Generalized Dispersion Relations for Electromagnetic Waves in Dusty Plasma

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Generalized Dispersion Relations for Electromagnetic Waves in Dusty Plasma

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Beginning with the name of ALLAH ALMIGHTY the most beneficent, merciful and the creator of whole universe.

Certificate

This is to certify that Ms. Shabana Bibi D/O Muhammad Nabi has accomplished the theoretical work successfully in this dissertation under my supervision at Department of Physics, Quaid-i-Azam University, Islamabad and satisfying the dissertation requirement for the degree of Master of Philosophy in Physics.

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DEDICATED

TO

My Hero Father, Supportive Mother

Whose affection is the reason of every success in my life.

And

TO

My Respected Supervisor

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All the praises to Almighty ALLAH, the most merciful and the sovereign power, who made me able to accomplish this research work successfully. I offer my humble and sincere words of thanks to His Holy Prophet Muhammad (S.A.W), who is forever a source of guidance and knowledge for humanity.

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Abstract

In this dissertation, we have studied the generalized linear theory for electromagnetic wave in a homogeneous dusty magnetoplasma. The generalized Hall-magnetohydrodynamic (GH-MHD) equations are derived by assuming massive charged dust macro-particles to be immobile. Due to the presence of immobile charged dust particles, various new electromagnetic modes are excited. Then linear properties of the Rao-dust-magnetohydrodynamic (R-D-MHD) waves are studied. In the linear regime, the existence of immobile dust grains produces the Rao cutoff frequency, which is proportional to the dust charge density and the ion gyrofrequency. The hydrodynamic equations for double dust particles in a quasineutral magnetoplasma, together with Ampere's law, are used to derive nonlinear equations for ultralow-frequency modes. In the linear approximation, the frequency of the double dust-acoustic waves (DDAWs) is obtained.

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Chapter 1

Introduction

1.1 INTRODUCTION

In essence a term ‘plasma’ is named for an ionized gas and also considered to be a fourth state of matter. As it is a common concept that a solid can be transformed into liquid by giving some amount of heat that breaks its intermolecular linkage. Likewise, sufficiently heating a liquid such that the rate of evaporation from its surface is faster than recondensation, generates a neutral gas. Similarly, plasma is formed by heating the neutral gas in a way that violent collisions between gaseous atoms results in detachment of electrons from their corresponding sites. In general, plasma is a quasi-neutral gas which consist of positive and negative charge carriers along neutral atoms which show collective behavior.

The behavior of this quasi neutral gas (containing various species) may change due to the masses of its charged and neutral particles. Without charge neutrality, an electric field is developed to make the plasma unstable. Hence, the quasi-charge neutrality is essentially vital. Therefore, the potential in the plasma is usually determined by the equations of motion for the plasma species and not through the Poisson’s equation. Coulomb’s force is responsible for the particle interaction in a plasma since it is electrically charged, and being the long range force, plasma possess collective behavior. Therefore the hottest of all the states of matter are the plasmas state, yet the long range Coulomb interactions of electrons and ions make them the richest in collective phenomena. Collective behavior means that motions depend on its local conditions as well as plasma in remote region. Three conditions a plasma system must satisfy are:

1) $\lambda_D \ll L$, L is the dimension/size of the system.

2) $N_D \gg 1$, N_D is the number density in a Debye sphere.

3) $\omega\tau \gg 1$, ω is the frequency of oscillations in plasma and τ is the mean time interval between collisions with atoms (neutral).

In other words, a plasma can be considered as a mixture of several fluids or a conducting fluid, with the number of particles (charged and neutral) that, accordance to the plasma models. Thus different asronomical objects have different values of number density like $10^{19}m^{-3}$ in the solar photosphere, $10^{12} - 10^{14}m^{-3}$ in the solar corona, and above $10^{20}m^{-3}$ amount of plasma density in the tokamak. Similar to number density, the values of the collision frequencies are also different in astrophysical environment, like $\sim 10^{-2}H_Z$ in the corona, $\sim 10^{7}H_Z$ in the photosphere, and $\sim 10^{4}H_Z$ in the tokamak for ions.

1.2 Brief History of Plasma Physics

Historically, the term plasma was used first by Czech physiologist Jan Evangelista Purkinje (in 1860s) to denote the remaining fluid after the clearing of many corpuscles from blood, this statement was suggested by Johannes Purkinje who was a medical scientist. Then in 1927, the American chemist and Nobel laureate Irving Langmuir proposed that a similar medium could be considered for electrons, ions and neutrals or for a an ionized gas. He linked the electrons and ions of an ionized gas with blood plasma which carries white and red cells. Langmuir was intended to increase the lifetime of light bulb, made of tungsten filament meanwhile he proposed the concept of 'plasma sheath' (layers that form between solid surface and plasmas). Even though, it turned out that there is no actual medium for particles like blood, so from then onwards plasma scientists have to tell others they are not working on blood. The research then started individually inspired by some practical problem, and lead to what is now called as **plasma physics**[1].

In result of this fundamental study, a vast area of research in plasma has opened the gateway for many other fields related to it, like the study of ionosphere, astrophysical processes, nuclear energy etc. Plasma is macroscopically defined by a neutral gas which consist of charged species (electrons and ions) with some neutral atoms. It is generally observed by previous knowledge that 99% of the visible universe is comprised of plasma, while the remaining 1% presents other matter like planets, interplanetary spaces accommodated dust species [2]. Density of plasma in spatial regime varies from lower to denser in astrophysical environment, which contains partially or fully ionized plasma.

Stars are glowing plasma body which contain high proportion of ionized particles. As the fusion of hydrogen and helium nuclei releases enormous amount of energy but this reaction needs high temperature and pressure that found only at the center of Sun. This process is easier for a gas which consists of heavy isotopes of hydrogen, but even then such a high temperature is needed. It is impossible for any laboratory container to hold the gas under this condition. However, gas at such a temperature become plasma. After a period of time, an idea came up to trap the gas in magnetic field, without touching material wall. Working on that idea, introduced us with controlled thermonuclear fusion processes. In this reaction, two high energetic ions are accelerated which defeated their repulsion of electrostatic force between them, in this way they come close to each other. Because of their fusion reaction, large amount of energy is released that works as an energy reservoir. Solar wind that originates from Sun comprising of a flow of charge particles that ionizes its surrounding plasma [3].

Aurora and other astrophysical perturbed plasma oscillations can be seen in the polar region of earth, sometime in night and also at the time of sunrise and sunset. Earth ionosphere having a massive amount of highly energetic particles results from excitation and ionization by solar wind . The ionosphere extends from 85 to 600**km**. It contains oxygen, nitrogen and nitric oxide in large proportion. The amount of ionization varies with radiation intensity of solar wind coming out from the Sun. In this way plasma is categorized according to the following distribution which can be seen through temperature and pressure variations of earth environment. The phenomena of the Lightning is due to the knocking of particles present in the cloud in upper atmosphere Fig. (1.1). It is because of the fact that both cloud and ground are oppositely charged. In result of the ionization of the gas molecules in the air, lightning is produced. The plasma physics become more and more famous after the study terms “sheath” [4] and “plasma” [5, 6] used by Tonks and Langmuir (20th century). Plasmas produced in libratory have many important applications in industries as well as in the field of technology.

Figure 1-1: Lightning: found on Earth is plasma surface [9]

Its examples include, outer surface treatments like plasma sprinkling, metal photoengraving and cutting [7] in electronics on microscopic level. While for coming period, plasma devices usage to control and enhance flight of vehicles in the atmosphere and other aerodynamic phenomena is under observations. The combination of physical plasma with other biological and medical sciences is an inventive and emerging field for the treatment and therapy of disorderliness [8].

This is known as plasma in which ionization and recombination balances so as to retain the gas to be neutral overall. Therefore the hottest of all the states of matter are the plasmas state,yet the long range Coulomb interactions of electrons and ions make them the richest in collective phenomena. Collectives behavior means that motions rely upon its nearby conditions and also just plasma in remote region. Three conditions a plasma framework must fulfill are:

1) $\lambda_D \ll L$, L is the dimension/size of the system.

2) $N_D \gg 1$, N_D is the number density in a Debye spherical region.

3) $\omega\tau \gg 1$, ω is the oscillations frequency in plasma and τ is the mean time interval of the collisions.

1.3 Dusty Plasma

The birth of a newly interesting field of research is came into being in 1996, by detection of dust particles in plasma while observing astronomical environment, known as dusty plasmas [10, 11]. The constituents of dusty plasmas are mainly electrons, ions and electrically charged or neutral dust particles. Because of more species, the parameters of plasma are modified and thus this is also called as complex plasma. Dust is neutral by nature, when in a plasma (highly ionized gases), dust particulates acquires either negative charge or sometime may positive charge depending upon the surrounding environment. Charge on a dust particles are usually negative, when a greater number of electrons are deposited on dust particles. There are different processes in which dust particles become negatively charged, for example collecting of electrons from environment around plasma, irradiation of ultraviolet radiation, sputtering of energetic ions etc. Each dust particles may have different size or dimension depending on its origin and nature. They can be micron sized or even less than this, while the mass the dust particle is greater than the mass of positive ion (The mass is of the order $10^6 - 10^{18} m_p$, where m_p denotes the proton mass). For instance, spherical dust grain having radius $1\mu m$ and mass density $2000\mathbf{kg}/m^3$ then mass of such particle is $8 \times 10^{-15} kg$ (roughly 5 trillion proton masses). Like their masses and charges, dust particles also have various shapes such as rods or spherical or may also irregular shaped. They are either consist of conducting materials or electrical insulator (dielectric), i.e. $\mathbf{Al_2O_3}$ or $\mathbf{S_iO_2}$. Even though the dust particles are mostly solid, but may also in form of ice crystals or liquid drops. Dust particles are affected by the electric and magnetic fields, and due to the presence of these fields the plasma behavior may vary. The presence of these dust particles differentiates itself from an ordinary plasma in three major categories:

- Density fluctuations and varying charge distribution.
- Generation of various instabilities in plasma.
- Arising of new dust driven modes.

1.4 Sources of Dusty Plasma

1.4.1 Dusty plasma in Earth atmosphere

Sun is the closest star to earth which is a huge luminous sphere of hot plasma. Its corona spreads out stream of charged particles in the space known as solar wind. It can be observable on earth when it triggers the phenomenon of geomagnetic storms or aurora. The high luminosity of very captivating phenomena aurora is appear as a result of the interaction between energetic particles of solar wind with Earth's ionosphere. The ionization of earth ionosphere is due to the emission of electromagnetic radiations from the Sun. Therefore radiations received by sun affects the amount of ionization. Thus, seasonal effects and geographical locations plays a significant role. Another source of plasma (dust is ubiquitous) in the Earth atmosphere is lightning which is triggered by the process of charging of vapors in the cloudy region. Lightning is thus produced is due to ionization of air, when electrical discharge occurs between the cloud and the ground having oppositely charged. One of the other common source of dusty plasma in the magnetosphere of Earth is pollution or contaminants produced by human beings. In the magnetosphere of Earth, there is also a source of extraterrestrial dust particles available, especially in the form of meteoroids, with sizes (radius) and number density are $5-10\times10^{-6}m$ and $10^{-15}-10^{-14}m^{-3}$ respectively. The most significant portion of the surface of Earth in which the charged dust particles is present is actually the polar summer mesopause which lies in between 80 to 90 km in elevation. Numerous important phenomenon detected in this region is the formation of special type of clouds that are called 'Noctilucent Cloud' (NLCs) shown in Fig (1.2). In 1885 NLCs were observed for the first time and considered to be different from other clouds. The peculiarity that was revealed about polar mesopause is that it is much cooler in summer than in winter.

1.4.2 Dusty plasma in space

In spatial environments a large amount of dusty plasma is available in different regions of universe like interstellar clouds, planetary rings, comet tails, and solar system etc. The existence of charged(negative and positive) dust particles in the astrophysical regions cause electrical complication for spacecraft due to this physical damage may also occur. While studying dust

Figure 1-2: Noctilucent clouds (NLC) [12].

particles present in space plasmas gives important explanation about the formation and structure of different astronomical objects (planetary rings, planets etc.). Dusty plasma present in the spatial environment may play a role for variety of regions, thus it is important to understand its properties. The "interstellar medium" is formed in region available between the stars due to the presence charged dust particles and gases such as hydrogen and helium. The major constituent of interstellar portion is hydrogen atoms, having average density is of the order of 1 atom per cm^3. While for its extremely lower density limit is about 0.1 atom per $cm^3$and as highest density limit as 1000 atoms per cm^3, have been detected in the region between the spiral arms and near the core of galaxies respectively. The other constituent of interstellar medium may cosmic dust particles, which are of bigger dimension than hydrogen atoms, therefore in the same volume of interstellar medium, there are large number of hydrogen atoms than the cosmic dust particles. It is evaluate that in the interstellar medium the cosmic dust density is thousand times lesser than the density of hydrogen atoms. The dust grains present in the interstellar medium are either dielectric or metal , or may both. It is believed that large amount of dusty plasma present in various parts of our solar system. Dusty plasma in different regions of our solar system and its origin and characteristics are discuss briefly in the following few sections.

1.4.3 Planetary rings

The outer layer of most of the planets consisting rings is made up of dust such as those in Jupiter, Saturn and Uranus sized from micron to submicron dust grains. In 80s for the first time the Voyager spacecraft detect the presence of spokes in Saturn's rings which are radial features. The formation and evolution of these spokes had been widely discussed in [13, 14, 15, 16]. Planetary rings of Saturn are the A, B and C while and the eleven rings of Uranus, these are consist mainly of large objects having dimension of 1 cm to 10 m. Each ring consist of a greater amount of number density of dust particles. In the following few sections we briefly describe about dusty plasma and dust particles, its origin and characteristics in planetary rings.

Jupiter In our solar system Jupiter is the fifth planet in numbering from the Sun and by far the largest of all planet. Many probes visited to Jupiter such as Pioneer 10 visited in December 1973, while the Pioneer 11 visited subsequently one year later. They capture some low resolution photographs of the planet, and provided new data about Jupiter's magnetosphere. After that in March 1979 Voyager 1 and in July Voyager 2 take flight to Jupiter, which improved our knowledge about the Galilean moons and observed Jupiter's rings. They also provide more information about planet's atmosphere from its photographs. In 2000, some of the high resolution pictures ever made of the planet is provided by Cassini probe, along the way to Saturn, flew by Jupiter. As the planetary ring Jupiter where very faint and consist of dust particles (like smoke) knocked off by meteor impacts from its moons. The dust particles provided by satellites Adrastea and Metis for the main ring of Jupiter. The main ring is encircle by two wide gossamer rings, and two satellites Thebe and Amalthe are responsible for its generation. Additionally an extremely thin and far away outermost ring is there that encircles Jupiter from reverse side. The origin of this outer ring is unpredictable, but it might be made of dust particles catched from interplanetary space. When the moon having low gravity struck by meteorites,it releases dust and particles from their atmospheres. Another prospective is that dust particles of this ring may also has been from comets that crashed into Jupiter.

Saturn In our solar system the sixth planet from the Sun is Saturn and is of the largest dimension planet after Jupiter. Many number of large rings in the Saturn which made primarily

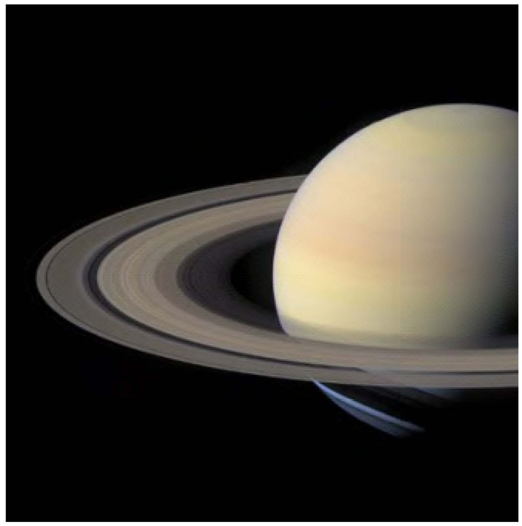

Figure 1-3: Saturn with its planetary rings [17].

out of ice and dust particles. Like Jupiter many probes also visited to the Saturn. After two years of Voyager I and Voyager II, in 1979 Pioneer 11 visit to Saturn for the first time. The information know about Saturn and its moon Titan is given by Cassini-Huygens orbiter in his visit to Saturn. After understanding knowledge of planetary rings, the Saturn is consider one of the important objects in the solar system. In 1610 Galileo Galilei were first observed the three rings of Saturn. With the passage of time many number of rings were discovered and are given names according to their discovery. The Ring System of Saturn are composed of D, C, B rings, Cassini Division, A, F, G and E rings, and some small intermediate rings are also present. The atmosphere of Saturn is filled with dust particles, silica rock and iron oxide and its extension above Saturn's equator are 6, 630 km to 120, 700 km.

Uranus Uranus is a gas giant, the third largest by diameter and fourth largest by mass. It is the seventh planet from Sun in our solar system . Uranus is composed mainly of rock and different ices (dust particles). The only spacecraft that have visited the planet Uranus is NASA's Voyager II . Launched in 1977, Voyager on January 24, 1986 made a high resolution images and interesting information of Uranus on its way visit to Neptune. In March 1977 the Voyager II was discovered this ring system. It believed that there are eleven dominant rings surrounded by

Uranus and its formation is a result dust originate from the breakdown of astrophysical objects. These rings are clearly different from the rings of Saturn and Jupiter. The outermost ring of Uranus is consist particularly of ice boulders (dust), having diameter of several feet.

Neptune It is the outermost gas giant, and the eighth planet from the sun in our solar system. It is third largest and fourth largest planet by mass and diameter respectively. In August 25, 1989, only one spacecraft, Voyager II has been visited to Neptune. In 20th century (1980's) astronomers found that the planets like Jupiter, Saturn, and Uranus all contain rings, but they will be speculate if Neptune also had rings. From the surface of Earth, there is no rings encircle Neptune could be observed. In 1989, the Voyager II reported that Neptune has spotted ring arcs (partial ring) about which scientists had suspected. When the probe become nearer, astronomers observed that the ring arcs were basically a portion the whole ring. Voyager II detected that the Neptune contain four rings, out of these the two major rings are glittering and limited in diameter. Astronomers observed that these rings are made up of fine dust particles but mores studies are required to understand accurately about its size and mass distribution because its still unknown. The two inner rings are wider in diameter and its have dark background, while the outermost ring consist of shiny clumps of particles.

1.4.4 Interplanetary space

The interplanetary region is full of dust particles that is known as 'interplanetary dust'. The presence of these dust grains was considered as zodiacal light. They are distributed in a wide area of inner solar system with a strong contribution for asteroids belt. All particles that are having the size smaller than 1 cm gradually spiral into the surface of sun for the timescale ranging from the period of thousands to millions of years. This is actually the manifestation of combined effect of solar radiations and Poynting–Robertson drag (orbital angular momentum loss of gyrating specie due to absorption and re-emission of solar wind). Astronomers observed that interplanetary space is looking empty at a distance point, but actually it consist of hot ionized plasma (positive and negative ions), electromagnetic radiation (photons), solar wind, micro dust particles, cosmic rays and also the magnetic fields. The dust particles present in interplanetary region are very smaller in diameter (of the order 0.001 cm) and are present in

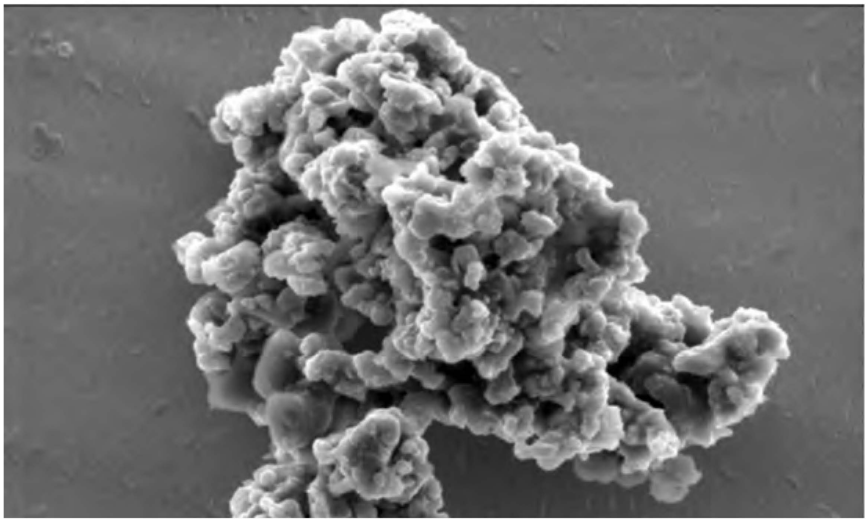

Figure 1-4: The fluffy appearence of interplanetary dust [20].

the most basic elements of our solar system. The surface of earth is constantly being rained upon by dust particles of the interplanetary space, having diameter is in order of micrometers (few to several hundred). The distribution of mass in interplanetary space of this dust grain flux peaks is approximately 200 μm [18]. These dust particles are considered to be originate from collision of astrophysical objects and from comets[19]. The interplanetary dust grains are often seems to be very fluffy and fragile shown in Fig (1.4). Mostly their sizes are about 5–20mm, that are found on collector. Some of these grains are fragile enough that they start disintegrating into hundreds of small fragments after colliding with the surface of collector. They are having high percentage of carbon.

1.4.5 Cometary dust

Dusty plasma in the form of grains and clusters is found widely in astrophysical environments for example cometary tails, planetary rings, asteroids, interstellar space, interplanetary medium, planetary nebulae etc. [13, 14, 15, 16]. A mixture of non-volatile particles and frozen gases having irregular appearance are called comets. A comet is in the shape of ball with a tail, looking like a star including ice and dust particles. Some comets looking like round spots of light and do not have tails. Mostly the body of comet is divided into three parts: the nucleus, head, and tail. Coma has a small bright nucleus (about 10km or less than this, in diameter) in its center, so the coma and nucleus joins to form the head of comet. Most parts of our solar system is filled with cloud of dust particles and gas. Many years ago, an astrophysical object were obliterated as a result of the collision of particles of this cloud, while rotating around the sun in our solar system, known as comets. Some of these particles grow up their size and become the planets. Some astronomers reported that these " dirty snowballs"(comets) are consists of dust particles, ice, and dirt. The cometary ice not only made of water, but also contains some gases like carbon dioxide(CO_2), ammonia(NH_3), and methane(CH_4). Large amount of ice in the frozen form, are responsible for preservation of the dust particles trapped inside for the birth of our solar system. The icy layers of comet start evaporation as it come near the Sun, as a result dust particles and dirt released to form a curved tail of dust particles. By heating the liberated gases glow and generate a luminous envelope around the nucleus. The Sun's radiation ionized these gases and the solar wind push it in backwards direction, the heated gases trails behind, while formation of second tail occur. The nucleus of comet get heats up as it come closer to the Sun. In all over the early time our solar system were mostly filled with comets, but with the passage of time, the density of comets became decreased. Many more comets are seen the by astronomers through their powerful telescopes, but simultanouly a rare number (15 or 20) of comets to be detected in the sky.

The increasing brightnesses of comets in a short time interval, it have been known to experience an "outbursts". This can be occur either due to blowout of new particles from the surface of comet or splitting of nucleus into two or more pieces (by heating its hidden parts for the first time from sun radiation). A lot of information about the cometary dust particles is provided by the Vega and Giotto spacecraft. The comet Hale-Bopp consists of two tails shown in the Figure (1.5) The blue tail always points in the direction away from the sun whereas the other white tail comprising dust (below one) is curved along its orbit. Photons emitted by the sun generates light pressure forming an angle between these tails.

1.4.6 Dusty plasmas in laboratory

Experiments were performed since 70s which observed dusty plasma. For example the effect of negative dust was studied in a DC positive column and electronegative discharges deducing

Figure 1-5: A bright view of comet [21].

that the negative dust particulates have a similar effect to negative ions.[22] In semiconductor industry, microelectronics, plasmas have a significant role. The presence of dust affects the fabrication process of wafer chips made-up of silicon. After switching off the discharge, dust falls on wafer chips and affects their performance. Studies were and are being developed in order to find the origin of this dust in discharges.[23] Figure (1.6) reflects such phenomenon. The bottom regions of mostly tokamaks and stellerators include a large quantity of small dust particles. Importance of these dust particles is because of its usage in fusion plasmas as a potential safety hazard.

Despite the fact that an enormous of articles of literature on dusty plasmas has concentrate attention on dust particles from a particle dynamics perspective [25], consideration of collective effects is additionally vital [26]. Studies of collective effects include motions of waves, instabilities of waves, and coherent structures formation etc. It is anticipated that the existence of charged dust particles can either change the current plasma modes or may present new eigenmod. The significance of dusty plasmas in the field of plasma research and other application are because of the following reasons:

- The dust particles are subject to both electromagnetic and gravitational forces because

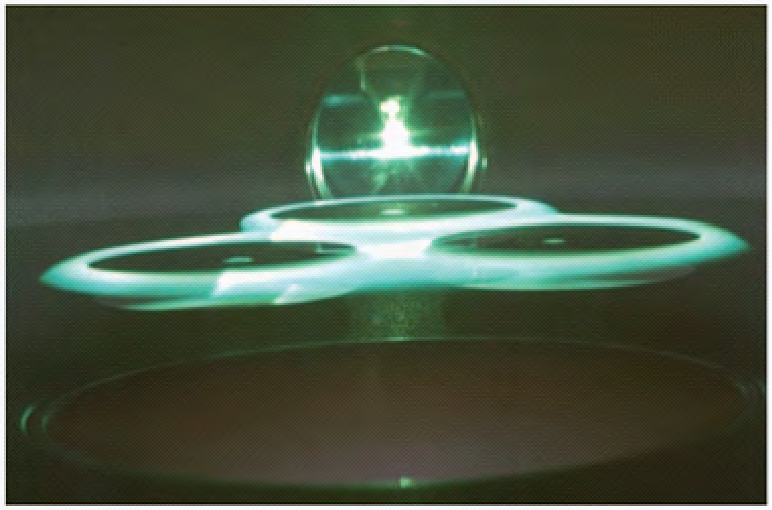

Figure 1-6: Rings of dust particles encircling silicon wafers in a plasma processing device[24]

the they acquire an electrical charge and have mass respectively.

- The charged dust particles act as active participant in the various collective plasma processes.

- As the dust particles are mostly of macroscopic dimension, therefore they can be visualized easily, and one get important observation about dust at the most fundamental level.

At the beginning of 20th century it was discovered that electromagnetic waves of low frequency have the ability of propagation in the conducting fluids, like plasmas, though they cannot able to propagate in rigid conductors. In 1942, Hannes Alfvén examined the characteristics of plasmas, supposing that he plasma medium is incompressible fluid which are to be a highly conducting and magnetized. He observed that a distinctive wave form appear in the fluid, whose direction of propagation were along the magnetic field. These waves mode is now known as the shear or torsional Alfvén waves. In 1949, Lundquist experimentally verified the presence of this shear Alfvén wave, in a conducting fluid of mercury. The interest of the waves discovered by Alfvén for plasma of the spatial and astronomical objects was realized very soon. In 1950, Herlofsen study the system of compressible plasma, and found the fast and slow magnetoacoustic waves along with the shear Alfvén wave. In 1990 Rao et al investigated a system of multicomponent of dusty plasma (in which collision of its contiuents are neglected) contain electrons, ions and negative charge dust particles, and predicted the Dust Acoustic.

It is observed that greater number of electrostatic and electromagnetic waves are propagating in unmagnetized and magnetized plasmas[27, 28]. For in excess of a solitary decade, it is observed and now established that the dispersion properties low frequency waves are altered within the sight of substantial charged dust particles in a plasma, and may likewise present novel waves [29, 30]. For example, by including the dust particle dynamics in an dusty plasma system in the absence of magnetic field ,the dust-acoustic waves[31] are generated, while the changing of constituents the plasma model, the dust ion-acoustic waves[32] is produced due to balancing of charges. Dusty plasma in presence of magnetic field, a large number of new modes have been appeared to exist, involving the possibilities of modified ion-cyclotron [33, 34] waves, modified lower-hybrid waves[35], dust drift waves[36], dust convective cells [37], dust magnetosonic waves[38], dust Alfven [39] and dust whistler [40] waves. The wave frequencies are significantly affected by the magnetic field only when the dust plasma particles are magnetized. D'Angelo has studied the characteristics of electrostatic dust-cyclotron waves[41] as well as dust-dust lower-hybrid waves[42] in magnetoplasmas system with double dust particles. And physical modeling of plasma system is characterized on the basis of study in which one are interested. The magnetohydrodynamic description is applicable, when the plasma processes are considered slow enough and the average particle separation and the mean free path of the particles is much less than the characteristic dimension of the plasma system.

1.5 Layout of the Thesis

This thesis is arranged in the way as chapter 1 deals with the basic introduction of plasma physics and particularly dusty plasma with brief history of plasma physics. Different aspects of dusty plasma and the dust contaminated systems have been discussed briefly from outside the atmospheric layers to the laboratories nowadays. chapter 2 deals with some important dusty Plasma Characteristics like charging processes of dust species and various types of waves in both magnetize and non magnetized dust ion-acoustic[32] dusty plasmas, here we have concentrate our attention on low-frequency electromagnetic waves. Chapter 3 deals with linear theory and a generalized dispersion relation of electromagnetic waves in a homogeneous, magnetized dusty plasma. The generalized equations for Hall-magnetohydrodynamic are calculated by supposing stationary heavy charged dust particles, and then by applying Fourier transformation we get the dispersion relation. This investigated to get it influence of stationary charged dust particles on different modes of electromagnetic wave. In the chapter 4, dusty plasma system in the presence of external magnetic field and linear characteristics of the R-D-MHD are studied. Chapter 5 deals with the hydrodynamic properties for double dust particles in a quasineutral magnetized plasma. This system is used to determine linear equations to calculate the ultra low frequency (comparing with gyrofrequency of dust particles) for dispersive dust Alfven waves (DDAWs). For DDAWs frequency is determined by linear approximation.

Chapter 2

Theoretical background

2.1 Dusty Plasma Characteristics

2.1.1 Quasi-neutrality

When there are no external disturbances, the overall charge must be neutral on any plasma system in order to sustain quasi-neutrality condition. Mathematically, we may write the condition for charge neutrality in the absence of dust as $n_{e0} = n_{i0}$ assuring that the positive and negative charge concentrations should be equal at equilibrium, where n_{e0}, n_{i0} is the equilibrium electron and densities, respectively. Whereas for dust particle the relation is modified as

$$\gamma Z_d n_{d0} + n_{i0} = n_{e0} \tag{2.1}$$

where $\gamma = +1$ (-1) for positive (negative) dust[43].

2.1.2 Debye shielding of dusty plasma

The basic characteristic of plasma is its capability to shield out electric field, that is supply to it. Suppose electric field is supplied to the plasma which consist on electrons, ions and dust particles which are either positively charged or negatively charged by putting a charged ball into dusty plasma. Opposite charges would attracted toward ball, if ball is positively charged then cloud of dust particles(which are negatively charged) and electrons would formed around it, if ball is negatively charged then cloud of dust particles(which are positively charged) and ions would formed around it. In the case when plasma is cold then there will be perfect shielding because thermal motion do not exist and number of charges on the surface of ball will be equal to charges inside the ball without producing electric field outside the cloud. Contrary, in case of finite temperature, particles residing on cloud corner would leave the cloud because of sufficient thermal energy and weak electric field. Thus there will be no perfect shielding. The boundary of cloud occurs at radius at which potential energy and thermal energy are approximately equal that is $K_B T_s$, K_B is Boltzmann constant and T_s temperature of species in plasma.

For the calculation of debye length, we assumed that the cloud potential $\varphi_s(r)$ at central point $(r = 0)$ is φ_{so}. Negatively charged heavy dust grains forms uniform background with electrons and ions having Boltzmann distribution, given as

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{k_B T_e}\right) \tag{2.2}$$

$$n_i = n_{i0} \exp\left(-\frac{e\varphi}{k_B T_i}\right) \tag{2.3}$$

where are $n_{e0}\,(n_{i0})$ electrons(ions)number densities away from the cloud. Poisson equation is in the form

$$\nabla^2 \varphi_s = 4\pi \left(en_e - en_i - q_d n_d\right) \tag{2.4}$$

where n_d number density of dust particles assuming that inside and outside of cloud dust number is same $q_d n_d = q_d n_{d0=} en_e - en_i$. Substituting Eq. (2.2) and Eq. (2.3) into Eq. (2.4) and assuming that $\frac{e\varphi}{k_B T_e} \ll 1$ and $\frac{e\varphi}{k_B T_i} \ll 1$,we obtained

$$\nabla^2 \varphi_s = \left(\frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2}\right) \varphi_s \tag{2.5}$$

where λ_{De} electron debye length and λ_{Di} ion debye length given as

$$\lambda_{De} = \sqrt{\frac{k_B T_e}{4\pi n_{e0} e^2}}, \tag{2.6}$$

$$\lambda_{Di} = \sqrt{\frac{k_B T_i}{4\pi n_{i0} e^2}} \tag{2.7}$$

We get deby radius of dusty plasma by assuming $\varphi_s = \varphi_{so} \exp\left(\frac{-r}{\lambda_D}\right)$

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}} \tag{2.8}$$

where sheath thickness is λ_D which depend upon temperature and number density.

2.1.3 Length scales

The dusty plasma parameters of length scale are effective debye length, dust radius, and the distance between dust particles on average, denoted by λ_D, ρ, Λ respectively. Size of dust is always smaller than λ_D and Λ. From the two cases, if (i) $\rho \ll \lambda_D \ll \Lambda$, debye shielding is formed since each particle is surrounded by electrons and if (ii) $\rho \ll \Lambda \ll \lambda_D$, the heavy dust particles plays a role in collective behavior for the distance between them is even less than the effective debye length as derived in the above $\lambda_D = \lambda_{De}\lambda_{Di}/\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}$ where λ_{De} and λ_{Di} are electron and ion debye lengths, respectively.[44] In a plasma, electric field is very large that pressure becomes smaller but it does not means that plasma is cold. Debye sphere will be formed everywhere it comes across charge fluctuations.

2.1.4 Characteristic frequencies of dusty plasma

At equilibrium, there is no electric field in the plasma but it sets up when plasma is perturbed from its mean position. The direction of field is as such to pull the charged particles towards their mean position but due to inertia, they resist and start oscillating about mean position. The characteristic frequency of plasma for general case k^{th} is given by

$$\omega_k^2 = \frac{4\pi q_k^2 n_{k0}}{m_k} \tag{2.9}$$

As dusty plasma contain species like electrons,ions and dust particles, all these have different oscillations frequencies depending upon the charge and mass of species. This can be explained as around the charged dust particles ions oscillate with ion plasma frequency

$$\omega_{pi} = \sqrt{\frac{4\pi n_{io} e^2}{m_i}}$$

electrons oscillate with electron plasma frequency

$$\omega_{pe} = \sqrt{\frac{4\pi n_{eo} e^2}{m_e}}$$

around the ions and dust grains oscillates about their mean position alongwith dust plasma frequency

$$\omega_{pd} = \sqrt{\frac{4\pi n_{do} z_d^2 e^2}{md}}$$

There are many important characteristics frequencies which are related to the collision of stationary neutrals with plasma species i.e. electrons,ions and dust particles. There is collision frequency of ions and neutrals ν_{in}, collision frequency of electrons and neutrals ν_{en} and collision frequency of dust and neutrals ν_{dn} respectively. Because of collision of stationary neutrals with plasma species their collective oscillations damped and their amplitudes decreases gradually. When plasma frequency ω_p greater then collision frequency then their will be slightly damping in oscillations.

2.1.5 Coulomb coupling parameter

The coulomb coupling parameter by which dusty plasma crystal formation possibility is demonstrate is special feature of dusty plasma, which is the ratio of potential energy to the thermal energy of dust. In shielding effect coulomb potential energy of dust is

$$U = \frac{q_d^2}{l} \exp\left(\frac{-|l|}{\lambda_D}\right) \tag{2.10}$$

and thermal energy of dust is given by $K_B T_D$. Thus coulomb coupling parameter is given by

$$\Gamma c = \frac{z_d^2 e^2}{l K_B T_D} \exp\left(\frac{-|l|}{\lambda_D}\right) \tag{2.11}$$

A dusty plasma is strongly coupled when $\Gamma c \gg 1$ and weakly coupled when $\Gamma c \ll 1$.

2.1.6 Charging of Dust grains

The central point in the story of dusty plasma is to know the charging processes of dust particles which are present in most of the plasma bodies. It mainly depends on the environment that surrounds the dust particulates, some important ways of charging are

(i) interaction that occurs between dust grains and gaseous plasma particles.

(ii) energetic particle's interaction with dust particle.

(iii) interaction of photons with dust particles.

The plasma particles are collected by dust particles which are immersed in a gaseous plasma. So dust grains act as probe for surrounding plasma environment. In this way, plasma particles that flow onto their surface, eventually charge these dust particles. In the case of energetic particles that are incident on dust grains, there are possibilities of their backscattering or refection and passing from these dust grains. In result, they may lose their energy partially or fully in passing through the dust and the portion of this lost energy goes into exciting other electrons, that may escape from this dust particle. The emitted electron are usually called secondary electron. So the positive dust particle results from the emission of these secondary electrons. The interaction of photons with dust grain surface causes the release of photoelectrons from their surface and the process is called photoemission. The dust grain becomes positively charged after the emission of these photoelectrons. The emitted electrons sometime captured by other dust grain and results into negatively charged dust particles. Dust particles tend to acquire a negative charge. Dust particles acquire charge according to the particle radius and the plasma density. The dust particles charge fluctuation is an essential feature of dusty plasma [45]. Hence the electric charge on the dust particles has to be determine as a dynamic variable because it is a time dependent quantity. In most of the cases, dust acquires a negative charge [46].

For dust particle in plasma having temperature denote by T_e for electrons and for ions T_i the fact that the flux of ions thermal speed are smaller than electrons thermal speed, this will form the dust particle charge Q and negative surface potential ϕ_s. On surface of dust the electrons and ion currents can also induce due to the impinging plasma particles on to a dust particle. Here the current is a functions of the ϕ_s of a dust particle. By adding these currents due to different charging calculate the charging rate $\frac{dQ}{dt}$of the dust particle. A neutral dust

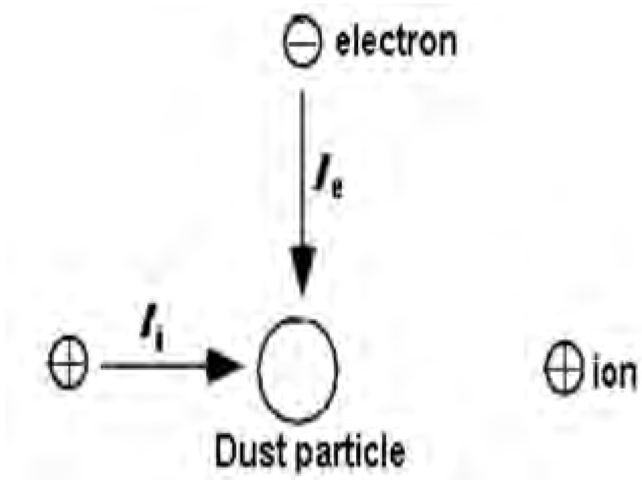

Figure 2-1: Charging of Dust particle [46]

particle when immersing in a plasma will become charging by gaining the electrons and ions from plasma according to the relation [47].

$$\frac{dQ}{dt} = I_e + I_i$$

For a plasma model whose contitunent are electron and ion, when $I_e + I_i = 0$, this show the charges are in the state of equilibrium. Fig(2.1) shows a simplest process of dust particle charging. The charge upon dust particles depend on the surface potential $Q = C\phi_s$, where C is capacitance constant for a dust particle in plasma [48].

When dust particle absorbed ions from plasma it become positively charging, and as a result its surface potential is increased. Thus it observed that the electron and ions currents are affects the surface potential. The negative surface potential is often attracting ions and repelling electrons. The current density electron is then decrease and the ion current increase ensuring that the net current at equilibrium is zero. Since from plasma potentials the dust charges are determined and can fluctuate, and dust particles may appear in all dimensions in an almost limit from micro molecules to rock fragments. So its not surprising that the research field of dusty plasma is wide and attractive now.

2.2 Types of Forces on dust particles

To understanding the motion and transport and collision of dust particles in a plasma, it is important to know the about various forces, act on it. When the dust particles are submerged in a plasma, it observed that several forces act on dust. From discharging process in laboratory,it is shown that a significant amount gravitational force experienced by massive dust particles having size the of order of micrometer. The electrostatic forces are also experience by dust particle, when it became charged. The main forces acting on a dust particle are the electrostatic force, force of gravity, neutral drag force, ion drag force and thermophoretic force. Explanation of these forces related closely to the treatment (transport of dust particles in discharging plasmas) shown in ref [49]. In the following sections, these forces are detailed.

2.2.1 The Force of gravity

In libratory experiments, massive dust particles are influenced by the force of gravity. This gravitational force is directly related to the mass of the dust particle, and hence to its mass density and to its cube of radius (volume).

$$\mathbf{F}_g = m_d\mathbf{g} = \frac{4}{3}\pi\rho r_d^3\mathbf{g} \tag{2.12}$$

This force negligible for dust particles whose size is less than micrometer($r_d \prec 1\mu m$) immersed in a plasma but not for particles ($r_d \geq 1\mu m$). Where g the acceleration due to gravity on the Earth surface, and ρ represent the density of the dust particles. Moreover, when the discharge is switched off, this force can not be neglected for any dimension of dust particles, therefore the force of static charges and the ion drag force disappeared.

2.2.2 The Electrostatic force

When a dust particles submerged in a plasma system it collect an electric charge, they are due to an electric force $\mathbf{F}_e$. Where this force produced by the electric field $\mathbf{E}$ that may existing in the framework of plasma.

$$\mathbf{F}_e=Q\mathbf{E} \tag{2.13}$$

The electric field is much larger in the sheaths adjacent to the wall boundary of the plasma and much smaller in the bulk of the plasma due to the quasineutrality. The negative charged particles are pushes by this force towards the center (core) plasma center from the electrodes, cause the trapping of charged dust particles in the discharge. The dominance of this force is only for nano sized particles while micro scale dimension particles can suspend at the region of the sheath, where the gravitational force is balances by electric field. The positive Debye sheath encompassing the dust particle, which shields its long range Coulomb field were discussed by Hamagushi et al in his work [50].

2.2.3 The neutral drag force

The most useful constituent is the neutral gas in libratory experiment and have small ionization fraction. Due to the collisions with the background neutral gas atoms or molecules a force produced called neutral drag force and therefore it is proportional to the pressure of gas. Utilizing the kinetic molecular gas theory presented by Graves et al. [51], the mathematical form of neutral drag force is approximated as:

$$\mathbf{F}_n = -\frac{4}{3}\pi\rho r_d^2 n_n m_n \mathbf{v}_{th_n} (\mathbf{v}_d - \mathbf{v}_n) \tag{2.15}$$

This force is proportional to r_d^2. where $\mathbf{v}_d$ and $\mathbf{v}_n$ denote the average velocity of the dust particles and the average velocity of the gas respectively. The neutral drag force is act as a resistance for a particle moving through a gas, and simply acts as a damping force on the velocity of the particles in case of $\mathbf{v}_n = 0$.

2.2.4 Ion drag force

The momentum transfer to the dust particles occur due to collisions with ions, as a result the ion drag force produced. Positive ions in the direction of the plasma sheaths can transferred their motion and some energy to the dust particles in the presence of electric field. The ions may be collected or may scattered. This force is the combination of two parts: the collection force $\mathbf{F}_{ic}$ and the orbit force $\mathbf{F}_{io}$ because of the immediate effect of positive ions with the dust particle and because of the electrostatic interactions between the ions and the dust particle respectively. This force is mathematically written as:

$$\mathbf{F}_i = \mathbf{F}_{ic} + \mathbf{F}_{io} \tag{2.16}$$

The condition to attained the ion drag force is that the ion interaction with dust particle outside the Debye length of the particle can be neglected. As $\mathbf{F}_{ic}$ is because of transfer motion from the ions and collect by the dust particles, and every ion transfers the amount of momentum $m_i\mathbf{v}_i$, so the force is given by

$$\mathbf{F}_{ic} = n_i m_i \mathbf{v}_i \mathbf{v}_s \pi b_c^2 \tag{2.17}$$

where n_i, m_i, $\mathbf{v}_i$, $\mathbf{v}_s$ are the density, mass, drift speed, and the mean speed of ions approaching the particle respectively given by

$$v_s = \left(\frac{8kT_i\pi m_n}{\pi m_i} + \mathbf{v}_i^2\right)^{1/2}$$

And

$$b_c = a\left(1 - \frac{2e\phi_s}{m_i\mathbf{v}_s^2}\right)^{1/2}$$

is the collision quantity based on orbit motion limited (OML) theory. The orbit force can be written as

$$\mathbf{F}_{io} = 4\pi n_i m_i \mathbf{v}_i \mathbf{v}_s b_{\pi/2}^2 \Gamma \tag{2.18}$$

where

$$b_{\pi/2} = \frac{eQ}{4\pi\varepsilon_0 m_i \mathbf{v}_s^2}$$

is the impact parameter for 90° deflections and

$$\Gamma = \frac{1}{2}\mathbf{In}\left(\frac{\lambda_D^2 + b_{\pi/2}^2}{b_c^2 + b_{\pi/2}^2}\right) \tag{2.19}$$

is the Coulomb logarithm integrated upon the limit from b_c to λ_D. This theory for for the explanation of the ion drag force was presented by Khrapak et al [52] assuming that outer region of the Debye length of the particle,no interaction with the dust particle can be occur.

2.2.5 Thermophoresis

The thermophoretic force in the neutral gas appear because of a temperature gradient. In neutral gas the momentum transfer side due gas atoms hitting the dust particles. In this case a larger amount of momentum from the hotter side hitting the dust than the colder side. So force in the direction of regions of colder gas is

$$\mathbf{F}_T = -\frac{32}{15}\left(\frac{\pi m_n}{8k_B T_n}\right)^{1/2} r_d^2 \mathbf{v} \kappa_{tr} \nabla T_n \tag{2.20}$$

where ∇T_n, κ_{tr}, and m_n represent temperature gradient , the translational part of the thermal conductivity and is the mass of the neutral gas respectively. Jellum et al. [53, 54] have studied the radio frequency glow discharge as a result he observed thermophoresis effects. By utilizing hot or cooled electrodes, the thermophoretic force has likewise been utilized to suspend to dust particles in discharges [55, 56].

2.3 Acoustic Waves in Dusty plasma

It is well known that there are two kinds of acoustic modes are support by uniform, unmagnetized, collisionless dusty plasma with a feeble Coulomb coupling between the charged dust particles. These are the low frequency electrostatic dust-acoustic(DA)[57] and dust-ion-acoustic waves (DIA) [58] which are two modes of electrostatics waves appear when charged dust particles are included to an ordinary e-i plasma system. In the accompanying, we characterize the fundamental physics as well as the mathematical explanation of these wave modes.

2.3.1 Dust acoustic waves

In 1990 Rao et al have been theoretically predicted the DA waves in a dusty plasma system ignoring collision, composed of electrons, ions and dust particles (have negative charged). A DA wave is a kind of longitudinal wave having low frequency, generated due to the compressions and rarefactions of propagation of dust density. This is considered like a fluid containing heavy dust particles, in which the propagation of sound wave in due to the oscillations of the dust particles. In this case the system collective electric fields of the plasma mediated the interactions between the charged dust particles. The collective motion of the dust particles originate the DA waves. The temporal and spatial scales on which the DA wave originate are new and were not present in the ordinary plasma (having electron and ion). The ion and electron thermal velocities are much greater than the DA wave phase velocity. And accordingly, inertia of ion and electron can be ignored and equilibrium is establish in the DA wave potential ϕ. The ion and electron (neglecting inertia) pressure provided restoring force for DA waves while the wave inertia is come from the mass of the dust particle. Here the electric force balanced the pressure gradient, leading to Boltzmann distribution for electron and ion perturbations of number density n_{J1} respectively, in the following

$$n_{e1} \approx n_{e0} \frac{e\phi}{k_B T_e} \tag{2.21}$$

and

$$n_{i1} \approx -n_{i0} \frac{e\phi}{k_B T_i} \tag{2.22}$$

The dust inertia play a very significant role in case DA waves. Consequently, from equation of continuity we derive the number density perturbation for dust particles as below

$$\frac{\partial n_{d1}}{\partial t} + n_{d0} \nabla \cdot \mathbf{u}_d = 0 \tag{2.23}$$

and the dust momentum equation

$$\frac{\partial \mathbf{u}_d}{\partial t} = -\frac{q_{d0}}{m_d} \nabla \phi - \frac{3k_B T_d}{m_d n_{d0}} \nabla n_{d1} \tag{2.24}$$

where n_{d1} is the number density perturbation and $\mathbf{u}_d$ are fluid velocity of dust particles. Eq. (2.21) to Eq. (2.24) are combined by Poisson's equation

$$\nabla^2 \phi = 4\pi \left(e n_{e1} - q_{d0} n_{d1} - e n_{i1} \right) \tag{2.25}$$

The dust charge q_{d0} is supposed to be constant for convenience. By combining Eq. (2.23) and Eq. (2.24) the dispersion relation of DA waves are derive as

$$\left(\frac{\partial^2}{\partial t^2} - 3V_{T_d}^2\nabla^2\right) n_{d1} = \frac{n_{d0}q_{d0}}{m_d}\nabla^2\phi \tag{2.26}$$

Substituting Eq. (2.21) and Eq. (2.22) into Eq. (2.25) we have

$$\nabla^2\phi = k_D^2\phi - 4\pi q_{d0}n_{d1} - en_{i1} \tag{2.27}$$

Assuming $n_{d1} = \hat{n}_{d1}\exp(-i\omega t + ik\cdot r)$ and $\phi = \hat{\phi}\exp(-i\omega t + ik\cdot r)$, where ω is the frequency and k are the wavevector. Using Fourier analysis (i.e. $\partial/\partial t = -i\omega$ and $\nabla = ik$) for Eq. (2.26) and Eq. (2.27), and the dispersion relation of the DA waves are obtain by combining the resultant equations.

$$1 + \frac{k_D^2}{k^2} - \frac{\omega_{pd}^2}{\omega^2 - 3k^2V_{T_d}^2} = 0 \tag{2.28}$$

which gives

$$\omega^2 = 3k^2V_{T_d}^2 + \frac{k^2C_D^2}{1 + k^2\lambda_D^2} \tag{2.29}$$

where $C_D = \omega_{pd}\lambda_D$ represent the DA speed. Since $\omega \gg kV_{T_d}$, we reduce Eq. (2.29) the frequency of DA wave

$$\omega = \frac{kC_D}{\left(1 + k^2\lambda_D^2\right)^{1/2}} \tag{2.30}$$

which in the long-wavelength limit (namely $k^2\lambda_D^2 \ll 1$) reduces to

$$\omega = kz_{d0}\left(\frac{n_{d0}}{n_{i0}}\right)^{1/2}\left(\frac{k_BT_i}{m_d}\right)^{1/2}\left[1 + \frac{T_i}{T_e}\left(1 - \frac{z_{d0}n_{d0}}{n_{i0}}\right)\right]^{-1/2} \tag{2.31}$$

for negatively charged dust particles. Eq. (2.31) show that the pressures of the electrons and ions (ignoring inertia) produce the restoring force for DA waves, and the mass of dust particle are responsible for inertia to support the DA waves. The plasma frequency of dust particles is much greater than the frequency of the DA waves. If the plasma and dust particle parameters are known then one can easily find the phase speed ($V_p = \omega/k$) of the DA wave by using Eq. (2.31). In many laboratory experiments, spectacularly detected waves are the DA waves [59]. The frequencies these DA waves are of the order of 10–20H_z.

2.3.2 Dust ion-acoustic waves

In 1992 Shukla and Silin predicted the DIA waves. The phase velocity of the DIA waves is much smaller than the electron thermal speed and much larger than the ion and dust thermal speeds. The propagation of DIA waves appear due to the ions collective motion that is affected by the existence of dust particles. The electron pressure (neglecting inertia) provided restoring force for DIA waves while the wave inertia is come from the ion mass. The DIA wave is mostly a fast wave having phase speed lying in between electron thermal speed and ion thermal speed. The stationary dust particles do not responding to the DIA wave because the dust plasma frequency is much smaller than the DIA wave frequency. For dust particles having negative charged ($n_{e0} \prec n_{i0}$), the phase speed ω/k of DIA wave is much greater than the ion acoustic speed. It tends to be clarified properly by enhancing the Debye length of the electron because of electron losses on dust particles. Accordingly, the electric field $\mathbf{E} = -\nabla\phi$ is larger. The number density perturbation of electron n_{e1} related with the DIA waves is represented by Eq. (2.21), while here the perturbation of ion number density n_{i1} is calculated from the equation of continuity as

$$\frac{\partial n_{i1}}{\partial t} + n_{i0}\nabla \cdot \mathbf{u}_i = 0 \tag{2.32}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} = -\frac{e}{m_i}\nabla\phi - \frac{3k_B T_i}{m_i n_{i0}}\nabla n_{i1} \tag{2.33}$$

where $\mathbf{u}_i$ is the ion fluid velocity. Combining Eq. (2.22) and Eq. (2.23) we obtain

$$\left(\frac{\partial^2}{\partial t^2} - 3V_{T_i}^2\nabla^2\right) n_{i1} = \frac{n_{i0}e}{m_i}\nabla^2\phi \tag{2.34}$$

Furthermore the number density perturbation of dust in Eq. (2.26) remains intact here in the case of DIA waves. Nonetheless, for dust particles in stationary state, we have $n_{d1} \approx 0$ and the dust plasma period ($= 2\pi/\omega_{pd}$) is much greater than the DIA waves shown on the time scale. Let assuming $\omega \gg kV_{T_i}$, kV_{Td}, using Fourier transformation on Eq. (2.21), Eq. (2.25), Eq. (2.26) and Eq. (2.24) and then combing these equations we get the dispersion relation of DIA wave.

$$1 + \frac{k_D^2}{k^2} - \frac{\omega_{pi}^2 + \omega_{pd}^2}{\omega^2} = 0 \tag{2.35}$$

Because of the massive dust particles(greater mass), the plasma frequency ω_{pd} of dust particles is much smaller than the plasma frequency ω_{pi} of ion. So, Eq. (2.35) gives

$$\omega = \frac{k^2 C_s^2}{\left(1 + k^2 \lambda_{De}^2\right)^{1/2}} \tag{2.36}$$

where $C_s = \omega_{pi}\lambda_{De} = (n_{i0}/n_{e0})^{1/2}\, c_s$ and $c_s = (k_B T_e/m_i)^{1/2}$. For long wavelength of limit i.e. $k^2\lambda_{De}^2 \ll 1$, the Eq. (2.36) reduces to

$$\omega = k\left(\frac{n_{i0}}{n_{e0}}\right)^{1/2} c_s \tag{2.37}$$

Eq. (2.37) shows that the phase speed ($V_p = \omega/k$) of the DIA waves in the model of a dusty plasma is larger than c_s because $n_{e0} \prec n_{i0}$ for negative charge dust particles. In the plasma system increasing in the phase speed is ascribed to the electron density depletion, that why the Debye radius of electron becomes larger. As a result, the DIA waves phase speed is enhancing due to appearance of a stronger charge electric field. Also in 1996, Barkan et al perform a laboratory experiments [60] in which he observed DIA waves. The typical values for frequencies of DIA waves in experimental plasma parameters are of the order tens of kHz. Because of the unbalance in number density of electrons and number density of ions existing in dusty plasmas system, the usual ion acoustic velocity ($C_s = (T_e/m_i)^{1/2}$) is smaller than the phase speed of DIA waves.

2.4 Electromagnetic Waves in Dusty plasma

In the framework of dusty plasma the charged dust particles are moving randomly, during this motion they colliding with one another due to their electromagnetic forces, and likewise responding to any external perturbations. Thus, many variety of waves phenomena generated due to the coherent motions of an constituents of plasma particles. From observation it is clear [61] that a plasma system whose constituents are ion and electron are supports longitudinal as well as transverse waves. In unmagnetized plasma system the Langmuir and ion acoustic waves accompany with number density and potential fluctuations are the examples of longitudinal waves. While, discussing transverse waves propagation in plasma system in the absence of magnetic field are purely electromagnetic waves and they are not accompanied with density fluctuations. Due to the presence external magnetic field in a plasma system, the chances of propagation of a greater variety of longitudinal and transverse waves is expected. So propagation of waves observed in a dusty plasmas is another collective behavior of dusty plasmas. Their are two major types of wave propagation are commonly observed in libratory experiments of dusty plasma are the DA wave and dust lattice wave. The electrons and ion pressure is responsible for dust acoustic wave propagation, traveling with frequency smaller than plasma frequency of ion and electron. The propagation of dust lattice wave on the other hand are detected in crystal state having different kind of modes, including compressional mode, shear or transverse mode depending on the motion of particle relative to the wave motion.

2.4.1 Alfvén waves

At the beginning of the 20th century it was discovered that electromagnetic waves having low frequency include the ability to propagate in conducting fluids, like a plasmas system, even though their propagation in rigid conductors is impossible. Any wave in nature is driven by some restoring force which opposes displacements in the system. In the context of the MHD model , two types of restoring forces are possible: one arising out of magnetic stresses; and the other arising out of pressure gradients. The ideal MHD theory can therefore support two basic types of magnetohydrodynamic waves, the Alfvén waves, called Torsional Alfvén Wave or Shear Alfvén Wave (SAW) and Compressional Alfvén Wave (CAW), also called magnetoacoustic or magnetosonic waves. The magnetoacoustic waves can be further divided into two types, the fast and slow magnetoacoustic waves. Below we give a short and intuitive description of Alfvén's shear and compressional waves features.

Shear Alfvén Wave

The well known basic principle of magnetohydrodynamics is that, when a conducting fluid moves in the presence of a external magnetic field, due to its motion electromagnetic force are raised which are responsible for the produces electric currents. Thus there is an interaction between the magnetic field and the motion of the fluid. The mechanical effects of a magnetic field are equivalent to a hydrostatic pressure, $\mathbf{B}^2/2\mu_0$,combined with a tension, $\mathbf{B}^2/\mu_0$,along the lines of force. In an incompressible fluid, only tension $(\mathbf{B}^2/2\mu_0)$ play its role because the hydrostatic pressure and pressure of the fluid are balance each other. This is a case which are analogue of the theory of stretched strings, which reported that this tension may lead to the possibility of transverse waves propagation along the lines of force, having a velocity $\mathbf{v}_A$ given by

$$\mathbf{v}_A^2 = \mathbf{B}^2 \left(\mu_0 \rho\right)^{-1} \tag{2.38}$$

where ρ is the density of mass of the fluid so it exists due to the balance between the magnetic field tension and ion inertia. The existence of such waves in a plasma was first theoretically predicted by **H. Alfven** in 1942. The name given by Alfven was "hydrodynamic electromagnetic waves" and are now called shear Alfven waves because the perturbations of the magnetic field are perpendicular to the ambient field and wave vector. Figure (1.8) illustrates the transverse nature of fluid motion and the frozen magnetic field lines. There aren't density or pressure fluctuations associated with this wave, the magnetic tension being the only restoring force for it. Because of this characteristics this wave is called "torsional" or "shear" Alfvén's wave.

Compressional Alfvén Wave

By extending the analogy, we expect to observe longitudinal oscillations due to pressure fluctuations. Those are called "magnetoacoustic", or "magnetosonic" or more simply "compressional" waves involving compressions and rarefactions of the plasma and magnetic field lines. Such a wave propagates with a speed v_M so to satisfy the relation.

$$\nabla \left(P + \frac{\mathbf{B}^2}{2\mu_0} \right) = v_M^2 \nabla \rho \tag{3.39}$$

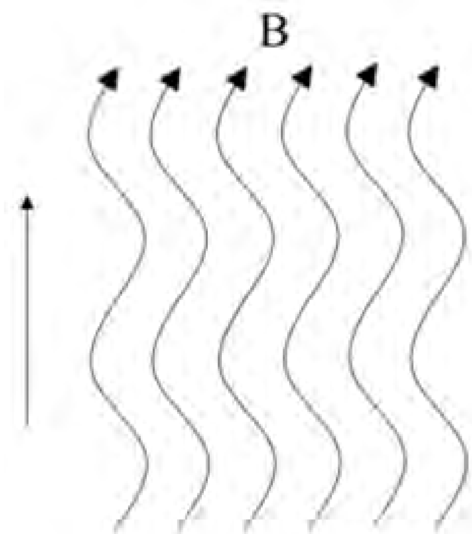

Figure 2-2: Shear Alfvén Wave [62].

which implies

$$v_M^2 = \frac{d}{d\rho}\left(P + \frac{\mathbf{B}^2}{2\mu_0}\right)_{\rho_0} = c_S^2 + \frac{d}{d\rho}\left(\frac{\mathbf{B}^2}{2\mu_0}\right)_{\rho_0} \tag{3.40}$$

where $c_S^2 = \gamma p_0/\rho_0$ is the sound speed and p_0, ρ_0 are respectively the pressure and density of the unperturbed plasma. Note that we have included the magnetic pressure in the restoring force. Because the particles are constrained to the field lines, we have $\mathbf{B}/\rho = \mathbf{B}_0/\rho_0$ and therefore

$$v_M^2 = c_S^2 + \frac{d}{d\rho}\left(\frac{\mathbf{B}_0^2\rho^2}{2\mu_0\rho_0^2}\right)_{\rho_0} = c_S^2 + \mathbf{v}_A^2 \tag{3.41}$$

where $\mathbf{v}_A$ is Alfvén's speed defined above and $\mathbf{B}_0$ is a uniform magnetic field. The nature of a magnetoacoustic wave is illustrated in Figure (1.9). This wave is a mixture of acoustic and magnetic waves, where both types of restoring forces are present. Furthermore it can be split up into two distinct modes, the fast and slow magnetoacoustic waves. For the first one of these modes, the pressure and magnetic restoring forces are roughly in phase, making the mode propagate fast, so it was called the fast mode. The other modes for which these restoring forces

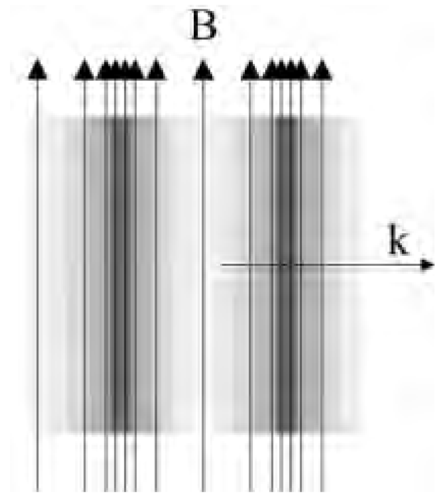

Figure 2-3: Compressional Alfvén Wave [62].

are roughly out of phase, is known as the slow mode. Any arbitrary disturbance in our system, described by ideal MHD theory, can be represented as a superposition of the Alfvén, fast and slow modes.

Chapter 3

Electromagnetic Waves in Hall MHD Dusty Plasma

3.1 Introduction Model

In this chapter, by investigating several kinds of the dispersion relation, a function that based on the wavevector and frequency, deals with various types of waves. For an ideal magneto-hydrodynamic (MHD) theory, the dispersion relation for Alfvén electromagnetic (EM) waves [63] is derived using the continuity as well as equations of motion for the plasmas mass flow, combine with the Faraday's law, were Ohm's law is using for relating the electric field and the mass flow velocity. Studying the Alfvén waves in a dusty plasma system, the restoring force is given by the magnetic pressure and the mass of ion provide the inertia for the waves. The relation shows dispersive properties [64] for Alfvén wave characterize by finite frequency ω, finite Larmor radius of ion , finite ion polarization, and finite electron inertia effects, while keeping the condition that $\omega < \omega_{ci}$, (where ω and ω_{ci} are the finite frequency and ion gyrofrequency respectively). For dispersive dust Alfvén waves, the frozen in field lines are breakdown, and a coupling (linear) show up between different modes [65, 66] for example, Alfvén waves are studying in magnetosonic mode, and the whistlerd may occur. In this plasma fluid model the dynamic of the dispersive Alfvén waves is administered by the equations of Hall MHD [67], in which the summed up structure of Ohm's law is used by including the $\mathbf{J} \times \mathbf{B}$ component, $\mathbf{J}$ and $\mathbf{B}$ represent the current density and magnitude of magnetic field in the framework of plasma repectivily. When the number density of ion is greater than the number density of electron while the charge on dust particles are negative in a dusty plasma system, then the modification in the wave range happens because of the condition of quasineutrality. Here, the dust plasma and dust gyroperiods are much longer than the time scales. While, a new mode of wave [68, 69], involving shear Alfvén waves, and magnetosonic waves of dust particles, which its possibilities is provides by the supposition of the dust particle dynamics. In this case the mass of dust particles provides the inertia, while the wave frequencies will be in the of range of the dust particle gyrofrequency or smaller than this. The application of this ultra low frequency electromagnetic waves is applicable during development of long wavelength Mach cones in rings of Saturn[70].The ultra low frequency fluctuations in astrophysical objects having low temperature and in cometary tails [71] may also account due to these.

Here the linear dispersive properties of electromagnetic waves of transitional frequency ($\omega_{p,d}$, $\omega_{c,d} < \omega < \omega_{c,e}$) in dusty magnetoplasma were studied. Its wavelength is long as compare with the gyroradius of ion and the skin depth of electron ($\lambda > \rho_{Li/e}$, $\omega_{p,e}/C$). We represent these EMW [63] in a dusty magnetoplasma system, having constituents electrons, ions, stationary dust particles with any charge either positive or negative. To derive a general momentum equation for ion and Faraday's law including Hall term too, for this purpose, combining equation of motion of electron (inertialess) with the equation of motion of ion and with Faraday's law and Ampére's law. The summed up set of equations for Hall-MHD are obtain from the close system of equations by combing continuity equation of ion and the condition of quasineutrality. For the derivation of a new dispersion relation, the above mentioned equations and laws for our required Hall-MHD plasma are linearized by applying Fourier transformation and then sum up these. This dispersion relation in a dusty magnetoplasma system shows a relation of linear coupling between the different modes like Alfvén wave, the modified magnetosonic mode, and the whistlers. Different limiting cases of this dispersion relation is analyzed and its showing its contact to earlier study of this work.

Let we are going to consider a dusty plasma system which is completely ionized contains the constituents electrons and ions, along with stationary charged particles of dust, having masses represented by m_e, m_i, m_d and the charges by $q_e = -e$, $q_i = Z_i e$, $q_d = Z_d e$ for electrons, ions and charged dust particles respectively, where e represent the charge magnitude on electron, Z_i

denote the state of charge of ion while density of electrons living on the surface of dust particle respectively. In dusty plasma system ion have mass mostly billion of times smaller than the dust particles mass in experimental and cosmic plasmas [72] where the dust particles dimension are the order of micron. let us assumed that our uniform dusty plasma system is submerged in a uniform magnetic field, denoted by $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ where B_0 shows the external applied magnetic field magnitude along with the $\hat{\mathbf{z}}$ representing unit vector in Cartesian coordinate system in the direction of z- axis. In this model heavy mass dust particles have supposed to be stationary (density n_d is steady while velocity $\mathbf{u}_d$ become zero of dust particles). By focusing on long wavelengths as mentioned above while using the equations of hydrodynamic for electrons with negligible inertia and ions have some inertia. Our calculation begins with the equation of motion in which the number densities for electrons and ions $n_{e,i}$ as well as velocities for electrons and ions $\mathbf{u}_{e,i}$ are represented by equation of continuity and equation of motion.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0 \tag{3.1}$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0 \tag{3.2}$$

$$0 = -\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_e \times \mathbf{B}\right) - \frac{1}{n_e}\nabla p_e \tag{3.3}$$

$$m_i D_i \mathbf{u}_i = Z_i e\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_i \times \mathbf{B}\right) - \frac{1}{n_i}\nabla p_i \tag{3.4}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = \frac{Z_i e}{m_i}\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_i \times \mathbf{B}\right) - \frac{1}{m_i n_i}\nabla p_i \qquad (3.4)$$

The electron inertia have ignored as the electron gyrofrequency $\omega_{ce} = eB_0/m_e c$ are much larger than the wave frequencies. Here we use the convective derivative. The electron and ion pressures denote by $P_{e,i}$ are considered to be obey adiabatic plasma compression. Therefore, by putting $\nabla P_{e,i} = \gamma_{e,i} T_{e,i} \nabla n_{e,i}$, where $T_{e,i}$ is the isothermal energy. Moreover, the electric field for waves is represented by $\mathbf{E}$ and the summation of static as well as wave magnetic fields are represented by $\mathbf{B}$, where $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$. By neglecting the displacement current, the framework is closing with the Maxwellian equations. The Ampere's law peruses as,

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \sum q_\alpha n_\alpha \mathbf{u}_\alpha = \frac{4\pi e}{c} \left(Z_i n_i \mathbf{u}_i - n_e \mathbf{u}_e \right) \tag{3.5}$$

and using Faraday's law of electromagnetism as

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \tag{3.6}$$

The qausineutrality condition at the state of equilibrium is

$$n_{e,0} - Z_i n_{i,0} + Z_d n_d = 0 \tag{3.7}$$

in the above qausineutrality condition, the unperturbed quantities are shown by the 0 subscript.

3.2 Reduced framework of equations

By dispensing with $\mathbf{E}$ from Eq. (3.4) and combing with Eq. (3.3) we obtain

$$\mathbf{E} = -\frac{1}{c} \left(\mathbf{u}_e \times \mathbf{B} \right) - \frac{1}{e n_e} \nabla p_e \tag{3.8}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = \frac{Z_i e}{m_i} \left\{ \left(-\frac{1}{c} \left(\mathbf{u}_e \times \mathbf{B} \right) - \frac{1}{e n_e} \nabla p_e \right) + \frac{1}{c} (\mathbf{u}_i \times \mathbf{B}) \right\} - \frac{1}{m_i n_i} \nabla p_i$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = \frac{Z_i e}{m_i c} \{ (\mathbf{u}_i \times \mathbf{B}) - (\mathbf{u}_e \times \mathbf{B}) \} - \left\{ \frac{z_i}{m_i n_{e,0}} \nabla p_e + \frac{1}{m_i n_i} \nabla p_i \right\} \tag{3.9}$$

which might be summed up with Eq. (3.5), viz.,

$$\mathbf{u}_e = \frac{z_i n_{i,0} \mathbf{u}_i}{n_e} - \frac{c}{4\pi e n_{e,0}} \left(\nabla \times \mathbf{B} \right) \tag{3.10}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = \frac{Z_i e}{m_i c} \left\{ (\mathbf{u}_i \times \mathbf{B}) - \left(\frac{Z_i n_{i,0} \mathbf{u}_i}{n_e} - \frac{c}{4\pi e n_{e,0}} (\nabla \times \mathbf{B}) \right) \times \mathbf{B} \right\} - \frac{1}{m_i n_{i,0}^2} \left\{ \frac{Z_i n_{i,0}^2}{n_{e,0}} \nabla p_e + n_{i,0} \nabla p_i \right\} \tag{3.11}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = \frac{Z_i e}{m_i c} \left\{ \left(1 - \frac{Z_i n_{i,0}}{n_e} \right) (\mathbf{u}_i \times \mathbf{B}) + \frac{c}{4\pi e n_{e,0}} (\nabla \times \mathbf{B})) \times \mathbf{B} \right\} - \frac{1}{m_i n_{i,0}^2} \left\{ \frac{Z_i n_{i,0}^2}{n_{e,0}} \nabla p_e + n_{i,0} \nabla p_i \right\}$$

using neutrality condition from Eq. (3.7)

$$\frac{\partial \mathbf{u}_i}{\partial t}+\mathbf{u}_i.\nabla\mathbf{u}_i = \frac{Z_i e}{m_i c}\left\{(\frac{-Z_{d,0}n_{d,0}}{n_{e,0}})(\mathbf{u}_i \times \mathbf{B}) + \frac{c}{4\pi e n_{e,0}}(\nabla \times \mathbf{B}) \times \mathbf{B}\right\} - \frac{1}{m_i n_{i,0}^2}\left\{\frac{Z_i n_{i,0}^2}{n_{e,0}}\nabla p_e + n_{i,0}\nabla p_i\right\} \tag{3.12}$$

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{-Z_i Z_d e n_d}{n_{e,0} m_i c}\left(\mathbf{v} \times \mathbf{B}_0\right) + \frac{Z_i}{4\pi n_{e,0} m_i}(\nabla \times \mathbf{b})) \times \mathbf{B}_0\} - \left(\frac{c_s^2}{n_{i,0}}\right)\nabla n_1$$

$$\frac{\partial \mathbf{v}}{\partial t} = -\Omega_R\left(\mathbf{v} \times \hat{\mathbf{z}}\right) + \frac{\alpha B_0}{4\pi n_{i,0} m_i}\left(\nabla \times \mathbf{b}\right) \times \hat{\mathbf{z}} - \left(\frac{c_s^2}{n_{i,0}}\right)\nabla n_1 \tag{3.13}$$

by eliminating $\mathbf{u}_e$ from Eq. (3.8), the $\mathbf{E}$ in Eq. (3.6) can then be eliminating using Eq. (3.3) and Eq. (3.9) to get

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\frac{Z_i n_i}{n_{e,0}}(\mathbf{u}_i \times \mathbf{B})\right] - \frac{c}{4\pi e n_{e,0}}\nabla \times \left[(\nabla \times \mathbf{B}) \times \mathbf{B}\right] \tag{3.14}$$

$$\frac{\partial \mathbf{b}}{\partial t} = \left(\frac{Z_i n_{i,0}}{n_{e,0}}\right)\left[\nabla \times (\mathbf{v} \times \mathbf{B}_0)\right] - \frac{c}{4\pi e n_{e,0}}\nabla \times \left[(\nabla \times \mathbf{b}) \times \mathbf{B}_0\right]$$

$$\frac{\partial \mathbf{b}}{\partial t} = \alpha\mathbf{B}_0\nabla \times (\mathbf{v} \times \hat{\mathbf{z}}) - \frac{\alpha c \mathbf{B}_0}{4\pi Z_i e n_{i,0}}\nabla \times \left[(\nabla \times \mathbf{b}) \times \hat{\mathbf{z}}\right] \tag{3.15}$$

$$\frac{\partial n_1}{\partial t} + n_{i,0}\nabla \cdot \mathbf{v} = \mathbf{0} \tag{3.16}$$

where in the above we utilized the condition of quasineutrality, given in Eq. (3.7), along with $Z_d < 0$ for positive and $Z_d > 0$ for negative charge dust particles, and also defined the documentations $\Omega_R = Z_d n_d \omega_{ci}/n_{e,0}$ while $\alpha = Z_i n_i/n_{e,0}$, and $\omega_{ce} = Z_i e\mathbf{B}_0/m_i c$ represent the ion gyrofrequency. Besides, the modified speed of sound for ion are presented mathematically as

$$c_s = \left[\frac{(\gamma_i n_{i,0} T_i + \gamma_e Z_i^2 n_{i,0}^2 T_e/n_{e,0})}{m_i n_{i,0}}\right]^{1/2}$$

Eq. (3.13) to Eq. (3.16) forming a whole framework for dust Hall MHD plasma, in which the advancement of smaller perturbations of the density of ion, the velocity of ion as well as the magnetic field are represented. In electron-ion plasma the ion rotation frequency Ω_R disappears with $\alpha = 1$ or $n_{e,0} = Z_i n_{i,0}$, and Eq. (3.13) without considering Ω_R, the acceleration of ions is provided by two forces $\mathbf{J} \times \mathbf{B}_0$ and $\nabla(p_{e1} + p_{i1})$, while Eq. (3.15) by taking $\alpha = 1$ basically develop the wave magnetic field within the sight of electric field $\mathbf{E} = -\mathbf{v}_e \times \mathbf{B}_0$. We have $\alpha > 1$, for our dusty plasma system of Hall MHD along with negative charge dust particles. Rotatory motion of ions around the negative charge stationary dust particles is because of the subsequent increment in electron fluid speed create another Lorentz force. This Lorentz force is due to the coming effects of two forces $\mathbf{J} \times \mathbf{B}_0$ and $\nabla(p_{e1} + p_{i1})$. The non minor coupling between different modes of waves in the system of dusty plasmas is because of to the rotational force following up on the ions, on the other hand because of $\alpha = 1$, we have increasing the phase speed of Alfvén wave as well as the skin depth of ion. Revolution of particles around the adversely charged static residue particles is

3.3 Linearization of Model Equations

Let us considering propagating of electromagnetic waves of smaller amplitude around the equilibrium state $\{n_{i,0}, 0, \mathbf{B}_0\}$, where the perturbations are $\{n_1, \mathbf{v}, \mathbf{b}\}$. In our uniform dusty magnetoplasma system, we subsequently linearizing the related equations of our system to find out the dispersion relation for the electromagnetic waves propagation. Thus our considered plasma framework has assumed to be uniform in both spatial and temporal regimes, for solving the above equations we have assumed that

$$\mathbf{E} = \mathbf{E}_1 e^{i(k.r - \omega t)}$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b} e^{i(k.r - \omega t)}$$

$$\mathbf{v} = \mathbf{v}_1 e^{i(k.r - \omega t)}$$

by applying Fourier transformation ($\frac{\partial}{\partial t} \to -i\omega$ and $\nabla = i\mathbf{K}$), the ω and $\mathbf{k} = \mathbf{k}_\perp + \hat{\mathbf{z}} k_z$ represents frequency and the wave vector of the waves respectively, then getting from Eq. (3.13), Eq. (3.15) and Eq. (3.16)

$$\omega n_1 - n_{i,0} \mathbf{k}.\mathbf{v} = 0 \tag{3.17}$$

$$\omega \mathbf{v} = -i\Omega_R(\mathbf{v}\times\mathbf{z}) - \frac{\alpha\mathbf{B}_0}{4\pi n_{i,0}m_i} i(k_z\mathbf{b} - b_z\mathbf{k}) + (\mathbf{k}\cdot\mathbf{v})\mathbf{k}\frac{c_s^2}{\omega} \tag{3.18}$$

$$\omega \mathbf{b} = -\alpha\mathbf{B}_0[k_z\mathbf{v} - (\mathbf{k}.\mathbf{v})\hat{\mathbf{z}}] + \frac{i\alpha c\mathbf{B}_0}{4\pi Z_i e n_{i,0}} k_z(\mathbf{k}\times\mathbf{b}) n_{i,0}\mathbf{k}\cdot\mathbf{v} = 0 \tag{3.19}$$

by applying the constraint that $\boldsymbol{\nabla}.\mathbf{B} = \mathbf{0}$, which gives $\mathbf{k}.\mathbf{b} = \mathbf{0}$, and taking dot product of $\mathbf{k}$ with Eq. (3.18) we obtain

$$\mathbf{k}.\mathbf{v} = \frac{\omega}{\omega^2 - k^2c_s^2}\left[\frac{\alpha\mathbf{B}_0}{4\pi n_{i,0}m_i}k^2 b_z - i\Omega_R(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}\right] \tag{3.20}$$

then combining with the $z-$ components of magnetic field of Eq. (3.18) and Eq. (3.19), form

$$\omega b_z = \frac{(\omega^2 - k_z^2c_s^2)k^2V_A^2}{\omega(\omega^2 - k^2c_s^2)}b_z + \frac{i\alpha c\mathbf{B}_0}{4\pi Z_i e n_{i,0}}k_z j_z - \alpha\mathbf{B}_0\Omega_R\frac{(\omega^2 - k_z^2c_s^2)}{\omega(\omega^2 - k^2c_s^2)}(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}} \tag{3.21}$$

where defining $j_z = (\mathbf{k}\times\mathbf{b}).\hat{\mathbf{z}}$, and also the Alfvén velocity of dust particles is $V_A = \alpha\mathbf{B}_0/\left(4\pi m_i n_{i,0}\right)^{1/2} \equiv \alpha v_A$, and $\alpha = Z_i n_{i,0}/n_{e,0}$.

By using Eq. (3.18) and Eq. (3.19), we get the below form

$$\omega j_z = -\alpha\mathbf{B}_0 k_z(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}} - \frac{i\alpha c\mathbf{B}_0}{4\pi Z_i e n_{i,0}}k^2 k_z b_z \tag{3.22}$$

and

$$\omega\left[1 - \frac{\Omega_R^2(\omega^2 - k_z^2c_s^2)}{\omega^2(\omega^2 - k^2c_s^2)}\right](\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}} = \frac{i\alpha c\mathbf{B}_0}{4\pi Z_i e n_{i,0}}k_z j_z + i\frac{\omega\Omega_R(\omega^2 - k_z^2c_s^2)}{\omega^2(\omega^2 - k^2c_s^2)}\frac{\alpha\mathbf{B}_0}{4\pi m_i n_{i,0}}k^2 b_z \tag{3.23}$$

Eqs.(3.21) to Eq. (3.23) form a framework of equation in form of components which are b_z, j_z, and $(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}$. The solution of last two equations [in term of last two and the solutions may then be substituted into Eq. (2.21)].

From the combination of Eq. (3.21) to Eq. (3.23) we derive

$$j_z = -i\left[\omega^2 - \frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)} - k^2V_A^2\right]^{-1} \times \left\{\left[1-\frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\right]\frac{1}{\alpha\omega_{ci}} + \frac{\omega\Omega_R(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\right\} \times k_z k^2 V_A^2 \omega b_z \tag{3.24}$$

and

$$(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}} = i\left[\omega^2 - \frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{(\omega^2-k^2c_s^2)} - k^2V_A^2\right]^{-1} \times \left[\frac{\Omega_R(\omega^2-k_z^2c_s^2)}{(\omega^2-k^2c_s^2)} + \frac{k_z^2V_A^2}{\alpha\omega_{ci}}\right]\frac{k^2V_A}{\sqrt{4\pi m_i n_{i,0}}} b_z \tag{3.25}$$

3.4 Waves Dispersion Relation

Substituting Eq. (3.24) and Eq. (3.25) into Eq. (3.21), we get a generalized dispersion relation

$$\begin{aligned}\left(\omega^2-k_z^2V_A^2\right)\left[\omega^2\left(\omega^2-k^2c_s^2\right)-\left(\omega^2-k_z^2c_s^2\right)k^2V_A^2\right] &= \omega^2\left(\omega^2-k^2c_s^2\right)\Omega_R^2 + \frac{2k_z^2k^2V_A^4\Omega_R}{\alpha\omega_{ci}} \\ &+ \frac{k_z^2k^2V_A^4}{\alpha^2\omega_{ci}^2}\left[\omega^2\left(\omega^2-k^2c_s^2\right)-\Omega_R^2\left(\omega^2-k_z^2c_s^2\right)\right]\end{aligned} \tag{3.26}$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$. From the first perception, we observed that the modification are because of the dust via Ω_R and α in $V_A = \alpha v_A$. The limit (i.e., $k_z = 0$) for perpendicular propagation, the same terms also disappear. So as to increase some understanding, we currently checking the mentioned outcomes in the disappearing dust particle range. While putting $\Omega_R = 0$, $\alpha = 1$and $V_A = v_A$ in Eq. (3.25), we subsequently get

$$\left(\omega^2-k_z^2v_A^2\right)\left[\omega^4-k^2\left(v_A^2+c_s^2\right)\omega^2+k_z^2k^2c_s^2v_A^2\right] = \left(\omega^2-k^2c_s^2\right)\frac{k_z^2k^2c_s^2v_A^4}{\omega_{ci}^2} \tag{3.27}$$

This dispersion relation was determined in [64] and examined in [65] . By substituting different limits, one obtain the electromagnetic ion-cyclotron Alfvén waves, fast as well as slow modes magnetosonic waves, the kinetic Alfvén waves, and whistlers of long wavelength as in following few section.

3.4.1 Electromagnetic ion-cyclotron Alfvén waves

For instance, by using limit $c_s^2 = 0$ and $\mathbf{k}_\perp = 0$ in the dispersion relation, we obtains the dispersive EM ion-cyclotron Alfvén wave with aligned magnetic field, in which the positive and negative (+/-) corresponding to right and left hand circularly polarized waves respectively.

$$\omega = k_z v_A \left(1 \pm \omega/\omega_{ci}\right)^{1/2}$$

3.4.2 Whistler frequency

The whistler frequency is recovered for limits $\omega >> kv_A$ and $c_s = 0$.

$$\omega = k_z k c^2 \omega_{ce}/\omega_{pe}^2$$

3.4.3 Kinetic Alfvén waves

The kinetic Alfvén waves are obtained by using the limits $v_A >> c_s$,$\mathbf{k}_\perp v_A$,$\mathbf{k}_\perp c_s, \omega_{ci} >> \omega >> k_z c_s$

$$\omega \approx k_z v_A \left(1 + \mathbf{k}_\perp c_s/\omega_{ci}^2\right)^{1/2}$$

3.4.4 Magnetosonic modes

For perpendicular wave propagation $\left(k^2 = k_x^2 + k_y^2 = \mathbf{k}_\perp^2\right)$, The fast magnetosonic wave mode may get

$$\omega = \mathbf{k}_\perp \left(v_A^2 + c_s^2\right)^{1/2}$$

For the perpendicular propagation of waves, $k_z = 0$ while $k = \mathbf{k}_\perp = \left(k_x^2 + k_y^2\right)^{1/2}$, we get from the Eq. (3.9) the modified magnetosonic waves mode.

$$\omega^2 = \Omega_R^2 + \mathbf{k}_\perp \left(c_s^2 + V_A^2\right)$$

Note that Rao determined the frequency cutoff $\omega\left(\mathbf{k}_{\perp}=0\right)=\Omega_{R}$ [67], which is not present in ordinary e-i plasmas. For the case when wave propagating in the direction of magnetic field ($k=k_{z}$), using Eq. (3.10) we get

$$\left(\omega^{2}-k_{z}^{2}V_{A}^{2}\right)^{2}=\omega^{2}\Omega_{R}^{2}+\frac{k_{z}^{4}V_{A}^{4}}{\alpha^{2}\omega_{ci}^{2}}\left(\omega^{2}-\Omega_{R}^{2}\right)+2k_{z}^{4}V_{A}^{4}\frac{\Omega_{R}}{\alpha\omega_{ci}} \tag{3.28}$$

3.5 Modified Dispersion Relation

The modified dispersion relaion can actually writing in the more simplest form as

$$k_{z}^{2}V_{A}^{2}=\frac{\omega^{2}\omega_{ci}}{\left(\omega_{ci}+\omega^{2}\right)}\mp\frac{Z_{d}n_{d}}{Z_{i}n_{i,0}}\omega\omega_{ci} \tag{3.29}$$

and which accurately matched with the widely recognized result [58] for the circularly polarized EMWs in a warm magnetoplasma in the direction of magnetic field. While for a cold dusty plasma system ($c_{s}=0$), the above relation Eq. (3.26) become

$$\left(\omega^{2}-k_{z}^{2}V_{A}^{2}\right)\left(\omega^{2}-\omega_{A}^{2}\right)=\omega^{2}\omega_{A}^{2}b+\Omega_{R}^{2}\left(\omega^{2}-\omega_{A}^{2}b\right)+2\alpha\Omega_{R}\omega_{ci}\omega_{A}^{2}b \tag{3.30}$$

the $\omega_{A}=kV_{A}$ while $b=k_{z}^{2}V_{A}^{2}/\alpha^{2}\omega_{ci}^{2}$. Here the limits are $\omega_{A}^{2}b<<\Omega_{R}^{2}$,$\omega^{2}\Omega_{R}/2\alpha\omega_{ci}$,$\omega^{2}$ and $k_{z}V_{A}$ $\omega_{A}<<\omega^{2}$, one obtains

$$\omega^{2}=\Omega_{R}^{2}+\left(k^{2}+k_{z}^{2}\right)$$

whereas in the limits $\omega_{A}^{2}b<<\Omega_{R}^{2}$,$\omega^{2}\Omega_{R}/2\alpha\omega_{ci}$,$\omega^{2}$ and $\omega^{2}<<k_{z}V_{A}\omega_{A}$, we have

$$\omega^{2}=k_{z}^{2}k^{2}V_{A}^{4}/\Omega_{R}^{2}$$

3.6 Conclusion

In this section we have studied the propagation of EMWs having intermediate-frequency ($\omega_{ce}>>\omega>>\omega_{cd}$) and long wavelength ($\lambda>\rho_{Li/e}$, $\omega_{p,e}/c$)) in a uniform warm Hall-MHD dusty plasma whose contitunent are electrons, ions, and stationary charged dust particles. From this investigation it is concluded that because of the existence of the last one the electron flow speed is increasing. The Rao cut-off frequency is obtained by [67] the partition charge increased due to the wave electric field, along with increasing the Alfvén velocity as well as for creating the ion rotation around the immobile charged dust particles. The new dispersion relation we derived is reveals that immobile dust particles automatically change the dispersion relation of coupled dust Alfvén ion-cyclotron-modified magnetosonic waves mode, whistlers in a nontrivial way. Here it is demonstrated the results derived in the previous work matched with the limiting cases of our generalized dispersion relation. The present results are an essential prerequisite to comprehend the dispersive properties of intermediate frequency, long wavelength EMWs in experimental plasma and spatial plasmas (presence of magnetic field), where a significant number density of charged dust particles is available.

Chapter 4

Rao-Dust-Alfvén Waves in Magnetized Plasmas

4.1 Introduction

In this part, the linear characteristics of the Rao-dust-magnetohydrodynamic (R-DMHD) waves [67] in 2-D spatial system will be discussing. Rao reported in his paper [67] that the dispersion properties of the 2-D R-D-MHD waves differ from the propagation of magnetosonic waves in ordinary e-i plasma system in the presence of magnetic field; as a result of the presence of the novel cutoff frequency, because of the availability of charged dust particles. Aside from being interesting from a major perspective, and not all that broadly contemplated up until now, the R-D-MHD modes have been as of late appear [74] to be exciting due to the upper-hybrid waves in a homogeneous dusty plasma associated with a magnetic field. In this chapter we will be study two fold: first to exhibit 2-D R-D-MHD waves, and secondly to discuss the finite amplitude modulation of 2-D R-D-MHD waves. Here we assume the presence of a homogeneous external magnetic field and depending on the 2-D fluid framework depiction, then we will determine scientifically the harmonic reaction of fluid framework to a little distance from point of equilibrium, attempting to understand the job of dust particle. In this chapter we describe the administering equations for the R-D waves in MHD plasma system, and then linearization of these equations and calculation of its harmonic solutions will be described.

We consider completely ionized dusty plasma framework containing electrons, ions, and massive dust particles which are charged, these having masses are m, m_i m_d and charges are e, $q_i = +Z_i e$, and $q_d = sZ_d e$, henceforward represented by e,i, and d, respectively. For simplicity the masses and charges of dust particles will be considered steady. Here negatively and positive charged dust particles, both cases are taking into account, differentiated by the sign of charge $s = sgn\ q_d = \pm 1$. This plasma system is is then submerging in a homogeneous magnetic field (external) with magnitude $\mathbf{B}_0 = B_0\hat{z}(B_0\text{=const})$ along the direction of $\hat{z}$axis.

4.2 Evolution equations

We are going to discuss the MHD framework of equations whose components are the electrons and the ions, while considering the heavy dust particles are considered as a stationary ($n_d \approx n_{d,0}$), whereas we are interested in time scales whose value is a lot shorter than the plasma period ($\sim \omega_{p,d}^{-1}$) dust particle values. While number density $n_{i.e}$ and velocity $\mathbf{u}_{i,e}$ of electron and ion are represented by the equation of continuity and equation of motion as in following.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0 \tag{4.1}$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0 \tag{4.2}$$

$$\mathbf{E} + \frac{1}{c}\mathbf{u}_e \times \mathbf{B} = \mathbf{0} \tag{4.3}$$

$$m_i D_i \mathbf{u}_i = Z_i e \left(\mathbf{E} + \frac{1}{c}\mathbf{u}_i \times \mathbf{B} \right) \tag{4.4}$$

where for our dusty plasma model inertia of electron and pressure (temperature) effects (for all species i.e., electron/ion) have completely neglected. The $\mathbf{E}$ and $\mathbf{B}$ of magnitude $\mathbf{E} = \mathbf{0} + \mathbf{E}_1$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$ represent the total electric and magnetic fields respectively. Where the index 0 and 1 in the subscript represent the external and wave field components respectively. In this whole chapter, we shall suppose that the components of electric field $\mathbf{E}_1 = (E_{1,x}, E_{1,y}, 0)$ and magnetic fields $\mathbf{B}_1 = (0, 0, B_1)$, where $E_{1,x/y}$ and B_1 have no restriction for dependence on $\{x, y, t\}$. The whole framework here closing with Maxwell's equations; using Ampere's law ignoring the displacement current part, we have

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \sum q_\alpha n_\alpha \mathbf{u}_\alpha = \frac{4\pi e}{c} \left(z_i n_i \mathbf{u}_i - n_e \mathbf{u}_e \right) \tag{4.5}$$

and Faraday's law electromagnetism are

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \tag{4.6}$$

It is important to explain that condition $\nabla \cdot \mathbf{B} = \mathbf{0}$ mean $\mathbf{k} \cdot \mathbf{b} = 0$, while diminishes to $\partial B / \partial z = 0$. Using the condition of quasineutrality that holds at state of equilibrium

$$n_{e,0} - z_i n_{i,0} + z_d n_d = 0 \tag{4.7}$$

Therefore we have interest in the perpendicular propagation of waves in the direction of external magnetic field. Likewise it is supposed that, all over investigation, the velocities, wavenumber, and the electric field all lie in the xy plane of Cartesian coordinate system. From Eq. (4.3) it is shown that $\mathbf{E}$ is perpendicular to $\mathbf{u}_e$ and $\mathbf{B}$.

4.3 Reduced system of equations

The elimination of $\mathbf{E}$ from Eq. (4.3) and Eq. (4.4), we get

$$m_i D_i \mathbf{u}_i = Z_i \frac{e}{c} \left(\mathbf{u}_i - \mathbf{u}_e \right) \tag{4.8}$$

combining with Eq. (4.5), to eliminate $\mathbf{u}_e$, such as

$$\mathbf{u}_e = Z_i \frac{n_i}{n_e} \mathbf{u}_i - \frac{c}{4\pi e n_e} \left(\nabla \times \mathbf{B} \right) \tag{4.9}$$

$$m_i D_i \mathbf{u}_i = Z_i \frac{e}{c} \left(\mathbf{u}_i - \left(Z_i \frac{n_i}{n_e} \mathbf{u}_i - \frac{c}{4\pi e n_e} \left(\nabla \times \mathbf{B} \right) \right) \right) \tag{4.10}$$

$$m_i D_i \mathbf{u}_i = Z_i \frac{e}{c} \left(1 - \frac{Z_i n_i}{n_e} \right) \left(\mathbf{u}_i \times \mathbf{B} \right) + \frac{Z_i}{4\pi n_e} \left(\nabla \times \mathbf{B} \right) \times \mathbf{B} \tag{4.11}$$

$$m_i D_i \mathbf{u}_i = Z_i \frac{q_d n_d}{n_e c} (\mathbf{u}_i \times \mathbf{B}) + \frac{Z_i}{4\pi n_e} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$= Z_i \frac{q_d n_d}{n_e c} (\mathbf{u}_i \times \mathbf{B}) + \frac{Z_i}{4\pi n_e} \left[\mathbf{B} \cdot \nabla \mathbf{B} - \frac{1}{2} \nabla \mathbf{B}^2 \right] \tag{4.12}$$

we have utilized the condition quasineutrality, $n_e - Z_i n_i - s Z_d n_d = 0$ in the above equation. In the first estimate, we considering that the strength of magnetic field were very weak and a nonuniform plasma system, the ions and the electrons [because of Eq. (4.9)] are exposed to a rotation because of existence of the charged dust particle, as additionally appeared in model [68] and [58]. The Lorentz force (centripetal) in Eq. (4.12), related with the frequency of rotation, is directly related to the charge q_d of dust particle (become zero lake of it). Presently, by the elimination of $\mathbf{E}$ in Eq. (4.3) and Eq. (4.6) and utilizing Eq. (4.9), we get

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\frac{Z_i n_i}{n_e} (\mathbf{u}_i \times \mathbf{B}) \right] - \frac{c}{4\pi e} \nabla \times \left[\frac{1}{n_e} (\nabla \times \mathbf{B}) \times \mathbf{B} \right] \tag{4.13}$$

Note that Eqs.(4.8)–(4.13) formed the novel electromagnetic waves of low-frequency, related to the existence of charged dust particles, as was latterly appeared in model[68]. Eq. (4.4), Eq. (4.6) to Eq. (4.8) in that.

The system of Eq. (4.10) and Eq. (4.11) is not end with $\mathbf{B}$ and $\mathbf{u}_i$, it likewise includes n_e and n_i, except if one constrains the examination to little perturbations of first order from equilibrium. Differently, for a consistent definition, one ought to utilize the complete system of Eq.(4.1) to Eq.(4.6).

The Eq. (4.1) to Eq. (4.6) formed a close framework depicting the development of the state vector $\mathbf{S} = (n_e, n_i, \mathbf{u}_e, \mathbf{u}_i, \mathbf{E}, \mathbf{B})$. Let supposing that magnetic field $\mathbf{B} = B\hat{z} = (B_0 + B_1)\,\hat{z}$ in the direction of unit vector $\hat{z}$, while no other quantity has components in this direction, viz,. $\mathbf{E} = \mathbf{0} + \mathbf{E}_1 = (E_x, E_y, 0)$, and $\mathbf{u}_{e/i} = \left(u_{e/i,x}, u_{e/i,y}, 0\right)$, where $E_{x/y}, u_{x/y}$ and B_1 are functions of $\{x, y, t\}$, we obtain

$$\frac{\partial n_e}{\partial t} + \frac{\partial \left(n_e \mathbf{u}_{e,x}\right)}{\partial x} + \frac{\partial \left(n_e \mathbf{u}_{e,y}\right)}{\partial y} = 0 \tag{4.14}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial \left(n_i \mathbf{u}_{i,x}\right)}{\partial x} + \frac{\partial \left(n_i \mathbf{u}_{i,y}\right)}{\partial y} = 0 \tag{4.15}$$

$$E_x = -\frac{1}{c}\mathbf{u}_{e,y}\mathbf{B} \tag{4.16}$$

$$E_y = +\frac{1}{c}\mathbf{u}_{e,x}\mathbf{B} \tag{4.17}$$

By substituting Eq. (4.16) and Eq. (4.17) in Eq. (4.4) and then combine with Eq. (4.9)we obtain

$$m_i \left(\frac{\partial}{\partial t} + \mathbf{u}_{i,x}\frac{\partial}{\partial x} + \mathbf{u}_{i,y}\frac{\partial}{\partial y}\right)\mathbf{u}_{i,x} = Z_i e \left(E_x + \frac{1}{c}\mathbf{u}_{i,y}\mathbf{B}\right)$$

$$m_i \left(\frac{\partial}{\partial t} + \mathbf{u}_{i,x}\frac{\partial}{\partial x} + \mathbf{u}_{i,y}\frac{\partial}{\partial y}\right)\mathbf{u}_{i,x} = \frac{Z_i e\mathbf{B}}{c}\left(\mathbf{u}_{i,y} - \mathbf{u}_{e,y}\right) \tag{4.18}$$

$$m_i \left(\frac{\partial}{\partial t} + \mathbf{u}_{i,x}\frac{\partial}{\partial x} + \mathbf{u}_{i,y}\frac{\partial}{\partial y}\right)\mathbf{u}_{i,y} = Z_i e \left(E_y - \frac{1}{c}\mathbf{u}_{i,x}\mathbf{B}\right)$$

$$m_i \left(\frac{\partial}{\partial t} + \mathbf{u}_{i,x}\frac{\partial}{\partial x} + \mathbf{u}_{i,y}\frac{\partial}{\partial y}\right)\mathbf{u}_{i,y} = -\frac{Z_i e\mathbf{B}}{c}\left(\mathbf{u}_{i,x} - \mathbf{u}_{e,x}\right) \tag{4.19}$$

Eq. (4.5) and Eq. (4.6) in components form

$$\frac{\partial B}{\partial y} = \frac{4\pi e}{c}\left(z_i n_i u_{i,x} - n_e u_{e,x}\right) \tag{4.20}$$

$$\frac{\partial B}{\partial x} = -\frac{4\pi e}{c}\left(z_i n_i u_{i,y} - n_e u_{e,y}\right) \tag{4.21}$$

and

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c}\frac{\partial B}{\partial t} \tag{4.22}$$

These equations explain the development of the nine physical quantities: n_e, n_i ,$u_{e,x/y}$

$,u_{i,x/y}$,$E_{x/y}$, and B. Eq. (4.16) and Eq. (4.17) are used for the elimination of $\mathbf{E}$ in Eq. (4.22), which at that point progresses toward becoming

$$\frac{\partial B}{\partial t} = -\frac{\partial (u_{e,x} B)}{\partial x} - \frac{\partial (\mathbf{u}_{e,y} B)}{\partial y} \tag{4.23}$$

Eq. (4.14) to Eq. (4.22) will be the basis of the analysis as in the following.

4.4 Linearized Equations: Harmonic Solution

The linearization of equation at equilibrium point is $S_0 = (n_{e,0}, n_{i,0}, 0, 0, 0, B_0)$ where $S = S_0 + S_1$ while by supposing linear first order perturbations are shown as

$$S_1 = \hat{S}_1 \exp i (\mathbf{kx} - \omega t) + c \cdot c$$

$$S_1 = \hat{S}_1 \exp i (kx + ky - \omega t) + c \cdot c$$

where "c.c." stand for complex conjugate, here a linear framework of equations of first order perturbation having amplitudes represented by $\left(\hat{S}_1\right)_J$. By applying Fourier Analysis Eq. (4.14) to Eq. (4.22) are transformed and we obtain

$$-i\omega \hat{n}_{e,1} + i\mathbf{k} (n_{e,0} \hat{\mathbf{u}}_{e1}) = 0 \tag{4.24}$$

$$-i\omega \hat{n}_{i,1} + i\mathbf{k} (n_{i,0} \hat{\mathbf{u}}_{i1}) = 0 \tag{4.25}$$

$$\hat{E}_{1,x} = -\frac{1}{c} \hat{u}_{e1,y} B_0 \tag{4.26}$$

$$\hat{E}_{1,y} = -\frac{1}{c} \hat{u}_{e1,x} B_0 \tag{4.27}$$

$$m_i (-i\omega) \hat{u}_{i1,x} = Z_i e \left(\hat{E}_{1x} + \frac{1}{c} \hat{u}_{i,y} B_0 \right) = \frac{Z_i e B_0}{c} (\hat{u}_{i,y} - \hat{u}_{e,y}) \tag{4.28}$$

$$m_i\left(-i\omega\right)\hat{u}_{i1,y} = Z_i e\left(\hat{E}_{1y} - \frac{1}{c}\hat{u}_{i,x}B_0\right) = -\frac{Z_i e B_0}{c}\left(\hat{u}_{i,x} - \hat{u}_{e,x}\right) \tag{4.29}$$

$$ik_y\hat{B}_1 = \frac{4\pi e}{c}\left(Z_i n_{i,0}\hat{u}_{i1,x} - n_{e,0}\hat{u}_{e1,x}\right) \tag{4.30}$$

$$ik_x\hat{B}_1 = -\frac{4\pi e}{c}\left(Z_i n_{i,0}\hat{u}_{i1,y} - n_{e,0}\hat{u}_{e1,y}\right) \tag{4.31}$$

And

$$ik_x\hat{E}_{1,y} - ik_y\hat{E}_{1,x} = \frac{1}{c}\left(i\omega\right)\hat{B}_1 \tag{4.32}$$

only first order harmonic terms are held here. Now, elimination of the amplitudes of velocity of electron from Eq. (4.28) to Eq. (4.31), for example calculating $\hat{u}_{e1,J}$ from the last two equation and then putting in the previous equation, we quickly acquires

$$i\omega\hat{u}_{i1,x} + s\Omega_{c,i}\frac{Z_d n_{d,0}}{n_{e,0}}\hat{u}_{i1,y} = i\Omega_{c,i}\frac{c}{4\pi e n_{e,0}}\hat{B}_1 k_x \tag{4.33}$$

$$i\omega\hat{u}_{i1,y} - s\Omega_{c,i}\frac{Z_d n_{d,0}}{n_{e,0}}\hat{u}_{i1,x} = i\Omega_{c,i}\frac{c}{4\pi e n_{e,0}}\hat{B}_1 k_y \tag{4.34}$$

Also, one may substitute from Eq. (4.26) and Eq. (4.27), into Eq. (4.32) in order to obtain

$$\mathbf{k}\cdot\hat{\mathbf{u}}_{e1} = \frac{\omega}{B_0}\hat{B}_1 \tag{4.35}$$

and, once more, use Eq. (4.28) and Eq. (4.29) to eliminate $\mathbf{u}_{e1}$ in it

$$\frac{\omega}{B_0}\hat{B}_1 = \frac{Z_i n_{i,0}}{n_{e,0}}\mathbf{k}\cdot\hat{\mathbf{u}}_{i1} \tag{4.36}$$

Now, Eq. (4.33), Eq. (4.34), and Eq. (4.36) form a framework, with respect to velocity and magnetic field $\hat{u}_{i1,J}$ $(J = x, y)$ and $\hat{B}_1$. Explicitly, we comprehend the last equation for $\hat{B}_1$ and substituting in previous two equation; we consequently acquires literally a closed model of equations, along side the definitions referenced below

$$\omega\left(i\omega v_x+\delta\Omega_{c,i}v_y\right)=i\Omega_{c,i}^2L^2k_x\left(k_xv_x+k_yv_y\right) \tag{4.37}$$

$$\omega\left(i\omega v_y+\delta\Omega_{c,i}v_x\right)=i\Omega_{c,i}^2L^2k_y\left(k_xv_x+k_yv_y\right) \tag{4.38}$$

in terms of the component of velocity having amplitudes $v_j=\mathbf{u}_{i1,j}(\ j=x,y)$ for the ion, where it is defined as

1) the gyrofrequency of ion is represented by $\Omega_{c,i}=Z_ieB_0/mc$.

2) the characteristic length is $L=\left(m_ic^2n_{i,0}/4\pi e^2n_{e,0}^2\right)^{1/2}$.

3) the dust parameter having no dimension is, $\delta=Z_dn_d/n_{e,0}=s\left(1-Z_in_{i,0}/n_{e,0}\right)$; see that δ drops in the dust free confinement.

Eq. (4.37) and Eq. (4.38) establish a 2×2 Cramer (linear) framework, in the form of components of velocities u_x, u_y, which have determinant that ought to disappear all together for a non-trivial solution that present; where Ω and $\mathbf{k}$ are frequency and wave vector of wave respectively in this manner are obeying the dispersion relation

$$\omega^2=\omega_g^2+C^2k^2 \tag{4.39}$$

where $k=\left(k_x^2+k_y^2\right)^{1/2}$, we defined the "gap frequency" ω_g

$$\omega_g=\frac{Z_dn_{d,0}Z_ieB_0}{n_{e,0}m_ic}=\delta\Omega_{c,i} \tag{4.40}$$

and the characteristic velocity $C=\Omega_{c,i}L$, represented as

$$C^2=\frac{Z_i^2B_0^2n_{i,0}}{4\pi n_{e,0}^2m_i}=\left(\frac{Z_in_{i,0}}{n_{e,0}}\right)^2\frac{B_0^2}{4\pi n_{i,0}m_i}\equiv\left(1-s\delta\right)^2V_A^2 \tag{4.41}$$

for example $C=\Omega_{c,i}L\equiv\left(1-s\delta\right)V_A$, where $V_A=B_0/\left(4\pi n_{i,0}m_i\right)^{1/2}$ is the Alfve´n velocity. The impact of the dust particle, which results in Eq. (4.3) a finite oscillational frequency with wavelength ($k\rightarrow0$)of infinite limit, while also due to the Eq. (4.4) with a modified phase velocity $v_{ph}=\omega/k$ ($\neq v_g=C^2k/\omega$, for $\delta\neq0$); as in actuality. The phase velocity v_g ($\approx C$ for $\omega\gg\omega_g$) is greater than the Alfve´n velocity V_A with the existence of negative dust particles, and lower than the Alfve´n speed V_A with the existence the positively dust particles.

It is also important to note that Eq. (4.39) coincides with Eq. (9) in Ref. [58]. Another observation here is the presence of the cutoff frequency v_g and the modified Alfve´n speed C, related with the magnetosonic waves of dust particle, which was observed and calculate by Rao for the very first time in his paper [68].

4.5 Conclusion

In this chapter, we considered the linear characteristics of R-D-MHD mode of waves propagating in a 2-D homogeneous cold dusty plasma system associated with magnetic field, where electrons, ions, and charged dust particles are its constituents. The ion rotation and a new cutoff frequency (absent in e-i plasma), is because of the existence of stationary charged dust particles which are also calculated in the famous paper [68] of Rao. The modified magnetoacoustic waves propagation of of dust particle is possible because of the finite inertial effects of ions. The modification in the phase velocity of magnetosonic waves is due to the modification in charged dust particles.

Chapter 5

Alfvén Waves in Plasmas with Double Dust

5.1 Introduction

In this chapter, we study the properties of linear propagation of dispersive dust Alfve´n waves (DDAWs) in a homogeneous plasma system associated with external magnetic field ignoring the particles collision, whose constituents are double dust particles (negative and positive charge particles). The external magnetic field is $B_0\hat{\mathbf{z}}$, in which B_0 and $\hat{\mathbf{z}}$ stands for the magnitude of external magnetic field and the unit vector respectively, unit vector shows the direction of magnetic field which is along the z axis. By comparison with the gyrofrequency of dust particles, the frequency of DDAWs called are Ultralow-frequency (ULF) joined by the finite fluctuations of density and perturbations of sheared magnetic field. Therefore, the magnitude of electric and magnetic fields of waves are presented as $\mathbf{E} = -\nabla\phi - \hat{\mathbf{z}}c^{-1}\partial_t A_z$ and $\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{z}}$, respectively, where ϕ and A_z denote the scalar potential, and the component of the vector potential in the direction of unit vector $\hat{\mathbf{z}}$.

5.2 Evolution equations

The dust particle number density n_σ and velocity $\mathbf{u}_\sigma$ are administered by the equation of continuity and equation of motion

$$\frac{\partial n_\sigma}{\partial t} + \nabla \cdot (n_\sigma \mathbf{u}_\sigma) = 0 \tag{5.1}$$

$$m_\sigma (\partial_t + \mathbf{u}_\sigma \cdot \nabla) \mathbf{u}_\sigma = Z_\sigma e \left(\mathbf{E} + \frac{1}{c} \mathbf{u}_\sigma \times \mathbf{B} \right) \tag{5.2}$$

The component normal to unit vector of the velocities of fluid for negative and positive charge dust particles of the DDAWs for low-frequency are respectively are below

$$\mathbf{V}_{1\perp} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla\phi + \frac{c}{B_0 \omega_{c1}} \left(\frac{\partial}{\partial t} + \frac{c}{B_0} \hat{z} \times \nabla\phi \cdot \nabla \right) \nabla_\perp \phi - v_{1z} \frac{\hat{z} \times \nabla A_z}{B_0} \tag{5.3}$$

$$\mathbf{V}_{2\perp} \approx \frac{c}{B_0} \hat{z} \times \nabla\phi + \frac{c}{B_0 \omega_{c2}} \left(\frac{\partial}{\partial t} + \frac{c}{B_0} \hat{z} \times \nabla\phi \cdot \nabla \right) \nabla_\perp \phi - v_{2z} \frac{\hat{z} \times \nabla A_z}{B_0} \tag{5.4}$$

where the subscript 1 and 2 represents the negative and positive charged dust particles respectively, while the gyrofrequency $\omega_{c\sigma} = Z_\sigma e B_0 / m_\sigma$ of contituent σ (σ stand for (1,2) negative and positive charge dust particles), Z_σ, m_σ, e, and $v_{\sigma z}$ represents the state of charge, mass, magnitude of the electron charge, and the magnitude of dust velocity in the direction of magnetic field, respectively. We have supposed that $|\hat{z} \times \nabla\phi \cdot \nabla| \gg (B_0/c)\, v_{\sigma z} \partial/\partial z$. Putting Eq. (5.3) and Eq. (5.4) into the equation of continuity we obtain

$$D_t n_1 + \frac{c n_{10}}{B_0 \omega_{c1}} D_t \nabla_\perp^2 \phi + n_{10} D_z v_{1z} = 0 \tag{5.5}$$

$$D_t n_2 + \frac{c n_{20}}{B_0 \omega_{c2}} D_t \nabla_\perp^2 \phi + n_{20} D_z v_{2z} = 0 \tag{5.6}$$

where the time and space derivatives are

$$D_t = (\partial/\partial t) + (c/B_0)\, \hat{z} \times \nabla\phi \cdot \nabla$$

$$D_z = (\partial/\partial z) - \left(\frac{\hat{z}\times\nabla A_z}{B_0}\right)\cdot\nabla$$

and $n_{12}\,(\ll n_{10}, n_{20})$ is a smaller perturbation of the dust density and its value in the state of equilibrium are $n_{10,20}$. At equilibrium, we have $Z_1 n_{10} = Z_2 n_{20}$.

Subtracting Eq. (5.6) from Eq. (5.5), using $Z_1 n_1 = Z_2 n_2$ and the parallel (to unit vector $\hat{z}$) component of Ampere's law

$$\begin{aligned} \nabla_\perp^2 A_z &\approx \frac{4\pi e}{c}\left(Z_1 n_{10} v_{1z} - Z_2 n_{20} v_{2z}\right) \\ &\equiv \frac{4\pi e Z_1 n_{10}}{c}\left(v_{1z} - v_{2z}\right) \end{aligned} \tag{5.7}$$

we acquire the determined vorticity equation

$$D_t \nabla_\perp^2 \phi + \frac{V_A^2}{c}\nabla_\perp^2 D_z A_z = 0 \tag{5.8}$$

where $V_A = B_0/\left(4\pi n_{10} m_*\right)^{1/2}$ is the Alfve´n speed for dust particle and the reduced mass $m_* = m_1\left(1 + Z_1 m_1/Z_2 m_2\right)^{1/2}$. In Eq. (5.7) we ignored the displacement current parallel component, while managing the DDAWs that its normal (to $\hat{z}$) phase speed is more smaller than light's speed in vacuum. It is essential to note that Eq. (5.8) may also applicable for a non uniform dusty plasma framework, therefore the current related with the $c\mathbf{E}_\perp \times \mathbf{B}_0/B_0^2$ convection of the density gradient $\partial n_0/\partial x$ at the state of equilibrium is zero. For the parallel components of dust particle, the eqution of motion are

$$D_t v_{1z} = \frac{Z_1 e}{m_1 c}\left(c D_z \phi + D_t A_z\right) \tag{5.9}$$

$$D_t v_{2z} = -\frac{Z_2 e}{m_2 c}\left(c D_z \phi + D_t A_z\right) \tag{5.10}$$

Subtracting Eq. (5.10) from Eq. (5.9) and using Eq. (5.7) we obtain

$$D_t\left(1 - \lambda^2 \nabla_\perp^2\right) A_z + c\partial_z \phi = 0 \tag{5.11}$$

The $\lambda = c/\omega_{pd}$ denote the skin depth of dust particle and

$$\omega_{pd} = \left[4\pi n_{10}e^2\left(Z_1^2 + Z_1Z_2m_1/m_2\right)/m_1\right]^{1/2}$$

it is the effective plasma frequency of dust particles. Eq. (5.8) and Eq. (5.11) consist of a pair for describing the characteristics of low-frequency DDAWs both linear and nonlinear in a dusty plasma system associated with magnetic field containing double dust particles. The nonlinearity in Eq. (5.8) and Eq. (5.11) can be disregarded during linear approximation. Supposing that scaler potential ϕ and component of vector potential A_z are related to $\exp\left(-i\omega t + ik\cdot r\right)$, while ω and $\mathbf{k} = \hat{z}k_z + \mathbf{k}_\perp$ are representing the frequency of waves and wavevector of waves respectively. By Fourier transformation Eq. (5.8) and Eq. (5.11) are transform and then combining these obtaining results of these equations to get the frequency for the ULF DDAWS

$$\omega = \frac{k_zV_A}{\left(1 + k_\perp^2\lambda^2\right)^{1/2}} \tag{5.12}$$

The dispersive characteristics of these derived ULF DDAWs in a dusty plasma system are altogether not quite the same as those of shear Alfve´n waves (SAWs) in an ordinary e-i plasma, and compressional dust Alfve´n waves (CDAWs) in a d-i plasma. The phase velocity of the ULF DDAWs and CDAWs is of the order of the dust particle Alfve´n velocity $B_0/\left(4\pi n_{10}m_*\right)^{1/2}\left(B_0/\left(4\pi n_{10}m_1\right)^{1/2}\right)$ respectively instead of those of Shear Alfven Waves which are of order $B_0/\left(4\pi n_{i0}m_i\right)^{1/2}$. The n_{i0} represent the number density of ion in unperturbed case, while m_i is the mass of ion. The parallel component of force of positive as well as negative charge dust particle caused the dispersion to the DDAWs, and the normal scale dimension of the waves would be denoted by c/ω_{pd}. It must be appeared differently in dispersive properties of the Shear Alfven Waves and Compressional Dust Alfvén Waves that originated from the parallel component of electrons force for the inertial Alfven´n waves and the ion inertial force, respectively, while normal component of ion force are for the kinetic Alfven´n waves and the ion inertial force, respectively. The normal scale sizes of the inertial, kinetic and CDAWs are represented by c/ω_{pe}, c_s/ω_{ci} , and c/ω_{pi}, respectively. The parameter $\omega_{pe}$and ω_{ci} denotes the plasma frequency of electron and ion respectively and c_s denote the speed of ion-acoustic waves. Eqn.(5.8) and (5.11) concede dust particle inertia driven tearing instability in a shear magnetic field $\hat{\mathbf{z}}B_0 + \hat{\mathbf{y}}B_{0y}(x)$, where $\hat{\mathbf{y}}$ is unit vector in the direction y axis, $B_{0y}(x) = -\partial_x A_0$ is the magnitude of sheared magnetic field, and A_0 is vector potential at the equilibrium point. For 2-D perturbations, we consider that

$$A_z = A_0(x) + A_1(x)\exp(\gamma t + iky)$$

and

$$\phi = \varphi(x)\exp(\gamma t + iky)$$

into Eq. (5.8) and Eq. (5.11) to determine

$$a\gamma\left(\partial_x^2 - k^2\right)\varphi + \frac{c}{B_0}ikB_{0y}(x)\left(\partial_x^2 - k^2\right)A_1 - \frac{4\pi}{B_0}ikJ_0'A_1 = 0 \tag{5.13}$$

and

$$\gamma\left[A_1 - \lambda^2\left(\partial_x^2 - k^2\right)A_1\right] + \frac{c}{B_0}ik\left[B_{0y}(x) - \lambda^2\frac{4\pi}{c}J_0'\right]\varphi = 0 \tag{5.14}$$

where $a = c^2/V_A^2$ and $J_0' = \partial_x J_0$. Eq. (5.13) and Eq. (5.14) are investigated in a standard form [74] to get the tearing mode growth rate

$$\gamma = \frac{kV_A\lambda^3\Delta'}{L_s I^2} \tag{5.15}$$

1) $\Delta' = \partial_x A_1/A_1 \succ 0$ is a jump over the internal layer of the width $\left(\gamma^2\lambda^2 L_s^2/k^2V_A^2\right)^{1/4}$.

2) $L_s = B_0/\partial_x B_{0y}$ is the length of the shear magnetic , and I shows the number of order unity.

Nonlinearity of tearing modes may also forming magnetic islands in plasmas systems of dusty plasmas.

5.3 Conclusion

In this last section, we have considered the, the linear characteristics of ULF DDAWs in magnatized plasma system consist of double dust particles. It is reasoned that the linear wave's frequency demonstrates that the phase speed (parallel) and the dust particle Alfve´n speed is of the same order, while the parallel scale diameter were in the order of or larger than the skin depth of dust particles. The existence of magnetic shear, provide the opportunity to calculate some one the particle inertia driven tearing modes, that may frame magnetic islands.

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