

INTERPRETATION OF BUTTERWORTH (BANDPASS) FILTERING



BY

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M.SC. GEOPHYSICS**

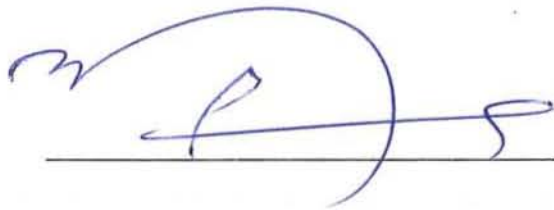
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FINAL APPROVAL OF THESIS

This dissertation submitted by Mr. Muhammad Aamir is accepted in its present form by the department of **Earth Sciences** as a requirement for the award of M.Sc degree in Geophysics.

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External Examiner

A handwritten signature in blue ink, consisting of a large, stylized 'D' shape with a horizontal line extending to the right, positioned above a horizontal line.

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A handwritten signature in blue ink, appearing to be 'Mubarik Ali', positioned above a horizontal line.

DEDICATED

TO

**MY CARING PARENTS
LOVING SISTER**

&

SINCERE FRIENDS

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ABSTRACT

The basic purpose of the present work is to understand the seismic filtering procedure with the special emphasis on the Butterworth (band pass) filtering.

The basic approach of interpretation is practical. Intuitive (and theoretically sound) arguments are used to explain the theory with extensive examples.

Implementation of Butterworth filtering was carried out both in the time domain and frequency domain.

The amplitude response improves as the order of Butterworth filter increases. Higher Butterworth order means higher slopes and ideal filtering can be achieved.

CHAPTER # 1

INTRODUCTION

INTRODUCTION

1.1 INTRODUCTION

Implementation of filtering was carried out in the program named SAP 2d. The SAP program is designed for training purposes. This program has some limitations listed below.

TRACE LENGTH: It is advisable to use data upto 2048 samples.

NUMBER of TRACES: There is no limit on number of traces.

PROCESSING: The plan is to have only those processes that can be applied after stack.

w85-8.sgy is the given seismic line. The seismic data in this line is filtered using Bandpass filtering, trapezoidal filtering and Butterworth filtering.

In the most general sense, a “filter” is a device or a system that alters in a prescribed way the input that passes through it. In essence, a filter converts inputs into outputs in such a fashion that certain desirable features of the inputs are retained in the outputs while undesirable features are suppressed.

The topic of my dissertation is Butterworth Bandpass filtering. In the dissertation Butterworth filter's responses are studied with the help of extensive examples. Magnitude function of Butterworth filters, Passband attenuation, Stopband attenuation, Phase characteristics of Butterworth filters, Passband amplitude response of different orders, Stopband amplitude response of different orders, Impulse responses, Step responses of different order butterworth filter are mentioned in figures to explain the Butterworth responses precisely.

Finally in the interpretation the input data convolved with a zero phase Butterworth filter, the result came in the fashion that overall amplitude decreases. In the SAP 1d minimum Butterworth order ($n=1$) produces side lobes. As the Butterworth order increases, side lobes are reduced and the wavelet is more compressed in time.

Butterworth Bandpass Filter has the following options

Filter corners

Four corner frequencies specified in Hz

Hamming Operator

Hamming operator check box when checked will apply the smoothing function.

Operator Length

Operator length in ms can be set as desired.

Order

Order of the filter is set to > 0 and < 10 . Higher order produces a sharper signal in time domain

CHAPTER # 2

SEISMIC DATA PROCESSING

SEISMIC DATA PROCESSING

2.1 SEISMIC DATA PROCESSING

Seismic data processing is the approach by which raw data recorded in the field is enhanced to the extent that it can be used for geological interpretation. This approach involves the sequence of operations for improving signal to noise ratio. Dobrin and Savit(1988), Yilmaz(1978), Keary and Brooks(1991), Sadi (1980), Sheriff(1989), Robinson and Coruh(1988) and OGTI Manual(1989) have discussed in detail the different sequences of data processing. The following is the processing sequences.

2.2 DEMULTIPLEXING

Data recorded on digital magnetic tape is not suitable for analysis therefore it is assembled from the digital tape by a sorting process. Thus "the process of sorting data from the magnetic tape into individual channel sequence is called demultiplexing."

2.3 VIBROSEIS CORRELATION

The signal generated by a vibroseis is not a short pulse but rather a sweep lasting seven to ten seconds. The sweep is transmitted through earth and is reflected from interfaces. Each reflection is a near duplicate of a sweep itself, so the reflections in vibroseis record overlap and are indistinguishable. To make it useable reflections are compressed into wavelets through cross-correlation of data with the original input sweep. After correlation each reflection on record looks similar to an impulsive source data. (Rehman, 1989).

2.4 EDITING

Editing mainly involves the removal of any obviously unusable data, for example, a record or part of it that may have come out too weak for one or other reason. The traces or the relevant parts of the traces are eliminated by setting all amplitudes to zero. (Parasnis, 1977)

2.5 AMPLITUDE ADJUSTMENT

In a seismic section the variations in the amplitudes of reflections are important factor in the interpretation. In some cases, the amplitude variations are so great that low level events become difficult to follow or even invisible. In order to raise the level of these weak events relative to the strong ones, so that geologic structure can be made visible, the analyst can apply a digital "Balance" or "AGC" (Automatic Gain Control). (Rehman, 1989)

2.6 DECONVOLUTION

When a dynamite is blasted, spike is produced that travels through earth, its amplitude decreases, higher frequencies are absorbed and it becomes a waveform of lower frequency and greater wavelength. This behavior of earth is termed as hi-cut filter. Deconvolution is a reverse process by which those lost higher frequencies are reproduced. "Deconvolution actually improves the temporal resolution of seismic data with compression of reflecting pulses, and suppresses the effect of multiples".

2.7 STATIC CORRECTION

Weathering layer and topographic effect influence the reflection times, which are independent of subsurface structure. The static correction which comprises weathering and elevation corrections, is applied to each trace for the removal of the effects of low velocity layer and surface undulations above the chosen datum plane.

2.8 NMO CORRECTION

Dynamic time correction or NMO correction converts the times of reflections into coincidence with those that would have been recorded at zero offset. It is a function of offset, velocity and reflector depth, and is derived from the velocity-time function. NMO correction is applied to each trace in each CDP gather to reduce the reflection times to two-way vertical time.

2.9 VELOCITY ANALYSIS

Velocity in seismic processing is an important parameter, which controls the stacking quality. Thus the proper velocity value gives the optimum dynamic correction which leads to efficient stacking process. The seismic traces of a common-depth-point-gather are basis for velocity analysis. Before velocity analysis suitable static correction and data enhancement procedures are applied to the data.

A series of normal moveout corrections, each based on arbitrary constant velocity are then applied to each trace of data set. Then NMO corrected traces are stacked to produce a single output trace. This calculation is repeated with incremented constant velocity, the range extends from minimum to maximum velocities to be encountered in the area. The velocity increments may not be uniform but may be rather small for application of slower velocities. A plot of velocities against recorded time for each analysis location represents the velocity function for that location.

2.10 STACKING

stacking is the reduction of data volume to a plane of mid point time at zero offsets by summing them along the offset axis. It enhances the reflections from true reflectors and attenuates unwanted interference or noise (Yilmaz, 1987; Badly, 1985)

2.11 MIGRATION

Migration is usually the last step applied to the processed seismic data for repositioning reflected energy from its common mid point position to its true map real picture of the subsurface structure.

CHAPTER # 3

FILTERING

FILTERING

3.1 PRINCIPLES OF DIGITAL FILTERING

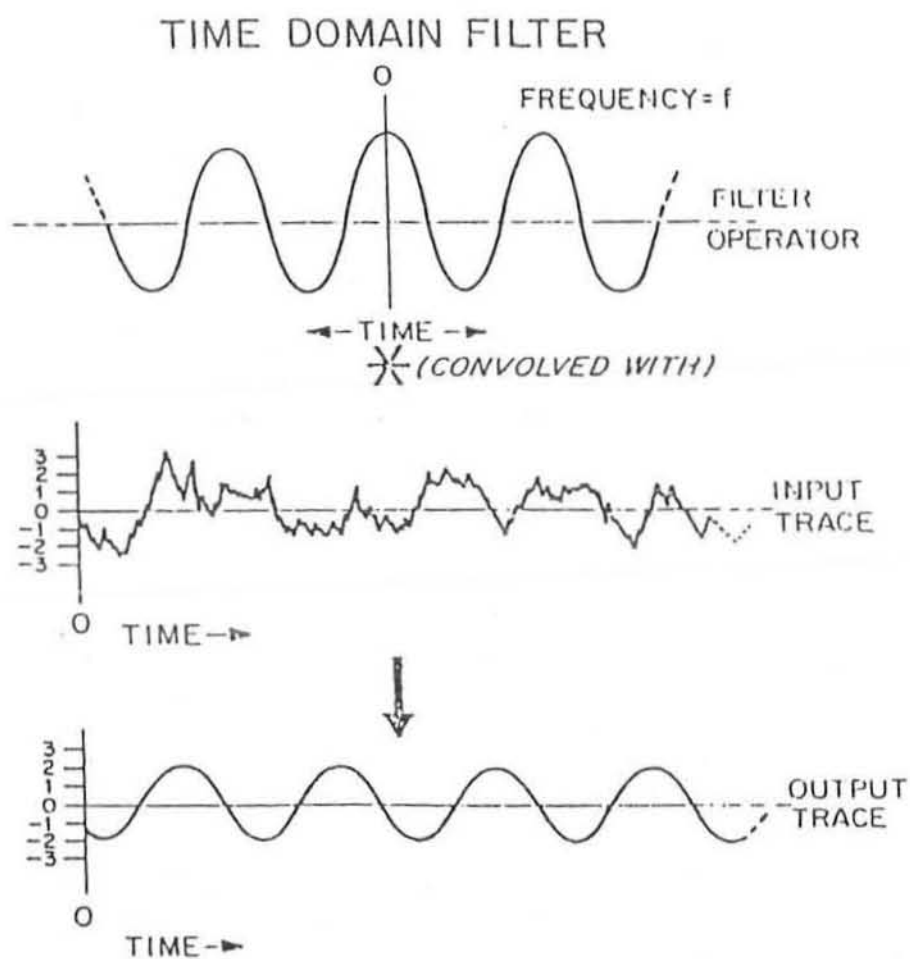
3.1.1 DIGITAL FILTER

The purpose of digital filter is to extract certain desired frequency components from a trace and attenuate all others. A filter can be designed and applied either in the time domain or in the frequency domain. The results are the same.

To extract signal from a trace in the time domain we convolve the trace with a cosine wave having the desired frequency. This cosine wave is the filter operator. The output of this filtering process will be a single frequency, (Fig.3.1). The amplitude and phase of the output are equal to that of the same frequency component in the original trace (Fig. 3.2) shows the same operation performed in the frequency domain. The amplitude spectrum of the operator is very simple; a spike at the desired frequency. The phase spectrum is simpler yet; zero at all frequencies. This is a “zero phase” filter. To filter the data in the time domain we convolve the trace with the filter operator. To perform the same operation in the frequency domain we multiply the amplitude spectra, frequency for frequency and add the phase spectra. In the output trace spectra we see that the amplitude spectrum contains only the frequency we selected and the phase spectrum is unchanged. (Only the phase of the selected frequency (f) is significant since the amplitudes of all other frequencies are zero). If we now convert the output trace spectra back to the time domain we will get a trace that is identical to the output trace in Fig. 3.1. In the following examples we will ignore the phase spectra since with zero phase filtering the phase is unchanged. (OGTI Manual, 1989)

3.2 TIME FILTERS

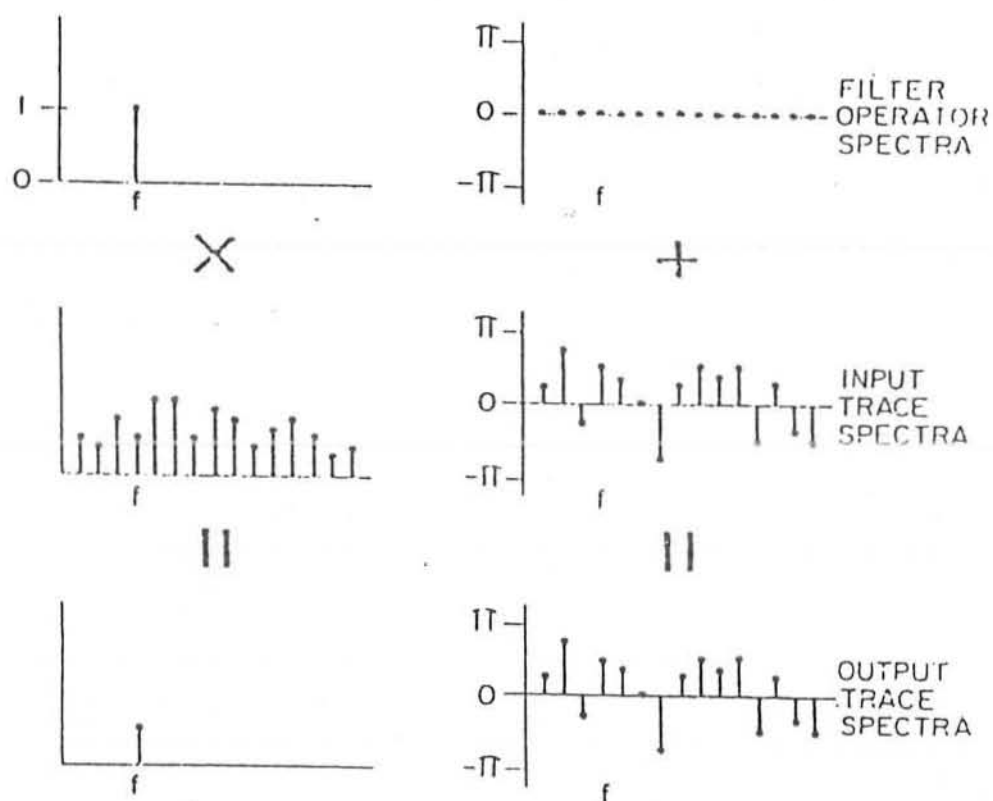
The standard practice in seismic data acquisition for exploration geophysics is to detect the signal by means of geophones (hydrophones) placed in the earth (water) and to digitize the analog signal output by the phone after passing it through certain electronic devices. The analog signal, which is digitized, will have important



Time Domain Filter

Fig 3.1

FREQUENCY DOMAIN FILTER



Frequency Domain Filter

Fig 3.2

limitations on its frequency spectrum, which is determined by the electronics as well as by the response function of the phone. Generally, the high frequencies are removed from the analog signal electronically in order to prevent aliasing upon digitization, while the very low frequencies which might be present in the original seismic signal are not recorded due to the insensitivity of the geophone (hydrophone) at low frequencies.

a) ALIAS FILTER

When we discuss the procedure of sampling a continuous wave form at discrete intervals, we learned that frequencies above the Nyquist frequency ($f_N = \frac{1}{2} dT$ where dT = sample rate) cannot be measured. If there are such frequencies present in the data, they will be recorded as if they had a lower frequency; we say the higher frequencies appear aliased in the digital signal. In order to prevent this effect from seismic data, the high frequencies are electronically removed from the signal before it is digitized. Suppose sampling interval in digitization is selected 4 milli seconds, f_N then would be 125 Hz, one must be sure to cut out in the field all frequencies above 125 Hz.

b) LOW-CUT FILTERS

In addition to high-cut filters used to prevent aliasing, low-cut filters are placed on the analog signals for specific reasons. In general the geophones and hydrophones approximate a flat frequency response curve for frequencies above a certain lower limit which varies with the instruments.

c) NOTCH FILTER

Another filter is ordinarily applied to the analog signal before it is digitized if the recording area is in the vicinity of any power lines. These lines may introduce large quantities of 60 Hz noise, and thus this frequency is removed by a filter whose frequency spectrum has a very narrow notch at 60 Hz.

3.3 SPATIAL FILTER-ARRAYS

The effective spatial filtering which take place when using an array of geophones (hydrophones) whose responses are summed to produce a seismic

trace. It will give that the geometry of the array determines the response of the system to different wavelengths of the seismic waves. Hence if the signal and noise have different wavelength spectra, by proper array design one may be able to enhance the signal relative to noise.

Before examining the response characteristics of various geophone arrays, it is necessary to make clear some important terminology. Arrays should be laid out in one plane (i.e. in the surface). Consider the propagation of a surface wave along the ground Fig 3.3-a. by measuring the spatial separation of adjacent peaks at a given time, one can determine the apparent wavelength (L_a) of this disturbance. In this case, this apparent wavelength is in fact equal to the actual wavelength (L) of the wave. On the other hand, suppose the wave is a reflection whose plane wave front strikes the earth's surface with an angle θ (Fig 3.3-b). Then, a measurement of the separation of two adjacent peaks at a given time will yield an apparent wavelength, which is greater than L (for $\theta < 90^\circ$). By inspection of the Fig 3.3-b

$$L_a = L / \sin \theta$$

As $\theta \rightarrow 90^\circ$, $L_a \rightarrow L$ as it should. Since this case corresponds to the surface wave example discussed above

As $\theta \rightarrow 0$, $L_a \rightarrow \infty$, i.e. the reflection energy is traveling nearly vertically. This means that the apparent wavelengths of vertically traveling events should be very large.

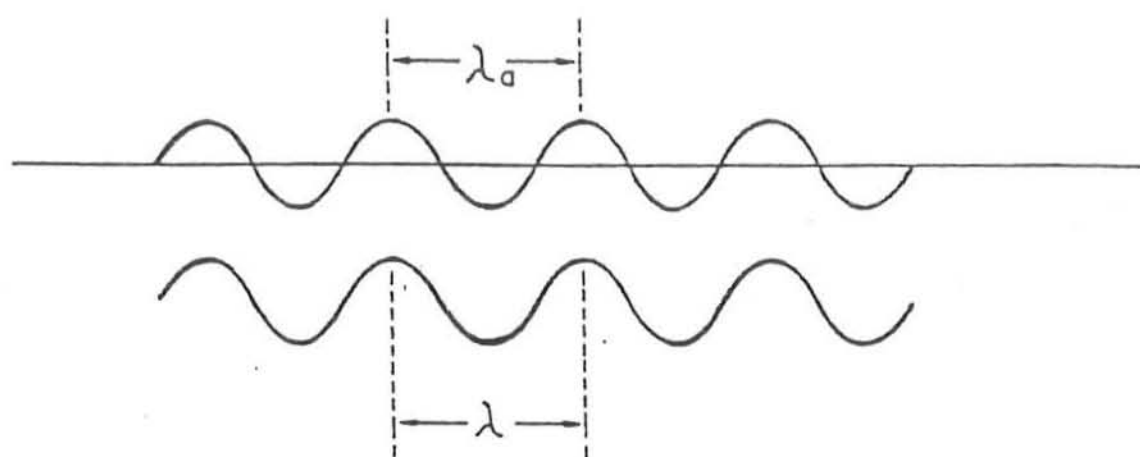
It is convenient in connection with these figures to also define the apparent velocity of the seismic wave. This (V_a) is defined to be the velocity with which the disturbance propagates along the earth's surface. V_a is in general not equal to the true seismic near-surface velocity V , but is larger than V . If we call the period of one seismic wave T , since a velocity is related to L and T by:

$$V = L / T \quad \text{then} \quad V_a = L_a / T$$

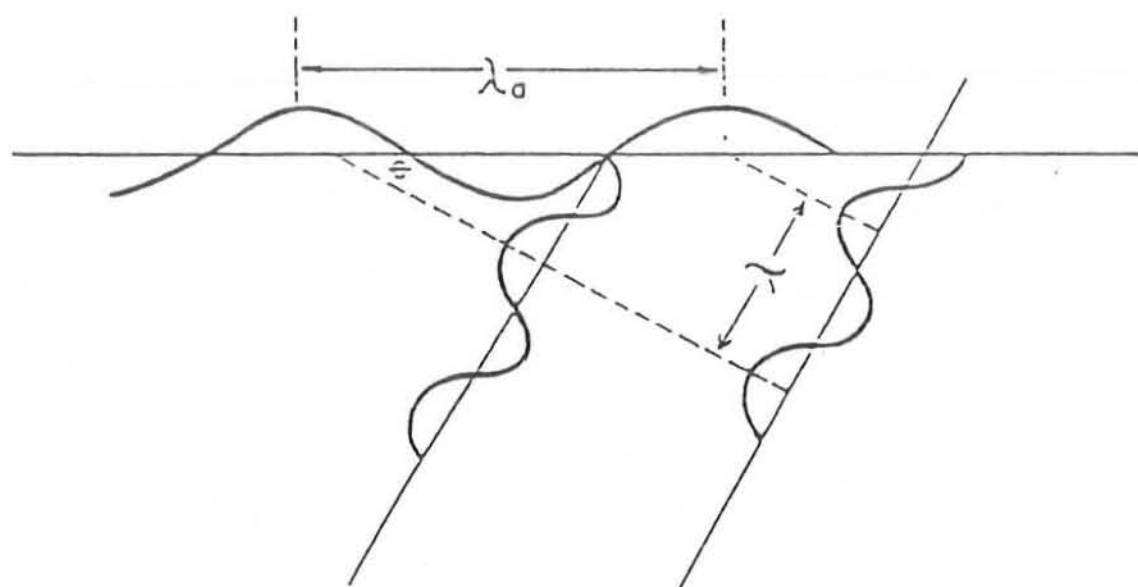
So the true and apparent velocities are related by:

$$V_a = V / \sin \theta$$

As $\theta = 90^\circ$, $V_a = V$ i.e. the waves are traveling horizontally



(a)



(b)

Wave Diagrams

Fig 3.3

As $\theta \rightarrow 0$, $V_a \rightarrow \infty$, a relation we can interpret by observing that vertically traveling energy will hit all the points on the surface simultaneously so that its apparent(horizontal) velocity will be infinitely large.

Spatial filter response is shown in Fig 3.4. When an array of geophones are used to record seismic data, we are in effect sampling the signal spatially. The geometry of the array will determine the (apparent) wavelength response of the system. An example of this is shown in Fig 3.5-a. here a linear array of 4 geophones are used to record

- a) A wave with short apparent wavelength
- b) One with large λ .

The spacing of geophones are approximately half the shorter wavelength. One sees from the figure that this tends to reduce the amplitude of the recorded signal for the shorter wavelength, due to the cancellation between peaks and troughs. On the other hand, the long wavelength signal reaches all the geophones with the same phase and hence it will be recorded with substantial amplitude.

From this simple example, one can see that if the apparent wavelength spectrum of the seismic reflection differs from that of noise, one may be able to devise an appropriate array which will significantly enhance the signal at the expense of the noise. In fact, this is a regular field procedure, which has led to study of a large number of arrays having various sizes, shapes and geometries. (OGTI Manual, 1989)

3.4 CONVOLUTION AND FILTERING

If a filter is linear and time invariant, its output can be computed by convolving the input signal with the filter's impulse response.

In the analog case, the procedure is described by:

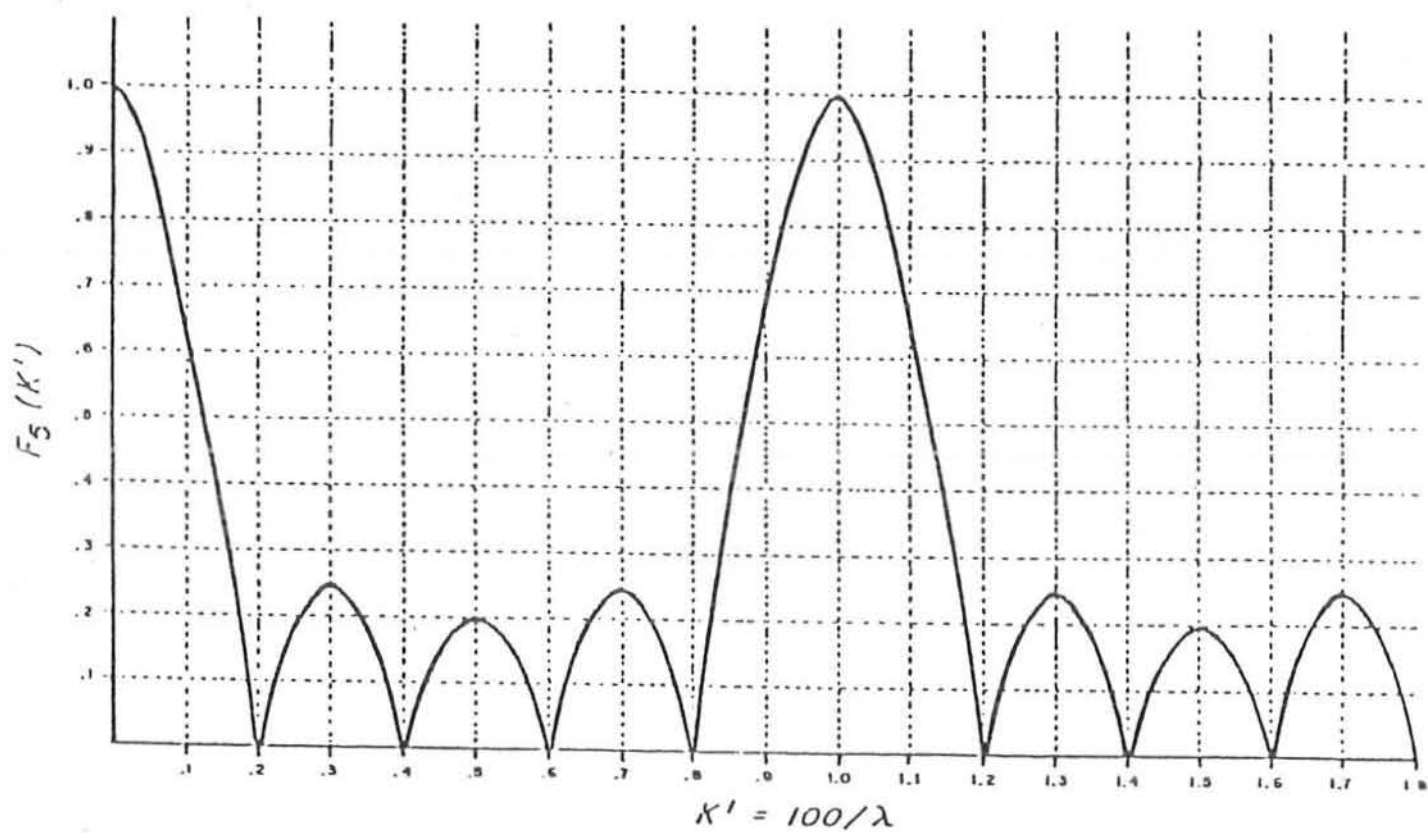
$$h(t) = f(t) * g(t) = \int_{-\infty}^{\infty} f(s) g(t-s) ds \dots\dots\dots \text{Eq-1}$$

Where $h(t)$ is the output

$f(t)$ is the input

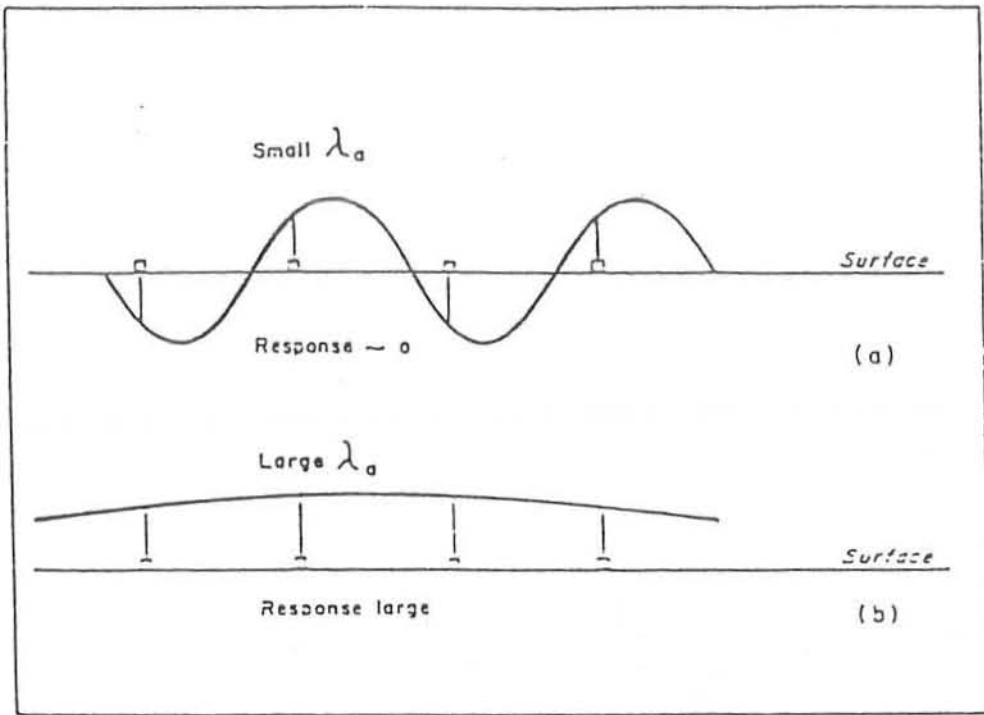
$g(t)$ is the filter's impulse response

$*$ is the symbol for convolution

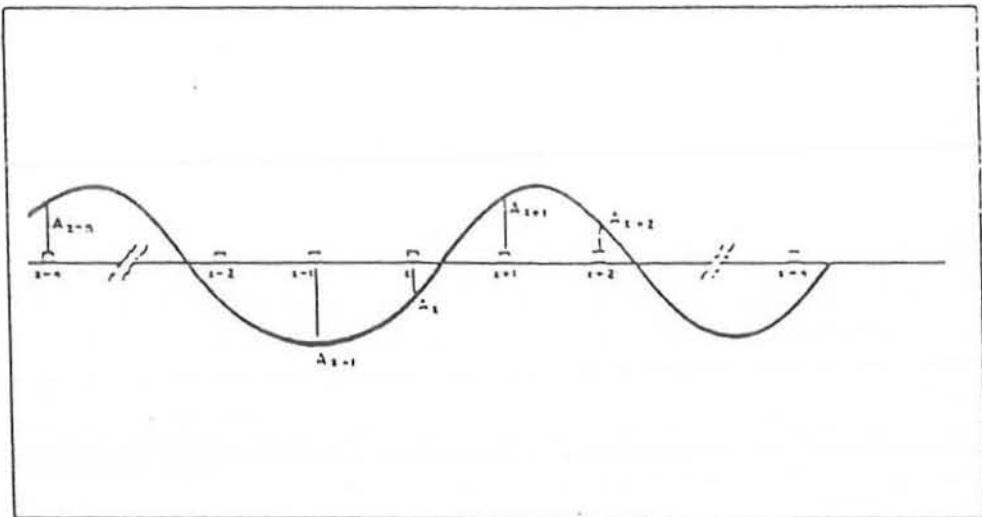


Spatial Filter Response

Fig 3.4



(a)



(b)

Wave Diagrams

Fig 3.5

I is integral symbol

In the digital case, the operation is:

$$h_t = f_t * g_t = \sum f_s \cdot g_{t-s}$$

3.4.1 Z-TRANSFORM

A discrete time array can be transformed into the Z-domain where it becomes a power series. Convolution in the time domain is equivalent to multiplication in the Z-domain. This equivalency is shown symbolically in Eq-3

$$f_t \leftrightarrow F(Z) \ ; \ g_t \leftrightarrow G(Z)$$
$$h_t = f_t * g_t \leftrightarrow H(Z) = F(Z) \cdot G(Z) \dots\dots\dots \text{Eq-3}$$

The similarity between the Z-domain and the frequency domain becomes apparent if we define the arbitrary variable Z as a complex exponential (Eq-4)

$$Z = e^{-i2\pi f dt} \qquad \text{where } I = (-1)^{1/2}$$

3.4.2 UNDESIRABLE NATURAL FILTERS

Some filtering occurs naturally and causes a loss in resolving power. The earth, for example, is a low-pass filter (Fig 3.6), which converts dynamite shot's impulsive signal into a wavelet, which may be 100 ± milliseconds long. This wavelet is the earth's impulse response its length makes it difficult to resolve closely spaced reflections.

Another example of unwelcome natural filter is the water layer in which marine seismic data are recorded. The water layer may be an effective echo chamber and cause signals both down going and up coming to reverberate strongly Fig 3.7. (OGTI Manual 1989)

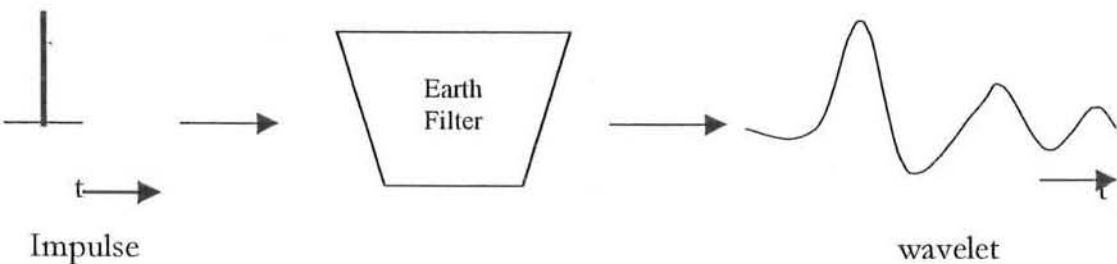


Fig 3.6 Earth's Impulse Response

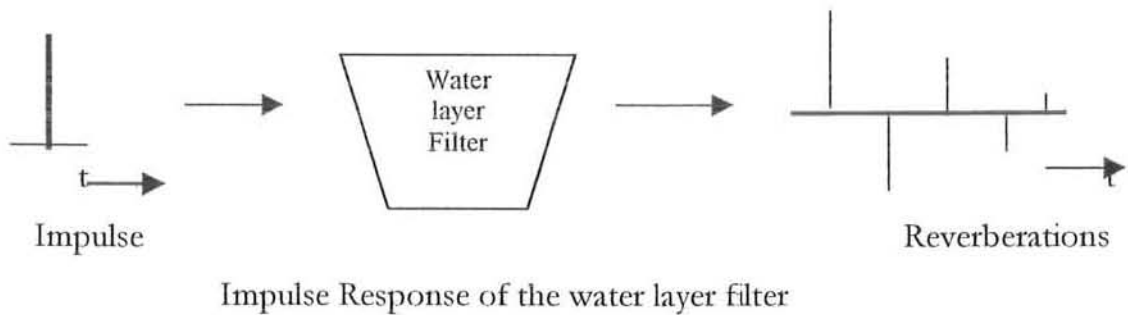


Fig (3.7)

3.5 INVERSE FILTERING

If the effect of a previous filter is to be undone, a new filter is to be designed such that, if the previous filter's impulse response is input, the new filter's output will be a unit impulse (Fig 3.8).

The process of applying an inverse filter to cancel the effect of a previous filter is called deconvolution. Deconvolution is usually done by convolution in the time domain. A convenient way of designing an inverse filter is to use Z-transform. if the previous filter's impulse response is $g(t)$ then:

$$g(t) \leftrightarrow G(Z) \dots\dots\dots \text{Eq-3.1}$$

We now wish to design a new filter, $h(t)$, which will reconvert $g(t)$ to the impulse $d(t)$ as shown in Eq-3.2

$$g(t) * h(t) = d(t) \dots\dots\dots \text{Eq-3.2}$$

the process in the Z-domain is :

$$G(Z) \cdot H(Z) = 1 \dots\dots\dots \text{Eq-3.3}$$

and so,

$$H(Z) = 1 / G(Z) \dots\dots\dots \text{Eq-3.4}$$

The Z-transform of the inverse filter's impulse response can thus be computed by a polynomial division.

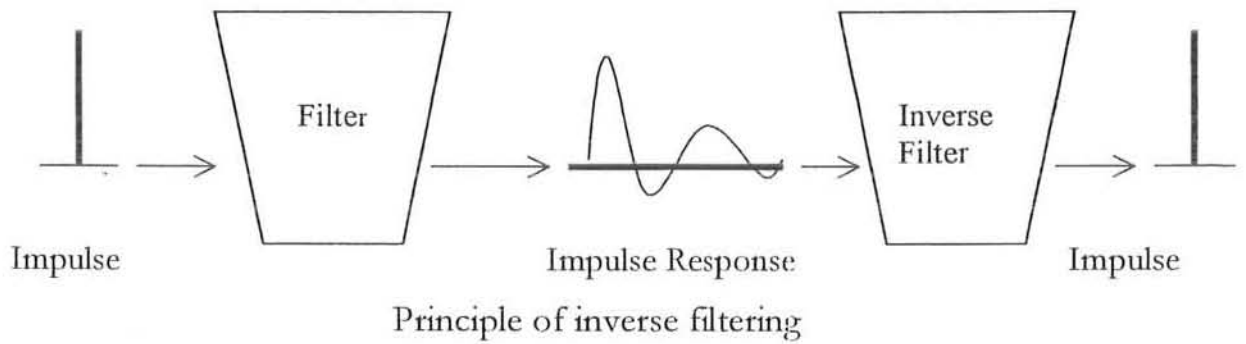


Fig 3.8

3.6 FREQUENCY FILTERING

Frequency filters discriminate against selected frequency components of an input wave form and may be

- Low-pass (LP)
- High-pass (HP)
- Band-reject (BR)
- Band-pass (BP)

in terms of their frequency response.

3.6.1 LOW-PASS FILTER RESPONSE

The pass band of the basic low-pass filter is defined to be 0 Hz (dc) up to the critical (cutoff) frequency, f_c at which the output voltage is 70.7% of the pass-band voltage, as indicated in Fig 3.1. The ideal pass-band, shown by the shaded region within the dashed lines, has an instantaneous roll-off at f_c . The band width of this filter is equal to f_c .

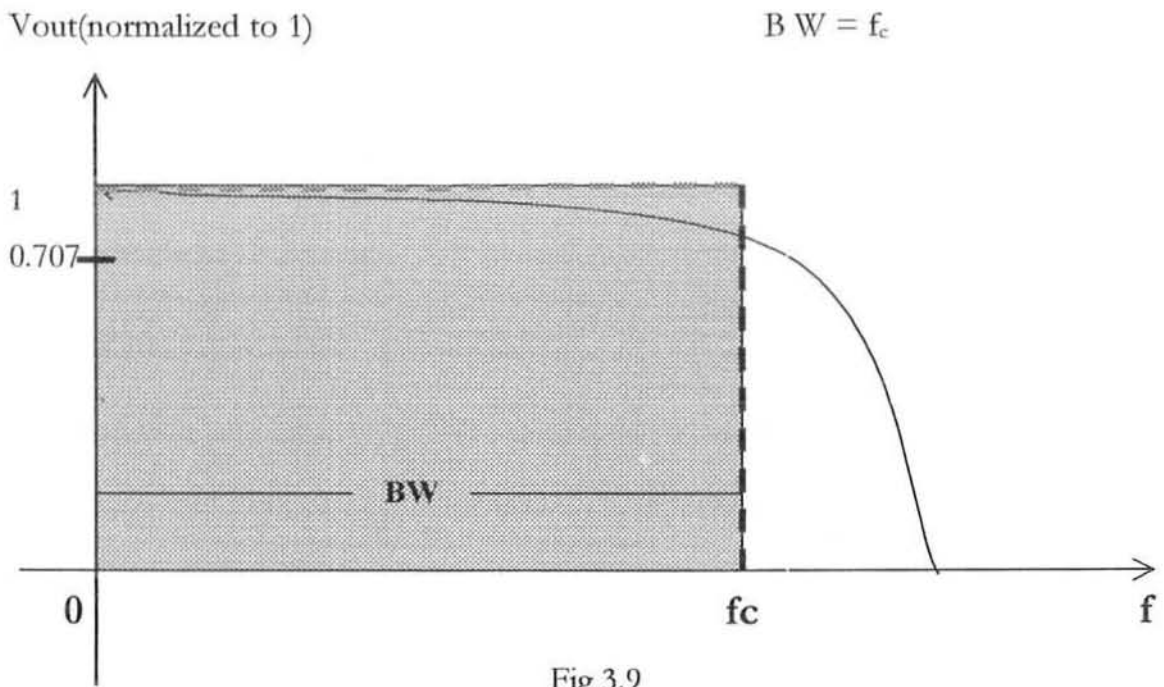


Fig 3.9

The magnitude response of practical lowpass filter shown in Figs.3.10 and 3.11

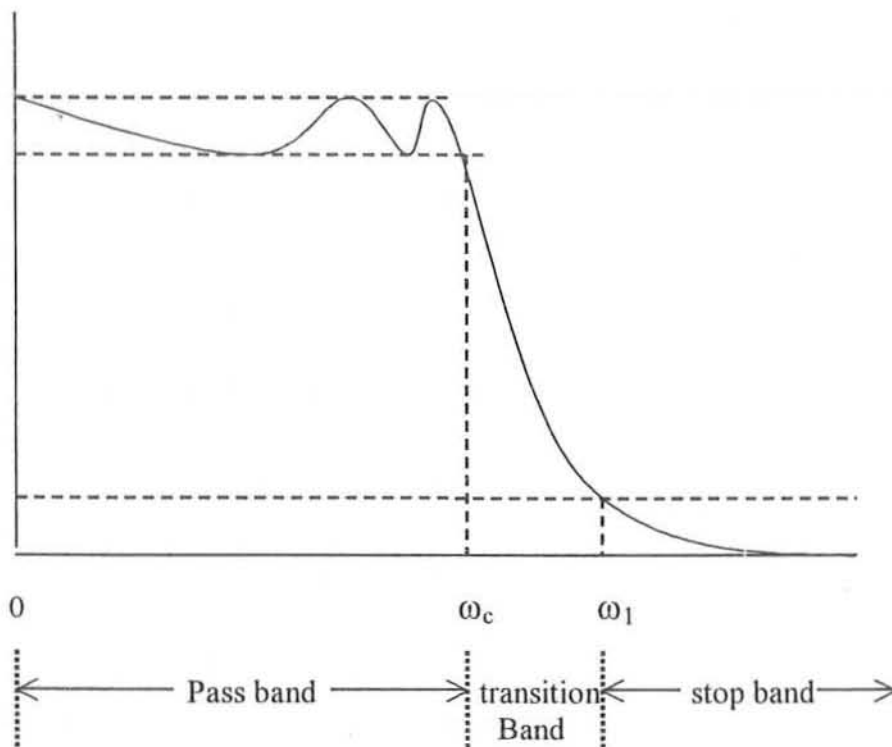


Fig 3.10. Magnitude response of a practical low pass filter with ripples in pass band.

The pass band extends from direct current up to the cut-off frequency ω_c . The transition band from ω_c up to the beginning of the stop band at ω_1 and stop band extends from ω_1 to infinity.

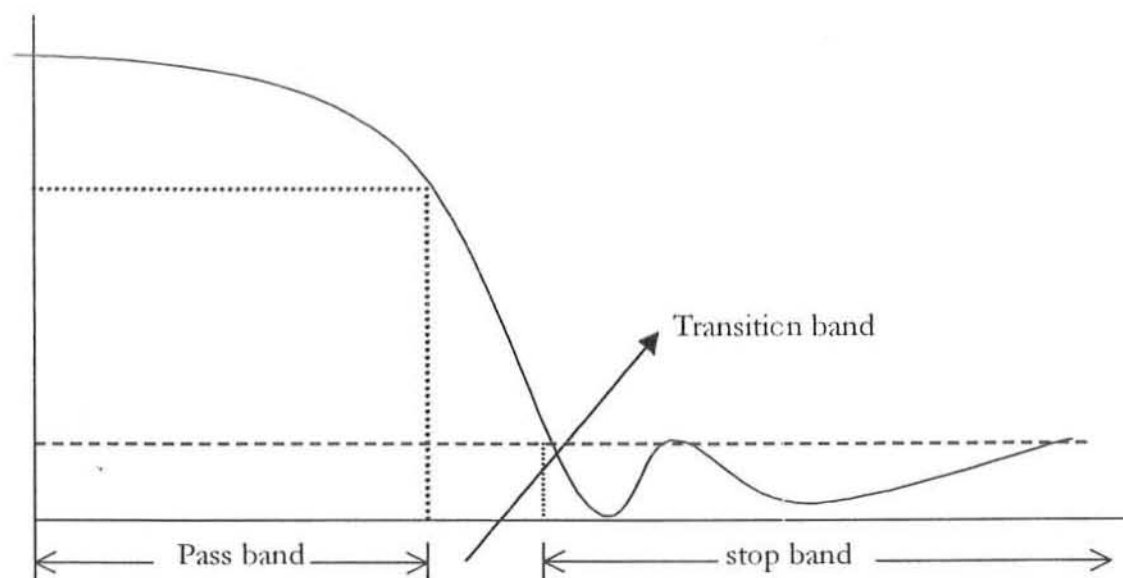


Fig 3.11. Magnitude response of a practical low pass filter with ripples in the stop band.

3.6.2 HIGH-PASS FILTER RESPONSE

A high-pass filter response is one that significantly attenuates all frequencies below f_c and passes all frequencies above f_c . The critical frequency is the frequency at which output voltage is 70.7% of the pass-band voltage as shown in Fig 3.12. The ideal response, shown by the shaded region within the dashed lines, has an instantaneous drop at f_c , which is not achievable.

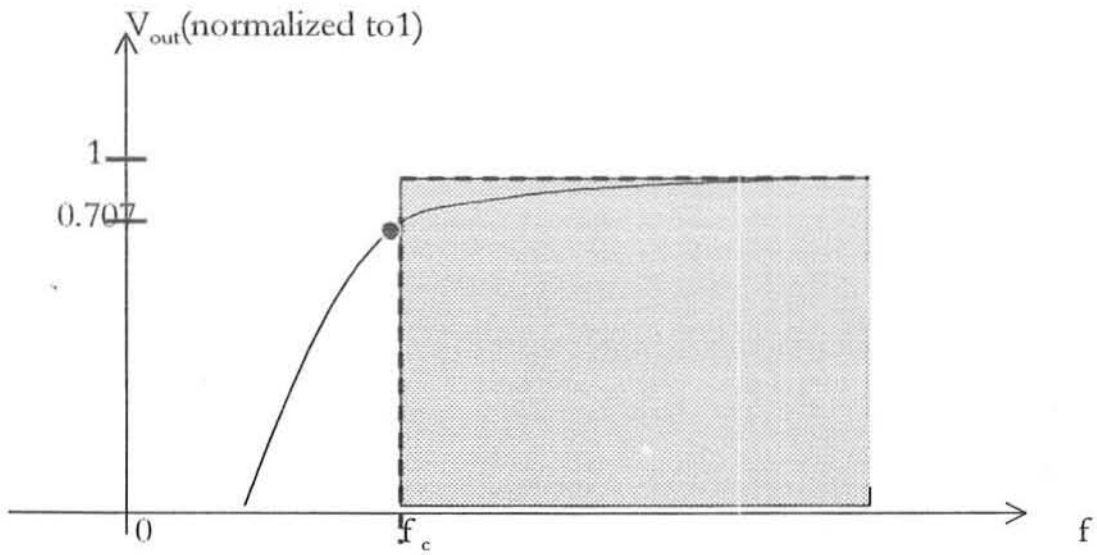


Fig 3.12

3.6.3 BAND-STOP FILTER RESPONSE

It is also known as *notch*, *band-reject* or *band-elimination*. It's operation is opposite to that of the band pass filter because frequencies within a certain bandwidth are rejected, and frequencies outside the bandwidth are passed. A general response curve for a band-stop filter is shown in Fig 3.13.

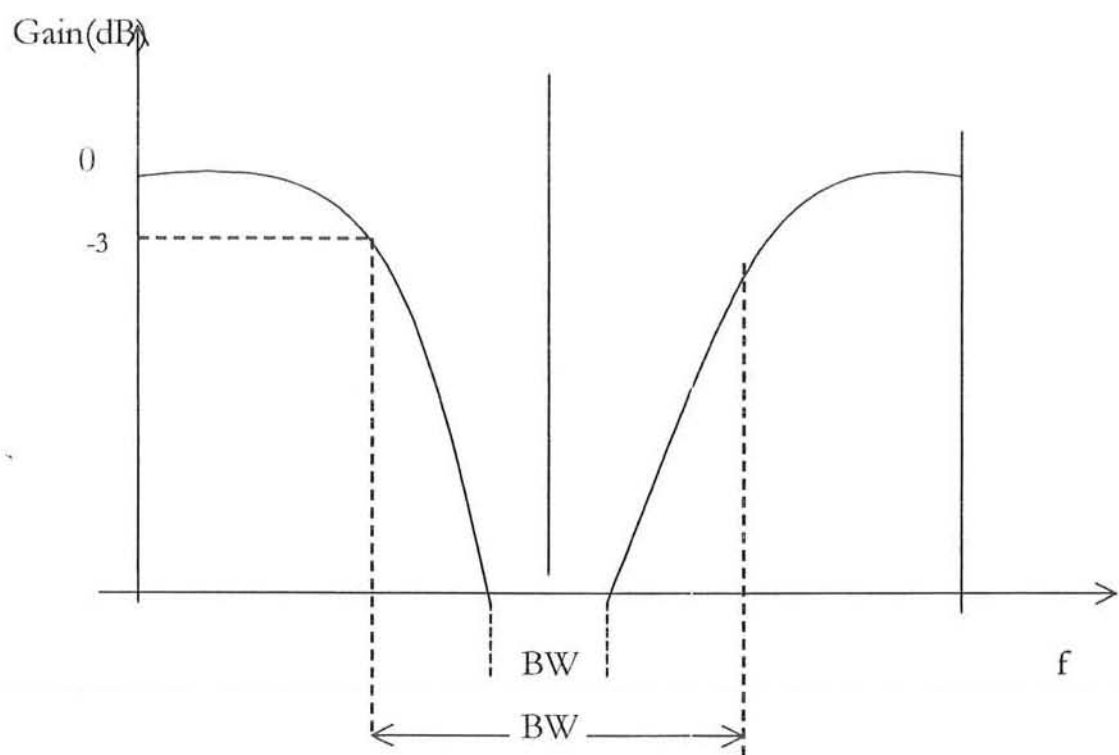


Fig 3.13

3.6.4 BAND-PASS FILTER RESPONSE

A band-pass filter passes all signals lying within a band between a lower-frequency and an upper-frequency limit and rejects all other frequencies that are outside this specified band. A generalized band-pass response curve is shown in Fig 3.14.

V_{out} (normalized to 1)

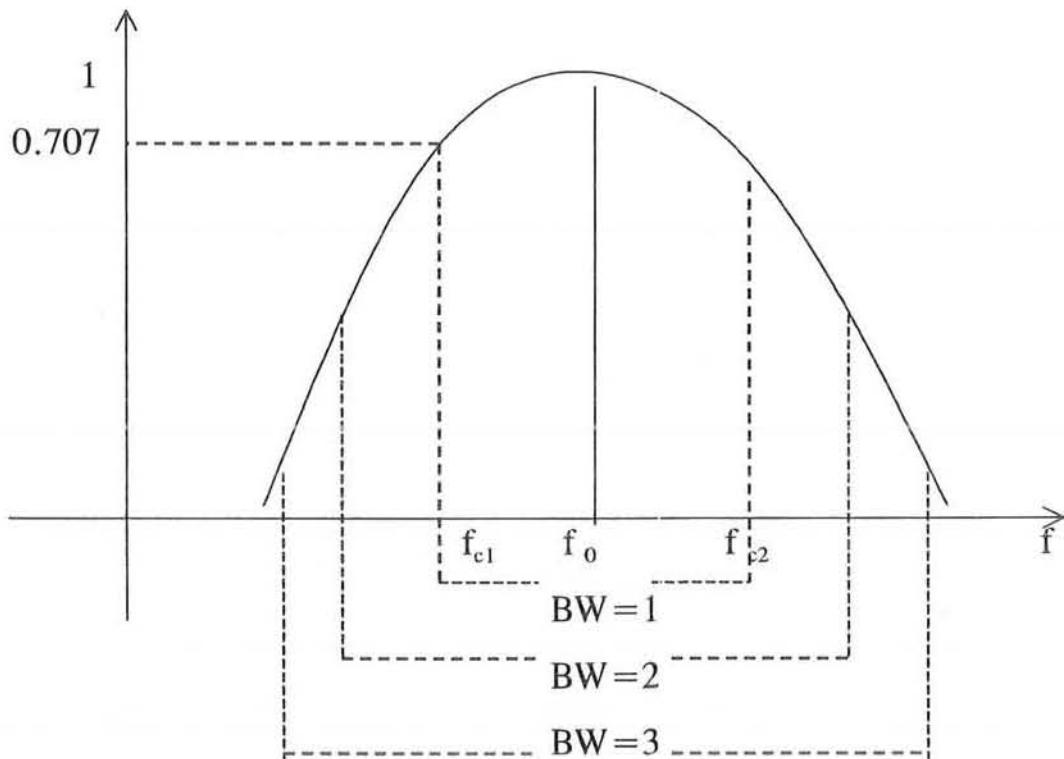


Fig 3.14

The bandwidth(BW) is defined as the difference between the upper critical frequency (f_{c2}) and the lower critical frequency (f_{c1}).

$$B W = f_{c2} - f_{c1}$$

The critical frequencies are, of course, the points at which the response curve is 70.7% of its maximum. These critical frequencies are also called 3 dB frequencies. The frequency about which the pass band is centered is called the central frequency, f_o and is a geometric mean of the critical frequencies.

$$f_o = (f_{c1} f_{c2})^{1/2}$$

The quality factor (Q) of band-pass filter is the ratio of the center frequency to the bandwidth i.e,

$$Q = f_o / B W$$

The value of Q is an indication of the selectivity of a band-pass filter. The higher the value of Q, the narrower the bandwidth and the better the selectivity for a given value f_o . Band-pass filters are some times classified as narrow-band ($Q > 10$) or wide-band ($Q < 10$).

An example of band-limited signal is in vibroseis data for which the frequency range of the input energy is known to lie for all practical purpose, within a certain band (e.g. 10-40 Hz)

CHAPTER # 4

BUTTERWORTH FILTERS

BUTTERWORTH FILTERS

The Butterworth characteristic provides a very flat amplitude response in the pass band and a roll-off rate of 20 dB/decade/pole. Butterworth filters are smooth filters. There is no ripple in the response in either the passband or the stopband and the transition between bands is monotonic. Filters with Butterworth response are normally use when all frequencies in the pass band must have the same gain. The Butterworth response is often referred to as a maximally flat response. (FLOYD, 1996)

Butterworth low pass filters (LPF) are designed to have an amplitude response characteristics that is as flat as possible at low frequencies and that is monotonically decreasing with increasing frequency. (Britton, 1993)

4.1 THE BUTTERWORTH APPROXIMATION

The n th-order Butterworth function is given by

$$B_n(\omega) = 1/(1+\omega^{2n}) \quad n = 1, 2, 3, \dots \quad \text{Eq 4.1}$$

For each value of n , the Butterworth function $B_n(\omega)$ has a squared magnitude function.

An n th-order normalized low-pass Butterworth filter has a magnitude function given by

$$|T(j\omega)|^2 = B_n(\omega) = 1/(1+\omega^{2n}) \quad \text{Eq 4.2}$$

A Graphical illustration of equation (4.2) is shown in fig 4.1. As $n \rightarrow \infty$, the Butterworth magnitude approaches the ideal magnitude response. As the order n of the Butterworth filter increases, the magnitude function is closer to unity in the passband, the transition band is narrow, and the magnitude function is closer to zero in the stop band. Hence, n is the parameter chosen to satisfy a set of prescribed pass band and stop band specifications. Fig 4.2 is another plot of equation (4.2) with the vertical scale given in dB

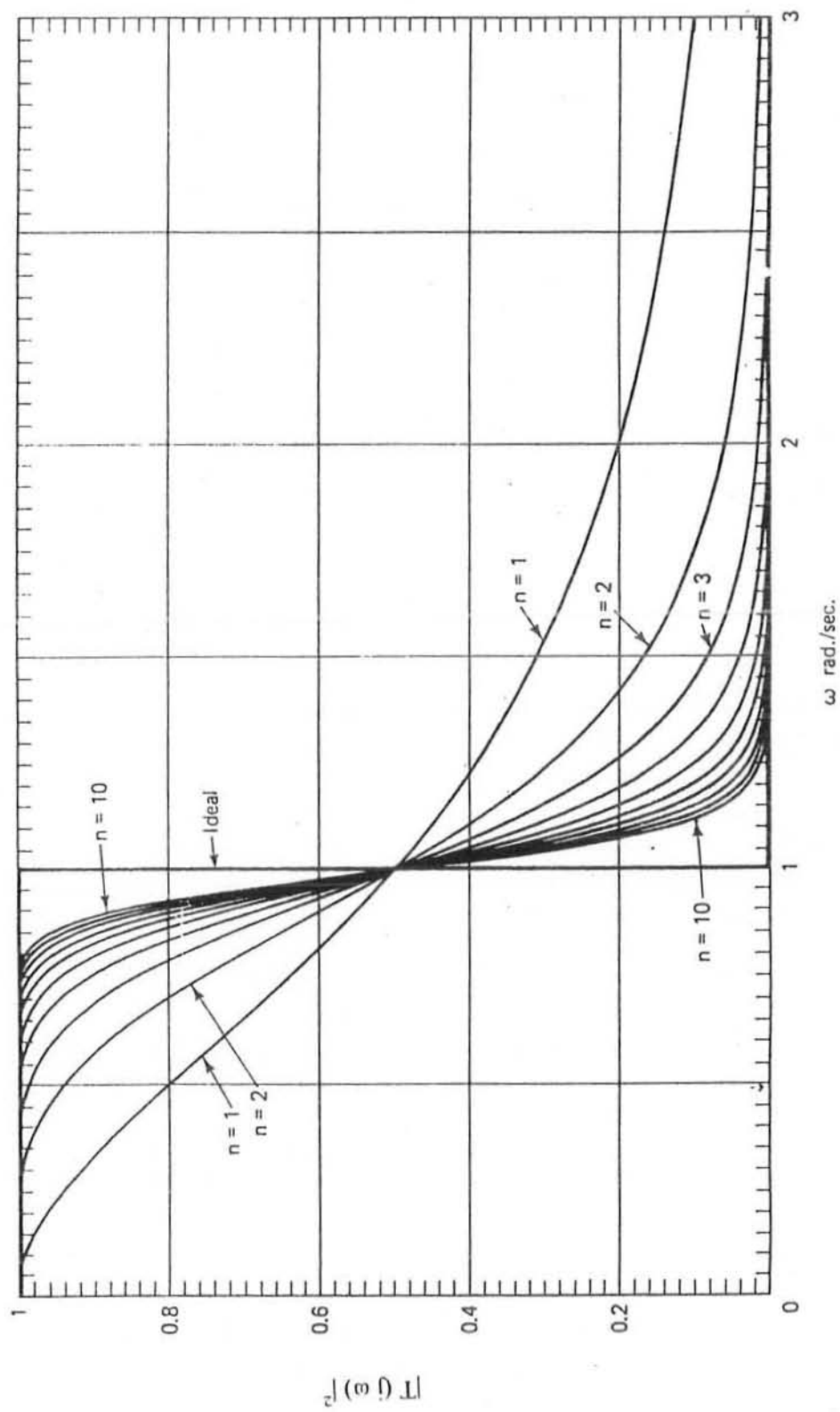


Fig 4.1 Magnitude Function of Butterworth Filters

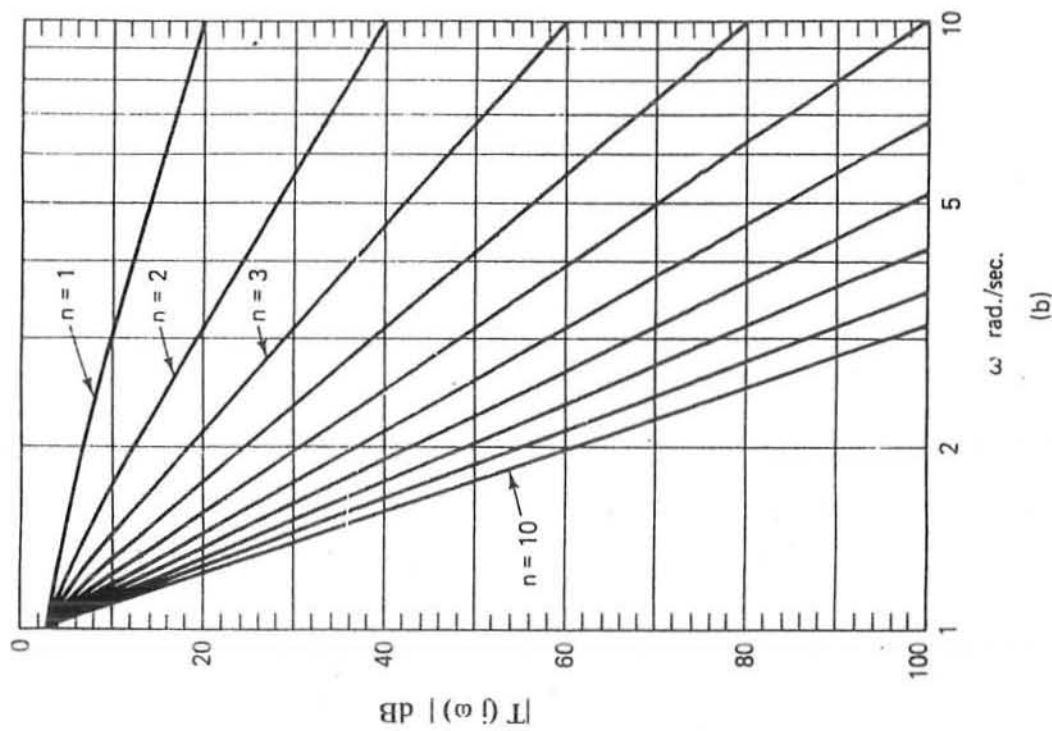
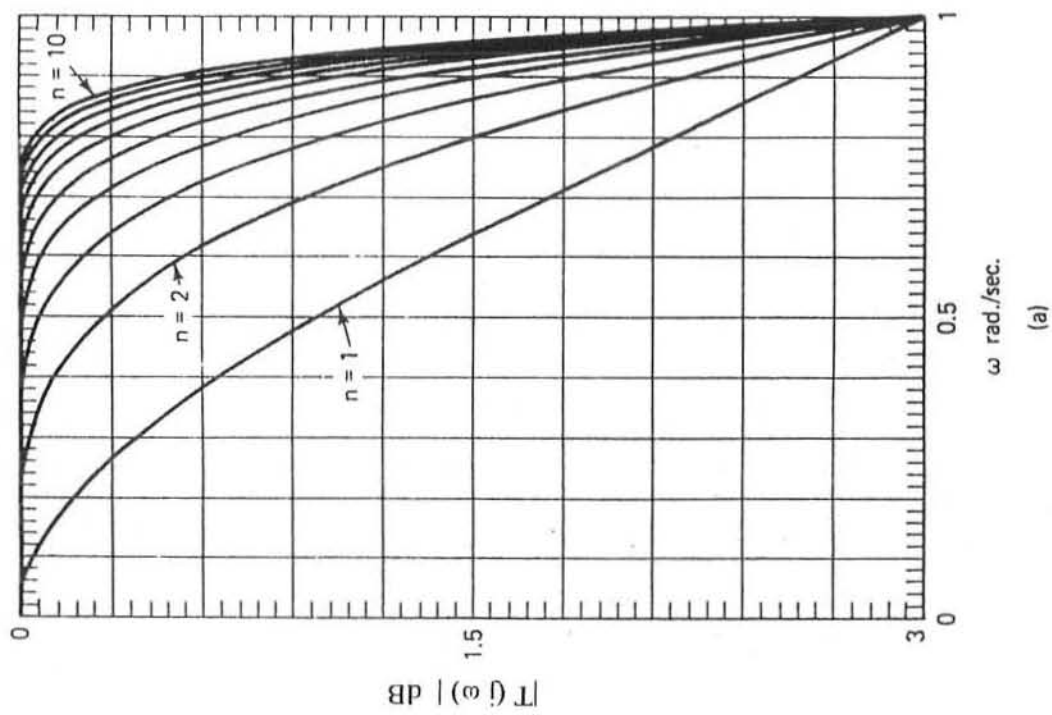


Fig 4.2 Magnitude Characteristic of Butterworth Filters
 (a) Pass-band attenuation (b) Stop-band attenuation

$$|T(j\omega)| \equiv -10 \log |T(j\omega)|^2, \text{ dB} \quad \text{Eq 4.3}$$

where the magnitude of transmittion is expressed in desibels in terms of the attenuation function.

The phase characteristics

$$\Phi(\omega) = -T(j\omega) \quad \text{Eq 4.4}$$

of an n th-order normalize low-pass Butterworth filter are shown in fig 4.3..For ω very small, the phase function behaves almost linearly, especially for low values of n . The pass-band magnitude response, the stop-band magnitude response, and the phase response of Butterworth filters of various orders are shown in fig 4.4, 4.5 and 4.6 respectively. These plots are normalized for a cut-off frequency of 1 Hz. To denormalize them, simply multiply the frequency axis by the desired cut-off frequency f_c .

4.2 BASIC PROPERTIES

Based on Eq (4.2) and fig 4.1, the normalized low-pass Butterworth filter has the following basic properties. (Harry, 1979)

BUTTERWORTH PROPERTY 1

For each n , we have

$$|T(j0)|^2 = 1, \quad |T(j1)|^2 = 0.5 \quad \text{and} \quad |T(j\infty)|^2 = 0 \quad \text{Eq 4.5}$$

this implies that the dc gain (the magnitude value at $\omega=0$) is 1 and the 3 dB cutoff frequency is at 1 rad./sec.

BUTTERWORTH PROPERTY 2

The magnitude function of butterworth filters are monotonically decreasing for $\omega \geq 0$. Hence $|T(j\omega)|$ has it's maximum value at $\omega=0$

BUTTERWORTH PROPERTY 3

The first $(2n-1)$ derivatives of n th-order Low-Pass Butterworth filter are zero at $\omega=0$. For this reason Butterworth filters are also called maximally flat magnitude filters.

BUTTERWORTH PROPERTY 4

The high frequency roll-off an n th-order butterworth filter is $20n$ dB / Decade.

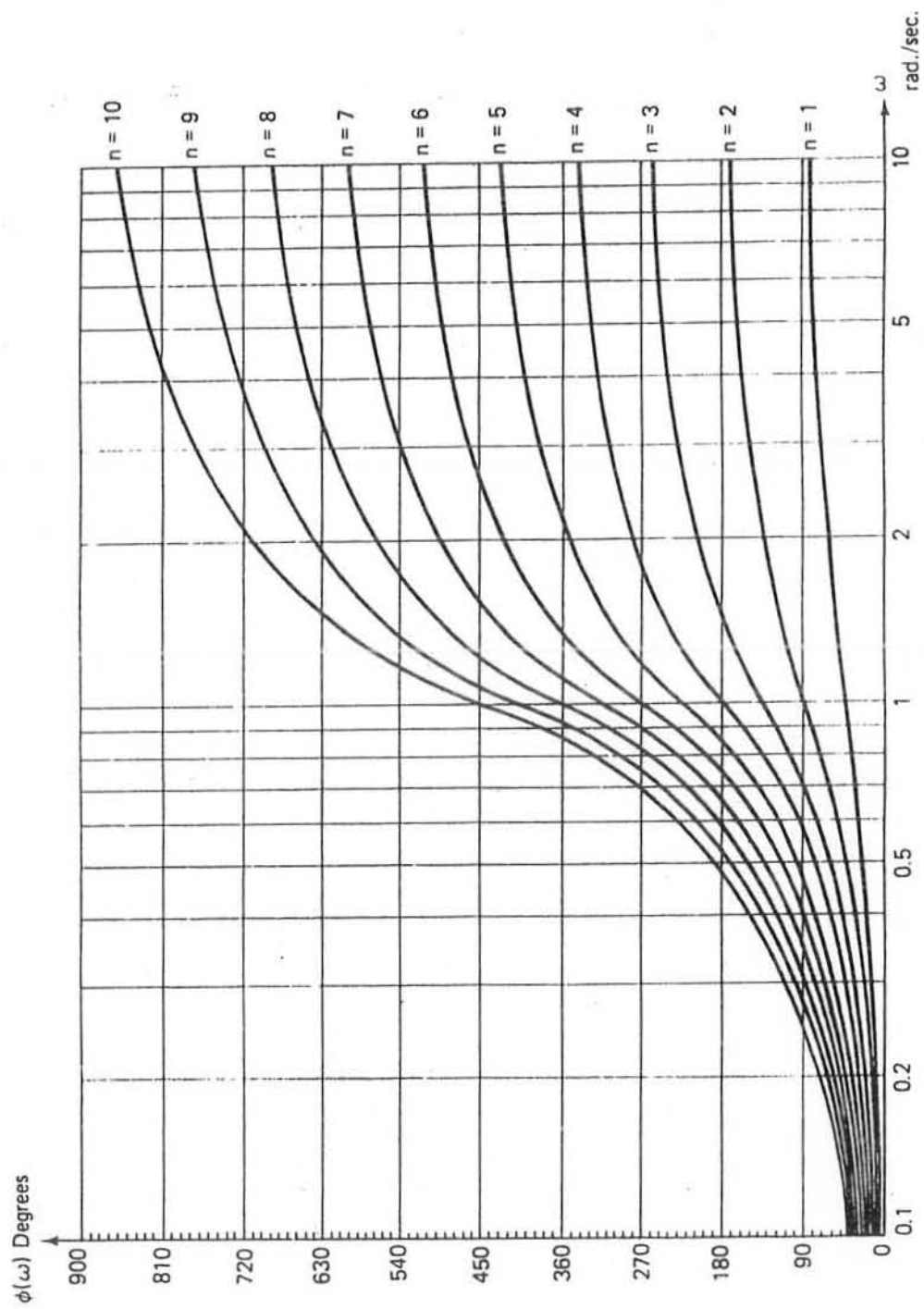


Fig 4.3 Phase Characteristics of Butterworth Filters

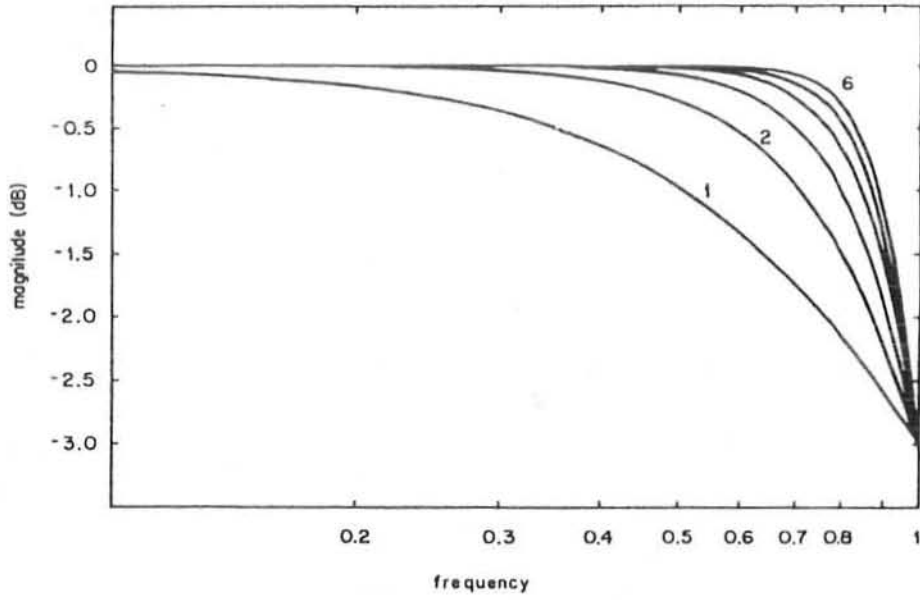


Fig 4.4 Pass-band amplitude response for Low-Pass Butterworth filters of orders 1 through 6

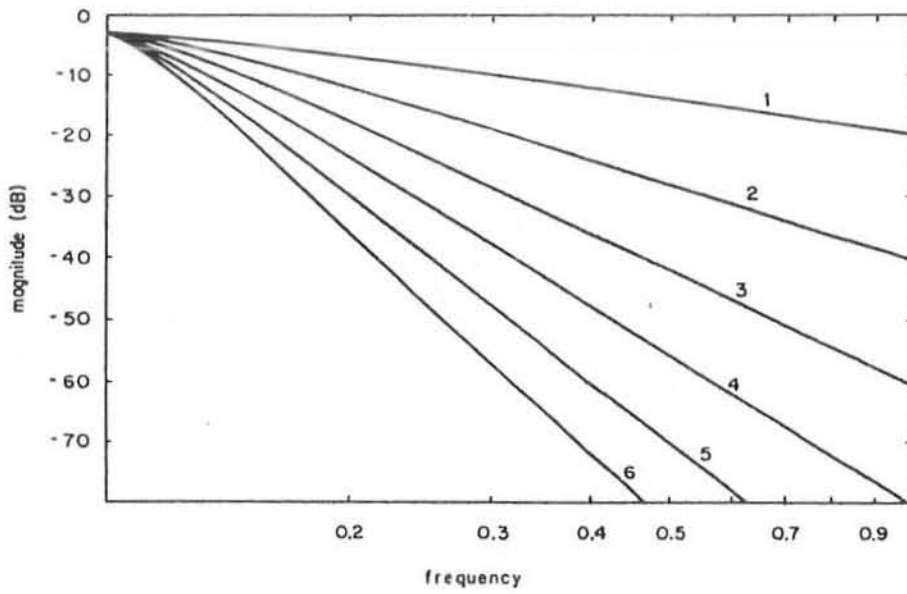


Fig 4.5 Stop-band amplitude response for Low-Pass Butterworth filters of orders 1 through 6

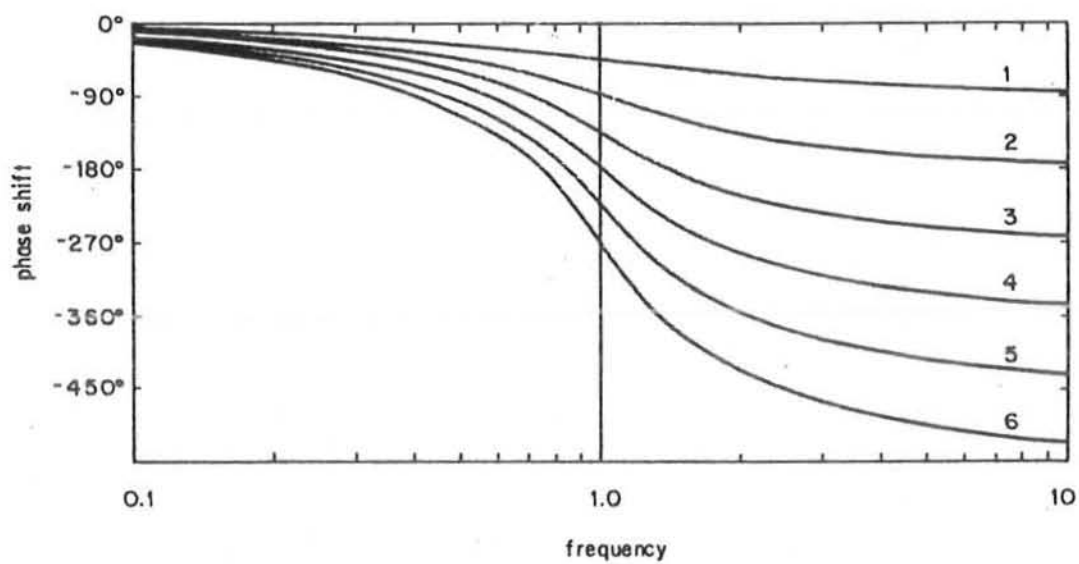


Fig 4.6 Phase response for Low-Pass Butterworth filters of orders 1 through 6

4.3 IMPULSE RESPONSE OF BUTTERWORTH FILTERS

The impulse response for the Low-Pass Butterworth filters shown in Figs. 4.7 and 4.8. These responses are normalized for Low-Pass filters having a cut-off frequency equal to 1 rad/s. To denormalize the response, divide the time axis by the desired cut-off frequency $\omega_c = 2\pi f_c$ and multiply the time axis by the same factor.

4.4 STEP RESPONSE of BUTTERWORTH FILTERS

The step response can be obtained by integrating the impulse response. Step responses for Low-Pass Butterworth filters are shown in Figs 4.9 and 4.10. These responses are normalized for Low-Pass filters having a cut-off frequency equal to 1 rad/sec. To denormalize the response, divide the time axis by the desired cut-off frequency $\omega_c = 2\pi f_c$. (Bratton, 1993)

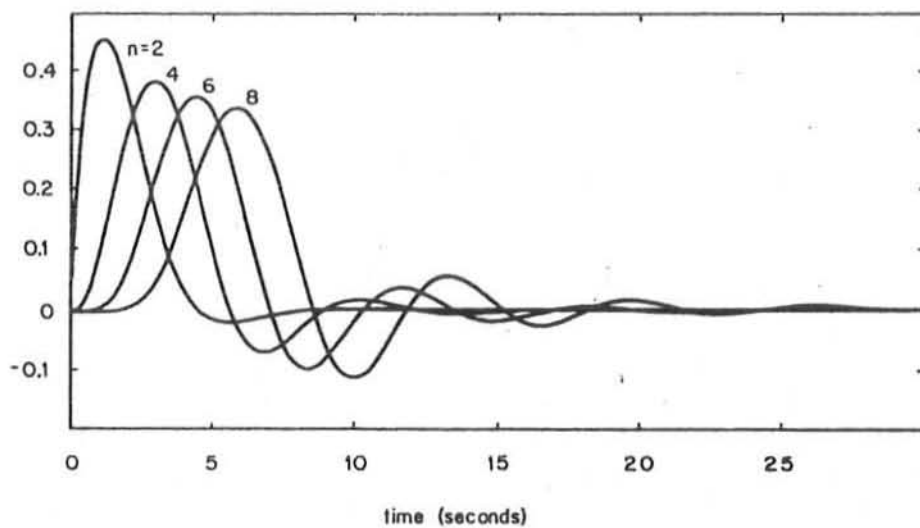


Fig 4.7 Impulse response of Even order Butterworth filters

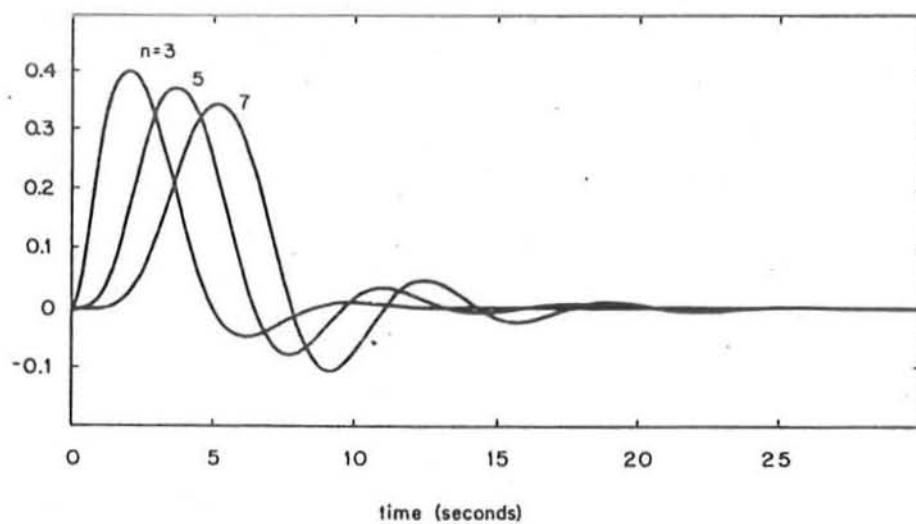


Fig 4.8 Impulse response of odd order Butterworth filters

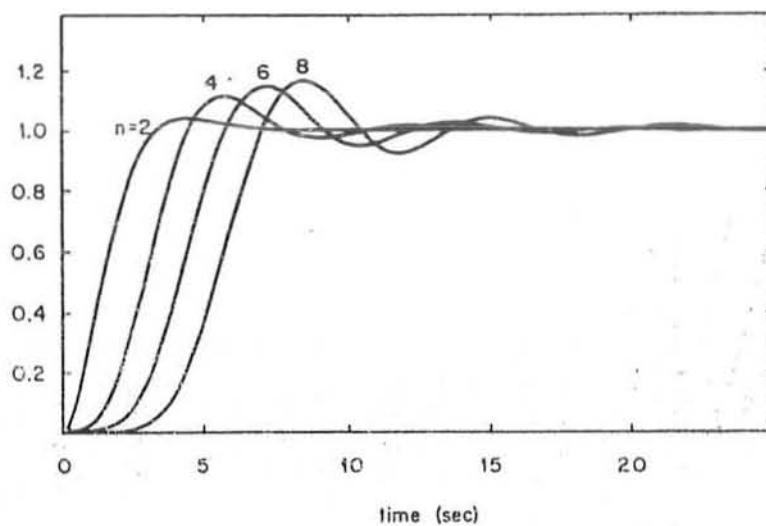


Fig 4.9 Step response of Even order Low-Pass Butterworth filters

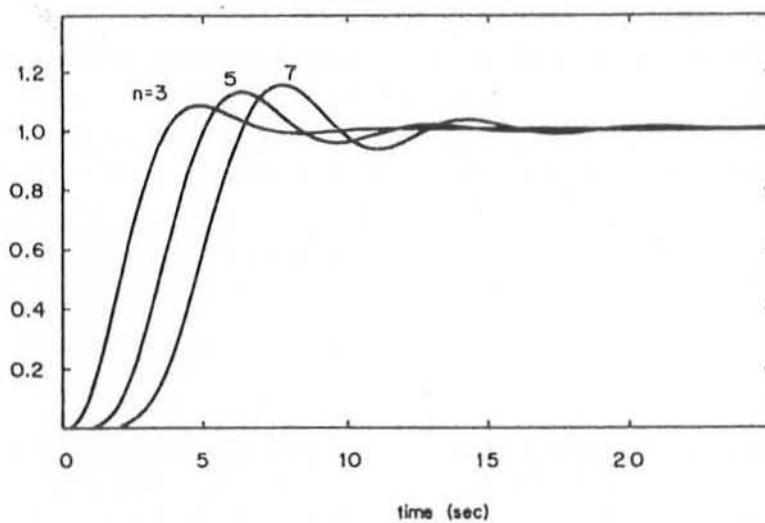


Fig 4.10 Step response of odd order Low-Pass Butterworth filters

CHAPTER #5

DATA INTERPRETATION

CONCLUSION#1

The original seismic section of w85-8.sgy is shown in fig. 5.1. The amplitude spectrum of this seismic section is demonstrated in fig. 5.1(a) which shows that the most of energy is concentrated within the frequency range 11-70Hz.

To observe the effectiveness of Butterworth filtering the original data convolved with Butterworth filter with frequencies 15-25-45-55Hz and having order $n=8$ is shown in fig. 5.2, where 15-55Hz is the Bandpass filter with 25dB/octave low roll-off and 45dB/octave high roll-off. 15-55Hz value corresponds to the 3dB point of the filter.

In the upper part of the Butterworth filtered seismic section some shallow reflectors of higher frequencies became visible, the frequencies are reduced from 950-1100ms and in the lower part of the section i.e. below 1250ms major frequencies have been reduced.

The amplitude spectrum of the Butterworth filtered section is shown in fig. 5.2(a).

Step: 1

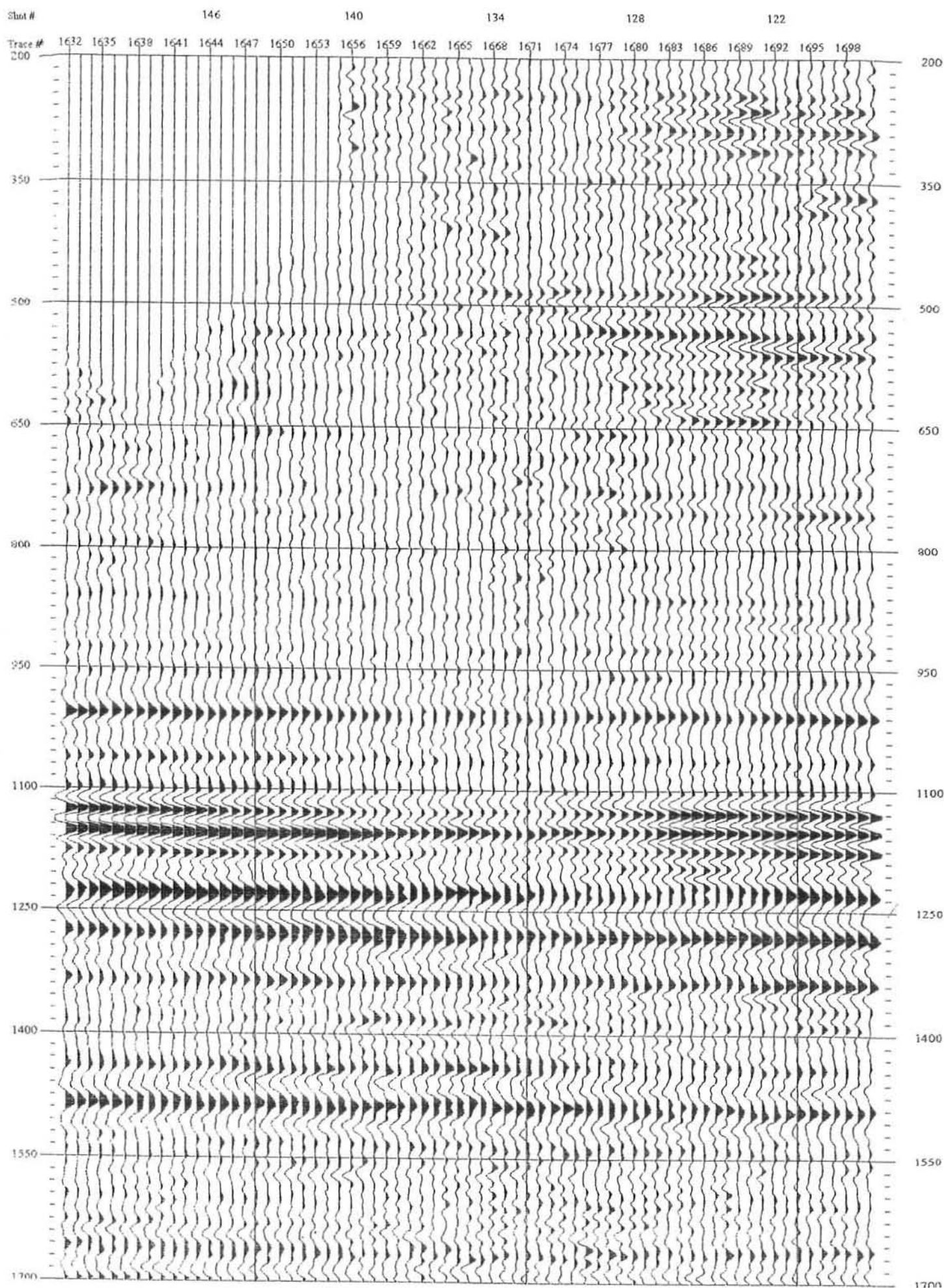


Fig. 5.1

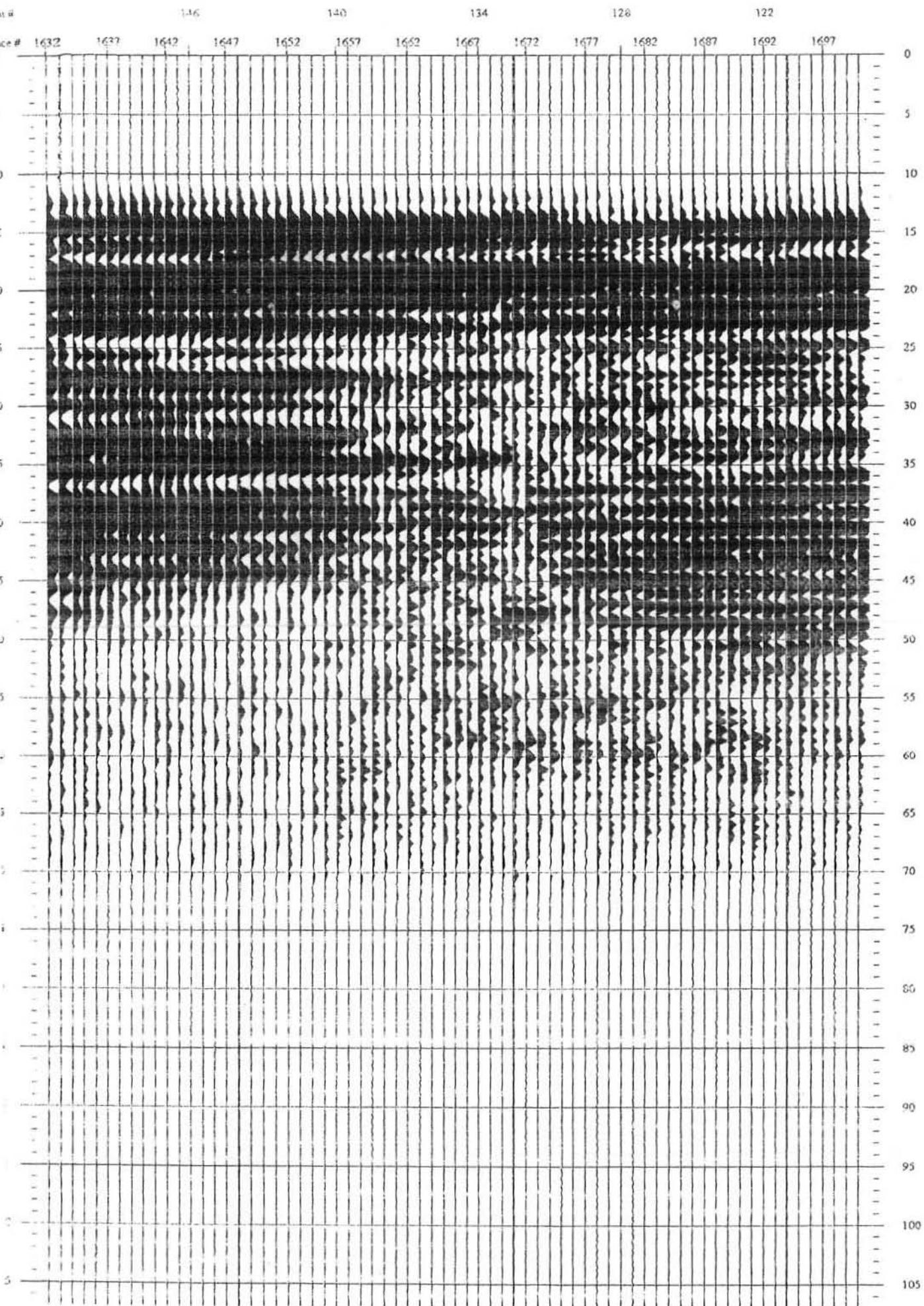


Fig 5.1(a)

Shot: 1

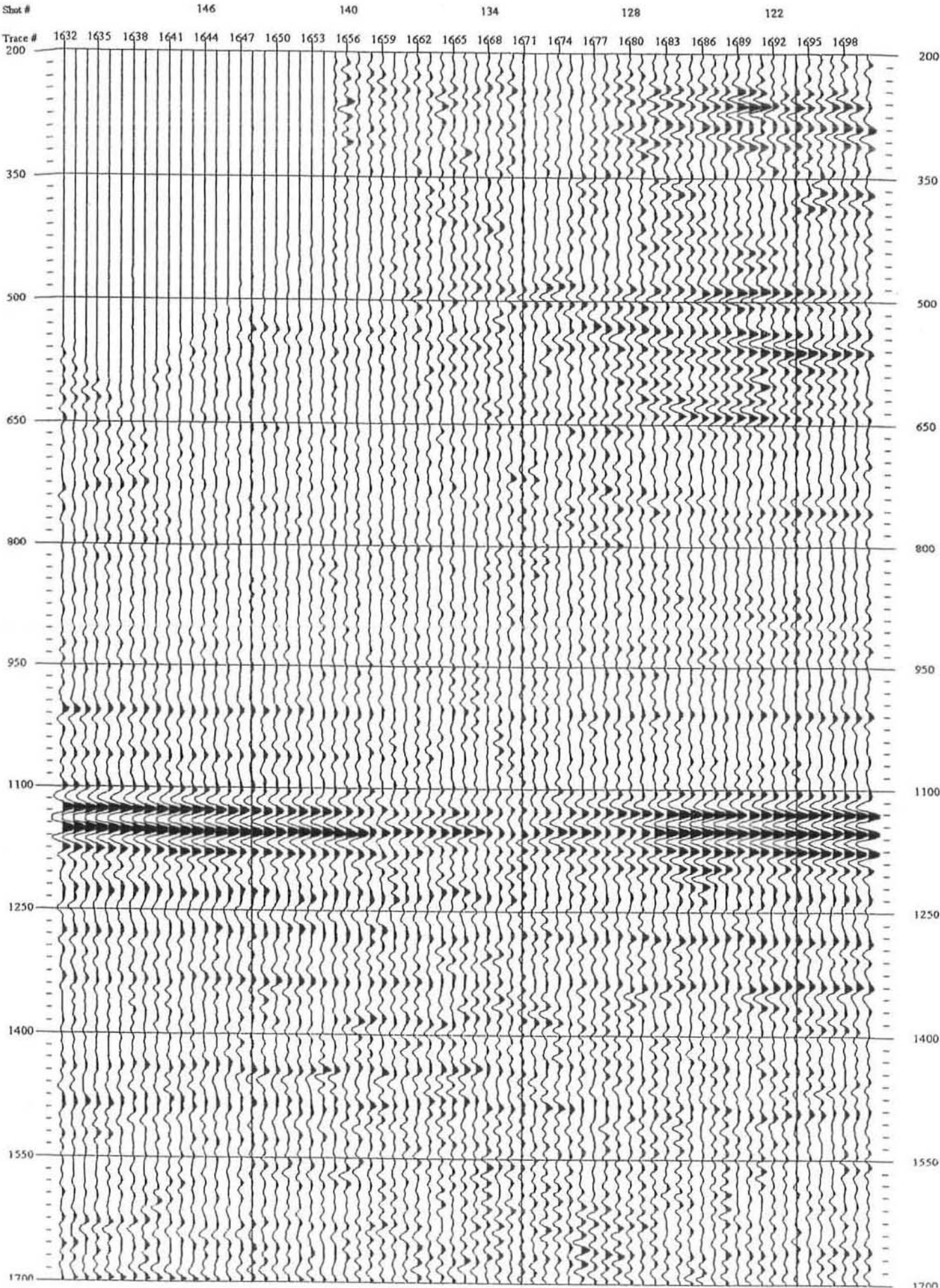


Fig. 5.2

Skip: 1

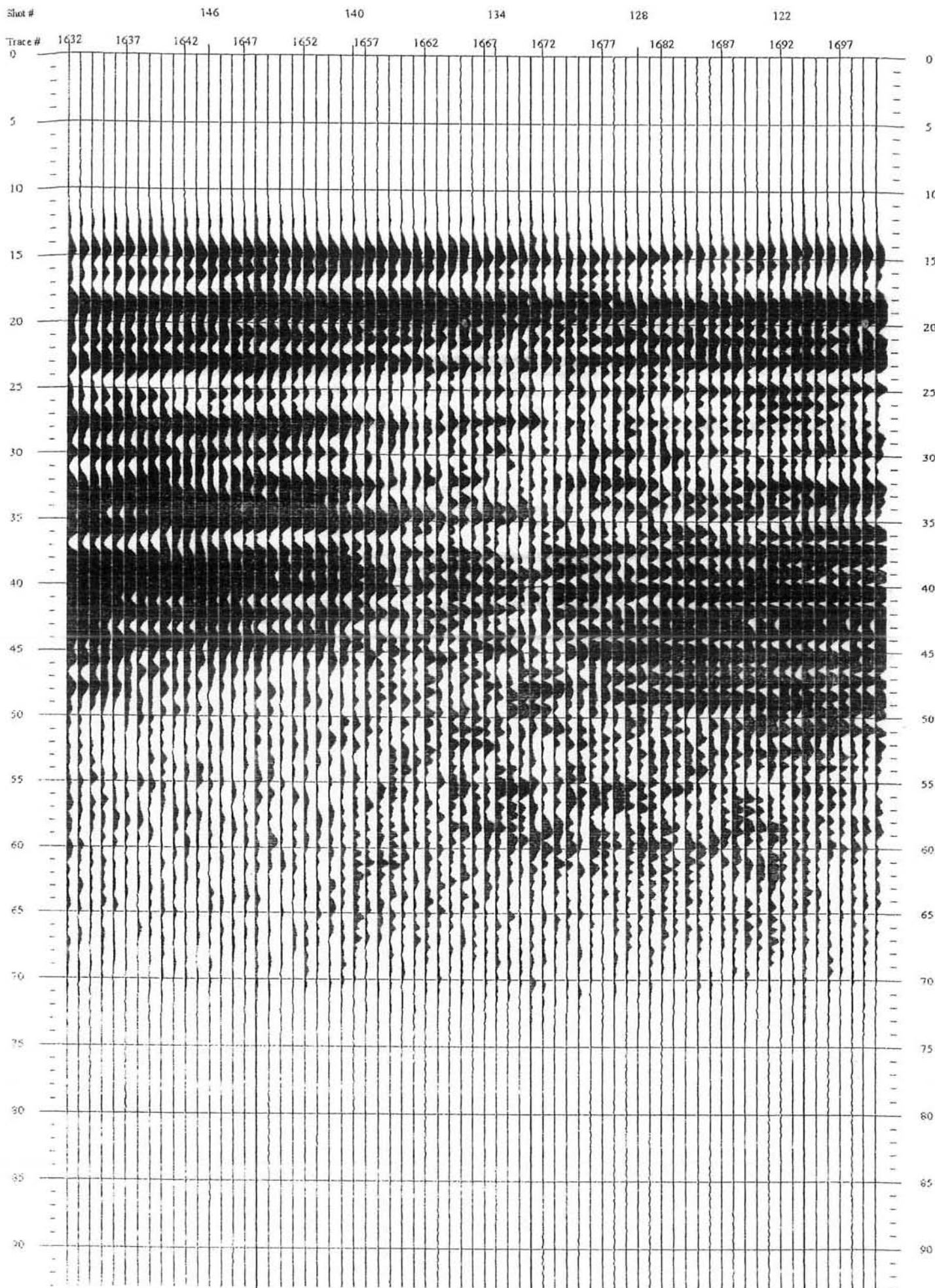


Fig 5.2 (a)

CONCLUSION # 2

The original seismic section shown in fig 5.3 convolved with Butterworth filter with frequencies 5-15-65-80 Hz. Shown in fig 5.4, where 5-80Hz is the Bandpass filter with 15db/octave is low roll-off and 65db/octave is high roll-off. Higher frequency bearing reflection amplitudes in the upper part (200-1100 m sec) are going to decrease and below (1250-1700 m sec) reflection amplitude almost decreased. And the main reflectors from 1100-1250 m sec are same. Overall the shape distortion of reflection amplitudes results in the decrease of temporal resolution. The amplitude spectrum of the Butterworth filtered section shown in fig 5.4 (a), which gives no desirable effect in comparison with the amplitude spectrum of the original seismic section fig. 5.3 (a).

Stop: 1

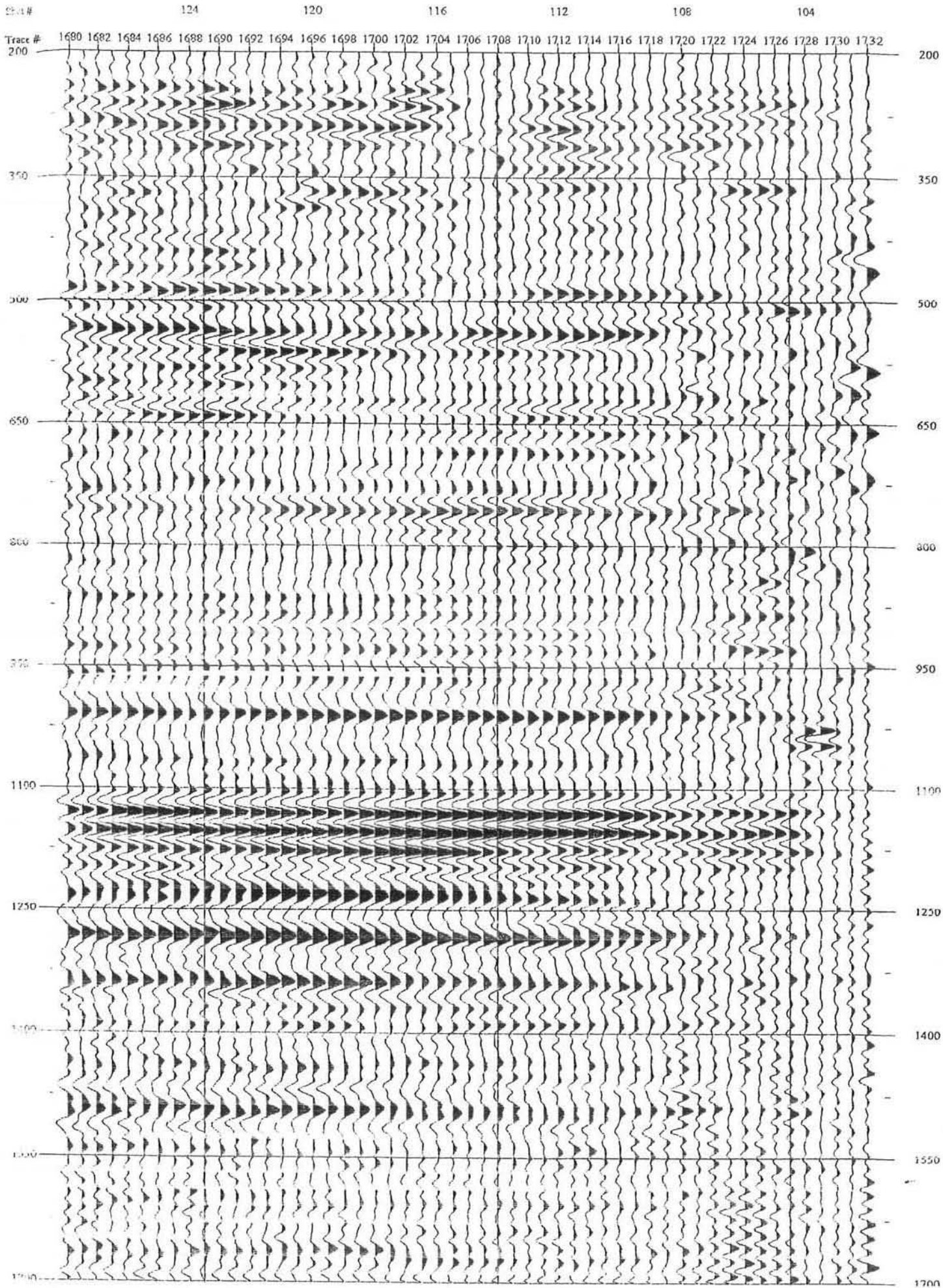


Fig. 5.3

Shot: 1

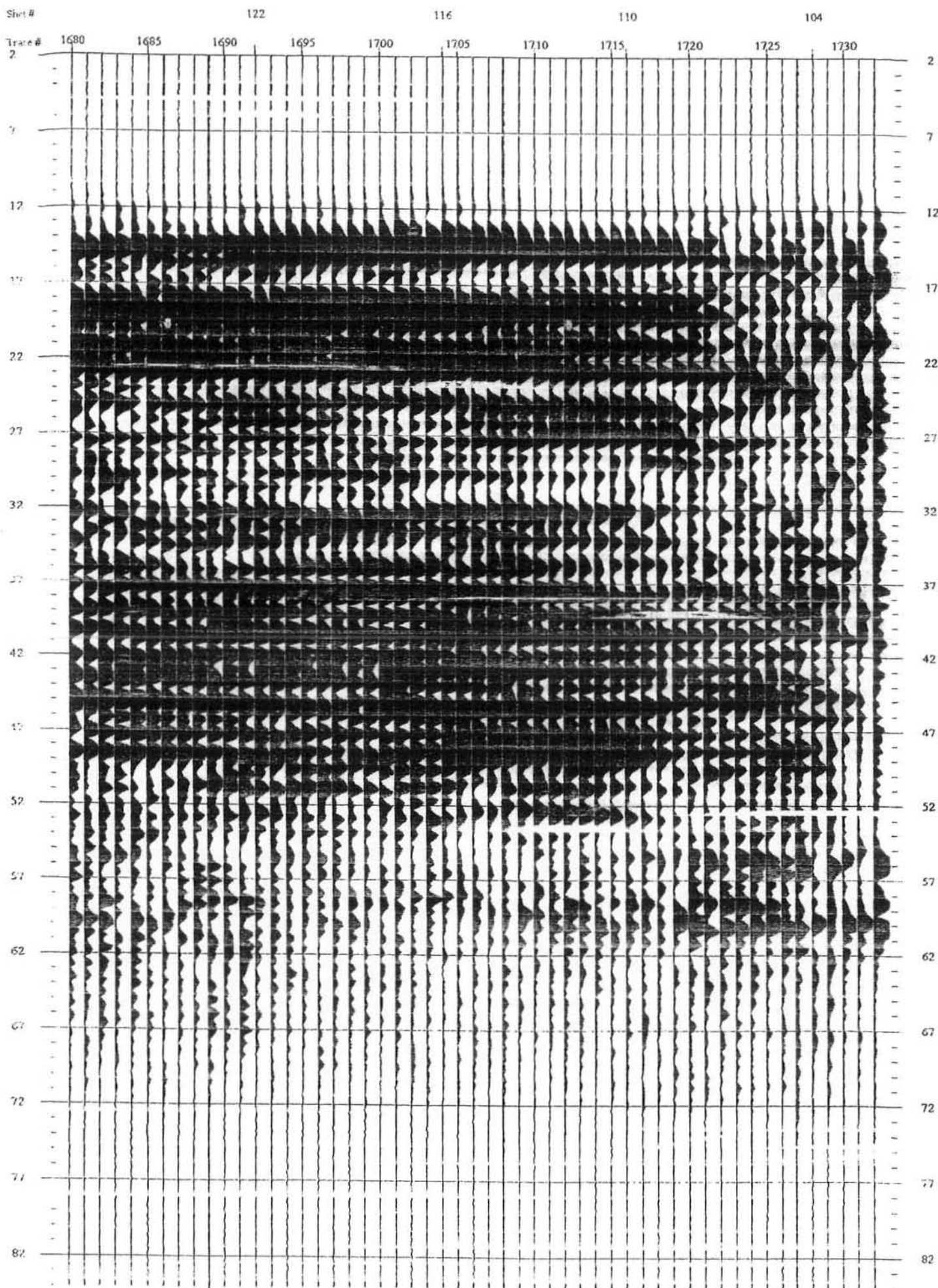


Fig. 5.3(1)

Step: 1

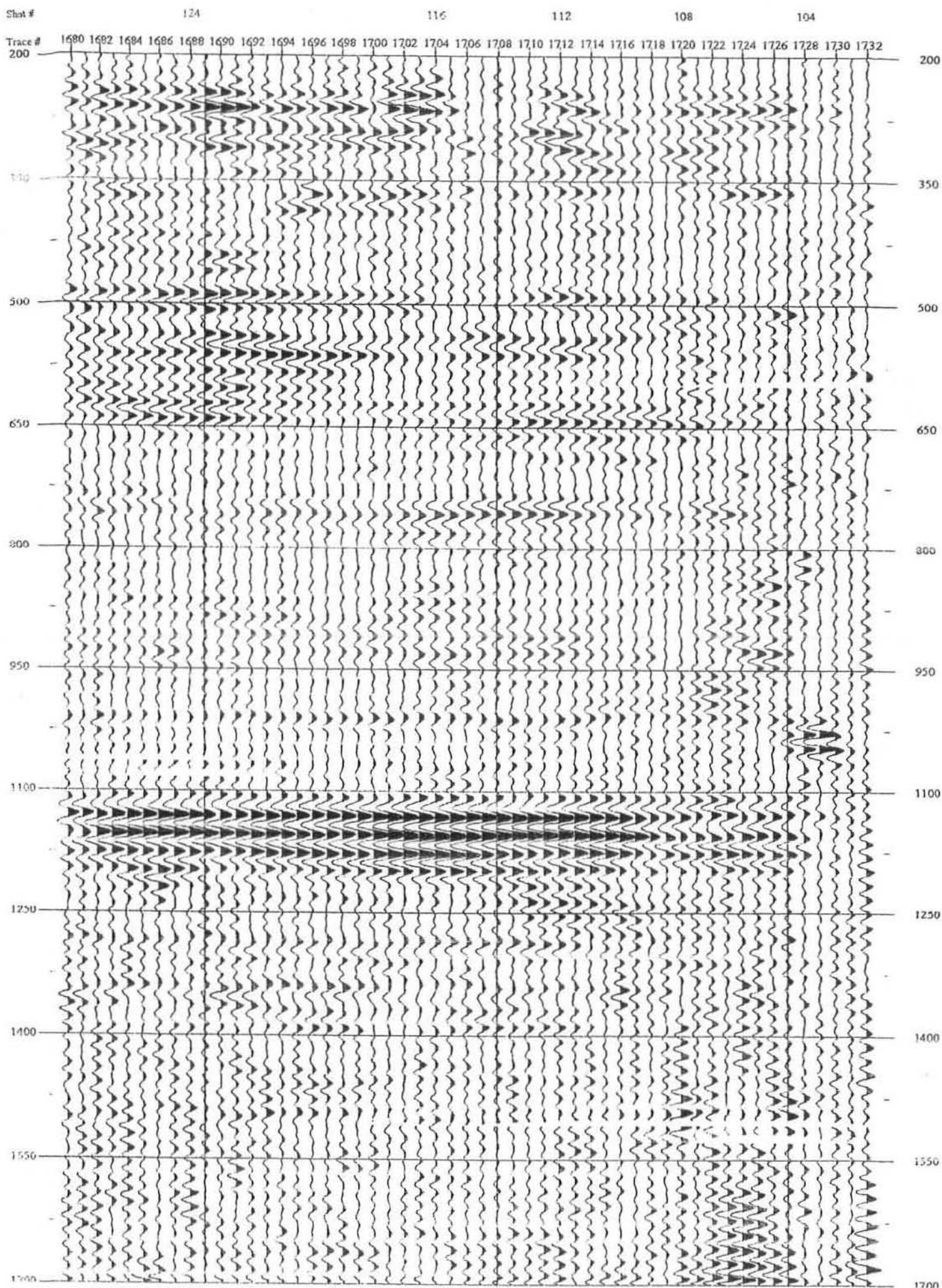


Fig. 5.4

Step: 1

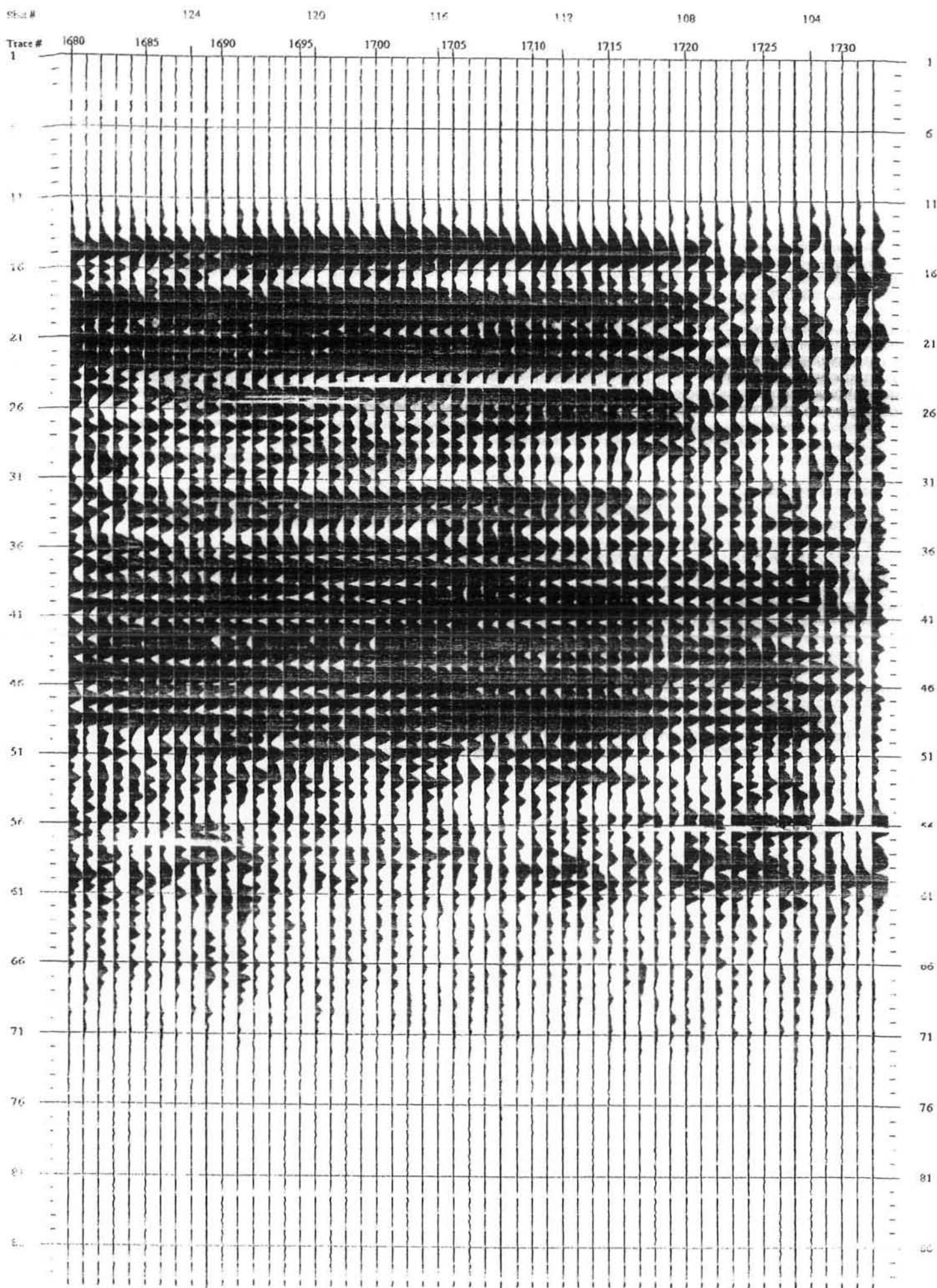


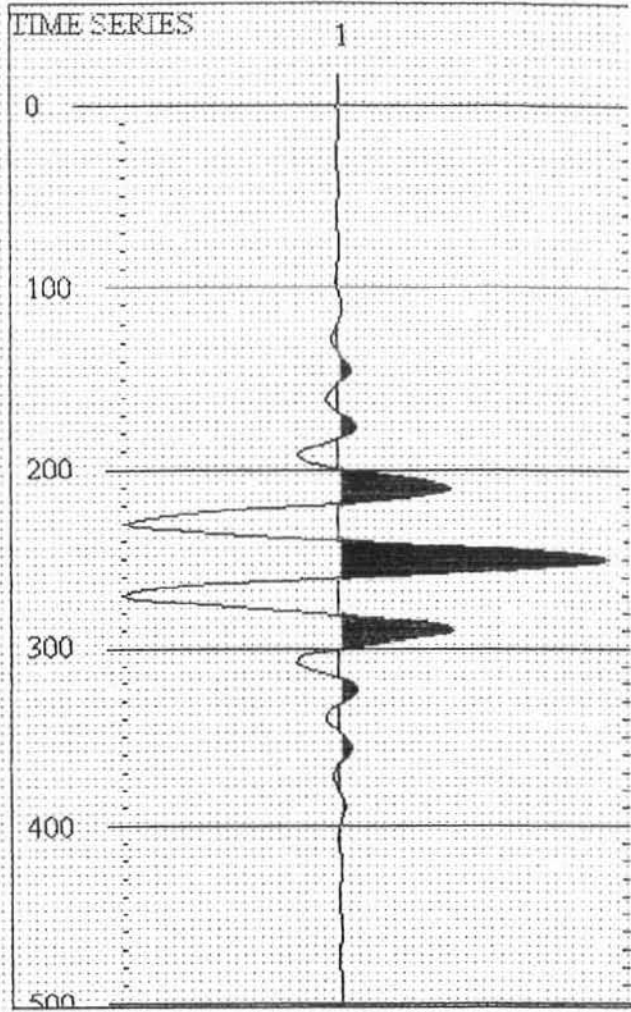
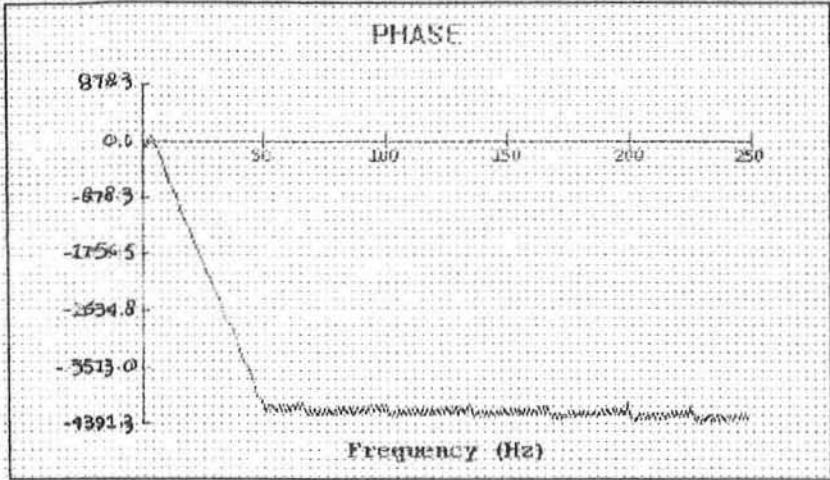
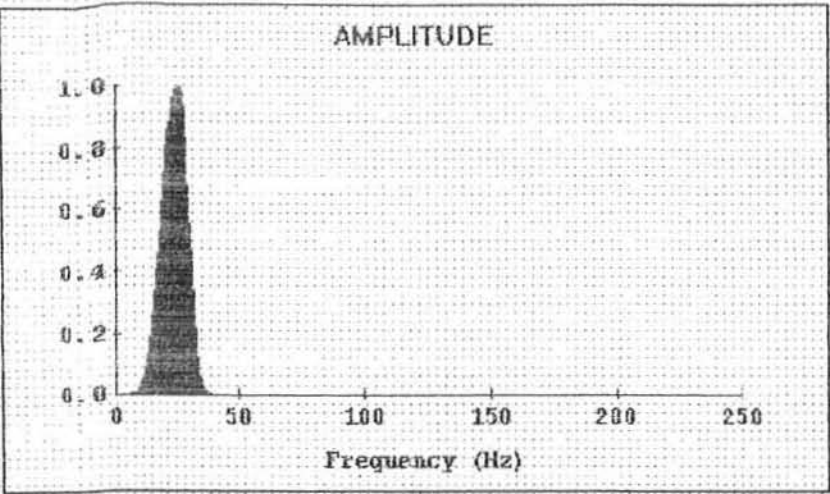
Fig. 5.4 (a)

CONCLUSION#3

The amplitude response, the phase response and the time series of Butterworth filter of order $n=1,2,3,4,6\&10$ are shown in fig. 5.5, 5.6, 5.7, 5.8, 5.9 & 5.10 respectively. In all these diagrams $F1=10$, $F2=20$, $F3=30$, $F4=40$ & operator length of 500 msec was applied.

The amplitude response of Butterworth filter increases towards higher frequencies, as Butterworth order n increases.

Minimum Butterworth order ($n=1$) produces more side lobes, as the order of Butterworth filter increases, the side lobes are going to reduce. The maximum Butterworth order ($n=10$) compressed the wavelet in time



$$n=1$$

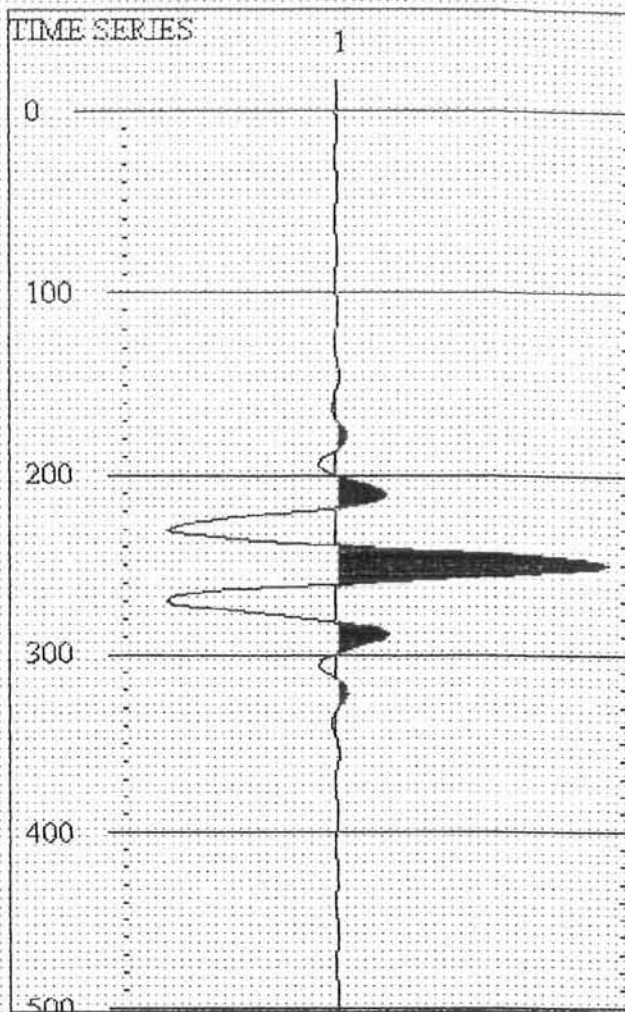
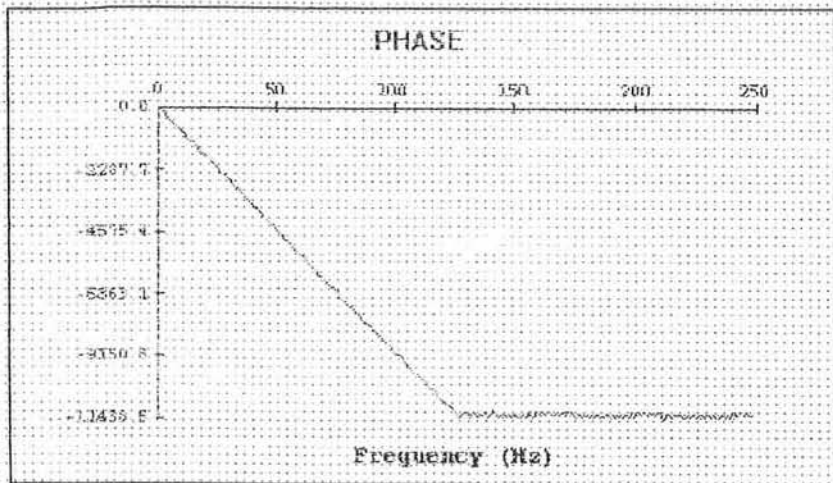
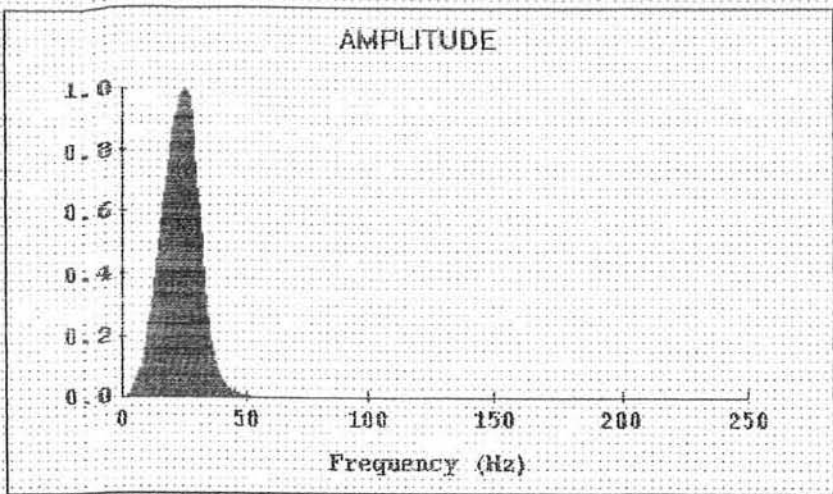
$$F_1=10 \text{ Hz}$$

$$F_2=20 \text{ Hz}$$

$$F_3=30 \text{ Hz}$$

$$F_4=40 \text{ Hz}$$

Fig 5.5



$$n=2$$

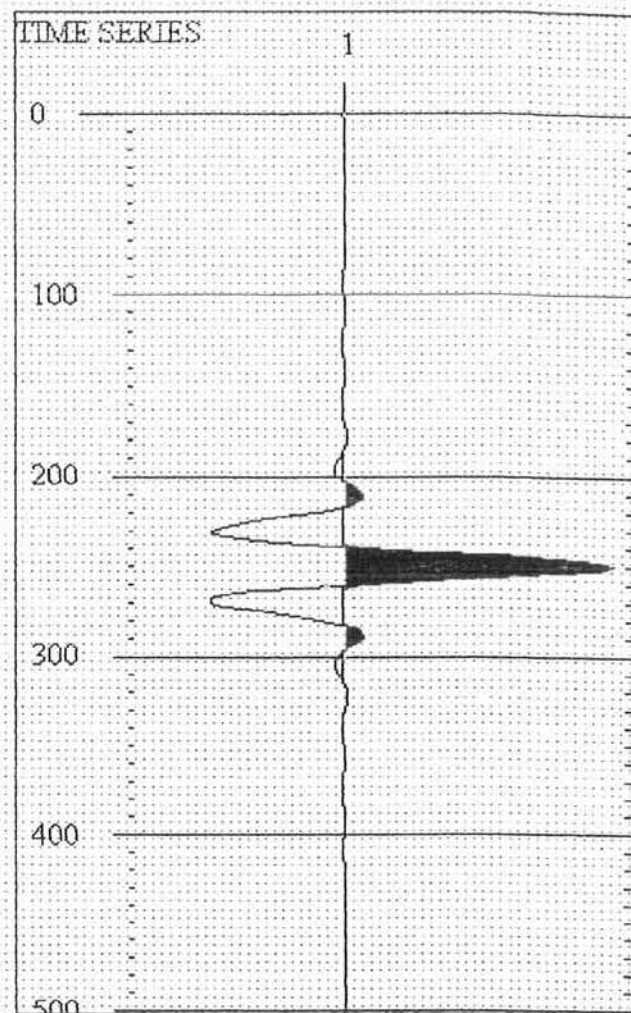
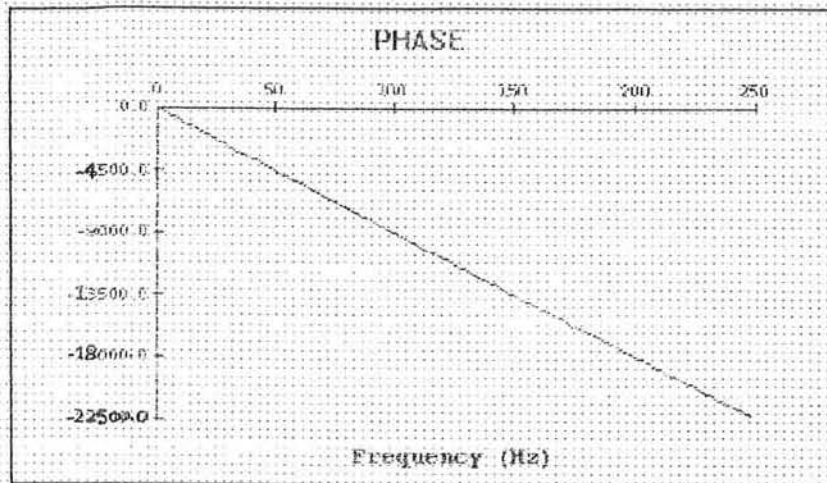
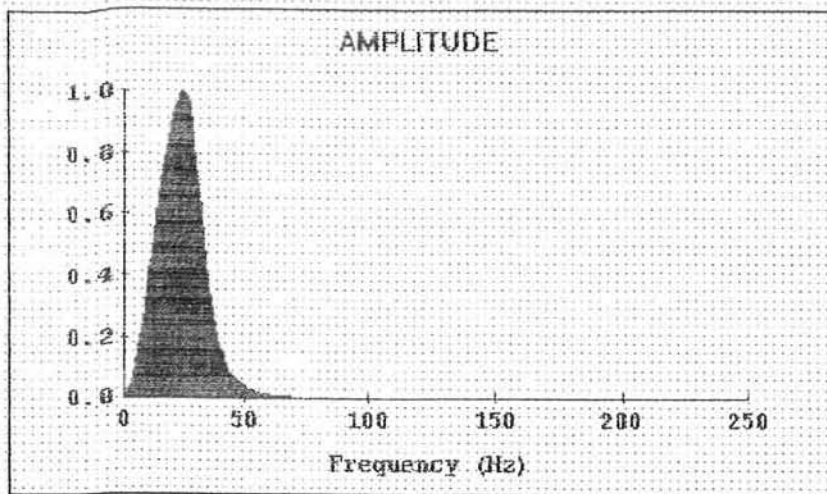
$$F_1=10 \text{ Hz}$$

$$F_2=20 \text{ Hz}$$

$$F_3=30 \text{ Hz}$$

$$F_4=40 \text{ Hz}$$

Fig 5.6



$n=3$

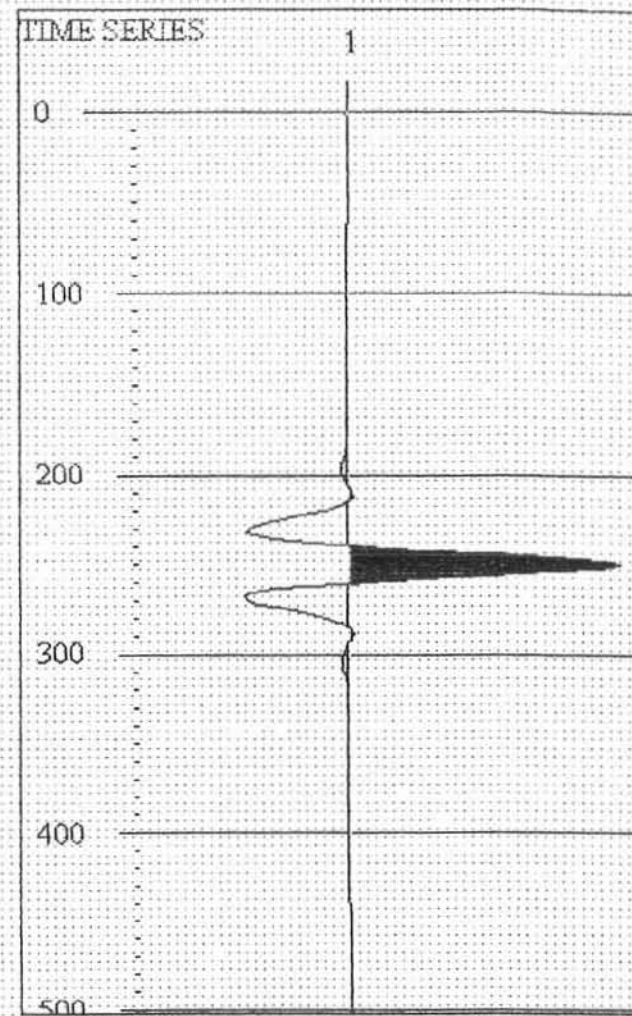
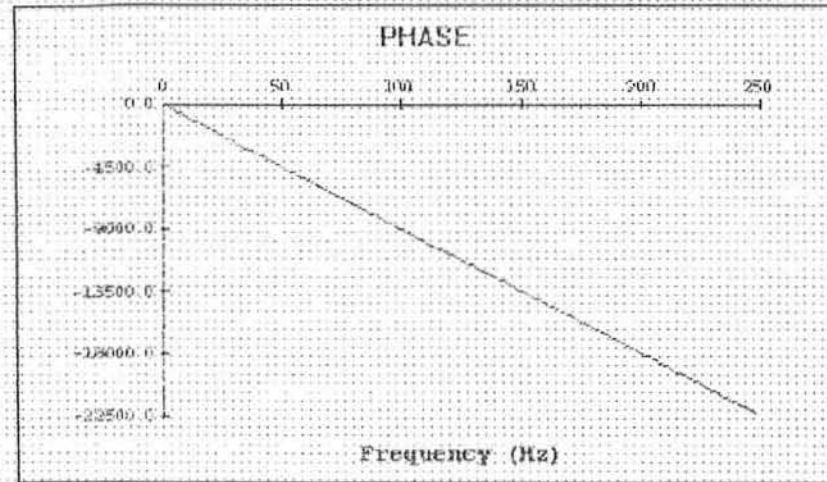
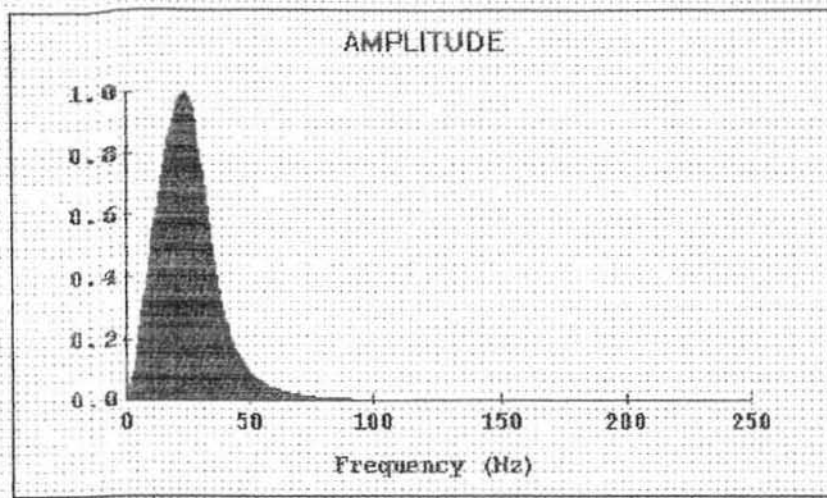
F 1=10 Hz

F 2=20 Hz

F 3=30 Hz

F 4=40 Hz

Fig 5.7



$n=4$

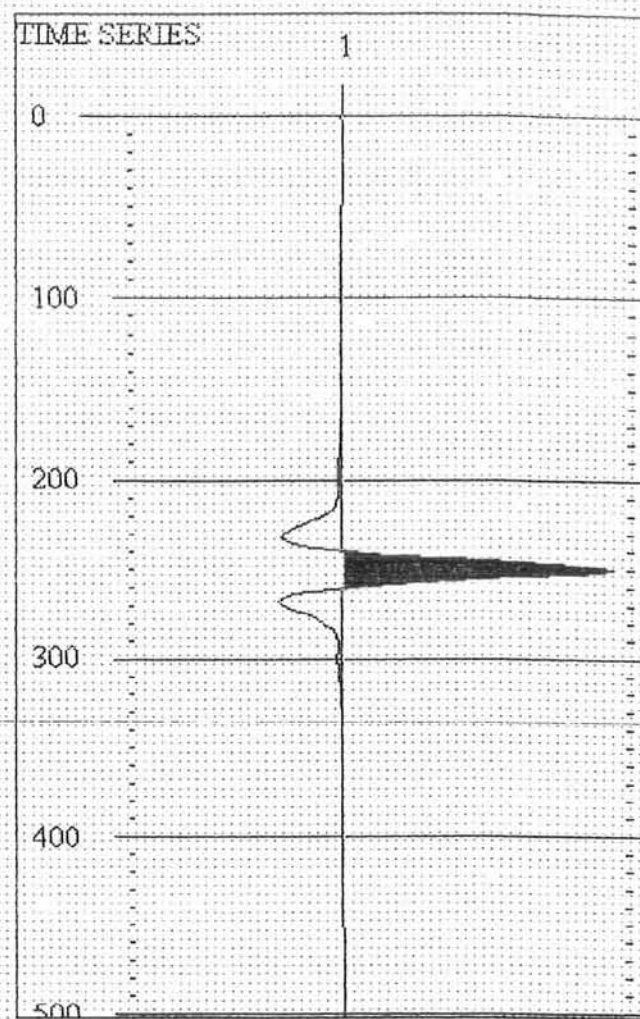
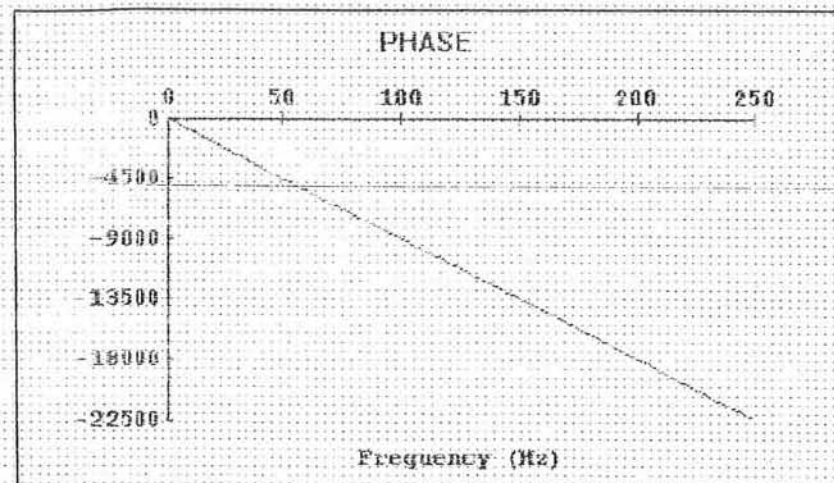
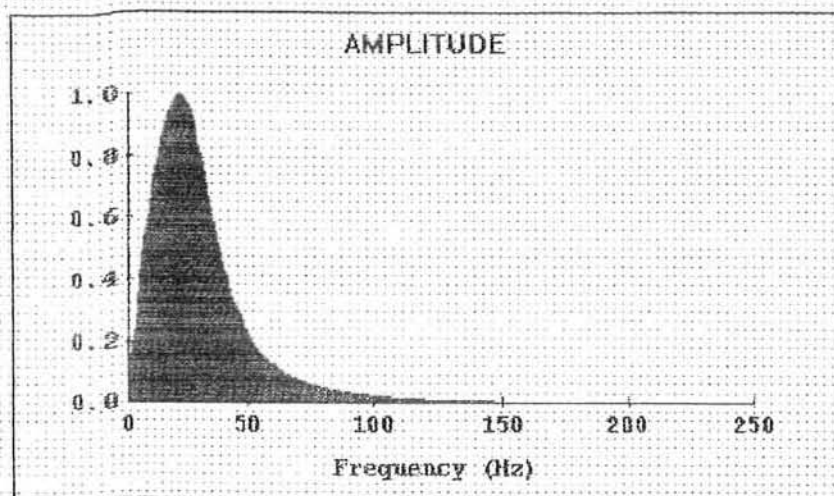
F 1=10 Hz

F 2=20 Hz

F 3=30 Hz

F 4=40 Hz

Fig 5.8



$n=6$

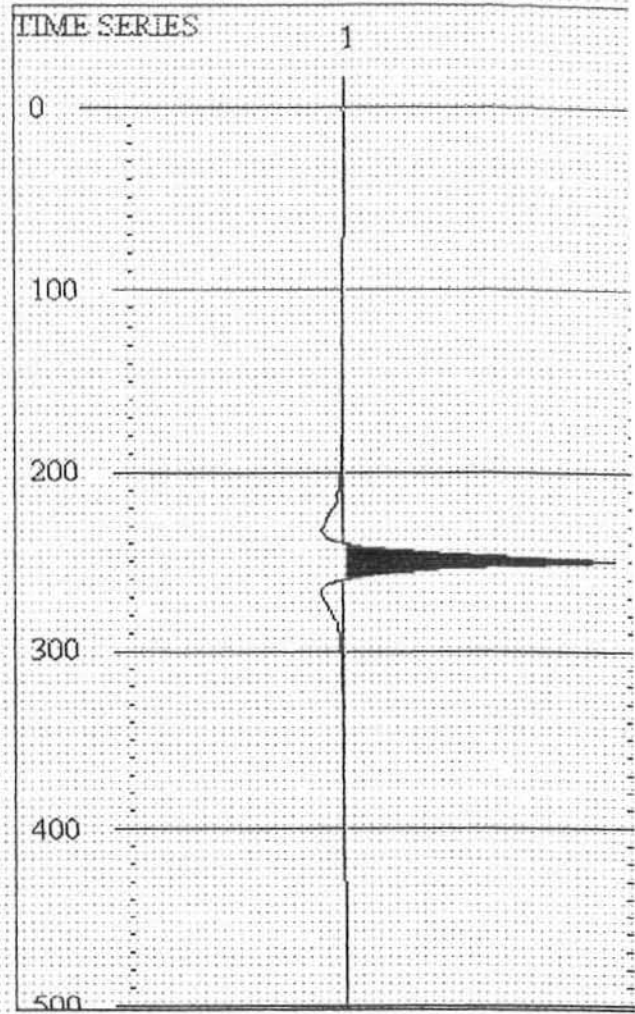
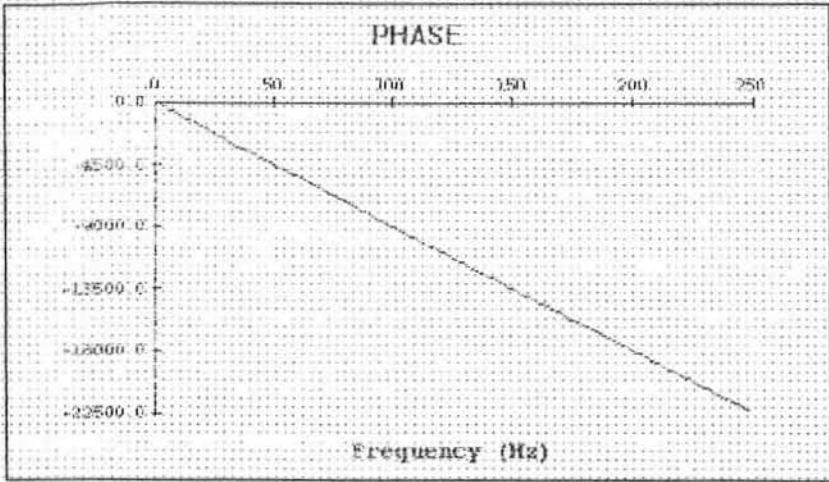
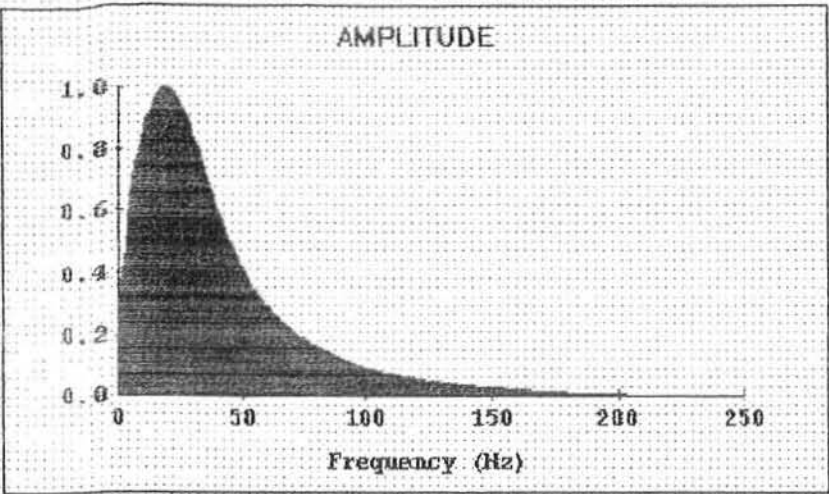
F 1=10 Hz

F 2=20 Hz

F 3=30 Hz

F 4=40 Hz

Fig 5.9



$n=10$

F 1=10 Hz

F 2=20 Hz

F 3=30 Hz

F 4=40 Hz

Fig 5.10