations remain frozen until the time of horizon re-entry, after which they evolve into anisotropies in the CMB.*

Fluctuations are created quantum mechanically and have a wave number $k$ (figure 3-5). Cosmologically relevant fluctuations are those which are born inside the horizon (Hubble radius) i.e. at the sub-horizon scales:

$$sub\text{-}horizon : k \gg aH \tag{3.11}$$

and in here

$$k^{-1} \ll (aH)^{-1}$$

i.e. wavelengths are within the Hubble radius.

However, while the comoving wave number is constant, the comoving Hubble radius shrinks during inflation. So the fluctuations exit the horizon eventually.

$$super - horizon : k < aH \tag{3.12}$$

### 3.7.2 Large Scale Structure from Quantum Fluctuations

Primordial fluctuations in the field grew with exponential expansion during inflation for about first 4 e-folds and after few e-folds, $\kappa > aH$ , the perturbations exit the horizon. Casual physics cannot act on super-horizon scales so they froze, forming a scale invariant spectra.

These fluctuations re-entered the horizon and when the photons decoupled, these fluctuations were imprinted on CMB as temperature anisotropy in the form of hotter and colder regions. These quantum fluctuations caused the time delay in the time at which inflation ended in different regions as a result of which temperature fluctuations $\delta T$ were developed during reheating. After the photon decoupling when the radiation era ended, these fluctuations $\delta T$ caused density fluctuations $\delta\rho$ which, under the action of gravity, amplified the inhomogeneities.

Inhomogeneities in CMB are small, revealing that at the time of photon decoupling universe was nearly homogeneous having small inhomogeneities. Therefore fluctuations can be analyzed as linear perturbations around a homogeneous back ground. Important fluctuations are in two quantities, matter and metric. Matter fluctuations are scalar and metric fluctuations are tensor in nature. They can be traded for each other by a gauge transformation. Scalar fluctuations are observed as density fluctuations and tensor fluctuations led to the formation of gravitational waves in the later universe [6]. After radiation-dominated era, when pressure was negligible and only gravitational force was at action, these density perturbations grew under the action of gravitational force and resulted in the formation of galaxies, superclusters, giant voids and all the large scale structures.

### 3.7.3 Power Spectrum of Perturbations

Power spectrum of the scalar fluctuations is given by the two point correlation function of the fluctuation's amplitude

$$\langle R_k R_{k'} \rangle = (2\pi)^3 \delta\left(k - k'\right) P_R\left(k\right) \tag{3.13}$$

and

$$\Delta_s^2 \equiv \Delta_R^2 = \frac{k^3}{2\pi^2} P_R\left(k\right) \simeq \frac{H_*^2}{\left(2\pi\right)^2} \frac{H_*^2}{\dot{\phi}_*^2} \tag{3.14}$$

Where the subscript $(..)_*$ denotes the values at the time of first horizon crossing or the horizon exit i.e. at scale $k = aH$.

The scale dependence of the power spectrum is defined by the scalar spectral index

$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln \kappa} \tag{3.15}$$

where scale invariance corresponds to the value $n_s = 1$.We my also define the running of the spectral index by

$$\alpha_s \equiv \frac{dn_s}{d \ln \kappa} \tag{3.16}$$

The tensor fluctuations contain two polarization modes $h_{ij}$ if $h = h^{\dagger}$, $h^{*}$ , their power spectrum is

$$\langle h_k h_{k'} \rangle = (2\pi)^3 \delta \left(k - k'\right) P_h \left(k\right) \tag{3.17}$$

and

$$\Delta_h^2 = \frac{k^3}{2\pi^2} P_h \left(k\right) \tag{3.18}$$

Power spectrum of the tensor perturbations is defined as the sum of the power spectra for the two polarizations

$$\Delta_t^2 \equiv 2\Delta_h^2 \simeq \frac{2}{\pi^2} \frac{H_*^2}{m_p^2} \tag{3.19}$$

It is customary to normalize the tensor fluctuations relative to the amplitude of scalar fluctuations. The *tensor-to-scalar ratio* $(r)$ is given as

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} \tag{3.20}$$

## 3.8 Slow-Roll Inflation

Slow-roll approximation is a technique used for the analysis of inflation. This technique is used in most of the inflationary models. The slow-roll approximation is based on the assumption of accelerated expansion.

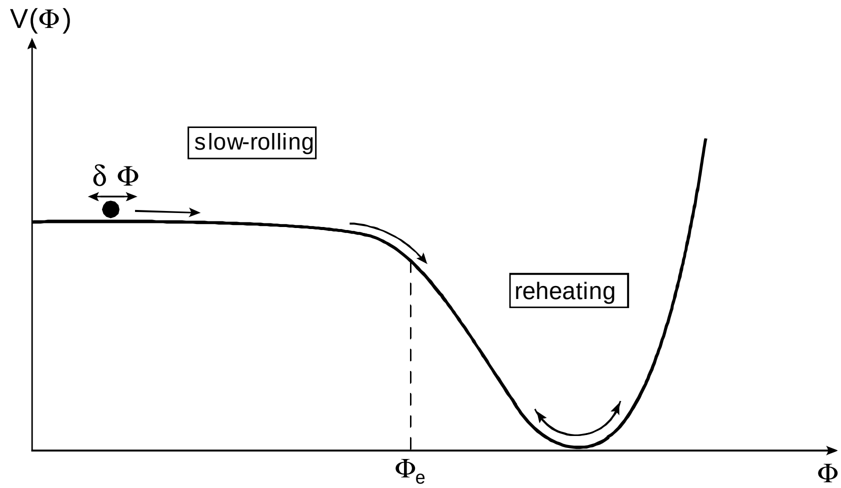



Figure 3-6: **Mechanism of slow-roll inflation**. *$\delta\phi$ represents quantum fluctuations in the inflaton field and $\phi_e$ is the value of field at which inflation ends.*

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### 3.8.1 Scalar Field Dynamics

Scalar fields are an important ingredient in the particle physics theories. The dynamics of an inflaton field, minimally coupled to gravity, have the following action integral:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right] \tag{3.21}$$

i.e. the Lagrangian is

$$\mathcal{L}_\phi = \frac{1}{2}\mathcal{R} + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi) \tag{3.22}$$

where $\mathcal{R}$ is the Ricci scalar of general relativity.

The *stress-energy tensor* is given by

$$T_{\alpha\beta} = -2\frac{\partial \mathcal{L}_\phi}{\partial g^{\alpha\beta}} + g_{\alpha\beta}\mathcal{L}_\phi \tag{3.23}$$

where

$T_{00} \equiv$ energy density

$T_{0i} = T_{i0} \equiv$ momentum density

$T_{ij} = T_{ji} \equiv$ stress tensor

The stress-energy tensor (3.23) for the above scalar field is

$$T_{\alpha\beta} = \partial_\alpha\phi\partial_\beta\phi - g_{\alpha\beta}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right] \tag{3.24}$$

Thus the energy density and the pressure of the inflaton field can be respectively described as

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{3.25}$$

$$P = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{3.26}$$

and from the definition of $\omega$, ($\omega \equiv p/\rho$ ), we have

$$\omega = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{3.27}$$

Substituting the expression for $\rho$, from equation (3.25), in the Friedmann equation (2.4) for flat universe ($\kappa = 0$), we get

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{3.28}$$

and the equation determining the dynamics of the scalar field is obtained as follows

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = 0 \tag{3.29}$$

### 3.8.2 Slow-Roll Parameters

Using the definition of $\omega$, the Friedmann second equation $(2.4)$, for universe dominated by homogeneous scalar field with $\kappa = 0$, is re-written as

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\rho\left(\frac{1}{3} + \omega\right) \tag{3.30}$$

As for flat universe, $\rho = \rho_{crit} = 3H^2$, so

$$\begin{aligned} \frac{\ddot{a}}{a} &= H^2\left[1 - \frac{3}{2}(\omega + 1)\right] \\ &= H^2(1 - \varepsilon) \end{aligned} \tag{3.31}$$

if we define

$$\varepsilon \equiv \frac{3}{2}(\omega_\phi + 1) \tag{3.32}$$

and this $\varepsilon$ is the first slow-roll parameter.

Comparing the two expressions for the Friedmann equation, $(2.5)$ and $(3.31)$, we get

$$\varepsilon = -\frac{\dot{H}}{H^2} \tag{3.33}$$

The various other ways of expressing $\varepsilon$ are

$$\varepsilon \equiv \frac{3}{2}\left(\omega_\phi + 1\right) = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} = \frac{\dot{\phi}^2}{2H^2} \tag{3.34}$$

The second slow-roll parameter, $\eta$, is defined as:

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \tag{3.35}$$

Using equation (3.34), $\eta$ can be written in terms of $\varepsilon$ as

$$\eta = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN} \tag{3.36}$$

### 3.8.3 Slow-Roll Conditions

Accelerated expansion requires $\ddot{a}(t)$ and so $H^2$ to be large, which implies that $\varepsilon < 1$. From equations (3.34) and (3.28) , it means that

$$\dot{\phi}^2 \ll H^2 \tag{3.37}$$

$$\ll V(\phi) \tag{3.38}$$

or in other words, the potential energy is dominant over the kinetic energy of the field.

Besides the accelerated expansion , the duration of this expansion must also be long enough, for which $\ddot{\phi}$ should be small. Equivalently, we get the condition

$$|\ddot{\phi}| \ll |3H\dot{\phi}|, |\partial_\phi V| \tag{3.39}$$

which from equation (3.35) implies that $|\eta| < 1$ and this means that the change in $\varepsilon$ over time is small.

The conditions $\varepsilon < 1$ and $|\eta| < 1$ are known as the *slow-roll conditions.*

### 3.8.4 Potential Slow-Roll Parameters

In addition to the Hubble slow-roll parameters, the potential slow-roll parameters are defined as $\varepsilon_v$ and $\eta_v$. They give the constraints on the shape of the inflationary potential [2]. They are given as (restoring $m_p$ temporarily):

$$\varepsilon_v = \frac{m_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \tag{3.40}$$

and:

$$\eta_v = m_p^2 \frac{V_{,\phi\phi}}{V} \tag{3.41}$$

In the slow-roll approximation, they are

$$\varepsilon_v \ll 1 \ , \ |\eta_v| \ll 1 \tag{3.42}$$

and relation between Hubble and potential slow-roll parameters is

$$\varepsilon \approx \varepsilon_v \ , \ \eta \approx \eta_v - \varepsilon_v \tag{3.43}$$

### 3.8.5 Number of e-Folds

If we define

$$dN = \ Hdt = dlna \tag{3.44}$$

which is the measure of the number of e-foldings $(N)$ of inflationary expansion, then $\varepsilon$ is redefined as

$$\varepsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} \tag{3.45}$$

So $N(\phi)$ becomes

$$N(\phi) = \int_t^{t_e} H dt = \int_\phi^{\phi_e} H d\phi \frac{dt}{d\phi} \tag{3.46}$$

Using inequality $(3.37)$ in equation $(3.28)$, we get

$$H^2 \simeq \frac{1}{3} V(\phi)$$

and using equations (3.38) in (3.29)

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H}$$

Putting the expressions for $H^2$ and $\left(\dot{\phi}\right)^{-1}$ in the above equation (3.46), it becomes

$$N(\phi) \approx -\int_\phi^{\phi_e} \frac{V}{V_{,\phi}} d\phi \tag{3.47}$$

Using the definition of slow-roll parameter $\varepsilon$, equation $(3.45)$, this expression for $N(\phi)$ modifies to

$$N(\phi) = \int_{\phi_e}^{\phi} \frac{1}{\sqrt{2\epsilon_v}} d\phi \approx \int_{\phi_e}^{\phi} \frac{1}{\sqrt{2\epsilon}} d\phi \tag{3.48}$$

where we find the value of the $\phi_e$ from the end of inflation condition.

### 3.8.6 Power Spectra Revisited

Power spectra of the scalar and tensor fluctuations in terms of slow-roll parameters are given as

$$\Delta_s^2 \equiv \Delta_R^2 = \frac{1}{8\pi^2}\frac{H^2}{m_p^2}\frac{1}{\varepsilon} \tag{3.49}$$

$$\Delta_t^2 \equiv 2\Delta_h^2 = \frac{2}{\pi^2}\frac{H^2}{m_p^2} \tag{3.50}$$

Both the expressions are valid at the horizon crossing condition i.e. $k = aH$.

The tensor-to-scalar ratio becomes

$$r = 16\varepsilon_* \tag{3.51}$$

where the subscript $(..)_*$ denotes $k = aH$.

## 3.9 Models of Inflation

There are varieties of inflationary models e.g. new, chaotic, extended, power-law, hybrid, natural, supernatural, eternal, D-term, F-term, brane, oscillating, trace-anomaly driven etc. These models are differentiated on the basis of the properties of scalar particle employed in the theory and by the form of their potentials. Inflationary models are classified as follows:

### 3.9.1 Single Field Models

#### Large Field Models

The first class is the "large field" models. In this class the initial value of the inflaton field is large $\sim \Delta\phi > m_p$ and it rolls down towards the potential minimum. Chaotic inflation [15] is also classified as a large field model.

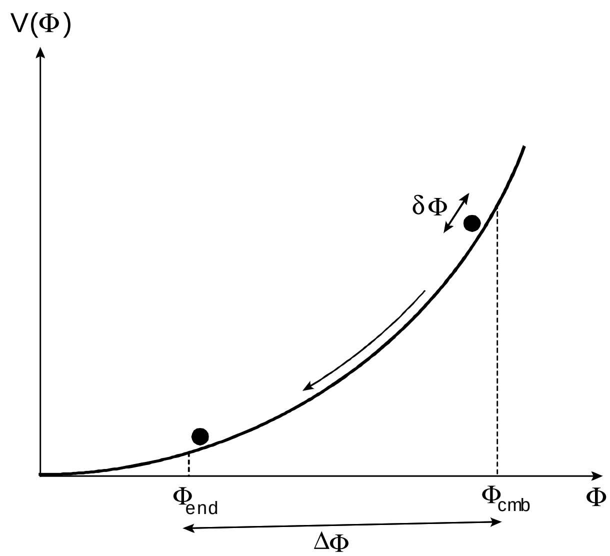


Figure 3-7: **An example of large-field inflation**. *Here* $\Delta\phi > M_{pl}$.

#### Chaotic Inflation

The chaotic inflation model is described by the potentials of the $\phi^n$ type, where a single monomial term dominates the potential [2]. Most commonly used potentials are quadratic

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

or quartic

$$V(\phi) = \frac{1}{4}\lambda\phi^4$$

inflaton potential.

The term "chaotic" means that the initial conditions of the inflaton field are distributed chaotically and there is no strict restriction upon the slope of the inflaton potential. In this scenario, the region which underwent sufficient amount of inflation gave rise to our universe. If the field evolution is super-Planckian, the gravitational waves produced by inflation should be observed in the near future.

### Small Field Models

The second class is the "small field" models. In this class the inflaton field is initially small i.e. $\Delta\phi < m_p$ and it slowly evolves toward the minimum of the potential at larger $\phi$. New inflation and natural inflation [16, 17] are the examples of this type.

### Natural Inflation

Natural inflation model is characterized by Pseudo Nambu-Goldstone Bosons (PNGBs), which appear when an approximate global symmetry is spontaneously broken. The PNGB potential is expressed as

$$v(\phi) = m^4\left[1 + \cos\left(\frac{\phi}{f}\right)\right] \tag{3.52}$$

where two mass scales $m$ and $f$ characterize the height and width of the potential, respectively. The typical mass scales are of order $f \sim m_{pl} \sim 10^{19} GeV$ and $m \sim m_{GUT}$ $\sim 10^{16}$ GeV.

### 3.9.2 Hybrid Inflation Model

In such a model, inflation ends by the phase transition that is triggered by the presence of another scalar field.

#### Linde's Hybrid Inflation

The inflationary model consisting of multiple scalar fields was also proposed by Linde [18, 19]. Linde's hybrid inflation model is described by the following potential:

$$V(\phi) = \frac{\lambda}{4}\left[\chi^2 - \frac{M^2}{\lambda}\right]^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}m^2\phi^2 \tag{3.53}$$

When $\phi^2$ is large, the field tends to roll down toward the potential minimum at $\chi = 0$.

# Chapter 4

# Standard Model Higgs as an Inflaton

Early attempts to model inflation using a self-interacting Higgs like scalar field minimally coupled to gravity faced the necessity, dictated by the amplitude of the primordial CMB perturbations, to assume an extremely small self-interaction quartic coupling constant $\lambda \sim 10^{-13}$. As the mass of the Higgs particle is related to the $\lambda$ by the relation $\sqrt{2\lambda}\upsilon$, therefore such small coupling leads to a small value of the Higgs mass. Such a model of inflation is ruled out by the present observational data. It was observed that the problem of an exceedingly small self-coupling can be solved by adding a non-minimal coupling term, $\xi\phi^2\mathcal{R}/2$, with a large non-minimal coupling constant $\xi$. In this case, the CMB anisotropy i.e. $\Delta T/T$ , is given by the ratio $\sqrt{\lambda}/\xi$ [20]. Therefore even for the value of $\lambda$ near 1, the small value of the anisotropy can be obtained.

## 4.1 Lagrangian

Consider the Standard Model (SM) Lagrangian non-minimally coupled to gravity

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{m^2}{2}\mathcal{R} + \xi\mathcal{H}^\dagger\mathcal{H}\mathcal{R}$$

where $\mathcal{H}$ is the Higgs doublet and $\xi$ is the non-minimal coupling constant between gravity and the Higgs field. The Higgs sector is described by the following Lagrangian:

$$\mathcal{L}_h = -\left|\partial\mathcal{H}\right| + \mu^2\mathcal{H}^\dagger\mathcal{H} - \lambda\left(\mathcal{H}^\dagger\mathcal{H}\right)^2 + \xi\mathcal{H}^\dagger\mathcal{H}\mathcal{R} \tag{4.1}$$

where $\lambda$ is self-coupling of Higgs and $\mu$ is the parameter for Higgs mass.

We consider $\phi$ as the neutral, real component of the Higgs doublet $\mathcal{H}$ that remains after the Higgs mechanism. The general form of the action integral for the scalar field non-minimally coupled to gravity through the Ricci scalar $\mathcal{R}$, in the Jordan frame is [21]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}m_p^2 f\left(\phi\right)\mathcal{R} - \frac{1}{2}k\left(\phi\right)\left(\partial\phi\right)^2 - V\left(\phi\right)\right] \tag{4.2}$$

where $f\left(\phi\right)$ is a general function of the scalar field, $k\left(\phi\right)$ is a general coefficient of kinetic energy and $V\left(\phi\right)$ is some general potential function.

## 4.2 Classical Calculations

### 4.2.1 Conformal Transformation

The minimal coupling between $\phi$ and $\mathcal{R}$ can be converted into a non-minimal one through a conformal transformation that will take the above action from the Jordan frame to the Einstein frame. In the Einstein frame, the gravity term is canonical i.e. it has the form $m_p\mathcal{R}/2$. The transformation is made by defining a new metric as

$$g^E_{\alpha\beta} = f\left(\phi\right)g_{\alpha\beta} \tag{4.3}$$

where $g^E_{\alpha\beta}$ is the metric in the Einstein frame.

Under this transformation, the determinant of metric transforms as [22]:

$$g^E = f(\phi)^4 g$$

and so

$$\sqrt{-g} \longrightarrow \frac{\sqrt{-g^E}}{f(\phi)^2} \tag{4.4}$$

and the Ricci scalar undergoes the following transformation [23]:

$$R = f(\phi)\mathcal{R}^E - 6g^E_{\alpha\beta}\partial^\alpha f(\phi)^{\frac{1}{2}}\partial^\beta f(\phi)^{\frac{1}{2}} \tag{4.5}$$

Calculating term-wise

$$\partial^\alpha f(\phi)^{\frac{1}{2}} = \frac{1}{2f(\phi)^{\frac{1}{2}}} f'(\phi)\partial^\alpha\phi$$

and so

$$g^E_{\alpha\beta}\partial^\alpha f(\phi)^{\frac{1}{2}}\partial^\beta f(\phi)^{\frac{1}{2}} = \frac{1}{4f}\left[f'(\phi)\right]^2(\partial\phi)^2$$

Substituting this in equation $(4.5)$, the transformation for the Ricci scalar becomes

$$R \rightarrow f(\phi)\mathcal{R}^E - \frac{3}{2} f'(\phi)^2(\partial\phi)^2 \tag{4.6}$$

Further, under the above transformation

$$(\partial\phi)^2 \rightarrow f(\phi)(\partial\phi)^2$$

Putting these transformations in the above action integral, equation (4.2) , we get the same action in the Einstein frame:

$$S^E = \int d^4x \frac{\sqrt{-g^E}}{f(\phi)^2} \left[ \frac{1}{2} m_p^2 f(\phi) \left( f(\phi) \mathcal{R}^E - \frac{3}{2} f'(\phi)^2 (\partial\phi)^2 \right) - \frac{1}{2} k(\phi) (\partial\phi)^2 - V(\phi) \right] \tag{4.7}$$

or

$$S^E = \int d^4x \sqrt{-g^E} \left[ \frac{1}{2} m_p^2 \mathcal{R}^E - \frac{3}{2} m_p^2 \frac{f'(\phi)^2}{f(\phi)^2} (\partial\phi)^2 - \frac{1}{2} \frac{k(\phi)}{f(\phi)} (\partial\phi)^2 - \frac{V(\phi)}{f(\phi)^2} \right] \tag{4.8}$$

Now we may define

$$V^E \equiv \frac{V(\phi)}{f(\phi)^2} \tag{4.9}$$

and a new field as

$$(d\Omega)^2 \equiv \frac{3}{2} m_p^2 \frac{f'(\phi)^2}{f(\phi)^2} (\partial\phi)^2 + \frac{k(\phi)}{f(\phi)} (\partial\phi)^2$$

or more explicitly

$$\left( \frac{d\Omega}{\partial\phi} \right)^2 = \frac{3}{2} m_p^2 \frac{f'(\phi)^2}{f(\phi)^2} + \frac{k(\phi)}{f(\phi)} \tag{4.10}$$

Substituting the expressions from equations (4.9) and (4.10) in equation (4.8) , we get

$$S^E = \int d^4x \sqrt{-g^E} \left[ \frac{1}{2} m_p^2 \mathcal{R}^E - \frac{1}{2} \left( \frac{d\Omega}{d\phi} \right)^2 - V^E \right] \tag{4.11}$$

This is the final form of the action integral in the Einstein frame.

### 4.2.2 Slow-Roll Parameters

The shape of the potential determines the inflationary dynamics and the cosmological predictions. As given in the previous chapter, the first and the second slow-roll parameters are defined respectively as

$$\varepsilon = \frac{1}{2} m_p^2 \left( \frac{V^E_{,\Omega}}{V^E} \right)^2 \tag{4.12}$$

$$\eta = m_p^2 \left( \frac{V^E_{,\Omega\Omega}}{V^E} \right) \tag{4.13}$$

For the present case, as we have defined the action and hence the Lagrangian in terms of another field $\Omega$, therefore

$$\frac{dV^E}{d\phi} = \frac{\partial V^E}{\partial \Omega} \frac{\partial \Omega}{\partial \phi} \qquad \Longrightarrow \qquad \frac{\partial V^E}{\partial \Omega} = \frac{dV^E}{d\phi} \left( \frac{\partial \Omega}{\partial \phi} \right)^{-1} \tag{4.14}$$

or

$$\frac{\partial V^E}{\partial \Omega} = V^{E'} \left( \frac{\partial \Omega}{\partial \phi} \right)^{-1} \tag{4.15}$$

where $V'$ is the derivative of $V$ with respect to $\phi$. In the similar way

$$\begin{aligned} \frac{d^2 V^E}{d\phi^2} &= \frac{\partial}{\partial \phi} \left[ \frac{\partial V^E}{\partial \Omega} \frac{\partial \Omega}{\partial \phi} \right] \\ &= \left( \frac{\partial^2 V^E}{\partial \Omega^2} \frac{\partial \Omega}{\partial \phi} \right) \frac{\partial \Omega}{\partial \phi} + \frac{\partial V^E}{\partial \Omega} \frac{\partial^2 \Omega}{\partial \phi^2} \end{aligned}$$

Plugging the expression for $(\partial V/\partial \Omega)$ from equation (4.15), we get

$$V^{E''} = \frac{d^2 V^E}{d\phi^2} = \frac{\partial^2 V^E}{\partial \Omega^2}\left(\frac{\partial \Omega}{\partial \phi}\right)^2 + V^{E'}\left(\frac{\partial \Omega}{\partial \phi}\right)^{-1}\frac{\partial^2 \Omega}{\partial \phi^2}$$

or

$$\frac{\partial^2 V^E}{\partial \Omega^2} = \left[V^{E''} - V^{E'}\left(\frac{\partial \Omega}{\partial \phi}\right)^{-1}\frac{\partial^2 \Omega}{\partial \phi^2}\right]\left(\frac{\partial \Omega}{\partial \phi}\right)^{-2} \tag{4.16}$$

Putting the expressions for $(\partial V/\partial \Omega)$ from equation (4.15) in equation (4.12) for $\varepsilon$ gives

$$\boxed{\varepsilon = \tfrac{1}{2} m_p^2 \left(\tfrac{V^{E'}}{V^E}\right)^2 \left(\tfrac{\partial \Omega}{\partial \phi}\right)^{-2}} \tag{4.17}$$

and on substituting the expression for $(\partial^2 V/\partial \Omega^2)$ from equation (4.16) in equation (4.12) for $\eta$, we get

$$\boxed{\eta = m_p^2 \left[\tfrac{V^{E''}}{V^E}\left(\tfrac{\partial \Omega}{\partial \phi}\right)^{-2} - \tfrac{V^{E''}}{V^E}\left(\tfrac{\partial \Omega}{\partial \phi}\right)^{-3}\tfrac{\partial^2 \Omega}{\partial \phi^2}\right]} \tag{4.18}$$

where $d\Omega/\partial \phi$ is given by equation (4.10).

### 4.2.3 Form of the Potential

The potential from the Higgs sector is

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \upsilon^2\right)^2 \approx \frac{\lambda}{4}\phi^4 \tag{4.19}$$

where $\lambda$ is the Higgs self-coupling and $\upsilon$ is the symmetry breaking scale. As we are assuming that inflation took place at GUT energy scale, therefore $\phi \gg \nu$ and so we ignore $\upsilon$. Specializing to gauge invariant, dimension less than 4 operators and without the higher derivatives, $f(\phi)$ must have the following form:

$$f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2} \tag{4.20}$$

where $\xi$ is the non-minimal coupling constant between gravity and the Higgs field.

Now from equations (4.9) and (4.20), we have

$$V^E = \frac{\frac{\lambda}{4}\phi^4}{\left(1 + \frac{\xi\phi^2}{m_p^2}\right)^2} \tag{4.21}$$

These expressions can be simplified if expressed in terms of a dimensionless variable $\psi$, defined as

$$\psi \equiv \frac{\sqrt{\xi}}{m_p}\phi \tag{4.22}$$

with this the above expressions become

$$f(\phi) = 1 + \psi^2 \tag{4.23}$$

and

$$V^E = \frac{\lambda m_p^4}{4\xi^2}\left[\frac{\psi^4}{\left(1 + \psi^2\right)^2}\right] \tag{4.24}$$

### 4.2.4 Calculation of Slow-Roll Parameters

**Calculation of $\varepsilon$**

Using the expression for $V^E$ in equation (4.24) , we have

$$V^{E'} = \frac{\lambda m_p^3}{4\xi^{\frac{3}{2}}}\left[\frac{4\psi^3\left(1+\psi^2\right)^2 - 4\psi^5\left(1+\psi^2\right)}{\left(1+\psi^2\right)^4}\right] \tag{4.25}$$

and

$$\frac{V^{E'}}{V^E} = \frac{\sqrt{\xi}}{m_p}\left[\frac{4}{\psi\left(1+\psi^2\right)}\right] \tag{4.26}$$

so

$$\left(\frac{V^{E'}}{V^E}\right)^2 = \frac{\xi}{m_p^2}\left[\frac{4}{\psi\left(1+\psi^2\right)}\right]^2 = \frac{\xi}{m_p^2}\left[\frac{16}{\psi^2\left(1+\psi^2\right)^2}\right] \tag{4.27}$$

The expression for $(d\Omega/\partial\phi)^2$ is given in equation (4.10) , by using which we get

$$\left(\frac{d\Omega}{\partial\phi}\right)^2 = \frac{3}{2}m_p^2\,\frac{f'\left(\phi\right)^2}{f\left(\phi\right)^2} + \frac{1}{f\left(\phi\right)} \tag{4.28}$$

Using the expression for $f\left(\phi\right)$ in equation (4.23)

$$f'(\phi) = 2\psi\left(\frac{\sqrt{\xi}}{m}\right)$$

we have

$$\frac{f'(\phi)}{f\left(\phi\right)} = \frac{2\psi}{\left(1+\psi^2\right)}\left(\frac{\sqrt{\xi}}{m}\right)$$

and hence

$$\left(\frac{f'(\phi)}{f(\phi)}\right)^2 = \frac{\xi}{m^2}\frac{4\psi^2}{\left(1+\psi^2\right)^2} \tag{4.29}$$

Plugging the expressions for $\left(f'(\phi)/f(\phi)\right)^2$and $f(\phi)$ in equation (4.28), the expression becomes

$$\left(\frac{d\Omega}{\partial\phi}\right)^2 = \frac{1+\psi^2+6\xi\psi^2}{\left(1+\psi^2\right)^2} \tag{4.30}$$

therefore

$$\left(\frac{d\Omega}{\partial\phi}\right)^{-2} = \frac{\left(1+\psi^2\right)^2}{1+\psi^2+6\xi\psi^2} \tag{4.31}$$

According to equation (4.17) , $\varepsilon$ is given by

$$\varepsilon = \frac{1}{2}m_p^2\left(\frac{V'}{V}\right)^2\left(\frac{\partial\Omega}{\partial\phi}\right)^{-2}$$

Putting the expressions for $(V'/V)^2$ and $(d\Omega/\partial\phi)^{-2}$ from equations (4.27) and (4.31) respectively, we get

$$\varepsilon = \frac{1}{2}m_p^2\left[\frac{\xi}{m^2}\left(\frac{16}{\psi^2\left(1+\psi^2\right)^2}\right)\frac{\left(1+\psi^2\right)^2}{1+\psi^2+6\xi\psi^2}\right]$$

which simplifies to

$$\varepsilon = 8\xi\frac{1}{\left(1+\psi^2+6\xi\psi^2\right)\psi^2}$$

The expression further simplifies if we consider the large $\xi$ limit which is the physical case, hence

$$\varepsilon_{\text{for large }\xi} \;\rightarrow\; \varepsilon \approx \frac{8\xi}{6\xi\psi^4}$$

or

$$\boxed{\varepsilon \approx \tfrac{4}{3\psi^4}} \tag{4.32}$$

**Calculation of $\eta$**

$\eta$ is given by equation (4.18)

$$\eta = m_p^2\left[\frac{V^{E''}}{V^E}\left(\frac{\partial\Omega}{\partial\phi}\right)^{-2} - \frac{V^{E'}}{V^E}\left(\frac{\partial\Omega}{\partial\phi}\right)^{-3}\frac{\partial^2\Omega}{\partial\phi^2}\right]$$

Now from the expression of $V^E$

$$V^{E''} = \frac{d}{d\phi}\left[\frac{\lambda m^3}{4\xi^{\frac{3}{2}}}\left(\frac{4\psi^3\left(1+\psi^2\right)^2 - 4\psi^5\left(1+\psi^2\right)}{\left(1+\psi^2\right)^4}\right)\right] \tag{4.33}$$

$$= \frac{\lambda m^2}{4\xi}\left[\frac{12\psi^2\left(1+\psi^2\right)^6 - 36\psi^4\left(1+\psi^2\right)^5 + 24\psi^6\left(1+\psi^2\right)^4}{\left(1+\psi^2\right)^8}\right] \tag{4.34}$$

so

$$\frac{V^{E''}}{V^E} = \frac{\xi}{m^2\psi^4}\left[\frac{12\psi^2\left(1+\psi^2\right)^6 - 36\psi^4\left(1+\psi^2\right)^5 + 24\psi^6\left(1+\psi^2\right)^4}{\left(1+\psi^2\right)^6}\right]$$

which simplifies to

$$\frac{V^{E''}}{V^E} = \frac{12\xi}{m^2}\left[\frac{\left(1-\psi^2\right)}{\psi^2\left(1+\psi^2\right)^2}\right] \tag{4.35}$$

Using the expression for $(d\Omega/\partial\phi)^{-2}$, we obtain

$$\left(\frac{\partial\Omega}{\partial\phi}\right)^{-3} = \frac{\left(1+\psi^2\right)^3}{\left(1+\psi^2+6\xi\psi^2\right)^{\frac{3}{2}}} \tag{4.36}$$

and $\left(\partial^2\Omega/\partial\phi^2\right)$ is calculated using equation (4.30) as follows:

$$\begin{aligned}\frac{\partial^2\Omega}{\partial\phi^2} &= \frac{d}{d\phi}\left[\frac{\sqrt{1+\psi^2+6\xi\psi^2}}{\left(1+\psi^2\right)}\right] \\ &= \frac{\frac{\sqrt{\xi}}{m}}{\left(1-\psi^2\right)^2}\left[\frac{\psi\left(6\xi-1\right)-\psi^3\left(6\xi+1\right)}{\left(1+\psi^2+6\xi\psi^2\right)^{\frac{1}{2}}}\right]\end{aligned} \tag{4.37}$$

Now, from equations (4.35) and (4.31)

$$\begin{aligned}\frac{V^{E''}}{V^E}\left(\frac{\partial\Omega}{\partial\phi}\right)^{-2} &= \frac{12\xi}{m^2}\left[\frac{\left(1-\psi^2\right)}{\psi^2\left(1+\psi^2\right)^2}\right]\frac{\left(1+\psi^2\right)^2}{1+\psi^2+6\xi\psi^2} \\ &= \frac{12\xi\left(1-\psi^2\right)}{m^2\psi^2\left(1+\psi^2+6\xi\psi^2\right)}\end{aligned} \tag{4.38}$$

and using equations (4.25), (4.36) and (4.37) , we get

$$\frac{V^{E''}}{V^E}\left(\frac{\partial\Omega}{\partial\phi}\right)^{-3}\frac{\partial^2\Omega}{\partial\phi^2} = \frac{\sqrt{\xi}}{m}\left[\frac{4}{\psi\left(1+\psi^2\right)}\right]\left[\frac{\left(1+\psi^2\right)^3}{\left(1+\psi^2+6\xi\psi^2\right)^{\frac{3}{2}}}\right]\times$$

$$\frac{\frac{\sqrt{\xi}}{m}}{\left(1-\psi^2\right)^2}\left[\frac{\psi\left(6\xi-1\right)-\psi^3\left(6\xi+1\right)}{\left(1+\psi^2+6\xi\psi^2\right)^{\frac{1}{2}}}\right]$$

which simplifies to

$$\frac{V^{E''}}{V^E}\left(\frac{\partial\Omega}{\partial\phi}\right)^{-3}\frac{\partial^2\Omega}{\partial\phi^2} = \frac{4\xi}{m^2}\left[\frac{\left(6\xi-1\right)-\psi^2\left(6\xi+1\right)}{\left(1+\psi^2+6\xi\psi^2\right)^2}\right] \tag{4.39}$$

Putting the expression from equations (4.38) and (4.39) in the expression for $\eta$, equation (4.18) , we get

$$\eta = m_p^2\left[\frac{12\xi\left(1-\psi^2\right)}{m^2\psi^2\left(1+\psi^2+6\xi\psi^2\right)} - \frac{4\xi}{m^2}\left[\frac{\left(6\xi-1\right)-\psi^2\left(6\xi+1\right)}{\left(1+\psi^2+6\xi\psi^2\right)^2}\right]\right]$$

simplification yields

$$\eta = 4\xi\left[\frac{3+18\xi\psi^2-3\psi^4-18\xi\psi^4-6\xi\psi^2+\psi^2+6\xi\psi^4}{\psi^2\left(1+\psi^2+6\xi\psi^2\right)^2}\right]$$

$$= \frac{4\xi}{\psi^2\left(1+\psi^2+6\xi\psi^2\right)^2}\left[3+12\xi\psi^2-12\xi\psi^4-3\psi^4\psi^2\right]$$

Again considering the large $\xi$ limit:

$$\eta_{\text{for large } \xi} \quad \rightarrow \quad \eta = \frac{4\xi}{36\xi^2\psi^6}\left[12\xi\psi^2 - 12\xi\psi^4\right]$$

$$= \quad \frac{4\psi^2}{3\psi^6}\left[1-\psi^2\right]$$

which can be put in a more illuminating form as:

$$\boxed{\eta = \tfrac{-4}{3\psi 2}\left[1 - \tfrac{1}{\psi^2}\right]} \tag{4.40}$$

### 4.2.5 Number of e-Folds

The number of e-folds during inflation is given by

$$N_e = \frac{1}{\sqrt{2}m}\int_{\phi_e}^{\phi}\frac{d\phi}{\sqrt{\varepsilon}}\left(\frac{d\Omega}{d\phi}\right) \tag{4.41}$$

where $\phi_e$ is the value of field at which inflation ends and $\phi$ is its value during inflation. $\phi_e$ is obtained using the slow-roll approximation i.e.

$$\varepsilon \approx 1$$

from equation (4.32)

$$\psi_e = \left(\frac{4}{3}\right)^{\frac{1}{4}} \tag{4.42}$$

Putting the expressions for $\varepsilon$ and $(d\Omega/\partial\phi)$ in equation (4.41)

$$N_e = \frac{1}{\sqrt{2}m}\int_{\phi_e}^{\phi}\frac{d\phi}{\sqrt{\frac{4}{3\psi^4}}}\frac{\sqrt{1+\psi^2+6\xi\psi^2}}{\left(1+\psi^2\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}m}\left(\frac{m}{\sqrt{\xi}}\right)\int_{\psi_e}^{\psi} d\psi\frac{\psi^2\sqrt{1+\psi^2+6\xi\psi^2}}{\left(1+\psi^2\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2\xi}}\int_{\psi_e}^{\psi} d\psi\frac{\psi^2\sqrt{1+\psi^2+6\xi\psi^2}}{\left(1+\psi^2\right)}$$

Solving this integral, the result obtained with all parameters to be positive is

$$N_e = \frac{3}{4}\left|[\psi^2 - \ln\left(1+\psi^2\right)]\right|_{\psi_e}^{\psi}$$

applying the limits

$$N_e = \frac{3}{4}[\psi^2 - \psi_e^2 - \ln\left(\frac{1+\psi^2}{1+\psi_e^2}\right) \tag{4.43}$$

If we take $N_e = 60$ , as is required by majority of the inflationary models, then this equation can give us the expression for the unknown $\psi$ during inflation. The equation (4.43) becomes

$$60 = \frac{3}{4}[\psi^2 - \left(\frac{4}{3}\right)^{\frac{1}{2}} - \ln\left(\frac{1+\psi^2}{1+\left(\frac{4}{3}\right)^{\frac{1}{2}}}\right)$$

or

$$\psi^2 - \ln\left(1+\psi^2\right) = 80 + 1.1155 - 0.76765$$

From this equation we get the value for $\psi^2$

$$\psi^2 = 84.32$$

and

$$\psi = 9.20 \tag{4.44}$$

### 4.2.6 Numerical Values of the Slow-Roll Parameters

Putting the value for $\psi$ in the expressions for $\varepsilon$ and $\eta$ yield their numerical values which are used to find the values of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$. The values of $n_s$ and $r$ are then compared with the observed values of these parameters from the WMAP satellite.

The value of $\varepsilon$ comes out to be

$$\varepsilon = \frac{4}{3\,(9.182995387)^4} = 1.875 \times 10^{-4} \tag{4.45}$$

and that of $\eta$ is

$$\begin{aligned}\eta &= \frac{-4}{3\,(9.182995387)^2}\left[1 - \frac{1}{(9.182995387)^2}\right] \\ &= -0.0156233888 \end{aligned} \tag{4.46}$$

### 4.2.7 Calculation of $n_s$ and $r$

The expressions for $n_s$ is

$$n_s = 1 - 6\varepsilon + 2\eta \tag{4.47}$$

and that for $r$ is

$$r = 16\varepsilon \tag{4.48}$$

Substituting the values of $\varepsilon$ and $\eta$ in equation (4.47)

$$\begin{aligned} n_s &= 1 - 6\left(1.875 \times 10^{-4}\right) + 2\left(-0.0156233888\right) \\ &= 0.968 \end{aligned} \tag{4.49}$$

Substituting the values of $\varepsilon$ in equation (4.48)

$$\begin{aligned} r &= 16\left(1.875 \times 10^{-4}\right) \\ &= 3.0 \times 10^{-3} \end{aligned} \tag{4.50}$$

Thus the classical results come out to be independent of the parameters of the Standard Model, in particular the Higgs mass and the Higgs self coupling $\lambda$. But these results are in good agreement with the cosmological data. They lie within the $1\sigma$ bound of the most recent WMAP 9 data [24].

## 4.3 Quantum Analysis

Now we consider how quantum corrections modify the classical results. For this we need to compute the action including the effects of the interactions of Higgs field with the particles of the Standard Model of particle physics through quantum loops. These calculations are done in the Jordan frame. The quantum corrections modify the expressions for $f(\phi)$, $k(\phi)$ and $V(\phi)$.

Lagrangian of the graviton-inflaton sector is

$$\mathcal{L} = \frac{1}{2} f(\phi)\,\mathcal{R} - \frac{k(\phi)}{2}(\partial\phi)^2 - V(\phi) \tag{4.51}$$

where

$$f(\phi) = 1 + \frac{\xi\phi^2}{m_p^2} \tag{4.52}$$

and

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \upsilon^2\right)^2 \tag{4.53}$$

and the Higgs field $\phi$ has a strong non-minimal coupling with $\xi \gg 1$.

The quantum corrections to the kinetic energy term, which is taken as $k(\phi) = 1$ in classical case, comes from wave-function renormalization and are approximately independent of $\xi$. Corrections to $k(\phi)$ comes with a factor of $1/\xi$ and is also suppressed by loop factors and couplings [21].

Quantum corrections to the potential are very important. They provide the upper bound to the Higgs mass. Higgs lighter than 230 $GeV$ can not act as an inflaton because the corresponding spectral index is ruled out by the WMAP data.

### 4.3.1 First Order Radiative Corrections

The quantum effective action, without the contribution of higher derivatives, reads

$$S=\int d^4x\sqrt{-g}\left(U(\phi)\mathcal{R}-\frac{1}{2}G\left(\phi\right)\left(\nabla\phi\right)^2-V\left(\phi\right)\right) \tag{4.54}$$

where

$$U(\phi)\equiv\frac{m_p^2}{2}f(\phi) \tag{4.55}$$

The *first-order* radiative correction to the potential in equation (4.54) is

$$V_{(1-loop)}\left(\phi\right)=\Sigma\left(\pm1\right)\frac{m^4(\phi)}{64\pi^2}\ln\frac{m^2(\phi)}{\mu^2}=\frac{\lambda A}{128\pi^2}\phi^4\ln\frac{\phi^2}{\mu^2}+.... \tag{4.56}$$

where $A$ is the *anomalous scaling factor* which appears in quantum treatment of the problem. It is a special combination of the coupling constants present in the Standard Model of particle physics. Due to the large value of the non-minimal coupling constant $\xi$, all the quantum corrections are determined by $A$. For $\xi\gg1$, the dominant correction to $U\left(\phi\right)$ is

$$U_{(1-loop)}\left(\phi\right)=\frac{3\xi\lambda}{32\pi^2}\phi^2\ln\frac{\phi^2}{\mu^2}+.... \tag{4.57}$$

The anomalous scaling $A$ and the corrections to $U\left(\phi\right)$ determine the inflationary dynamics.

As inflation is easy to analyze in the Einstein frame, so we transform the quantum results in the Einstein frame, where the new potential takes the form

$$V = \frac{\lambda m_p^4}{4\xi^2}\left(1 - \frac{2m_p^2}{\xi\phi^2} + \frac{A_i}{16\pi^2}\ln\frac{\phi}{\mu}\right) \tag{4.58}$$

where $A_i$ is the *inflationary anomalous scaling*, which arises as a result of the modification in $A$ due to quantum corrections.

The expressions for inflationary parameters i.e. $\varepsilon$ and $\eta$ include $A_i$. This quantity $A_i$ also enters in the expressions for the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ and therefore from the observational constraints over the values of these parameters we can calculate the CMB compatible range for the scaling $A_i$,which is

$$-12 < A_i < 14 \tag{4.59}$$

In the Standard Model, the scale factor $A$ is given in terms of the coupling constants. Using the *1-loop* corrections to the potential, equation (4.56), the expression for $A$ comes out to be

$$A = \frac{3}{8\lambda}[2g^4 + (g^2 + g'^2)^2 - 16y_t^4] + 6\lambda \tag{4.60}$$

For the accepted range of the Higgs mass i.e. [25]

$$115 \leqslant m_H \leqslant 180 \tag{4.61}$$

$A$ belongs to the following range

$$-48 < A < -20 \tag{4.62}$$

at the electroweak scale. Comparison of this range with the range obtained using the CMB observational constraints (4.59) shows that they strongly contradict each other.

This implies that the radiative corrections alone can not make the theory compatible with the known mass range of the Higgs field. In order to add more precision to the theory, Renormalization Group (RG) Improvement is added.

### 4.3.2 Renormalization Group (RG) Improvement

According to the technique used by Coleman and Weinberg [26], the action with *1-loop* RG improvement is given by

$$S = \int d^4 x g^{\frac{1}{2}} \left( -V(\phi) = U(\phi)\,\mathcal{R} - \frac{1}{2} G(\phi) \left(\nabla \phi\right)^2 + .... \right) \tag{4.63}$$

with $V(\phi), U(\phi)$ and $G(\phi)$ are given as:

$$V(\phi) = \frac{\lambda(t)}{4} Z^4(s)\,\phi^4 \tag{4.64}$$

$$U(\phi) = \frac{1}{2}\left(m_p^2 + \xi(s)\, Z^2 \phi^2\right) \tag{4.65}$$

$$G(\phi) = Z^2(s) \tag{4.66}$$

where $Z(s)$ is the field renormalization. Here the running scale $s = ln(\phi/\mu)$ is normalized at the top quark mass, $\mu = m_{top}$.

The couplings $\lambda(t)$, $\xi(t)$ and $Z(t)$ belong to the solution of RG equations

$$\frac{dg_i}{dt} = \beta_{g_g} \tag{4.67}$$

$$\frac{dZ}{dt} = \gamma Z \tag{4.68}$$

Considering only the Higgs coupling to the other fields, the *1-loop* beta functions are obtained in terms of the suppression factor $\omega$ as [27]

$$\beta_\lambda = \frac{\lambda}{16\pi^2}\left(18\omega^2\lambda + A\left(t\right)\right) - 4\gamma\lambda \tag{4.69}$$

$$\beta_\xi = \frac{6\xi}{16\pi^2}\left(1+\omega^2\right)\lambda - 2\gamma\xi \tag{4.70}$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(\frac{-2}{3}g'^2 - 8g_s^2 + (1+\frac{\omega}{2})y_t^2\right) - \gamma y_t \tag{4.71}$$

$$\beta_g = -\frac{\left(39-\omega\right)g^3}{\left(12\right)16\pi^2} \tag{4.72}$$

$$\beta_{g'} = \frac{\left(18+\omega\right)g'^3}{\left(12\right)16\pi^2} \tag{4.73}$$

$$\beta_{g_s} = -\frac{7g_s}{16\pi^2} \tag{4.74}$$

where $\gamma$ is the *anomalous dimension* of the Higgs field and is given by a standard expression

$$\gamma = \frac{1}{16\pi^2}\left(\frac{9g^2}{4} + \frac{3g'^2}{4} - 3y_t^2\right) \tag{4.75}$$

### RG Improved Results

The RG improvement of the action reveals that this action coincides with the tree level action for a new field

$$\varphi = Z(s)\phi \tag{4.76}$$

Then from equations (4.64) and (4.65), the RG improved potentials take the form of *1-loop* potential, equation (4.58), for $\varphi$ with normalization being at point $\mu = \varphi_{end.}$ In this case, the parameters of the theory are determined by the running anomalous scaling $A_i(s)$ taken at $s_{end}$ and the running of $A(s)$ depends upon $\lambda(s)$.

$A_i(s)$ varies from large negative values at the electroweak scale towards the smaller negative values at inflation. This makes the observational data from the CMB compatible with the generally accepted range of the Higgs mass. The RG flow allows to calculate the power spectrum as a function of Higgs mass $m_H$. The scalar spectral index drops below $n_s = 0.94$ for more negative values of the running scale, which happens only when $m_H$ approaches the instability bound or $m_H > 180GeV$. So we get the bounds on $m_H$. Both the lower and upper bound is obtained from the lower bound on the CMB data. The range obtained from the numerical analysis, for $m_H$, is [27]

$$135.62GeV \leqslant m_H \leqslant 184.49GeV \tag{4.77}$$

## 4.3.3 Quantum Results

It is seen that the inflationary parameters in the quantum case varies with the Higgs mass. Radiative corrections are enhanced by a large value of $\xi$. In the RG improved theory, the scalar spectral index $n_s$ depends upon the Standard Model parameters, in particular on the Higgs mass and the mass of the top quark. This is shown in figure 4-1.

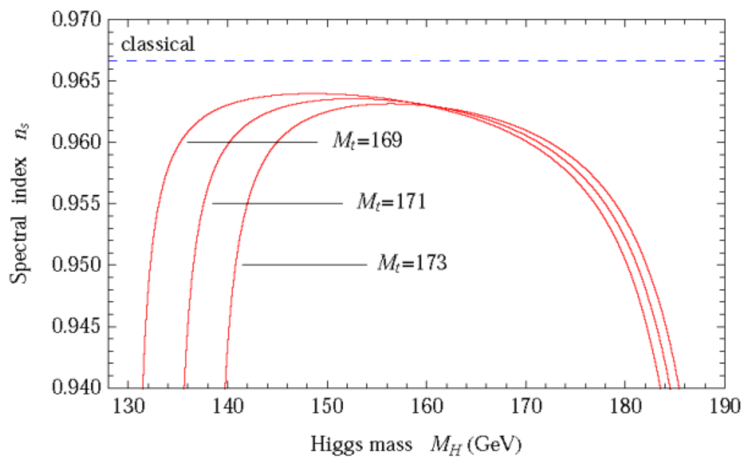


Figure 4-1: **Plot of the scalar spectral index** $n_s$ *as a function of Higgs mass* $m_H$ *for three different values of top quark mass* $m_t$ [28].

## 4.4 Cutoff Problem In Quantum Calculations

The energy scale at which inflation took place is approximately

$$E_i \approx \frac{m_p}{\sqrt{\xi}} \tag{4.78}$$

Considering the term in the Lagrangian, $(\xi/m_p)\,\phi^2\Box\phi$, amplitude for the $2\phi \to 2\phi$ scattering in high energy limit is

$$A \sim \left(\frac{\xi}{m_p}\right)^2 E^2$$

where the factor $(\xi/m_p)^2$ comes from the vertex contribution and as amplitude is dimensionless, therefore the other factor is energy. Now as the amplitudes sum up to unity so we have

$$A \sim \left(\frac{\xi}{m_p}\right)^2 E^2 \leqslant 1$$

this implies the energy scale to be

$$E \leqslant \frac{m_p}{\xi} \tag{4.79}$$

which is defined as the cutoff point of the theory i.e. [29]

$$\Lambda \equiv \frac{m_p}{\xi} \tag{4.80}$$

above which the Standard Model has to be replaced by a more fundamental theory. This makes Higgs inflation unnatural [27]. As $\xi$ is large, this cutoff $\Lambda$ is far below the Planck mass $m_p$ and so it is considerably smaller than the value of the Higgs field $\phi$ during inflation. Comparing equations (4.79) and (4.80)

$$\Lambda \ll E_i \tag{4.81}$$

This arises a question on the self-consistency of Higgs inflation.

### 4.4.1 Cutoff Problem Revisited

To get rid of this cutoff problem, one needs to determine the energy domain in which the theory is valid with large non-minimal coupling. The energy domain $E < \Lambda$ depends on the background value of the Higgs field $\phi$ and for $\phi = 0$, it has the upper bound $\Lambda \equiv m_p/\xi$. A cutoff that is obtained by a non-zero background value of the field has the upper bound higher than the energy scales characterizing the dynamics of inflation. Furthermore, $\Lambda$ coincides with $m_p$ during inflation.

### 4.4.2 Calculation of Cutoff in the Jordan Frame

To obtain the cutoff scale, scalar field $\phi$ and metric $g_{\alpha\beta}$ are expanded around their background values having the subscript $(...)_b$

$$\phi = \phi_b + \delta\phi \tag{4.82}$$

$$g_{\alpha\beta} = (g_{\alpha\beta})_b + h_{\alpha\beta} \tag{4.83}$$

where $\delta\phi$ and $h_{\alpha\beta}$ are the perturbations. The second order Lagrangian for the perturbations has the form [29]

$$\begin{aligned}\mathcal{L} \quad &= - \quad \frac{m_p^2 + \xi\phi_b^2}{8} \left(h^{\alpha\beta}\Box h_{\alpha\beta} + 2\partial_\beta h^{\alpha\beta}\partial^\gamma h_{\alpha\gamma} - 2\partial_\beta h^{\alpha\beta}\partial_\alpha h - h\Box h\right) + \\ &\frac{1}{2}\left(\partial_\alpha \delta\phi\right)^2 + \xi\phi_b \left(\Box h - \partial\sigma\partial\gamma h^{\sigma\gamma}\right)\delta\phi \end{aligned} \tag{4.84}$$

The terms up to two derivatives are retained because they determine the high energy scattering amplitudes and hence the cutoff scale. In this approach of non-trivial background field, there is a large mixing of scalar perturbations and the metric. The kinetic term can be diagonalized by the following change of variables

$$\delta\phi = \sqrt{\frac{m_p^2 + \xi\phi_b^2}{m_p^2 + \xi\phi_b^2 + 6\xi^2\phi_b^2}}\delta\hat{\phi}$$

and

$$h_{\alpha\beta} = \frac{1}{\sqrt{m_p^2 + \xi\phi_b^2}}\hat{h}_{\alpha\beta} - \frac{2\xi\phi_b}{\sqrt{\left(m_p^2 + \xi\phi_b^2\right)\left(m_p^2 + \xi\phi_b^2 + 6\xi^2\phi_b^2\right)}}\left(g_{\alpha\beta}\right)_b \delta\hat{\phi}$$

The cutoff scale is now read out of the operators with dimensions greater than four. The leading order term is the interaction $\xi\Box h\left(\delta\phi\right)^2$, which has the form

$$\frac{\xi\sqrt{m_p^2+\xi\phi_b^2}}{\left(m_p^2+\xi\phi_b^2+6\xi^2\phi_b^2\right)}\left(\delta\hat{\phi}\right)^2\Box\hat{h}$$

The inverse of the coefficient is identified as the cutoff in the Jordan frame

$$\Lambda_j\left(\phi_b\right)=\frac{\left(m_p^2+\xi\phi_b^2+6\xi^2\phi_b^2\right)}{\xi\sqrt{m_p^2+\xi\phi_b^2}} \tag{4.85}$$

This expression for cutoff simplifies differently for three different regions of the background field:

### $\phi_b \ll \frac{m_p}{\xi}$ or the Low Field Region

This small field region corresponds to the present universe. In this region the cutoff is

$$\Lambda_j\simeq\frac{m_p}{\xi} \tag{4.86}$$

which is similar to the previous result with zero background. It is smaller than the Planck mass $m_p$, but it is way above the present energy content.

### $\frac{m_p}{\xi} \ll \phi_b \ll \frac{m_p}{\sqrt{\xi}}$ or the Intermediate Region

This intermediate region is relevant for the *reheating era.* The cutoff scale in this region is

$$\Lambda_j\simeq\frac{\xi\phi_b^2}{m_p} \tag{4.87}$$

which is below the Planck mass $m_p$, but starts to grow further.

### $\phi_b \gg \frac{m_p}{\sqrt{\xi}}$ or the Large Field Region

The large field region corresponds to the inflationary period. Here the cutoff becomes

$$\Lambda_j \simeq \sqrt{\xi}\phi \tag{4.88}$$

This value of the cutoff is much higher than its value in the case when the background field was absent. So the problem of cutoff is resolved by expanding the field around a non zero background.

## 4.5 Conclusion

In contrast to the classical analysis, the quantum mechanical study of the case tells that the inflationary parameters depend on the Higgs mass. Radiative corrections are enhanced by the large value of $\xi$. In the RG improved theory, the scalar spectral index $n_s$ depends upon the Standard Model parameters, particularly on the masses of Higgs particle and the top quark. This fact is illustrated in the figure 4-1, showing the behaviour of the scalar spectral index with the change in the Higgs mass.

Now as the results from LHC have been arrived, we know that the possible Higgs particle has the mass value of 125.6 GeV [30, 31]. When we compare this value to the CMB compatible range of the Higgs mass, it is apparent that the LHC value falls short of the range. Moreover, recent results from the ESA's **Planck** [32] do not alter the existing value of the scalar spectral index $n_s$ and so with the current parameters of the Standard Model and the LHC mass value of the Higgs particle, this model of inflation does not seem to be successful.

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