Peristaltic Transport with wall properties and Partial Slip Effects

By

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MASTER OF PHILOSOPHY

IN

MATHEMATICS

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CERTIFICATE

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

1. Prof. Dr. Muhammad Ayub
(Chairman)

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2013

DEDICATED TO

MY BELOVED PARENTS

PREFACE

The process of peristaltic transport of fluid is very common in several industrial and physiological applications. In physiology such process is particularly involved in movement of chime in the intestine, swallowing of food through esophagus, bile transport, urine transport from kidney to bladder etc. roller and finger pumps also operate under the principle of peristalsis. This activity is involved in corrosive fluid transport. A large number of theoretical investigations dealing with peristaltic flows of viscous and non-Newtonian fluids have been analyzed since the early attempts by Latham [1]. Some recent studies on the title may be seen in the refs. [2-15]. In all these investigations, the flow analysis have been conducted using one or more assumptions of long wavelength, small wave number, low Reynolds number, small amplitude ratio etc. In [15] the authors have developed a mathematical model for the influence of wall properties on the peristaltic transport of viscous fluid in a channel. The constructed model is realistic physiologically from neuron-muscular properties of any smooth muscle. Motivated by such fact, the present dissertation is arranged as follows.

Chapter one includes the basic definitions and laws relevant to the analysis of chapters two and three. Chapter two describes the peristaltic transport of viscous fluid in a channel with wall properties. Heat and mass transfer are present. The first order chemical reaction effects are considered. Chapter three extends the flow analysis in chapter two for the partial slip effects. Partial slip effect is formulated in terms of shear stress. Graphical results are displayed and discussed. Important conclusions have been pointed out.

Contents

Chapter 1

Basic definitions and equations

This chapter includes fundamental definitions and equations for the better understanding of the flow analysis in the subsequent chapters

1.1 Basics of peristaltic transport

1.1.1 Peristalsis

The term peristalsis is coined from the Greek word "peristaltikos", which means clasping and compressing. Therefore it is defined as a wave of relaxation and contraction to the walls of a flexible conduit, thereby pumping the enclosed content.

1.1.2 Peristaltic transport

It is a sort of material transport induced by a progressive wave of area expansion or contraction along the length of a distensible tube containing some content. It is defined as successive waves of involuntary contraction relaxation passing along the walls of hollow tubular structures and pumping the enclosed content onward.

1.1.3 Pumping

One specific feature of peristaltic transport is pumping phenomenon. The operation of a pump of moving liquids from lower pressure to higher pressure under certain conditions is called pumping.

Positive and negative pumping

The pumping is termed as positive or negative depending on whether the mean flow rate θ is positive or negative.

Adverse and favorable pressure gradient

If the pressure rise per wavelength (ΔP_λ) is negative then pressure gradient is called favorable otherwise adverse.

Peristaltic pumping

In this case pressure rise is adverse ($\Delta P_\lambda > 0$) and flow rate is positive ($\theta > 0$).

Augmented pumping

Here pressure rise is favorable ($\Delta P_\lambda < 0$) and flow rate is positive ($\theta > 0$).

Retrograde pumping

In this case the pressure rise is adverse ($\Delta P_\lambda > 0$) and flow rate is negative ($\theta < 0$).

Free pumping

Here the flow rate is positive ($\theta > 0$) but pressure rise is neither adverse nor favorable. In other words $\Delta P_\lambda = 0$.

Free pumping flux

The critical value of mean flow rate θ corresponding to $\Delta P_\lambda = 0$ is called free pumping flux.

1.1.4 Bolus

Bolus is the volume of fluid confined in a closed streamline in the moving frame with the speed of propagation i.e. wave frame.

1.1.5 Trapping

Generally the shape of streamlines is approximately similar to that of the boundary wall in the wave frame. However, under certain circumstances some of the streamlines driven and enclose a bolus which is pushed forward along with the peristaltic wave with the wave speed. This phenomenon is known as trapping.

1.2 Some basic equations

In this dissertation the flow is governed by the following fundamental expressions:

- Equation of continuity
- Equation of motion
- Energy equation
- Concentration equation

1.2.1 Continuity equation

The continuity equation is derived from the law of conservation of mass which says that matter cannot be created or destroyed. Mathematically it can be stated into the form

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{V}) = 0. \tag{1.1}$$

Here ρ denotes the density of fluid, $\mathbf{V}$ the velocity field and t the time. In the case of incompressible fluid the Eq. (1.1) is reduced to

$$\nabla.\mathbf{V} = \mathbf{0}. \tag{1.2}$$

The above equation holds when the source/sink is absent in the control volume.

1.2.2 Equation of motion

This equation is the outcome of law of conservation of linear momentum which states that the amount of momentum don't alter inside a problem domain and can be changed only through the action of forces described by laws of motion. Its mathematical expression is

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div}\mathbf{S} + \boldsymbol{\rho}\mathbf{f}, \tag{1.3}$$

where $\mathbf{S}$ is the Cauchy stress tensor and the second term on the R.H.S. is the body force. In general the material derivative $\frac{d}{dt}$ can be expressed as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}. \tag{1.4}$$

1.2.3 Energy equation

The main idea of this equation is that when the fluid element moves along with the fluid, its temperature changes as a result of heat conduction and heat production because of viscous heating. We can express mathematically it by

$$\rho C_p \left(\frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}\right) T = \kappa \boldsymbol{\nabla}^2 T + \mathbf{S}.\mathbf{L}. \tag{1.5}$$

In above expression ρ the density, C_p the specific heat, κ the thermal conductivity, L the rate of strain tensor and T the temperature of fluid.

1.2.4 Concentration equation

Mass diffusion is due to the concentration gradient and change in mass concentration is due to diffusivity of mass and chemical reaction. We can express it as follows

$$\left(\frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}\right) C = D\boldsymbol{\nabla}^2 V + k_1 \left(C - C_0\right), \tag{1.6}$$

where C depicts the concentration, D the coefficient of mass diffusivity and k_1 the chemical reaction parameter respectively.

1.3 Some dimensionless numbers

1.3.1 Reynolds number

The Reynolds number expresses the ratio of inertial forces to viscous forces (due to viscosity of the fluid). The importance of Reynolds number is that it helps in determining whether the flow is laminar or turbulent. Low Reynolds number gives laminar flow where prominent forces are viscous and flow is characterized by smooth fluid motion while at high Reynolds number where inertial forces are dominating turbulent flow occurs. For $Re < 2300$, the flow is laminar and turbulent for $Re > 4000$. Flow is in transition when its value is between the mentioned values. It is denoted by Re and can be expressed as

$$\mathrm{Re} = \frac{inertial\ forces}{viscous\ forces} = \frac{cd}{\nu}, \tag{1.7}$$

where c denotes the velocity, d the length scale and ν the kinematic viscosity.

1.3.2 Grashof number

Grashof number, symbolized as Gr, is the ratio of the buoyancy forces (caused by the spatial variation in fluid density due to temperature gradient) to viscous acting on the fluid. Free convection is the tendency of a substance to migrate due to buoyant force. It commonly arises in the study of situations involving natural convection. It can be written as

$$Gr = \frac{buoyant\ forces}{viscous\ forces} = \frac{g\beta_T\,(T_1 - T_0)\,d^2}{\nu c}, \tag{1.8}$$

in which g denotes the gravitational acceleration, β_T thermal expansion coefficient and $(T_1 - T_0)$ is the temperature difference. Note that for $Gr >> 1$, turbulent flow occurs since the viscous forces are negligible and buoyant forces become dominant.

1.3.3 Prandtl number

It is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. Mathematically we have

$$\mathrm{Pr} = \frac{momentum\ diffusivity}{thermal\ diffusivity} = \frac{\mu C_p}{\kappa}, \tag{1.9}$$

where μ denotes the dynamic viscosity, C_p the specific heat and κ the thermal conductivity.

1.3.4 Eckert number

Eckert number is the ratio of kinetic energy to enthalpy of the flow. It can be written as

$$Ec = \frac{kinetic\ energy}{enthalpy} = \frac{c^2}{C_p(T_1 - T_0)}. \tag{1.10}$$

1.3.5 Brinkman number

Brinkman number is a measure of the significance of the viscous heating (due to viscous dissipation) comparative to the conductive heat transfer. It arises in circumstances where large velocity changes occur over short distances such as lubricant flow. Mathematically we have

$$Br = Ec * \mathrm{Pr} = \frac{\mu c^2}{\kappa(T_1 - T_0)}. \tag{1.11}$$

1.3.6 Amplitude ratio

It is the ratio of amplitude of peristaltic wave to the width of the channel.

$$\epsilon = \frac{a}{d}. \tag{1.12}$$

1.3.7 Wave number

Wave number is the property of a wave and interpreted as the ratio of the width of the channel to the wavelength. Its mathematical form is given by

$$\delta = \frac{d}{\lambda}. \tag{1.13}$$

1.4 Mechanisms of heat transfer

Heat transfer mechanism can be grouped into following categories:

1.4.1 Conduction

Regions with high molecular kinetic energy will pass their thermal energy through direct molecular collisions. This process is known as conduction.

1.4.2 Convection

The process in which the bulk motion of the fluid increases the heat transfer between the solid surface and the fluid. In case of free convection, when heat conducts into a fluid at rest it leads to a volumetric expansion. This results in a gravity-induced pressure gradient. The expanded fluid then becomes buoyant and moves, thereby transporting heat by fluid motion (i.e. convection) in addition to conduction. It is also called natural convection. Rising of hot air is natural convection. When the fluid is forced to flow over the surface by external means, forced convection occurs.

1.4.3 Radiation

It is the simplest way of heat transfer by electromagnetic waves not by moving molecules (as in conduction and convection). Radiation is the only way through which heat can be moved through a vacuum. In liquids and gasses, convection and radiation play vital role when heat transfer is under consideration but convection is absent in case of solids and radiation is negligible.

1.5 Mass transfer

In mass transfer, energy (including thermal energy) is moved from one place to another by the physical transfer of a hot or cold object.

1.6 No slip vs partial slip condition

No slip condition for fluid in contact with the solid boundary states that the velocity of the fluid relative to the boundary is zero. This is due to the viscous property of the fluids. We can define this phenomenon physically as the particles which are adjacent to the boundary do not move along with the flow. This means that adhesion is stronger than cohesion. However there are situations where this condition cannot be taken into account like fluid past a permeable wall, rough and coated surfaces, emulsion, foam, slotted plates, polymer solutions, gas and liquid flow in micro devices etc. In such cases slip is appropriate condition to be used.

Chapter 2

Peristaltic motion in a compliant walls channel with heat transfer and chemical reaction

2.1 Introduction

The peristaltic transport of viscous fluid in a vertical symmetric channel is discussed. the channel walls are compliant. Mathematical analysis is presented in the presence of heat and mass transfer. The first order chemical reaction is considered. Analysis has been carried out not for only long wavelength but also for low Reynolds number assumptions. Solution expressions are developed for small Grashof number. The graphs are sketched for different parameters appearing in the solution expressions. The contents of this chapter are a detailed review of paper by Hayat et al. [15].

2.2 Physical model

We consider the two dimensional flow of an incompressible viscous fluid in a symmetric vertical channel of width $2d$. Sinusoidal waves of small amplitude a and long wavelength λ induces flow in a channel. Moreover the channel walls are compliant in nature. We select rectangular coordinates (x, y) with $x-axis$ along the wave propagation and $y-axis$ normal to it ($see\ Fig.$

2.1.).

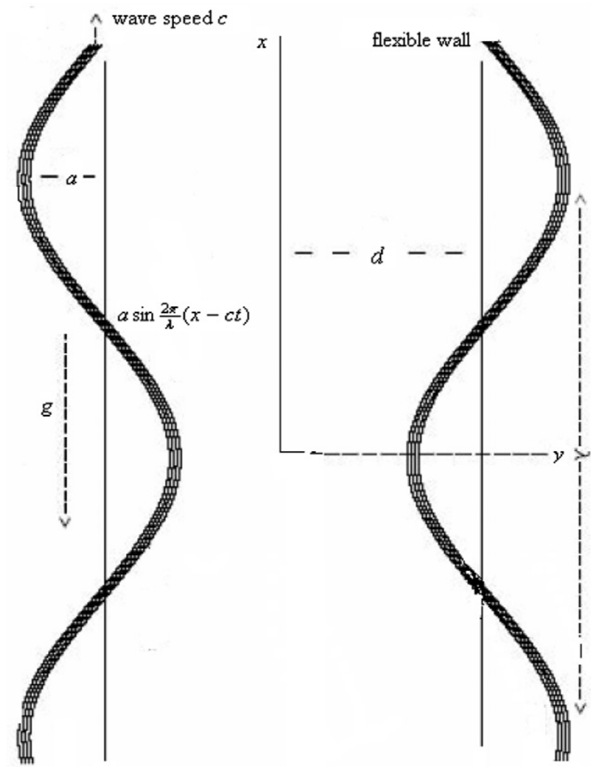


Fig. 2.1. *Schematic diagram of the problem.*

The channel walls can be expressed as

$$y = \eta(x,t) = +\left[d + a\sin\frac{2\pi}{\lambda}(x-ct)\right] \quad \text{at} \quad \text{right wall}, \tag{2.1}$$

$$y = \eta(x,t) = -\left[d + a\sin\frac{2\pi}{\lambda}(x-ct)\right] \quad \text{at} \quad \text{left wall}. \tag{2.2}$$

Here η is the wall displacement, d the mean half width of the channel, a the amplitude, wavelength and speed of the wave are denoted by λ and c respectively. The velocity profile $\mathbf{V}$ for two dimensional flow is

$$\mathbf{V} = (u(x,y,t), v(x,y,t), 0), \tag{2.3}$$

where the velocity components in the fixed frame of reference in the longitudinal and transverse directions are designated by $u(x,y,t)$ and $v(x,y,t)$ respectively.

2.3 Problem formulation

The fundamental equations for the incompressible fluid are

$$\operatorname{div} \mathbf{V} = 0, \tag{2.4}$$

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div} \mathbf{S} + \rho g \beta_T (T - T_0) + \rho g \beta_C (C - C_0), \tag{2.5}$$

$$\rho C_p \frac{dT}{dt} = \kappa \nabla^2 T + \mathbf{S}.\mathbf{L}, \tag{2.6}$$

$$\frac{d\mathbf{C}}{dt} = D \nabla^2 C - k_1 (C - C_0), \tag{2.7}$$

in which $\mathbf{V}$ is the velocity, ρ density of the fluid, $\mathbf{S}$ the Cauchy stress tensor, g the gravitational acceleration, β_T the coefficient of thermal expansion, β_C the coefficient of concentration expansion, C_p the specific heat at constant volume, κ the thermal conductivity, L the rate of strain tensor, T, C and k_1 denotes the temperature, concentration and chemical reaction parameter and $\frac{d}{dt}$ the material time derivative given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}. \tag{2.8}$$

The expression of Cauchy stress tensor $\mathbf{S}$ in viscous incompressible fluid is

$$\mathbf{S} = -p\mathbf{I} + \mu \mathbf{A}_1, \tag{2.9}$$

in which p is the pressure, $\mathbf{I}$ the identity tensor, μ the dynamic viscosity and $\mathbf{A}_1$ represents the first Rivlin-Ericksen tensor given by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^*, \tag{2.10}$$

with

$$\mathbf{L} = (\operatorname{grad} \mathbf{V}), \tag{2.11}$$

where superscript $*$ indicates the matrix transpose. By Eqs. (2.4) and (2.11) one has

$$\mathbf{L} = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{L}^* = \begin{bmatrix} u_x & v_x & 0 \\ u_y & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2.12}$$

where $\mathbf{L}$ is the velocity gradient and asterisk denotes transpose of matrix. Thus Eq. (2.10) reduces to

$$\mathbf{A}_1 = \begin{bmatrix} 2u_x & u_y + v_x & 0 \\ v_x + u_y & 2v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2.13}$$

Using (2.13) in (2.9) we get

$$\mathbf{S} = \begin{bmatrix} -p + 2\mu u_x & \mu (u_y + v_x) & 0 \\ \mu (u_y + v_x) & -p + 2\mu v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2.14}$$

Further we define

$$\mathbf{S}.\mathbf{L} = tr(\mathbf{SL}) = S_{xx} + S_{xy}(u_y + v_x) + S_{yy}. \tag{2.15}$$

Substituting Eqs. (2.9)−(2.15) , the continuity, momentum, energy and concentration equations in the presence of body force become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.16}$$

$$\rho \left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right] u = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \rho g \beta_T (T - T_0) + \rho g \beta_C (C - C_0), \tag{2.17}$$

$$\rho \left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right] v = -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right], \tag{2.18}$$

$$\rho C_p \left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right] T = \kappa \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu \left[\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2\right], \tag{2.19}$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right] C = D \left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] - k_1 (C - C_0). \tag{2.20}$$

The corresponding boundary conditions for the flow are given by

$$u = 0,\ v = \pm\eta_t \qquad \text{at}\ \ y = \pm\eta = \pm\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right], \tag{2.21}$$

$$T = \begin{Bmatrix} T_1 \\ T_0 \end{Bmatrix},\ \ C = \begin{Bmatrix} C_1 \\ C_0 \end{Bmatrix}\ \ \text{at}\ \ (y = \pm\eta), \tag{2.22}$$

$$\frac{\partial}{\partial x}L'(\eta) = \frac{\partial p}{\partial x} = \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \rho g\beta_T(T - T_0) + \rho g\beta_C(C - C_0) - \rho\frac{du}{dt}\ \text{at}\ \ (y = \pm\eta), \tag{2.23}$$

where an operator L' is used to represents the motion of compliant walls with viscous damping forces as follows:

$$L' = -\tau'\frac{\partial^2}{\partial x^2} + m_1\frac{\partial^2}{\partial t^2} + d'\frac{\partial}{\partial t}. \tag{2.24}$$

In the above expression τ' the longitudinal tension per unit width, m_1 the mass per unit area and d' is the wall damping coefficient. Eliminating p from Eqs. (2.17) and (2.18) we get

$$\rho\frac{d}{dt}\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right] = \mu\left[\frac{\partial}{\partial y}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial y^2}\right)\right] + \rho g\beta_T\frac{\partial T}{\partial y} + \rho g\beta_C\frac{\partial C}{\partial y}. \tag{2.25}$$

2.3.1 Non-dimensionalization

If $\Psi(x, y, t)$ is the stream function then

$$u = \frac{\partial\Psi}{\partial y},\ \ v = -\frac{\partial\Psi}{\partial x}. \tag{2.26}$$

Now continuity equation is identically satisfied. We introduce the following non-dimensional variables and parameters as follows:

$$\begin{aligned} \bar{x} &= \frac{x}{\lambda},\ \bar{y} = \frac{y}{d},\ \bar{\Psi} = \frac{\Psi}{cd},\ \bar{p} = \frac{d^2 p}{c\mu\lambda},\ \bar{t} = \frac{ct}{\lambda}, \\ \gamma &= k_1\frac{d^2}{v},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \phi = \frac{C - C_0}{C_1 - C_0},\ \bar{\eta} = \frac{\eta}{d}. \end{aligned} \tag{2.27}$$

Using the above transformations and dropping bars we get non-dimensional form of Eqs. (2.19) – (2.25)

$$\delta\,\mathrm{Re}\frac{d}{dt}\left[\delta^2\Psi_{xx} + \Psi_{yy}\right] = \Psi_{yyyy} + 2\delta^2\Psi_{xxyy} + \delta^4\Psi_{xxxx} + Gr\left[\theta_y + N\phi_y\right], \tag{2.28}$$

$$\delta \Pr \operatorname{Re} \frac{d\theta}{dt} = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Br \left[4\delta^2 \left(\Psi_{xy}\right)^2 + \left(\Psi_{yy} - \delta^2 \Psi_{xx}\right)^2 \right], \tag{2.29}$$

$$\delta \operatorname{Re} \frac{d\phi}{dt} = \frac{1}{Sc} \left[\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right] - \gamma \phi, \tag{2.30}$$

$$\Psi_y = 0, \ \theta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \ \phi = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ at } y = \pm \eta, \tag{2.31}$$

$$\left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t} \right] \eta = \frac{\partial^3 \Psi}{\partial y^3} + \delta^2 \frac{\partial^3 \Psi}{\partial x^2 \partial y} - \delta \operatorname{Re} \frac{d\Psi_y}{dt} + Gr \left[\theta + N\phi \right] \text{ at } y = \pm \eta. \tag{2.32}$$

The dimensionless forms of η is

$$\eta(x) = \left(1 + \epsilon \sin 2\pi \left(x - t\right)\right). \tag{2.33}$$

Here δ $(= d/\lambda)$ is the wave number, ϵ $(= a/d)$ the amplitude ratio, γ $\left(= k_1 d^2/\nu\right)$ the chemical reaction parameter ($\gamma < 0$ shows the generative chemical reaction and $\gamma > 0$ for destructive chemical reaction), Re $(= cd/\nu)$ the Reynolds number, Gr $\left(= g\beta_T \left(T_1 - T_0\right) d^2/\nu c\right)$ depicts the Grashof number, Br $(= Ec \Pr)$ shows the Brinkman number, $\Pr$ $(= \mu C_P/\kappa)$ the Prandtl number, N $\left(= \beta_C \left(C_1 - C_0\right)/\beta_T \left(T_1 - T_0\right)\right)$ represents the buoyancy ratio parameter, Ec $\left(= c^2/C_P \left(T_1 - T_0\right)\right)$ the Eckert number, Sc $(= \mu/\rho D)$ the Schmidt number and E_1 $\left(= -\tau d^3/\lambda^3 \mu c\right)$, E_2 $\left(= m_1 c d^3/\lambda^3 \mu\right)$ and E_3 $\left(= d' d^3/\lambda^2 \mu\right)$ are the non-dimensional elasticity parameters respectively.

Invoking long wavelength and low Reynolds number assumptions Eqs. (2.28) – (2.32) ultimately take the following forms

$$\Psi_{yyyy} + Gr \left[\theta_y + N\phi_y\right] = 0, \tag{2.34}$$

$$\frac{\partial^2 \theta}{\partial y^2} + Br \left(\Psi_{yy}\right)^2 = 0, \tag{2.35}$$

$$\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \gamma \phi = 0, \tag{2.36}$$

$$\Psi_y = 0, \ \theta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \ \phi = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ at } y = \pm \eta = \pm \left(1 + \epsilon \sin 2\pi \left(x - t\right)\right), \tag{2.37}$$

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right]\eta=\frac{\partial^3\Psi}{\partial y^3}+Gr\left[\theta+N\phi\right]\quad \text{at } y=\pm\eta. \tag{2.38}$$

2.4 Perturbation solution

We note that the resulting Eqs. are non-linear. It seems difficult to obtain the general solution in closed form for arbitrary values of all parameters arising in these equations. Here our interest lies in seeking the perturbation solution. Therefore we expand the flow quantities as follows:

$$\begin{aligned}
\Psi &= \Psi_0+Gr\Psi_1+O\left(Gr^2\right)+...,\\
\theta &= \theta_0+Gr\theta_1+O\left(Gr^2\right)+...,\\
Z &= Z_0+GrZ_1+O\left(Gr^2\right)+...,\\
\phi &= \phi_0+Gr\phi_1+O\left(Gr^2\right)+....
\end{aligned}\tag{2.39}$$

We seek the solution of the problem as a power series expansion in the small parameter Gr. Using these expressions into Eqs. (2.34)−(2.38) and then comparing the coefficients one obtains the following systems:

2.4.1 Zeroth order system

$$\Psi_{0yyyy}=0, \tag{2.40}$$

$$\theta_{0yy}+Br\left(\Psi_{0yy}\right)^2=0, \tag{2.41}$$

$$\frac{1}{Sc}\phi_{0yy}-\gamma\phi_0=0, \tag{2.42}$$

$$\Psi_{0y}=0,\ \phi_0=\begin{Bmatrix}1\\0\end{Bmatrix},\ \theta_0=\begin{Bmatrix}1\\0\end{Bmatrix},\ \text{at}\ \ y=\pm\eta, \tag{2.43}$$

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right]\eta=\Psi_{0yyy}\quad \text{at } y=\pm\eta. \tag{2.44}$$

2.4.2 First order system

$$\Psi_{1yyyy}=-\left[\theta_{0y}+N\phi_{0y}\right], \tag{2.45}$$

$$\theta_{1yy} = -2Br\Psi_{0yy}\Psi_{1yy}, \tag{2.46}$$

$$\frac{1}{Sc}\frac{\partial^2\phi_1}{\partial y^2} - \gamma\phi = 0, \tag{2.47}$$

$$\Psi_{1y} = 0, \quad \theta_1 = 0, \quad \phi_1 = 0,$$

$$\Psi_{1yyy} + [\theta_0 + N\phi_0] = 0 \ \text{ at } y = \pm\eta. \tag{2.48}$$

2.4.3 Zeroth order solution

The solution of the problem is given by

$$\Psi_0 = \frac{Ly}{2}\left[\frac{y^2}{3} - \eta^2\right], \tag{2.49}$$

$$\theta_0 = \frac{1}{2}\left[1 + \frac{y}{\eta}\right] + \frac{BrL^2}{12}\left[\eta^4 - y^4\right], \tag{2.50}$$

$$\phi_0 = \frac{1}{2}\left[\frac{\cosh N_1 y}{\cosh N_1\eta} + \frac{\sinh N_1 y}{\sinh N_1\eta}\right], \tag{2.51}$$

where

$$N_1 = \sqrt{\gamma Sc}, \ L = 8\epsilon\pi^3\left[\frac{E_3}{2\pi}\sin 2\pi(x - ct) - (E_1 + E_2)\cos 2\pi(x - ct)\right].$$

2.4.4 First order solution

The solution of the first order system is

$$\Psi_1 = L_1 y + L_2\frac{y^2}{2} + L_3\frac{y^3}{6} - \frac{y^4}{48\eta} + \frac{BrL^2y^7}{2520} - \frac{N}{2N_1^3} \times \left[\frac{\sinh N_1 y}{\cosh N_1\eta} + \frac{\cosh N_1 y}{\sinh N_1\eta}\right], \tag{2.52}$$

$$\begin{aligned}\theta_1 \;=\;& A_1 + A_2 y - BrL\left[L_2\frac{y^3}{3} + L_3\frac{y^4}{6} - \frac{y^5}{40\eta} + \frac{BrL^2y^8}{1680}\right] + \frac{BrLN}{N_1^3\cosh N_1\eta} \times \\ & \left(y\sinh N_1 y - \frac{2\cosh N_1 y}{N_1}\right) + \frac{BrLN}{N_1^3\sinh N_1\eta} \times \left(y\cosh N_1 y - \frac{2\sinh N_1 y}{N_1}\right),\end{aligned} \tag{2.53}$$

$$\phi_1 = 0, \tag{2.54}$$

where

$$\begin{aligned}
L_1 &= \frac{\eta^2}{4}+\frac{7BrL^2\eta^6}{180}+\frac{\beta_2 BrL^2\eta^5}{6}+\frac{N}{2N_1^2},\\
L_2 &= \frac{1}{2\eta}*\left[\frac{N}{N_1^2}+\frac{\eta^2}{6}\right],\\
L_3 &= -\frac{1}{2}\left(1+\frac{BrL^2\eta^4}{6}\right),\\
A_1 &= 2BrL\left[\frac{L_3\eta^4}{12}+\frac{BrL^2\eta^8}{3360}\right]-\frac{2BrLN}{2N_1^3\left(\cosh N_1\eta\right)}\times\left[\eta\sinh N_1\eta-\frac{2\cosh N_1\eta}{N_1}\right],\\
A_2 &= \frac{2BrL}{\eta}\left[\frac{L_2\eta^3}{6}-\frac{\eta^4}{80}\right]-\frac{2BrLN}{2\eta N_1^3\left(\sinh N_1\eta\right)}\times\left[\eta\cosh N_1\eta-\frac{2\sinh N_1\eta}{N_1}\right].
\end{aligned}$$

Heat transfer coefficients at the zeroth and first orders are given by

$$Z_0 = \theta_{0y}\left(\eta\right)\eta_x = \eta_x\left(\frac{1}{2\eta}-\frac{BrL^2\eta^3}{3}\right), \tag{2.55}$$

$$\begin{aligned}
Z_1 &= \eta_x\theta_{1y}\left(\eta\right),\\
&= \eta_x\left[A_2-2BrL\left\{L_2\frac{\eta^2}{2}+L_3\frac{\eta^3}{3}-\frac{\eta^3}{16}+\frac{BrL^2\eta^7}{420}\right\}\right]+\frac{\eta_x BrLN}{N_1^2\cosh N_1\eta}\\
&\quad\times\left[\eta\cosh N_1\eta-\frac{\sinh N_1\eta}{N_1}\right]+\frac{\eta_x BrLN}{N_1^2\sinh N_1\eta}\times\left[\eta\sinh N_1\eta-\frac{\cosh N_1\eta}{N_1}\right].
\end{aligned} \tag{2.56}$$

2.5 Discussion

Graphical results have been displayed in order to explore the quantitative effects of sundry parameters which the expressions of stream function Ψ, longitudinal velocity $u=\Psi_0+Gr\Psi_{1y}$, temperature θ, heat transfer coefficient Z and mass concentration ϕ includes. Particulary, the role of compliant wall parameters, i.e. E_1 the elastic tension in the membrane, E_2 the mass per unit area, and E_3 the coefficient of viscous damping are described.

2.5.1 Analysis of velocity profile

Effect of different parameters on velocity profile is displayed in the $Figs.$ 2.2 $-$ 2.6. Here the velocity increases when we increase the values of Gr and Br $(shown\ in\ Figs.\ 2.2\ and\ 2.3)$. $Fig.$ 2.4 indicates the effect of Schmidt number on the velocity. Velocity decreases for increasing values of Sc. For the description of the effects of wall parameters, $Fig.$ 2.5 is plotted. It shows that velocity increases as we increase elastic tension E_1 and mass per unit area E_2 whereas it decreases as the viscous damping E_3 increases. To observe the effect of chemical reaction parameter γ, $Fig.$ 2.6 is plotted. Obviously the velocity decrease when γ is increased.

2.5.2 Analysis of temperature profile

$Figs.$ 2.7−2.11 depict the effect of different parameters on the temperature. $Fig.$ 2.7 shows that for increasing values of Gr the temperature increases. Similar behavior is noticed for increasing values of Br ($Fig.$ 2.8). $Fig.$ 2.9 studies the effect of Schmidt number on temperature. It shows that temperature decreases with the increasing values of Sc. The effects of elastic tension E_1, mass per unit area E_2 and viscous damping E_3 are explained in $Fig.$ 2.10. The graph shows that by increasing the value of E_1 the temperature rises. Similar behavior is shown by E_2. It reduces for increasing value of E_3. To investigate the behavior of temperature for different values of chemical reaction parameter γ, $Fig.$ 2.11 is displayed. The temperature decreases for increasing values of γ. From the comparison of the results we observe that parameters show similar behavior for the velocity and temperature profiles.

2.5.3 Analysis of concentration distribution

The results in $Figs.$ 2.12 − 2.14 indicate the concentration distribution for the variations of Sc, γ and ϵ. Fig 2.12 displays that concentration field decreases with an increase in Sc. $Fig.$ 2.13 illustrates that by increasing value of chemical reaction parameter γ, the concentration field decreases. Effect of increasing value of ϵ on the mass concentration is displayed in $Fig.$ 2.14. It is observed that mass concentration decreases for large values of ϵ.

2.5.4 Heat transfer coefficient

To study the role of different parameters on heat transfer coefficient we have plotted $Figs.$ 2.15 − 2.17. $Fig.$ 2.15 shows that value of heat transfer coefficient decreases with the increase of Gr. Whereas it shows opposite behavior in the case of Br ($Fig.$ 2.16). Wall properties effect on heat transfer coefficient is illustrated in $Fig.$ 2.17. For increasing values of E_1 the heat transfer coefficient. Similar effect is shown by E_2. Whereas it decreases with the increase of

E_3. It is observed that effect of all parameters on heat transfer coefficient are similar to that of temperature.

2.5.5 Trapping phenomenon

The formation of an internally circulating bolus of fluid by closed streamlines is shown in $Figs$ 2.18 − 2.20. $Fig.$ 2.18 (a) and 2.18 (b) show that the behavior is increasing behavior for $x < 0.25$ and it decreases for $x > 0.25$ when Gr increases. $Figs$ 2.19 (a) and 2.19 (b) depict the effect of Br on streamlines. We observed that effect is qualitatively similar to that of Gr. $Fig.$ 2.20 illustrates that size of trapped bolus increases for increasing value of $E_1$and E_2.However such size decreases for increasing value of E_3.

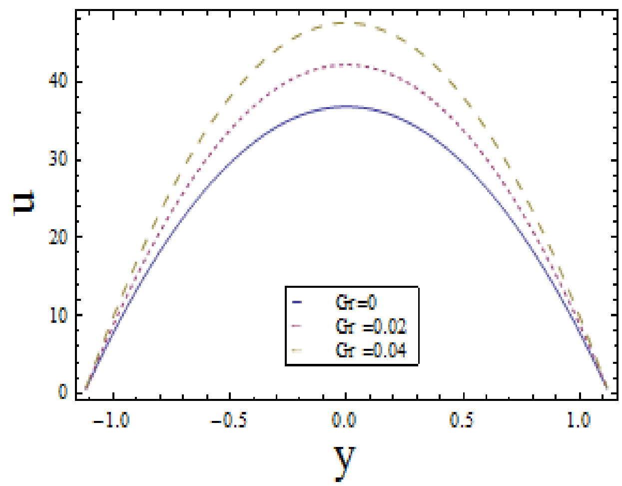


Fig. 2.2. Effect of Gr on u when $Br = 1$, $\gamma = 0.1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.5$, $Sc = 1$, $N = 1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

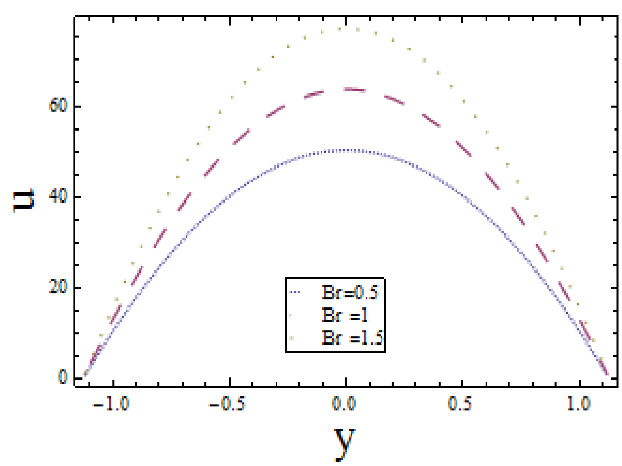
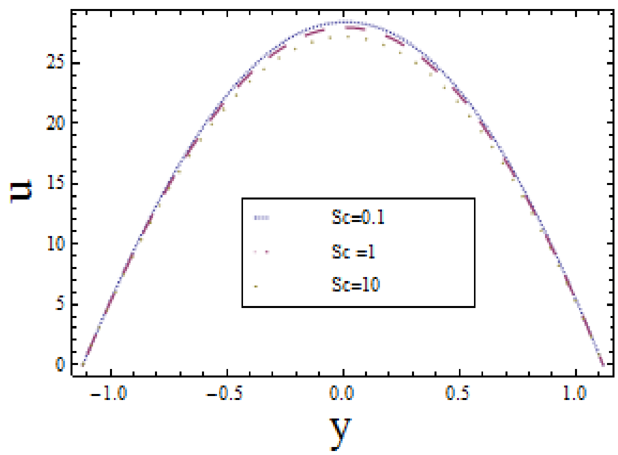
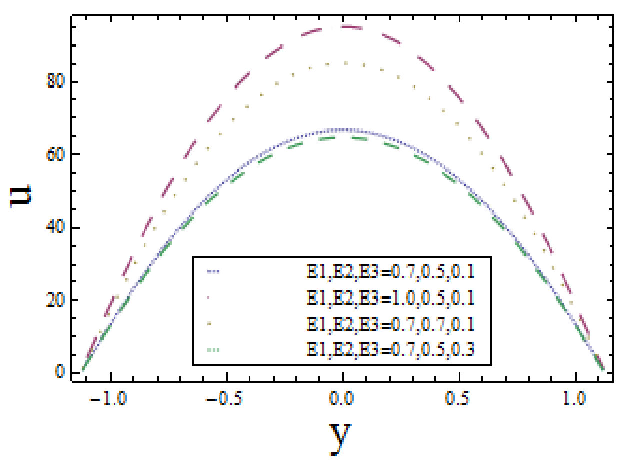


Fig. 2.3. Effect of Br on u when $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.5$, $Gr = 0.1$, $Sc = 1$, $N = 1$, $\gamma = 0.1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.



$Fig.$ 2.4. Effect of Sc on u when $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $Br = 1$, $Gr = 0.5$, $N = 10$, $\gamma = 1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.



$Fig.$ 2.5. Effect of parameters of wall properties on u when $Gr = 0.2$, $Br = 1$, $Sc = 1$, $N = 1$, $\gamma = 0.1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

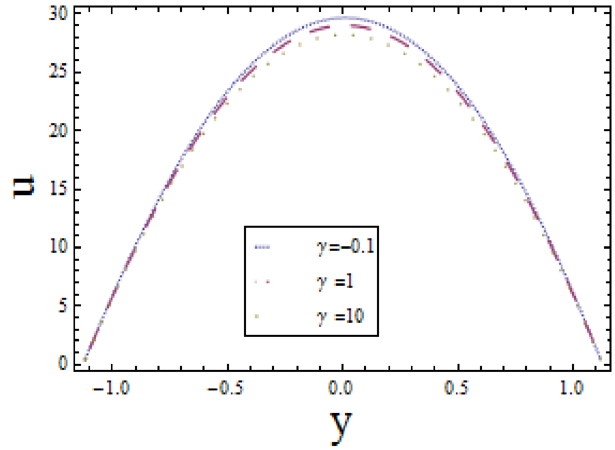


Fig. 2.6. Effect of γ on u when $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $Br = 1$, $Sc = 1$, $N = 10$, $Gr = 0.5$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

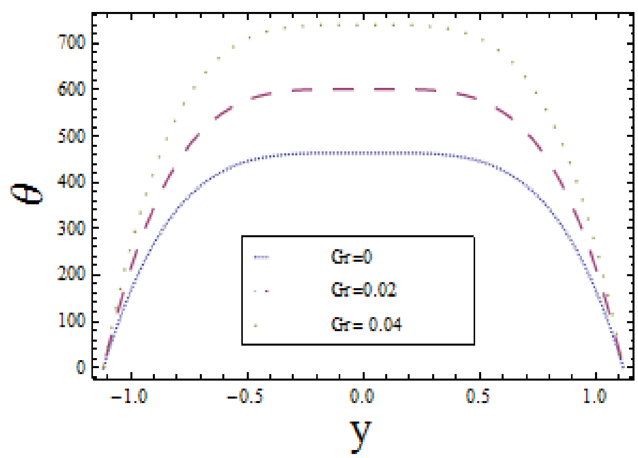


Fig. 2.7. Effect of Gr on θ when $Sc = 1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $Br = 1$, $N = 1$, $\gamma = 1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

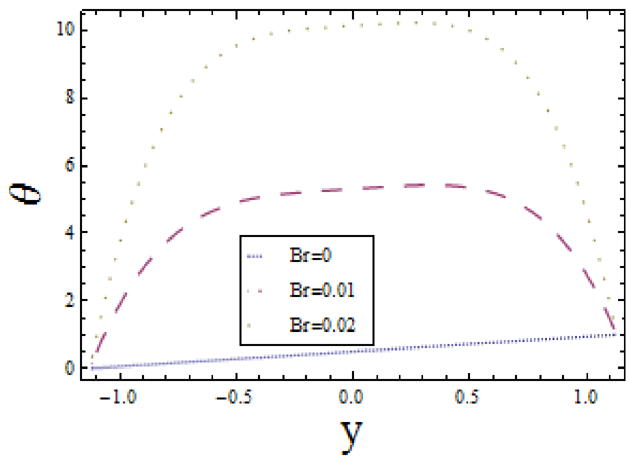


Fig. 2.8. Effect of Br on θ when $Gr = 0.01$. $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $Sc = 1$, $N = 0.5$, $\gamma = 0.1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

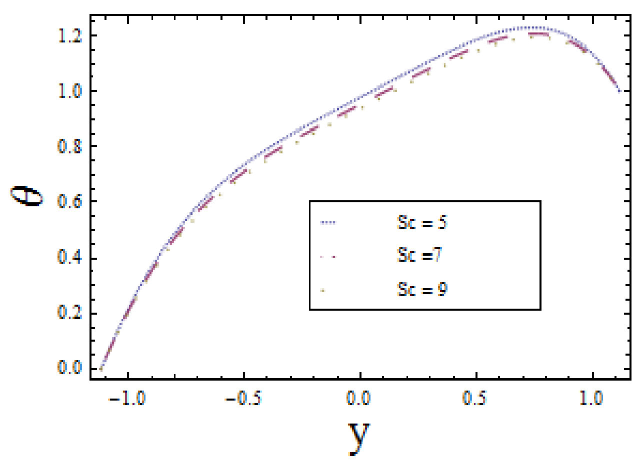


Fig. 2.9. Effect of Sc on u when $E_1 = 0.03$, $E_2 = 0.01$, $E_3 = 0.01$, $Br = 1$, $Gr = 0.2$, $N = 10$, $\gamma = 1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

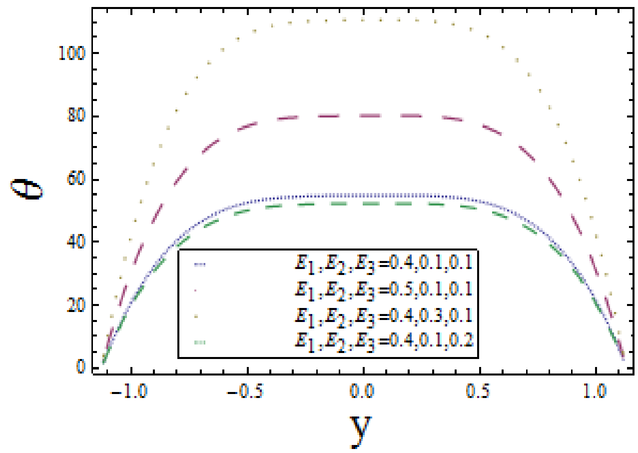


Fig. 2.10. Effect of wall properties parameters on u when $Gr = 0.01$, $Br = 1$, $Sc = 1$, $N = 0.5$, $\gamma = 0.1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

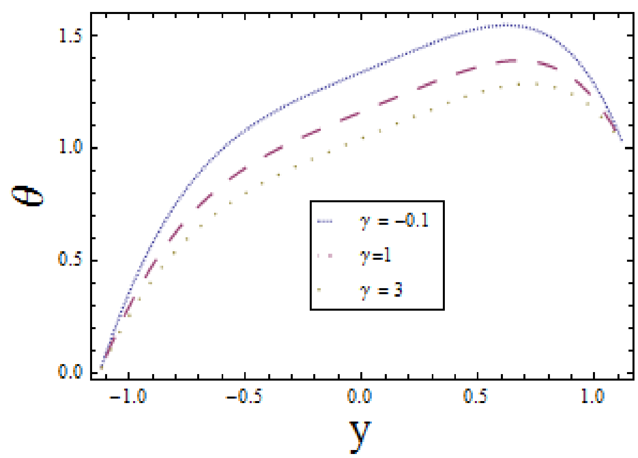


Fig. 2.11. Effect of γ on u when $N = 10$, $E_1 = 0.03$, $E_2 = 0.01$, $E_3 = 0.01$, $Br = 1$, $Sc = 1$, $Gr = 0.2$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

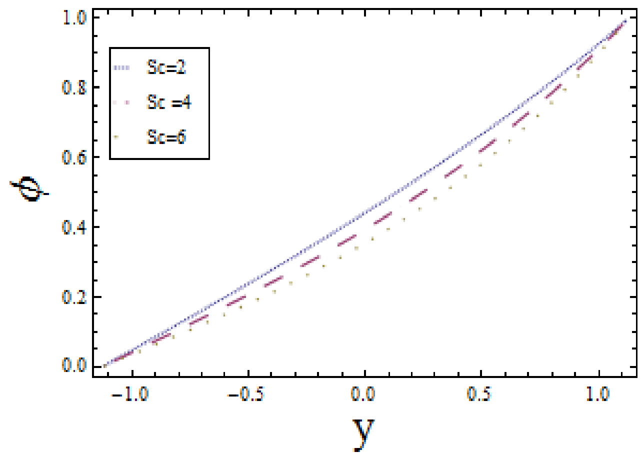


Fig. 2.12. Effect of Sc on ϕ when $\gamma = 0.1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

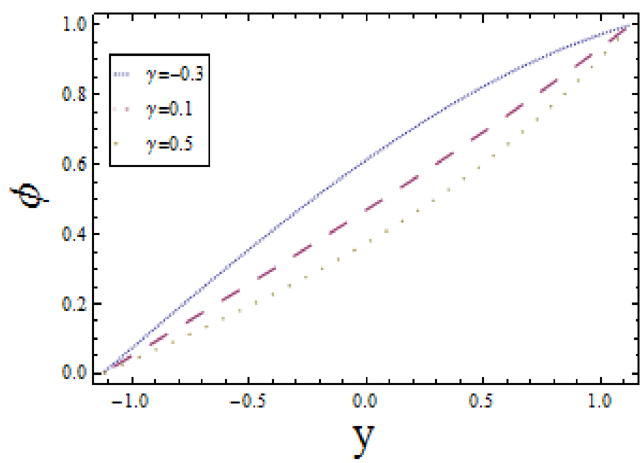


Fig. 2.13. Effect of γ on ϕ when $Sc = 1$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

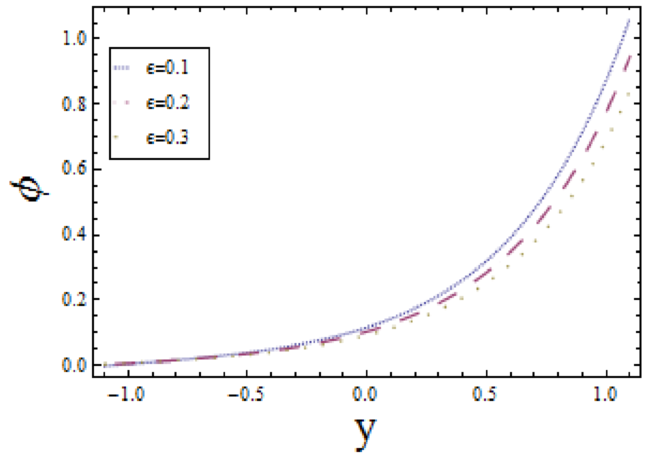


Fig. 2.14. Effect of ϵ on ϕ when $\gamma = 2$, $Sc = 2$, $x = 0.2$, $t = 0.1$.

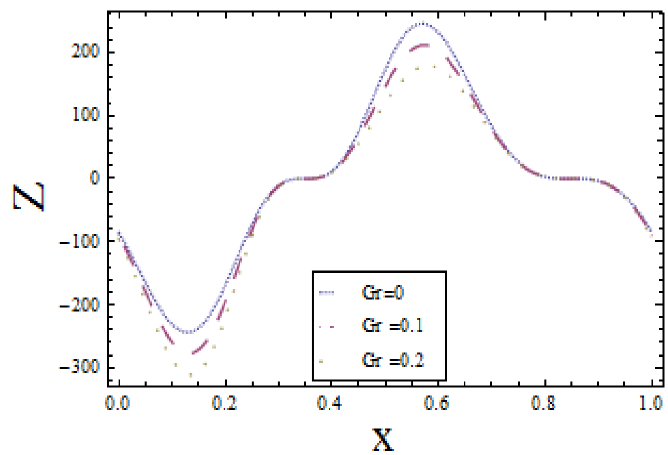


Fig. 2.15. Variation of Gr on Z when $\gamma = 1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $Br = 0.1$, $Sc = 1$, $N = 1$, $t = 0.1$, $\epsilon = 0.2$.

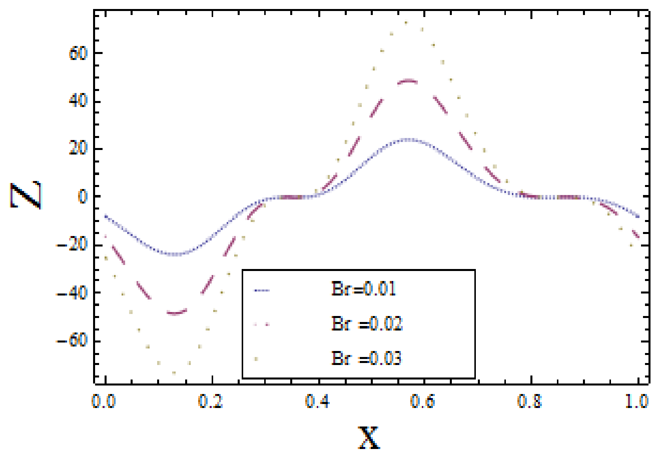


Fig. 2.16. Variation of Br on Z when $Gr = 0.01$, $Sc = 1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $N = 1$, $\gamma = 1$, $t = 0.1$, $\epsilon = 0.2$.

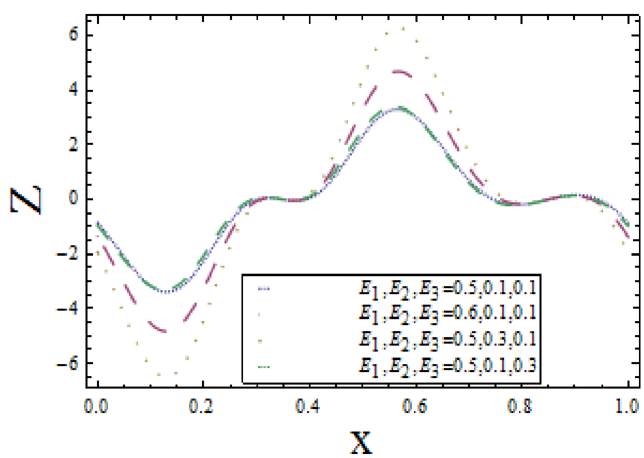
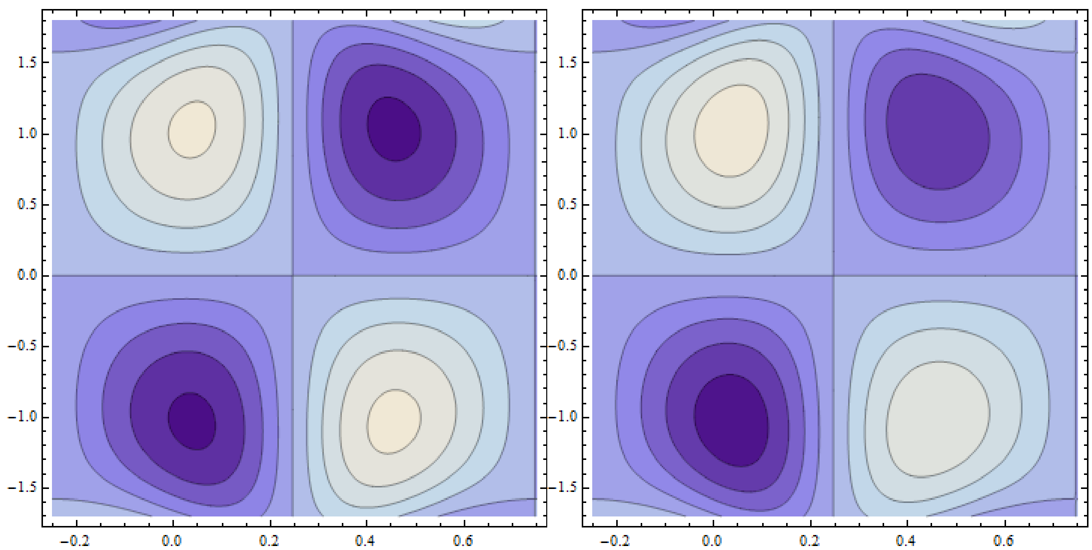
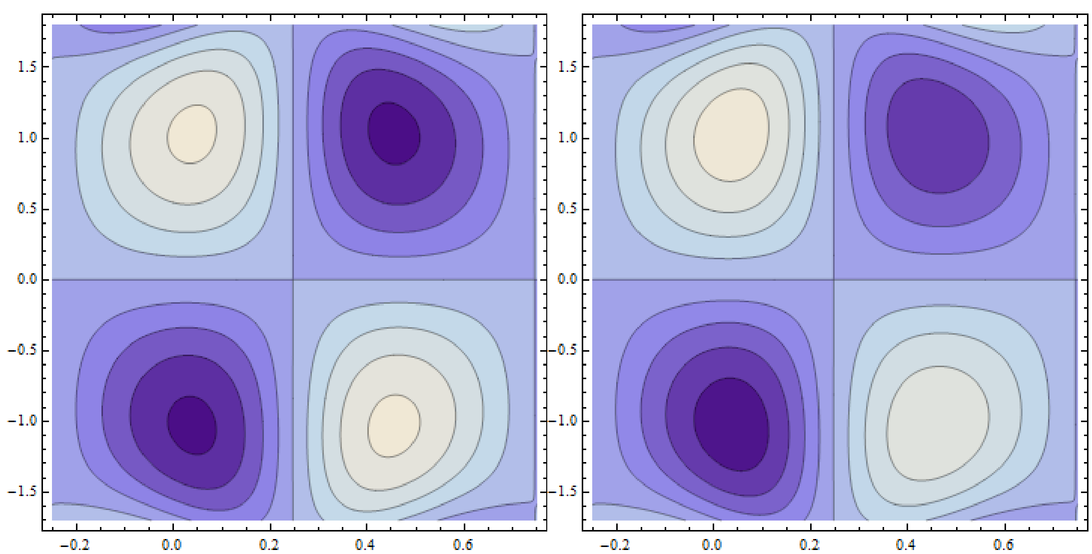


Fig. 2.17. Variation of wall properties on Z when $Br = 0.01$, $Gr = 0.2$, $Sc = 2$, $N = 1$, $\gamma = 1$, $t = 0.1$, $\epsilon = 0.2$.



(a) (b)

Fig. 2.18. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Br = 2$, $Sc = 2$, $N = 0.1$, $\gamma = 2$, $t = 0$, $\epsilon = 0.1$, (a) $Gr = 0$, (b) $Gr = 0.02$.



(a) (b)

Fig. 2.19. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Gr = 0.2$, $Sc = 2$, $N = 0.1$, $\gamma = 2$, $t = 0$, $\epsilon = 0.1$, (a) $Br = 0$, (b) $Br = 0.2$.

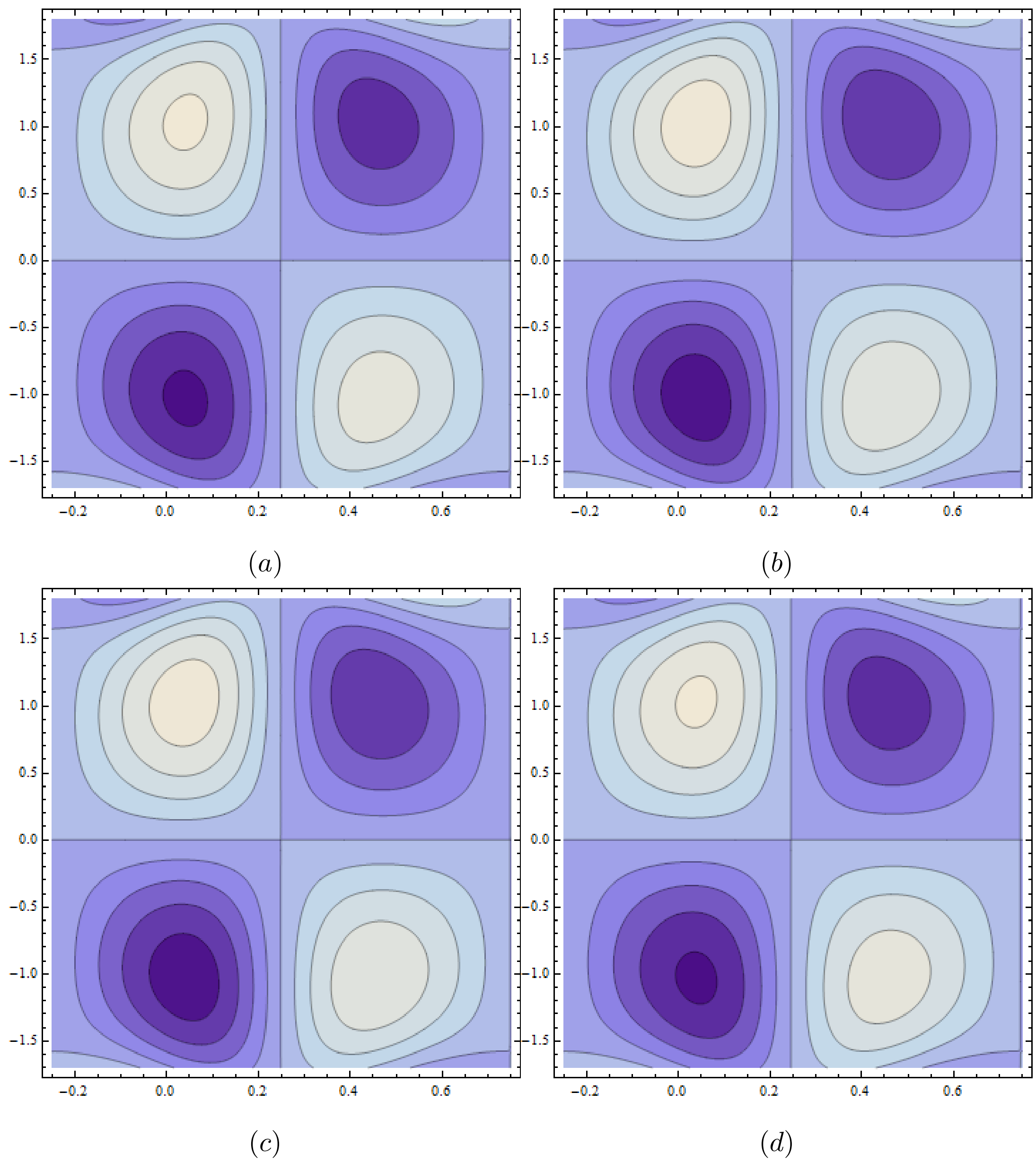


Fig. 2.20. Streamlines for $Gr = 0.02$, $Sc = 2$, $Br = 2$, $N = 0.1$, $t = 0$, $\gamma = 2$, $\epsilon = 0.1$, (a) $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.1$ (b) $E_1 = 1.1$, $E_2 = 0.1$, $E_3 = 0.1$ (c) $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.1$ (d) $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.2$.

Chapter 3

Partial slip effect on the peristaltic motion in a compliant wall channel with heat transfer and chemical reaction

3.1 Introduction

The purpose of this chapter is to address the influence of partial slip condition on peristaltic transport of viscous fluid in a vertical channel. Resulting equations are solved using the perturbation technique. Series expressions for velocity, temperature and heat transfer coefficient are obtained. The effects of various embedded parameters on the velocity, temperature and heat transfer coefficient have been pointed out. Streamlines are plotted and trapping phenomenon is discussed.

3.2 Mathematical formulation

We investigate the two-dimensional flow of an incompressible viscous fluid in a vertical symmetric channel of width $2d$. The waves are propagating on the channel walls with speed c. The wave shapes of the walls are expressed as

$$y = \eta(x,t) = +\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right] \quad \text{at} \quad \text{right wall}, \tag{3.1}$$

$$y = \eta(x,t) = -\left[d + a\sin\frac{2\pi}{\lambda}(x - ct)\right] \quad \text{at} \quad \text{left wall}, \tag{3.2}$$

where a is the wave amplitudes, λ is the wavelength and t the time.

In the static frame of reference the slip conditions at the walls are defined as

$$\begin{aligned} u_w - u(x,\eta,t) &= \beta_1 * \tau_{xy}, \quad \text{at} \quad y = +\eta, \\ u(x,-\eta,t) - u_w &= \beta_1 * \tau_{xy}, \quad \text{at} \quad y = -\eta, \end{aligned} \tag{3.3}$$

where $u(x,\eta,t)$ and u_w are defined as longitudinal velocity and wall velocity respectively. τ_{xy} is the shear stress, β_1 is the dimensional slip parameter. Since the waves are travelling along the distensible walls of the channel therefore

$$u_w = 0,$$

and thus $Eqs.$ (3.3) become

$$\begin{aligned} u(x,\eta,t) + \beta_1 * \tau_{xy} &= 0, \quad \text{at} \quad y = +\eta, \\ u(x,-\eta,t) - \beta_1 * \tau_{xy} &= 0, \quad \text{at} \quad y = -\eta. \end{aligned} \tag{3.4}$$

The thermal and concentration slip conditions can be defined as

$$T \pm \beta_2 * \frac{\partial T}{\partial y} = \begin{Bmatrix} T_1 \\ T_0 \end{Bmatrix} \quad \text{at } y = \pm\eta, \tag{3.5}$$

$$C \pm \beta_3 * \frac{\partial C}{\partial y} = \begin{Bmatrix} C_1 \\ C_0 \end{Bmatrix} \quad \text{at} \quad y = \pm\eta, \tag{3.6}$$

where β_2 and β_3 are the dimensional thermal and concentration slip parameters and T_1 and T_0 are the temperature and C_1 and C_0 are the concentrations of the walls respectively.

If $\Psi(x, y, t)$ is the stream function then

$$u = \frac{\partial \Psi}{\partial y}, \ v = -\frac{\partial \Psi}{\partial x}, \tag{3.7}$$

Now we define the following non-dimensional variables and parameters

$$\begin{aligned}
\bar{x} &= \frac{x}{\lambda}, \ \bar{y} = \frac{y}{d}, \ \bar{\Psi} = \frac{\Psi}{cd}, \ \bar{p} = \frac{d^2 p}{c\mu\lambda}, \ \bar{t} = \frac{ct}{\lambda}, \ \bar{\eta} = \frac{\eta}{d}, \\
\gamma &= k_1 \frac{d^2}{v}, \ \theta = \frac{T - T_0}{T_1 - T_0}, \ \phi = \frac{C - C_0}{C_1 - C_0}, \ \bar{\beta}_1 = \frac{\beta_1}{d}, \ \bar{\beta}_2 = \frac{\beta_2}{d}, \\
\bar{\beta}_3 &= \frac{\beta_3}{d}, \ \delta = \frac{d}{\lambda}, \ \epsilon = \frac{a}{d}, \ \gamma = \frac{k_1 d^2}{\nu}, \ Gr = \frac{g\beta_T (T_1 - T_0) d^2}{\nu c}, \\
\mathrm{Re} &= \frac{cd}{\nu}, \ Br = Ec * \mathrm{Pr}, \ \ \mathrm{Pr} = \mu C_P/\kappa, \ N = \frac{\beta_C (C_1 - C_0)}{\beta_T (T_1 - T_0)}, \ Sc = \frac{\mu}{\rho D}, \\
Ec &= \frac{c^2}{C_P (T_1 - T_0)}, \ E_1 = \frac{-\tau d^3}{\lambda^3 \mu c}, \ E_2 = \frac{m_1 c d^3}{\lambda^3 \mu}, \ E_3 = \frac{d' d^3}{\lambda^2 \mu}.
\end{aligned} \tag{3.8}$$

Here δ is the wave number, ϵ is the amplitude ratio, γ is the chemical reaction parameter , Gr depicts the perturbation parameter, Pr the Prandtl number, Re the Reynolds number, Br shows the Brinkman number, N represents the buoyancy ratio parameter, Ec is the Eckert number, Sc is the Schmidt number and E_1, E_2 and E_3 are the non-dimensional elasticity parameter. $\bar{\beta}_1$ $\bar{\beta}_2$ and $\bar{\beta}_3$ are the non-dimensional velocity, thermal and concentration slip parameters respectively.

Using non-dimensional variables, the governing mathematical problems are

$$\delta \, \mathrm{Re} \frac{d}{dt} \left[\delta^2 \Psi_{xx} + \Psi_{yy}\right] = \Psi_{yyyy} + 2\delta^2 \Psi_{xxyy} + \delta^4 \Psi_{xxxx} + Gr \left[\theta_y + N\phi_y\right], \tag{3.9}$$

$$\delta \, \mathrm{Pr} \, \mathrm{Re} \frac{d\theta}{dt} = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Br \left[4\delta^2 (\Psi_{xy})^2 + \left(\Psi_{yy} - \delta^2 \Psi_{xx}\right)^2\right], \tag{3.10}$$

$$\delta \, \mathrm{Re} \frac{d\phi}{dt} = \frac{1}{Sc} \left[\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right] - \gamma\phi, \tag{3.11}$$

$$\Psi_y \pm \beta_1 \left[\frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2}\right] = 0, \ \theta \pm \beta_2 \frac{\partial \theta}{\partial y} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \ \phi \pm \beta_3 \frac{\partial \phi}{\partial y} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix},$$

$$\left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t}\right] \eta = \frac{\partial^3 \Psi}{\partial y^3} + \delta^2 \frac{\partial^3 \Psi}{\partial x^2 \partial y} - \delta \, \mathrm{Re} \frac{d\Psi_y}{dt} + Gr \left[\theta + N\phi\right] \ \text{at} \ y = \pm\eta, \tag{3.12}$$

where the dimensionless form of η is written as

$$\eta = (1 + \epsilon \sin 2\pi (x - t)).$$

Using long wavelength and low Reynolds number approximations, Eqs. (3.9) – (3.12) take the forms

$$\Psi_{yyyy} + Gr\left[\theta_y + N\phi_y\right] = 0, \tag{3.13}$$

$$\frac{\partial^2\theta}{\partial y^2} + Br\left(\Psi_{yy}\right)^2 = 0, \tag{3.14}$$

$$\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} - \gamma\phi = 0, \tag{3.15}$$

$$\Psi_y \pm \beta_1\frac{\partial^2\Psi}{\partial y^2} = 0,\ \theta \pm \beta_2\frac{\partial\theta}{\partial y} = \begin{Bmatrix}1\\0\end{Bmatrix},\ \phi \pm \beta_3\frac{\partial\phi}{\partial y} = \begin{Bmatrix}1\\0\end{Bmatrix},$$

$$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial x\partial t}\right]\eta = \frac{\partial^3\Psi}{\partial y^3} + Gr\left[\theta + N\phi\right] \text{ at } y = \pm\eta. \tag{3.16}$$

3.3 Solution procedure

We are interested to find the solutions of flow quantities in terms of small Grashof number Gr. Hence it is reasonable to expand the quantities as follows:

$$\begin{aligned}
\Psi &= \Psi_0 + Gr\Psi_1 + O\left(Gr^2\right) + ...,\\
\theta &= \theta_0 + Gr\theta_1 + O\left(Gr^2\right) + ...,\\
Z &= Z_0 + GrZ_1 + O\left(Gr^2\right) + ...,\\
\phi &= \phi_0 + Gr\phi_1 + O\left(Gr^2\right) + ...
\end{aligned} \tag{3.17}$$

Substituting above expressions into Eqs. (3.13) – (3.16), we get two systems of equations for Gr^0 and Gr^1 as follows.

3.3.1 Zeroth order system

The relevant problem at this order is

$$\Psi_{0yyyy} = 0, \tag{3.18}$$

$$\theta_{0yy} + Br\left(\Psi_{0yy}\right)^2 = 0, \tag{3.19}$$

$$\frac{1}{Sc}\phi_{0yy} - \gamma\phi_0 = 0, \tag{3.20}$$

$$\Psi_{0y} \pm \beta_1\Psi_{0yy} = 0,\ \theta_0 \pm \beta_2\theta_{0y} = \begin{Bmatrix}1\\0\end{Bmatrix},\ \phi_0 \pm \beta_3\phi_{0y} = \begin{Bmatrix}1\\0\end{Bmatrix},\ \text{ at }\ y = \pm\eta,$$

$$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial x\partial t}\right]\eta = \Psi_{0yyy} \quad \text{at } y = \pm\eta. \tag{3.21}$$

3.3.2 First order system

At this order of system the subjected problem can be expressed as

$$\Psi_{1yyyy} = -\left[\theta_{0y} + N\phi_{0y}\right], \tag{3.22}$$

$$\theta_{1yy} = -2Br\psi_{0yy}\Psi_{1yy}, \tag{3.23}$$

$$\frac{1}{Sc}\frac{\partial^2\phi_1}{\partial y^2} - \gamma\phi = 0, \tag{3.24}$$

$$\Psi_{1y} \pm \beta_1\Psi_{1yy} = 0,\ \theta_1 \pm \beta_2\theta_{1y} = 0,\ \phi_1 \pm \beta_3\phi_{1y} = 0,$$

$$\Psi_{1yyy} + [\theta_0 + N\phi_0] = 0 \ \text{ at } y = \pm\eta. \tag{3.25}$$

3.3.3 Zeroth order solution

Here the solution expression are

$$\Psi_0 = \frac{Ly}{2}\left[\frac{y^2}{3} - \eta^2\right] - L\beta_1 y\eta, \tag{3.26}$$

$$\theta_0 = \frac{1}{2}\left[1 + \frac{y}{\eta + \beta_2}\right] + \frac{BrL^2}{12}\left[\eta^4 - y^4\right] + \beta_2 BrL^2\frac{\eta^3}{3}, \tag{3.27}$$

$$\phi_0 = \frac{1}{2}\left[\frac{\cosh N_1 y}{(\cosh N_1\eta + \beta_3 N_1 \sinh N_1\eta)} + \frac{\sinh N_1 y}{(\sinh N_1\eta + \beta_3 N_1 \cosh N_1\eta)}\right], \tag{3.28}$$

$$N_1 = \sqrt{\gamma Sc},\ L = 8\epsilon\pi^3\left[\frac{E_3}{2\pi}\sin 2\pi\,(x-ct) - (E_1+E_2)\cos 2\pi\,(x-ct)\right].$$

3.3.4 First order solution

Here the resulting expressions are

$$\begin{aligned}\Psi_1 \quad = \quad & L_1 y + L_2\frac{y^2}{2} + L_3\frac{y^3}{6} - \frac{y^4}{48\,(\eta+\beta_2)} + \frac{BrL^2y^7}{2520} \\ & -\frac{N}{2N_1^3}\times\left[\frac{\sinh N_1 y}{(\cosh N_1\eta + \beta_3 N_1 \sinh N_1\eta)} + \frac{\cosh N_1 y}{(\sinh N_1\eta + \beta_3 N_1 \cosh N_1\eta)}\right],\end{aligned} \tag{3.29}$$

$$\begin{aligned}\theta_1 \quad = \quad & A_1 + A_2 y - BrL\left[L_2\frac{y^3}{3} + L_3\frac{y^4}{6} - \frac{y^5}{40\,(\eta+\beta_2)} + \frac{BrL^2y^8}{1680}\right] \\ & + \frac{BrLN}{N_1^3\,(\cosh N_1\eta + \beta_3 N_1 \sinh N_1\eta)}\times\left(y\sinh N_1 y - \frac{2\cosh N_1 y}{N_1}\right) \\ & + \frac{BrLN}{N_1^3\,(\sinh N_1\eta + \beta_3 N_1 \cosh N_1\eta)}\times\left(y\cosh N_1 y - \frac{2\sinh N_1 y}{N_1}\right),\end{aligned} \tag{3.30}$$

$$\phi_1 = 0, \tag{3.31}$$

with

$$
\begin{aligned}
L_1 &= \frac{\eta^2}{4} + \frac{7BrL^2\eta^6}{180} + \frac{\beta_2 BrL^2\eta^5}{6} + 2\beta_1\eta\left(\frac{1}{4} + \frac{7BrL^2\eta^4}{24} + \frac{\beta_2 BrL^2\eta^3}{6}\right) - \frac{\beta_1 BrL^2\eta^6}{60} \\
&\quad + \frac{N}{2N_1\left(\cosh N_1\eta + \beta_3 N_1 \sinh N_1\eta\right)}\left[\frac{\cosh N_1\eta}{N_1} + \beta_1 \sinh N_1\eta\right], \\
L_2 &= \frac{\eta^3}{12\left(\eta+\beta_1\right)\left(\eta+\beta_2\right)} + \frac{\eta^2\beta_1}{4\left(\eta+\beta_1\right)\left(\eta+\beta_2\right)} + \frac{N}{2N_1\left(\eta+\beta_1\right)\left(\sinh N_1\eta + \beta_3 N_1\cosh N_1\eta\right)} \\
&\quad \times\left[\frac{\sinh N_1\eta}{N_1} + \beta_1\cosh N_1\eta\right], \\
L_3 &= -\frac{1}{2}\left(1 + \frac{BrL^2\eta^4}{6} + \frac{2\beta_2 BrL^2\eta^3}{3}\right), \\
A_1 &= 2BrL\left[\frac{L_3\eta^4}{12} + \frac{BrL^2\eta^8}{3360}\right] - \frac{2BrLN}{2N_1^3\left(\cosh N_1\eta + \beta_3 N_1\sinh N_1\eta\right)} \\
&\quad \times\left[\eta\sinh N_1\eta - \frac{2\cosh N_1\eta}{N_1}\right] - \beta_2\left[\begin{array}{c} -\frac{2BrLL_3\eta^3}{3} - \frac{Br^2L^2\eta^7}{210} + \frac{N}{2N_1^2\left(\cosh N_1\eta+\beta_3 N_1\sinh N_1\eta\right)} \\ \times\left(\eta\cosh N_1\eta - \frac{\sinh N_1\eta}{N_1}\right)\end{array}\right],
\end{aligned}
$$

$$
\begin{aligned}
A_2 &= \frac{2BrL}{\left(\eta+\beta_2\right)}\left[\frac{L_2\eta^3}{6} - \frac{\eta^5}{80\left(\eta+\beta_2\right)}\right] - \frac{2BrLN}{2N_1^3\left(\eta+\beta_2\right)\left(\sinh N_1\eta + \beta_3 N_1\cosh N_1\eta\right)} \\
&\quad \times\left[\eta\cosh N_1\eta - \frac{2\sinh N_1\eta}{N_1}\right] - \frac{\beta_2}{\left(\eta+\beta_2\right)} \\
&\quad \times\left[-BrLL_2\eta^2 + \frac{BrL\eta^4}{8\left(\eta+\beta_2\right)} + \frac{2BrLN}{2N_1^2\left(\cosh N_1\eta + \beta_3 N_1\sinh N_1\eta\right)} \times\left(\eta\sinh N_1\eta - \frac{\cosh N_1\eta}{N_1}\right)\right].
\end{aligned}
$$

The heat transfer coefficients at the walls are given by

$$
\begin{aligned}
Z_0 &= \eta_x\theta_{0y}(\eta) = \eta_x\left(\frac{1}{2\left(\eta+\beta_2\right)} - \frac{BrL^2\eta^3}{3}\right), \qquad (3.32) \\
Z_1 &= \eta_x\theta_{1y}(\eta), \\
&= \eta_x\left[A_2 - 2BrL\left\{L_2\frac{\eta^2}{2} + L_3\frac{\eta^3}{3} - \frac{\eta^4}{16\left(\eta+\beta_2\right)} + \frac{BrL^2\eta^7}{420}\right\}\right] \\
&\quad + \frac{\eta_x BrLN}{N_1^2\left(\cosh N_1\eta + \beta_3 N_1\sinh N_1\eta\right)} \times\left[\eta\cosh N_1\eta - \frac{\sinh N_1\eta}{N_1}\right] \\
&\quad + \frac{\eta_x BrLN}{N_1^2\left(\sinh N_1\eta + \beta_3 N_1\cosh N_1\eta\right)} \times\left[\eta\sinh N_1\eta - \frac{\cosh N_1\eta}{N_1}\right]. \qquad (3.33)
\end{aligned}
$$

3.4 Graphical results and discussion

This section is arrenged to scrutinize the quantitative effects of pertinent parameters which include perturbation parameter Gr, Brinkman number Br, Schmidt number Sc, slip parameters β_1, β_2 and β_3, chemical reaction parameter γ and occlusion parameter ε respectively. In particular the roles of wall parameters which are elastic tension in membrane E_1, mass per unit area E_2 and the coefficient of viscous damping E_3 are explained. This section is further divided into five subsections consisting of the results for velocity, temperature, concentration, heat transfer coefficient and trapping phenomenon.

3.4.1 Analysis of velocity profile

The velocity profile for various parameters of interest are displayed in the $Figs$ $3.1-3.8$. It is also interesting to note that the velocity profile is parabolic for fixed values of the parameters and its magnitude is maximum near the centre of the channel. Fig 3.1 depicts the behavior of velocity for different values of Gr where all other parameters are kept fixed. It is observed that velocity increases for increasing value of Gr. The effect of Br on velocity is shown in Fig 3.2. It can be noticed that behavior is similar to Gr. $Fig.$ 3.3 examines the effect of Sc on velocity which shows decrease in velocity for increasing values of Sc. The variations of elastic parameters E_1, E_2 and E_3 are shown in $Fig.$ 3.4. For increasing values of E_1 and E_2 the velocity increases and for increasing values of E_3, it decreases . $Fig.$ 3.5 is plotted to see the effect of γ on velocity profile. Decrease in velocity is observed for increasing values of γ. $Figs.$ $3.6-3.8$ are prepared to see the effect of slip parameters on velocity. Similar behavior is shown by β_1 and β_2 i.e. an increase in velocity is noted for increasing values whereas opposite behavior is shown by β_3 for increasing values.

3.4.2 Analysis of temperature profile

For the examination of the influence of different parameters on the temperature, we plotted the $Figs.$ $3.9-3.16$. $Fig.$ 3.9 explains the effect of Gr on temperature. Rise in temperature is noticed for increasing values of Gr. Similar effect is shown by Br in $Fig.$ 3.10. To study the influences of Sc and γ on temperature, $Figs.$ 3.11 and 3.12 are prepared. One can observe that temperature decreases for increasing values of Sc and γ. $Fig.$ 3.13 explores the effect of wall parameters on temperature. It is noticed that temperature increases for larger values of E_1 and E_2 whereas it decreases by increasing E_3. $Figs.$ 3.14 − 3.16 are sketched to observe the effects of slip parameters. Graphs reveal that for increasing value of β_1 the temperature increases in left part of channel and it decreases in right part. It is noticed that temperature increases for increasing values of β_2 and decreases for increasing values of β_3 respectively.

3.4.3 Analysis of concentration distribution

$Figs.$ 3.17 − 3.20 discuss the effect of sundry parameters on concentration distribution. It is noticed from $Figs.$ 3.17 and 3.18 that the concentration decreases as we increase the values of Sc and γ. $Figs.$ 3.19 and 3.20 are drawn to observe the effects of variation in ε and slip parameters. It is seen that for large values of ε and β_3, the concentration field decreases.

3.4.4 Heat transfer coefficient

In order to discuss the effects of different physical parameters on heat transfer coefficient Z, $Figs.$ 3.21 − 3.26 are sketched. $Figs.$ 3.21 and 3.22 depict that absolute value of heat transfer coefficient decreases with an increase in Gr. However it increases with Br. It is seen that value of heat transfer coefficient increases with the increase in the values of E_1 and E_2 whereas it decreases by increasing E_3 (*see* $Fig.$ 3.23). Effects of slip parameters are displayed in $Figs.$ 3.24 − 3.26. Heat transfer coefficient increases with the increase in the values of β_1 and β_3 whereas it decreases by increasing β_2.

3.4.5 Trapping phenomenon

$Figs.$ 3.27 − 3.32 displays the effect of different physical parameters on the streamlines. Fig. 3.27 is drawn to study the effect of perturbation parameter. It is concluded that the size of left trapped bolus increases with increase in Gr whereas size of right trapped bolus decreases. Similar behavior is seen for Br ($Fig.$ 3.28). The effect of elastic parameters on trapping can be seen in $Fig.$ 3.29. We notice increase in size of trapped bolus by increasing values of E_1 and E_2 whereas trapping bolus decreases for increasing values of E_3. Impact of slip parameters on trapping is illustrated in the $Figs.$ 3.30 − 3.32. It is shown in $Fig.$ 3.30 that size and number of trapping bolus increase with increasing value of β_1. Similar effect is seen for β_2 $(Fig.\ 3.31)$. Mixed behavior is observed for β_3. i.e. size of left bolus increases and right bolus decreases with the increase of β_3.

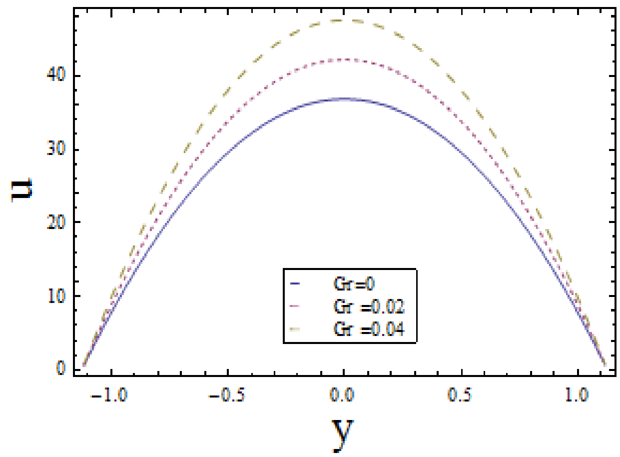


Fig. 3.1. Effect of Gr on u when $\gamma = 0.1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.5$, $Br = 1$, $Sc = 1$, $N = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

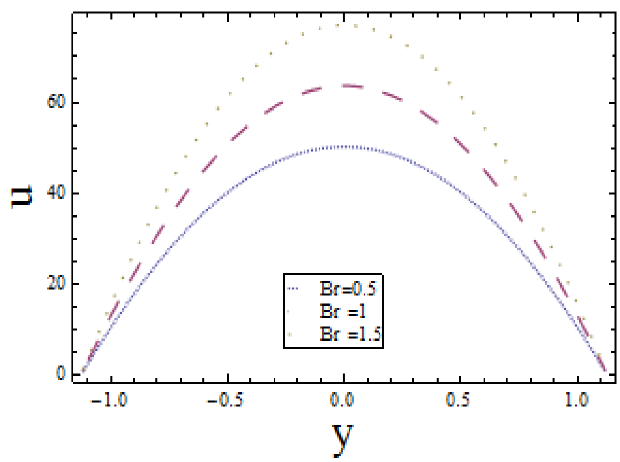


Fig. 3.2. Effect of Br on u when $Gr = 0.1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.5$, $Sc = 1$, $N = 1$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

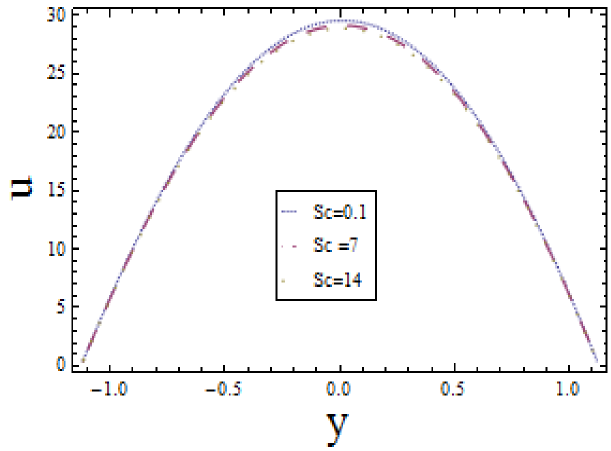


Fig. 3.3. Effect of Sc on u when $N = 10$, $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $Br = 1$, $Gr = 0.5$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

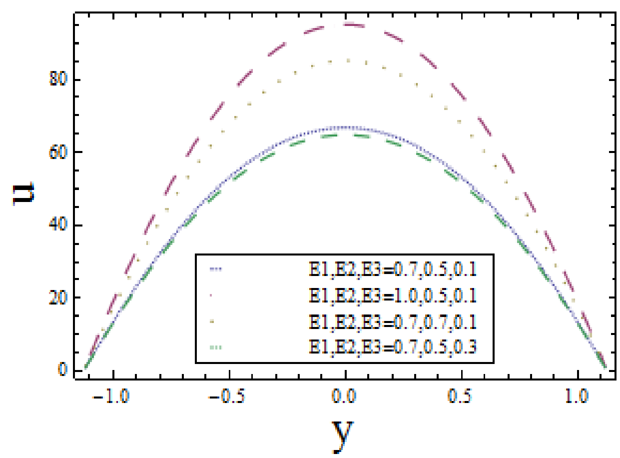


Fig. 3.4. Effect of parameters of wall properties on u when $Gr = 0.2$, $Br = 1$, $Sc = 1$, $N = 1$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

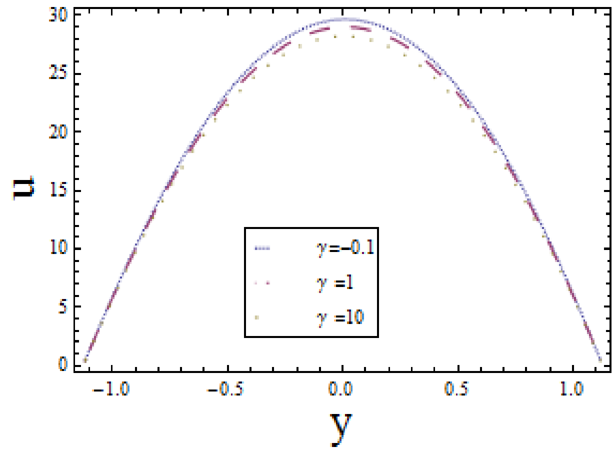


Fig. 3.5. Effect of γ on u when $E_1 = 0.3$, $E_2 = 0.2$, $E_3 = 0.1$, $Br = 1$, $Sc = 1$, $N = 10$, $Gr = 0.5$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

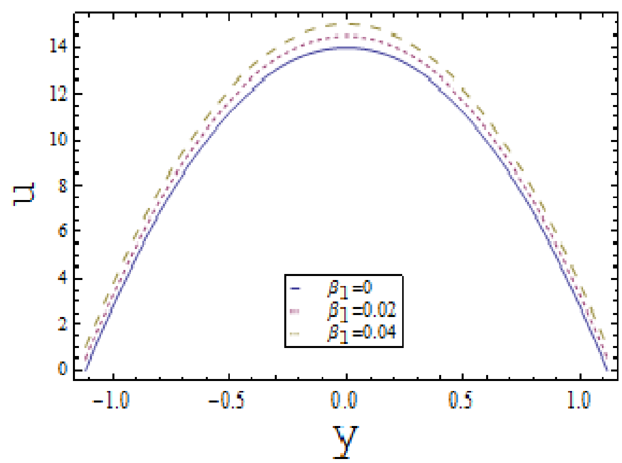


Fig. 3.6. Effect of β_1 on u when $E_1 = 0.1$, $E_2 = 0.5$, $E_3 = 0.5$, $Br = 1$, $Sc = 2$, $N = 0.1$, $Gr = 0.01$, $\gamma = 0.1$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

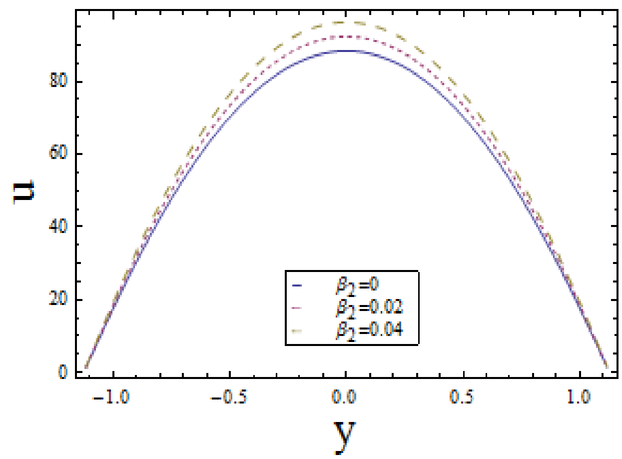


Fig. 3.7. Effect of β_2 on u when $Br = 2$, $Sc = 1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.5$, $N = 0.1$, $Gr = 0.1$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

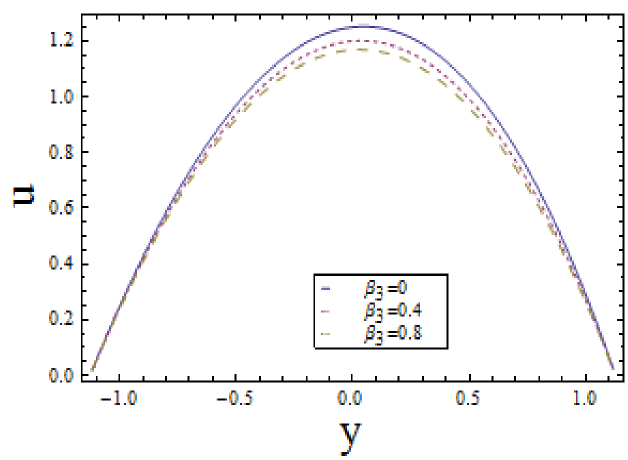


Fig. 3.8. Effect of β_3 on u when $Gr = 0.1$, $\gamma = 1$, $E_1 = 0.03$, $E_2 = 0.01$, $E_3 = 0.01$, $Br = 1$, $Sc = 1, N = 10$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

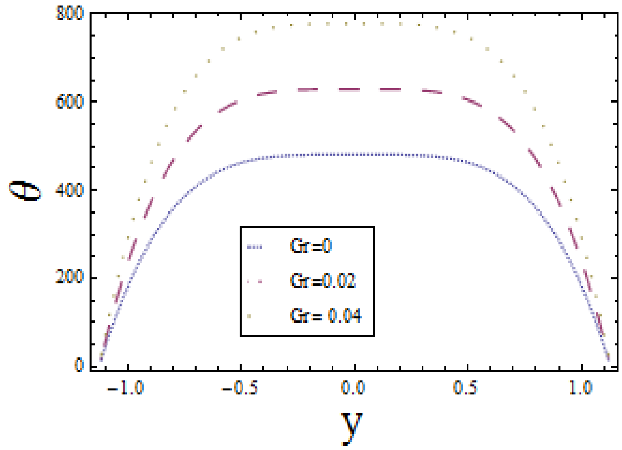


Fig. 3.9. Effect of Gr on θ when $Br = 1$, $Sc = 2$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $N = 1$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

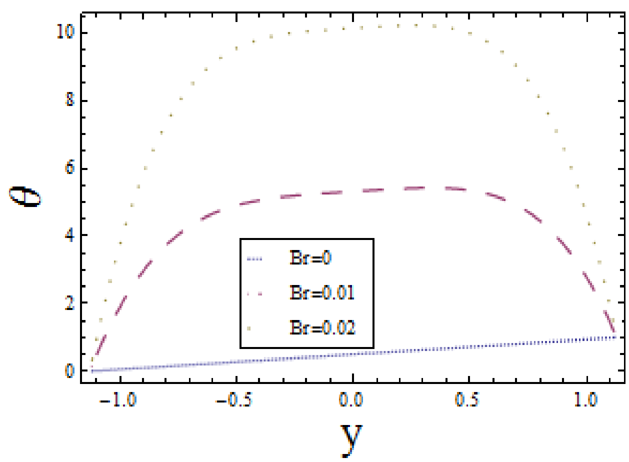


Fig. 3.10. Effect of Br on θ when $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $Gr = 0.01$, $Sc = 1$, $N = 0.5$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

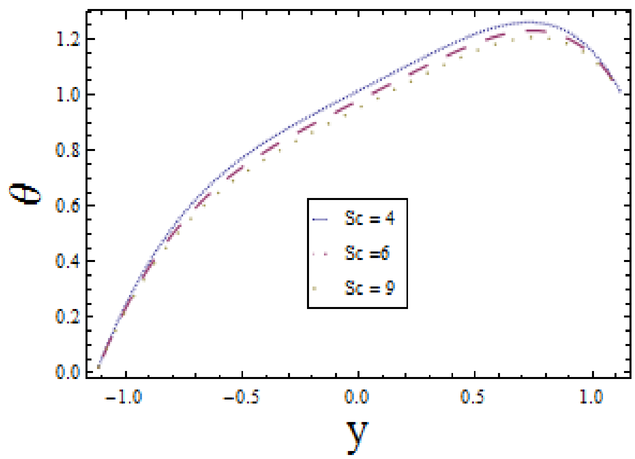


Fig. 3.11. Effect of Sc on θ when $E_1 = 0.03$, $E_2 = 0.01$, $E_3 = 0.01$, $Br = 1$, $Gr = 0.2$, $N = 10$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

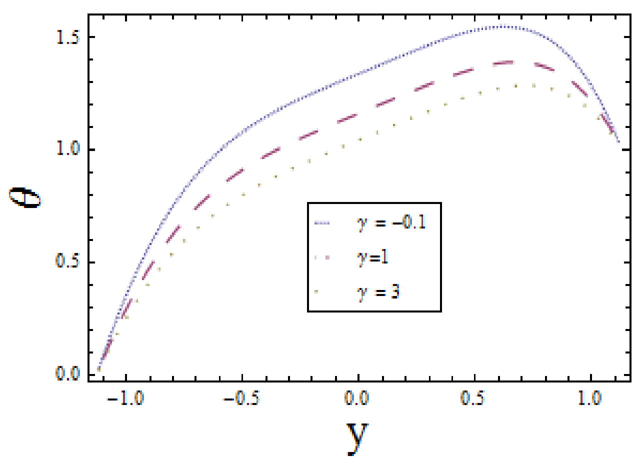


Fig. 3.12. Effect of γ on θ when $E_1 = 0.03$, $E_2 = 0.01$, $E_3 = 0.01$, $Br = 1$, $Sc = 1$, $N = 10$, $Gr = 0.2$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

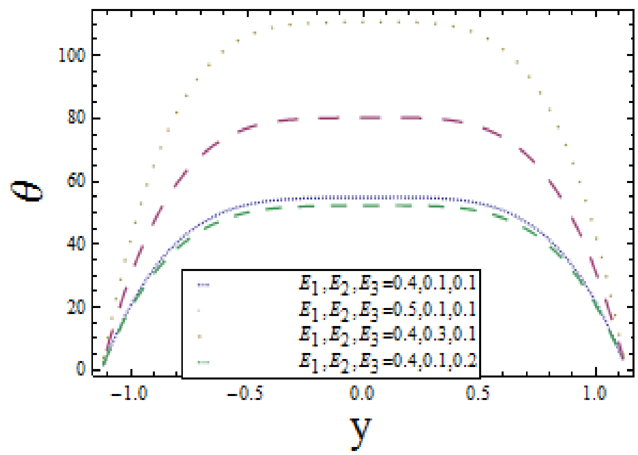


Fig. 3.13. Effect of parameters of wall properties on θ when $Gr = 0.01$, $Br = 1$, $Sc = 1$, $N = 0.1$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $\epsilon = 0.2$, $x = 0.2$, $t = 0.1$.

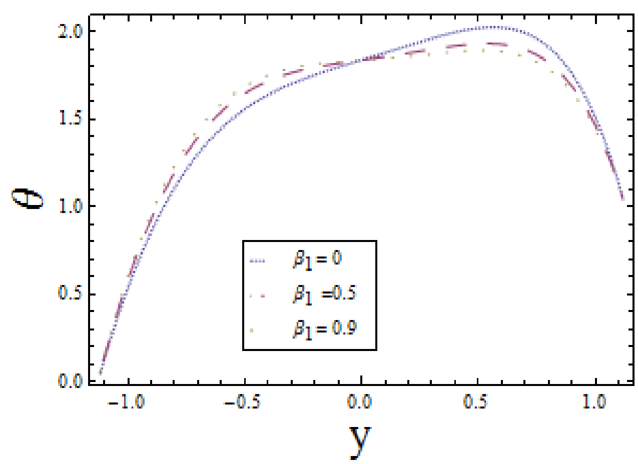


Fig. 3.14. Effect of β_1 on θ when $E_1 = 0.04$, $E_2 = 0.01$, $E_3 = 0.02$, $Br = 1$, $Sc = 2$, $N = 10$, $Gr = 0.5$, $\gamma = 1$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

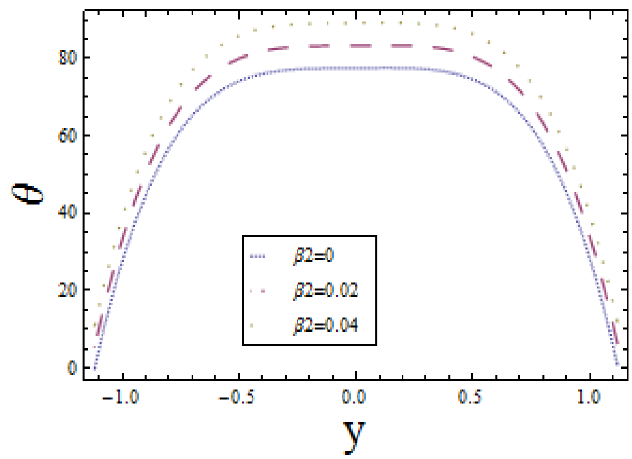


Fig. 3.15. Effect of β_2 on θ when $E_1 = 0.5$, $E_2 = 0.1$, $E_3 = 0.2$, $Br = 1$, $Sc = 2$, $N = 10$, $Gr = 0.01$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

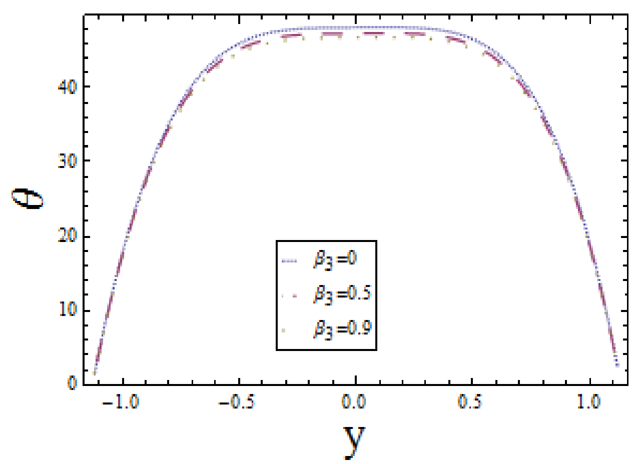


Fig. 3.16. Effect of β_3 on θ when $Sc = 3$, $N = 10$, $E_1 = 0.1$, $E_2 = 0.2$, $E_3 = 0.2$, $Br = 2$, $Gr = 0.5$, $\gamma = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

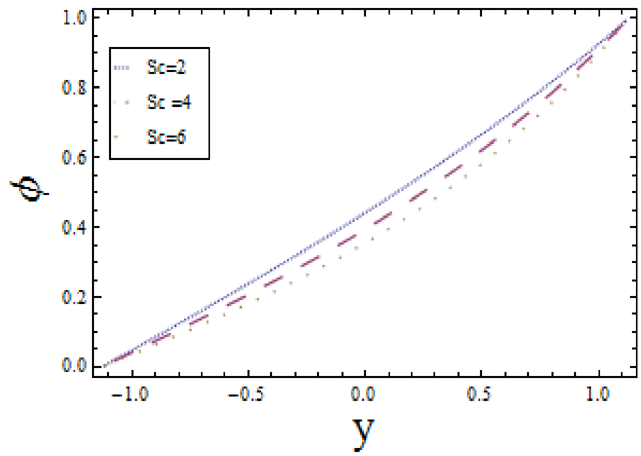


Fig. 3.17. Effect of Sc on ϕ when $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$, $\gamma = 0.1$, $\beta_3 = 0.01$.

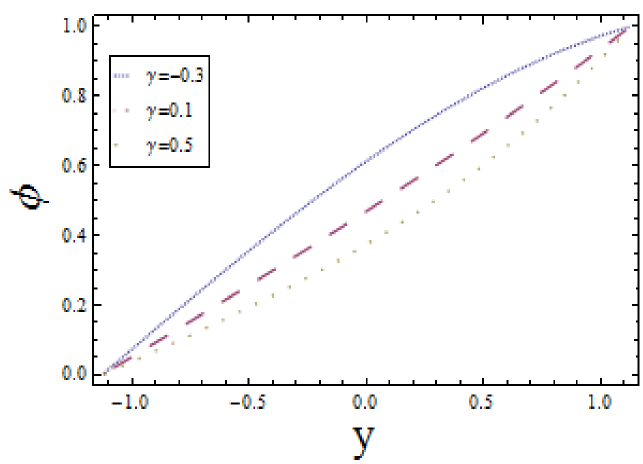


Fig. 3.18. Effect of γ on ϕ when $Sc = 1$, $\epsilon = 0.2$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$.

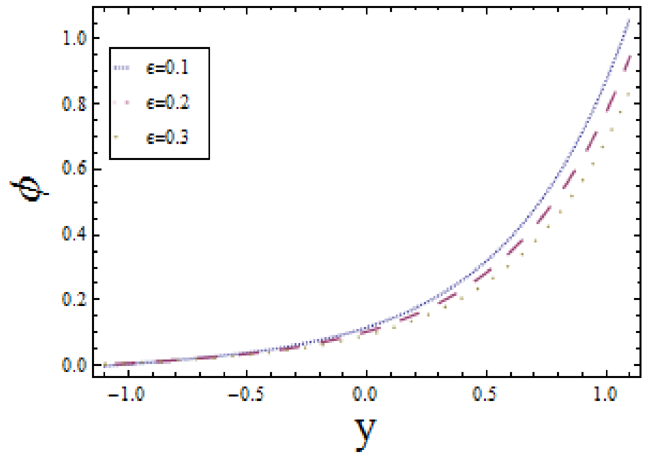


Fig. 3.19. Effect of ϵ on ϕ when $\gamma = 2$, $Sc = 2$, $\beta_3 = 0.01$, $x = 0.2$, $t = 0.1$.

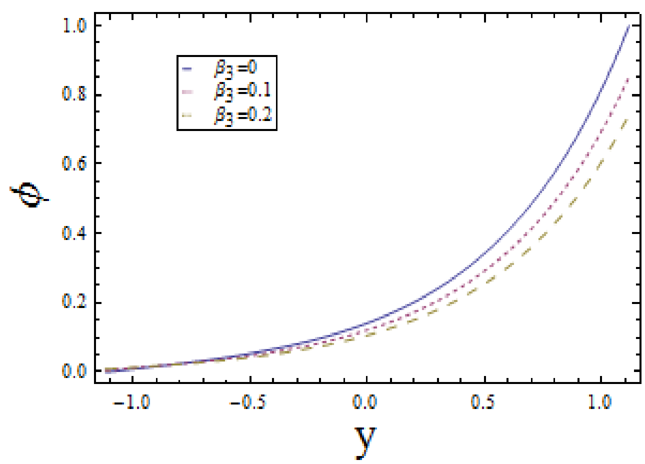


Fig. 3.20. Effect of β_3 on ϕ when $\gamma = 1$, $Sc = 3$, $x = 0.2$, $t = 0.1$, $\epsilon = 0.2$.

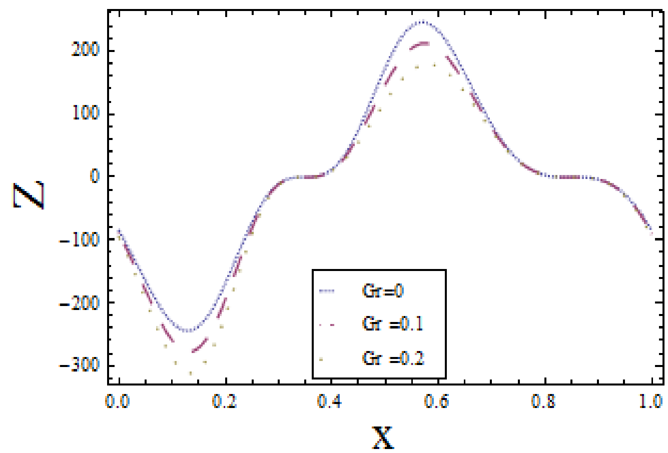


Fig. 3.21. Effect of Gr on Z when $Sc = 1$, $N = 1$, $E_1 = 1$, $E_2 = 0.5$, $E_3 = 0.1$, $Br = 0.1$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0.1$, $\epsilon = 0.2$.

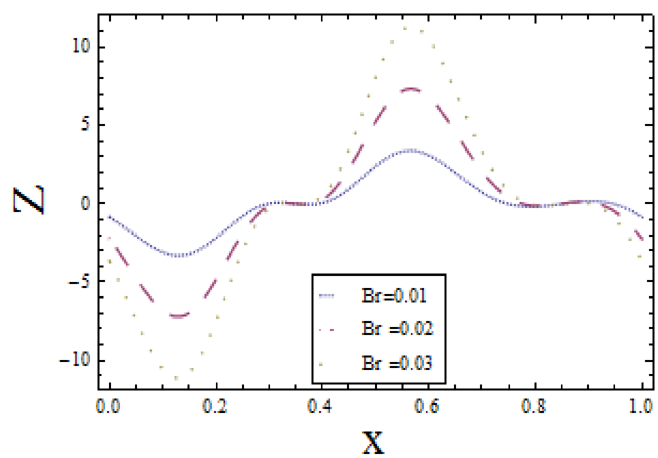


Fig. 3.22. Effect of Br on Z when $E_1 = 0.1$, $E_2 = 0.5$, $E_3 = 0.1$, $Gr = 0.01$, $Sc = 1$, $N = 1$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0.1$, $\epsilon = 0.2$.

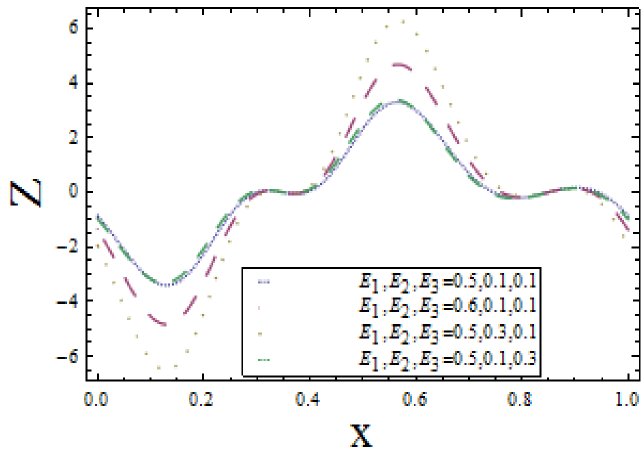


Fig. 3.23. Effect of wall properties on Z when $Br = 0.01$, $Gr = 0.2$, $Sc = 2$, $N = 1$, $\gamma = 1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0.1$, $\epsilon = 0.2$.

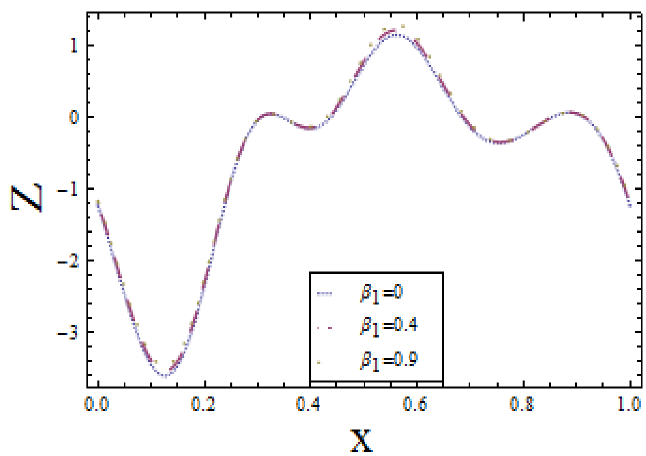


Fig. 3.24. Effect of β_1 on Z when $E_1 = 0.02$, $E_2 = 0.01$, $E_3 = 0.02$, $Br = 3$, $Sc = 1$, $N = 10$, $\gamma = 0.1$, $Gr = 0.05$, $\beta_2 = 0.1$, $\beta_3 = 0.1$, $t = 0.1$, $\epsilon = 0.2$.

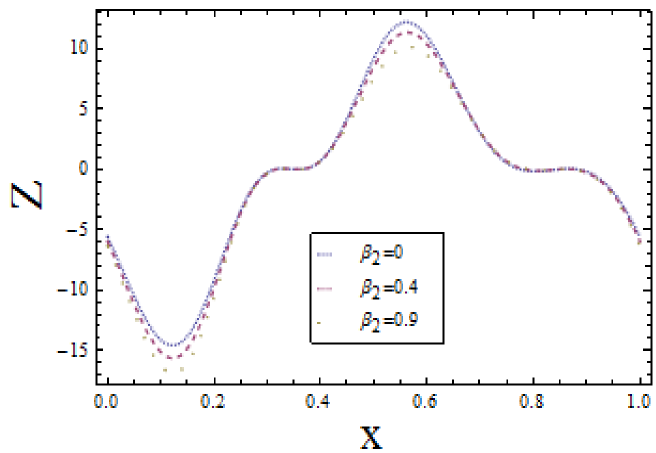


Fig. 3.25. Effect of β_2 on Z when $E_1 = 0.01$, $E_2 = 0.07$, $E_3 = 0.05$, $Br = 2$, $Gr = 0.03$, $Sc = 3$, $N = 10$, $\gamma = 0.1$, $\beta_1 = 0.1$, $\beta_3 = 0.1$, $t = 0.1$, $\epsilon = 0.2$.

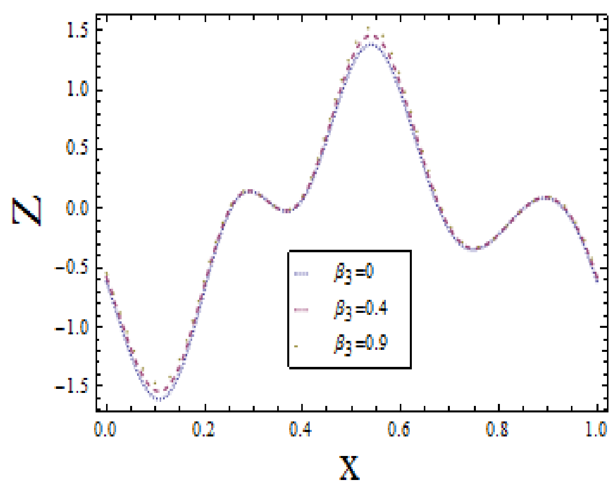
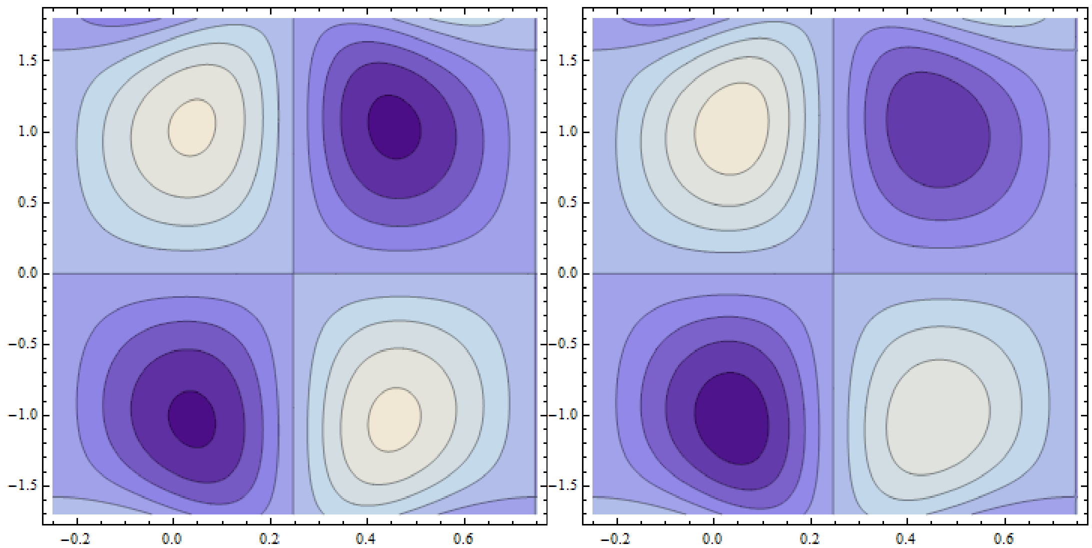
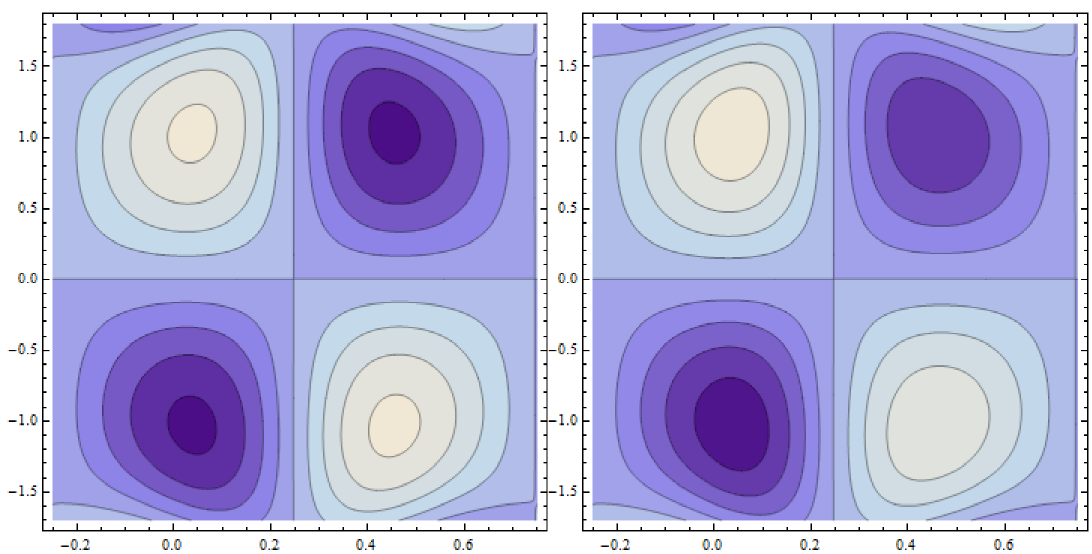


Fig. 3.26. Effect of β_3 on Z when $E_1 = 0.01$, $E_2 = 0.02$, $E_3 = 0.05$, $Br = 3$, $Gr = 0.03$, $Sc = 2$, $N = 10$, $\gamma = 1$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $t = 0.1$, $\epsilon = 0.2$.



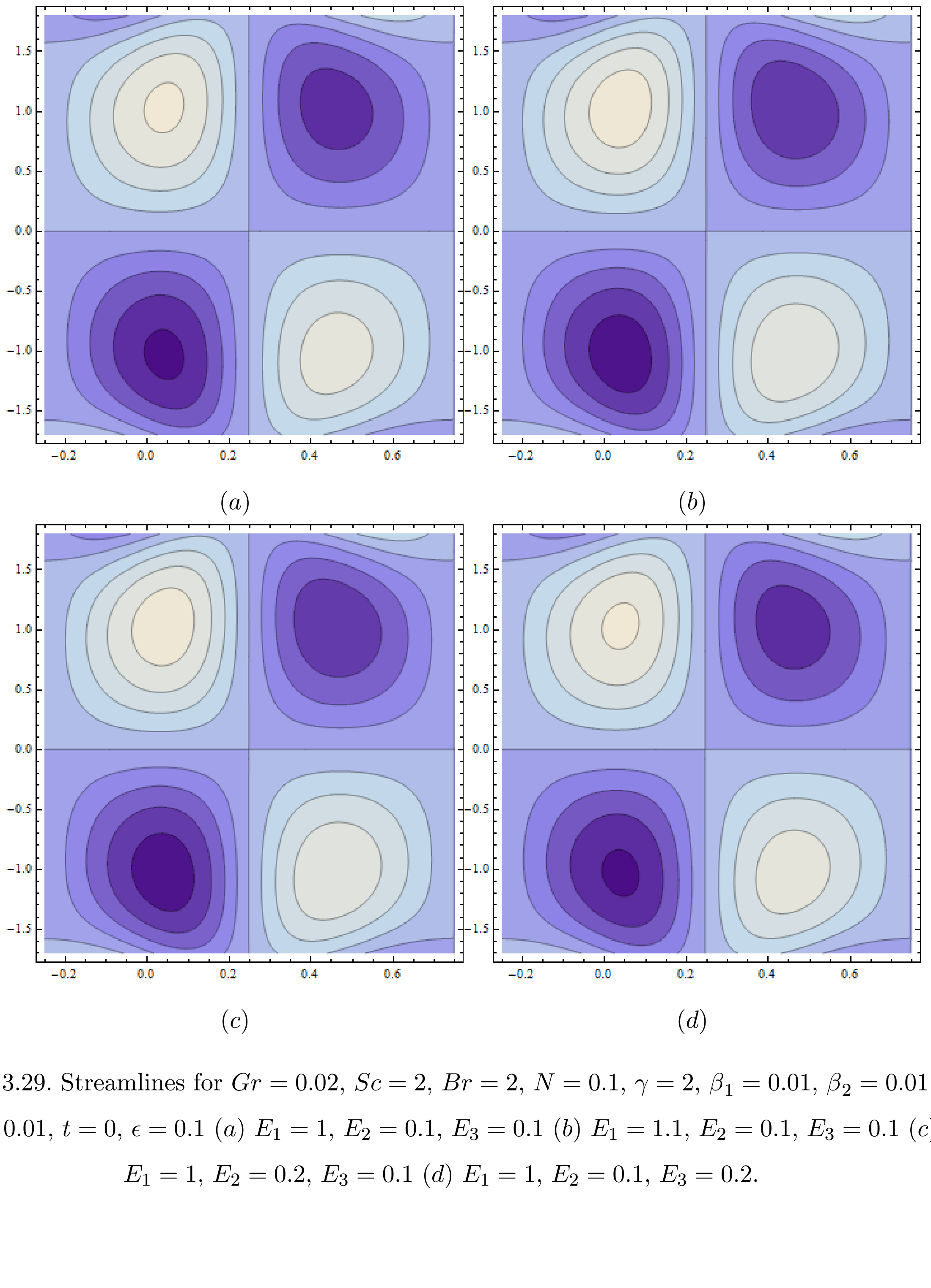
(a) (b)

Fig. 3.27. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Br = 2$, $N = 0.1$, $\gamma = 2$, $Sc = 2$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0$, $\epsilon = 0.1$, (a) $Gr = 0$, (b) $Gr = 0.02$.



(a) (b)

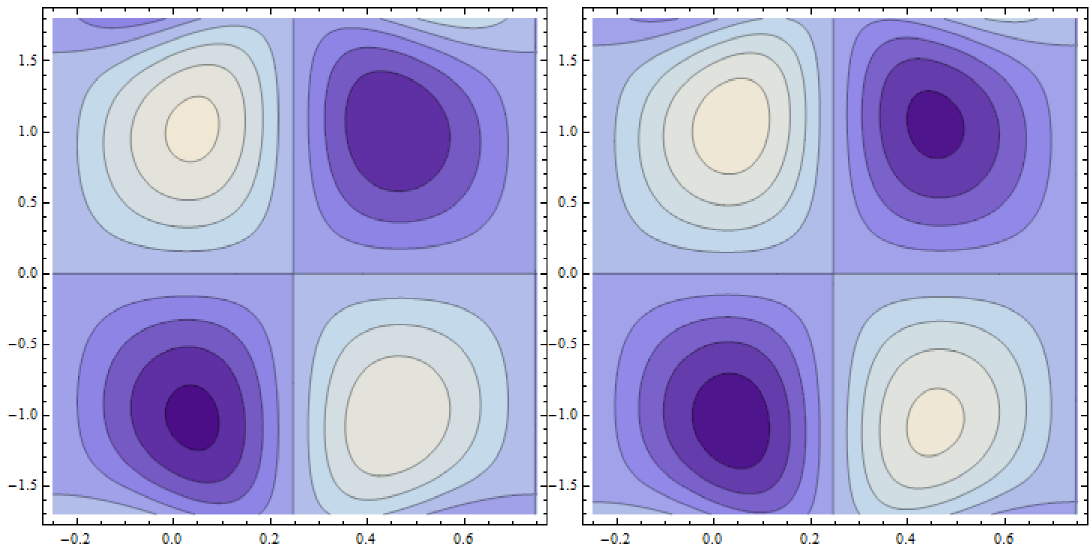
Fig. 3.28. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Gr = 0.2$, $Sc = 2$, $N = 0.1$, $\gamma = 2$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0$, $\epsilon = 0.1$, (a) $Br = 0$, (b) $Br = 0.2$.



(a) (b)

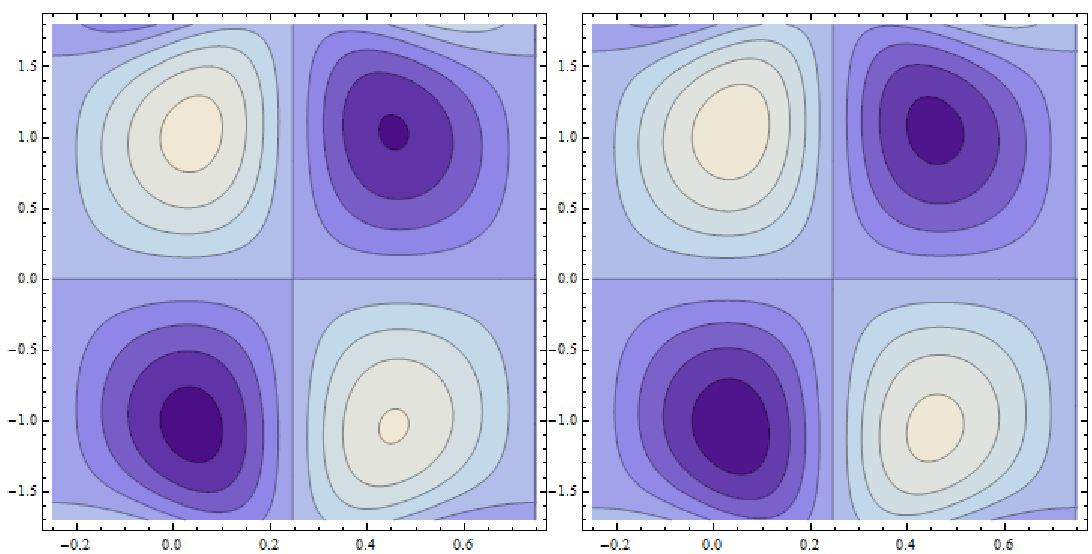
(c) (d)

Fig. 3.29. Streamlines for $Gr = 0.02$, $Sc = 2$, $Br = 2$, $N = 0.1$, $\gamma = 2$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, $t = 0$, $\epsilon = 0.1$ (a) $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.1$ (b) $E_1 = 1.1$, $E_2 = 0.1$, $E_3 = 0.1$ (c) $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.1$ (d) $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.2$.



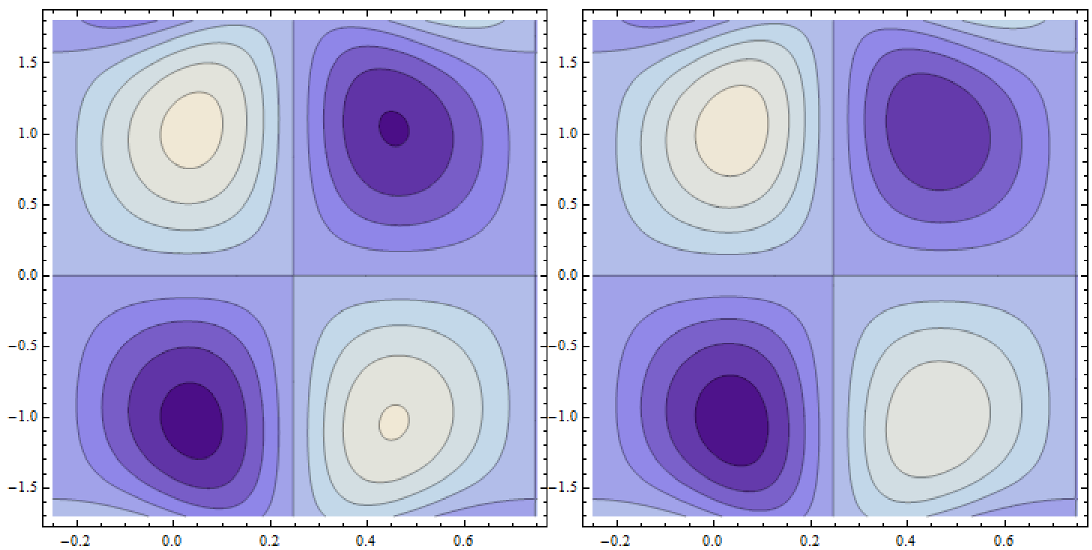
(a) (b)

Fig. 3.30. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Gr = 0.01$, $Sc = 2$, $Br = 2$, $N = 0.1$, $\gamma = 2$, $t = 0$, $\epsilon = 0.1$, $\beta_2 = 0.01$, $\beta_3 = 0.01$, (a) $\beta_1 = 0$, (b) $\beta_1 = 0.03$.



(a) (b)

Fig. 3.31. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Gr = 0.01$, $Sc = 2$, $Br = 2$, $N = 0.1$, $\gamma = 2$, $t = 0$, $\epsilon = 0.1$, $\beta_1 = 0.01$, $\beta_3 = 0.01$, (a) $\beta_2 = 0$, (b) $\beta_2 = 0.03$.



(a) (b)

Fig. 3.32. Streamlines for $E_1 = 1$, $E_2 = 0.2$, $E_3 = 0.2$, $Gr = 0.01$, $Sc = 2$, $Br = 2$, $N = 0.1$, $\gamma = 2$, $t = 0$, $\epsilon = 0.1$, $\beta_1 = 0.01$, $\beta_2 = 0.01$, (a) $\beta_3 = 0$, (b) $\beta_3 = 0.03$.

3.5 Conclusions

We have discussed the partial slip effects on peristaltic transport of viscous fluid in a vertical symmetric channel. Effects of slip parameter on the longitudinal velocity, temperature, concentration and trapping are investigated. The main points are listed below.

- The longitudinal velocity increases by increasing β_1 and β_2.
- Behavior of β_3 on longitudinal velocity is quite opposite to that of β_1 and β_2.
- Temperature increases in the left part and decreases in the right part of the channel for increasing values of β_1.
- Temperature increases for increasing values of β_2 and decreases by increasing β_3.
- Concentration field decreases when slip parameter β_3 is increased.
- The size of trapped bolus increases by increasing $\beta_1$and β_2.

- Trapped bolus shows mixed behavior for β_3 i.e. it increases for $x < 0.25$ and decreases for $x > 0.25$.

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