Peristaltic flow of Non-Newtonian Nano fluid in an asymmetric channel

By

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MASTER OF PHILOSOPHY

IN

<u>MATHEMATICS</u>

Supervised By

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CERTIFICATE

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

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Prof. Dr. Muhammad Ayub (Chairman)

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2012

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Preface

Peristaltic is a mechanism to pump a fluid by means of progressive area of contraction or expansion on the length of a distensible tube or channel containing fluid. This mechanism has large number applications in physiology, industry and biosciences. Some typical applications include urine transport from kidney to bladder through ureter, vasomotorin small blood vessels. Some recent studies dealing the peristaltic flows different flow geometries are given in the refs.[1-10].

Recently, the peristaltic flows of non-Newtonian fluids have gained considerable attention. In nature there is not a single model which exhibits all the properties of fluids. Therefore, numerous models have been reported to discuss different aspects of the fluid. Some new fluid models are reported in the refs.[11-15].

Very recently, the flow of Williamson fluid model has given much importance due to its large number of applications. Some important studies on the Williamson fluid are reported in the paper [15-20].

The purpose of the present dissertation is to examine the peristaltic flow of Williamson fluid model in an asymmetric channel with or without Nanoparticle volume fraction. The solutions are constructed with the help of regular perturbation method and Homotopy perturbation method.

In chapter one we have examined the peristaltic flow a Williamson fluid in an asymmetric channel. The analytical solutions are found under the assumption of long wavelength and low Reynolds approximations.

Chapter two is devoted to study peristaltic flow of Nano non-Newtonian fluid in an asymmetric channel. The assumption of long wavelength and low Reynolds approximations and then solved analytically with the help of HPM.

Contents

Chapter 1

Peristaltic flow of a Williamson fluid in an asymmetric channel

1.1 Introduction

In this chapter, we have presented the peristaltic flow of a Williamson fluid in an asymmetric channel. The flow equations of Williamson fluid for two dimensional channel are modelled under the assumptions of long wavelength and low Reynolds number approximation. The reduced equations are solved analytically with the help of regular perturbation method. The flow analysis is characterized in detail. This work is the review of Akbar and Nadeem [26], however, the essential details missing in their work are incorporated.

1.2 Fluid model

For an incompressible fluid the balance of mass and momentum are defined as

$$\operatorname{div}\mathbf{U} = 0, \tag{1.1}$$

$$\varsigma\frac{d\mathbf{U}}{dt} = \operatorname{div} + \varsigma\mathbf{f}, \tag{1.2}$$

where ς is the density, $\mathbf{U}$ is the velocity vector, $\mathbf{S}$ is the Cauchy stress tensor, $\mathbf{f}$ represents the specific body force and d/dt represents the material time derivative. The constitutive equation for Williamson fluid is charecterized as

$$\mathbf{S} = -p\mathbf{I} + \boldsymbol{\epsilon}, \tag{1.3}$$
$$\boldsymbol{\epsilon} = -[\mu_\infty + (\mu_{0+}\mu_\infty)(1 - \Gamma\mathring{\gamma})^{-1}]\mathring{\gamma},$$

in which - $p\mathbf{I}$ is the spherical part of the stress due to constraint of incompressibility, $\boldsymbol{\epsilon}$ is the extra stress tensor, μ_∞ is the infinite shear rate viscosity, μ_0 is the zero shear rate viscosity, Γ is the time constant and $\mathring{\boldsymbol{\gamma}}$ is defined as

$$\mathring{\boldsymbol{\gamma}} = \sqrt{\frac{1}{2}\sum\sum \mathring{\boldsymbol{\gamma}}_{ij}\mathring{\boldsymbol{\gamma}}_{ji}} = \sqrt{\frac{\Pi}{2}}. \tag{1.4}$$

Here $\boldsymbol{\Pi}$ is the second invariant strain tensor. We consider the constitutive Eq. (1.4) , the case for which $\mu_\infty = 0$ and $\Gamma\gamma‹1$. The component of extra stress tensor therefore, can be written as

$$\boldsymbol{\epsilon} = -\mu_0[(1 + \Gamma\mathring{\boldsymbol{\gamma}})]\mathring{\boldsymbol{\gamma}}. \tag{1.5}$$

1.3 Mathematical formulation

Consider the peristaltic flow of an incompressible Williamson fluid in a two dimensional channel of width $d_1 + d_2$. Along the channel walls the flow is generated by sinusoidal wave trains propagating with constant speed c . The geometry of the wall surface is defined as

$$Y = N_1 = \bar{d}_1 + \bar{a}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right], \tag{1.6}$$

$$Y = N_2 = -\bar{d}_2 - \bar{b}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \Phi\right], \tag{1.7}$$

where the amplitudes of the waves are a_1 and b_1, λ is the wave length, the width of the channel is $\bar{d}_1 + \bar{d}_2$, c is the velocity of propagation, t is the time and $\bar{X}$ is the direction of wave propagation. The phase difference Φ varies in the range $0 \leq \Phi \leq \pi$ in which $\Phi = 0$ corresponds to symmetric channel with waves out of phase and $\Phi = \pi$, the waves are in phase, further $\bar{a}_1$, $\bar{a}_2$, $\bar{d}_1$, $\bar{d}_2$ and Φ satisfies the condition

$$\bar{a}_1^2 + \bar{b}_1^2 + 2\bar{a}_1\bar{b}_1 \cos\Phi \leq (\bar{d}_1 + \bar{d}_2)^2. \tag{1.8}$$

The equations governing the flow of a Williamson fluid are given by

$$\begin{aligned}
\frac{\partial \bar{U}}{\partial X} + \frac{\partial \bar{V}}{\partial Y} &= 0, & (1.9)\\
\varsigma(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}) &= -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial Y}, & (1.10)\\
\varsigma(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}} &= -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}}. & (1.11)
\end{aligned}$$

Introducing a wave frame(x, y) moving with velocity c away from the fixed frame $(\bar{X}, \bar{Y})$ by the transformation

$$\bar{x} = \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c,\ \bar{v} = \bar{V}\ ,\ \bar{P}(x) = \bar{P}(X, t). \tag{1.12}$$

Also, we introduce the following nondimensional parameters to reduce the number of parameters in the given equations

$$\begin{aligned}
x &= \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c},\ t = \frac{c\bar{t}}{\lambda},\ h_1 = \frac{\bar{h}_1}{d_1},\ h_2 = \frac{\bar{h}_2}{\bar{d}_2}, & (1.13)\\
\tau_{xx} &= \frac{\lambda}{\mu_0 c}\bar{\tau}_{xx},\ \tau_{xy} = \frac{\bar{d}_1}{\mu_0 c}\bar{\tau}_{xy},\ \tau_{yy} = \frac{\bar{d}_1}{\mu_0 c}\bar{\tau}_{\bar{y}\bar{y}},\\
\delta &= \frac{\bar{d}_1}{\lambda},\ \mathrm{Re} = \frac{\varsigma c\bar{d}_1}{\mu_0},\ We = \frac{\Gamma c}{\bar{d}_1},\ P = \frac{d_1^2}{c\lambda\mu_0}\bar{P},\ \gamma = \frac{\mathring{\bar{\gamma}}\bar{d}_1}{c}.
\end{aligned}$$

Making use of Eqs. (1.12) and (1.13) into Eqs. (1.9) to (1.11), the resulting equations interms of stream function $\psi(u = \frac{\partial\psi}{\partial y}, v = -\delta\frac{\partial\psi}{\partial x})$ take the following form

$$\delta\,\mathrm{Re}\left[\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right]\frac{\partial\psi}{\partial y} = -\frac{\partial p}{\partial x} - \delta^2\frac{\partial\boldsymbol{\tau}_{xx}}{\partial x} - \frac{\partial\boldsymbol{\tau}_{xy}}{\partial y}, \tag{1.14}$$

$$\delta^3 \operatorname{Re}\left[\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\right]\frac{\partial \psi}{\partial x} = -\frac{\partial p}{\partial y} - \delta^2 \frac{\partial \boldsymbol{\tau}_{xy}}{\partial x} - \delta \frac{\partial \boldsymbol{\tau}_{yy}}{\partial y}, \tag{1.15}$$

where

$$\boldsymbol{\epsilon}_{xx} = -2[1 + We\mathring{\gamma}]\frac{\partial^2 \Psi}{\partial x \partial y}, \tag{1.16}$$

$$\boldsymbol{\epsilon}_{xy} = -[1 + We\mathring{\gamma}](\frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2}), \tag{1.17}$$

$$\boldsymbol{\epsilon}_{yy} = 2\delta[1 + We\mathring{\gamma}]\frac{\partial^2 \Psi}{\partial x \partial y}, \tag{1.18}$$

$$\mathring{\boldsymbol{\gamma}} = \left[2\delta^2(\frac{\partial^2 \Psi}{\partial x \partial y})^2 + (\frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2})^2 + 2\delta^2(\frac{\partial^2 \Psi}{\partial x \partial y})^2\right]^{\frac{1}{2}}, \tag{1.19}$$

in which δ, Re, We represents the wave, Reynolds and Weissenberg numbers, respectively. Under the assumptions of long wavelength $\delta << 1$ and low Reynold number, neglecting the terms of order δ and higher, Eqs. (1.14) and (1.15) along with (1.16) to (1.19) take the following form

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y}[(1 + We\frac{\partial^2 \Psi}{\partial y^2})\frac{\partial^2 \Psi}{\partial y^2}], \tag{1.20}$$

$$\frac{\partial P}{\partial y} = 0. \tag{1.21}$$

Elimination of pressure from above two equations, yields

$$\frac{\partial^2}{\partial y^2}\left[(1 + We\frac{\partial^2 \Psi}{\partial y^2})\frac{\partial^2 \Psi}{\partial y^2}\right] = 0. \tag{1.22}$$

The dimensionless mean flow Θ is defined by

$$\Theta = L + 1 + d, \tag{1.23}$$

in which

$$L = \int_{h_2(x)}^{h_1(x)} \frac{\partial \Psi}{\partial y} dy = \Psi(h_1(x) - h_2(x)), \tag{1.24}$$

where

$$h_1(x) = 1 + a\cos 2\pi x, h_2(x) = -d - b\cos(2\pi x + \Phi). \tag{1.25}$$

The boundary conditions in terms of stream function Ψ are defined as

$$\Psi = \frac{L}{2}, \frac{\partial \Psi}{\partial y} = -1 \text{ for } y = h_1(x), \tag{1.26}$$

$$\Psi = -\frac{L}{2}, \frac{\partial \Psi}{\partial y} = -1 \text{ for } y = h_2(x). \tag{1.27}$$

1.4 Perturbation solution

Since $Eq.$(1.22) is non-linear equation, its exact solution seems to be impossible subject to these conditions, therefore, we are interested to calculate the analytical solution with the help of famous regular perturbation method.

For perturbation solution, we expand Ψ, L and P as,

$$\Psi = \Psi_0 + \Psi_1 + 0(We^2), \tag{1.28}$$

$$L = L_0 + L_1 + 0(We^2), \tag{1.29}$$

$$P = P_0 + P_1 + 0(We^2). \tag{1.30}$$

Making use $Eqs.$(1.28) to (1.29) into $Eq.$(1.22) and boundary conditions (1.26) and (1.27), and equating the like powers of We, we obtain the following systems

1.4.1 Zeroth order system

$$\frac{\partial^4 \Psi_0}{\partial y^4} = 0, \tag{1.31}$$

$$\frac{\partial P_o}{\partial x} = \frac{\partial^3 \Psi_0}{\partial y^3}, \tag{1.32}$$

$$\Psi_0 = \frac{L_o}{2}, \frac{\partial \Psi_0}{\partial y} = -1 \text{ on } h_1(x), \tag{1.34}$$

$$\Psi_0 = -\frac{L_o}{2}, \frac{\partial \Psi_0}{\partial y} = -1 \text{ on } h_2(x). \tag{1.35}$$

1.4.2 First order system

$$\frac{\partial^4 \Psi_1}{\partial y^4} = -\frac{\partial^2}{\partial y^2}(\frac{\partial \Psi_0}{\partial y})^2, \tag{1.36}$$

$$\frac{\partial P_1}{\partial x} = \frac{\partial^3 \Psi_1}{\partial y^3} + \frac{\partial}{\partial y}(\frac{\partial \Psi_0}{\partial y})^2, \tag{1.37}$$

$$\Psi_1 = \frac{L_1}{2}, \frac{\partial \Psi_1}{\partial y} = 0 \text{ on } h_1(x), \tag{1.38}$$

$$\Psi_1 = -\frac{L_1}{2}, \frac{\partial \Psi_1}{\partial y} = 0 \text{ on } h_2(x). \tag{1.39}$$

1.4.3 Solution of zeroth order system

the solution of Eqs. (1.31) and (1.32) subject to boundary conditions (1.34) and (1.35) are directly defined as

$$\begin{aligned} \Psi_0 = {} & \frac{L_0 + h_1 - h_2}{(h_2 - h_1)^3}(2y^3 - 3(h_1 + h_2)y^2 + 6h_1h_2y) - y + \\ & \frac{1}{(h_2 - h_1)^3}((\frac{L_0}{2} + h_1)(h_2^3 - 3h_1h_2^2) \\ & -(h_2 - \frac{F_0}{2})(h_1^3 - 3h_2h_1^2)), \end{aligned} \tag{1.40}$$

$$\frac{dP_0}{dx} = \frac{12(L_0 + h_1 - h_2)}{(h_2 - h_1)^3}. \tag{1.41}$$

For one wavelength the integration of $Eq.(41)$, yeilds

$$\Delta P_0 = \int_0^1 \frac{dP_0}{dx} dx, \tag{1.42}$$

where $\frac{dP_0}{dx}$ is defined in Eq.(1.41).

1.4.4 Solution for the first order system

Substituting the zeroth-order solution (1.40)into (1.36), the solution of the resulting problem satisfing the boundary conditions take the following form:

$$\Psi_1 = C_0 + C_1 y + C_2 \frac{y^2}{2!} + C_3 \frac{y^3}{3!} - 288(\frac{L_0 + h_1 - h_2}{(h_2 - h_1)^3})^2 \frac{y^4}{4!}, \tag{1.43}$$

where

$$\begin{aligned}
C_0 &= -\frac{6}{(h_2-h_1)^3}(L_1 - \frac{A}{4!}(h_1^3(2h_2-h_1) - \\
&\quad h_2^3(2h_1-h_2)))(\frac{h_1 h_2^2}{2} - \frac{h_2^3}{6}) - \frac{A}{3!}(\frac{h_1^2 h_2^2}{2} + \frac{h_1 h_2^3}{2} - \frac{h_2^4}{4}) - \frac{L_1}{2}, && (1.44)\\
C_1 &= \frac{A h_1 h_2}{2}(\frac{(h_1+h_2)}{3} - \\
&\quad \frac{1}{2(h_2-h_1)^3}(h_1^3(2h_2-h_1) - h_2^3(2h_1-h_2))) + \frac{6 h_1 h_2 L_1}{(h_2-h_1)^3}, && (1.45)\\
C_2 &= A(\frac{(h_1+h_2)}{4(h_2-h_1)^3}(h_1^3(2h_2-h_1) - h_2^3(2h_1-h_2)) + \\
&\quad \frac{(h_1^2 + h_1 h_2 + h_2^2)}{3!}) - \frac{6L_1(h_1+h_2)}{(h_2-h_1)^3}, && (1.46)\\
C_3 &= \frac{12}{(h_2-h_1)^3}(F_1 - \frac{A}{4!}(h_1^3(2h_2-h_1) - h_2^3(2h_1-h_2))), && (1.47)\\
A &= -288(\frac{L_0 + h_1 - h_2}{(h_2-h_1)^3}). && (1.48)
\end{aligned}$$

The axial pressure gradient at this order is

$$\Delta P_1 = \int_0^1 \frac{dP_0}{dx} dx. \tag{1.49}$$

Summerizing the perturbation results for small parameter We, the expression for stream funcions and pressure gradient can be written as

$$\Psi = \frac{L_1(h_1+h_2)}{(h_2-h_1)^3}(2y^3-3(h_1+h_2)y^2+6h_1h_2y)-y+\frac{1}{(h_2-h_1)^3}((\frac{L}{2}+h_1)(h_2^3-3h_1h_2^2)$$
$$-(h_2-\frac{L}{2})(h_1^3-3h_2h2_1)+We(B+Cy+D\frac{y^2}{2!}+E\frac{y^4}{3!}+A_1\frac{y^4}{4!}), \quad (1.50)$$
$$\frac{dP}{dx} = \frac{12(L+h_1+h_2)}{(h_2-h_1)^3}+We(-\frac{12}{(h_2-h_1)^3}\frac{A_1}{4!}(h_1^3(2h_2-h_1)-h_2^3(2h_1-h_2))$$
$$-144(h_1+h_2)[\frac{(L+h_1+h_2)}{(h_2-h_1)^3}]^2. \quad (1.51)$$

1.5 Results and discussion

Using mathematics software expression for pressure rise ΔP is calculated. The effects of various parameters on the pressure rise ΔP are shown in $Figs.$1 to 4, for various values of Weissenberg number We , channel width d and wave amplitudes a, b. It is observed from $Fig.$1 that pressure rise decreases for small values of θ (0$\leq$ θ $\leq$4.4) with the increase in We and for large θ (4.4$\leq$ θ $\leq$6.4), the pressure rise remains constant and then again decreases for $\theta = 6.4$. It is observed that the pressure rise increases with the increase in a and b for small θ (0$\leq$ θ $\leq$40) and for large θ (40$\leq$ θ $\leq$80), the results are opposite (see $Fig.$2 and $Fig.$3). It is also observed that the pressure rise decreases with the increase in d for small θ (0$\leq$ θ $\leq$30) and for large θ (30$\leq$ θ $\leq$80), the results are opposite (see $Fig.$4). $Fig.$5 and $Fig.$6 represent that for $[70, 100]$ the pressure gradient is small, we say that the flow can easily pass without imposition of large pressure gradient, while in the narrow part of the channel, to retain same flux large pressure gradient is required. Moreover in the narrow part of the channel, the pressure gradient decreases with the increase in We and d. It is also observed that the behavior of We and d on the pressure gradient are similar. The pressure rise ΔP for different values of b are shown in $Fig.$7. It is seen that the curves for the pressure rise are not linear and in the region, the pressure rise decreases with the increase in b while in the region $\theta \in [0, 40]$.

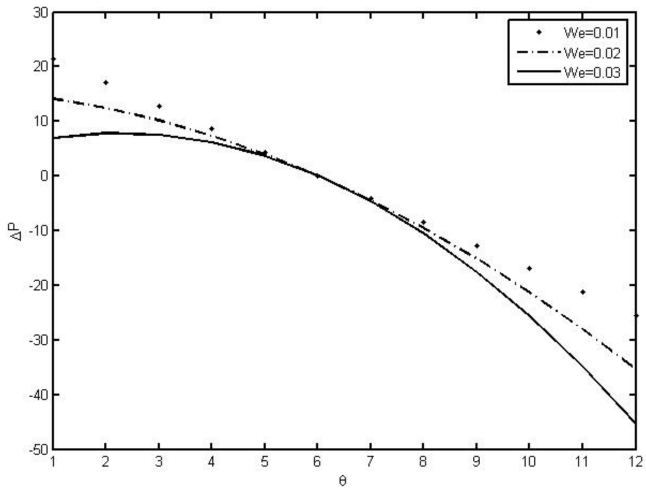

Figure 1-1: Variation of ΔP with θ for different values of We at $a = 0.01$, $b = 0.01$,$d = 0.2$ and $\phi = 0.8$.

1.6 Trapping phenomena

Trapping is another interesting phenomenon in peristaltic motion. It is basically the formation of an internally circulating bolus of fluid by closed stream lines. This trapped bolus pushed a head along peristaltic waves. $Figs.$8 to15 illustrate the stream lines for different values of We, Q and a. The stream lines for different values of We are shown in $Figs.$8, 9 and 10. It is found that with the increase in Weissenberg number We the size of the trapping bolus decreases in the upper half of the channel and increases in the lower half of the channel. In $Figs.$11, 12 and 13 the stream lines are prepared for different values of volume flow rate Q. It is depicted that the size of the trapped bolus increases in the upper half of the channel with the increase in Q, while the size and the number of the trapped bolus increases in the lower half of the channel. It is observed from $Figs.$ 14, 15 that the size of the trapping bolus increases in the lower and upper half of the channel with the increase in amplitude of the wave a.

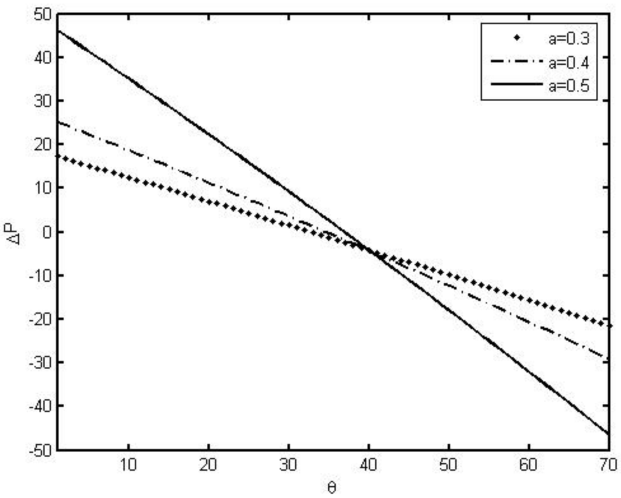

Figure 1-2: Variation of ΔP with θ for different values of a at $We = 0.001$, $b = 0.1$, $d = 0.4$ and $\phi = 0.3$.

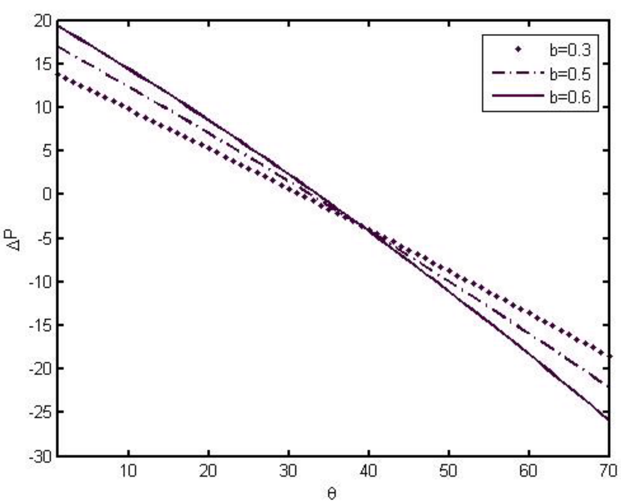

Figure 1-3: . Variation of ΔP with θ for different values of b at $We = 0.001$, $a = 0.1$, $d = 0.4$ and $\phi = \pi$.

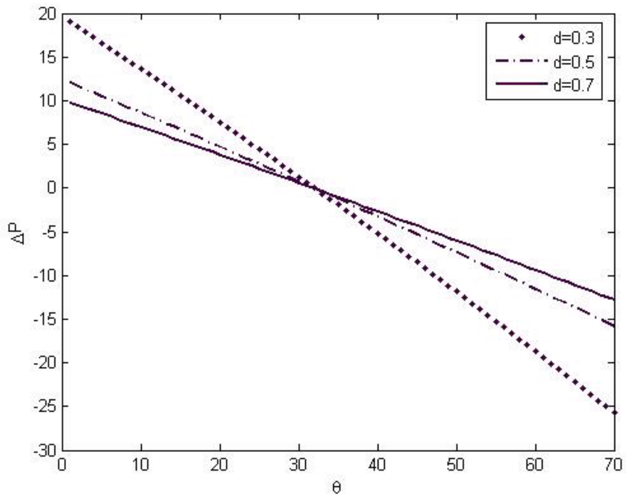

Figure 1-4: Variation of ΔP with θ for different values of d at $We = 0.001$, $a = 0.1$,$b = 0.4$, and $\phi = \pi/6$.

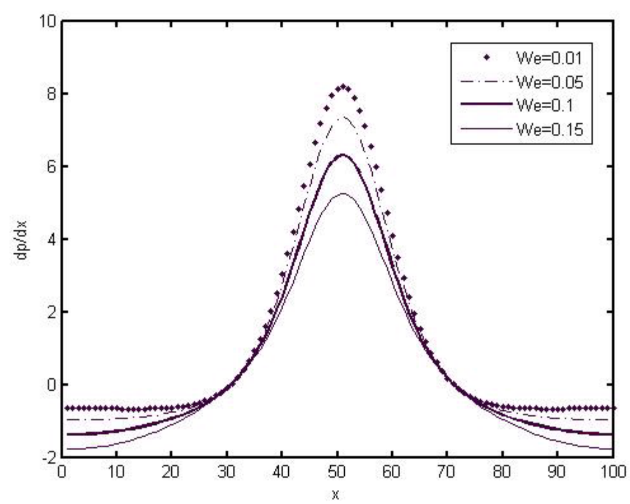

Figure 1-5: Variation of dP/dx with x for different values of We at $a = 0.5$, $b = 0.5$, $d = 0.4$ and $\phi = 0.01$.

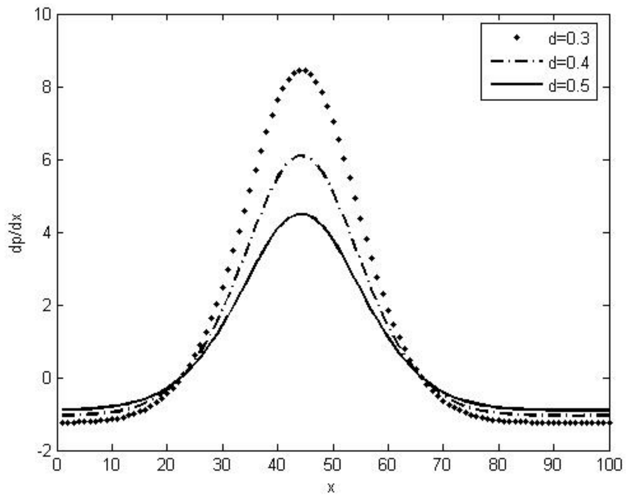

Figure 1-6: Variation of dP/dx with x for different values of d at $a = 0.5$, $b = 0.5$, $We = 0.4$ and $\phi = 0.01$.

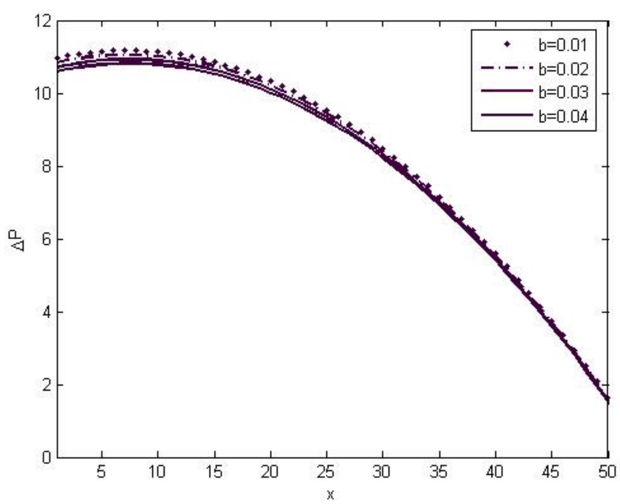

Figure 1-7: . Variation of ΔP with θ for different values of We at $a = 0.5$, $b = 0.5, d = 0.4$ *and* $\phi = 0.01$.

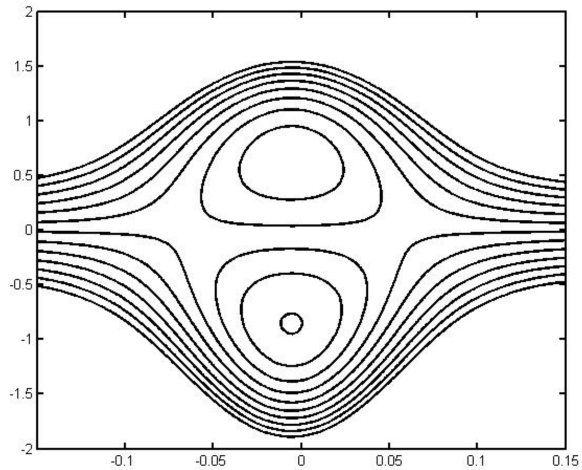

Figure 1-8: stream lines for $We = 0.53$.

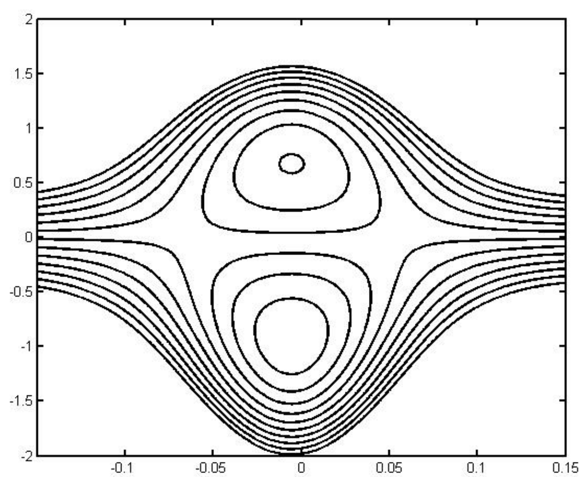

Figure 1-9: stream lines for $We = 0.62$.

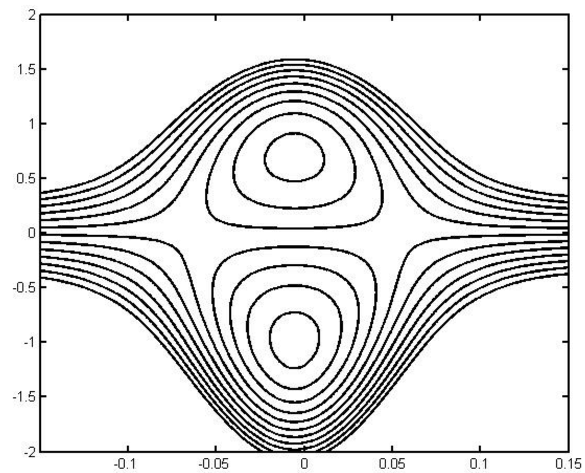

Figure 1-10: stream lines for $We = 0.72$

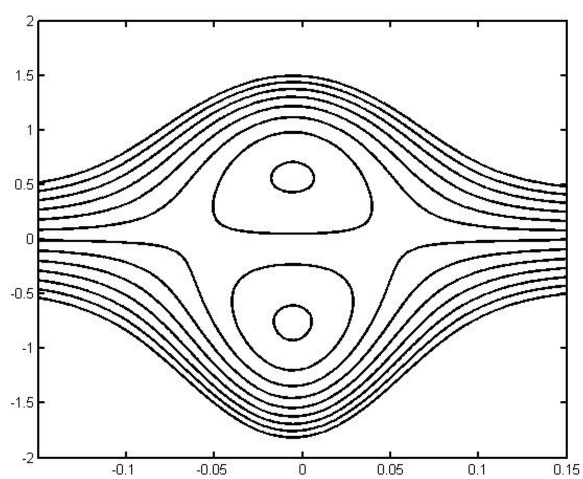

Figure 1-11: stream lines for $Q = 0.8$

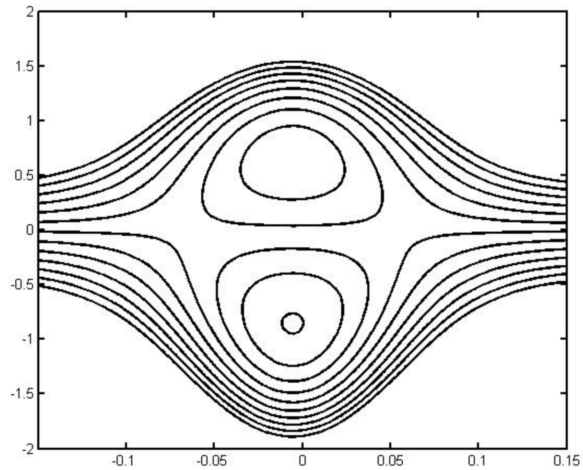

Figure 1-12: stream lines for $Q = 0.9$.

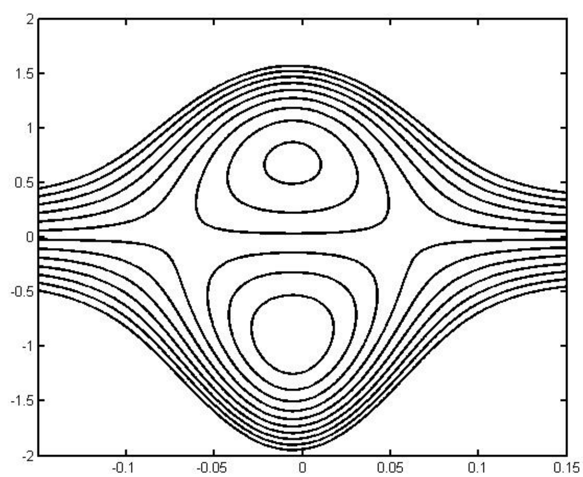

Figure 1-13: stream lines for $Q = 0.98$.

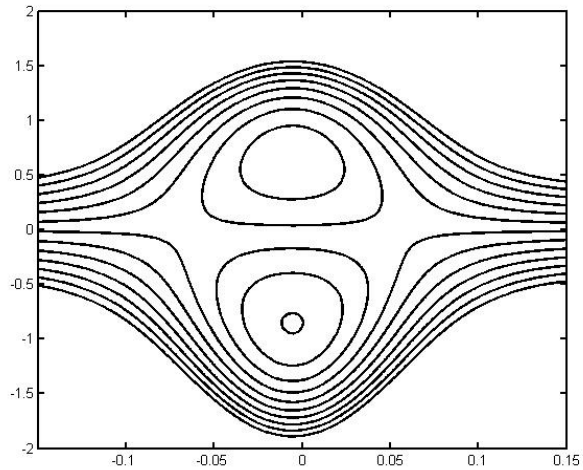

Figure 1-14: stream lines for $a = 0.1$.

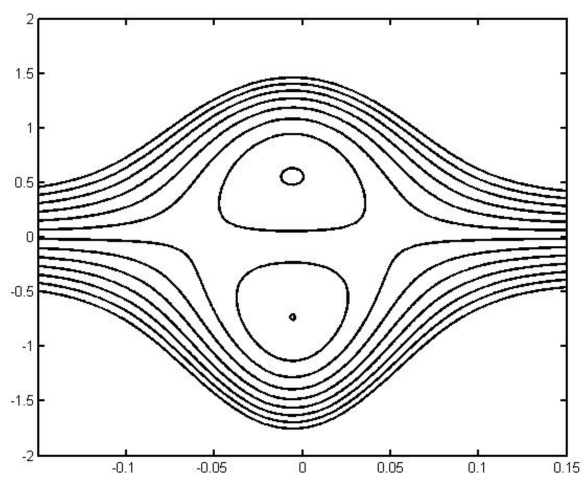

Figure 1-15: stream lines for $a = 0.13$.

Chapter 2

Peristaltic flow of Non-Newtonain nanofluid in an asymmetric channel

2.1 Introduction

This chapter deals with the study of peristaltic flow of non-Newtonian fluid in asymmetric channel. The governing equations of nano-Williamson fluid model are presented and simplified with the help of long wavelength and low Reynolds number approximations. The simplified problem is solved analytically with the help of homotopy perturbation method (HPM). The physical features of pertinent parameters are disscussed through graphs. Finally, the trapping phenomena is aslo presented through ploting stream lines.

2.2 Mathematical formulation

Let us consider the peristaltic flow of an incompressible non-Newtonian fluid in a two dimensional vertical channel of width $d_1 + d_2$. The flow is generated due to sinusoidal wave trains propagating with constant speed c along the channel walls. The geometry of the wall surface is defined as

$$Y = N_1 = \bar{d}_1 + \bar{a}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right], \tag{2.1}$$

$$Y = N_2 = -\bar{d}_2 - \bar{b}_1 \cos\left[\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \Phi\right], \tag{2.2}$$

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, the width of the channel is $\bar{d}_1 + \bar{d}_2$, the velocity of propagation is c, $\bar{t}$ is the time and $\bar{X}$ is the direction of wave propagation. The phase difference Φ varies in the range $0 \leq \Phi \leq \pi$ in which $\Phi = 0$ corresponds to symmetric channel with waves out of phase and $\Phi = \pi$, the waves are in phase, further $\bar{a}_1$, $\bar{a}_2$, $\bar{d}_1$, $\bar{d}_2 and \Phi$ satisfies the condition

$$\bar{a}_1^2 + \bar{b}_1^2 + 2\bar{a}_1\bar{b}_1 \cos\Phi \leq (\bar{d}_1 + \bar{d}_2)^2. \tag{2.3}$$

The governing equations of nano non-Newtonian fluid for vertical asymmetric channel are defined as

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0, \tag{2.4}$$

$$\varsigma(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}) = -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} + \varsigma g\alpha(\bar{T} - \bar{T}_0) + \varsigma g d(\bar{C} - \bar{C}_0), \tag{2.5}$$

$$\varsigma(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}} = -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}}, \tag{2.6}$$

$$\begin{aligned} \varsigma c_p(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}) &= \kappa[\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}] + \tau\{D_B(\frac{\partial \bar{C}}{\partial \bar{x}}\frac{\partial \bar{T}}{\partial \bar{x}} + \frac{\partial \bar{C}}{\partial \bar{y}}\frac{\partial \bar{T}}{\partial \bar{y}}) \\ &+ \frac{D_{\bar{T}}}{\bar{T}_0}[(\frac{\partial \bar{T}}{\partial \bar{x}})^2 + (\frac{\partial \bar{T}}{\partial \bar{y}})^2]\}, \end{aligned} \tag{2.7}$$

$$(\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}) = D_B(\frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}) + \frac{D_{\bar{T}}}{\bar{T}_0}(\frac{\partial^2 \bar{T}}{\partial \bar{x}} + \frac{\partial^2 \bar{T}}{\partial \bar{y}}), \tag{2.8}$$

Introducing the nondimensional variables

$$\begin{aligned}
x &= \frac{2\pi\bar{x}}{\lambda}, y = \frac{\bar{y}}{d_1}, \upsilon = \frac{\bar{\upsilon}}{\lambda}, t = \frac{2\pi\bar{t}}{\lambda}, \delta = \frac{2\pi d_1}{\lambda}, d = \frac{d_2}{d_1}, P = \frac{2\pi d_1^2 P}{\mu c_1 \lambda}, \\
h_1 &= \frac{\bar{h}_1}{d_1}, h_2 = \frac{\bar{h}_2}{d_2}, \mathrm{Re} = \frac{\rho c_1 d_1}{\mu}, a = \frac{a_1}{d_1}, b = \frac{a_2}{d_1}, S = \frac{\bar{S} d_1}{\mu c_1}, \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0}, \\
\sigma &= \frac{\bar{C} - \bar{C}_0}{\bar{C}_1 - \bar{C}_0}, \alpha = \frac{\kappa}{(\varsigma c)_f}, N_b = \frac{(\varsigma c)_p D_B (\bar{C}_1 - \bar{C}_0)(\bar{T}_1 - \bar{T}_0)}{(\varsigma c)_f \alpha}, N_t = \frac{(\varsigma c)_p D_T (\bar{T}_1 - \bar{T}_0)^2}{\bar{T}_0 (\varsigma c)_f \alpha}, \\
P_r &= \frac{\nu}{\alpha}, G_r = \frac{g\alpha d_1^2 (\bar{T}_1 - \bar{T}_0)}{\nu c_1}, B_r = \frac{g\alpha d_1^2 (\bar{C}_1 - \bar{C}_0)}{\nu c_1}. \qquad (2.9)
\end{aligned}$$

Using the above non dimensional quantities and the resulting equations in terms of stream function $\psi(u = \frac{\partial \boldsymbol{\psi}}{\partial y}, v = -\delta \frac{\partial \boldsymbol{\psi}}{\partial x})$ can be written as

$$\begin{aligned}
\delta \,\mathrm{Re} \left[\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right] \frac{\partial \psi}{\partial y} &= -\frac{\partial P}{\partial x} - \delta^2 \frac{\partial \boldsymbol{\tau}_{xx}}{\partial x} - \frac{\partial \boldsymbol{\tau}_{xy}}{\partial y} + \\
& \quad G_r \frac{\partial \theta}{\partial y} + B_r \frac{\partial \sigma}{\partial y}, \qquad (2.10)
\end{aligned}$$

$$\delta^3 \,\mathrm{Re} \left[\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right] \frac{\partial \psi}{\partial x} = -\frac{\partial P}{\partial y} - \delta^2 \frac{\partial \boldsymbol{\tau}_{xy}}{\partial x} - \delta \frac{\partial \boldsymbol{\tau}_{yy}}{\partial y}, \qquad (2.11)$$

$$\frac{\partial^2 \theta}{\partial y^2} + N_b \frac{\partial \theta}{\partial y} \frac{\partial \sigma}{\partial y} + (\frac{\partial \theta}{\partial y})^2 = 0, \qquad (2.12)$$

$$\frac{\partial^2 \sigma}{\partial y^2} + \frac{N_t}{N_b} (\frac{\partial \theta}{\partial y})^2 = 0. \qquad (2.13)$$

where

$$\boldsymbol{\tau}_{xx} = -2[1 + We\mathring{\gamma}]\frac{\partial^2\Psi}{\partial x\partial y}, \tag{2.14}$$

$$\boldsymbol{\tau}_{xy} = -[1 + We\mathring{\gamma}](\frac{\partial^2\Psi}{\partial y^2} - \delta^2\frac{\partial^2\Psi}{\partial x^2}), \tag{2.15}$$

$$\boldsymbol{\tau}_{yy} = 2\delta[1 + We\mathring{\gamma}]\frac{\partial^2\Psi}{\partial x\partial y}, \tag{2.16}$$

$$\mathring{\boldsymbol{\gamma}} = \left[2\delta^2(\frac{\partial^2\Psi}{\partial x\partial y})^2 + (\frac{\partial^2\Psi}{\partial y^2} - \delta^2\frac{\partial^2\Psi}{\partial x^2})^2 + 2\delta^2(\frac{\partial^2\Psi}{\partial x\partial y})^2\right]^{\frac{1}{2}}. \tag{2.17}$$

in which δ, Re, We represents the wave, Reynolds and Weissenberg numbers, respectively. Under the assumptions of long wavelength $\delta << 1$ and low Reynolds number, neglecting the terms of order δ and higher,

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y}[(1 + We\frac{\partial^2\Psi}{\partial y^2})\frac{\partial^2\Psi}{\partial y^2}] + G_r\theta + B_r\sigma, \tag{2.18}$$

$$\frac{\partial P}{\partial y} = 0. \tag{2.19}$$

Elimination of pressure from equations, give

$$\frac{\partial^2}{\partial y^2}[(1 + We\frac{\partial^2\Psi}{\partial y^2})\frac{\partial^2\Psi}{\partial y^2}] + G_r\frac{\partial\theta}{\partial y} + B_r\frac{\partial\sigma}{\partial y} = 0, \tag{2.20}$$

in which

$$L = \int_{h_2(x)}^{h_1(x)}\frac{\partial\Psi}{\partial y}dy = \Psi(h_1(x) - h_2(x)), \tag{2.21}$$

$$h_1(x) = 1 + a\cos 2\pi x,\ \ h_2(x) = -d - b\cos(2\pi x + \Phi). \tag{2.22}$$

The bounddary conditions in terms of stream function Ψ can be defined as

$$\Psi = \frac{L}{2}, \frac{\partial \Psi}{\partial y} = -1 \text{ for } y = h_1(x), \tag{2.23}$$

$$\Psi = -\frac{L}{2}, \frac{\partial \Psi}{\partial y} = -1 \text{ for } y = h_2(x). \tag{2.24}$$

2.3 Homotopy perturbation method

Since, the above equations of above boundry value problem is highly nonlinear and coupled differential equations their exact solutions are not possible, therefore we are intersted to compute the solution with the help of homotopy perturbation method. The homotopy perturbation method is defined as

$$\begin{aligned} H(\Psi, q) &= (1-q)[L(\Psi) - L(\Psi_0)] + \\ &\quad q[\frac{\partial^4 \Psi}{\partial y^4} + 2We(\frac{\partial^3 \Psi}{\partial y^3})^2 + 2We\frac{\partial^2 \Psi}{\partial y^2}\frac{\partial^4 \Psi}{\partial y^4} + G_r\frac{\partial \theta}{\partial y} + \\ B_r\frac{\partial \sigma}{\partial y}] &= 0, \end{aligned} \tag{2.25}$$

$$H(\theta, q) = (1-q)[L(\theta) - L(\theta_0)] + q[\frac{\partial^2 \theta}{\partial y^2} + N_b\frac{\partial \theta}{\partial y}\frac{\partial \sigma}{\partial y} + N_t(\frac{\partial \theta}{\partial y})^2], \tag{2.26}$$

$$H(\sigma, q) = (1-q)[L(\sigma) - L(\sigma_0)] + q[\frac{\partial^2 \sigma}{\partial y^2} + \frac{N_t}{N_b}(\frac{\partial^2 \theta}{\partial y^2})^2] = 0, \tag{2.27}$$

$$\begin{aligned} H(p, q) &= (1-q)[\frac{\partial P}{\partial x} - \frac{\partial^3 \Psi}{\partial y^3}] + q[\frac{\partial P}{\partial x} - \frac{\partial^3 \Psi}{\partial y^3} - 2We\frac{\partial^2 \Psi}{\partial y^2}\frac{\partial^3 \Psi}{\partial y^3} - G_r\theta \\ -B_r\sigma] &= 0. \end{aligned} \tag{2.28}$$

According to HPM, we choose the following linear operaters

$$L_\Psi = \frac{\partial^4}{\partial y^4}, \tag{2.29}$$

$$L_\theta = \frac{\partial^2}{\partial y^2}, \tag{2.30}$$

$$L_\sigma = \frac{\partial^2}{\partial y^2}. \tag{2.31}$$

Using

$$\Psi = \Psi_0 + q\Psi_1 + ... \tag{2.32}$$

$$\theta = \theta_0 + q\theta_1 + ... \tag{2.33}$$

$$\sigma = \sigma_0 + q\sigma_1 + ... \tag{2.34}$$

Making use of Eqs. (2.29) to (2.34) and equating the like powers of q, we obtain the following systems

2.3.1 Zeroth order system

$$\frac{\partial^2\theta_0}{\partial y^2} = 0, \tag{2.35}$$

$$\theta_0 = 0 \; at \; y = h_1, \tag{2.36}$$

$$\theta_0 = 1 \; at \; y = h_2, \tag{2.37}$$

$$\frac{\partial^2\sigma_0}{\partial y^2} = 0, \tag{2.38}$$

$$\sigma_0 = 0 \; at \; y = h_1, \tag{2.39}$$

$$\sigma_0 = 1 \; at \; y = h_2, \tag{2.40}$$

$$\frac{\partial^4\Psi_0}{\partial y^4} = 0, \tag{2.41}$$

$$\Psi_0 = \frac{L}{2}, \frac{\partial \Psi_0}{\partial y} = -1 \ at \ y = h_1, \tag{2.42}$$

$$\Psi_0 = -\frac{L}{2}, \frac{\partial \Psi_0}{\partial y} = -1 \ at \ y = h_2, \tag{2.43}$$

$$\frac{\partial P_0}{\partial x} = \frac{\partial^3 \Psi_0}{\partial y^3}. \tag{2.44}$$

2.3.2 First order system

$$\frac{\partial^2 \theta_1}{\partial y^2} + N_b \frac{\partial \theta_0}{\partial y}\frac{\partial \sigma_0}{\partial y} + N_t (\frac{\partial \theta_0}{\partial y})^2 = 0, \tag{2.45}$$

$$\theta_1 = 0 \ at \ y = h_1, \tag{2.46}$$

$$\theta_1 = 0 \ at \ y = h_2, \tag{2.47}$$

$$\frac{\partial^2 \sigma_1}{\partial y^2} + \frac{N_t}{N_b}(\frac{\partial^2 \theta_0}{\partial y^2})^2 = 0, \tag{2.48}$$

$$\sigma_1 = 0 \ at \ y = h_1, \tag{2.49}$$

$$\sigma_1 = 0 \ at \ y = h_2, \tag{2.50}$$

$$\frac{\partial^4 \Psi_1}{\partial y^4} + 2We(\frac{\partial^3 \Psi_0}{\partial y^3})^2 + 2We\frac{\partial^2 \Psi_0}{\partial y^2}\frac{\partial^4 \Psi_0}{\partial y^4} + G_r \frac{\partial \theta_0}{\partial y} + B_r \frac{\partial \sigma_0}{\partial y} = 0, \tag{2.51}$$

$$\Psi_0 = 0, \frac{\partial \Psi_0}{\partial y} = 0 \ at \ y = h_1, \tag{2.52}$$

$$\Psi_0 = 0, \frac{\partial \Psi_0}{\partial y} = 0 \ at \ y = h_2, \tag{2.53}$$

$$\frac{\partial P_1}{\partial x} = \frac{\partial^3 \Psi_1}{\partial y^3} + 2We \frac{\partial^2 \Psi_0}{\partial y^2} \frac{\partial^3 \Psi_0}{\partial y^3} + G_r \theta_0 + B_r \sigma_0. \tag{2.54}$$

2.3.3 Solution of zeroth order system

The solution of the zeroth order system satisfing the boundry conditions straight forward written as

$$\theta_0 = \frac{y}{h_2 - h_1} + \frac{h_1}{h_1 - h_2}, \tag{2.55}$$

$$\sigma_0 = \frac{y}{h_2 - h_1} + \frac{h_1}{h_1 - h_2}, \tag{2.56}$$

$$\begin{aligned} \Psi_0 &= \frac{L(h_1 + h_2)}{(h_2 - h_1)^3}(2y^3 - 3(h_1 + h_2)y^2 + 6h_1 h_2 y) - y + \\ & \frac{1}{(h_2 - h_1)^3}((\frac{L}{2} + h_1)(h_2^3 - 3h_1 h_2^2) - (h_2 - \frac{L}{2})(h_1^3 - 3h_2 h_1), \end{aligned} \tag{2.57}$$

$$\frac{\partial P_0}{\partial x} = \frac{12(L + h_1 - h_2)}{(h_2 - h_1)^3}. \tag{2.58}$$

2.3.4 Solution of first order system

With the help of zeroth order system, the solution of first order system is defined as

$$\begin{aligned} \theta_1 &= \frac{1}{2(h_1 - h_2)^2}(-h_1 h_2 N_b + h_1 y N_b + h_2 y N_b - y^2 N_b \\ & -h_1 h_2 N_t + h_1 y N_t + h_2 y N_t - y^2 N_t), \end{aligned} \tag{2.59}$$

$$\sigma_1 = 0, \tag{2.60}$$

$$\Psi_1 = Ch_1^2h_2^2 + 2Ch_1^2h_2y + 2Ch_1h_2^2y - Ch_1^2y^2 - 4Ch_1h_2y^2 - Ch_2^2y^2 + 2Ch_2y^3 - Cy, \quad (2.61)$$

where

$$288We\frac{L(h_1+h_2)^2}{(h_2-h_1)^6} + \frac{G_r}{(h_2-h_1)} + \frac{B_r}{(h_2-h_1)} = C, \quad (2.62)$$

$$\begin{aligned}\frac{\partial P_1}{\partial x} &= \frac{144We(L+h_1-h_2)^2}{(h_2-h_1)^6} - \frac{144h_2(L+h_1-h_2)^2}{(h_2-h_1)^6} \\ &\quad -\frac{G_r(h_1+h_2)}{2(h_1-h_2)} - \frac{B_r(h_1+h_2)}{2(h_1-h_2)}. \quad (2.63)\end{aligned}$$

Finally, substituting zeroth order and first order solutions into Eqs. (2.32) to (2.34) when $q \to 1$, take the form

$$\begin{aligned}\Psi &= \frac{L(h_1+h_2)}{(h_2-h_1)^3}(2y^3 - 3(h_1+h_2)y^2 + 6h_1h_2y) - y + \\ &\quad \frac{1}{(h_2-h_1)^3}((\frac{L}{2}+h_1)(h_2^3 - 3h_1h_2^2) - (h_2 - \frac{L}{2})(h_1^3 - 3h_2h2_1) + \\ &\quad Ch_1^2h_2^2 + 2Ch_1^2h_2y + 2Ch_1h_2^2y - Ch_1^2y^2 - 4Ch_1h_2y^2 - Ch_2^2y^2 + \\ &\quad 2Ch_2y^3 - Cy, \quad (2.64)\end{aligned}$$

$$\begin{aligned}
\frac{\partial P}{\partial x} &= \frac{12(L+h_1-h_2)}{(h_2-h_1)^3}+\frac{144We(L+h_1-h_2)^2}{(h_2-h_1)^6} \\
&-\frac{144h_2(F+h_1-h2)^2}{(h_2-h_1)^6}-\frac{G_r(h_1+h_2)}{2(h_1-h_2)}-\frac{B_r(h_1+h_2)}{2(h_1-h_2)}+ \\
&\frac{1}{20(h_2-h_1)^9N_b}(5(h_2-h_1)^7((-1+h_1)h_1+(-1+h_2)h_2)G_rN_b^2-2(h_1-h_2)^9B_rN_t^2 \\
&-2N_b(2304(L+h_1-h_2)^3(11h_1^2+8h_1h_2+11h_2^2)We^2 \\
&-(h_1-h_2)^5(8(L+h_1-h_2) \\
&(11h_1^2+8h_1h_2+11h_2^2)We(B_r+G_r)+(h_1-h_2)^4G_rN_t. \qquad (2.65)
\end{aligned}$$

2.4 Results and discussion

Using mathematics software the expression for pressure rise ΔP is calculated numerically . The effects of various parameters on the pressure rise ΔP are shown in $Fig.$1to 4 for various values of Weissenberg number We , channel width d and wave amplitudes a, b. It is observed from $Fig.$1 that pressure rise increases for small values of θ (0≤ θ ≤24) with the increase in We and for θ (24≤ θ ≤48), the pressure rise remains constant and then decreases for θ ≥48 . It is observed that the pressure rise increases with the increase in a for small θ (0≤ θ ≤25) for θ (25≤ θ ≤45), it remains constant (see $Fig.$2) and for θ (25≤ θ ≤70) pressure rise decreases. It is observed that the pressure rise increases with the increase in b for small θ (0≤ θ ≤23) and then for θ ≥23 pressure rise shows opposite behaviour. It is also observed that the pressure rise decreases with the increase in d for small θ (0≤ θ ≤12.5) and for large θ (12.5≤ θ ≤25), the results are opposite (see $Fig.$4). $Fig.$5 represent that for and $[60, 100]$ the pressure gradient is small. It is seen that with the increase in the Brownian motion parameter N_b concentration profile decreases in the region (0≤y≤1). For thermophoresis parameter N_t concentration profile increases in the region (0≤y≤1.6). It is seen that with the increase in the Brownian motion parameter N_b temperature profile increases in the region (0≤y≤1). For thermophoresis parameter N_t temprature profile increases in the range (0≤y≤1). It is observed that pressure rise increases for small values of x with the increase in Brownian parameter ,thermophoresis parameter, local temprature Grashaf number and local nanopartical Grashaf number.

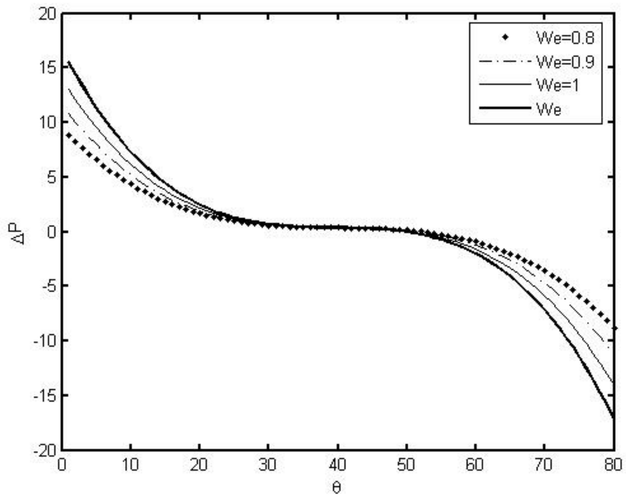

Figure 2-1: Variation of ΔP with θ for different values of We at $a = 0.2$, $b = 1$, $d = 3$, $\phi = 0.5$,$G_r = 0.5$, $B_r = 0.5$, $N_b = 1$, $N_t = 1$

2.5 Trapping phenomena

Trapping is another interesting phenomenon in peristaltic motion. It is basically the formation of an internally circulating bolus of fluid by closed stream lines. This trapped bolus pushed a head along peristaltic waves. $Figs.$ (14) to (19) illustrate the stream lines for different values of We and a. The stream lines for different values of We are shown in $Figs.(14),(15),(16)$. It is found that with the increase in Weissenberg number We the size of the trapping bolus decreases in the upper half of the channel and increases in the lower half of the channel. It is observed from $Figs.(17),(18),(19)$ that the size of the trapping bolus increases in the lower and upper half of the channel with the increase in amplitude of the wave a.

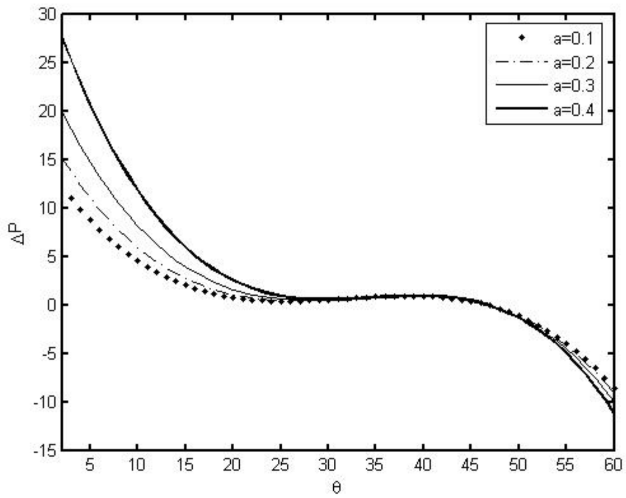

Figure 2-2: Variation of ΔP with θ for different values of a at $We = 0.98$, $b = 0.2$, $d = 3$, $\phi = 0.5$,$G_r = 1$, $B_r = 2$, $N_b = 1$, $N_t = 1$

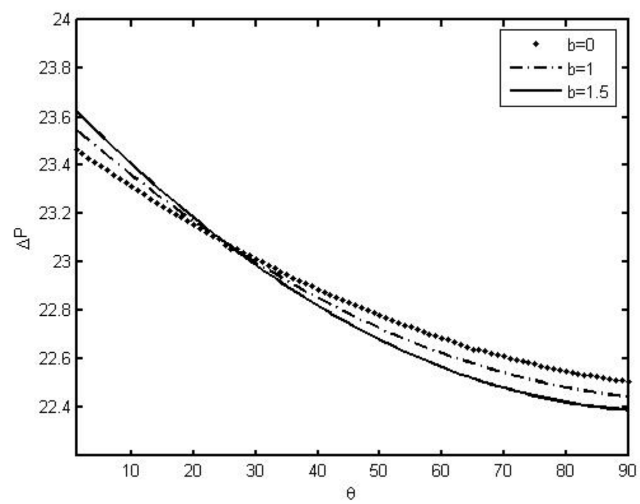

Figure 2-3: Variation of ΔP with θ for different values of b at $a = 0.1$, $We = 0.01$, $d = 7.5$, $\phi = 0.5$,$G_r = 0.91$, $B_r = 0.1$, $N_b = 1$, $N_t = 1$

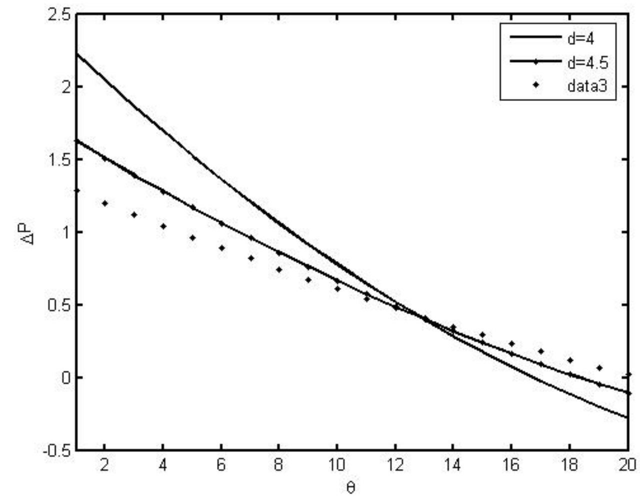

Figure 2-4: Variation of ΔP with θ for different values of d at $a = 0.1$, $b = 1$, $d = 7.5$, $\phi = 0.5$, $G_r = 0.91$, $B_r = 0.1$, $N_b = 1$, $N_t = 1$, $We = 0.01$

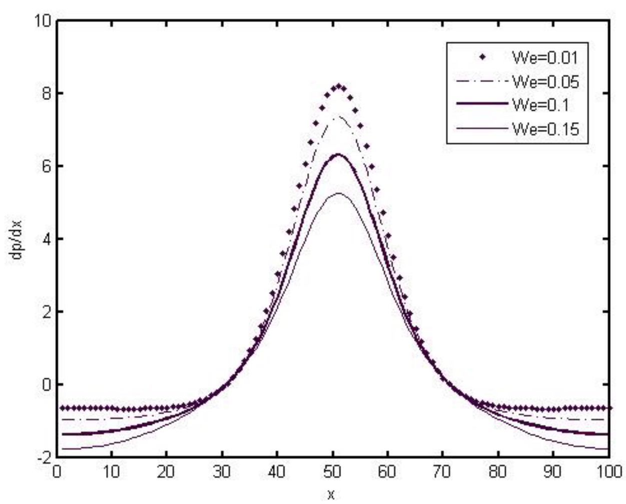

Figure 2-5: Variation of dP/dx with x for different values of We at $a = 0.5$, $b = 0.5$, $d = 0.4$ and $\phi = 0.01$.

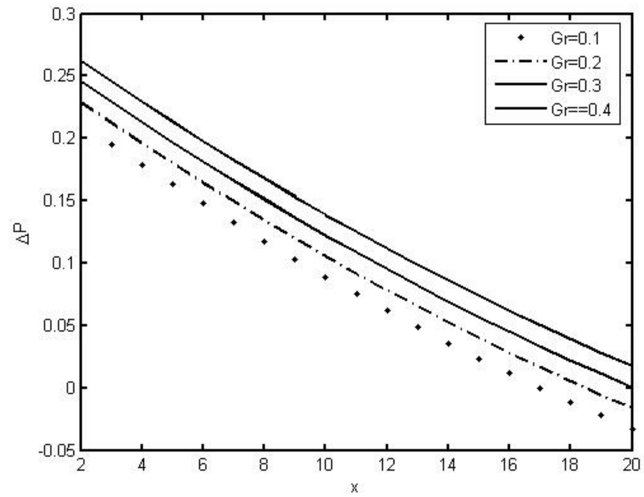

Figure 2-6: Variation of ΔP with x for different values of G_r at $We = 0.01$, $b = 1$, $d = 3$, $\phi = 0.5$,$a = 0.2$, $B_r = 0.05$, $N_b = 1$, $N_t = 1$

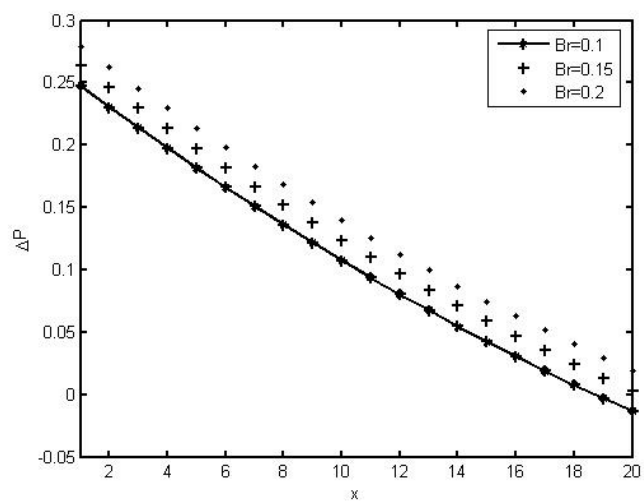

Figure 2-7: Variation of ΔP with x for different values of B_r at $We = 0.01$, $b = 1$, $d = 5$, $\phi = 0.5$,$a = 0.4$, $G_r = 0.1$, $N_b = 0.5$, $N_t = 0.5$

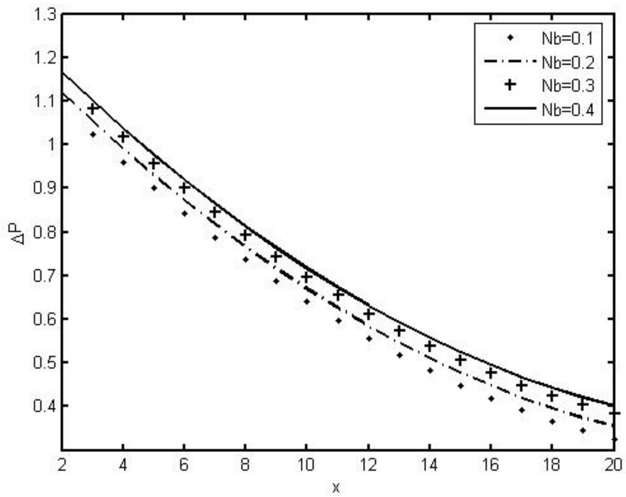

Figure 2-8: Variation of ΔP with x for different values of N_b at $We = 0.01$, $b = 1$, $d = 3.5$, $\phi = 0.5$,$a = 0.2$, $G_r = 1$, $N_t = 1$

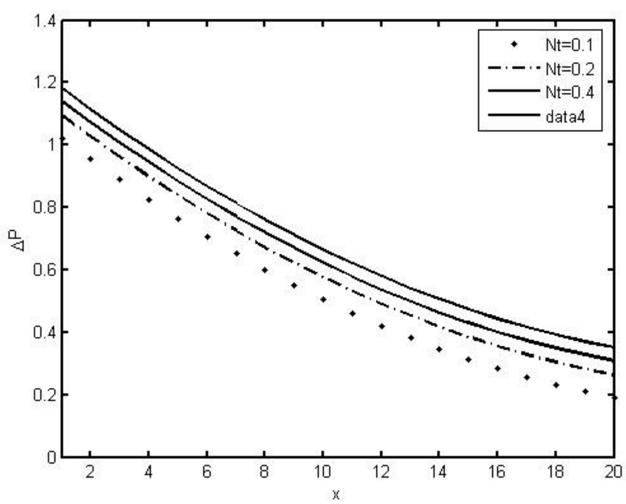

Figure 2-9: Variation of ΔP with x for different values of N_t at $We = 0.01$, $b = 1$, $d = 3.5$, $\phi = 0.5$,$a = 0.2$, $G_r = 1$, $N_b = 1$

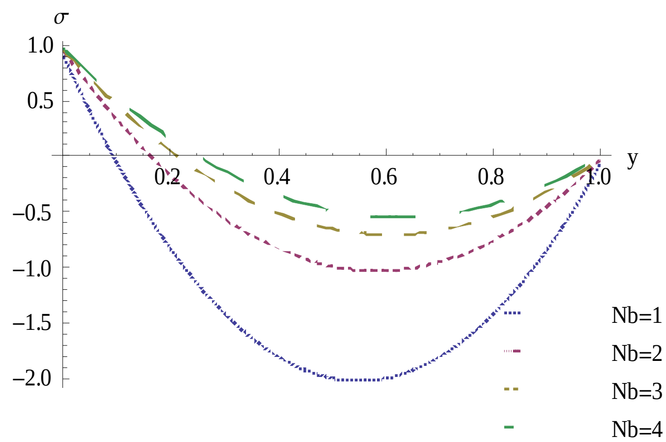

Figure 2-10: Variation of σ with x for different values of N_b at $b = 0.5$, $d = 0.1$, $\phi = 1.5$,$a = 0.6$, $N_t = 1$

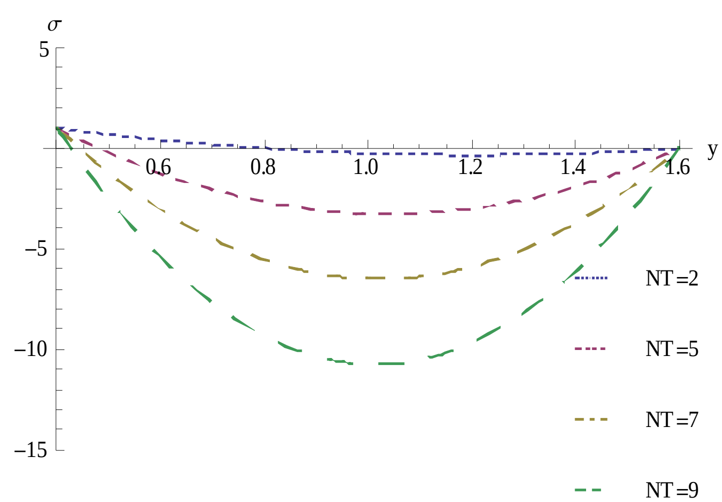

Figure 2-11: Variation of σ with x for different values of N_t at $b = 0.5$, $d = 0.1$, $\phi = 1.5$,$a = 0.6$, $N_b = 1.5$

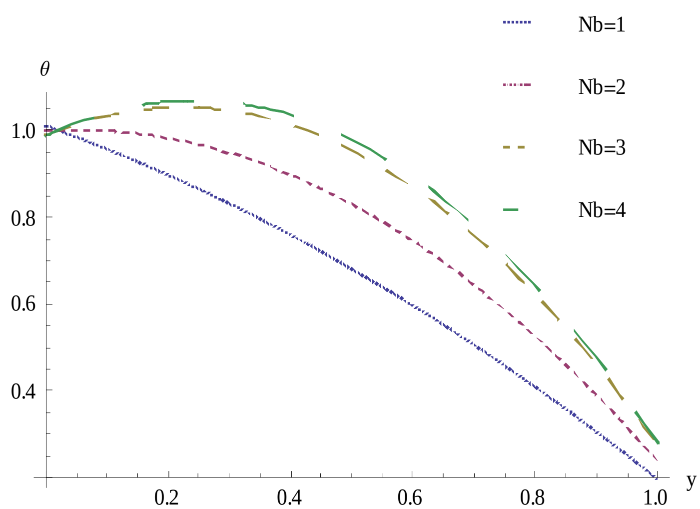

Figure 2-12: Variation of σ with x for different values of N_b at $b = 0.5$, $d = 0.2$, $\phi = 3.5$,$a = 0.6$, $N_t = 4$

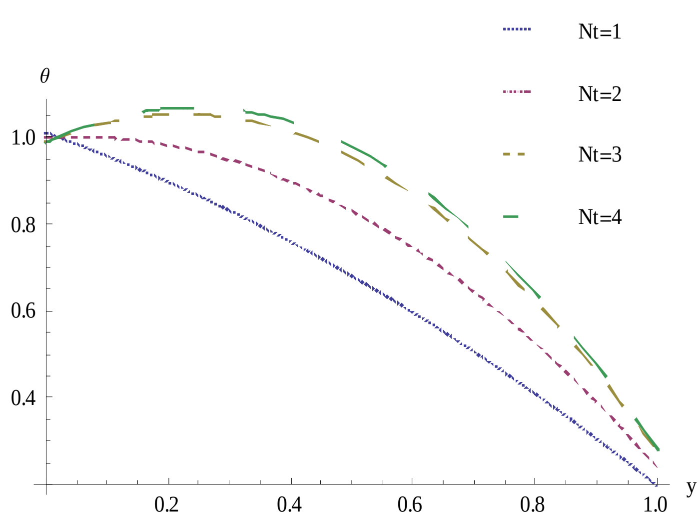

Figure 2-13: Variation of σ with x for different values of N_t at $b = 0.5$, $d = 0.1$, $\phi = 3.5$,$a = 0.6$, $N_b = 8$

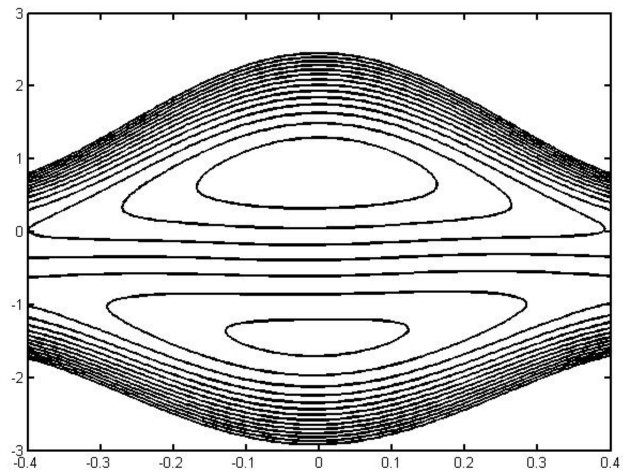

Figure 2-14: stream lines for $We = 0.06$

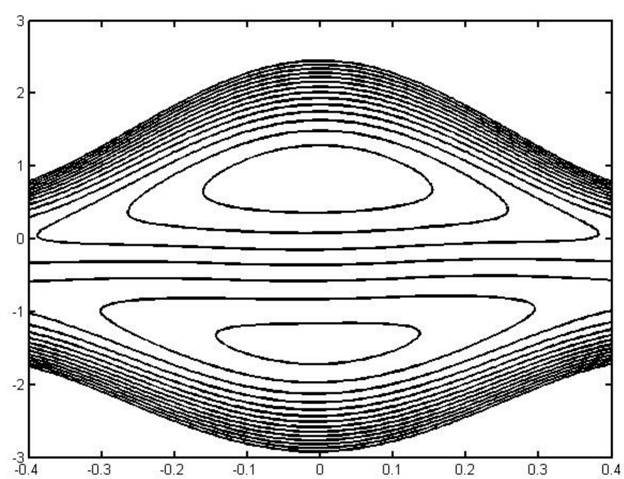

Figure 2-15: stream lines for $We = 0.08$.

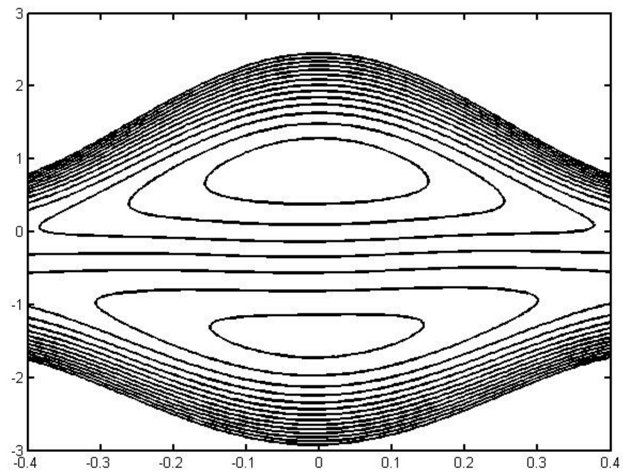

Figure 2-16: stream lines for $We = 0.09$.

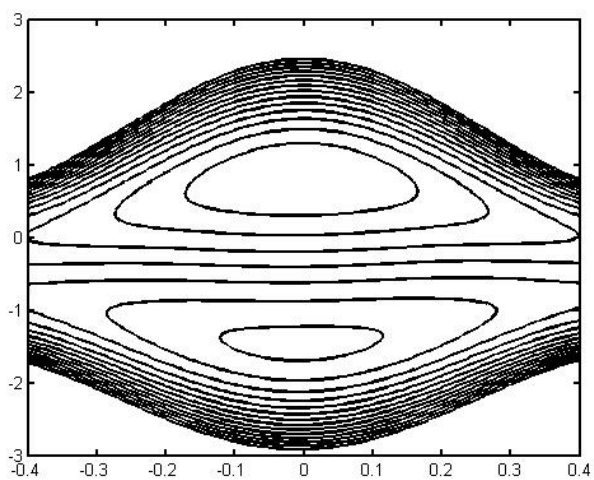

Figure 2-17: stream lines for $a = 0.8$.

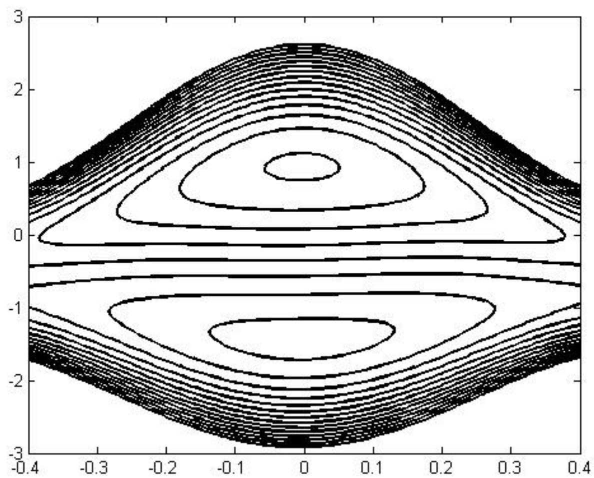

Figure 2-18: stream lines for $a = 0.9$.

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