Randall-Sundrum Model and Applications

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By

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Declaration

I hereby declare that the material contained in this thesis is my original work. I have not previously presented any part of this work elsewhere for any other degree.

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Certificate

It is certified that the work contained in this dissertation was carried out by **Azad Hussain** under my supervision.

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Dedicated

To

My Parents and My
Advisor, Dr. M. Jamil Aslam

Acknowledgment

Praise is to **Allah**, the Cherisher and Sustainer of the worlds, Who showed His countless blessings upon us. Almighty Allah enabled me to compile my humble endeavors in the shape of this thesis. All respect goes to the **Holy Prophet Hazrat Muhammad (PBUH)** whose teachings strengthened our faith in Allah, Who is the source of knowledge and wisdom for entire humanity, who clarified that pursuit of knowledge is a divine commandment.

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Azad Hussain

Abstract

The Randall Sundrum model (RS) was presented in 1999 to solve the Higgs hierarchy problem of the particle physics. This model created interest in the phenomenologist and theoreticians to work in the extra dimensions scenarios. In the present dissertation we will examine the rare decay $\Lambda_b \rightarrow \Lambda l^+ l^-$ in the context of SM and RS model with custodial protection. We compute the physical observables like, differential branching ratio, forward-backward asymmetries and polarization asymmetries of Λ baryon in SM and RS_c model. By using low energy effective Hamiltonian the hadronic matrix elements parametrized by form factors that are calculated by QCD sum rules. We compare the results of these observables obtained in the SM and RS_c models.

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Chapter 1

Introduction

Since from its evolution in 1960 the Standard Model (SM) of particle physics provides the opportunity to physicist to learn about the fundamental processes that are occurring in nature. The SM is extraordinarily precise in its predictions and steady to most experimental results. However, there are some deficiencies in the SM. One of them that is important is the hierarchy problem (small Higgs mass) which interrogates that why the weak force is 10^{32} times stronger than the gravity. The SM explains the weak, electromagnetic and strong forces successfully but fails to explain gravity, which is very unsatisfactory and indicates that there is a theory beyond the SM. So it is considered as an incomplete theory till now.

According to the SM neutrinos are massless but now the experiments have given the evidence for the neutrinos to have mass. These small neutrino's masses can not be incorporated in the mathematical framework of standard model. The non zero mass of the neutrinos is the direct experimental evidence for incompleteness of the SM.

There are also some indirect evidences for physics beyond the SM, such as dark matter. Most part of the universe is made of dark matter, but it is very difficult to detect it and it can be detected only by the gravitational effects. The SM also does not explain the nature of the dark matter. The particle contents of the SM has been completed after the Higgs boson's discovery in 2012 in the collider experiments. The characteristics of the particles of the SM were disclosed slowly. The top quark and Higgs boson masses are important among these characteristics because they are used to determine the response of Higgs quartic coupling.

According to recent experimental results, the Higgs boson mass was found to be 125.09 ± 0.21 GeV [1] and the top quark mass is 173.34 ± 0.76 GeV [2] .

To solve the problems that are mentioned above, many extensions in the SM have been proposed such as supersymmetry (SUSY) in which the bosonic and fermionic degree of freedom have been treated equally. The hierarchy problem and unification of three couplings can be solved by supersymmetry. Grand unified theory (GUT) is also the extension of the SM in which the gauge sector of the SM is extended. By this theory the tree gauge couplings are unified at a scale about 10^{16} GeV that is called GUT scale.

There are also many others different extensions of the SM like extra dimensions, string theory, effective theory etc. All these theories (beyond the SM) are used to fill up the deficits of the SM but there are no signs for these theories in the experiments till now.

In present dissertation, we will take the extension of SM in extra dimensions in the form of Randall Sundrum (RS) model [3] which has one compact extra dimension along the non factorisable anti-deSitter (AdS_5) metric. In this model there are two three-branes which act as the boundries for the warped extra dimensions. There is a five dimensional bulk (5D bulk) between these branes. An exponential hierarchy is generated in the energy scales by the background AdS_5 metric. As the Planck's scale is at one brane (UV brane) and other at second brane (IR brane). The plank's scale is much larger than the other scale i.e., $\Lambda_{UV} >> \Lambda_{IR}$.
Then, we wil study the Randall Sundrum model with custodial protection (RS_c) in which the gauge group $SU(2)_L \times U(1)_Y$ is enlarged to $SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$ by which the harmful contributions to T-parameter (peskin takeuchi parameter) can be cured and also the $Z\bar{b}_L b_L$ vertex can be protected from extra correction. Later, we will study the implications of RS_c model in flavor sector.

This dissertation is organized as follows. In chapter 2, of this dissertation we will discuss SM including mathematical framework, Higgs mechanism and the limitations of the SM. In chapter 3, we will discuss briefly the RS model, Custodial symmetry and the RS model with custodial protection. In chapter 4, we will study the the effective field theory (EFT) which is the theory that includes the suitable degrees of freedom to explain the physical phenomenon which is occurring at a particular (chosen) energy scale or length scale and other degrees of freedom are ignored that are out of this selected scale. We will also study about different approaches of EFT like top down and bottom up approach with the examples like SM as an effective theory and then we will give some introduction of the renormalization theory and the divergences. At the end of this chapter we will discuss the matching conditions with examples. In chapter 5, we will study the rare decay $\Lambda_b \to \Lambda l^+ l^-$ and find out the different observables like differential decay width, branching ratio, forward-backward asymmetries and polarization asymmetries for this decay in the SM and RS_c model by using the effective Hamiltonian and the parametrization of hadronic matrix elements in terms of form factors calculated by QCD sum rule and compare the results of both models . At the end of this chapter we will also describe the conclusion.

Chapter 2

The Standard Model (SM)

2.1 History

The first approach towards the SM was given by Glashow in 1961 to combine the weak and electromagnetic interactions at an energy scale. This prediction require that there should exist four vector bosons $W^{\pm}$, Z and γ which are acquired by the rotation of weak mixing angle θ_W. In addition the accurate structure of weak neutral current that is mediated by the Z boson was also acquired. The $W^{\pm}$ and Z bosons are taken as the mediators of weak forces. There is a serious problem with this model in the case of awarding masses to the $W^{\pm}$ and Z bosons because according to gauge symmetry's prediction they should have zero masses. In the interaction Lagrangian, the parameters M_W and M_Z for the vector bosons were put by hand. The gauge symmetry and normalizibility is spoiled by introducing the mass term in the Lagrangian for vector bosons. Another approach to build the SM was made by Nambu in the form of Goldstone theorem in 1960. According to this theorem there exist a particle with zero spin and zero mass due to which the spontaneous breaking of global symmetries take place. In 1964 P. Higgs, Englert and Brout Kibble and Guralnic studied the spontaneous breaking of local gauge symmetries, that required for the electroweak symmetry breaking.

The formulation of electroweak theory was done by Weinberg and A. Salam who include the gauge group $SU(2) \times U(1)$ initiated by Glashow. This theory is known as Glashow-Weinberg and Salam model or the Standard Model (SM) of particle physics, that was established by the assistance of gauge principle and intermediate vector boson theory. Actually, the SM is a gauge theory related to the electroweak interactions whose basis lies on $SU(2) \times U(1)$ gauge group and the intermediate vector bosons, $W^{\pm}$, γ and Z are the related four gauge bosons. The masses of gauge bosons $W^{\pm}$ and Z are created by the Higgs mechanism. The discovery of weak neutral currents in 1973 was the first proof of the SM to be accurate theory of electroweak (EW) interactions. The SM also predicted the masses for W and Z bosons that was confirmed experimentally in 1983. With the discovery of Higgs boson, we have now all the contents of the SM.

2.2 Particle Contents

There are many elementary particles that are present in the SM and can be distinguished from each other by some properties like color charge. The SM have following classes of particles. The elementary particles in the SM are divided into three classes as shown in the figure 2.1 . Now we explain these classes of particles one by one.

(i) Fermions

There are twelve spin half elementary particles that are known as fermions. These particles obey the Pauli exclusion principle. For every fermion there is a corresponding anti-fermion.
The fermions are divided into two classes based on their interactions that are quarks and leptons. There are six quarks and six leptons. The property that define the quarks is color charge of quarks, so they interact with each other by the strong force. The quarks form the color-neutral particles known as hadrons by color confinement process. Hadrons are further classified into mesons and baryons. Mesons are formed by the combination of a quark and an anti-quark while the baryons are formed by the combination of three quarks.
The other six fermions do not have any color charge and are known as leptons. Neutrinos do not have electric charge, so it is very difficult to detect them. While the electron, muon and tau carry electric charge so they interact electromagnetically. There are three generations of quarks

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \tag{2.1}$$

and also three generations for leptons.

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L \tag{2.2}$$

All the ordinary matter is formed by the first generation of charged particles because these particles do not decay. While the particles of second and third generations have very short lifetime and only observed at high energy scale.

(ii) Gauge bosons

Gauge bosons are used as mediators of fundamental interactions in the SM. In physics, the particles effect the other particles with the help of interactions. The SM describes that these forces results from matter particles by exchanging other particles. All the gauge bosons of the SM have spin 1(integer spin), so these are bosons and do not obey the Pauli exclusion principle. The different types of gauge bosons are explained below

- The force between the charged particles is the electromagnetic force that is mediated by photon. The photon is massless particle and have spin 1.
- The weak interactions are mediated by W^+, W^- and Z gauge bosons. These gauge bosons are massive, W^+, W^- have relatively less mass than Z. The weak forces mediated by $W^\pm$ act upon the left handed particles and the right handed anti-particles. The Z boson which is electrically neutral and interact with the both the left-handed particles and right handed anti-particles.
- The strong interactions (between the quarks) are mediated by the eight massless gluons. Gluons can also interact with each other due to their effectual color charge. The interactions between all the standard model particles are summarized in the figure 2.2.

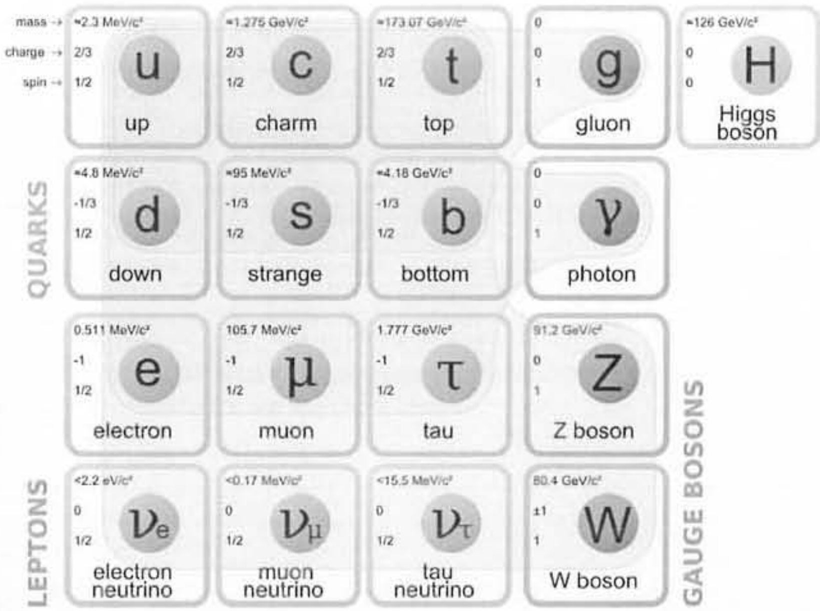

Figure 2.1: Classification of elementary particles of the SM

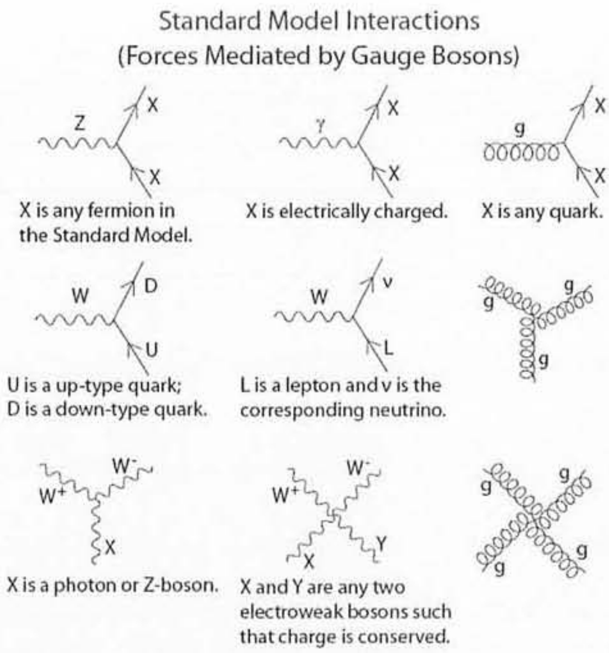

Figure 2.2: Fundamental interactions between elementary particles of the SM.

(iii) The Higgs boson
The Higgs particle is a single particle in the SM which has spin 0 and it has large mass compared to other standard particles. Due to spin zero Higgs is classified as boson. The Higgs boson explains that how the other massive particles get their masses excluding the gluons and photon. Furthermore it also gives the explanation for photon to be massless. The elementary particles get their masses by interacting with Higgs particle. Because of large mass, Higgs also interact with itself. The Higgs particle can be observed and recorded in a particle accelerator with very high energy because of its large mass and immediate decay.

For the confirmation of Higgs boson experiments were started in LHC in 2010 at CERN. The Higgs boson confirmed experimentally on 04 July 2012 by the two major experiments at LHC (CMS and ATLAS). These experiments reported that the mass of Higgs boson is about 125 GeV/c^2.

2.3 Mathematical framework of SM

We have divided the mathematical framework of the SM into two sectors, the quantum chromodynamics sector and the electroweak sector. Now we will explain both sectors below.

2.3.1 The Quantum chromodynamics sector of SM

The gauge group of the standard model is $SU(3)_C \times SU(2)_L \times U(1)_Y$,Where C represents color, L represents left-handed and Y denotes hyper charge. The QCD sector of the SM explain the strong interactions . The QCD is based upon the gauge group $SU(3)_C$ and is a non Abelian gauge theory. Under $SU(3)_C$ quarks transform as triplet and belong to fundamental representation, while the gluons which mediate the strong interactions belong to the adjoint representation of $SU(3)_C$.
The Lagrangian for the QCD sector is [4].

$$L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{\Psi}_i (i\gamma^\mu D_\mu - m) \Psi_i \ , \qquad (2.3)$$

where

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G_{b\mu} G_{c\nu} \ , \qquad (2.4)$$

is the field strength tensor, and

$$D_\mu = \partial_\mu \delta - i g_s T_a G^a_\mu \ , \qquad (2.5)$$

is the covariant derivative. g_s is the coupling constant of strong interactions and a=1,2,3....8 runs over the color. T_a are considered as generators of the gauge group and satisfy the following relation.

$$[T_a, T_b] = if_{abc}T_c ,$$

Where f_{abc} represents the structure constant of the group. For $SU(3)_C$ the relation of T_a generators with the 3×3 Gell-Mann matrices [5] is given below

$$T_a = \frac{\lambda_a}{2} .$$

The corresponding Lagrangian does not change under $SU(3)_C$ infinitesimal local gauge transformations.

2.3.2 The Electroweak sector of SM

The electroweak sector of the SM include the weak and electromagnetic forces [6]. The gauge fields are the mediators of these forces. This concept can be enlarged to massive gauge fields by launching the Higgs mechanism that gives masses to the particles keeping gauge symmetries invariant. We will describe here some theoretical aspects of electroweak sector of the SM

(I) Gauge sector

Gauge theories remain invariant under the global gauge transformations i.e.,

$$\Psi \longrightarrow U\Psi ,$$

where U is a unitary matrix for non-Abelian gauge transformation that will act upon the fermion field Ψ.
To make the gauge theory invariant under local gauge transformation (which involve the dependence of space time coordinate x). We replace the space time derivative ∂_μ by the covariant derivative D_μ in which an additional vector field V_μ is included

$$i\partial_\mu \longrightarrow iD_\mu - gV_\mu ,$$

where g is the universal gauge coupling constant. The gauge field V_μ transform by a rotation plus a shift under the local gauge transformations as given below.

$$V_\mu \longrightarrow UV_\mu U^{-1} + ig^{-1}[\partial_\mu U]U^{-1} .$$

In comparison with this, the curl of V_μ represented by F.

$$F_{\mu\nu} = -ig^{-1}[D_\mu, D_\nu] ,$$

only rotates under the gauge transformation. The Lagrangian for spin half particles and for vector fields of massless paticles is written below:

$$L[\Psi, V] = \bar{\Psi} i\gamma^\mu D_\mu \Psi - \frac{1}{2} TrF^2 .$$

This include the following interactions.
Fermion-gauge bosons:

$$-g\bar{\Psi}V\Psi .$$

Three boson coupling:

$$igTr(\partial_\nu V_\mu - \partial_\mu V_\nu)[V_\mu, V_\nu] .$$

Four boson coupling:

$$\frac{1}{2} g^2 Tr[V_\mu, V_\nu]^2 .$$

(II) **The Higgs mechanism** :

In gauge invariant theory to generate the masses for the vector bosns the spontaneous symmetry breaking is extended to a mechanism that is known as Higgs mechanism. The gauge invariance of the SM gauge group $SU(2) \times U(1)$ requires that the masses of gauge bosons should be zero but the Lagrangian has the mass term that spoils the gauge invariance. The Higgs mechanism avoids this restriction by initiating with a theory that is gauge invariant and has the massless gauge bosons. The vector bosons $W^\pm$ and Z^0 attains their masses from the spontaneous symmetry breaking of local gauge symmetry by

$$SU(2) \times U(1) \longrightarrow U(1)_{em} .$$

that is achieved by introducing a complex scalar field that is self interacting and is denoted by Φ, and this field transform as $SU(2)$ doublet. Four independent fields are incorporated by the field Φ and its complex conjugate $\Phi^\dagger$.
The spontaneous symmetry breaking is applicable if one of the four fundamental fields in the Lagrangian have non zero vacuum expectation value.

$$\langle \Phi \rangle = \langle 0|\Phi|0 \rangle = \frac{v}{2} \neq 0 ,$$

before the spontaneous symmetry breaking, the three fields related to $W^\pm$ and Z^0 become the longitudinal degree of freedom while the photon combined with the symmetry group $U(1)_{em}$ and remain massless.

2.3.3 Formulation of EW sector of the SM

(1) The matter sector

The left handed fermions with isospin doublets and right handed fermions with isospin singlet appear in the fundamental representation of the gauge group $SU(2) \times U(1)$. It is perceived that the first, second and third generation of fermions have the same symmetry pattern:

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L \begin{bmatrix} \nu_{eR} \\ e_R^- \end{bmatrix}; \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}_L \begin{bmatrix} \nu_{\mu R} \\ \mu_R^- \end{bmatrix}; \begin{bmatrix} \tau^- \\ \nu_\tau \end{bmatrix}_L \begin{bmatrix} \nu_{\tau R} \\ \tau_R^- \end{bmatrix}$$

We can not derive the symmetry structure in the SM. It is confirmed experimentally that the parity conservation is violated in weak interactions. The violation of parity in weak interactions is due to the different isospin of the left handed and right handed fermion fields. Hence the experimental observation is included in the natural way.
The relation between the basic quantum numbers and the electric charge Q is reported by Gell-Mann-Nishijima relation.

$$Q = I_3 + \frac{Y}{2} .$$

(2) Interactions

The Lagrangian of electroweak sector of the SM is given below:

$$L = L_G + L_F + L_H . \tag{2.6}$$

These three terms of the Lagrangian represents the fundamental interactions of the SM. These terms are described one by one as follows.

(i) Gauge fields

The non-Abelian gauge group $SU(2) \times U(1)$ is generated by the isospin operators (I_1, I_2, I_3) and hypercharge Y. The iso singlet B_μ and iso triplet W_μ^a, where a=1, 2, 3 give the following field strength tensors.

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a + \partial_\nu W_\mu^a + g_2 \epsilon_{abc} W_\mu^b W_\nu^c , \tag{2.7}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu , \tag{2.8}$$

where g_2 is known as non-Abelian $SU(2)$ gauge group coupling constant. Hence the Lagrangian for gauge fields has the following form

$$L_G = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} . \tag{2.9}$$

This Lagrangian remains invariant under the $SU(2)\times U(1)$ non-Abelian gauge transformations.

(ii) Fermion fields and fermion – gauge interactions

Each lepton family has the following left handed fermion fields,

$$\Psi_j^L = \begin{pmatrix} \Psi_{j+}^L \\ \Psi_{j-}^L \end{pmatrix} ,$$

which are grouped into $SU(2)$ doublet with family index j with component index $\sigma = \pm$, while the right handed fields are grouped into singlet

$$\Psi_j^R = \Psi_{j\sigma}^R .$$

If the relatins given in Eq. (2.7) and Eq. (2.8) are fulfilled then each right and left handed multiplet are the eigen state of Y (weak hyper charge). The fermion gauge field interactions are included in the covariant derivative that is given below:

$$D_\mu = \partial_\mu - ig_2 I_a W_\mu^a + ig_1 \frac{Y}{2} B_\mu . \tag{2.10}$$

The fermion gauge field interactions are given by the following Lagrangian

$$L_F = \sum_j \bar{\Psi}_j^L i\gamma^\mu D_\mu \Psi_j^L + \sum_{j,\sigma} \bar{\Psi}_{j\sigma}^R i\gamma^\mu D_\mu \Psi_{j\sigma}^R , \tag{2.11}$$

where g_1 defines the coupling constant of Abelian gauge group $U(1)$.

(iii) Higgs field and the Higgs interactions

For the spontaneous symmetry breaking of the gauge group $SU(2)\times U(1)$ to the group $U(1)_{em}$ that remains unbroken , the gauge fields are coupled to a single complex scalar field that is doublet.

$$\Phi(x) = \begin{pmatrix} \Phi^+(x) \\ \Phi^-(x) \end{pmatrix} , \tag{2.12}$$

with hypercharge $Y = 1$ through

$$L_H = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) , \tag{2.13}$$

where the covariant derivative is defined as

$$D_\mu = \partial_\mu - ig_2 I_a W_\mu^a + ig_1 \frac{B_\mu}{2} .$$

The self interactions of Higgs field

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 \tag{2.14}$$

are constructed in such a manner that Φ has non zero vacuum expectation value, i.e. ,

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

where

$$v = \frac{2\mu}{\sqrt{\lambda}} \tag{2.15}$$

We can write the field $\Phi(x)$ that is given in Eq. (2.12) as follow

$$\Phi(x) = \begin{pmatrix} \Phi^+(x) \\ \frac{(v+H(x)+i\chi(x))}{\sqrt{2}} \end{pmatrix} , \tag{2.16}$$

The vacuum expectation values of the Φ^+, H, χ components of field are zero.

The invariance of Lagrangian implies that Φ^+, χ components can be gauged away which makes them unphysical (Higgs sector or would be Goldstone bosons). For such particular gauge the form of Higgs field is given below

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} .$$

The physical Higgs is defined by the real $H(x)$ that explain the small vibrations around the ground state.
The fermions get their masses by interacting with Higgs through Yukawa coupling that are given below

$$L_{Yukawa} = g_l(\nu_L\Phi^+ l_R + \bar{l}_R\Phi^-\nu_L + \bar{l}_L\Phi^0 l_R + \bar{l}_R\Phi^{0*} l_L) . \tag{2.17}$$

The mass term of fermion follows from the v part of Φ^0 [6]. The physics laws related to the weak and electromagnetic interactions between the leptons are summarized in the Lagrangian L. It also gives the prediction about the self interaction form of gauge fields. Furthermore, the fundamental particles like gauge bosons, fermions and Higgs itself get their masses through Higgs mechanism [7].

2.3.4 Masses and mass eigen states of particles

To get mass from Higgs boson, we replace

$$\left[\Phi \longrightarrow 0, \frac{v}{\sqrt{2}}\right]$$

in the Lagrangian of Higgs boson given in Eq. (2.13). By doing this it appears that the symmetry SU(2) is lost but this is only apparent and exist in hidden form. The local gauge symmetry $U(1)_{em}$ is preserved in the resulting Lagrangian [7].

$$SU(2) \times U(1) \longrightarrow U(1)_{em}$$

(i) Mass of gauge bosons

In the basis $(B, \overrightarrow{W})$, the mass matrix for gauge bosons is

$$M_V^2 = \frac{1}{4}v^2 \begin{pmatrix} g_W^2 & & & \\ & g_W^2 & & \\ & & g_W^2 & g_W g_W' \\ & & g_W g_W' & g_W'^2 \end{pmatrix} .$$

In the non diagonal form this matrix gives mass to the vector bosons. The charged weak gauge bosons receive the mass by the following equation

$$M_{W^\pm}^2 = \frac{1}{4} g_W^2 v^2 . \tag{2.18}$$

The boson state $W^\pm$ can also be defined as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}[W_\mu^1 \pm W_\mu^2] . \tag{2.19}$$

The mass term for the neutral gauge bosons gives the following matrix

$$M_{V_N}^2 = \frac{1}{4} \begin{pmatrix} g_W^2 & g_W g_W' \\ g_W g_W' & g_W'^2 \end{pmatrix} . \tag{2.20}$$

As the determinant of above matrix is zero, so one eigen value of $M_{V_N}^2$ is obviously zero. The diagonalization of the above matrix by the following definition of fields Z_μ, A_μ gives

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 , \tag{2.21}$$

and

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3 . \tag{2.22}$$

The above Eq. in matrix form can also be written as

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} , \qquad (2.23)$$

Then we got

$$M_A^2 = 0 .$$

This implies that A_μ is representing the photon (massless particle).

$$M_Z^2 = \frac{1}{4}(g_W^2 + g_W'^2)\upsilon^2$$

$$M_Z^2 = \frac{1}{4}g_W^2\upsilon^2(\frac{1}{\cos\theta_W^2}) . \qquad (2.24)$$

Here

$$\tan\theta_W = \frac{g_W'}{g_W} \qquad (2.25)$$

i.e., the ratio of couplings of $SU(2)$ and $U(1)$ define the mixing angle θ_W.

By introducing a parameter

$$\rho = \frac{M_W^2}{M_Z^2\cos\theta_W^2}$$

and using the expression for M_Z^2, we obtain

$$\rho = 1 .$$

This shows that Higgs field is doublet under $SU(2)_L$ [8].

The mixing angle appear to have large value experimentally i.e., $\sin^2\theta_W \simeq 0.23$. This value is far away from limits 0 or 1, which shows that mixing effect is large. This result explains that the weak and electromagnetic interactions are actually the demonstration of unified electroweak interactions. From here we can also conclude that the weak and electromagnetic interactions are unified truly in the electroweak sector of the SM. The relation between the value of ground state of Higgs field and Fermi coupling constant is

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} .$$

In the β decay and by combining with the following mass relation

$$M_W^{\pm} = \frac{1}{4} g_W^2 \upsilon^2$$

we can derive the value of υ

$$\upsilon = \sqrt{\frac{1}{\sqrt{2} G_F}}$$

$$\upsilon \simeq 246 \; GeV$$

(ii) The masses of fermions

The fermions i.e., leptons attain their mass by interacting with ground state of Higgs field through Yukawa interactions.

$$M_f = g_f \frac{\upsilon}{\sqrt{2}} ,$$

where g_f is the coupling constant for Yukawa interactions and υ is the ground state (vacuum expectation) value of Higgs.

(iii) The mass of Higgs boson

The real field $H(x)$ that illustrates the small vibrations around the ground state of Higgs inform us about the physical spin 0 neutral particle having mass

$$M_H = \frac{\mu}{\sqrt{2}} = \sqrt{\lambda} \upsilon$$

where λ is the coupling constant. This session is conclude by the remarks given below

1) The presence of weak neutral current having the same effectual coupling constant as that of charged give explicit prediction of the unification of electromagnetic and weak interactions. This current is confirmed experimentally.

2) There is only one free parameter in the theory that is $\sin^2 \theta_W$.

2.4 Limitations of the Standard Model

The SM is a successful model as it well match with the experimental data obtained from particle accelerators. The evidences for the physics beyond the SM are very small from the experimental point of view but the SM has many theoretical deficits that should not be appear in a basic theory. Some deficits are as follow:

- As SM has only left handed neutrinos and not right handed, so the Higgs mechanism can not give mass to neutrinos. So generally mass for neutrino is a problem in the SM.

- Asymmetry between matter -antimatter can not be explained by the SM.

- There is no quantum interpretation of gravity, we have no way that the SM include general relativity in terms of QFT.

- Also the gauge hierarchy problem is not explained by the SM i.e., the large energy gap between the Planck scale related to gravity and electroweak scale at which the electromagnetic and weak forces combine.

- Without some restrictions the SM can not be an applicant for the unification of all forces at a certain energy if such a theory exist.

So the above points shows that the SM is a low energy demonstration of a theory which is more fundamental. There are some theories which are proposed for physics beyond the SM like grand unification theories (GUTS), technicolor, supersymmetry (SUSY) and extra dimensions to name the few. In present dissertation we will focus on the extra dimension framework specially Randall Sundrum model (RS), that is explained in detail in the next chapter.

Chapter 3

The Randall Sundrum model

The hierarchy between the Planck scale and weak scale receive the new explanation in the model given in [10, 11] by Lisa Randall and Raman Sundrum in 1999, which is known as Randall Sundrum (RS) mdel.

3.1 Setup

This model considers the presence of an extra dimension that is compactified on a circle. The above and below halves of the circle are identified in figure 3.1. given below.

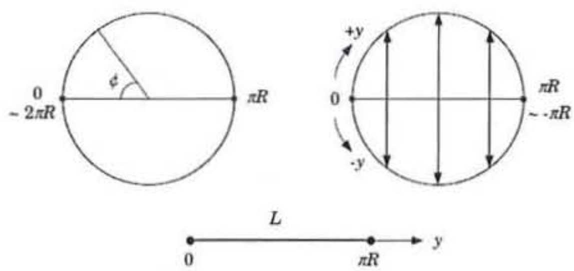

Figure 3.1: S^1/Z_2 orbifold

This means that work is done in S^1/Z_2 orbifold. Where Z_2 is multiplicative group $\{-1, 1\}$ and S^1 is representing the sphere having one dimension (i.e., circle). This construction require two points which are fixed, one at $y = 0$ (origin) and other at $y = \pi R = L$. There is four dimensional world on each of these points. These worlds having (3+1) dimensions bounding the five dimensional (5D) bulk are known as 3-branes. These two 3-branes are separated from each other by a distance L as shown in Fig. 3.2

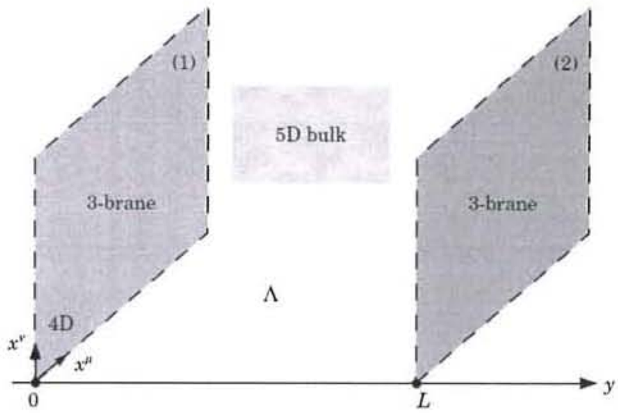

Figure 3.2: Randall Sundrum setup

3.2 The Einstein Hilbert Action:

The Einstein Hilbert action in general theory of relativity give the Einstein field equation.

$$S = \frac{1}{2\kappa^2} \int R\sqrt{-g}d^4x \tag{3.1}$$

$$\kappa^2 = 8\pi G c^{-4},$$

where G is gravitational constant and c is speed of light and R is Ricci scalar.

3.3 The warped metric

First of all we search for a metric that corresponds to the above setup. As we are searching for that solutions of Einstein equations which may suitable for the real world.

If we derive 4D universe from this theory then it should be flat and static. This gives following Ansatz

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \ , \tag{3.2}$$

where $\eta_{\mu\nu}$ =diag(-1,1,1,1) is the 4D Minkowski metric and the factor $e^{-2A(y)}$ is known as warp factor. The above metric is non-factorizable because it depends upon the y which is extra dimension coordinate. This implies that, it can not be written as the multiple of a manifold extra dimension and Minkowski metric. To determine $A(y)$ we have to determine 5D Einstein field equations.

3.4 Derivation Of Einstein Field Equations

Let's start with the action

$$S = S_H + S_M \ ,$$

where S_H and S_M are representing Eisenstein Hilbert action and matter part of action respectively.

$$S = \int d^4X \int_{-L}^{+L} dy\sqrt{-g}(\frac{1}{2\kappa^2}R + L_M) \ . \tag{3.3}$$

The action principle give us information that variation of above action with respect to inverse metric is giving zero, i.e.,

$$\delta S = 0 \ ,$$

$$0 = \int [\frac{1}{2\kappa^2}\frac{\delta(\sqrt{-g}R)}{\delta g^{\mu\nu}} + \frac{\delta(\sqrt{-g}L_M}{\delta g^{\mu\nu}}]\delta g^{\mu\nu} d^4X \ ,$$

$$0 = \int [\frac{1}{2\kappa^2}(\frac{\sqrt{-g}\delta R}{\delta g^{\mu\nu}} + \frac{R\delta\sqrt{-g}}{\delta g^{\mu\nu}}) + \frac{\delta(\sqrt{-g}L_M)}{\delta g^{\mu\nu}}]\delta g^{\mu\nu} d^4X \ ,$$

$$0 = \int [\frac{1}{2\kappa^2}(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R\delta(\sqrt{-g})}{\sqrt{-g}\delta g^{\mu\nu}}) + \frac{\delta(\sqrt{-g}L_M)}{\sqrt{-g}\delta g^{\mu\nu}}]\delta g^{\mu\nu}\sqrt{-g} d^4X \ ,$$

Above equation is true for every $g^{\mu\nu}$. So

$$\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R\delta\sqrt{-g}}{\sqrt{-g}\delta g^{\mu\nu}} = \frac{-2\kappa^2\delta(\sqrt{-g}L_M)}{\sqrt{-g}\delta g^{\mu\nu}} \tag{3.4}$$

The above Eq. (3.4) is the equation of motion for the metric field. Its right hand side is directly proportional to Stress-energy tensor i.e.,

$$T_{\mu\nu} = \frac{-2\delta(\sqrt{-g}L_M)}{\sqrt{-g}\delta g^{\mu\nu}} = \frac{-2\delta(\sqrt{-g}S_M)}{\delta g^{\mu\nu}} ,$$

$$T_{\mu\nu} = \frac{-2\sqrt{-g}\delta(L_M)}{\sqrt{-g}\delta g^{\mu\nu}} + g_{\mu\nu}L_M ,$$

$$T_{\mu\nu} = \frac{-2\delta L_M}{\delta g^{\mu\nu}} + g_{\mu\nu}L_M .$$

3.4.1 Variation of Riemann tensor, the Ricci tensor and Ricci scalar

We define Riemmann tensor as follow

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu\Gamma^{\rho}_{\nu\sigma} - \partial_\nu\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} .$$

As Riemann Carvature tensor is the function of Levi-Civita ($\Gamma^{\lambda}_{\mu\nu}$), so its variations can be determined as

$$\delta R^{\rho}_{\sigma\mu\nu} = \partial\mu\delta\Gamma^{\rho}_{\nu\sigma} - \partial_\nu\delta\Gamma^{\rho}_{\mu\nu} + \delta\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} + \Gamma^{\rho}_{\mu\lambda}\delta\Gamma^{\lambda}_{\nu\sigma} - \delta\Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\rho}_{\nu\lambda}\delta\Gamma^{\lambda}_{\mu\sigma} .$$

The variation in Christoffel symbols ($\delta\Gamma^{\rho}_{\nu\mu}$) can be obtained by the difference of two connections, and this is the tensor, so its covariant derivative can be determined as

$$\nabla_\lambda(\delta\Gamma^{\rho}_{\nu\mu}) = \partial_\lambda(\delta\Gamma^{\rho}_{\nu\mu}) + \Gamma^{\rho}_{\sigma\lambda}\delta\Gamma^{\sigma}_{\nu\mu} - \Gamma^{\sigma}_{\nu\lambda}\delta\Gamma^{\lambda}_{\mu\sigma} - \Gamma^{\rho}_{\nu\lambda}\delta\Gamma^{\lambda}_{\mu\sigma} .$$

Correspondingly the variation in Riemann curvature tensor can be expressed as below

$$\delta R^{\rho}_{\sigma\mu\nu} = \nabla_\mu(\delta\Gamma^{\rho}_{\mu\sigma}) - \nabla_\nu(\delta\Gamma^{\rho}_{\mu\sigma}) . \tag{3.5}$$

Defining the Ricci Scalar as follow

$$R = g^{\mu\nu}R_{\mu\nu} ,$$

The variation in Ricci Scalar by the inverse metric is given as

$$\delta R = R_{\mu\nu}\delta g^{\mu\nu} + g^{\mu\nu}\delta R_{\mu\nu} , \tag{3.6}$$

$$\delta R^{\rho}_{\sigma\mu\nu} = \nabla_\mu(\delta\Gamma^{\rho}_{\nu\sigma}) - \nabla_\nu(\delta\Gamma^{\rho}_{\mu\sigma}) .$$

By contraction of indices we can simplify the change in curvature tensor, as follows

$$\delta R_{\mu\nu} = \delta R^{\rho}_{\mu\rho\nu} = \nabla_{\rho}(\delta\Gamma^{\rho}_{\nu\mu}) - \nabla_{\nu}(\delta\Gamma^{\rho}_{\rho\mu}) \ ,$$

$$\delta R = R_{\mu\nu}\delta g^{\mu\nu} + \nabla_{\sigma}(g^{\mu\nu}\delta\Gamma^{\sigma}_{\nu\mu} - g^{\mu\sigma}\delta\Gamma^{\rho}_{\rho\mu}) \ .$$

As

$$\Delta_{\sigma} g^{\mu\nu} = 0 \tag{3.7}$$

and by multiplying above equation by$\sqrt{-g}$, we get total derivative as

$$\sqrt{-g}\nabla_{\mu}A^{\mu} = \partial_{\mu}(\sqrt{-g}A^{\mu}) \tag{3.8}$$

Integrating and using Strokes Theorem will give us the boundary term only. The change in metric $\delta g^{\mu\nu}$ disappear at infinity. therefore, this term does not take part in the variation of action. This gives.

$$\delta R = R_{\mu\nu}\delta g^{\mu\nu} \ ,$$

and so

$$\frac{\delta R}{\delta g^{\mu\nu}} = R_{\mu\nu} \ . \tag{3.9}$$

3.4.2 Variation of the determinant

$$\delta g = \delta det(g_{\mu\nu}) = g g^{\mu\nu}\delta g_{\mu\nu}$$

$$\delta\sqrt{-g} = \frac{-\delta g}{2\sqrt{-g}} = \frac{\sqrt{-g}(g^{\mu\nu}\delta g_{\mu\nu})}{2} = \frac{-\sqrt{-g}(g_{\mu\nu}\delta g^{\mu\nu})}{2} \tag{3.10}$$

In above equations we used

$$g_{\mu\nu}\delta g^{\mu\nu} = -g^{\mu\nu}\delta g^{\mu\nu} \ .$$

Using the rules of differentiation, the inverse of the metric is

$$\delta g^{\mu\nu} = -g^{\mu\alpha}(\delta g_{\alpha\beta})g^{\beta\nu} \ ,$$

$$\frac{\delta\sqrt{-g}}{\sqrt{-g}\delta g^{\mu\nu}} = \frac{-g_{\mu\nu}}{2} \ . \tag{3.11}$$

The eqution of motion is

$$R_{\mu\nu} - \frac{g_{\mu\nu}R}{2} = \frac{8\pi G T_{\mu\nu}}{c^4}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$$

$$G_{MN} = R_{MN} - \frac{1}{2}g_{MN}R = \kappa^2 T_{MN} \ , \tag{3.12}$$

where the M and N have values 0, 1, 2, 3 and 5 and 5D Newton constant is defined as

$$\kappa^2 = \frac{1}{2M^3} \ .$$

The energy momentum tensor is defined as

$$T_{MN} = \frac{-2\delta S_M}{\sqrt{-g}\delta g^{MN}}$$

so in the action the term which is like $\sqrt{-g}V$ where V is constant related to an energy momentum tensor equal to $V g_{MN}$.
The Einstein tensor corresponds to the metric whose parametrization is done in the Eq. (3.2) and has been done in next section.

3.5 Einstein tensor

We want to determine the Einstein tensor for the following metric,

$$\begin{aligned} ds^2 = &\ e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \\ ds^2 = &\ \qquad g_{MN}(y)dx^\mu dx^\nu \end{aligned}$$

where

$$g_{MN}(y) \ = \ e^{-2A(y)}\eta_{\mu\nu} + \delta^5_M \delta^5_N \tag{3.13}$$

Its inverse is

$$g^{MN}(y) \ = \ e^{2A(y)}\eta_{\mu\nu} + \delta^M_5 \delta^N_5 \tag{3.14}$$

3.5.1 Christoffel Symbol

$$\Gamma^P_{MN} = \frac{1}{2}g^{PR}(\partial_M g_{NR} + \partial_N g_{RM} - \partial_R g_{MN}) \tag{3.15}$$

g_{MN} only depends on extra dimension.

$$\partial_L g_{MN} = \partial_5 g_{MN} = \partial_5 g_{\mu\nu} \tag{3.16}$$

so non vanishing Chrostoffel symbols are of two types only.
Take $M = \mu, N = \nu, P = 5$. So Eq. (3.15) can be written as follow

$$\Gamma^5_{\mu\nu} = \tfrac{1}{2}g^{5R}(-\partial_R g_{\mu\nu}) \ ,$$

Take $R = 5$, one gets

$$\Gamma^5_{\mu\nu} = \tfrac{1}{2}g^{55}(-\partial_5 g_{\mu\nu}) \ , \tag{3.17}$$

then

$$\begin{aligned} \partial_5 g_{\mu\nu} &= \partial_5(e^{-2A(y)}\eta_{\mu\nu}) \\ &= -2e^{-2A(y)}\partial_5(A(y))\eta_{\mu\nu} \ . \end{aligned}$$

putting it in Eq. (3.17), we get

$$\Gamma^5_{\mu\nu} = A'(y)e^{-2A(y)}\eta_{\mu\nu} \tag{3.18}$$

using $P = \nu, M = \mu, N = 5$, the Christoffel symbols becomes

$$\Gamma^\nu_{\mu 5} = \tfrac{1}{2}g^{\nu R}(\partial_5 g_{R5}) \ ,$$

Now take $R = \rho$

$$\Gamma^\nu_{\mu 5} = \tfrac{1}{2}(e^{2A}\eta^{\nu\rho})(\partial_5 g_{\rho\mu}) \ . \tag{3.19}$$

But

$$g_{\rho\mu} = e^{-2A(y)}\eta_{\rho\mu}$$

Therefore

$$\partial_5(g_{\rho\mu}) = \partial_5(e^{-2A(y)}\eta_{\rho\mu}) = -2A'(y)e^{-2A(y)}\eta_{\rho\mu}$$

and inserting it in Eq. (3.19), we get

$$\begin{aligned} \Gamma^\nu_{\mu 5} &= -A'\eta^{\nu\rho}\eta_{\rho\mu} \ , \\ \Gamma^\nu_{\mu 5} &= -A'\delta^\nu_\mu \ . \end{aligned} \tag{3.20}$$

3.5.2 Ricci Tensor

The Ricci tensor in $5d.$ can be expressed as

$$R_{MN} = \partial_P \Gamma^P_{MN} - \partial_N \Gamma^P_{MP} + \Gamma^P_{PQ}\Gamma^Q_{MN} - \Gamma^P_{NQ}\Gamma^Q_{MP}$$

Take $R = \sigma, Q = 5, P = 5, Q = \sigma, M = \mu, N = \nu$, we get

$$R_{\mu\nu} = \partial_5 \Gamma^5_{\mu\nu} - \partial_5 \Gamma^\sigma_{\mu\sigma} + \Gamma^\sigma_{\sigma 5}\Gamma^5_{\mu\nu} - \Gamma^5_{\nu\sigma}\Gamma^\sigma_{\mu 5} \tag{3.21}$$

Before proceeding, one has to keep in mind that

$$\partial_5 \Gamma^\sigma_{\mu\sigma} = 0$$

Now use the values of Christoffel symbols calculated previously

$$\begin{aligned} R_{\mu\nu} &= \partial_5(A'e^{-2A}\eta_{\mu\nu}) + (-A'\delta^\sigma_\sigma)(A'e^{-2A}\eta_{\mu\nu}) - (A'e^{-2A}\eta_{\nu\sigma})(-A'\delta^\sigma_\mu) \\ R_{\mu\nu} &= (A'' - 2A'^2)e^{-2A}\eta_{\mu\nu} - A'^2 e^{-2A}\eta_{\mu\nu} - (A'e^{-2A}\eta_{\nu\sigma})(-A'\delta^\sigma_\mu) \\ &= (A'' - 2A'^2)e^{-2A}\eta_{\mu\nu} - A'^2 e^{-2A}\eta_{\mu\nu} + A'^2 e^{-2A}\eta_{\nu\mu} \end{aligned}$$

As

$$\eta_{\mu\nu} = -\eta_{\nu\mu}$$

Therefore,

$$\begin{aligned} R_{\mu\nu} &= (A'' - 2A'^2)e^{-2A}\eta_{\mu\nu} - A'^2 e^{-2A}\eta_{\mu\nu} - A'^2 e^{-2A}\eta_{\mu\nu} \\ R_{\mu\nu} &= (A'' - 4A'^2)e^{-2A}\eta_{\mu\nu} . \end{aligned}$$

We have

$$g_{\mu\nu} = e^{-2A}\eta_{\mu\nu}$$

By using above expression, we get

$$\begin{aligned} R_{\mu\nu} &= (A'' - 4A'^2)g_{\mu\nu} \\ R_{\mu 5} &= 0 \\ \Rightarrow g_{\mu 5} &= 0 \end{aligned} \tag{3.22}$$

$$\begin{aligned} R_{55} &= -\partial_5 \Gamma^\sigma_{5\sigma} - \Gamma^\sigma_{5\rho}\Gamma^\rho_{5\sigma} \\ &= 4A'' - 4A'^2 \end{aligned}$$

3.5.3 Ricci scalar

With Ricci tensor in hand, we can define the Ricci scalar in $5d$ as

$$\begin{aligned} R &= g^{MN}R_{MN} \\ R &= g^{\mu\nu}R_{\mu\nu} + g^{55}R_{55} \ . \end{aligned} \tag{3.23}$$

Using the expression for $R_{\mu\nu}$ and R_{55}

$$\begin{aligned} R &= g^{\mu\nu}(A'' - 4A'^2)g_{\mu\nu} + g^{55}(4A'' - 4A'^2) \\ R &= g^{\mu\nu}g_{\mu\nu}(A'' - 4A'^2) + g^{55}(4A'' - 4'^2) \ . \end{aligned} \tag{3.24}$$

As $g^{\mu\nu}g_{\mu\nu} = 4$ and $g^{55} = 1$, therefore,

$$\begin{aligned} R &= 4(A'' - 4A'^2) + 4A'' - 4A'^2 \\ &= 8A'' - 20A'^2 \ . \end{aligned} \tag{3.25}$$

3.5.4 Derivation of Einstein tensor

We know that

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{3.26}$$

using the values of $R_{\mu\nu}$ from Eq. (3.22) and of R from Eq. (3.25) in above expression, we get

$$G_{\mu\nu} = (6A'^2 - 3A'')g_{\mu\nu} \ ,$$

and

$$G_{55} = R_{55} - \frac{1}{2}g_{55}R \ .$$

Using values R and R_{55} from Eq. (3.25) and Eq. (3.22) respectively in above expression, we get

$$G_{55} = 6A'^2 \tag{3.27}$$

As

$$G_{MN} = \ R_{MN} - \tfrac{1}{2}g_{MN}R = \kappa^2 T_{MN}$$

therefore

$$\Rightarrow G_{55} = \ \kappa^2 T_{55} = \tfrac{1}{2M^3}(T_{55}) \ , \tag{3.28}$$

and

$$T_{55} = \ -\Lambda \ .$$

So we can write

$$G_{55} = \frac{-\Lambda}{2M^3} \tag{3.29}$$

Above equation shows that the real solution is possible only for A if the value of cosmological constant is negative. So the space which lies between the branes is anti-desitter space (AdS5) i.e., a space having negative curvature.
By equating Eq. (3.27) and Eq. (3.29), we get

$$A'^2 = \frac{-\Lambda}{12M^3} = \kappa^2 \tag{3.30}$$

Taking square root on both sides, we get

$$A' = \pm\kappa \ .$$

Therefore

$$\frac{dA}{dy} = \pm\kappa \tag{3.31}$$

integrate both sides w.r.t y, we get

$$\begin{aligned} \int dA &= \pm\kappa \int dy \ , \\ A(y) &= \pm\kappa y \ . \end{aligned} \tag{3.32}$$

We require a solution that is invariant if we replace y by $-y$, so we take

$$A(y) = \kappa|y| \ .$$

Eventually, the parametrization of metric in the RS model can be done as

$$ds^2 = e^{-2\kappa|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{3.33}$$

Where$-L \leq y \leq L$. The expression for Einstein tensor that we have found is

$$G_{\mu\nu} = (6A'^2 - 3A'')g_{\mu\nu} \ ,$$

Also we have

$$A(y) = \pm\kappa y \ .$$

Differentiating above w.r.t y, we get,

$$A'(y) = sgn(y)\kappa \ . \tag{3.34}$$

By using the heaviside function we can write the term sgn (y) as

$$sgn(y) = \theta(y) - \theta(-y)$$

Taking derivative in Eq. (3.34), we have

$$A'' = 2\kappa\delta(y) \ ,$$

The above delta function comes due to twist of A at the brane that is placed at $y = 0$ and there also comes another delta that arises due to twist at the second brane that is placed at $y = L$, so the expression for A'' becomes

$$A'' = 2\kappa(\delta(y) - \delta(y - L)) \ .$$

Inserting these expressions in the Einstein tensor, we get

$$G_{\mu\nu} = 6\kappa^2 g_{\mu\nu} - 6\kappa(\delta(y) - \delta(y - L))g_{\mu\nu} \ , \tag{3.35}$$

therefore, we have

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu} \ .$$

The leading term of Eq. (3.35) corresponds to energy momentum tensor. This implies that

$$\kappa^2 T_{\mu\nu} = \frac{-\Lambda}{2M^3} g_{\mu\nu} = 6\kappa^2 g_{\mu\nu}$$

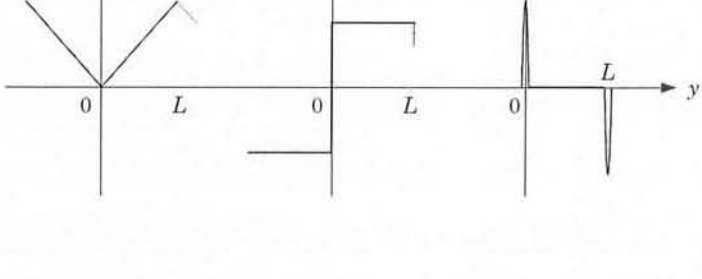

Figure 3.3: The function $A(y)$ and its first and second derivatives.

The second term of Eq. (3.35) does not match to any term, so this problem is solved by contributing the energy densities of both branes one at $y = 0$ and other at $y = L$. These are also known as brane tensions. We can do it by by contributing a term for each brane in the action. The energy densities of brane first ($y = 0$) and brane second ($y = L$) are λ_1 and λ_2, respectively. Writing

$$\begin{aligned} S_1 &= -\int d^4x\sqrt{-g_1}\lambda_1 = -\int d^4x dy\sqrt{-g}\lambda_1\delta(y) \\ S_2 &= -\int d^4x\sqrt{-g_2}\lambda_2 = -\int d^4x dy\sqrt{-g}\lambda_2\delta(y-L) \end{aligned} \tag{3.36}$$

where g_1 and g_2 are representing the determinant of metrics corresponding to first and second brane, respectively.
Distances across the branes can be defined by the matrices induced on the branes, i.e.,

$$ds^2 = g^i_{\mu\nu}dx^\mu dx^\nu$$

$$ds^2 = g_{\mu\nu}(x, y_i)dx^\mu dx^\nu$$

where $i = 1, 2$ and $y_1 = 0$, $y_2 = L$. Comparing above equation with metric of Eq. (3.33), we get$g_1 = g\delta(y)$ and $g_2 = g\delta(y - L)$.
As

$$g_{55} = 1 ,$$

to assure Einstein equations, we require to apply following relation

$$\lambda_1 = -\lambda_2 = 12\kappa M^3 . \tag{3.37}$$

Squaring both sides, we get

$$\begin{aligned} \lambda_1^2 &= (12M^3)^2\kappa^2 \\ \kappa^2 &= \frac{-\Lambda}{12M^3} \end{aligned}$$

by using it in above equation, we get

$$\Lambda = -\frac{\lambda^2}{12M^3} \tag{3.38}$$

From Eq. (3.37) and Eq. (3.38) we are able to know that 4D universe is smooth and steady. To obtain a disappearing effectual 4D cosmological constant the 4D brane origins are compensated by 5D cosmological constant

3.6 Exponential Hierarchy

The next task is to know about the physical scales for case when all the matter fields limited on the second brane. Let's take a scalar field (say Higgs). The Lagrangian for Higgs field is

$$L = (D^{\nu}\phi^{\dagger})(D_{\nu}\phi) - \lambda(\phi^{\dagger}\phi - v^2)^2$$

$$\phi \longrightarrow H, \phi^{\dagger} \longrightarrow H^{\dagger}$$

$$\begin{aligned} L &= D^{\nu}H^{\dagger}D_{\nu}H - \lambda(H^{\dagger}H - v^2)^2 \\ &= g_2^{\mu\nu}D_{\mu}H^{\dagger}D_{\nu}H - \lambda(H^{\dagger}H - v^2)^2 \end{aligned}$$

Action for Higgs scalar field is

$$S_{Higgs} = \int d^4x\sqrt{-g_2}L \tag{3.39}$$

Here $\sqrt{-g_2}$ is used so that volume did not change under Lorentz Transformation. Using value of L, we get

$$S_{Higgs} = \int d^4x\sqrt{-g_2}\Big[g_2^{\mu\nu}D_{\mu}H^{\dagger}D_{\nu}H - \lambda(H^{\dagger}H - v^2)^2\Big] .$$

We have

$$g^{\mu\nu} = e^{2\kappa|y|}\eta^{\mu\nu} ,$$

for brane at $y = L$, it becomes

$$g^{\mu\nu} = e^{2\kappa L}\eta^{\mu\nu} .$$

So the action becomes

$$\begin{aligned} S_{Higgs} &= \int d^4x\sqrt{-g_2}\Big[e^{2\kappa L}\eta^{\mu\nu}D_{\mu}H^{\dagger}D_{\nu}H - \lambda(H^{\dagger}H - v^2)^2\Big] \\ &= \int d^4x e^{-4\kappa L}\Big[e^{2\kappa L}\eta^{\mu\nu}D_{\mu}H^{\dagger}D_{\nu}H - \lambda(H^{\dagger}H - v^2)^2\Big] \end{aligned}$$

For canonically normalized action, use

$$H = e^{\kappa L}\widetilde{H} ,$$

$$S_{Higgs} = \int d^4x e^{-4\kappa L}\Big[e^{4\kappa L}\eta^{\mu\nu}D_{\mu}\widetilde{H}^{\dagger}D_{\nu}\widetilde{H} - \lambda(e^{2\kappa L}\widetilde{H}^{\dagger}\widetilde{H} - v^2)^2\Big] ,$$

after simplification, we get

$$S_{Higgs} = \int d^4x \left[\eta^{\mu\nu} D_\mu \widetilde{H}^\dagger D_\nu \widetilde{H} - \lambda(\widetilde{H}^\dagger \widetilde{H} - (e^{-\kappa L})^2)^2 \right] . \quad (3.40)$$

The above action is done for normalized Higgs scalar field, The vacuum expectation value can be expressed exponentially as

$$v_{eff} = e^{-\kappa L} v . \quad (3.41)$$

Since all the mass parameters get mass from the vacuum expectation value of Higgs in the SM, so all the mass parameters have suppressed exponentially on the brane that is present at $y = L$ (second brane). The apparent Higgs mass will goes to weak scale for the magnitude of bare Higgs mass of Planck scale order. Due to this, the brane that is present at $y = 0$ is known as "Planck brane" and brane that is at $y = L$ is known as "TeV" brane. $M_W = 10^{-16} M_{Planck}$, the suitable magnitude of extra dimension's size is

$$\kappa L \simeq ln 10^{16} \simeq 35 .$$

To know that this exponential expression is fruitful to solve the hierarchy problem or not, it must be known that what is the effect of extra dimension on the gravity's effective scale. We can get this information by the process that the 5D action gives the 4D action.

If we perturbed the action of Eq. (3.1) about the background metric that is used in Eq. (3.2), we get an expression which has following sechamatic form, as

$$S \ni M^3 \int d^4x \int_{-L}^{L} dy e^{-2\kappa|y|} \sqrt{-g^{(0)}} R^{4D}(h^{(0)}_{\mu\nu})$$

$$= M^3 \int_{-L}^{L} e^{-2\kappa|y|} dy \int d^4x \sqrt{-g^{(0)}} R^{4D}(h^{(0)}_{\mu\nu})$$

Since $\int_{-L}^{L} dy e^{-2\kappa|y|}$ is symmetric and is an even integral. So

$$\int_{-L}^{L} dy e^{-2\kappa|y|} = \; 2 \int_0^L dy e^{-2\kappa|y|}$$

Therefore

$$\begin{aligned} \int_{-L}^{L} dy e^{-2\kappa|y|} &= 2 \frac{e^{-2\kappa|y|}}{-2\kappa} \bigg|_0^L \\ &= -\frac{1}{\kappa}(e^{-2\kappa L} - e^0) \\ &= \frac{1 - e^{-2\kappa L}}{2} \end{aligned}$$

Hence,

$$S = M^3(\frac{1-e^{-2\kappa L}}{\kappa}) \int d^4x \sqrt{-g^{(0)}} R^4 D(h_{\mu\nu}(0)) \ , \tag{3.42}$$

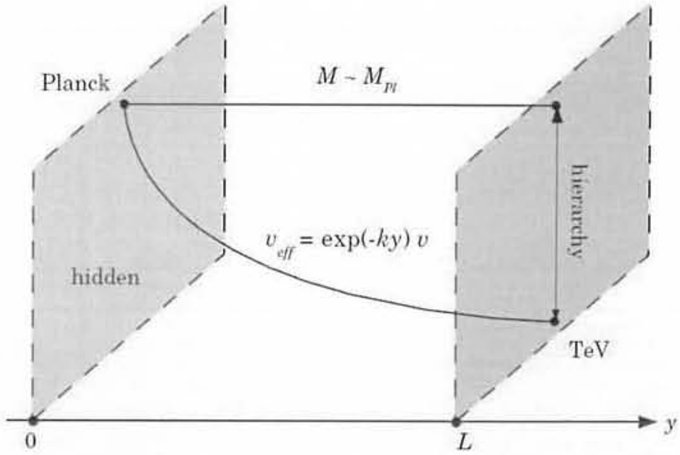

Figure 3.4: The generation of exponential hierarchy

Above expression is related with the 4D action. From above expression the effecetive 4D Planck mass has value

$$M_{pl}^2 = M^3\left(\frac{1-e^{-2\kappa L}}{\kappa}\right) \tag{3.43}$$

This shows that if value of κL is large, the Planck's mass depends weakly on the extra dimension's size.
From two results that are given in Eq. (3.41) and Eq. (3.43), we observe that the weak scale is suppressed exponentially down the extra dimensions. However gravity scale mainly did not depend on it, as shown in Fig. 3.4.
In conclusion, an exponential hierarchy could be created among the weak scale and gravity scale in a theory in which magnitude of all bare parameters could be calculated by Plank scale. Hence the RS model gives actual solution for hierarchy problem. Effective Planks mass lasts finite also for decompactification limit $L \longrightarrow \infty$. For the limit $L \longrightarrow \infty$ we have only single brane and is called RS Model II.

3.7 Graviton modes

To know about the working of gravity in the RS model, we should first know about the gravitons and their mathematical expression that is related with the little fluctuations $h_{MN}(x,y)$ about the following back ground metric

$$ds^2 = e^{-2\kappa|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \ .$$

This can be obtained by calculating the solution for linearized Einstein equation.

3.8 Conformally flat metric

Conformally flat metric is a metric that is proportionate with the flat space. To get it we relate the previous extra dimension variable"y" with a new variable "z"by

$$dy^2 = e^{-2\kappa|y|}dz^2 \ .$$

By integrating above equation, we get a constant that we set in such a way that for zero value of "z" we get zero value of "y". This leads to the following result

$$\kappa|z| = e^{\kappa|y|} - 1$$

$$\kappa|z| + 1 = e^{\kappa|y|}$$

$$\frac{1}{\kappa|z|+1} = e^{-\kappa|y|}$$

$$e^{-2\kappa|y|} = \frac{1}{(\kappa|z|+1)^2}$$

The metric for this new variable becomes

$$ds^2 = \frac{1}{(\kappa|z|+1)^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2) \ .$$

So the conformally flat metric is written as follow

$$ds^2 = e^{-2A(z)}\eta_{MN}dx^M dx^N \ , \tag{3.44}$$

where $x^5 = z$ and $A(z)$ is specified as

$$e^{-2A(z)} = \frac{1}{(\kappa|z|+1)^2} \ ,$$

$$
\begin{aligned}
-2A(z) &= ln(\kappa|z|+1)^2 \ , \\
&= -2ln(\kappa|z|+1) \ , \\
A(z) &= ln(\kappa|z|+1) \ .
\end{aligned}
$$

Now, we find derivative of $A(z)$ w.r.t z

$$
A'(z) = \frac{sgn(z)\kappa}{\kappa|z|+1} \ , \tag{3.45}
$$

and by taking double derivative of $A(z)$, we get

$$
A'' = \frac{sgn'(z)\kappa}{\kappa|z|+1} - \frac{\kappa^2}{(\kappa(z)+1)^2} \ .
$$

Finally

$$
A''(z) = \frac{2\kappa(\delta(z) - \delta(z - L_z))}{\kappa|z|+1} - \frac{\kappa^2}{(\kappa|z|+1)^2} \ , \tag{3.46}
$$

where we used

$$
sgn'(z) = 2\delta(z) \ .
$$

3.8.1 Linearized Einstein Equations

If the conformal transformation of a metric g_{MN} is the metric $\widetilde{g}_{MN}$, i.e.,

$$
g_{MN} = \ e^{-2A}\widetilde{g}_{MN} \ ,
$$

or

$$
\widetilde{g}_{MN} = \ e^{2A} g_{MN} \ .
$$

Then the specific Einstein Tensors are affiliated by

$$
\begin{aligned}
G_{MN}(g_{MN}) = \widetilde{G}_{MN}(\widetilde{g}_{MN}) + (n-2)[\widetilde{\nabla}_M A \widetilde{\nabla}_N A + \widetilde{\nabla}_M A \widetilde{\nabla}_N A \\
-\widetilde{g}_{MN}(\widetilde{\nabla}_R \widetilde{\nabla}^R - \frac{n-3}{2}\widetilde{\nabla}_R A \widetilde{\nabla}_R A)
\end{aligned} \tag{3.47}
$$

where n represents number of space time dimensions. Now we derive the expression given in Eq. (3.47). We know that

$$
g_{MN} = \ e^{-2A}\widetilde{g}_{MN} \ ,
$$

$$
\widetilde{g}^{MN}\widetilde{g}_{NP} = \ g^{MN}_{NP} \ .
$$

In above eqs. ∇_M corresponds to g_{MN} and $\widetilde{\nabla}_M$ corresponds to $\widetilde{g}_{MN}$. ∇_M and$\widetilde{\nabla}_M$ are related by following equation

$$\nabla_M \omega_N = \widetilde{\nabla}_M \omega_N - \Gamma^R_{MN} \omega_R \tag{3.48}$$

where

$$\Gamma^R_{MN} = \frac{1}{2} g^{RS} [\widetilde{\nabla}_M g_{NS} + \widetilde{\nabla}_N g_{MS} - \widetilde{\nabla}_S g_{MN}] \ . \tag{3.49}$$

If we reverse the rules for $\widetilde{\nabla}$ and ∇ in above equations, then we get

$$\widetilde{\nabla}_M \omega_N = \nabla_M \omega_N - \Gamma^R_{MN} \omega_R \ ,$$

or

$$\Gamma^R_{MN} = \frac{1}{2} \widetilde{g}^{RS} [\nabla_M \widetilde{g}_{NS} + \nabla_N \widetilde{g}_{MS} - \nabla_S \widetilde{g}_{MN}]. \tag{3.50}$$

As

$$\nabla_M g_{NR} = 0 \ ,$$

and

$$\nabla_M (\widetilde{g}_{NR}) = \nabla_M (e^{2A} g_{NR}) = 2e^{2A} g_{NR} \nabla_M A \ ,$$

we have

$$\widetilde{g}^{RS} = e^{-2A} g^{RS} \ .$$

Using all these values in Eq. (3.50), we get

$$\Gamma^R_{MN} = g^{RS} g_{NS} \nabla_M A + g^{RS} g_{MS} \nabla_N A - g^{RS} g_{MS} \nabla_S A \ ,$$

as $g^{RS} g_{NS} = \delta^R_N$ so above equation finally becomes

$$\Gamma^R_{MN} = \delta^R_N \nabla_M A + \delta^R_M \nabla_N A - g^{RS} g_{MN} \nabla_S A \ . \tag{3.51}$$

It can also be written as

$$\Gamma^R_{MN} = 2\delta^R (_M \nabla_N) A - g^{RS} g_{MN} \nabla_S A \ .$$

Now we will to perceive the relation for R^S_{MNR}. We have

$$\nabla_M \nabla_N \omega_R - \nabla_N \nabla_M \omega_R = R^S_{MNR} \omega_S \ . \tag{3.52}$$

As

$$\nabla_N \omega_R = \partial_N \omega_R - \Gamma^S_{NR} \omega_S \ ,$$

so

$$\nabla_M(\nabla_N\omega_R) = \partial_M(\nabla_N\omega_R) - \Gamma_{M(NR)}(\nabla_N\omega_R)$$

Inserting the value of $\nabla_N\omega_R$, we get

$$\begin{aligned}\nabla_M\nabla_N\omega_R &= \partial_M(\partial_N\omega_R - \Gamma^S_{NR}\omega_S) - \Gamma^T_{MN}(\partial_T\omega_R - \Gamma^S_{TR}\omega_S) \\ &\quad -\Gamma^T_{MR}(\partial_N\omega_T - \Gamma^S_{NT}\omega_S)\end{aligned}$$

Similarly

$$\begin{aligned}\nabla_N\nabla_M\omega_R &= \partial_N(\partial_M\omega_R - \Gamma^S_{MR}\omega_S) - \Gamma^T_{NM}(\partial_T\omega_R - \Gamma^S_{TR}\omega_S) \\ &\quad -\Gamma^T_{NR}(\partial_M\omega_T - \Gamma^S_{MT}\omega_S)\end{aligned}$$

Inserting these expressions in Eq. (3.52) and using $\Gamma^T_{MN} = \Gamma^T_{NM}$. After simplification, we get

$$R^S_{MNR}\omega_S = -\partial_M\Gamma^S_{NR}\omega_S + \partial_N\Gamma^S_{MR}\omega_S + \Gamma^T_{MR}\Gamma^S_{NT}\omega_S - \Gamma^T_{NR}\Gamma^S_{MT}\omega_S\ ,$$

and it can also be written as

$$R^S_{MNR} = R^S_{MNR} - \nabla_M\Gamma^S_{NR} + \nabla_N\Gamma^S_{MR} + \Gamma^T_{MR}\Gamma^S_{NT} - \Gamma^T_{NR}\Gamma^S_{MT}$$

Using Eq. (3.51), we get

$$\begin{aligned}\widetilde{R}^S_{MNR} &= R^S_{MNR} - \nabla_M(\delta^S_R\nabla_N A + \delta^S_N\nabla_R A - g^{ST}g_{NR}\nabla_T A) \\ &\quad +\nabla_N(\delta^S_R\nabla_M A + \delta^S_M\nabla_R A - g^{ST}g_{MR}\nabla_T A) \\ &\quad +(\delta^T_R\nabla_M A + \delta^T_M\nabla_R A - g^{TF}g_{MR}\nabla_F A) \\ &\quad (\delta^S_T\nabla_N A + \delta^S_N\nabla_T A - g^{SF}g_{NT}\nabla_F A) \\ &\quad -(\delta^T_R\nabla_N A + \delta^T_N\nabla_R A - g^{TF}g_{NR}\nabla_F A) \\ &\quad (\delta^S_T\nabla_M A + \delta^S_M\nabla_T A - g^{SF}g_{MT}\nabla_F A) \qquad (3.53)\end{aligned}$$

$$\begin{aligned}\widetilde{R}^S_{MNR} &= R^S_{MNR} - \delta^S_R\nabla_M\nabla_N A - \delta^S_N\nabla_M\nabla_R A + g^{ST}g_{NR}\nabla_M\nabla_T A + \delta^S_R\nabla_N\nabla_M A \\ &\quad +\delta^S_M\nabla_N\nabla_R A - g^{ST}g_{MR}\nabla_N\nabla_T A + \delta^T_R\delta^S_T\nabla_M A\nabla_N A + \delta^T_R\delta^S_N\nabla_M A\nabla_T A \\ &\quad -\delta^T_R g^{SF}g_{NT}\nabla_M A\nabla_F A + \delta^T_M\delta^S_T\nabla_R A\nabla_N A + \delta_M T\delta^S_N\nabla_R A\delta_T A \\ &\quad -\delta^T_M g^{SF}g_{NT}\nabla_R A\nabla_F A - g^{TF}g_{MR}\nabla_F A\nabla_N A\delta^S_T - g^{TF}g_{MR}\nabla_F A\nabla_T A\delta^S_N \\ &\quad -\delta^T_R\delta^S_T\nabla_N A\nabla_M A - \delta^T_R\delta^S_M\nabla_N A\nabla_T A + \delta^T_R g^{SF}g_{MT}\nabla_N A\nabla_F A \\ &\quad -\delta^T_N\delta^S_T\nabla_R A\nabla_M A - \delta^T_N\delta_M S\nabla_R A\nabla_T A + \delta^T_N g^{SF}g_{MR}\nabla_R A\nabla_F A \\ &\quad +g^{TF}g_{NR}\nabla_F A\nabla_M A\delta^S_T + g^{TF}g_{NR}\nabla_F A\nabla_T A\delta^S_M - g^{TF}g_{NR}g^{SV}g_{MT}\nabla_F A\nabla_V A \\ &\quad +g^{TF}g_{MR}g^{SV}g_{NT}\nabla_F A\nabla_V A \qquad (3.54)\end{aligned}$$

In above equation we interchange $N \leftrightarrow M$ in term (22) and $N \leftrightarrow R$ in term (24) and then simply we obtain the following equation.

$$\begin{aligned}
\widetilde{R}^S_{MNR} &= R^S_{MNR} - \delta^S_N \nabla_M \nabla_R A + g^{ST} g_{NR} \nabla_M \nabla_T A + \delta^S_M \nabla_N \nabla_R A \\
&\quad - g^{ST} g_{MR} \nabla_N \nabla_T A + \delta^S_N \nabla_M A \nabla_R A - g^{SF} g_{NR} \nabla_M A \nabla_F A + \delta^S_M \nabla_R A \nabla_N A \\
&\quad - g^{SF} g_{NR} \nabla_R A \nabla_F A + \delta^S_M \nabla_R A \nabla_N A + \delta^S_N \nabla_R A \nabla_M A - g^{SF} g_{NM} \nabla_R A \nabla_F A \\
&\quad - g^{TF} g_{MR} \nabla_F A \nabla_N A \delta^S_N - \delta^S_M \nabla_N A \nabla_R A + g^{SF} g_{MR} \nabla_N A \nabla_F A - \delta^S_N \nabla_R A \nabla_M A \\
&\quad - \nabla^S_M \nabla_R A \nabla_N A + g^{SF} g_{MN} \nabla_R A \nabla_F A + g^{TF} g_{NR} \nabla_F A \nabla_T A \delta^S_M \ .
\end{aligned}$$

Again by doing some simplifications, we obtain

$$\begin{aligned}
\widetilde{R}^S_{MNR} &= R^S_{MNR} + \delta^S_M \nabla_N \nabla_R A - \delta^S_N \nabla_M \nabla_R A - g^{ST} g_{RM} \nabla_N \nabla_T A \\
&\quad + g^{ST} g_{RN} \nabla_M \nabla_T A + \nabla_M A \delta^S_N \nabla_R A - \nabla_N A \nabla^S_M \nabla_R A \\
&\quad - \nabla_M A g_{NR} g^{SF} \nabla_F A + \nabla_N A g_{MR} g^{SF} \nabla_F A \\
&\quad - g_{RM} \delta^S_N g^{TF} \nabla_F A \nabla_N A + g_{RN} \delta^S_M g^{TF} \nabla_F A \nabla_T A \ .
\end{aligned}$$

By contracting over N and S, we obtain

$$\begin{aligned}
\widetilde{R}_{MR} &= R_{MR} + \delta^S_M \nabla_N \nabla_R A - \delta^S_S \nabla_M \nabla_R A - g^{ST} g_{RM} \nabla_S \nabla_T A \\
&\quad + g^{ST} g_{RS} \nabla_M \nabla_T A + \nabla_M A \delta^S_S \nabla_R A - \nabla_N A \nabla^S_M \nabla_R A \\
&\quad g_{SR} g^{SF} \nabla_M A \nabla_F A + \nabla_S A g_{MR} g^{SF} \nabla_F A \\
&\quad - g_{RM} \delta^S_S g^{TF} \nabla_F A \nabla_N A + g_{RS} \delta^S_M g^{TF} \nabla_F A \nabla_T A \ .
\end{aligned}$$

AS $\delta^S_S = n$

$$\begin{aligned}
\widetilde{R}_{MR} &= R_{MR} + \nabla_M \nabla_R A - n \nabla_M \nabla_R A - g^{ST} g_{RM} \nabla_S \nabla_T A \\
&\quad + \delta^T_R \nabla_M \nabla_T A + n \nabla_M A \nabla_R A - \nabla_M A \nabla_R A \\
&\quad - \delta^F_R \nabla_M A \nabla_F A + g_{MR} g^{SF} \nabla_S A \nabla_F A \\
&\quad - n g_{RM} g^{TF} \nabla_F A \nabla_T A + g_{RM} g^{TF} \nabla_F A \nabla_T A \ .
\end{aligned}$$

or

$$\begin{aligned}
\widetilde{R}_{MR} &= R_{MR} + \nabla_M \nabla_R A - n \nabla_M \nabla_R A - g^{ST} g_{RM} \nabla_S \nabla_T A \\
&\quad + \nabla_M \nabla_R A + n \nabla_M A \nabla_R A - \nabla_M A \nabla_R A \\
&\quad - \nabla_M A \nabla_R A + g_{MR} g^{TF} \nabla_T A \nabla_F A \\
&\quad - n g_{RM} g^{TF} \nabla_F A \nabla_T A + g_{RM} g^{TF} \nabla_F A \nabla_T A \ .
\end{aligned}$$

After more simplification, we get

$$\begin{aligned}
\widetilde{R}_{MR} &= R_{MR} - (n-2) \nabla_M \nabla_R A + (n-2) \nabla_M A \nabla_R A - g^{ST} g_{MR} \nabla_S \nabla_T A \\
&\quad - (n-2) g_{MR} g^{ST} \nabla_S A \nabla_T A \ .
\end{aligned}$$

We have

$$\widetilde{g}^{MR} = e^{-2A} g^{MR} ,$$

so

$$\widetilde{g}^{MR}\widetilde{R}_{MR} = e^{-2A} g^{MR}\Big\{ R_{MR} - (n-2)\nabla_M\nabla_R A + (n-2)\nabla_M A\nabla_R A$$
$$-g^{ST} g_{RM}\nabla_S\nabla_T A - (n-2) g_{RM} g^{ST}\nabla_S A\nabla_T A\Big\} .$$

We know that

$$\widetilde{g}^{MR}\widetilde{R}_{MR} = \widetilde{R} ,$$

therefore,

$$\widetilde{R} = e^{-2A}\Big\{ g^{MR} R_{MR} - (n-2) g^{MR}\nabla_M\nabla_R A + (n-2) g^{MR}\nabla_M A\nabla_R A$$
$$-g^{ST} g^{MR} g_{MR}\nabla_S\nabla_T A - (n-2) g^{MR} g_{MR} g^{ST}\nabla_S A\nabla_T A\Big\} .$$

Also

$$g^{MR} g_{MR} = n ,$$

$$\widetilde{R} = e^{-2A}\Big\{ R - (n-2) g^{MR}\nabla_M\nabla_R A + (n-2) g^{MR}\nabla_M A\nabla_R A$$
$$-n g^{ST}\nabla_S\nabla_T A - n(n-2) g^{ST}\nabla_S A\nabla_T A\Big\}$$

. By changing $T \to R$ and $S \to M$, we have

$$\widetilde{R} = e^{-2A}\Big\{ R - (n-2) g^{MR}\nabla_M\nabla_R A + (n-2) g^{MR}\nabla_M A\nabla_R A$$
$$-n g^{MR}\nabla_M\nabla_R A - n(n-2) g^{MR}\nabla_M A\nabla_R A\Big\} .$$

By simplifying, we obtain

$$\widetilde{R} = e^{-2A}\Big\{ R - 2(n-2) g^{MR}\nabla_M\nabla_R A + (n-1)(n-2) g^{MR}\nabla_M A\nabla_R A\Big\} . \tag{3.55}$$

The Einstein equation is

$$\widetilde{G}_{MR} = \widetilde{R}_{MR} - \frac{1}{2}\widetilde{g}_{MR}\widetilde{R}\,. \tag{3.56}$$

Using the definitions of these conformal transformations, we have

$$\begin{aligned}\widetilde{G}_{MR} &= R_{MR} + (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A\nabla_R A\Big] - g_{MR}g^{ST}\nabla_S\nabla_T A \\ &\quad -(n-2)g_{MR}g^{ST}\nabla_S A\nabla_T A \\ &\quad -\frac{1}{2}g_{MR}e^{2A}e^{-2A}\Big\{R - 2(n-1)g^{MR}\nabla_M\nabla_R A - (n-2)(n-1)g^{MR}\nabla_M A\nabla_R A\Big\}\end{aligned} \tag{3.57}$$

By rearranging the above equation and replacing $M \to S$ and $R \to T$, we get

$$\begin{aligned}\widetilde{G}_{MR} &= R_{MR} - \frac{1}{2}g_{MR}R + (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A\nabla_R A\Big] - g^{ST}\nabla^T\nabla_T A \\ &\quad -(n-2)g_{MR}\nabla^T A\nabla_T A + g_{MR}g^{ST}(n-1)\nabla_S\nabla_T A \\ &\quad +\frac{(n-2)(n-1)}{2}g_{MR}g^{ST}\nabla_S A\nabla_T A\end{aligned}$$

$$\begin{aligned}&= G_{MR} + (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A + \nabla_R A\Big] - g_{MR}\nabla^T\nabla_T A \\ &\quad -(n-2)g_{MR}\nabla^T A\nabla_T A + g_{MR}\nabla^T\nabla_T A(n-1) + \frac{(n-2)(n-1)}{2}g_{MR}\nabla_S A\nabla_T A\end{aligned}$$

$$\begin{aligned}&= G_{MR} + (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A\nabla_R A\Big] + (n-1-1)g_{MR}\nabla^T\nabla_T A \\ &\quad +\Big[\frac{(n-2)(n-1)}{2} - (n-2)\Big]g_{MR}\nabla^T A\nabla_T A\,,\end{aligned}$$

$$\begin{aligned}&= G_{MR} + (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A\nabla_R A + g_{MR}\nabla^T\nabla_T A \\ &\quad +\frac{n-3}{2}g_{MR}\nabla^T A\nabla_T A\Big]\,,\end{aligned}$$

$$G_{MR} = \widetilde{G}_{MR} - (n-2)\Big[-\nabla_M\nabla_R A + \nabla_M A\nabla_R A - g_{MR}\{-\nabla^T\nabla_T A - \frac{n-3}{2}\nabla^T A\nabla_T A\}\Big]\,.$$

As

$$\nabla_M A = -\widetilde{\nabla}_M A$$

and

$$\nabla_M \nabla_N A = -\widetilde{\nabla}_M \widetilde{\nabla}_N A$$

$$\nabla_M A \nabla_N A = (-\widetilde{\nabla}_M A)(-\widetilde{\nabla}_N A)$$

By using above definition, we get

$$\begin{aligned} G_{MR} &= \widetilde{G}_{MR} - (n-2)\Big[\widetilde{\nabla}_M \widetilde{\nabla}_R A + \widetilde{\nabla}_M A \widetilde{\nabla}_R A \\ &\quad -\widetilde{g} MR\{\widetilde{\nabla}^T \widetilde{\nabla}_T A - \frac{n-3}{2}\widetilde{\nabla}^T A \widetilde{\nabla}_T A\}\Big] \end{aligned}$$

The RS model described above is the simplest RS model in which there is a great problem for the electroweak precision parameters because the mass of lowest lying KK modes is of the order of 10 TeV which is beyond the reach of LHC. This problem can be solved with enlarged custodial symmetry by which the RS model become steady with the electroweak precision data for KK scales even as low as (2-3) TeV. The increament in the mass of lowest lying KK modes in the case of simplest RS model is due to harmful contribution to T parameter and contribution of excessively large corrections to left handed $Zb\bar{b}$ vertex.So with the enlarge custodial symmetry both the $Zb\bar{b}$ vertex and T-parameter are protected.

3.9 Custodial Symmetry

The Higgs Lagrangian is

$$L_{Higgs} = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \ , \tag{3.58}$$

where

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \ , \tag{3.59}$$

is the Higgs potential and

$$D_\mu(\phi) = (\partial_\mu + i\frac{g}{2}\sigma.W_\mu + i\frac{g'}{2}B_\mu)\phi \tag{3.60}$$

is the covariant derivative.
The Higgs Lagragian does not change under $SU(2)_L \times U(1)_Y$ symmetry, but there can be an accidental (approximate) global symmetry in the Higgs Lagrangian also. To check it we use the following process

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \tag{3.61}$$

where ϕ is the doublet field of the Higgs and ϕ^+ and ϕ^- are its components and

$$\phi = \begin{pmatrix} \phi^{0^*} \\ -\phi^- \end{pmatrix} \tag{3.62}$$

is also doublet under $SU(2)_L$. The Higgs field in terms of matrix is

$$\Phi = \frac{1}{\sqrt{2}}(\epsilon\phi^*, \phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{0^*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} . \tag{3.63}$$

So the Higgs Lagrangian corresponding to the matrix field is

$$L_{Higgs} = Tr(D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) . \tag{3.64}$$

Here

$$V(\Phi) = -\mu^2 Tr\Phi^\dagger\Phi + \lambda(Tr\Phi^\dagger\Phi)^2 , \tag{3.65}$$

and

$$D_\mu\Phi = (\partial_\mu\Phi + i\frac{g}{2}\sigma.W_\mu\Phi - i\frac{g'}{2}B_\mu\Phi\sigma_3) . \tag{3.66}$$

The third term in the last equation have σ_3 which is due to opposite hypercharge of ϕ and $\epsilon\phi^*$. The Higgs matrix field under gauge symmetry $SU(2)_L \times U(1)_Y$ is written as follow:

$$SU(2)_L : \qquad \Phi \to L\Phi \tag{3.67}$$

$$U(1)_Y : \qquad \Phi \to \Phi e^{-i\sigma_3\theta} \tag{3.68}$$

the σ_3 in the exponential appear because ϕ and $\epsilon\phi^*$ have opposite hypercharges. So the term

$$Tr(D_\mu\Phi)^\dagger D^\mu\Phi \to Tr(D_\mu\Phi)^\dagger L^\dagger L D^\mu\Phi = Tr(D_\mu\Phi)^\dagger D^\mu\Phi \tag{3.69}$$

is invariant under $SU(2)_L$. To obtain the approximate global symmetry obviously, it is necessary that the hypercharge coupling disappear $g' \to$

0. By this limit the Higgs Lagrangian remains in the same form as given in Eq. (3.58), but the gauge covariant derivative changes to following form

$$D_\mu \Phi = (\partial_\mu + i\frac{g}{2}\sigma.W_\mu)\Phi \ . \tag{3.70}$$

Therefore, in this limit, we obtain that the Lagrangian has global $SU(2)_R$ symmetry. It can be written as follows

$$SU(2)_R : \quad \Phi \to \Phi R^\dagger \ . \tag{3.71}$$

This implies the term

$$Tr(D_\mu \Phi)^\dagger (D^\mu \Phi) \to Tr R(D_\mu \Phi)^\dagger (D^\mu \Phi) R^\dagger = Tr(D_\mu \Phi)^\dagger (D^\mu \Phi) \tag{3.72}$$

not changes under $SU(2)_R$. Hence there is a global symmetry $SU(2)_L \times SU(2)_R$ in the Higgs sector for limit $g' \to 0$

$$SU(2)_L \times SU(2)_R : \quad \Phi \to L\Phi R^\dagger \ . \tag{3.73}$$

In terms of vacuum expectation value,

$$< \Phi > = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \tag{3.74}$$

Both $SU(2)_L$ and $SU(2)_R$ are broken by the vacuum expectation value

$$L < \Phi > \neq < \Phi > \qquad < \Phi > R^\dagger \neq < \Phi > \ , \tag{3.75}$$

but the subgroup $SU(2)_{L+R}$ is not broken by vacuum expectation value, associated simultaneously to the transformation of $SU(2)_L$ and $SU(2)_R$ with $L = R$,

$$L < \Phi > L^\dagger = < \Phi > \ . \tag{3.76}$$

Hence the global symmetry is broken by the vacuum expectation value in the following way

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R} \tag{3.77}$$

This is known as "custodial symmetry". There are 3 generators that are broken (3+3-3=3) , because $SU(2)$ is a three dimensional group and due to these three generators there arises three Goldstone bosons which are eaten by the massless W^+,W^- and Z bosons to attain masses through Higgs mechanism.

$$\begin{aligned} M_W^2 &= \frac{1}{2} g^2 v^2 \\ , M_Z^2 &= \frac{1}{4}(g^2 + g'^2) v^2 \ , \end{aligned} \tag{3.78}$$

this implies

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2} = \cos^2\theta_W \ . \tag{3.79}$$

Hence

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \tag{3.80}$$

at tree level. We can also know about the characteristics of a theory that is beyond the tree level by the assist of custodial symmetry because of the symmetry $SU(2)_{L+R}$ that remain unbroken for $g' \rightarrow 0$. The radiative corrections because of the Higgs and gauge bosons to the ρ parameter have to be proportionate to g'^2. e.g. because of Higgs boson's loops there is leading correction in the the ρ parameter in the $\overline{MS}$ scheme.

$$\hat{\rho} \approx 1 - \frac{G_F M_Z^2 \sin^2\theta_W}{24\sqrt{2}\pi^2} ln \frac{m_H^2}{M_Z^2} \tag{3.81}$$

for $g' \rightarrow 0$ ($\sin^2\theta_W \rightarrow 0$), the above correction disappear. So the parameter $\rho = 1$ is protected by the custodial symmetry from radiative corrections.

3.10 The Randall Sundrum model with custodial protection (RS_c)

There are many changes that have been suggested in the model and each one add some new characteristics in the simplest RS model. In present case we take the scenario where the SM gauge group is extended to the the following gauge group

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{L,R} \tag{3.82}$$

The Randall Sundrum model with custodial protection (RS_c) [12–14] is defined by above group together with metric given in Eq (3.2).
The custodial protection is perceived by forcing the discrete $P_{L,R}$ symmetry, that gives the mirror actions of two $SU(2)_{L,R}$ groups which protects the Z couplings to left handed fermions [15] from the excessively large corrections. Also this enlargement of gauge group made RS model steady with electroweak precision observables for lightest KK excitation's masses that are of the order of few TeV, that can be achieved by LHC. Here two symmetry breaking take place, first one is

this that the gauge group given in Eq. (3.82) breaks to SM gauge group and enforce some acceptable boundary condition on the UV brane [16]

$$SU(2)_R \times U(1)_X \to U(1)_Y$$

The second one is the spontaneous symmetry breaking that takes place by the Higgs on the IR brane.

$$SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$$

This gives the custodial symmetry that secure the T parameter[17, 18]. Here All the SM fields are given the permission to move in the bulk excluding the Higgs field that is restricted to IR brane at $y = L$.

Chapter 4

Effective Field Theory (EFT)

4.1 Introduction

In our living world there are many amazing things and one of them is the existence of fascinating physics at every scale. When we meditate on that scales of energy, time or distance which are not explored already, we discover new physical phenomena. From the lifetime of Z or W ($10^{-25}sec$) to the age of the universe that is nearly $10^{18}sec$ there exist different physical phenomena. If we want to study a particular phenomenon then it is convenient to separate that from all others. For this purpose the possibility is to divide the whole space into different areas. So in each area there is a different suitable explanation of important physics which is known as "effective theory".
The word "important" is used as a key-word because the relevant physical processes differ from each other place by place in space. The word "suitable" is also used as a key-word because there is not any single explanation of physics which is applicable in whole space.
The simple idea is this that if we have parameters which are extremely large or extremely small to our interesting physical quantities then we can achieve the simple illustration of physics by setting the parameters which are extremely large to infinity or extremely small to zero. Then the effects of parameters which are near about to the point of our interest are taken to be as a small perturbation about that point. This trick is old, but without this trick the understanding of current physics would be very hard if not impossible. This trick is used without thinking about it. For example, the Newtonian mechanics is still taught as separate subject without taking it as limit of the relativistic mechanics for small velocities. The relativity can be ignored in that region of space where the velocities are very small as compared to speed of light. This does not mean that treating the mechanics in the full relativistic

fashion is wrong but it became easy if we do not include relativity in the case where it is of no need. It is not compulsory to use effective theory if full theory is known. Anything can be calculated in full theory by cleverness. However, the use of effective theory is convenient. The calculations become easy by using effective field theory because we only concentrate on the physics of interest in EFT. When we use the EFT in particle physics then the parameter that is relevant is a "distance scale". Trying a procedure where the parameters which are small as compared to "distance scale" are shrinked down to zero. By this procedure, we get the simple and useful shape of important physics. Just to solve as an example, one such procedure has already been used while studying the multi-pole expansion in the electrodynamics. However, the construction of the EFT is interesting in the case of relativistic quantum field theory where the creation and annihilation of particles occur. In general the EFT are used in two distinct approaches.

1)Top down
2) Bottom up

4.1.1 Top-down

The top down approach begins by a known theory (high energy theory) and then by removing the degrees of freedom related to energies above some particular energy scale, say E_0. Our goal is to get a low energy theory which authorize one to calculate the required observables related to the energies below E_0 more easily than the original high energy theory. There are a few parameters to deal in calculations of low energy effective theory than the high energy theory. Hence the calculations are more easy in low energy effective theory than the known high energy theory. However, the establishment of such a low energy effective theory which fulfils this is done simply by removing the high energy degrees of freedom but they can be entangled up in non trivial way with the corresponding low energy degrees of freedom. One way to disentangle the low energy and high energy degree of freedom was given by Wilson and some others in 1970. This method is known as the Wilsonian approach of effective field theory. It include two steps: (i) identify the high energy degree of freedom and remove them from the action. The degrees of freedom of high energy refer to the heavy fields or high momenta. By integrating out high energy degree of freedom an effective action is obtained which describes the non local interactions between low energy degrees of freedom i.e., light fields or low momenta. (ii) The second step is to get a local effective action from the the effective action of first step by expanding it in local operators. These steps

are described below in more detail [19].
(1): Suppose we have a theory that has a particular energy scale E_0 and is described by an action (say S) and we want to know about physics at an energy scale E that is lower as compared to E_0 i.e, $E << E_0$. First of all we select a cutoff Λ at or just below E_0 and split the field ϕ w.r.t Λ into low and high momenta components such as

$$\phi = \phi_H + \phi_L \ ,$$

where the momenta of ϕ_H is $k > \Lambda$ and ϕ_L have momenta $k < \Lambda$. Now we integrate out (remove) the fields of higher momenta by using path integral formalism

$$\int \mathcal{D}\phi_L \int \mathcal{D}\phi_H e^{iS[\phi_H,\phi_L]} = \int \mathcal{D}\phi_L e^{iS_\Lambda[\phi_L]} \ .$$

Here, the Wilsonian effective action is stated by

$$e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_H e^{iS[\phi_L,\phi_H]} \ .$$

Hence the Lagrangian density is stated as

$$S_\Lambda[\phi_L] = \int d^D x \mathcal{L}_{eff}[\phi_L] \ ,$$

where D is representing the space time dimension.

(2): Typically, integrating out the heavy fields give the non local effective action which we expand in terms of local operators as

$$S_\Lambda = S_0(\Lambda, g^*) + \sum_i \int d^D x g^i \mathcal{O}_i \ ,$$

where $\mathcal{O}_i$'s are the local operators and g^i's are coupling constants. For weak coupling, expansion point S_0 can be considered as free action of initial theory, such that $g^* = 0$.

Examples :

i) One example is in the QCD. If we take the QCD just for any process, some parts are perturbative and some are non perturbative. While working for such kind of theories we can construct the low energy theory which only have non perturbative scale and remove all the perturbative scales. By this simple procedure one can figure out what is perturbative and what is non perturbative physics.
ii) Integrating out heavy particles like W, Z and top quark to separate

the perturbative and non perturbative physics is an example of "top down"effective theory. Heavy quark effective theory (for bottom and charm quarks) is also an example of "top-down" EFT.
iii) Non relativistic QED and non relativistic QCD and SCEFT (Soft Collinear Effective Field Theory) are also examples of the top down approach.

4.1.2 Bottom-Up

In this perspective, the high energy theory is not known but still we can construct an effective theory. For this purpose we can use the idea that there exist important interactions at different energy scales, few of them are very large so that they can not be seen till now and may not be seen in near future. However, an effective field theory could be used to describe physics at a particular energy scale, say E with a given accuracy ϵ in terms of QFT having finite set of parameters. Without knowing what is going on at high energies we can construct an EFT. We have listen about the finite set of parameters in renormalizibility but the new features in effective field theory like dependence on energy scale E and accuracy ϵ appears because we do not know what is going on at high energies. At high energy scale E, the effect of physics can be explained by the tower of interactions with integral mass dimensions from two to infinity, starting from ordinary interactions and going on to incorporate non renormalizable interactions of high dimensions.
The tower of interactions are governed by the following principles.
1) We have finite set of parameters to describe the interactions having dimension $k-4$.
2) The interaction terms with dimensions $k-4$ have the co-efficients that are less or of the order of

$$\frac{1}{M^k},$$

where $E < M$ for mass M that is independent of k. The above two states are the principles of effective field theory. They make sure that the calculations of the physical observables at a particular energy scale E with an accuracy ϵ require a finite set of parameters because the contribution of interactions with dimension k is proportional to

$$\left(\frac{E}{M}\right)^k.$$

Hence, we require to contribute terms only upto dimension k_ϵ. i.e.,

$$\left(\frac{E}{M}\right)^k \approx \epsilon$$

$$\Rightarrow k_\epsilon \approx \frac{\ln(1/\epsilon)}{\ln(M/E)} .$$

So as we move up in energy the non re-normalizable interactions become more important as k_ϵ increases. That is the signal for approaching to new physics. Before we reach the energies that are of the order of M, the non re-normalizable interactions vanish and are disclosed as re-normalizable or less non re-normalizable interactions still having higher scale say M'. The examples of bottom up approach are the low energy Fermi theory of weak interactions and according to some physicist the SM also falls in this category. Now we discuss it in detail in the next section

4.2 Standard Model (SM) as an EFT

Let's try to understand SM as an effective theory. Following the method of expansion

$$\sum_n L_{low}^{(n)} = L^{(0)} + L^{(1)} + ...$$

where $L^{(0)}$ is the SM Lagrangian and $L^1, ...$ are the corresponding corrections. In SM, we have massive gauge bosons and fermions in addition the photon and gluons which are massless.

4.2.1 Fermions

For the fermions there is a broad mass spectrum. The masses of quarks and leptons are given in tables 4.1 and 4.2 respectively.

Quarks	Masses
u_L, u_R	1.5-3.3 MeV
d_L, d_R	3.5-6.0 MeV
s_L, s_R	100 ± 30 MeV
c_L, c_R	1.37 ± 0.03 GeV
b_L, b_R	4.20 ± 0.12 GeV
t_L, t_R	173.34 ± 0.76 GeV

Table 4.1: Masses of qaurks of the SM

The corresponding gauge couplings are different for left and right handed fermions.

Leptons	Masses
e_L, e_R	0.511 MeV
μ_L, μ_R	105.6 MeV
τ_L, τ_R	171.7 MeV
ν_{e_L}	$< 1 \times 10^{-8}$ GeV
ν_{μ_L},	< 0.0002 GeV
ν_{τ_L}	< 0.02 GeV

Table 4.2: Masses of leptons of the SM

So even in the SM there are large number of different mass scales and if we think from the EFT point of view then the suitable choice is the top down approach. Firstly, we will integrate the top quark and then the W and Z boson and we proceed down to the bottom quark etc. We can construct the EFT by integrating out one degree of freedom and we get new effective field theory by integrating out another degree of freedom every time.

Let's think in a different context. We have all the stuff (masses of the SM particles) and we are interested in physics at high energy scale beyond the scale of weak bosons and top quarks. For this purpose we use bottom-up approach. Writing.

$$L^{(0)} = L_{Gauge} + L_{Fermi} + L_{Higgs} + L_{N_R} \ , \tag{4.1}$$

where L_{N_R} is the Lagrangian for right handed neutrinos and L_{Higgs} is for Higgs particle. The different L's can be written as:

$$\begin{aligned} L_{Gauge} &= -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A \ , \\ L_{Fermi} &= \sum_{\psi_L}\bar{\psi}_L i\gamma^\mu D_\mu \psi_L + \sum_{\psi_R}\bar{\psi}_R i\gamma^\mu D_\mu \psi_R \ , \\ iD_\mu &= i\partial_\mu + g_1 B_\mu Y + g_2 W^a_\mu T^a + g A^a_\mu T^a \ , \end{aligned}$$

where D_μ is covariant derivative for gauge fields and g_1 represents the gauge coupling for hypercharge and g_2 for $SU(2)_{weak}$ and g for $SU(3)_c$. Power counting in this down-up approach corresponds to what we left out.
Define

$$\epsilon = \frac{M_{SM}}{\Lambda_{new}}$$

Λ_{new} represents the things we left over to describe. The expansion is made in this ϵ and the power counting depends upon the powers of Λ_{new}. In the numerator there comes the things like the top quark mass, W boson mass, Z boson mass and the Higgs mass. In the denominator we have something like $M_{Planck}, M_{GUT}, and M_{SUSY}$ which are scales corresponding to high energy. So from the EFT point of view anything that is left out of the SM description (generate at high energy scale) goes in the denominator of expansion.

The term $L^{(1)}$ describes the corrections that corresponds to the physics beyond the SM. Physics at high energy scale is described by the higher dimensional operator (operator having dimensions greater than four) which are built from the SM fields.

4.3 Marginal, Relevant, Irrelevant Operators:

Consider an effective field theory for scalar in d-dimensions

$$S[\phi] = \int d^dX(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda\phi^4}{4!} - \frac{\tau\phi^6}{6!} + ...) \, , \qquad (4.2)$$

where $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ represents the standard kinetic term and $\frac{1}{2}m^2\phi^2$ represents the mass term.

We can look at the mass dimensions of various operators here. The action method is

$$[\phi] = \frac{d-2}{2}$$

So the mass dimensions of field in d-dimensions is $\frac{d-2}{2}$

$$[d^dX] = -d \qquad [\lambda] = 4 - d \qquad [m^2] = 2 \qquad [\tau] = 6 - 2d \, .$$

As an example we want to study the correlation function

$$< \phi(X_1).......\phi(X_n) >$$

at long distance (small momenta). Define

$$\phi'(X') = s^{\frac{(d-2)}{2}}\phi(X) \qquad (4.3)$$

$\phi'(X')$ is the real field but rescaled by a s parameter. The corresponding action in terms of ϕ' field becomes

$$S'(\phi') = \int d^dX'[\frac{1}{2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{2}m^2s^2\phi'^2 - \frac{\lambda}{4}s^{4-d}\phi'^4 - \frac{\tau}{6!}s^{6-2d}\phi'^6 + ...](4.4)$$

Let's look at the correlation function in terms of ϕ'

$$< \phi(sX')........\phi(sX'_n) >= s^{\frac{n(2-d)}{2}} < \phi'(X'_1)........\phi'(X'_n) > \quad . \tag{4.5}$$

We could study this in various dimensions. For simplicity take the most common case of our interest i.e.,"$d = 4$" and ask the question what happened when s gets large . This is something we often do while studying EFT to figure out how we can study the large distance behavior.
In the limit $s \to \infty$:

- The term $\frac{1}{2}m^2 s^2 \phi^2$ becomes more and more important because we have explicitly s^2 in this term. So ϕ^2 is called relevant.

- τ is less important because the power of s is in negative and λ term is equal important as before.

So we could say that ϕ^2 is the relevant operator, ϕ^4 is marginal and ϕ^6 is irrelevant operator.

$\phi^2 \to$ is relevant	dimensions$< d$	$[m^2] > 0$
$\phi^4 \to$ is marginal	dimensions$=d$	$[\lambda] = 0$
$\phi^6 \to$ is irrelevant	dimensions$> d$	$[\tau] < 0$

For Finite s (but large) the dimensions of operators (parameters) tells us about the importance of different terms. For power counting we see the dimensions of parameters

$$(m^2) \sim (\Lambda^2_{new}) \quad , \quad \lambda \sim (\Lambda^0_{new}) \quad , \quad \tau \sim (\Lambda_{new})^{-2}$$

and we can do power counting in this Λ_{new}. Long distance $s \to \infty$ means small momenta, therefore, in terms of momentum.

$$p << \Lambda_{new}.$$

As the power of Λ_{new} is negative for τ, hence it corresponds to irrelevant operator.

4.4 Renormalization

Renormalization is a technique which is used to deal with infinities appearing in the calculated quantities. There is a large number of infinities in relativistic field theory, so these infinities should be removed before comparing the theoretical and experimental predictions.

The renormalization is a skill that systematically isolates and then eliminates all such infinities from the observables which are physically measurable. Keep in mind that renormalization is not just only for relativistic field theory but it a general theory. For example , suppose an electron is moving in a solid and if the electron is weakly interacting with the solid lattice then an effective mass m^* can be used to describe its reaction to an external force and m^* (effective mass) is surely different from the mass m which is measured outside the solid. In such a straightforward case both mass m^* and m are measurable and so are finite. The condition is same for the field theory but there are two major differences. (i) Because of interactions there appear to be infinities relating to the divergent loop diagrams. Such infinities are appearing due to the contribution of high momenta for weak interactions. (ii) The interactions between the particles and observables can not be switched off because it is impossible to measure bare quantities in the absence of these interactions. In renormalization all divergences are shuffled into bare quantities. i.e. ,the non-measurable quantities can be re-defined. In this way we can absorb these divergences to different parameters to make those observables finite which are physically measurable. So, the theories in which all divergences can be absorbed by redefining some physical parameters are known as re-normalizable theories. The theories which do not have this property are known as non-re-normalizable theories. By this criteria we can select a right theory easily.

4.4.1 Divergences:

We start from Feynman diagrams that generally have divergences. Let's have two four point interaction which are labeled by λ.

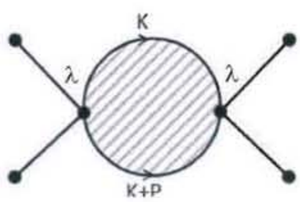

Figure 4.1: Two-Two particle scattering.

The corresponding loop integral can be written as

$$I \sim \lambda^2 \int \frac{d^d k}{(k^2 - m^2 + io)(k^2 - m^2 + io)} . \tag{4.6}$$

This integral diverges as Λ^{d-4}, where $d-4$ is the degree of divergence. In $d = 4$ dimension

$$I \sim \int \frac{d^d k}{k^4} \sim \int \frac{dk}{k} \sim ln\Lambda \leftrightarrow \frac{1}{\epsilon} \tag{4.7}$$

in dimensional regularization $d = 4 - 2\epsilon$ and if we think about what it does ? The the answer is that it is some thing that renormalizes the $\lambda\phi^4$ operator and we add the counter term

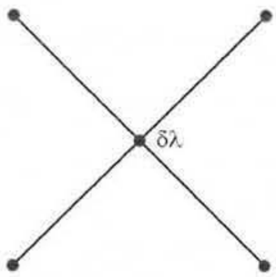

Figure 4.2

Now, what happen if we keep the τ term and a λ term simultaneously.

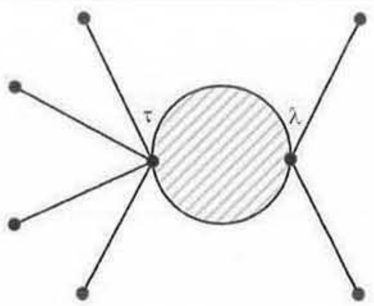

Figure 4.3: Four-Two particles scattering

The loop integral in this case becomes

$$I \sim \lambda\tau \int \frac{d^d k}{()()} \,. \tag{4.8}$$

which renormalizes $\tau\phi^6$. It gets the same divergence but now the renormalizing operator is an operator with the 6-point in the outside. Inserting one λ and one τ we get back the renormalization of τ.

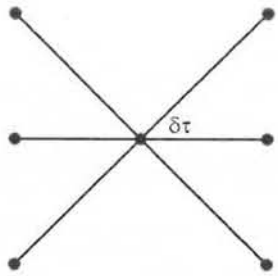

Figure 4.4

Now we can also include two $\tau' s$

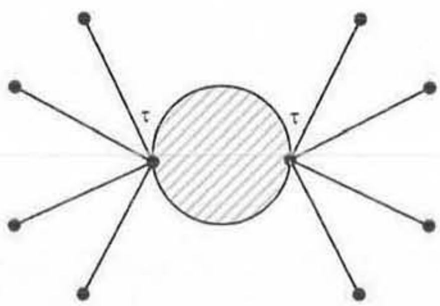

Figure 4.5: Four-Four particles scattering

This leads to

$$I \sim \tau^2 \int \frac{d^d k}{()()} \, . \tag{4.9}$$

which renormalizes ϕ^8.

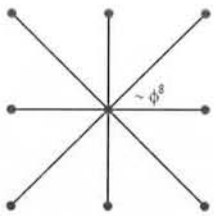

Figure 4.6

Since ϕ^8 is not in $S[\phi]$ (without +.....) so the theory can not be renormalized in traditional sense that is classical way of thinking. In the effective theory way of thinking , other operators we have in this diagram will become relevant.
If

$$\tau \sim \frac{1}{\Lambda_{new}^2}$$

the $p^2\tau$ is very small, so theory can be renormalized systematically in $\frac{1}{\Lambda_{new}}$ but it is necessary to add ϕ^8operator at order Λ_{new}^{-4}. To contribute all corrections to $\frac{1}{\Lambda_{new}^r}$or $\frac{1}{S^r}$ we require to contribute all operators with dimensions $[O] \leq d + r$.
So here we can place more powerful argument that power counting is always connected to dimensions.

4.5 Matching Condition

The connection between the coupling enforced by the demand that the two EFTs describe the same physics is known as matching condition. These conditions are assessed in both theories by the renormalization scale μ of the order of boundary mass to remove the large logarithms. Suppose we have two theories, one is high energy theory and other is low energy theory and we are interested to do matching at some scale $\mu = \mu_m = M$. The requirement for the matching at this scale is that the parameters of both theories should be related with each other in such a way that the description of physics above and below the boundary line (Matching line shown in diagram given below) remain the same i.e., physics remains same in both theories. In lowest order matching for the QCD this condition is elaborated as the coupling constants of the interactions having light fields should be continuous across the matching line (boundary) so that the theory above and below the threshold (matching line) is same.

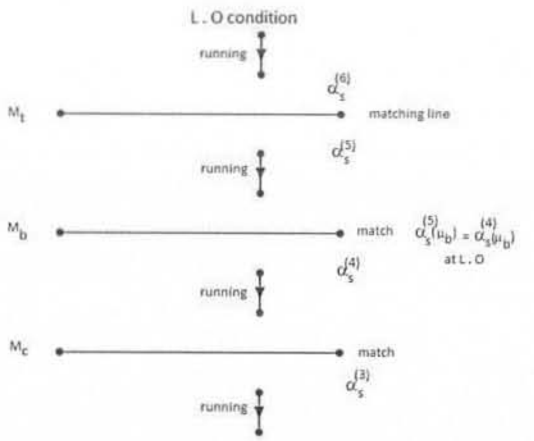

Figure 4.7: Matching condition at lowest order

Whenever we reach the threshold we do matching, we switch the field contents and get a new effective field theory with new coupling constants as shown in above diagram. In the above figure the arrows are representing the running of coupling constants while the lines are representing the matching between the two theories.
The matching condition at b-quark scale is

$$\alpha_s^{(5)}(\mu_b){=}\mu_s^{(4)}(\mu_b) \text{ at leading order (L.O).}$$

That is a leading order matching condition. Where $\mu_b \sim M_b$ or $\mu_b = M_b, \frac{M_b}{2}, 2M_b$
The above matching condition is not true at higher orders.

$$\alpha_s^{(4)}(\mu_b) = \alpha_s^{(5)}(\mu_b)[1 + \frac{\alpha_s^{(5)}}{\pi}(\frac{-1}{6}ln\frac{\mu_b^2}{M_b^2}) + (\frac{\alpha_s^{(5)}}{\pi})^2(\frac{11}{72} - \frac{11}{24}ln\frac{\mu_b}{M_b} + \frac{1}{36}ln^2\frac{\mu_b}{M_b}) + ...] \,.$$

We ensure that the theory with five flavor (with coupling constant $\alpha_s^{(5)}$) has the same description of physics as the theory with four flavors (with coupling constant $\alpha_s^{(4)}$) , but the diagrams differ in both theories.
Now we discuss the general procedure of matching related to masses here.

4.5.1 General Procedure

Let suppose we have n particles having masses in the following order

$$M_1 >> M_2 >> M_3 >> .... >> M_n$$

and we want to pass through $L_1, L_2, L_3, ...L_n$. This process include the following steps.
1) First of all we match L_1 at scale $\mu_1 = M_1$ with L_2.
This matching conditions determines the corresponding parameters.
2) Then we require to determine the β-function and irregular dimensions in L_2 and evolve the couplings down.
(3)Now we do matching at scale $\mu = M_2$ between L_2 and L_3.and so on. We stop at the order where we are interested in i.e. ,at $\mu \sim M_n$.

Everything up to that stage is just to determine the theory L_n and the value of coupling constants by knowing the information at higher scale.i.e, knowing the information in high energy theory we propagate the knowledge down to the low energies consistently. Then we find the parameters in theory L_n.

4.5.2 Example

For the explanation of the matching condition we consider the example of $b \to c\bar{u}d$ in the SM. The Lagrangian corresponding to this transition is

$$L_{SM} = \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{u}_L \gamma^\mu V_{CKM} d_L.$$

The tree diagram of $b \to c\bar{u}d$ is

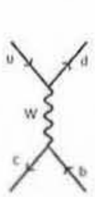

Figure 4.8: Tree diagram of $b \to c\bar{u}d$

and the tree level matching is

$$\mathcal{A} = (\frac{ig_2}{\sqrt{2}})^2(-i)V_{cb}V_{ud}^*(g^{\mu\nu} - \frac{k^\mu k^\nu}{M_W^2})(\frac{1}{k^2 - M_W^2})[\bar{u}^c\gamma_\mu P_l u^b][\bar{u}^d\gamma_\mu P_l u^u] \quad (4.10)$$

where $k^\mu = p_b^\mu - p_c^\mu = p_u^\mu + p_d^\mu$ is the momentum transfer.

The momentum going through propagator is of the order of mass of bottom quark,

$$\text{momenta} \sim M_b$$

As the momentum is small as compared to the mass of W-boson, so we can expand propagator and the leading term of the propagator is

$$\sim \frac{(-ig^{\mu\nu})}{(-M_W^2)} + O(\frac{M_b^2}{M_W^4})$$

We drop the higher order terms like $O(\frac{M_b^2}{M_W^4})$

The Feynman rules for the effective theory after integrating the W boson from the above high energy theory (SM) are.

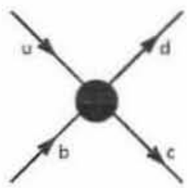

Figure 4.9: Point diagram of $b \to c\bar{u}d$

$$\mathcal{A} = -i4\frac{G_F}{\sqrt{2}}[\bar{u}^c\gamma_\mu P_L u_L][\bar{u}_d\gamma^\mu P_L u^u] \tag{4.11}$$

It means that by integrating out the heavy particles from the Standard Model (high energy theory) we obtain the Fermi theory (low energy effective theory). So by matching the SM and the low energy EFT (Fermi theory), we can find the value of coupling constant in terms of the parameters of high energy theory (SM).i.e.,

$$G_F = \frac{\sqrt{2}g_2^2}{8M_W^2}$$

Chapter 5

Analysis of $\Lambda_b \to \Lambda l^+ l^-$ in SM and RS_c Model

Rare decays occur through flavor changing neutral current (FCNC) $b \to s, d$ transitions. SM can be tested at loop level by these decays. Due to this cause, the B-factories Belle, BaBar [20] and the LHCb have included these decays in their major research directions. We can get important information concerning the earlier studied features of the SM, like Cabibo-Kobayashi-Maskawa matrix elements.
After radiative decay $b \to s\gamma$ measured by CLEO [21], the focal point is the rare decays that are induced by $b \to sl^+l^-$ transitions . These transitions have comparatively large branching ratio in the SM. Rare decays are much sensitive to the new physics effects beyond the SM and hence constitutes a reasonable method to look about such effects. As the NP in semileptonic b-decays can can come through two ways, (i) The operator basis remains the same as that of the SM and NP comes only through modification of Wilson coefficients of the SM e.g., [22–24, 27]. RS_c model belong to this class of models. (ii) The new operators along with the new Wilson coefficients also appear in addition to the modification of Wilson coefficients already present in the SM. SUSY is a common example of this class of models. The interesting question which follow next is what happens in the case of heavy baryon to light baryon transitions and which physical quantity is more sensitive to the effects of NP.
In this chapter we will study the baryonic $\Lambda_b \to \Lambda l^+ l^-$ decay where the particular forms is the study of physical observables such as , differential decay width,differential branching ratio, lepton forward-backward asymmetry and Λ polarization in the SM and Randall Sundrum model with custodial protection.

5.1 Effective Hamiltonian

After integrating out the heavy particles like top quark, Z and $W^{\pm}$ for a scale above $\mu = O(m_b)$, we get the effective Hamiltonian for $b \to sl^+l^-$ in the SM [26]

$$\begin{aligned} H_{eff}(b \to sl^+l^-) &= -\frac{G_F}{2\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)O_i(\mu) + C_{7\gamma}(\mu)O_{7\gamma}(\mu) \right. \\ &\left. + C_{8G}(\mu)O_{8G}(\mu) + C_9(\mu)O_9(\mu) + C_{10}(\mu)O_{10}(\mu) \right], \end{aligned} \tag{5.1}$$

where $O_i(\mu)$ are the four-quark operators and $C_i(\mu)$ are the related Wilson coefficients. Here the terms proportional to $V_{ub}V_{us}^*$ are neglected for $\left|\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}\right| < 0.02$. The operators that are used in the $\Lambda_b \to \Lambda l^+l^-$ transitions are given below

Current-current (tree) operators :

$$O_1^{\mu} = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha), \qquad O_2^{\mu} = (\bar{s}_\alpha c_\alpha)V - A(\bar{c}_\beta b_\beta) \tag{5.2}$$

The QCD penguin operators :

$$\begin{aligned} O_3 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A} \\ O_4 &= (\bar{s}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V-A} \\ O_5 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \\ O_6 &= (\bar{s}_\beta b_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\beta)_{V+A} \end{aligned} \tag{5.3}$$

The magnetic penguin operators :

$$\begin{aligned} O_{7\gamma} &= \frac{e}{8\pi^2} \bar{s}\sigma_{\mu\nu}(m_b R + m_s L) b F^{\mu\nu} \\ O_{8G} &= \frac{g}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(m_b R + m_s L) b G^{\mu\nu} \end{aligned} \tag{5.4}$$

The Semi leptonic operators :

$$O_{9V} = (\bar{s}_\alpha b_\alpha)_{V-A}(\bar{l}l)_V, \qquad O_{10A} = (\bar{s}_\alpha b_\alpha)_{V-A}(\bar{l}l)_A . \quad (5.5)$$

Here $R = (1+\gamma_5)$, $L = (1-\gamma_5)$, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and, α and β are representing the color indices. The electromagnetic interactions are represented by coupling constant e and strong interactions by coupling constant g, q represents the quark at scale $\mu = O(m_b)$ and$(\bar{q}_\alpha q_\beta)_{V-A} = \bar{q}_\alpha \gamma_\nu (1-\gamma_5) q_\beta$ is the definition of left handed current while the expression $(\bar{q}_\alpha q_\beta)_{V+A} = \bar{q}_\alpha \gamma_\nu (1+\gamma_5) q_\beta$ define the right handed current.

The decay amplitude of free quarks by using the above definitions can be written as follow

$$\mathcal{M}(b \to sl^+l^-) = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Big\{ C_9^{eff}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l) - \frac{2i}{q^2} C_7^{eff} \bar{s} i\sigma_{\mu\nu} \frac{q^\nu}{s}(m_b P_R + m_s P_L) b(\bar{l}\gamma^\mu l) \Big\} , \quad (5.6)$$

where we have used $P_L = \frac{L}{2}$ and $P_R = \frac{R}{2}$ and the elements V_{tb} , V_{ts}^* belongs to Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, q^2 is the squared momentum transfer, G_F is the fermi constant, while α represents fine structure constant and C_7^{eff} , C_9^{eff} and C_{10} are Wilson's coefficients belonging to different interactions. The Wilson's coefficient C_9^{eff} is given by [27, 28]

$$C_9^{eff}(\mu) = C_9^{NDR}(\mu) + Y_{SD}(z, s'), \quad (5.7)$$

where $z = m_c/m_b$, $s' = q^2/m_b^2$ and $Y_{SD}(z, s')$ is defined as follow

$$\begin{aligned} Y_{SD}(z,s') &= h(z,s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ &\quad -\frac{1}{2}h(1,s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ &\quad -\frac{1}{2}h(0,s')(C_3(\mu) + 3C_4(\mu)) + \frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)), \end{aligned} \quad (5.8)$$

with

$$\begin{aligned} h(z,s') &= -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2} \begin{cases} \ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi & \text{for } x \equiv 4z^2/s' < 1 \\ 2\arctan\frac{1}{\sqrt{x-1}} & \text{for } x \equiv 4z^2/s' > 1 \end{cases} , \\ h(0,s') &= \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}i\pi . \end{aligned} \quad (5.9)$$

In naive dimensional regularization (NDR) scheme C_9^{NDR} is written as

$$C_9^{NDR} = P_0^{NDR} + \frac{Y^{SM}}{\sin^2\theta_W} - 4Z^{SM} + P_E E^{SM} , \qquad (5.10)$$

where

$$\begin{aligned} Y^{SM}(x_t) &= Y_0(t) + \sum_{n=1}^{\infty} C_n(x_t, x_n), \\ Z^{SM}(x_t) &= Z_0(t) + \sum_{n=1}^{\infty} C_n(x_t, x_n). \end{aligned} \qquad (5.11)$$

Now the the parameters of Eq. (5.11) and are specified below

$$\begin{aligned} Y_0^{SM}(x_t) &= \frac{x_t}{8}\left[\frac{x_t-4}{x_t-1} + \frac{3x_t}{(x_t-1)^2}\right] \\ Z_0^{SM}(x_t) &= \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \left[\frac{32x_t^4 - 38x_t^3 + 25x_t^2 - 18x_t}{72(x_t-1)^4} - \frac{1}{9}\right]\ln x_t \end{aligned}$$

$$C_n(x_t, x_n) = \frac{x_t}{8(x_t-1)^2}\left[x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_t x_n)\ln\frac{x_t + x_n}{1 + x_n}\right] \qquad (5.12)$$

As P_E is of very small order so we neglect $P_E E^{SM}$ term. and we have $P_0^{NDR} = 2.60 \pm 0.25$, $Y^{SM} = 0.98$, $Z^{SM} = 0.679$ and $\sin^2\theta_W = 0.23$ [29–31]. The expression for $\eta(\hat{s'})$ is as follow

$$\eta(\hat{s'}) = 1 + \frac{\alpha_s(\mu_b)}{\pi}\omega(\hat{s'}) \qquad (5.13)$$

where

$$\omega(\hat{s'}) = -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(\hat{s'}) - \frac{2}{3}ln\hat{s'}ln(1-\hat{s'}) - \frac{5+4\hat{s'}}{3(1+2\hat{s'})}ln(1-\hat{s'}) - \frac{2\hat{s'}(1+\hat{s'})(1-2\hat{s'})}{3(1-\hat{s'})^2(1+2\hat{s'})}ln\hat{s'} + \frac{5+9\hat{s'}-6\hat{s'}^2}{(1-\hat{s'})(1+2\hat{s'})} \qquad (5.14)$$

and

$$\alpha_s(x) = \frac{\alpha_s(Z)}{1 - \beta_0\frac{\alpha_s(m_z)}{2\pi}ln(\frac{m_Z}{x})} \qquad (5.15)$$

with $\alpha_s(m_Z) = 0.118$ and $\beta_0 = \frac{23}{3}$. The Wilsons's coefficients that have been appeared in Eq. (5.8) can be found by following expression:

$$C_j = \sum_{i=1}^{8} k_{ji}\eta^{a_i} \qquad (j = 1, ...6), \tag{5.16}$$

k_{ji} are given as

$$\begin{aligned}
k_{1i} &= (0\,,\ 0\,,\ \frac{1}{2}\,,\ -\frac{1}{2}\,,\ 0\,,\ 0\,,\ 0\,,\ 0\ ),\\
k_{2i} &= (0\,,\ 0\,,\ \frac{1}{2}\,,\ \frac{1}{2}\,,\ 0\,,\ 0\,,\ 0\,,\ 0\ ),\\
k_{3i} &= (0\,,\ 0\,,\ -\frac{1}{14}\,,\ \frac{1}{6}\,,\ 0.0510\,,\ -\ 0.1403\,,\ -\ 0.0113\,,\ 0.0054\ ),\\
k_{4i} &= (0\,,\ 0\,,\ -\frac{1}{14}\,,\ -\frac{1}{6}\,,\ 0.0984\,,\ 0.1214\,,\ 0.0156\,,\ 0.0026\ ),\\
k_{5i} &= (0\,,\ 0\,,\ 0\,,\ 0\,,-0.0397\,,\ 0.0117\,,\ -\ 0.0025\,,\ 0.0304\ ),\\
k_{6i} &= (0\,,\ 0\,,\ 0\,,\ 0\,,\ 0.0335\,,\ 0.0239\,,\ -\ 0.0462\,,-0.0112\ ),
\end{aligned} \tag{5.17}$$

The Wilson's coefficient C_{10} is specified as follow

$$C_{10}^{SM} = -\frac{Y^{SM}}{\sin^2\theta_W}\ . \tag{5.18}$$

Finally the effective coefficient C_7^{eff} is defined as

$$C_7^{eff}(\mu_b) = \eta^{\frac{16}{17}} C_7(\mu_W) + \frac{8}{3}(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}})C_8(\mu_W) + C_2(\mu_W)\sum_{i=1}^{8} h_i\eta^{a_i} \tag{5.19}$$

where

$$\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}\ , \tag{5.20}$$

and

$$C_2(\mu_W) = 1 \qquad C_7(\mu_W) = -\frac{1}{2}D_0'(x_t) \qquad C_8(\mu_W) = -\frac{1}{2}E_0'(x_t) \tag{5.21}$$

The wilson's coeffients that are in above equation are defined in leading logarithm approximation. The μ_W corresponds to the scale of W-boson mass. The parameters that are appeared in the expression of C_7^{eff} are given below

$$
\begin{aligned}
h_i &= (2.2996\,,\ -1.0880\,,\ -\frac{3}{7}\,,\ -\frac{1}{14}\,,\ -0.6494\,,\ -0.0380\,,\ -0.0186\,,\ -0.0057 \\
a_i &= (\ \frac{14}{23}\,,\ \frac{16}{23}\,,\ -\frac{6}{13}\,,-\frac{12}{13}\,,\ 0.4086\,,\ -0.4230\,,\ -0.8994\,,\ 0.1456
\end{aligned}
\quad (5.2
$$

The functions $D'_0(x_t)$ and$E'_0(x_t)$ that are only defined for extra dimensions scenario can be found in [32–34] and are written as

$$D'_0(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4}\ln x_t \tag{5.23}$$

$$E'_0(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^4}\ln x_t \tag{5.24}$$

Now we study the decay $\Lambda_b \to \Lambda l^+ l^-$ in RS_c model. The effective Hamiltonian corresponding to RS_c model can be expressed as

$$
\begin{aligned}
\mathcal{H}^{eff}_{RS_c} = -\frac{G_F\alpha_{em}}{\sqrt{2}\pi}V_{tb}V^*_{ts}\Big\{ & C_9^{eff,RS_c}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l) + C_9^{\prime eff,RS_c}(\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l) \\
& + C_{10}^{eff,RS_c}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l) + C_{10}^{\prime eff,RS_c}(\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu \gamma_5 l) \\
& - \frac{2i}{q^2}C_7^{eff,RS_c}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{s}(m_b P_R + m_s P_L)(\bar{l}\gamma^\mu l) \\
& - \frac{2i}{q^2}C_7^{\prime eff,RS_c}\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{s}(m_b P_L + m_s P_R)(\bar{l}\gamma^\mu l)\Big\}
\end{aligned}
\tag{5.25}
$$

In the case of Randall Sundrum model the Wilson's coefficients are modified, and the corresponding modification in the Wilson's coefficients is given below

$$C_i^{(\prime)RS_c} = C_i^{(\prime)SM} + \Delta C_i^{(\prime)} \qquad i = 7, 9, 10, \tag{5.26}$$

Here

$$
\begin{aligned}
\Delta C_9 &= \left[\frac{\Delta Y_s}{\sin^2\theta_w} - 4\Delta Z_s\right], \\
\Delta C'_9 &= \left[\frac{\Delta Y'_s}{\sin^2\theta_w} - 4\Delta Z'_s\right],
\end{aligned}
\tag{5.27}
$$

$$\Delta C_{10} = -\frac{\Delta Y_s}{\sin^2\theta_w}, \tag{5.28}$$

and

$$\Delta C'_{10} = -\frac{\Delta Y'_s}{\sin^2\theta_w}, \tag{5.29}$$

with

$$\begin{aligned}
\Delta Y_s &= -\frac{1}{V_{tb}V^*_{ts}}\sum_X \frac{\Delta^{ll}_L(X)-\Delta^{ll}_R(X)}{4M^2_X g^2_{SM}}\Delta^{bs}_L(X) \\
\Delta Y'_s &= -\frac{1}{V_{tb}V^*_{ts}}\sum_X \frac{\Delta^{ll}_L(X)-\Delta^{ll}_R(X)}{4M^2_X g^2_{SM}}\Delta^{bs}_R(X) \\
\Delta Z_s &= +\frac{1}{V_{tb}V^*_{ts}}\sum_X \frac{\Delta^{ll}_R(X)}{8M^2_X g^2_{SM}\sin^2\theta_{(w)}}\Delta^{bs}_L(X)
\end{aligned} \tag{5.30}$$

and

$$\Delta Z'_s = +\frac{1}{V_{tb}V^*_{ts}}\sum_X \frac{\Delta^{ll}_R(X)}{8M^2_X g^2_{SM}\sin^2\theta_{(w)}}\Delta^{bs}_R(X)\,. \tag{5.31}$$

Also $X = Z, Z_H, Z'$ and $A^{(1)}$ and $g^2_{SM} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_w}$. With θ_w is the Weinberg angle. The functions that are used above in the expressions of $\Delta Y_s, \Delta Y'_s, \Delta Z_s$ and $\Delta Z'_s$ are given in references [35–43] .
For $\Delta C^{(\prime)}_7$, $\Delta C^{(\prime)}_7(\mu_b) = 0.429\Delta C^{(\prime)}_7(M_{KK}) + 0.128\Delta C^{(\prime)}_8(M_{KK})$ is used which include the following three contributions [37]

$$(\Delta C_7)_1 = iQ_u\, r\sum_{F=u,c,t}[A + 2m^2_F(A'+B')]\Big[\mathcal{D}^\dagger_L Y^u(Y^u)^\dagger Y^d\mathcal{D}_R\Big]_{23}$$

$$\begin{aligned}
(\Delta C_7)_2 = {}& -iQ_d\, r\frac{8}{3}(g^{4D}_s)^2\sum_{F=d,s,b}[I_0 + A + B + 4m^2_F(I'_0 + A' + B')] \\
& \Big[\mathcal{D}^\dagger_L\mathcal{R}_L Y^d\mathcal{R}_R\mathcal{D}_R\Big]_{23}
\end{aligned}$$

$$\begin{aligned}
(\Delta C_7)_3 = {}& iQ_d\, r\frac{8}{3}(g^{4D}_s)^2\sum_{F=d,s,b}m_F[I_0 + A + B]\Big\{\Big[\mathcal{D}^\dagger_L\mathcal{R}_L\mathcal{R}_L Y^d\mathcal{D}_R\Big] \\
& +\frac{m_b}{m_s}\Big[\mathcal{D}^\dagger_L Y^d\mathcal{R}_R\mathcal{R}_R\mathcal{D}_R\Big]_{23}\Big\}
\end{aligned}$$

$$
\begin{aligned}
(\Delta C_7')_1 &= iQ_u\, r \sum_{F=u,c,t} [A + 2m_F^2(A'+B')]\Big[\mathcal{D}_R^\dagger (Y^d)^\dagger Y^u (Y^u)^\dagger \mathcal{D}_L\Big]_{23} \\
(\Delta C_7')_2 &= -iQ_d\, r\frac{8}{3}(g_s^{4D})^2 \sum_{F=d,s,b} [I_0 + A + B + 4m_F^2(I_0' + A' + B')] \\
&\quad \Big[\mathcal{D}_R^\dagger \mathcal{R}_R (Y^d)^\dagger \mathcal{R}_L \mathcal{D}_L\Big]_{23} \qquad (5.32)
\end{aligned}
$$

$$
\begin{aligned}
(\Delta C_7')_3 &= iQ_d\, r\frac{8}{3}(g_s^{4D})^2 \sum_{F=d,s,b} m_F[I_0 + A + B]\Big\{\Big[\mathcal{D}_R^\dagger \mathcal{R}_R \mathcal{R}_R (Y^d)^\dagger \mathcal{D}_L\Big] \\
&\quad + \frac{m_b}{m_s}\Big[\mathcal{D}_R^\dagger (Y^d)^\dagger \mathcal{R}_L \mathcal{R}_L \mathcal{D}_L\Big]_{23}\Big\} \qquad (5.33)
\end{aligned}
$$

$$
\begin{aligned}
(\Delta C_8)_1 &= ir \sum_{F=u,c,t} [A + 2m_F^2(A'+B')]\Big[\mathcal{D}_L^\dagger Y^u (Y^u)^\dagger Y^d \mathcal{D}_R\Big]_{23} \\
(\Delta C_8)_2 &= -ir\frac{9}{8}(g_s^{4D})^2 \frac{\nu^2}{m_b m_s}\mathcal{T}_3 \sum_{F=d,s,b} [\bar{A} + \bar{B} + 2m_F^2(\bar{A}' + \bar{B}')] \\
&\quad \Big[\mathcal{D}_L^\dagger Y^d \mathcal{R}_R (Y^d)^\dagger \mathcal{R}_L Y^d \mathcal{D}_R\Big]_{23} \\
(\Delta C_8)_3 &= -ir\frac{9}{4}(g_s^{4D})^2 \mathcal{T}_3 \sum_{F=d,s,b} [\bar{A} + \bar{B} + 2m_F^2(\bar{A}' + \bar{B}')] \\
&\quad \Big[\mathcal{D}_L^\dagger \mathcal{R}_L Y^d \mathcal{R}_R \mathcal{D}_R\Big]_{23} \qquad (5.34)
\end{aligned}
$$

$$
\begin{aligned}
(\Delta C_8')_1 &= ir \sum_{F=u,c,t} [A + 2m_F^2(A'+B')]\Big[\mathcal{D}_R^\dagger (Y^d)^\dagger Y^u (Y^u)^\dagger \mathcal{D}_L\Big]_{23} \\
(\Delta C_8')_2 &= -ir\frac{9}{8}(g_s^{4D})^2 \frac{\nu^2}{m_b m_s}\mathcal{T}_3 \sum_{F=d,s,b} [\bar{A} + \bar{B} + 2m_F^2(\bar{A}' + \bar{B}')] \\
&\quad \Big[\mathcal{D}_R^\dagger (Y^d)^\dagger \mathcal{R}_L (Y^d) \mathcal{R}_R (Y^d)^\dagger \mathcal{D}_L\Big]_{23} \\
(\Delta C_8')_3 &= -ir\frac{9}{4}(g_s^{4D})^2 \mathcal{T}_3 \sum_{F=d,s,b} [\bar{A} + \bar{B} + 2m_F^2(\bar{A}' + \bar{B}')] \\
&\quad \Big[\mathcal{D}_R^\dagger \mathcal{R}_R (Y^d)^\dagger \mathcal{R}_L \mathcal{D}_L\Big]_{23} \qquad (5.35)
\end{aligned}
$$

the parameters $r = \frac{\nu}{\frac{G_F}{4\pi^2} V_{tb} V_{ts}^* m_b}$ and $\mathcal{T}_3 = \frac{1}{L}\int_0^L dy[g(y)]^2$. Q_u expresses the charge of up quark while Q_d represents the electric charge of down quark. The functions $I_0^{(\prime)}$,$A^{(\prime)}$, $B^{(\prime)}$ can be written as

$$
\begin{aligned}
I_0(t) &= \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^2}\left(-\frac{1}{t-1}+\frac{\ln(t)}{(t-1)^2}\right), \\
I_0(t)' &= \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^4}\left(\frac{1+t}{2t(t-1)^2}-\frac{\ln(t)}{(t-1)^3}\right), \\
A(t) &= B(t) = \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^2}\left(\frac{t-3}{(t-1)^2}+\frac{2\ln(t)}{(t-1)^3}\right), \\
A(t)' &= 2B'(t) = \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^4}\left(-\frac{t^2-5t-2}{6t(t-1)^3}-\frac{\ln(t)}{(t-1)^4}\right), \\
\bar{A}(t) &= \bar{B}(t) = \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^2}\left(-\frac{3t-1}{(t-1)^2}+\frac{2t^2\ln(t)}{(t-1)^3}\right), \\
\bar{A}'(t) &= \bar{B}'(t) = \frac{i}{(4\pi)^2}\frac{1}{M_{KK}^4}\left(\frac{5t+1}{(t-1)^3}-\frac{2t(2+t)\ln(t)}{(t-1)^4}\right),
\end{aligned}
\tag{5.36}
$$

where, $t = \frac{m_F^2}{M_{KK}^2}$. By fitting the parameters for the $B \to K^*\mu^+\mu^-$,The modified Wilson's Coefficient are given in the Table 5.1.

	ΔC_7	$\Delta C_7'$	ΔC_9	$\Delta C_9'$	ΔC_{10}	$\Delta C_{10}'$
Values	0.046	0.05	0.0023	0.038	0.030	0.50

Table 5.1: The Values of modifications in Wilson's Coefficients in RS_c model used in the analysis.

5.2 Parametrization of hadronic matrix elements

If we know the decay amplitude for the free quark, then we can determine the decay amplitude for the decay $\Lambda_b \to \Lambda l^+ l^-$ at hadron level by inserting the amplitude of free quarks between the final and initial states of baryons.There are four hadronic matrix elements corresponding to our amplitude given below, i.e.,

$$
\begin{aligned}
&\langle\Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P+q)\rangle \quad , \qquad \langle\Lambda(P)|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P+q)\rangle, \\
&\langle\Lambda(P)|\bar{s}\sigma_{\mu\nu} b|\Lambda_b(P+q)\rangle \quad , \qquad \langle\Lambda(P)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\Lambda_b(P+q)\rangle,
\end{aligned}
\tag{5.37}
$$

The above matrix elements are usually parametrized in terms of a series of form factors [44–48]. These can be written as

$$\langle\Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(g_1\gamma_\mu + g_2 i\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\Lambda_b(P+q), \tag{5.38}$$

$$\langle\Lambda(P)|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(G_1\gamma_\mu + G_2 i\sigma_{\mu\nu}q^\nu + G_3 q_\mu)\gamma_5\Lambda_b(P+q), \tag{5.39}$$

$$\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}q^\nu b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(f_1\gamma_\mu + f_2 i\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b(P+q), \tag{5.40}$$

$$\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\Lambda_b(P+q)\rangle = \overline{\Lambda}(P)(F_1\gamma_\mu + F_2 i\sigma_{\mu\nu}q^\nu + F_3 q_\mu)\gamma_5\Lambda_b(P+q), \tag{5.41}$$

The parameters g_i, G_i, f_i and F_i are representing the form factors and are the functions of q^2 (momentum transfer square).Because of conservation of vector current, as $q^\mu \bar{l}\gamma_\mu l = 0$. the form factors f_3 and F_3 do not take part in the decay amplitude of $\Lambda_b \to \Lambda l^+ l^-$.
The matrix elements including the scalar $\bar{s}b$ and pseudo-scalar $\bar{s}\gamma_5 b$ currents are also parametrized in terms of form factors,which we can be get by contracting q^μ to the both sides of Eq. (5.38) and Eq. (5.39).

$$\begin{aligned}\langle\Lambda(P)|\bar{s}b|\Lambda_b(P+q)\rangle &= \frac{1}{m_b+m_s}\overline{\Lambda}(P)[g_1(m_{\Lambda_b}-m_\Lambda) + g_3 q^2]\Lambda_b(P+q),\\ \langle\Lambda(P)|\bar{s}\gamma_5 b|\Lambda_b(P+q)\rangle &= \frac{1}{m_b-m_s}\overline{\Lambda}(P)[G_1(m_{\Lambda_b}+m_\Lambda) - G_3 q^2]\gamma_5\Lambda_b(P+q).\end{aligned} \tag{5.42}$$

We can also combine the above matrix elements with each other according to our requirement.

$$\begin{aligned}\langle\Lambda(P)|\bar{s}\gamma_\mu(1-\gamma_5)b|\Lambda_b(P+q)\rangle &= \overline{\Lambda}(P)[\gamma_\mu(g_1 - G_1\gamma_5) + i\sigma_{\mu\nu}(g_2 - G_2\gamma_5)q^\nu\\ &\quad + (g_3 - G_3\gamma_5)q_\mu]\Lambda_b(P+q),\end{aligned} \tag{5.43}$$

$$\begin{aligned}\langle\Lambda(P)|\bar{s}\gamma_\mu(1+\gamma_5)b|\Lambda_b(P+q)\rangle &= \overline{\Lambda}(P)[\gamma_\mu(g_1 + G_1\gamma_5) + i\sigma_{\mu\nu}(g_2 + G_2\gamma_5)q^\nu\\ &\quad + (g_3 + G_3\gamma_5)q_\mu]\Lambda_b(P+q),\end{aligned} \tag{5.44}$$

$$\begin{aligned}\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}q^\nu(1-\gamma_5)b|\Lambda_b(P+q)\rangle &= \overline{\Lambda}(P)[\gamma_\mu(f_1 - F_1\gamma_5) + i\sigma_{\mu\nu}(f_2 - F_2\gamma_5)\\ &\quad q^\nu + (f_3 - F_3\gamma_5)q_\mu]\Lambda_b(P+q),\end{aligned} \tag{5.45}$$

$$\begin{aligned}\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\Lambda_b(P+q)\rangle &= \overline{\Lambda}(P)[\gamma_\mu(f_1 + F_1\gamma_5) + i\sigma_{\mu\nu}(f_2 + F_2\gamma_5)\\ &\quad q^\nu + (f_3 + F_3\gamma_5)q_\mu]\Lambda_b(P+q),\end{aligned} \tag{5.46}$$

$$\begin{aligned}\langle\Lambda(P)|\bar{s}(1-\gamma_5)b|\Lambda_b(P+q)\rangle &= \frac{1}{m_b+m_s}\overline{\Lambda}(P)[(g_1 + G_1\gamma_5)m_{\Lambda_b} + (-g_1 + G_1\gamma_5)\\ &\quad m_\Lambda + (g_3 + G_3\gamma_5)q^2]\Lambda_b(P+q),\end{aligned} \tag{5.47}$$

5.3 Formula for Observables

In this section we will present the formulas for different physical observable. In order to do that the decay amplitude corresponding to $\Lambda_b \to \Lambda l^+ l^-$ in the SM can be expressed as

$$\mathcal{M}_{\Lambda_b \to \Lambda l^+ l^-} = -\frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[T_\mu^1 (\bar{l}\gamma^\mu l) + T_\mu^2 (\bar{l}\gamma^\mu \gamma_5 l) \right], \quad (5.48)$$

where the functions T_μ^1, T_μ^2 can be written as

$$\begin{aligned} T_\mu^1 &= \overline{\Lambda}(P) \Big[\{\gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu\} C_9^{eff,SM} \\ &+ \{\gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) + (g_3 + G_3\gamma_5) q_\mu\} C_9'^{eff,SM} \\ &- \frac{2i}{q^2} \{\gamma_\mu [f_1(m_b + m_s) + F_1(m_b - m_s)\gamma_5] + i\sigma_{\mu\nu} q^\nu [f_2(m_b + m_s) \\ &+ F_2(m_b - m_s)\gamma_5]\} C_7^{eff,SM} - \frac{2i}{q^2} \{\gamma_\mu [f_1(m_b + m_s) - F_1(m_b - m_s)\gamma_5] \\ &+ i\sigma_{\mu\nu} q^\nu [f_2(m_b + m_s) - F_2(m_b - m_s)\gamma_5]\} C_7'^{eff,SM} \Big] \Lambda_b (P + q), \end{aligned} \quad (5.49)$$

$$\begin{aligned} T_\mu^2 &= \overline{\Lambda}(P) \Big[\{\gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu\} C_{10}^{SM} \\ &+ \{\gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) + (g_3 + G_3\gamma_5) q_\mu\} C_{10}'^{SM} \Big] \Lambda_b (P + q), \end{aligned} \quad (5.50)$$

For simplicity, the decay amplitude can also be written in the following form.

$$\mathcal{M}_{\Lambda_b \to \Lambda l^+ l^-} = -\frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[T_\mu^1 (\bar{l}\gamma^\mu l) + T_\mu^2 (\bar{l}\gamma^\mu \gamma_5 l) \right],$$

Here

$$\begin{aligned} T_\mu^1 &= \bar{\Lambda}(P)[\gamma_\mu(A_1 + A_2\gamma_5) + i\sigma_{\mu\nu} q^\nu (B_1 + B_2\gamma_5) + q_\mu(D_1 + D_2\gamma_5)]\Lambda_b(P + q), \\ T_\mu^2 &= \bar{\Lambda}(P)[\gamma_\mu(A_3 + A_4\gamma_5) + i\sigma_{\mu\nu} q^\nu (B_3 + B_4\gamma_5) + q_\mu(D_3 + D_4\gamma_5)]\Lambda_b(P + q), \end{aligned} \quad (5.51)$$

The functions A_i, B_i and D_i are given by

$$
\begin{aligned}
A_1 &= g_1(C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} f_1(C_7^{eff,SM} + C_7^{\prime eff,SM}),\\
A_2 &= G_1(-C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} f_1(C_7^{eff,SM} - C_7^{\prime eff,SM}),\\
B_1 &= g_2(C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} f_2(C_7^{eff,SM} + C_7^{\prime eff,SM}),\\
B_2 &= G_2(-C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} F_2(C_7^{eff,SM} - C_7^{\prime eff,SM}),\\
D_1 &= g_3(C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} f_3(C_7^{eff,SM} + C_7^{\prime eff,SM}),\\
D_2 &= G_3(-C_9^{eff,SM} + C_9^{\prime eff,SM}) - \frac{2(m_b+m_s)}{q^2} F_3(C_7^{eff,SM} - C_7^{\prime eff,SM}),\\
A_3 &= g_1(C_{10}^{SM} + C_{10}^{\prime SM}),\\
A_4 &= G_1(-C_{10}^{SM} + C_{10}^{\prime SM}),\\
B_3 &= g_2(C_{10}^{SM} + C_{10}^{\prime SM}),\\
B_4 &= G_2(-C_{10}^{SM} + C_{10}^{\prime SM}),
\end{aligned}
\tag{5.52}
$$

where $C_7^{\prime eff,SM}=0$, $C_9^{\prime eff,SM}=0$ and $C_{10}^{\prime SM}=0$.
Now the Decay amplitude for $\Lambda_b \to \Lambda l^+ l^-$ in context of RS_c is

$$
\mathcal{M}_{RS_c}^{\Lambda_b\to\Lambda l^+l^-} = \frac{G_F \alpha_{em}}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[M_\mu^1 (\bar{l}\gamma^\mu l) + M_\mu^2 (\bar{l}\gamma^\mu \gamma_5 l) \right], \tag{5.53}
$$

where the functions M_μ^1, M_μ^2 are defined below.

$$
\begin{aligned}
M_\mu^1 &= \overline{\Lambda}(P)\Big[\{\gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu\} C_9^{eff,RS_c} \\
&\quad + \{\gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) + (g_3 + G_3\gamma_5) q_\mu\} C_9^{\prime eff,RS_c} \\
&\quad - \frac{2i}{q^2} \{\gamma_\mu [f_1(m_b+m_s) + F_1(m_b-m_s)\gamma_5] + i\sigma_{\mu\nu} q^\nu [f_2(m_b+m_s) \\
&\quad + F_2(m_b-m_s)\gamma_5]\} C_7^{eff,RS_c} - \frac{2i}{q^2} \{\gamma_\mu [f_1(m_b+m_s) - F_1(m_b-m_s)\gamma_5] \\
&\quad + i\sigma_{\mu\nu} q^\nu [f_2(m_b+m_s) - F_2(m_b-m_s)\gamma_5]\} C_7^{\prime eff,RS_c} \Big] \Lambda_b(P+q),
\end{aligned}
\tag{5.54}
$$

$$
\begin{aligned}
M_\mu^2 &= \overline{\Lambda}(P)\bigg[\left\{\gamma_\mu\left(g_1 - G_1\gamma_5\right) + i\sigma_{\mu\nu}q^\nu\left(g_2 - G_2\gamma_5\right) + \left(g_3 - G_3\gamma_5\right)q_\mu\right\}C_{10}^{RS_c} \\
&\quad + \left\{\gamma_\mu\left(g_1 + G_1\gamma_5\right) + i\sigma_{\mu\nu}q^\nu\left(g_2 + G_2\gamma_5\right) + \left(g_3 + G_3\gamma_5\right)q_\mu\right\}C_{10}^{\prime RS_c}\bigg] \\
&\quad \Lambda_b\left(P + q\right),
\end{aligned} \tag{5.55}
$$

where

$$
\begin{aligned}
C_7^{eff,RS_c} &= C_7^{eff,SM} + \Delta C_7 \ , \\
C_7^{\prime eff,RS_c} &= C_7^{\prime eff,SM} + \Delta C_7' \ , \\
C_9^{eff,RS_c} &= C_9^{eff,SM} + \Delta C_9 \ , \\
C_9^{\prime eff,RS_c} &= C_9^{\prime eff,SM} + \Delta C_9' \ , \\
C_{10}^{RS_c} &= C_{10}^{SM} + \Delta C_{10} \ , \\
C_{10}^{\prime RS_c} &= C_{10}^{\prime SM} + \Delta C_{10}' \ .
\end{aligned}
$$

For the ease, the decay amplitude can also be written as follow.

$$
\mathcal{M}_{\Lambda_b \to \Lambda l^+ l^-} = -\frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^*\left[M_\mu^1(\bar{l}\gamma^\mu l) + M_\mu^2(\bar{l}\gamma^\mu\gamma_5 l)\right],
$$

with

$$
\begin{aligned}
M_\mu^1 &= \bar{\Lambda}(P)[\gamma_\mu(\mathcal{A}_1 + \mathcal{A}_2\gamma_5) + i\sigma_{\mu\nu}q^\nu(\mathcal{B}_1 + \mathcal{B}_2\gamma_5) + q_\mu(\mathcal{D}_1 + \mathcal{D}_2\gamma_5)]\Lambda_b(P+q), \\
M_\mu^2 &= \bar{\Lambda}(P)[\gamma_\mu(\mathcal{A}_3 + \mathcal{A}_4\gamma_5) + i\sigma_{\mu\nu}q^\nu(\mathcal{B}_3 + \mathcal{B}_4\gamma_5) + q_\mu(\mathcal{D}_3 + \mathcal{D}_4\gamma_5)]\Lambda_b(P+q),
\end{aligned} \tag{5.56}
$$

The functions $\mathcal{A}_i$, $\mathcal{B}_i$ and $\mathcal{D}_i$ are given by

$$
\begin{aligned}
\mathcal{A}_1 &= g_1(C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}) - \frac{2(m_b + m_s)}{q^2}f_1(C_7^{eff,RS_c} + C_7^{\prime eff,RS_c}), \\
\mathcal{A}_2 &= G_1(-C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}) - \frac{2}{q^2}f_1(C_7^{eff,RS_c} - C_7^{\prime eff,RS_c}), \\
\mathcal{B}_1 &= g_2(C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}) - \frac{2(m_b - m_s)}{q^2}f_2(C_7^{eff,RS_c} + C_7^{\prime eff,RS_c}), \\
\mathcal{B}_2 &= G_2(-C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}) - \frac{2(m_b - m_s)}{q^2}F_2(C_7^{eff,RS_c} - C_7^{\prime eff,RS_c}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_1 &= g_3(C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}), \\
\mathcal{D}_2 &= G_3(-C_9^{eff,RS_c} + C_9^{\prime eff,RS_c}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_3 &= g_1(C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
\mathcal{A}_4 &= G_1(-C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
\mathcal{B}_3 &= g_2(C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
\mathcal{B}_4 &= G_2(-C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
\mathcal{D}_3 &= g_3(C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
\mathcal{D}_4 &= G_3(-C_{10}^{RS_c} + C_{10}'^{RS_c}), \\
&. \qquad (5.57)
\end{aligned}
$$

5.3.1 The differential decay rates of $\Lambda_b \to \Lambda l^+ l^-$

We can write the differential decay rate of $\Lambda_b \to \Lambda l^+ l^-$ in the rest frame of Λ_b baryon [49] as:

$$
\frac{d\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{dq^2} = \frac{1}{(2\pi)^3} \frac{1}{32 m_{\Lambda_b}^3} \int_{u_{min}}^{u_{max}} |\widetilde{M}_{\Lambda_b \to \Lambda l^+ l^-}|^2 du, \qquad (5.58)
$$

where

$$
u = (p_\Lambda + p_{l^-})^2 ,
$$

and

$$
s = (p_{l^+} + p_{l^-})^2 = q^2 ,
$$

with p_Λ representing the four momentum of Λ,while p_{l^+} and p_{l^-} represent the four-momenta vectors of l^+ and l^- respectively. $\widetilde{M}_{\Lambda_b \to \Lambda l^+ l^-}$ represents the decay amplitude after performing the integration over the angle between the Λ baryon and l^-. The lower and upper limits of u are written as

$$
\begin{aligned}
u_{max} &= (E_\Lambda^* + E_l^*)^2 - \left(\sqrt{E_\Lambda^{*2} - m_\Lambda^2} - \sqrt{E_l^{*2} - m_l^2}\right)^2, \\
u_{min} &= (E_\Lambda^* + E_l^*)^2 - \left(\sqrt{E_\Lambda^{*2} - m_\Lambda^2} + \sqrt{E_l^{*2} - m_l^2}\right)^2, \qquad (5.59)
\end{aligned}
$$

with E_Λ^* representing the energies of Λ and E_l^* denote the energies of l^- in the rest frame of $l^+ l^-$,

$$
E_\Lambda^* = \frac{m_{\Lambda_b}^2 - m_\Lambda^2 - q^2}{2\sqrt{q^2}}, \qquad E_l^* = \frac{\sqrt{q^2}}{2}. \qquad (5.60)
$$

Using everything, we can get the decay rate of $\Lambda_b \to \Lambda l^+ l^-$ as

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{\alpha^2 G_F^2 \left|V_{tb}V_{ts}^*\right|^2}{128 m_{\Lambda_b}^3 \pi^4} \sqrt{1-\frac{4m_l^2}{q^2}} \sqrt{\lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2)} \times \\
& \Bigg\{ + m_{\Lambda_b} m_b (1+2m_l^2/q^2)((m_{\Lambda_b}^2 - m_\Lambda^2 - t)(f_2^2 + g_2^2 q^2) - 4 f_2 g_2 q^2 m_{\Lambda_b}) \\
& (C_9^{*eff} C_7^{eff} + C_7^{*eff} C_9^{eff}) + q^2 \frac{m_{\Lambda_b}^2}{2m_b^2}((m_\Lambda + \frac{2}{3q^2} m_b^2 (1+2m_l^2/q^2)(\lambda(2f_2^2 + g_2^2 q^2) \\
& +3q^2(m_{\Lambda_b}^2 + m_\Lambda^2 - q^2)(f_2^2 + g_2^2 q^2) + 6 f_2 g_2 q^2 m_\Lambda (m_{\Lambda_b}^2 - m_\Lambda^2 + q^2)) \left|C_7^{eff}\right|^2 \\
& +\frac{1}{6}(1+2m_l^2/q^2)(\lambda(f_2^2 + 2g_2^2 q^2) + 3q^2(m_{\Lambda_b}^2 + m_\Lambda^2 - q^2)(f_2^2 + g_2^2 q^2) \\
& +6 f_2 g_2 q^2 m_\Lambda (m_{\Lambda_b}^2 - m_\Lambda^2 + q^2)) \left|C_9^{eff}\right|^2 \\
& +\frac{1}{6}(((1+2m_l^2/q^2)\lambda + 3(1-2m_l^2/q^2)(m_{\Lambda_b}^2 + m_\Lambda^2 - q^2)) f_2^2 - g_2^2 q^2 (1-4m_l^2/q^2) \\
& (\lambda - 3((m_{\Lambda_b}^2 - m_\Lambda^2)^2 + (m_{\Lambda_b}^2 + m_\Lambda^2) q^2)) + 6 f_2 g_2 q^2 m_\Lambda (1-4m_l^2/q^2)(m_{\Lambda_b}^2 - m_\Lambda^2 + q^2)) \\
& \left|C_{10}\right|^2 \Bigg\}. \qquad (5.61)
\end{aligned}$$

Here,

$$\lambda = \lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2) = m_{\Lambda_b}^4 + m_\Lambda^4 + q^4 - 2m_{\Lambda_b}^2 m_\Lambda^2 - 2m_\Lambda^2 q^2 - 2q^2 m_{\Lambda_b}^2. \qquad (5.62)$$

Some input parameters	Values
m_μ	0.10565 GeV
m_τ	1.77682 GeV
m_c	1.275±0.025 GeV
m_b	4.18 ± 0.03 GeV
m_t	$173.21 \pm 0.51 \pm 0.71$ GeV
m_W	80.385 ± 0.015 GeV
m_{Λ_b}	5.6195 ± 0.0004 GeV
m_Λ	1.11568 GeV
τ_{Λ_b}	$(1.451 \pm 0.013) \times 10^{-12}$ s
$\hbar$	$6.85 \times 10^{-25} GeV^{-2}$
α_{em}	$\frac{1}{137}$
$\lvert V_{tb}V_{ts}^*\rvert$	0.040

Table 5.2: The values of input parameters that are required in our calculations.

parameter	COZ	FZOZ	QCDSR	twist-3	up to twist-6
$f_2(0)$	$0.74^{+0.06}_{-0.06}$	$0.87^{+0.07}_{-0.07}$	0.45	$0.14^{+0.02}_{-0.01}$	$0.15^{+0.02}_{-0.02}$
a_1	$2.01^{+0.17}_{-0.10}$	$2.08^{+0.15}_{-0.09}$	0.57	$2.91^{+0.10}_{-0.07}$	$2.94^{+0.11}_{-0.06}$
a_2	$1.32^{+0.14}_{-0.08}$	$1.41^{+0.11}_{-0.08}$	-0.18	$2.26^{+0.13}_{-0.08}$	$2.31^{+0.14}_{-0.10}$
$g_2(0)(10^{-2}\text{GeV}^{-1})$	$-2.4^{+0.3}_{-0.2}$	$-2.8^{+0.4}_{-0.2}$	-1.4	$-0.47^{+0.06}_{-0.06}$	$1.3^{+0.2}_{-0.4}$
a_1	$2.76^{+0.16}_{-0.13}$	$2.80^{+0.16}_{-0.11}$	2.16	$3.40^{+0.06}_{-0.05}$	$2.91^{+0.12}_{-0.09}$
a_2	$2.05^{+0.23}_{-0.13}$	$2.12^{+0.21}_{-0.13}$	1.46	$2.98^{+0.09}_{-0.08}$	$2.24^{+0.17}_{-0.13}$

Table 5.3: Numerical values of the form factors used in our calculations

In order to calculate the numerical values of the branching ratio, the numerical values of form factors are given in Table 5.3. and the values of the parameters are described in Table 5.2.
The branching ratio of the decay $\Lambda_b \to \Lambda l^+ l^-$ as a function of di-lepton momentum transfered square q^2 in the SM and RS_c model is given in the following figure.

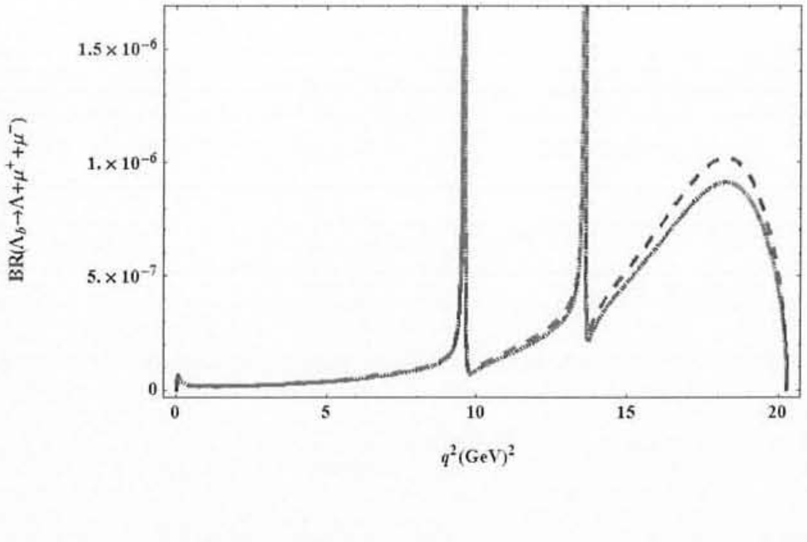

Figure 5.1: The branching ratio of $\Lambda_b \to \Lambda\mu^+\mu^-$ in SM and RS model. The solid line corresponds to the SM results with central values of form factors. The dashed line is for RS_c.

From the above graphs we can say that the predictions of the SM and the Randall Sundrum model with custodial protection are almost same excluding some values of q^2 where the predictions of SM and RS_c have little difference in the case of branching ratio, which is not enough for the NP predictions.

5.3.2 FBAs of $\Lambda_b \to \Lambda l^+ l^-$

Now we can traverse the FBAs of $\Lambda_b \to \Lambda l^+ l^-$, which is an important hint for the possible NP. To determine the forward-backward asymmetry, we take the double differential decay rate formula given below for the process $\Lambda_b \to \Lambda l^+ l^-$

$$\frac{d^2\Gamma(q^2,z)}{dq^2dz} = \frac{1}{(2\pi)^3}\frac{1}{64m^3_{\Lambda_b}}\lambda^{1/2}(m^2_{\Lambda_b}, m^2_{\Lambda}, q^2)\sqrt{1-\frac{4m_l^2}{q^2}}|\widetilde{M}_{\Lambda_b\to\Lambda l^+l^-}|^2 , \tag{5.63}$$

Following references [50, 51], the normalized and differential FBAs for the semi-leptonic decay $\Lambda_b \to \Lambda l^+ l^-$ are defined as

$$\frac{dA_{FB}(q^2)}{dq^2} = \int_0^1 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz} - \int_{-1}^0 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz} \tag{5.64}$$

and

$$A_{FB}(q^2) = \frac{\int_0^1 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz} - \int_{-1}^0 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz}}{\int_0^1 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz} + \int_{-1}^0 dz\frac{d^2\Gamma(q^2,z)}{dq^2dz}}. \tag{5.65}$$

Using the decay amplitude that is given in Eq. (5.6), the forward backward asymmetries for the $\Lambda_b \to \Lambda$ transition can be determined as

$$\frac{dA_{FB}(q^2)}{dq^2} = \frac{G_F^2\alpha_{em}^2|V_{tb}V_{ts}^*|^2}{256m^3_{\Lambda_b}\pi^5}\lambda(m^2_{\Lambda_b}, m^2_{\Lambda}, q^2)(1-\frac{4m_l^2}{q^2})R_{FB}(q^2) \tag{5.66}$$

where

$$\begin{aligned} R_{FB}(q^2) &= 2\Big[(m_s m_\Lambda + m_b m_{\Lambda_b})f_2^2 - m_s(m_\Lambda^2 - m_{\Lambda_b}^2 + q^2)f_2 g_2 \\ &\quad + (m_s m_\Lambda - m_b m_{\Lambda_b})q^2 g_2^2\Big] Re(C_7^{eff}C_{10}^*) \\ &\quad + \Big[(f_2 - g_2 m_\Lambda)^2 - g_2^2 m_{\Lambda_b}^2\Big] Re(C_9^{eff}C_{10}^*) \end{aligned} \tag{5.67}$$

By taking form factors along their uncertainties and plotting the forward backward asymmetry of $\Lambda_b \to \Lambda\mu^+\mu^-$ dependence on q^2 for the under consideration decay in SM and RS_c model.

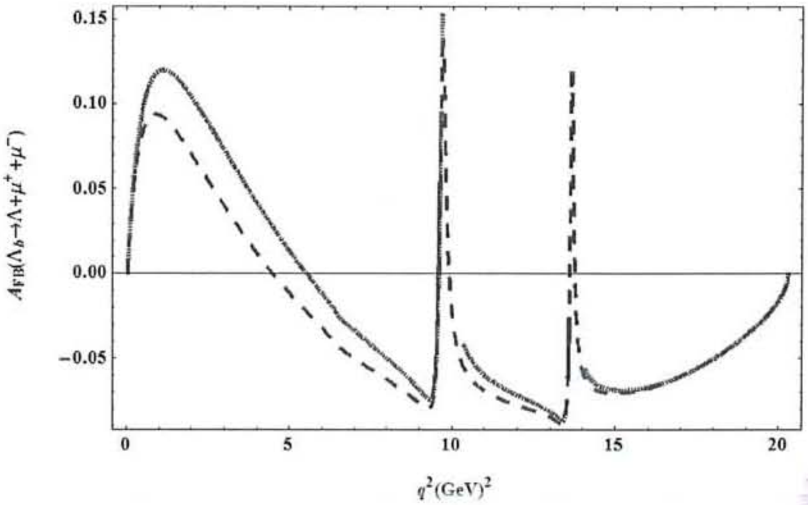

Figure 5.2: The forward backward asymmetry of $\Lambda_b \to \Lambda\mu^+\mu^-$ in SM and RS model. The legends are same as in Fig. 5.1.

The zero position of q^2 is given as [53]

$$Re\left(C_9^{eff}(q_0^2) \right) = -\frac{m_b}{q^2} C_7^{eff} \left\{ \frac{f_2(q_0^2)}{g_2(q_0^2)}(1 - m_\Lambda) + \frac{f_1(q_0^2)}{g_1(q_0^2)}(1 + m_\Lambda) \right\} ,$$

that depends upon the m_b, the ratio of effective Wilson's coefficients $\frac{C_7^{eff}(q^2)}{Re\left(C_9^{eff}(q^2) \right)}$ and ratio of form factors as shown in the above equation. The ratios of form factors present in the above equation are independent of hadronic uncertainty. Hence, the accuracy of the zero position of forward-backward asymmetry is calculated by the accuracy of ratio of Wilson's effective coefficients and m_b. The above graphs show the dependence of forward-backward asymmetry (A_{FB}) on the dilepton momentum transfer q^2. From these graphs we can see that the zero position of A_{FB} for $\Lambda_b \to \Lambda\mu^+\mu^-$ is sensitive to the extra dimensions i.e., the zero position of A_{FB} is moved to the left as compared to the SM predictions. So A_{FB} can be used as the probe for NP.

5.3.3 Λ polarization in $\Lambda_b \to \Lambda l^+l^-$

If we want to know the spin polarization of Λ, then it is necessary to write the four spin vector Λ in terms of a unit vector $\hat{\xi}$ in the direction of the Λ spin in its rest frame, like [52].

$$s_0 = \frac{\vec{p}_\Lambda \cdot \vec{\xi}}{m_\Lambda}, \; \vec{s} = \vec{\xi} + \frac{s_0}{E_\Lambda + m_\Lambda}\vec{p}_\Lambda, \tag{5.68}$$

where the unit vectors are chosen in the direction of transverse, longitudinal and the normal components of the polarization of the Λ ,

$$\begin{aligned} \hat{e}_L &= \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|}, \\ \hat{e}_N &= \frac{\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)}{|\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)|}, \\ \hat{e}_T &= \frac{\vec{p}_- \times \vec{p}_\Lambda}{|\vec{p}_- \times \vec{p}_\Lambda|}. \end{aligned}$$

The polarization asymmetries of Λ baryon in $\Lambda_b \to \Lambda l^+l^-$ are defined as

$$P_i^{(\mp)}(s) = \frac{\frac{d\Gamma}{dq^2}(\vec{\xi} = \hat{e}) - \frac{d\Gamma}{dq^2}(\vec{\xi} = -\hat{e})}{\frac{d\Gamma}{dq^2}(\vec{\xi} = \hat{e}) + \frac{d\Gamma}{dq^2}(\vec{\xi} = -\hat{e})} \tag{5.69}$$

where $i = L,\ N,\ T$ and $\vec{\xi}$ are representing the direction of spin along the Λ. The polarized differential decay rate of Λ_b baryon in the decay $\Lambda_b \to \Lambda l^+l^-$ along any spin direction $\vec{\xi}$ is associated to the unpolarized differential decay rate Eq. (5.58) by the relation given below

$$\frac{d\Gamma(\vec{\xi})}{dq^2} = \frac{1}{2}\left(\frac{d\Gamma}{dq^2}\right)[1 + (P_L\vec{e}_L + P_N\vec{e}_N + P_T\vec{e}_T)\cdot\vec{\xi}]. \tag{5.70}$$

The expressions of the longitudinal and normal polarizations of Λ baryon are given below :

$$
\begin{aligned}
P_L(q^2) &= (1/\frac{d\Gamma}{dq^2})\frac{\alpha^2 G_F^2 \left|V_{tb}V_{ts}^*\right|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2)}{64 m_\Lambda m_{\Lambda_b}^3 \pi^5 q^3}\sqrt{1-\frac{4m_l^2}{q^2}} \times \Big\{ \\
&+m_\Lambda m_{\Lambda_b} m_l (2m_l^2+s)(g_2^2 s - f_2^2)\sqrt{s}(C_7^{*eff}C_9^{eff} + C_7^{eff}C_9^{*eff}) \\
&+\frac{m_b^2}{3}\Big[\frac{m_\Lambda}{\sqrt{q^2}}((12m_l^2((m_\Lambda^2 - m_{\Lambda_b}^2)f_2^2 + g_2^2 q^4)) - 3q^2(q^2 - m_\Lambda^2 + m_{\Lambda_b}^2)(f_2^2 - g_2^2 q^2) \\
&+(1-\frac{4m_l^2}{q^2})(q^2 + m_\Lambda^2 - m_{\Lambda_b}^2)(f_2^2 + g_2^2 q^2))\Big] \Big|C_7^{eff}\Big|^2 \\
&+\frac{q^2}{12}\Big[12 m_l^2 m_\Lambda \sqrt{q^2}((\left|C_{10}\right|^2 - \Big|C_9^{eff}\Big|^2) f_2^2 + g_2^2((m_{\Lambda_b}^2 - m_\Lambda^2)\left|C_{10}\right|^2 - (q^2 - m_\Lambda^2 + m_{\Lambda_{\cdot}}^2 \\
&+m_\Lambda\sqrt{q^2}(f_2^2 + g_2^2 t)(3(q^2 - m_\Lambda^2 + m_{\Lambda_b}^2) - (1-\frac{4m_l^2}{q^2})(m_\Lambda^2 - m_{\Lambda_b}^2 + q^2))(\Big|C_9^{eff}\Big|^2 + |(\\
P_N(q^2) &= (1/\frac{d\Gamma}{dq^2})\frac{\alpha^2 G_F^2 \left|V_{tb}V_{ts}^*\right|^2 \sqrt{\lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2)}}{512 m_b m_{\Lambda_b}^3 \pi^4 \sqrt{q^2}}(1-\frac{4m_l^2}{q^2}) \times \\
&\Big\{ - m_b q^2 (m_{\Lambda_b}^2 - m_\Lambda^2 + q^2)(m_\Lambda f_2^2 - g_2(m_\Lambda^2 - m_{\Lambda_b}^2 + q^2) f_2 + g_2^2 m_\Lambda t)(C_{10}C_9^{*eff} + C_1 \\
&+4 m_b m_{\Lambda_b} q^2 (-m_\Lambda f_2^2 + g_2(m_\Lambda f_2^2 - g_2(m_\Lambda^2 - m_{\Lambda_b}^2 + q^2) f_2 + g_2^2 m_\Lambda t)(C_{10}C_9^{*eff} + C_{10}^* \\
&+4 m_b m_{\Lambda_b} q^2 (-m_\Lambda f_2^2 + g_2(m_\Lambda^2 - m_{\Lambda_b}^2 + q^2) f_2 - g_2^2 m_\Lambda t)(C_{10}C_7^{*eff} + C_{10}^* C_7^{eff})\Big\},
\end{aligned}
$$

where the expression for the $\lambda(m_{\Lambda_b}^2, m_\Lambda^2, q^2)$ is same as we defined before in Eq. (5.62) and for more compact expression the mass of strange quark is neglected .
In present case $P_T(q^2) = 0$ because here the scalar currents are not possible. Now we plot the dependence of longitudinal polarization of Λ baryon on dilepton momentum transfer q^2 in the SM and RS_c model scenario.

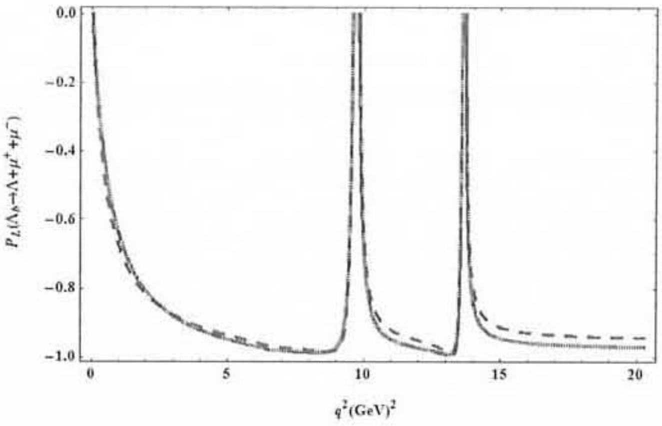

Figure 5.3: The Longitudinal Polarization asymmetry of Λ baryon in the SM and RS_c model. The legends are same as in Fig. 5.1.

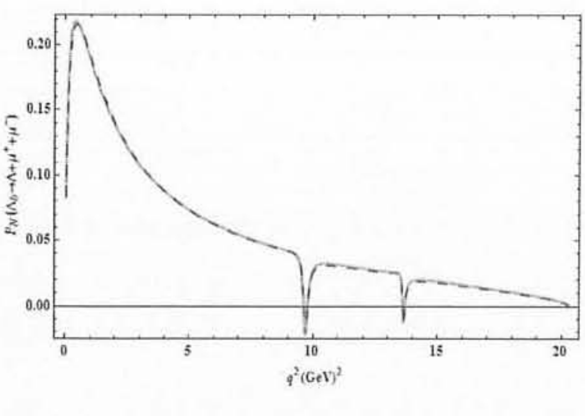

Figure 5.4: The Transverse Polarization asymmetry of Λ baryon in the SM and RS_c model. The legends are same as in Fig. 5.1.

The above diagrams show the longitudinal and normal polarization asymmetries of Λ baryon both in RS_c model and SM. From these graphs we can observe that there is no reasonable difference between the predictions of the SM and the RS_c model. So, we can say that the polarization asymmetries are not useful for establishing NP.

5.4 Conclusion

In this dissertation, we examine the rare $\Lambda_b \to \Lambda l^+ l^-$ decay channel in the SM and the RS_c model. We prefer to examine the bottom decays because these decays are sensitive to the flavor structure which leads to very rich phenomenology. The hadronic matrix elements involved in the decay $\Lambda_b \to \Lambda l^+ l^-$ are parametrize in terms of form factors which are calculated by light cone sum rules (LCSR). It is found that there is a reasonable difference between the zero position of A_{FB} of the SM and RS_c model which is due to the ratio of effective Wilson's coefficients and the mass of bottom quark m_b. So, forward-backward asymmetry is sensitive to the extra dimensions and it can be used for the exploration of NP effects. However, the polarizations of Λ baryon are not sensitive to the extra dimensions. So, polarization gives no signal for the NP effects. In the case of branching ratio the predictions of the SM and the RS_c are also almost the same except some values of q^2 where a very small difference has been observed, which is not enough for the NP predictions.

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