# SHIELDING OF TEST CHARGE PROJECTILES IN PLASMA

MASTER OF PHILOSOPHY

IN

PHYSICS

BY

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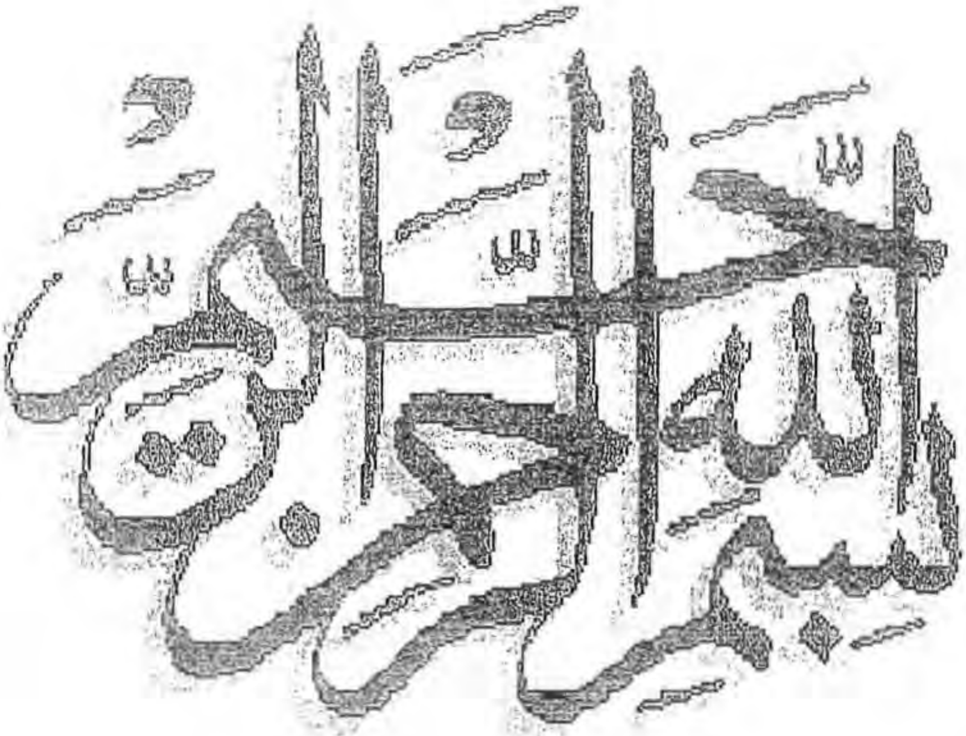


IN THE NAME OF
ALLAH
THE BENEFICENT
THE MERCIFUL

This work is submitted as a dissertation in partial fulfillment for the award of the

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**2009**

TO

MY SWEET PARENTS

## CERTIFICATE

It is certified that the work contained in this dissertation was carried out by **Mr. Muhammad Shoaib** under my supervision.

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Shoaib

*MUHAMMAD SHOAIB BALOCH*

# ABSTRACT

In this work, we have studied the shielding potential ($\phi$) of a test charge in plasma by using the kinetic theory approach. We found that for slowly moving test charge (compared with the thermal velocity of ions and electrons), the test charge is shielded both by electrons and ions. For a fast moving test charge (compared with the velocity of electrons), no shielding was observed. For a test charge moving with the intermediate speed (greater than ions but less than electrons thermal velocity), only electrons were found to take part in the shielding. Next, we have calculated the shielding potential in dusty plasma by using the linear dielectric theory. The electrostatic potential for two projectiles is computed for different values of $K_D$ (normalize effective wave number) and R (the separation between the two projectiles) retaining two ion correlation effects and then it is compared with that of single ion projectile case. Further, we extend it for multi-component plasma. Here we also employ the test charge approach to calculate the shielding potential. The constituents of multi-component plasma are the Boltzmann distributed electrons, mobile positive and negative ions, and immobile positive/negative dust particles. The shielded potential is found to be modified due to the presence of negative ions in plasma. We have calculated the Debye screening and wake potential. It is found that the presence of negative ions significantly modify the dust-ion-acoustic speed.

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# Chapter 1

# Introduction

*In this chapter, we discuss the basic parameters of plasma and the historical background of test charge projectiles in plasma, potential produced, wake potential and energy loss in plasma.*

## 1.1 What is Plasma

The term plasma is a Greek word which means formed or molded. This term was introduced by Czech Physiologist Jan. Evangelista Purkinje. He used the word plasma to denote a clear fluid which remains after the removal of all the corpuscular material in blood [1]. In 1922 American scientist Irving Langmuir proposed that the electrons, ions and neutrals in an ionized gas could similarly be considered as corpuscular material entertained in some kind of fluid medium and called this entertaining "plasma". Any ionized gas cannot be called a plasma, there is always some degree of ionization required for it. "A plasma is a quasineutral gas of charged and neutral particles which shows collective behavior"[2]. The plasma is "quasineutral" that is neutral enough that one can say that $n_i \simeq n_e \simeq n$, where $n$ is the plasma density, but plasma is not so neutral that all interesting electromagnetic forces vanish. Plasma contains the charged particles, as these charged particles moves around, they can generate local concentrations of positive or negative charges, which give rise to electric fields. Motion of charged particles also generates current, which produces magnetic field. These fields affect the motion of other charged particles far away. These forces explains the phenomena of collective behavior. The plasma represents a macroscopically neutral gas containing many interacting particles (electrons and ions ) and neutrals. It is likely that 99% of matter in our universe is in the form of a plasma. The term forth state of matter follows from the idea that as the heat is added to a solid, it undergoes a phase transition to a new state, usually liquid. If heat is added to a liquid, it undergoes a phase transition to gaseous state. The addition of still more energy to the gas results in the ionization of some atoms. At a temperature above $100,000^{\circ} K$, most of the matter exist in an ionized state, this ionized state of matter is called the forth state. A plasma state can exist at temperature lower than $100,000^{\circ} K$ provided there is a mechanism for ionizing the gas, and if the density is low enough so that recombination is not rapid.

## 1.2 Debye Shielding

It is well known that a fundamental characteristics of a plasma is its ability to shield out the electric field of an individual charged particle or of a surface that is at some non-zero potential. This characteristic provides a measure of the distance (called Debye radius) over which the influence of the electric field of an individual charged particle (or of a surface that has a non-zero potential) is felt by other charged particles inside the plasma. The charged particles arrange themselves in such a way as to effectively shield any electrostatic field within a distance of the order of the Debye length. This shielding of electrostatic field is a consequence of the collective effects of the plasma particles. The Debye length can also be regarded as a measure of the distance over which fluctuating electric potentials may appear in a plasma, corresponding to conversion of thermal particle kinetic energy into electrostatic potential energy [3].

Let us assume that an electric field is applied by inserting a charged ball inside a plasma whose constituents are electrons and ions. The ball would attract particles of opposite charges, i.e., if it is positive, a cloud of electrons and if it is negative, a cloud of positive ions. We also assume that recombination of the plasma particles do not occur on the surface of the ball. If the plasma were cold means that there were no agitations of charged particles, there would be just as many charges in the cloud as in the ball. This case corresponds to a perfect shielding i.e., no electric field would be present in the body of the plasma outside the cloud. On the other hand, if the temperature is finite, those particles which are at the edge of the cloud where the electric field is weak, would have enough thermal energy to escape from the cloud. The edge of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy $KT_\alpha$ of the particles ($K$ is the Boltzmann constant and $T_\alpha$ is the temperature of the plasma species $\alpha$). This corresponds to an incomplete shielding and a finite electric potential. We now calculate an approximate thickness of such a charged cloud (sheath) [2].

The Poison equation can be written as

$$\epsilon_0 \nabla^2 \phi = -e(n_i - n_e), \tag{1.1}$$

where $n_i, n_e, \phi, \epsilon_0$ are the ions number density, electrons number density, electrostatic potential and the permittivity of free space.

Assuming electron follows Boltzmann distribution

$$n_e = n_0 \exp(e\phi/kT) \tag{1.2}$$

Eq. (1.1) in one-dimension can be written as

$$\epsilon_0 \frac{d^2\phi}{dx^2} = en_0 \left[\exp\left(\frac{e\phi}{KT_e}\right) - 1\right] \tag{1.3}$$

Expanding the exponential term in the region where the potential energy is very small that is $|e\phi/KT_e| \ll 1$, we get

$$\epsilon_0 \frac{d^2\phi}{dx^2} = en_0 \left[\left(\frac{e\phi}{KT_e}\right) + \frac{1}{2}\left(\frac{e\phi}{KT_e}\right)^2 + ...\right] \tag{1.4}$$

Keeping only the linear terms in the above Eq.(1.4), we get

$$\epsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_0 e^2}{KT_e}\phi \tag{1.5}$$

This is a second order linear differential equation. Let us define a term $\lambda_D$ such that

$$\lambda_D = \left(\frac{\epsilon_0 KT_e}{n_0 e^2}\right)^{1/2} \tag{1.6}$$

The solution of Eq.(1.5) is

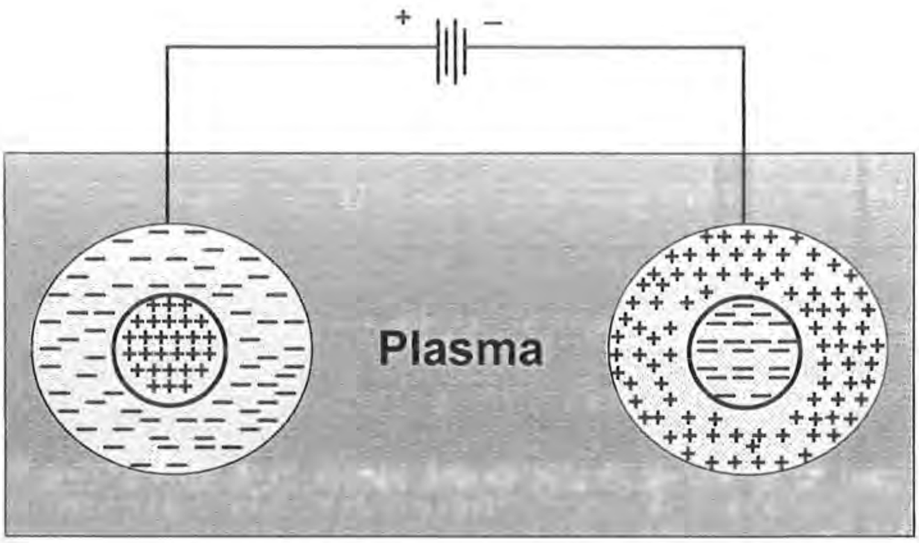


Figure 1-1: Debye shielding in plasma

$$\phi = \phi_0 \exp(-|x|/\lambda_D) \tag{1.7}$$

It may be noted here that we have ignored the exponentially growing solution because it represent unphysical result. The quantity $\lambda_D$, called the Debye length, is a measure of the shielding distance or thickness of the sheath.

When a boundary surface is introduced in a plasma the perturbation produced extends only upto a distance of the order of $\lambda_D$ from the surface. In the neighborhood of any surface inside the plasma there is layer of width of order of $\lambda_D$, known as plasma sheath, inside which the condition of macroscopic electrical neutrality needs not to be satisfied. Beyond the plasma sheath region there is the plasma region where the macroscopic neutrality is maintained [3].

## 1.3 Plasma Criterion

## First criterion

The Debye shielding effect is a characteristic of all plasmas, although it does not occur in every media that contains charged particles. A necessary and obvious requirement for the existence of plasma is that the physical dimensions of the system be large compared to $\lambda_D$. Otherwise the shielding would not be perfect. If $L$ is the characteristic dimension of the plasma, the "first criterion" for a plasma is [3].

$$L \gg \lambda_D \tag{1.8}$$

## Second criterion

Since the shielding process is the result of collective particle behavior inside a Debye sphere, it is also necessary that the number of electrons participating in the shielding process be very large. If there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept. We can calculate the number of electrons $N_D$, inside a Debye sphere as

$$N_D = \frac{4}{3}\pi\lambda_D^3 n_e = \frac{4}{3}\pi\left(\frac{\epsilon_0 kT}{n_e^{1/3}e^2}\right)^{3/2} \tag{1.9}$$

The second criterion for plasma is

$$n_e\lambda_D^3 \gg 1 \tag{1.10}$$

This means that there must be large number of charged particles in a Debye sphere to shield out the opposite charge.

## Third criterion

We can also consider macroscopic charge neutrality as a "*third criterion*" for the existence of a plasma. Although it is not an independent one, and can be expressed as

$$n_e = \sum_j n_j \tag{1.11}$$

## Fourth criterion

### Plasma frequency

Macroscopic space charge neutrality is an important property of plasma. When a plasma is disturbed from equilibrium condition, the resulting internal space charge field give rise to collective particle motion which tend to restore the original space charge neutrality. Because of their inertia, electrons will overshoot and oscillate around their equilibrium position with frequency known as " *plasma frequency*"[3]

$$\omega_{pe} = \left(\frac{n_0 e^2}{\epsilon_0 m_e}\right)^{1/2} \tag{1.12}$$

Collision between electron and neutral particles tend to stop these oscillations and gradually decrease their amplitude. It is necessary that the electron neutral collision frequency, $\nu_{en}$ be smaller than the plasma frequency

$$\omega_{pe} > \nu_{en} \tag{1.13}$$

Eq. (1.13) is known as the " *fourth criterion*" for the existence of plasma. This can also be written in another form

$$\omega\tau > 1,$$

where $\tau = 1/\nu_{en}$ is the average time of collision between electron and neutral particles.

## 1.4 What is Dusty Plasma

Dusty plasma consists of plasma particles (electrons, ions and neutrals) and dust grains-small solid particles which can be dielectric or conductors. The main difference of dusty plasma from many component plasmas is that the dust charge is not fixed but is determined by the plasma parameters in their surroundings [4]. Dust represents much of the solid matter in the universe; on the other hand, plasmas (the statistical system comprising a mixture of electrons, ions, and the neutrals) almost cover 99% of the universe. Thus the dust co-exist with plasma and forms a "Dusty Plasma", also known as "Complex Plasma" [5, 6]. A dusty plasma is normal electron-ion plasma with an additional charged component of small micron sized particulates. This extra component, which increases the complexity of the system even further, is responsible for the name "Complex Plasma". The interaction between dust and plasma leads to variety of physical and dynamical consequences. The presence of small dust particles in plasma, besides the usual negative and positive ions and mobile electrons, leads to new types of plasma waves [7]. The presence of these dust particles in the plasma leads to change the basic plasma parameters such as electron density and temperature. Depending on the plasma parameters, Coulomb crystal [8, 9] can be formed.

Dust grains of various sizes, origin and occur in nature as well as in the space environments [10, 11, 12, 13]. The dust grains are extremely massive as compared to electrons and ions. Mass $m_d$ of the dust grains is typically $m_d > \left(10^8 - 10^{12}\right) m_p$, ($m_p$ is the mass of proton) and the dust mass $m_d \sim \left(10^{-2} - 10^{-15}\right) g$. The size of the grain is in the range of $1\mu m - 1cm$. In reference [12] this range is given as $10nm - 100\mu m$. The charge on the dust grain is also large, compared to the electron charge. Typically, it is of the order of $Z_d e \sim \left(10^3 - 10^6\right) e$, $e$ is the electronic charge.

### 1.4.1 Macroscopic Charge neutrality

Dusty plasmas are characterized as a low-temperature ionized gas whose constituents are electrons, ions and micron-sized dust particulates. The presence of dust particles (grains) changes the plasma parameters and affects the collective processes in such plasma systems. In particular, the charged dust grains can effectively collect electrons and ions from the background plasma. Thus in the state of equilibrium the electron and ion densities are determined by the neutrality condition which is given by [14].

$$en_{i0} - en_{e0} + q_d n_{d0} = 0 \tag{1.14}$$

where $n_{e,i,d}$ is the concentration of plasma electrons, ions and dust particles, respectively, e is the magnitude of electron charge and $q_d = Z_{de}(-Z_{de})$ is the amount of charge present on the dust grain surface when the grains are positively (negatively) charged with $Z_d$ being the number of charges residing on the surface of dust grain. Note that the charge of the dust particle can vary significantly depending on plasma parameters. In studying the basic physics of dusty plasmas, this third term in equation (1.14) carries very interesting implications. The presence of the dust particles in the plasma, alters the local plasma potential profile, modifies the transport of particles in the plasma, modifies certain types of ion plasma waves, and introduces new type of dust plasma modes.

### 1.4.2 Debye Shielding in Dusty Plasma

The phenomenon of Debye shielding is investigated in a dusty plasma consisting of Boltzmann electrons and ions, and negatively charged, massive dust grains [15]. It is well known that one of the fundamental characteristics of a plasma is its ability to shield out the electric field of an individual charged particle or of a surface that is inserted in plasma. This characteristic provides a measure of the distance (called Debye radius) over which the influence of the electric field of an individual charged particle (or of a surface that has a non-zero potential) is felt by other charged particles inside the plasma. Let us assume that an electric field is applied by inserting a charged ball inside a dusty plasma whose constituents are electrons, ions and positively or negatively charged dust particles. The ball would attract particles of opposite charges, i.e., if it is positive, a cloud of electrons and dust particles (if they are negatively charged) would surround it and vice versa. We also assume that recombination of the plasma particles does not occur on the surface of the ball. If the plasma were cold means that there were no agitations of charged particles, there would be just as many charges in the cloud as in the ball. This case corresponds to a perfect shielding, i.e., no electric field would be present in the body of plasma outside the cloud. On the other hand, if the temperature is finite, those particles which are at the edge of the cloud where the electric field is weak, would have enough thermal energy to escape from the cloud. The edge of the cloud then occurs at a radius where the potential energy is approximately equal to the thermal energy $KT_\alpha$ of the particles ($K$ is the Boltzmann constant and $T_\alpha$ is the temperature of the plasma species $\alpha$). This corresponds to an incomplete shielding and a finite electric potential. We now calculate an approximate thickness of such a charged cloud (sheath).

We also assume that the dust-ion mass ratio $m_d/m_i$ is also large that the inertia of the dust particles prevents them from moving significantly. The massive dust particles form only a uniform background of negative charges. The electrons and ions are assumed to be in local thermodynamics equilibrium, and their number densities, $n_e$ and $n_i$, obey the Boltzmann distribution, namely

$$n_e = n_{e0} \exp\left(\frac{e\phi_\alpha}{k_B T_e}\right) \tag{1.15}$$

and

$$n_i = n_{i0} \exp\left(-\frac{e\phi_\alpha}{k_B T_i}\right) \tag{1.16}$$

where $n_{e0}$and $n_{i0}$ are the electron and ion number densities, respectively, far from the cloud.

For our present system the Poisson's equation can be written as

$$\nabla^2 \phi_\alpha = 4\pi(en_e - en_i - q_d n_d) \tag{1.17}$$

where $n_d$ is the dust particle number density. In equilibrium, the charge neutrality condition for dusty plasma can be written as $q_d n_{d0} = en_{eo} - en_{io}$. Substituting equations (1.15) and (1.16) into equation (1.17) and assuming $e\phi_\alpha/k_B T_e \ll 1$ and $e\phi_\alpha/k_B T_i \ll 1$, we get

$$\nabla^2 \phi_\alpha = \left(\frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2}\right) \phi_\alpha, \tag{1.18}$$

where $\lambda_{De} = \sqrt{k_B T_e / 4\pi n_{eo} e^2}$ and $\lambda_{Di} = \sqrt{k_B T_i / 4\pi n_{io} e^2}$ are the electron and ion Debye radii, respectively. For spherical symmetric case, the solution of Eq. (1.18) in one-dimensional case can be written as $\phi_\alpha = \phi_{\alpha 0} \exp(-r/\lambda_D)$ with

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}} \tag{1.19}$$

The quantity $\lambda_D$ is a measure of the shielding distance or the thickness of the sheath. For a dusty plasma with negatively charged dust grains, we have $n_{eo} << n_{io}$ and $T_e \geq T_i$, i.e. $\lambda_{De} >> \lambda_{Di}$. Accordingly, we have $\lambda_D \simeq \lambda_{Di}$. This means that shielding distance or the thickness of the sheath in a dusty plasma is mainly determined by the temperature and number density of the ions. However, when the dust particles are positively charged and most of the ions are attached onto the dust grain surface, i.e. when $T_e n_{io} << T_i n_{eo}$ , we have $\lambda_{De} << \lambda_{Di}$. This corresponds to $\lambda_D \simeq \lambda_{De}$ . In this case the shielding distance or the thickness of the sheath is mainly determined by the temperature and density of the electrons.

## 1.5 Dusty Plasma in Space and Laboratory

The physics of dusty plasmas has appeared as one of the rapidly growing fields of science. Dusty plasmas are important in technological applications and in astrophysical situations. They can be formed under laboratory conditions (e.g., plasma processing reactors, laboratory experiments, fusion experiments) with sizes of the dust cloud of a few millimeters to astrophysical systems (e.g., planetary rings, comet tails, nebula) that are millions of kilometers across. In the following, we shall discuss briefly about the presence of dusty plasmas in space and laboratory.

### 1.5.1 Dusty Plasma in Space

We now focus our attention on dusty plasmas in space. It is well known that the dusty plasmas are ionized gases embedded with charged fine dust particles in space and occur in a wide variety of environments. Dust is present in such diverse objects as interstellar clouds, solar system, cometary tails, planetary rings. Since, charged dust grains are common in low earth orbit and in the interplanetary medium, the presence of this charged material can cause both physical damage and electrical problems for spacecraft. However, charged dust particles in space plasmas may also help to explain the formation of planetary rings, comet tails and nebulae. The dust is contained in streams of particles that flows through our solar system, and scientists are anxious to study it so they can learn more about the formation of earth, other planets and life. Because of the variety of areas in which a dusty plasma may play a role, it is important to understand the physical properties of this plasma system in space. Outer space is divided into many levels and the one that separates the stars is called interstellar space. It is often a misconception that space is a vacuum or simply empty. Space is a nearly perfect vacuum, even better than the best ones made in labs on earth, but it is not void. The fact is that space is filled with tiny particles called cosmic dust and elements like hydrogen and helium. This applies for interstellar space also and all the mentioned particles make up what is known as the "interstellar medium". The interstellar medium is mainly made of hydrogen atoms. The actual density of hydrogen as it exist in interstellar space is on the average of about 1 atom per cubic centimeter. In the extremes, as low as 0.1 atom per cubic centimeter has been found in the space between the spiral arms and as high as 1000 atoms per cubic centimeter are known to exist near the galactic core. The interstellar medium also contains cosmic dust. These particles are much bigger than hydrogen atoms. However, there are far fewer particles of cosmic dust than there are hydrogen atoms in the same volume of space. It is estimated that cosmic dust is 1000 times less common than hydrogen atoms in the interstellar medium. The dust grains in the interstellar or circumstellar clouds are dielectric (ices, silicates, etc.) and metallic (graphite, magnetite, amorphous carbons etc.).

Our solar system which consists of the sun and all the objects that orbit around it, is also believed to contain a large amount of dust grain and the origins of these dust grains in the solar system could possibly be, for example, micrometeoroids, space debris, man-made pollution and lunar ejecta, etc.

### 1.5.2 Dusty Plasma in Laboratory

Since our work in this chapter is mainly concerned with the theoretical studies of test projectiles in dusty plasmas and it seems unreasonable to discuss laboratory dusty plasmas. So, we are giving a brief introduction to the laboratory dusty plasma.

Dusty plasma research involves the study of the interaction between the charged dust particles and plasma in which the dust particles are suspended. The presence of dusty plasmas in space as discussed above can be considered as a starting point for the understanding of laboratory dusty plasmas in the sense that there are two main features which differentiate laboratory dusty plasmas from space and astrophysical plasmas. Firstly laboratory discharge have geometric boundaries whose properties like structure, composition, temperature, conductivity etc. can have an influence on the formation and transport of the dust grains and secondly, the external circuit which maintains the dusty plasma, imposes varying boundary conditions on the dusty discharge. In the following we shall discuss briefly about the presence/occurrence of dusty plasmas in the laboratory devices, particularly in direct current (dc) and radio-frequency (rf) discharges, plasma processing reactors and fusion plasma devices.

Historically, many of the first dusty plasma investigations were performed using rf glow plasma. In these systems, the suspended microparticles often form regular two-dimensional lattice structures called plasma crystals. In dc glow discharge dusty plasma experiments, the regular 2-D crystalline structures are often not observed. The suspended microparticles generally remain in a more fluid-like state. Experiments have focused on e.g., dust acoustic waves and dust ion-acoustic waves, vortices, and three dimensional particle transport.

The main aim of early dusty plasma investigations was obtaining a good control of contamination in plasma-processing reactors, either by eliminating dust particles from the gas phase or by preventing them from getting into contact with the surface. This task is almost been accomplished, and the knowledge gained in the course of elaborate studies can now be utilized in new research directions. Application of macroscopic grains is one of the recent developments in the material science. The common use of low pressure plasma processing reactors and the availability of the laser light scattering diagnostics showed that many of these discharges produced and trapped large quantities of macroscopic dust grains. These dust particles in the plasma are not anymore considered as unwanted pollutants. Contrariwise, at present they have turned into production goods. Low-pressure plasmas have the unique property of dust trapping, so that the position and residence time of particles in the reactor can be controlled. This offers numerous opportunities of particle processing, like surface modification (coating, etching), bulk modification (melting, crystallization), and many others. Particles generated in low-pressure discharges have typically very well-defined size and shape. Scanning electron microscopes (SEMs) of the dust using a low-energy probe reveal narrow size distributions.

Significant amount of small dust particles with sizes between a few nanometers and a few $10\mu m$ are found in several fusion devices. Though it is not a major problem today, dust is considered a problem that could arise in future long pulse fusion devices. This is primarily due to its radioactivity and due to its very high chemical reactivity. These dust particles are also believed to be heavier than the hydrogen isotopes which are the fuel in the fusion devices. Though some mechanisms leading to the formation of dust in tokamaks such as desorption, sputtering and evaporation have been identified, their relative importance is not yet adequately understood. Very little is known about the transport and fate of dust particles, e.g., whether they interact repetitively with the fusion plasma. Recently, the dust particles were collected from the TEXTOR-94 which is a medium sized tokamak [20]. It is not known yet that how quickly dust particles can be transported in the tokamak interior.

## 1.6 Historical Background

The test charge approach is the most appropriate technique used to calculate the shielding potential of a projectile moving through a plasma and the energy of the test charge projectile through the plasma. The shielding of moving test charge particles through a plasma has been the subject of several theoretical investigations ever since the beginning of the century. In 1940 Fermi [16] pointed out that the shielding of a fast particle due to the ionization of the material through which it is passing, considerably affected by the density of the material. This is due to the alteration of the electric field of the passing particle by the electric polarization of the medium. In 1948 [17] N. Bohr extended the study by incorporating the ion dynamics. They also calculated the structure of far-wake behind a charged particle moving through a rarefied, uniform plasma. In particular, the excitation of electrostatic ion waves. After the Bohr, Neufeld and Ritchi [18] were the first to calculate the potential distribution of test particle in an electron-ion plasma with fixed ion background. In 1971 Sanmartin and Lam [19] have studied theoretically, the shielding potential in electron-ion plasma. Then the potential of a slowly moving test charge in a collisional plasma, studied by the Stenflo and Shukla [20], and found that far field potential may fall off as the inverse square of the distance when the collision frequency is larger than the plasma frequency. In 1973 Chen *et al.* [21], have studied the wake potential due to a charge moving faster than the ion acoustic velocity. The energy loss of streaming beam particles to the background plasma by exciting a wake plasma wave in inertial confinement fusion (ICF) and magnetic confinement fusion (MCF) has also been discussed. In recent years, number of experiments have been performed in which micron sized dust particulates were artificially grown from ion cluster in the plasma to produce a dusty plasma [22, 23] and the scientist calculated the shielding potential of test charge projectile in the dusty plasma. The analytical and numerical results for the shielding potential of two heavy projectile ions passing through a multicomponent dusty plasma has also been reported recently [24], which is useful to explain the crystallization of dust grains in astrophysical and laboratory plasmas. Nasim *et al.* [25, 26], calculated the shielding potential of a test charge in dusty plasma with dust charge fluctuations both analytically as well as numerically, which is helpful for understanding coagulation of dust grains in space and laboratory plasmas. Further more the shielding potential for a slowly moving test charged projectile has also been investigated [27]. A large amplitude wake field is observed which propagates ahead of the test charge in the large dust-neutral collision frequency limit. The energy loss of a test charge in an unmagnetized dusty plasma is estimated for different dust parameters (such as dust charge state, dust number density, dust charge fluctuations and dust-neutral collisions) using Krook and BGK-type collisional models [28]. For higher the dust-charge-state, the more pronounced wake-field (which extends up to several Debye lengths) is produced. The variation of the dust number density shows a similar behavior. For large dust-neutral collisions a weakly damped large amplitude wake-field ahead of the test charge position is observed for higher collision frequencies. The dust neutral collisions are also found to enhance the energy loss for test charge velocities greater than the dust acoustic speeds in contrast with the dust charge fluctuations which enhances the energy loss only for test charge velocities smaller than the dust acoustic speeds [29]. By employing a dielectric approach, the energy loss of a pair of test charged projectiles passing through a multicomponent dusty plasma, retaining two ion correlation effects is computed. On the other hand, Yaqoob *et al.* [30], calculated the shielded potential and the energy loss of $N^2$ projectiles propagating through a multicomponent dusty plasma. Analytical expressions for the shielded potential as well as energy loss have been obtained by taking into account the two-body correlation effects. It is found that the correlation effect causes distortion in the potential profile depending upon the separation between the two projectiles. The distortion becomes pronounced for separation smaller than the Debye length. They also calculated the dielectric response function for modified dust acoustic waves [31] by incorporating the dipole moment and moment of inertia of finite sized elongated dust grains. Using this dielectric constant, generalized expressions for the Debye potential and for the wake potential are obtained due to a cluster of $N^2$ dust grain projectiles moving with a constant velocity along the z-axis through a multi-component dusty plasma. A negative wake potential is observed behind each projectile in different geometries. Ali *et al.* [32], calculated the analytical and numerical results for the slowing down of a pair of heavy test charge projectiles through a multicomponent, dust-contaminated plasma. The correlation and interference effects of two collinear and noncollinear projectiles on electrostatic potential and energy loss are studied for a Maxwellian distribution and for a special class of physically reasonable size distributions. The energy loss behavior versus projectile velocity of noncollinear projectiles is also examined for various orientations. It is found that the energy loss for Maxwellian distribution for large value of spectral index ($\kappa$) is larger compared to that for generalized Lorentzian distribution. It is also observed that for smaller values of $\kappa$, the test charge projectile gains energy instead of losing. These results would be useful for understanding the energy loss mechanism, which might be responsible for the coagulation of dust particles in molecular clouds, in the ion-beam driven inertial confinement fusion scheme and in dust plasma crystal formation, etc. The shielded potential and the energy loss for a variety of arrays of dust grain projectiles, arranged in different orientations and separation distance moving with a constant velocity are calculated [33]. By employing the dielectric theory, the Debye and wake potentials are calculated for the said system. It is found that a projectile moving with high speed forms a negative wake behind and a wave front ahead of it. A generalized expression for the Debye potential and the wake field potential due to an axial propagation of dust grain has also been calculated [34]. The dust grain projectiles are assumed to lie on sets of concentric circles propagating through the dusty plasma with a constant velocity along the z axis. Some specific cases of electrostatic potential due to four and eight projectiles are studied both analytically and numerically. Then the electrostatic potential and the energy loss for $N\times M$ projectiles propagating through a dusty plasma are studied using the dielectric theory for different dust parameters of interest [35]. These results are useful to explain the coagulation of dust grains in laboratory and space plasma as well as in the field of ion-beam driven ICF scheme. They also calculated the shielded potential and the energy loss by $N \times M$ projectiles passing through a collisional dust-contaminated plasma with dust-charge fluctuations and grain-size distribution. Then the general expressions are obtained for the shielded potential and for the energy loss by considering two-body correlation effects. An interference contribution of these projectiles to the potential and energy loss was observed which depends upon their orientation and separation distance. The dust-charge fluctuation produces a potential well instead of Coulomb-type potential for a slowly moving test charge with slow charge relaxation rate and energy is gained by the charged projectiles. However, fast charge relaxation enhances the energy loss and some peaks are observed showing the excitation of some electrostatic modes. On the other hand, the dust neutral collisions also enhance the energy loss for projectile velocities greater than the dust acoustic speed for a Maxwellian plasma. They also included the effects of self gravitation of massive dust grains and shielded potential and the energy loss of pair of charged projectiles passing through a dust-contaminated plasma.

It is found that for two collinear projectiles the potential is enhanced by increasing the dust Jeans frequency for separation less than Debye length and the energy loss decreases with the increase of Jeans frequency for arbitrary separation. The present investigation would be useful to explain the coagulation of dust particles in the molecular clouds and in the ion-beam-driven inertial confinement fusion approach [36, 37]. The shielding potential by the pair of test charge projectiles passing through a multicomponent, self-gravitating, dusty plasma with a generalized Lorentzian distribution is also presented in [38]. Ali *et al.* [39], calculated the electrostatic potential for a test charge in a multicomponent dusty plasma, whose constituents are the Boltzmann distributed electrons, mobile positive and negative ions, and immobile positive/negative charged dust particles. By using the modified dielectric constant of the dust-ion-acoustic (DIA) waves, the Debye screening and wake potentials are obtained. It is found that the presence of mobile negative ions significantly modify the DIA wave and the wake potential. Then Shukla *et al.* [40], extend the work to calculate the electrostatic potential in dense plasma. By using the dielectric response function of quantum electron plasmas, potential distributions around a moving test charge are calculated. The near-field potential follows the modified Debye-Hückel potential, while the far-field potential turns out to be oscillatory. Both the Debye-Hückel and wake potentials strongly depend on the Fermi energy and the electron quantum correlation strength. The relevance of the present investigation to semiconductor plasmas is discussed. Then Shukla and Stenflo [41] have calculated the shielded potential of a slowly moving test charge in a quantum plasma. It is shown that, besides the Debye Hückel near-field potential, there is a far-field potential which decays with the inverse cube of the distance between the origin of the test charge and the location of the observer. They discussed the screening and wake potentials around a test charge in an electron ion quantum plasma, by using the linear dielectric response formalism. The short range screening potential in quantum plasma is found to be significantly different from the Debye Hückel shielding potential, while the wake potential has a long-range oscillatory behavior. Both short and long range potentials may lead to the trapping of other charges of the same polarity [42]. Shukla *et al.* [43], calculated the test charge potential involving the electron dust acoustic oscillations in a two-component plasma whose constituents are hot electrons and positive nanoparticles. The hot degenerate inertialess electrons are assumed to follow the Thomas Fermi distribution, while positive nanoparticles are inertial. The expressions for the Debye-Hückel and wake potentials due to a moving test charge are obtained. Furthermore, the effects of the Fermi energy, the number density of hot degenerate electrons, and the test charge speed on the potential profiles are numerically examined.

The thermodynamic properties of plasma in equilibrium can be calculated from the knowledge of two-particle correlation function. Extensive work has been done to evaluate the two-particle correlation function in variety of plasmas using various methods. Experimentally measured pair correlation functions are used to determine the charge, the screening radius, and the interaction potential [44].

## 1.7 Motivations

Shielding in plasma is very important area of research. One of the main objective of this study is to improve the existing analytical work for the calculation of the electrostatic potential, and extend it for a multicomponent plasma. We have used Vlasov and Poisson's system of equations for the calculation of electrostatic potential, with the help of space time Fourier transform. In recent years, there has been renewed interest in the charge particle interaction with the dense plasma [45, 46] due to its importance in the context of intertial confinement fusion (ICF) approach where one may use fast heavy ion beams to drive D-T fuel target. The most important process in the interaction between the test charge particle and plasma is the phenomena of energy loss of test charge, due to localized collisions and the excitation of wake field and the collective modes in the plasma. It is shown that charged particulates can attract each other due to collective interactions involving the dust acoustic waves.

In this thesis we reviewed earlier work on the shielding potential of test charge in dusty plasma. The correlation effects of the two projectiles on the electrostatic potential are also presented. A comparison has been made for correlated and un-correlated numerical results of electrostatic potential. We also studied the electrostatic potential and wake potential in a multicomponent dusty plasma with the help of kinetic theory of plasma, which consists of inertialess Boltzmannian electrons, positive-negative ions, and positive and negative dust.

It is important to mention here that the relevance of such type of work, calculating the electrostatic potential of test charge, is not limited to ICF alone but has applications in space, astrophysical as well as in laboratory plasmas.

## 1.8 Layout of Dissertation

The present chapter contains introduction to plasma and some basic parameters of plasma such as macroscopic neutrality, Debye shielding and characteristic frequency. Historical background of the test charge projectiles in plasma and their interaction with the plasma and wake potential generated by the test charge projectile in plasma. The next chapter is devoted to the development of theoretical background for understanding the dielectric theory and its role in the calculation of the electrostatic potential. A brief introduction of the test charge approach and the Debye screening of the test charge in the plasma and theoretical calculation of the dielectric constant is also given. In chapter 3, a general overview of the presence of dusty plasmas in space and laboratory is presented. We calculate the electrostatic potential of test charge projectile in dusty plasma and presents numerical results of shielding in dusty plasma. Chapter 4 contains the theoretical background which is helpful in finding the electrostatic potential due to the motion of test charge projectile in multi-component plasma. We have calculated the wake potential due the motion of test charge projectile in multi-component plasma.

# Chapter 2

# Shielding in Plasma

*"In this chapter, we have developed a theoretical background and to calculate the electrostatic potential by using test charge approach with the help of kinetic theory of plasma".*

## 2.1 Why Kinetic Theory

In any macroscopic physical system containing many individual particles, there are basically three levels of description, one is the " Exact microscopic description", second one is the " Kinetic theory", and the last one is " the Macroscopic or fluid description".

In microscopic description, one may start writing down Newton's law ($\mathbf{F} = m\mathbf{a}$) for large number of plasma particles (typically of the order of $10^{20}$ particles). To define a state of $10^{20}$ particles and to keep the track of all these particles is a hopeless task. Even a supercomputer can not do this job.

The next technique is to use fluid theory, the fluid variables are number density, fluid velocity and pressure, which are functions of $\mathbf{x}$ and $t$ only. This is possible because the velocity distribution of each species about some mean velocity can be written as Maxwellian, which is uniquely specified by the two parameters, namely the density and temperature. In the hydrodynamics of ordinary fluids and gases, interparticle collisions are usually sufficiently frequent to maintain Maxwellian distribution of particles everywhere in the fluid. In the high temperature plasma, however, interparticle collisions are relatively infrequent, and deviation from local thermodynamic equilibrium can be maintained for long times.

In the case where there are no collisions, the plasma particles will freely stream to large distances along the field. To treat such problems, we need kinetic theory, in which individual particle velocities are taken into account. Such a theory will be needed to treat problems involving flow across the magnetic field, especially when the magnetic field is very weak. In this case, the gyration period and gyration radius are not small compared with characteristic time-scales and length-scales of the flow [47].

In summary, therefore, kinetic theory is needed to treat (i) problems involving flow along a magnetic field ( or in the absence of magnetic field) in the case of long mean-free path and (ii) problems of high-frequency and/or short-wavelength flow across a magnetic field.

## 2.2 Plasma as Dielectric Medium

The plasma has been defined as a statistical ensemble of mobile charged particles. These charged particles move randomly in the system, interact with each other through electromagnetic forces, and respond to the electromagnetic disturbance which might be externally applied or self-generated. A plasma is therefore inherently capable of sustaining rich classes of electromagnetic phenomena.

A proper description of such electromagnetic phenomena may be obtained if we know the macroscopic plasma response to a given electromagnetic disturbance. The function which characterizes these responses is known as plasma response function. All the macroscopic properties of the plasma (as a medium) are contained in these response functions. If the external disturbance to the plasma is a time dependent electric field E, then the response function appears in the form of the displacement D. The relationship between the displacement and field given by the dielectric constant such that $D = \varepsilon E$.

The dielectric function $\varepsilon$ is complete in the sense that it contains all the information about the linear electromagnetic properties of the plasma. This dielectric response function describes essentially the longitudinal properties of the plasma, and effectively describe the collective behavior of the plasma. In the next section, we shall calculate this dielectric response function (dielectric constant), using a kinetic treatment of the plasma.

## 2.3 Phase Space

Considering a system of N-particles moving according to the laws of classical mechanics. "A state of the system at any instant of time $t$ is specified by means of 3N-position coordinates $q_1, q_2, q_3, ...q_{3N}$ and 3N-momentum coordinates $p_1, p_2, p_3, ...p_{3N}$ . The 7N-dimensional space is known as the **phase space** of the system".

A point represented by $(q_i, p_i)$ where $i = 1, 2, 3, ....3N$ represents a particular state of the system in phase space. The point $(q_i, p_i)$ is called a phase point or representative point. Some times 3N-position components are referred to separately as configuration space and 3N-momentum components as the momentum space. Thus the complete phase space is the sum of the configuration space and momentum space.

## 2.4 Distribution Function

The basic element in the kinetic description of a plasma is the distribution function $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ which describes the particle distribution both in physical and velocity space. A plasma in thermal equilibrium is characterized by a homogeneous, isotropic and time independent distribution function [48]. A single particle distribution function or simply distribution function $f_\alpha(\mathbf{r}, \mathbf{v}, t)$, is defined as the number of particles per unit volume or phase space that are present, at time $t$, in any infinitesimally small volume of space, centered at the point in the phase space $(\mathbf{r}, \mathbf{v})$. Most of the essential information about the plasma is contained in the single particle distribution function, $f_\alpha(\mathbf{r}, \mathbf{v}, t)$, where the position vector $\mathbf{r}$, and velocity vector $\mathbf{v}$, and time $t$ all are independent variables,

$$f_\alpha(\mathbf{r}, \mathbf{v}, t) = \lim_{\Delta v \to 0} \frac{\text{number of particles of type } \alpha \text{ in } \Delta V}{\Delta V} \tag{2.1}$$

In other words, $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ is the number of type-$\alpha$, situated between $\mathbf{r}$ and $\mathbf{r} + \Delta\mathbf{r}$, and between $\mathbf{v}$ and $\mathbf{v} + \Delta\mathbf{v}$ as shown in the Fig. 2.1.

The single particle distribution function can be found by solving Boltzmann equation , which can itself be derived from the individual particle picture by using kinetic theory.

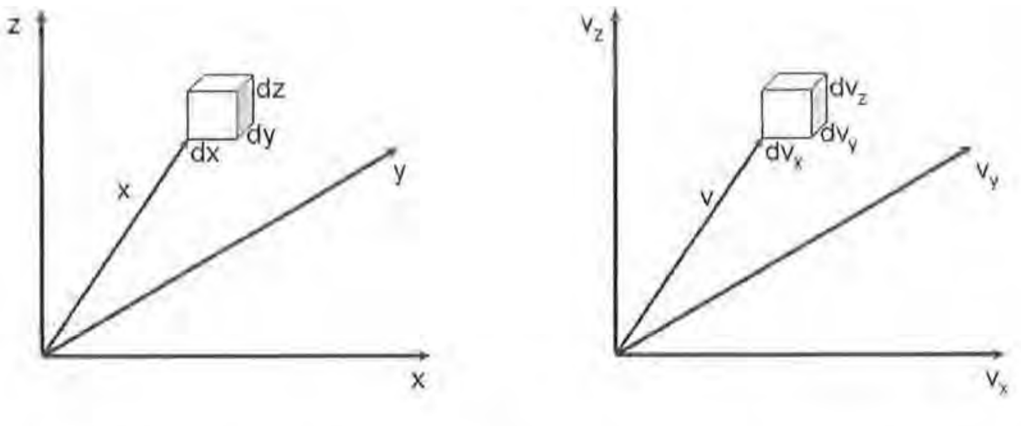


Figure 2-1: Left: A configuration space volume element $d^3x = dxdydz$ at spatial position x. Right: equivalent velocity space volume element.

## 2.5 Boltzmann Equation

### 2.5.1 Collisionless Boltzmann Equation

The differential equation that governs the temporal and spatial variation of the distribution function under the action of external forces and collisions, known as the Boltzmann equation. When we calculate the average value of the particle physical property that is a macroscopic variable of interest, it is very important to have information about the distribution function for the system under consideration. The dependence of the distribution function on the independent variables $\mathbf{r}$, $\mathbf{v}$ and $t$ is also governed by the Boltzmann equation. We are going to discuss here a derivation of the collisionless Boltzmann equation and the general form it takes when the effects of the particle interactions are not taken into account [3]. Let $d^6n_\alpha(\mathbf{r},\mathbf{v},t)$ represents the number of particles of type $\alpha$ within a volume element $d^3rd^3v$ of phase space about $(\mathbf{r},\mathbf{v})$

$$d^6n_\alpha(\mathbf{r},\mathbf{v},t) = f_\alpha(\mathbf{r},\mathbf{v},t)\, d^3rd^3v \tag{2.2}$$

Suppose there is no interaction between the particles and a particle of type $\alpha$ in the phase space, is present at $(\mathbf{r},\mathbf{v})$ in time $t$ and after time $dt$ the particle will be found at new position $(\mathbf{r}',\mathbf{v}')$ in the phase space such that

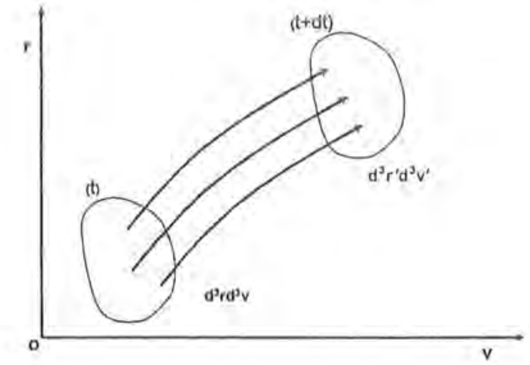


Figure 2-2: When there is no collision, the paticles in volume element $d^3rd^3v$ in phase space at time $t$ will be same in a new volume element $d^3r'd^3v'$ after time $dt$

$$\mathbf{r}'\left(t+dt\right)=\mathbf{r}\left(t\right)+\mathbf{v}dt \tag{2.3}$$

$$\mathbf{v}'\left(t+dt\right)=\mathbf{v}\left(t\right)+\mathbf{a}dt \tag{2.4}$$

In the presence of external force $\mathbf{F}$, all the particles of type $\alpha$ which are present inside a volume element $d^3rd^3v$ in the phase space at time $t$ will occupy a new volume element $d^3r'd^3v'$ after time $dt$. Since we are considering the same particles at $t$ and $t+dt$, so we can write

$$f_\alpha\left(\mathbf{r}+\mathbf{v}dt,\mathbf{v}+\mathbf{a}dt,t+dt\right)d^3rd^3v=f_\alpha\left(\mathbf{r},\mathbf{v},t\right)d^3rd^3v \tag{2.5}$$

$$\left[f_\alpha\left(\mathbf{r}+\mathbf{v}dt,\mathbf{v}+\mathbf{a}dt,t+dt\right)-f_\alpha\left(\mathbf{r},\mathbf{v},t\right)\right]d^3rd^3v=0 \tag{2.6}$$

Using Taylor series expansion to expand the first term of the Eq. (2.6), we get

$$f\left(x_1,x_2,x_3....x_n\right)=\sum_{j=0}^{\infty}\left[\frac{1}{j!}\left(\sum_{k=1}^{n}\left(x_k-a_k\right)\frac{\partial}{\partial x'}\right)^j f\left(x_1',x_2'...x_n'\right)\right]_{x_1'=a_1,x_2'=a_2,....x_n'=a_n} \tag{2.7}$$

Where Eq. (2.7) is the generalized expression of the Taylor series with n variables. After using the Taylor series, we obtain the simplified form of Eq. (2.6) given by.

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \mathbf{a} \cdot \nabla_\mathbf{v} f_\alpha = 0 \tag{2.8}$$

Eq.(2.8) is known as the Boltzmann equation in the absence of collisions, where $\mathbf{a}$ is the acceleration of the particle of type $\alpha$, located at a position $\mathbf{x}$ and possessing a velocity $\mathbf{v}$. Note that $\mathbf{a}$ includes the effect of all noncollisional forces on the particles. Where $\nabla$ and $\nabla_\mathbf{v}$ are Del. operators in the configuration space and in the velocity space respectively. The acceleration $\mathbf{a}$ can be written in terms of external force $\mathbf{F} = m\mathbf{a}$ acting or applied on the particle species. For plasmas, the dominant force is electromagnetic, which is Lorentz force,

$$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) = m\mathbf{a} \tag{2.9}$$

So, finally collisionless Boltzmann equation can be written as

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q}{m}\left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right] \cdot \nabla_\mathbf{v} f_\alpha = 0, \tag{2.10}$$

where

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{2.11}$$

and

$$\nabla_\mathbf{v} = \hat{\mathbf{x}}\frac{\partial}{\partial v_x} + \hat{\mathbf{y}}\frac{\partial}{\partial v_y} + \hat{\mathbf{z}}\frac{\partial}{\partial v_z}. \tag{2.12}$$

### 2.5.2 Collisional Boltzmann Equation

Now we are taking into account the effects, due to the interaction of the particles. With the effect of collisions some of the particles of type $\alpha$ which exists initially in the volume element $d^3rd^3v$ of the phase space may leave that volume element. Some particles which were initially outside the volume element $d^3rd^3v$ can come in the volume element [3] as shown in the Fig.2 .

So the number of particles are different in the two volume elements of phase space due to the interaction of particles. We can express this net gain or loss of particles of type $\alpha$, as a

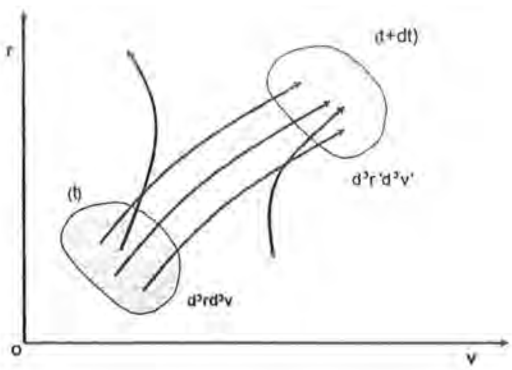


Figure 2-3: When collisions are taking into acount, particles at time $t$ in a volume element $d^3rd^3v$ in phase space, are different in a new volume element $d^3r'd^3v'$ after time $dt$

result of collisions during the time interval $dt$, in the volume element $d^3rd^3v$, by

$$\left(\frac{\delta f_\alpha}{\delta t}\right)_C d^3rd^3vdt, \tag{2.13}$$

where $(\delta f/\delta t)_C$ shows the rate of change of $f_\alpha$ due to collisions of the particles. So, when the collisions are considered Eq.(2.6) will becomes

$$\left[f_\alpha\left(\mathbf{r}+\mathbf{v}dt, \mathbf{v}+\mathbf{a}dt, t+dt\right)-f_\alpha\left(\mathbf{r}, \mathbf{v}, \mathbf{t}\right)\right]d^3rd^3v=\left(\frac{\delta f_\alpha}{\delta t}\right)_C d^3rd^3vdt \tag{2.14}$$

Again using Taylor series expansion, Eq. (2.14) can be written as

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\cdot\nabla f_\alpha+\mathbf{a}\cdot\nabla_{\mathbf{v}}f_\alpha=\left(\frac{\delta f_\alpha}{\delta t}\right)_C \tag{2.15}$$

This is the collisional Boltzmann equation, which takes into account the collision effects of plasma particles

## 2.6 Vlasov Equation

In the Vlasov theory of plasmas, the microfields produced by plasma particles are replaced by the average field that the particles produce at a given space point [48]. The distribution function of plasma particles is calculated in the presence of self-generated fields.

The Vlasov equation, would correctly describe the behavior of collisionless plasma

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}}{c}\right]\cdot\nabla_{\mathbf{v}} f_\alpha = 0 \tag{2.16}$$

This is the well known Vlasov equation for the evolution of distribution function $f_\alpha(\mathbf{x},\mathbf{v},t)$ in a collisionless plasma.

### 2.6.1 Maxwell-Vlasov Model

In the presence of electric (**E**) and magnetic (**B**) fields, the net force acting on a charged particle $q$ is given by $q\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}\right]$. These fields could be externally applied fields or self-generated fields. In order to have a closed set of equations, we must find some way of finding the self-generated electric and magnetic fields derivable from the distribution function that describes the plasma particles [47]. The Maxwell equation in terms of distribution function can be written as

$$\nabla\cdot\mathbf{E} = 4\pi\sum_\alpha n_\alpha q_\alpha \int f_\alpha d\mathbf{v} + 4\pi\rho_{ext} \tag{2.17}$$

$$\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial\,\mathbf{E}}{\partial t} + \frac{4\pi}{c}\sum_\alpha n_\alpha q_\alpha \int \mathbf{v} f_\alpha d\mathbf{v} + \frac{4\pi}{c}\mathbf{j}_{ext} \tag{2.18}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \tag{2.19}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{2.20}$$

where $\rho_{ext}$ and $\mathbf{j}_{ext}$ are external charge and current densities. We can write the charge and current density of the plasma in terms of the distribution function $f_\alpha(\mathbf{x},\mathbf{v},t)$ [49] given by

$$\rho = \sum_\alpha q_\alpha \int f_\alpha d\mathbf{v}, \tag{2.21}$$

$$\mathbf{j} = \sum_\alpha q_\alpha \int \mathbf{v} f_\alpha d\mathbf{v}, \tag{2.22}$$

where $\rho$ is the volume charge density of plasma, $\mathbf{j}$ is the current density of plasma, and $q_\alpha$

is the charge of the $\alpha th$-species.

## 2.7 The Linearized Vlasov Equation

We are assuming a small perturbation from equilibrium value. The electron and ion equilibrium distribution functions $f_{\alpha 0}$ must be chosen such that the electron and ion number densities are equal, so that they correspond to a physical system in which the plasma is quasineutral. There will be then no electric field in equilibrium state. The electric field will arise only when the plasma became perturbed. The electric **E** and magnetic **B** fields depends upon the distribution function $f_\alpha$, so the Vlasov equation (2.16) is a nonlinear partial differential equation for $f_\alpha$. It is difficult to solve this nonlinear Vlasov equation. Instead of solving the Vlasov equation for the exact distribution function $f_\alpha$, the behavior of small perturbation $f_{\alpha 1}$ from some plasma equilibrium state $f_{\alpha 0}$ can be calculated using the linearized Vlasov equation. After solving these linearized Vlasov equation, we get linearized solution, these solution must satisfy the conditions imposed in driving the approximate equations.

When the plasma is in the equilibrium state, the Vlasov and Maxwell equations can be written as

$$\frac{\partial f_{\alpha 0}}{\partial t} + \mathbf{v} \cdot \nabla f_{\alpha 0} + \frac{q_\alpha}{m_\alpha}\left[\mathbf{E}_0 + \frac{\mathbf{v} \times \mathbf{B}_0}{c}\right] \cdot \nabla_{\mathbf{v}} f_{\alpha 0} = 0 \tag{2.23}$$

$$\nabla \cdot \mathbf{E}_0 = 4\pi \sum_\alpha n_\alpha q_\alpha \int f_{\alpha 0} d\mathbf{v} + 4\pi \rho_{0,ext} \tag{2.24}$$

$$\nabla \times \mathbf{B}_0 = \frac{1}{c}\frac{\partial\, \mathbf{E}_0}{\partial t} + \frac{4\pi}{c} \sum_\alpha n_\alpha q_\alpha \int \mathbf{v} f_{\alpha 0} d\mathbf{v} \tag{2.25}$$

These are the equations when plasma is in equilibrium state and $\mathbf{j}_{0,ext}$, $\rho_{0,ext}$ are the equilibrium current densities and charged densities, respectively.

Suppose that there is a small perturbation in the distribution function $f_\alpha$ from the equilibrium state such as $f_{\alpha 1}$. There are small perturbations in the electric **E** and magnetic **B** fields from their equilibrium state. These perturbations are expressed as

$$f_\alpha(\mathbf{x},\mathbf{v},t) = f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t), \tag{2.26}$$

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0(\mathbf{x},t)+\varepsilon\mathbf{E}_1(\mathbf{x},t), \tag{2.27}$$

$$\mathbf{B}(\mathbf{x},t)= \mathbf{B}_0(\mathbf{x},t)+\varepsilon\mathbf{B}_1(\mathbf{x},t), \tag{2.28}$$

where $\varepsilon$ is the perturbation coefficient, which measures the weakness of the perturbation. Substituting Eqs. (2.26) – (2.28) in Eqs. (2.16) – (2.19) and neglecting the terms of the order of $\varepsilon^2$, we get

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)[f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t)]\\ &+\frac{q_\alpha}{m_\alpha}[\{\mathbf{E}_0(\mathbf{x},t) + \varepsilon\mathbf{E}_1(\mathbf{x},t)\}]\cdot\nabla_v\left(f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t)\right)\\ &+\frac{q_\alpha}{m_\alpha}\left[\frac{\mathbf{v}\times(\mathbf{B}_0(\mathbf{x},t) + \varepsilon\mathbf{B}_1(\mathbf{x},t))}{c}\right]\cdot\nabla_v\left(f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t)\right) = 0\end{aligned} \tag{2.29}$$

$$\begin{aligned}&\nabla\cdot[\mathbf{E}_0(\mathbf{x},t) + \varepsilon\mathbf{E}_1(\mathbf{x},t)] = 4\pi\sum_\alpha n_\alpha q_\alpha\int[f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t)]\,d\mathbf{v}\\ &+4\pi\left[\rho_{0,ext} + \varepsilon\rho_{1,ext}\right]\end{aligned} \tag{2.30}$$

$$\begin{aligned}&\nabla\times[\mathbf{B}_0(\mathbf{x},t) + \varepsilon\mathbf{B}_1(\mathbf{x},t)] = \frac{1}{c}\frac{\partial\ [\mathbf{E}_0(\mathbf{x},t) + \varepsilon\mathbf{E}_1(\mathbf{x},t)]}{\partial t}\\ &+\frac{4\pi}{c}\sum_\alpha n_\alpha q_\alpha\int\mathbf{v}\,[f_{\alpha 0}(\mathbf{x},\mathbf{v},t) + \varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t)]\,d\mathbf{v} + \frac{4\pi}{c}\left[\mathbf{j}_{0,ext} + \varepsilon\mathbf{j}_{1,ext}\right]\end{aligned} \tag{2.31}$$

The linearized Vlasov equations for the perturbed distribution function $f_{\alpha 1}$ and the perturbed $\mathbf{E}_1$ and $\mathbf{B}_1$ fields can be expressed as,

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)f_{\alpha 1}+\frac{q_\alpha}{m_\alpha}\left(\mathbf{E}_1+\frac{\mathbf{v}\times\mathbf{B}_1}{c}\right)\cdot\nabla_v f_{\alpha 0}+\frac{q_\alpha}{m_\alpha}\left(\mathbf{E}_0+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right)\cdot\nabla_v f_{\alpha 1}=0 \quad (2.32)$$

$$\nabla\cdot\mathbf{E}_1=4\pi\sum_\alpha n_\alpha q_\alpha\int f_{\alpha 1}d\mathbf{v}+4\pi\rho_{1,ext} \quad (2.33)$$

$$\nabla\times\mathbf{B}_1=\frac{1}{c}\frac{\partial\,\mathbf{E}_1}{\partial t}+\frac{4\pi}{c}\sum_\alpha n_\alpha q_\alpha\int\mathbf{v}f_{\alpha 1}d\mathbf{v}+\frac{4\pi}{c}\mathbf{j}_{1,ext} \quad (2.34)$$

where $\mathbf{j}_{1,ext}$, $\rho_{1,ext}$ are the perturbed current densities and charge densities in the plasma due to small perturbations. The above set of linearized equations can be solved by conventional methods so as to investigate the plasma properties.

### 2.7.1 Solution of the Linearized Vlasov Equation

Consider a uniform field-free plasma that obeys the equilibrium Vlasov equations, that is, $\mathbf{E}_0=\mathbf{B}_0=\mathbf{0}$ and $f_{\alpha 0}(v_x,v_y,v_z)$. This statement implies that there is no net charge and current in the plasma system.

Suppose that at time $t=0$, if a small charge is projected in plasma, then the total distribution function can be written as

$$f_\alpha(\mathbf{x},\mathbf{v},t=0)=f_{\alpha 0}(v_x,v_y,v_z)+\varepsilon f_{\alpha 1}(\mathbf{x},\mathbf{v},t=0). \quad (2.35)$$

For electrostatic perturbations, the charge imbalance may give rise a perturbed electric field, such that

$$\nabla\times\mathbf{E}_1=0 \quad (2.36)$$

If we define $\phi_0(\mathbf{x},t=0)$ as equilibrium electrostatic potential and $\phi_1(\mathbf{x},t)$ as perturbed electrostatic potential then the total electrostatic potential $\phi(\mathbf{x},t)$ can be written as

$$\phi(\mathbf{x},t)=\phi_0(\mathbf{x},t=0)+\phi_1(\mathbf{x},t). \quad (2.37)$$

The perturbed electrostatic field and potential satisfy the following relation

$$\mathbf{E}_1 = -\nabla\phi_1 \tag{2.38}$$

For electrostatic perturbations, the linearized Vlasov equation becomes

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) f_{\alpha 1} = \frac{q_\alpha}{m_\alpha}\nabla\phi_1 \cdot \nabla_v f_{\alpha 0} \tag{2.39}$$

If we assume that $\rho_{\text{ext}}$ is also a perturbed quantity due to projected charge, then Eq. (2.33) can be written as

$$\nabla\cdot\mathbf{E}_1{=}4\pi\sum_\alpha n_\alpha q_\alpha\int f_{\alpha 1}d\mathbf{v}{+}4\pi\rho_{1,ext}.$$

or

$$-\nabla^2\phi_1 = 4\pi\sum_\alpha n_\alpha q_\alpha\int f_{\alpha 1}d\mathbf{v}{+}4\pi\rho_{\text{ext}}. \tag{2.40}$$

the above equation is known as Poisson's equation.

The system of Eqs. (2.39) and (2.40) is called linearized Vlasov-Poisson's system of equations. The electrostatic potential can be obtained by solving the linearized Vlasov-Poisson's system of equations.

We shall use the method of space-time Fourier transform to solve the linearized Vlasov-Poisson's system of equations, which would reduce to simple algebraic equations. The space-time Fourier transform of a function $\Psi(\mathbf{x},t)$ can be defined as

$$\tilde{\Psi}(\mathbf{K},\omega) \equiv \int_{-\infty}^{\infty} dt\int_{-\infty}^{\infty} d\mathbf{x}\Psi(\mathbf{x},t)\exp\left[-i\left(\mathbf{K}\cdot\mathbf{x}{-}\omega t\right)\right], \tag{2.41}$$

and the inverse transform as

$$\Psi(\mathbf{x},t) = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} d\mathbf{K}\tilde{\Psi}(\mathbf{K},\omega)\exp\left[i\left(\mathbf{K}\cdot\mathbf{x}{-}\omega t\right)\right]. \tag{2.42}$$

Taking the space-time Fourier transform of Eq. (2.39), we get

$$\begin{aligned}
&\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\mathbf{x} \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) f_{\alpha 1}(\mathbf{x},\mathbf{v},t) \exp\left[-i\left(\mathbf{K}\cdot\mathbf{x}-\omega t\right)\right] \\
= &\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\mathbf{x} \frac{q_\alpha}{m_\alpha} \nabla\phi_1(\mathbf{x},t)\cdot\nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{v}) \exp\left[-i\left(\mathbf{K}\cdot\mathbf{x}-\omega t\right)\right].
\end{aligned} \tag{2.43}$$

From Eq. (2.43), we obtain

$$\begin{aligned}
&(\omega - \mathbf{K}\cdot\mathbf{v}) \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\mathbf{x} f_{\alpha 1}(\mathbf{x},\mathbf{v},t) \exp\left[-i\left(\mathbf{K}\cdot\mathbf{x}-\omega t\right)\right] \\
= &-\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\mathbf{x} \frac{q_\alpha}{m_\alpha} \phi_1(\mathbf{x},t)\, \mathbf{K}\cdot\nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{v}) \exp\left[-i\left(\mathbf{K}\cdot\mathbf{x}-\omega t\right)\right],
\end{aligned}$$

Further simplification gives

$$\widetilde{f}_{\alpha 1}(\mathbf{K},\mathbf{v},\omega) = -\frac{q_\alpha}{m_\alpha} \frac{\mathbf{K}\cdot\nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{v})}{(\omega - \mathbf{K}\cdot\mathbf{v})} \widetilde{\phi}_1(\mathbf{K},\omega). \tag{2.44}$$

Similarly, by taking the space-time Fourier transform of Eq. (2.40), we get

$$K^2 \widetilde{\phi}_1(\mathbf{K},\omega) = 4\pi \sum_\alpha q_\alpha n_{\alpha 0} \int \widetilde{f}_{\alpha 1}(\mathbf{K},\mathbf{v},\omega)\, d\mathbf{v} + 4\pi \widetilde{\rho}_{\mathbf{ext}}(\mathbf{K},\omega), \tag{2.45}$$

where $\mathbf{K}$ is the wave vector, $\omega$ is the wave frequency and $\widetilde{\rho}_{\mathbf{ext}}$ is the Fourier transform of $\rho_{\mathbf{ext}}$. Substituting the value of $\widetilde{f}_{\alpha 1}(\mathbf{K},\mathbf{v},\omega)$ from Eq. (2.44) in Eq.(2.45), we get

$$K^2 \widetilde{\phi}_1(\mathbf{K},\omega) = \sum_\alpha \frac{4\pi q_\alpha^2 n_{\alpha 0}}{m_\alpha} \int d\mathbf{v} \frac{\mathbf{K}\cdot\nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{v})}{(\omega - \mathbf{K}\cdot\mathbf{v})} \widetilde{\phi}_1(\mathbf{K},\omega) + 4\pi \widetilde{\rho}_{\mathbf{ext}},$$

or

$$\widetilde{\phi}_1(\mathbf{K},\omega) = \frac{4\pi \widetilde{\rho}_{\mathbf{ext}}}{\epsilon(K,\omega) K^2}, \tag{2.46}$$

where $\epsilon(K,\omega)$ is the dielectric response function, which plays a very important role to describe the behavior of the plasma and is written as

$$\epsilon(\mathbf{K},\omega) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int d\mathbf{v} \frac{\mathbf{K}\cdot\nabla_{\mathbf{v}} f_{\alpha 0}(\mathbf{v})}{(\omega - \mathbf{K}\cdot\mathbf{v})}, \tag{2.47}$$

where $\omega_{p\alpha}^2 = 4\pi q_\alpha^2 n_{j0}/m_\alpha$ is the plasma frequency of the $\alpha th$ species.

We assume that at $t \to \infty$, the plasma is homogeneous and in a stationary state, the particle distribution function is Maxwellian

$$f_{\alpha 0}(\mathrm{v}) = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} \exp\left(\frac{-m_\alpha \mathrm{v}^2}{2T_\alpha}\right), \tag{2.48}$$

where $T_\alpha$ is the temperature in energy units. The one-dimensional dielectric response function and the equilibrium Maxwellian distribution function can be written as

$$\epsilon(K,\omega) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int dv_x \frac{K}{(\omega - Kv_x)} \frac{\partial f_{\alpha 0}(v_x)}{\partial v_x},. \tag{2.49}$$

with

$$f_{\alpha 0}(v_x) = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{1/2} \exp\left(\frac{-m_\alpha v_x^2}{2T_\alpha}\right),$$

Equation (2.49), can also be rewritten as

$$\begin{aligned} \epsilon(K,\omega) &= 1 + \sum_\alpha \frac{K_{D\alpha}^2}{K^2} \frac{1}{\sqrt{2\pi}} \int dv_x \frac{Kv_x}{(Kv_x - \omega)} \exp\left(\frac{-m_\alpha v_x^2}{2T_\alpha}\right) \\ &= 1 + \sum_\alpha \frac{K_{D\alpha}^2}{K^2} W(Z_\alpha) \\ &= 1 + \sum_\alpha \chi_\alpha(K,\omega), \end{aligned} \tag{2.50}$$

where $K_{D\alpha} = \omega_{p\alpha}/V_{t\alpha}$ is the Debye wave-number and is the inverse of Debye-length $\lambda_{D\alpha} = \sqrt{T_\alpha/4\pi q_\alpha^2 n_{\alpha 0}}$; $V_{t\alpha} = \sqrt{T_\alpha/m_\alpha}$ is the thermal velocity, $Z_\alpha = \omega/(KV_{t\alpha})$, $\chi_\alpha(K,\omega) = \left(K_{D\alpha}^2/K^2\right)$ $\times W(Z_\alpha)$ is the susceptibility of the plasma and $W(Z)$ is the plasma dispersion function, such that

$$\begin{aligned} W(Z) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \frac{x}{x - Z} \exp\left(-\frac{x^2}{2}\right) \\ &= 1 + Z \exp\left(\frac{-Z^2}{2}\right) \left[i\sqrt{\frac{\pi}{2}} - \int_0^Z dy \exp\left(\frac{y^2}{2}\right)\right] \end{aligned} \tag{2.51}$$

## 2.8 Shielding of a Particle using Test Charge Approach

Plasma consists of free electrons and ions both randomly moving because of their thermal motion. Due to mobility of electrons and ions, any excess charge created in a plasma tend to be rapidly neutralized and plasma can maintain its charge neutrality. Whenever electrons and ions diffuse at different rates, an electric field will be setup to ensure that both electrons and ions diffuse at the same rate. An important consequence of the mobility of electrons and ions is the effective screening of the electric field due to a charge placed in plasma.

The idea of calculating the shielding potential of charged particle is very much interesting and is very large area of the research in plasma physics [50, 51, 52, 53]. In this topic, we discuss the shielding potential of a test charge projectile by using the test charge approach, when

1. The particle is stationary
2. The particle is moving

In the first case, we shall calculate the shielded potential of a stationary test charge particle and shall show that it produces the usual Debye shielded potential. In the second case, we shall calculate the shielded potential of a moving test charge particle and shall show that the effective field of a moving test charge in an isotropic plasma changes with the speed of the test charge. In order to calculate shielded potential of test charge, we may consider a point test charge moving with a uniform speed $V_t$ through a Vlasov plasma and calculate the shielding potential. We assume that in the absence of test charge, the plasma is uniform and field free. We also assume that the test charge is moving in a straight line with velocity $V_t$ and its location at time t is given by

$$\mathbf{x}' = \mathbf{x}'_0 + \mathbf{V}_t t \tag{2.52}$$

where $\mathbf{x}'_0$ is the position of the test charge at time $t = 0$. The charge density of the point test charge [49] can be represented as:

$$\rho_t = q_t \delta(\mathbf{x} - \mathbf{x}'),$$

$$\rho_t = q_t \delta(\mathbf{x} - \mathbf{x}'_0 - \mathbf{V}_t t), \tag{2.53}$$

where $\delta(\mathrm{x}-\mathrm{x}_0'-\mathrm{v}_t't)$ is a three dimensional Dirac delta function and $q_t$ is the test charge. This test charge can either be one of the plasma particle or a particle from the external beam, singled out as a test particle. When the test charge is moving through a plasma with velocity $V_t$ then the space-charge density associated with it is represented by Eq. (2.53), so that its Fourier transformed charged density can be expressed as

$$\tilde{\rho}_{ext}(\mathbf{K},\omega)=q_t\int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}d\mathrm{x}\delta(\mathrm{x}-\mathrm{x}_0'-\mathbf{V}_t t)\exp\left[-i\left(\mathbf{K}\cdot\mathrm{x}-\omega t\right)\right] \tag{2.54}$$

$$\tilde{\rho}_{ext}(\mathbf{K},\omega)=2\pi q_t\exp\left[-i\mathbf{K}\cdot\mathrm{x}_0'\right]\delta(\omega-\mathbf{K}\cdot\mathbf{V}_t) \tag{2.55}$$

Using this value of space charge density in Eq. (2.46), the perturbed electrostatic potential would take the following form,

$$\tilde{\phi}_1\left(\mathbf{K},\omega\right)=\frac{8\pi^2 q_t\exp\left[-i\mathbf{K}\cdot\mathrm{x}_0'\right]\delta(\omega-\mathbf{K}\cdot\mathbf{V}_t)}{\epsilon\left(K,\omega\right)K^2}, \tag{2.56}$$

where

$$\epsilon\left(\mathbf{K},\omega\right)=1+\sum_{\alpha}\frac{\omega_{p\alpha}^2}{K^2}\int d\mathrm{v}\frac{\mathbf{K}\cdot\nabla_{\mathrm{v}}f_{\alpha 0}\left(\mathrm{v}\right)}{\left(\omega-\mathbf{K}\cdot\mathrm{v}\right)} \tag{2.57}$$

is known as the dielectric response function. For a moving test charge, plasma behaves as a dielectric medium with dielectric function $\epsilon\left(\mathbf{K},\omega\right)$. Taking the inverse transformation of Eq. (2.56), we obtain the following result

$$\phi_1\left(\mathrm{x},t\right)=\frac{8\pi^2 q_t}{(2\pi)^4}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\mathbf{K}\frac{\exp\left[-i\mathbf{K}\cdot\mathrm{x}_0'\right]\exp\left[i\left(\mathbf{K}\cdot\mathrm{x}-\omega t\right)\right]\delta(\omega-\mathbf{K}\cdot\mathbf{V}_t)}{K^2\epsilon\left(K,\omega\right)} \tag{2.58}$$

Performing $\omega-$integration, we get

$$\phi_1\left(\mathrm{x},t\right)=\frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}d\mathbf{K}\frac{\exp\left[-i\mathbf{K}\cdot\mathrm{x}_0'\right]\exp\left[i\mathbf{K}\cdot\left(\mathrm{x}-\mathbf{V}_t t\right)\right]}{K^2\epsilon\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)} \tag{2.59}$$

Using Eq. (2.52), we get the potential in the following form

$$\phi_1\left(\mathrm{x},t\right)=\frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}d\mathbf{K}\frac{\exp\left[i\mathbf{K}\cdot\left(\mathrm{x}-\mathrm{x}'\right)\right]}{K^2\epsilon\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)} \tag{2.60}$$

The above equation is the general expression for the shielded potential of a test charge in plasma. In the next section, we shall study the shielding effect on a stationary test charge and calculate the shielded potential of a stationary test charge.

### 2.8.1 Shielding of Stationary Test Charge

In the previous section, we have calculated the general expression for the shielded potential of a test charge $q_t$ in plasma moving with velocity $V_t$ which is located at a position $\mathbf{x}'$, that is

$$\phi_1(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp\left[i\mathbf{K}\cdot(\mathbf{x}-\mathbf{x}')\right]}{K^2\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t)}, \tag{2.61}$$

where the dielectric function $\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t)$ is given by

$$\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t) = 1+\sum_\alpha \chi_\alpha(K,\mathbf{K}\cdot\mathbf{V}_t). \tag{2.62}$$

Now we consider that test charge particle is stationary located at origin, so we put $\mathbf{x}'=0$ and $V_t = 0$ in Eq. (2.61) so as to calculate the shielded potential of a stationary test charge, that is

$$\phi_1(\mathbf{x}) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp(i\mathbf{K}\cdot\mathbf{x})}{K^2\epsilon(K)}, \tag{2.63}$$

and response (*dielectric*) function can be written as

$$\epsilon(K) = 1+\sum_\alpha \frac{K_{D\alpha}^2}{K^2},$$

or

$$\epsilon(K) = 1+\frac{K_D^2}{K^2}, \tag{2.64}$$

where

$$K_D = \lambda_D^{-1} = \sqrt{\sum_\alpha K_{D\alpha}^2}$$

is the effective Debye wave-number and is the inverse of Debye-length. Using spherical coordinates, we can write Eq. (2.63) as:

$$\phi_1(\mathbf{x}) = \frac{q_t}{2\pi^2}\int_0^\infty \frac{K^2 dK}{\left(K^2+\lambda_D^{-2}\right)}\int_0^\pi \sin\theta d\theta \exp\left(iKx\cos\theta\right)\int_0^{2\pi} d\varphi.$$

Performing the $\varphi$ integration, we get

$$\phi_1(\mathbf{x}) = \frac{q_t}{\pi}\int_0^\infty \frac{K^2 dK}{\left(K^2+\lambda_D^{-2}\right)}\int_{-1}^1 d\mu \exp\left(iKx\mu\right),$$

where $\mu = \cos\theta$ and $\theta$ is angle between $\mathbf{K}$ and $\mathbf{x}$. Performing the $\mu$ and $K$ integrations, we get

$$\phi_1(\mathbf{x}) = \frac{q_t}{|\mathbf{x}|}\exp\left[-\frac{|\mathbf{x}|}{\lambda_D}\right]. \tag{2.65}$$

This potential is called Debye-Hückel potential and can also be rewritten as:

$$\phi_D(\mathbf{x}) = \frac{q_t}{|\mathbf{x}|}\exp\left[-\frac{|\mathbf{x}|}{\lambda_D}\right] \tag{2.66}$$

The Debye-Hückel potential along-with Coulomb potential $\left(\phi_c(\mathbf{x}) = q_t/|\mathbf{x}|\right)$ is shown in Fig.(2.3). The space-charge distribution induced in the plasma is determined not only from the Coulomb potential $\phi_c(\mathbf{x})$, but also from the effective potential in a self consistent fashion. For distances much smaller than the Debye length, the effective potential is essentially equivalent to the Coulomb potential, while for distance larger than the Debye length, the potential field decrease exponentially. The potential field around a test point charge is effectively shielded by the induced space-charge field in the one component plasma.

### 2.8.2 Shielding of a Slowly Moving Test Charge

In this section, we shall investigate the shielding effect on a slowly moving test charge and shall calculate the shielded potential of a slowly moving test charge. If we consider a two component electron-ion $(\alpha = e, i)$ plasma through which a test charge is moving slowly, such that

$$V_t \ll V_{ti},$$

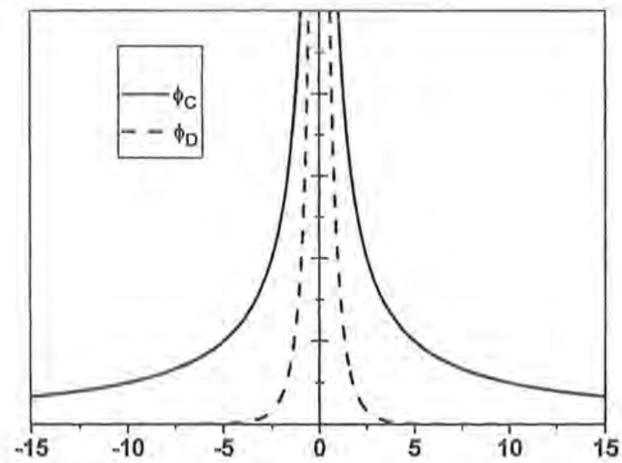


Figure 2-4: Debye-Hückel potential

where $V_{ti}$ is the thermal velocity of the ions, Then the dielectric function becomes

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_{\alpha}\chi_{\alpha}(K, \mathbf{K}\cdot\mathbf{V}_t),$$

where $\chi_{\alpha}(K, \mathbf{K}\cdot\mathbf{V}_t) = \left(K_{D\alpha}^2/K^2\right) W(Z_{\alpha})$ and $Z_{\alpha} = \mathbf{K}\cdot\mathbf{V}_t/KV_{t\alpha}$. Since $V_t \ll V_{ti}$, then $Z_{\alpha} \simeq 0$, and $W(Z\alpha)$ is obtained from Eq. (2.52) such that

$$W(Z_{\alpha}) = 1.$$

So we can write dielectric function as:

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_{\alpha}\left(\frac{K_{D\alpha}^2}{K^2}\right)$$

or

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \frac{\lambda_D^{-2}}{K^2}. \tag{2.67}$$

Substituting the above value of dielectric function into Eq. (2.61), we get

$$\phi_1(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp\left[i\mathbf{K}\cdot(\mathbf{x}-\mathbf{x}')\right]}{\left(K^2+\lambda_D^{-2}\right)}. \tag{2.68}$$

Here, $\mathbf{x}' = \mathbf{x}'_0 + \mathbf{V}_t t$. By writing $d\mathbf{K}$ in spherical coordinates and performing $\mu$ integration, we get

$$\phi_1(\mathbf{x},t) = \frac{2q_t}{\pi}\int_0^{\infty}\frac{K^2 dK}{\left(K^2+\lambda_D^{-2}\right)}\left(\frac{\sin\left[K\left|\mathbf{x}-\mathbf{x}'\right|\right]}{K\left|\mathbf{x}-\mathbf{x}'\right|}\right).$$

Next, we perform $K$ integration, and obtained the following result

$$\phi_1(\mathbf{x},t) = \frac{q_t}{\left|\mathbf{x}-\mathbf{x}'\right|}\exp\left[-\frac{\left|\mathbf{x}-\mathbf{x}'\right|}{\lambda_D}\right], \tag{2.69}$$

where

$$\lambda_D^{-1} = \sqrt{\sum_\alpha K_{D_\alpha}^2}.$$

Equation (2.69) is the expression for the shielded potential of a test charge moving slowly in an electron-ion plasma. It is clear from equation (2.69) that both electrons and ions equally participate in the shielding process. In the next section, we shall calculate the shielded potential of a fast moving test charge in plasma by assuming that the test charge velocity is much larger than the electron thermal velocity.

### 2.8.3 Shielding of a Fast Moving Test Charge

In this section, we shall calculate the shielded potential of a fast moving test charge. To calculate the shielded potential of a fast moving test charge, we shall start with the general expression for the shielded potential given by Eq. (2.60)

$$\phi_1(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp\left[iK\left|\mathbf{x}-\mathbf{x}'\right|\mu\right]}{K^2\epsilon\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)},$$

where the expression for the dielectric function is

$$\epsilon\left(K,\mathbf{K}\cdot\mathbf{V}_t\right) = 1 + \sum_\alpha \chi_\alpha\left(K,\mathbf{K}\cdot\mathbf{V}_t\right),$$

or

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int d\mathbf{v} \frac{\mathbf{K}\cdot\nabla_\mathbf{v} f_{\alpha 0}(\mathbf{v})}{(\mathbf{K}\cdot\mathbf{V}_t - \mathbf{K}\cdot\mathbf{v})} \tag{2.70}$$

Let us consider a two component electron-ion ($\alpha = e, i$) plasma through which a fast moving test charge propagates, such that

$$V_t \gg V_{te},$$

where $V_{te}$ is the thermal velocity of the electrons. As $V_t \gg V_{te}$ then we can write Eq. (2.71) as

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int d\mathbf{v} \frac{\mathbf{K}\cdot\nabla_\mathbf{v} f_{\alpha 0}(\mathbf{v})}{\mathbf{K}\cdot\mathbf{V}_t\left(1 - \frac{\mathbf{K}\cdot\mathbf{v}}{\mathbf{K}\cdot\mathbf{V}_t}\right)}.$$

Further simplification gives

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int d\mathbf{v} \frac{\mathbf{K}\cdot\nabla_\mathbf{v} f_{\alpha 0}(\mathbf{v})}{\mathbf{K}\cdot\mathbf{V}_t}\left(1 - \frac{\mathbf{K}\cdot\mathbf{v}}{\mathbf{K}\cdot\mathbf{V}_t}\right)^{-1}. \tag{2.71}$$

Using the binomial series expansion, we can write the above equation as

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{K^2} \int \mathbf{K}\cdot\nabla_\mathbf{v} f_{\alpha 0}(\mathbf{v}) \left(\frac{1}{\mathbf{K}\cdot\mathbf{V}_t} - \frac{\mathbf{K}\cdot\mathbf{v}}{(\mathbf{K}\cdot\mathbf{V}_t)^2} + \cdots\right) d\mathbf{v}.$$

For $V_t \gg V_{te}$, the above expression becomes

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) \simeq 1.$$

Therefore, we can write the expression for the shielded potential of a fast moving test charge as:

$$\begin{aligned}
\phi_1(\mathbf{x},t) &= \frac{q_t}{2\pi^2}\int_0^\infty K^2 dK \int_{-1}^1 d\mu \int_0^{2\pi} d\varphi \frac{\exp\left[iK\left|\mathbf{x}-\mathbf{x}'\right|\mu\right]}{K^2} \\
&= \frac{2q_t}{\pi}\int_0^\infty dK \frac{\sin\left[K\left|\mathbf{x}-\mathbf{x}'\right|\right]}{K\left|\mathbf{x}-\mathbf{x}'\right|}.
\end{aligned}$$

Performing the $K$-integration we get

$$\phi_1(\mathbf{x},t) = \frac{q_t}{\left|\mathbf{x}-\mathbf{x}_0' - \mathbf{V}_t t\right|}. \tag{2.72}$$

This is the desired expression for the shielded potential of a fast moving test charge. It is evident from the above expression that there is no shielding at all when the test charge moves much more faster than the electron thermal velocity since the plasma particles don't get enough time to shield out the test charge.

### 2.8.4 Shielding of a Test Charge of Intermediate Speed

In the previous two subsections, we have calculated the shielded potential of a slowly moving, and fast moving test charge respectively. Now we want to calculate the shielded potential of a test charge which is moving with a speed greater than the ion thermal speed and less than the electron thermal speed, that is moving with intermediate speed. The range of the speed of the particle will be

$$V_{ti} \ll V_t \ll V_{te}.$$

We shall start with Eq. (2.59) for the shielded potential

$$\phi_1(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp\left[iK\left|\mathbf{x}-\mathbf{x}'\right|\mu\right]}{K^2\epsilon\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)}, \tag{2.73}$$

where the expression for the dielectric function is

$$\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t) = 1+\sum_{\alpha}\chi_\alpha\left(K,\mathbf{K}\cdot\mathbf{V}_t\right)$$

or

$$\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t) = 1+\sum_{\alpha}\frac{\omega_{p\alpha}^2}{K^2}\int d\mathbf{v}\frac{\mathbf{K}\cdot\nabla_{\mathbf{v}}f_{j0}(\mathbf{v})}{(\mathbf{K}\cdot\mathbf{V}_t-\mathbf{K}\cdot\mathbf{v})}. \tag{2.74}$$

Since $V_{ti} \ll V_t$, then above equation can be written as

$$\begin{aligned}\epsilon(K,\mathbf{K}\cdot\mathbf{V}_t) &= 1+\frac{\omega_{pe}^2}{K^2}\int d\mathbf{v}\frac{\mathbf{K}\cdot\nabla_{\mathbf{v}}f_{e0}(\mathbf{v})}{(\mathbf{K}\cdot\mathbf{V}_t-\mathbf{K}\cdot\mathbf{v})} \\ &\quad +\frac{\omega_{pi}^2}{K^2}\int d\mathbf{v}\mathbf{K}\cdot\nabla_{\mathbf{v}}f_{i0}(\mathbf{v})\left(\frac{1}{\mathbf{K}\cdot\mathbf{V}_t}-\frac{\mathbf{K}\cdot\mathbf{v}}{(\mathbf{K}\cdot\mathbf{V}_t)^2}+\cdots\right)d\mathbf{v}.\end{aligned}$$

Since $V_{ti} \ll V_t$, we can write the above equation as

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \frac{\omega_{pe}^2}{K^2}\int d\mathbf{v}\frac{\mathbf{K}\cdot\nabla_{\mathbf{v}} f_{j0}(\mathbf{v})}{(\mathbf{K}\cdot\mathbf{V}_t - \mathbf{K}\cdot\mathbf{v})}.$$

Proceeding steps from Eq. (2.57) to Eq. (2.60), we get

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \chi_e(K, \mathbf{K}\cdot\mathbf{V}_t), \tag{2.75}$$

where $\chi_e(K, \mathbf{K}\cdot\mathbf{V}_t) = \left(K_{De}^2/K^2\right) W(Z_e)$ is the susceptibility of the electron, where $Z_e = (\mathbf{K}\cdot\mathbf{V}_t)/(KV_{te})$, and

$$W(Z_e) = 1 + Z_e \exp\left(\frac{-Z_e^2}{2}\right)\left[i\sqrt{\frac{\pi}{2}} - \int_0^{Z_e} dy \exp\left(\frac{y^2}{2}\right)\right].$$

Since $V_t \ll V_{te}$, so Eq. (2.75) can be written as

$$\epsilon(K, \mathbf{K}\cdot\mathbf{V}_t) = 1 + \frac{\lambda_{De}^{-2}}{K^2}, \tag{2.76}$$

with

$$\lambda_{De}^{-1} = K_{De} = \omega_{pe}/V_{te},$$

where $\omega_{pe}^2 = 4\pi e^2 n_{e0}/m_e$ is the electron plasma frequency.

Equations (2.73) and (2.76) give the following result for the shielded potential,

$$\phi_1(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{K}\frac{\exp\left[i\mathbf{K}\cdot(\mathbf{x}-\mathbf{x}')\right]}{K^2 + \lambda_D^{-2}}. \tag{2.77}$$

Here, $\mathbf{x}' = \mathbf{x}_0' + \mathbf{V}_t t$. By writing $d\mathbf{K}$ in spherical coordinates and performing $\mu$ integration, we get

$$\phi_1(\mathbf{x},t) = \frac{2q_t}{\pi}\int_0^{\infty}\frac{K^2 dK}{K^2 + \lambda_D^{-2}}\left(\frac{\sin\left[K\left|\mathbf{x}-\mathbf{x}'\right|\right]}{K\left|\mathbf{x}-\mathbf{x}'\right|}\right).$$

Next, we perform $K$ integration, and obtained the following result

$$\phi_1(\mathbf{x},t) = \frac{q_t}{\left|\mathbf{x}-\mathbf{x}'\right|}\exp\left[-\left|\mathbf{x}-\mathbf{x}'\right|(\omega_{pe}/V_{te})\right]. \tag{2.78}$$

The Equation (2.78) is the expression for the shielded potential of a test charge moving with intermediate speed. It is clear from this expression that if the test particle is moving with intermediate speed then the shielding is done predominantly by the electrons.

## 2.9 Summary

Since plasma is a collection of very large number of microscopic particles and we are studying the interaction of these microscopic particle and how these charge particles are shielded in plasma. That is the reason, we discuss the kinetic theory of plasma in this chapter. We also discuss in this chapter, how plasma behave as a dielectric medium. In this chapter we have calculated shielded potential of a test charge projectile in electron-ion plasma by using Vlasov-Poisson's model. Since in most of the space and experimental plasma, the presence of dust charge particles is well known. In the next chapter, we shall discuss the shielding of test charges in the dust-contaminated plasma.

# Chapter 3

# Shielding in Dusty Plasma

*"In this chapter, we shall discuss the shielded potential of single and double test-charge projectiles in dust contaminated plasma".*

## 3.1 Shielded Potential of Two Test Charges

Let us consider a multicomponent dusty plasma composed of electrons, singly ionized positive ions, and negatively charged dust grains with a fixed charge state $Z_d$ and the mass $m_d$. The dusty plasma is characterized by the equilibrium number density $n_{\alpha 0}$ and the temperature $T_\alpha$, where $\alpha$ equals $e$ for electrons, $i$ for ions, and $d$ for the dust, through which two-point projectile ions with effective positive charges $Z_1 e$ and $Z_2 e$, and the masses $M_1$ and $M_2$ move with the same velocity $V_p$ along the x-axis. The equilibrium quasineutrality condition is $n_{i0} = n_{e0} + n_{d0} Z_d$. For axial symmetric case, the projectile trajectories $X_k\,(k = 1, 2)$ can be written as $X_1\,(t) = V_p t \hat{e}_x$ and $X_2\,(t) = V_p\,(t - \tau)\,\hat{e}_x$, where $\tau$ is the delay time of the second projectile and $\hat{e}_x$ is the unit vector along the x-axis.

The dynamics of negatively charged dust grains is governed by the Vlasov equation

$$\frac{\partial f_d\,(\mathbf{x}, \mathbf{v}, t)}{\partial t} + \mathbf{v} \cdot \nabla f_d\,(\mathbf{x}, \mathbf{v}, t) + \frac{Z_d e}{m_d} \nabla \phi \cdot \nabla_{\mathbf{v}} f_d\,(\mathbf{x}, \mathbf{v}, t) = 0 \tag{3.1}$$

where $f_d$ is the dust distribution function, $e$ is the magnitude of the electron charge, and $\phi$ is the shielded potential.

To close the system of equations, we may use Gauss's law

$$\nabla \cdot \mathbf{E} = 4\pi\rho_p + 4\pi\rho_t \tag{3.2}$$

where $\rho_p = -en_e + en_i - Z_d e \int f_d(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v}$ is the charge density in plasma and $\rho_t = Z_1 e\delta(\mathbf{x} - \mathbf{V}_p t)$ $+ Z_2 e\delta(\mathbf{x} - \mathbf{V}_p(t-\tau))$ is the charge density of test charge projectile in dusty plasma, and $\nabla \cdot \mathbf{E} = -\nabla^2\phi$, then Eq. (3.2) can be written as:

$$-\nabla^2\phi = 4\pi e(n_i - n_e) - 4\pi Z_d e \int f_d(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{v} + 4\pi Z_1 e\delta(\mathbf{x} - V_p t) + 4\pi Z_2 e\delta(\mathbf{x} - V_p(t-\tau)), \tag{3.3}$$

where $\delta(\mathbf{x} - \mathbf{V}_p t)$ and $\delta(\mathbf{x} - \mathbf{V}_p(t-\tau))$ are the Dirac delta functions for the two-point projectiles. Equations (3.1) – (3.3) form the basis of dielectric theory. We solve these equations for the electrostatic potential in order to study the shielding effect on the two projectiles.

Now we calculate the electrostatic (ES) potential for one dimensional (1D) case by using dielectric theory. We perform a preliminary analysis of the ES potential of multispecies plasma by assuming that the system remains uniform and unperturbed in the $(y, z)$ plane perpendicular to the direction of motion of projectile (i.e., along the x-axis). The one-dimensional distribution function for dust particle is $f_d(x, v_x, t)\,\delta(y)\,\delta(z)$ which satisfies the Vlasov equation. We have used the following normalization:

$$\begin{aligned}
x' &\longrightarrow \frac{x}{\lambda_{Dd}}, & v_x' &\longrightarrow \frac{v_x}{V_{td}}, & & \\
f_d'\left(x', v_x', t'\right) &\longrightarrow V_{td} f_d(x, v_x, t)/n_{d0}, & t' &\longrightarrow t\omega_{pd}, & n_e' &= n_e/Z_d n_{d0}, \\
\phi' &= Z_d e\phi/T_d, & Z_k' &= Z_k/N_d Z_d \ll 1, & n_e' &= n_e/Z_d n_{d0}
\end{aligned}$$

where $Z_k'$ is the normalized effective charged state of $k$th projectile (i.e., $k = 1, 2$), $\omega_{pd} = \sqrt{4\pi Z_d^2 e^2 n_{d0}/m_d}$ is the dust plasma frequency, $V_{td} = \sqrt{T_d/m_d}$ is the dust thermal speed, $N_d = n_{d0}\lambda_{Dd}$ is the number of dust particles in Debye sphere, $\lambda_{Dd} = \sqrt{T_d/4\pi Z_d^2 e^2 n_{d0}}$ is the dust Debye length. The one-dimensional form of Eq. (3.1) is given by

$$\frac{\partial f_d}{\partial t} + v_x\frac{\partial f_d}{\partial x} + \frac{Z_d e}{m_d}\frac{\partial \phi}{\partial x}\frac{\partial f_d}{\partial v_x} = 0.$$

By normalizing the above equation, we get the following result

$$\left(\frac{\partial}{\partial t'}+v_x'\frac{\partial}{\partial x'}\right)f_d'\left(x',v_x',t'\right)+\frac{\partial\phi'}{\partial x'}\frac{\partial f_d'\left(x',v_x',t'\right)}{\partial v_x'}=0. \tag{3.4}$$

In the absence of test charged projectiles, the plasma is uniform and field-free. The plasma is slightly perturbed by the charged projectiles, then Eq. (3.4) can be linearized as:

$$\left(\frac{\partial}{\partial t'}+v_x'\frac{\partial}{\partial x'}\right)f_{d1}'\left(x',v_x',t'\right)+\frac{\partial\phi_1'}{\partial x'}\frac{\partial f_{d0}'\left(v_x'\right)}{\partial v_x'}=0, \tag{3.5}$$

where $f_{d0}(v_x)$ is the distribution function at equilibrium. The normalized and linearized form of Boltzmann density distribution for electrons and ions can be written as

$$n_{e1}' \approx \left(\frac{K_{De}'}{K_{Dd}}\right)^2\phi_1'\left(x',t'\right), \tag{3.6}$$

$$n_{i1}' \approx -\left(\frac{K_{Di}'}{K_{Dd}}\right)^2\phi_1'\left(x',t'\right), \tag{3.7}$$

where $K_{De}'=\sqrt{4\pi n_{e0}e^2/T_e}$, $K_{Di}'=\sqrt{4\pi n_{i0}e^2/T_i}$ and $K_{Dd}=\sqrt{4\pi Z_d^2 n_{d0}e^2/T_d}$ is the wave number of electron, ion and dust grain respectively. The normalized form of Eq. (3.3) after linearization is

$$-\frac{\partial^2\phi_1'}{\partial x^2}=n_{i1}'-n_{e1}'-\int f_{d1}'\left(x',v_x',t'\right)dv_x'+Z_1'\delta\left(x'-V_pt'\right)+Z_2'\delta\left(x'-V_p\left(t'-\tau\right)\right). \tag{3.8}$$

The electrostatic potential in one-dimensional approximation can be solved by using space-time Fourier transform of Eqs. (3.5) – (3.8), and then taking the inverse transform. Fourier transform of Eq. (3.5) is

$$\int_0^\infty dt'\int_{-\infty}^\infty dx'\left(\frac{\partial}{\partial t'}+v_x'\frac{\partial}{\partial x'}\right)f_{d1}'\left(x',v_x',t'\right)\exp\left[-i\left(Kx'-\omega t'\right)\right]$$
$$+\int_0^\infty dt'\int_{-\infty}^\infty dx'\frac{\partial\phi_1'}{\partial x'}\frac{\partial f_{d0}'\left(v_x'\right)}{\partial v_x'}\exp\left[-i\left(Kx'-\omega t'\right)\right]=0\,. \tag{3.9}$$

Eq. (3.9) would take the following form:

$$\begin{aligned}&\left(\omega - Kv_x^{'}\right)\int_0^{\infty} dt^{'}\int_{-\infty}^{\infty} dx^{'} f_{d1}^{'}\left(x^{'},v_x^{'},t^{'}\right)\exp\left[-i\left(Kx^{'}-\omega t^{'}\right)\right]\\ = &\int_0^{\infty} dt^{'}\int_{-\infty}^{\infty} dx^{'}\phi_1^{'}\left(x^{'},t^{'}\right)K\frac{\partial f_{d0}^{'}\left(v_x^{'}\right)}{\partial v_x^{'}}\exp\left[-i\left(Kx^{'}-\omega t^{'}\right)\right].\end{aligned}$$

Further simplification gives

$$\tilde{f}_{d1}^{'}\left(K,v_x^{'},\omega\right) = \frac{\partial f_{d0}^{'}\left(v_x^{'}\right)}{\partial v_x^{'}}\frac{K}{\left(\omega - Kv_x^{'}\right)}\tilde{\phi}_1^{'}\left(K,\omega\right). \tag{3.10}$$

By taking Fourier transform of Eqs. (3.6) , (3.7) and (3.8) , we obtain the following results

$$n_{e1}^{'} \approx \left(\frac{K_{De}^{'}}{K_{Dd}}\right)^2 \tilde{\phi}_1^{'}\left(K,\omega\right). \tag{3.11}$$

$$n_{i1}^{'} \approx -\left(\frac{K_{Di}^{'}}{K_{Dd}}\right)^2 \tilde{\phi}_1^{'}\left(K,\omega\right). \tag{3.12}$$

$$\begin{aligned}K^2\tilde{\phi}_1^{'}\left(K,\omega\right) = \; & n_{i1}^{'} - n_{e1}^{'} - \int f_{d1}^{'}\left(K,v_x^{'},\omega\right)dv_x^{'} + 2\pi Z_1^{'}\delta\left(\omega - KV_p\right)\\ & + 2\pi Z_2^{'}\delta\left(\omega - KV_p\right)\exp\left(iKV_p\tau\right).\end{aligned} \tag{3.13}$$

From Eqs. (3.10) – (3.13), we get

$$\begin{aligned}&K^2\tilde{\phi}_1^{'}\left(K,\omega\right) + K_D^{'2}\tilde{\phi}_1^{'}\left(K,\omega\right) + \left(\int \frac{\partial f_{d0}^{'}\left(v_x^{'}\right)}{\partial v_x^{'}}\frac{K}{\left(\omega - Kv_x^{'}\right)}dv_x^{'}\right)\tilde{\phi}_1^{'}\left(K,\omega\right)\\ &= 2\pi\left(Z_1^{'} + Z_2^{'}\exp\left(iKV_p\tau\right)\right)\delta\left(\omega - KV_p\right),\end{aligned}$$

where $K_D^{'2}$ is the square of effective normalized wave number of electron and ion plasma, given by

$$K_D^{'2} = \frac{K_{Di}^{'2} + K_{De}^{'2}}{K_{Dd}^2}. \tag{3.14}$$

Further simplification gives

$$\tilde{\phi}_1^{'}\left(K,\omega\right) = \frac{2\pi\left(Z_1^{'} + Z_2^{'}\exp\left(iKV_p\tau\right)\right)\delta\left(\omega - KV_p\right)}{K^2\varepsilon\left(K,\omega\right)}, \tag{3.15}$$

where

$$\varepsilon(K,\omega) = 1 + \frac{K_D'^2}{K^2} + \frac{1}{K^2}\int \frac{\partial f_{d0}'\left(v_x'\right)}{\partial v_x'} \frac{K}{(\omega - K v_x')} dv_x' \tag{3.16}$$

is the dielectric function. By taking the inverse Fourier transformation of Eq. (3.15), the electrostatic potential would take the following form

$$\phi_1'\left(x',t'\right) = \frac{1}{2\pi}\int dK \int \frac{\exp\left[i\left(Kx' - \omega t'\right)\right]}{K^2\varepsilon(K,\omega)} \left(Z_1' + Z_2' \exp\left(iKV_p\tau\right)\right) \delta\left(\omega - KV_p\right) d\omega. \tag{3.17}$$

By performing the $\omega$−integration, we get

$$\phi_1'\left(x',t'\right) = \frac{1}{2\pi}\int dK \frac{\exp\left[iK\left(x' - V_p t'\right)\right]}{K^2\varepsilon(K,KV_p)} \left(Z_1' + Z_2' \exp\left(iKV_p\tau\right)\right). \tag{3.18}$$

We obtain the electrostatic potential in a reference frame in which both the projectiles are assumed to be at rest and the leading projectile is considered to be at the origin:

$$\phi_1'\left(x'\right) = \frac{1}{2\pi}\int dK \frac{\exp\left[iKx'\right]}{K^2\varepsilon(K,KV_p)} \left(Z_1' + Z_2' \exp\left(iKV_p\tau\right)\right). \tag{3.19}$$

If we assume that the distribution function of dust particle is Maxwellian,

$$f_{d0}(\mathbf{v}) = \left(\frac{1}{2\pi V_{td}^2}\right)^{3/2} \exp\left[-\frac{1}{2}\left(\frac{\mathbf{v}}{V_{td}}\right)^2\right] \tag{3.20}$$

The normalized and one-dimensional form of above equation can be written as:

$$f_{d0}'\left(v_x'\right) = \frac{1}{n_{d0}}\left(\frac{1}{2\pi}\right)^{1/2} \exp\left[-\frac{1}{2}\left(v_x'\right)^2\right].$$

By differentiating this equation with respect to $v_x$, we get

$$\frac{\partial f_{d0}'\left(v_x'\right)}{\partial v_x'} = -\frac{v_x'}{\sqrt{2\pi}}\frac{1}{n_{d0}} \exp\left[-\frac{1}{2}\left(v_x'\right)^2\right]. \tag{3.21}$$

Then the integral in the dielectric constant is simply the plasma dispersion function and is given as

$$\int \frac{\partial f'_{d0}\left(v'_x\right)}{\partial v'_x}\frac{K}{\left(\omega-Kv'_x\right)}dv'_x = \frac{1}{n_{d0}\sqrt{2\pi}}\int \frac{v'_x\exp\left[-v'^2_x/2\right]}{\left(v'_x-\omega/K\right)}dv'_x$$
$$= \frac{1}{n_{d0}}W\left(\frac{\omega}{|K|}\right) \tag{3.22}$$
$$\equiv \frac{1}{n_{d0}}\left[X\left(\frac{\omega}{|K|}\right)+iY\left(\frac{\omega}{|K|}\right)\right], \tag{3.23}$$

where $X\left(\frac{\omega}{|K|}\right)$ and $Y\left(\frac{\omega}{|K|}\right)$ are the real and imaginary parts of the plasma dispersion function, respectively. The dielectric function (3.16) can be written as

$$\varepsilon\left(K,\omega\right)=1+\frac{K'^2_D}{K^2}+\frac{1}{K^2}\frac{1}{n_{d0}}W\left(\frac{\omega}{|K|}\right), \tag{3.24}$$

where

$$W\left(\frac{\omega}{|K|}\right)=W\left(Z\right)=\frac{1}{\sqrt{2\pi}}\int \frac{y\exp\left[-y^2/2\right]}{\left(y-Z\right)}dy \tag{3.25}$$

is the plasma dispersion function, where $Z=\frac{\omega}{|K|}$. By further solving the Eq. (3.25), the following result will be obtained

$$W\left(Z\right)=1+Z\exp\left(\frac{-Z^2}{2}\right)\left[i\sqrt{\frac{\pi}{2}}-\int_0^Z dy\exp\left(\frac{y^2}{2}\right)\right]. \tag{3.26}$$

A cutoff parameter $K_{\max}$ must be introduced to avoid logarithmic divergence at large $K$ in Eq. (3.15). This divergence corresponds to the incapability of linearized Vlasov theory to treat close encounters between the projectile ions and the plasma species (electron, ions, and dust grains), where $K_{\max}=1/b_{\min}$ and $b_{\min}=Z_k e^2/m_r\left(V_p^2+V_{td}^2\right)$ is the minimum impact parameter, $Z_k\left(k=1,2\right)$ is the charge state of the $k$th projectile and $m_r$ is the reduced mass. It is instructive to first calculate the electrostatic shielded potential due to a single projectile, which is given by

$$\phi_1^{sp'}\left(x'\right)=\frac{Z'_1}{2\pi}\int_0^{K_{\max}} dK\frac{\exp\left[iKx'\right]}{K^2\varepsilon\left(K,KV_p\right)}, \tag{3.27}$$

where $\phi_1^{sp'}\left(x'\right)$ is the electrostatic shielded potential due to a single projectile.

### 3.1.1 Electrostatic Potential for 3D Case

In this section, we shall study the behavior of the electrostatic potential in a three-dimension in which the dust distribution function $f_d(\mathbf{x},\mathbf{v},t)$ would satisfy the Vlasov Maxwell system of equations. We normalize the parameters for three-dimensional case as:

$$\mathbf{x}' \longrightarrow \frac{\mathbf{x}}{\lambda_{Dd}}, \qquad \mathbf{v}' \longrightarrow \frac{\mathbf{v}}{V_{td}},$$
$$f_d'\left(\mathbf{x}',\mathbf{v}',t'\right) \longrightarrow V_{td}^3 f(\mathbf{x},\mathbf{v},t)/n_{d0}, \qquad t' \longrightarrow t\omega_{pd},$$

and choose the frame in which both the projectile ions are at rest, and the leading projectile is at origin. The linearized and dimensionless form of Eq.(3.1) is

$$\left(\frac{\partial}{\partial t'}+\mathbf{v}'\cdot\frac{\partial}{\partial \mathbf{x}'}\right)f_{d1}'\left(\mathbf{x}',\mathbf{v}',t'\right)+\nabla'\phi_1'\cdot\nabla_{\mathbf{v}}'f_{d0}'\left(\mathbf{v}'\right)=0. \tag{3.28}$$

The normalized and linearized form of Boltzmann density distribution for electrons and ions can be written as

$$n_{e1}' \approx \left(\frac{K_{De}'}{K_{Dd}}\right)^2\phi_1'\left(x',t'\right), \tag{3.29}$$

$$n_{i1}' \approx -\left(\frac{K_{Di}'}{K_{Dd}}\right)^2\phi_1'\left(x',t'\right). \tag{3.30}$$

The linearized and dimensionless form of Eq. (3.3) is

$$-\nabla'^2\phi_1'=n_{i1}'-n_{e1}'-\int f_{d1}'\left(\mathbf{x}',\mathbf{v}',t'\right)d\mathbf{v}'+Z_1'\delta\left(\mathbf{x}'-\mathbf{V}_pt'\right)+Z_2'\delta\left(\mathbf{x}'-\mathbf{V}_p\left(t'-\tau\right)\right). \tag{3.31}$$

The Fourier transform of Eqs. (3.28) − (3.31) can be written as:

$$\tilde{f}_{d1}'\left(\mathbf{K},\mathbf{v}',\omega\right)=\frac{\mathbf{K}\cdot\nabla_{\mathbf{v}}'f_{d0}'\left(\mathbf{v}'\right)}{\left(\omega-\mathbf{K}\cdot\mathbf{v}'\right)}\tilde{\phi}_1'(K,\omega), \tag{3.32}$$

$$n_{e1}'\approx\left(\frac{K_{De}'}{K_{Dd}}\right)^2\tilde{\phi}_1'(K,\omega), \tag{3.33}$$

$$n_{i1}^{'} \approx -\left(\frac{K_{Di}^{'}}{K_{Dd}}\right)^{2}\widetilde{\phi_{1}^{'}}(K,\omega), \tag{3.34}$$

and

$$\begin{aligned} K^{2}\widetilde{\phi_{1}^{'}}(K,\omega) &= n_{i1}^{'} - n_{e1}^{'} - \int \widetilde{f_{d1}^{'}}\left(\mathbf{K},\mathbf{v}^{'},\omega\right)d\mathbf{v}^{'} + 2\pi Z_{1}^{'}\delta\left(\omega - \mathbf{K}\cdot\mathbf{V}_{p}\right) \\ &\quad + 2\pi Z_{2}^{'}\delta\left(\omega - \mathbf{K}\cdot\mathbf{V}_{p}\right)\exp\left(i\mathbf{K}\cdot\mathbf{V}_{p}\tau\right). \end{aligned} \tag{3.35}$$

Substituting $\widetilde{f_{d1}^{'}}, n_{e1}^{'}$ and $n_{i1}^{'}$ by using Eqs. (3.32) – (3.34) in Eq. (3.35), we get

$$\widetilde{\phi_{1}^{'}}(K,\omega) = \frac{2\pi\left(Z_{1}^{'} + Z_{2}^{'}\exp\left(i\mathbf{K}\cdot\mathbf{V}_{p}\tau\right)\right)\delta\left(\omega - \mathbf{K}\cdot\mathbf{V}_{p}\right)}{K^{2}\varepsilon\left(K,\omega\right)}. \tag{3.36}$$

By taking the inverse Fourier transform of Eq. (3.36), the electrostatic potential becomes

$$\phi_{1}^{'}\left(x^{'},t^{'}\right) = \frac{2\pi}{(2\pi)^{4}}\int d\mathbf{K}\int \frac{\exp\left[i\left(\mathbf{K}\cdot\mathbf{x}^{'} - \omega t^{'}\right)\right]}{K^{2}\varepsilon\left(K,\omega\right)}\left(Z_{1}^{'} + Z_{2}^{'}\exp\left(i\mathbf{K}\cdot\mathbf{V}_{p}\tau\right)\right)\delta\left(\omega - \mathbf{K}\cdot\mathbf{V}_{p}\right)d\omega. \tag{3.37}$$

Performing $\omega$−integration, we obtain

$$\phi_{1}^{'}\left(x^{'},t^{'}\right) = \frac{1}{(2\pi)^{3}}\int d\mathbf{K}\frac{\exp\left[i\mathbf{K}\cdot\left(\mathbf{x}^{'} - \mathbf{V}_{p}t^{'}\right)\right]}{K^{2}\varepsilon\left(K,\mathbf{K}\cdot\mathbf{V}_{p}\right)}\left(Z_{1}^{'} + Z_{2}^{'}\exp\left(i\mathbf{K}\cdot\mathbf{V}_{p}\tau\right)\right). \tag{3.38}$$

Writing $\mathbf{x}^{'}$ and $\mathbf{K}$ in spherical coordinates;

$$\begin{aligned} \mathbf{x}^{'} &= \left(x^{'}\sin\theta_{1}\cos\varphi_{1},\ x^{'}\sin\theta_{1}\sin\varphi_{1},\ x^{'}\cos\theta_{1}\right) \\ \mathbf{K} &= \left(K\sin\theta\cos\varphi,\ K\sin\theta\sin\varphi,\ K\cos\theta\right) \end{aligned}$$

then

$$\begin{aligned} \mathbf{K}\cdot\left(\mathbf{x}^{'} - \mathbf{V}_{p}t^{'}\right) &= Kx^{'}\sin\theta_{1}\sin\theta\cos\left(\varphi - \varphi_{1}\right) + K\left(X - V_{p}t^{'}\right)\cos\theta \\ &= K\rho\sqrt{1-\mu^{2}}\cos\left(\varphi - \varphi_{1}\right) + K\mu\xi, \end{aligned}$$

where $\mu = \cos\theta = \cos\left(K, V_{p}\right)$, $X = x^{'}\cos\theta_{1}$, $\rho = x^{'}\sin\theta_{1}$, and $\xi = X - V_{p}t^{'}$ is the position in a reference frame in which both the projectile ions are at rest. The electrostatic potential would takes the following form:

$$\phi_1'\left(\mathbf{x}'\right)=\frac{1}{(2\pi)^3}\int_0^{K_{\max}}dK\int_{-1}^{1}\frac{d\mu e^{(iK\mu\xi)}}{\varepsilon\left(K,\mathbf{K}\cdot\mathbf{V}_p\right)}\int_0^{2\pi}e^{iK\rho\sqrt{1-\mu^2}\cos\varphi}d\varphi\left(Z_1'+Z_2'\exp\left(i\mathbf{K}\cdot\mathbf{V}_p\tau\right)\right), \tag{3.39}$$

where $\varphi-\varphi_1=\varphi$. For azimuthal symmetric case, the zeroth order Bessel function has the following form:

$$\begin{aligned}J_0\left(K\rho\sqrt{1-\mu^2}\right) &= \frac{1}{2\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}\int_0^{2\pi}e^{\pm iK\rho\sqrt{1-\mu^2}\cos\varphi}d\varphi\\ &= \frac{1}{2\pi}\int_0^{2\pi}e^{\pm iK\rho\sqrt{1-\mu^2}\cos\varphi}d\varphi,\end{aligned}$$

where $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. Therefore electrostatic potential can be re-written as:

$$\phi_1'\left(\mathbf{x}'\right)=\frac{1}{(2\pi)^2}\int_0^{K_{\max}}dK\int_{-1}^{1}\frac{d\mu e^{(iK\mu\xi)}}{\varepsilon\left(K,\mathbf{K}\cdot\mathbf{V}_p\right)}J_0\left(K\rho\sqrt{1-\mu^2}\right)\left(Z_1'+Z_2'\exp\left(i\mathbf{K}\cdot\mathbf{R}\right)\right), \tag{3.40}$$

where $\mathbf{R}=\mathbf{V}_p\tau$ is the separation between the two projectiles. For a single projectile, we can write electrostatic potential as:

$$\phi_1'^{Sp}\left(\mathbf{x}'\right) = \frac{Z_1'}{(2\pi)^2}\int_0^{K_{\max}}dK\int_{-1}^{1}\frac{d\mu e^{(iK\mu\xi)}}{\varepsilon\left(K,\mathbf{K}\cdot\mathbf{V}_p\right)}J_0\left(K\rho\sqrt{1-\mu^2}\right). \tag{3.41}$$

## 3.2 Numerical Results and Discussions

In this dissertation, we also presents the numerical results of the electrostatic shielding potential of test charge projectiles and discuss the modification in the shielding potential. Propagating velocity of test charge projectiles is, $V_p=2V_{td}$, $n_i=10^{10}cm^{-3}, n_d=10^5cm^{-3}, Z_d=10^3-10^4$. We first plot the normalized electrostatic potential of Eq.(3.41) versus the normalized axial $(-20<\xi<20)$ and the radial distance $(-20<\rho<20)$ of a single projectile shown in the Figure 3-1. This shielded potential is symmetric both in radial and axial direction as viewed

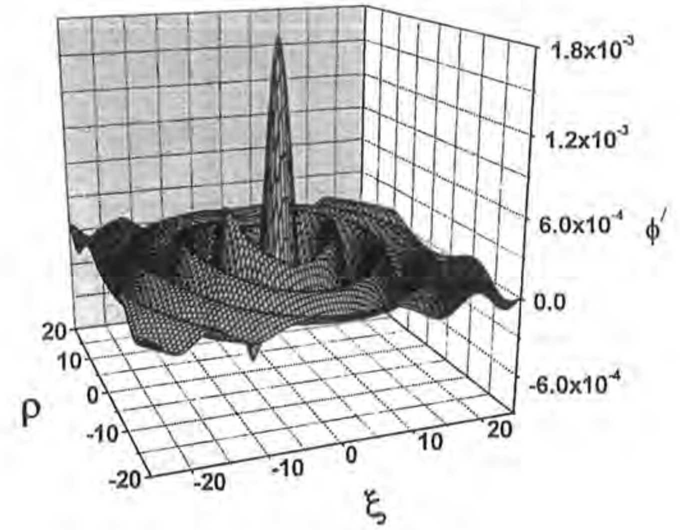


Figure 3-1: Shielding of a single test charge projectile

from a frame moving with the velocity of the projectile. We also presents numerical results of the shielded potential for the different values of $R$ and $V_p$. We numerically solve Eqs. (3.42 ) and (3.43) for the potential of two collinear and non-collinear projectiles and the results are shown in Figs. (3-5)–(3-7).

### 3.2.1 Shielding Potential for Collinear Case

When the two projectiles are moving in a straight line i.e., one behind the other with the same velocity $\mathbf{V}_p$ along the *z-axis,* having $\mathbf{B} = \mathbf{V}_p\tau$ fixed separation between the two projectiles , the potential is given as

$$\phi_1(\mathbf{r}, \mathbf{t}) = \frac{1}{(2\pi)^2} \int_0^{K_{\max}} \int_{-1}^{1} \frac{J_0\left(K\rho\sqrt{1-\mu^2}\right) \mathrm{e}^{(iK\mu\xi)}}{\varepsilon\left(K, \mathbf{K}\cdot\mathbf{V}_p\right)} \left(Z_1^{'} + Z_2^{'}\mathrm{e}^{iK\mu\xi}\right) dK d\mu \qquad (3.42)$$

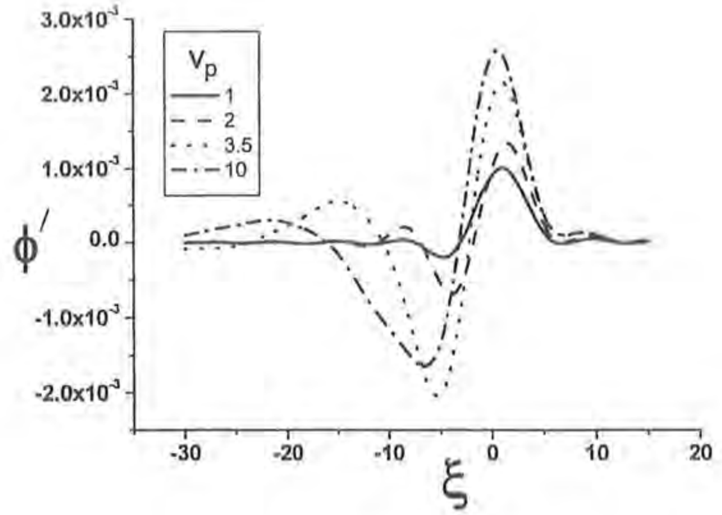


Figure 3-2: Shielded potential of a test charge projectile with different velocities.

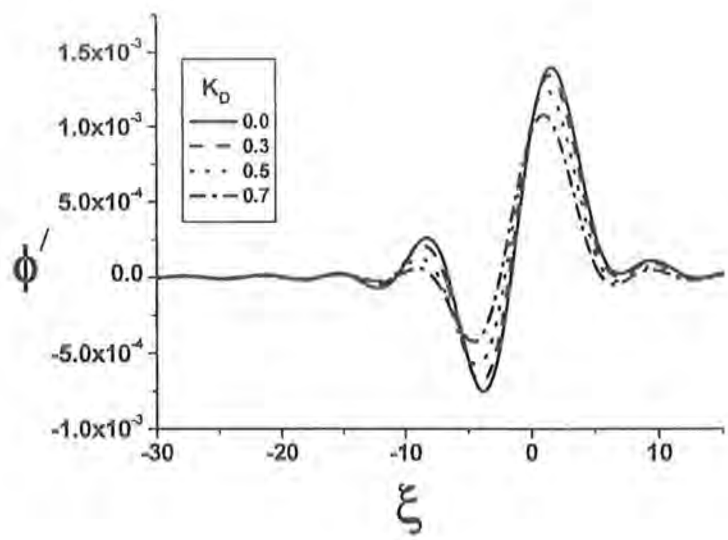


Figure 3-3: Shielded potential with different values of $K_D$.

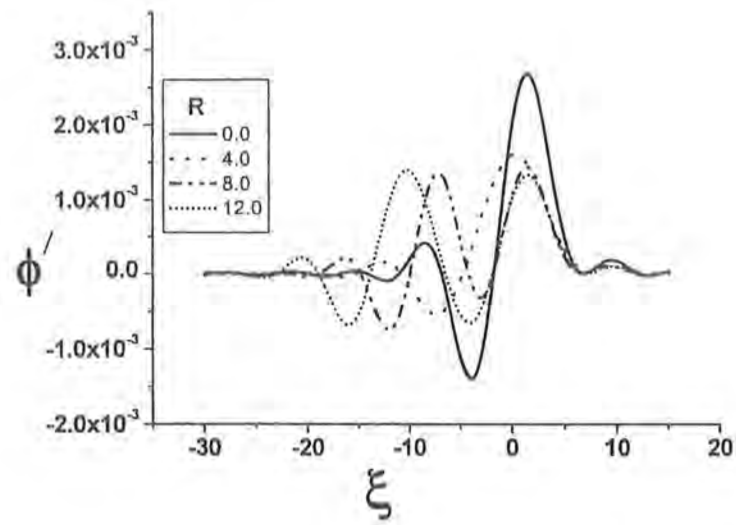


Figure 3-4: Shielded potential with different values of separation distance between test charge projectiles

### 3.2.2 Shielding Potential for Non-collinear Case

When the two projectiles noncollinear motion, the separation vector **R** between them can be resolved in two components **B** and **D**, along and perpendicular to the direction of propagation of projectiles. The vector **R** is given by

$$\mathbf{R} = \mathbf{B} - \mathbf{D}$$

The shielding potential for the non-collinear case can be written as

$$\phi_1(\mathbf{r}, \mathbf{t}) = \frac{Z_1'}{(2\pi)^2} \int_0^{K_{\max}} \int_{-1}^{1} \frac{1}{\varepsilon(K, \mathbf{K} \cdot \mathbf{V}_p)} J_0\left(K\rho\sqrt{1-\mu^2}\right) \mathrm{e}^{(iK\mu\xi)} dK d\mu$$
$$+ \frac{Z_2'}{(2\pi)^2} \int_0^{K_{\max}} \int_{-1}^{1} \frac{1}{\varepsilon(K, \mathbf{K} \cdot \mathbf{V}_p)} J_0\left(K(\rho - \mathbf{D})\sqrt{1-\mu^2}\right) e^{iK\mu(\xi + \mathbf{B})} dK d\mu \quad (3.43)$$

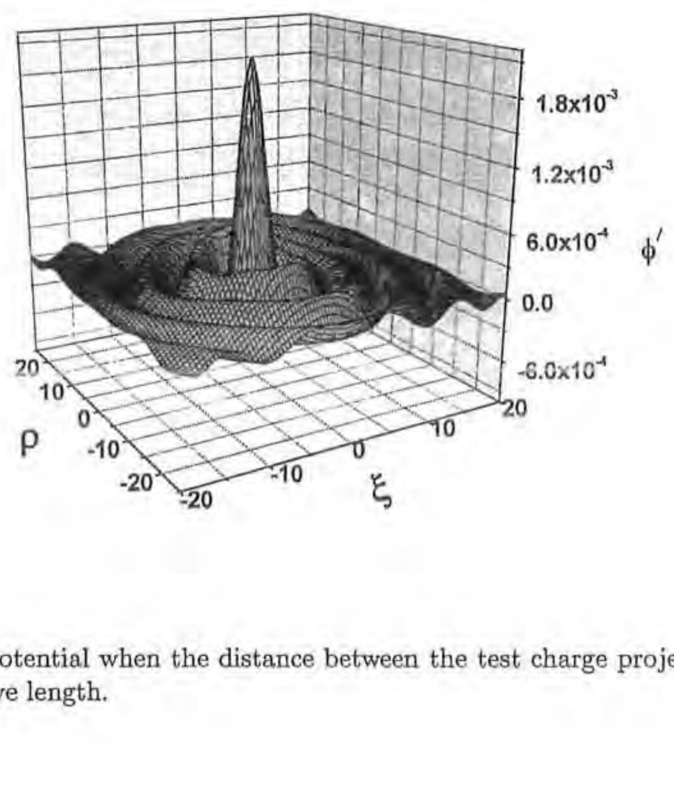


Figure 3-5: Shieding potential when the distance between the test charge projectles is smaller than the effective Debye length.

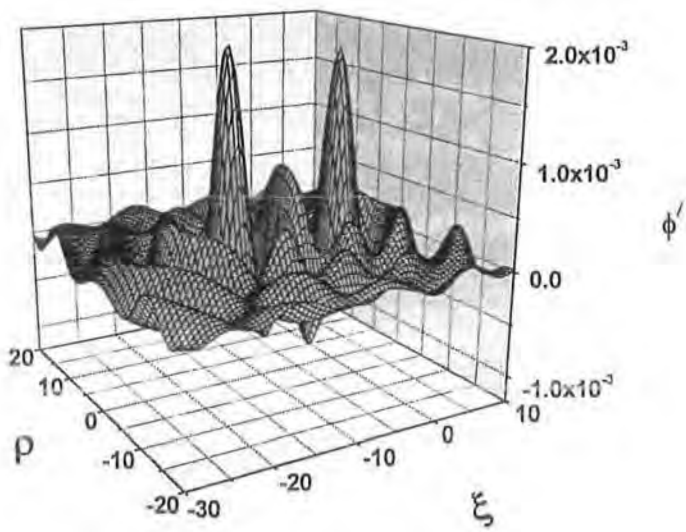


Figure 3-6: When the two projectiles are collinear and the separation distance is larger than the effective Debye length.

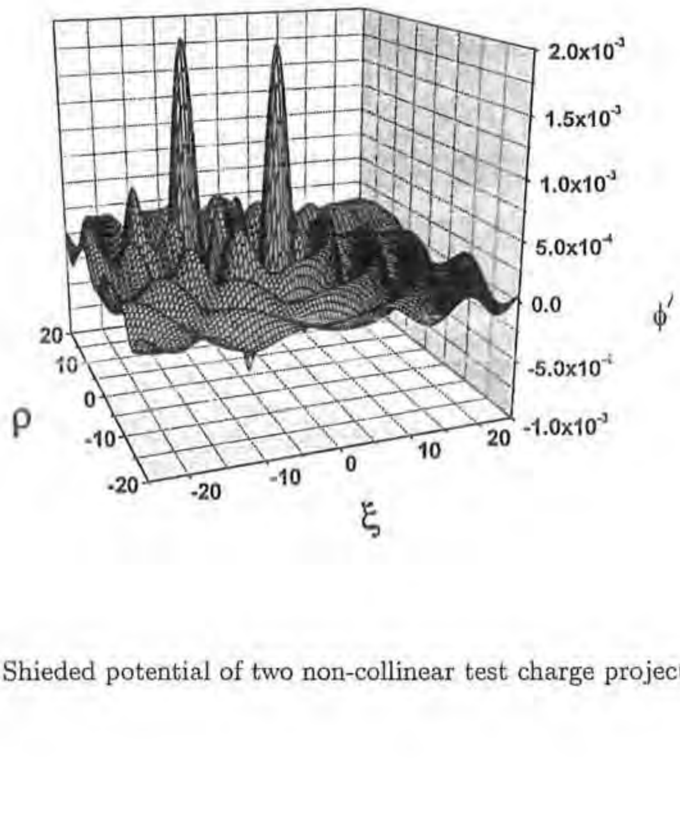


Figure 3-7: Shieded potential of two non-collinear test charge projectiles

## 3.3 Summary

In this chapter we studied, how the plasma shield out two test charge projectiles. What are effects of these two test charges on the shielding, is also studied in this chapter. How the shielding potential modify when these test charge projectiles are collinear is also presented here. We also discuss the shielding when the test charge projectiles makes some angle with each other. Numerical results of shielding in plasma are also shown in this chapter. In the next chapter we shall study the shielding in a multicomponent plasma.

# Chapter 4

# Shielding in Multicomponent Plasma

*" This chapter is devoted to a study of multicomponent plasma, and the shielded potential due to moving test charge projectile and the wake potential of moving test charge projectile".*

## 4.1 Introduction

The calculation of electrostatic potential of a moving test charge in a multicomponent plasma is one of the most fundamental and extensively studied problem in plasma physics. When a test charge is introduced in a plasma, it polarizes the plasma and produces a shield cloud around the charge. If the plasma is cold and there is no thermal agitation then the shielding would be perfect and potential of a test charge drops to zero outside the cloud. The test charge would be electrically neutral. However, if the plasma is not cold, there would few particles at the edge of the cloud that would have enough thermal energy to escape from the cloud, so that the shielding would not be complete.

In this chapter we have considered an unmagnetized collisionless multicomponent plasma consisting of the Boltzmann distributed electrons, mobile positive and negative ions, and immobile positive/negative charged dust grains. By employing the Vlasov–Poisson system containing a test charge density, we calculate the electrostatic potential for the test charge moving with a constant speed along the z-axis. Further, the Debye screening and wake potentials are obtained in the presence of mobile negative and positive ions. Here, the occurrence of negative ions is very much important [55, 56, 57, 58, 59, 60, 61, 62]. It is worth mentioning here that the dust-ion-acoustic speed (DIA) and oscillatory wake potential are significantly modified due to the negative ions. The mobile negative ions could play a significant role in attracting the same polarity charges in the wake potential regions and in making the ordered crystalline structures. The results should be useful in the context of charged particle repulsion and attraction in microelectronic plasmas and in polar mesosphere [54]. We are discussing here the wake field and Debye screening in a multicomponent plasma which can be produced in the laboratory [55, 62].

## 4.2 Mathematical Model

Since we are dealing with the small amplitude wave, it is appropriate to use linearize Vlasov equation (2.16)

$$\frac{\partial f_{j1}}{\partial t} + \mathbf{V} \cdot \nabla f_{j1} + \frac{q_j}{m_j}\mathbf{E}_1 \cdot \nabla_v f_{j0}(\mathbf{V}) = 0, \tag{4.1}$$

where $f_{j1}$ is the perturbed part of the distribution function and $E(-\nabla\phi)$ is the induced electric field, $q_j$ and $m_j$ are the charge and mass.

The electrostatic potential $\phi_1$ satisfy the Poisson's equation

$$\begin{aligned} -\nabla^2\phi_1 &= 4\pi(\rho_{plasma} + \rho_{test}) \\ -\nabla^2\phi_1 &= 4\pi\sum_j q_j n_{j0}\int f_{j1} d\mathbf{V} + 4\pi q_t\,\delta(\mathbf{r} - \mathbf{V}_t t) \end{aligned} \tag{4.2}$$

Taking the space time Fourier transform of Eq.(4.1), we get

$$\begin{aligned} &\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr} \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right) f_{j1} \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right] \\ &-\frac{q_j}{m_j}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr}\left[\nabla\phi_1 \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]\right] \cdot \nabla_v f_{j0}(\mathbf{V}) = 0 \end{aligned} \tag{4.3}$$

or

$$\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr} \left[\iota\left(\omega-\mathbf{k}\cdot\mathbf{v}\right)\right] f_{j1} \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]$$
$$-\frac{q_j}{m_j}\int_{-\infty}^{\infty} dt \left[\int_{-\infty}^{\infty} \mathbf{dr}\nabla\phi_1 \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]\right]\cdot\nabla_v f_{j0}\left(\mathbf{V}\right)=0 \tag{4.4}$$

Integrating by parts,we get the following result

$$\left[\iota\left(\omega-\mathbf{k}\cdot\mathbf{v}\right)\right]\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} f_{j1} \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]\mathbf{dr}$$
$$=-\frac{q_j}{m_j}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr}\phi_1 \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]\left[\iota\mathbf{k}\cdot\nabla_v f_{j0}\left(V\right)\right] \tag{4.5}$$

$$\tilde{f}_{j1}\left(\omega,k\right)=-\frac{q_j}{m_j}\frac{\left[\mathbf{k}\cdot\nabla_v f_{j0}\left(V\right)\right]\tilde{\phi}_1(w,k)}{\left(\omega-\mathbf{k}\cdot\mathbf{v}\right)} \tag{4.6}$$

Now taking the space time Fourier transform of Eq. (4.2), we have

$$-\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr}\left[\nabla^2\phi_1 \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]\right]=$$
$$4\pi\sum_j q_j n_{j0}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr}\left[\int f_{j1} dV\right]\exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right]$$
$$+4\pi q_t\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} \mathbf{dr}\left[\delta(\mathbf{r}-\mathbf{V}_t t)\right]\exp\left[\iota\left(\mathbf{k}\cdot\mathbf{r}-\omega t\right)\right] \tag{4.7}$$

Integrating by parts the right hand side of Eq. (4.7), we get

$$k^2\tilde{\phi}_1\left(\mathbf{k},\omega\right)=4\pi\sum_j q_j n_{j0}\int \tilde{f}_{j1}\left(\mathbf{k},\omega\right)d\mathbf{V}+4\pi q_t\int_{-\infty}^{\infty} dt \exp\left[\iota\left(\mathbf{k}\cdot\mathbf{V}_t t-\omega t\right)\right] \tag{4.8}$$

or

$$k^2\tilde{\phi}_1\left(\mathbf{k},\omega\right)-4\pi\sum_j q_j n_{j0}\int \tilde{f}_{j1}\left(\mathbf{k},\omega\right)d\mathbf{V}=4\pi q_t\int_{-\infty}^{\infty} dt \exp\left[-\iota\left(\omega-\mathbf{k}\cdot\mathbf{V}_t\right)t\right] \tag{4.9}$$

Substituting the perturbed distribution function $\tilde{f}_{j1}$ into Eq.(4.9),

$$k^2\tilde{\phi}_1(\mathbf{k},\omega)+4\pi\sum_j\frac{n_{j0}q_j{}^2}{m_j}\int\frac{[\mathbf{k}\cdot\nabla_v f_{j0}(\mathbf{V})]\,\tilde{\phi}_1(\mathbf{k},\omega)}{(\omega-\mathbf{k}\cdot\mathbf{v})}d\mathbf{V}$$
$$=4\pi q_t\int_{-\infty}^{\infty}dt\exp\left[-\iota(\omega-\mathbf{k}\cdot\mathbf{V}_t)t\right] \tag{4.10}$$

or

$$k^2\tilde{\phi}_1(\mathbf{k},\omega)\left[1+\sum_j 4\pi\frac{n_{j0}q_j{}^2}{m_j}\int\frac{[\mathbf{k}\cdot\nabla_v f_{j0}(\mathbf{V})]}{k^2(\omega-\mathbf{k}\cdot\mathbf{v})}d\mathbf{V}\right]=8\pi^2q_t\delta(\omega-\mathbf{k}\cdot\mathbf{V}_t) \tag{4.11}$$

where

$$\varepsilon(k,\omega)=1+\sum_j 4\pi\frac{n_{j0}q_j{}^2}{m_j}\int\frac{[\mathbf{k}\cdot\nabla_v f_{j0}(\mathbf{V})]}{k^2(\omega-\mathbf{k}\cdot\mathbf{v})}d\mathbf{V}$$

and

$$\omega_{pj}=\sqrt{\left(4\pi q_j^2n_{j0}\right)/m_j}.$$

Equation (4.11) becomes

$$\tilde{\phi}_1(\mathbf{k},\omega)=\frac{8\pi^2q_t\delta(\omega-\mathbf{k}\cdot\mathbf{V}_t)}{k^2\varepsilon(k,\omega)} \tag{4.12}$$

Taking the inverse space time Fourier transform of Eq. (4.12), we get

$$\phi_1(\mathbf{r},t)=\frac{8\pi^2q_t}{(2\pi)^4}\int_{-\infty}^{\infty}d\mathbf{k}\int_{-\infty}^{\infty}d\omega\frac{\delta(\omega-\mathbf{k}\cdot\mathbf{V}_t)}{k^2\varepsilon(k,\mathbf{k}\cdot\mathbf{V}_p)}\exp\left[-\iota(\mathbf{k}\cdot\mathbf{r}-\omega t)\right] \tag{4.13}$$

Now performing $\omega$-integration, we get

$$\phi_1(\mathbf{r},t)=\frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}d\mathbf{k}\frac{\exp\left[-\iota\mathbf{k}\cdot(\mathbf{r}-\mathbf{V}_tt)\right]}{k^2\varepsilon(k,\mathbf{k}\cdot\mathbf{V}_t)} \tag{4.14}$$

where the dielectric function can be written as

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{\omega_{pj}^2}{k^2}\int\frac{\left[\mathbf{k}\cdot\nabla_v f_{j0}\left(V\right)\right]}{\left(\mathbf{k}\cdot\mathbf{V}_t-\mathbf{k}\cdot\mathbf{V}\right)}dV \tag{4.15}$$

or

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{K_{Dj}^2}{k^2}W\left(C_j\right) \tag{4.16}$$

Maxwellian distribution in on dimension is

$$f_{j0}\left(v_x\right)=\left(\frac{m_j}{2\pi T_j}\right)^{1/2}\exp\left(\frac{-m_jv_x^2}{2T_j}\right), \tag{4.17}$$

and

$$\nabla_v f_{j0}\left(v_x\right)=\left(\frac{1}{2\pi}\right)^{1/2}\left(\frac{1}{V_{tj}}\right)\exp\left(\frac{-m_jv_x^2}{2T_j}\right)\left(\frac{-v_x}{V_{tj}^2}\right)$$

Now the dielectric response function can be written as

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{\omega_{pj}^2}{k^2}\left(\frac{1}{2\pi}\right)^{1/2}\left(\frac{1}{V_{tj}}\right)\int\frac{k\exp(-m_jv_x^2/2T_j)(-v_x/V_{tj}^2)}{(\omega-\mathbf{k}\cdot\mathbf{V})}dv_x \tag{4.18}$$

Let us define $q^2=v_x^2/2V_{tj}^2$, then Eq. (4.18) can be written as

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{\omega_{pj}^2}{k^2}\left(\frac{1}{V_{tj}}\right)\left(\frac{1}{2\pi}\right)^{1/2}\int\frac{k\exp(-q^2)(-v_x/V_{tj}^2)}{(\omega-\sqrt{2}kqV_{tj})}(\sqrt{2}V_{tj}dq)$$

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{\omega_{pj}^2}{k^2}\left(\frac{1}{V_{tj}^2}\right)\left(\frac{1}{2\pi}\right)^{1/2}\int\frac{\exp(-q^2)(-q)dq}{(\omega/\sqrt{2}kV_{tj}-q)},$$

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\left(\frac{\omega_{pj}^2}{V_{tj}^2}\right)\frac{1}{k^2}\left(\frac{1}{\pi}\right)^{1/2}\int\frac{q\exp(-q^2)dq}{(q-C_j)}, \tag{4.19}$$

or

$$\varepsilon\left(k,\mathbf{k}\cdot\mathbf{V}_t\right)=1+\sum_j\frac{K_{Dj}^2}{k^2}W(C_j) \tag{4.20}$$

Here, $C_j = \omega/\sqrt{2}kV_{tj}, K_{Dj}^2 = \left(\omega_{pj}^2/V_{tj}^2\right)$, $W(C_j)$ is the plasma dispersion function and can be written as

$$W(C_j) = \left(\frac{1}{\pi}\right)^{1/2} \int \frac{q\exp(-q^2)dq}{(q - C_j)}.$$

We are using $W(C_j)$ in the double factorial form

For $Z \ll 1$, we can write the plasma dispersion function as

$$W(Z) = \iota\sqrt{\frac{\pi}{2}} Z \exp\left(\frac{-Z^2}{2}\right) + 1 - Z^2 + \frac{Z^4}{3} + ....\frac{(-1)^{n+1} Z^{2n+2}}{(2n+1)!!}$$

For $Z \gg 1$,

$$W(Z) = \iota\sqrt{\frac{\pi}{2}} Z \exp\left(\frac{-Z^2}{2}\right) - \frac{1}{Z^2} + \frac{3}{Z^4} + ....\frac{(2n+1)!!}{Z^{2n}}$$

For $kV_{tp}, kV_{tn} \ll \omega \ll kV_{te}$, the dielectric function (4.20) can be written as

$$\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t) = 1 + \frac{K_{De}^2}{k^2} W(C_j) + \frac{K_{Dp}^2}{k^2} W(C_p) + \frac{K_{Dn}^2}{k^2} W(C_n),$$

$$\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t) = 1 + \frac{K_{De}^2}{k^2} - \frac{K_{Dp}^2}{k^2}\frac{1}{2C_p^2} - \frac{K_{Dn}^2}{k^2}\frac{1}{2C_n^2},$$

$$\begin{aligned}
\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t) &= 1 + \frac{1}{k^2\lambda_{De}^2} - \frac{\omega_{pp}^2}{\omega^2} - \frac{\omega_{pn}^2}{\omega^2}, \\
\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t) &= \frac{1 + k^2\lambda_{De}^2}{k^2\lambda_{De}^2}\left[1 - \frac{k^2\lambda_{De}^2\left(\omega_{pp}^2 + \omega_{pn}^2\right)}{\left(1 + k^2\lambda_{De}^2\right)\omega^2}\right] \\
\frac{1}{\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t)} &= \frac{k^2\lambda_{De}^2}{1 + k^2\lambda_{De}^2}\left[1 + \frac{\omega_k^2}{\omega^2 - \omega_k^2}\right] \qquad (4.21)
\end{aligned}$$

$$\frac{1}{\varepsilon(k, \mathbf{k}\cdot\mathbf{V}_t)} = \frac{k^2\lambda_{De}^2}{1 + k^2\lambda_{De}^2} + \frac{k^2\lambda_{De}^2}{1 + k^2\lambda_{De}^2}\frac{\omega_k^2}{\left(\omega^2 - \omega_k^2\right)} \qquad (4.22)$$

where $\omega_{pd}^2 = K_{Dd}^2 V_{tj}^2$ is the plasma frequency of the dust and $\omega_k^2 = k^2\lambda_{De}^2\left(\omega_{pp}^2 + \omega_{pn}^2\right) / \left(1 + k^2\lambda_{De}^2\right)\omega^2$ is dispersion relation for the DIA wave which is modified with $\omega_{pn}$. Here, $\omega_{pn} = \left(4\pi n_{n0} Z_n^2 e^2/m_n\right)^{1/2}$

is the negative ion frequency and $\omega_{pp} = \left(4\pi n_{p0} Z_p^2 e^2/m_p\right)^{1/2}$ is the positive ion plasma frequency, $m_n(m_p)$ is the mass of negative (positive) ions, and $\lambda_{De} = \left(T_e/4\pi n_{e0} e^2\right)^{1/2}$ is the electron Debye length.

Electrostatic potential of the test charge projectile become

$$\phi_1(\mathbf{r},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}\frac{d\mathbf{k}}{k^2}\left[\frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}\right]\exp\left[-\iota\mathbf{k}.\left(\mathbf{r}-\mathbf{V}_t t\right)\right]$$
$$+\frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}\frac{d\mathbf{k}}{k^2}\left[\left(\frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}\right)\times\frac{\omega_k^2}{\left(\mathbf{k}\cdot\mathbf{V}_t\right)^2-\omega_k^2}\right]\exp\left[-\iota\mathbf{k}.\left(\mathbf{r}-\mathbf{V}_t t\right)\right] \quad (4.23)$$

or

$$\phi_1(\mathbf{r},t) = \phi_D(\mathbf{r},t) + \phi_W(\mathbf{r},t) \quad (4.24)$$

Here $\phi_D(\mathbf{r},t)$ represents the Debye part of electrostatic potential and $\phi_W(r,t)$ represents the wake part of the electrostatic potential. Next we want to simplify the wake part of the electrostatic potential and the results would be presented in the next section.

### 4.2.1 Debye Potential in a Multi-component Plasma

When the test charge projectile moves in a multi-component plasma, the charged particles try to shield the test charge. In the previous section, we have obtained the Debye shielded potential as

$$\phi_D(\mathbf{r},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}d\mathbf{k}\left[\frac{\lambda_{De}^2}{1+k^2\lambda_{De}^2}\right]\exp\left[-\iota\mathbf{k}.\left(\mathbf{r}-\mathbf{V}_t t\right)\right] \quad (4.25)$$

This expression for the Debye shielding can be simplified by using the spherical polar coordinates and we can obtain the expression of standard Debye potential.

### 4.2.2 Wake Potential in a Multi-component Plasma

$$\phi_W(\mathbf{r},t) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty}d\mathbf{k}\left[\left(\frac{\lambda_{De}^2}{1+k^2\lambda_{De}^2}\right)\frac{\omega_k^2}{\left(\mathbf{k}\cdot\mathbf{V}_t\right)^2-\omega_k^2}\right]\exp\left[-\iota\mathbf{k}.\left(\mathbf{r}-\mathbf{V}_t t\right)\right] \quad (4.26)$$

For simplifying the wake part of potential, we use the cylindrical co-ordinates

$$\mathbf{k} = \left(k_\perp \cos\phi_k, k_\perp \sin\phi_k, k_\parallel\right)$$

$$\mathbf{r} = \left(\rho\cos\phi_r, \rho\sin\phi_r, r_\parallel\right)$$

Since we are using single test charge projectile approach moving along the z-axis, such that

$$\iota\mathbf{k}.\left(\mathbf{r} - \mathbf{V}_t t\right) = \iota k_\perp \rho\cos\phi_k + \iota k_\parallel \xi$$

where

$$\xi = r_\parallel - V_t t$$

The wake potential can be written as

$$\phi_W\left(\mathbf{r}, t\right) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{k}\left[\frac{\lambda_{De}^2}{1 + k^2\lambda_{De}^2}\frac{\omega_k^2}{\left(\omega^2 - \omega_k^2\right)}\right]\exp\left[-\iota k_\perp \rho\cos\phi_k - \iota k_\parallel \xi\right]$$

or

$$\phi_W\left(\mathbf{r}, t\right) = \frac{q_t}{2\pi^2}\int_{-\infty}^{\infty} d\mathbf{k} M \exp\left[-\iota k_\perp \rho\cos\phi_k - \iota k_\parallel \xi\right] \tag{4.27}$$

where

$$M = \left(\frac{\lambda_{De}^2}{1 + k^2\lambda_{De}^2}\right)\frac{\omega_k^2}{\left(\omega^2 - \omega_k^2\right)} \tag{4.28}$$

Now simplifying the Eq. (4.28) as

$$
\begin{aligned}
M &= \frac{\lambda_{De}^2}{1+k^2\lambda_{De}^2}\frac{\omega_k^2}{\left(\omega^2-\omega_k^2\right)} \\
&= \frac{\lambda_{De}^2\omega_k^2}{\left(K_\parallel^2+K_\perp^2+1\right)\left(k_\parallel^2V_t^2-\omega_k^2\right)} \\
&= \frac{\lambda_{De}^2\omega_k^2}{\left(K_\parallel^2+K_\perp^2+1\right)\left(V_t^2/\lambda_{De}^2\right)\left[K_\parallel^2-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)/\left(K_\parallel^2+K_\perp^2+1\right)\left(\lambda_{De}^2/V_t^2\right)\right]} \\
&= \frac{\lambda_{De}^4\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)}{\left(V_t^2\right)\left[K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)\right]\left(K_\parallel^2+K_\perp^2+1\right)} \qquad (4.29)
\end{aligned}
$$

Now substituting the value of $M$ in Eq. (4.27), the expression of the wake potential will become

$$
\begin{aligned}
\phi_W(\mathbf{r},t) &= \frac{q_t\lambda_{De}^4\left(\omega_{pp}^2+\omega_{pp}^2\right)}{2\pi^2\left(V_t^2\right)}\int_{-\infty}^{\infty}\frac{d\mathbf{k}\exp\left[-\iota k_\perp\rho\cos\phi_k-\iota k_\parallel\xi\right]}{\left(K_\parallel^2+K_\perp^2+1\right)} \\
&\times\left[\frac{\left(K_\parallel^2+K\perp^2\right)}{\left[K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)\right]}\right] \qquad (4.30)
\end{aligned}
$$

Expressing $d\mathbf{k}$ in cylindrical co-ordinates, and writing $d\mathbf{k} = k_\perp dk_\perp dk_\parallel d\phi$ in the above equation, we get

$$
\phi_W(\mathbf{r},t) = \left(\frac{q_t}{2\pi^2}\right)\frac{\lambda_{De}\left(\omega_{pp}^2+\omega_{pp}^2\right)}{\left(V_t^2\right)}\int_{-\infty}^{\infty}k_\perp dk_\perp dk_\parallel\exp\left[-\iota k_\parallel\xi/\lambda_{De}\right]\int_{-\pi}^{+\pi}d\phi\exp\left[\frac{-\iota k_\perp\rho\cos\phi_k}{\lambda_{De}}\right]
$$
$$
\frac{\left(K_\parallel^2+K_\perp^2\right)}{\left[K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)\right]} \qquad (4.31)
$$

Performing the $\phi$ integration and using the zeroth order Bessel function $J_0\left[K_\perp\rho/\lambda_{De}\right]$, we get

$$\phi_W(\mathbf{r},t) = \left(\frac{q_t}{\pi}\right)\frac{\lambda_{De}\left(\omega_{pp}^2+\omega_{pp}^2\right)}{\left(V_t^2\right)}\int_{-\infty}^{\infty} K_\perp dK \perp dK_\parallel \exp\left[-\iota k_\parallel \xi/\lambda_{De}\right]\times$$
$$\frac{J_0\left[K_\perp\rho/\lambda_{De}\right]\left(K_\parallel^2+K_\perp^2\right)}{\left[K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)\right]} \tag{4.32}$$

or

$$\phi_W(\mathbf{r},t) = \left(\frac{q_t}{\pi}\right)\frac{\lambda_{De}\left(\omega_{pp}^2+\omega_{pp}^2\right)}{\left(V_t^2\right)}\int_{-\infty}^{\infty} K_\perp dK \perp dK_\parallel \exp\left[-\iota k_\parallel \xi/\lambda_{De}\right]\times$$
$$\frac{J_0\left[K_\perp\rho/\lambda_{De}\right]\left(K_\parallel^2+K_\perp^2\right)}{N} \tag{4.33}$$

where

$$N=\left[K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pp}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)\right] \tag{4.34}$$

where $J_0\left[K_\perp\rho/\lambda_{De}\right]$ is the zeroth order Bessel function defined below,

$$J_0\left[K_\perp\rho/\lambda_{De}\right]=\frac{1}{2\pi}\int_{-\pi}^{+\pi} d\phi \exp\left[\frac{-\iota K_\perp \rho\cos\phi_k}{\lambda_{De}}\right] \tag{4.35}$$

Let us first simplify the Eq. (4.34), we get

$$\begin{aligned} N \equiv\ & K_\parallel^2\left(K_\parallel^2+K_\perp^2+1\right)-\left(\omega_{pn}^2+\omega_{pp}^2\right)\left(K_\parallel^2+K_\perp^2\right)\left(\lambda_{De}^2/V_t^2\right)= \\ & K_\parallel^4+K_\parallel^2K_\perp^2+K_\parallel^2-\left(\omega_{pn}^2+\omega_{pp}^2\right)K_\perp^2\left(\lambda_{De}^2/V_t^2\right) \\ & -\left(\omega_{pp}^2+\omega_{pp}^2\right)K_\parallel^2\left(\lambda_{De}^2/V_t^2\right) \end{aligned}$$

or

$$
\begin{aligned}
N &= K_{\|}^{4} + K_{\|}^{2}\left[1 + K_{\perp}^{2} - \left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)\left(\lambda_{De}^{2}/V_{t}^{2}\right)\right] \\
&\quad - \left(\omega_{pn}^{2} + \omega_{pp}^{2}\right) K_{\perp}^{2}\left(\lambda_{De}^{2}/V_{t}^{2}\right)
\end{aligned} \tag{4.36}
$$

Eq. (4.36) is quadratic in $K_{\|}^{4}$, the roots of the quadratic equation are

$$
\begin{aligned}
&K_{\pm}^{2} = \pm\frac{1}{2}\left[1 + K_{\perp}^{2} - \frac{\left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)\lambda_{De}^{2}}{V_{t}^{2}}\right] \\
&+\frac{1}{2}\sqrt{\left[1 + K_{\perp}^{2} - \frac{\left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)\lambda_{De}^{2}}{V_{t}^{2}}\right]^{2} + 4K_{\perp}^{2}\frac{\left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)\lambda_{De}^{2}}{V_{t}^{2}}}
\end{aligned} \tag{4.37}
$$

Now substituting Eq. (4.36) into Eq. (4.33), we get

$$
\begin{aligned}
\phi_{W}(r,t) &= \left(\frac{q_t}{\pi}\right)\frac{\lambda_{De}\left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)}{\left(V_{t}^{2}\right)}\int_{-\infty}^{\infty} K_{\perp}dK \perp dK_{\|}J_{0}\left[K_{\perp}\rho/\lambda_{De}\right]\exp\left[-\iota k_{\|}\xi/\lambda_{De}\right] \\
&\quad\times\frac{\left(K_{\|}^{2} + K_{\perp}^{2}\right)}{\left(K_{\|}^{2} - K_{+}^{2}\right)\left(K_{\|}^{2} - K_{-}^{2}\right)}
\end{aligned} \tag{4.38}
$$

Performing $K_{\|}$ integration and using the residue theorem, we get

$$
\begin{aligned}
&\phi_{W}(\mathbf{r},t) = (-2q_t)\frac{\lambda_{De}\left(\omega_{pp}^{2} + \omega_{pp}^{2}\right)}{\left(V_{t}^{2}\right)}\int_{-\infty}^{\infty} K_{\perp}dK \perp dK_{\|}J_{0}\left[K_{\perp}\rho/\lambda_{De}\right] \\
&\times\frac{\left(K_{\perp}^{2} + K_{+}^{2}\right)\sin\left(K_{+}\xi/\lambda_{De}\right)}{\left(K_{\perp}^{2} + K_{+}^{2} + 1\right)\left(K_{\perp}^{2} + K_{-}^{2}\right)K_{+}}
\end{aligned} \tag{4.39}
$$

Here we have obtained the wake potential in $K_{\perp}$, for simplifying equation (4.38), we used assumption $K_{\perp}^{2} \ll 1$. Where $K_{-}$ and $K_{+}$ can be written as

$$
K_{-}^{2} = 1 - \frac{\lambda_{De}^{2}\left(\omega_{pn}^{2} + \omega_{pp}^{2}\right)}{\left(V_{t}^{2}\right)},
$$

$$K_+^2 = \frac{\lambda_{De}^2 K_\perp^2 \left(\omega_{pn}^2 + \omega_{pp}^2\right)}{\left(V_t^2\right)},$$

and

$$J_0\left[K_\perp\rho/\lambda_{De}\right] = [1 - \frac{K_\perp^2\rho^2}{4\lambda_{De}^2} + \frac{1}{64}\left(\frac{K_\perp\rho}{\lambda_{De}}\right)^4 ...]$$

Here, we have also used the series of the zeroth order Bessel function in terms of Gamma function,

$$J_n(x) = \frac{x^n}{2^n\Gamma(n+1)}\left[1 - \frac{x^2}{2(2n+1)} + \frac{x^4}{2.4(2n+2)(2n+4)}...\right] \tag{4.40}$$

The wake potential takes the following form:

$$\phi_W(\mathbf{r},t) = (-2q_t)\frac{\left(\omega_{pn}^2 + \omega_{pp}^2\right)^{1/2}}{V_t}\left[1 + \frac{\left(\omega_{pp}^2 + \omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]$$
$$\times\left[1 - \frac{\left(\omega_{pn}^2 + \omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]^{-1}\int_0^1 K_\perp^2 \sin\left[K_+\xi/\lambda_{De}\right] dK_\perp \tag{4.41}$$

or

$$\phi_W(\mathbf{r},t) = (-2q_t)\frac{\left(\omega_{pn}^2 + \omega_{pp}^2\right)^{1/2}}{V_t}\left[1 + \frac{\left(\omega_{pn}^2 + \omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]\left[1 - \frac{\left(\omega_{pn}^2 + \omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]^{-1}$$
$$\times\left[-\frac{\cos(\theta)}{\theta} + \frac{2\sin(\theta)}{\theta^2} + \frac{2\cos(\theta)}{\theta^3} - \frac{2}{\theta^3}\right] \tag{4.42}$$

with

$$\theta = \left(\omega_{pn}^2 + \omega_{pp}^2\right)^{1/2}\xi/V_t \tag{4.43}$$

Using the condition $\theta \gg 1$, we obtain the required result for the wake potential [38].

$$\phi_W(\mathbf{r},t) = \left(\frac{2q_t}{\xi}\right)\left[1+\frac{\left(\omega_{pn}^2+\omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]$$
$$\times\left[1-\frac{\left(\omega_{pn}^2+\omega_{pp}^2\right)\lambda_{De}^2}{V_t^2}\right]^{-1}\cos\left[\frac{\left(\omega_{pn}^2+\omega_{pp}^2\right)\xi}{V_t}\right] \tag{4.44}$$

## 4.3 Summary

In this dissertation, we present the concept of shielding in plasma. In chapter 1, basic parameters of plasma are discussed. Some basic concepts of dusty plasma are presented. We also discuss in this chapter the history of shielding in plasma. In chapter 2, we have studied the shielding of test charge propagated through the plasma using the Vlasov-Poisson model. Our numerical calculation shows that electrostatic potential is Debye Hückel type when the test charge velocity ($V_p$) is greater than the dust thermal speed ($V_{td}$) and becomes Coulomb type when $V_p \gg V_{td}$. Next, in the chapter 3, we present the shielding in dusty plasma and the correlation effects of two test charge projectile case. These two projectiles are moving with the same velocity but with a time delay ($\tau$). Two cases are considered here (i) the two projectiles moving one behind the other along the same direction (ii) they are moving along the same direction making some angle with each other. We observed that when the separation distance between the two projectiles is smaller than the $\lambda_D$, the projectiles behave as a single projectile having charge equal to the sum of the charges of the two projectiles. When R is very large, the motion of the two projectiles is completely uncorrelated. In chapter 4, we have considered a multicomponent plasma consisting of Boltzmann distributed electrons, mobile positive and negative ions, and immobile positive/negative charged dust grains. By employing the Vlasov-Poisson system of equations containing a test charge, We calculate the electrostatic potential for the test charge moving with a constant speed along the z-axis. Further, the Debye screening and wake potentials are calculated in the presence of mobile negative and positive ions. It is important to note here that the DIA speed and oscillatory wake potential are significantly modified due to the presence of negative ions. The mobile negative ions could play a significant role in attracting the same polarity charges in the wake potential regions and in making regular crystalline structure.

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