A New Approach to Decision Support System Through Application of Spherical Cubic Fuzzy Numbers

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A Thesis Submitted to the Department of Mathematics,
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in

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By

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Quaid-i-Azam University

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I, **Tehreem,** hereby state that my PhD thesis titled **A New Approach to Decision Support System Through Application of Spherical Cubic Fuzzy Numbers** is my own work and has not been submitted previously by me for taking any degree from the Quaid-I-Azam University Islamabad, Pakistan or anywhere else in the country/world.

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By

Tehreem

CERTIFICATE

A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE

DOCTOR OF PHILOSOPHY IN MATHEMATICS

We accept this thesis as conforming to the required standard.

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Certificate of Approval

This is to certify that the research work presented in this thesis entitled **A New Approch to Decision Support System Through Application of Spherical Cubic Fuzzy Numbers** was conducted by **Ms. Tehreem** under the kind supervision of **Dr. Amjad Hussain**. No part of this thesis has been submitted anywhere else for any other degree. This thesis is submitted to the Department of Mathematics, Quaid-i-Azam University, Islamabad in partial fulfillment of the requirements for the degree of Doctor of Philosophy in field of Mathematics from Department of Mathematics, Quaid-i-Azam University Islamabad, Pakistan.

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Dedicated

To

My

Father (LATE)

Acknowledgement

All praises to almighty **"ALLAH"** the creator of the universe, who blessed me with the knowledge and enabled me to complete the dissertation. All respects to **Holy Prophet MUHAMMAD (S.A.W),** who is the last messenger, whose life is a perfect model for the whole humanity.

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Tehreem

Abstract: This study discusses the methods used to deal with vagueness such as fuzzy sets (FSs), intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PyFSs), picture fuzzy sets (PFSs), spherical fuzzy sets (SFSs), intuitionistic cubic fuzzy sets (ICFS) and Pythagorean cubic fuzzy sets (PCFSs). Moreover, the main contribution of this study is the introduction of spherical cubic fuzzy sets (SCFSs). In addition, score function, accuracy function and some operators and distance measures are defined for SCFSs that are handy in the processes of decision making. Different Hamacher operators are used to established multi-criteria decision-making methods for the assessment of business execution with spherical fuzzy information. Furthermore, several new operations are characterized through Dombi $\tilde{t}$-norm and Dombi $\tilde{t}$-conorms to get the best results during the decision criteria. Also, various characteristics of such operators are examined. The operators defined are averaging operators, geometric operators, Hamacher averaging operators, Hamacher geometric operators, Dombi averaging operators and Dombi geometric operatos which include simple fuzzy weighted operators, ordered weighted operators and hybrid weighted operators. Additionally, a new methodology with incomplete weight information for spherical cubic fuzzy (SCF) multi-criteria decision making (MCDM) is suggested using TOPSIS method. The maximum deviation model is also put forward to determine the criteria of weight values. Finally, each of the proposed operators, methods and models are compared to the existing methods and techniques. Thus, the proposed methods are verified to be more effective in different types of decision making processes.

Contents

Chapter 1

Introduction

1.1 Literature survey

We often face imprecision or uncertainty in real-life situations due to a variety of factors. The concept of FS [1] firstly presented by Zadeh for uncertain situations, which provides a degree of membership degree. Membership degree 0 indicates complete dissatisfaction, while membership degree 1 indicates complete satisfaction. Other values in the unit interval are used to denote partial satisfaction depending on their level of satisfaction. Yager et al. [2], Pedrycz et al.[3], Maji [4], Trabia [5], Beni [6], and others have applied FS theory to a variety of contexts, including intelligent structures, pattern recognition, soft sets, traffic and transportation, and clustering. [7, 8, 9] is a good place to start significant progress in the concept of FS and other uncertainty methods, as well as their application domains.

Although the idea of FS was a success, it could not be handled in some situations, After that, Atanassov introduced the concept of IFS [10], which is an extended form of FS theory which handles more effectively with uncertain conditions because its structure is not limited to only membership degree. De et al. discuss medical diagnosis using IFSs in [11], and Xu described some aggregation operators for IFSs in [12], which were applied by Li to multi-attribute decision making (MADM) in [13]. Szmidt and Kacprzyk analyzed and applied some IFS similarity measures to medical diagnostics issues in [14].

The limitation of Atanassov's IFS structure is that the membership and non-membership functions can only be expressed as a sum of in unit interval. As consequence, an IFS is unable to properly explain a human's perspective in some cases. Yager [15] suggested the PyFS as an enlarged form of IFS to dealing with uncertainty in these situations. In addition, Fei et al. [16] applied Pythagorean fuzzy sets in a multi-attribute decision support technique. Several studies are based on the aggregation methodology of PyFS as well as its formulation in MADM. In [17, 18, 19, 20, 21], showing that PyFS analysis seems to be very worth to its broader scope. Garg developed the idea of linguistic PyFSs and analysed a MADM situation in [22]. In [23], Garg used the PyFSs system to explore the structural decision making with some probability, and in [24], the popular TOPSIS technique is utilized in the PyFSs framework.

Whenever a decision maker presents (.87.89), the IFSs and PyFSs are unable to handle appropriately, i.e. $.87 + .89 = 1.76 \notin [0, 1]$ and $(.87)^2 + (.89)^2 = .7569 + .7921 = 1.549 \notin [0, 1]$. Yager [25] proposed the concept of a q-rung set to overcome similar apprehensions.. Afterthat PyFS modified version q-rung works with uncetainities. The combination of the q^{th}order of memberships in q-ROFS cannot exceeds the unit interval $[0, 1]$. q -rung, a more widespread and practical alternative, has taken the place of PyFS, IFS, and FS. Figure 1 depicts the geometrical description of q-ROFSs and their existing techniques.

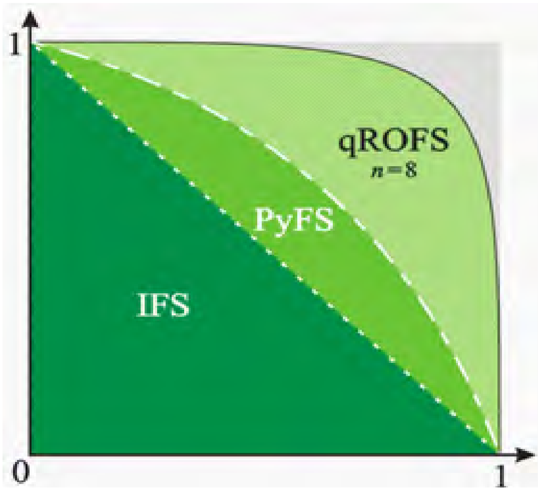

Figure 1: (Geometrical description of IFS, PyFS and q-ROFS)

Zadeh proposed the idea of IVFS [26], which is generalized form of a fuzzy set. However, Gargov [27] defined the concept of an IVIFS to extend an idea of IVFS. [28, 29, 30, 31] discuss the principle of IVIFS aggregation and its applications in MADM. IVIFSs were preferred to IFSs even though memberships degrees are defined as a finite interval $[0, 1]$ instead of a specific value. Furthermore, in [32], the concepts of the IVPyFS are introduced. Jun [33] came up with the concept of cubic set (CS), which is a generalised version of IVFS and IFS. Non-membership is a fuzzily defined set, whereas membership is expressed as an interval. Additional similar studies can be found in [34, 35, 36].

In situations where the human decision is not either yes or no but also has a degree of abstinence or refusal, the concepts of FS, IFS, PyFS, and Q- ROFS may not be used. Cuong [37] in Figure 2, defined the PFS (picture fuzzy set) to meet with such demands and model a concept close to human nature, which is based on four possible situations: satisfied, absence, dissatisfied, and refusing degrees. [38, 39] contain other simple analysis on PFSs.

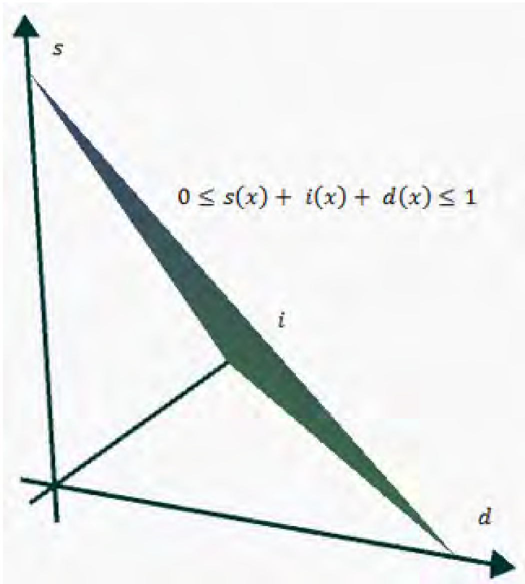

Figure 2 : (Geometrical representation of PFS)

Wei et al. [40] and Garg in [41] presented the aggregated operators in multi-attribute group decision support system using idea of PFS and it has been extensively used in [42] established clustering technique on the basis of computational intelligence. [43, 44, 45] can be reviewed for additional related research.

It is concluded from observing the structure of PFS that generalization of IFSs and can therefore handle data and circumstances more accurately than other structures. Because of some limitation on PFSs, applying principles to membership, abstinence and non-membership through self-choice is difficult. (T-SFSs) and (SFSs) were defined by Mahmood et al. [46] to improve the structure of PFSs. T-SFSs with this form of structure shape both human attitude and opinion, as well as yes/no type matters, and can handle any type of data without limitations. When we look at the shortcomings of PFSs and SFSs, it becomes clear that T-SFS has no restrictions in its structure.

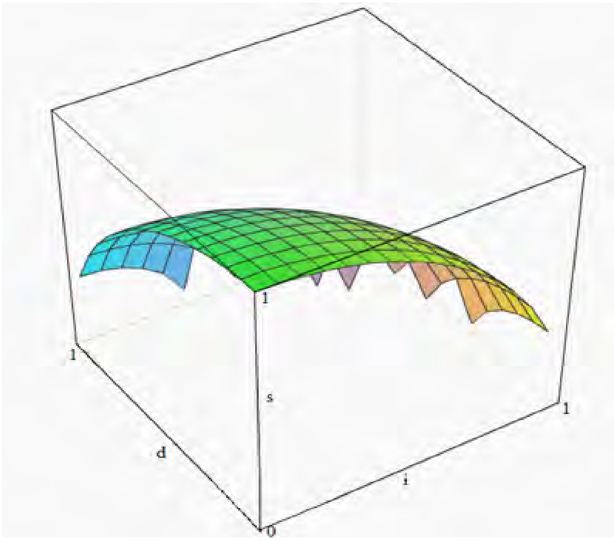

Figure 3 (Spherical fuzzy space)

By the range of PFS, Mahmood et al. [46] suggested a method for the limited structure of PFS and established the spherical fuzzy set (SFS). SFS is further generalized by adding a parameter "t" to the TSFS, decision makers choose membership degree values anywhere in the interval $[0, 1]$. A TSFS is the most generalized fuzzy framework for representing human opinion about any imprecise occurrence in a flexible and unlimited manner. Figure 3 depicts a geometrical comparison of the PFS, SFS, and TSFS, demonstrating the dominance of TSFS over the other fuzzy frameworks. The comprehensive structure and originality of SFSs are evident from the relationship with existing systems and their constraints. [47, 48, 49, 50, 51] contain some additional SFS-related research.

Multi-criteria group decision making process widely known idea in fuzzy set theoretic. It is considered one of the most influential subjects, and is discussed in almost every framework of fuzzy sets. Aggregation operators, as well as distance, similarity, and entropy scales are commonly used in the MADM method. Several aggregated operators, i.e, the average and geometric aggregated operators of IFSs have been developed so far and are used in MADM problems [52, 53]. MADM has designed and implemented several forms of aggregation operators for IFSs [54, 55, 56]. [57, 58] suggested average and geometric aggregated for PyFSs, which we will used in MADM problems. Garg [59] developed the linguistic PyFS and applied it to MADM problems. [60, 61, 62] contain some other related work on PyFS aggregation theory and its applications in MADM. The WA and WG aggregated for q-rung was presented by Liu and Wang [63], and their MADM's applications were examined.

Garg [64] and Wang et al. [65] developed the WA and WG aggregation operators of PFSs, respectively, which were later used in criteria decison making. The practicality of these aggregated operations is investigated in attribute decision making. Some collaborative aggregation operators for TSFSs and their applications in criterion decision support systems are investigated in [66]. In [67] Ullah et al. developed numerous TSFS similarity measures and investigated their application in pattern recognition challenges. Similarity tests can be used in MADM problems as well. [68] develops and applies the definition of T-spherical fuzzy Muirhead mean operators to the MADM problem. Xia and Xu [69] and Wang and Liu [70] used the averaging and geometric aggregation operators of hesitant fuzzy sets (HFSs) in MADM, respectively.

Data processing for operators has generally been a fascinating study topic, particularly for Hamacher operators. The Hamacher operators, as described by Hamacher presented in [71]. Jana et al. [72] suggested using picture fuzzy Hamacher aggregated operators to analyse organizational efficiency. Hamacher aggregation and its applications in decision support system were presented by Huang in [73]. The Hamacher aggregated operator link with interval-valued set was applied to multi-attribute decision support issues by Li et al.[74]. Xiao et al. offered the idea of an ordering weighting geometrical operators based on IVF in problem [75]. Garg [76] presented about the Hamacher aggregated operators using the structure of intuitionistic with entropy and these are used in multi-attribute decision support system. Wei et al. [77] suggested picture fuzzy Hamacher aggregated operators. Zhu et al. [78] established the notion of fuzzified clustering algorithms based on the Hamacher operations. Harish Garg researched generalized geometric operators for complex intuitionistic fuzzy sets [79]. The notion of the extended Hamacher family was defined by Roychowdhury et al. [80]. By applying the entropy measurement under spherical fuzzy data, [81] Barkub et al. suggested a methodology to the TOPSIS technique. Many studies have predicted climate and evaluated time series using new fuzzy methods based on SFS circumstances. They considered a variety of Hamacher aggregated operations for PFHA and PFHG operators under a picture fuzzy set, as well as an MCDM problem with the presented strategy's utility and flexibility.

In 1982, Dombi operations introduced and play a vital role on the pre-defined operators. Many researchers given the concept of norm by utilizing the Dombi operators in [82, 83, 84, 85]. They have a tendency to fluctuate in response to the operation of criteria. Lin used IFSs and combine them with Dombi operators to develop the Bonferroni operator [86] utilizing IFSs to tackle real-world challenges. Shi used them in decision support system and adjusted Dombi operations to neutrosophic sets in [87]. In [88] Lu proposed Dombi aggregation operation and linguistically cubic sets to address complex difficulties in multi-criteria decision support system. Afterthat, the Dombi prioritized aggregated operators proposed by Wei [89]. In multi-attribute decision support system, Zhang et al. [90] suggested use of picture Dombi Heronian operators. Jana et al. introduced numerous Dombi aggregated operators under the bipolar fuzzy set in [91]. Jana et al. [92] proposed a novel concept of Dombi picture aggregated operators and its use in the multicriteria technique. Later on, Ashraf et al. [93] expand the idea of picture fuzzy Dombi aggregated and offer a new concept of spherical Dombi aggregation operators, as well as applications in decision support system.

TOPSIS method play a vital role in decision support system. Khan et al. [94, 95] employed an integral choquet TOPSIS technology to address multi-attribute decision supoort difficulties and the IVPF GRA method. [96] is another important study that combines TOPSIS generalisations with the MCDM theory. Using hesitant fuzzy linguistic information, Wu et al. established a VIKOR and TOPSIS-based MCGDM technique in [97]. Additional significant details are available at [98, 99, 100, 101, 102, 103].

1.2 Chapter wise research study

This thesis consists of six chapters. $\underset{\sim}{X}$ denotes the universal set throughout the study, unless otherwise stated.

In chapter 1, we will discuss some motivational literature review.

In chapter 2, we will discuss some of the very earliest ideas FSs, IVFSs, IFSs, IVIFSs, PyFSs, IVPyFSs, PFSs, IVPFSs, SFSs, IVSFSs, ICFSs and PCFSs. The relationship between each concept's definitions is demonstrated. These ideas are useful when starting new research studies.

In chapter 3, We will define a spherical cubic fuzzy set. Afterthat, compare two spherical cubic fuzzy numbers, we define several essential operators and construct scoring function. The distance between two spherical cubic fuzzy numbers is defined as well. We introduced various aggregation operators i.e, SCFWA, SCFOWA, SCFHWA, SCFWG, SCFOWG and the SCFHWG operators are proposed based on the specified operators. We discuss some of the existing operators and propose a multi-attribute decision support system by utilizing these operators.

We defined decision support system to evaluating marketing performance with spherical fuzzy data in chapter 4. We used Hamacher aggregation operators like SCFHWA operator, SCFHOWA operator, SCFHHA operator, SCFHWG operator, SCFHOWG operator and SCFHHG operator. Finally, we supported the proposed strategy by comparing it to existing solutions for feasibility and adequacy.

In Chapter 5, we use Dombi $\tilde{t}$-norm and conorms which characterize various novel procedures in order to come at the optimum choice criteria. We proposed the SCFDWA, SCFDOWA, SCFDHWA, SCFDHWG, SCFDOWG and SCFDHWG operators. These previously stated operators are extremely helpful in successfully arranging selection difficulties. Then, using the spherical cubic fuzzy set, a computation is developed, and this methodology is used to decision-making problems to illustrate its importance and usefulness. The comparative study of different approaches also being carried out to highlight the benefits of our methodology. The findings indicate that the proposed technique is both rational and effective in the given situation.

In Chapter 6, we utilize the TOPSIS technique to develop a new methodology for multi-aatribute decision support system with partial weighted data using spherical cubic fuzzy (SCF). To begin, the maximum deviation model for establishing weight value criterion is proposed. On the basis of the provided technique, an MCDM methodology based on SCF data is offered. A numeric illustration is also presented. At last, the new study is compared to earlier research in a systematic and structured manner.

1.3 Research profile

1. Tehreem, Hussain, A. and Khan, M. S. A. “Average operators based on spherical cubic fuzzy number and their application in multi-attribute decision making” Annals of optimization theory and practice, 3(4), 83-111, (2020).

2. Tehreem, Al-Shomrani, M. M., Abdullah, S. and Hussain, A. “Evaluation of enterprise production based on spherical cubic Hamacher aggregation operators” Mathematics, 8(10), 1761, (2020).

3. Tehreem, Hussain, A. and Alsanad, A. “Novel Dombi aggregation operators in spherical cubic fuzzy information with applications in multiple attribute decision-making” Mathematical Problems in Engineering, (2021).

4. Tehreem, Hussain, A., Alsanad, A. and AA Mosleh, M. “Spherical cubic fuzzy extended TOPSIS method and its application in multicriteria decision-making” Mathematical Problems in Engineering, (2021).

Research Contributions: Chapter 3, published in the Journal of "Annals of Optimization Theory and Practice". Chapter 4, published in the Journal of "Mathematics, MDPI". Chapter 5, published in the journal of "Mathematical Problems in Engineering". Chapter 6, published in "Mathematical Problems in Engineering".

Chapter 2

Preliminaries

The goal of this chapter is to express key definitions i.e, FS IVFS, IFS, IVIFS, PyFS, IVPyFS, PFS, IVPFS, SFS, IVSFS, CS, ICFS and PyCFS and their characteristics.

2.1 Operations on FS

The notion of FS, IVFS, and their operations will be discussed.

Definition 2.1.1 *[1] Consider $\underset{\sim}{X} \neq \emptyset$ then a FS $\underset{\sim}{F}$ is expressed below:*

$$\underset{\sim}{F} = \langle \underset{\sim}{x}, \breve{\alpha}_{\underset{\sim}{F}}(\underset{\sim}{x}) \mid \underset{\sim}{x} \in \underset{\sim}{X} \rangle,$$

here $\breve{\alpha}_{\underset{\sim}{F}}$ denote memebership function.

Definition 2.1.2 *[104] Consider a set $\underset{\sim}{X} \neq \emptyset$ then an IVFS $\underset{\sim}{F}_t$ is expressed below:*

$$\underset{\sim}{F}_t = \left\{ \left[\breve{\alpha}^{-}_{\underset{\sim}{F}_t}(\underset{\sim}{x}), \breve{\alpha}^{+}_{\underset{\sim}{F}_t}(\underset{\sim}{x}) \right] \mid \underset{\sim}{x} \in \underset{\sim}{X} \right\},$$

subject to the condition $\sup \left[\breve{\alpha}^{-}_{\underset{\sim}{F}_t}(\underset{\sim}{x}), \breve{\alpha}^{+}_{\underset{\sim}{F}_t}(\underset{\sim}{x}) \right] \leq 1$.

2.2 Operations on IFS

The concepts of IFS and IVIFS, as well as their operation, will be explored.

Definition 2.2.1 *[10] Consider a set $\underset{\sim}{X} \neq \emptyset$ then IFS $\underset{\sim}{I}$ is expressed below:*

$$\underset{\sim}{I} = \left\langle \breve{\alpha}_{\underset{\sim}{I}}(\underset{\sim}{x}), \breve{\beta}_{\underset{\sim}{I}}(\underset{\sim}{x}) \right\rangle,$$

here $\breve{\alpha}_{\underset{\sim}{I}}(\underset{\sim}{x})$ and $\breve{\beta}_{\underset{\sim}{I}}(\underset{\sim}{x})$ are membership fuction and non-membership function with $0 \leq \breve{\alpha}_{\underset{\sim}{I}} + \breve{\beta}_{\underset{\sim}{I}} \leq 1$.

Definition 2.2.2 *[105] Consider a set $\underset{\sim}{X} \neq \emptyset$ then an IVIFS $\tilde{I}_t$ is expressed below:*

$$\underset{\sim}{I}_t = \left\{ \left[\check{\alpha}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\alpha}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right], \left[\check{\beta}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\beta}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right] \right\},$$

here $\left[\check{\alpha}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\alpha}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right]$ and $\left[\check{\beta}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\beta}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right]$ are membership function and non-membership function, subject to the following condition

$$0 \leq \sup \left[\check{\alpha}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\alpha}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right] + \sup \left[\check{\beta}^-_{\underset{\sim}{I}_t}(\underset{\sim}{x}), \check{\beta}^+_{\underset{\sim}{I}_t}(\underset{\sim}{x}) \right] \leq 1.$$

2.3 Operations on PyFS

The idea of PyFS, IVPyFS, and their operations will be discussed.

Definition 2.3.1 *[15] Consider a set $\underset{\sim}{X} \neq \emptyset$ then a PyFS $\underset{\sim}{P}$ is expressed below:*

$$\underset{\sim}{P} = \left\langle \check{\alpha}_{\underset{\sim}{P}}(\underset{\sim}{x}), \check{\beta}_{\underset{\sim}{P}}(\underset{\sim}{x}) \right\rangle,$$

here $\check{\alpha}_{\underset{\sim}{P}}(\underset{\sim}{x})$ and $\check{\beta}_{\underset{\sim}{P}}(\underset{\sim}{x})$ are membership function and non-membership function with $0 \leq (\check{\alpha}_{\underset{\sim}{P}})^2 + (\check{\beta}_{\underset{\sim}{P}})^2 \leq 1$.

Definition 2.3.2 *[106] Consider a set $\underset{\sim}{X} \neq \emptyset$ then an IVPyFS $\underset{\sim}{P}_t$ is expressed below:*

$$\underset{\sim}{P}_t = \left\{ \left[\check{\alpha}^-_{\underset{\sim}{P}_t}(\underset{\sim}{x}), \check{\alpha}^+_{\underset{\sim}{P}_t}(\underset{\sim}{x}) \right], \left[\check{\beta}^-_{\underset{\sim}{P}_t}(\underset{\sim}{x}), \check{\beta}^+_{\underset{\sim}{P}_t}(\underset{\sim}{x}) \right] \right\},$$

subject to the following condition

$$0 \leq \left(\sup \left[\check{\alpha}^-_{\underset{\sim}{P}_t}(\underset{\sim}{x}), \check{\alpha}^+_{\underset{\sim}{P}_t}(\underset{\sim}{x}) \right] \right)^2 + \left(\sup \left[\check{\beta}^-_{\underset{\sim}{P}_t}(\underset{\sim}{x}), \check{\beta}^+_{\underset{\sim}{P}_t}(\underset{\sim}{x}) \right] \right)^2 \leq 1.$$

2.4 Operations on SFS

Now we will examine at the concepts of SFS and IVSFS, as well as their properties.

Definition 2.4.1 *[108] Consider a set $\underset{\sim}{X} \neq \emptyset$ then a SFS $\underset{\sim}{S}$ is defined below:*

$$\underset{\sim}{S} = \left\langle \check{\alpha}_{\underset{\sim}{S}}(\underset{\sim}{x}), \check{\eta}_{\underset{\sim}{S}}(\underset{\sim}{x}), \check{\beta}_{\underset{\sim}{S}}(\underset{\sim}{x}) \right\rangle,$$

here membership function $\check{\alpha}_{\underset{\sim}{S}}(\underset{\sim}{x})$, neutral $\check{\eta}_{\underset{\sim}{S}}(\underset{\sim}{x})$ and non-membership is $\check{\beta}_{\underset{\sim}{S}}(\underset{\sim}{x})$ with $0 \leq \left(\check{\alpha}_{\underset{\sim}{S}}\right)^2 + \left(\check{\eta}_{\underset{\sim}{S}}\right)^2 + \left(\check{\beta}_{\underset{\sim}{S}}\right)^2 \leq 1$.

SFNs ranking :

Now we will discuss some characteristics that will help us evaluate SFNs.

Definition 2.4.2 *[108] Consider a SFN $\underset{\sim}{S}_1 = \left\langle \check{\alpha}_{\underset{\sim}{S}_1}(\underset{\sim}{x}), \check{\eta}_{\underset{\sim}{S}_1}(\underset{\sim}{x}), \check{\beta}_{\underset{\sim}{S}_1}(\underset{\sim}{x}) \right\rangle$. The following are the score and accuracy functions:*

1. $\check{\check{S}}_c(\underset{\sim}{S}_1) = \frac{\left(\check{\alpha}_{\underset{\sim}{S}_1} + 1 - \check{\eta}_{\underset{\sim}{S}_1} + 1 - \check{\beta}_{\underset{\sim}{S}_1}\right)}{3} = \frac{\left(2 + \check{\alpha}_{\underset{\sim}{S}_1} - \check{\eta}_{\underset{\sim}{S}_l} - \check{\beta}_{\underset{\sim}{S}_l}\right)}{3}$
2. *accuracy*$(\underset{\sim}{S}_1) = \check{\alpha}_{\underset{\sim}{S}_1} - \check{\beta}_{\underset{\sim}{S}_1}$.

2.5 Cubic sets

The notion of CS, ICFS and PCFS as well as their main characteristics, will be discussed.

Definition 2.5.1 *[33] Consider $\underset{\sim}{X} \neq \emptyset$, then a CS $\c{C}$ is expressed as below:*

$$\c{C} = \left\langle \overset{*}{\breve{\alpha}}(\underset{\sim}{x}), \breve{\beta}(\underset{\sim}{x}) \right\rangle,$$

here $\overset{*}{\breve{\alpha}}(\underset{\sim}{x})$ represent an IVFS and $\breve{\beta}(\underset{\sim}{x})$ represent the FS in $\underset{\sim}{X}$.

Intuitionistic cubic fuzzy sets:

Definition 2.5.2 *[109] Consider $\underset{\sim}{X} \neq \emptyset$, then an ICFS $\underset{\sim}{I}_c$ is expressed below:*

$$\underset{\sim}{I}_c = \left\langle \breve{\alpha}_{\underset{\sim}{I}_c}(\underset{\sim}{x}), \breve{\beta}_{\underset{\sim}{I}_c}(\underset{\sim}{x}) \right\rangle,$$

here $\breve{\alpha}_{\underset{\sim}{I}_c}(\underset{\sim}{x})$ and $\breve{\beta}_{\underset{\sim}{I}_c}(\underset{\sim}{x})$ are membership function and non-membership function respectively.

Operations on ICFSs:
The following operations for ICFSs holds:

1. **Multiplication** : The multiplication of $\tilde{I}_{c_1}$ and $\tilde{I}_{c_2}$ is an ICFS $\tilde{I}_{c_1} \otimes \tilde{I}_{c_2}$ defined as

$$\underset{\sim}{I}_{c_1} \otimes \underset{\sim}{I}_{c_2} = \left\langle \begin{array}{c} \left(\left[\breve{a}_1^- \breve{a}_2^-, \breve{a}_1^+ \breve{a}_2^+\right], \breve{\lambda}_1 \breve{\lambda}_2\right), \\ \left(\left[\breve{b}_1^- + \breve{b}_2^- - \breve{b}_1^- \breve{b}_2^-, \breve{b}_1^+ + \breve{b}_2^+ - \breve{b}_1^+ \breve{b}_2^+\right], \breve{\mu}_1 + \breve{\mu}_2 - \breve{\mu}_1 \breve{\mu}_2\right) \end{array} \right\rangle,$$

2. **Exponent** :For $\gamma \geq 0$, exponent of $\underset{\sim}{I}_{c_1}$ is an ICFS $\underset{\sim}{I}_{c_1}^{\gamma}$ defined as

$$\underset{\sim}{I}_{c_1}^{\gamma} = \left\langle \begin{array}{c} \left(\left[\left(\breve{a}_1^-\right)^{\gamma}, \left(\breve{a}_1^+\right)^{\gamma}\right], \left(\breve{\lambda}_1\right)^{\gamma}\right), \\ \left(\left[1 - \left(1 - \breve{b}_1^-\right)^{\gamma}, 1 - \left(1 - \breve{b}_1^+\right)^{\gamma}\right], 1 - \left(1 - \breve{\mu}_1\right)^{\gamma}\right) \end{array} \right\rangle,$$

3. **Scalar multiplication** :For $\gamma \geq 0$, scalar multiplication of $\underset{\sim}{I}_{c_1}$ is an ICFS $\gamma.\underset{\sim}{I}_{c_1}$ defined as

$$\gamma.\underset{\sim}{I}_{c_1} = \left\langle \begin{array}{c} \left(\left[1 - \left(1 - \breve{a}_1^-\right)^{\gamma}, 1 - \left(1 - \breve{a}_1^+\right)^{\gamma}\right], 1 - \left(1 - \breve{\lambda}_1\right)^{\gamma}\right), \\ \left(\left[\left(\breve{b}_1^-\right)^{\gamma}, \left(\breve{b}_1^+\right)^{\gamma}\right], \left(\breve{\mu}_1\right)^{\gamma}\right) \end{array} \right\rangle.$$

Pythagorean cubic fuzzy sets:

Definition 2.5.3 *[110] Consider $\underset{\sim}{X} \neq \emptyset$, then a PyCFS $\underset{\sim}{P}_c$ is expressed below:*

$$\underset{\sim}{P}_c = \left\langle \breve{\alpha}_{\underset{\sim}{P}_c}(\underset{\sim}{x}), \breve{\beta}_{\underset{\sim}{P}_c}(\underset{\sim}{x}) \right\rangle,$$

here $\breve{\alpha}_{\underset{\sim}{P}_c}(\underset{\sim}{x}) = \left\langle [\breve{a}^-, \breve{a}^+], \breve{\lambda} \right\rangle$, $\breve{\beta}_{\underset{\sim}{P}_c}(\underset{\sim}{x}) = \left\langle [\breve{b}^-, \breve{b}^+], \breve{\mu} \right\rangle$ *are memebership function and non-membership function respectively.*

Operations on PyCFSs

The following operations for PyCFSs holds:

1. $$\underset{\sim}{P}_{c_1} \oplus \underset{\sim}{P}_{c_2} = \left\langle \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{(\breve{a}_1^-)^2 + (\breve{a}_2^-)^2 - (\breve{a}_1^-)^2 (\breve{a}_2^-)^2}, \\ \sqrt{(\breve{a}_1^+)^2 + (\breve{a}_2^+)^2 - (\breve{a}_1^+)^2 (\breve{a}_2^+)^2} \end{array} \right], \\ \sqrt{(\breve{\lambda}_1)^2 + (\breve{\lambda}_2)^2 - (\breve{\lambda}_1)^2 (\breve{\lambda}_2)^2} \end{array} \right), \\ \left([\breve{b}_1^- \breve{b}_2^-, \breve{b}_1^+ \breve{b}_2^+], \breve{\mu}_1 \breve{\mu}_2 \right) \end{array} \right\rangle,$$

2. $$\underset{\sim}{P}_{c_1} \otimes \underset{\sim}{P}_{c_2} = \left\langle \begin{array}{c} \left([\breve{a}_1^- \breve{a}_2^-, \breve{a}_1^+ \breve{a}_2^+], \breve{\lambda}_1 \breve{\lambda}_2 \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{(\breve{b}_1^-)^2 + (\breve{b}_2^-)^2 - (\breve{b}_1^-)^2 (\breve{b}_2^-)^2}, \\ \sqrt{(\breve{b}_1^+)^2 + (\breve{b}_2^+)^2 - (\breve{b}_1^+)^2 (\breve{b}_2^+)^2} \end{array} \right], \\ \sqrt{(\breve{\mu}_1)^2 + (\breve{\mu}_2)^2 - (\breve{\mu}_1)^2 (\breve{\mu}_2)^2} \end{array} \right) \end{array} \right\rangle,$$

3. $$\underset{\sim}{P}_{c_1}^{\gamma} = \left\langle \begin{array}{c} \left([(\breve{a}_1^-)^{\gamma}, (\breve{a}_1^+)^{\gamma}], (\breve{\lambda}_1)^{\gamma} \right), \\ \left(\left[\sqrt{1 - (1 - \breve{b}_1^-)^{\gamma}}, \sqrt{1 - (1 - \breve{b}_1^+)^{\gamma}} \right], \sqrt{1 - (1 - \breve{\mu}_1)^{\gamma}} \right) \end{array} \right\rangle,$$

4. $$\gamma . \underset{\sim}{P}_{c_1} = \left\langle \begin{array}{c} \left(\left[\sqrt{1 - (1 - \breve{a}_1^-)^{\gamma}}, \sqrt{1 - (1 - \breve{a}_1^+)^{\gamma}} \right], \sqrt{1 - (1 - \breve{\lambda}_1)^{\gamma}} \right), \\ \left(\left[(\breve{b}_1^-)^{\gamma}, (\breve{b}_1^+)^{\gamma} \right], (\breve{\mu}_1)^{\gamma} \right) \end{array} \right\rangle.$$

Chapter 3

Spherical cubic aggregated operators and their application

We define spherical cubic fuzzy sets in this chapter as sets whose membership, neutrality, and nonmembership degrees are all cubic fuzzy numbers and the square sum of their membership, neutrality, nonmembership are not greater than one. We define numerous important operators and develop score functions to comparing two spherical cubic fuzzy numbers. Also described is the distance between two spherical cubic fuzzy numbers. we defined various aggregation operators i.e, SCFWA, SCFOWA, SCFHWA, SCFWG, SCFOWG, SCFHWG. We investigate the operating principles of a few existing operators and suggest multi-attributedecision support technique. At last, an illustrated model is offered to demonstrate authenticity, effectiveness, and productivity by demonstrating the decision-making phases in detail.

3.1 Spherical cubic fuzzy sets

We will examine the spherical cubic fuzzy set in this section, as well as its fundamental relations and methods.

Definition 3.1.1 *Consider a set* $X \neq \emptyset$*, a SCFS* S_c *expressed below:*

$$S_c = \left\{ x, \left\langle \check{\alpha}_{S_c}(x), \check{\eta}_{S_c}(x), \check{\beta}_{S_c}(x) \right\rangle \mid x \in X \right\},$$

here $\check{\alpha}_{S_c}(x) = \left\langle [\check{a}^-, \check{a}^+], \check{\lambda} \right\rangle$ *is the membership,* $\check{\eta}_{S_c}(x) = \left\langle [\check{n}^-, \check{n}^+], \check{\delta} \right\rangle$ *is the neutral and* $\check{\beta}_{S_c}(x) = \left\langle [\check{b}^-, \check{b}^+], \check{\mu} \right\rangle$ *is the non-membership degree respectively.*

The extended form of FS are defined such as IFS, PyFS, q-ROFS, PFS. Moreover, each of these notions are extended to interval valued sets and for cubic sets as shown in below Figure 4.

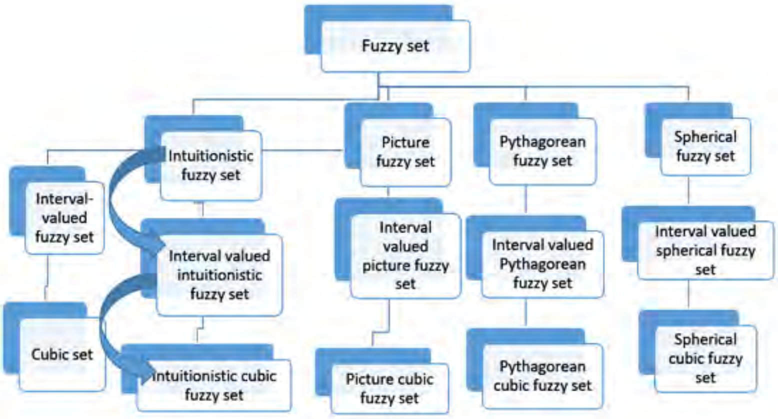

Figure 4 (Flow chart of spherical cubic fuzzy set)

Definition 3.1.2 *Let* $\mathcal{S}_c = \left(\left\langle [\breve{a}^-, \breve{a}^+], \check{\lambda}\right\rangle, \left\langle [\breve{n}^-, \breve{n}^+], \check{\delta}\right\rangle, \left\langle [\check{b}^-, \check{b}^+], \breve{\mu}\right\rangle\right)$ *SCFN. The score function score* $(\mathcal{S}_c)$ *is determined as below:*

$$\acute{\acute{S}}_c\left(\mathcal{S}_c\right) = \frac{\left[(\breve{a}^- + \breve{a}^+ + \check{\lambda})^2 + (\breve{n}^- + \breve{n}^+ + \check{\delta})^2 - (\check{b}^- + \check{b}^+ + \breve{\mu})^2\right]}{9} \tag{1}$$

$\acute{\acute{S}}_c\left(\mathcal{S}_c\right) \in [-1, 1]$.

Definition 3.1.3 *The accuracy function of SCFNs* $ac(\mathcal{S}_c)$ *is expressed as below:*

$$ac\left(\mathcal{S}_c\right) = \frac{\left[(\breve{a}^- + \breve{a}^+ + \check{\lambda})^2 + (\breve{n}^- + \breve{n}^+ + \check{\delta})^2 + (\check{b}^- + \check{b}^+ + \breve{\mu})^2\right]}{9} \tag{2}$$

Proposition 3.1.4 *Let* $\mathcal{S}_{c_1}$ *and* $\mathcal{S}_{c_2}$ *be two SCFNs and* $\gamma \geq 0$ *be any constant. The operations are defined as under:*

1. $\mathcal{S}_{c_1} \subseteq \mathcal{S}_{c_2} \Longleftrightarrow \check{\alpha}_{\mathcal{S}_{c_1}} \leq \check{\alpha}_{\mathcal{S}_{c_2}}, \check{\eta}_{\mathcal{S}_{c_1}} \leq \check{\eta}_{\mathcal{S}_{c_2}}, \check{\beta}_{\mathcal{S}_{c_1}} \geq \check{\beta}_{\mathcal{S}_{c_2}}$

2. $\mathcal{S}_{c_1} = \mathcal{S}_{c_2} \Longleftrightarrow \mathcal{S}_{c_1} \subseteq \mathcal{S}_{c_2}$ *and* $\mathcal{S}_{c_2} \subseteq \mathcal{S}_{c_1}$

3. $\left(\mathcal{S}_{c_1}\right)^c = \left\{\left(x, \left\langle [\check{b}_1^-, \check{b}_1^+], \breve{\mu}_1\right\rangle, \left\langle [\breve{n}_1^-, \breve{n}_1^+], \check{\delta}_1\right\rangle, \left\langle [\breve{a}_1^-, \breve{a}_1^+], \check{\lambda}_1\right\rangle\right)\right\}.$

4. $$\mathcal{S}_{c_1} \cup \mathcal{S}_{c_2} = \left\{\begin{array}{c} x, \left(\max\left\{[\breve{a}_1^-, \breve{a}_1^+], [\breve{a}_2^-, \breve{a}_2^+]\right\}, \min\left\{\check{\lambda}_1, \check{\lambda}_2\right\}\right), \\ \left(\min\left\{[\breve{n}_1^-, \breve{n}_1^+], [\breve{n}_2^-, \breve{n}_2^+]\right\}, \max\left\{\check{\delta}_1, \check{\delta}_2\right\}\right), \\ \left(\min\left\{[\check{b}_1^-, \check{b}_1^+], [\check{b}_2^-, \check{b}_2^+]\right\}, \max\left\{\breve{\mu}_1, \breve{\mu}_2\right\}\right) \end{array}\right\}$$

5. $$\mathcal{S}_{c_1} \cap \mathcal{S}_{c_2} = \left\{\begin{array}{c} x, \left(\min\left\{[\breve{a}_1^-, \breve{a}_1^+], [\breve{a}_2^-, \breve{a}_2^+]\right\}, \max\left\{\check{\lambda}_1, \check{\lambda}_2\right\}\right), \\ \left(\min\left\{[\breve{n}_1^-, \breve{n}_1^+], [\breve{n}_2^-, \breve{n}_2^+]\right\}, \max\left\{\check{\delta}_1, \check{\delta}_2\right\}\right), \\ \left(\max\left\{[\check{b}_1^-, \check{b}_1^+], [\check{b}_2^-, \check{b}_2^+]\right\}, \min\left\{\breve{\mu}_1, \breve{\mu}_2\right\}\right) \end{array}\right\}$$

6. $$\text{Ş}_{c_1} \oplus \text{Ş}_{c_2} = \left\{ \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\left(\breve{a}_1^-\right)^2 + \left(\breve{a}_2^-\right)^2 - \left(\breve{a}_1^-\right)^2 \left(\breve{a}_2^-\right)^2}, \\ \sqrt{\left(\breve{a}_1^+\right)^2 + \left(\breve{a}_2^+\right)^2 - \left(\breve{a}_1^+\right)^2 \left(\breve{a}_2^+\right)^2} \end{array} \right], \\ \sqrt{\left(\breve{\lambda}_1\right)^2 + \left(\breve{\lambda}_2\right)^2 - \left(\breve{\lambda}_1\right)^2 \left(\breve{\lambda}_2\right)^2} \end{array} \right), \\ \left(\left[\breve{n}_1^-\breve{n}_2^-, \breve{n}_1^+\breve{n}_2^+\right], \breve{\delta}_1\breve{\delta}_2\right), \left(\left[\breve{b}_1^-\breve{b}_2^-, \breve{b}_1^+\breve{b}_2^+\right], \breve{\mu}_1\breve{\mu}_2\right) \end{array} \right\}$$

7. $$\text{Ş}_{c_1} \otimes \text{Ş}_{c_2} = \left\{ \begin{array}{c} \left(\left[\breve{a}_1^-\breve{a}_2^-, \breve{a}_1^+\breve{a}_2^+\right], \breve{\lambda}_1\breve{\lambda}_2\right), \left(\left[\breve{n}_1^-\breve{n}_2^-, \breve{n}_1^+\breve{n}_2^+\right], \breve{\delta}_1\breve{\delta}_2\right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\left(\breve{b}_1^-\right)^2 + \left(\breve{b}_2^-\right)^2 - \left(\breve{b}_1^-\right)^2 \left(\breve{b}_2^-\right)^2}, \\ \sqrt{\left(\breve{b}_1^+\right)^2 + \left(\breve{b}_2^+\right)^2 - \left(\breve{b}_1^+\right)^2 \left(\breve{b}_2^+\right)^2} \end{array} \right], \\ \sqrt{\left(\breve{\mu}_1\right)^2 + \left(\breve{\mu}_2\right)^2 - \left(\breve{\mu}_1\right)^2 \left(\breve{\mu}_2\right)^2} \end{array} \right) \end{array} \right\}$$

8. $$\gamma\tilde{S}_c = \left\{ \begin{array}{c} \left(\begin{array}{c} \left[\sqrt{1-\left(1-(\breve{a}^-)^2\right)^\gamma}, \sqrt{1-\left(1-(\breve{a}^+)^2\right)^\gamma}\right], \\ \sqrt{1-\left(1-(\breve{\lambda})^2\right)^\gamma} \end{array} \right), \\ \left(\left[(\breve{n}^-)^\gamma, (\breve{n}^+)^\gamma\right], (\breve{\delta})^\gamma\right), \left(\left[(\breve{b}^-)^\gamma, (\breve{b}^+)^\gamma\right], (\breve{\mu})^\gamma\right) \end{array} \right\}$$

9. $$\text{Ş}_c^\gamma = \left\{ \begin{array}{c} \left(\left[(\breve{a}^-)^\gamma, (\breve{a}^+)^\gamma\right], (\breve{\lambda})^\gamma\right), \left(\left[(\breve{n}^-)^\gamma, (\breve{n}^+)^\gamma\right], (\breve{\delta})^\gamma\right), \\ \left(\begin{array}{c} \left[\sqrt{1-\left(1-(\breve{b}^-)^2\right)^\gamma}, \sqrt{1-\left(1-(\breve{b}^+)^2\right)^\gamma}\right], \\ \sqrt{1-\left(1-(\breve{\mu})^2\right)^\gamma} \end{array} \right) \end{array} \right\}.$$

The following proposition contains a few of the properties of spherical cubic fuzzy sets that can be easily determined.

Proposition 3.1.5 *Let Ş_{c_1} and Ş_{c_2} be two SCFNs in Ẋ and $\gamma, \gamma_1, \gamma_2 \geq 0$. Then*

1. $(\gamma_1 \oplus \gamma_2)\text{Ş}_{c_1} = \gamma_1\text{Ş}_{c_1} \oplus \gamma_2\text{Ş}_{c_1}$
2. $(\gamma_1 \otimes \gamma_2)\text{Ş}_{c_1} = \gamma_1\text{Ş}_{c_1} \otimes \gamma_2\text{Ş}_{c_1}$
3. $(\text{Ş}_{c_1} \oplus \text{Ş}_{c_2})^\gamma = \text{Ş}_{c_1}^\gamma \oplus \text{Ş}_{c_2}^\gamma$
4. $(\text{Ş}_{c_1} \otimes \text{Ş}_{c_2})^\gamma = \text{Ş}_{c_1}^\gamma \otimes \text{Ş}_{c_2}^\gamma$
5. $\text{Ş}_{c_1}^{\gamma_1+\gamma_2} = \text{Ş}_{c_1}^{\gamma_1} \otimes \text{Ş}_{c_1}^{\gamma_2}$
6. $(\text{Ş}_{c_1}^\gamma)^n = (\text{Ş}_{c_1}^n)^\gamma$.

3.2 SCFWA operators

Now we will define the SCFWA operator and look at its most important characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.2.1 *Let* $\S_{c_i} = \left\langle c_{\S_{c_i}}, \dot{c}_{\S_{c_i}}, \ddot{c}_{\S_{c_i}} \right\rangle$ *be the collection and SCFWA mapping* $SCFWA : \Psi^n \rightarrow \Psi$ *defined as below:*

$$SCFWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \breve{\omega}_1 \S_{c_1} \oplus \breve{\omega}_2 \S_{c_2} \oplus, ..., \oplus \breve{\omega}_n \S_{c_n}.$$

For SCFWA Operator

By utilizing the property (6) and (8) of Proposition 3.1.4 of SCFNs, we get aggregated value of SCFWA operator.

Theorem 3.2.2 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection and the aggregated value of SCFWA is determined as below:*

$$SCFWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) =$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{a}_i^{-}\right)^{\breve{\omega}_i}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{a}_i^{+}\right)^{\breve{\omega}_i}} \right], \\ \sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{\lambda}_i\right)^{\breve{\omega}_i}} \end{array} \right), \\ \left(\left[\prod_{i=1}^{n} \left(\breve{n}_i^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n} \left(\breve{n}_i^{+}\right)^{\breve{\omega}_i} \right], \prod_{i=1}^{n} \left(\breve{\delta}_i\right)^{\breve{\omega}_i} \right), \\ \left(\left[\prod_{i=1}^{n} \left(\breve{b}_i^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n} \left(\breve{b}_i^{+}\right)^{\breve{\omega}_i} \right], \prod_{i=1}^{n} \left(\breve{\mu}_i\right)^{\breve{\omega}_i} \right) \end{array} \right\}.$$

Theorem 3.2.3 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection then SCFWA aggregation operator satisfied the following characteristics.*

Idempotency:

For $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ are equal i.e $\S_{c_i} = \S_c$, so

$$SCFWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \S_c.$$

Boundary: For all $\breve{\omega}$,

$$\S_{c_i}^{-} \leq SCFWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq \S_{c_i}^{+},$$

Monotonicity: Let

$$\S_{c_i}^{*} = \left[\left\langle [a_i^{*-}, a_i^{*+}], \breve{\lambda}_i^{*} \right\rangle, \left\langle [n_i^{*-}, n_i^{*+}], \breve{\delta}_i^{*} \right\rangle, \left\langle [b_i^{*-}, b_i^{*+}], \breve{\mu}_i^{*} \right\rangle\right]$$

then

$$SCFWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq SCFWA_{\breve{\omega}}\left(\S_{c_1}^{*}, \S_{c_2}^{*}, ..., \S_{c_n}^{*}\right).$$

3.3 SCFOWA operators

Now, we will introduce the SCFOWA operator and establish its important characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.3.1 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection, then SCFOWA operator is determined as below:*

$$SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \breve{\omega}_1 \S_{c_{\sigma(1)}} \oplus \breve{\omega}_2 \S_{c_{\sigma(2)}} \oplus, ..., \oplus \breve{\omega}_n \S_{c_{\sigma(n)}}.$$

Moreover, the SCFOWA is also SCFA operator determined as:

$$SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \frac{1}{n}(\S_{c_1} \oplus \S_{c_2} \oplus, ..., \oplus \S_{c_n}).$$

By utilizing the property (6) *and* (8) *of Proposition 3.1.4, we get the following SCFOWA aggregation operator.*

Theorem 3.3.2 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection and the aggregated value of SCFOWA operator is determined as,*

$$SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) =$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{a}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{a}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}} \right], \\ \sqrt{1 - \prod_{i=1}^{n} \left(1 - \breve{\lambda}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}} \end{array} \right), \\ \left(\left[\prod_{i=1}^{n} \left(\breve{n}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n} \left(\breve{n}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i} \right], \prod_{i=1}^{n} \left(\breve{\delta}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i} \right), \\ \left(\left[\prod_{i=1}^{n} \left(\breve{b}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n} \left(\breve{b}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i} \right], \prod_{i=1}^{n} \left(\breve{\mu}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i} \right) \end{array} \right\}.$$

Theorem 3.3.3 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection the SCFOWA aggregation operator satisfied the following characteristics.*

Idempotency:
For $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ are equal i.e $\S_{c_i} = \S_c$ then

$$SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \S_c.$$

Boundary:
For all $\breve{\omega}$,

$$\S_{c_i}^{-} \leq SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq \S_{c_i}^{+}$$

Monotonicity:
Let

$$\S_c^{*} = \left[\left\langle [\mathfrak{a}_{\tilde{\sigma}(i)}^{*-}, \mathfrak{a}_{\tilde{\sigma}(i)}^{*+}], \breve{\lambda}_{\tilde{\sigma}(i)}^{*}\right\rangle, \left\langle [\mathfrak{n}_{\tilde{\sigma}(i)}^{*-}, \mathfrak{n}_{\tilde{\sigma}(i)}^{*+}], \breve{\delta}_{\tilde{\sigma}(i)}^{*}\right\rangle, \left\langle [\mathfrak{b}_{\tilde{\sigma}(i)}^{*-}, \mathfrak{b}_{\tilde{\sigma}(i)}^{*+}], \breve{\mu}_{\tilde{\sigma}(i)}^{*}\right\rangle\right]$$

then

$$SCFOWA_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq SCFOWA_{\breve{\omega}}\left(\S_{c_1}^{*}, \S_{c_2}^{*}, ..., \S_{c_n}^{*}\right).$$

3.4 SCFHWA operators

We will introduce the SCFHWA operator and discuss its essential characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.4.1 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}}\right\rangle$ *be the collection, then the SCFHWA operator is determined as below:*

$$SCFHWA_{\omega,\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \bigoplus_{i=1}^{n} \breve{\omega}_i \hat{S}_{c_{\sigma(i)}}.$$

here the weighting vector is $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$, *where* $\hat{S}_{c_{\tilde{\sigma}(i)}} = n\omega_i \S_{c_i}$.

Theorem 3.4.2 *Consider the collection* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}}\right\rangle$ *then the SCFHWA aggregation operator is expressed as below:*

$$SCFHWA_{\omega,\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) =$$

$$\left\{\begin{array}{c} \left(\begin{array}{c} \left[\sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\alpha}_{\hat{\sigma}(i)}^{-}\right)^{\breve{\omega}_i}}, \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\alpha}_{\hat{\sigma}(i)}^{+}\right)^{\breve{\omega}_i}}\right], \\ \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\lambda}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}} \end{array}\right), \\ \left(\left[\prod_{i=1}^{n}\left(\breve{n}_{\hat{\sigma}(i)}^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{n}_{\hat{\sigma}(i)}^{+}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\delta}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right), \\ \left(\left[\prod_{i=1}^{n}\left(\breve{b}_{\hat{\sigma}(i)}^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{b}_{\hat{\sigma}(i)}^{+}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\mu}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right) \end{array}\right\}.$$

Idempotency:

Here, $Ş_{c_i} = \left\langle c_{Ş_{c_i}}, \dot{c}_{Ş_{c_i}}, \ddot{c}_{Ş_{c_i}} \right\rangle$ are equal i.e $Ş_{c_i}$ =$Ş_c$ then

$$SCFHWA_{\omega,\breve{\omega}}\left(Ş_{c1}, Ş_{c2}, ..., Ş_{cn}\right) = Ş_c.$$

Boundary:

For all $\breve{\omega}$,

$$Ş_{c_i}^{-} \leq SCFHWA_{\omega,\breve{\omega}}\left(Ş_{c1}, Ş_{c2}, ..., Ş_{cn}\right) \leq Ş_{c_i}^{+},$$

Monotonicity:

Let

$$Ş_c^* = \left[\left\langle \left[\breve{a}_{\breve{\sigma}(i)}^{*-}, \breve{a}_{\breve{\sigma}(i)}^{*+}\right], \breve{\lambda}_{\breve{\sigma}(i)}^{*} \right\rangle, \left\langle \left[\breve{n}_{\breve{\sigma}(i)}^{*-}, \breve{n}_{\breve{\sigma}(i)}^{*+}\right], \breve{\delta}_{\breve{\sigma}(i)}^{*} \right\rangle, \left\langle \left[\breve{b}_{\breve{\sigma}(i)}^{*-}, \breve{b}_{\breve{\sigma}(i)}^{*+}\right], \breve{\mu}_{\breve{\sigma}(i)}^{*} \right\rangle\right]$$

then

$$\left(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}\right) \leq \left(Ş_{c_1}^*, Ş_{c_2}^*, ..., Ş_{c_n}^*\right).$$

3.5 SCFWG operators

Now, we will introduce the idea of the SCFWG operator and main characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.5.1 *Let* $Ş_{c_i} = \left\langle \breve{\alpha}_{Ş_{c_i}}, \breve{\eta}_{Ş_{c_i}}, \breve{\beta}_{Ş_{c_i}} \right\rangle$ *be the collection, then the aggregated value of SCFWG operator is expressed as below:*

$$SCFWG_{\breve{\omega}}(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = \breve{\omega}_1 Ş_{c_1} \otimes \breve{\omega}_2 Ş_{c_2} \otimes, ..., \otimes \breve{\omega}_n Ş_{c_n}.$$

The laws define (7) and (9) in Proposition 3.1.4, we the following aggregated value of SCFWG operator.

Theorem 3.5.2 *Let* $Ş_{c_i} = \left\langle \breve{\alpha}_{Ş_{c_i}}, \breve{\eta}_{Ş_{c_i}}, \breve{\beta}_{Ş_{c_i}} \right\rangle$ *be the collection, then the aggregated value of SCFWG operator is expressed as below:*

$$SCFWG_{\breve{\omega}}\left(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}\right) =$$

$$\left\{\begin{array}{c} \left(\left[\prod_{i=1}^{n}\left(\breve{a}_i^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{a}_i^{+}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\lambda}_i\right)^{\breve{\omega}_i}\right), \\ \left(\left[\prod_{i=1}^{n}\left(\breve{n}_i^{-}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{n}_i^{+}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\delta}_i\right)^{\breve{\omega}_i}\right), \\ \left(\begin{array}{c} \left[\sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}_i^{-}\right)^{\breve{\omega}_i}}, \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}_i^{+}\right)^{\breve{\omega}_i}}\right], \\ \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\mu}_i\right)^{\breve{\omega}_i}} \end{array}\right) \end{array}\right\}.$$

Theorem 3.5.3 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection, then the SCFWG aggregation operator satisfied the following characteristics.*

Idempotency:
For all $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ are equal i.e $\S_{c_i} = \S_c$ then

$$SCFWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \S_c.$$

Boundary:
For all $\breve{\omega}$,

$$\S_{c_i}^{-} \leq SCFWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq \S_{c_i}^{+},$$

Monotonicity:
Assume that

$$\S_c^{*} = \left\{ \left(\left[\breve{a}_i^{*-}, \breve{a}_i^{*+} \right], \breve{\lambda}_i^{*} \right), \left(\left[\breve{n}_i^{*-}, \breve{n}_i^{*+} \right], \breve{\delta}_i^{*} \right), \left(\left[\breve{b}_i^{*-}, \breve{b}_i^{*+} \right], \breve{\mu}_i^{*} \right) \right\}$$

then

$$SCFWG_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq SCFWG_{\breve{\omega}}\left(\S_{c_1}^{*}, \S_{c_2}^{*}, ..., \S_{c_n}^{*}\right).$$

3.6 SCFOWG operator

Now, we will introduce the idea of the SCFOWG operator and its essential characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.6.1 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection, then the aggregated value of SCFOWG operator is expressed as below: .*

$$SCFOWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \prod_{i=1}^{n} \breve{\omega}_i \hat{S}_{c_{\breve{\sigma}(i)}}.$$

Now, by property (7) *and* (9) *of Proposition 3.1.4, we get the following aggregated value of SCFOWG operator.*

Theorem 3.6.2 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection and the aggregated SCFOWG operator is expressed as below:*

$$SCFOWG_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) =$$

$$\left\{ \begin{array}{c} \left(\left[\prod_{i=1}^{n}\left(\breve{a}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{a}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\lambda}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}\right), \\ \left(\left[\prod_{i=1}^{n}\left(\breve{n}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{n}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\delta}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}\right), \\ \left(\begin{array}{c} \left[\sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}^{-}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}}, \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}^{+}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}}\right], \\ \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\mu}_{\tilde{\sigma}(i)}\right)^{\breve{\omega}_i}} \end{array} \right) \end{array} \right\}.$$

Theorem 3.6.3 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection the aggregated value of SCFOWG operator satisfied the following characteristics.*

Idempotency:
Here, $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ are equal i.e $\S_{c_i} = \S_c$ then

$$SCFOWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \S_c.$$

Boundary:
For all $\breve{\omega}$,

$$\S^{-}_{c_i} \leq SCFOWG_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq \S^{+}_{c_i},$$

Monotonicity:
Let

$$\S^{*}_{c} = \left[\left\langle [\mathfrak{a}^{*+}_{\tilde{\sigma}(i)}, \mathfrak{a}^{*+}_{\tilde{\sigma}(i)}], \breve{\lambda}^{*}_{\tilde{\sigma}(i)} \right\rangle, \left\langle [\mathfrak{n}^{*-}_{\tilde{\sigma}(i)}, \mathfrak{n}^{*+}_{\tilde{\sigma}(i)}], \breve{\delta}^{*}_{\tilde{\sigma}(i)} \right\rangle, \left\langle [\mathfrak{b}^{*-}_{\tilde{\sigma}(i)}, \mathfrak{b}^{*+}_{\tilde{\sigma}(i)}], \breve{\mu}^{*}_{\tilde{\sigma}(i)} \right\rangle\right]$$

then

$$SCFOWG_{\breve{\omega}}\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) \leq SCFOWG_{\breve{\omega}}\left(\S^{*}_{c_1}, \S^{*}_{c_2}, ..., \S^{*}_{c_n}\right).$$

3.7 SCFHWG operators

We will introduce the SCFHWG operator and discuss its essential characteristics, such as idempotency, boundedness, and monotonicity.

Definition 3.7.1 *Let* $\S_{c_i} = \left\langle \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} \right\rangle$ *be the collection, then the aggregated value of SCFHWG operator is expressed below:*

$$SCFHWG_{\omega,\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \prod_{i=1}^{n} \breve{\omega}_i \hat{S}_{c_{\tilde{\sigma}(i)}}.$$

where $\hat{S}_{c_{\tilde{\sigma}(i)}} = n\omega_i \S_{c_i}$.

Theorem 3.7.2 *Let* $\c{S}_{c_i} = \left\langle \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} \right\rangle$ *be the collection, then the aggregated value of SCFHWG operator is expressed as below:*

$$SCFHWG_{\omega,\breve{\omega}}\left(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}\right) =$$

$$\left\{ \begin{array}{c} \left(\left[\prod_{i=1}^{n}\left(\breve{a}^{-}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{a}^{+}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\lambda}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right), \\ \left(\left[\prod_{i=1}^{n}\left(\breve{n}^{-}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}, \prod_{i=1}^{n}\left(\breve{n}^{+}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right], \prod_{i=1}^{n}\left(\breve{\delta}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}\right), \\ \left(\begin{array}{c} \left[\sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}^{-}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}}, \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{b}^{+}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}}\right], \\ \sqrt{1-\prod_{i=1}^{n}\left(1-\breve{\mu}_{\hat{\sigma}(i)}\right)^{\breve{\omega}_i}} \end{array} \right), \end{array} \right\}.$$

Idempotency:

For $\c{S}_{c_i} = \left\langle \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} \right\rangle$ are equal i.e $\c{S}_{c_i} = \c{S}_c$ then

$$SCFHG_{\omega,\breve{\omega}}\left(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}\right) = \c{S}_c.$$

Boundary:

For all $\breve{\omega}$,

$$\c{S}^{-}_{c_i} \leq SCFHG_{\omega,\breve{\omega}}\left(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}\right) \leq \c{S}^{+}_{c_i},$$

Monotonicity:

Let

$$\c{S}^{*}_{c_i} = \left\{\left(\left[\breve{a}^{*-}_{\hat{\sigma}(i)}, \breve{a}^{*+}_{\hat{\sigma}(i)}\right], \breve{\lambda}^{*}_{\hat{\sigma}(i)}\right), \left(\left[\breve{n}^{*-}_{\hat{\sigma}(i)}, \breve{n}^{*+}_{\hat{\sigma}(i)}\right], \breve{\delta}^{*}_{\hat{\sigma}(i)}\right), \left(\left[\breve{b}^{*-}_{\hat{\sigma}(i)}, \breve{b}^{*+}_{\hat{\sigma}(i)}\right], \breve{\mu}^{*}_{\hat{\sigma}(i)}\right)\right\}$$

then

$$SCFHG_{\omega,\breve{\omega}}\left(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}\right) \leq SCFHG_{\omega,\breve{\omega}}\left(\c{S}^{*}_{c_1}, \c{S}^{*}_{c_2}, ..., \c{S}^{*}_{c_n}\right).$$

3.8 Multi-criteria decision - making process utilizing spherical cubic fuzzy weighting aggregated operators

Now, we will use the MCGDM approach to apply the structure of spherical cubic weighting aggregated operators. Suppose we obtain q different alternatives $A = \{A_1, A_2, ..., A_q\}$ and according to that s different attributes are choosen like $C = \{C_1, C_2, ..., C_s\}$ to be determined by utilizing the weighted vector $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$

with the condition that $\sum_{i=1}^{n}\breve{\omega}_i = 1$. Assume that decision maker using SCFNs to evaluate alternative based on various criteria in $\c{X}$: $\c{S}_{c_{lm}} = \left(\breve{\alpha}_{\c{S}_{c_{lm}}}, \breve{\eta}_{\c{S}_{c_{lm}}}, \breve{\beta}_{\c{S}_{c_{lm}}}\right)$ $(l = 1,2,...,n)$ $(m = 1,2,...,t)$. Suppose that $\breve{\alpha}_{\c{S}_{c_{ip}}}$ represent the alternatives degree which satisfy the criteria $\c{S}_{c_{lm}}$ i.e $\breve{\alpha}_{\c{S}_{c_{lm}}} = \left(\left[\breve{a}^-_{lm}, \breve{a}^+_{lm}\right], \breve{\lambda}_{lm}\right)$, $\breve{\eta}_{\c{S}_{c_{lm}}}$ represent the alternatives degree which neutral the criteria $\c{S}_{c_{lm}}$ i.e $\breve{\eta}_{\c{S}_{c_{lm}}} = \left([\breve{n}^-_{lm}, \breve{n}^+_{lm}], \breve{\delta}_{lm}\right)$ and $\breve{\beta}_{\c{S}_{c_{ip}}}$ represent the alternatives degree which does not satisfy the criteria $\c{S}_{c_{lm}}$ i.e $\breve{\beta}_{\c{S}_{c_{lm}}} = \langle[\breve{b}^-_{lm}, \breve{b}^+_{lm}], \breve{\mu}_{lm}\rangle$ with the condition that $\left[\breve{a}^-_{lm}, \breve{a}^+_{lm}\right] \subset [0,1]$, $[\breve{n}^-_{lm}, \breve{n}^+_{lm}] \subset [0,1]$, $[\breve{b}^-_{lm}, \breve{b}^+_{lm}] \subset [0,1]$ with the mappings $\breve{\lambda}_{lm} : \c{X} \to [0,1]$, $\breve{\delta}_{lm} : \c{X} \to [0,1]$ and $\breve{\mu}_{lm} : \c{X} \to [0,1]$. Through generating a $\left(sup\left[\breve{a}^-_{lm}, \breve{a}^+_{lm}\right]\right)^2 + \left(sup[\breve{n}^-_{lm}, \breve{n}^+_{lm}]\right)^2 + \left(sup[\breve{b}^-_{lm}, \breve{b}^+_{lm}]\right)^2 \leq 1$ and $\left(\breve{\lambda}_{lm}\right)^2 + \left(\breve{\delta}_{lm}\right)^2 + \left(\breve{\mu}_{lm}\right)^2 \leq 1$, $(l = 1,2,...,n)$ $(m = 1,2,...,t)$. As a result, a SCF decision matrix might be used to represent MCGDM concerns.

$$\c{M} = (\c{S}_{c_{lm}})_{n\times t} = \left(\left\langle \breve{\alpha}_{\c{S}_{c_{lm}}}, \breve{\eta}_{\c{S}_{c_{lm}}}, \breve{\beta}_{\c{S}_{c_{lm}}} \right\rangle\right)_{n\times t}.$$

Step 1:

Make a spherical cubic fuzzy structure out of the decision matrix $\c{M} = (\c{S}_{c_{lm}})_{n\times t} = \left(\left\langle \breve{\alpha}_{\c{S}_{c_{lm}}}, \breve{\eta}_{\c{S}_{c_{lm}}}, \breve{\beta}_{\c{S}_{c_{lm}}} \right\rangle\right)_{n\times t}$. The first sort of criterion is cost criteria, whereas the second is profit criteria. There is no need to normalize the rating values of same-class criterion. To convert cost rating data to profit rating values, apply the normalization formula below.

$$\varrho_{lm} = (\c{x}_{lm}, \c{y}_{lm}) = \left\{ \begin{array}{c} \c{\varsigma}_{lm}, \text{ criteria for the type of benefit} \\ \c{\varsigma}^c_{lm} \text{ type of cost criterion} \end{array} \right\}$$

here $\c{\varsigma}^c_{lm}$ shows the complement of $\c{\varsigma}_{lm}$. Here, $\c{M}^{\,l} = (\varrho_{lm})_{n\times t} = (\c{x}_{lm}, \c{y}_{lm})_{n\times t}$. The SCFWA, SCFOWA, SCFWG, SCFOWG, SCFHA, SCFHG operators would be introduced to MCGDM in the main procedures.

Step 2:

Use the indicated aggregation operators to compute SCFNs $\c{S}_{c_l}$ for distinct selections $\c{A}_l$ with weights $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$.

Step 3:

We compute the scores $\H{S}_c(\c{S}_{c_i})$

Step 4:

After ranking, we will select the best option.

3.9 Numeric Illustration

In the manufacturing industry, supply chain is an important factor of the planning process. Though choosing a supplier is difficult, making the optimal choice will promote economic growth and customer satisfaction. The proposed approach will come in handy when choosing a supplier. The below are the arguments that support the proposed supplier evaluation technique:

The buying administrator selects a supplier for the products based on four characteristics: organizational context, action plan, and economics. The supplier model is a crucial phase in the industry's organizing. The best decision will enhance your company's productivity, but finding an ideal provider is extremely challenging. As a result, the proposed methodology will be utilized to assess and select the best supplier for an organization in Pakistan's eastern province. The following is how the proposed supplier evaluation accessibility has been made:

The approach for locating an appropriate source for component purchases. The following four parameters are taken into account by the decision maker. The four criteria are marked by the characters $\{C_1, C_2, C_3, C_4\}$. $\breve{\omega} = (.35, .4, .25)^T$ is the weight vector of four criterion. Four providers should be further evaluated, according to a committee of three decision makers. The four suppliers are denoted by the characters $\{A_1, A_2, A_3, A_4\}$. To categorize the suppliers, the rankings criteria are needed. The decision matrices are SCFNs, as shown below.

Step 1:

Tables 1, 2, and 3 show the selections of decision makers.

	C_1	C_2	C_3	C_4
A_1	([0.2, 0.3]; 0.4) ([0.5, 0.6]; 0.5) ([0.2, 0.4]; 0.2)	([0.2, 0.4]; 0.4) ([0.1, 0.5]; 0.4) ([0.2, 0.4]; 0.2)	([0.4, 0.6]; 0.6) ([0.2, 0.6]; 0.6) ([0.2, 0.4]; 0.2)	([0.2, 0.5]; 0.5) ([0.1, 0.5]; 0.3) ([0.4, 0.6]; 0.6)
A_2	([0.3, 0.5]; 0.2) ([0.2, 0.5]; 0.4) ([0.2, 0.6]; 0.7)	([0.4, 0.7]; 0.6) ([0.1, 0.3]; 0.3) ([0.1, 0.5]; 0.3)	([0.1, 0.3]; 0.2) ([0.2, 0.5]; 0.3) ([0.2, 0.4]; 0.4)	([0.4, 0.6]; 0.6) ([0.1, 0.5]; 0.4) ([0.1, 0.2]; 0.3)
A_3	([0.1, 0.3]; 0.2) ([0.2, 0.5]; 0.3) ([0.4, 0.7]; 0.6)	([0.3, 0.6]; 0.6) ([0.5, 0.6]; 0.3) ([0.3, 0.4]; 0.2)	([0.3, 0.5]; 0.2) ([0.2, 0.5]; 0.4) ([0.3, 0.5]; 0.8)	([0.1, 0.2]; 0.3) ([0.2, 0.4]; 0.5) ([0.2, 0.3]; 0.1)
A_4	([0.2, 0.3]; 0.4) ([0.1, 0.4]; 0.8) ([0.2, 0.6]; 0.3)	([0.1, 0.2]; 0.4) ([0.3, 0.5]; 0.7) ([0.4, 0.8]; 0.3)	([0.5, 0.6]; 0.9) ([0.2, 0.3]; 0.3) ([0.1, 0.3]; 0.1)	([0.2, 0.3]; 0.1) ([0.1, 0.2]; 0.4) ([0.2, 0.3]; 0.7)

Table 1 (Spherical cubic fuzzy information of 1st decision maker)

	C_1	C_2	C_3	C_4
A_1	([0.1, 0.3]; 0.6) ([0.2, 0.5]; 0.3) ([0.2, 0.4]; 0.6)	([0.1, 0.4]; 0.3) ([0.2, 0.5]; 0.2) ([0.1, 0.5]; 0.4)	([0.2, 0.6]; 0.7) ([0.2, 0.4]; 0.2) ([0.2, 0.4]; 0.3)	([0.1, 0.3]; 0.6) ([0.4, 0.7]; 0.6) ([0.2, 0.5]; 0.3)
A_2	([0.1, 0.5]; 0.4) ([0.2, 0.4]; 0.5) ([0.1, 0.7]; 0.6)	([0.2, 0.5]; 0.3) ([0.1, 0.3]; 0.2) ([0.3, 0.7]; 0.2)	([0.1, 0.4]; 0.3) ([0.2, 0.5]; 0.6) ([0.1, 0.5]; 0.3)	([0.3, 0.7]; 0.2) ([0.1, 0.3]; 0.7) ([0.2, 0.5]; 0.2)
A_3	([0.2, 0.4]; 0.4) ([0.2, 0.6]; 0.6) ([0.1, 0.3]; 0.2)	([0.4, 0.5]; 0.6) ([0.7, 0.8]; 0.2) ([0.1, 0.3]; 0.7)	([0.2, 0.4]; 0.3) ([0.2, 0.6]; 0.6) ([0.1, 0.3]; 0.4)	([0.1, 0.4]; 0.3) ([0.2, 0.5]; 0.6) ([0.2, 0.5]; 0.6)
A_4	([0.2, 0.4]; 0.4) ([0.5, 0.6]; 0.6) ([0.4, 0.5]; 0.3)	([0.7, 0.8]; 0.6) ([0.1, 0.2]; 0.2) ([0.3, 0.5]; 0.4)	([0.5, 0.6]; 0.9) ([0.2, 0.3]; 0.3) ([0.1, 0.3]; 0.1)	([0.3, 0.4]; 0.2) ([0.1, 0.3]; 0.4) ([0.5, 0.8]; 0.3)

Table 2 Spherical cubic fuzzy information of 2nd decision maker

	C_1	C_2	C_3	C_4
A_1	([0.2, 0.4]; 0.1) ([0.1, 0.3]; 0.4) ([0.2, 0.3]; 0.6)	([0.1, 0.3]; 0.2) ([0.2, 0.5]; 0.3) ([0.3, 0.4]; 0.6)	([0.2, 0.6]; 0.7) ([0.2, 0.4]; 0.2) ([0.2, 0.4]; 0.4)	([0.1, 0.3]; 0.6) ([0.2, 0.5]; 0.3) ([0.4, 0.7]; 0.6)
A_2	([0.1, 0.4]; 0.2) ([0.1, 0.2]; 0.4) ([0.3, 0.5]; 0.3)	([0.1, 0.5]; 0.3) ([0.2, 0.5]; 0.5) ([0.5, 0.7]; 0.3)	([0.2, 0.4]; 0.2) ([0.1, 0.2]; 0.1) ([0.5, 0.6]; 0.4)	([0.1, 0.2]; 0.3) ([0.4, 0.5]; 0.6) ([0.1, 0.3]; 0.3)
A_3	([0.3, 0.7]; 0.2) ([0.2, 0.4]; 0.2) ([0.2, 0.3]; 0.6)	([0.2, 0.4]; 0.2) ([0.3, 0.5]; 0.6) ([0.2, 0.4]; 0.3)	([0.2, 0.4]; 0.7) ([0.1, 0.5]; 0.2) ([0.6, 0.7]; 0.5)	([0.1, 0.3]; 0.2) ([0.1, 0.3]; 0.2) ([0.4, 0.5]; 0.2)
A_4	([0.1, 0.3]; 0.2) ([0.5, 0.6]; 0.3) ([0.3, 0.4]; 0.4)	([0.2, 0.3]; 0.5) ([0.6, 0.8]; 0.1) ([0.3, 0.4]; 0.2)	([0.5, 0.6]; 0.9) ([0.2, 0.3]; 0.3) ([0.1, 0.3]; 0.1)	([0.2, 0.4]; 0.4) ([0.1, 0.2]; 0.5) ([0.1, 0.3]; 0.4)

Table 3 Spherical cubic fuzzy information of 3rd decision maker

Using the SCFWA operator, the decision maker's weights $\breve{\omega} = (.35, .4, .25)^T$. The combined findings are shown in Table 4 below.

	C_1	C_2	C_3	C_4
A_1	$([0.17,0.34];0.46)$ $([0.23,0.47];0.39)$ $([0.2,0.37];0.41)$	$([0.14,0.38];0.32)$ $([0.16,0.5];0.28)$ $([0.17,0.44];0.35)$	$([0.29,0.6];0.67)$ $([0.2,0.46];0.29)$ $([0.2,0.4];0.28)$	$([0.14,0.3];0.6)$ $([0.2,0.5];0.40)$ $([0.30,0.58];0.45)$
A_2	$([0.20,0.48];0.30)$ $([0.17,0.36];0.44)$ $([0.17,0.61];0.53)$	$([0.28,0.59];0.44)$ $([0.12,0.34];0.29)$ $([0.23,0.62];0.26)$	$([0.13,0.37];0.25)$ $([0.17,0.40];0.30)$ $([0.19,0.48];0.36)$	$([0.31,0.59];0.42)$ $([0.14,0.40];0.55)$ $([0.17,0.44];0.31)$
A_3	$([0.21,0.49];0.30)$ $([0.2,0.51];0.36)$ $([0.19,0.40];0.39)$	$([0.32,0.52];0.54)$ $([0.50,0.64];0.30)$ $([0.17,0.36];0.37)$	$([0.24,0.43];0.44)$ $([0.17,0.54];0.40)$ $([0.23,0.44];0.53)$	$([0.1,0.31];0.28)$ $([0.24,0.46];0.43)$ $([0.21,0.52];0.51)$
A_4	$([0.18,0.34];0.36)$ $([0.28,0.52];0.56)$ $([0.29,0.50];0.32)$	$([0.49,0.60];0.51)$ $([0.23,0.39];0.26)$ $([0.33,0.56];0.30)$	$([0.31,0.56];0.69)$ $([0.2,0.36];0.52)$ $([0.15,0.42];0.28)$	$([0.24,0.37];0.25)$ $([0.1,0.24];0.42)$ $([0.24,0.44];0.43)$

Table 4 (Aggregation by spherical cubic decision maker)

We used the SCFWA operator with $\breve{\omega} = (.1, .2, .25, .45)^T$as the criterion vector to get the aggregate SCFNs for the $Ą_i$ alternatives.

$A_1 =$	$([0.19,0.45];0.56)$	$([0.19,0.52];0.34)$	$([0.23,0.48];0.38)$
$A_2 =$	$([0.26,0.53];0.38)$	$([0.15,0.39];0.41)$	$([0.18,0.50];0.32)$
$A_3 =$	$([0.21,0.42];0.40)$	$([0.25,0.52];0.38)$	$([0.20,0.45];0.47)$
$A_4 =$	$([0.33,0.48];0.48)$	$([0.16,0.31];0.42)$	$([0.23,0.46];0.35)$

Step 3:

We may find the $\text{S̋}_c(Ą_i)$ of every $Ą_i$ and used the Definition 3.1.2, as illustrated below:

$$\text{S̋}_c(Ą_1) = .15,\ \text{S̋}_c(Ą_2) = .14,\ \text{S̋}_c(Ą_3) = .12,\ \text{S̋}_c(Ą_4) = .11.$$

Step 4:

To choose the best options, we must first rearrange the SCFNs in order of decreasing: $Ą_1 > Ą_2 > Ą_3 > Ą_4$. As a result, $Ą_1$ is the best option.

For SCFOWA Operator

Step 1:

The obtained data is provided by three significant in Table 4 according to the various relevance of all the decision-makers.

Step 2:

To use the SCFOWA operator, we get for all options $Ą_i$, the collection SCFNs as s weight vector $\breve{\omega} = (.1, .2, .25, .45)^T$ shown as below:

$A_1 =$	$\left(\ ([0.17, 0.43]; 0.42)\ \right)$	$\left(\ ([0.23, 0.51]; 0.30)\ \right)$	$\left(\ ([0.27, 0.50]; 0.41)\ \right)$
$A_2 =$	$\left(\ ([0.23, 0.49]; 0.28)\ \right)$	$\left(\ ([0.17, 0.41]; 0.32)\ \right)$	$\left(\ ([0.29, 0.60]; 0.37)\ \right)$
$A_3 =$	$\left(\ ([0.20, 0.42]; 0.31)\ \right)$	$\left(\ ([0.28, 0.53]; 0.38)\ \right)$	$\left(\ ([0.33, 0.39]; 0.48)\ \right)$
$A_4 =$	$\left(\ ([0.27, 0.43]; 0.27)\ \right)$	$\left(\ ([0.27, 0.41]; 0.47)\ \right)$	$\left(\ ([0.33, 0.60]; 0.38)\ \right)$

Step 3:

We may find the $\check{S}_c(Ą_i)$ of every $Ą_i$ and used the Definition 3.1.2, as illustrated below:

$$\check{S}_c(Ą_1) = .08,\ \check{S}_c(Ą_2) = .02,\ \check{S}_c(Ą_3) = .05,\ \check{S}_c(Ą_4) = .06.$$

Step 4:

To choose the best options, we must first rearrange the SCFNs in order of decreasing: $A_1 > A_4 > A_2 > A_3$. As a result, A_1 is the best option.

For SCFWG Operator

Step 1:

The obtained data is provided by three significant in Table 4 according to the various relevance of all the decision-makers.

Step 2:

To use the SCFWG operator, we get for all options $Ą_i$, the collection SCFNs as s weight vector $\breve{\omega} = (.1, .2, .25, .45)^T$ shown as below:

$A_1 =$	$\left(\ ([0.19, 0.14]; 0.56)\ \right)$	$\left(\ ([0.23, 0.30]; 0.42)\ \right)$	$\left(\ ([0.26, 0.37]; 0.45)\ \right)$
$A_2 =$	$\left(\ ([0.28, 0.39]; 0.38)\ \right)$	$\left(\ ([0.19, 0.22]; 0.51)\ \right)$	$\left(\ ([0.23, 0.25]; 0.38)\ \right)$
$A_3 =$	$\left(\ ([0.19, 0.17]; 0.40)\ \right)$	$\left(\ ([0.33, 0.28]; 0.47)\ \right)$	$\left(\ ([0.26, 0.28]; 0.55)\ \right)$
$A_4 =$	$\left(\ ([0.31, 0.21]; 0.48)\ \right)$	$\left(\ ([0.21, 0.18]; 0.53)\ \right)$	$\left(\ ([0.31, 0.32]; 0.45)\ \right)$

Step 3:

We may find the $\check{S}_c(Ą_i)$ of every $Ą_i$ and used the Definition 3.1.2, as illustrated below:

$$\check{S}_c(Ą_1) = .13,\ \check{S}_c(Ą_2) = .06,\ \check{S}_c(Ą_3) = .04,\ \check{S}_c(Ą_4) = .08.$$

Step 4:

To choose the best options, we must first rearrange the SCFNs in order of decreasing: $Ą_1 > Ą_4 > Ą_2 > Ą_3$. As a result, $Ą_1$ is the best option.

For SCFOWG Operator

Step 1:

The obtained data is provided by three significant in Table 4 according to the various relevance of all the decision-makers.

Step 2:

To use the SCFOWG operator, we get for all options $Ą_i$, the collection SCFNs as s weight vector $\breve{\omega} = (.1, .2, .25, .45)^T$ shown as below:

$A_1 =$	$\left(([0.22, 0.27]; 0.50) \right)$	$\left(([0.25, 0.22]; 0.35) \right)$	$\left(([0.25, 0.37]; 0.49) \right)$
$A_2 =$	$\left(([0.27, 0.27]; 0.33) \right)$	$\left(([0.18, 0.22]; 0.41) \right)$	$\left(([0.29, 0.23]; 0.46) \right)$
$A_3 =$	$\left(([0.23, 0.29]; 0.40) \right)$	$\left(([0.24, 0.28]; 0.46) \right)$	$\left(([0.33, 0.30]; 0.55) \right)$
$A_4 =$	$\left(([0.33, 0.27]; 0.40) \right)$	$\left(([0.18, 0.17]; 0.57) \right)$	$\left(([0.22, 0.24]; 0.45) \right)$

Step 3:

We may find the $\H{\check{S}}_c(Ą_i)$ of every $Ą_i$ and used the Definition 3.1.2, as illustrated below:

$$\H{\check{S}}_c(Ą_1) = .11,\ \H{\check{S}}_c(Ą_2) = .05,\ \H{\check{S}}_c(Ą_3) = .04,\ \H{\check{S}}_c(Ą_4) = .06.$$

Step 4:

To choose the best options, we must first rearrange the SCFNs in order of decreasing: $Ą_1 > Ą_4 > Ą_2 > Ą_3$. As a result, $Ą_1$ is the best option.

For SCFHA Operator

Step 1:

The obtained data is provided by three significant in Table 4 according to the various relevance of all the decision-makers.

Step 2:

By utilizing, $\hat{S}_{c_i} = qw_i Ş_{c_i}$ using the data in Table 4 and a weight of $\breve{\omega} = (.1, .2, .25, .45)^T$ of $Ą_i$, the following results are shown in Table 5:

	$\widetilde{C}_1$	$\widetilde{C}_2$	$\widetilde{C}_3$	$\widetilde{C}_4$
$\widetilde{A}_1$	([0.26, 0.38]; 0.46) ([0.04, 0.19]; 0.06) ([0.05, 0.10]; 0.22)	([0.24, 0.42]; 0.39) ([0.05, 0.22]; 0.04) ([0.03, 0.15]; 0.13)	([0.38, 0.63]; 0.68) ([0.06, 0.18]; 0.03) ([0.05, 0.11]; 0.11)	([0.68, 0.25]; 0.49) ([0.13, 0.28]; 0.19) ([0.10, 0.32]; 0.18)
$\widetilde{A}_2$	([0.31, 0.52]; 0.30) ([0.03, 0.21]; 0.06) ([0.05, 0.14]; 0.12)	([0.37, 0.64]; 0.53) ([0.03, 0.15]; 0.09) ([0.18, 0.36]; 0.05)	([0.19, 0.40]; 0.28) ([0.01, 0.13]; 0.06) ([0.04, 0.13]; 0.07)	([0.19, 0.45]; 0.32) ([0.07, 0.17]; 0.36) ([0.04, 0.19]; 0.09)
$\widetilde{A}_3$	([0.27, 0.49]; 0.29) ([0.03, 0.09]; 0.12) ([0.03, 0.38]; 0.13)	([0.37, 0.63]; 0.52) ([0.02, 0.11]; 0.06) ([0.12, 0.36]; 0.04)	([0.20, 0.41]; 0.29) ([0.03, 0.11]; 0.06) ([0.03, 0.24]; 0.08)	([0.38, 0.57]; 0.53) ([0.07, 0.17]; 0.36) ([0.04, 0.19]; 0.09)
$\widetilde{A}_4$	([0.25, 0.37]; 0.41) ([0.22, 0.39]; 0.19) ([0.16, 0.17]; 0.11)	([0.43, 0.56]; 0.52) ([0.04, 0.11]; 0.02) ([0.10, 0.17]; 0.06)	([0.43, 0.63]; 0.74) ([0.06, 0.15]; 0.22) ([0.05, 0.12]; 0.18)	([0.29, 0.42]; 0.31) ([0.02, 0.09]; 0.21) ([0.08, 0.33]; 0.19)

Table 5 (Spherical cubic fuzzy hybrid aggregation decision maker)

Step 3:

We may find the $\text{S̋}_c(Ą_i)$ of every $Ą_i$ and used the Definition 3.1.2, as illustrated below:

$$\text{S̋}_c(Ą_1) = .11,\ \text{S̋}_c(Ą_2) = .05,\ \text{S̋}_c(Ą_3) = .04, \text{S̋}_c(Ą_4) = .08.$$

Step 4:

To choose the best options, we must first rearrange the SCFNs in order of decreasing: $Ą_1 > Ą_4 > Ą_2 > Ą_3$. As a result, $Ą_1$ is the best option.

For SCFHG Operator

Using the (SCGHG) operator, we produced the following scoring functions.

$$\text{S̋}_c(Ą_1) = .14,\ \text{S̋}_c(Ą_2) = .04,\ \text{S̋}_c(Ą_3) = .01,\ \text{S̋}_c(Ą_4) = .09.$$

$$Ą_1 > Ą_4 > Ą_2 > Ą_3.$$

As a result, $Ą_1$ is the best option.

Alternatives rank:

Table 6 shows the order in which the various choices are ranked.

operators	Ranking
SCFWA operator	$A_1 > A_2 > A_3 > A_4$
SCFOWA operator	$A_1 > A_4 > A_2 > A_3$
SCFWG operator	$A_1 > A_4 > A_2 > A_3$
SCFOWG operator	$A_1 > A_4 > A_2 > A_3$
SCFHA operator	$A_1 > A_4 > A_2 > A_3$
SCFHG operator	$A > A_4 > A_2 > A_3$

Table 6 (**Alternatives ranks**)

Figure 5 illustrates the supplier selection comparative study.

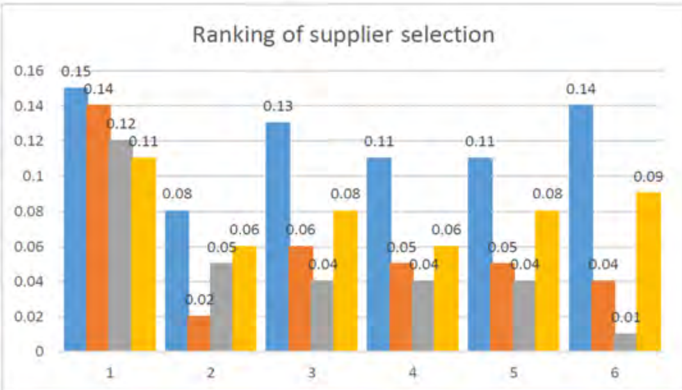

Figure 5 (Ranking of alternatives)

3.10 Validity and Reliability Test

3.10.1 Verify by VIKOR Technique

We demonstrate the results of SCFWA operators using the VIKOR approach. Table 4 shows the expected data of each decision - makers using the SCFWA operator.

We use the vikor approach to the data in Table 4 by selecting the weight vector $\breve{\omega} = (.1, .2, .25, .45)^T$ as the criteria weight. The steps for using the VIKOR approach to validate the example are as follows.

Step 1: We normalize the Table 4.

Step 2 : Compute PIS R^+ and NIS R^-

Step 3 : Calculate P_i, Q_i, R_i as below:

$$\text{\c{P}}_i = \sum_{j=1}^{m} \frac{\breve{\omega}_j d(\zeta_{ij}, \zeta\xi_j^{+})}{d(\zeta_j^{+}, \zeta_j^{-})}$$

$$\text{\c{Q}}_i = \max_{1 \leq j \leq m} \frac{\breve{\omega}_j d(\zeta_{ij}, \zeta_j^{+})}{d(\zeta_j^{+}, \zeta_j^{-})}$$

$$\text{\c{R}}_i = \frac{\tilde{x}(\text{\c{P}}_i - \text{\c{P}}^*)}{\text{\c{P}}^- - \text{\c{P}}^*} + \frac{(1-\tilde{x})(\text{\c{Q}}_i - \text{\c{Q}}^*)}{\text{\c{Q}}^- - \text{\c{Q}}^*}$$

Suppose $\tilde{x} = 0.4$ and that the results in Table 7 are valid.

Step 4 : Rank all the alternatives of $\text{\c{R}}_i$.

Step 5 : Based on the ranking outcomes, we may deduce that $\text{\c{R}}_1$ is the lowest, meaning that A_1 is the best supplier of all.

i	$\text{\c{P}}_i$	$\text{\c{Q}}_i$	$\text{\c{R}}_i$
1	.56	.12	.01
2	.4	.3	.5
3	.92	.42	0.6
4	.82	.2	.91

Table 7

3.11 Comparative Analysis

We evaluated our extended fuzzy aggregation operators against pre-defined fuzzy aggregation operators in this part and came to a conclusion. Despite the fact that SFSs concept is extremely important in a variety of domains, there are many challenges that SFSs does not address. In which the functions of membership, neutrality, and non-membership are all cubic fuzzy numbers. The numeric study in Section 3.9, which was addressed by SCFS, is a novel concept.It is not possible to tackle the problem described in this article due to the prior aggregation operators' restricted methodology. SCFNs, on the other hand, can readily fix the problem. As a result, SCFA operators are more reliable in solving ambiguous situations.

We have not used the concept of scoring function in the numerical problem mentioned in Section 3.9. By treating membership, neutrality, and non-membership as cubic fuzzy numbers, we can consider SCFN _c as a collection of three cubic numbers. Table 8 provides the outcomes of the alternatives, which are compared to Table 5 using the SCF score function. Finally, as shown in Figure 6, we find that A_1 is the best option among all possibilities.

In MCGDM issues, applying IF and ICF aggregation operators has various shortcomings, and we can't solve the problems under certain conditions. SCF aggregation operators, on the other hand, do not have these constraints, thus we receive more accurate data.

operators	Ranking of alternatives
SCFWA operator	$A_1 = A_2 = A_4 > A_3$
SCFOWA operator	$A_1 = A_3 > A_4 > A_2$
SCFWG operator	$A_2 > A_4 > A_1 > A_3$
SCFOWG operator	$A_1 > A_2 = A_3 > A_4$
SCFHA operator	$A_1 > A_2 > A_4 > A_3$
SCFHG operator	$A_1 > A_3 > A_2 > A_4$

Table 8 (Ranking Scheme)

The detailed comparison is depicted in Figure 6, which follows.

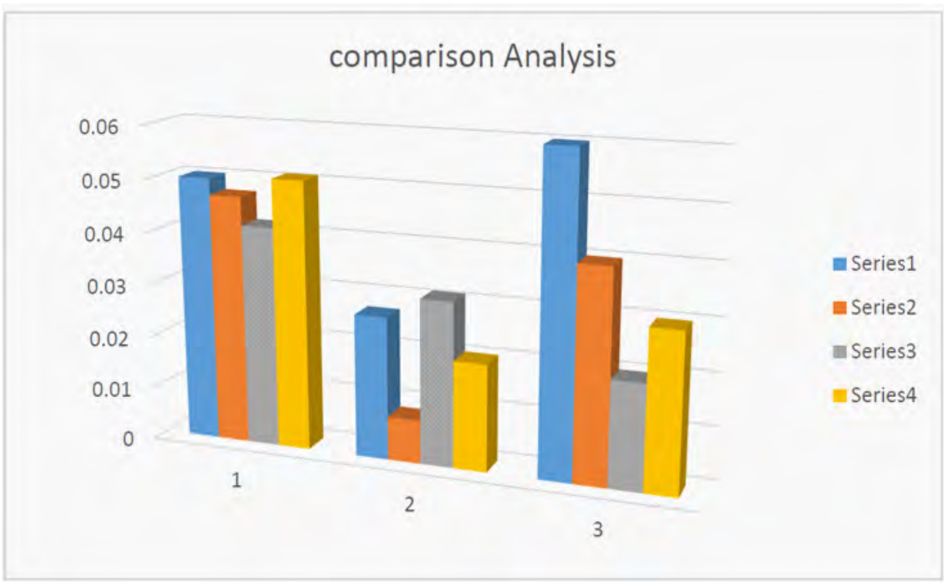

Figure 6 Comparison Analysis

3.12 Conclusion

The definition of the spherical cubic fuzzy set, which is the generalization of the interval valued spherical fuzzy set, was introduced in this chapter. Several spherical cubic fuzzy operational laws were developed. For the better comparison, of spherical cubic sets, we established a score and accuracy degree. The spherical cubic fuzzy distance between spherical cubic fuzzy numbers was also described. Aggregating the spherical cubic fuzzy information, we proposed (SCFWA), (SCFOWA), (SCFHWA) (SCFWG), (SCFOWG) and (SCFHWG) operators. In addition, to clarify the decision-making issues, we have applied the existing aggregation operators. A numeric representation has been presented that shows how the initially proposed operators can solve the decision-making procedure in a more efficient way. Finally, we have made several comparisons with existing operators to discussed the validity, practicality, and effectiveness of the suggested methods.

Chapter 4

Applications of spherical cubic fuzzy Hamacher weighting aggregatedoperators

In this chapter, we have established multi-attribute decision support system for the assessment of business execution with spherical fuzzy information. We have applied Hamacher aggregation operators such as the SCFHWA operator, SCFHOWA operator, SCFHHA operator, SCFHWG operator, SCFHOWG operator and SCFHHG operator for the appraisal of the best choice of enterprise. We ultimately defend the proposed approach with the existing strategies for possibility and adequacy.

4.1 Hamacher operators for spherical fuzzy set

Now, we will discussed about Hamacher aggregation operators on spherical fuzzy set. We will defined the aggregation operators on SFHWA, SFHOWA and SFHHA and so on.

Definition 4.1.1 *Let* $\c{S}_i = \left\langle \breve{\alpha}_{\c{S}_i}, \breve{\eta}_{\c{S}_i}, \breve{\beta}_{\c{S}_i} \right\rangle$ *be the collection then SFHWA operator is defined as* $SFHWA_{\breve{\omega}}(\c{S}_1,\c{S}_2,...,\c{S}_n) = \bigoplus_{i=1}^{n} \breve{\omega}_i \c{S}_i$ *and the weights* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$*. Now by Definition* **??** *spherical fuzzy weighted average (SFWA) operator by the induction on* n *converted to the form mentioned below.*

$$SFHWA_{\breve{\omega}}(\c{S}_1, \c{S}_2, ..., \c{S}_n) =$$

$$\left\{\begin{array}{c}\sqrt{\left(\frac{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(\check{\alpha}_{\c{S}_i}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\check{\alpha}_{\c{S}_i}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(\check{\alpha}_{\c{S}_i}\right)^2\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(1-\left(\check{\alpha}_{\c{S}_i}\right)^2\right)^{\breve{\omega}_i}}\right)}, \\ \left(\frac{\sqrt{\hat{f}}\prod_{i=1}^{n}\left(\check{\eta}_{\c{S}_i}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(1-\left(\check{\eta}_{\c{S}_i}\right)^2\right)\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(\left(\check{\eta}_{\c{S}_i}\right)^2\right)^{2\varpi_i}}}\right), \\ \left(\frac{\sqrt{\hat{f}}\prod_{i=1}^{n}\left(\check{\beta}_{\c{S}_i}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(1-\left(\check{\beta}_{\c{S}_i}\right)^2\right)\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(\left(\check{\beta}_{\c{S}_i}\right)^2\right)^{2\varpi_i}}}\right)\end{array}\right\} \tag{9}$$

Definition 4.1.2 *Let* $\c{S}_i =< \check{\alpha}_{\c{S}_i}, \check{\eta}_{\c{S}_i}, \check{\beta}_{\c{S}_i} >$ *be a collection then SFHOWA operator is defined as follows:*

$$SFHOWA_{\breve{\omega}}(\c{S}_1, \c{S}_2, ..., \c{S}_n) =$$

$$= \left\{\begin{array}{c}\sqrt{\left(\frac{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(\check{\alpha}_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\check{\alpha}_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(\check{\alpha}_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(1-\left(\check{\alpha}_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}}\right)}, \\ \left(\frac{\sqrt{\hat{f}}\prod_{i=1}^{n}\left(\check{\eta}_{\c{S}_{\sigma(i)}}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(1-\left(\check{\eta}_{\c{S}_{\sigma(i)}}\right)^2\right)\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(\left(\check{\eta}_{\c{S}_{\sigma(i)}}\right)^2\right)^{2\varpi_i}}}\right), \\ \left(\frac{\sqrt{\hat{f}}\prod_{i=1}^{n}\left(\check{\beta}_{\c{S}_{\sigma(i)}}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+\left(\hat{f}-1\right)\left(1-\left(\check{\beta}_{\c{S}_{\sigma(i)}}\right)^2\right)\right)^{\breve{\omega}_i}+\left(\hat{f}-1\right)\prod_{i=1}^{n}\left(\left(\check{\beta}_{\c{S}_{\sigma(i)}}\right)^2\right)^{2\varpi_i}}}\right)\end{array}\right\} \tag{10}$$

Definition 4.1.3 *Let* $\c{S}_i =< \check{\alpha}_{\c{S}_i}, \check{\eta}_{\c{S}_i}, \check{\beta}_{\c{S}_i} >$ *be the collection then SFHHA operator is defined as follows,* $SFHHA_{\breve{\omega}}(\c{S}_1, \c{S}_2, ..., \c{S}_n) = \bigoplus_{i=1}^{n} \breve{\omega}_i \c{S}_i$ *and the weights* $\breve{\omega} =$

$(\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$. *Where the* i^{th} *largest weighted SFNs* $\c{S}_i$ *is* $\c{S}_{\sigma(i)}$ *and* $\c{S}_{\sigma(i)} = m\varpi_i \c{S}_i(x)$.

$$SFHHA_{\breve{\omega}}(\c{S}_1, \c{S}_2, ..., \c{S}_n) =$$

$$\left\{ \begin{array}{c} \sqrt{\left(\dfrac{\prod_{i=1}^{n} \left(1 + \left(\hat{f} - 1\right)\left(\breve{\alpha}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i} - \prod_{i=1}^{n} \left(1 - \left(\breve{\alpha}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n} \left(1 + \left(\hat{f} - 1\right)\left(\breve{\alpha}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i} + \left(\hat{f} - 1\right)\prod_{i=1}^{n} \left(1 - \left(\breve{\alpha}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{\breve{\omega}_i}} \right)}, \\ \left(\dfrac{\sqrt{\hat{f}} \prod_{i=1}^{n} \left(\breve{\eta}'_{\c{S}_{\sigma(i)}}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n} \left(1 + \left(\hat{f} - 1\right)\left(1 - \left(\breve{\eta}'_{\c{S}_{\sigma(i)}}\right)^2\right)\right)^{\breve{\omega}_i} + \left(\hat{f} - 1\right)\prod_{i=1}^{n} \left(\left(\breve{\eta}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{2\varpi_i}}} \right), \\ \left(\dfrac{\sqrt{\hat{f}} \prod_{i=1}^{n} \left(\breve{\beta}'_{\c{S}_{\sigma(i)}}\right)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n} \left(1 + \left(\hat{f} - 1\right)\left(1 - \left(\breve{\beta}'_{\c{S}_{\sigma(i)}}\right)^2\right)\right)^{\breve{\omega}_i} + \left(\hat{f} - 1\right)\prod_{i=1}^{n} \left(\left(\breve{\beta}'_{\c{S}_{\sigma(i)}}\right)^2\right)^{2\varpi_i}}} \right) \end{array} \right\} \quad (11)$$

4.2 Hamacher operations in SCFS

We will define several new Hamacher operations and SCFS characteristics in this section..

Definition 4.2.1 *Let* $\c{S}_{c_1}$ *and* $\c{S}_{c_2}$ *be two SCFS in* $\c{X}$ *and* $\gamma > 0$, *we determine the following basic Hamacher operators in SCFS.*

$$
1.\ \text{\c{S}}_{c_1} \oplus \text{\c{S}}_{c_2} = \left\{ \begin{array}{c}
\left(\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\frac{\left(\check{a}_1^-\right)^2+(\check{a}_2^-)^2-\left(\check{a}_1^-\right)^2(\check{a}_2^-)^2-(1-\hat{f})\left(\check{a}_1^-\right)^2\left(\check{a}_2^-\right)^2}{1-(1-\hat{f})\left(\check{a}_1^-\right)^2(\check{a}_2^-)^2}}, \\
\sqrt{\frac{\left(\check{a}_1^+\right)^2+(\check{a}_2^+)^2-\left(\check{a}_1^+\right)^2(\check{a}_2^+)^2-(1-\hat{f})\left(\check{a}_1^+\right)^2\left(\check{a}_2^+\right)^2}{1-(1-\hat{f})\left(\check{a}_1^+\right)^2(\check{a}_2^+)^2}}
\end{array} \right], \\
\sqrt{\frac{\left(\check{\lambda}_1\right)^2+(\check{\lambda}_2)^2-\left(\check{\lambda}_1\right)^2(\check{\lambda}_2)^2-(1-\hat{f})\left(\check{\lambda}_1\right)^2\left(\check{\lambda}_2\right)^2}{1-(1-\hat{f})\left(\check{\lambda}_1\right)^2(\check{\lambda}_2)^2}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{(\check{n}_1^-)(\check{n}_2^+)}{\sqrt{\hat{f}+(1-\hat{f})\left((\check{n}_1^-)^2+(\check{n}_2^+)^2-(\check{n}_1^-)^2(\check{n}_2^+)^2\right)}}, \\
\frac{(\check{n}_1^+)(\check{n}_2^+)}{\sqrt{\hat{f}+(1-\hat{f})\left((\check{n}_1^+)^2+(\check{n}_2^+)^2-(\check{n}_1^+)^2(\check{n}_2^+)^2\right)}}
\end{array} \right], \\
\frac{(\check{\delta}_1)(\check{\delta}_2)}{\sqrt{\hat{f}+(1-\hat{f})\left((\check{\delta}_1)^2+(\check{\delta}_2)^2-(\check{\delta}_1)^2(\check{\delta}_2)^2\right)}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{(\check{b}_1^-)(\check{b}_2^-)}{\sqrt{\hat{f}+(1-\hat{f})\left((\check{b}_1^-)^2+(\check{b}_2^-)^2-(\check{b}_1^-)^2(\check{b}_2^-)^2\right)}}, \\
\frac{(\check{b}_1^+)(\check{b}_2^+)}{\sqrt{\hat{f}+(1-\hat{f})\left((\check{b}_1^+)^2+(\check{b}_2^+)^2-(\check{b}_1^+)^2(\check{b}_2^+)^2\right)}}
\end{array} \right], \\
\frac{(\check{\mu}_1)(\check{\mu}_2)}{\sqrt{\hat{f}+(1-\hat{f})((\check{\mu}_1)^2+(\check{\mu}_2)^2-(\check{\mu}_1)^2(\check{\mu}_2)^2)}}
\end{array} \right)
\end{array} \right\}
$$

$$
2.\ \text{\c{S}}_{c_1} \otimes \text{\c{S}}_{c_2} = \left\{ \begin{array}{c}
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{\left(\check{a}_1^-\right)(\check{a}_2^-)}{\sqrt{\hat{f}+(1-\hat{f})\left(\left(\check{a}_1^-\right)^2+(\check{a}_2^-)^2-\left(\check{a}_1^-\right)^2(\check{a}_2^-)^2\right)}}, \\
\frac{\left(\check{a}_1^+\right)(\check{a}_2^+)}{\sqrt{\hat{f}+(1-\hat{f})\left(\left(\check{a}_1^+\right)^2+(\check{a}_2^+)^2-\left(\check{a}_1^+\right)^2(\check{a}_2^+)^2\right)}}
\end{array} \right], \\
\frac{\left(\check{\lambda}_1\right)(\check{\lambda}_2)}{\sqrt{\hat{f}+(1-\hat{f})\left(\left(\check{\lambda}_1\right)^2+(\check{\lambda}_2)^2-\left(\check{\lambda}_1\right)^2(\check{\lambda}_2)^2\right)}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\frac{\left(\check{n}_1^-\right)^2+(\check{n}_2^+)^2-\left(\check{n}_1^-\right)^2(\check{n}_2^+)^2-(1-\hat{f})\left(\check{n}_1^-\right)^2\left(\check{n}_2^+\right)^2}{1-(1-\hat{f})\left(\check{n}_1^-\right)^2(\check{n}_2^+)^2}}, \\
\sqrt{\frac{\left(\check{n}_1^+\right)^2+(\check{n}_2^+)^2-\left(\check{n}_1^+\right)^2(\check{n}_2^+)^2-(1-\hat{f})\left(\check{n}_1^+\right)^2\left(\check{n}_2^+\right)^2}{1-(1-\hat{f})\left(\check{n}_1^+\right)^2(\check{n}_2^+)^2}}
\end{array} \right], \\
\sqrt{\frac{\left(\check{\delta}_1\right)^2+(\check{\delta}_2)^2-\left(\check{\delta}_1\right)^2(\check{\delta}_2)^2-(1-\hat{f})\left(\check{\delta}_1\right)^2\left(\check{\delta}_2\right)^2}{1-(1-\hat{f})\left(\check{\delta}_1\right)^2(\check{\delta}_2)^2}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\frac{\left(\check{b}_1^-\right)^2+(\check{b}_2^-)^2-\left(\check{b}_1^-\right)^2(\check{b}_2^-)^2-(1-\hat{f})\left(\check{b}_1^-\right)^2\left(\check{b}_2^-\right)^2}{1-(1-\hat{f})\left(\check{b}_1^-\right)^2(\check{b}_2^-)^2}}, \\
\sqrt{\frac{\left(\check{b}_1^+\right)^2+(\check{b}_2^+)^2-\left(\check{b}_1^+\right)^2(\check{b}_2^+)^2-(1-\hat{f})\left(\check{b}_1^+\right)^2\left(\check{b}_2^+\right)^2}{1-(1-\hat{f})\left(\check{b}_1^+\right)^2(\check{b}_2^+)^2}}
\end{array} \right], \\
\sqrt{\frac{(\check{\mu}_1)^2+(\check{\mu}_2)^2-(\check{\mu}_1)^2(\check{\mu}_2)^2-(1-\hat{f})(\check{\mu}_1)^2(\check{\mu}_2)^2}{1-(1-\hat{f})(\check{\mu}_1)^2(\check{\mu}_2)^2}}
\end{array} \right),
\end{array} \right\}
$$

$$3.\ \gamma\tilde{S}_{c_1}=\left\{\begin{array}{l}\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{a}_1^-\right)^2\right)^\gamma-\left(1-\left(\check{a}_1^-\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{a}_1^-\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{a}_1^-\right)^2\right)^\gamma}},\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{a}_1^+\right)^2\right)^\gamma-\left(1-\left(\check{a}_1^+\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{a}_1^+\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{a}_1^+\right)^2\right)^\gamma}}\end{array}\right],\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{\lambda}_1\right)^2\right)^\gamma-\left(1-\left(\check{\lambda}_1\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{\lambda}_1\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{\lambda}_1\right)^2\right)^\gamma}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\check{n}_1^-)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{n}_1^-)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{n}_1^-\right)^{2\gamma}\right.}},\\ \frac{\sqrt{\hat{f}}(\check{n}_1^+)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{n}_1^+)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{n}_1^+\right)^{2\gamma}\right.}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}(\check{\delta}_1)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{\delta}_1)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{\delta}_1\right)^{2\gamma}\right.}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\check{b}_1^-)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{b}_1^-)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{b}_1^-\right)^{2\gamma}\right.}},\\ \frac{\sqrt{\hat{f}}(\check{b}_1^+)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{b}_1^+)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{b}_1^+\right)^{2\gamma}\right.}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}(\check{\mu}_1)^\gamma}{\sqrt{(1+(\hat{f}-1)((1-(\check{\mu}_1)^2))^\gamma+(\hat{f}-1)(\check{\mu}_1)^{2\gamma}}}\end{array}\right)\end{array}\right\}$$

$$4.\ \left(\c{S}_{c_1}\right)^\gamma=\left\{\begin{array}{l}\left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\check{a}_1^-)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{a}_1^-)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{a}_1^-\right)^{2\gamma}\right.}},\\ \frac{\sqrt{\hat{f}}(\check{a}_1^+)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{a}_1^+)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{a}_1^+\right)^{2\gamma}\right.}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}(\check{\lambda}_1)^\gamma}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\check{\lambda}_1)^2\right)\right)^\gamma+(\hat{f}-1)\left(\check{\lambda}_1\right)^{2\gamma}\right.}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{n}_1^-\right)^2\right)^\gamma-\left(1-\left(\check{n}_1^-\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{n}_1^-\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{n}_1^-\right)^2\right)^\gamma}},\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{n}_1^+\right)^2\right)^\gamma-\left(1-\left(\check{n}_1^+\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{n}_1^+\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{n}_1^+\right)^2\right)^\gamma}}\end{array}\right],\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{\delta}_1\right)^2\right)^\gamma-\left(1-\left(\check{\delta}_1\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{\delta}_1\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{\delta}_1\right)^2\right)^\gamma}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{b}_1^-\right)^2\right)^\gamma-\left(1-\left(\check{b}_1^-\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{b}_1^-\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{b}_1^-\right)^2\right)^\gamma}},\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\check{b}_1^+\right)^2\right)^\gamma-\left(1-\left(\check{b}_1^+\right)^2\right)^\gamma}{\left(1+(\hat{f}-1)\left(\check{b}_1^+\right)^2\right)^\gamma+(\hat{f}-1)\left(1-\left(\check{b}_1^+\right)^2\right)^\gamma}}\end{array}\right],\\ \sqrt{\frac{(1+(\hat{f}-1)(\check{\mu}_1)^2)^\gamma-(1-(\check{\mu}_1)^2)^\gamma}{(1+(\hat{f}-1)(\check{\mu}_1)^2)^\gamma+(\hat{f}-1)(1-(\check{\mu}_1)^2)^\gamma}}\end{array}\right)\end{array}\right\}$$

Proposition 4.2.2 *Let* $\c{S}_{c_1}$ *and* $\c{S}_{c_2}$ *be two SCFS in* $\c{X}$ *and* $\gamma_1, \gamma_2 > 0$ *.We have the* *following properties.*

1. $Ş_{c_2} \oplus Ş_{c_1} = Ş_{c_1} \oplus Ş_{c_2}$,
2. $Ş_{c_2} \otimes Ş_{c_1} = Ş_{c_1} \otimes Ş_{c_2}$,
3. $\gamma(Ş_{c_2} \oplus Ş_{c_1}) = \gamma\tilde{S}_{c_2} \oplus \gamma\tilde{S}_{c_1}$,
4. $\gamma(Ş_{c_2} \otimes Ş_{c_1}) = \gamma\tilde{S}_{c_2} \otimes \gamma\tilde{S}_{c_1}$,
5. $(\gamma_1 \oplus \gamma_2)Ş_{c_1} = \gamma_1 Ş_{c_1} \oplus \gamma_2 Ş_{c_1}$,
6. $(\gamma_1 \otimes \gamma_2)Ş_{c_1} = \gamma_1 Ş_{c_1} \otimes \gamma_2 Ş_{c_1}$,
7. $(Ş_{c_1} \otimes Ş_{c_2})^{\gamma} = Ş_{c_1}^{\gamma} \otimes Ş_{c_2}^{\gamma}$,
8. $(Ş_{c_1} \oplus Ş_{c_2})^{\gamma} = Ş_{c_1}^{\gamma} \oplus Ş_{c_2}^{\gamma}$,
9. $Ş_{c_1}^{\gamma_1+\gamma_2} = Ş_{c_1}^{\gamma_1} \otimes Ş_{c_1}^{\gamma_2}$,
10. $(Ş_{c_1}^{\gamma})^n = (Ş_{c_1}^{n})^{\gamma}$.

4.3 SCFHWA aggregation operator

We will now analyse the features of several spherical cubic fuzzy Hamacher aggregation operators.

Definition 4.3.1 *Let* $Ş_{c_i} =< \breve{\alpha}_{Ş_i}, \check{\eta}_{Ş_i}, \check{\beta}_{Ş_i} >$ *be a SCFN in* $\underset{\cdot}{X}$ *and let the SCFHWA operator is a mapping of* $\Phi^n \rightarrow \Phi$ *such that* $SCFHWA_{\breve{\omega}}(Ş_{c_1}, Ş_{c_2}, ..., Ş_n) = \overset{n}{\underset{i=1}{\oplus}} \breve{\omega}_i Ş_{c_i}$, *and the weights* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$. *Now we get the following results related to SCFNs and Hamacher operators.*

Theorem 4.3.2 *Let* $Ş_{c_i} =< \breve{\alpha}_{Ş_{c_i}}, \check{\eta}_{Ş_{c_i}}, \check{\beta}_{Ş_{c_i}} >$ *be a collection then SCFHWA operator is defined as follows:*

$$SCFHWA_{\breve{\omega}}(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) =$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}_i^{-}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{a}_i^{-}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}_i^{-}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{a}_i^{-}\right)^2\right)^{\breve{\omega}_i}}},\\ \sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}_i^{+}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{a}_i^{+}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}_i^{+}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{a}_i^{+}\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right],\\ \sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}_i\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\check{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}_i\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\check{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{n}_i^{-})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}_i^{-})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{n}_i^{-}\right)^{2\varpi_i}}},\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{n}_i^{+})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}_i^{+})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{n}_i^{+}\right)^{2\varpi_i}}}\end{array}\right],\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{\delta}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{\delta}_i)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{\delta}_i\right)^{2\varpi_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{b}_i^{-})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^{-})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{b}_i^{-}\right)^{2\varpi_i}}},\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{b}_i^{+})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^{+})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{b}_i^{+}\right)^{2\varpi_i}}}\end{array}\right],\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{\mu}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_i)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}(\breve{\mu}_i)^{2\varpi_i}}}\end{array}\right)\end{array}\right\} \tag{12}$$

Proof. We will use mathematical induction to prove that. When $n = 2$ is used, we obtain

$$\breve{\omega}_1 \text{Ş}_{c_1} \oplus \breve{\omega}_2 \text{Ş}_{c_2} =$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_1^-\right)^2\right)^{\breve{\omega}_1}-\left(1-\left(\breve{a}_1^-\right)^2\right)^{\breve{\omega}_1}}{\left(1+(\hat{f}-1)\left(\breve{a}_1^-\right)^2\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(1-\left(\breve{a}_1^-\right)^2\right)^{\breve{\omega}_1}}},\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_1^+\right)^2\right)^{\breve{\omega}_1}-\left(1-\left(\breve{a}_1^+\right)^2\right)^{\breve{\omega}_1}}{\left(1+(\hat{f}-1)\left(\breve{a}_1^+\right)^2\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(1-\left(\breve{a}_1^+\right)^2\right)^{\breve{\omega}_1}}}\end{array}\right],\\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_1}-\left(1-\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_1}}{\left(1+(\hat{f}-1)\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(1-\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_1}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\breve{n}_1^-)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_1^-)^2\right)\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(\breve{n}_1^-\right)^{2\varpi_1}\right.}},\\ \frac{\sqrt{\hat{f}}(\breve{n}_1^+)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_1^+)^2\right)\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(\breve{n}_1^+\right)^{2\varpi_1}\right.}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}(\breve{\delta}_1)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_1)^2\right)\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(\breve{\delta}_1\right)^{2\varpi_1}\right.}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\breve{b}_1^-)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_1^-)^2\right)\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(\breve{b}_1^-\right)^{2\varpi_1}\right.}},\\ \frac{\sqrt{\hat{f}}(\breve{b}_1^+)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_1^+)^2\right)\right)^{\breve{\omega}_1}+(\hat{f}-1)\left(\breve{b}_1^+\right)^{2\varpi_1}\right.}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}(\breve{\mu}_1)^{\breve{\omega}_1}}{\sqrt{\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_1)^2)\right)^{\breve{\omega}_1}+(\hat{f}-1)(\breve{\mu}_1)^{2\varpi_1}\right.}}\end{array}\right)\end{array}\right\}$$

$$\oplus\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_2^{-}\right)^2\right)^{\breve{\omega}_2}-\left(1-\left(\breve{a}_2^{-}\right)^2\right)^{\breve{\omega}_2}}{\left(1+(\hat{f}-1)\left(\breve{a}_2^{-}\right)^2\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(1-\left(\breve{a}_2^{-}\right)^2\right)^{\breve{\omega}_2}}}, \\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_2^{+}\right)^2\right)^{\breve{\omega}_2}-\left(1-\left(\breve{a}_2^{+}\right)^2\right)^{\breve{\omega}_2}}{\left(1+(\hat{f}-1)\left(\breve{a}_2^{+}\right)^2\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(1-\left(\breve{a}_2^{+}\right)^2\right)^{\breve{\omega}_2}}}\end{array}\right], \\ \sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{\lambda}_2\right)^2\right)^{\breve{\omega}_2}-\left(1-\left(\breve{\lambda}_2\right)^2\right)^{\breve{\omega}_2}}{\left(1+(\hat{f}-1)\left(\breve{\lambda}_2\right)^2\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(1-\left(\breve{\lambda}_2\right)^2\right)^{\breve{\omega}_2}}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\breve{n}_2^{+})^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_2^{+})^2\right)\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(\breve{n}_2^{+}\right)^{2\varpi_2}\right.}}, \\ \frac{\sqrt{\hat{f}}(\breve{n}_2^{+})^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_2^{+})^2\right)\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(\breve{n}_2^{+}\right)^{2\varpi_2}\right.}}\end{array}\right], \\ \frac{\sqrt{\hat{f}}(\breve{\delta}_2)^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_2)^2\right)\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(\breve{\delta}_2\right)^{2\varpi_2}\right.}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}(\breve{b}_2^{-})^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_2^{-})^2\right)\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(\breve{b}_2^{-}\right)^{2\varpi_2}\right.}}, \\ \frac{\sqrt{\hat{f}}(\breve{b}_2^{+})^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_2^{+})^2\right)\right)^{\breve{\omega}_2}+(\hat{f}-1)\left(\breve{b}_2^{+}\right)^{2\varpi_2}\right.}}\end{array}\right], \\ \frac{\sqrt{\hat{f}}(\breve{\mu}_2)^{\breve{\omega}_2}}{\sqrt{\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_2)^2)\right)^{\breve{\omega}_2}+(\hat{f}-1)(\breve{\mu}_2)^{2\varpi_2}\right.}}\end{array}\right)\end{array}\right\}$$

$$\breve{\omega}_1 \S_{c_1} \oplus \breve{\omega}_2 \S_{c_2} =$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{2}\left(1-\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(1-\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{2}\left(1-\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(1-\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}}} \end{array} \right], \\ \sqrt{\frac{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{2}\left(1-\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(1-\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{n}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_i^-)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(\breve{n}_i^-\right)^{2\varpi_i}\right.}}, \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{n}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_i^+)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(\breve{n}_i^+\right)^{2\varpi_i}\right.}} \end{array} \right], \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{\delta}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_i)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(\breve{\delta}_i\right)^{2\varpi_i}\right.}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{b}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^-)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(\breve{b}_i^-\right)^{2\varpi_i}\right.}}, \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{b}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^+)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}\left(\breve{b}_i^+\right)^{2\varpi_i}\right.}} \end{array} \right], \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{2}(\breve{\mu}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{2}\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_i)^2)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{2}(\breve{\mu}_i)^{2\varpi_i}\right.}} \end{array} \right) \end{array} \right\}$$

Hence the Equation 12 hold for $n = 2$.

(ii) Assuming that Equation [**?**] is valid for $n = k$, we have

$$SCFHWA_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}...\c{S}_{c_k}) =$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{a}_i^-\right)^2\right)^{\breve{\omega}_i}}},\\ \sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{a}_i^+\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right],\\ \sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{\lambda}_i\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{n}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_i^-)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{n}_i^-\right)^{2\varpi_i}}},\\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{n}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_i^+)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{n}_i^+\right)^{2\varpi_i}}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{\delta}_1)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_i)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{\delta}_i\right)^{2\varpi_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{b}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^-)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{b}_i^-\right)^{2\varpi_i}}},\\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{b}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_i^+)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{b}_i^+\right)^{2\varpi_i}}}\end{array}\right],\\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{\mu}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_i)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}(\breve{\mu}_i)^{2\varpi_i}}}\end{array}\right)\end{array}\right\}$$

Now for $n = k + 1$ we have,

$SCFHWA_{\breve{\omega}}(\c{S}_{c_1},\c{S}_{c_2}...\c{S}_{c_k},\c{S}_{c_{k+1}}) = SCFHWA_{\breve{\omega}}(\c{S}_{c_1},\c{S}_{c_2}...\c{S}_{c_k})\oplus\c{S}_{c_{k+1}}$

$$
= \left\{ \begin{array}{c}
\left(\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_1^{-}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{a}_1^{-}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_1^{-}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{a}_1^{-}\right)^2\right)^{\breve{\omega}_i}}}, \\
\sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_1^{+}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{a}_1^{+}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{a}_1^{+}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{a}_1^{+}\right)^2\right)^{\breve{\omega}_i}}}
\end{array} \right], \\
\sqrt{\frac{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k}\left(1-\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(1-\left(\breve{\lambda}_1\right)^2\right)^{\breve{\omega}_i}}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{n}_1^{-})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_1^{-})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{n}_1^{-}\right)^{2\varpi_i}}}, \\
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{n}_1^{+})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_1^{+})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{n}_1^{+}\right)^{2\varpi_i}}}
\end{array} \right], \\
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{\delta}_1)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_1)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{\delta}_1\right)^{2\varpi_i}}}
\end{array} \right), \\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{b}_1^{-})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_1^{-})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{b}_1^{-}\right)^{2\varpi_i}}}, \\
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{b}_1^{+})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_1^{+})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}\left(\breve{b}_1^{+}\right)^{2\varpi_i}}}
\end{array} \right], \\
\frac{\sqrt{\hat{f}}\prod_{i=1}^{k}(\breve{\mu}_1)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k}\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_1)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k}(\breve{\mu}_1)^{2\varpi_i}}}
\end{array} \right)
\end{array} \right\}
$$

$$
\oplus\left\{\begin{array}{c}
\left(\begin{array}{c}
\left[\begin{array}{c}
\sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_{k+1}^{-}\right)^{2}\right)^{\breve{\omega}_{k+1}}-\left(1-\left(\breve{a}_{k+1}^{-}\right)^{2}\right)^{\breve{\omega}_{k+1}}}{\left(1+(\hat{f}-1)\left(\breve{a}_{k+1}^{-}\right)^{2}\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(1-\left(\breve{a}_{k+1}^{-}\right)^{2}\right)^{\breve{\omega}_{k+1}}}},\\
\sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{a}_{k+1}^{+}\right)^{2}\right)^{\breve{\omega}_{k+1}}-\left(1-\left(\breve{a}_{k+1}^{+}\right)^{2}\right)^{\breve{\omega}_{k+1}}}{\left(1+(\hat{f}-1)\left(\breve{a}_{k+1}^{+}\right)^{2}\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(1-\left(\breve{a}_{k+1}^{+}\right)^{2}\right)^{\breve{\omega}_{k+1}}}}
\end{array}\right],\\
\sqrt{\frac{\left(1+(\hat{f}-1)\left(\breve{\lambda}_{k+1}\right)^{2}\right)^{\breve{\omega}_{k+1}}-\left(1-\left(\breve{\lambda}_{k+1}\right)^{2}\right)^{\breve{\omega}_{k+1}}}{\left(1+(\hat{f}-1)\left(\breve{\lambda}_{k+1}\right)^{2}\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(1-\left(\breve{\lambda}_{k+1}\right)^{2}\right)^{\breve{\omega}_{k+1}}}}
\end{array}\right),\\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{\sqrt{\hat{f}}(\breve{n}_{k+1}^{-})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_{k+1}^{-})^{2}\right)\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(\breve{n}_{k+1}^{-}\right)^{2\varpi_{k+1}}\right.}},\\
\frac{\sqrt{\hat{f}}(\breve{n}_{k+1}^{+})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{n}_{k+1}^{+})^{2}\right)\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(\breve{n}_{k+1}^{+}\right)^{2\varpi_{k+1}}\right.}}
\end{array}\right],\\
\frac{\sqrt{\hat{f}}(\breve{\delta}_{1})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{\delta}_{k+1})^{2}\right)\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(\breve{\delta}_{k+1}\right)^{2\varpi_{k+1}}\right.}}
\end{array}\right),\\
\left(\begin{array}{c}
\left[\begin{array}{c}
\frac{\sqrt{\hat{f}}(\breve{b}_{k+1}^{-})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_{k+1}^{-})^{2}\right)\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(\breve{b}_{k+1}^{-}\right)^{2\varpi_{k+1}}\right.}},\\
\frac{\sqrt{\hat{f}}(\breve{b}_{k+1}^{+})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left(\left(1-(\breve{b}_{k+1}^{+})^{2}\right)\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)\left(\breve{b}_{k+1}^{+}\right)^{2\varpi_{k+1}}\right.}}
\end{array}\right],\\
\frac{\sqrt{\hat{f}}(\breve{\mu}_{k+1})^{\breve{\omega}_{k+1}}}{\sqrt{\left(1+(\hat{f}-1)\left((1-(\breve{\mu}_{k+1})^{2})\right)^{\breve{\omega}_{k+1}}+(\hat{f}-1)(\breve{\mu}_{k+1})^{2\varpi_{k+1}}\right.}}
\end{array}\right)
\end{array}\right\}
$$

$$
SCFHWA_{\breve{\omega}}(\c{S}_{c_1},\c{S}_{c_2}...\c{S}_{c_{k+1}})=
$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{a}_i^-)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k+1}\left(1-(\breve{a}_i^-)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{a}_i^-)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(1-(\breve{a}_i^-)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{a}_i^+)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k+1}\left(1-(\breve{a}_i^+)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{a}_i^+)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(1-(\breve{a}_i^+)^2\right)^{\breve{\omega}_i}}} \end{array} \right], \\ \sqrt{\frac{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{\lambda}_i)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{k+1}\left(1-(\breve{\lambda}_i)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)(\breve{\lambda}_i)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(1-(\breve{\lambda}_i)^2\right)^{\breve{\omega}_i}}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{n}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)\left((1-(\breve{n}_i^-)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(\breve{n}_i^-\right)^{2\varpi_i}}}, \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{n}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)\left((1-(\breve{n}_i^+)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(\breve{n}_i^+\right)^{2\varpi_i}}} \end{array} \right], \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{\delta}_1)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)\left((1-(\breve{\delta}_i)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(\breve{\delta}_i\right)^{2\varpi_i}}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{b}_i^-)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)\left((1-(\breve{b}_i^-)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(\breve{b}_i^-\right)^{2\varpi_i}}}, \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{b}_i^+)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)\left((1-(\breve{b}_i^+)^2)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}\left(\breve{b}_i^+\right)^{2\varpi_i}}} \end{array} \right], \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{k+1}(\breve{\mu}_i)^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{k+1}\left(1+(\hat{f}-1)((1-(\breve{\mu}_i)^2))\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{k+1}(\breve{\mu}_i)^{2\varpi_i}}} \end{array} \right) \end{array} \right\}$$

So it is true for $n = k+1$. Hence it's hold for all values of n. ■

Proposition 4.3.3 *Let* $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ *be a colllection and the weights of* $\c{S}_{c_i}$ *be* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$ *and* $\breve{\omega}_i > 0$, *then we established the characteristics as below.*

Boundary: For every $\breve{\omega}$,

$$\c{S}_{c_i}^- \leq SCFHWA_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) \leq \c{S}_{c_i}^+$$

Idempotency: For every $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ are same then

$$SCFHWA_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) = \c{S}_c.$$

Monotonicity: Let $\S_c^*$ be a collection of SCFN, then

$$SCFHWA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq SCFHWA_{\breve{\omega}}(\S^*_{c_1}, \S^*_{c_2}, ..., \S^*_{c_n}).$$

The concept of SCFHWA aggregation operators evaluates SFHWA only. In the MCGDM issue, there are conditions when the ordering situation of the SCFN resolves issues.

4.4 SCFHOWA aggregation operator

Now we will discuss the SCFHOWA operator and its fundamental features.

Definition 4.4.1 *Let $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ be a collection . Then the SCFHOWA operator is mapping $SCFHOWA : \Phi^n \to \Phi$ and the weights $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ with $\sum_{i=1}^{n} \breve{\omega}_i = 1$ and $\breve{\omega}_i \in [0, 1]$ defined as follows:*

$$SCFHOWA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \overset{n}{\underset{i=1}{\oplus}} \breve{\omega}_i \S_{c_{\sigma(i)}}$$

for all i, $\S_{c_{\sigma(i-1)}} \geq \S_{c_{\sigma(i)}}$ and the permutation $(\sigma_1, \sigma_2, ..., \sigma_n)$ so we get the following result related to SCFNs and Hamacher operator.

Theorem 4.4.2 *Let $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ be a collection. Then the aggregation of SCFHOWA is also a SCFN and it is defined as follows:*

$$SCFHOWA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) =$$

$$
\left\{
\begin{array}{c}
\left(
\begin{array}{c}
\left[
\begin{array}{c}
\sqrt{\dfrac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}^{-}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\breve{a}^{-}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}^{-}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\breve{a}^{-}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}}, \\
\sqrt{\dfrac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}^{+}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\breve{a}^{+}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{a}^{+}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\breve{a}^{+}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}}
\end{array}
\right], \\
\sqrt{\dfrac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\check{\lambda}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\check{\lambda}_{\sigma(i)}\right)^{2}\right)^{\breve{\omega}_i}}}
\end{array}
\right), \\
\left(
\begin{array}{c}
\left[
\begin{array}{c}
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{n}^{-}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}^{-}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{n}^{-}_{\sigma(i)}\right)^{2\varpi_i}}}, \\
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{n}^{+}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}^{+}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{n}^{+}_{\sigma(i)}\right)^{2\varpi_i}}}
\end{array}
\right], \\
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{\delta}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{\delta}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{\delta}_{\sigma(i)}\right)^{2\varpi_i}}}
\end{array}
\right), \\
\left(
\begin{array}{c}
\left[
\begin{array}{c}
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{b}^{-}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{b}^{-}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{b}^{-}_{\sigma(i)}\right)^{2\varpi_i}}}, \\
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{b}^{+}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{b}^{+}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{b}^{+}_{\sigma(i)}\right)^{2\varpi_i}}}
\end{array}
\right], \\
\dfrac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\check{\mu}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{\mu}_{\sigma(i)})^{2}\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\check{\mu}_{\sigma(i)}\right)^{2\varpi_i}}}
\end{array}
\right)
\end{array}
\right\} \tag{13}
$$

Here the weight vector is $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ under the specified conditions $\sum_{i=1}^{n}\breve{\omega}_i = 1$ and $\breve{\omega}_i > 0$.

Proposition 4.4.3 *Let* $Ş_{c_i} =< \check{\alpha}_{Ş_{c_i}}, \check{\eta}_{Ş_{c_i}}, \check{\beta}_{Ş_{c_i}} >$ *be a collection and the weights of* $Ş_{c_i}$ *be* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{\hat{n}}\breve{\omega}_i = 1$ *and* $\breve{\omega}_i > 0$, *then we established the characteristics as below.*

Idempotency: For every $Ş_{c_i} =< \check{\alpha}_{Ş_{c_i}}, \check{\eta}_{Ş_{c_i}}, \check{\beta}_{Ş_{c_i}} >$ are same i.e, $Ş_{c_i} = Ş_c$ then $SCFHOWA_{\breve{\omega}}(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = Ş_c$.

Boundary: For every $\breve{\omega}$,

$$
Ş_c^{-} \leq SCFHOWA_{\breve{\omega}}(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \leq Ş_c^{+}
$$

Monotonicity: Let

$$S^* = \{([\breve{a}^{*-}_{\tilde{\sigma}(i)}, \breve{a}^{*+}_{\tilde{\sigma}(i)}], \check{\lambda}^*_{\tilde{\sigma}(i)}), ([\breve{n}^{*-}_{\tilde{\sigma}(i)}, \breve{n}^{*+}_{\tilde{\sigma}(i)}], \check{\delta}^*_{\tilde{\sigma}(i)}), ([\check{b}^{*-}_{\tilde{\sigma}(i)}, \check{b}^{*+}_{\tilde{\sigma}(i)}], \breve{\mu}^*_{\tilde{\sigma}(i)})\}$$

then

$$SCFHOWA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq SCFHOWA_{\breve{\omega}}(\S^*_{c_1}, \S^*_{2}, ..., \S_{c_n}).$$

4.5 SCFHHA aggregation operator

Now we will examine at the SCFHHA operator and its basic features in more detail.

Definition 4.5.1 *Let* $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ *be a collection of SCFNs in* X. *Then the SCFHHA operator is mapping* $SCFHHA : \Phi^n \rightarrow \Phi$ *and the weights* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$ *and* $\breve{\omega}_i \in [0,1]$ *as* $SCFHHA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \bigoplus_{i=1}^{n} \breve{\omega}_i S^*_{\sigma(i)}$ *where* $S^*_{\sigma(i)}$ *represented the* i^{th} *highest weight SFNs* $\S_{c_i}$. *And the weights of* $\S_{c_i}$ *is* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$. *Here,* $\S^*_{c_{\sigma(i)}} = mw_i \S_{c_{\sigma(i)}}$ *where* n *represents the balancing coefficient when* $\breve{\omega} = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})^T$ *then SCFHWA and SCFOHWA operators are considered as special case of SCFHHA operator. Now we will get the following result related to SCFN and Hamacher operator.*

Theorem 4.5.2 *Let* $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ *be a collection. Then the aggregation of SCFHHA is also a SCFN and it is defined as follows:*

$$SCFHHA_{\omega,\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) =$$

$$\left\{\begin{array}{c}\left(\left[\begin{array}{c}\sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\tilde{e}^{*-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\tilde{e}^{*-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\tilde{e}^{*-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\tilde{e}^{*-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}},\\ \sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\tilde{e}^{*+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\tilde{e}^{*+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\tilde{e}^{*+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\tilde{e}^{*+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right],\\ \sqrt{\dfrac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}^{*}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\check{\lambda}^{*}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{\lambda}^{*}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\check{\lambda}^{*}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}}\right),\\ \left(\left[\begin{array}{c}\dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{n}^{*-}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}^{*-}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{n}^{*-}_{\sigma(i)}\right)^{2\varpi_i}}},\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{n}^{*+}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{n}^{*+}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{n}^{*+}_{\sigma(i)}\right)^{2\varpi_i}}}\end{array}\right],\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{\mu}^{*}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{\mu}^{*}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{\mu}^{*}_{\sigma(i)}\right)^{2\varpi_i}}}\right),\\ \left(\left[\begin{array}{c}\dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{b}^{*-}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{b}^{*-}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{b}^{*-}_{\sigma(i)}\right)^{2\varpi_i}}},\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{b}^{*+}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{b}^{*+}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{b}^{*+}_{\sigma(i)}\right)^{2\varpi_i}}}\end{array}\right],\\ \dfrac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\check{\delta}^{*}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\check{\delta}^{*}_{\sigma(i)}\right)^2\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\check{\delta}^{*}_{\sigma(i)}\right)^{2\varpi_i}}}\right)\end{array}\right\} \tag{14}$$

Proposition 4.5.3 *Let* $\text{Ş}_{c_i} =< \check{\alpha}_{\text{Ş}_{c_i}}, \check{\eta}_{\text{Ş}_{c_i}}, \check{\beta}_{\text{Ş}_{c_i}} >$ *be a collection and the weights of* Ş_{c_i} *be* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$ *and* $\breve{\omega}_i > 0$, *then we established thecharacteristics as below.*

Idempotency: For every $\text{Ş}_{c_i} =< \check{\alpha}_{\text{Ş}_{c_i}}, \check{\eta}_{\text{Ş}_{c_i}}, \check{\beta}_{\text{Ş}_{c_i}} >$ are same i.e $\text{Ş}_{c_i} = \text{Ş}_c$ then

$$SCFHHA_{\breve{\omega}}(\text{Ş}_{c_1}, \text{Ş}_{c_2}, ..., \text{Ş}_{c_n}) = \text{Ş}_c.$$

Boundary:

For every $\breve{\omega}$,

$$\text{Ş}^{-}_{c_i} \leq SCFHHA_{\omega,\breve{\omega}}(\text{Ş}_{c_1}, \text{Ş}_{c_2}, ..., \text{Ş}_{c_n}) \leq \text{Ş}^{+}_{c_i}.$$

Monotonicity: Let

$$\text{Ş}^{*}_{c} = \{([\check{a}^{*-}_{\tilde{\sigma}(i)}, \check{a}^{*+}_{\tilde{\sigma}(i)}], \check{\lambda}^{*}_{\tilde{\sigma}(i)}), ([\check{n}^{*-}_{\tilde{\sigma}(i)}, \check{n}^{*+}_{\tilde{\sigma}(i)}], \check{\mu}^{*}_{\tilde{\sigma}(i)}), ([\check{b}^{*-}_{\tilde{\sigma}(i)}, \check{b}^{*+}_{\tilde{\sigma}(i)}], \check{\delta}^{*}_{\tilde{\sigma}(i)})\}$$

then

$$SCFHHA_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq SCFHHA_{\breve{\omega}}(\S^*_{c_1}, \S^*_{2}, ..., \S_{c_n}).$$

4.6 SCFHWG aggregation operator

We will now define some geometric aggregation operators based on Hamacher spherical cubic operations.

Definition 4.6.1 *Let $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ be a SCFN in X and let the SCFHWG operator is a mapping of $\Phi^n \rightarrow \Phi$ such that*

$$SCFHWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \bigotimes_{i=1}^{n} \breve{\omega}_i \S_{c_i},$$

and the weights represented by $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ with $\sum_{i=1}^{n} \breve{\omega}_i = 1$. Now we get the following results related to SCFNs and Hamacher operators.

Theorem 4.6.2 *Let $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ be a SCFN in X then the aggregation value of them using the SCFHWG operator is also SCFN and is defined as follows:*

$$SCFHWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) =$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\breve{a}_i^{-})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{a}_i^{-})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\breve{a}_i^{-}\right)^{2\varpi_i}}}, \\ \frac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\breve{a}_i^{+})^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{a}_i^{+})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\breve{a}_i^{+}\right)^{2\varpi_i}}}\end{array}\right], \\ \frac{\sqrt{\hat{f}}\prod\limits_{i=1}^{n}(\breve{\lambda}_i)^{\breve{\omega}_i}}{\sqrt{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{\lambda}_i)^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(\breve{\lambda}_i\right)^{2\varpi_i}}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{n}_i^{-}\right)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\check{n}_i^{-}\right)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{n}_i^{-}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\check{n}_i^{-}\right)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{n}_i^{+}\right)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\check{n}_i^{+}\right)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\check{n}_i^{+}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\check{n}_i^{+}\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right], \\ \sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{\delta}_i\right)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\breve{\delta}_i\right)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{\delta}_i\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\breve{\delta}_i\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}_i^{-}\right)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\breve{b}_i^{-}\right)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}_i^{-}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\breve{b}_i^{-}\right)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}_i^{+}\right)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-\left(\breve{b}_i^{+}\right)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}_i^{+}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-\left(\check{n}_i^{+}\right)^2\right)^{\breve{\omega}_i}}}\end{array}\right], \\ \sqrt{\frac{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\mu}_i)^2\right)^{\breve{\omega}_i}-\prod\limits_{i=1}^{n}\left(1-(\breve{\mu}_i)^2\right)^{\breve{\omega}_i}}{\prod\limits_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\mu}_i)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod\limits_{i=1}^{n}\left(1-(\breve{\mu}_i)^2\right)^{\breve{\omega}_i}}}\end{array}\right)\end{array}\right\} \tag{15}$$

Proof. We will neglect the proof here due to similarity. ■

4.7 SCFHOWG aggregation operator

The concept of the SCFHOWG operator and its basic features will now be discussed.

Definition 4.7.1 *Let* $\S_{c_i} =< \breve{\alpha}_{\S_{c_i}}, \breve{\eta}_{\S_{c_i}}, \breve{\beta}_{\S_{c_i}} >$ *be a collection of SCFNs in* $\underset{\sim}{X}$*. Then the SCFHOWG operator is mapping* $SCFHOWG : \Phi^n \rightarrow \Phi$ *and the weights* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum\limits_{i=1}^{n} \breve{\omega}_i = 1$ *defined as follows:*

$$SCFHOWG_{\breve{\omega}}(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \overset{n}{\underset{i=1}{\otimes}} \breve{\omega}_i S_{\sigma(i)}$$

for all i, $S_{\sigma(i-1)} \geq S_{\sigma(i)}$ *and the permutation* $(\sigma_1, \sigma_2, ..., \sigma_n)$ *so we get the following result related to SCFNs and Hamacher operator.*

Theorem 4.7.2 *Let* $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ *be a collection. Then the aggregation of SCFHOWG is also a SCFN and it is defined as follows:*

$$SCFHOWG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) =$$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} \left[\begin{array}{c} \frac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{a}^{-}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{a}^{-}_{\sigma(i)})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{a}^{-}_{\sigma(i)}\right)^{2\varpi_i}}}, \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{a}^{+}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{a}^{+}_{\sigma(i)})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{a}^{+}_{\sigma(i)}\right)^{2\varpi_i}}} \end{array} \right], \\ \frac{\sqrt{\hat{f}}\prod_{i=1}^{n}(\breve{\lambda}_{\sigma(i)})^{\breve{\omega}_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\left(1-(\breve{\lambda}_{\sigma(i)})^2\right)\right)\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(\breve{\lambda}_{\sigma(i)}\right)^{2\varpi_i}}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{n}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{n}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{n}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{n}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{n}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{n}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{n}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{n}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}} \end{array} \right], \\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\delta}_{\sigma(i)})^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-(\breve{\delta}_{\sigma(i)})^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\delta}_{\sigma(i)})^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-(\breve{\delta}_{\sigma(i)})^2\right)^{\breve{\omega}_i}}} \end{array} \right), \\ \left(\begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{b}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{b}^{-}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}}, \\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-\left(\breve{b}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)\left(\breve{b}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-\left(\breve{b}^{+}_{\sigma(i)}\right)^2\right)^{\breve{\omega}_i}}} \end{array} \right], \\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\mu}_{\sigma(i)})^2\right)^{\breve{\omega}_i}-\prod_{i=1}^{n}\left(1-(\breve{\mu}_{\sigma(i)})^2\right)^{\breve{\omega}_i}}{\prod_{i=1}^{n}\left(1+(\hat{f}-1)(\breve{\mu}_{\sigma(i)})^2\right)^{\breve{\omega}_i}+(\hat{f}-1)\prod_{i=1}^{n}\left(1-(\breve{\mu}_{\sigma(i)})^2\right)^{\breve{\omega}_i}}} \end{array} \right) \end{array} \right\} \quad (16)$$

Here the weight vector is $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ under the specified conditions $\sum_{i=1}^{n}\breve{\omega}_i = 1$ and $\breve{\omega}_i > 0$.

Proposition 4.7.3 *Let* $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ *be a collection and the weights of* $\c{S}_{c_i}$ *be* $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ *with* $\sum_{i=1}^{\hat{n}}\breve{\omega}_i = 1$, *then we established the characteristics as below.*

Idempotency: For every $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ $(i = 1, 2, ..., n)$ are same i.e, $\c{S}_{c_i} = S$ then

$$SCFHOWG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) = S.$$

Boundary: For every $\breve{\omega}$,

$$\c{S}_c^- \leq SCFHOWG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) \leq \c{S}_c^+.$$

Monotonicity: Let

$$S^* = \{([\breve{a}^{*-}_{\tilde{\sigma}(i)}, \breve{a}^{*+}_{\tilde{\sigma}(i)}], \breve{\lambda}^*_{\tilde{\sigma}(i)}), ([\breve{n}^{*-}_{\tilde{\sigma}(i)}, \breve{n}^{*+}_{\tilde{\sigma}(i)}], \breve{\delta}^*_{\tilde{\sigma}(i)}), ([\breve{b}^{*-}_{\tilde{\sigma}(i)}, \breve{b}^{*+}_{\tilde{\sigma}(i)}], \breve{\mu}^*_{\tilde{\sigma}(i)})\}$$

then

$$SCFHOWG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) \leq SCFHOWG_{\breve{\omega}}(\c{S}^*_{c_1}, \c{S}^*_{c_2}, ..., \c{S}^*_{c_n}).$$

4.8 SCFHHG aggregation operator

Now we will discuss SCFHHG operator and further discuss their basic properties.

Definition 4.8.1 *Let $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ be a collection of SCFNs in $\c{X}$. Then the SCFHHG operator is mapping $SCFHHG : \Phi^n \to \Phi$ and the weight vector $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ under the specified conditions $\sum_{i=1}^{n} \breve{\omega}_i = 1$ and $\breve{\omega}_i \in [0, 1]$ defined as*

$$SCFHHG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) = \overset{n}{\underset{i=1}{\oplus}} \breve{\omega}_i \c{S}^*_{c_{\sigma(i)}}$$

where $\c{S}^*_{c_{\sigma(i)}}$ represented the i^{th} highest weight SCFNs $\c{S}_{c_i}$.

$$\c{S}^*_{c_{\sigma(i)}} = mw_i \c{S}_{c_{\sigma(i)}} = \left\{([\breve{a}^{*-}_{\sigma(i)}, \breve{a}^{*+}_{\sigma(i)}], \breve{\lambda}^*_{\sigma(i)}), ([\breve{n}^{*-}_{\sigma(i)}, \breve{n}^{*+}_{\sigma(i)}], \breve{\delta}^*_{\sigma(i)}), ([\breve{b}^{*-}_{\sigma(i)}, \breve{b}^{*+}_{\sigma(i)}], \breve{\mu}^*_{\sigma(i)})\right\}$$

Theorem 4.8.2 *Let $\c{S}_{c_i} =< \breve{\alpha}_{\c{S}_{c_i}}, \breve{\eta}_{\c{S}_{c_i}}, \breve{\beta}_{\c{S}_{c_i}} >$ be a collection of SCFNs in $\c{X}$. Then the aggregation of SCFHHG is also a SCFN and it is defined as follows:*

$$SCFHHG_{\omega,\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) =$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\frac{\sqrt{\hat f}\prod_{i=1}^{n}(\breve a^{*-}_{\sigma(i)})^{\breve\omega_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\left(1-(\breve a^{*-}_{\sigma(i)})^2\right)\right)\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(\breve a^{*-}_{\sigma(i)}\right)^{2\varpi_i}}},\\ \frac{\sqrt{\hat f}\prod_{i=1}^{n}(\breve a^{*+}_{\sigma(i)})^{\breve\omega_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\left(1-(\breve a^{*+}_{\sigma(i)})^2\right)\right)\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(\breve a^{*+}_{\sigma(i)}\right)^{2\varpi_i}}}\end{array}\right],\\ \frac{\sqrt{\hat f}\prod_{i=1}^{n}(\check\lambda^{*}_{\sigma(i)})^{\breve\omega_i}}{\sqrt{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\left(1-(\check\lambda^{*}_{\sigma(i)})^2\right)\right)\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(\check\lambda^{*}_{\sigma(i)}\right)^{2\varpi_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check n^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check n^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check n^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check n^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}},\\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check n^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check n^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check n^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check n^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}}\end{array}\right],\\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check\delta^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check\delta^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check\delta^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check\delta^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check b^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check b^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check b^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check b^{*-}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}},\\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check b^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check b^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check b^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check b^{*+}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}}\end{array}\right],\\ \sqrt{\frac{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check\mu^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}-\prod_{i=1}^{n}\left(1-\left(\check\mu^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}{\prod_{i=1}^{n}\left(1+(\hat f-1)\left(\check\mu^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}+(\hat f-1)\prod_{i=1}^{n}\left(1-\left(\check\mu^{*}_{\sigma(i)}\right)^2\right)^{\breve\omega_i}}}\end{array}\right)\end{array}\right\} \quad (17)$$

Proposition 4.8.3 *Let* $\text{Ş}_{c_i} =< \breve\alpha_{\text{Ş}_{c_i}}, \breve\eta_{\text{Ş}_{c_i}}, \breve\beta_{\text{Ş}_{c_i}} >$ *be a collection then we established the characteristics as below.*

Idempotency: For every $\text{Ş}_{c_i} =< \breve\alpha_{\text{Ş}_{c_i}}, \breve\eta_{\text{Ş}_{c_i}}, \breve\beta_{\text{Ş}_{c_i}} >$ are same i.e $\text{Ş}_{c_i} = \text{Ş}_c$ so, $SCFHHG_{\breve\omega}(\text{Ş}_{c_1}, \text{Ş}_{c_2}, ..., \text{Ş}_{c_n}) = \text{Ş}_c$.

Boundary:

For every $\breve\omega$,

$$\text{Ş}_c^- \leq SCFHHG_{\omega,\breve\omega}(\text{Ş}_{c_1}, \text{Ş}_{c_2}, ..., \text{Ş}_{c_n}) \leq \text{Ş}_c^+.$$

Monotonicity: Let

$$\text{Ş}_c^* = \left\{([\breve a^{*-}_{\sigma(i)}, \breve a^{*+}_{\sigma(i)}], \check\lambda^*_{\sigma(i)}), ([\check n^{*-}_{\sigma(i)}, \check n^{*+}_{\sigma(i)}], \check\delta^*_{\sigma(i)}), ([\check b^{*-}_{\sigma(i)}, \check b^{*+}_{\sigma(i)}], \check\mu^*_{\sigma(i)})\right\}$$

for collection of SCFN if $[\check{a}^{-}_{\breve{\sigma}(i)}, \check{a}^{+}_{\breve{\sigma}(i)}] \leq [\check{a}^{*-}_{\breve{\sigma}(i)}, \check{a}^{*+}_{\breve{\sigma}(i)}]$, $\check{\lambda}_i \leq \check{\lambda}^{*}_{\breve{\sigma}(i)}$; $[\check{n}^{-}_{\breve{\sigma}(i)}, \check{n}^{+}_{\breve{\sigma}(i)}] \leq [\check{n}^{*-}_{\breve{\sigma}(i)}, \check{n}^{*+}_{\breve{\sigma}(i)}]$, $\check{\mu}_i \leq \check{\mu}^{*}_{\breve{\sigma}(i)}$; $[\check{b}^{-}_{\breve{\sigma}(i)}, \check{b}^{+}_{\breve{\sigma}(i)}] \leq [\check{b}^{*-}_{\breve{\sigma}(i)}, \check{b}^{*+}_{\breve{\sigma}(i)}]$, $\check{\delta}_i \leq \check{\delta}^{*}_{\breve{\sigma}(i)}$.So,

$$SCFHHG_{\breve{\omega}}(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) \leq SCFHHG_{\breve{\omega}}(\c{S}^{*}_{c_1}, \c{S}^{*}_{c_2}, ..., \c{S}^{*}_{c_n}).$$

4.9 Model of MCGDM based on using spherical cubic Hamacher aggregation operators

In this section, SCFHWA operator are used to MCGDM method. Assume that there are m alternatives $\c{A}= \{\c{A}_1, \c{A}_2, ..., \c{A}_m\}$ and n criteria $\c{C}= \{\c{C}_1, \c{C}_2, ..., \c{C}_n\}$ be assessed with the weights $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_n)^T$ with $\sum_{i=1}^{n} \breve{\omega}_i = 1$. To assess the achievement based on the criteria C_p of the alternative $\c{A}_i$, the decision maker need to give not just the statistics about the alternative $\c{A}_i$, not fulfilling the criteria C_p .The ratings of alternatives $\c{A}_i$ on criteria $\c{C}_p$ given by choice producer be SCFNs in $\c{X}$. $\c{S}_{c_{ip}} = \left\langle \breve{\alpha}_{\c{S}_{c_{ip}}}, \check{\eta}_{\c{S}_{c_{ip}}}, \check{\beta}_{\c{S}_{c_{ip}}} \right\rangle$

Step 1: Settle on MCDM decision matrix $D = (\c{S}_{c_{ip}})_{m\times n} = \left(\left\langle \breve{\alpha}_{\c{S}_{c_{ip}}}, \check{\eta}_{\c{S}_{c_{ip}}}, \check{\beta}_{\c{S}_{c_{ip}}} \right\rangle\right)_{m\times n}$ normally, the cost and benefit criteria are assigned. If the decision matrix contains both cost and benefit criteria, there is no need to normalize the rating values because all of the criteria are of the same type. However, if the decision matrix contains both cost and benefit criteria, the rating estimations of the cost type can be changed over to the rating estimations of the benefit type using the following normalized formula.

$$\c{S}_{ip} =< \c{c}_{ip}, \c{d}_{ip} >= \left\{ \begin{array}{c} v_{ip}, \text{ if the criterion is for benefit} \\ v^{c}_{ip}, \text{ if the criterion is for cost} \end{array} \right\}$$

Here v^{c}_{ip} represents the complement of v_{ip}. Along these lines we get the normalization of SCF decision matrix, which is represented by D^m and is given by $D^m = (\hat{S}_{ip})_{m\times n} = (<\c{c}_{ip},\c{d}_{ip} >)_{m\times n}$.

Next we will apply the SCFHWA, SCFHOWA and SCFHHA operators to MCDM , which further requires the following steps.

Step 2: Utilize the recommended aggregation operators to find the SCFNs $\c{S}_{c_i}$ for the alternatives A_i. The developed operators were designed to keep the system from becoming too reliant on general preferences of the alternatives A_i here the weight vector of criteria is $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, ..., \breve{\omega}_m)^T$.

Step 3: Utilizing the score functions of SCFNs we find the score $sc(\c{S}_{c_i})$.

Step 4: To choose the best option, rank all of the alternatives.

4.10 Numerical Analysis

Because of these challenges, the enterprise's long-term stability has been affected. As a result of the development of creation, natural pollution, low-quality creation, asset mismanagement, and the lack of security of the premiums of the representatives, investors lose interest in contributing wealth and the propensity to make speculative investments in the organisation. The key administration slowly understood it is little disapproved of conduct for enterprises on the off chance that they need to accomplish the objective on investor esteem in the creation of procedure, paying little head to the enthusiasm of different partners necessities. The entire dynamic procedure is presented by a stream graph in the following Figure 7. The investment company takes a decision relying upon the following four characteristics.

C_1:Financial Execution

C_2:Customer Execution

C_3:Internal Procedures Execution

C_4:Staff Execution

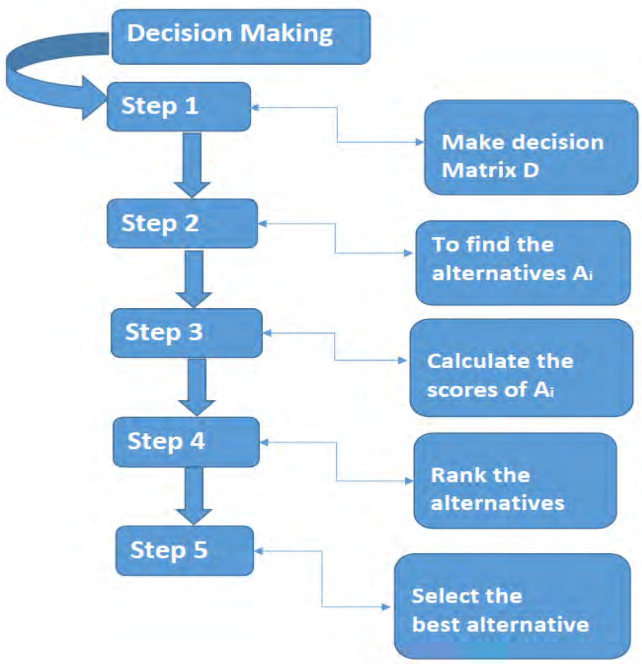

Figure 7 (Flow chart of algorithm)

To avoid dominant one another, decision makers must omit the four prospective enterprises A_i from the considered qualities whose weight vector $(.2, .3, .5)^T$ and criteria weighting vector $(.1, .2, .3, .4)^T$ the decision maker presented from Tables 9, 10, and 11.

	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix} ([0.2,0.4],0.1), \\ ([0.2,0.1],0.4), \\ ([0.2,0.1],0.5) \end{pmatrix}$	$\begin{pmatrix} ([0.5,0.2],0.6), \\ ([0.3,0.4],0.1), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.5],0.2), \\ ([0.1,0.2],0.5), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.4), \\ ([0.3,0.2],0.3), \\ ([0.1,0.2],0.2) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.2,0.4],0.1), \\ ([0.2,0.1],0.2), \\ ([0.2,0.3],0.5) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.4), \\ ([0.3,0.2],0.2), \\ ([0.4,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.5,0.1],0.6), \\ ([0.1,0.2],0.1), \\ ([0.1,0.3],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.2],0.2), \\ ([0.1,0.5],0.4) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.1,0.2],0.2), \\ ([0.1,0.4],0.1), \\ ([0.3,0.2],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.2), \\ ([0.1,0.2],0.2), \\ ([0.1,0.6],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.4), \\ ([0.1,0.2],0.4), \\ ([0.1,0.4],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.5],0.4), \\ ([0.2,0.1],0.3), \\ ([0.1,0.3],0.1) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.1,0.4],0.1), \\ ([0.1,0.3],0.2), \\ ([0.1,0.2],0.5) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.3), \\ ([0.1,0.2],0.4), \\ ([0.1,0.3],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.1), \\ ([0.2,0.1],0.2), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.3],0.4), \\ ([0.1,0.2],0.2), \\ ([0.1,0.3],0.2) \end{pmatrix}$

Table 9 (First decision maker data)

	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix} ([0.1,0.5],0.4), \\ ([0.1,0.2],0.1), \\ ([0.2,0.3],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.6],0.3), \\ ([0.1,0.2],0.3), \\ ([0.1,0.2],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.5],0.4), \\ ([0.1,0.2],0.3), \\ ([0.1,0.2],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.5],0.4), \\ ([0.1,0.2],0.2), \\ ([0.1,0.2],0.1) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.1,0.5],0.1), \\ ([0.1,0.3],0.2), \\ ([0.2,0.1],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.1],0.4), \\ ([0.5,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.4), \\ ([0.2,0.1],0.2), \\ ([0.3,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.3],0.4), \\ ([0.1,0.2],0.3), \\ ([0.2,0.1],0.1) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.4,0.2],0.5), \\ ([0.1,0.2],0.4), \\ ([0.1,0.3],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.1), \\ ([0.1,0.2],0.2), \\ ([0.3,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.3],0.5), \\ ([0.2,0.4],0.4), \\ ([0.1,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.1), \\ ([0.2,0.3],0.2), \\ ([0.3,0.4],0.3) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.1,0.3],0.5), \\ ([0.3,0.4],0.4), \\ ([0.2,0.1],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.1), \\ ([0.1,0.2],0.4), \\ ([0.3,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.2), \\ ([0.2,0.4],0.3), \\ ([0.3,0.1],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.5), \\ ([0.1,0.3],0.2), \\ ([0.1,0.2],0.2) \end{pmatrix}$

Table 10 (Second decision maker data)

	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix} ([0.1,0.3],0.2), \\ ([0.2,0.3],0.1), \\ ([0.1,0.4],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.4],0.1), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.4],0.3), \\ ([0.3,0.2],0.2), \\ ([0.1,0.4],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.1],0.2), \\ ([0.1,0.4],0.3) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.1,0.2],0.5), \\ ([0.3,0.1],0.2), \\ ([0.2,0.4],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.4), \\ ([0.3,0.1],0.3), \\ ([0.2,0.4],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.4), \\ ([0.3,0.2],0.1), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.4), \\ ([0.4,0.1],0.2), \\ ([0.2,0.3],0.3) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.5,0.1],0.6), \\ ([0.2,0.1],0.2), \\ ([0.1,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.3],0.4), \\ ([0.4,0.5],0.2), \\ ([0.2,0.1],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.4],0.2), \\ ([0.2,0.1],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.3,0.4],0.5), \\ ([0.4,0.3],0.2) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.1,0.2],0.3), \\ ([0.2,0.3],0.4), \\ ([0.4,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.3],0.4), \\ ([0.2,0.1],0.3), \\ ([0.5,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.2), \\ ([0.3,0.4],0.5), \\ ([0.1,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.2],0.2), \\ ([0.3,0.4],0.2), \\ ([0.4,0.1],0.5) \end{pmatrix}$

Table 11 (Third decision maker data)

By Spherical cubic fuzzy Hamacher weighted average operator

Step 1: The data of decision makers provided in the Table 9, 10 and 11. No need to normalize the data. Assume $\hat{f} = 2$, and weight vector $\breve{\omega} = (0.2, 0.3, 0.5)^T$, using SCFHWA operator, the aggregated information of the data in Table 9, 10 and 11 of all the decision makers are represented in Table 12. By using SCFHWA operator defined in Equation 12, taking $\hat{f} = 2$, utilizing the aggregate data in Table 12, the weights $\breve{\omega} = (.1, .2, .3, .4)^T$, we get the alternatives.

$$A_1 = (([.18, .4], .34), ([.07, .06], .07), ([.3, .12], .1)$$

$$A_2 = (([.19, .22], .4), ([.1, .04], .08), ([.06, .08], .08)$$

$$A_3 = (([.17, .26], .35), ([.08, .14], .13), ([.07, .09], .05)$$

$$A_4 = (([.11, .29], .31), ([.06, .12], .13), ([.07, .05], .1)$$

Step 2: Using the Equation 1, of score function $score(A_i)$ of A_i the calculated scores are as below:

$$\check{S}_c(A_1) = .08,\ \check{S}_c(A_2) = .1,\ \check{S}_c(A_3) = .1,\ \check{S}_c(A_4) = .07.$$

Step 3: Rank all the scores and choose the best alternative.

$$\underset{\sim}{A}_3 = \underset{\sim}{A}_2 > \underset{\sim}{A}_1 > \underset{\sim}{A}_4.$$

Step 4: $\underset{\sim}{A}_3$ is the best choice.

C_1	C_2	C_3	C_4
$\begin{pmatrix} ([0.13,0.39],0.27), \\ ([0.11,0.15],0.08), \\ ([0.09,0.22],0.13) \end{pmatrix}$	$\begin{pmatrix} ([0.26,0.37],0.38), \\ ([0.16,0.27],0.09), \\ ([0.06,0.14],0.28) \end{pmatrix}$	$\begin{pmatrix} ([0.22,0.45],0.32), \\ ([0.12,0.11],0.21), \\ ([0.06,0.23],0.21) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.36],0.35), \\ ([0.16,0.09],0.16), \\ ([0.06,0.23],0.14) \end{pmatrix}$
$\begin{pmatrix} ([0.13,0.36],0.36), \\ ([0.14,0.09],0.14), \\ ([0.14,0.19],0.15) \end{pmatrix}$	$\begin{pmatrix} ([0.13,0.18],0.37), \\ ([0.24,0.07],0.28), \\ ([0.25,0.14],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.27,0.14],0.45), \\ ([0.15,0.11],0.09), \\ ([0.09,0.12],0.17) \end{pmatrix}$	$\begin{pmatrix} ([0.14,0.23],0.38), \\ ([0.19,0.09],0.17), \\ ([0.12,0.18],0.17) \end{pmatrix}$
$\begin{pmatrix} ([0.42,0.16],0.52), \\ ([0.09,0.11],0.16), \\ ([0.08,0.17],0.08) \end{pmatrix}$	$\begin{pmatrix} ([0.13,0.31],0.3), \\ ([0.14,0.26],0.14), \\ ([0.14,0.12],0.23) \end{pmatrix}$	$\begin{pmatrix} ([0.13,0.22],0.39), \\ ([0.15,0.3],0.23), \\ ([0.09,0.11],0.06) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.29],0.28), \\ ([0.18,0.22],0.3), \\ ([0.22,0.28],0.14) \end{pmatrix}$
$\begin{pmatrix} ([0.1,0.28],0.35), \\ ([0.14,0.28],0.3), \\ ([0.19,0.07],0.14) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.35],0.32), \\ ([0.09,0.09],0.3), \\ ([0.26,0.18],0.09) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.25],0.18), \\ ([0.18,0.25],0.31), \\ ([0.09,0.11],0.13) \end{pmatrix}$	$\begin{pmatrix} ([0.13,0.29],0.36), \\ ([0.12,0.27],0.14), \\ ([0.14,0.1],0.26) \end{pmatrix}$

Table 12 (Aggregation of all decision makers data)

By spherical cubic fuzzy Hamacher ordered weighted average operator

Step 1: The aggregated data of three decision makers provided in Table 12. Using SCFHOWA operator defined in Equation 13, taking $\hat{f} = 2$, utilizing the aggregated data in Table 12, and the weight vector is $\breve{\omega} = (.1, .2, .3, .4)^T$, we get the collective SCFN of the alternatives.

$$\underset{\sim}{A}_1 = (([.16, .39], .32), ([.06, .06], .06), ([.03, .12], .1)$$

$$\underset{\sim}{A}_2 = (([.15, .26], .38), ([.1, .04], .09), ([.07, .09], .07)$$

$$\underset{\sim}{A}_3 = (([.22, .23], .34), ([.06, .13], .1), ([.07, .09], .05)$$

$$\underset{\sim}{A}_4 = (([.11, .3], .29), ([.06, .1], .13), ([.1, .04], .13)$$

Step 2: Using the Equation 1, of score function $\ddot{S}_c(\underset{\sim}{A}_i)$ the calculated scores are as follows.

$$\ddot{S}_c(\underset{\sim}{A}_1) = .05,\ \ddot{S}_c(\underset{\sim}{A}_2) = .06,\ \ddot{S}_c(\underset{\sim}{A}_3) = .06,\ \ddot{S}_c(\underset{\sim}{A}_4) = .03.$$

Step 3: Rank all the scores and choose the best alternative.

$$\text{\c{A}}_3 = \text{\c{A}}_2 > \text{\c{A}}_1 > \text{\c{A}}_4.$$

Step 4: $\text{\c{A}}_3$ is the best choice.

C_1	C_2	C_3	C_4
$([0.26, 0.37], 0.38)$, $([0.16, 0.27], 0.09)$, $([0.06, 0.14], 0.28)$	$([0.2, 0.45], 0.32)$, $([0.1, 0.14], 0.18)$, $([0.06, 0.23], 0.16)$	$([0.14, 0.39], 0.32)$, $([0.12, 0.14], 0.12)$, $([0.06, 0.23], 0.09)$	$([0.13, 0.36], 0.31)$, $([0.14, 0.08], 0.13)$, $([0.09, 0.022], 0.28)$
$([0.27, 0.14], 0.43)$, $([0.18, 0.11], 0.1)$, $([0.11, 0.12], 0.14)$	$([0.13, 0.31], 0.4)$, $([0.16, 0.11], 0.16)$, $([0.17, 0.14], 0.13)$	$([0.1, 0.2], 0.38)$, $([0.21, 0.07], 0.22)$, $([0.14, 0.22], 0.13)$	$([0.16, 0.28], 0.36)$, $([0.17, 0.08], 0.17)$, $([0.14, 0.16], 0.18)$
$([0.1, 0.35], 0.43)$, $([0.23, 0.29], 0.21)$, $([0.09, 0.1], 0.06)$	$([0.43, 0.14], 0.54)$, $([0.09, 0.09], 0.23)$, $([0.06, 0.2], 0.06)$	$([0.1, 0.28], 0.24)$, $([0.12, 0.27], 0.22)$, $([0.3, 0.18], 0.2)$	$([0.13, 0.18], 0.24)$, $([0.15, 0.27], 0.14)$, $([0.14, 0.16], 0.13)$
$([0.1, 0.28], 0.31)$, $([0.18, 0.34], 0.36)$, $([0.08, 0.11], 0.09)$	$([0.13, 0.29], 0.39)$, $([0.09, 0.22], 0.23)$, $([0.14, 0.1], 0.14)$	$([0.1, 0.3], 0.33)$, $([0.12, 0.12], 0.26)$, $([0.26, 0.08], 0.21)$	$([0.1, 0.32], 0.16)$, $([0.14, 0.19], 0.19)$, $([0.22, 0.09], 0.35)$

Table 13 (Aggregation of all decision makers data by SCFHOWA operator)

By spherical cubic fuzzy Hamacher Hybird aggregation operator

The decision making data is given in the Table 9, 10 and 11. Applied $\text{\c{S}}_{c_i} = m\omega_i \hat{S}_{c_i}$ to the data given in the Table 9 to 11, by using the weight vector $\breve{\omega} = (.2, .3, .5)^T$ of all the alternatives $\text{\c{A}}_i$. The aggregated data with the weight $\breve{\omega} = (.2, .4, .4)$ by using SCFHHA operator defined in Equation 14, is given in Table 13.

Step 1: Using $\text{\c{S}}_{c_i} = m\omega_i \hat{S}_{c_i}$to the given data in Table 13, using the weights are $\breve{\omega} = (.1, .2, .3, .4)^T$,the calculated values are given in the Table 14. Again using SCFHHA operator and weights are $\breve{\omega} = (.15, .25, .31, .29)^T$,we get the collective alterantives.

$$\text{\c{A}}_1 = (([.18, .31], .36), ([.04, .03], .03), ([.03, .05], .03)$$

$$\text{\c{A}}_2 = (([.16, .2], .37), ([.05, .03], .04), ([.04, .04], .04)$$

$$\text{\c{A}}_3 = (([.18, .24], .33), ([.04, .05], .05), ([.03, .04], .03)$$

$$\text{\c{A}}_4 = (([.1, .28], .27), ([.03, .04], .05), ([.04, .03], .04)$$

Step 2 : Using the Definition 1 of score function $\ddot{S}_c(A_i)$, the calculated scores are as below.

$$\ddot{S}_c(A_1) = .06,\ \ddot{S}_c(A_2) = .08,\ \ddot{S}_c(A_3) = .08,\ \ddot{S}_c(A_4) = .05.$$

Step 3: Rank all the scores and choose the best alternative.

$$A_3 = A_2 > A_1 > A_4.$$

Step 4: A_3 is the best choice.

C_1	C_2	C_3	C_4
([0.33, 0.21], 0.49) ([0.11, 0.04], 0.03) ([0.02, 0.05], 0.05)	([0.13, 0.42], 0.44) ([0.01, 0.01], 0.01) ([0.01, 0.02], 0.01)	([0.19, 0.3], 0.33) ([0.16, 0.09], 0.11) ([0.11, 0.16], 0.07)	([0.08, 0.25], 0.16) ([0.32, 0.35], 0.29) ([0.3, 0.38], 0.33)
([0.16, 0.29], 0.47) ([0.02, 0.01], 0.02) ([0.01, 0.02], 0.02)	([0.26, 0.14], 0.48) ([0.04, 0.03], 0.03) ([0.03, 0.03], 0.04)	([0.11, 0.16], 0.33) ([0.15, 0.08], 0.16) ([0.15, 0.12], 0.09)	([0.08, 0.22], 0.2) ([0.34, 0.3], 0.34) ([0.34, 0.37], 0.34)
([0.13, 0.23], 0.42) ([0.04, 0.07], 0.05) ([0.03, 0.03], 0.02)	([0.13, 0.33], 0.33) ([0.02, 0.02], 0.03) ([0.02, 0.02], 0.01)	([0.26, 0.1], 0.33) ([0.31, 0.31], 0.35) ([0.29, 0.35], 0.29)	([0.11, 0.26], 0.25) ([0.12, 0.16], 0.11) ([0.11, 0.1], 0.11)
([0.15, 0.35], 0.42) ([0.01, 0.02], 0.01) ([0.01, 0.01], 0.02)	([0.06, 0.17], 0.21) ([0.34, 0.39], 0.4) ([0.37, 0.28], 0.34)	([0.09, 0.32], 0.26) ([0.09, 0.09], 0.16) ([0.16, 0.09], 0.13)	([0.11, 0.26], 0.2) ([0.05, 0.06], 0.07) ([0.03, 0.03], 0.04)

Table 14 (Aggregation of all decision makers data by SCFHHWA operator)

By Spherical cubic fuzzy Hamacher weighted geometric operator

Step 1: The decision makers are provided in the Table 9, 10 and 11. No need to normalize the data. Assume $\hat{f} = 2$, and weights are $\breve{\omega} = (.2, .3, .5)^T$, using SCFHWG operator defined in Equation 15, the aggregated information of the data in Table 9, 10 and 11 of all the decision makers and represented in Table 12. By using SCFHWG operator, taking $\hat{f} = 2$, utilizing the aggregate data in Table 12, the weighting are as $\breve{\omega} = (.1, .2, .3, .4)^T$, we get the alternatives.

$$A_1 = (([.04, .19], .16), ([.24, .22], .25), ([.11, .3], .27)$$

$$A_2 = ((([.04, .06], .23), ([.29, .16], .24), ([.23, .28], .26)$$

$$A_3 = (([.03, .08], .13), ([.25, .34], .33), ([.25, .29], .2)$$

$$Ą_4 = (([.03, .12], .1), ([.22, .32], .32), ([.29, .18], .32)$$

Step 2: Using the Equation 1of score function $\H{S}_c(Ą_i)$ the calculated scores are as below:

$$\H{S}_c(Ą_1) = .06,\ \H{S}_c(Ą_2) = .08,\ \H{S}_c(Ą_3) = .08,\ \H{S}_c(Ą_4) = .05.$$

Step 3: Rank all the scores and choose the best alternative.

$$Ą_3 = Ą_2 > Ą_1 > Ą_4.$$

Step 4: A_3 is the best choice.

By SCFHOWG operator

Step 1: The aggregated data of three decison makers is given in Table 12. Using SCFHOWG operator defined in Equation 16, taking $\hat{f} = 2$, utilizing the aggregated data in Table 12, and the weight vector is $\breve{\omega} = (.1, .2, .3, .4)^T$, we get the collective SCFN for the alternatives.

$$Ą_1 = (([.03, .2], .13), ([.23, .21], .24), ([.13, .31], .28)$$

$$Ą_2 = (([.03, .08], .18), ([.28, .15], .25), ([.23, .3], .28)$$

$$Ą_3 = (([.04, .07], .34), ([.06, .13], .1), ([.07, .09], .05)$$

$$Ą_4 = (([.03, .13], .09), ([.21, .29], .32), ([.33, .17], .33)$$

Step 2: Using the Equation 1 of score function $\H{S}_c(Ą_i)$ of $Ą_i$ the calculated scores are as below.

$$\H{S}_c(Ą_1) = .02,\ \H{S}_c(Ą_2) = .03,\ \H{S}_c(Ą_3) = .03,\ \H{S}_c(Ą_4) = .01.$$

Step 3: Rank all the scores and choose the best alternative.

$$Ą_3 = Ą_2 > Ą_1 > Ą_4.$$

Step 4: $Ą_3$ is the best choice.

By spherical cubic fuzzy Hamacher Hybird Aggregation operator

The decision making data is given in the Table 9, 10 and 11. Applied $Ş_{c_i} = m\omega_i\hat{S}_{c_i}$ to the data given in the Table 9 to Table 11, by using the weighting $\breve{\omega} = (.2, .3, .5)^T$

of all the alternatives A_i. The aggregated data with the weight vector $\breve{\omega} = (.2, .4, .4)$ by using SCFHHG operator defined in Equation 17, is presented in Table 13.

Step 1: By using, $Ş_{c_i} = m\omega_i \hat{S}_{c_i}$ to the given data in Table 13, using the weight vector as $\breve{\omega} = (.1, .2, .3, .4)^T$,the calculated values are given in the Table 14. Again using SCFHHG operator and weight vector $\breve{\omega} = (.15, .25, .31, .29)^T$,we get the collective alterantives $Ą_i$ as below.

$$Ą_1 = (([.07, .19], .2), ([.2, .2], .17), ([.17, .22], .18)$$

$$Ą_2 = (([.06, .1], .23), ([.2, .17], .2), ([.2, .21], .19)$$

$$Ą_3 = (([.08, .11], .21), ([.19, .2], .21), ([.17, .2], .17)$$

$$Ą_4 = (([.04, .16], .15), ([.18, .2], .22), ([.21, .15], .19)$$

Step 2 : Using the Definition of score function $\H{S}_c(Ą_i)$ of $Ą_i$ the calculated score are as follows.

$$\H{S}_c(Ą_1) = .03,\ \H{S}_c(Ą_2) = .04,\ \H{S}_c(Ą_3) = .04,\ \H{S}_c(Ą_4) = .01.$$

Step 3: Rank all the scores and choose the best alternative.

$$Ą_3 = Ą_2 > Ą_1 > Ą_4.$$

Step 4: $Ą_3$ consider as best one.

It is clear from the comparision Analysis and Ranking by using the score function that $Ą_3$ has larger score. The graphical results shown in Figure 8.

4.11 Comparison Analysis

We disscused numerical problem, as SCFN is most propelled structure so it is not feasible for the currents fuzzy aggregation operators to resolve the data contained in said issue, which shows that the curent aggregation operators have constarined methodology. In any case, that we think about any issue under the TSFS and ICFS data we can understand it effectively by adding over the information as neutral membership in ICFS. So SCFS is more general idea than TSFS and gives more accurate results than TSFS. In this manner SCFHA operator are all the most impressive to determine the eccentric issues.

Operators	$\breve{S}_c(A_1)$	$\breve{S}_c(A_2)$	$\breve{S}_c(A_3)$	$\breve{S}_c(A_4)$	Ranking
SCFWA operator	.08	.1	.1	.07	$A_3 = A_2 > A_1 > A_4$
SCFOWA operator	.05	.06	0.06	.03	$A_3 = A_2 > A_1 > A_4$
SCFHHA operator	.06	.08	.08	.05	$A_3 = A_2 > A_1 > A_4$
SCFWG operator	.02	.05	.05	.01	$A_3 = A_2 > A_1 > A_4$
SCFOWG operator	.02	.03	.03	.01	$A_3 = A_2 > A_1 > A_4$
SCFHHG operator	.03	.04	.04	.01	$A_3 = A_2 > A_1 > A_4$

Table 15 (Comparison of all alternatives)

Operators	$\breve{S}_c(A_1)$	$\breve{S}_c(A_2)$	$\breve{S}_c(A_3)$	$\breve{S}_c(A_4)$
SCFWA operator	.03	.08	.06	.09
SCFOWA operator	.02	.07	.06	.08
SCFHHA operator	.05	.09	.08	.10
SCFWG operator	.01	.08	.05	.14
SCFOWG operator	.01	.03	.02	.09
SCFHHG operator	.03	.05	.04	.07

Table 16 Comparison Anaysis

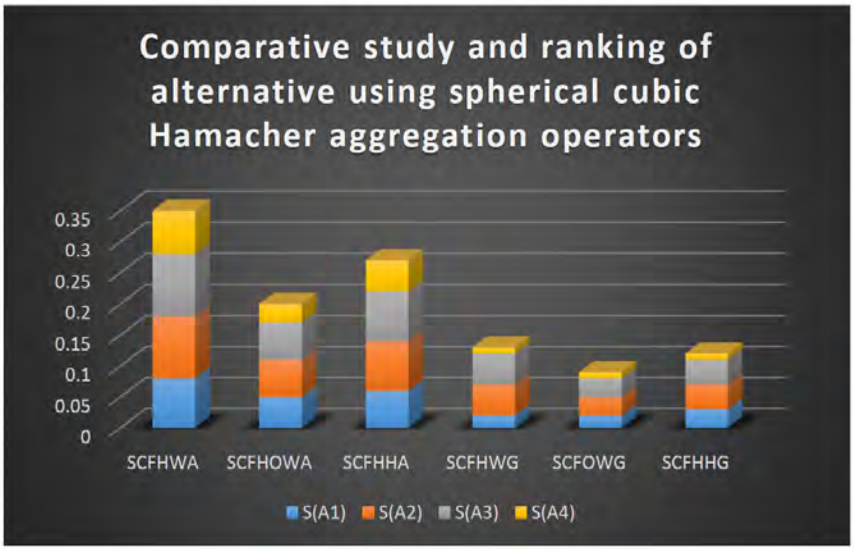

Figure 8 (Comparison study and ranking of alternatives. (SCFHWA) (SCFHOWA) , (SCFHHA), (SCFHWG), (SCFHOWG), (SCFHHG))

In Section 4.10, we look at the numeric problems, instead of using the score of SCFNs, we have use the score function of cubic fuzzy numbers by taking the membership, neutral and non-membership function of SCFN as individual CFNs, i.e, consider SCFN as the group three CFNs $C_1 = \langle[\check{a}^-, \check{a}^+], \check{\lambda}\rangle$, $C_2 = \langle[\check{n}^-, \check{n}^+], \check{\delta}\rangle$, $C_3 = \langle[\check{b}^-, \check{b}^+], \check{\mu}\rangle$ and then calculate the score function indiviually by the score function of CFNs,and then calculate the average by the formula $= \frac{1}{3}(sc(C_1) + sc(C_2) + sc(C_3))$. The ranking result of the alternative given in the Table 15 and 16, and graphically results shown in Figure 9, we get the same outcome get from Table 15, which obtained from TSFN scoring function i.e, A_3 is the best choice in all the alternatives as shown in Figure 8 and 9. The calculation by utilizing TSFS for MCGDM issues has a few implements and can't deal with the issues under the same uncertain circumstances, so their proposed calculations may not deliver the precise outcomes in spite the fact that SCFHWA operators do not have such impediments and in this manner can give progressively precise outcomes. The results demonstrate that the proposed system is continuously capable of dealing with the defenselessness of dynamic appraisal given by the decision-maker. For the given solution of enterprise in the certified enterprises in the determination problem, I can make an objection ranking result and suit conditions in which decision-makers show constrained insight.

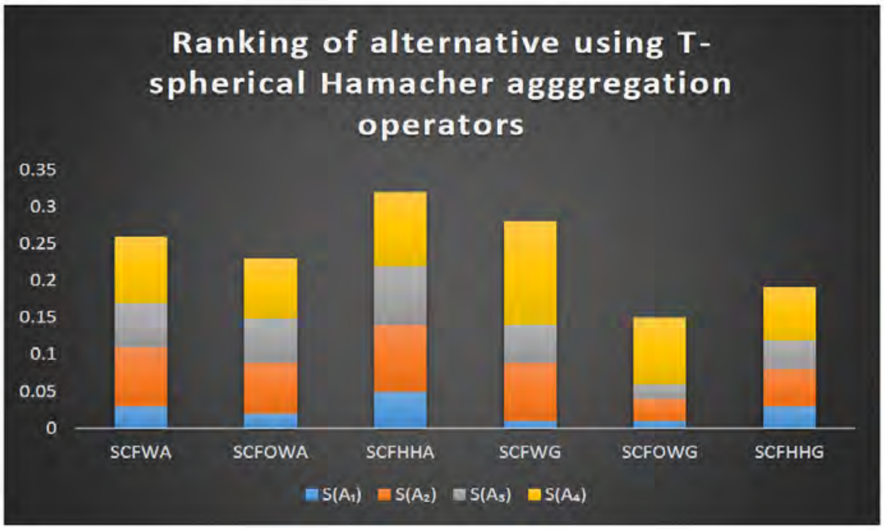

Figure 9 (Comparison analysis (TSFHWA) (TSFHOWA) , (TSFHHA), (TSFHWG), (TSFHOWG) and (TSFHHG))

4.12 Conclusion

Enterprises are the significant factor of customers, stocker holders, government, creditors, and other stakeholders. In the production of enterprises, two attributes should be under consideration. Monetary and society, subsequently we should think about the entirely of partner's advantage in the execution of enterprise assessing time. We set up a presentation assessing framework based on partner benefits. We utilize the concept of spherical cubic fuzzy Hamacher weighted average (SCFHWA) operator, spherical cubic fuzzy Hamacher ordered weight average (SCFHOWA), spherical cubic fuzzy Hamacher hybrid average (SCFHHA) operator to assess the best enterprise based on performance.

Chapter 5

Application in decision-making using spherical cubic Dombi aggregation operators

In this Chapter, we use Dombi $\tilde{t}$-norm and conorms which characterize various novel procedures in order to come at the optimum choice criteria. We proposed the SCFDWA, SCFDOWA, SCFDHWA, SCFDHWG, SCFDOWG and SCFDHWG operators. These previously stated operators are extremely helpful in successfully arranging selection difficulties. Then, using the spherical cubic fuzzy set, a computation is developed, and this methodology is used to decision-making problems to illustrate its importance and usefulness. We have demonstrated that pre-defined method is appropriate and provides decision makers with expanding numerical data before making decisions on their options throughout the calculation. A comparison study with different approaches is also being carried out to highlight the benefits of our methodology. The findings indicate that the proposed technique is both rational and effective in the given situation.

5.1 Spherical cubic fuzzy Dombi aggregated operators

In this chapter, we introduced the aggregate Dombi operations. We provides several discussions on the applications of the approach suggested. The proposed Dombi aggregate operations are analyzed. We will define the SCFDWA operator, SCFDOWA operator, SCFDWG operator, SCFDOWG operator, and SCFDHWG operator.

Definition 5.1.1 *Let $\S_{c_1}$ and $\S_{c_2}$ be two SCFNs in $\c{X}$ and $\gamma \geq 0$. Following are the Dombi operations by using the spherical cubic fuzzy sets:*

$$1.\ \breve{S}_{c_1} \oplus_D \breve{S}_{c_2} = \left\{ \begin{array}{c} \left\langle \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\breve{a}_1^-\right)^2}{1-\left(\breve{a}_1^-\right)^2} \right)^\gamma + \left(\frac{\left(\breve{a}_2^-\right)^2}{1-\left(\breve{a}_2^-\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{ \left(\frac{\left(\breve{a}_1^+\right)^2}{1-\left(\breve{a}_1^+\right)^2} \right)^\gamma + \left(\frac{\left(\breve{a}_2^+\right)^2}{1-\left(\breve{a}_2^+\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \right\rangle, \\ \sqrt{1 - \frac{1}{1+\left\{ \left(\frac{(\breve{\lambda}_1)^2}{1-(\breve{\lambda}_1)^2} \right)^\gamma + \left(\frac{(\breve{\lambda}_2)^2}{1-(\breve{\lambda}_2)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \\ \left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\breve{n}_1^-\right)^2}{\left(\breve{n}_1^-\right)^2} \right)^\gamma + \left(\frac{1-\left(\breve{n}_2^-\right)^2}{\left(\breve{n}_2^-\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\breve{n}_1^+\right)^2}{\left(\breve{n}_1^+\right)^2} \right)^\gamma + \left(\frac{1-\left(\breve{n}_2^+\right)^2}{\left(\breve{n}_2^+\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \right\rangle, \\ \sqrt{\frac{1}{1+\left\{ \left(\frac{1-(\breve{\delta}_1)^2}{(\breve{\delta}_1)^2} \right)^\gamma + \left(\frac{1-(\breve{\delta}_2)^2}{(\breve{\delta}_2)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \\ \left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\breve{b}_1^-\right)^2}{\left(\breve{b}_1^-\right)^2} \right)^\gamma + \left(\frac{1-\left(\breve{b}_2^-\right)^2}{\left(\breve{b}_2^-\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{ \left(\frac{1-\left(\breve{b}_1^+\right)^2}{\left(\breve{b}_1^+\right)^2} \right)^\gamma + \left(\frac{1-\left(\breve{b}_2^+\right)^2}{\left(\breve{b}_2^+\right)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \right\rangle, \\ \sqrt{\frac{1}{1+\left\{ \left(\frac{1-(\breve{\mu}_1)^2}{(\breve{\mu}_1)^2} \right)^\gamma + \left(\frac{1-(\breve{\mu}_2)^2}{(\breve{\mu}_2)^2} \right)^\gamma \right\}^{\frac{1}{\gamma}}}} \end{array} \right\}$$

$$\textit{2.}\ \ \text{Ş}_{c_1}\otimes_D \text{Ş}_{c_2} = \left\{ \begin{array}{c}
\left\langle \left[\begin{array}{c} \sqrt{\dfrac{1}{1+\left\{\left(\frac{1-\left(\check{a}_1^-\right)^2}{\left(\check{a}_1^-\right)^2}\right)^{\gamma}+\left(\frac{1-\left(\check{a}_2^-\right)^2}{\left(\check{a}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\dfrac{1}{1+\left\{\left(\frac{1-\left(\check{a}_1^+\right)^2}{\left(\check{a}_1^+\right)^2}\right)^{\gamma}+\left(\frac{1-\left(\check{a}_2^+\right)^2}{\left(\check{a}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \begin{array}{c} \\ \\ \\ \end{array} \right\rangle, \\
\sqrt{\dfrac{1}{1+\left\{\left(\frac{1-\left(\check{\lambda}_1\right)^2}{\left(\check{\lambda}_1\right)^2}\right)^{\gamma}+\left(\frac{1-\left(\check{\lambda}_2\right)^2}{\left(\check{\lambda}_2\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\
\left\langle \left[\begin{array}{c} \sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{n}_1^-\right)^2}{1-\left(\check{n}_1^-\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{n}_2^-\right)^2}{1-\left(\check{n}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{n}_1^+\right)^2}{1-\left(\check{n}_1^+\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{n}_2^+\right)^2}{1-\left(\check{n}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \begin{array}{c} \\ \\ \\ \end{array} \right\rangle, \\
\sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{\delta}_1\right)^2}{1-\left(\check{\delta}_1\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{\delta}_2\right)^2}{1-\left(\check{\delta}_2\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\
\left\langle \left[\begin{array}{c} \sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{b}_1^-\right)^2}{1-\left(\check{b}_1^-\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{b}_2^-\right)^2}{1-\left(\check{b}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{b}_1^+\right)^2}{1-\left(\check{b}_1^+\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{b}_2^+\right)^2}{1-\left(\check{b}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \begin{array}{c} \\ \\ \\ \end{array} \right\rangle, \\
\sqrt{1-\dfrac{1}{1+\left\{\left(\frac{\left(\check{\mu}_1\right)^2}{1-\left(\check{\mu}_1\right)^2}\right)^{\gamma}+\left(\frac{\left(\check{\mu}_2\right)^2}{1-\left(\check{\mu}_2\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}
\end{array} \right\}$$

$$3.\ \rho \cdot_D \mathfrak{S}_{c_1} = \left\{ \begin{array}{c} \left\langle \begin{array}{c} \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{\rho\left(\frac{\left(\breve{a}_1^-\right)^2}{1-\left(\breve{a}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{\rho\left(\frac{\left(\breve{a}_1^+\right)^2}{1-\left(\breve{a}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{1 - \frac{1}{1+\left\{\rho\left(\frac{(\breve{\lambda}_1)^2}{1-(\breve{\lambda}_1)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{n}_1^-\right)^2}{\left(\breve{n}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{n}_1^+\right)^2}{\left(\breve{n}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-(\breve{\delta}_1)^2}{(\breve{\delta}_1)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{b}_1^-\right)^2}{\left(\breve{b}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{b}_1^+\right)^2}{\left(\breve{b}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-(\breve{\mu}_1)^2}{(\breve{\mu}_1)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \end{array} \right\}$$

4. $$(\mathcal{S}_{c_1})^{\cdot_D(\rho)} = \left\{ \begin{array}{c} \left\langle \left[\sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{a}_1^-\right)^2}{\left(\breve{a}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-\left(\breve{a}_1^+\right)^2}{\left(\breve{a}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{\frac{1}{1+\left\{\rho\left(\frac{1-(\breve{\lambda}_1)^2}{(\breve{\lambda}_1)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{\left(\breve{n}_1^-\right)^2}{1-\left(\breve{n}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{\left(\breve{n}_1^+\right)^2}{1-\left(\breve{n}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{(\breve{\delta}_1)^2}{1-(\breve{\delta}_1)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{\left(\breve{b}_1^-\right)^2}{1-\left(\breve{b}_1^-\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{\left(\breve{b}_1^+\right)^2}{1-\left(\breve{b}_1^+\right)^2}\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{\rho\left(\frac{(\breve{\mu}_1)^2}{1-(\breve{\mu}_1)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \end{array} \right\}$$

5.2 SCFDWA operator

We explain the following weighted average aggregated operators in light of defined Dombi operations of SCFNs.

Definition 5.2.1 *Let $\mathcal{S}_{c_i}$ be a collection of SCFNs in $\mathcal{X}$. The structure of SCFDWA operator is determined as below:*

$$SCFDWA(\mathcal{S}_{c_1}, \mathcal{S}_{c_2}, ..., \mathcal{S}_{c_n}) = \sum_{i=1}^{n} \breve{\omega}_i \mathcal{S}_{c_i}.$$

Theorem 5.2.2 *Let $\mathcal{S}_{c_i}$ be a collection of SCFNs in $\mathcal{X}$. The SCFDWA operator de-*

termined as follows,

$$SCFDWA\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right) = \tag{18}$$

$$\left\{\begin{array}{c}
\left\langle \left[\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{a}_i^-\right)^2}{1-\left(\breve{a}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{a}_i^+\right)^2}{1-\left(\breve{a}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right]; \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{(\breve{\lambda}_i)^2}{1-(\breve{\lambda}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle, \\
\left\langle \left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{n}_i^-\right)^2}{\left(\breve{n}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{n}_i^+\right)^2}{\left(\breve{n}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-(\breve{\delta}_i)^2}{(\breve{\delta}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle, \\
\left\langle \left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{b}_i^-\right)^2}{\left(\breve{b}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{b}_i^+\right)^2}{\left(\breve{b}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-(\breve{\mu}_i)^2}{(\breve{\mu}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle
\end{array}\right\}$$

Proof. We will prove it by mathematical induction, so Theorem 5.2.2 is true for $n = 2$.

$$SCFDWA(\S_{c_1} + \S_{c_2}) = \gamma_1 \S_{c_1} + \gamma_2 \S_{c_2}$$

$$
= \left\{ \begin{array}{c}
\left\langle \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{\gamma_1\left(\frac{\left(\breve{a}_1^-\right)^2}{1-\left(\breve{a}_1^-\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{\left(\breve{a}_2^-\right)^2}{1-\left(\breve{a}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{\gamma_1\left(\frac{\left(\breve{a}_1^+\right)^2}{1-\left(\breve{a}_1^+\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{\left(\breve{a}_2^+\right)^2}{1-\left(\breve{a}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{1 - \frac{1}{1+\left\{\gamma_1\left(\frac{(\breve{\lambda}_1)^2}{1-(\breve{\lambda}_1)^2}\right)^{\gamma}+\gamma_1\left(\frac{(\breve{\lambda}_2)^2}{1-(\breve{\lambda}_2)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\
\left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-\left(\breve{n}_1^-\right)^2}{\left(\breve{n}_1^-\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-\left(\breve{n}_2^-\right)^2}{\left(\breve{n}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-\left(\breve{n}_1^+\right)^2}{\left(\breve{n}_1^+\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-\left(\breve{n}_2^+\right)^2}{\left(\breve{n}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-(\breve{\delta}_1)^2}{(\breve{\delta}_1)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-(\breve{\delta}_2)^2}{(\breve{\delta}_2)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\
\left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-\left(\breve{b}_i^-\right)^2}{\left(\breve{b}_i^-\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-\left(\breve{b}_2^-\right)^2}{\left(\breve{b}_2^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-\left(\breve{b}_1^+\right)^2}{\left(\breve{b}_1^+\right)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-\left(\breve{b}_2^+\right)^2}{\left(\breve{b}_2^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \\ \sqrt{\frac{1}{1+\left\{\gamma_1\left(\frac{1-(\breve{\mu}_1)^2}{(\breve{\mu}_1)^2}\right)^{\gamma}+\gamma_2\left(\frac{1-(\breve{\mu}_2)^2}{(\breve{\mu}_2)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle,
\end{array} \right\}
$$

Now assume that Equation 18 is true for $n = k$.

Now prove for Equation 18 in which $n = k + 1$ i.e.,

$$SCFDWA\left(\S_{c_1},\S_{c_2},...,\S_{c_{k+1}}\right)=\sum_{i=1}^{k}\gamma_k\S_{c_k}+\gamma_{k+1}\S_{c_{k+1}}$$

$$SCFDWA\left(\S_{c_1},\S_{c_2},...,\S_{c_{k+1}}\right)$$

$$=\left\{\begin{array}{c}\left\langle\left[\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{\left(\breve{a}_i^-\right)^2}{1-\left(\breve{a}_i^-\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{\left(\breve{a}_i^+\right)^2}{1-\left(\breve{a}_i^+\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{(\breve{\lambda}_i)^2}{1-(\breve{\lambda}_i)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-\left(\breve{n}_i^-\right)^2}{\left(\breve{n}_i^-\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-\left(\breve{n}_i^+\right)^2}{\left(\breve{n}_i^+\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-(\breve{\delta}_i)^2}{(\breve{\delta}_i)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-\left(\breve{b}_i^-\right)^2}{\left(\breve{b}_i^-\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-\left(\breve{b}_i^+\right)^2}{\left(\breve{b}_i^+\right)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k}\breve{\omega}_i\left(\frac{1-(\breve{\mu}_i)^2}{(\breve{\mu}_i)^2}\right)^\gamma\right\}^{\frac{1}{\gamma}}}}\right\rangle,\end{array}\right\}$$

$$
\oplus \left\{
\begin{array}{c}
\left\langle \left[\begin{array}{c}
\sqrt{1-\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{\left(\breve{a}_{k+1}^{-}\right)^{2}}{1-\left(\breve{a}_{k+1}^{-}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\
\sqrt{1-\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{\left(\breve{a}_{k+1}^{+}\right)^{2}}{1-\left(\breve{a}_{k+1}^{+}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}
\end{array} \right]; \right\rangle, \\
\sqrt{1-\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{(\breve{\lambda}_{k+1})^{2}}{1-(\breve{\lambda}_{k+1})^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\
\left\langle \left[\begin{array}{c}
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-\left(\breve{n}_{k+1}^{-}\right)^{2}}{\left(\breve{n}_{k+1}^{-}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-\left(\breve{n}_{k+1}^{+}\right)^{2}}{\left(\breve{n}_{k+1}^{+}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}
\end{array} \right]; \right\rangle, \\
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-(\breve{\delta}_{k+1})^{2}}{(\breve{\delta}_{k+1})^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \\
\left\langle \left[\begin{array}{c}
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-\left(\breve{b}_{k+1}^{-}\right)^{2}}{\left(\breve{b}_{k+1}^{-}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-\left(\breve{b}_{k+1}^{+}\right)^{2}}{\left(\breve{b}_{k+1}^{+}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}
\end{array} \right]; \right\rangle, \\
\sqrt{\frac{1}{1+\left\{\gamma_{k+1}\left(\frac{1-(\breve{\mu}_{k+1})^{2}}{(\breve{\mu}_{k+1})^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}
\end{array}
\right\}
$$

$$
= \left\{ \begin{array}{c}
\left\langle \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{\left(\breve{a}_i^-\right)^2}{1-\left(\breve{a}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{\left(\breve{a}_i^+\right)^2}{1-\left(\breve{a}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{(\breve{\lambda}_i)^2}{1-(\breve{\lambda}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle, \\
\left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-\left(\breve{n}_i^-\right)^2}{\left(\breve{n}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-\left(\breve{n}_i^+\right)^2}{\left(\breve{n}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-(\breve{\delta}_i)^2}{(\breve{\delta}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle, \\
\left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-\left(\breve{b}_i^-\right)^2}{\left(\breve{b}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-\left(\breve{b}_i^+\right)^2}{\left(\breve{b}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{k+1} \breve{\omega}_i \left(\frac{1-(\breve{\mu}_i)^2}{(\breve{\mu}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle
\end{array} \right\}
$$

So by the mathematical induction, it is true for all n.

$$SCFDWA\left(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}\right)$$
$$= \left\{ \begin{array}{c} \left\langle \left[\begin{array}{c} \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\breve{a}_i^-\right)^2}{1-\left(\breve{a}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\breve{a}_i^+\right)^2}{1-\left(\breve{a}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{(\check{\lambda}_i)^2}{1-(\check{\lambda}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\check{n}_i^-\right)^2}{\left(\check{n}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\check{n}_i^+\right)^2}{\left(\check{n}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-(\check{\delta}_i)^2}{(\check{\delta}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\begin{array}{c} \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\check{b}_i^-\right)^2}{\left(\check{b}_i^-\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\check{b}_i^+\right)^2}{\left(\check{b}_i^+\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-(\check{\mu}_i)^2}{(\check{\mu}_i)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \end{array} \right\}$$

Proved. ■

Properties: The characteristics of SCFDWA are listed below.

1. **Idempotancy:** Let $\S_{c_i}$ be a collection in $\breve{X}$. Then the collection of SCFN's $\S_{c_i}$ are equal. i.e,
$$SCFDWA(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \S_c.$$

2. **Boundary:** Let $\S_{c_i}$ be a collection in $\breve{X}$.
$$\S_{c_i}^- = \left\langle \begin{array}{c} \left(\left[\min \breve{a}_i^-, \min \breve{a}_i^+\right], \min \check{\lambda}_i\right), \left(\left[\max \check{n}_i^-, \max \check{n}_i^+\right], \max \check{\delta}_i\right), \\ \left(\left[\max \check{b}_i^-, \max \check{b}_i^+\right], \max \check{\mu}_i\right) \end{array} \right\rangle$$
$$\S_{c_i}^+ = \left\langle \begin{array}{c} \left(\left[\max \breve{a}_i^-, \max \breve{a}_i^+\right], \max \check{\lambda}_i\right), \left(\left[\min \check{n}_i^-, \min \check{n}_i^+\right], \min \check{\delta}_i\right), \\ \left(\left[\min \check{b}_i^-, \min \check{b}_i^+\right], \min \check{\mu}_i\right) \end{array} \right\rangle$$

Thus
$$\S_{c_i}^- \leq SCFDWA(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq \S_{c_i}^+.$$

3. **Monotonicity:** Let $Ş_{c_i}$ be a collection in $\c{X}$. $Ş_{c_i} \subseteq Ş^{k}_{c_i}$ then

$$SCFDWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \subseteq SCFDWA\left(Ş^{k}_{c_1}, Ş^{k}_{c_2}, ..., Ş^{k}_{c_n}\right).$$

5.3 SCFDOWA operator

We explain the following ordered weighted averaging aggregated operators in light of defined Dombi operations of SCFNs.

Definition 5.3.1 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCDOWA operator is ddetermined as below:*

$$SCFDOWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = \sum_{i=1}^{n} \breve{\omega}_i Ş_{c_{\eta(i)}}.$$

Theorem 5.3.2 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCDOWA operator is deetermined as below, where $\gamma > 0$*

$$SCFDOWA\left(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}\right) =$$

$$\left\{\begin{array}{c}\left\langle\left[\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{a}^-_{\eta(i)}\right)^2}{1-\left(\breve{a}^-_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{a}^+_{\eta(i)}\right)^2}{1-\left(\breve{a}^+_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{\lambda}_{\eta(i)}\right)^2}{1-\left(\breve{\lambda}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{n}^-_{\eta(i)}\right)^2}{\left(\breve{n}^-_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{n}^+_{\eta(i)}\right)^2}{\left(\breve{n}^+_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{\delta}_{\eta(i)}\right)^2}{\left(\breve{\delta}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{b}^-_{\eta(i)}\right)^2}{\left(\breve{b}^-_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{b}^+_{\eta(i)}\right)^2}{\left(\breve{b}^+_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{\mu}_{\eta(i)}\right)^2}{\left(\breve{\mu}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle\end{array}\right\} \quad (19)$$

Proof. Similar to Theorem 5.2.2 proof. ■

Following are the properties of $SCFDOWA$.

1. **Idempotancy:** Let $\c{S}_{c_i}$ be a collection in $\c{X}$. Then we say collection of SCFN's $\c{S}_{c_i}$ are equal. i.e,
$$SCFDOWA(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) = \c{S}_{c_i}.$$

2. **Boundary:** Let $\c{S}_{c_i}$ be a collection in $\c{X}$.
$$\c{S}^-_{c_i} = \left\langle\begin{array}{c}\left(\left[\min\breve{a}^-_i, \min\breve{a}^+_i\right], \min\breve{\lambda}_i\right), \left(\left[\max\breve{n}^-_i, \max\breve{n}^+_i\right], \max\breve{\delta}_i\right),\\ \left(\left[\max\breve{b}^-_i, \max\breve{b}^+_i\right], \max\breve{\mu}_i\right)\end{array}\right\rangle$$
$$\c{S}^+_{c_i} = \left\langle\begin{array}{c}\left(\left[\max\breve{a}^-_i, \max\breve{a}^+_i\right], \max\breve{\lambda}_i\right), \left(\left[\min\breve{n}^-_i, \min\breve{n}^+_i\right], \min\breve{\delta}_i\right),\\ \left(\left[\min\breve{b}^-_i, \min\breve{b}^+_i\right], \min\breve{\mu}_i\right)\end{array}\right\rangle$$

Thus

$$Ş_{c_i}^{-} \leq SCFDOWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \leq Ş_{c_i}^{+}.$$

3. **Monotonicity:** Let $Ş_{c_i}$ be a collection of SCFNs in $X̧$. $Ş_{c_i} \subseteq Ş_{c_i}^{k}$ then

$$SCFDOWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \subseteq SCFDOWA\left(Ş_{c_1}^{k}, Ş_{c_2}^{k}, ..., Ş_{c_n}^{k}\right).$$

5.4 SCFDHWA operator

We define the main hybrid weighted averaging aggregated operators in light of defined Dombi operations of SCFNs.

Definition 5.4.1 *Let $Ş_{c_i}$ be a collection in $X̧$. The SCFDHWA operator is*

$$SCFDHWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = \sum_{i=1}^{n} \breve{\omega}_i Ş_{c_{\eta(i)}}^{*},$$

here $\breve{\omega}_i$ shows the weight vectors with $\sum_{i=1}^{n} \breve{\omega}_i = 1$ and $(i = 1, 2, ..., n)$ and i^{th} largest value is $Ş_{c_{\eta(i)}}^{*}$ $\left(Ş_{c_{\eta(i)}}^{*} = n\gamma_i Ş_{c_{\eta(i)}}, i \in N\right)$ and the total order $Ş_{c_{\eta(1)}}^{*} \geq Ş_{c_{\eta(2)}}^{*} \geq, ..., \geq Ş_{c_{\eta(n)}}^{*}$.

Theorem 5.4.2 *Let $Ş_{c_i}$ be a collection in $X̧$. The SCFDHWA operator is determined as below:*

$$SCFDHWA\left(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}\right) =$$

$$\left\{\begin{array}{c}\left\langle\left[\begin{array}{c}\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{\left(\check{a}_{\eta(i)}^{-}\right)^2}{1-\left(\check{a}_{\eta(i)}^{-}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{\left(\check{a}_{\eta(i)}^{+}\right)^2}{1-\left(\check{a}_{\eta(i)}^{+}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\ \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{\left(\check{\lambda}_{\eta(i)}\right)^2}{1-\left(\check{\lambda}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{1-\left(\check{n}_{\eta(i)}^{-}\right)^2}{\left(\check{n}_{\eta(i)}^{-}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{1-\left(\check{n}_{\eta(i)}^{+}\right)^2}{\left(\check{n}_{\eta(i)}^{+}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{1-\left(\check{\delta}_{\eta(i)}\right)^2}{\left(\check{\delta}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,\\ \left\langle\left[\begin{array}{c}\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\check{b}_{\eta(i)}^{-}\right)^2}{\left(\check{b}_{\eta(i)}^{-}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{1-\left(\check{b}_{\eta(i)}^{+}\right)^2}{\left(\check{b}_{\eta(i)}^{+}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right];\ \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}w_i^*\left(\frac{1-\left(\check{\mu}_{\eta(i)}\right)^2}{\left(\check{\mu}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,\end{array}\right\} \tag{20}$$

Proof. Same as proof of Theorem 5.2.2. ■

The characteristics of SCFDHWA are listed below.

1. **Idempotancy:** Let $Ş_{c_i}$ be a collection in $X̦$. Then we say collection of SCFN's $Ş_{c_i}$ are equal. i.e,
$$SCFDHWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = Ş_c.$$

2. **Boundary:** Let $Ş_{c_i}$ be a collection in $X̦$.

$$Ş_{c_i}^{-} = \left\{\begin{array}{c}\left(\left[\min\check{a}_i^-, \min\check{a}_i^+\right], \min\check{\lambda}_i\right), \left(\left[\max\check{n}_i^-, \max\check{n}_i^+\right], \max_i\check{\delta}_i\right),\\ \left(\left[\max\check{b}_i^-, \max\check{b}_i^+\right], \max\check{\mu}_i\right)\end{array}\right\}$$
$$Ş_{c_i}^{+} = \left\{\begin{array}{c}\left(\left[\max\check{a}_i^-, \max\check{a}_i^+\right], \max\check{\lambda}_i\right), \left(\left[\min\check{n}_i^-, \min\check{n}_i^+\right], \min\check{\delta}_i\right),\\ \left(\left[\min\check{b}_i^-, \min\check{b}_i^+\right], \min\check{\mu}_i\right)\end{array}\right\}$$

Thus
$$Ş_{c_i}^{-} \leq SCFDHWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \leq Ş_{c_i}^{+}.$$

3. **Monotonicity:** Let $Ş_{c_i}$ be a collection in $\c{X}$. $Ş_{c_i} \subseteq Ş^k_{c_i}$ then

$$SCFDHWA(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \subseteq SCFDHWA(Ş^k_{c_1}, Ş^k_{c_2}, ..., Ş^k_{c_n}).$$

5.5 SCFDWG operator

We define the main weighted geometric aggregated operators in light of characterized Dombi operations of SCFNs.

Definition 5.5.1 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCFDWG operator is determined as below:*

$$(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = \prod_{i=1}^{n} (Ş_{c_i})^{\breve{\omega}_i}.$$

Theorem 5.5.2 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCFDWG operator with $\gamma > 0$ is determined as below:*

$$SCFDWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) =$$

$$\left\{ \begin{array}{c} \left\langle \begin{array}{c} \left[\sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\breve{a}_i^-\right)^2}{\left(\breve{a}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-\left(\breve{a}_i^+\right)^2}{\left(\breve{a}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \\ \sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{1-(\check{\lambda}_i)^2}{(\check{\lambda}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\check{n}_i^-\right)^2}{1-\left(\check{n}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\check{n}_i^+\right)^2}{1-\left(\check{n}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \\ \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{(\check{\delta}_i)^2}{1-(\check{\delta}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \\ \left\langle \begin{array}{c} \left[\sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\check{b}_i^-\right)^2}{1-\left(\check{b}_i^-\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{\left(\check{b}_i^+\right)^2}{1-\left(\check{b}_i^+\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \\ \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} \breve{\omega}_i \left(\frac{(\check{\mu}_i)^2}{1-(\check{\mu}_i)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \end{array} \right\rangle, \end{array} \right\} \quad (21)$$

Proof. Proof is identical with Theorem 5.2.2. ■
Following are the properties of SCFDWG.

1. **Idempotancy:** Let $Ş_{c_i}$ be a collection in $\c{X}$. Then we say collection of SCFN's $Ş_{c_i}$ are equal i.e,

$$SCFDWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = Ş_c.$$

2. **Boundary:** Let $Ş_{c_i}$ be a collection in $\c{X}$.

$$Ş_{c_i}^{-} = \left\{ \begin{array}{c} \left(\left[\min \breve{a}_i^{-}, \min \breve{a}_i^{+}\right], \min \breve{\lambda}_i\right), \left(\left[\max \breve{n}_i^{-}, \max \breve{n}_i^{+}\right], \max \breve{\delta}_i\right), \\ \left(\left[\max \breve{b}_i^{-}, \max \breve{b}_i^{+}\right], \max \breve{\mu}_i\right) \end{array} \right\}$$

$$Ş_{c_i}^{+} = \left\{ \begin{array}{c} \left(\left[\max \breve{a}_i^{-}, \max \breve{a}_i^{+}\right], \max \breve{\lambda}_i\right), \left(\left[\min \breve{n}_i^{-}, \min \breve{n}_i^{+}\right], \min \breve{\delta}_i\right), \\ \left(\left[\min \breve{b}_i^{-}, \min \breve{b}_i^{+}\right], \min \breve{\mu}_i\right) \end{array} \right\}$$

Thus,

$$Ş_{c_i}^{-} \leq SCFDWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \leq Ş_{c_i}^{+}.$$

2. **Monotonicity:** Let $Ş_{c_i}$ be a collection in $\c{X}$. $Ş_{c_i} \subseteq Ş_{c_i}^{k}$ then

$$SCFDWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \subseteq SCFDWG\left(Ş_{c_1}^{k}, Ş_{c_2}^{k}, ..., Ş_{c_n}^{k}\right).$$

5.6 SCFDOWG operator

We explain the following ordered weighted geometric aggregated operators in light of characterized Dombi operations of SCFNs.

Definition 5.6.1 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCFDOWG operator is determined as below:*

$$SCFDOWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = \prod_{i=1}^{n} \left(Ş_{c_{\eta_{(i)}}}\right)^{\breve{\omega}_i}.$$

Theorem 5.6.2 *Let $Ş_{c_i}$ be a collection in $\c{X}$. The SCFDOWG operator is determined as below:*

$$SCFDOWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) =$$

$$\left\{\begin{array}{c} \left\langle \left[\sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{a}^{-}_{\eta(i)}\right)^2}{\left(\breve{a}^{-}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{a}^{+}_{\eta(i)}\right)^2}{\left(\breve{a}^{+}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{1-\left(\breve{\lambda}_{\eta(i)}\right)^2}{\left(\breve{\lambda}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{n}^{-}_{\eta(i)}\right)^2}{1-\left(\breve{n}^{-}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{n}^{+}_{\eta(i)}\right)^2}{1-\left(\breve{n}^{+}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{\delta}_{\eta(i)}\right)^2}{1-\left(\breve{\delta}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{b}^{-}_{\eta(i)}\right)^2}{1-\left(\breve{b}^{-}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{b}^{+}_{\eta(i)}\right)^2}{1-\left(\breve{b}^{+}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^{n}\breve{\omega}_i\left(\frac{\left(\breve{\mu}_{\eta(i)}\right)^2}{1-\left(\breve{\mu}_{\eta(i)}\right)^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \end{array}\right\} \tag{22}$$

here the weight vector is $\breve{\omega}_i$ *with* $\sum_{i=1}^{n}\breve{\omega}_i = 1$.

Proof. Proof is identical with Theorem 5.2.2. ■

The characteristics of SCFDOWG are listed below.

1. **Idempotancy:** Let $Ş_{c_i}$ be a collection in $\c{X}$. Then we say collection of SCFN's $Ş_{c_i}$ are equal i.e,
$$SCFDOWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) = Ş_c.$$

2. **Boundary:** Let $Ş_{c_i}$ be a collection in $\c{X}$.
$$Ş^{-}_{c_i} = \left\{\begin{array}{c} \left(\left[\min \breve{a}^{-}_i, \min \breve{a}^{+}_i\right], \min \breve{\lambda}_i\right), \left(\left[\max \breve{n}^{-}_i, \max \breve{n}^{+}_i\right], \max \breve{\delta}_i\right), \\ \left(\left[\max \breve{b}^{-}_i, \max \breve{b}^{+}_i\right], \max \breve{\mu}_i\right) \end{array}\right\}$$
$$Ş^{+}_{c_i} = \left\{\begin{array}{c} \left(\left[\max \breve{a}^{-}_i, \max \breve{a}^{+}_i\right], \max \breve{\lambda}_i\right), \left(\left[\min \breve{n}^{-}_i, \min \breve{n}^{+}_i\right], \min \breve{\delta}_i\right), \\ \left(\left[\min \breve{b}^{-}_i, \min \breve{b}^{+}_i\right], \min \breve{\mu}_i\right) \end{array}\right\}.$$

Thus

$$\S_{c_i}^{-} \leq SCFDOWG(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \leq \S_{c_i}^{+}.$$

2. **Monotonicity:** Let $\S_{c_i}$ be a collection in $\c{X}$. $\S_{c_i} \subseteq \S_{c_i}^{k}$ then

$$SCFDOWG(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) \subseteq SCFDOWG(\S_{c_1}^{k}, \S_{c_2}^{k}, ..., \S_{c_n}^{k}).$$

5.7 SCFDHWG operator

The following hybrid weighted geometric aggregated operators are described in light of characterised Dombi operations of SCFNs.

Definition 5.7.1 *Let $\S_{c_i}$ be a collection in $\c{X}$. The SCFDHWG operator is determined as below,*

$$(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) = \prod_{i=1}^{n} \left(\S_{c_i}^{*}\right)^{\breve{\omega}_i},$$

here $\breve{\omega}_i$ shows the weight vector with $\sum_{i=1}^{n} \breve{\omega}_i = 1$ and the i^{th} largest weight value is $\S_{c_{\eta(i)}}^{*}$ $\left(\S_{c_{\eta(i)}}^{*} = n\gamma_i \S_{c_{\eta(i)}}, i \in N\right)$ and the total order is $\S_{c_{\eta(1)}}^{*} \geq \S_{c_{\eta(2)}}^{*} \geq, ..., \geq \S_{c_{\eta(n)}}^{*}$ and weights with $\sum_{i=1}^{n} w_i^{*} = 1$, $w_i^{*} \geq 0$.

Theorem 5.7.2 *Let $\S_{c_i}$ be a collection in $\c{X}$. The SCFDHWG operator is determined as below:*

$$SCFDHWG(\S_{c_1}, \S_{c_2}, ..., \S_{c_n}) =$$

$$\left\{ \begin{array}{l} \left\langle \left[\sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{1-\left(\breve{a}^-_{\eta(i)}\right)^2}{\left(\breve{a}^-_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{1-\left(\breve{a}^+_{\eta(i)}\right)^2}{\left(\breve{a}^+_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{1-\left(\breve{\lambda}_{\eta(i)}\right)^2}{\left(\breve{\lambda}_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{n}^-_{\eta(i)}\right)^2}{1-\left(\breve{n}^-_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{n}^+_{\eta(i)}\right)^2}{1-\left(\breve{n}^+_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{\delta}_{\eta(i)}\right)^2}{1-\left(\breve{\delta}_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle, \\ \left\langle \left[\sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{b}^-_{\eta(i)}\right)^2}{1-\left(\breve{b}^-_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{b}^+_{\eta(i)}\right)^2}{1-\left(\breve{b}^+_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right]; \sqrt{1-\frac{1}{1+\left\{ \sum_{i=1}^{n} w_i^* \left(\frac{\left(\breve{\mu}_{\eta(i)}\right)^2}{1-\left(\breve{\mu}_{\eta(i)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle, \end{array} \right\} \quad (23)$$

where $\breve{\omega}_i$ *represents the weight vector with* $\sum_{i=1}^{n} \breve{\omega}_i = 1$ *and the* i^{th} *largest weight value is* $\c{S}^*_{c_{\eta(i)}} \left(\c{S}^*_{c_{\eta(i)}} = n\gamma_i \c{S}_{c_{\eta(i)}}, i \in N \right)$ *and the total order is* $\breve{S}_{\tilde{x}_{\eta(1)}} \geq \breve{S}_{\tilde{x}_{\eta(2)}} \geq, ..., \geq \breve{S}_{\tilde{x}_{\eta(n)}}$ *and* $\sum_{i=1}^{n} w_i^* = 1$, $w_i^* = (w_1^*, w_2^*, ..., w_n^*)$.

Proof. Proof is same as Theorem 5.2.2. ■

The characteristics of SCFDHWG are listed below.

1. **Idempotancy:** Let $\c{S}_{c_i}$ be a collection in $\c{X}$. Then we say collection of SCFN's $\c{S}_{c_i}$ are equal. i.e,

$$SCFDHWG(\c{S}_{c_1}, \c{S}_{c_2}, ..., \c{S}_{c_n}) = \c{S}_c.$$

2. **Boundary:** Let $\c{S}_{c_i}$ be a collection in $\c{X}$.

$$Ş_{c_i}^{-} = \left\{ \begin{array}{c} \left(\left[\min \breve{a}_i^{-}, \min \breve{a}_i^{+}\right], \min \check{\lambda}_i\right), \left(\left[\max \breve{n}_i^{-}, \max \breve{n}_i^{+}\right], \max \check{\delta}_i\right), \\ \left(\left[\max \breve{b}_i^{-}, \max \breve{b}_i^{+}\right], \max \breve{\mu}_i\right) \end{array} \right\}$$

$$Ş_{c_i}^{+} = \left\{ \begin{array}{c} \left(\left[\max \breve{a}_i^{-}, \max \breve{a}_i^{+}\right], \max \check{\lambda}_i\right), \left(\left[\min \breve{n}_i^{-}, \min \breve{n}_i^{+}\right], \min \check{\delta}_i\right), \\ \left(\left[\min \breve{b}_i^{-}, \min \breve{b}_i^{+}\right], \min \breve{\mu}_i\right) \end{array} \right\}$$

Thus,

$$Ş_{c_i}^{-} \leq SCFDHWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \leq Ş_{c_i}^{+}.$$

3. **Monotonicity:** Let $Ş_{c_i}$ be a collection in $Ẋ$. $Ş_{c_i} \subseteq Ş_{c_i}^{k}$ then

$$SCFDHWG(Ş_{c_1}, Ş_{c_2}, ..., Ş_{c_n}) \subseteq SCFDHWG(Ş_{c_1}^{k}, Ş_{c_2}^{k}, ..., Ş_{c_n}^{k}).$$

5.8 MAGDM computations with spherical cubic Dombi aggregated operators

We now introduce a new decision-making technique based on the concept of a spherical cubic fuzzy set. This strategy will use information that is specified by the given problem and will not handle with any additional data given by the decision makers in order to minimize the influence of information on decision outcomes.

Step 1: Make $D = (E_{ip})_{m\times n} = \left[\left(E_{ip}^{-(s)}, E_{ip}^{+(s)}\right), N_{ip}^{-(s)}\right]_{m\times n}$ as spherical cubic fuzzy decision matrix and then used the idea of SCFWA/SCFWG operator and the aggregated spherical cubic fuzzy value γ_i^k of the alternative A_i defined as: $.\vartheta_i^k = SCFWA_\gamma \left[\left(\vartheta_{ip}^{k}, \vartheta_{ip}^{\prime k}\right), \vartheta_{ip}^{\prime\prime -(s)}\right]_{m\times n}$ the weight vector is given as $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_n)$ we apply the SCFWA, SCFOWA, SCFWG, SCFOWG, SCFHWA, SCFHWG operations described as below.

Step 2: As a result, we use the spherical cubic fuzzy data to construct the desirable value of the alternatives with the weighting using the defined Dombi operators.

Step 3: The scoring and precision function are calculated.

Step 4: Using the scoring function definition, rank the options and choose the best option with the highest score function value.

5.9 Numeric Interpretation

Suppose Mr. A, a business administration at a wealth management corporation, is selecting between four investment options: $Ą_1$, $Ą_2$, $Ą_3$, and $Ą_4$. The financial administrator, according to the firm, must consider the following three aspects. $Ḃ_1$, $Ḃ_2$, $Ḃ_3$, where $ḅ_1$ indicates "high-risk", $ḅ_2$ indicates "progress" and $ḅ_3$ indicates "surrounding impacts". In light of $ḅ_i$, Mr. A has requested three expert groups to determine if $Ą_j$ is the best chance of a viable investment, and the evaluator has the weighting $(0.2, 0.3, 0.5)$.The spherical cubic fuzzy set will be established, and the decision matrices will be depicted in Tables 17, 18 and 19. Normalization of data given in Table 20, 21 and 22.

	$ƅ_1$	$ƅ_2$	$ƅ_3$
$Ą_1$	$\begin{pmatrix} ([0.3,0.4],0.5), \\ ([0.1,0.3],0.2), \\ ([0.1,0.2],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.4],0.3), \\ ([0.1,0.2],0.4), \\ ([0.2,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.5],0.4), \\ ([0.2,0.1],0.2), \\ ([0.2,0.1],0.4) \end{pmatrix}$
$Ą_2$	$\begin{pmatrix} ([0.2,0.6],0.1), \\ ([0.2,0.1],0.2), \\ ([0.1,0.2],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.1],0.1), \\ ([0.3,0.2],0.2), \\ ([0.2,0.4],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.3],0.4), \\ ([0.3,0.4],0.2), \\ ([0.1,0.3],0.2) \end{pmatrix}$
$Ą_3$	$\begin{pmatrix} ([0.4,0.4],0.2), \\ ([0.2,0.1],0.5), \\ ([0.1,0.4],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.5,0.1],0.2), \\ ([0.1,0.2],0.3), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.3],0.4), \\ ([0.1,0.2],0.3), \\ ([0.1,0.4],0.2) \end{pmatrix}$
$Ą_4$	$\begin{pmatrix} ([0.3,0.3],0.4), \\ ([0.3,0.4],0.3), \\ ([0.3,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.5,0.4],0.2), \\ ([0.1,0.2],0.4), \\ ([0.3,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.4],0.5), \\ ([0.2,0.1],0.3), \\ ([0.3,0.2],0.1) \end{pmatrix}$

Table 17 Investing capacity in a wealth administration firm D^1

	$ƅ_1$	$ƅ_2$	$ƅ_3$
$Ą_1$	$\begin{pmatrix} ([0.1,0.2],0.2), \\ ([0.2,0.3],0.5), \\ ([0.4,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.3],0.3), \\ ([0.4,0.1],0.1), \\ ([0.2,0.2],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.4],0.6), \\ ([0.1,0.3],0.2), \\ ([0.2,0.2],0.1) \end{pmatrix}$
$Ą_2$	$\begin{pmatrix} ([0.4,0.2],0.6), \\ ([0.3,0.1],0.1), \\ ([0.2,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.3],0.1), \\ ([0.2,0.1],0.2), \\ (0.4,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.5],0.8), \\ ([0.1,0.3],0.1), \\ ([0.1,0.2],0.1) \end{pmatrix}$
$Ą_3$	$\begin{pmatrix} ([0.2,0.4],0.3), \\ ([0.3,0.1],0.2), \\ ([0.3,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.4), \\ ([0.3,0.1],0.3), \\ ([0.1,0.2],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.5],0.4), \\ ([0.2,0.1],0.1), \\ ([0.3,0.2],0.3) \end{pmatrix}$
$Ą_4$	$\begin{pmatrix} ([0.3,0.4],0.3), \\ ([0.1,0.2],0.1), \\ ([0.3,0.1],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.6,0.1],0.6), \\ ([0.1,0.2],0.1), \\ ([0.2,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.1],0.2), \\ ([0.1,0.4],0.3), \\ ([0.1,0.2],0.3) \end{pmatrix}$

Table 18 Investing capacity in a wealth administration firm D^2

	$\mathfrak{b}_1$	$\mathfrak{b}_2$	$\mathfrak{b}_3$
$\underset{\sim}{A}_1$	$\left(\begin{array}{c}([0.4, 0.2], 0.3),\\ ([0.1, 0.2], 0.2),\\ ([0.4, 0.1], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.4], 0.3),\\ ([0.2, 0.4], 0.4),\\ ([0.2, 0.1], 0.2)\end{array}\right)$	$\left(\begin{array}{c}([0.1, 0.4], 0.6),\\ ([0.2, 0.3], 0.2),\\ ([0.1, 0.3], 0.1)\end{array}\right)$
$\underset{\sim}{A}_2$	$\left(\begin{array}{c}([0.2, 0.5], 0.3),\\ ([0.2, 0.1], 0.1),\\ ([0.1, 0.2], 0.4)\end{array}\right)$	$\left(\begin{array}{c}([0.1, 0.5], 0.4),\\ ([0.3, 0.2], 0.1),\\ ([0.2, 0.1], 0.3)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.2], 0.7),\\ ([0.4, 0.3], 0.1),\\ ([0.2.0.1], 0.2)\end{array}\right)$
$\underset{\sim}{A}_3$	$\left(\begin{array}{c}([0.5, 0.4], 0.6),\\ ([0.2, 0.3], 0.1),\\ ([\ 0.2, 0.1], 0.2)\end{array}\right)$	$\left(\begin{array}{c}([0.2, 0.3], 0.1),\\ ([0.3, 0.4], 0.1),\\ ([0.3, 0.1], 0.3)\end{array}\right)$	$\left(\begin{array}{c}([0.3, 0.1], 0.2),\\ ([0.3, 0.2], 0.4),\\ ([0.2, 0.1], 0.1)\end{array}\right)$
$\underset{\sim}{A}_4$	$\left(\begin{array}{c}([0.2, 0.5], 0.3),\\ ([0.3, 0.1], 0.3),\\ ([0.1, 0.1], 0.2)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.4], 0.4),\\ ([0.3, 0.2], 0.1),\\ ([0.1, 0.2], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.5, 0.3], 0.3),\\ ([0.2, 0.1], 0.2),\\ ([0.1, 0.2], 0.2)\end{array}\right)$

Table 19 (Investing capacity in a wealth administration firm D^3)

	$\mathfrak{b}_1$	$\mathfrak{b}_2$	$\mathfrak{b}_3$
$\underset{\sim}{A}_1$	$\left(\begin{array}{c}([0.5, 0.4], 0.3),\\ ([0.2, 0.3], 0.1),\\ ([0.2, 0.2], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.3, 0.4], 0.3),\\ ([0.4, 0.2], 0.1),\\ ([0.2, 0.1], 0.2)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.5], 0.4),\\ ([0.2, 0.1], 0.2),\\ ([0.4, 0.1], 0.2)\end{array}\right)$
$\underset{\sim}{A}_2$	$\left(\begin{array}{c}([0.1, 0.6], 0.2),\\ ([0.2, 0.1], 0.2),\\ ([0.4, 0.2], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.1], 0.3),\\ ([0.1, 0.2], 0.3),\\ ([0.3, 0.4], 0.2)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.3], 0.2),\\ ([0.2, 0.4], 0.3),\\ ([0.2, 0.3], 0.1)\end{array}\right)$
$\underset{\sim}{A}_3$	$\left(\begin{array}{c}([0.2, 0.4], 0.4),\\ ([0.5, 0.1], 0.2),\\ ([0.1, 0.4], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.1, 0.1], 0.5),\\ ([0.1, 0.2], 0.1),\\ ([0.3, 0.2], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.3], 0.2),\\ ([0.3, 0.2], 0.1),\\ ([0.2, 0.4], 0.1)\end{array}\right)$
$\underset{\sim}{A}_4$	$\left(\begin{array}{c}([0.4, 0.3], 0.3),\\ ([0.3, 0.4], 0.3),\\ ([0.2, 0.1], 0.3)\end{array}\right)$	$\left(\begin{array}{c}([0.4, 0.4], 0.5),\\ ([0.1, 0.2], 0.1),\\ ([0.1, 0.1], 0.3)\end{array}\right)$	$\left(\begin{array}{c}([0.5, 0.4], 0.2),\\ ([0.3, 0.1], 0.2),\\ ([0.1, 0.2], 0.3)\end{array}\right)$

Table 20 (Normalized investing capacity in a wealth administration R^1)

	$\underset{\sim}{b}_1$	$\underset{\sim}{b}_2$	$\underset{\sim}{b}_3$
$\underset{\sim}{A}_1$	$\begin{pmatrix} ([0.2,0.2],0.1), \\ ([0.5,0.3],0.2), \\ ([0.2,0.1],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.3],0.2), \\ ([0.3,0.1],0.4), \\ ([0.1,0.2],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.6,0.4],0.3), \\ ([0.2,0.3],0.1), \\ ([0.1,0.2],0.2) \end{pmatrix}$
$\underset{\sim}{A}_2$	$\begin{pmatrix} ([0.6,0.2],0.4), \\ ([0.1,0.1],0.3), \\ ([0.2,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.3],0.4), \\ ([0.2,0.1],0.2), \\ ([0.2,0.1],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.7,0.2],0.4), \\ ([0.1,0.3],0.4), \\ ([0.2,0.1],0.2) \end{pmatrix}$
$\underset{\sim}{A}_3$	$\begin{pmatrix} ([0.3,0.4],0.2), \\ ([0.2,0.1],0.3), \\ ([0.1,0.2],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.2],0.2), \\ ([0.3,0.1],0.3), \\ ([0.2,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.5],0.3), \\ ([0.1,0.1],0.2), \\ ([0.3,0.2],0.3) \end{pmatrix}$
$\underset{\sim}{A}_4$	$\begin{pmatrix} ([0.2,0.4],0.3), \\ ([0.1,0.2],0.1), \\ ([0.4,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.6,0.1],0.6), \\ ([0.1,0.2],0.1), \\ ([0.3,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.4), \\ ([0.3,0.4],0.1), \\ ([0.3,0.2],0.1) \end{pmatrix}$

Table 21 (Normalized investing capacity in a wealth administration R^2)

	$\underset{\sim}{b}_1$	$\underset{\sim}{b}_2$	$\underset{\sim}{b}_3$
$\underset{\sim}{A}_1$	$\begin{pmatrix} ([0.3,0.2],0.4), \\ ([0.2,0.2],0.1), \\ ([0.1,0.4],0.4) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.4],0.4), \\ ([0.4,0.4],0.2), \\ ([0.2,0.4],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.6,0.4],0.1), \\ ([0.2,0.3],0.2), \\ ([0.1,0.3],0.1) \end{pmatrix}$
$\underset{\sim}{A}_2$	$\begin{pmatrix} ([0.3,0.5],0.2), \\ ([0.1,0.1],0.2), \\ ([0.4,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.5],0.1), \\ ([0.1,0.5],0.3), \\ ([0.3,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.8,0.5],0.2), \\ ([0.1,0.3],0.1), \\ ([0.1,0.2],0.1) \end{pmatrix}$
$\underset{\sim}{A}_3$	$\begin{pmatrix} ([0.6,0.4],0.5), \\ ([0.1,0.3],0.2), \\ ([0.2,0.1],0.2) \end{pmatrix}$	$\begin{pmatrix} ([0.1,0.3],0.2), \\ ([0.1,0.4],0.3), \\ ([0.3,0.1],0.3) \end{pmatrix}$	$\begin{pmatrix} ([0.2,0.1],0.3), \\ ([0.4,0.2],0.3), \\ ([0.1,0.1],0.2) \end{pmatrix}$
$\underset{\sim}{A}_4$	$\begin{pmatrix} ([0.3,0.5],0.2), \\ ([0.3,0.1],0.3), \\ ([0.2,0.1],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.4,0.4],0.4), \\ ([0.1,0.2],0.3), \\ ([0.1,0.2],0.1) \end{pmatrix}$	$\begin{pmatrix} ([0.3,0.3],0.5), \\ ([0.2,0.1],0.2), \\ ([0.2,0.2],0.1) \end{pmatrix}$

Table 22 (Normalized investing capacity in a wealth administration R^3)

	b_1	b_2	b_3
A_1	$\left(\begin{array}{c}([0.2056, 0.1469], 0.1852),\\ ([0.1537, 0.158], 0.1),\\ ([0.1, 0.1948], 0.2505)\end{array}\right)$	$\left(\begin{array}{c}([0.2268, 0.2646], 0.1852),\\ ([0.2646, 0.1453], 0.1406),\\ ([0.1, 0.1948], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.4181, 0.3318], 0.1821),\\ ([0.1, 0.1569], 0.1),\\ ([0.1969, 0.1425], 0.1)\end{array}\right)$
A_2	$\left(\begin{array}{c}([0.2085, 0.2814], 0.1406),\\ ([0.1, 0.1], 0.124),\\ ([0.2567, 0.1], 0.1)\end{array}\right)$	$\left(\begin{array}{c}([0.2134, 0.1986], 0.1792),\\ ([0.1, 0.1602], 0.1613),\\ ([0.1755, 0.1969], 0.1885)\end{array}\right)$	$\left(\begin{array}{c}([0.4961, 0.2042], 0.1406),\\ ([0.1, 0.2305], 0.1792),\\ ([0.1, 0.1435], 0.1)\end{array}\right)$
A_3	$\left(\begin{array}{c}([0.2143, 0.3], 0.2353),\\ ([0.1625, 0.1266], 0.124),\\ ([0.1, 0.1969], 0.1393)\end{array}\right)$	$\left(\begin{array}{c}([0.1406, 0.1266], 0.625),\\ ([0.1240, 0.1453], 0.1569),\\ ([0.1755, 0.1], 0.1425)\end{array}\right)$	$\left(\begin{array}{c}([0.2065, 0.1959], 0.1569),\\ ([0.1852, 0.1], 0.1266),\\ ([0.1393, 0.1969], 0.1393)\end{array}\right)$
A_4	$\left(\begin{array}{c}([0.1859, 0.2871], 0.158),\\ ([0.1613, 0.1469], 0.1613),\\ ([0.1885, 0.1], 0.1729)\end{array}\right)$	$\left(\begin{array}{c}([0.3515, 0.2134], 0.3887),\\ ([0.1, 0.1], 0.1266),\\ ([0.1392, 0.1], 0.1435)\end{array}\right)$	$\left(\begin{array}{c}([0.2056, 0.1859], 0.2252),\\ ([0.158, 0.1406], 0.1),\\ ([0.1393, 0.1], 0.435)\end{array}\right)$

Table 23 (Aggregated spherical cubic fuzzy decision information matrix R^3)

A_1	$\left(\begin{array}{c}([0.2587, 0.213], 0.1856),\\ ([\ 0.9859, 0.988], 0.994),\\ ([0.9927, 0.983], 0.9816),\end{array}\right)$
A_2	$\left(\begin{array}{c}([0.2782, 0.25], 0.1494),\\ ([0.9949, 0.99], 0.9896),\\ ([0.9777, 0.992], 0.9929),\end{array}\right)$
A_3	$\left(\begin{array}{c}([0.1988, 0.246], 0.2057),\\ ([\ 0.9871, 0.992], 0.9912),\\ ([0.9922, 0.984], 0.99),\end{array}\right)$
A_4	$\left(\begin{array}{c}([0.2271, 0.253], 0.224),\\ ([0.9888, 0.991], 0.9898),\\ ([0.9856, 0.995], 0.9868),\end{array}\right)$

Table 24 (Aggregated SCFDWA data)

A_1	$\begin{pmatrix} ([0.4161, 0.373], 0.2874), \\ ([0.9636, 0.961], 0.9883), \\ ([0.9812, 0.964], 0.9774) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.4865, 0.321], 0.2467), \\ ([0.9918, 0.965], 0.972), \\ ([0.972, 0.976], 0.9849) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.3106, 0.344], 0.3407), \\ ([0.9798, 0.987], 0.9792), \\ ([0.9765, 0.967], 0.9781) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.3977, 0.274], 0.3784), \\ ([0.9858, 0.976], 0.9818), \\ ([0.9743, 0.989], 0.9749) \end{pmatrix}$

Table 25 (Aggregated SCFDOWA data)

A_1	$\begin{pmatrix} ([0.4954, 0.39], 0.2529), \\ ([0.9624, 0.958], 0.9867), \\ ([0.9817, 0.964], 0.975) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.5013, 0.342], 0.2631), \\ ([0.9899, 0.959], 0.9686), \\ ([0.9684, 0.971], 0.983) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.3303, 0.368], 0.3559), \\ ([0.9777, 0.985], 0.9761), \\ ([0.9738, 0.964], 0.9735) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.3905, 0.272], 0.3587), \\ ([0.9801, 0.981], 0.9828), \\ ([0.9699, 0.99], 0.9725) \end{pmatrix}$

Table 26 (Aggregated SCFDHWA data)

A_1	$\begin{pmatrix} ([0.9767, 0.9873], 0.989), \\ ([0.1672, 0.156], 0.109), \\ ([0.121, 0.1852], 0.1911) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.9714, 0.9773], 0.9936), \\ ([0.1006, 0.1407], 0.144), \\ ([0.2101, 0.13], 0.1192) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.9853, 0.9751], 0.9848), \\ ([0.1602, 0.1257], 0.1321), \\ ([0.1245, 0.1783], 0.1407) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.9881, 0.9774], 0.99), \\ ([0.149, 0.1368], 0.1425), \\ ([0.1693, 0.1006], 0.1618) \end{pmatrix}$

Table 27 (Aggregated SCFDWG data)

A_1	$\begin{pmatrix} ([0.9093, 0.9278], 0.9578), \\ ([0.2674, 0.275], 0.1525), \\ ([0.1929, 0.2675], 0.2112) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.8737, 0.9471], 0.9691), \\ ([0.1275, 0.2616], 0.2352), \\ ([0.2351, 0.2176], 0.1732) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.9505, 0.9389], 0.9402), \\ ([0.1999, 0.1618], 0.2029), \\ ([0.2154, 0.2539], 0.208) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.9175, 0.9619], 0.9257), \\ ([0.1678, 0.2178], 0.19), \\ ([0.2253, 0.1495], 0.2225) \end{pmatrix}$

Table 28 (Aggregated SCFDOWG data)

A_1	$\begin{pmatrix} ([0.8686, 0.9207], 0.9675), \\ ([0.2715, 0.2851], 0.1623), \\ ([0.1902, 0.2665], 0.2221) \end{pmatrix}$
A_2	$\begin{pmatrix} ([0.8653, 0.9396], 0.9648), \\ ([0.1416, 0.2820], 0.2487), \\ ([0.2493, 0.2383, 0.1836) \end{pmatrix}$
A_3	$\begin{pmatrix} ([0.9439, 0.9299], 0.9345), \\ ([0.2098, 0.1745], 0.2172), \\ ([0.2272, 0.2675], 0.2287) \end{pmatrix}$
A_4	$\begin{pmatrix} ([0.9206, 0.9624], 0.9335), \\ ([0.1985, 0.1937], 0.1846), \\ ([0.2435, 0.1386], 0.2329) \end{pmatrix}$

Table 29 (Aggregated SCFDHWG data)

Step 1: To combine all spherical cubic fuzzy decision matrices normalised independently, we utilized the following the idea of the SCFWG operator. Table 23 shows the aggregated spherical cubic fuzzy matrix.

Step 2:

1. SCFDWA, as described in Equation [18], will evolve their efficiency separately, as shown in Table 24.

2. SCFDOWA, as described in Equation [19], will evolve their efficiency separately, as shown in Table 25.

3. SCFDHWA, as described in Equation [20], will evolve their efficiency separately, as shown in Table 26.

4. SCFDWG, as described in Equation [21], will evolve their efficiency separately, as shown in Table 27.

5. SCFDOWG, as described in Equation [22], will evolve their efficiency separately, as shown in Table 28.

6. SCFDHWG, as described in Equation [23], will evolve their efficiency separately, as shown in Table 29.

Step 3: Following are the score of each alternative given in Table 30.

	$\underset{\smile}{A}_1$	$\underset{\smile}{A}_2$	$\underset{\smile}{A}_3$	$\underset{\smile}{A}_4$
SCFDWA operator	.0564	.0493	.055	.0593
SCFDOWA operator	.1257	.1209	.123	.1261
SCFDHWA operator	.1357	.1231	.133	.1414
SCFDWG operator	.9622	.955	.9614	.9702
SCFDOWG operator	.8715	.8646	.8714	.8740
SCFDHWG operator	.8583	.8506	.8525	.8763

Table 30 (Ranking of alternatives using SCFD operators)

Step 4: Rank criteria of alternatives given in Table 31.

	Ranking
SCFDWA operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$
SCFDOWA operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$
SCFDHWA operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$
SCFDWG operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$
SCFDOWG operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$
SCFDHWG operator	$\underset{\smile}{A}_4 > \underset{\smile}{A}_1 > \underset{\smile}{A}_3 > \underset{\smile}{A}_2$

Table 31 (Criteria for ranking)

5.10 Comparison Analysis

We present two comparative studies that demonstrate that our suggested operators are both accurate and appropriate at aggregating spherical cubic data.

1. Jana et al. proposed picture Dombi operators in 2019. In this work, we compare proposed spherical cubic Dombi operators to existing Dombi operators (Table 32). Figure 10 shows an illustration of ranking evaluation is based on spherical cubic fuzzy Dombi operators.

$\check{Q}_1$	$(.56, .34, .10)$	$(.90, .07, .03)$	$(.40, .33, .19)$	$(.09, .79, .03)$
$\check{Q}_2$	$(.70, .10, .09)$	$(.10, .66, .20)$	$(.06, .81, .12)$	$(.72, .14, .09)$
$\check{Q}_3$	$(.88, .09, .03)$	$(.08, .10, .06)$	$(.05, .83, .09)$	$(.65, .25, .07)$
$\check{Q}_4$	$(.80, .07, .04)$	$(.70, .15, .11)$	$(.03, .88, .05)$	$(.07, .82, .05)$
$\check{Q}_5$	$(.85, .06, .03)$	$(.64, .07, .22)$	$(.06, .88, .05)$	$(.13, .77, .09)$

Table 32 ((Jana et al. 2019) Picture fuzzy matrix)

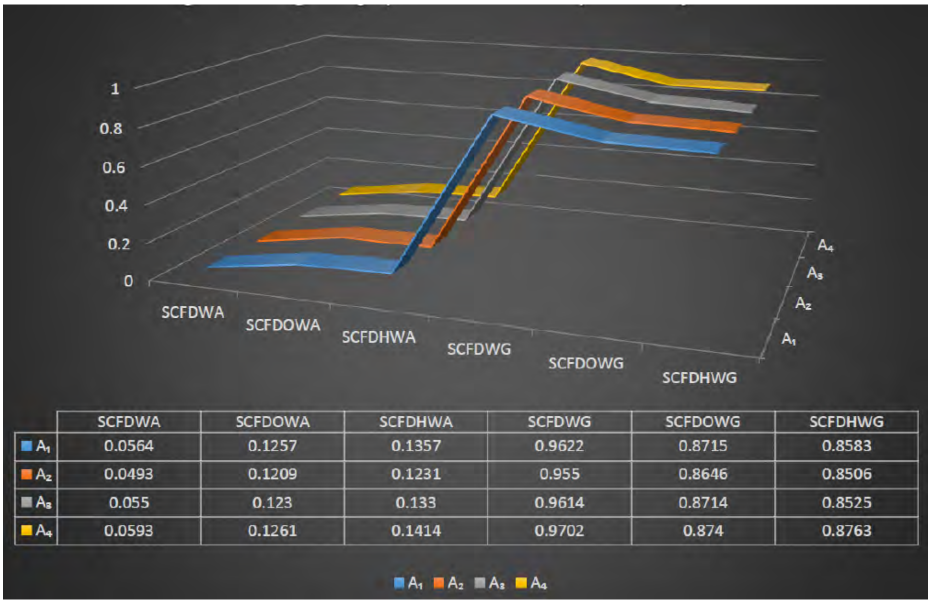

Figure 10 (Ranking using spherical cubic fuzzy Dombi operators)

Now, using the concept of the spherical Dombi operator, choose the optimum option as shown in Table 33.

Table 33 Alternatives are ranked using SFD operators

ρ	$sc(\check{Q}_1)$	$sc(\check{Q}_2)$	$sc(\check{Q}_3)$	$sc(\check{Q}_4)$	$sc(\check{Q}_5)$	Ranking
1	.9977	.9935	.9967	.9975	.9887	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$
2	.9978	.9931	.9963	.9971	.9878	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$
3	.9976	.9931	.9962	.9975	.9877	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$
4	.9976	.9932	.9961	.9975	.9877	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$
5	.9975	.9933	.9960	.9975	.9879	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$
6	.9975	.9933	.9961	.9975	.9880	$\check{Q}_1 > \check{Q}_4 > \check{Q}_3 > \check{Q}_2 > \check{Q}_5$

$\check{Q}_1$ is the optimal alternative. The results are comparable to those described by Jana et al. [92]. Jana et al. [92] propose a strategy for dealing with picture fuzzy sets, but it fails to handle with spherical cubic fuzzy sets. As a result, the new strategy suggested in this chapter can be used to deal with a wider range of uncertainty in decision support system. As a result, when compared to previous Dombi operators, novel spherical cubic Dombi operators are more accurate in resolving decision problems.

2.The SFD operators were presented by Ashraf et al. [93], and in this section, we compare the proposed and novel spherical cubic Dombi aggregating operators. Table 34 and Table 35 show the spherical Dombi results collected from Ashraf et al. [93].

$\underset{\sim}{A}_1$	$(.6582, .4279, .2947)$	$(.5742, .3611, .3398)$	$(.6297, .4954, .4093)$
$\underset{\sim}{A}_2$	$(.7339, .4891, .2905)$	$(.4523, .6776, .2498)$	$(.6582, .3076, .4993)$
$\underset{\sim}{A}_3$	$(.5134, .5334, .3894)$	$(.6844, .2763, .2739)$	$(.6236, .2667, .2731)$
$\underset{\sim}{A}_4$	$(.6435, .3934, .2715)$	$(.4954, .2445, .4523)$	$(.6603, .2223, .4353)$

Table 34 ((Ashraf et al. 2020) Collective SDF information matrix)

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$	$\underset{\sim}{A}_4$
SFDWA operator	.8242	.8936	.8845	.8809
SFDOWA operator	.8245	.8905	.8853	.8537
SFDHWA operator	.8653	.9757	.8734	.8731
SFDWG operator	.3930	.4427	.3770	.3518
SFDOWG operator	.4393	.4532	.4374	.3674
SFDHWG operator	.7945	.9044	.7335	.7163

Table 35 Ranking of alternatives through SFD aggregation operators

The best alternative is $\underset{\sim}{A}_2$.

Table 36, shows the calculation of best alternative using the spherical cubic Dombi aggregation operators.

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$	$\underset{\sim}{A}_4$
SCFDWA operator	.0564	.0493	.055	.0593
SCFDOWA operator	.1257	.1209	.123	.1261
SCFDHWA operator	.1357	.1231	.133	.1414
SCFDWG operator	.9622	.955	.9614	.9702
SCFDOWG operator	.8715	.8646	.8714	.8740
SCFDHWG operator	.8583	.8506	.8525	.8763

Table 36 Ranking of alternatives through SCFD operator

$\underset{\sim}{A}_4$ is the greatest option. The findings are comparable to those of Ashraf et al. [93]. Ashraf et al. methodology is described in [93]. We start with a spherical fuzzy set and extend it to a sphere cubic fuzzy set to acquire more precise results. This chapter methodology is more extensive in addressing the ambiguity in decision-making situations. As a consequence, Dombi aggregated operators under the structure of spherical cubic are more consistent and productive in resolving selection problems than existing Dombi operators.

As presented in Figure 11, the data obtained using the notion of spherical cubic fuzzy Dombi operators produce the closest outcomes in terms of ranking of spherical Dombi operators, as well as more appropriate and precise approaches to decision support system.

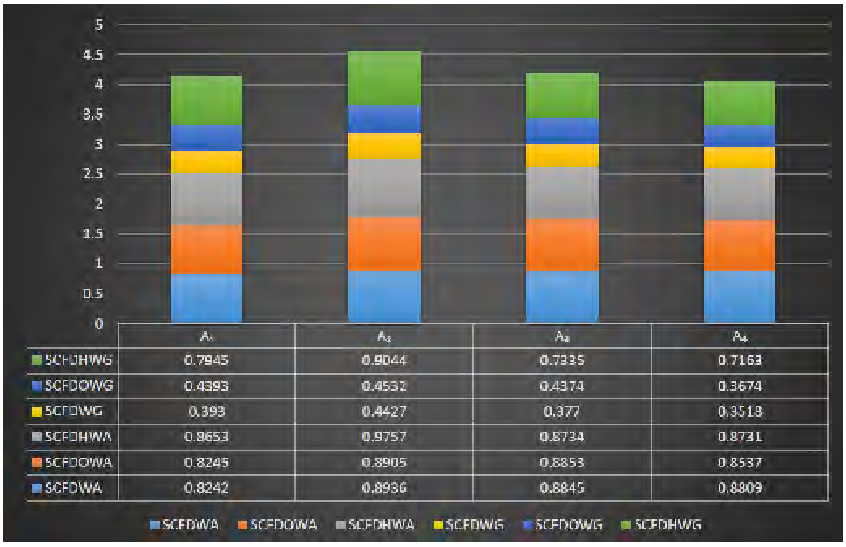

Figure 11 (Ranking using spherical Dombi operators)

5.11 Conclusion

The concept of a spherical cubic fuzzy set, which is a modified version of the spherical fuzzy set, was introduced in this chapter. Some spherical cubic fuzzy operating laws have been developed. We have a set scoring function for comparing spherical cubic fuzzy numbers. The notion of Dombi aggregated operators with SCF framework has been presented. The basic properties of spherical cubic fuzzy Dombi aggregated operators are defined. For aggregation of spherical cubic fuzzy sets, we proposed (SCFDWA), (SCFDOWA), (SCFDHWA), (SCFDWG), (SCFDOWG), (SCFDHWG) under the spherical cubic fuzzy information. We examined idempotency, boundary, and monotonocity, as well as a relationship between these well-known operators. In addition, to demonstrate the effectiveness of the proposed operators, we developed a multi-attribute decision support system. A numeric representation was given to illustrate that the specified operators have a different approach to solving the decision support system. Finally, we compared the effective methodology to current operators to verify its reliability and effectiveness.

Chapter 6

Application of spherical cubic fuzzy extended TOPSIS in decision support system

This chapter's goal is to provide a new approach with incomplete weight information for spherical cubic fuzzy (SCF) multi-attribute decision support system using TOPSIS method. For this, the maximum deviation model is suggested to determine the criteria of weight values. A multi-attribute decision support system is introduced using SCF information, based on suggested scheme. Furthermore, to ensure that the provided knowledge is appropriate, a numerical example is given. Finally, a systematic and structured analysis is given for the comparison of present work with the existing work.

6.1 Multi-attribute decision support system using extended TOPSIS

This section discusses a multi-attribute decision support system strategy based on the Pythagorean cubic fuzzy TOPSIS algorithm with undetermined weight.

Formulation of the problem

The MCDM problems are described as a decision-making mechanism that provides the attributes with ranking information in relation to the criteria. We suggest a spherical cubic fuzzy decision-making mechanism that not only describes the data on the Z_i alternatives that fulfill the A_j, criterion, the data on the Z_i alternatives that remain unchanged the A_j, criterion, and the degree where Z_i fails to meet the requirement of A_j. Suppose we have an MCDM function with $Z = \{Z_1, Z_2, ..., Z_m\}$ of m alternatives. and $A = \{A_1, A_2, ..., A_n\}$ be the set of criteria. In order to measure the efficiency of the i^{th} alternative Z_i in the j^{th} criterion A_j, the decision-maker must use knowledge of the fulfillment of criteria A_j by alternative Z_i's but of its non-fulfillment of A_j and remain unchanged of A_j. The $\check{\alpha}_{Ş_{c_{ij}}}, \check{\eta}_{Ş_{c_{ij}}}$ and $\check{\beta}_{Ş_{c_{ij}}}$, which represent the membership function, neutrality and non-membership function. Now alternative efficiency based on criteria A_j is represented by $Ş_{c_{ij}} = \left\langle \check{\alpha}_{Ş_{c_{ij}}}, \check{\eta}_{Ş_{c_{ij}}}, \check{\beta}_{Ş_{c_{ij}}} \right\rangle = \left(\left\langle \left[\check{a}_{ij}^{-}, \check{a}_{ij}^{+}\right], \check{\lambda}_{ij} \right\rangle, \left\langle \left[\check{n}_{ij}^{-}, \check{n}_{ij}^{+}\right], \check{\delta}_{ij} \right\rangle, \left\langle \left[\check{b}_{ij}^{-}, \check{b}_{ij}^{+}\right], \check{\mu}_{ij} \right\rangle \right)$ with the specified conditions $0 \leq \check{\lambda}_{ij}^2 + \check{\delta}_{ij}^2 + \check{\mu}_{ij}^2 \leq 1$ and $0 \leq \left(sup\left(\left[\check{a}_{ij}^{-}, \check{a}_{ij}^{+}\right] \right) \right)^2 + \left(sup\left(\left[\check{n}_{ij}^{-}, \check{n}_{ij}^{+}\right] \right) \right)^2 + \left(sup\left(\left[\check{b}_{ij}^{-}, \check{b}_{ij}^{+}\right] \right) \right)^2 \leq 1$. The decision matrix $\tilde{D}$ of SCFN is shown below:

$$\tilde{D} = \begin{bmatrix} Ş_{c_{11}} & Ş_{c_{12}} & \cdots & Ş_{c_{1n}} \\ Ş_{c_{21}} & Ş_{c_{22}} & \cdots & Ş_{c2n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ Ş_{c_{m1}} & Ş_{c_{m2}} & \cdots & Ş_{mn} \end{bmatrix}$$

Taking the different degrees attributes, the weight vector given in decision matrix satisfied the condition $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$. Due to uncertainty in practical decision making problems and inherent human thinking nature the knowledge about the weight attribute is unknown. For simplicity, let $\tilde{\Delta}$ represent the weight information , where the construction of $\tilde{\Delta}$ for $i \neq j$ shown below:

1. Weak rank criteria $\{\omega_i \geq \omega_j\}$;
2. Strictly ranking criteria $\left\{\omega_i - \omega_j \geq \left(\check{\lambda}_i > 0\right)\right\}$;
3. Ranking criteria with scaling $\left\{\omega_i \geq \check{\lambda}_i \omega_j\right\}, \check{\lambda}_i \in [0,1]$;

6.2 Maximum deviation methodology for optimal weight

In multi-criteria decision making process, optimal weight plays a vital role. Motivated by the above discussion in this section, we present a technique of maximizing deviation to define the criteria weights to illustrate MCDM problem with numerical details, a higher weight must be allocated to the criteria with a higher deviation value compared to the alternatives, while the criteria with a small distance value compared to the alternatives would be as if in order to determine the optimal weight parameters, as a result, we design an optimization method using the maximizing deviation approach. For $A_j \in A$ criteria , the distance of Z_i's alternatives can be described as:

$$\tilde{D}_{ij}(\omega) = \sum_{z=1}^{m} \omega_i \tilde{d}\left(\c{S}_{c_{ij}}, \c{S}_{c_{zj}}\right).$$

Where

$$\tilde{d}\left(\c{S}_{c_{ij}}, \c{S}_{c_{zj}}\right) = \frac{1}{9}\left[\begin{array}{c} \left|\left(\breve{a}_{ij}^{-}\right)^2 - \left(\breve{a}_{zj}^{-}\right)^2\right| + \left|\left(\breve{a}_{ij}^{+}\right)^2 - \left(\breve{a}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\lambda}_{ij}\right)^2 - \left(\breve{\lambda}_{zj}\right)^2\right| + \\ \left|\left(\breve{n}_{ij}^{-}\right)^2 - \left(\breve{n}_{zj}^{-}\right)^2\right| + \left|\left(\breve{n}_{ij}^{+}\right)^2 - \left(\breve{n}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\delta}_{ij}\right)^2 - \left(\breve{\delta}_{zj}\right)^2\right| + \\ \left|\left(\breve{b}_{ij}^{-}\right)^2 - \left(\breve{b}_{zj}^{-}\right)^2\right| + \left|\left(\breve{b}_{ij}^{+}\right)^2 - \left(\breve{b}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\mu}_{ij}\right)^2 - \left(\breve{\mu}_{zj}\right)^2\right| + \end{array}\right].$$

represent the spherical cubic fuzzy distance measure between $\c{S}_{c_{ij}}$ and $\c{S}_{c_{zj}}$.

Definition 6.2.1 *Let* $\tilde{D}_j(\omega) = \sum_{i=1}^{m} \tilde{D}_{ij}(\omega)$,

$$\sum_{i=1}^{m}\sum_{z=1}^{m} \omega_j \left(\frac{1}{9}\left[\begin{array}{c} \left|\left(\breve{a}_{ij}^{-}\right)^2 - \left(\breve{a}_{zj}^{-}\right)^2\right| + \left|\left(\breve{a}_{ij}^{+}\right)^2 - \left(\breve{a}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\lambda}_{ij}\right)^2 - \left(\breve{\lambda}_{zj}\right)^2\right| + \\ \left|\left(\breve{n}_{ij}^{-}\right)^2 - \left(\breve{n}_{zj}^{-}\right)^2\right| + \left|\left(\breve{n}_{ij}^{+}\right)^2 - \left(\breve{n}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\delta}_{ij}\right)^2 - \left(\breve{\delta}_{zj}\right)^2\right| + \\ \left|\left(\breve{b}_{ij}^{-}\right)^2 - \left(\breve{b}_{zj}^{-}\right)^2\right| + \left|\left(\breve{b}_{ij}^{+}\right)^2 - \left(\breve{b}_{zj}^{+}\right)^2\right| + \left|\left(\breve{\mu}_{ij}\right)^2 - \left(\breve{\mu}_{zj}\right)^2\right| + \end{array}\right]\right)$$

j=1,2,3,...,n.

$\tilde{D}_j(\omega)$ then denotes the distance for the parameters $A_j \in A$, from the alternatives. The choice of the weight vector ω, which maximizes the deviation, is based on the proposed model to describe a non-linear model:

First Model:

$$max\tilde{D}_j(\omega) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m} \omega_j \tilde{d}\left(\c{S}_{c_{ij}}, \c{S}_{c_{zj}}\right) \text{ s.that } \sum_{j=1}^{n} \omega_j = 1.$$

We have this model to clarify,

$$\tilde{L}(\omega, \varrho) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m} \omega_j \tilde{d}\left(\c{S}_{c_{ij}}, \c{S}_{c_{zj}}\right) + \frac{\varrho}{2}\left(\sum_{j=1}^{n} \omega_j^2 - 1\right) = 0.$$

which shows the Lagrange function of the problem of restricted optimization of first model where ϱ is a real number, denoting the variable of Lagrange multiplier. Now $\tilde{L}'$s partial derivatives are determined as:

$$\frac{\partial \tilde{L}(\omega, \varrho)}{\partial \omega_j} = \sum_{i=1}^{m}\sum_{z=1}^{m} \omega_j \tilde{d}\left(\c{S}_{c_{ij}}, \c{S}_{c_{zj}}\right) + \varrho \sum_{j=1}^{n} \omega_j - 1 = 0 \quad (24)$$

$$\frac{\partial \tilde{L}(\omega, \varrho)}{\partial \varrho} = \frac{1}{2}\left(\sum_{j=1}^{n} \omega_j^2 - 1\right) = 0 \tag{25}$$

We get,

$$\omega_j = \frac{1}{\varrho}\left(\sum_{i=1}^{m}\sum_{z=1}^{m} \omega_j \tilde{d}\left(Ş_{c_{ij}}, Ş_{c_{zj}}\right)\right) \tag{26}$$

Using the above Equations, we get

$$\varrho = \sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\tilde{d}\left(Ş_{c_{ij}}, Ş_{c_{zj}}\right)\right)^2} \tag{27}$$

Where $\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\tilde{d}\left(Ş_{c_{ij}}, Ş_{c_{zj}}\right)\right)^2}$ means the sum of deviation of all the alternatives with respect to all the criteria.

From Equations 26 and 27, we get

$$\omega_j = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m} \breve{\omega}_j \tilde{d}\left(Ş_{c_{ij}}, Ş_{c_{zj}}\right)}{\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\tilde{d}\left(Ş_{c_{ij}}, Ş_{c_{zj}}\right)\right)^2}} \tag{28}$$

There are, however, real cases where the weight vector information is not totally unknown, but slightly modified. For these examples, based on the following constrained optimization was constructed on the known weight information set. The weight value ω_j is also a set of restricted conditions where $\tilde{\Delta}$ is the criteria should be satisfied. The second model given in Equation 29 is a linear programming model which can be implemented using the software LINGO 11.0.

By normalization ω_j, we make sum into unity, and we get

$$= \frac{\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\frac{1}{9}\left[\begin{array}{c}\left|\left(\check{a}_{ij}^{-}\right)^{2}-\left(\check{a}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{a}_{ij}^{+}\right)^{2}-\left(\check{a}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\lambda}_{ij}\right)^{2}-\left(\check{\lambda}_{zj}\right)^{2}\right|+\\ \left|\left(\check{n}_{ij}^{-}\right)^{2}-\left(\check{n}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{n}_{ij}^{+}\right)^{2}-\left(\check{n}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\delta}_{ij}\right)^{2}-\left(\check{\delta}_{zj}\right)^{2}\right|+\\ \left|\left(\check{b}_{ij}^{-}\right)^{2}-\left(\check{b}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{b}_{ij}^{+}\right)^{2}-\left(\check{b}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\mu}_{ij}\right)^{2}-\left(\check{\mu}_{zj}\right)^{2}\right|+\end{array}\right]\right)}{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\frac{1}{9}\left[\begin{array}{c}\left|\left(\check{a}_{ij}^{-}\right)^{2}-\left(\check{a}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{a}_{ij}^{+}\right)^{2}-\left(\check{a}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\lambda}_{ij}\right)^{2}-\left(\check{\lambda}_{zj}\right)^{2}\right|+\\ \left|\left(\check{n}_{ij}^{-}\right)^{2}-\left(\check{n}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{n}_{ij}^{+}\right)^{2}-\left(\check{n}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\delta}_{ij}\right)^{2}-\left(\check{\delta}_{zj}\right)^{2}\right|+\\ \left|\left(\check{b}_{ij}^{-}\right)^{2}-\left(\check{b}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{b}_{ij}^{+}\right)^{2}-\left(\check{b}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\mu}_{ij}\right)^{2}-\left(\check{\mu}_{zj}\right)^{2}\right|+\end{array}\right]\right)\right)} \tag{29}$$

Second Model:

$$max\tilde{D}_{j}\left(\omega\right) =$$

$$= \sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{z=1}^{m}\left(\frac{1}{9}\left[\begin{array}{c}\left|\left(\check{a}_{ij}^{-}\right)^{2}-\left(\check{a}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{a}_{ij}^{+}\right)^{2}-\left(\check{a}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\lambda}_{ij}\right)^{2}-\left(\check{\lambda}_{zj}\right)^{2}\right|+\\ \left|\left(\check{n}_{ij}^{-}\right)^{2}-\left(\check{n}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{n}_{ij}^{+}\right)^{2}-\left(\check{n}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\delta}_{ij}\right)^{2}-\left(\check{\delta}_{zj}\right)^{2}\right|+\\ \left|\left(\check{b}_{ij}^{-}\right)^{2}-\left(\check{b}_{zj}^{-}\right)^{2}\right|+\left|\left(\check{b}_{ij}^{+}\right)^{2}-\left(\check{b}_{zj}^{+}\right)^{2}\right|+\left|\left(\check{\mu}_{ij}\right)^{2}-\left(\check{\mu}_{zj}\right)^{2}\right|\end{array}\right]\right)\right)$$

where $\sum_{j=1}^{n}\omega_j = 1.$

6.3 Proposed technique

In the spherical cubic fuzzy aggregation operator process [108], too much information is lost due to the difficulty of the spherical cubic fuzzy aggregating process, which means a lack of consistency in the final results. We have therefore expanded the TOPSIS approach to take into account spherical cubic information in order to address this limitation and have used the distance measurements of SCFNs to obtain the final ranking of the alternatives. TOPSIS is a strategy for solving decision support problems that selects the alternative with the smallest distance from the positive ideal solution (PIS). The greatest distance from the negative ideal solution (NIS) and is utilized generally in practical situations to solve ranking problems. Under the notation for SCF, the spherical cubic fuzzy positive ideal solution ($SCF\check{P}IS$) is expressed by $\check{p}^{+}$, and it is possible to write the spherical negative ideal solution with ($SCF\check{N}IS$) is expressed by $\check{p}^{-}$:

Let two SCFSs are $\text{Ş}_{c_1} = \left(\left\langle\left[\check{a}_1^-, \check{a}_1^+\right], \check{\lambda}_1\right\rangle, \left\langle\left[\check{n}_1^-, \check{n}_1^+\right], \check{\delta}_1\right\rangle, \left\langle\left[\check{b}_1^-, \check{b}_1^+\right], \check{\mu}_1\right\rangle\right)$ and $\text{Ş}_{c_2} = \left(\left\langle\left[\check{a}_2^-, \check{a}_2^+\right], \check{\lambda}_2\right\rangle, \left\langle\left[\check{n}_2^-, \check{n}_2^+\right], \check{\delta}_2\right\rangle, \left\langle\left[\check{b}_2^-, \check{b}_2^+\right], \check{\mu}_2\right\rangle\right)$ then

$$\check{p}^+ = \left\langle \begin{array}{c} \max\left\{\left[\check{a}_1^-, \check{a}_1^+\right], \left[\check{a}_2^-, \check{a}_2^+\right]\right\}, \max\left\{\left[\check{n}_1^-, \check{n}_1^+\right], \left[\check{n}_2^-, \check{n}_2^+\right]\right\}, \\ \max\left\{\left[\check{b}_1^-, \check{b}_1^+\right], \left[\check{b}_2^-, \check{b}_2^+\right]\right\}, \min\left\{\check{\lambda}_1, \check{\lambda}_2\right\}, \min\left\{\check{\delta}_1, \check{\delta}_2\right\}, \min\left\{\check{\mu}_1, \check{\mu}_2\right\} \end{array} \right\rangle \tag{30}$$

$$\check{p}^- = \left\langle \begin{array}{c} min\left\{\left[\check{a}_1^-, \check{a}_1^+\right], \left[\check{a}_2^-, \check{a}_2^+\right]\right\}, \min\left\{\left[\check{n}_1^-, \check{n}_1^+\right], \left[\check{n}_2^-, \check{n}_2^+\right]\right\}, \\ \min\left\{\left[\check{b}_1^-, \check{b}_1^+\right], \left[\check{b}_2^-, \check{b}_2^+\right]\right\}, \max\left\{\check{\lambda}_1, \check{\lambda}_2\right\}, \max\left\{\check{\delta}_1, \check{\delta}_2\right\}, \max\left\{\check{\mu}_1, \check{\mu}_2\right\} \end{array} \right\rangle \tag{31}$$

The separated distance measure $\check{d}^+$ and $\check{d}^-$ for each alternative of $(SCF\check{P}IS)$ $\check{p}^+$ and $(SCF\check{N}IS)$ $\check{p}^-$ formulated as:

$$\check{d}^+ = \sum_{j=1}^{n} \omega_j \tilde{d}\left(\text{Ş}_{c_{ij}}, \text{Ş}_{c_j}^+\right)$$

$$= \frac{1}{9}\sum_{j=1}^{n}\omega_j \left[\begin{array}{c} \left|\left(\check{a}_{ij}^-\right)^2 - \left(\overset{+}{\overline{\check{a}_j^-}}\right)^2\right| + \left|\left(\check{a}_{ij}^+\right)^2 - \left(\overset{+}{\overline{\check{a}_j^+}}\right)^2\right| + \left|\left(\check{\lambda}_{ij}\right)^2 - \left(\overset{+}{\overline{\check{\lambda}_j}}\right)^2\right| + \\ \left|\left(\check{n}_{ij}^-\right)^2 - \left(\overset{+}{\overline{\check{n}_j^-}}\right)^2\right| + \left|\left(\check{n}_{ij}^+\right)^2 - \left(\overset{+}{\overline{\check{n}_j^+}}\right)^2\right| + \left|\left(\check{\delta}_{ij}\right)^2 - \left(\overset{+}{\overline{\check{\delta}_j}}\right)^2\right| + \\ \left|\left(\check{b}_{ij}^-\right)^2 - \left(\overset{+}{\overline{\check{b}_j^-}}\right)^2\right| + \left|\left(\check{b}_{ij}^+\right)^2 - \left(\overset{+}{\overline{\check{b}_j^+}}\right)^2\right| + \left|\left(\check{\mu}_{ij}\right)^2 - \left(\overset{+}{\overline{\check{\mu}_j}}\right)^2\right| \end{array} \right] \tag{32}$$

$$\check{d}^- = \sum_{j=1}^{n} \omega_j \tilde{d}\left(\text{Ş}_{c_{ij}}, \text{Ş}_{c_j}^-\right)$$

$$= \frac{1}{9}\sum_{j=1}^{n}\omega_j \left[\begin{array}{c} \left|\left(\check{a}_{ij}^-\right)^2 - \left(\overset{-}{\overline{\check{a}_j^-}}\right)^2\right| + \left|\left(\check{a}_{ij}^+\right)^2 - \left(\overset{-}{\overline{\check{a}_j^+}}\right)^2\right| + \left|\left(\check{\lambda}_{ij}\right)^2 - \left(\overset{-}{\overline{\check{\lambda}_j}}\right)^2\right| + \\ \left|\left(\check{n}_{ij}^-\right)^2 - \left(\overset{-}{\overline{\check{n}_j^-}}\right)^2\right| + \left|\left(\check{n}_{ij}^+\right)^2 - \left(\overset{-}{\overline{\check{n}_j^+}}\right)^2\right| + \left|\left(\check{\delta}_{ij}\right)^2 - \left(\overset{-}{\overline{\check{\delta}_j}}\right)^2\right| + \\ \left|\left(\check{b}_{ij}^-\right)^2 - \left(\overset{-}{\overline{\check{b}_j^-}}\right)^2\right| + \left|\left(\check{b}_{ij}^+\right)^2 - \left(\overset{-}{\overline{\check{b}_j^+}}\right)^2\right| + \left|\left(\check{\mu}_{ij}\right)^2 - \left(\overset{-}{\overline{\check{\mu}_j}}\right)^2\right| \end{array} \right] \tag{33}$$

Relative coefficient of closeness of Z_i to $(SCF\check{P}IS)$ $\check{p}^+$:

$$\bar{C}_i = \frac{\check{d}^-}{\check{d}^+ + \check{d}^-} \tag{34}$$

Where $\bar{C}_i \in [0,1]$. Alternative Z_i is clearly similar to the $(SCF\check{P}IS)$ $\check{p}^+$ and further from the $(SCF\check{N}IS)$ $\check{p}^-$as $\bar{C}_i$ approaches 1. As a result, we determine the ranking orders of all individuals depending on the closeness correlation $\bar{C}_i$ and select the best option from a collection of viable options. Based on the preceding methods, in which the attribute weighting information is inadequate or undetermined, and the attribute values take the form of SCFNs, we will create an appropriate strategy to solving multi-criteria decision support problems.

Following are steps of our proposed technique.

Step 1: First of all we will construct the decision matrices $\tilde{D} = \left(\S_{c_{ij}}\right)_{m\times n} = \left(\left\langle \left[\check{a}_{ij}^-, \check{a}_{ij}^+\right], \check{\lambda}_{ij}\right\rangle, \left\langle \left[\check{n}_{ij}^-, \check{n}_{ij}^+\right], \check{\delta}_{ij}\right\rangle, \left\langle \left[\check{b}_{ij}^-, \check{b}_{ij}^+\right], \check{\mu}_{ij}\right\rangle\right)_{m\times n}$ are SCFNs, for the alternative Z_i and the criteria A_j.

Step2: To combine all the spherical cubic fuzzy matrices, use the SCFWG operator.

Step 3: If the knowledge of the criteria weights is absolutely unknown, the first model can be used to obtain them, if the knowledge of the criteria weights is not completely known but partially known, then the criteria weights can be determined using the second model.

Step 4: Using the Equations (17) and (18), we will find $(SCF\check{P}IS)$ $\check{p}^+$ and $(SCF\check{N}IS)$ $\check{p}^-$.

Step 5: Using the Equations (20) and (21), we will find $\check{d}^+$ and $\check{d}^-$.

Step 6: Rank all the alternatives Z_i and select the best one.

6.4 Illustrative description

To illustrate the approach proposed in this chapter, we will present a mathematical formulation to represent the possible evaluation of the marketing of emerging technology using spherical cubic fuzzy information in this section. There's a panel with four possible choices Z_i $(i = 1, 2, 3, 4)$ new technology companies to select. To assess the three potential emerging technology enterprises, the experts select three attributes:(1) A_1 is technical progress; (2) A_2 is growing market and market risk; (3) A_3 is industrial capacity (4) A_4 is the human economic and financial conditions. The weight vector is $(.25, .30, .45)^T$ and the spherical cubic fuzzy decision matrices are provided in Table 37, 38 and 39 should evaluate the four potential emerging technology companies using the spherical cubic fuzzy information.

	A_1	A_2	A_3	A_4
Z_1	$\begin{pmatrix} \langle [0.6, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.3], 0.4 \rangle, \\ \langle [0.2, 0.4], 0.4 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.5], 0.4 \rangle, \\ \langle [0.2, 0.5], 0.1 \rangle, \\ \langle [0.3, 0.4], 0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.1], 0.6 \rangle, \\ \langle [0.4, 0.5], 0.4 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.5], 0.6 \rangle, \\ \langle [0.2, 0.3], 0.5 \rangle, \\ \langle [0.3, 0.5], 0.2 \rangle \end{pmatrix}$
Z_2	$\begin{pmatrix} \langle [0.4, 0.7], 0.1 \rangle, \\ \langle [0.5, 0.2], 0.2 \rangle, \\ \langle [0.4, 0.5], 0.8 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.6], 0.2 \rangle, \\ \langle [0.2, 0.5], 0.4 \rangle, \\ \langle [0.2, 0.3], 0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.6], 0.7 \rangle, \\ \langle [0.5, 0.3], 0.2 \rangle, \\ \langle [0.4, 0.5], 0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.5], 0.7 \rangle, \\ \langle [0.5, 0.2], 0.1 \rangle, \\ \langle [0.3, 0.4], 0.1 \rangle \end{pmatrix}$
Z_3	$\begin{pmatrix} \langle [0.6, 0.7], 0.4 \rangle, \\ \langle [0.2, 0.4], 0.3 \rangle, \\ \langle [0.3, 0.4], 0.6 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.5], 0.6 \rangle, \\ \langle [0.2, 0.1], 0.5 \rangle, \\ \langle [0.3, 0.4], 0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.6, 0.7], 0.7 \rangle, \\ \langle [0.2, 0.1], 0.4 \rangle, \\ \langle [0.3, 0.5], 0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.5], 0.2 \rangle, \\ \langle [0.2, 0.1], 0.4 \rangle, \\ \langle [0.2, 0.4], 0.6 \rangle \end{pmatrix}$
Z_4	$\begin{pmatrix} \langle [0.6, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.3], 0.4 \rangle, \\ \langle [0.3, 0.4], 0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.6], 0.4 \rangle, \\ \langle [0.5, 0.6], 0.4 \rangle, \\ \langle [0.2, 0.3], 0.5 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.6, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.4], 0.1 \rangle, \\ \langle [0.3, 0.4], 0.3 \rangle \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.6], 0.7 \rangle, \\ \langle [0.2, 0.5], 0.6 \rangle, \\ \langle [0.2, 0.3], 0.1 \rangle \end{pmatrix}$

Table 37 (1st spherical cubic fuzzy decision making)

	A_1	A_2	A_3	A_4
Z_1	$\begin{pmatrix} \langle [0.3, 0.4], 0.4 \rangle, \\ \langle [0.2, 0.3], 0.5 \rangle, \\ \langle [0.6, 0.7], 0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.2, 0.4], 0.4 \rangle, \\ \langle [0.5, 0.6], 0.3 \rangle, \\ \langle [0.5, 0.6], 0.3 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.7, 0.8], 0.5 \rangle, \\ \langle [0.6, 0.4], 0.2 \rangle, \\ \langle [0.2, 0.3], 0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.6], 0.5 \rangle, \\ \langle [0.5, 0.3], 0.1 \rangle, \\ \langle [0.2, 0.3], 0.2 \rangle, \end{pmatrix}$
Z_2	$\begin{pmatrix} \langle [0.3, 0.4], 0.7 \rangle, \\ \langle [0.5, 0.7], 0.2 \rangle, \\ \langle [0.6, 0.7], 0.4 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.1, 0.2], 0.6 \rangle, \\ \langle [0.5, 0.4], 0.2 \rangle, \\ \langle [0.5, 0.6], 0.3 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.7], 0.6 \rangle, \\ \langle [0.8, 0.5], 0.2 \rangle, \\ \langle [0.5, 0.6], 0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.2, 0.3], 0.1 \rangle, \\ \langle [0.4, 0.5], 0.4 \rangle, \\ \langle [0.3, 0.4], 0.3 \rangle, \end{pmatrix}$
Z_3	$\begin{pmatrix} \langle [0.6, 0.7], 0.3 \rangle, \\ \langle [0.2, 0.4], 0.1 \rangle, \\ \langle [0.3, 0.4], 0.6 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.2, 0.3], 0.5 \rangle, \\ \langle [0.6, 0.2], 0.1 \rangle, \\ \langle [0.5, 0.6], 0.3 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.4], 0.6 \rangle, \\ \langle [0.5, 0.6], 0.3 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.2, 0.3], 0.7 \rangle, \\ \langle [0.4, 0.5], 0.4 \rangle, \\ \langle [0.3, 0.4], 0.2 \rangle, \end{pmatrix}$
Z_4	$\begin{pmatrix} \langle [0.6, 0.7], 0.6 \rangle, \\ \langle [0.2, 0.8], 0.2 \rangle, \\ \langle [0.3, 0.4], 0.5 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.4, 0.6], 0.3 \rangle, \\ \langle [0.1, 0.2], 0.8 \rangle, \\ \langle [0.3, 0.4], 0.4 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.5, 0.7], 0.7 \rangle, \\ \langle [0.1, 0.6], 0.2 \rangle, \\ \langle [0.4, 0.6], 0.3 \rangle, \end{pmatrix}$	$\begin{pmatrix} \langle [0.2, 0.3], 0.6 \rangle, \\ \langle [0.3, 0.6], 0.2 \rangle, \\ \langle [0.5, 0.6], 0.1 \rangle, \end{pmatrix}$

Table 38 (2nd spherical cubic fuzzy decision making)

	A_1	A_2	A_3	A_4
Z_1	$\begin{pmatrix} \langle[0.6,0.7],0.6\rangle, \\ \langle[0.2,0.3],0.4\rangle, \\ \langle[0.2,0.4],0.4\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4,0.5],0.4\rangle, \\ \langle[0.2,0.5],0.1\rangle, \\ \langle[0.3,0.4],0.3\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5,0.7],0.6\rangle, \\ \langle[0.2,0.1],0.6\rangle, \\ \langle[0.4,0.5],0.4\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4,0.5],0.6\rangle, \\ \langle[0.2,0.5],0.3\rangle, \\ \langle[0.3,0.5],0.2\rangle, \end{pmatrix}$
Z_2	$\begin{pmatrix} \langle[0.4,0.7],0.1\rangle, \\ \langle[0.5,0.2],0.2\rangle, \\ \langle[0.4,0.5],0.8\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5,0.6],0.2\rangle, \\ \langle[0.2,0.5],0.4\rangle, \\ \langle[0.2,0.3],0.6\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5,0.6],0.7\rangle, \\ \langle[0.5,0.3],0.2\rangle, \\ \langle[0.4,0.5],0.3\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4,0.5],0.7\rangle, \\ \langle[0.5,0.2],0.1\rangle, \\ \langle[0.3,0.4],0.1\rangle, \end{pmatrix}$
Z_3	$\begin{pmatrix} \langle[0.6,0.7],0.4\rangle, \\ \langle[0.2,0.4],0.3\rangle, \\ \langle[0.3,0.4],0.6\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4,0.5],0.6\rangle, \\ \langle[0.2,0.1],0.5\rangle, \\ \langle[0.3,0.4],0.3\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.6,0.7],0.7\rangle, \\ \langle[0.2,0.1],0.4\rangle, \\ \langle[0.3,0.5],0.3\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4,0.5],0.2\rangle, \\ \langle[0.2,0.1],0.4\rangle, \\ \langle[0.2,0.4],0.6\rangle, \end{pmatrix}$
Z_4	$\begin{pmatrix} \langle[0.6,0.7],0.6\rangle, \\ \langle[0.2,0.3],0.4\rangle, \\ \langle[0.3,0.4],0.5\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5,0.6],0.4\rangle, \\ \langle[0.5,0.6],0.4\rangle, \\ \langle[0.2,0.3],0.5\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.6,0.7],0.6\rangle, \\ \langle[0.2,0.4],0.1\rangle, \\ \langle[0.3,0.4],0.3\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5,0.6],0.7\rangle, \\ \langle[0.2,0.5],0.6\rangle, \\ \langle[0.2,0.3],0.1\rangle, \end{pmatrix}$

Table 39 (3rd spherical cubic fuzzy decision making)

	A_1	A_2	A_3	A_4
Z_1	$\begin{pmatrix} \langle[0.4874,0.5918],0.5313\rangle, \\ \langle[0.2000,0.3000],0.4277\rangle, \\ \langle[0.3872,0.5261],0.4752\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3249,0.4676],0.4000\rangle, \\ \langle[0.2633,0.5281],0.1390\rangle, \\ \langle[0.3776,0.4752],0.3000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5531,0.7286],0.5681\rangle, \\ \langle[0.2781,0.1516],0.4315\rangle, \\ \langle[0.3545,0.4530],0.4752\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4277,0.5281],0.5681\rangle, \\ \langle[0.2633,0.4290],0.2158\rangle, \\ \langle[0.2744,0.4530],0.2000\rangle, \end{pmatrix}$
Z_2	$\begin{pmatrix} \langle[0.3669,0.5918],0.1793\rangle, \\ \langle[0.5000,0.2912],0.2000\rangle, \\ \langle[0.4752,0.5761],0.7320\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3085,0.4315],0.2781\rangle, \\ \langle[0.2633,0.4676],0.3249\rangle, \\ \langle[0.3294,0.4275],0.5373\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4676,0.6284],0.6684\rangle, \\ \langle[0.5757,0.3497],0.2000\rangle, \\ \langle[0.4337,0.5337],0.4257\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3249,0.4290],0.3905\rangle, \\ \langle[0.4676,0.2633],0.1516\rangle, \\ \langle[0.3000,0.4000],0.1863\rangle, \end{pmatrix}$
Z_3	$\begin{pmatrix} \langle[0.6000,0.7000],0.3669\rangle, \\ \langle[0.2000,0.4000],0.2158\rangle, \\ \langle[0.3000,0.4000],0.6000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3249,0.4290],0.5681\rangle, \\ \langle[0.2781,0.1231],0.3085\rangle, \\ \langle[0.3759,0.4],0.3000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5313,0.7000],0.6684\rangle, \\ \langle[0.2000,0.1516],0.4517\rangle, \\ \langle[0.3759,0.5337],0.3000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3249,0.4290],0.2912\rangle, \\ \langle[0.2624,0.1621],0.4000\rangle, \\ \langle[0.2351,0.4000],0.5265\rangle, \end{pmatrix}$
Z_4	$\begin{pmatrix} \langle[0.6000,0.7000],0.6000\rangle, \\ \langle[0.2000,0.4026],0.3249\rangle, \\ \langle[0.3000,0.4000],0.5000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.4676,0.6000],0.3669\rangle, \\ \langle[0.3085,0.4315],0.4925\rangle, \\ \langle[0.2351,0.3341],0.4734\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.5681,0.7000],0.6284\rangle, \\ \langle[0.1625,0.4517],0.1231\rangle, \\ \langle[0.3341,0.4752],0.3000\rangle, \end{pmatrix}$	$\begin{pmatrix} \langle[0.3789,0.4874],0.6684\rangle, \\ \langle[0.2259,0.5281],0.4315\rangle, \\ \langle[0.3294,0.4257],0.1000\rangle, \end{pmatrix}$

Table 40 (Aggregative spherical fuzzy decision making)

Suppose that the data about the attribute weighting is entirely unknown. Using the procedures below, we may get the best option (s).

Step 1: In Table 37, 38 and 39 the decision makers have decision.

Step 2: To combine all the spherical cubic fuzzy matrices, use the SCFWG operator.

Step 3: We obtain the weight vector by using Equation 29 with the matrix in Table 40.

$$\breve{\omega} = (.2110, .2884, .2670, .2336)^T$$

Step 4: Now, PIS p^+ and NIS p^- are given by equation (18) and equation (19):

$$p^+ = \left\{ \begin{array}{c} \begin{pmatrix} \langle [0.6000, 0.7000], 0.6000 \rangle, \\ \langle [0.2000, 0.4026], 0.3249 \rangle, \\ \langle [0.3000, 0.4000], 0.5000 \rangle, \end{pmatrix}, \begin{pmatrix} \langle [0.4676, 0.6000], 0.3669 \rangle, \\ \langle [0.3085, 0.4315], 0.4925 \rangle, \\ \langle [0.2351, 0.3341], 0.4734 \rangle, \end{pmatrix}, \\ \begin{pmatrix} \langle [0.5681, 0.7000], 0.6284 \rangle, \\ \langle [0.1625, 0.4517], 0.1231 \rangle, \\ \langle [0.3341, 0.4752], 0.3000 \rangle, \end{pmatrix}, \begin{pmatrix} \langle [0.4277, 0.5281], 0.5681 \rangle, \\ \langle [0.2633, 0.4290], 0.2158 \rangle, \\ \langle [0.2744, 0.4530], 0.2000 \rangle, \end{pmatrix} \end{array} \right\}$$

$$p^- = \left\{ \begin{array}{c} \begin{pmatrix} \langle [0.3669, 0.5918], 0.1793 \rangle, \\ \langle [0.5000, 0.2912], 0.2000 \rangle, \\ \langle [0.4752, 0.5761], 0.7320 \rangle, \end{pmatrix}, \begin{pmatrix} \langle [0.3085, 0.4315], 0.2781 \rangle, \\ \langle [0.2633, 0.4676], 0.3249 \rangle, \\ \langle [0.3294, 0.4275], 0.5373 \rangle, \end{pmatrix}, \\ \begin{pmatrix} \langle [0.4676, 0.6284], 0.6684 \rangle, \\ \langle [0.5757, 0.3497], 0.2000 \rangle, \\ \langle [0.4337, 0.5337], 0.4257 \rangle, \end{pmatrix}, \begin{pmatrix} \langle [0.3249, 0.4290], 0.2912 \rangle, \\ \langle [0.2624, 0.1621], 0.4000 \rangle, \\ \langle [0.2351, 0.4000], 0.5265 \rangle, \end{pmatrix} \end{array} \right\}$$

Step 5: For calculating d_i^+ and d_i^- use Equation 32 and Equation 33.

$d_1^+ = .0818$, $d_2^+ = .0987$, $d_3^+ = .0925$, $d_4^+ = .1108$

$d_1^- = .1273$, $d_2^- = .0799$, $d_3^- = .1074$, $d_4^- = .1326$

Step 6: Calculate the $Ç_i$ by using Equation 34.

$Ç_1 = .609$, $Ç_2 = .4473$, $Ç_3 = .5372$, $Ç_4 = .5448$

Step 7: Ranking all the alternatives Z_i $(i = 1, 2, 3, 4)$ according to $Ç_i$

$$Z_1 > Z_4 > Z_3 > Z_2.$$

6.5 Comparison

The method proposed is compared and shown to be more general while achieving the same results as existing technique. We convert the more general SCFN to IVSFN to do this. In order to achieve this non-membership values, spherical fuzzy numbers are omitted (SFNs). Examples of this are given in the following subsections.

Comparison with interval valued spherical fuzzy sets

SCFNs can be changed to IVSFNs by deleting the non-membership . The interval valued of spherical fuzzy data is shown in Table 41.

By using IVSF TOPSIS methodology, PIS p^+ and NIS p^- for IVSF are in Table 41 as follows:

	A_1	A_2	A_3	A_4
Z_1	$\begin{pmatrix}[0.4874, 0.5918], \\ [0.2000, 0.3000], \\ [0.3872, 0.5261]\end{pmatrix}$	$\begin{pmatrix}[0.3249, 0.4676], \\ [0.2633, 0.5281], \\ [0.3776, 0.4752]\end{pmatrix}$	$\begin{pmatrix}[0.5531, 0.7286], \\ [0.2781, 0.1516], \\ [0.3545, 0.4530]\end{pmatrix}$	$\begin{pmatrix}[0.4277, 0.5281], \\ [0.2633, 0.4290], \\ [0.2744, 0.4530]\end{pmatrix}$
Z_2	$\begin{pmatrix}[0.3669, 0.5918], \\ [0.5000, 0.2912], \\ [0.4752, 0.5761]\end{pmatrix}$	$\begin{pmatrix}[0.3085, 0.4315], \\ [0.2633, 0.4676], \\ [0.3294, 0.4275]\end{pmatrix}$	$\begin{pmatrix}[0.4676, 0.6284], \\ [0.5757, 0.3497], \\ [0.4337, 0.5337]\end{pmatrix}$	$\begin{pmatrix}[0.3249, 0.4290], \\ [0.4676, 0.2633], \\ [0.3000, 0.4000]\end{pmatrix}$
Z_3	$\begin{pmatrix}[0.6000, 0.7000], \\ [0.2000, 0.4000], \\ [0.3000, 0.4000]\end{pmatrix}$	$\begin{pmatrix}[0.3249, 0.4290], \\ [0.2781, 0.1231], \\ [0.3759, 0.4752]\end{pmatrix}$	$\begin{pmatrix}[0.5313, 0.7000], \\ [0.2000, 0.1516], \\ [0.3759, 0.5337]\end{pmatrix}$	$\begin{pmatrix}[0.3249, 0.4290], \\ [0.2624, 0.1621], \\ [0.2351, 0.4000]\end{pmatrix}$
Z_4	$\begin{pmatrix}[0.6000, 0.7000], \\ [0.2000, 0.4026], \\ [0.3000, 0.4000]\end{pmatrix}$	$\begin{pmatrix}[0.4676, 0.6000], \\ [0.3085, 0.4315], \\ [0.2351, 0.3341]\end{pmatrix}$	$\begin{pmatrix}[0.5681, 0.7000], \\ [0.1625, 0.4517], \\ [0.3341, 0.4752]\end{pmatrix}$	$\begin{pmatrix}[0.3798, 0.4874], \\ [0.2259, 0.5281], \\ [0.3294, 0.4257]\end{pmatrix}$

Table 41 (IVSFD matrix)

$$p^+ = \left\{ \begin{array}{c} \begin{pmatrix}[0.6000, 0.7000], \\ [0.2000, 0.4026], \\ [0.3000, 0.4000]\end{pmatrix}, \begin{pmatrix}[0.4676, 0.6000], \\ [0.3085, 0.4315], \\ [0.2351, 0.3341]\end{pmatrix}, \\ \begin{pmatrix}[0.5681, 0.7000], \\ [0.1625, 0.4517], \\ [0.3341, 0.4752]\end{pmatrix}, \begin{pmatrix}[0.4277, 0.5281], \\ [0.2633, 0.4290], \\ [0.2744, 0.4530]\end{pmatrix} \end{array} \right\}$$

$$p^- = \left\{ \begin{array}{c} \begin{pmatrix}[0.3669, 0.5918], \\ [0.5000, 0.2912], \\ [0.4752, 0.5761]\end{pmatrix}, \begin{pmatrix}[0.3085, 0.4315], \\ [0.2633, 0.4676], \\ [0.3294, 0.4275]\end{pmatrix}, \\ \begin{pmatrix}[0.4676, 0.6284], \\ [0.5757, 0.3497], \\ [0.4337, 0.5337]\end{pmatrix}, \begin{pmatrix}[0.3249, 0.4290], \\ [0.2624, 0.1621], \\ [0.2351, 0.4000]\end{pmatrix} \end{array} \right\}$$

The distance measures $ḑ_i^-$ and $ḑ_i^+$ IVSF p^+ and the IVSF p^- are as follows:

$ḑ_1^+ = .0411$, $ḑ_2^+ = .0703$, $ḑ_3^+ = .0489$, $ḑ_4^+ = .0633$

$ḑ_1^- = .0693$, $ḑ_2^- = .0464$, $ḑ_3^- = .0666$, $ḑ_4^- = .0649$

Step 6: Calculate the $Ç_i$ by using Equation (22):

$Ç_1 = .6277$, $Ç_2 = .3974$, $Ç_3 = .5768$, $Ç_4 = .5065$

Step 7: Ranking all the alternatives Z_i $(i = 1, 2, 3, 4)$ according to $Ç_i$

$$Z_1 > Z_3 > Z_4 > Z_2.$$

Z_1 is best option.

This paper applied the TOPSIS approach to spherical cubic fuzzy sets. This approach has also been shown to provide more general knowledge than previous techniques. If several contradictory and/or unknown variables characterize the information needed for decision-making, this approach is able to decide the best decision and handle some uncertainty which other methods cannot, thus enabling decision-makers to take more informed decisions.

	A_1	A_2	A_3	A_4
Z_1	$(.5313, .4277, .4752)$	$(.4000, .139, .3000)$	$(.5681, .4315, .4752)$	$(.5681, .2158, .2000)$
Z_2	$(.1793, .2000, .7320)$	$(.2781, .3249, .5373)$	$(.6684, .2000, .4257)$	$(.3905, .1516, .1863)$
Z_3	$(.3669, .2158, .6000)$	$(.5681, .3085, .3000)$	$(.6684, .4517, .3000)$	$(.2912, .4000, .5265)$
Z_4	$(.6000, .3249, .5000)$	$(.3669, .4925, .4734)$	$(.6284, .1231, .3000)$	$(.6684, .4315, .1000)$

Table 42 (Selection through spherical fuzzy aggregation)

Spherical fuzzy comparative analysis

SFNs are special types of SCFNs in which decision-makers only determine the roles of membership function, neutral function and non-membership function. Table 42 demonstrates the membership function, neutral function and non-membership function of a SCFN converted to SFN by removing the interval portion of SCFN.

Based on Table 42, Spherical fuzzy TOPSIS is utilizied to calculate SF (PIS p^+) and SF (PIS p^-) as:

$$p^+ = \left\{ \begin{array}{c} (0.6000, 0.3249, 0.5000), (0.3669, 0.4925, 0.4734), \\ (0.6284, 0.1231, 0.3000), (0.5681, 0.2158, 0.2000), \end{array} \right\}$$

$$p^- = \left\{ \begin{array}{c} (0.1793, 0.2000, 0.7320), (0.2781, 0.3249, 0.5373), \\ (0.6684, 0.2000, 0.4257), (0.2912, 0.4000, 0.5265), \end{array} \right\}$$

The distance measures $ḑ_i^-$ and $ḑ_i^+$ IVSF p^+ and the IVSF p^- are as follows:

$ḑ_1^+ = .3657$, $ḑ_2^+ = .0284$, $ḑ_3^+ = .0532$, $ḑ_4^+ = .0426$

$ḑ_1^- = .0545$, $ḑ_2^- = .0336$, $ḑ_3^- = .0425$, $ḑ_4^- = .0659$

Step 6: Calculate the $Ç_i$ by using equation (22):

$Ç_1 = .1298$, $Ç_2 = .5413$, $Ç_3 = .4437$, $Ç_4 = .6075$

Step 7: Ranking all the alternatives Z_i $(i = 1, 2, 3, 4)$ according to $Ç_i$

$$Z_4 > Z_2 > Z_3 > Z_1.$$

Z_4 is best option.

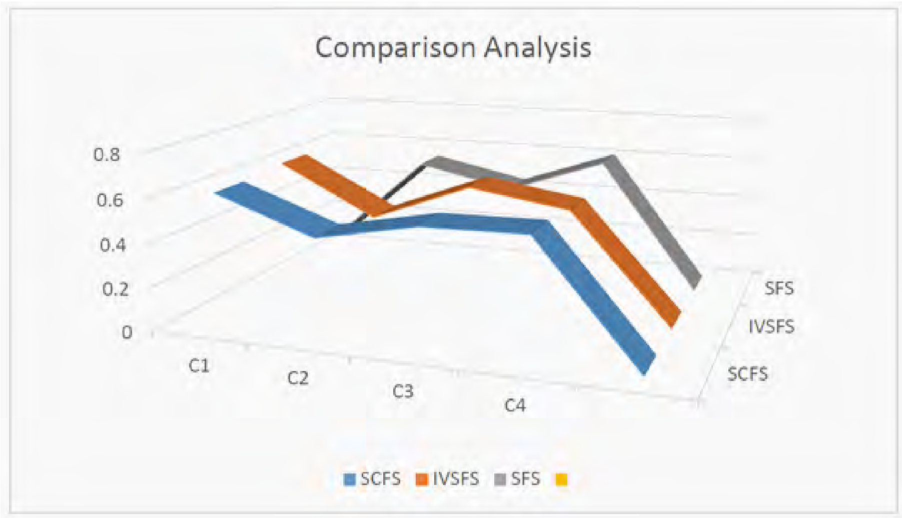

Figure 12 (SFS comparative study with SCFS TOPSIS technique)

The comparison analysis approach varies from the previous method in this paper in the order of the list of decisions. In particular, Z_1 and Z_3 alternatives switched positions. Since SFN does not contain as much details as just membership and non-membership that can result in loss of membership data, which resulted in a different result. In this case, despite variations in rank, the best choice in all the studied cases was the same and ranking in 1 out of 3 approaches is different. The graphically representation in Figure 12, shows the comparison analysis of extended Topsis SCFS with IVSFS and SFS.

We have the following benefits from the above analysis:

1: SCFNs can convey uncertainty in the MCDM more accurately than IVSFNs. In other words, SCFNs are the IVSFN extension. We should, therefore, know that the SCFNs have a greater prospect of application than the IVSFNs.

2: The approach proposed combines decision-maker expectations and intuition, decreasing the possibility of MCDM problems.

3: The method presented in this document is a modern extension of an existing technique that can solve a greater variety of MCDM problems than before TOPSIS.

6.6 Conclusion

As many realistic MCGDM problems arise in a dynamic setting and frequently conform to incomplete data and ambiguity. The SCFS is a very powerful method to tackle the fuzziness of the experts' decisions on alternative parameters. We first introduced a process in this paper

The maximization deviation method was named to evaluate the optimal relative criteria weight based on the spherical cubic fuzzy environment. An important thing the benefit of the proposed approach is its ability to minimize the effect of the experts' subjectivity and to remain adequate knowledge on the original decision at the same time. Then we suggested an expanded TOPSIS-based approach to solving the spherical cubic fuzzy knowledge MCGDM problems.

The method is based on the relative similarity of each alternative for determining the ranking order of all alternatives, which avoids the loss of too much information in the process of aggregating information. Finally, an illustration shows the efficiency and applicability of the proposed process. Our solution tends to be straightforward and to have less knowledge loss and can easily be extended to other management decisions in a hesitant spherical environment.

In future, under spherical cubic fuzzy, we will implement the principle of TODIM methods. We will also describe the spherical cubic fuzzy Linguistic sets and proposes the TOPSIS and TODIM MCGDM-based methods in spherical cubic fuzzy linguistic environment.

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