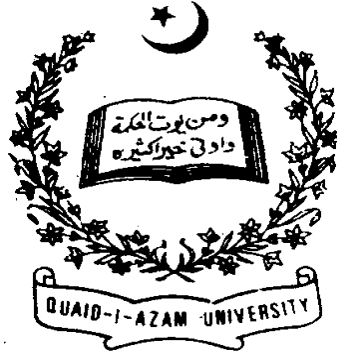# Interaction of heat transfer and variable viscosity with peristaltic motion

*By*

# *Fahad Munir Abbasi*

**Department of Mathematics**
**Quaid-i-Azam University**
**Islamabad, Pakistan**
**2014**

# Interaction of heat transfer and variable viscosity with peristaltic motion

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## Department of Mathematics
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## Islamabad, Pakistan
## 2014

# Interaction of heat transfer and variable viscosity with peristaltic motion

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A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENT
FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

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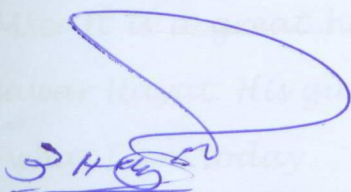# Interaction of heat transfer and variable viscosity with peristaltic motion

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### CERTIFICATE

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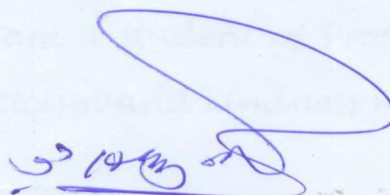
**We accept this dissertation as conforming to the required standard**
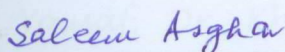
1. _____
   Prof. Dr. Tasawar Hayat
   (Supervisor)

2. _____
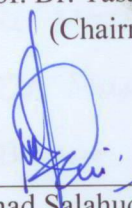   Prof. Dr. Tasawar Hayat
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## 2014

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Fahad Munir Abbasi

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# Preface

It is commonly accepted now that peristaltic flows are caused by the propagation of waves along the flexible boundaries of channel or tube. Such flows in physiology are represented by food movement in the digestive tract, urine transport from kidney to bladder, semen movement in vas deferens, movement of lymphatic fluids in lymph vessels, bile flow from the gall bladder into the duodenum, vasomotion of blood cells, movement of ovum in the female fallopian tube, transport of spermatozoa in the ductus efferents etc. Peristaltic motion in the industrial applications are employed in the transport of corrosive and noxious fluids, roller and finger pumps, hose pumps, tube pumps, dialysis machines and heart-lung machines. The concept of heat transfer has key role in the analysis of tissues, hemodialysis and oxygenation. The recent progress in the application of heat (hyperthermia), radiation (laser therapy) and coldness (cryosurgery), as means to destroy undesirable tissues including cancer have stimulated much interest in mathematical modeling for properties of tissue. Moreover, radiofrequency therapy has significance in the treatment of diseases like tissue coagulation, the liver cancer, the lung cancer and reflux of stomach acid. Moreover the energy flux is not only induced due to temperature gradient but by compositions gradient as well. Phenomena of heat and mass transfer taking place simultaneously can perturb each other's efficiency by producing the Soret and Dufour effects. Such effects are especially significant in chemical engineering and geosciences. Specifically the Soret effect is useful for isotope separation and in mixtures between gases having light and medium molecular weights, the Dufour effect is very significant. The concept of variable viscosity of fluid is now very well recognized. Some of the processes in which viscosity of the fluids depends on temperature include nuclear power plants, pumps operated at high temperatures, in turbines, rockets, missile technologies and space vehicles. Even in blood, the

living ingredient for animals is very sensitive to temperature changes and slight change in temperature may cause irreversible damage during dialysis or in heart lung machines. Magnetohydrodynamic (MHD) flow in channels/tubes has wide applications in purification of crude oil, petroleum industry, fluid droplets sprays, electrostatic precipitation, designing cooling systems, magnetic resonance imaging (MRI), blood pumps etc. Inability of the Newtonian fluid model to predict the viscoelastic behavior of several fluids in industry and physiology lead to the development of several non-Newtonian models. Third order fluid model is one of such models which can easily describe both the shear thickening and shear thinning properties even in the steady flows. The present thesis in view of afore mentioned facts is arranged as follows:

Review of available literature on peristalsis under various flow configurations is included in Chapter one. The fundamental equations are also presented here. The MHD mixed convection peristaltic flow with variable viscosity and thermal conductivity is analyzed in chapter two. Joule heating effect here is also taken into account. Viscosity and thermal conductivity of fluid vary linearly with temperature. Resulting nonlinear system is solved numerically. Velocity, pressure gradient, temperature and streamlines are examined through graphs. Numerical values of heat transfer rate at the wall are computed and analyzed for variation in certain embedded parameters. The main observations of this chapter are summarized at the end of chapter. Material of this chapter is accepted for publication in **"Sains Malasiayana"**.

Hydromagnetic peristaltic transport of fluid with variable viscosity is studied in chapter three. Here fluid fills the porous space in a channel. Heat transfer analysis is carried out in the presence of viscous dissipation and Joule heating. Viscosity of the fluid is taken temperature dependent. Both channel and magnetic field have the inclined considerations.  The resulting problems are solved numerically. A parametric study is performed to predict the impact of embedded

variables. Results of this chapter are compiled in the Key findings of this chapter. The main observations of this chapter have been submitted for publication in **Journal of Mechanical Engineering Science**.

Impact of convective boundary conditions on the peristaltic transport in an asymmetric channel is examined in chapter four. Joule heating effect is included in the analysis of heat transfer. Cases of hydrodynamic and magnetohydrodynamic (MHD) fluids are considered separately. Analytic solutions for stream function and temperature are constructed. Numerical integration is carried out for pressure rise per wavelength. Effects of influential flow parameters are pointed out through graphs. Findings of this study are accepted for publication in **Journal of Central South University, 21 (2014) 1411-1416"**.

Simultaneous effects of heat and mass transfer on the peristaltic transport of viscous fluid are examined in chapter five. The channel is assumed to be axially symmetric. Soret and Dufour effects along with Ohmic heating are considered. Closed form solutions for temperature and concentration are analyzed by plots for variation of embedded parameters. Numerical values of heat and mass transfer rates at the wall are also tabulated. Material of this study is published in **Magnetohydrodynamics, 47 (2011) 295-302.**

Effects of heat and mass transfer on the peristaltic transport of variable viscosity fluid are studied in chapter six. The fluid viscosity is considered space dependent. Thermo-diffusion and diffusion-thermo effects are taken into account. Series solutions for the nonlinear equations are obtained using the perturbation technique.  Results for the stream function, temperature and concentration are constructed. The variations of sundry parameters are analyzed. Results of this chapter are published in **Applied Mathematics and Information Sciences, 8 (2014) 211-219**.

Influences of velocity, thermal and concentration slips on the peristaltic transport in an asymmetric channel are analyzed in chapter seven. Fluid is electrically conducting under the effect of a constant applied magnetic field. Soret, Dufour and Joule heating effects are taken into account. Close form solutions for stream function, temperature, concentration and heat transfer coefficient are obtained. Plots of physical quantities of interest are given and explored. Research presented in this chapter is accepted for publication in **Arabian Journal for Science and Engineering, 39 (2014) 4341–4349**.

MHD mixed convective peristaltic transport in a vertical channel is studied in chapter eight. Simultaneous effects of heat and mass transfer for peristaltic transport of viscous fluid are addressed in presence of Joule heating and Soret and Dufour effects. Perturbation solution is prepared for small Brinkman number. Flow quantities of interest are plotted and analyzed. Material of this chapter is accepted for publication in **Applied Mechanics and Technical Physics**.

Mixed convective heat and mass transfer analysis for peristaltic transport in an asymmetric channel has been carried out in chapter nine. Mathematical modeling and analysis is presented in presence of Soret and Dufour effects. The convective conditions for both temperature and concentration are used in the analysis of heat and mass transfer. Resulting problems are solved for the series solutions. Numerical values of heat and mass transfer rates are displayed and studied. Main observations of this chapter are accepted for publication in **Journal of Central South University**.

The effect of variable viscosity on the peristaltic motion of MHD third order fluid in a channel is studied in chapter ten. Mathematical formulation includes the concept of slip condition. Both the governing equations and boundary conditions are nonlinear even after long wavelength and low

Reynolds number assumptions. The series solutions of stream function, longitudinal velocity and pressure gradient are first derived and then discussed in detail. The pressure rise and frictional forces are monitored through numerical integration. This material is published in **International Journal for Numerical Methods in Fluids, 67 (2011) 1500-1515**.

Heat transfer analysis for the peristaltic transport of third order fluid is carried out in chapter eleven. Viscosity of third order fluid is considered space dependent. The fluid is electrically conducting in the presence of uniform applied magnetic field. Analysis is performed when no-slip condition at the boundaries does not hold. The velocity and thermal slip effects are thus taken into account. Perturbation solution is discussed and a comparative study between the cases of constant and variable viscosities is presented and analyzed. The observations of this study are published in **Chinese Physics Letters, 28 (2011) 044701.**

Soret and Dufour effects on peristaltic transport of third order fluid in a symmetric channel are reported in chapter twelve. Joule heating effect is also taken in to account. The governing nonlinear problem is solved using perturbation approach. Graphical results are reported and discussed for various parameters of interest entering into the problem. Findings of this chapter are accepted for publication in **Heat Transfer Research, 45 (2014) 589-603**.

# Contents

# Chapter 1

# Introduction

The aim of this chapter is to provide a material for the better understanding of the subsequent chapters. Here the literature review about peristaltic flows, and some definitions and fundamental equations are presented.

## 1.1 Peristaltic transport

The peristaltic transport of fluid is caused by a progressive wave of area contraction/expansion travelling along the length of a distensible tube. Such wave in turn propels the fluid in the direction of propagation of wave. This type of transport mechanism finds its application in the physiological and industrial processes. These include the locomotion of worms, urine passage from kidney to bladder, food swallowing by the esophagus, vasomotion of small blood vessels, sanitary and corrosive fluids transport, movement of bio fluids (chyme in gastrointestinal tract, bile in the bile duct, ovum in the fallopian tube and spermatozoa in cervical canal), in heart lung machine and roller and finger pumps.

## 1.2 Literature review

Despite of the fact that the mechanism of peristalsis was known to the physiologist since long, mechanical analysis of peristalsis begin in later half of the twentieth century. Initial investigation of peristaltic pumping was inspired by its application in human ureter system. Studies of

Kiil [1] and Boyarsky [2] are fundamentals in this direction. Latham [3] in his MS thesis carried out the experimental investigation of peristaltic pump. Shapiro [4] discussed the inertia free two-dimensional peristaltic transport when the wavelength of peristaltic wave is large compared to the half channel width. Under these assumptions there is a steady flow in a wave frame. Later on these two considerations (the long wavelength and low Reynolds number approximation) were widely utilized by the researchers in the analysis of peristaltic flows. The theoretical investigation of Shapiro [4] was found in good agreement with the experimental work of Latham [3]. These pioneering studies of Latham [3] and Shapiro [4] were first precise efforts to mechanically investigate peristaltic flows and triggered extensive interest in peristaltic flows. Later on several investigations were made to study peristaltic flows under different flow configurations. More realistic models for the ureteral waves were proposed by Lykoudis [5] and Weinberg et al. [6]. Time elapsed during the chyme motion through small intestine was calculated by Barton and Raynor [7] using the concept of peristaltic motion. Inertia free peristaltic flow in a roller pump was studied by Maginniss [8]. Shapiro et al. [9] examined the peristaltic flow of Newtonian fluid in a planner channel and circular tube. Long wavelength and low Reynolds number approximations were used in this investigation. In continuation Jaffrin [10] studied the effects of inertia and streamline curvature on peristaltic pumping. Yin and Fung [11] and Fung and Yih [12] studied the peristaltic flow for arbitrary Reynolds and wave numbers in axisymmetric tube and planar channel respectively. In these studies the authors performed the analysis for small amplitude ratio. Jaffrin and Shapiro [13] made review for studies available on peristaltic pumping. Mittra and Prasad [14] studied the interaction of peristalsis with poiseuille flow. Wilson and Perel [15] analyzed the interaction of peristalsis with pulsatile flow. Brown and Hung [16] compiled the computational and experimental investigations on two-dimensional peristaltic flows up to 1977. Srivastava and Srivastava [17] discussed the interaction of peristalsis with pulsatile flow in a cylindrical tube by assuming that the flow is generated by an arbitrary pressure gradient and peristaltic waves. Peristaltic transport of blood was seen by Srivastava and Srivastava [18]. In this study they used the Casson fluid model to describe the non-Newtonian properties of blood. Numerical solutions of two-dimensional peristaltic flow problem were obtained by Takabatake and Ayukawa [19]. Takabatake et al. [20] studied the peristaltic pumping in circular cylindrical tubes. They obtained the numerical solutions of the

5

problem and discussed the efficiency of such flow. Fluid mechanics of gastrointestinal tract was studied by Mishra and Rao [21]. They considered the peristaltic transport in a channel with porous peripheral layer analogous to the flow in gastrointestinal tract.

The magnetohydrodynamic character of fluid has a pivotal role in solidification processes of metals and metal alloys, study of nuclear fuel debris, control of underground spreading of chemical wastes and pollution control, design of MHD power generators, blood pump machines, diagnose and treatment of cancer tumors. Interaction of peristaltic flow with pulsatile magneto-fluid through a porous medium was analyzed by Afifi and Gad [22]. Elshehawey et al. [23] examined the effects of inclined magnetic field on magneto fluid flow between two inclined wavy plates. In this study they further assumed the boundaries to be porous. Peristaltic flow of blood through a non-uniform channel was inspected by Mekheimer [24]. Blood was assumed to flow under the influence of a uniform applied magnetic field. Mekheimer [25] carried out the study of magnetohydrodynamic nonlinear peristaltic transport in an inclined planar channel. Magnetic fluid model induced by peristaltic waves was proposed by Siddiqui et al. [26]. Hayat and Ali [27] investigated the peristaltically induced motion of MHD third grade fluid in a deformable tube. Abd El Naby et al. [28] analyzed the effects of magnetic field on trapping while studying peristaltic motion of a generalized Newtonian fluid. Nonlinear peristaltic transport in a planar channel under the influence of applied magnetic field was studied by Hayat et al. [29]. Hayat and Ali [30] provided the mathematical description of peristaltic hydromagnetic flow in a tube. Effect of applied magnetic field on the peristaltic transport of micropolar fluid was discussed by Wang et al. [31]. Hayat et al. [32] studied the influence of inclined magnetic field on peristaltic transport of fourth grade fluid in an inclined asymmetric channel.

In several physical situations, the no-slip condition between the fluid and solid boundary is not valid. Such situation arises in fluid mechanics within the body of living beings where the internal linings of the tubular organs are coated with mucus/secretions. Also in the case of artificial heart the no-slip condition is no longer effective. In such situations one may take into account the slip effects while analyzing the flow phenomena. Mandiviwalla and Archer [33] inspected the effects of velocity slip on the peristaltic pumping in a rectangular channel. Hayat et al. [34] studied the peristaltic transport through a porous medium under the influence of partial slip. Hayat et al. [35] studied the peristaltic transport with simultaneous effects of

heat transfer and velocity slip. Effects of partial slip and heat transfer on the peristaltic flow of MHD Newtonian fluid in an asymmetric channel were examined by Yildirim and Sezer [36]. Kumari and Radhakrishnamachrya [37] studied the effects of slip on heat transfer for peristaltic transport in the presence of magnetic field.

Processes that take place at high temperature e.g. in nuclear reactors, turbines, rockets, missile technologies, pumps operated at high temperatures and space vehicles may face variation in fluid viscosity with temperature. Moreover the dependence of fluid viscosity is justified physiologically because normal person or animal of similar size takes 1-2L of the fluid every day. Also 6-7L of the fluid is received by the small intestine as secretions from salivary glands, stomach, pancreas, liver and small intestine itself. Ali et al. [38] carried out the study to examine the influence of velocity slip on the peristaltic transport in a planar channel. Fluid viscosity was assumed to be space dependent and a uniform magnetic field was also taken into account. El Naby et al. [39] studied the effects of an endoscope and variable viscosity on peristaltic motion. Elshehawey and Gharsseldien [40] examined the peristaltic transport of three-layered flow with variable viscosity. Hayat and Ali [41] examined the effect of variable viscosity on the peristaltic transport of Newtonian fluid in an asymmetric channel. Ebaid [42] computed numerical solution for the MHD peristaltic flow of bio-fluid with variable viscosity in a circular cylindrical tube via Adomian decomposition method. Effects of heat transfer on the MHD peristaltic transport of a variable viscosity fluid were studied by Nadeem and Akbar [43]. The fluid viscosity in this study was considered to vary linearly with temperature and Adomian decomposition Method (ADM) was used to obtain the solution of the coupled equations. Nadeem et al. [44] discussed the influence of heat transfer on the peristaltic transport of fluid with variable viscosity. Nadeem and Akbar [45] presented the study for the effects of temperature dependent viscosity on peristaltic flow of Jeffrey-six constant fluid in a non-uniform vertical tube. Nadeem and Akbar [46] examined the influence of temperature dependent viscosity on peristaltic transport of a Newtonian fluid.

The concept of heat transfer has a key role in the analysis of tissues, hemodialysis and oxygenation. Radhakrishnamachrya and Murty [47] studied influence of heat transfer on peristaltic transport in a non-uniform channel. Vajravelu et al [48] performed the heat transfer analysis of the peristaltic transport in a vertical porous annulus under the long wavelength consideration.

Simultaneous effects of heat transfer and applied magnetic field on the peristaltic transport in a vertical annulus were studied by Mekheimer and Elmaboud [49]. Srinivas and Kothandapani [50] examined the peristaltic transport in an asymmetric channel with heat transfer. Hayat et al. [51] performed the heat transfer analysis for the peristaltic transport of an electrically conducting fluid in porous space. Srinivas and Kothandapani [52] studied the impact of heat and mass transfer on the MHD peristaltic motion with complaint walls. Hayat and Hina [53] investigated the heat and mass transfer effects on the peristaltic motion of Maxwell fluid. Mekheimer et al. [54] analyzed the peristaltic transport under the influence of heat transfer and space porosity. Nadeem and Akbar [55] analyzed the effect of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer. Ali et al. [56] studied the heat transfer effect on peristaltic transport in a curved channel. Hayat and Noreen [57] examined the peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field. Effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel were analyzed by Srinivas and Muthuraj [58]. Vajravelu et al. [59] discussed the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum. Hayat et al. [60] analyzed the heat transfer effects on the peristaltic transport in a curved channel with complaint walls. Mixed convection heat and mass transfer in an asymmetric channel with peristalsis was discussed by Srinivas et al. [61]. Effects of heat and mass transfer on the peristaltic flow of Johnson Segalman fluid in a vertical asymmetric channel with induced MHD was examined by Nadeem and Akbar [62]. Tripathi [63] proposed a mathematical model for swallowing of food bolus through the oesophagus under the influence of heat transfer. Akbar et al. [64] computed the effects of heat and mass transfer on the peristaltic transport of hyperbolic tangent fluid. Akbar and Nadeem [65] investigated the heat and mass transfer effects in the peristaltic transport of Johnson-Segalman fluid. Effects of induced magnetic field were also taken into account in this study. Effects of induced magnetic field and heat transfer on peristaltic transport were also discussed by Hayat et al. [66]. This study was performed for the Carreau fluid model. Peristaltic transport of third order fluid under the effect of applied magnetic field was studied by Hayat et al. [67]. Hayat et al. [68] also analyzed the peristaltic flow of a third order fluid in an asymmetric channel with slip effects. Heat and mass transfer analysis for the peristaltic transport of third order fluid in a diverging tube was carried out by Nadeem et al. [69]. Akbar

et al. [70] studied the effects of slip and heat transfer on the peristaltic flow of third order fluid in an inclined asymmetric channel.

## 1.3 Fundamental laws

### 1.3.1 Law of conservation of mass

Mathematical form of law of conservation of mass is known as continuity equation which for unsteady flow is given by

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{V}) = 0, \tag{1.1}$$

in which $\rho$ denotes the density of the fluid and $\mathbf{V}$ the velocity. Eq. (1.1) for an incompressible fluid is

$$\boldsymbol{\nabla} \cdot \mathbf{V} = 0. \tag{1.2}$$

### 1.3.2 Law of conservation of linear momentum

Newton's second law of motion states that time rate of change of linear momentum is equal to the total force acting on the fluid element. Mathematically it is expressed as

$$\rho \frac{d\mathbf{V}}{dt} = \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \rho \mathbf{b}, \tag{1.3}$$

for an incompressible flow. Here $\boldsymbol{\tau}$ is the Cauchy stress tensor which is different for different fluids, $\boldsymbol{\nabla} . \boldsymbol{\tau}$ the surface force and $\rho \mathbf{b}$ the body force. The body force $\rho \mathbf{b}$ is different for different flow situations. For instance the expression of body force $\rho \mathbf{b}$ for the flow of an electrically conducting fluid in the presence of magnetic field and buoyancy effects is

$$\rho \mathbf{b} = \rho \mathbf{g} \left[ \beta_T \left( T - T_0 \right) + \beta_c \left( C - C_0 \right) \right] + \mathbf{J} \times \mathbf{B} \tag{1.4}$$

where $\mathbf{g}$ is the gravitational acceleration, $T$ and $T_0$ the temperatures of fluid and boundary respectively, $C$ and $C_0$ the concentrations of fluid and boundary respectively, $\beta_T$ and $\beta_c$ the

volumetric expansion coefficients, **J** the current density and **B** the applied magnetic field. Furthermore first two terms on the right hand side of Eq. (1.4) represent the body force due to density differences caused by temperature difference and composition difference. The last term of Eq. (1.4) is the Lorentz force.

The Cauchy stress tensor $\boldsymbol{\tau}$ is expressed as

$$
\boldsymbol{\tau} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \text{ or } \boldsymbol{\tau} = \begin{bmatrix} \sigma_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \sigma_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \sigma_{zz} \end{bmatrix}, \tag{1.5}
$$

where $\sigma_{xx}\left(\sigma_{rr}\right)$, $\sigma_{yy}\left(\sigma_{\theta\theta}\right)$ and $\sigma_{zz}$ are the normal stresses while all others are shear stresses. In cartesian coordinates $(x, y, z)$ Eq. (1.3) can be written as

$$
\rho \left( \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right) = \frac{\partial\left(\sigma_{xx}\right)}{\partial x} + \frac{\partial\left(\tau_{xy}\right)}{\partial y} + \frac{\partial\left(\tau_{xz}\right)}{\partial z} + \rho b_x, \tag{1.6}
$$

$$
\rho \left( \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) = \frac{\partial\left(\tau_{yx}\right)}{\partial x} + \frac{\partial\left(\sigma_{yy}\right)}{\partial y} + \frac{\partial\left(\tau_{yz}\right)}{\partial z} + \rho b_y, \tag{1.7}
$$

$$
\rho \left( \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right) = \frac{\partial\left(\tau_{zx}\right)}{\partial x} + \frac{\partial\left(\tau_{zy}\right)}{\partial y} + \frac{\partial\left(\sigma_{zz}\right)}{\partial z} + \rho b_z. \tag{1.8}
$$

### 1.3.3 First law of thermodynamics

The law of conservation of energy is also known as first law of thermodynamics and can be expressed as

$$
\rho \frac{d(C_p T)}{dt} = \boldsymbol{\tau} \cdot \boldsymbol{\nabla}\mathbf{V} - \boldsymbol{\nabla} \cdot (-K\boldsymbol{\nabla}T) + \frac{1}{\sigma}\mathbf{J} \cdot \mathbf{J}. \tag{1.9}
$$

Here $C_p$ is the internal energy, $\boldsymbol{\tau} \cdot \boldsymbol{\nabla}\mathbf{V}$ the viscous dissipation, $K\boldsymbol{\nabla}T$ the heat flux, $T$ the temperature field, $C_p$ the specific heat, $\sigma$ the electrical conductivity of fluid, $K$ the thermal conductivity, $D$ the diffusion coefficient and the last term on RHS of Eq. (1.9) is due to the Joule heating.

### 1.3.4 Advection diffusion equation

The diffusion of chemically reacting species is governed by Fick's second law. It is given by

$$\frac{dC}{dt} = D\boldsymbol{\nabla}^2 C - K_n(C - C_0)^n, \tag{1.10}$$

where $C$ is the concentration field of the diffusing species, $n$ the order of chemical reaction, $D$ the diffusion coefficient and $K_n$ the reaction rate.

### 1.3.5 Dufour and Soret effects

It has been experimentally verified that the diffusion of energy can be caused by a composition gradient. This fact is known as Dufour effect or diffusion-thermo effect. The diffusion of diffusing species by temperature gradient is termed as Soret effect or thermal diffusion effect. In most of the studies dealing with the transfer of heat and mass, Dufour and Soret effects are neglected under the assumptions that these effects have smaller order of magnitude when compared to the effects described by Fourier's and Fick's laws. However recent developments show that these effects are significant when transfer of heat and mass occur in the flow of mixture of gases with very light molecular weight ($H_2$, $He$) and gases with medium molecular weight ($N_2$, air). Eqs. (1.9) and (1.10) can be modified to make them capable of describing Dufour and Soret effects as follows:

$$\rho c_p \frac{dT}{dt} = K_c \boldsymbol{\nabla}^2 T + \overline{\tau}.\overline{L} + \frac{DK_T}{C_s}\nabla^2 C + \frac{1}{\sigma}\mathbf{J} \cdot \mathbf{J}, \tag{1.11}$$

$$\frac{dC}{dt} = D\boldsymbol{\nabla}^2 C + \frac{DK_T}{T_m}\nabla^2 T, \tag{1.12}$$

where $\overline{\tau}.\overline{L}$ is the viscous dissipation term, $\overline{\tau}$ the stress tensor, $\overline{L}$ the velocity gradient, $C_s$ the concentration susceptibility, $K_T$ the thermal-diffusion ratio and $T_m$ the mean fluid temperature.

### 1.3.6 Maxwell's equations

Equations which mathematically state a set of laws namely Gauss' law of electricity, Gauss' law of magnetism, Faraday's law and Ampere-Maxwell law are known as Maxwell equations after

the name of James Clerk Maxwell. These equations are

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0}, \qquad \text{(Gauss' law for electricity)}, \qquad (1.13)$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \text{(Gauss' law for magnetism)}, \qquad (1.14)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \text{(Faraday's law)}, \qquad (1.15)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_e \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad \text{(Ampere-Maxwell law)}. \qquad (1.16)$$

Here $\rho_c$ indicates the charge density, $\varepsilon_0$ the permittivity of free space, $\mathbf{B}$ the magnetic field, $\mathbf{E}$ the electric field, $\mathbf{J}$ the current density and $\mu_e$ the electric constant.

## 1.4 Constitutive equations of a third order fluid

The Cauchy stress tensor $\boldsymbol{\tau}$ in an incompressible homogeneous third order fluid is [67-70]:

$$\overline{\boldsymbol{\tau}} = -p\mathbf{I} + \left(\mu + \beta_3 tr \overline{\mathbf{A}}_1^2\right) \mathbf{A}_1 + \alpha_1 \overline{\mathbf{A}}_2 + \alpha_2 \overline{\mathbf{A}}_1^2 + \beta_1 \overline{\mathbf{A}}_3 + \beta_2 \left(\overline{\mathbf{A}}_1 \overline{\mathbf{A}}_2 + \overline{\mathbf{A}}_2 \overline{\mathbf{A}}_1\right), \qquad (1.17)$$

in which $\mu$, $\alpha_i \, (i = 1, 2)$ and $\beta_i \, (i = 1, 2.3)$ are the material constants. The Rivlin-Ericksen tensors can be represented as follows

$$\overline{\mathbf{A}}_1 = \overline{\mathbf{L}} + \overline{\mathbf{L}}^t, \qquad (1.18)$$

$$\overline{\mathbf{A}}_{n+1} = \frac{d\overline{\mathbf{A}}_n}{dt} + \overline{\mathbf{A}}_n \overline{\mathbf{L}} + \overline{\mathbf{L}}^t \overline{\mathbf{A}}_n \qquad n = 1, 2. \qquad (1.19)$$

# Chapter 2

# MHD mixed convection peristaltic flow with variable viscosity and thermal conductivity

This chapter concerns with mixed convection peristaltic flow of an electrically conducting fluid in an inclined channel. Analysis has been carried out in the presence of Joule heating. The fluid viscosity and thermal conductivity are assumed to vary with respect to temperature. A nonlinear coupled governing system is computed. Numerical results are presented for the velocity, pressure gradient, temperature and streamlines. Heat transfer rate at the wall is computed and analyzed. Graphs reflecting the contributions of embedded parameters are discussed.

## 2.1 Mathematical formulation

We examine the peristaltic transport of viscous fluid in an inclined asymmetric channel of width $(d_1 + d_2)$. The channel is inclined at an angle $\alpha_1$. The $\overline{X}-$axis is chosen along the length of the channel and $\overline{Y}-$axis is taken normal to the $\overline{X}-$axis. A uniform magnetic field of strength $B_0$ acts parallel to the $\overline{Y}-$axis. Effects of induced magnetic and electric fields are ignored. The viscosity and thermal conductivity are temperature dependent. The following forms of waves propagate along the channel walls

$$\overline{H}_1(\overline{X}, \overline{t}) = d_1 + \xi_1,$$

$$\overline{H}_2(\overline{X}, \overline{t}) = -d_2 - \xi_2,$$

$$\xi_1 = a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right),$$

$$\xi_2 = b_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t}) + \gamma\right),$$

in which $\overline{t}$ is time, $\overline{H}_1$ and $\overline{H}_2$ represent the upper and lower walls of channel, $a_1$ and $b_1$ are the amplitude of the waves at respective walls, $\gamma$ the phase difference of two waves and $c$ and $\lambda$ are the speed and wavelength of the waves respectively. Appropriate velocity field for this problem is $\overline{V} = \left[\overline{U}(\overline{X}, \overline{Y}, \overline{t}), \overline{V}(\overline{X}, \overline{Y}, \overline{t}), 0\right]$. Laws of conservation of mass and linear momentum give

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{2.1}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + 2\frac{\partial}{\partial \overline{X}}\left(\overline{\mu}(T)\frac{\partial \overline{U}}{\partial \overline{X}}\right) + \frac{\partial}{\partial \overline{Y}}\left[\overline{\mu}(T)\left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}}\right)\right]$$
$$-\sigma B_0^2 \overline{U} + \rho g \alpha^* (T - T_m)\sin\alpha_1 + \rho g \sin\alpha_1, \tag{2.2}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + 2\frac{\partial}{\partial \overline{Y}}\left(\overline{\mu}(T)\frac{\partial \overline{V}}{\partial \overline{Y}}\right) + \frac{\partial}{\partial \overline{X}}\left[\overline{\mu}(T)\left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}}\right)\right]$$
$$-\rho g \alpha^* (T - T_m)\cos\alpha_1 - \rho g \cos\alpha_1. \tag{2.3}$$

In above equations $\rho$ is the density of fluid, $\sigma$ the electric conductivity, $\overline{\mu}(T)$ the temperature dependent dynamic viscosity, $g$ the acceleration due to gravity, $T_m$ the mean value of temperature of both the channel walls and $\alpha^*$ the thermal expansion coefficient. We consider $(\overline{u}, \overline{v})$ and $\overline{p}$ as the velocity components and pressure in the wave frame $(\overline{x}, \overline{y})$. The transformations between laboratory and wave frames are

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}), \tag{2.4}$$

14

where $(\overline{U}, \overline{V})$ and $\overline{P}$ are the velocity components and pressure in the laboratory frame. The conservation laws of mass and momentum in wave frame can be expressed as

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{2.5}$$

$$\rho\left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right) (\overline{u} + c) = -\frac{\partial \overline{p}}{\partial \overline{x}} + 2 \frac{\partial}{\partial \overline{x}} \left( \overline{\mu}(T) \frac{\partial \overline{u}}{\partial \overline{x}} \right) + \frac{\partial}{\partial \overline{y}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right) \right]$$
$$-\sigma B_0^2 (\overline{u} + c) + \rho g \alpha^* (T - T_m) \sin \alpha_1 + \rho g \sin \alpha_1, \tag{2.6}$$

$$\rho\left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right) (\overline{v}) = -\frac{\partial \overline{p}}{\partial \overline{y}} + 2 \frac{\partial}{\partial \overline{y}} \left( \overline{\mu}(T) \frac{\partial \overline{v}}{\partial \overline{y}} \right) + \frac{\partial}{\partial \overline{x}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right) \right]$$
$$-\rho g \alpha^* (T - T_m) \cos \alpha_1 - \rho g \cos \alpha_1. \tag{2.7}$$

Defining the dimensionless quantities

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \ y = \frac{\overline{y}}{d_1}, \ u = \frac{\overline{u}}{c}, \ v = \frac{\overline{v}}{c\delta}, \ \delta = \frac{d_1}{\lambda}, \ h_1 = \frac{\overline{H_1}}{d_1}, \ h_2 = \frac{\overline{H_2}}{d_1}, \ d = \frac{d_2}{d_1}, \ p = \frac{d_1^2 \overline{p}}{c\lambda\mu_0}, \\
\upsilon &= \frac{\mu_0}{\rho}, \ \mu(\theta) = \frac{\overline{\mu}(T)}{\mu_0}, \ M^2 = \left( \frac{\sigma}{\mu_0} \right)^2 B_0^2 d_1^2, \ t = \frac{c\overline{t}}{\lambda}, \ G_t = \frac{\rho g \alpha^* (T_1 - T_0) d_1^2}{\mu_0 c}, \ \theta = \frac{T - T_m}{T_1 - T_0}, \\
Re &= \frac{\rho c d_1}{\mu_0}, \ Fr = \frac{c^2}{g d_1}, \ u = \psi_y, \ v = -\psi_x
\end{aligned} \tag{2.8}
$$

and adopting long wavelength and low Reynolds number approach we have in terms of stream function $\psi$ the following equations

$$p_x = \frac{\partial}{\partial y} \left[ \mu(\theta) \frac{\partial^2 \psi}{\partial y^2} \right] - M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right)$$
$$+ G_t \theta \operatorname{Si} n\alpha_1 + \frac{Re \operatorname{Si} n\alpha_1}{Fr}, \tag{2.9}$$

$$p_y = 0, \tag{2.10}$$

and the continuity equation is identically satisfied. Here, $h_{1,2}$ are the dimensionless wall shapes, M the Hartman number, $G_t$ the Grashoff number, Re the Reynolds number, Fr the Froud number and $\psi$ the stream function. Equation (2.10) indicates that $p \neq p(y)$ and compatibility

15

equation from Eqs. (2.9) and (2.10) has the form

$$0 = \frac{\partial^2}{\partial y^2}\left[\mu\left(\theta\right)\frac{\partial^2 \psi}{\partial y^2}\right] - M^2\left(\frac{\partial^2 \psi}{\partial y^2}\right) + G_t\frac{\partial \theta}{\partial y}\sin\alpha. \tag{2.11}$$

The volume flow rate in fixed frame is defined as

$$Q = \int_{\overline{H}_2}^{\overline{H}_1}\overline{U}(\overline{X},\overline{Y},\overline{t})dY \tag{2.12}$$

and in wave frame we have

$$q = \int_{h_2}^{h_1}\overline{u}(x,y)dy, \tag{2.13}$$

in which $h_{1,2}$ are functions of $x$ alone. From Eqs. (2.12), (2.13) and (2.8) one has

$$Q = q + ch_1(x) - ch_2(x).$$

The time averaged flow over a period $T_f$ is

$$\overline{Q} = \frac{1}{T}\int_0^{T_f}Qdt,$$

which implies that

$$\overline{Q} = q + cd_1 + cd_2.$$

Defining $\eta$ and $F$ as the dimensionless mean flows in laboratory and wave frames by

$$\eta = \frac{\overline{Q}}{cd_1}, F = \frac{q}{cd_1}$$

one can write

$$\eta = F + 1 + d, \tag{2.14}$$

where

$$F = \int_{h_2}^{h_1}\frac{\partial \psi}{\partial y}dy = \psi(h_1) - \psi(h_2). \tag{2.15}$$

16

Dimensionless boundary conditions in terms of stream function are

$$\psi = \frac{F}{2}, \quad \psi_y = -1 \text{ at } y = h_1,$$

$$\psi = -\frac{F}{2}, \quad \psi_y = -1 \text{ at } y = h_2. \tag{2.16}$$

## 2.2  Heat transfer analysis

We have assumed that the walls at $h_1$ and $h_2$ have temperature $T_0$ and $T_1$ respectively. Energy equation in presence of viscous dissipation and Joule heating effects in laboratory frame is

$$\rho C_p \left( T_{\bar{t}} + \overline{U} T_{\overline{X}} + \overline{V} T_{\overline{Y}} \right) = \overline{\nabla}. \left[ \overline{K}(T) (\nabla T) \right] + \overline{\mu}(T) \left[ 2 \left\{ \left( \frac{\partial \overline{U}}{\partial \overline{X}} \right)^2 + \left( \frac{\partial \overline{V}}{\partial \overline{Y}} \right)^2 \right\} + \left( \frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right)^2 \right]$$
$$+ \sigma B_0^2 \overline{U}^2. \tag{2.17}$$

In wave frame above equation takes the form

$$\rho C_p \left( (\overline{u} + c) T_{\overline{x}} + \overline{v} T_y \right) = \overline{\nabla}. \left[ \overline{K}(T) (\nabla T) \right] + \overline{\mu}(T) \left[ 2 \left\{ \left( \frac{\partial \overline{u}}{\partial \overline{x}} \right)^2 + \left( \frac{\partial \overline{v}}{\partial \overline{y}} \right)^2 \right\} + \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right)^2 \right]$$
$$+ \sigma B_0^2 (\overline{u} + c)^2, \tag{2.18}$$

in which $T$ is the fluid temperature, $C_p$ the specific heat, $K(T)$ the dimensional temperature dependent thermal conductivity and $\overline{\mu}(T)$ is the temperature dependent viscosity. The dimensionless energy equation in view of afore mentioned approximations becomes

$$\frac{\partial}{\partial y} \left\{ K(\theta) \theta_y \right\} + Br \mu(\theta) \left( \psi_{yy} \right)^2 + Br M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 = 0, \tag{2.19}$$

where $Br$ is the Brinkman number, $K(\theta)$ the dimensionless temperature dependent thermal conductivity, $\theta$ the dimensionless temperature, Pr the Prandtle number and E the Eckert number. These involved quantities are defined as follows:

$$\theta = \frac{T - T_m}{T_1 - T_0}, \quad K(\theta) = \frac{\overline{K}(T)}{K_0}, \quad Br = \Pr E, \quad Pr = \frac{\mu_0 C_p}{K_0}, \quad E = \frac{c^2}{C_p (T_1 - T_0)}. \tag{2.20}$$

17

Dimensionless boundary conditions for the temperature are given by

$$\theta = -\frac{1}{2} \text{ at } y = h_1 \text{ and } \theta = \frac{1}{2} \text{ at } y = h_2. \tag{2.21}$$

Temperature dependent viscosity and thermal conductivity are taken in the form

$$\mu(\theta) = 1 - \alpha\theta, \ \ K(\theta) = 1 + \epsilon\theta, \ \ (\alpha, \ \epsilon) \ll 1. \tag{2.22}$$

Note that $\mu_0$ and $K_0$ are the viscosity and thermal conductivity of the fluid when the temperature of the fluid approaches the mean temperature. Also the cases of constant viscosity and thermal conductivity can be recovered by choosing $\alpha$ and $\epsilon$ equal to zero. The values of $h_1$ and $h_2$ are

$$h_1(x) = 1 + a\cos(2\pi x), \ h_2(x) = -d - b\cos(2\pi x + \gamma). \tag{2.23}$$

We note that Eq. (2.19) is nonlinear in the present analysis. Hence in this study we have nonlinear coupled system. Analytic solution of the resulting nonlinear coupled system seems difficult. Hence the numerical solution through **Mathematica** is preferred here. The step sizes for x and y are chosen 0.01. The flow quantities of interest will be analyzed in the next section.

## 2.3   Graphical analysis

This section aims to analyze the numerical results for influential parameters. Plots of pressure gradient, axial velocity, temperature and stream function are obtained and examined. Numerical values of the heat transfer rate are given in Table 2.1.

It is obvious from Figs. 2.1 that pressure gradient for constant viscosity fluid is less when compared with variable viscosity fluid. Temperature dependent thermal conductivity also increases the value of pressure gradient. Such results are obvious from the Figs. 2.1a-b. An increase in $\alpha$ and $\epsilon$ also increases the pressure gradient. Pressure gradient decreases by increasing Hartman and Froud numbers. However this change in pressure gradient due to M and Fr is sufficiently large when compared with that of $\alpha$ and $\epsilon$ (see Figs. 2.1d and f). As the value of $\alpha_1$ is varied from 0 to $\pi/2$ the pressure gradient tends to attain significantly large value which highlights the fact that pressure gradient is higher in a vertical channel when compared

18

to horizontal channel (Fig. 2.1 c). For assisting flow (+ve value of acceleration due to gravity g) pressure gradient is large than that of opposing flow (see Fig. 2.1e).

Plots of axial velocity are given in Figs. 2.2 a-d. These Figs. showed that velocity tends to follow parabolic path with maximum value occurring near the center of the channel. Fig. 2.1a shows that when the viscosity parameter $\alpha$ is taken zero i.e. the velocity for constant viscosity fluid is seen to have maximum value near the center of the channel whereas it tends to shift towards the lower wall for non zero values of the viscosity parameter. Moreover a slight increase in the maximum value of velocity is observed when $\alpha$ increases. Channel inclination angle $\alpha_1$ is seen to have almost similar effect on the velocity as that of $\alpha$ (see Fig. 2.2b). Maximum value of velocity in inclined channel is shifted towards the lower wall. Increase in the strength of applied magnetic field decreases the velocity. This change in velocity is maximum when the Hartman number is taken between 1 and 2 (see Fig. 2.2c). Change in the value of Grashoff number varies the symmetry of the velocity profile relative to the channel i.e. velocity has maximum value near the center for opposing flow and this change occurs when $G_t$ has +ve value. Effect of Grashoff number on the velocity also depends on the channel inclination which is well justified physically (see Fig. 2.2d). It is further noted that $\epsilon$ has no effect on the axial velocity.

Behavior of the dimensionless temperature through $\alpha$, $\epsilon$, $M$, $\alpha_1$, $G_t$ and Br are examined in the Figs. 2.3 a-f. Fig. 2.3a depicts that an increase in the viscosity parameter decreases the temperature. This decrease in temperature subject to decrease in $\alpha$ is uniform throughout the channel. Fig. 2.3 b analyzes the impact of variable thermal conductivity parameter $\epsilon$ on the temperature. Effect of $\epsilon$ on the temperature is noteworthy in the sense that it is not uniform throughout the channel. It means that near the lower wall ($h_2$) an increase in the value of $\epsilon$ decreases the value of temperature of the fluid whereas near the upper wall ($h_1$) it increases the value of $\theta$. Since $\epsilon$ yields the perturbation in the thermal conductivity of the fluid so increase in this parameter gives rise to increase in the ability of the fluid to dissipate or absorb heat. Hence when temperature of the fluid is higher than the temperature of the boundary then increase in $\epsilon$ results in reduction of the temperature of fluid and vice versa.

Increase in the Hartman number increases the value of $\theta$. This fact is primarily due to consideration of Joule heating (see Fig. 2.3c). Channel inclination also has non-uniform effect on the temperature. Increase in $\alpha_1$ near the lower wall increases the temperature but such

19

increase although is not very significant. Similarly the Grashoff number also has very less and non-uniform effect on the temperature (see Figs. 2.3d and e). Fig. 2.3 f shows that an increase in Br increases the temperature $\theta$ significantly. Also when Br is increased then the temperature plots become non-linear. Streamlines for $\alpha$ and $\epsilon$ are plotted in the Figs. 2.4 and 2.5. It is seen that the size of trapped bolus decreases with an increase in $\alpha$. However $\epsilon$ has no significant effect on the size of the bolus.

Table 2.1 gives the numerical values of heat transfer rate at the wall $\theta'(h_1)$ for different values of the embedded parameters. Results showed that an increase in the value of viscosity parameter $\alpha$ reduces the transfer rate but heat transfer rate increases through increase in $\epsilon$. The transfer rate decreases via inclined nature of channel. Further an increase in Hartman number increases the transfer rate.

## 2.4   Main points

Effects of variable viscosity and thermal conductivity on the MHD peristaltic transport of fluid in an inclined asymmetric channel are analyzed. Mixed convection and Joule heating effects are present. Main results are reported below.

- Variable viscosity and thermal conductivity tend to increase the pressure gradient.

- Viscosity parameter, Grashoff number and inclination angle have similar effects on the velocity. However velocity is unperturbed due to change in thermal conductivity parameter.

- Variable thermal conductivity has no significant change on the bolus size.

- Variable viscosity tends to reduce the temperature of fluid.

- Viscosity parameter and inclination angle have similar effects on heat transfer rate at the wall.

The following values are used in the plots

Figs. 2.1 (a-g), $\alpha = 0.1$, $\alpha_1 = \pi/6$, $\epsilon = 0.1$, $Re = 3$, $Fr = 2$, $Br = 0.2$, $M = 0.5$, $G_t = 2$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\gamma = \pi/4$, $d = 0.7$ and $\eta = 0.7$.

Figs. 2.2 (a-d), $\alpha = 0.1$, $\alpha_1 = \pi/6$, $\epsilon = 0.1$, $x = 0$, $Re = 3$, $Fr = 2$, $Br = 0.4$, $M = 0.5$, $G_t = 2$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\gamma = \pi/4$, $d = 0.7$ and $\eta = 0.9$.

Figs. 2.3 (a-f), $\alpha = 0.1$, $\alpha_1 = \pi/6$, $\epsilon = 0.1$, $x = 0$, $Re = 5$, $Fr = 2$, $Br = 0.5$, $M = 1$, $G_t = 1$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\gamma = \pi/4$, $d = 0.7$ and $\eta = 1.5$.

Figs. 2.4 and 2.5, $\alpha = 0.1$, $\alpha_1 = \pi/6$, $\epsilon = 0.1$, $Re = 3$, $Fr = 2$, $Br = 0.2$, $M = 0.5$, $G_t = 2$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\gamma = \pi/4$, $d = 0.7$ and $\eta = 1.2$.
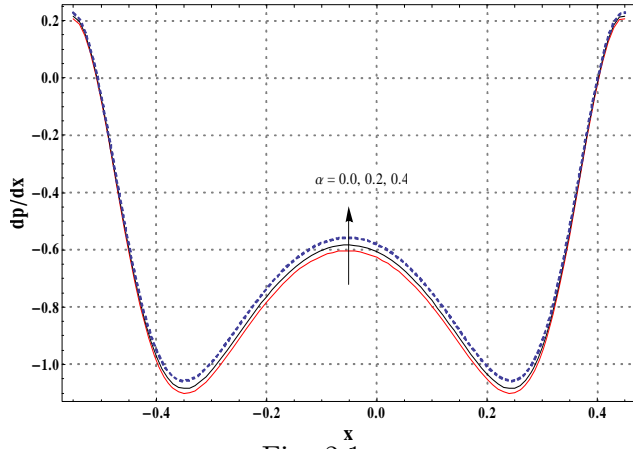
Fig. 2.1 a
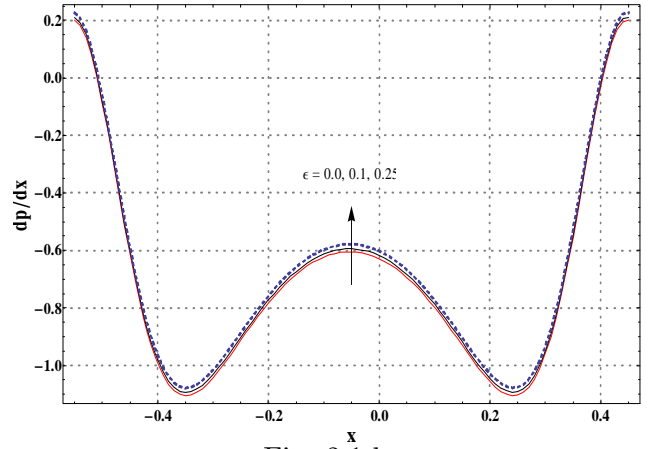


Fig. 2.1 b



Fig. 2.1 c



Fig. 2.1 d

22
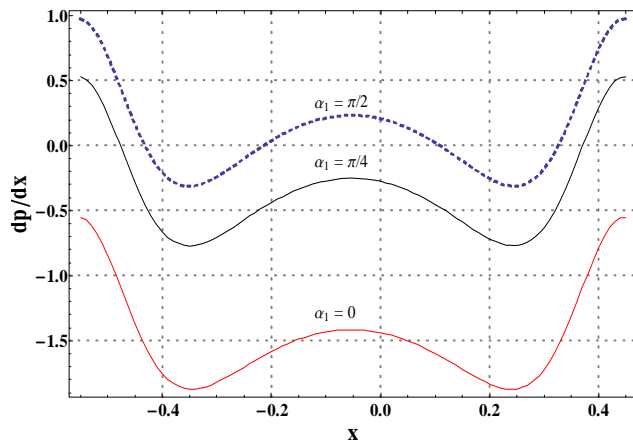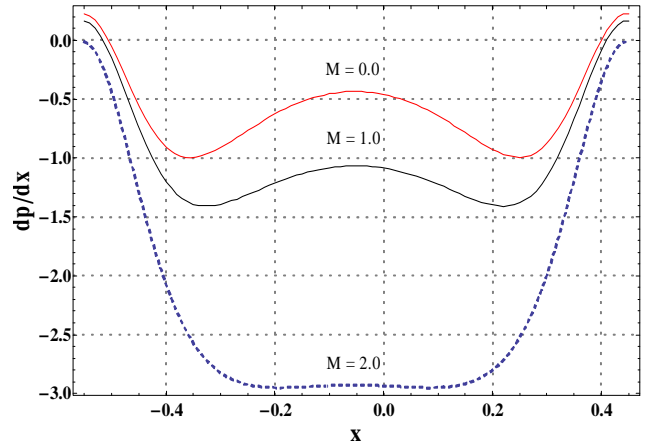
Fig. 2.1 e



Fig. 2.1 f
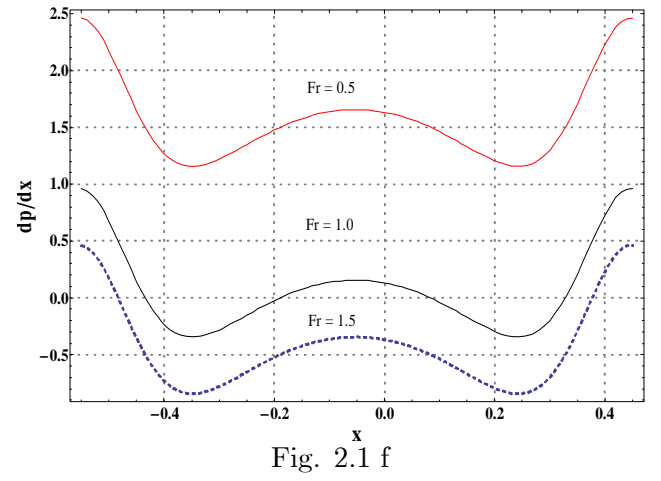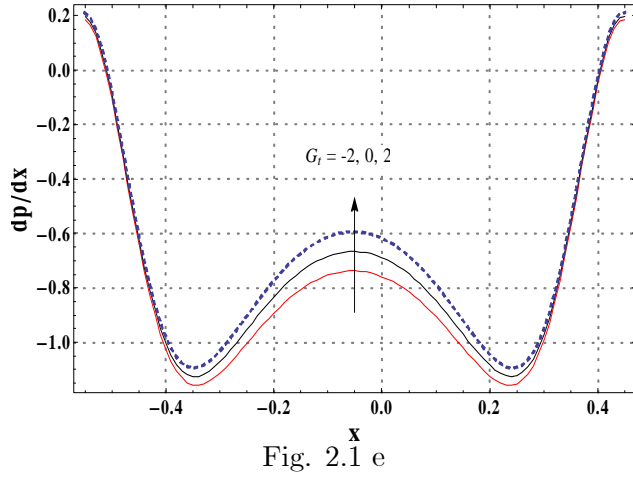
Figs. 2.1 a-f. Development of pressure gradient in the axial direction.
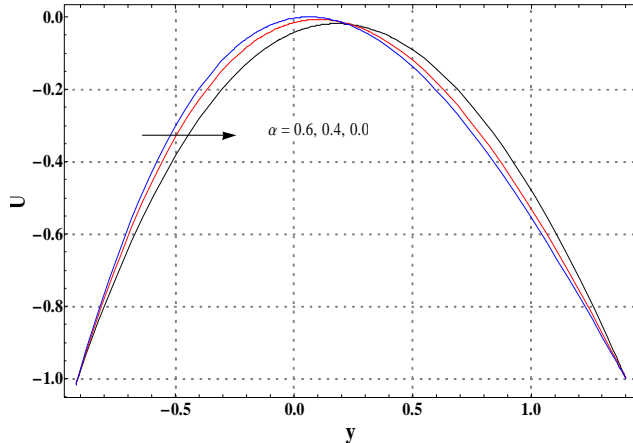
Fig. 2.2 a
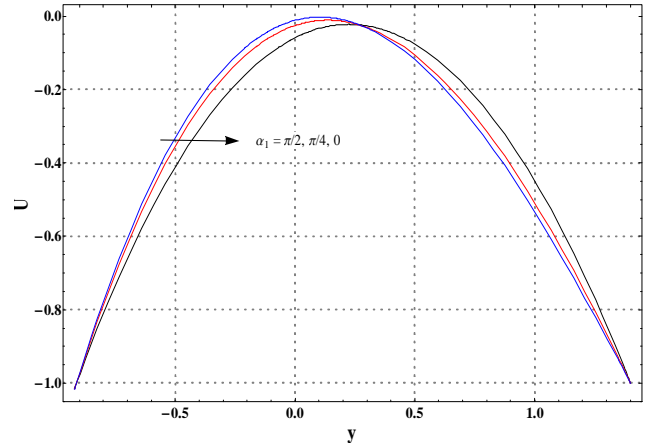
Fig. 2.2 b
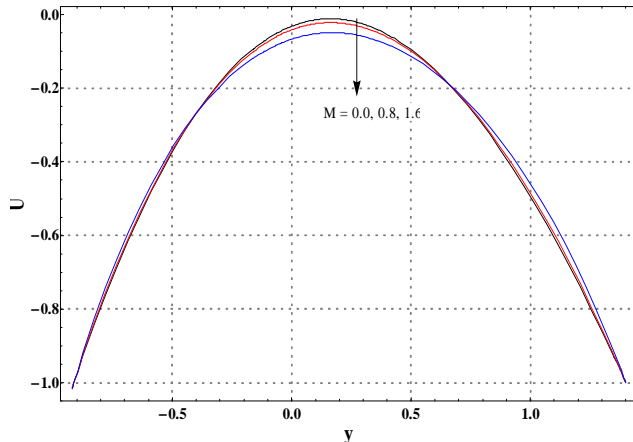
Fig. 2.2 c
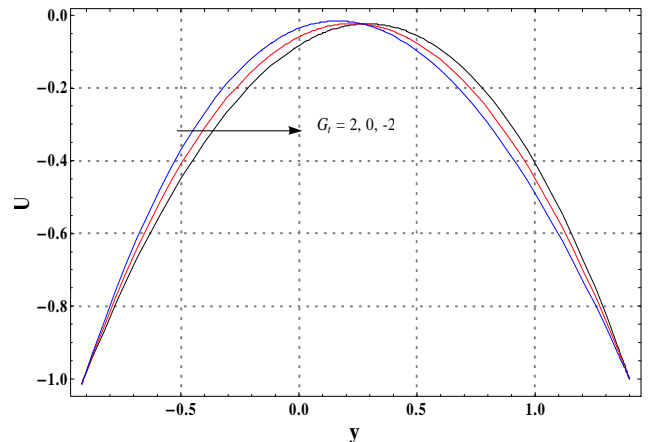
Fig. 2.2 d

Fig. 2.2 a-d Axial velocity for different parameters.
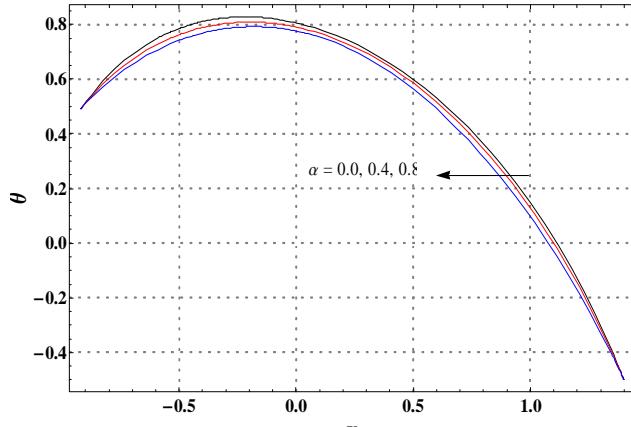
Fig. 2.3 a
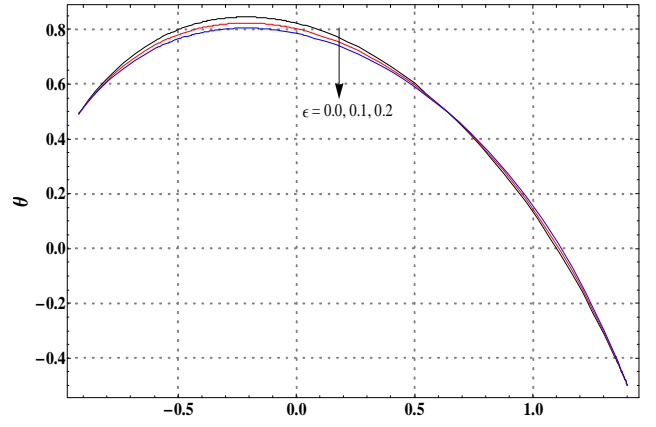
Fig. 2.3 b
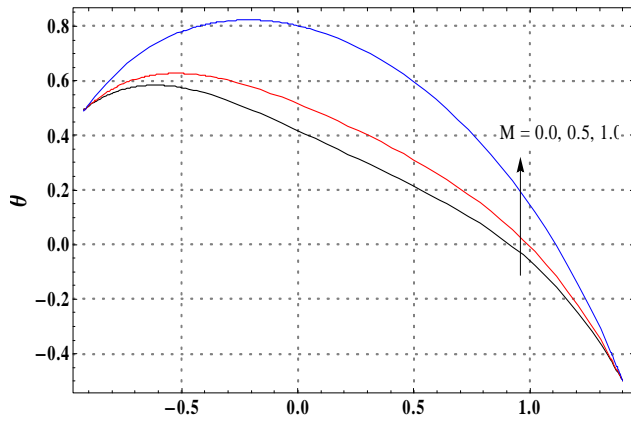
Fig. 2.3 c
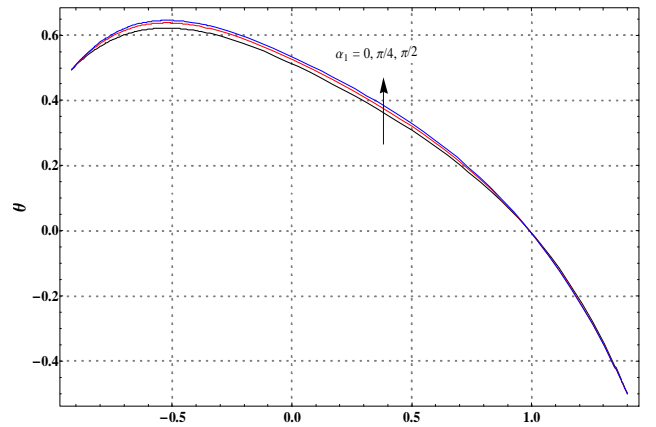
Fig. 2.3 d
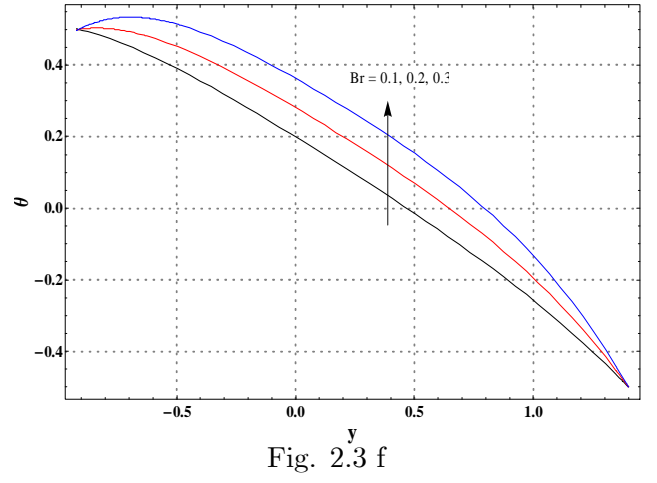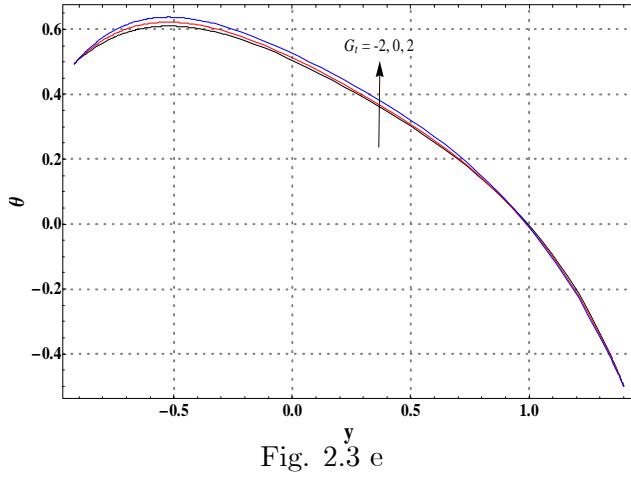
Fig. 2.3 e
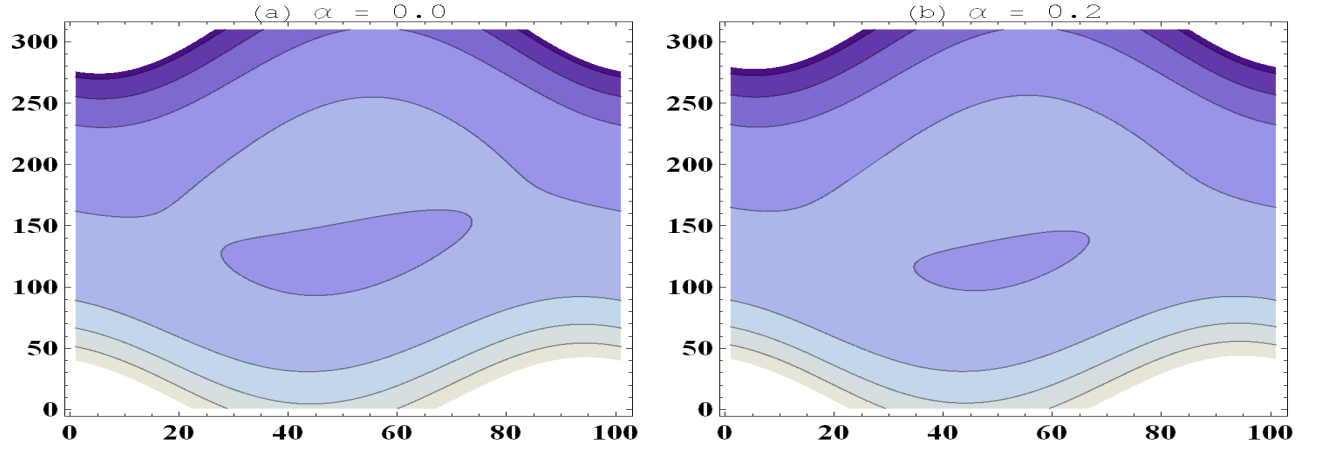


Fig. 2.3 f

Figs. 2.3 a-f. Dimensionless temperature for various parameters.

Fig. 2.4 Streamlines for viscosity parameter $\alpha$.
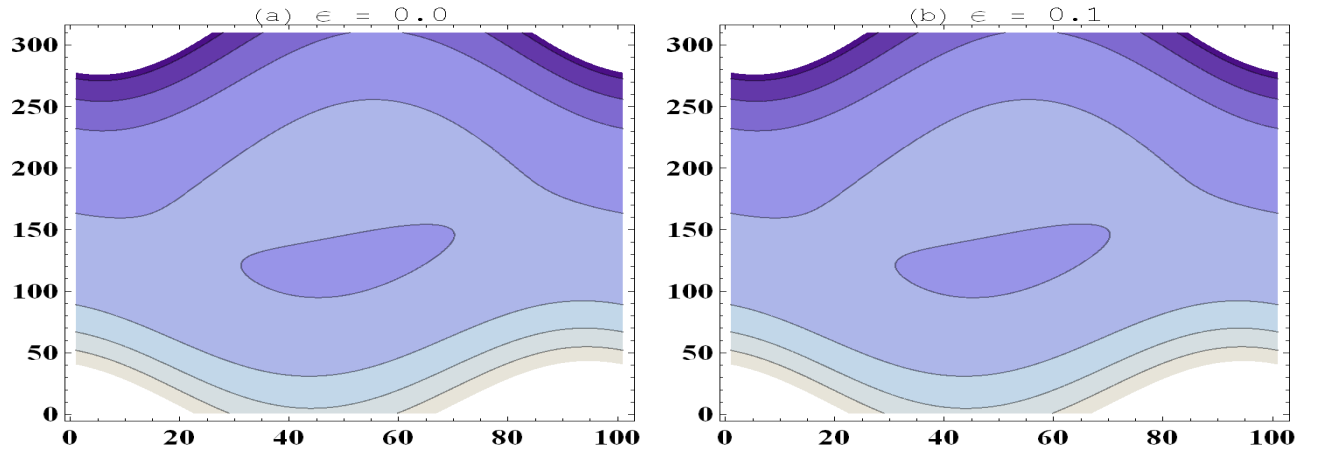


Fig. 2.5 Streamlines for thermal conductivity parameter $\epsilon$.

| $\alpha$ | $\epsilon$ | $\alpha_1$ | $M$ | $-\theta'(h_1)$ |
|---|---|---|---|---|
| 0.0 | 0.1 | $\pi/6$ | 0.5 | 1.0603 |
| 0.4 | | | | 1.0257 |
| 0.6 | | | | 1.0051 |
| 0.1 | 0.0 | | | 0.9893 |
| | 0.1 | | | 1.0524 |
| | 0.2 | | | 1.1258 |
| | | 0.1 | 0 | 1.1278 |
| | | $\pi/4$ | | 1.0247 |
| | | $\pi/2$ | | 0.9892 |
| | | $\pi/6$ | 0.0 | 0.9857 |
| | | | 0.4 | 1.0284 |
| | | | 0.8 | 1.1567 |

Table 2.1. Numerical values of heat transfer rate at the wall $(h_1)$ when $Re = 3$, $Fr = 2$, $Br = 0.2$, $x = 0$, $G_t = 2$, $Pr = 0.5$, $a = 0.4$, $b = 0.3$, $\gamma = \pi/4$, $d = 0.7$ and $\eta = 1.2$.

# Chapter 3

# Hydromagnetic peristaltic transport of variable viscosity fluid with heat transfer and porous medium

This chapter examines the peristaltic transport of variable viscosity fluid in a planar channel with heat transfer. The fluid viscosity is taken temperature dependent. The fluid is electrically conducting in the presence of a constant applied magnetic field. Both channel and magnetic field are considered inclined. An incompressible fluid saturates the porous space. Heat transfer analysis is carried out in the presence of viscous dissipation and Joule heating. The resulting problems are solved numerically. A parametric study is performed to predict the impact of embedded parameters. Important results have been pointed out in the key findings.

## 3.1   Mathematical analysis

Let us examine the peristaltic transport of an incompressible viscous fluid in a symmetric channel. The channel is taken inclined at an angle $\alpha_1$ to the vertical. The fluid is electrically conducting in the presence of an inclined magnetic field with constant strength $\mathbf{B}_0$. The flow generated is due to peristaltic waves travelling along the channel walls. The wall geometry is chosen in the form

$$\overline{H}(\overline{X}, \overline{t}) = d_1 + \xi,$$

where $d_1$ is the half channel width and $\xi$ is the disturbance produced due to propagation of peristaltic waves at the walls. This disturbance can be written in the form

$$\xi = a_1 cos \left( \frac{2\pi}{\lambda} (\overline{X} - c\overline{t}) \right),$$

in which $\overline{t}$ is time, $a_1$ is the amplitude of the peristaltic wave, $c$ and $\lambda$ are the speed and wavelength of the waves respectively. Appropriate velocity field for this problem is $\overline{V} = \left[ U(\overline{X}, \overline{Y}, \overline{t}), V(\overline{X}, \overline{Y}, \overline{t}), 0 \right]$. Laws of conservation of mass, momentum and energy can be represented as follows:

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{3.1}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + 2 \frac{\partial}{\partial \overline{X}} \left( \overline{\mu}(T) \frac{\partial \overline{U}}{\partial \overline{X}} \right) + \frac{\partial}{\partial \overline{Y}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right) \right]$$
$$- \sigma B_0^2 Cos\beta \left( \overline{U} Cos\beta - \overline{V} \operatorname{Si} n\beta \right) + \rho g \alpha^* (T - T_0) \operatorname{Si} n\alpha_1 + \rho g \operatorname{Si} n\alpha_1 - \frac{\overline{\mu}(T)\overline{U}}{\overline{k}}, \tag{3.2}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + 2 \frac{\partial}{\partial \overline{Y}} \left( \overline{\mu}(T) \frac{\partial \overline{V}}{\partial \overline{Y}} \right) + \frac{\partial}{\partial \overline{x}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{V}}{\partial \overline{x}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right) \right]$$
$$- \sigma B_0^2 \operatorname{Si} n\beta \left( \overline{U} Cos\beta - \overline{V} \operatorname{Si} n\beta \right) - \rho g \alpha^* (T - T_0) Cos\alpha_1 - \rho g Cos\alpha_1 - \frac{\overline{\mu}(T)\overline{V}}{\overline{k}}, \tag{3.3}$$

$$\rho C_p \left( T_{\overline{t}} + \overline{U} T_{\overline{X}} + \overline{V} T_{\overline{Y}} \right) = K \left[ T_{\overline{X}\overline{X}} + T_{\overline{Y}\overline{Y}} \right] + \overline{\mu}(T) \left[ 2 \left\{ \left( \frac{\partial \overline{U}}{\partial \overline{X}} \right)^2 + \left( \frac{\partial \overline{V}}{\partial \overline{Y}} \right)^2 \right\} + \left( \frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right)^2 \right]$$
$$+ \sigma B_0^2 \left( \overline{U} Cos\beta - \overline{V} \operatorname{Si} n\beta \right)^2 + \frac{\overline{\mu}(T) \overline{U}^2}{\overline{k}}. \tag{3.4}$$

Here $\overline{U}$, $\overline{V}$ and $\overline{P}$ are the velocity components and pressure in the laboratory frame $(\overline{X}, \overline{Y}, \overline{t})$, $\rho$ the density of fluid, $\mu$ the dynamic viscosity, $g$ the acceleration due to gravity, $\overline{k}$ permeability of the porous medium parameter, $\beta$ the inclination of applied magnetic field and $\alpha^*$ the thermal expansion coefficient. We also denote $C_p$ as the specific heat, $K$ the thermal conductivity and $T$ the temperature of fluid. We further adopt $(\overline{u}, \overline{v})$ and $\overline{p}$ as the velocity components and

pressure in the wave frame $(\overline{x}, \overline{y})$. The transformations between laboratory and wave frames are

$$\overline{x} = \overline{X} - c\overline{t}, \ \ \overline{y} = \overline{Y}, \ \ \overline{u} = \overline{U} - c, \ \ \ \overline{v} = \overline{V}, \ \ \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}). \tag{3.5}$$

The governing equations in the wave frame are given by

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{3.6}$$

$$\rho \left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right)(\overline{u} + c) = -\frac{\partial \overline{p}}{\partial \overline{x}} + 2 \frac{\partial}{\partial \overline{x}} \left( \overline{\mu}(T) \frac{\partial \overline{u}}{\partial \overline{x}} \right) + \frac{\partial}{\partial \overline{y}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right) \right]$$
$$- \sigma B_0^2 Cos\beta \left( (\overline{u} + c) Cos\beta - \overline{v} \operatorname{Si} n\beta \right) + \rho g \alpha^* (T - T_0) \operatorname{Si} n\alpha_1 + \rho g \operatorname{Si} n\alpha_1 - \frac{\overline{\mu}(T)(\overline{u} + c)}{\overline{k}}, \tag{3.7}$$

$$\rho \left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right)(\overline{v}) = -\frac{\partial \overline{p}}{\partial \overline{y}} + 2 \frac{\partial}{\partial \overline{y}} \left( \overline{\mu}(T) \frac{\partial \overline{v}}{\partial \overline{y}} \right) + \frac{\partial}{\partial \overline{x}} \left[ \overline{\mu}(T) \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right) \right]$$
$$- \sigma B_0^2 \operatorname{Si} n\beta \left( (\overline{u} + c) Cos\beta - \overline{v} \operatorname{Si} n\beta \right) - \rho g \alpha^* (T - T_0) Cos\alpha_1 - \rho g Cos\alpha_1 - \frac{\overline{\mu}(T)(\overline{v})}{\overline{k}}, \tag{3.8}$$

$$\rho C_p \left( (\overline{u} + c) T_{\overline{x}} + \overline{v} T_y \right) = K \left[ T_{\overline{x}\overline{x}} + T_{\overline{y}\overline{y}} \right] + \overline{\mu}(T) \left[ 2 \left\{ \left( \frac{\partial \overline{u}}{\partial \overline{x}} \right)^2 + \left( \frac{\partial \overline{v}}{\partial \overline{y}} \right)^2 \right\} + \left( \frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}} \right)^2 \right]$$
$$+ \sigma B_0^2 \left( (\overline{u} + c) Cos\beta - \overline{v} \operatorname{Si} n\beta \right)^2 + \frac{\overline{\mu}(T)(\overline{u} + c)^2}{\overline{k}}, \tag{3.9}$$

Defining the dimensionless quantities

$$x = \frac{\overline{x}}{\lambda}, \ y = \frac{\overline{y}}{d_1}, \ u = \frac{\overline{u}}{c}, \ v = \frac{\overline{v}}{c\delta}, \ \delta = \frac{d_1}{\lambda}, \ h = \frac{\overline{H}}{d_1}, \ a = \frac{a_1}{d_1}, \ p = \frac{d_1^2 \overline{p}}{c\lambda \mu_0}, \ \upsilon = \frac{\mu_0}{\rho},$$

$$\mu(\theta) = \frac{\overline{\mu}(T)}{\mu_0}, \ M^2 = \left( \frac{\sigma}{\mu_0} \right)^2 B_0^2 d_1^2, \ Re = \frac{\rho c d_1}{\mu_0}, \ t = \frac{c\overline{t}}{\lambda}, \ \theta = \frac{T - T_0}{T_0}, \ G_t = \frac{\rho g \alpha^* T_0 d_1^2}{\mu_0 c},$$

$$Fr = \frac{c^2}{g d_1}, \ k = \frac{\overline{k}}{d_1^2}, \ Br = \Pr E, \ E = \frac{c^2}{C_p T_0}, \ Pr = \frac{\mu_0 C_p}{K}, \ u = \psi_y, \ v = -\psi_x, \tag{3.10}$$

31

and adopting long wavelength and low Reynolds number approach, the equations in terms of stream function $\psi$ are

$$p_x = \frac{\partial}{\partial y}\left[\mu\left(\theta\right)\frac{\partial^2\psi}{\partial y^2}\right] - \left\{M^2 Cos^2\beta + \frac{\mu\left(\theta\right)}{k}\right\}\left(\frac{\partial\psi}{\partial y} + 1\right) +$$
$$G_t\theta\,Si\,n\alpha_1 + \frac{\mathrm{Re}\,Si\,n\alpha_1}{Fr}, \tag{3.11}$$

$$p_y = 0, \tag{3.12}$$

$$\theta_{yy} + Br\mu\left(\theta\right)\left(\psi_{yy}\right)^2 + Br\left[M^2 Cos^2\beta + \frac{\mu\left(\theta\right)}{k}\right]\left(\frac{\partial\psi}{\partial y} + 1\right)^2 = 0, \tag{3.13}$$

Note that the incompressibility condition is automatically satisfied and Eq. (3.11) witnesses that $p \neq p(y)$. From Eqs. (3.10) and (3.11) we have

$$0 = \frac{\partial^2}{\partial y^2}\left[\mu\left(\theta\right)\frac{\partial^2\psi}{\partial y^2}\right] - \left\{M^2 Cos^2\beta + \frac{\mu\left(\theta\right)}{k}\right\}\left(\frac{\partial^2\psi}{\partial y^2}\right) + G_t\frac{\partial\theta}{\partial y}\,Si\,n\alpha_1. \tag{3.14}$$

Here $v$ denotes the kinematic viscosity, Re the Reynolds number, $Br$ the Brinkman number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the wave number, $Gr$ the Grashoff number, $Fr$ the Froud number, $k$ the dimensionless permeability parameter and $\theta$ the dimensionless temperature.

We consider the temperature dependent viscosity in the form

$$\mu(\theta) = e^{-\alpha\theta}$$

which gives

$$\mu(\theta) = 1 - \alpha\theta, \qquad \text{for } \alpha \ll 1.$$

where $\alpha$ is the viscosity parameter. Note that for $\alpha = 0$ our problem reduces to the constant viscosity case.

The dimensionless volume flow rate in laboratory frame is defined by

$$Q = \int_0^{\overline{H}} \overline{U}(\overline{X}, \overline{Y}, \overline{t})dY \tag{3.15}$$

and in wave frame we have

$$q = \int_0^h \overline{u}(x, y) dy \tag{3.16}$$

in which $h$ is function of $x$ alone. From Eqs. (3.14), (3.15) and (3.6) one has

$$Q = q + ch(x).$$

The time averaged flow over a period $T_f$ is

$$\overline{Q} = \frac{1}{\overline{T}} \int_0^{T_f} Q dt$$

which implies that

$$\overline{Q} = q + ca.$$

Defining $\eta$ and F as the dimensionless mean flows in the laboratory and wave frames by

$$\eta = \frac{\overline{Q}}{ca}, \; F = \frac{q}{ca} \tag{3.17}$$

one can write

$$\eta = F + 1, \tag{3.18}$$

where

$$F = \int_0^h \frac{\partial \psi}{\partial y} dy. \tag{3.19}$$

The corresponding boundary conditions for the present flow configuration are

$$\begin{aligned}
\psi &= 0, \quad \psi_{yy} = 0, \quad \theta_y = 0, \quad y = 0, \\
\psi &= F, \; \psi_y = -1, \; \theta = 0, \quad y = h,
\end{aligned} \tag{3.20}$$

with

$$h(x) = 1 + a\cos(2\pi x). \tag{3.21}$$

## 3.2 Solution and analysis

Numerical solutions of the derived problems are obtained. For that we have taken 0.01 as the step size for the variations of $x$ and $y$. The obtained results are analyzed graphically in this section.

The graphs of pressure gradient, stream function, velocity and temperature are examined. Moreover, the numerical values of heat transfer rate at the wall are computed and analyzed through Table 3.1. Hence Figs. 3.1(a-h) are plotted to analyze the pressure gradient variation with respect to different embedded parameters. Figs. 3.2-3.8 depict the behavior of streamlines, Figs. 3.9(a-f) for velocity and Figs. 3.10(a-f) for the temperature profile.

It is seen that the pressure gradient varies and has maximum value at the wider part of the channel. Minimum value of the pressure gradient is found near the more occlude part of the channel. It is observed that value of the pressure gradient increases with an increase in the variable viscosity parameter i.e. the pressure gradient has a higher value for the variable viscosity fluid than that of the fluid with constant viscosity (see Fig. 3.1a). Presence of porous medium decreases the value of pressure gradient (see Fig. 3.1b). Pressure gradient is larger when we move from the horizontal to the vertical channel. Increase in the Hartman number and decrease in the magnetic field inclination tend to decrease the pressure gradient (Figs. 3.1d and f). An increase in Froud number decreases the pressure gradient while the Brinkman number increases its value (Figs. 3.1e and h). Corresponding value of the pressure gradient is higher in the case of resisting (or opposing) flow when compared to that of the assisting flow (Fig. 3.1g).

The volume of the fluid trapped within a streamline is usually termed as bolus. This study showed that the size of the trapped bolus decreases with an increase in the magnetic field parameter M (see Fig. 3.2). Bolus size is smaller in the case of fluid with constant viscosity (see Fig. 3.3). Size of the trapped bolus increases with an increase in the value of the porosity parameter k. It means that the bolus size gets reduced in case of flow through porous medium (Fig. 3.4). From Fig. 3.5 it is observed that the impact of channel inclination on the size of the trapped bolus is not very significant. However an increase in channel inclination enhances the bolus size by a very small amount. An increase in both magnetic field inclination and Brinkman number give rise to an increase in the size of the trapped bolus (see Figs. 3.6 and 3.7). Bolus is found to have a comparatively larger size in the case of assisting flow (+ value of g) (see Fig.

3.8).

Velocity profile is seen to follow a parabolic trajectory with maximum value near the center of the channel. Such value of the velocity increases when we move from constant viscosity fluid to the fluid with temperature dependent viscosity. It is seen from Fig. 3.9b that the velocity decreases in the case of porous medium. Increase in the Hartrman number decreases the velocity near the center line whereas the case is opposite for an increase in Brinkman number (see Figs. 3.9d and f). Inclination of the channel results in the increase of the velocity for both assisting and resisting flows (see Fig. 3.9c). Fig. 3.9e revealed that the velocity has a higher value for assisting flow.

Dimensionless temperature is analyzed in the Figs. 3.10(a-f). Fig. 3.10a showed that the temperature is higher for the case of constant viscosity fluid when compared with the fluid of variable viscosity. Fig. 3.10b depicts that an increase in the value of permeability parameter decreases the value of the dimensionless temperature. Dimensionless temperature increases with an increase in the value of the Hartman number (Fig. 3.10 c). Similarly temperature is higher for the case of assisting flow (see Fig. 3.10 d). Increase in inclination of channel gives higher value of the temperature whereas the case is opposite for the inclination of magnetic field (see Figs. 3.10 e and f).

Numerical values of the heat transfer rate at the wall for different parameters are plotted in Table 3.1. It is noted that the heat transfer rate decreases by increasing the viscosity parameter $\alpha$, permeability parameter k, Grashoff number $G_t$ and channel and magnetic field inclinations ($\alpha_1$ and $\beta$ respectively). However it increases due to increase in Hartman number (M) and Brinkman number Br.

## 3.3   Key findings

Numerical solution for the peristaltic transport of the variable viscosity fluid with porous medium and heat transfer in an inclined channel is obtained and analyzed. The main points of present attempt are listed below.

- Pressure gradient has a higher value for variable viscosity fluid than constant viscosity fluid.

- There is a decrease of pressure gradient in presence of porous medium.

- Pressure gradient has higher value when channel is vertical and there is resisting flow.

- Size of the trapped bolus increases with an increase in viscosity and permeability parameters.

- Trapped bolus has a larger size for assisting flow case.

- Fluid velocity with constant viscosity is less when compared to that of variable viscosity.

- Effects of porous medium on pressure gradient and velocity are qualitatively similar.

- Unlike the velocity, the temperature of the fluid is higher for constant viscosity case.

- Temperature is higher for the case of assisting flow.

- Heat transfer rate at the wall decreases with the increase in channel and magnetic field inclinations.

The used values of the parameters here are $\alpha = 0.2, k = 1.0, \alpha_1, \beta = \pi/4, \mathrm{Re} = 5, Fr = 2, Br = 0.5, M = 0.5, G_t = 0.5, a = 0.3$ and $\eta = 0.3$ (unless stated otherwise).
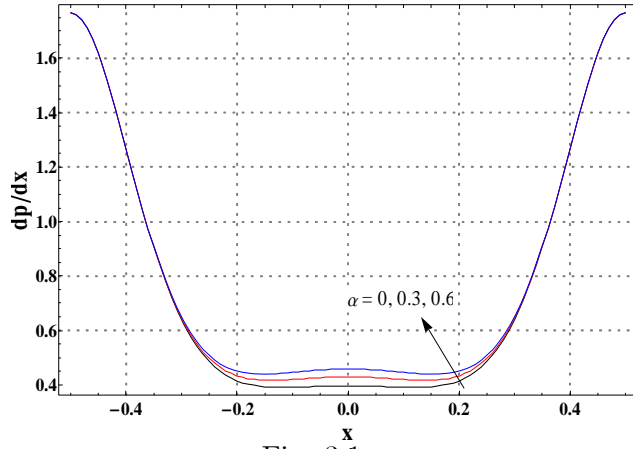
Fig. 3.1 a
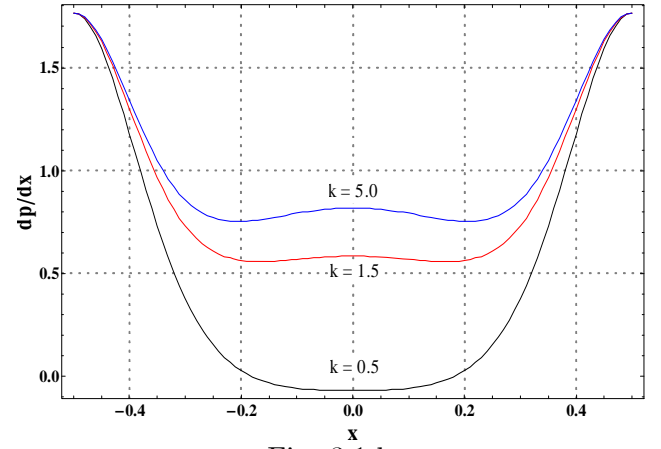

Fig. 3.1 b


Fig. 3.1 c


Fig. 3.1 d

37
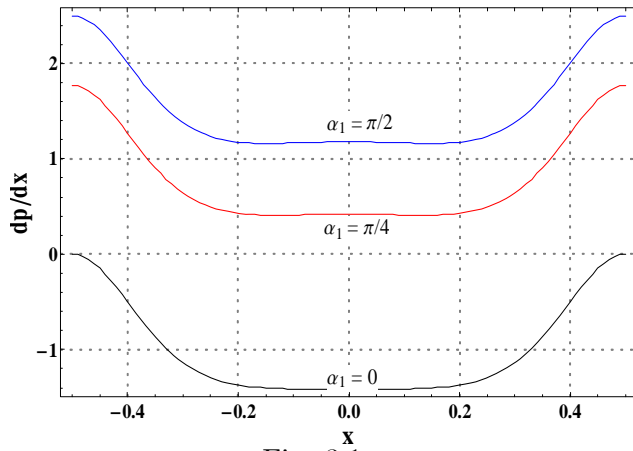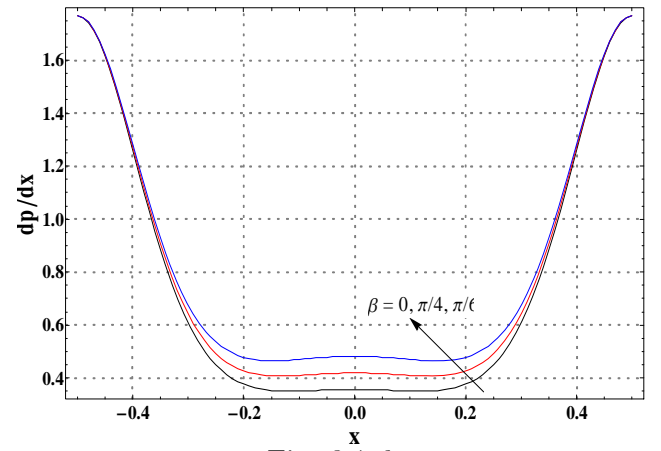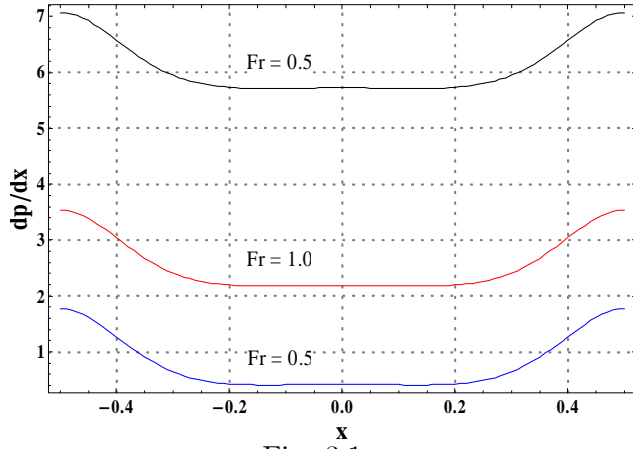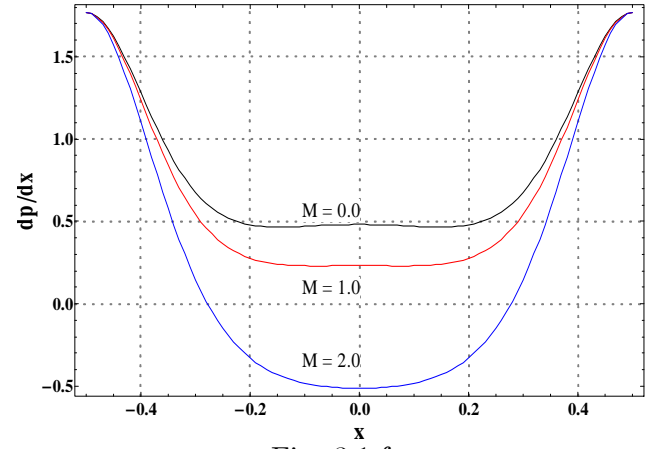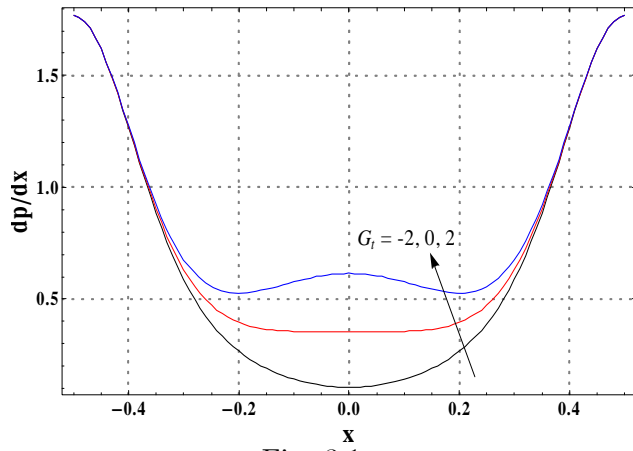
Fig. 3.1 e

Fig. 3.1 f

Fig. 3.1 g
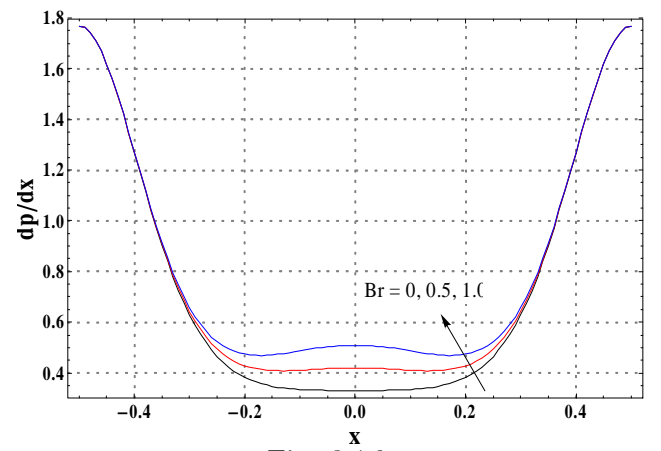
Fig. 3.1 h

Figs. 3.1 (a-h) Pressure gradient for the various parameters.
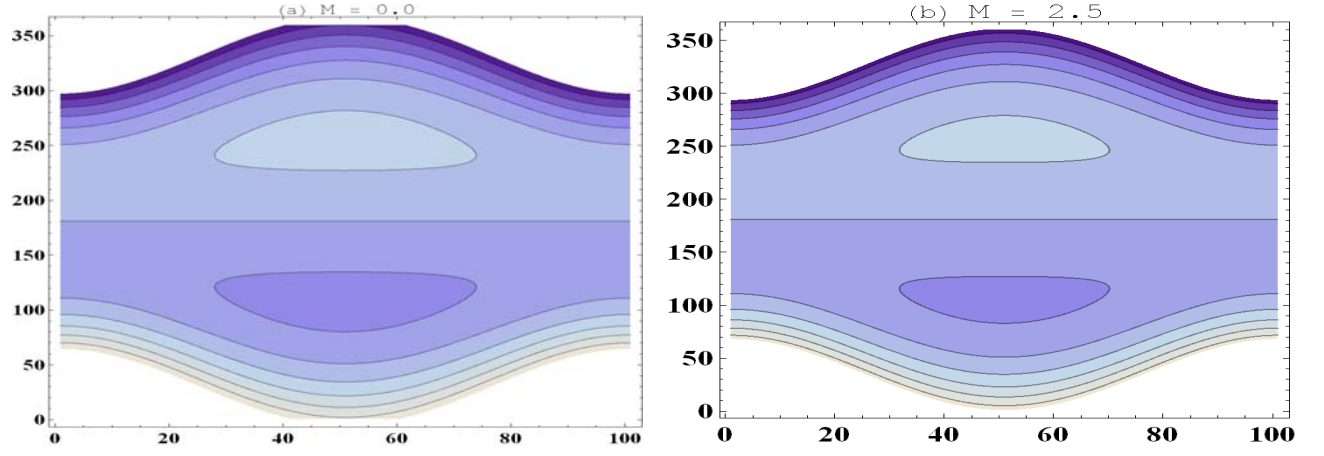
Fig. 3.2 Streamlines for variation of Hartman number.



Fig. 3.3 Streamlines for variation of viscosity parameter.
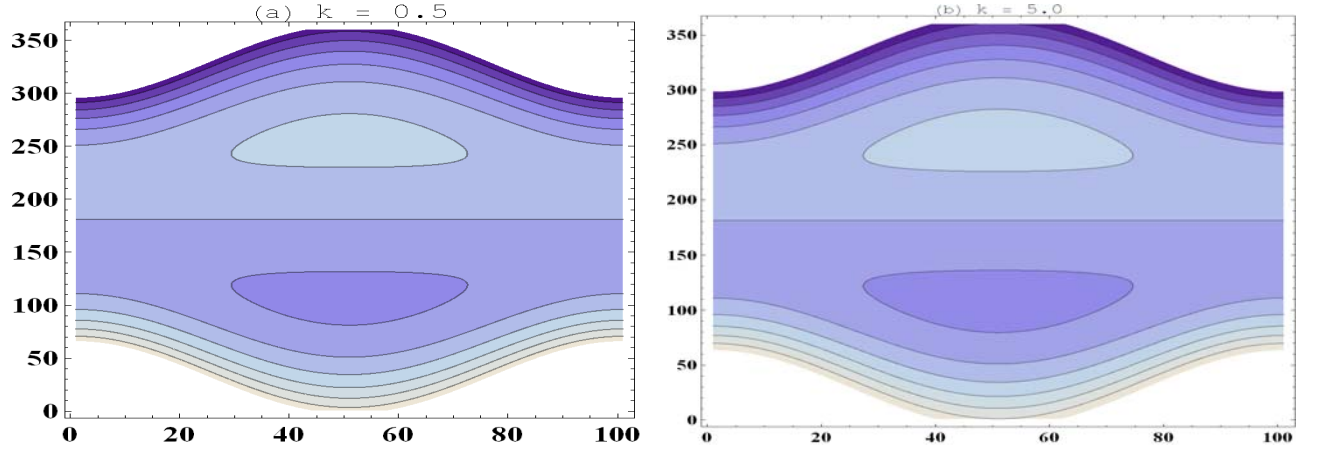
Fig. 3.4 Streamlines for variation of porosity parameter.
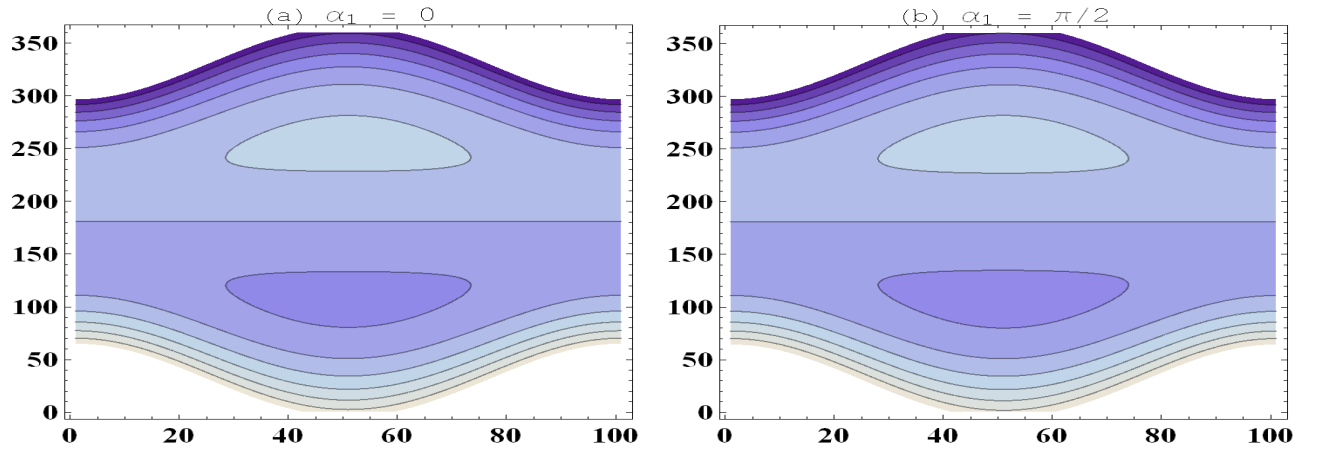


Fig. 3.5 Streamlines for variation of channel inclination.
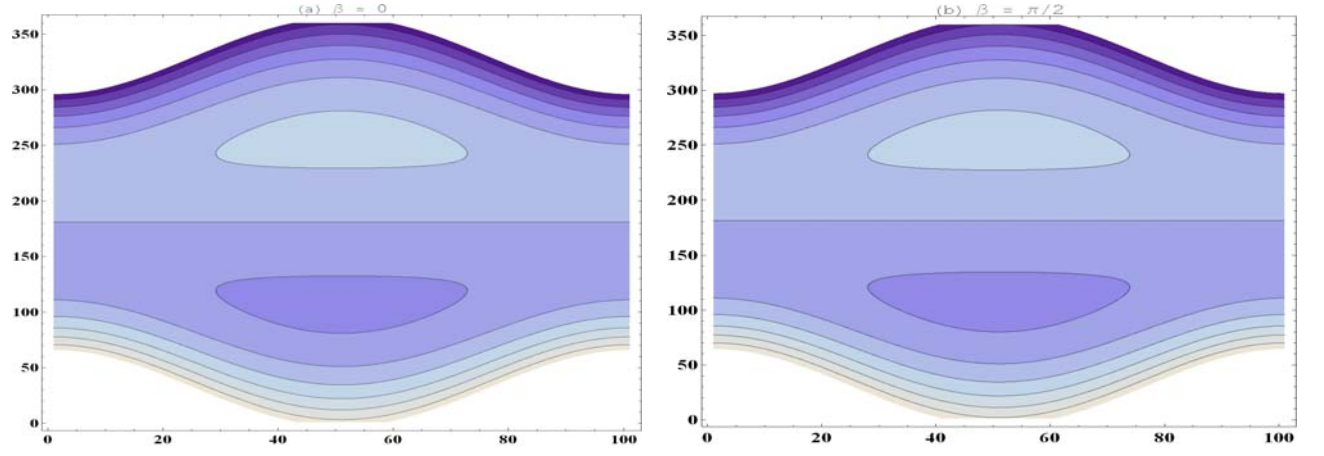
Fig. 3.6 Streamlines for different inclinations of magnetic field.
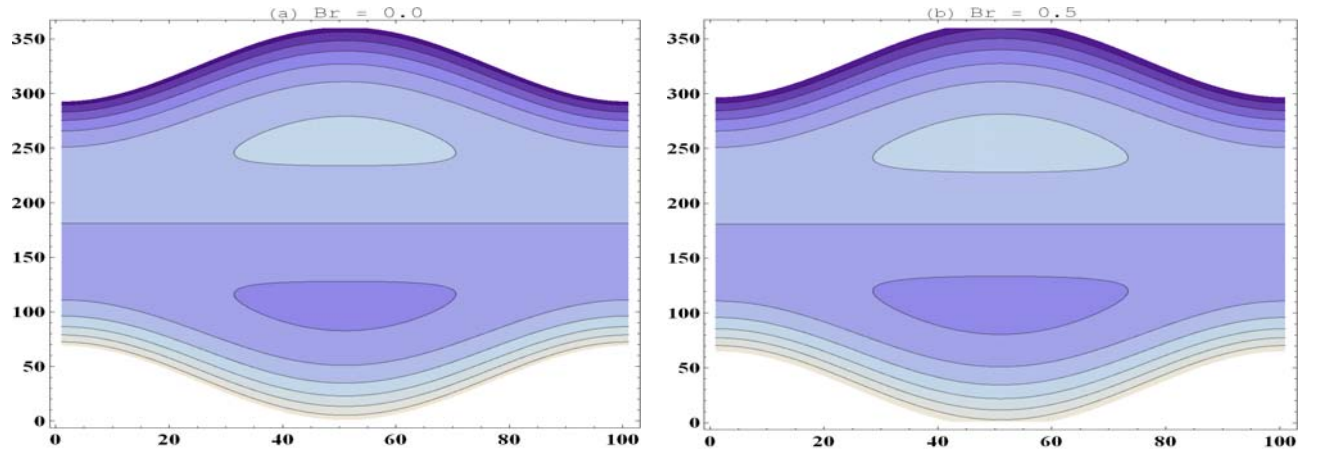


Fig. 3.7 Streamlines for different Brinkman number.
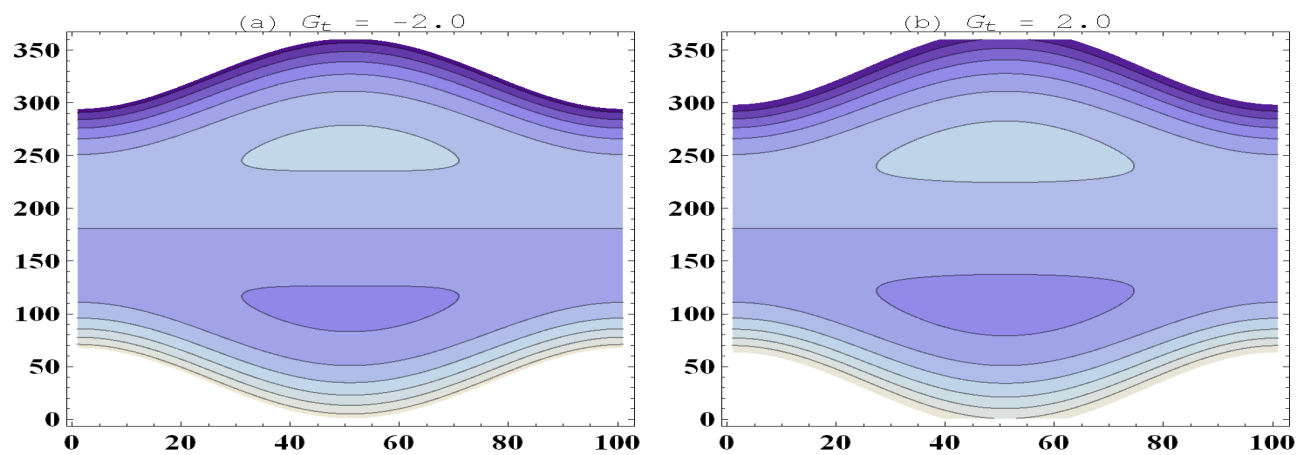
Fig. 3.8 Streamlines for different Grashoff number.
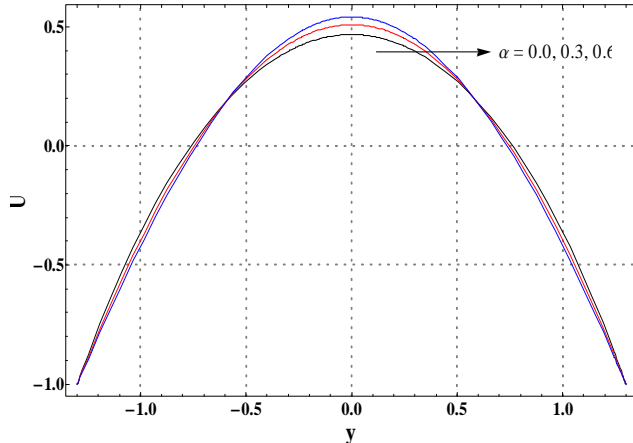
Fig. 3.9 a
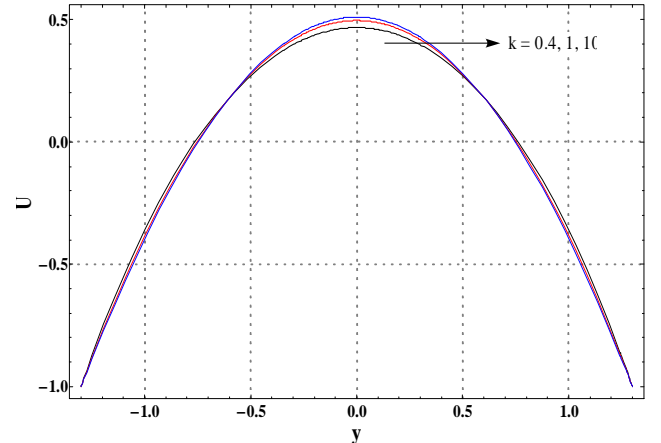

Fig. 3.9 b


Fig. 3.9 c


Fig. 3.9 d

43
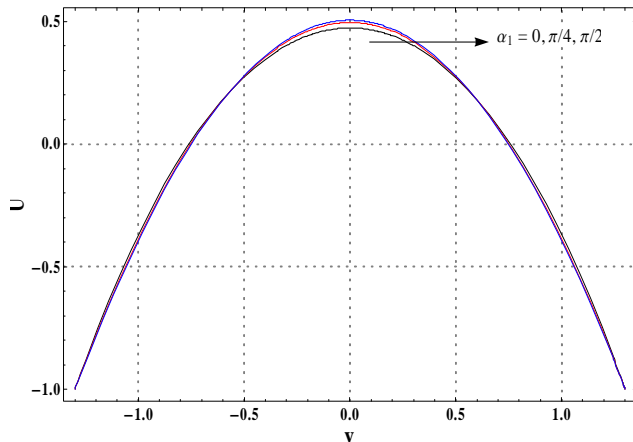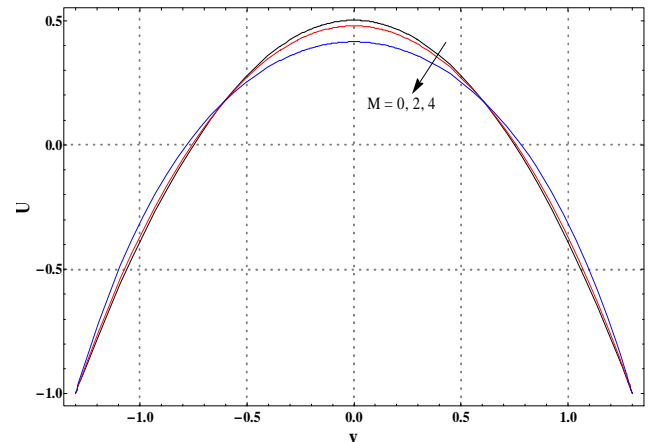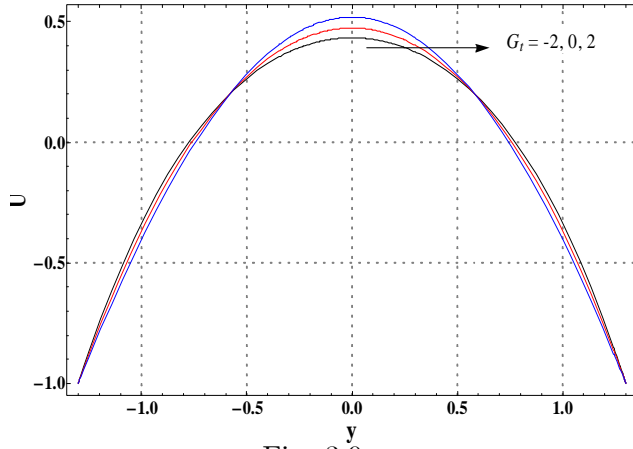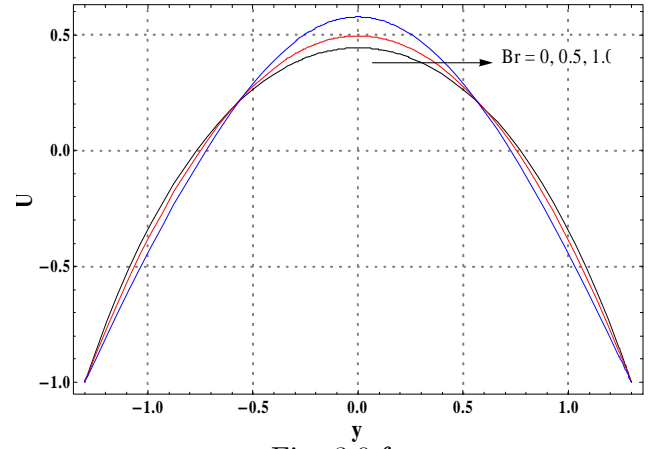
Fig. 3.9 e



Fig. 3.9 f

Figs. 3.9 (a-f) Velocity profile for different parameters.
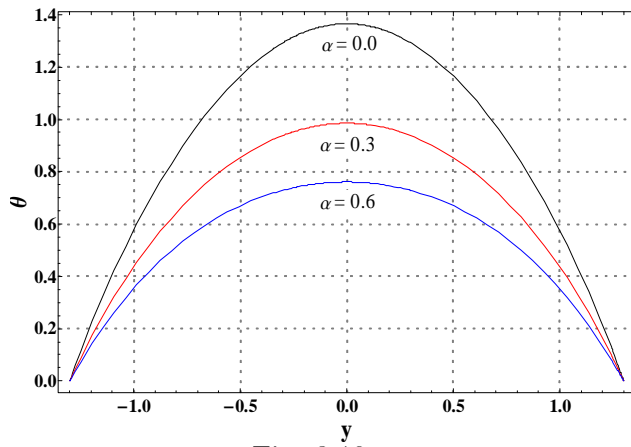
Fig. 3.10 a
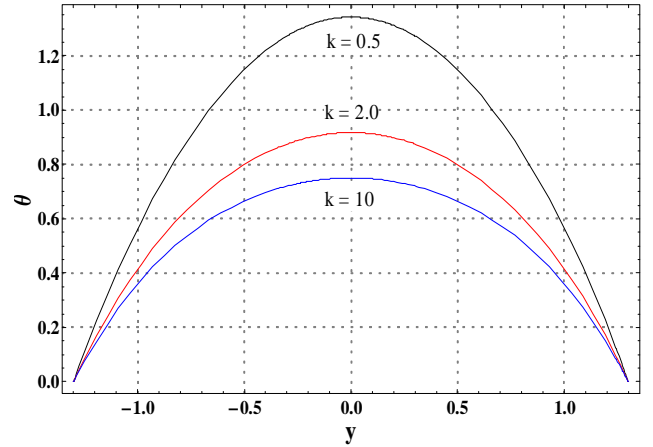


Fig. 3.10 b
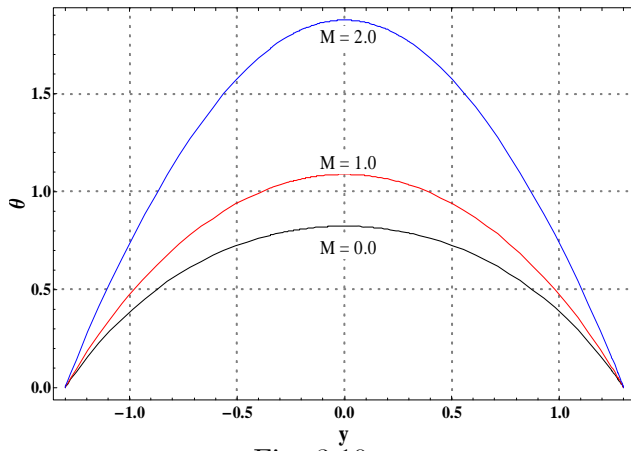


Fig. 3.10 c
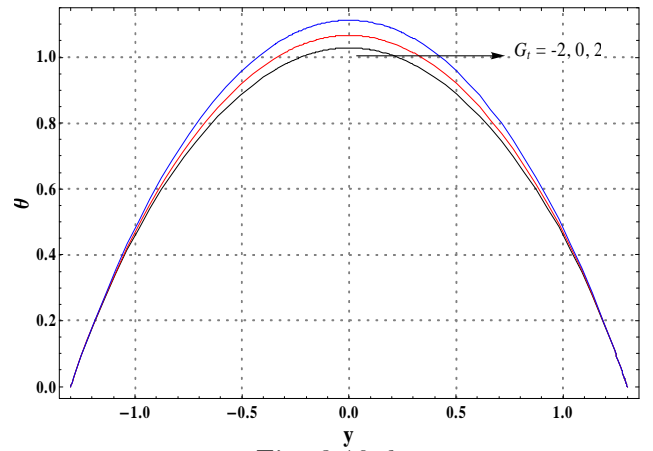


Fig. 3.10 d
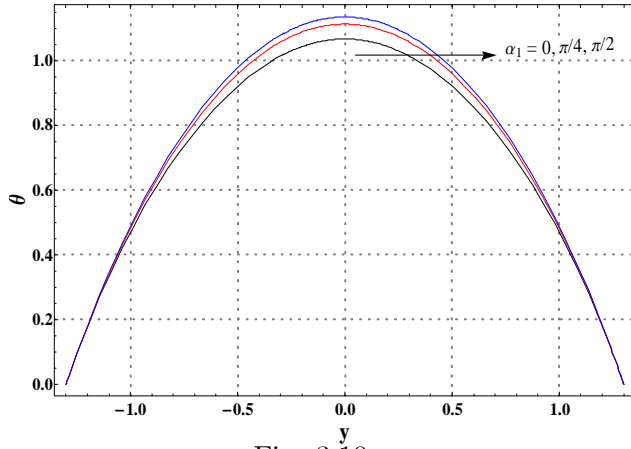
Fig. 3.10 e
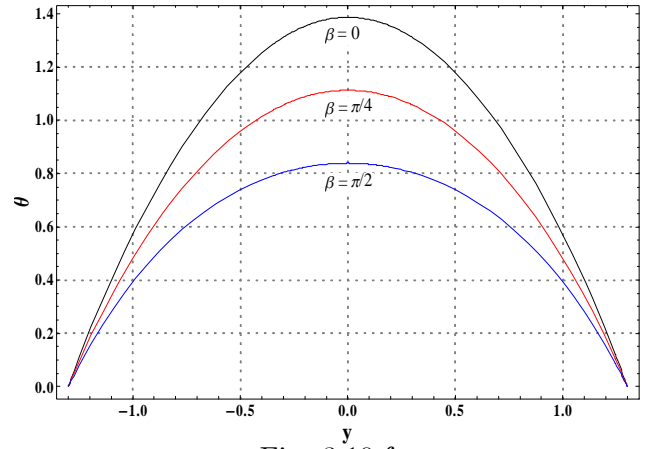
Fig. 3.10 f

Figs. 3.10 (a-f) Temperature profile for different parameters.

| $\alpha$ | $k$ | $M$ | $G_t$ | $\alpha_1$ | $\beta$ | $Br$ | $\theta'(h)$ |
|---|---|---|---|---|---|---|---|
| 0.0 | 1.0 | 1.0 | 2 | $\pi/4$ | $\pi/4$ | 0.5 | 2.32081 |
| 0.3 | | | | | | | 1.76871 |
| 0.6 | | | | | | | 1.44276 |
| 0.2 | 0.5 | | | | | | 2.24094 |
| | 2.0 | | | | | | 1.71568 |
| | 10.0 | | | | | | 1.52373 |
| | 1.0 | 0.0 | | | | | 1.61917 |
| | | 1.0 | | | | | 1.91839 |
| | | 2.0 | | | | | 2.8159 |
| | | 1.0 | $-2.0$ | | | | 1.95425 |
| | | | 0.0 | | | | 1.93242 |
| | | | 2.0 | | | | 1.91839 |
| | | | 1.0 | 0.0 | | | 1.93242 |
| | | | | $\pi/4$ | | | 1.92427 |
| | | | | $\pi/2$ | | | 1.92153 |
| | | | | $\pi/4$ | 0.0 | | 2.22499 |
| | | | | | $\pi/4$ | | 1.92427 |
| | | | | | $\pi/2$ | | 1.62345 |
| | | | | | $\pi/4$ | 0.1 | 0.401872 |
| | | | | | | 0.2 | 0.795321 |
| | | | | | | 0.3 | 1.18027 |

Table 3.1. Numerical values of heat transfer rate at the wall for different parameters.

# Chapter 4

# Peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating

This chapter concentrates on the peristaltic transport of viscous fluid in an asymmetric channel. The channel walls exhibit convective type boundary conditions. Both cases of hydrodynamic and magnetohydrodynamic (MHD) fluids are considered. Analytic solutions for stream function and temperature are constructed. Numerical integration is carried out for pressure rise per wavelength. Effects of influential flow parameters are pointed out through graphs.

## 4.1 Mathematical analysis

Consider the peristaltic transport of an incompressible MHD viscous fluid in an asymmetric channel of width $d_1 + d_2$. The $\overline{X}$-axis is taken along the length of the channel whereas the $\overline{Y}$-axis is taken normal to the $\overline{X}$-axis. The flow is due to the waves travelling on the walls of channel. A constant magnetic field $B_0$ is applied in the $\overline{Y}$- direction. The fluid is electrically conducting while the channel walls are non-conducting. The upper $\overline{H}_1$ and lower $\overline{H}_2$ walls are maintained at the temperature $T_0$ and $T_1$ respectively. The geometry of the walls is given as follows

$$\overline{H}_1(\overline{X}, \overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right),$$

$$\overline{H}_2(\overline{X}, \overline{t}) = -d_2 - b_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t}) + \gamma\right), \tag{4.1}$$

where $a_1$ and $b_1$ are the amplitudes of the waves travelling on the upper and lower walls, $\lambda$ is the wavelength, $c$ is the wave speed, $\gamma$ is the phase difference and $\overline{t}$ is the time. The laws of mass, momentum and energy yield

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{4.2}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \mu\left[\frac{\partial^2 \overline{U}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{U}}{\partial \overline{Y}^2}\right] - \sigma B_0^2 \overline{U}, \tag{4.3}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \mu\left[\frac{\partial^2 \overline{V}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{V}}{\partial \overline{Y}^2}\right], \tag{4.4}$$

$$\rho C_p\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)T = K\left[\frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2}\right]$$
$$+\mu\left[2\left(\left(\frac{\partial \overline{U}}{\partial \overline{X}}\right)^2 + \left(\frac{\partial \overline{V}}{\partial \overline{Y}}\right)^2\right) + \left(\frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}}\right)^2\right] + \sigma B_0^2 \overline{U}^2, \tag{4.5}$$

where $(\overline{U}, \overline{V})$ are the velocity components in $(\overline{X}, \overline{Y})$ directions respectively, $P$ the pressure, $\mu$ the dynamic viscosity, $\rho$ the density of fluid, $\sigma$ the electrical conductivity, $K$ the thermal conductivity and $C_p$ the specific heat. We have an interest to transform our problem from the fixed frame $(\overline{X}, \overline{Y})$ to the moving/wave frame $(\overline{x}, \overline{y})$. For that we define

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}). \tag{4.6}$$

On setting

$$x = \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d_1}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c\delta}, \quad \delta = \frac{d_1}{\lambda}, \quad h_1 = \frac{\overline{H}_1}{d_1},$$

$$h_2 = \frac{\overline{H}_2}{d_1}, \quad d = \frac{d_2}{d_1}, \quad a = \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1}, \quad p = \frac{d_1^2 \overline{p}}{c\lambda\mu}, \quad \upsilon = \frac{\mu}{\rho},$$

$$Re = \frac{\rho c d_1}{\mu}, \quad t = \frac{c\overline{t}}{\lambda}, \quad M = \left(\frac{\sigma}{\mu}\right)^{1/2} B_0 d_1, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad Br = \Pr E,$$

$$E = \frac{c^2}{C_p(T_1 - T_0)}, \quad \Pr = \frac{\mu C_p}{K}, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{4.7}$$

dimensionless form of Eqs. $(4.2 - 4.5)$ take the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4.8}$$

$$\delta \operatorname{Re}\left((u+1)\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - M^2(u+1), \tag{4.9}$$

$$\delta^3 \operatorname{Re}\left((u+1)\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{4.10}$$

$$\delta \Pr \operatorname{Re}\left((u+1)\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}$$

$$+ Br\left[2\delta^2\left(\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2\right) + \left(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] + BrM^2(u+1)^2. \tag{4.11}$$

Here $\upsilon$ denotes the kinematic viscosity, $M$ the Hartman number, Re the Reynolds number, $Br$ the Brinkman number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the wave number and $\theta$ the dimensionless temperature. Making use of the long wavelength and low Reynolds number approximation we have

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - M^2\left(\frac{\partial \psi}{\partial y} + 1\right), \tag{4.12}$$

$$\frac{\partial p}{\partial y} = 0, \tag{4.13}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Br\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 + BrM^2\left(\frac{\partial \psi}{\partial y} + 1\right)^2, \tag{4.14}$$

where Eq. $(4.12)$ indicates that $p \neq p(y)$.

Defining $\eta$ and F as the dimensionless mean flows in laboratory and wave frames by

$$\eta = \frac{\overline{Q}}{cd_1}, \quad F = \frac{q}{cd_1}, \tag{4.15}$$

which after using $Q = q + c(d_1 + d_2)$ give

$$\eta = F + 1 + d, \tag{4.16}$$

where

$$F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} dy. \tag{4.17}$$

The convective boundary condition for the temperature can be expressed as follows

$$-K\frac{\partial T}{\partial Y} = l(T - T_w),$$

in which $K$ is the thermal conductivity, $l$ the wall heat transfer coefficient and $T_w$ the temperature of the wall. Physically this condition states that the rate of heat transfer from the wall to the fluid (or vice versa) is proportional to the difference of their temperature. Having in mind the asymmetric nature of considered channel, we choose $l_1$ and $l_2$ the respective heat transfer coefficients of upper and lower walls. Hence the dimensionless boundary conditions are

$$\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \frac{\partial \theta}{\partial y} + Bi_1\theta = 0, \quad at \ y = H_1,$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \frac{\partial \theta}{\partial y} - Bi_2(\theta - 1) = 0, \quad at \ y = H_2, \tag{4.18}$$

$$h_1(x) = 1 + a\cos(2\pi x), \ h_2(x) = -d - b\cos(2\pi x + \phi), \tag{4.19}$$

$$Bi_1 = \frac{l_1 d_1}{K}, Bi_2 = \frac{l_2 d_1}{K}, \tag{4.20}$$

where $Bi_{1,2}$ are the Biot-numbers for the upper and lower walls. Pressure rise per wavelength is given by

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx} dx. \tag{4.21}$$

## 4.2  Solution expressions

The solutions of Eqs. (4.12) and (4.14) subject to the conditions (4.18) are

$$\psi = \frac{-e^{-My}}{2L_1} \left( L_2 - L_3 \right),$$
(4.22)

$$\theta = \frac{1}{L_4} \left( \frac{-L_5}{A_1} + \left( 1 + Bi_1 h_1 \right) L_6 + \frac{L_7}{\left( A_1 \right)} \right).$$
(4.23)

For $M = 0$ one has

$$\psi = -\frac{\left( h_1 + h_2 - 2y \right) \left( -2 \left( h_1 - h_2 \right) \left( h_1 - y \right) \left( h_2 - y \right) + FL_8 \right)}{2 \left( h_1 - h_2 \right)^3},$$
(4.24)

$$\theta = 1 + \frac{6Br \left( F + h_1 - h_2 \right)^2}{Bi_2 \left( h_1 - h_2 \right)^3} + \frac{3Bi_2 Br \left( F + h_1 - h_2 \right)^2}{4Bi_2 \left( h_1 - h_2 \right)^2} - \frac{C_1}{Bi_2 C_2}$$
$$- \frac{3Br \left( F + h_1 - h_2 \right)^2 \left( h_1 + h_2 - 2y \right)^4}{4 \left( h_1 - h_2 \right)^6} + \frac{C_3 y}{C_2}.$$
(4.25)

and L$_i$s, A$_i$s and C$_i$s are given as follows:

$$C_1 = \left(Bi_1\left(Bi_2\left(h_1-h_2\right)^3+6Br\left(F+h_1-h_2\right)^2\right)+6Bi_2Br\left(F+h_1-h_2\right)^2\right)\left(-1+Bi_2h_2\right),$$

$$C_2 = \left(h_1-h_2\right)^3\left(Bi_1+Bi_2+Bi_2Bi_1h_1-Bi_2Bi_1h_2\right),$$

$$C_3 = \left(-Bi_1\left(Bi_2\left(h_1-h_2\right)^3+6Br\left(F+h_1-h_2\right)^2\right)+6Bi_2Br\left(F+h_1-h_2\right)^2\right),$$

$$L_1 = \left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right),$$

$$L_2 = \left(F+h_1-h_2\right)\left(2e^{\left(h_1+h_2\right)M}-2e^{2My}\right),$$

$$L_3 = e^{My}\left(h_1+h_2-2y\right)\left(e^{Mh_2}\left(-2+FM\right)-e^{Mh_1}\left(2+FM\right)\right),$$

$$L_4 = Bi_1+Bi_2+Bi_2Bi_1h_1-Bi_2Bi_1h_2,$$

$$L_5 = Br\left(1-Bi_2+h_2\right)M^2\left(MA_2+A_3-A_4-Bi_1\left(A_5+A_6\right)\right),$$

$$L_6 = Bi_2-\frac{1}{\left(A_1\right)}\times\left(BrM^2\left(-A_2-A_7\right)\right)+\left(\frac{BrBi_2M^2}{\left(A_1\right)}\times A_8\right)-A_9,$$

$$L_7 = BrM^2\left(A_{10}-A_{11}y^2\right),$$

$$L_8 = \left(h_1^2-4h_1h_2+h_2^2+2\left(h_1+h_2\right)y-2y^2\right).$$

$$A_1 = 4\left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right)^2,$$

$$A_2 = -2e^{2h_1M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)+2e^{2h_2M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)$$
$$-8e^{h_1M}\left(F+h_1-h_2\right)\left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right),$$

$$A_3 = 8e^{h_2M}\left(F+h_1-h_2\right)M\left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right)$$
$$-4h_1\left(e^{2h_2M}\left(2+h_1M-h_2M\right)^2+e^{2h_1M}\left(2+h_1M-h_2M\right)^2\right),$$

$$A_4 = 2e^{\left(h_1+h_2\right)M}A_{13},$$

$$A_5 = e^{2h_1M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)+2e^{2h_2M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)$$
$$-8e^{h_1M}\left(F+h_1-h_2\right)\left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right)$$
$$+8e^{h_2M}\left(F+h_1-h_2\right),$$

$$A_6 = 2h_1^2\left(e^{2h_2M}\left(2+h_1M-h_2M\right)^2+e^{2h_1M}\left(2-h_1M+h_2M\right)^2-2e^{\left(h_1+h_2\right)M}A_{13}\right),$$

$$A_7 = 8e^{h_2M}\left(F+h_1-h_2\right)M\left(e^{h_1M}\left(-2+h_1M-h_2M\right)+e^{h_2M}\left(2+h_1M-h_2M\right)\right)$$
$$+4h_2A_{12},$$

$$A_8 = e^{2h_1M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)+2e^{2h_2M}\left(F+h_1-h_2\right)^2\left(1+M^2\right)+8A_{14},$$

$$A_9 = 8e^{h_2 M}(F + h_1 - h_2)\left(e^{h_1 M}(-2 + h_1 M - h_2 M) + e^{h_2 M}(2 + h_1 M - h_2 M)\right) + 2h_2^2 A_{12},$$

$$A_{10} = e^{2M(h_1 + h_2 - y)}(F + h_1 - h_2)^2(1 + M^2) + 2e^{2yM}(F + h_1 - h_2)^2(1 + M^2)$$
$$+ 8e^{M(h_1 + h_2 - y)}(F + h_1 - h_2)A_{14},$$

$$A_{11} = 8e^{yM}(F + h_1 - h_2)M\left(e^{h_1 M}(-2 + h_1 M - h_2 M) + e^{h_2 M}(2 + h_1 M - h_2 M)\right) + 2A_{12},$$

$$A_{12} = e^{2h_2 M}(2 + h_1 M - h_2 M)^2 + e^{2h_1 M}(2 - h_1 M + h_2 M)^2 - 2e^{(h_1 + h_2)M}A_{13},$$

$$A_{13} = 4 + h_1^2 M^4 - 2h_1 h_2 M^4 + h_2^2 M^4 + 2F(h_1 - h_2)M^2(1 + M^2) + F^2(M^2 + M^4),$$

$$A_{14} = e^{h_1 M}(F + h_1 - h_2)\left(e^{h_1 M}(-2 + h_1 M - h_2 M) + e^{h_2 M}(2 + h_1 M - h_2 M)\right).$$

## 4.3 Graphical analysis

This section is concerned with the discussion of streamlines, pressure gradient and pressure rise per wavelength. This objective is achieved through Figs. 4.1-4.4. Dimensionless temperature profile is observed in the Figs. 4.5-4.7 for various parameters. In addition the heat transfer rate at the upper wall is numerically tabulated. Such numerical values are presented in Table 4.1.

Figs. 4.1 (a and b) examine the streamlines plotted against rate of volume flow $\eta$ when $M = 0$. It is observed that there is a lower limit of flow rate below which no trapping occurs. This shows that without sufficient volume of the fluid flowing per unit time through the channel, trapping is unlikely to occur. The "threshold" or critical value of flow rate in present problem is observed which approximately is equal to 1.6. It is noticed that when the value of flow rate is increased from the critical value then the size of the trapped bolus not only increases but also it gets elongated (see Figs. 4.1a & 1b). Figs. 4.2 showed that the size of the trapped bolus decreases for a given value of flow rate and eventually vanishes when Hartman number increases.

Fig. 4.3 plots the pressure gradient. It is seen that the pressure gradient is maximum at the center of the channel i.e. in the wider part of the channel. Moreover the corresponding value of the pressure gradient is decreased when $M$ has higher values. Pressure rise per wavelength decreases with an increase in flow rate and for a fixed value of flow rate the pressure rise also decreases by increasing the value of $M$.

Figs. 4.5a and 4.5b examine the temperature profile. Fig 4.5a includes the Joule heating

effect whereas Fig. 4.5b is without such effect. It is shown that there is no significant effect of $M$ on the temperature profile when there is no Joule heating. However the temperature in presence of Joule heating effect increases significantly for an increase in M.

Figs. 4.6 (a and b) analyze the effects of $Bi_1$ and $Bi_2$ separately on the temperature. It is found that variation of $Bi_1$ has significant effect near the upper wall and hardly has any effect near the lower wall. Similarly the variation of $Bi_2$ is significant near the lower wall. Further the temperature decreases by increasing $Bi_{1,2}$. Fig. 4.7 is for the case when both the walls have same heat transfer coefficient i.e. $Bi_1 = Bi_2 = Bi$. Here the temperature also decreases with an increase in $Bi$. However such decrease is initially large.

Numerical values of heat transfer rate at the upper wall is given in Table 4.1. Here $\theta'(h_1)$ is the heat transfer rate for $\theta$ given in (4.21) and $(\theta^*)'(h_1)$ is one when we neglect the Joule heating effect. It is seen that the transfer rate increases when M and $Bi$'s are increased. This increase in heat transfer differs a lot in the two cases when Joule heating is considered and neglected. It is seen that $Bi$ for one wall has very little or no effect (for large value of Bi) on transfer rate at the other wall.

## 4.4  Concluding remarks

Effects of convective boundary conditions and Joule heating on the peristaltic transport of MHD viscous fluid are examined. The key findings of the presented analysis are mentioned below.

- There is a critical value of the flow rate below which trapping is not likely to occur.

- The applied magnetic field tends to decrease the size of trapped bolus.

- Magnetic field decreases the magnitude of pressure gradient and pressure rise.

- Temperature rises in presence of Joule heating.

- Biot number decreases the dimensionless temperature.

- Effects of Hartman and Biot numbers on the heat transfer rate at the wall are opposite.
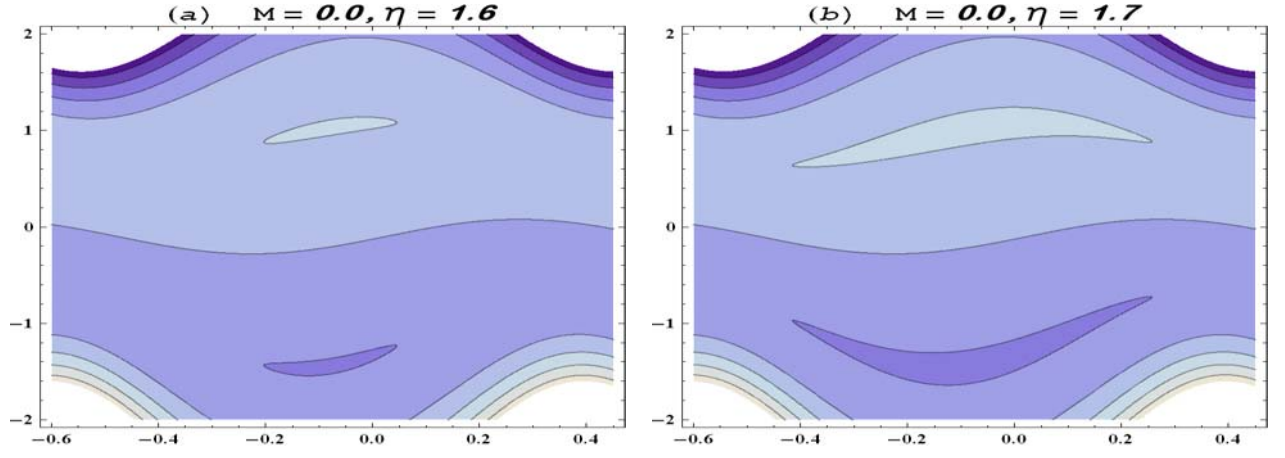
Fig. 4.1 Streamlines for M = 0.
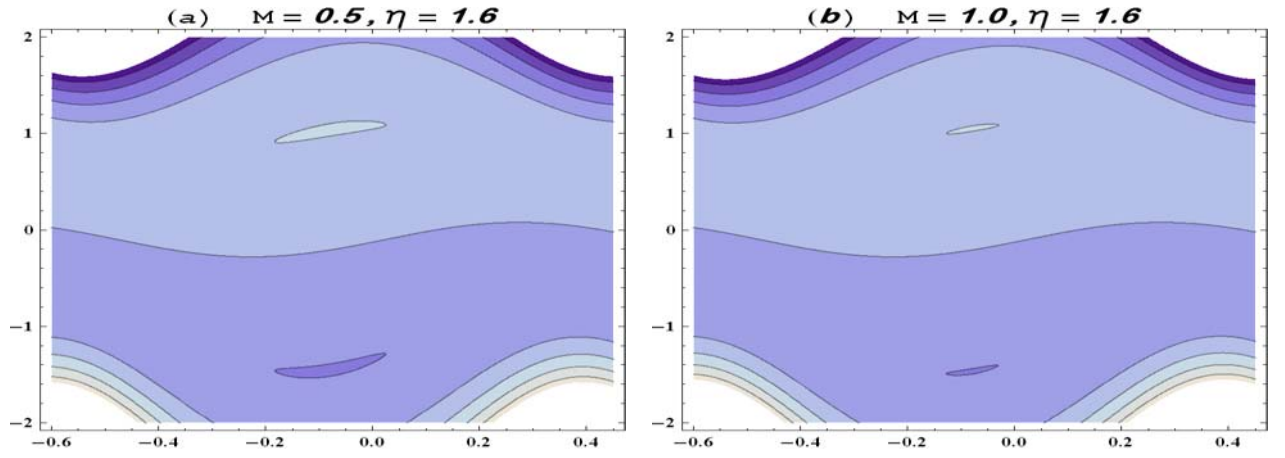


Fig. 4.2 Streamlines for M ≠ 0.
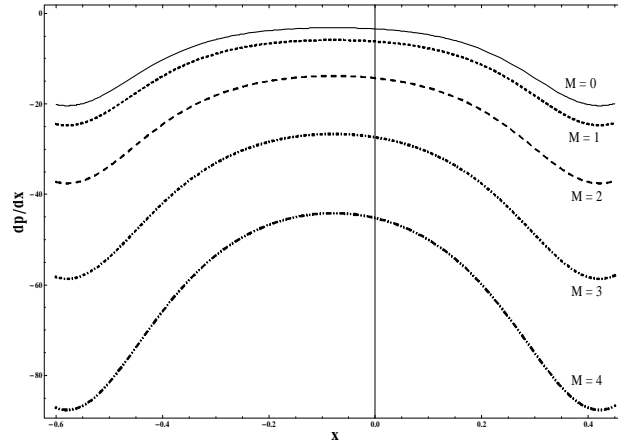
Fig. 4.3 Pressure gradient $dp/dx$ for different values of M.
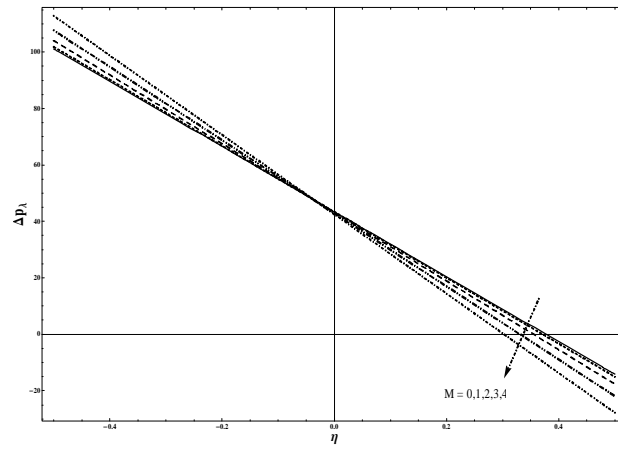


Fig. 4.4 Pressure rise $\Delta p_\lambda$ for different values of M.

57
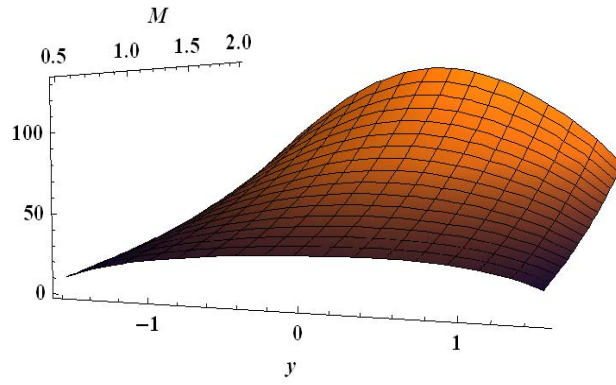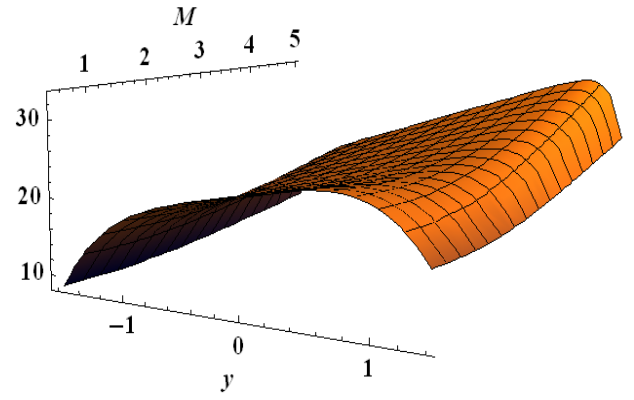
Fig. 4.5 (a)                                        Fig. 4.5 (b)

Fig. 4.5 Dimensionless temperature with variation in M.
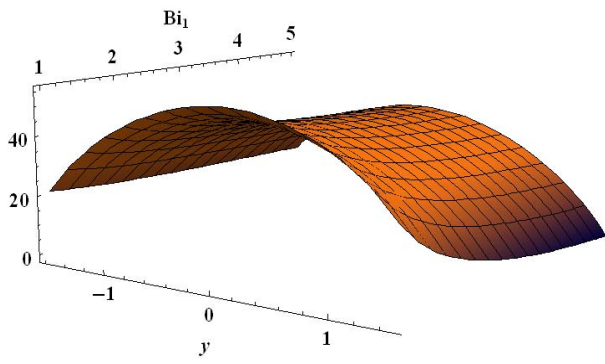


Fig. 4.6 (a)                                        Fig. 4.6 (b)
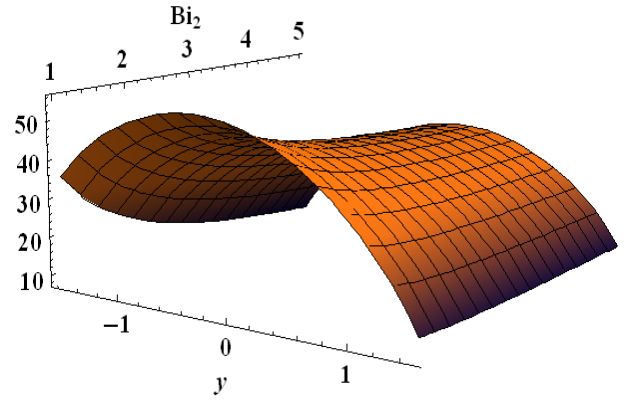
Fig. 4.6 Dimensionless temperature with variation in $Bi_{1,2}$.
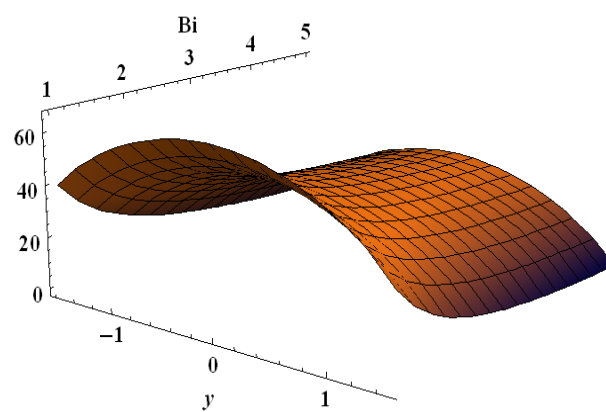
Fig. 4.7 Dimensionless temperature with $Bi_1 = Bi_2 = Bi$.

| $M$ | $Bi_1$ | $Bi_2$ | $Br$ | $-\theta'(h_1)$ | $-(\theta^*)'(h_1)$ |
|-----|--------|--------|------|-----------------|---------------------|
| 0.5 | 1.0 | 2.0 | 2.0 | 12.1396 | 10.1291 |
| 1.0 | | | | 18.1183 | 10.1291 |
| 1.5 | | | | 27.9366 | 10.1291 |
| 2.0 | | | | 41.4483 | 10.1291 |
| | | | 0.5 | 10.5343 | 2.70455 |
| | | | 1.0 | 20.839 | 5.17941 |
| | | | 1.5 | 31.1436 | 7.65427 |
| | | | 2.0 | 41.4483 | 10.1291 |
| | 0.5 | 0.5 | | 46.7126 | 11.3298 |
| | 1.0 | 1.0 | | 46.7727 | 11.3899 |
| | 1.5 | 1.5 | | 46.8055 | 11.4227 |
| | 2.0 | 2.0 | | 46.8262 | 11.4434 |

Table: 4.1. Effect of various parameters on heat transfer rate at the upper wall.

# Chapter 5

# Peristaltic motion with Soret and Dufour effects

This chapter examines the magnetohydrodynamic (MHD) peristaltic flow in a symmetric channel in the presence of Ohmic heating. Effects of Soret and Dufour are retained. Long wavelength and low Reynolds number assumptions are employed. Attention is mainly focused to analyze the flow quantities for various parameters of interest.

## 5.1   Mathematical analysis

We consider the flow of an incompressible MHD fluid in a channel of width $2d_1$. The $\overline{X}-$ axis is chosen along the walls of channel and $\overline{Y}-$ axis is selected normal to it. A uniform magnetic field $\mathbf{B}_0$ is applied in the $Y$ direction and induced magnetic field is neglected. The channel walls are represented by

$$H(\overline{X}, \overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right),\tag{5.1}$$

where $a_1$ is the wave amplitude, $\lambda$ is the wavelength, c is the wave velocity and $\overline{t}$ is the time. The equations governing the two-dimensional flow are

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0,\tag{5.2}$$

$$\rho \left( \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \mu \left[ \frac{\partial^2 \overline{U}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{U}}{\partial \overline{Y}^2} \right] - \sigma B_0^2 \overline{U}, \tag{5.3}$$

$$\rho \left( \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \mu \left[ \frac{\partial^2 \overline{V}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{V}}{\partial \overline{Y}^2} \right], \tag{5.4}$$

$$\rho C_p \left( \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{T} = K \left[ \frac{\partial^2 \overline{T}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{Y}^2} \right] +$$
$$\mu \left[ 2 \left( \left( \frac{\partial \overline{U}}{\partial \overline{X}} \right)^2 + \left( \frac{\partial \overline{V}}{\partial \overline{Y}} \right)^2 \right) + \left( \frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}} \right)^2 \right] + \sigma B_0^2 \overline{U}^2 + \frac{DK_T}{C_s} \left[ \frac{\partial^2 \overline{C}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{Y}^2} \right] \tag{5.5}$$

$$\left[ \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right] \overline{C} = D \left[ \frac{\partial^2 \overline{C}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{Y}^2} \right] + \frac{DK_T}{T_m} \left[ \frac{\partial^2 \overline{T}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{Y}^2} \right], \tag{5.6}$$

where $(\overline{U}, \overline{V})$ are velocity components in the laboratory frame $(\overline{X}, \overline{Y})$, $\overline{P}$ is pressure, $\overline{C}$ the concentration field, $\overline{T}$ the temperature field, $D$ mass diffusivity, $K_T$ thermal diffusion ratio, $C_p$ specific heat, $C_s$ concentration susceptibility, $C_0$ concentration at boundary, $T_m$ and $T_0$ the fluid mean temperature and temperature at boundary respectively.

If $(\overline{u}, \overline{v})$ are the velocity components in the wave frame $(\overline{x}, \overline{y})$ then introducing

$$\overline{x} = \overline{X} - c\bar{t}, \ \ \overline{y} = \overline{Y}, \ \ \overline{u} = \overline{U} - c, \ \ \overline{v} = \overline{V}, \ \ \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \bar{t}) \tag{5.7}$$

equations (5.2)-(5.6) in wave frame are reduced as follows:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{5.8}$$

$$\rho \left( (\overline{u} + c) \frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v} \frac{\partial \overline{u}}{\partial \overline{y}} \right) = -\frac{\partial \overline{p}}{\partial \overline{x}} + \mu \left[ \frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} \right] - \sigma B_0^2 (\overline{u} + c), \tag{5.9}$$

$$\rho \left( (\overline{u} + c) \frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v} \frac{\partial \overline{v}}{\partial \overline{y}} \right) = -\frac{\partial \overline{p}}{\partial \overline{y}} + \mu \left[ \frac{\partial^2 \overline{v}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{y}^2} \right], \tag{5.10}$$

$$\rho C_p \left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right) \overline{T} = K \left[ \frac{\partial^2 \overline{T}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{y}^2} \right] +$$
$$\mu \left[ 2 \left( \left( \frac{\partial \overline{u}}{\partial \overline{x}} \right)^2 + \left( \frac{\partial \overline{v}}{\partial \overline{Y}} \right)^2 \right) + \left( \frac{\partial \overline{u}}{\partial \overline{y}} + \frac{\partial \overline{v}}{\partial \overline{x}} \right)^2 \right] + \sigma B_0^2 (\overline{u} + c)^2 + \frac{DK_T}{C_s} \left[ \frac{\partial^2 \overline{C}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{y}^2} \right] \tag{5.11}$$

$$\left[ (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right] \overline{C} = D \left[ \frac{\partial^2 \overline{C}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{y}^2} \right] + \frac{DK_T}{T_m} \left[ \frac{\partial^2 \overline{T}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{y}^2} \right]. \tag{5.12}$$

Defining

$$x = \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d_1}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c\delta}, \quad \delta = \frac{d_1}{\lambda}, \quad h = \frac{H}{d_1}, \quad a = \frac{a_1}{d_1}, \quad p = \frac{d_1^2 \overline{p}}{c\lambda \mu},$$

$$\theta = \frac{T - T_0}{T_0}, \phi = \frac{C - C_0}{C_0}, M = \left(\frac{\sigma}{\mu}\right)^{1/2} B_0 d_1, \boldsymbol{v} = \frac{\mu}{\rho}, Re = \frac{\rho c d_1}{\mu},$$

$$t = \frac{c\overline{t}}{\lambda}, \quad Br = \Pr E, \quad Du = \frac{DC_0 K_T}{C_s C_p \mu T_0}, \quad Sr = \frac{\rho D K_T T_0}{\mu T_m C_0},$$

$$Sc = \frac{\mu}{\rho D}, \quad E = \frac{c^2}{C_p T_0}, \quad \Pr = \frac{\mu C_p}{K}, \quad u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}, \tag{5.13}$$

the equations $(5.8) - (5.12)$ in dimensionless form are stated as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5.14}$$

$$\delta \operatorname{Re} \left( (u+1)\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \left( \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - M^2(u+1), \tag{5.15}$$

$$\delta^3 \operatorname{Re} \left( (u+1)\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \left( \delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \tag{5.16}$$

$$\delta \Pr \operatorname{Re} \left( (u+1)\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} \right) = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Br \left[ 2\delta^2 \left( \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial x}\right)^2 \right) + \left( \delta^2 \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right]$$

$$+ BrM^2(u+1)^2 + Du \Pr \left( \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right). \tag{5.17}$$

$$\delta \left( (u+1)\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} \right) = \frac{1}{Sc} \left( \delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + Sr \left( \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right). \tag{5.18}$$

Making use of the long wavelength and low Reynolds number approximations we obtain

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right), \tag{5.19}$$

$$\frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial \psi^2}{\partial y^2} = 0, \tag{5.20}$$

$$\frac{\partial p}{\partial y} = 0, \tag{5.21}$$

63

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Br \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + Br M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 + \Pr Du \frac{\partial^2 \phi}{\partial y^2}, \tag{5.22}$$

$$0 = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2}, \tag{5.23}$$

in which $p \neq p(y)$, $\boldsymbol{v}$ denotes the kinematic viscosity, $M$ the Hartman number, Re the Reynolds number, $Br$ the Brinkman number, $Du$ the Dufour number, $Sr$ the Soret number, $Sc$ the Schmidt number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the wave number and the dimensionless temperature and concentration are $\theta$ and $\phi$ respectively. The boundary conditions are

$$
\begin{aligned}
\psi &= 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0, \quad \frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = 0, \quad at \ y = 0, \\
\psi &= F, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \phi = 0, \quad at \ y = h,
\end{aligned} \tag{5.24}
$$

$$h(x) = 1 + a\cos(2\pi x), \quad F = \int_0^h \frac{\partial \psi}{\partial y} dy. \tag{5.25}$$

Pressure rise per wavelength $\Delta p_\lambda$ at this stage is

$$\Delta p_\lambda = \int\limits_0^1 \frac{\partial p}{\partial x} dx. \tag{5.26}$$

Heat transfer coefficient $Z$ for this problem is defined as

$$Z = h_x \theta_y.$$

## 5.2 Solutions expressions

Solving the above equations subject to the corresponding boundary conditions one finds that

$$\psi = \frac{FMy \cosh(hM) + y \sinh(hM) - (F+h)\sinh(My)}{hM \cosh(hM) - \sinh(hM)}, \tag{5.27}$$

$$\frac{dp}{dx} = -\frac{(F+h)M^3 \cosh(hM)}{hM \cosh(hM) - \sinh(hM)}, \tag{5.28}$$

64

$$\theta = \frac{BrM^2 \left(2A_1 - A_2 \cosh[(hM) + A_3 + A_4\right)}{8\left(-1 + \Pr ScSrDu\right)\left(-hM \cosh\left(hM\right) + \sinh\left(hM\right)\right)^2}, \tag{5.29}$$

$$\phi = \frac{BrM^2 ScSr \left(B_1 + B_2 \cosh[(2hM) - B_3 + B_4\right)}{8\left(-1 + \Pr ScSrDu\right)\left(-hM \cosh\left(hM\right) + \sinh\left(hM\right)\right)^2}, \tag{5.30}$$

where

$A_1 = h\left(4 + h + h^3 M^4\right) - \left(1 + h^2 M^4\right)y^2 + F^2 M^2\left(1 + M^2\right)\left(h - y\right)\left(h + y\right) + 2F$

$\left(2 + hM^2\left(1 + M^2\right)\left(h - y\right)\left(h + y\right)\right), A_2 = F^2\left(1 + M^2\right) + 2F\left(4 + h + hM^2\right) - 2y^2$

$+h\left(8 + h\left(3 + M^2\left(1 + 2h^2 - 2y^2\right)\right)\right), A_3 = \left(F + h\right)^2\left(1 + M^2\right)\cosh[2My] + 4hM$

$\left(2F + h\left(2 + h\right) - y^2\right)\sinh[2hM] - 16h\left(F + h\right)M \cosh[hM]\left(M\left(h - y\right) + \sinh[My]\right),$

$A_4 = 16\left(F + h\right)\sinh[hM]\left(M\left(h - y\right) + \sinh[My]\right)$

$B_1 = -8F - 2h\left(4 + h\right) - 2Fh^2\left(F + 2h\right)M^2 - 2h^2\left(F + h\right)^2 M^4$

$+2\left(1 + F\left(F + 2h\right)M^2 + \left(F + h\right)^2 M^4\right)y^2, B_2 = F^2\left(1 + M^2\right) +$

$2F\left(4 + h + hM^2\right) - 2y^2 + h\left(8 + h\left(3 + M^2\left(1 + 2h^2 - 2y^2\right)\right)\right),$

$B_3 = \left(F + h\right)^2\left(1 + M^2\right)\cosh[2My] - 4hM\left(2F + h\left(2 + h\right) - y^2\right)\sinh[2hM],$

$B_4 = 16h\left(F + h\right)M \cosh[hM]\left(M\left(h - y\right) + \sinh[My]\right) - 16\left(F + h\right)$

$\sinh[hM]\left(M\left(h - y\right) + \sinh[My]\right).$

## 5.3 Discussion

Here Figs. 5.1(a-e) are drawn for the temperature, Figs. 5.2(a-e) for concentration and Figs. 5.3(a-e) for the heat transfer coefficient. Tables 5.1 and 5.2 describe the temperature and concentration gradients. It is observed that with an increase in M the temperature increases. An increase in temperature becomes more apparent with an increase in M (see Fig. 5.1a). This fact agrees well to the expected results of Ohmic heating i.e. the temperature increases with an increase in the magnetic field strength. This fact is quite opposite to the observed results of behavior of M on temperature in the previous attempts by several other authors who considered the magnetic field but ignored the effects of Ohmic heating which in turn gives the lesser accurate prediction about the behavior of temperature for different values of M. From Figs. 5.1(b-d), the temperature increases with an increase in Schmidt, Dufour and Soret numbers but contrary to the previous case this increase is not much. Fig. 5.1(e) shows that there is rapid increase in temperature with an increase in the Brinkman number. From Figs.

5.2(a-e), we can clearly see that there is decrease in the concentration with an increase in M, Sr, Sc and Br. This decrease is more in case of Hartman number M and Brinkman number Br when compared with the other parameters. The result that concentration decreases effectively with an increase in $M$ is of considerable importance. This means that we can use the heating effect of applied magnetic field to develop small thermal gradients which in turn can be used to control the concentration of any particular substance in the fluid. This result has wide range of applications especially for cure of several diseases, for instance clearing several type of blockages in many arteries that exhibit peristaltic mechanism. The thermal gradients can be used more efficiently in place of radiations with lesser side effects. These thermal gradients can also be used for the transport of several minerals and medicine within the human body on the desired location.

Heat transfer coefficient has its own physical significance in problems related to heat transfer. It is always helpful to examine the behavior of heat transfer coefficient ($Z$ in this case) for a given problem. Keeping this in mind the Figs. 5.3(a-e) are explicitly plotted to examine how $Z$ develops throughout the flow field. In all the Figs. 5.3(a-e), Z has oscillatory behavior which is due to peristalsis. In all these Figs. the absolute value of heat transfer coefficient is positive. Fig. 5.3(a) shows a rapid fluctuation in Z for changing values of M. From Figs. 5.3(b-d), it is quite obvious that the Du, Sc and Sr effect the heat transfer coefficient but this effect is relatively smaller when compared with the effects of other parameters like Br, M and d. Since Soret and Dufour effects have gained reasonable interest in convective heat and mass transfer. One cannot neglect such effects when there is appreciable density difference in the flow regime. The concentration gradient in fact yields the thermal energy flux (referred as Dufour effect) whereas the temperature gradients contribute to the mass fluxes (referred as Soret effect). One can neglect Soret and Dufour effects in heat and mass transfer mechanism when such effects are small in comparison to the effects by Fourier's or Fick's laws. Motivated by this fact we computed the temperature and concentration gradients at $y = h$ in the Tables 5.1 and 5.2. It is noticed from Table 1 that magnitude of temperature gradient increases when $M$, Du and Br are increased. However the situation is reverse when there is an increase in a. Table 5.2 depicts that concentration gradient is an increasing function of M, Du, Br and Sr. Further the concentration gradient is decreasing function of a.
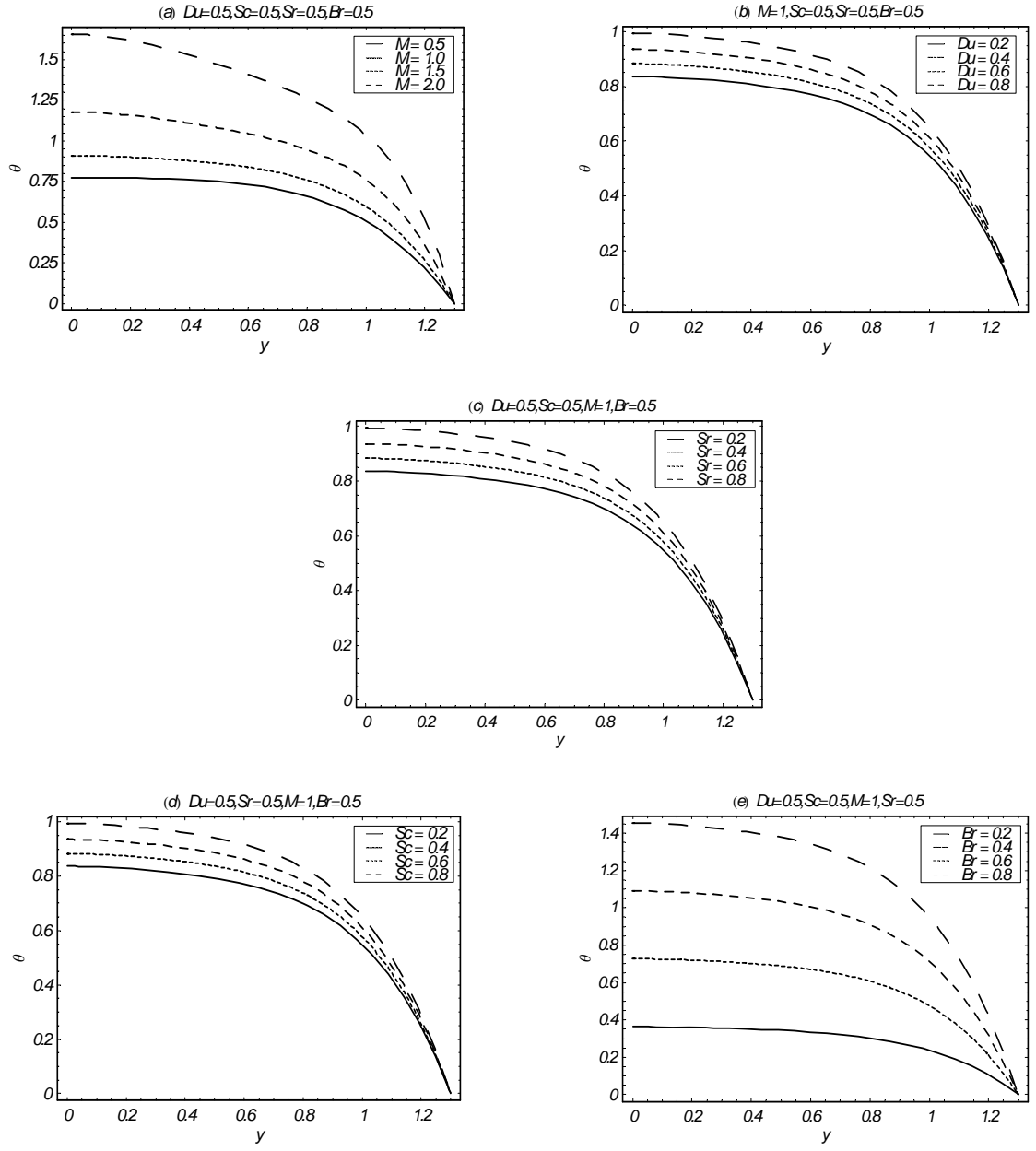
Fig. 5.1(a-e) Variation of temperature when $a = 0.3$, $\eta = 1.4$, and $x = 0$.
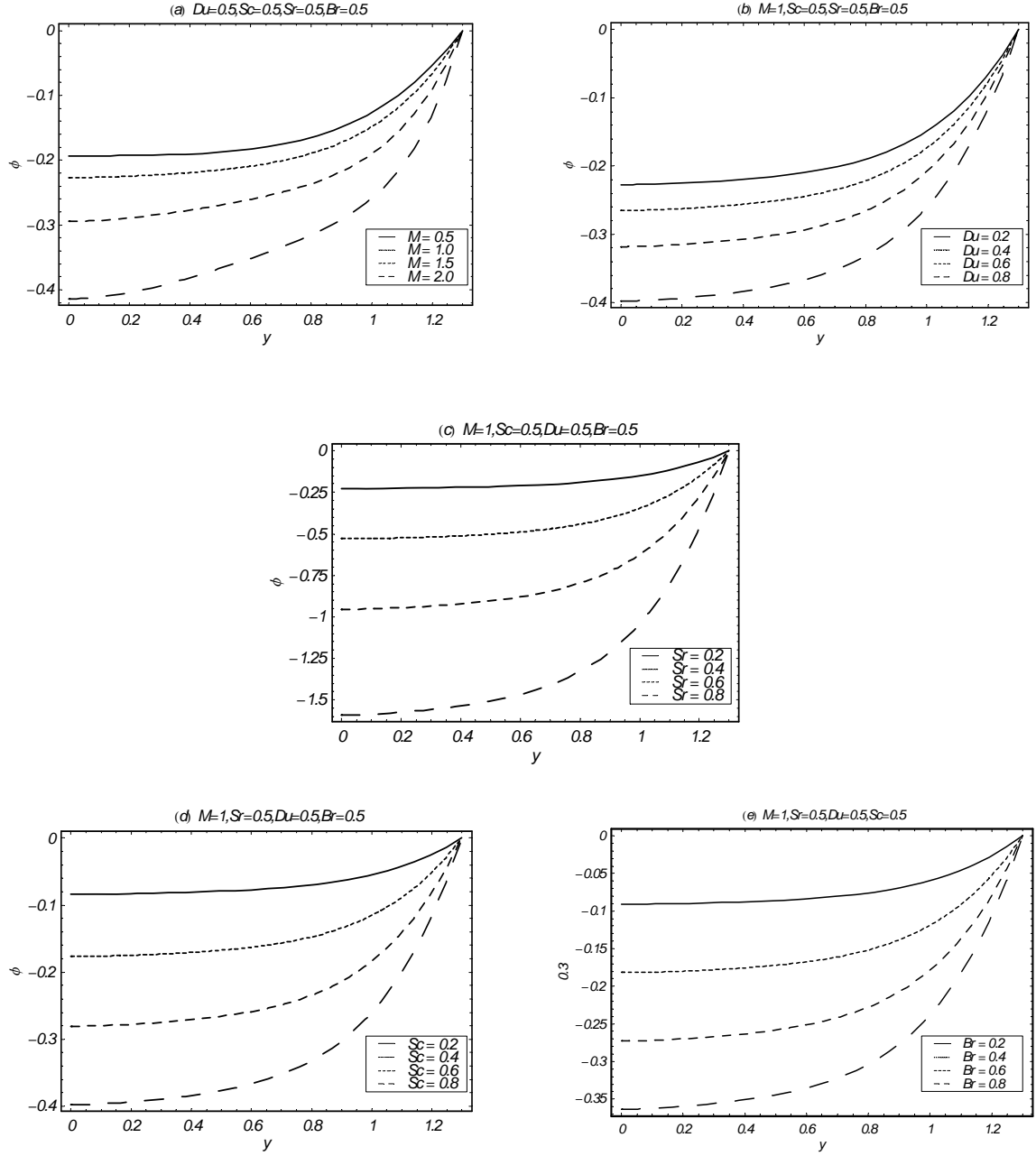
Fig. 5.2(a-e) Variation of concentration when $a = 0.3$, $\eta = 1.4$, and $x = 0$.
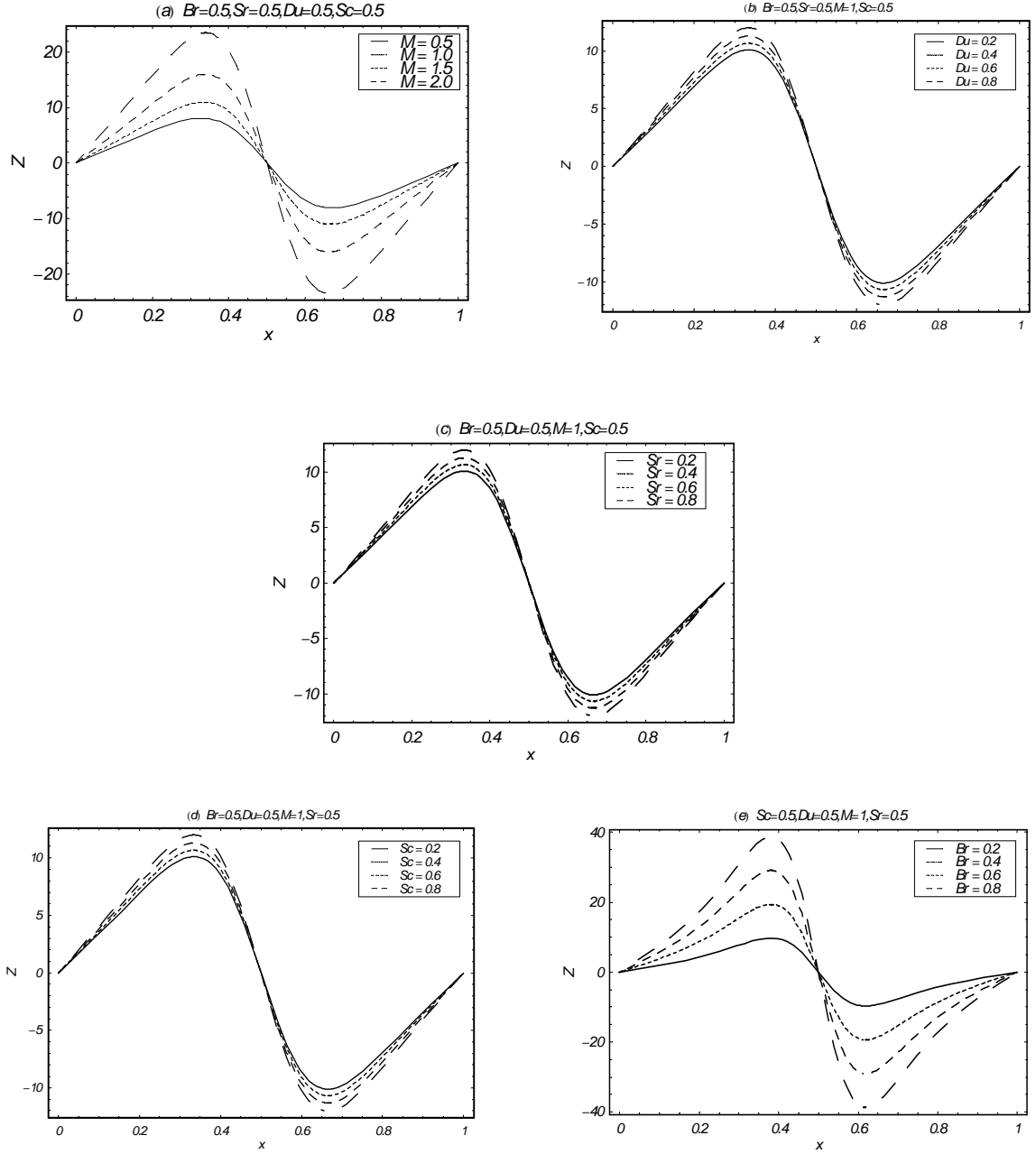
Fig. 5.3(a-e) Variation of Z when $a = 0.3$, $\eta = 1.4$, and $x = 0$.

| $M$ | $Du$ | $Br$ | $a$ | $\theta'(h)$ |
|-----|------|------|-----|--------------|
| 0.5 | 1.0 | 0.5 | 0.3 | $-2.8569$ |
| 1.0 | | | | $-3.6054$ |
| 1.5 | | | | $-5.0997$ |
| 2.0 | | | | $-7.7157$ |
| | 0.0 | | | $-2.7040$ |
| | 0.5 | | | $-3.0903$ |
| | 1.0 | | | $-3.6054$ |
| | 1.5 | | | $-4.3265$ |
| | | 0.0 | | $0.0$ |
| | | 0.5 | | $-3.0903$ |
| | | 1.0 | | $-6.1807$ |
| | | 1.5 | | $-9.2711$ |
| | | | 0.2 | $-3.3563$ |
| | | | 0.4 | $-2.5868$ |
| | | | 0.6 | $-2.11325$ |
| | | | 0.8 | $-1.81531$ |

Table 5.1: Variation of $\frac{\partial \theta}{\partial y}_{y=h}$ via different parameters.

| M | Du | Br | a | Sr | $\phi'(h)$ |
|---|---|---|---|---|---|
| 0.5 | 1.0 | 0.5 | 0.3 | 0.5 | 0.71424 |
| 1.0 | | | | | 0.90136 |
| 1.5 | | | | | 1.2749 |
| 2.0 | | | | | 1.9289 |
| | 0.0 | | | | 0.6760 |
| | 0.5 | | | | 0.7725 |
| | 1.0 | | | | 0.9013 |
| | 1.5 | | | | 1.0816 |
| | | 0.0 | | | 0.0 |
| | | 0.5 | | | 0.7725 |
| | | 1.0 | | | 1.5452 |
| | | 1.5 | | | 2.3178 |
| | | | 0.2 | | 0.8390 |
| | | | 0.4 | | 0.6467 |
| | | | 0.6 | | 0.5283 |
| | | | 0.8 | | 0.4538 |
| | | | | 0.2 | 0.2787 |
| | | | | 0.4 | 0.5753 |
| | | | | 0.6 | 0.8914 |
| | | | | 0.8 | 1.2291 |

Table 5.2: Variation of $\frac{\partial \phi}{\partial y}_{y=h}$ via different parameters.

# Chapter 6

# Soret and Dufour effects on peristaltic transport of MHD fluid with variable viscosity

This chapter looks at the Soret and Dufour effects on the magnetohydrodynamic (MHD) peristaltic flow of variable viscosity fluid in a symmetric channel. Modeling is presented in the presence of Ohmic heating. Mathematical analysis is carried out through large wavelength and small Reynolds number. Results for the stream function, temperature and concentration are developed. The variations of sundry parameters on the flow quantities of interest are sketched and examined.

## 6.1   Mathematical analysis

Let us investigate the magnetohydrodynamic flow of an incompressible viscous fluid in a channel with width $2d_1$. The $\overline{X}-$ axis is chosen along the walls of channel and $\overline{Y} -$ axis is taken normal to the $\overline{X}-$ axis. A constant magnetic field of strength $\mathbf{B}_0$ is exerted in the $\overline{Y} -$ direction. Induced magnetic field is not accounted. Sinusoidal wave propagates along the channel walls with constant wave speed $c$ represented by

$$H(\overline{X}, \overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right). \qquad (6.1)$$

If $(\overline{U}, \overline{V})$ and $(\overline{u}, \overline{v})$ are the velocity components in the laboratory $(\overline{X}, \overline{Y})$ and wave $(\overline{x}, \overline{y})$ frames then transformations between laboratory and wave frames are

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}). \tag{6.2}$$

In above expressions $a_1$ is the wave amplitude, $\lambda$ is the wavelength, $\overline{t}$ is the time and $\overline{P}$ and $\overline{p}$ are the pressures in the laboratory and wave frames respectively. Introducing the variables in the forms

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d_1}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c\delta}, \quad \delta = \frac{d_1}{\lambda}, \quad h = \frac{H}{a}, \quad a = \frac{a_1}{d_1}, \quad p = \frac{d_1^2 \overline{p}}{c\lambda\mu_0}, \\
\theta &= \frac{T - T_0}{T_0}, \quad \phi = \frac{C - C_0}{C_0}, \quad M = \left(\frac{\sigma}{\mu_0}\right)^{1/2} B_0 d_1, \quad \boldsymbol{v} = \frac{\mu_0}{\rho}, Re = \frac{\rho c d_1}{\mu_0}, \\
t &= \frac{c\overline{t}}{\lambda}, u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \quad Br = \Pr E, \quad Du = \frac{DC_0 K_T}{C_s C_p \mu_0 T_0}, \quad Sr = \frac{\rho D K_T T_0}{\mu_0 T_m C_0}, \\
Sc &= \frac{\mu_0}{\rho D}, \quad E = \frac{c^2}{C_p T_0}, \quad \Pr = \frac{\mu C_p}{K}, \text{ and } \mu(y) = \frac{\bar{\mu}(\bar{y})}{\mu_0}.
\end{aligned}
\tag{6.3}
$$

the governing equations after adopting long wavelength and low Reynolds number approximation become

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - M^2\left(\frac{\partial\psi}{\partial y} + 1\right), \tag{6.4}$$

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - M^2\frac{\partial\psi^2}{\partial y^2} = 0, \tag{6.5}$$

$$0 = \frac{\partial^2\theta}{\partial y^2} + Br\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + BrM^2\left(\frac{\partial\psi}{\partial y} + 1\right)^2 + \Pr Du\frac{\partial^2\phi}{\partial y^2}, \tag{6.6}$$

$$0 = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} + Sr\frac{\partial^2\theta}{\partial y^2}, \tag{6.7}$$

where $p$ is the pressure, $\overline{C}$ the concentration field, $\overline{T}$ the temperature field, $D$ mass diffusivity, $K_T$ the thermal diffusion ratio, $C_p$ the specific heat, $C_s$ the concentration susceptibility, $\mu(y)$ the variable viscosity, $T_m$ the fluid mean temperature, $\boldsymbol{v}$ the kinematic viscosity, $M$ the Hartman number, Re the Reynolds number, $Br$ the Brinkman number, $Du$ the Dufour parameter, $Sr$ the Soret parameter, $Sc$ the Schmidt number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the

wave number, $\psi$ the stream function, $\theta$ the dimensionless temperature and $\phi$ the concentration. The boundary conditions are

$$\psi = 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0, \quad \frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = 0, \quad at \ \ y = 0,$$

$$\psi = F, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \phi = 0, \quad at \ \ y = h, \tag{6.8}$$

$$h(x)=1 + a\cos(2\pi x), \quad F = \int_0^h \frac{\partial \psi}{\partial y} dy, \tag{6.9}$$

Pressure rise per wavelength $\Delta p_\lambda$ is

$$\Delta p_\lambda = \int\limits_0^1 \frac{dp}{dx} dx. \tag{6.10}$$

The dimensionless expression of space dependent viscosity is

$$\mu(y) = e^{-\alpha y} = 1 - \alpha y \qquad \alpha \ll 1,$$

where "$\alpha$" is the viscosity parameter.

We look for solutions in the series form represented below

$$\psi = \psi_0 + \alpha\psi_1 + \dots.$$

$$F = F_0 + \alpha F_1 + \dots.$$

$$p = p_0 + \alpha p_1 + \dots.$$

Employing the procedure of perturbation method and retaining the results up to order $o(\alpha)$ we have

$$\psi = A_1 + \frac{(F + h)\alpha A_2}{8\left(-hM\cosh\left(hM\right) + \sinh\left(hM\right)\right)^2}, \tag{6.11}$$

$$\theta = -\frac{BrM^2 B_1}{8\left(-1 + \Pr ScSrDu\right)\left(hM\cosh\left(hM\right) - \sinh\left(hM\right)\right)^2} \tag{6.12}$$
$$-\frac{BrM\alpha\left[B_2 + B_3 + B_4 - B_5 + B_6 - B_7 + B_8\right]}{32\left(-1 + \Pr ScSrDu\right)\left(hM\cosh\left(hM\right) - \sinh\left(hM\right)\right)^3},$$

74

$$\phi = -\frac{BrM^2 ScSr[C_1 - C_2 + C_3]}{8(-1 + \text{Pr } ScSrDu)(hM\cosh(hM) - \sinh(hM))^2}$$

$$+\frac{BrMScSr\alpha\left[C_4 + C_5 + 2\left(C_6 + C_7 + \frac{1}{4}M(C_8 + C_9 - C_{10}) - C_{11} - C_{12} + C_{13}\right)\right]}{64(-1 + \text{Pr } ScSrDu)(hM\cosh(hM) - \sinh(hM))^3}. \qquad (6.13)$$

Heat transfer coefficient is defined as follows:

$$Z = h_x \theta_y.$$

The involved $A_i(i = 1, 2)$, $B_j(j = 1 - 8)$ and $C_k(k = 1 - 13)$ are

$A_1 = \frac{FMy\cosh(hM)+y\sinh(hM)-(F+h)\sinh(My)}{hM\cosh(hM)-\sinh(hM)}$, $A_2 = yA_3 + (hMy\cosh(hM) + (h^3M^2 - y)$

$\sinh(hM))^2 \sinh(My)$, $A_3 = -1 + 2h^2M^2 + \cosh(2hM) + 2My\cosh(My)(-hM$

$\cosh(hM) + \sinh(hM)) - 2hM\sinh(2hM)$, $B_1 = -8F - 2h(4+h) - 2F + h^2(F+2h)$

$M^2 - 2h^2(F+h)^2 M^4 + 2(1 + F(F+2h)M^2 + (F+h)^2 M^4)y^2 + (F^2(1+M^2)$

$+2F(4+h+hM^2) - 2y^2 + h(8 + h(3 + M^2(1 + 2h^2 - 2y^2)))) \cosh(2hM) - (F+h)^2$

$(1+M^2)\cosh(2My) - 4hM(2F + h(2+h) - y^2)\sinh(2hM) + 16h(F+h)M$

$\cosh(hM)(M(h-y) + \sinh(My)) - 16(F+h)\sinh(hM)(M(h-y) + \sinh(My))$,

$B_2 = -8h^3(F+h)(h-y)\sinh(2hM)M^4 - \frac{1}{2}(F+h)B_9\cosh(hM) - 16h^4(F+h)$

$M^4\cosh(hM)^3 + 8h^3(F+h)M^4(h-y)\cosh(2hM)$, $B_3 = \frac{1}{2}(F+h)(3(F+h) + 3$

$(F + h - 8h^2)M^2 + 8h^4M^4)\cosh(3hM) + 16h^2(F+h)M^4y^2\cosh(hM)^2\cosh(My)$

$-2h(F+h)^2 M^2(1+M^2)y\cosh(hM)\cosh(2My)$, $B_4 = (F+h)MB_{10}\sinh(hM)$,

$B_5 = 64F1h^2M^3\cosh(hM)^2\sinh(hM) + 2h^3(F+h)^2 M^3(1+M^2)\cosh(2My)$

$\sinh(hM) + 2(F+h)^2 M(1+M^2)y\cosh(2My)\sinh(hM) + 16(F+h)M^2y^2\cosh(My)$

$\sinh(hM)^2 - 8h^4(F+h)M^5(h-y)\sinh(2hM)$, $B_6 = hMB_{11}\cosh(hM)\sinh(2hM)$,

$B_7 = hM(-4F + F^2 - 4h + 2Fh + h^2 + (F+h)^2(1+h^2)M^2 + h^2(F+h)^2 M^4)$

$\sinh(3hM) - 16h^2(F+h)M^3y\cosh(hM)^2\sinh(My) + 16h^3(F+h)M^3\sinh(hM)^2$

$\sinh(My) - 16(F+h)My\sinh(hM)^2\sinh(My) - 8h^4(F+h)M^4\sinh(2hM)\sinh(My)$,

$B_8 = 16h(F+h)M^2y\sinh(2hM)\sinh(My) + (F+h)^2(1+M^2)(-3 + 2M^2y^2)$

$\sinh(hM)\sinh(2My) - h(F+h)M\cosh(hM)B_{12}$, $B_9 = F(1+M^2)(3 + 4hM^2(-3h$

$+2h^3M^2 + 2y - 2M^2y^3)) + h(3 + M^2(3 + 4h(h(-3 + M^2(-3 + 2h(1 + h + hM^2)))$

$+2(-3+y) + 2M^2y - 2M^2(1+M^2)y^3)))$, $B_{10} = -3h(4+F+h) + h(F(-3+5h^2)$

$+h(-3 + h(16 + 5h)))M^2 + h^3(F+h)(5 + 4h^2)M^4 + 4h^5(F+h)M^6 + 4(F+h)$

$(1+M^2)y - 4h^3(F+h)M^4(1+M^2)y^2 - 4(F+h)M^2(1+M^2)y^3$, $B_{11} = -3(F+h)^2$

$+(-3F^2 - 6Fh + 32f1h + (-3 + 2F(8+F))h^2 + 4(4+F)h^3 + 2h^4)M^2 + 2h^2(F+h)^2$

$M^4$, $B_{12} = 16M^2y^2\sinh(M(h-y)) + (F+h)(1+M^2)(-3 + 2M^2y^2)\sinh(2My) + 16$

$M^2y^2\sinh(M(h+y))$, $C_1 = 2C_{14} - 16h(F+h)M^2(h-y)\cosh(hM) - (F^2(1+M^2)$

$+2F(4+h+hM^2) - 2y^2 + h(8 + h(3 + M^2(1 + 2h^2 - 2y^2)))) \cosh(2hM)$, $C_2 = 8F$

$\cosh(M(h-y)) - 8h\cosh(M(h-y)) + F^2\cosh(2My) + 2Fh\cosh(2My) + h^2$

$\cosh(2My) + F^2M^2\cosh(2My) + 2FhM^2\cosh(2My) + h^2M^2\cosh(2My) + 8(F+h)$

$\cosh\left(M\left(h+y\right)\right)+16FhM\sinh\left(hM\right)+16h^{2}M\sinh\left(hM\right)-16FMy\sinh\left(hM\right)-16hMy$

$\sinh\left(hM\right),C_{3}=8FhM\sinh\left(2hM\right)+8h^{2}M\sinh\left(2hM\right)+4h^{3}M\sinh\left(2hM\right)-4hMy^{2}$

$\sinh\left(2hM\right)+8FhM\sinh\left(M\left(h-y\right)\right)+8h^{2}M\sinh\left(M\left(h-y\right)\right)-8h\left(F+h\right)M$

$\sinh\left(M\left(h+y\right)\right),C_{4}=\left(3\left(F+h\right)^{2}+\left(3F^{2}+6F\left(1-4h\right)h+h\left(64f1+3\left(1-8h\right)h\right)\right)\right.$

$\left.M^{2}\right)\cosh\left(3hM\right),C_{5}=2M\cosh\left(hM\right)^{2}C_{15},C_{6}=-8h^{3}\left(F+h\right)M^{4}\left(h-y\right)+2\left(F+h\right)^{2}$

$M\left(1+M^{2}\right)\left(h^{3}M^{2}+y\right)\cosh(My),C_{7}=2My\sinh\left(hM\right)+16\left(F+h\right)M^{2}y^{2}\cosh\left(My\right)$

$\sinh\left(hM\right)^{2},C_{8}=64F1hM\cosh\left(hM\right)+32h^{3}\left(F+h\right)M^{3}\left(h-y\right)\cosh\left(2hM\right)-64F1h$

$M\cosh\left(3hM\right),C_{9}=C_{16}\sinh\left(hM\right),C_{10}=32h^{4}\left(F+h\right)M^{4}\left(h-y\right)\sinh\left(2hM\right)+(-h$

$\left(F+h\right)\left(-8+5F+5h+\left(5F+\left(5-16h\right)h\right)M^{2}\right)+8F1\left(4+F+h+\left(F+h-4h^{2}\right)M^{2}\right))$

$\sinh\left(3hM\right),C_{11}=4MF1\left(4-4h^{2}M^{2}\right)+4M\left(F+h\right)\left(-3y+h^{2}M^{2}\left(3h+y\right)\right)+4M(F1$

$\left(4-4h^{2}M^{2}\right)+4M\left(F+h\right)\left(y+h^{2}M^{2}\left(-h+y\right)\right))\cosh\left(2hM\right)+4M2h\left(F+h\right)M(h^{3}M^{2}$

$-2y)\sinh\left(2hM\right)\sinh\left(My\right)+4M\left(F+h\right)^{2}\left(1+M^{2}\right)\left(-3+2M^{2}y^{2}\right)\sinh\left(hM\right)\sinh\left(2My\right),$

$C_{12}=\cosh\left(hM\right)\left(F+h\right)\left(F\left(1+M^{2}\right)\left(3+4hM^{2}\left(-3h+2h^{3}M^{2}+2y-2M^{2}y^{3}\right)\right)\right.$

$+\cosh\left(hM\right)h(3+M^{2}(3+4h^{2}(-3+M^{2}\left(-3+2h\left(4+h+hM^{2}\right)\right)+2\cosh\left(hM\right)\left(-3+y\right)$

$+2M^{2}y-2M^{2}\left(1+M^{2}\right)y^{3}))),C_{13}=64F1hM^{2}\cosh\left(2hM\right)+4h\left(F+h\right)^{2}M^{2}\left(1+M^{2}\right)$

$y\cosh\left(2My\right)+2hM\left(-32F1hM^{2}\sinh\left(2hM\right)+\left(F+h\right)C_{18}\right),C_{14}=h\left(4+h+h^{3}M^{4}\right)$

$-\left(1+h^{2}M^{4}\right)y^{2}+F^{2}M^{2}\left(1+M^{2}\right)\left(h-y\right)\left(h+y\right)+2F(2+hM^{2}\left(1+M^{2}\right)\left(h-y\right)\left(h+y\right)),$

$C_{15}=16h^{2}\left(F+h\right)M^{3}y^{2}\cosh\left(My\right)-\left(h\left(F+h\right)\left(-8+5F+5h+\left(5F+\left(5-16h\right)h\right)M^{2}\right)\right.$

$+8F1\left(4+F+h+\left(F+h+4h^{2}\right)M^{2}\right))\sinh\left(hM\right)-8((4f1\left(-1+h^{2}M^{2}\right)+\left(F+h\right)\left(y+h^{2}\right.$

$\left.M^{2}\left(-h+y\right)\right))))\sinh\left(My\right),C_{16}=8F1\left(4+F+h+\left(F+h-4h^{2}\right)M^{2}\right)+\left(F+h\right)C_{17},$

$C_{17}=h(-56+F\left(1+M^{2}\right)\left(-13+24h^{2}M^{2}+16h^{4}M^{4}\right)+h(-13+M^{2}(-13+8h(10+h$

$\left(1+M^{2}\right)\left(3+2h^{2}M^{2}\right)))))+16\left(F+h\right)\left(1+M^{2}\right)y-16h^{3}\left(F+h\right)M^{4}\left(1+M^{2}\right)y^{2}-16$

$\left(F+h\right)M^{2}\left(1+M^{2}\right)y^{3},C_{18}=16M^{2}y^{2}\sinh\left(M\left(h-y\right)\right)+\left(F+h\right)\left(1+M^{2}\right)\left(-3+2M^{2}y^{2}\right)$

$\sinh\left(2My\right)+16M^{2}y^{2}\sinh\left(M\left(h+y\right)\right).$

## 6.2   Discussion

This section describes the impacts of pertinent parameters on the temperature and concentration. We recall that theme of this study is to point out the influences of Soret and Dufour. Therefore, the results of various parameters associated with velocity are not included. In order to achieve the desired objective, we present the plots in such a way that the left panels are for

constant viscosity (i.e. for $\alpha = 0$) and the right ones are for variable viscosity ($\alpha = 0.4$). It is observed in all the graphs related to temperature that the rise in temperature for variable viscosity fluid is relatively slower when compared with fluid having constant viscosity. Here Figs. 6.1-6.5 are drawn for temperature whereas the Figs. 6.6-6.10 show variation of concentration. Temperature increases with an increase in M (Fig. 6.1). Further an increase in temperature is abrupt in view of Ohmic heating. The variations of Du, Sr, Sc and Br on the temperature are displayed in the Figs. 6.2-6.5. These Figs. indicate that there is an increase in temperature by increasing Du, Sr, Sc and Br. It is also found that an increase in temperature is more for Br when compared with the other parameters.

Figs. 6.6-6.10 are presented to examine the behavior of embedded parameters on the concentration. Increase in concentration is observed when M and Du are increased (see Figs. 6.6 & 6.7). Concentration also increases when Sr, Sc and Br are increased (Figs. 6.8-6.10).

Behavior of heat transfer coefficient Z for various parameters is shown in the Figs. 6.11-6.15. As expected Z shows an oscillatory behavior which is because of peristalsis. It is also noted that there is no variation in Z for amplitude ratio ($d$) between 0 and 0.3. It is observed from Fig. 6.11 that Z increases when M is increased. The absolute values of Z in variable viscosity fluid are more than the constant viscosity fluid. Figs 6.12-6.15 show that Z increases with the increase in Du, Sr, Sc and Br. Effects of Du, Sr, Sc and Br on Z are opposite to that of M.
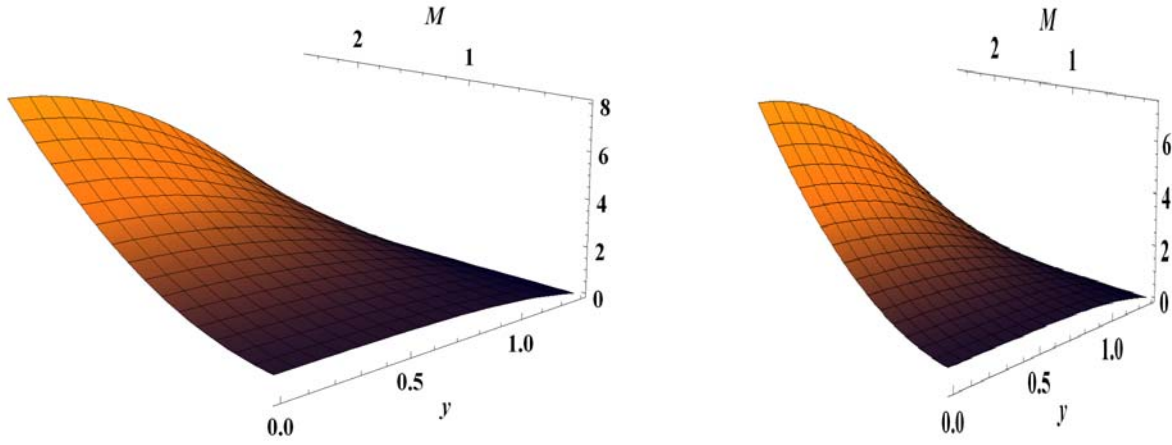
Fig. 6.1 Effect of M on $\theta$ when $Du = 0.5$, $Sr = 0.5$, $Sc = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
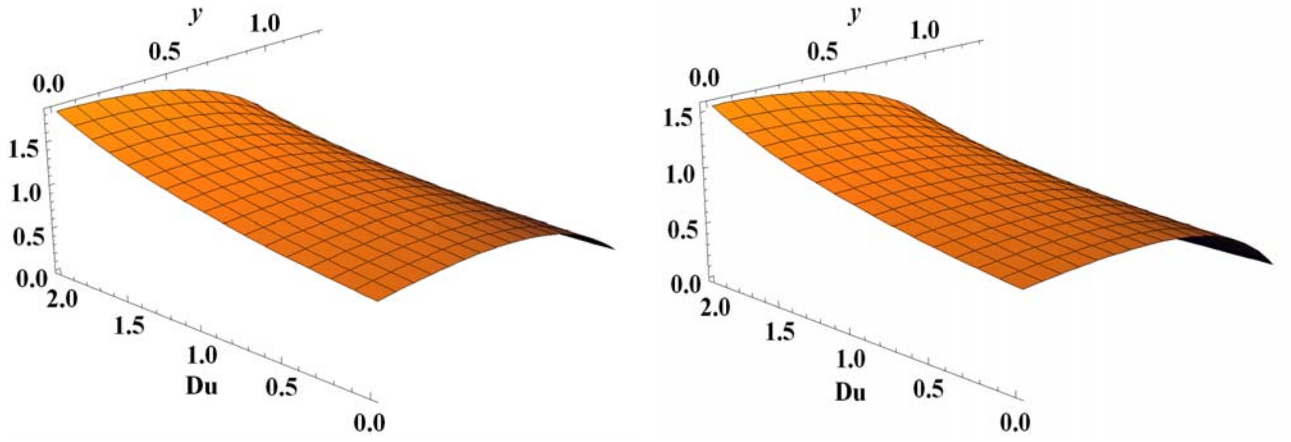


Fig. 6.2 Effect of Du on $\theta$ when $M = 0.5$, $Sr = 0.5$, $Sc = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
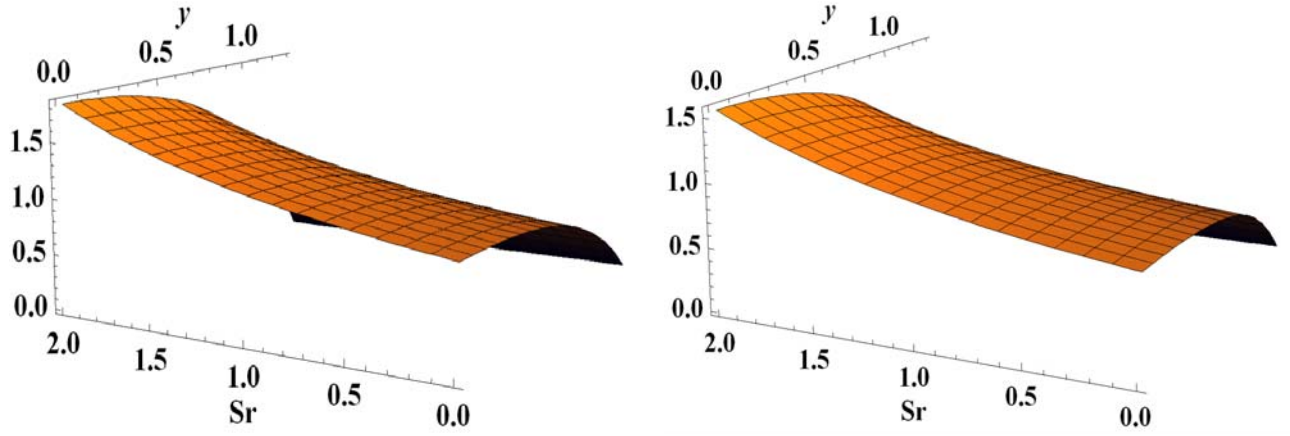
Fig. 6.3 Effect of Sr on $\theta$ when $Du = 0.5$, $M = 0.5$, $Sc = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
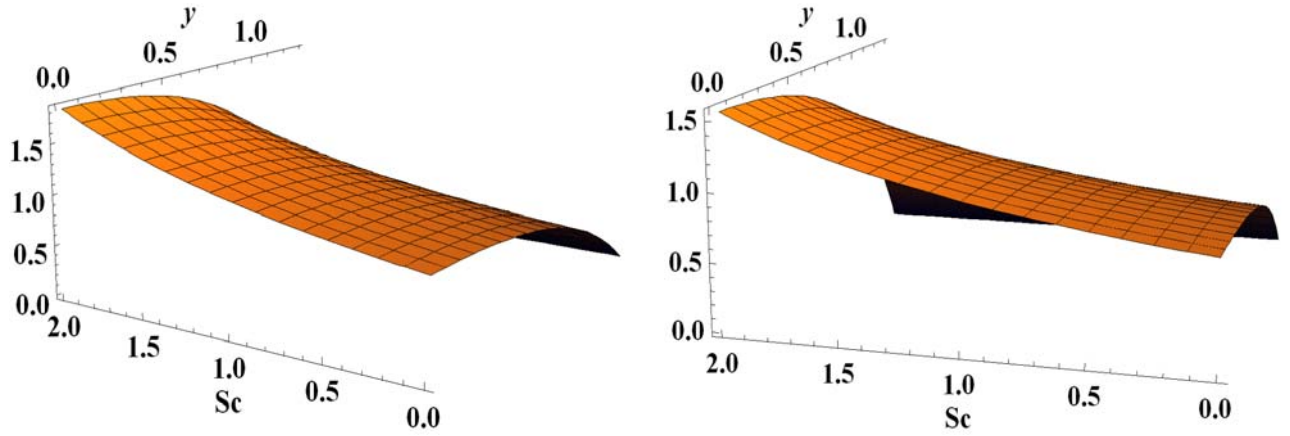


Fig. 6.4 Effect of Sc on $\theta$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
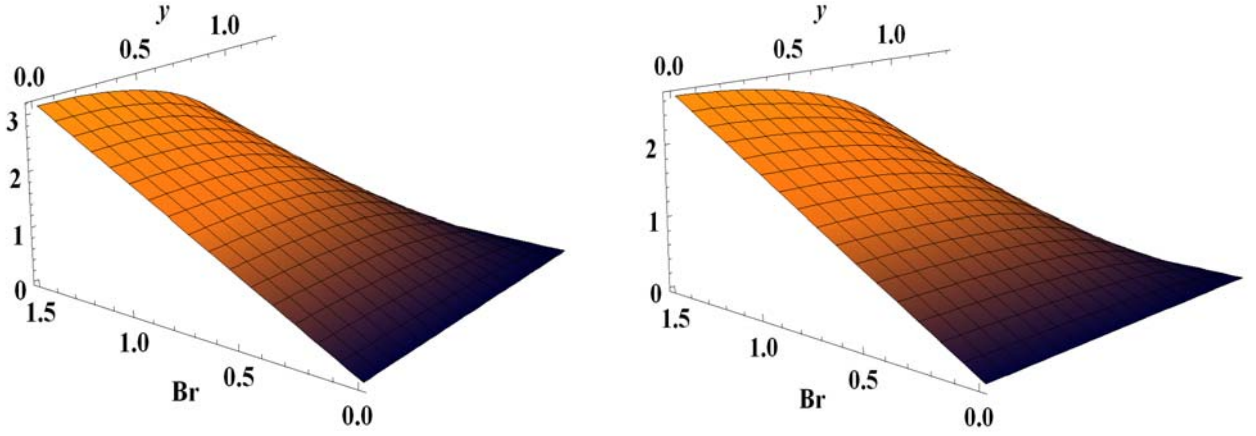
Fig. 6.5 Effect of Br on $\theta$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Sc = 0.5$ and $x = 0$.
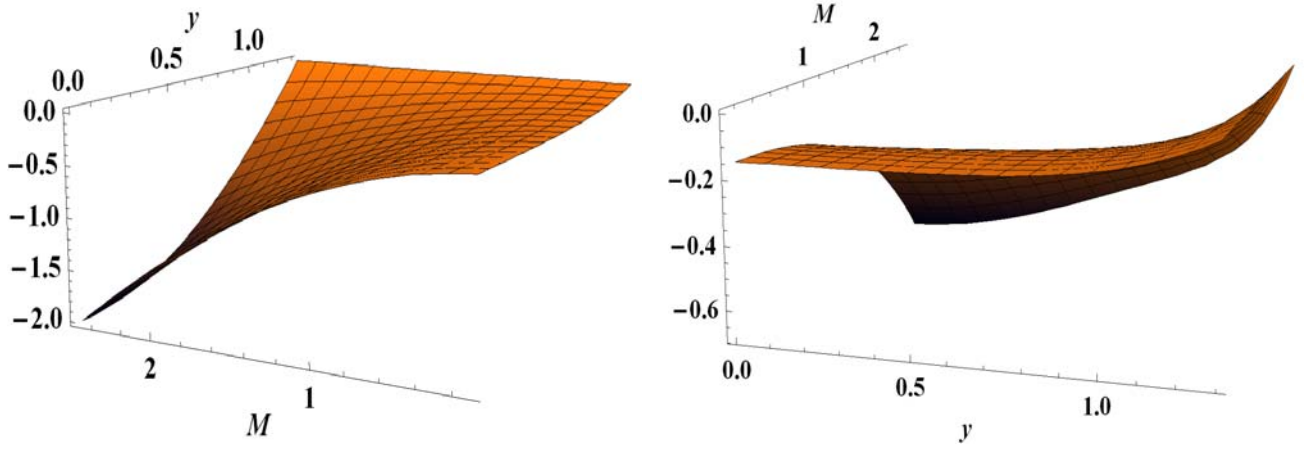


Fig. 6.6 Effect of M on $\phi$ when $Du = 0.5$, $Sr = 0.5$, $Sc = 0.5$, $d = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
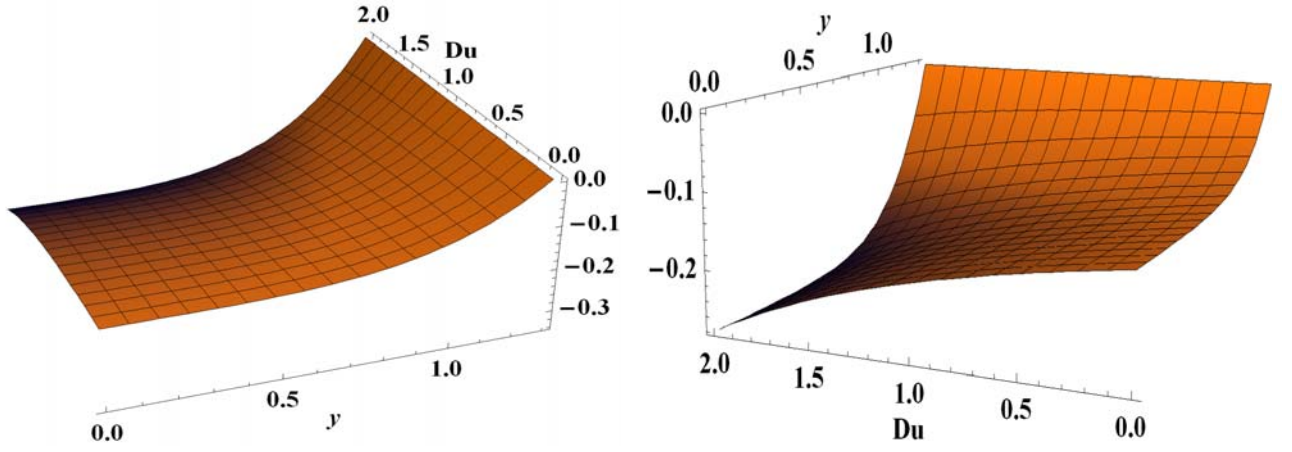
Fig. 6.7 Effect of Du on $\phi$ when $M = 0.5$, $Sr = 0.5$, $Sc = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
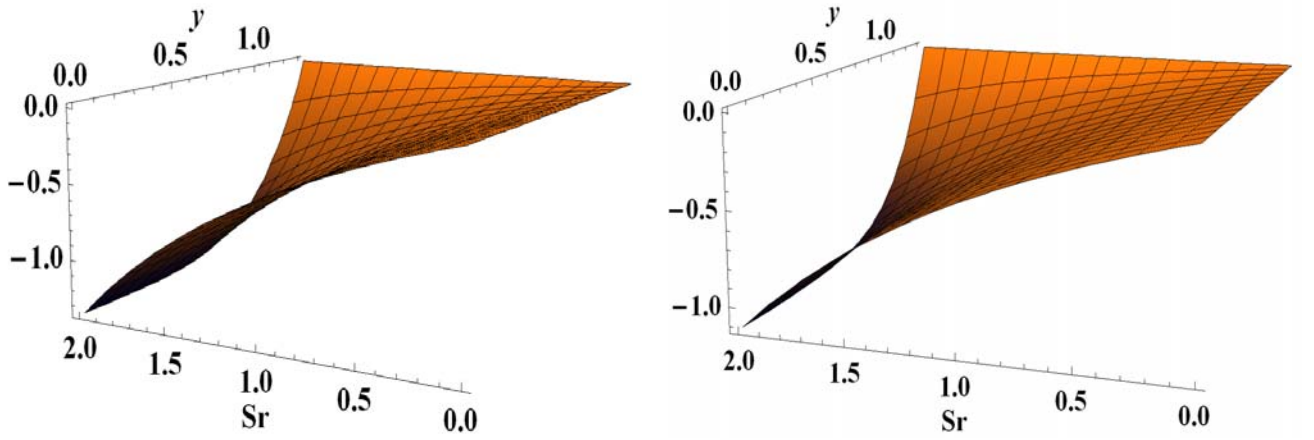


Fig. 6.8 Effect of Sr on $\phi$ when $Du = 0.5$, $M = 0.5$, $Sc = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Br = 0.5$ and $x = 0$.
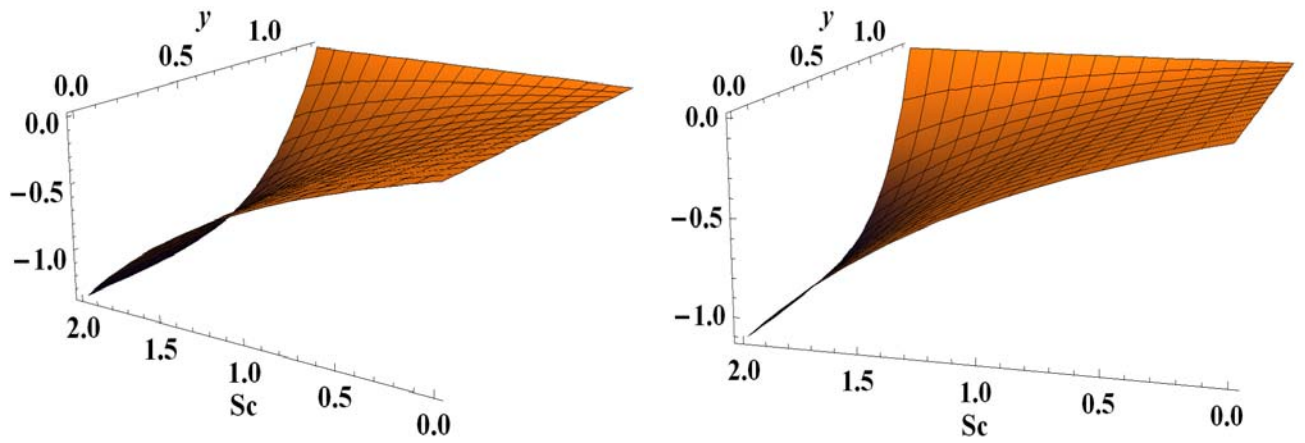
Fig. 6.9 Effect of Sc on $\phi$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$, $Br = 0.5$ and $x = 0$.
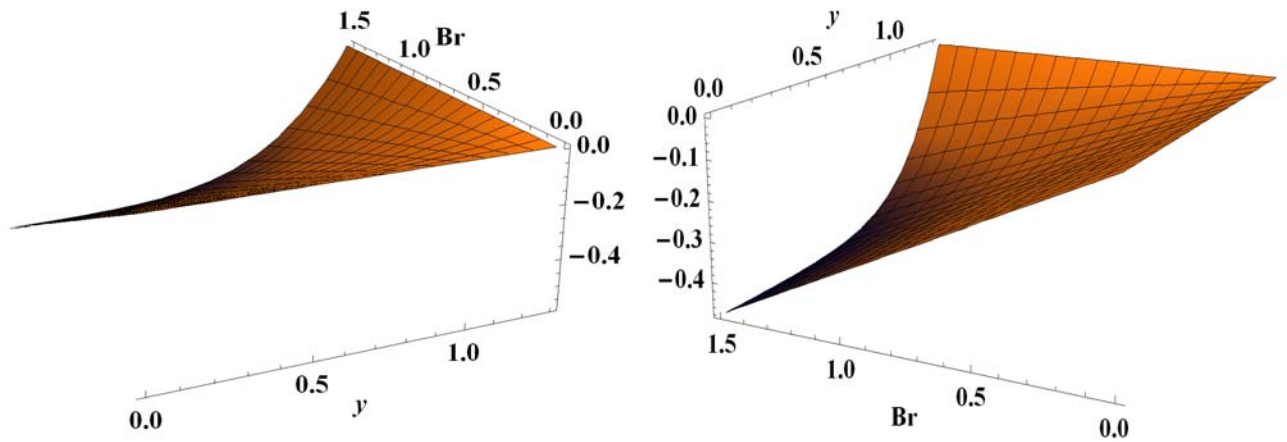


Fig. 6.10 Effect of Br on $\phi$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ , $Sc = 0.5$ and $x = 0$.
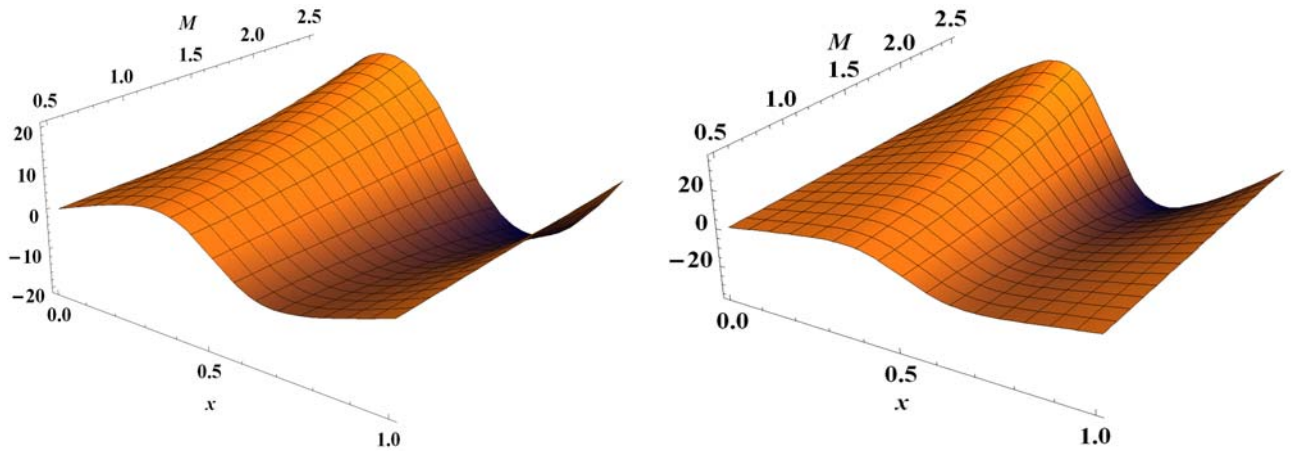
Fig. 6.11 Effect of M on $Z$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ and $Br = 0.5$ and $x = 0$.
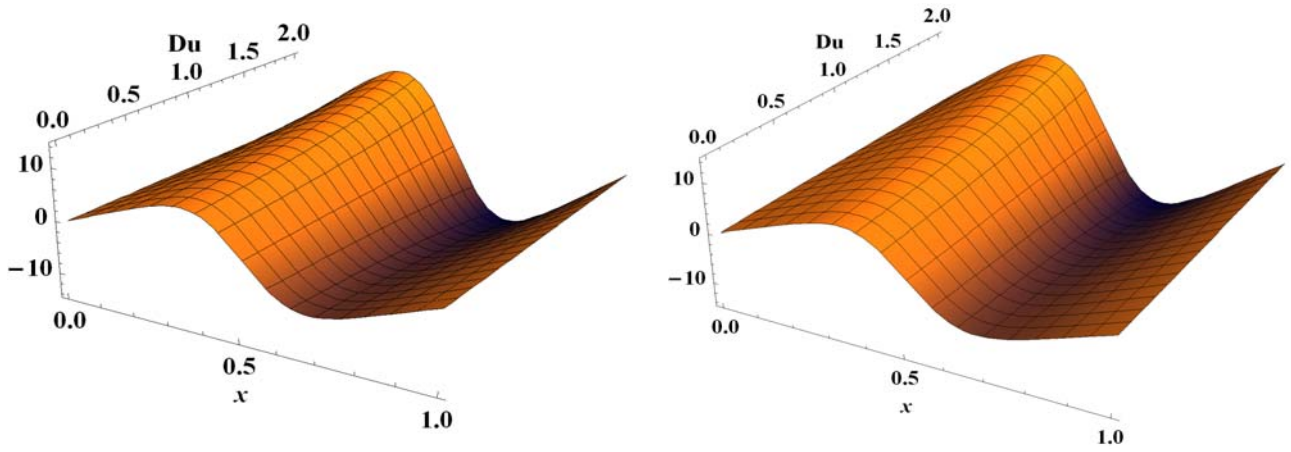
Fig. 6.12 Effect of Du on $Z$ when $Sc = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ and $Br = 0.5$.
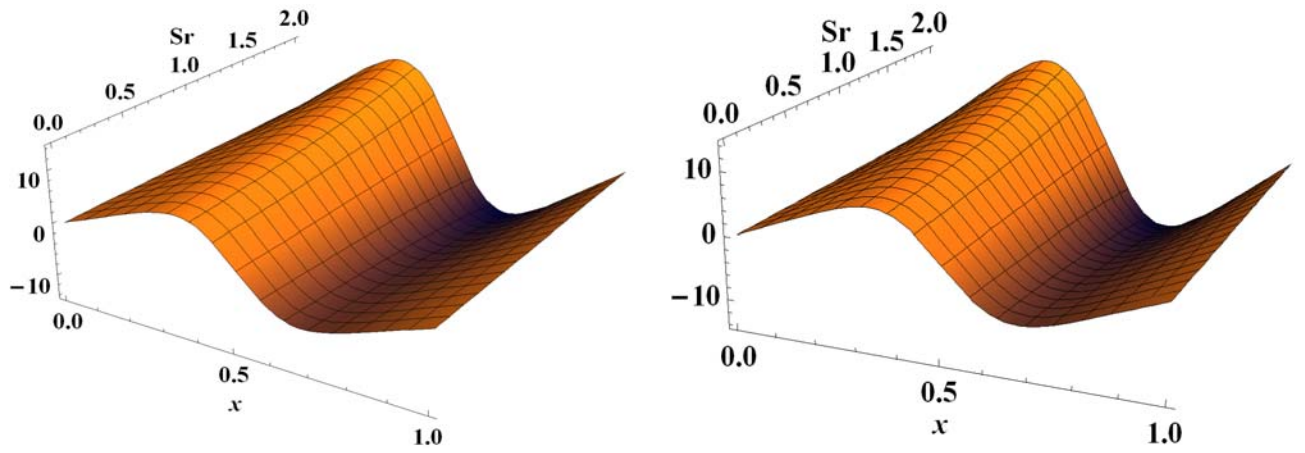
Fig. 6.13 Effect of Sr on $Z$ when $Sc = 0.5$, $Du = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ and $Br = 0.5$.
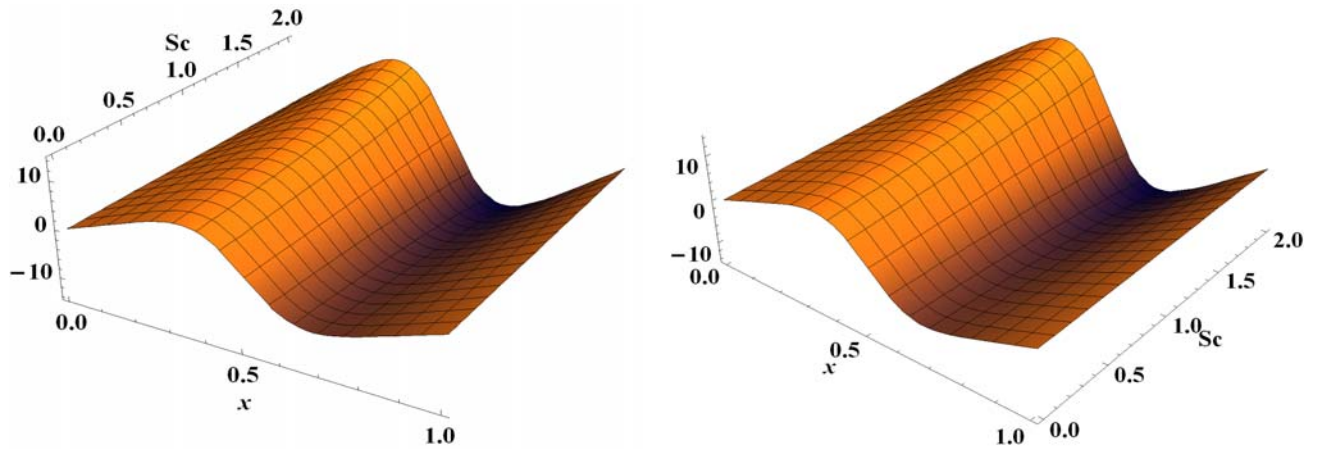


Fig. 6.14 Effect of Sc on $Z$ when $Du = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ and $Br = 0.5$.
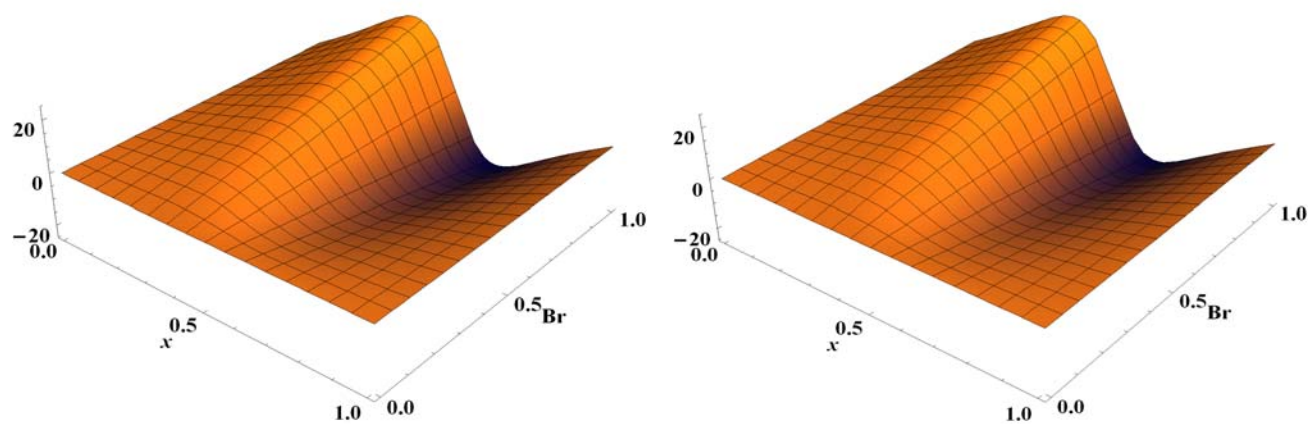
Fig. 6.15 Effect of Br on $Z$ when $Sc = 0.5$, $Sr = 0.5$, $M = 0.5$, $a = 0.3$, $\eta = 1.4$ and $Du = 0.5$.

# Chapter 7

# Soret and Dufour effects on peristaltic flow in an asymmetric channel

This chapter studies the combined effects of heat and mass transfer on the peristaltic flow of magnetohydrodynamic (MHD) fluid in an asymmetric channel. Asymmetry in channel is induced because of the wave amplitude and phases. Analysis has been performed in the presence of slip conditions. Attention has been given to the Soret and Dufour effects. The obtained results are plotted and discussed.

## 7.1 Mathematical analysis

We consider MHD flow of incompressible fluid in an asymmetric channel. The $\overline{X}$-axis is selected along the channel walls and $\overline{Y}-$axis perpendicular to the $\overline{X}$-axis. The flow is due to the sinusoidal wave trains travelling on the walls of the channel with speed $c$. The geometry of the waves is given below

$$H_1(\overline{X}, \overline{t}) \;=\; d_1 + a_1 \cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right), \tag{7.1}$$

$$H_2(\overline{X}, \overline{t}) \;=\; -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t}) + \gamma\right), \tag{7.2}$$

in which $a_1$ and $b_1$ are the amplitudes of the waves travelling on the upper and lower walls respectively, $\gamma$ is the phase difference of two waves and $d_1 + d_2$ is the width of the channel. The phase difference $\gamma$ ranges $0 \leq \gamma \leq \pi$. Here $\gamma = 0$ corresponds to the symmetric channel with waves out of phase and for $\gamma = \pi$ the waves are in phase. Moreover $d_1$, $d_2$, $a_1$, $b_1$ and $\gamma$ satisfy the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos(\gamma) \leq (d_1 + d_2)^2.$$

A uniform magnetic field $\mathbf{B}_0$ is applied in the transverse direction to the flow. Effects of induced magnetic field are not considered by choosing low magnetic Reynolds number. However, Joule heating and Soret and Dufour effects are taken into account. The equations of mass, momentum, energy and concentration in the laboratory frame $(\overline{X}, \overline{Y})$ are

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{7.3}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial}{\partial \overline{X}}\left(2\mu\frac{\partial \overline{U}}{\partial \overline{X}}\right) + \frac{\partial}{\partial \overline{Y}}\left[\mu\left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}}\right)\right] - \sigma B_0^2 \overline{U}, \tag{7.4}$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \frac{\partial}{\partial \overline{X}}\left[\mu\left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}}\right)\right] + \frac{\partial}{\partial \overline{Y}}\left(2\mu\frac{\partial \overline{V}}{\partial \overline{Y}}\right), \tag{7.5}$$

$$\rho C_p\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)T = K\left[\frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2}\right]$$
$$+ \mu\left[2\left\{\left(\frac{\partial \overline{U}}{\partial \overline{X}}\right)^2 + \left(\frac{\partial \overline{V}}{\partial \overline{Y}}\right)^2\right\} + \left(\frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}}\right)^2\right] + \sigma B_0^2 \overline{U}^2 + \frac{DK_T}{C_s}\left[\frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2}\right], \tag{7.6}$$

$$\left[\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right]C = D\left[\frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2}\right] + \frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2}\right], \tag{7.7}$$

where $\overline{U}, \overline{V}$ are the velocity components in the $\overline{X}$ and $\overline{Y}$ directions respectively, $\overline{P}$ the pressure, $T$ the temperature, $C$ the concentration, $\rho$ the density of fluid, $\mu$ the dynamic viscosity, $C_p$ the specific heat, $K$ thermal conductivity, $T_m$ fluid mean temperature, $D$ the mass diffusivity, $C_s$ the concentration susceptibility and $K_T$ the thermal diffusion ratio. Using the following transformations in order to transform our problem in to wave frame $(\overline{x}, \overline{y})$

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}). \tag{7.8}$$

We define the following dimensionless quantities

$$x = \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d_1}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c\delta}, \quad \delta = \frac{d_1}{\lambda}, \quad h_1 = \frac{H_1}{d_1}, \quad h_2 = \frac{H_2}{d_1}, \quad d = \frac{d_2}{d_1}, \quad p = \frac{d_1^2 \overline{p}}{c\lambda\mu},$$

$$\theta = \frac{T - T_0}{T_1 - T_0}, \phi = \frac{C - C_0}{C_1 - C_0}, M = \left(\frac{\sigma}{\mu}\right)^{1/2} B_0 d_1, \boldsymbol{v} = \frac{\mu}{\rho}, Re = \frac{\rho c d_1}{\mu}, \quad a = \frac{a_1}{d_1}, \quad b = \frac{b_1}{d_1},$$

$$t = \frac{c\overline{t}}{\lambda}, \quad Br = \Pr E, \quad Du = \frac{D(C_1 - C_0) K_T}{C_s C_p \mu (T_1 - T_0)}, \quad Sr = \frac{\rho D K_T (T_1 - T_0)}{\mu T_m (C_1 - C_0)},$$

$$Sc = \frac{\mu}{\rho D}, \quad E = \frac{c^2}{C_p (T_1 - T_0)}, \quad Pr = \frac{\mu C_p}{K}, \quad u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}, \tag{7.9}$$

in which $\delta$ is the wave number, $\theta$ the non-dimensional temperature, $\phi$ the dimensionless concentration, $M$ the Hartman number, Re the Reynolds number, $Br$ the Brinkman number, $T_0$ and $C_0$ the temperature and concentration at upper wall, $T_1$ and $C_1$ the temperature and concentration at lower wall, Pr the Prandtl number, $E$ the Eckert number, $Sr$ the Soret number and $Sc$ the Schmidt number. We find that $p$ does not depend upon $y$ i.e. $\partial p/\partial y = 0$ and $x-$component of momentum equation reads to

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - M^2 \left(\frac{\partial \psi}{\partial y} + 1\right), \tag{7.10}$$

where $(u, v)$ are the velocity components in the wave frame, $p$ the pressure in wave frame and $\psi$ the stream function.

Now the compatibility, energy and concentration equations are given by

$$\frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial \psi^2}{\partial y^2} = 0, \tag{7.11}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Br \left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 + Br M^2 \left(\frac{\partial \psi}{\partial y} + 1\right)^2 + \Pr Du \frac{\partial^2 \Phi}{\partial y^2}, \tag{7.12}$$

$$0 = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2}. \tag{7.13}$$

89

The non-dimensional boundary conditions in terms of slip are

$$\psi = \frac{F}{2}, \quad \frac{\partial\psi}{\partial y} + \beta_1\frac{\partial^2\psi}{\partial y^2} = -1, \quad \theta + \beta_2\frac{\partial\theta}{\partial y} = 0, \quad \phi + \beta_3\frac{\partial\Phi}{\partial y} = 0, \qquad at \quad y = h_1,$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial\psi}{\partial y} - \beta_1\frac{\partial^2\psi}{\partial y^2} = -1, \quad \theta - \beta_2\frac{\partial\theta}{\partial y} = 1, \quad \phi - \beta_3\frac{\partial\Phi}{\partial y} = 1, \qquad at \quad y = h_2, (7.14)$$

in which $\beta_1$, $\beta_2$ and $\beta_3$ are the dimensionless velocity, temperature and concentration slip parameters respectively. These are defined as

$$\beta_1 = \frac{\mu\beta_1^*}{d_1}, \beta_2 = \frac{\beta_2^*}{d_1}, \ \beta_3 = \frac{\beta_3^*}{d_1},$$

where $\beta_1^*$, $\beta_2^*$ and $\beta_3^*$ are the dimensional velocity, temperature and concentration slip parameters respectively.

Now the dimensionless expressions of wave shapes are

$$h_1(x){=}1 + a\cos(2\pi x), \qquad h_2(x){=} - d - b\cos(2\pi x + \gamma). \tag{7.15}$$

Flow rate and pressure rise per wavelength $\Delta p_\lambda$ are defined as follows:

$$F = \int_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy, \tag{7.16}$$

$$\Delta p_\lambda = \int_0^1\frac{\partial p}{\partial x}dx. \tag{7.17}$$

Heat transfer coefficient $Z_1$ at the upper wall is defined by

$$Z_1 = h_{1x}\theta_y.$$

The solution of the involved problems through **Mathematica** are analyzed in the next section.

## 7.2   Discussion

This section is especially prepared to examine the effects of pertinent parameters. Here we arrange the variations of such parameters in the Figs. 7.1-7.4. These Figs. depict the plots of

stream lines, temperature, concentration and heat transfer coefficient at the upper wall. Figs. 7.1(a-d) illustrate the streamlines for an increase in velocity slip parameter $\beta_1$. The volume of fluid trapped within a streamline is termed as bolus. It is observed that the trapped bolus gets elongated and its size increases when the slip parameter increases.

Figs. 7.2(a-i) exhibit the temperature variation. It is seen that the temperature increases due to increase in M, $\beta_2$, Du, Sr, Sc, Pr and $\gamma$. However it is found that the temperature decreases when d and $\beta_1$ are increased. Increase in $\theta$ is rapid and much in cases of M and Pr (see Figs. 7.2a and 7.2f) but slower and small for $\beta_2$, Du, Sr and Sc (see Figs. 7.2b, 2c,2d and 2e).

Figs. 7.3(a-h) demonstrate how concentration field develops when certain parameters of interest are varied. Key observations from these Figs. are that concentration decreases with the increase in M, $\beta_3$, Du, Sr, Sc and Pr whereas it increases by increasing $\beta_1$ and $\gamma$. Here decrease in $\phi$ is much for M, $\beta_3$, Sr, Sc, Pr and small for Du. Moreover the concentration remains invariant for thermal slip parameter $\beta_2$.

Figs. 7.4(a-e) are plotted to see the behavior of heat transfer coefficient $Z_1$ at the upper wall. Because of peristalsis it is found that, $Z_1$ has an oscillatory behavior. Moreover $Z_1$ has maximum value near the upper wall. It increases with the increase in M and $\gamma$. Further $Z_1$ decreases with increase in d and $\beta_1$. An interesting and noteworthy fact is that the thermal slip parameter $\beta_2$ has no meaningful variation on $Z_1$.

## 7.3   Main points

Simultaneous effects of heat and mass transfer on the peristaltic transport of viscous fluid in an asymmetric channel with slip effects are analyzed. They main observations are summarized as follows:

- The trapped bolus gets elongated as the value of viscosity parameter is increased.

- Effects of Hartman number M, Dufour number Du and temperature slip parameter $\beta_2$ on the temperature are opposite to that of velocity slip parameter $\beta_1$.

- Increase in temperature subject to an increase in Hartman number M is rapid when

compared to that of other parameters.

- Increase in applied magnetic field results in decrease of concentration.

- Thermal slip parameter has no effect on the fluid concentration.

- Absolute value of the heat transfer coefficient increases with increase in the Hartman number M.
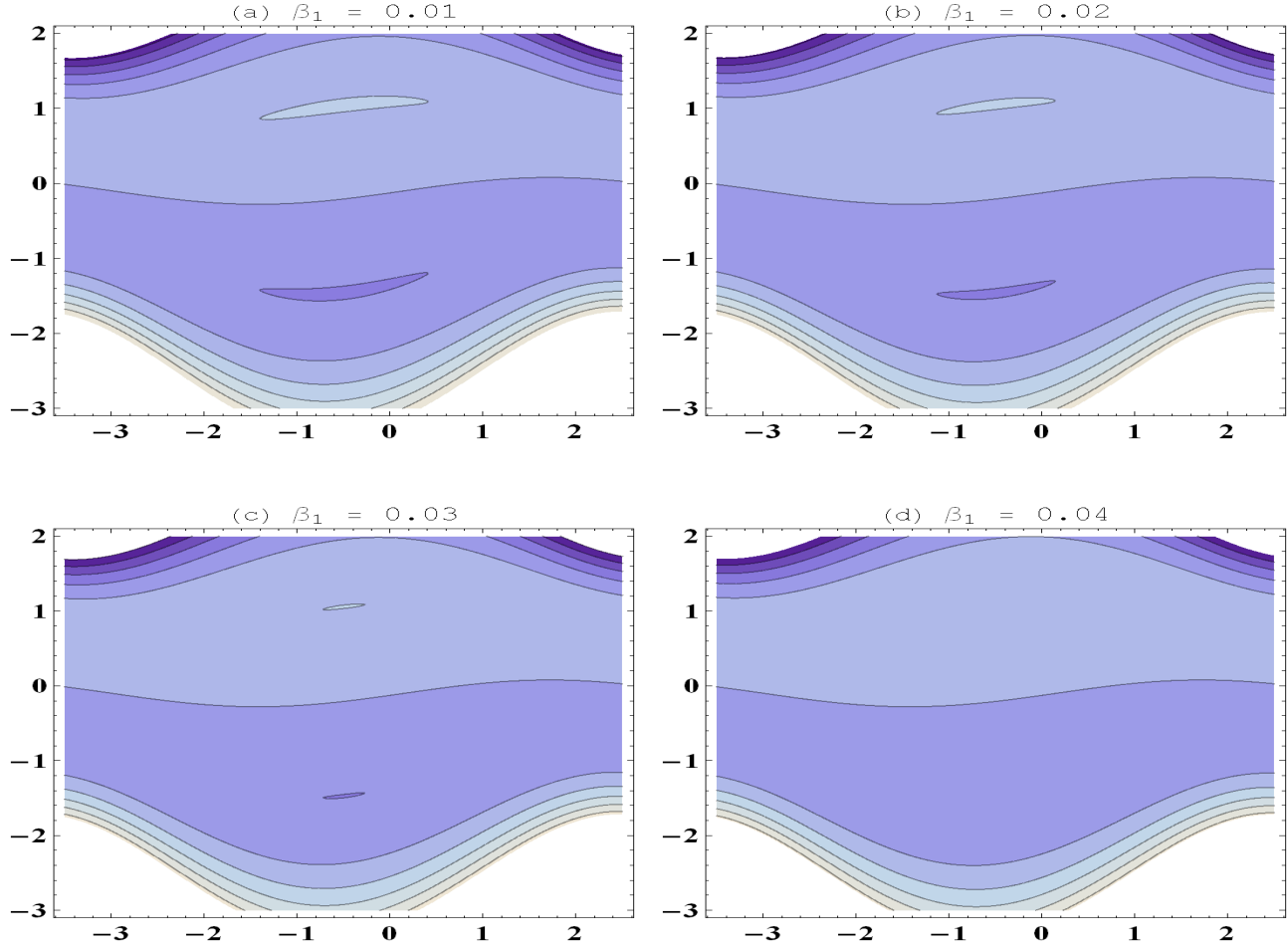
Fig. 7.1. Effect of velocity slip parameter $\beta_1$ on stream lines.
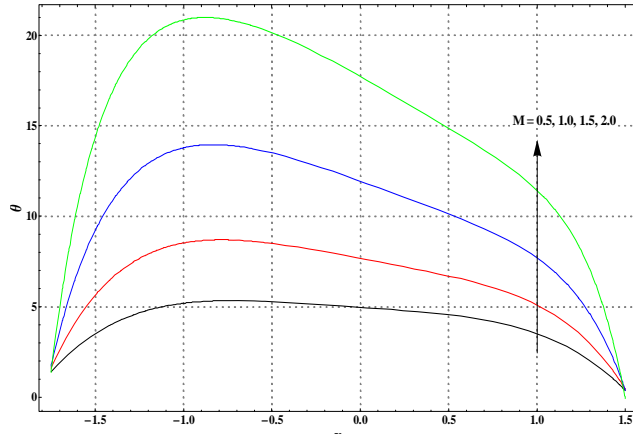
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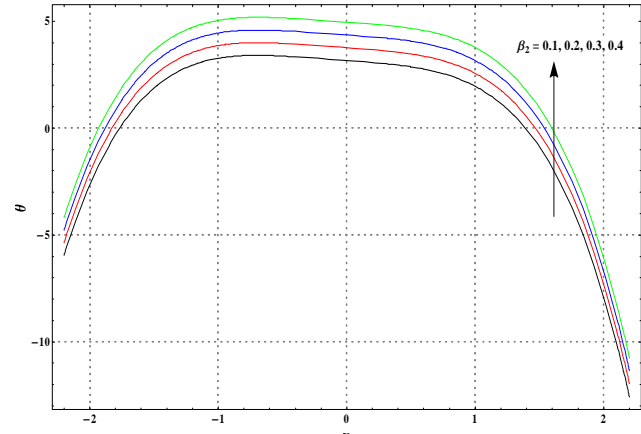
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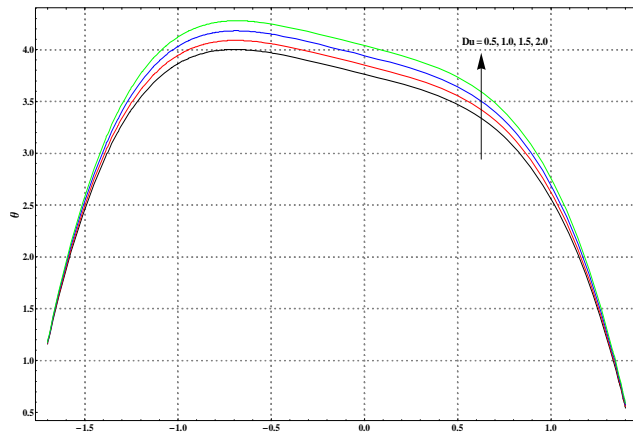
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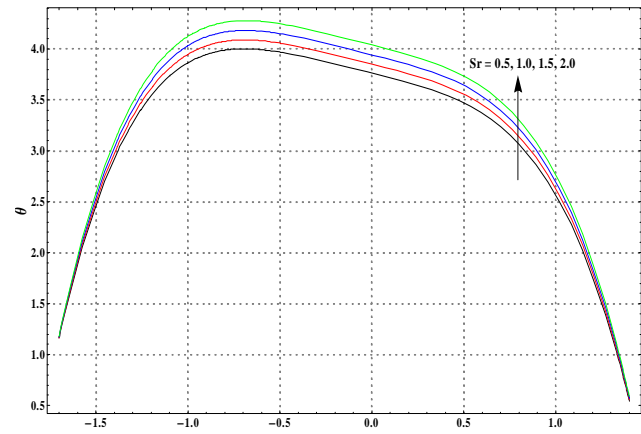
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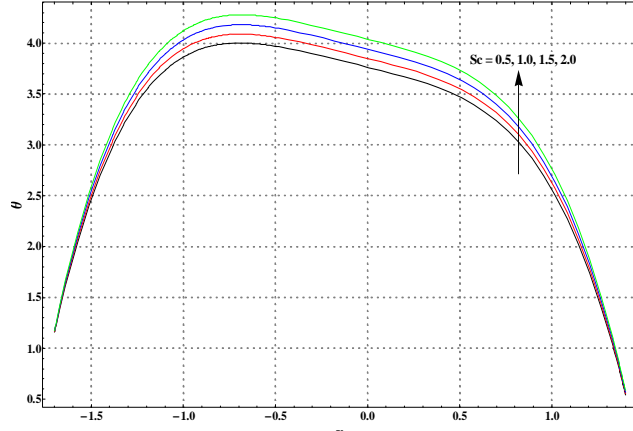
Fig. 7.2 e
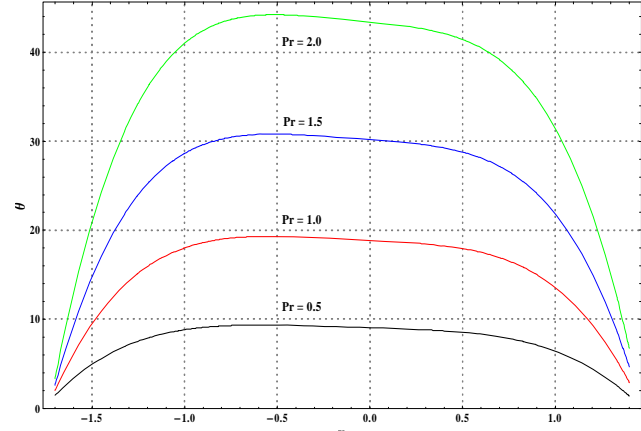


Fig. 7.2 f
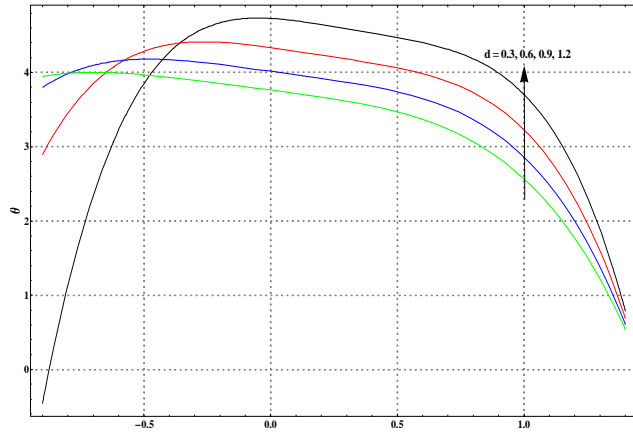


Fig. 7.2 g
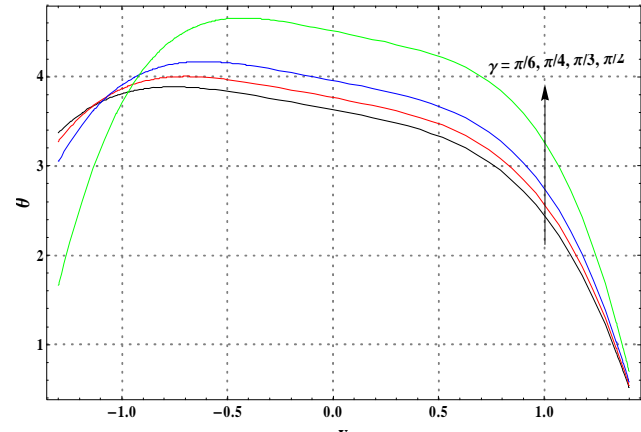


Fig. 7.2 h
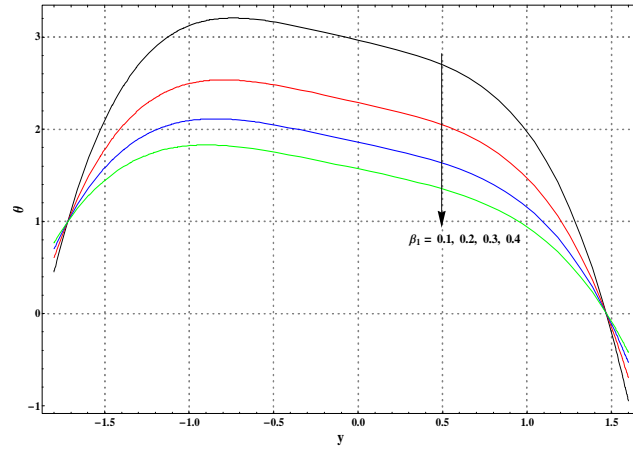


Fig. 7.2 i

Figs. 7.2(a-i). Effects of various parameters on temperature $\theta$.
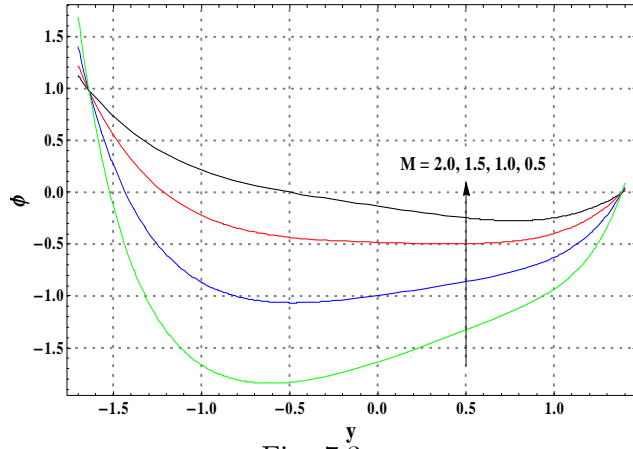
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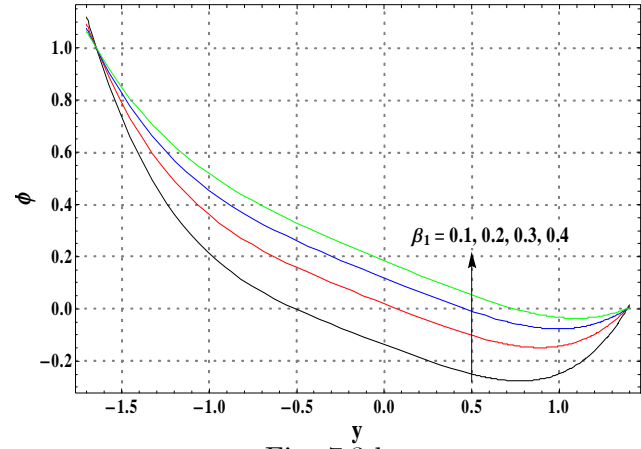
Fig. 7.3 b

Fig. 7.3 c

Fig. 7.3 d
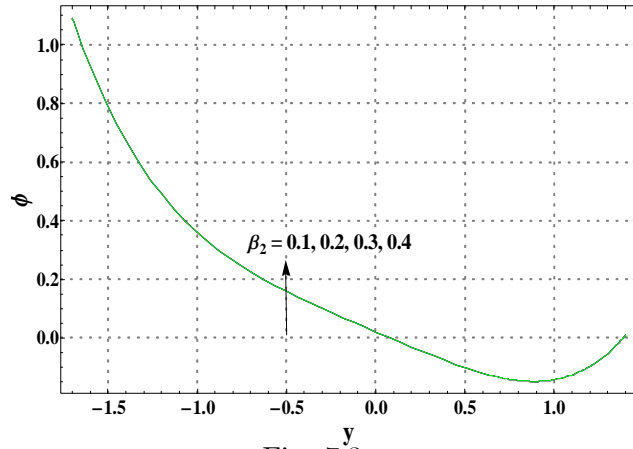
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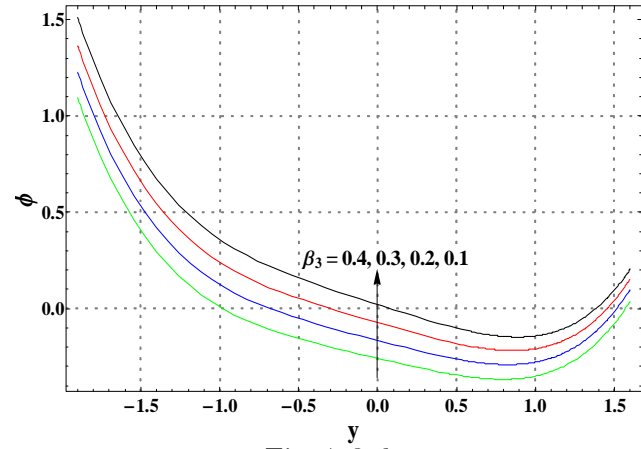
Fig. 7.3 f

Fig. 7.3 g

Fig. 7.3 h
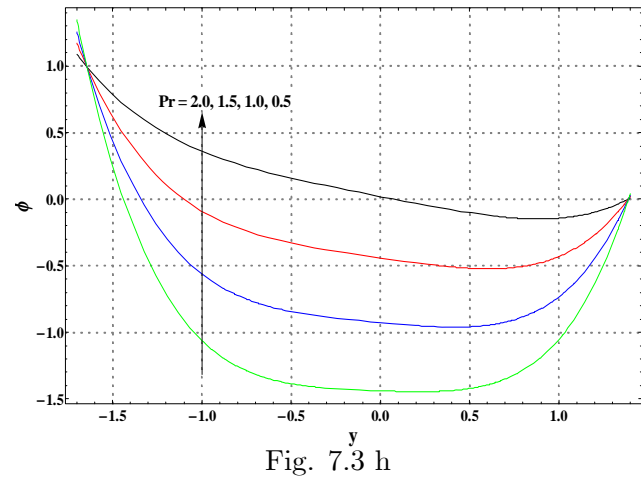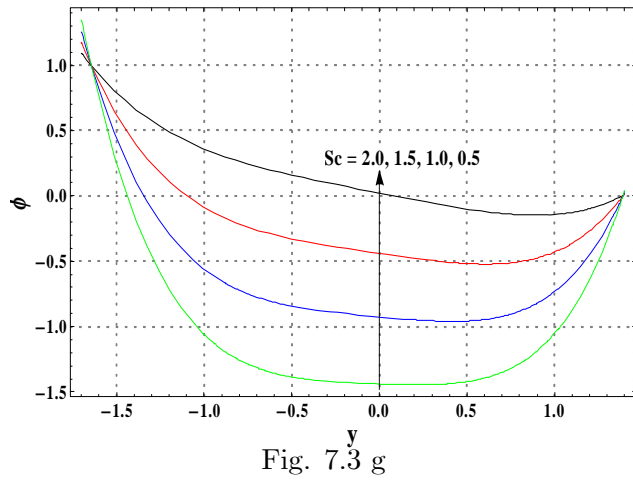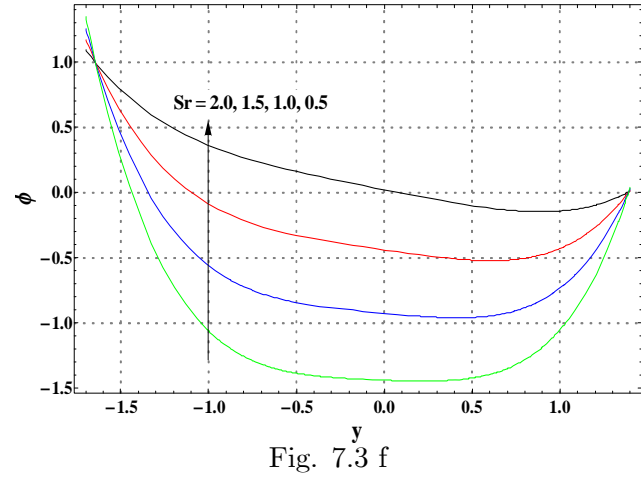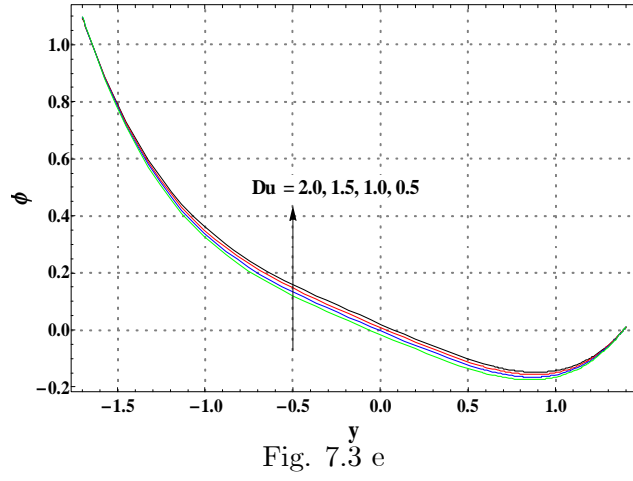
Figs. 7.3(a-h). Effects of various parameters on concentration.
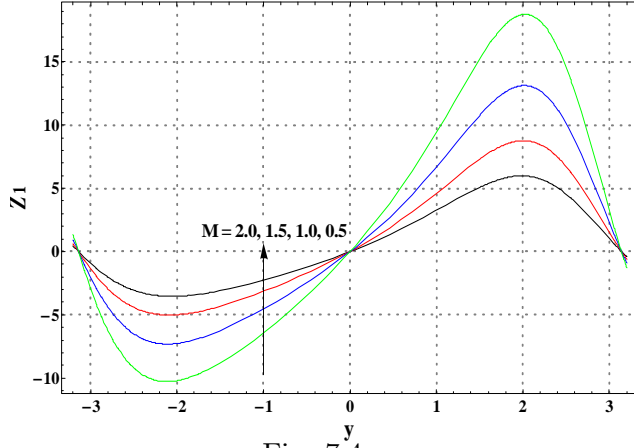
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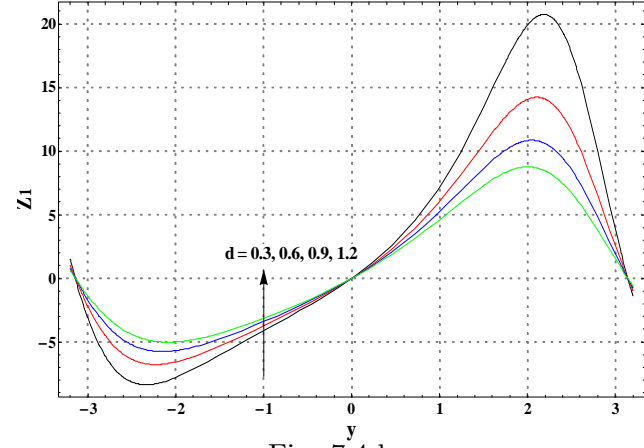
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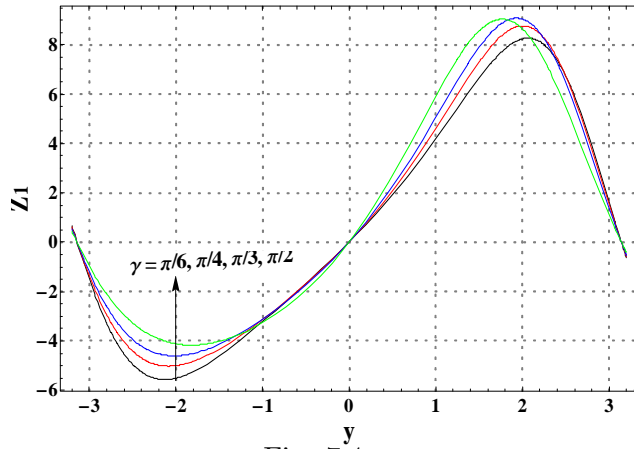
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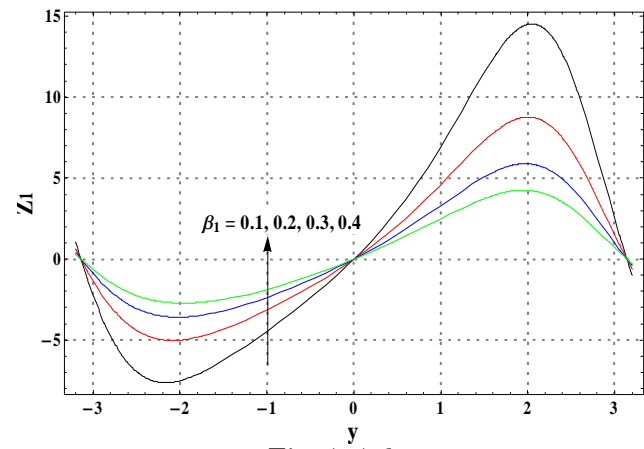
Fig. 7.4 d
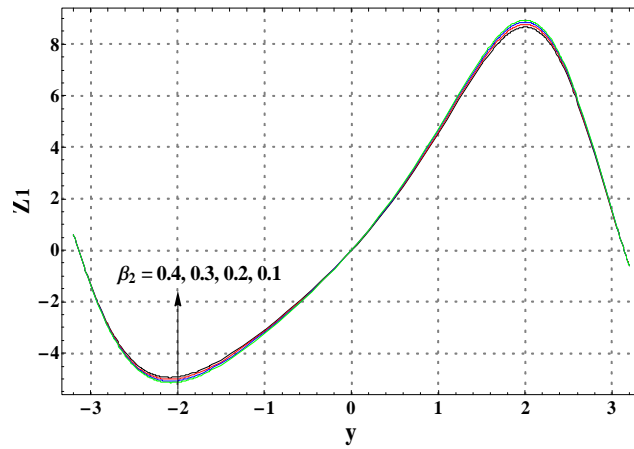
Fig. 7.4 e

Figs. 7.4(a-e). Behavior of $Z_1$ for different parameters.

98

# Chapter 8

# Peristaltic transport in a vertical channel subject to Soret and Dufour effects

This chapter looks at the simultaneous effects of heat and mass transfer in peristaltic transport of viscous fluid. Mathematical modelling is provided in the presence of Joule heating and Soret and Dufour effects. Perturbation solution is prepared for small Brinkman number. Flow quantities of interest are plotted and analyzed.

## 8.1   Mathematical formulation

Here our aim is to develop a model for the peristaltic transport of viscous fluid in a vertical channel. An incompressible fluid is considered in a vertical channel. Magnetohydrodynamic (MHD) character of fluid is accounted. The flow is generated by sinusoidal waves having constant speed $c$. The channel width is taken as $2d_1$. An incompressible and magnetohydrodynamic (MHD) fluid fills the region in the channel. The fluid is conducting through uniform applied magnetic field $\mathbf{B}_0$. Induced magnetic field is not taken into account. Electric field is absent.

Geometry of flow problem is given below

The wall shape is taken as

$$H(\overline{X}, \overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right),\qquad(8.1)$$

where $a_1$ is the wave amplitude, $\lambda$ the wavelength and $\overline{t}$ the time. The velocity field in two-dimensional flow is given by

$$V = \left[\overline{U}(\overline{X}, \overline{Y}, \overline{t}), \overline{V}(\overline{X}, \overline{Y}, \overline{t}), 0\right].$$

Conservation laws of mass, momentum and energy in the present flow consideration are reduced in the following expressions

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0,\qquad(8.2)$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \mu\left[\frac{\partial^2 \overline{U}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{U}}{\partial \overline{Y}^2}\right] - \sigma B_0^2 \overline{U} + \rho g \alpha^* (T - T_0)$$
$$+ \rho g \alpha^{**} (C - C_0),\qquad(8.3)$$

$$\rho\left(\frac{\partial}{\partial \overline{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \mu\left[\frac{\partial^2 \overline{V}}{\partial \overline{X}^2} + \frac{\partial^2 \overline{V}}{\partial \overline{Y}^2}\right],\qquad(8.4)$$

$$\rho C_p \left( \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) T = K \left[ \frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2} \right] +$$

$$\mu \left[ 2 \left( \left( \frac{\partial \overline{U}}{\partial \overline{X}} \right)^2 + \left( \frac{\partial \overline{V}}{\partial \overline{Y}} \right)^2 \right) + \left( \frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}} \right)^2 \right] + \sigma B_0^2 \overline{U}^2 + \frac{DK_T}{C_s} \left[ \frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2} \right] + \Phi^* \qquad (8.5)$$

$$\left[ \frac{\partial}{\partial \bar{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right] C = D \left[ \frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2} \right] + \frac{DK_T}{T_m} \left[ \frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2} \right]. \qquad (8.6)$$

In above expressions $(\overline{U}, \overline{V})$ are the velocity components in the laboratory frame $(\overline{X}, \overline{Y})$, $\overline{P}$ the pressure, $g$ the acceleration due to gravity, $\alpha^*$ the coefficient of thermal expansion, $\alpha^{**}$ the coefficient of expansion due to concentration, $K$ the thermal conductivity, $C$ the concentration field, $T$ the temperature field, $D$ the mass diffusivity, $K_T$ the thermal diffusion ratio, $\Phi^*$ the constant heat addition/absorption, $C_p$ the specific heat, $C_s$ the concentration susceptibility, $C_0$ the concentration at boundary, $T_m$ and $T_0$ the fluid mean temperature and temperature at the boundary respectively.

If $(\overline{u}, \overline{v})$ are the velocity components in the wave frame $(\overline{x}, \overline{y})$ then using the transformations

$$\overline{x} = \overline{X} - c\bar{t}, \ \ \overline{y} = \overline{Y}, \ \ \overline{u} = \overline{U} - c, \ \ \ \overline{v} = \overline{V}, \ \ \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \bar{t}) \qquad (8.7)$$

and taking

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \ \ y = \frac{\overline{y}}{d_1}, \ \ u = \frac{\overline{u}}{c}, \ \ v = \frac{\overline{v}}{c\delta}, \ \ \delta = \frac{d_1}{\lambda}, \ \ h = \frac{H}{d_1}, \ \ a = \frac{a_1}{d_1}, \ \ p = \frac{d_1^2 \overline{p}}{c\lambda\mu}, \\
\theta &= \frac{T - T_0}{T_0}, \phi = \frac{C - C_0}{C_0}, M^2 = \left( \frac{\sigma}{\mu} \right) B_0^2 d_1^2, \boldsymbol{v} = \frac{\mu}{\rho}, Re = \frac{\rho c d_1}{\mu}, \\
t &= \frac{c\bar{t}}{\lambda}, \ \ Br = \Pr E, \ \ Du = \frac{DC_0 K_T}{C_s C_p \mu T_0}, \ \ Sr = \frac{\rho D K_T T_0}{\mu T_m C_0}, \ \ Sc = \frac{\mu}{\rho D}, \\
E &= \frac{c^2}{C_p T_0}, \ \ Pr = \frac{\mu C_p}{K}, \ \ G_t = \frac{\rho g \alpha^* T_0 d_1^2}{\mu c}, \ \ G_c = \frac{\rho g \alpha^{**} C_0 d_1^2}{\mu c}, \\
\Phi &= \frac{a^2 \Phi^*}{k(T - T_0)}, \ \ u = \frac{\partial \psi}{\partial y}, \ \ v = -\frac{\partial \psi}{\partial x},
\end{aligned}
\qquad (8.8)
$$

incompressibility condition (8.2) is clearly satisfied while Eqs. (8.3)-(8.6) through standard definition of stream function $\psi$ can be written as

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right) + G_t \theta + G_c \phi, \qquad (8.9)$$

101

$$\frac{\partial^4 \psi}{\partial y^4} - M^2 \frac{\partial \psi^2}{\partial y^2} + G_t \frac{\partial \theta}{\partial y} + G_c \frac{\partial \phi}{\partial y} = 0, \tag{8.10}$$

$$\frac{\partial p}{\partial y} = 0, \tag{8.11}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Br \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + Br M^2 \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 + \Pr Du \frac{\partial^2 \phi}{\partial y^2} + \Phi, \tag{8.12}$$

$$0 = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2}. \tag{8.13}$$

From Eq (8.11) $p \neq p(y)$. Here $\boldsymbol{v}$ denotes the kinematic viscosity, $M$ the Hartman number, Re the Reynolds number, $G_t$ the local temperature Grashoff number, $G_c$ the local concentration Grashoff number, $\Phi$ the dimensionless heat source/sink parameter, $Br$ the Brinkman number, $Du$ the Dufour number, $Sr$ the Soret number, $Sc$ the Schmidt number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the wave number and the dimensionless temperature and concentration are denoted by $\theta$ and $\phi$ respectively. The boundary conditions are prescribed in the following forms

$$\begin{aligned} \psi &= 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0, \ \frac{\partial \theta}{\partial y} = 0, \ \frac{\partial \phi}{\partial y} = 0, \quad at \ \ y = 0, \\ \psi &= F, \quad \frac{\partial \psi}{\partial y} = -1, \ \theta = 0, \ \phi = 0, \quad at \ \ y = h, \end{aligned} \tag{8.14}$$

$$h(x) = 1 + a \cos(2\pi x), \ \ F = \int_0^h \frac{\partial \psi}{\partial y} dy, \tag{8.15}$$

where $h$ is the dimensionless form of the wall shape. The dimensionless mean flow rates in laboratory $(\eta)$ and wave $(F)$ frames are related by

$$\eta = F + 1.$$

Pressure rise per wavelength $\Delta p_\lambda$ can be put into the following relation

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx} dx. \tag{8.16}$$

## 8.2   Series expressions

We are interested to develop the series solutions of the resulting problems. For that we expand the stream function in terms of small Brinkman number and write

$$\psi = \psi_0 + Br\psi_1 + ....$$

Similarly the other physical quantities like $\theta$, $\phi$, $F$ and $P$ are also expanded. It is worth mentioning to point out that the condition $Br \ll 1$ is not an unrealistic condition. For example, the most common Newtonian fluid i.e. water, at $25^o c$ (298 Kelvin $\approx$ room temperature) has dynamic viscosity $\mu = 0.89$ cP (centi poise) and thermal conductivity 0.58 W/mK (watt per meter Kelvin). If we consider the characteristic speed 1m/s (meter per second) and wall temperature to be 310 Kelvin ($\approx$ equal to body temperature), then the Brinkman number comes out to be $1.2787 \times 10^{-4} \ll 1$. This provides us a confidence regarding the selection of Brinkman number $Br$ as a small quantity. Resulting zeroth and first order systems are given by

$$\frac{dp_0}{dx} = \frac{\partial^3 \psi_0}{\partial y^3} - M^2 \left( \frac{\partial \psi_0}{\partial y} + 1 \right) + G_t \theta_0 + G_c \phi_0,$$

$$0 = \frac{\partial^2 \theta_0}{\partial y^2} + \Pr Du \frac{\partial^2 \phi_0}{\partial y^2} + \Phi,$$

$$0 = \frac{1}{Sc} \frac{\partial^2 \phi_0}{\partial y^2} + Sr \frac{\partial^2 \theta_0}{\partial y^2},$$

$$\psi_0 = 0, \quad \frac{\partial^2 \psi_0}{\partial y^2} = 0, \quad \frac{\partial \theta_0}{\partial y} = 0, \quad \frac{\partial \phi_0}{\partial y} = 0, \quad at \ y = 0,$$

$$\psi_0 = F_0, \quad \frac{\partial \psi_0}{\partial y} = -1, \quad \theta_0 = 0, \quad \phi_0 = 0, \quad at \ y = h.$$

First order system

$$\frac{dp_1}{dx} = \frac{\partial^3 \psi_1}{\partial y^3} - M^2 \frac{\partial \psi_1}{\partial y} + G_t \theta_1 + G_c \phi_1,$$

$$0 = \frac{\partial^2 \theta_1}{\partial y^2} + \Pr Du \frac{\partial^2 \phi_1}{\partial y^2} + \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 + M^2 \left( \frac{\partial \psi_0}{\partial y} + 1 \right)^2,$$

$$0 = \frac{1}{Sc}\frac{\partial^2 \phi_1}{\partial y^2} + Sr\frac{\partial^2 \theta_1}{\partial y^2},$$

$$\psi_1 = 0, \quad \frac{\partial^2 \psi_1}{\partial y^2} = 0, \quad \frac{\partial \theta_1}{\partial y} = 0, \quad \frac{\partial \phi_1}{\partial y} = 0, \quad at \ y = 0,$$

$$\psi_1 = F_1, \quad \frac{\partial \psi_1}{\partial y} = 0, \quad \theta_1 = 0, \quad \phi_1 = 0, \quad at \ y = h.$$

The solutions of above systems yield the following expressions

$$\psi = \frac{e^{-My}e^{M(h+2y)}}{6L_2}\left(-6\left(-1+A\right)A_1\left(y\right) + 2e^{hM}L_1 y\left(6A_3\right)\right)\beta$$

$$+ \quad Br\left[-\frac{e^{5Mh+(y)}}{L_{11}}A_4\left(y\right) + L_{12}\right],$$

$$\theta = \frac{\left(-h^2+y^2\right)\beta}{2\left(-1+A\right)} - \frac{BrL_7}{360L_6^2},$$

$$\phi = \frac{L_8}{360\left(-1+A\right)^3} + \frac{BrL_{10}}{L_9^2}.$$

Where

$$L_1 = \left(3\left(-1+A\right)\left(F+h\right)M^2 A_2\left(y\right)\right)e^{M(h+2y)},$$

$$L_2 = \left(-1+A\right)M^2\left(1+hM+e^{2hM}\left(-1+hM\right)\right),$$

$$L_3 = 16e^{-2My+3M(h+y)}h\left(-1+h^2M^2\right)\left(G_t - G_cScSr\right)\Phi^2,$$

$$L_4 = A_6\left(y\right)+A_7\left(y\right)\left(-1+A\right)\left(F+h\right)+A_8\left(y\right)+A_9\left(y\right),$$

$$L_5 = \left(-1+A\right)^3 M^{10}\left(1+hM+e^{2hM}\left(-1+hM\right)\right),$$

$$L_6 = \left(-1+A\right)^3\left(1+hM+e^{2hM}\left(-1+hM\right)\right),$$

$$L_7 = 720\left(-1+A\right)^2 e^{M(h-y)F^2M^2}B_1\left(y\right)+B_2\left(y\right)+B_3\left(y\right)+B_4\left(y\right)+B_5\left(y\right),$$

$$L_8 = ScSr\left(180\left(-1+A\right)^2\left(h-y\right)\left(h+y\right)\Phi\right),$$

$$L_9 = \left(360\left(-1+A\right)^3\right)\left(1+hM+e^{2hM}\left(-1+hM\right)\right),$$

$$L_{10} = C_1\left(y\right)+C_2\left(y\right)+C_3\left(y\right)+C_4\left(y\right)+C_5\left(y\right),$$

$$L_{11} = \left(1+hM+e^{2hM}\left(-1+hM\right)\right),$$

$$L_{12} = A_5\left(y\right)+\frac{L_3}{L_5^5}\left\{e^{M(h+2y)}L_4\right\}+A_{10}\left(y\right),$$

$A = PrScSrDu$, and $A_is$, $B_is$ and $C_is$ are given below

$$A_1 = \frac{FMy\cosh(hM) + y\sinh(hM) - (F+h)\sinh(My)}{(hM\cosh(hM) - \sinh(hM))^2},$$

$$A_2 = A_1(y^2 A_3 + G_c(hMy\cosh(hM) + (h^3 M^2 - y))\cosh(hM))^2 \cosh(My),$$

$$+ \sinh(hM)) - 2hM\sinh(2hM),$$

$$A_3 = (F+h)^2\left(1+h^2\right)M^2 + h^2\left(F+h\right)^2 M^4) + G_t(2\left(F+h\right)^2 M\left(1+M^2\right)y$$

$$\cosh(2My)\sinh(hM) + 16\left(F+h\right)M^2 y^2 \cosh(My)),$$

$$A_4 = \left(F+h-8h^2\right)M^2 + 8h^4 M^4)\cosh(3hM) + 16h^2\left(F+h\right)M^4 y^2 \cosh(hM)^2 \cosh(My)$$

$$G_t(\sinh(My) - 16\left(F+h\right)My\sinh(hM)^2 \sinh(My)),$$

$$A_5 = 2F\left(4+h+hM^2\right) - 2y^2 + G_c(h\left(8+h\left(3+M^2\left(1+2h^2-2y^2\right)\right)\right)))\cosh(2hM),$$

$$A_6 = 2y^2(h+y)(2F\left(4+h+hM^2\right) - G_t 2y^2 + h\left(8+h\left(3+M^2\left(1+2h^2-2y^2\right)\right)))),$$

$$A_7 = (y+2h^3)M^2\cosh(2My) + 2FhM^2\cosh(2My) + h^2 M^2\cosh(2My) + 8\left(F+h\right),$$

$$A_8 = e^{()y+2h}G_t(16\left(F+h\right)\sinh(hM)\left(M\left(h-y\right) + \sinh(My))),$$

$$A_9 = 16M^2 y^2 \sinh(M\left(h-y\right)) + (F+h)\left(1+M^2\right)\left(-3+2M^2 y^2\right)\sinh(2My) + 16G_c$$

$$+M^2 y^2 \sinh(M\left(h+y\right)),$$

$$A_{10} = \left(1+M^2\right)y - 4h^3\left(F+h\right)M^4\left(1+M^2\right)y^2 - 4\left(F+h\right)M^2\left(1+M^2\right)y^3,$$

$$B_1 = A_1(-8F^2 - 2h\left(4+h+3y\right) - 2Fy + h^2\left(F+2h\right)M^2 - 2h^2\left(F+h\right)^2 M^4)$$

$$+2(1+F\left(F+2h\right)M^2 + \left(F+h\right)^2 M^4)y^2 + (F^2\left(1+M^2\right) + 2F\left(4+h+hM^2\right)$$

$$-2y^2 + h\left(8+h\left(3+M^2\left(1+2h^2-2y^2\right)\right)))) \cosh(2hM) - (F+h)^2\left(1+M^2\right)\cosh(2My).$$

$$B_2 = 16h\left(F+h\right)MG_c y^3(h+M^2 y)\cosh(hM)\left(M\left(h-y\right) + \sinh(My)) - 16\left(F+h\right)$$

$$h\sinh(hM)\left(M\left(h-y\right) + y\sinh(My)),$$

$$B_3 = \tfrac{1}{2}\left(F+h\right)B_9 \cosh(hM) - 16h^4\left(F+h\right)A3 + 8h^3\left(F+h\right)M^4\left(h-y\right)\cosh(2hM),$$

$$B_4 = 128F1h^3 M^4 \cosh(hM)^2 \sinh(hM) + 2h^3\left(F+h\right)^2 M^3\left(1+M^2\right)\cosh(2My) + 8h^3$$

$$\left(F+h\right)M^4\left(h-y\right)A_5,$$

$$B_5 = h^4 My(-4Fy + F^2 - 4h + 2y^2 Fh + h^2 + \left(F+h\right)^2\left(1+h^2\right)M^2 + h^2\left(F+h\right)^2 M^4),$$

$$C_1 = 16h\left(F+h\right)M^2\left(h-y\right)\cosh(hM)yG_c - (F^2\left(1+M^2\right) + 2F\left(4+h+hM^2\right)$$

$$-2y^2 + h\left(8+h\left(3+M^2\left(1+2h^2-2y^2\right)\right))))\cosh(2hM),$$

$$C_2 = F^2 M^2\cosh(2My) + 2FhM^2\cosh(2My) + h^2 M^2\cosh(2My) + 8\left(F+h\right)F^2$$

$$\cosh(2My) + 2Fh\cosh(2My),$$

$$C_3 = 32y^2\left(F+h\right)M^3 y^2 \cosh(My)B_2 - (h\left(F+h\right)\left(-8+5F+5h+\left(5F+\left(5-16h\right)h\right)M^2\right)),$$

$$C_4 = 8F\left(4+F+h+\left(F+h+4h^2\right)M^2\right))\sinh(hM) - 8((4y\left(-1+h^2 M^2\right) + (F+h),$$

$$C_5 = h(-64+Fy\left(1+M^2\right)A_6\left(-13+24h^2 M^2 + 16h^4 M^4\right) + hB_3(-13+M^2(-13+8h(10+h)))).$$

## 8.3 Graphical results and analysis

This section emphasizes the impact of embedding flow parameters in the solution expressions. Here plots are prepared in the two and three dimensions. Arrangement of the graphs is made in such a way that Figs. 8.1 (a-d) show the pressure gradient, Figs. 8.2 (a-d) the pressure rise per wavelength, Figs. 8.3 (a-d) the 3-D plots for velocity profile, Figs. 8.4-8.9 the stream lines and Figs. 8.10 and 8.11 the 3D plots for temperature and concentration fields respectively.

Figs. 8.1 (a-d) reveal that pressure gradient decreases by increasing $M$ and $Sr$. However, the pressure gradient increases with an increase in $\Phi$ and Du. It is further noticed that the change in dp/dx is larger in the cases of $\Phi$ and M when compared with Du and Sr.

Pressure rise per wavelength via mean flow rate $\eta$ is displayed in the Figs. 8.2 (a-d). These Figs. show that the pressure rise decreases when flow rate increases. For a given constant value of the mean flow rate, the pressure rise increases with an increase in $\Phi$ and $G_t$ but it decreases with increase in $G_c$ (see Figs. 8.2 a,c and d). Behavior of pressure rise for M is however different. The pressure rise increases for negative value of mean flow rate when $M$ increases. However, the behavior is opposite for positive values of mean flow rate (Fig. 8.2b).

We displayed the 3D plots for velocity, temperature and concentration. Here it is observed from Fig. 8.3 that the velocity decreases when $M$ is increased. Such decrease largely depends upon the values of $M$. Physically the decrease in velocity is due to the reason that the Lorentz force has a role of retarding force. Consequently it increases the friction resistance which opposes the motion of fluid.

Figs. 8.3 (b-d) depict that the velocity is increasing function of $\Phi$ and $G_t$. However reverse situation is observed for $G_c$ i.e. the velocity decreases when $G_c$ increases.

Trapping is considered interesting feature of the peristaltic transport. The volume of fluid that gets trapped in a stream line during peristaltic motion is known as bolus. Figs. 8.4-8.9 depict that the size of trapped bolus decreases when $M$, $G_c$ and $Sr$ are larger. It is also found that the results are opposite for the cases of $G_t$, $\Phi$ and $Du$.

The variations of temperature and concentration fields can be seen through Figs. 8.10 and 8.11. These Figs show that the temperature increases when $M$, $G_t$, $\Phi$ and Du are increased. Clearly the rise in temperature with an increase in M and Du is more than $\Phi$ and $G_t$. Effects of $G_c$ on the temperature are not significant.

Figs. 8.11 plot the influences of $M$, $\Phi$, $Du$, $G_t$ and $G_c$ on the concentration field. Here concentration field decreases when there is an increase in $M$, $\Phi$ and $Du$. Further, the variations of concentration field for $G_t$ and $G_c$ are insignificant. It is also concluded that temperature increases and concentration decreases when $M$ increases.
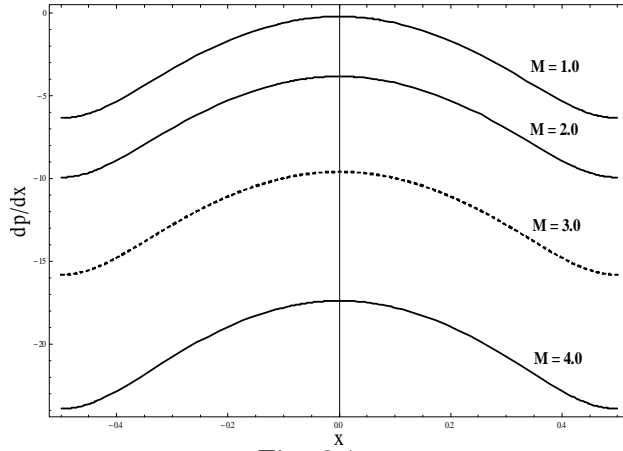
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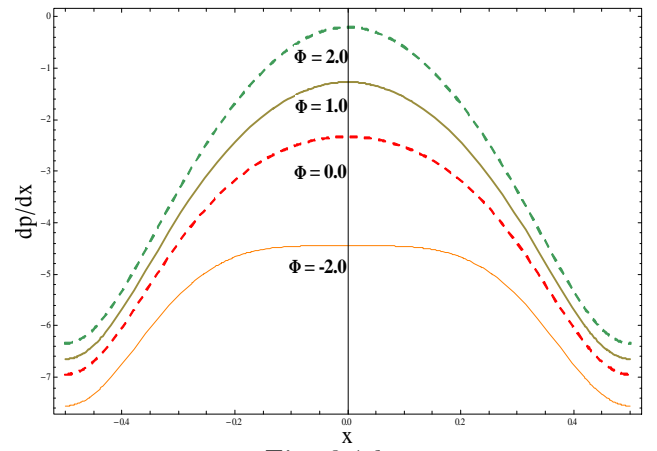
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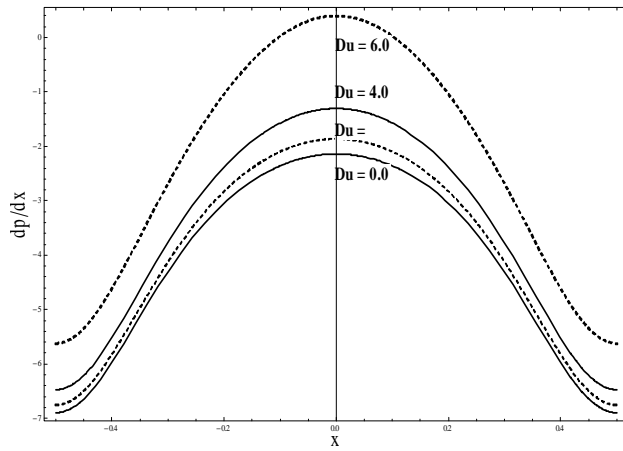
Fig. 8.1 c
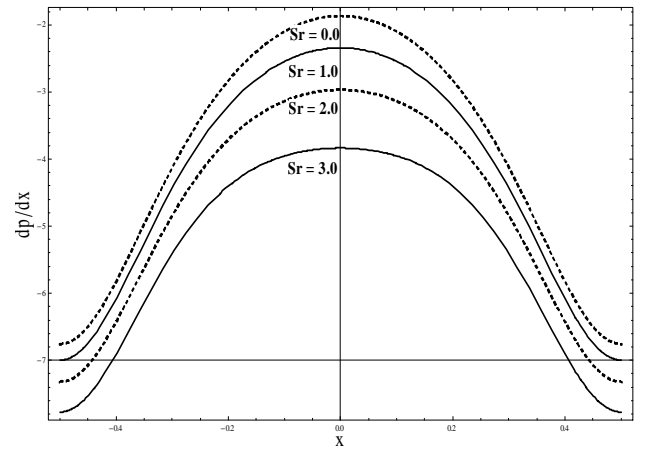
Fig. 8.1 d

Figs. 8.1 (a-d) Behavior of pressure gradient for variations in M, $\Phi$, Du and Sr.
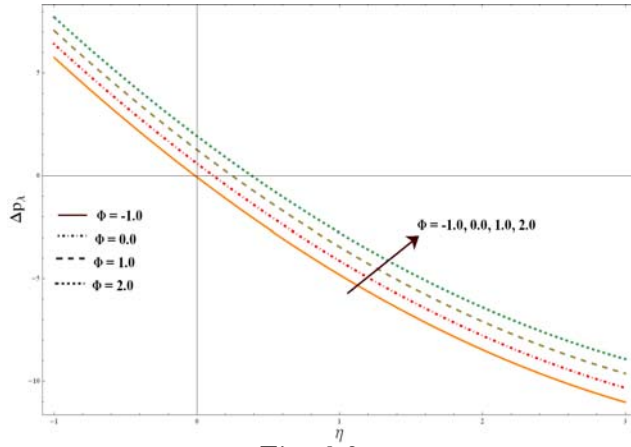
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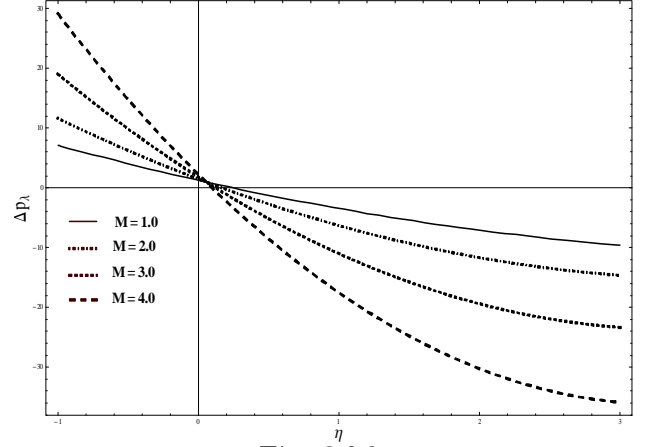
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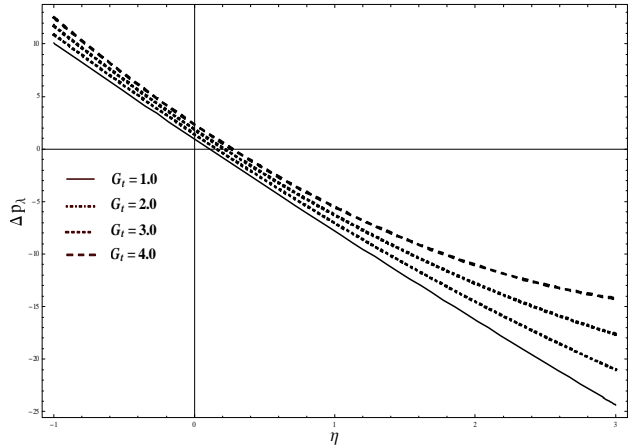
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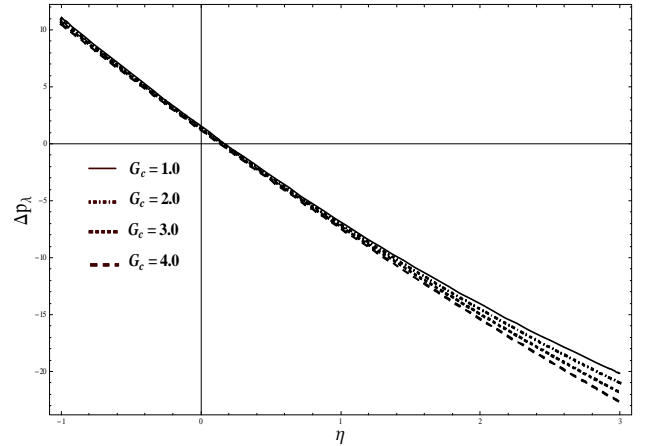
Fig. 8.2 d

Fig. 8.2 (a-d) Pressure rise per wavelength.

110
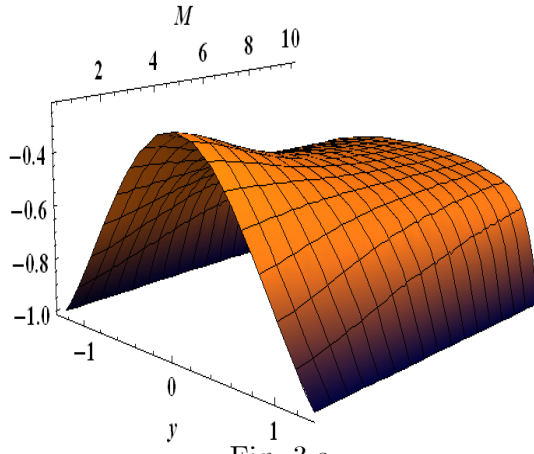
Fig. 3 a
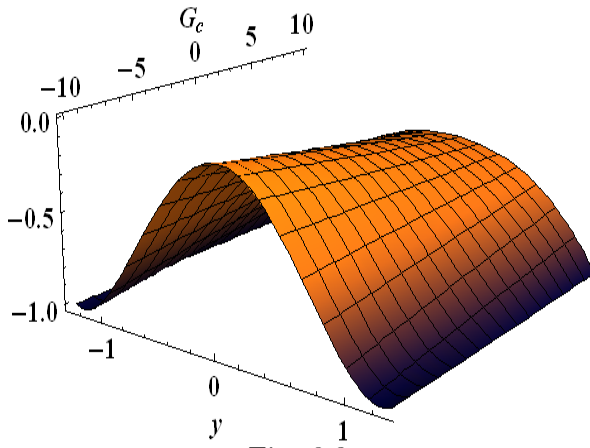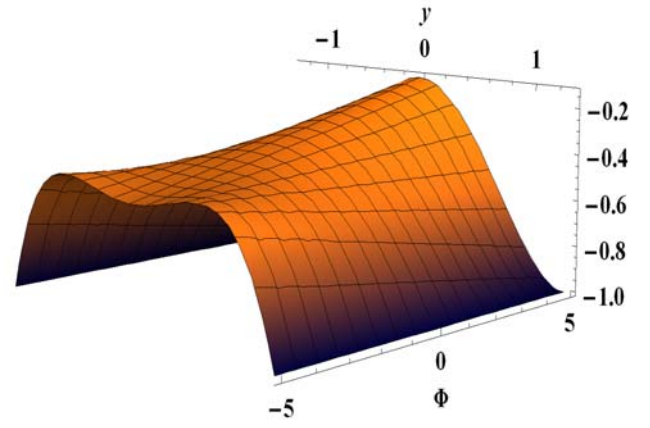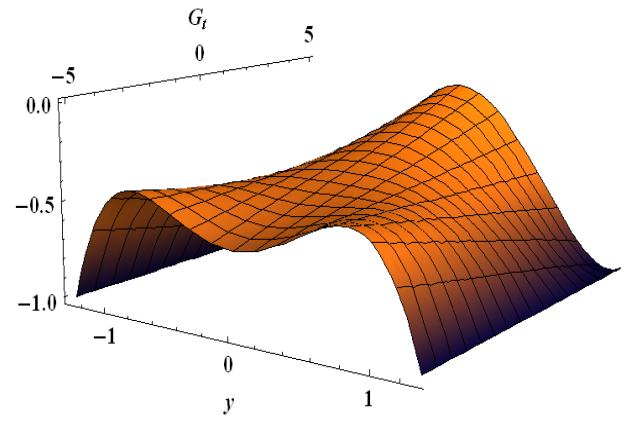
Fig. 8.3 c

Fig. 8.3 d

Fig. 8.3(a-d) Velocity profile with variations of $M$, $\Phi$, $G_c$ and $G_t$.

111
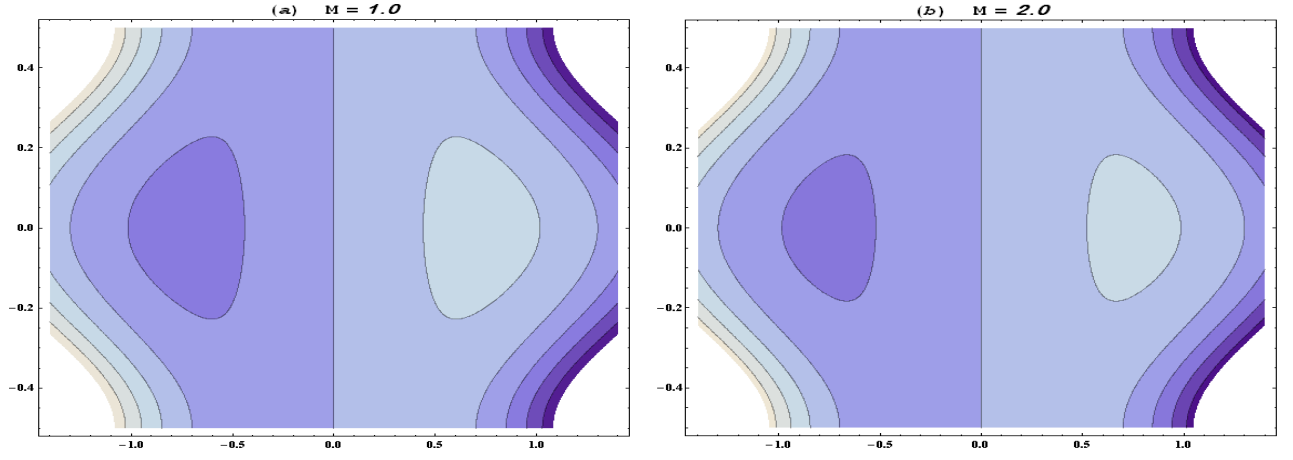
Fig. 8.4 Effect of Hartman number $(M)$ on stream lines when $\Phi = 1$, $Sr = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_c = 2$ and $G_t = 1$.
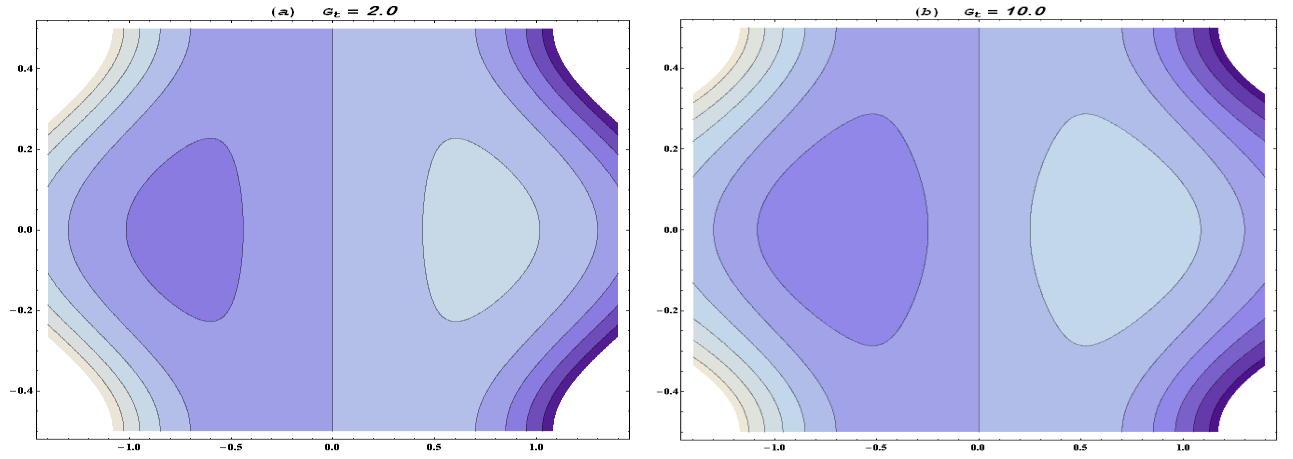


Fig. 8.5 Effect of $G_t$ on stream lines for $\Phi = 1$, $Sr = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_c = 2$ and $M = 1$.

112
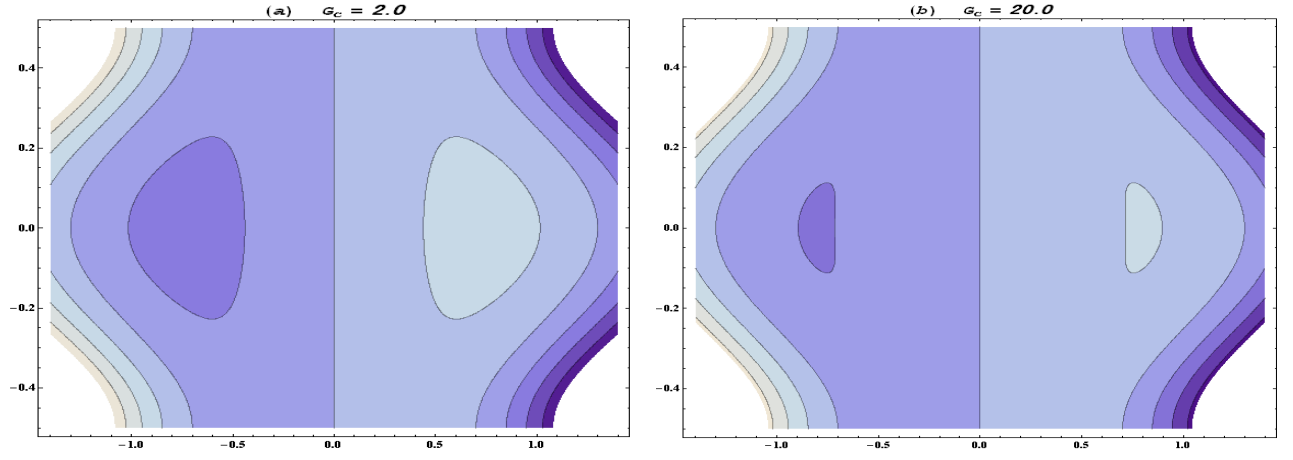
Fig. 8.6 Effect of $G_c$ on stream lines when $\Phi = 1$, $Sr = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_t = 2$ and $M = 1$.
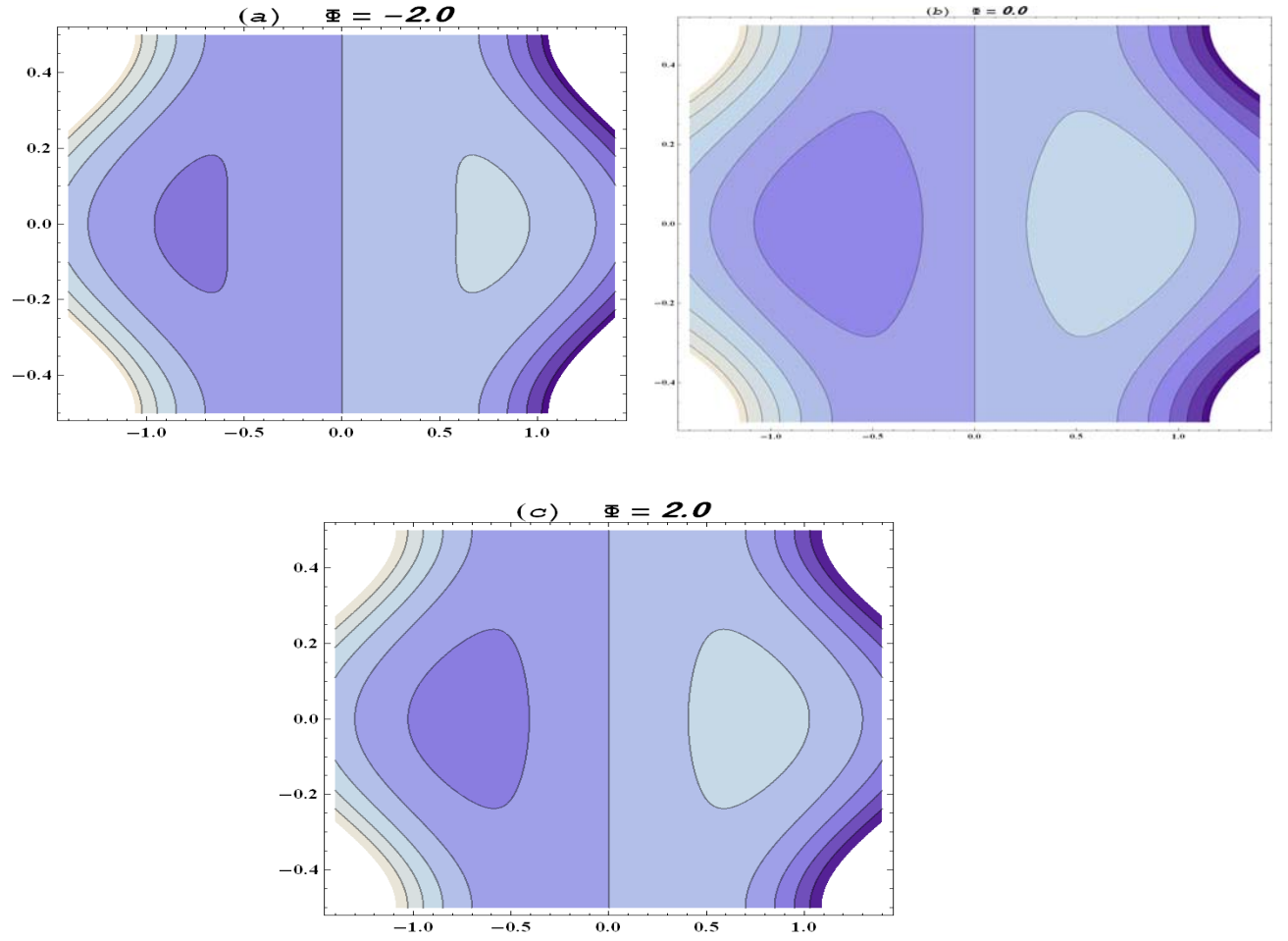
Fig. 8.7 Effect of $\Phi$ on stream lines when $G_c = 2$, $Sr = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_t = 2$ and $M = 1$.

114
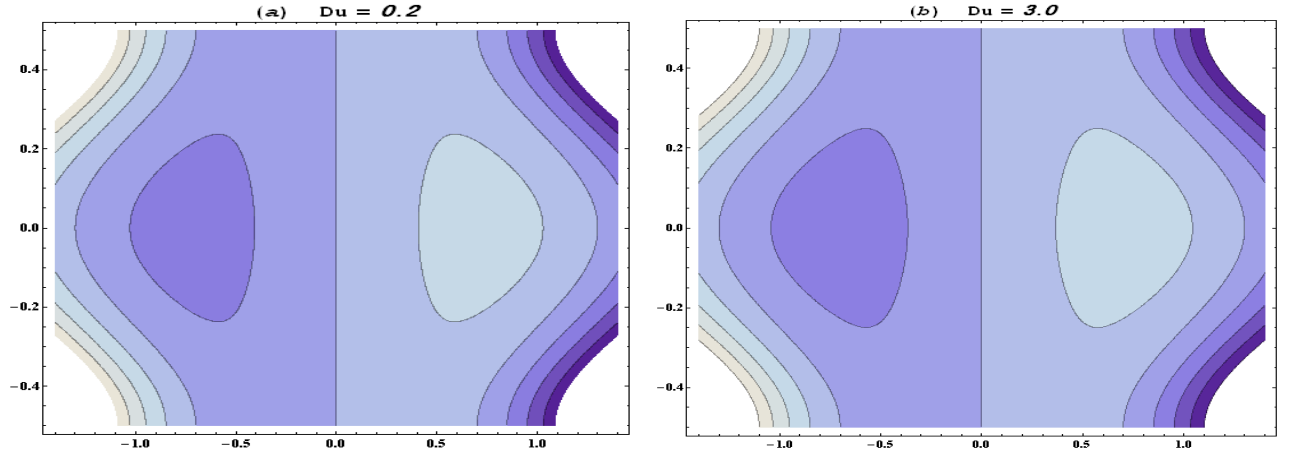
Fig. 8.8 Effect of $Du$ on stream lines for $\Phi = 1$, $G_c = 2$, $Sr = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_t = 2$ and $M = 1$.
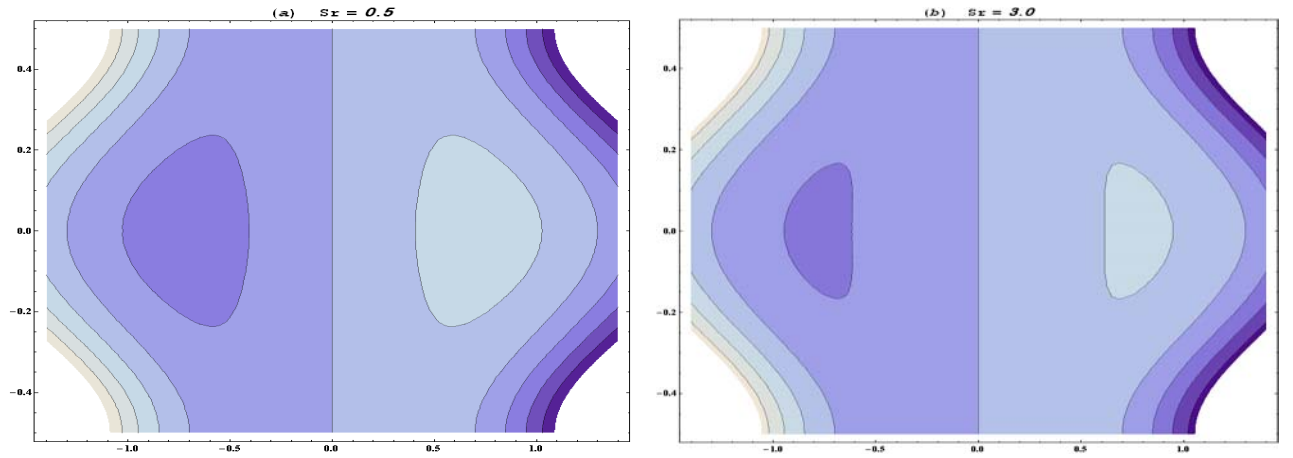


Fig. 8.9 Effect of $Sr$ on stream lines when $\Phi = 1$, $G_c = 2$, $Du = 0.5$, $Sc = 0.5$, $Br = 0.25$, $G_t = 2$ and $M = 1$.
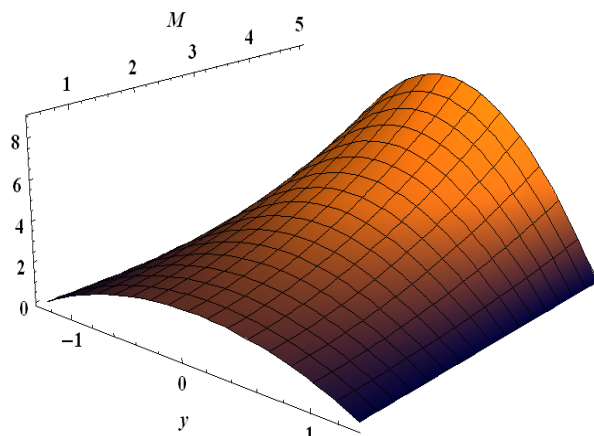
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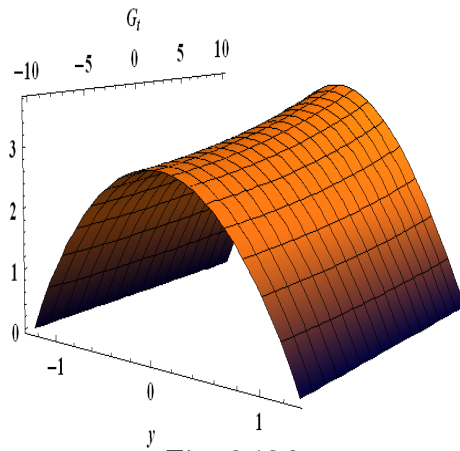

Fig. 8.10 b


Fig. 8.10 c


Fig. 8.10 d


Fig. 8.10 e

116
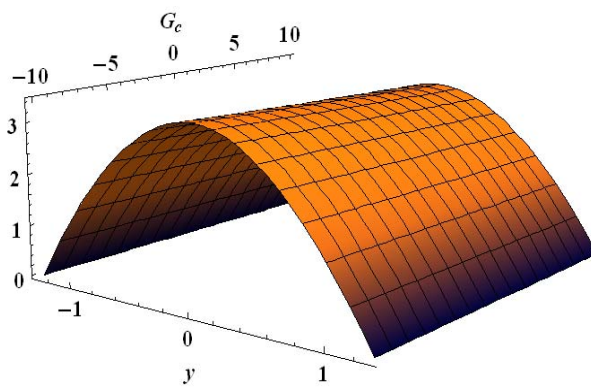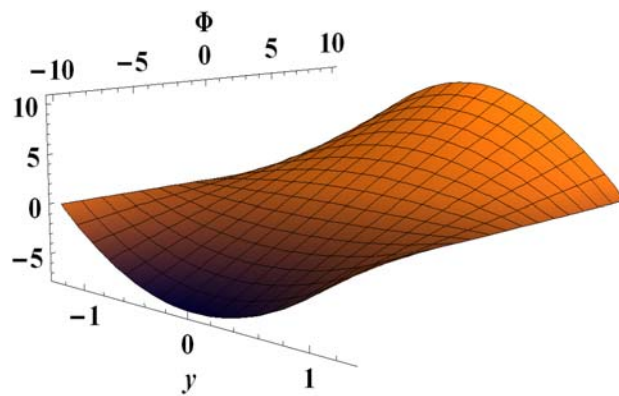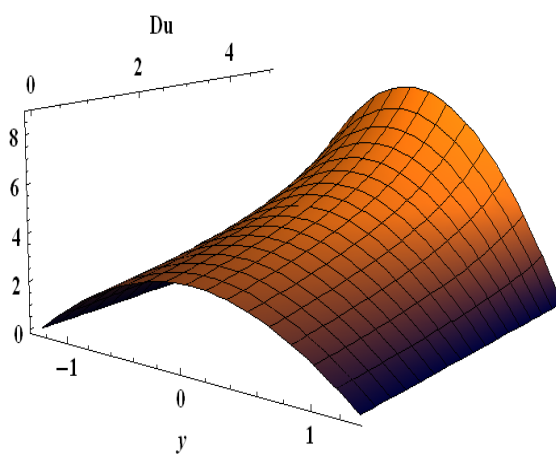
Figs. 8.10 (a-e) Variations of $M$, $G_t$, $G_c$, $\Phi$ and $Du$ on temperature.
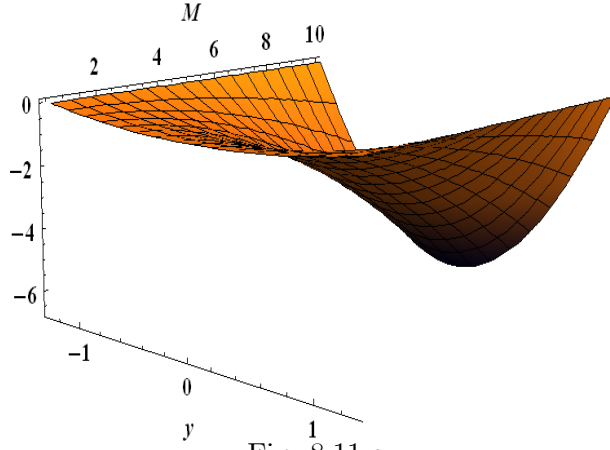
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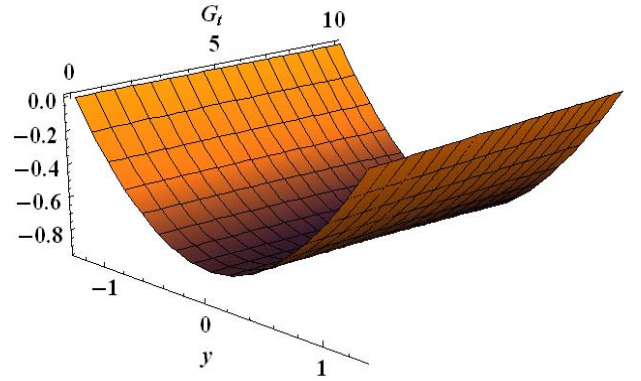
Fig. 8.11 b
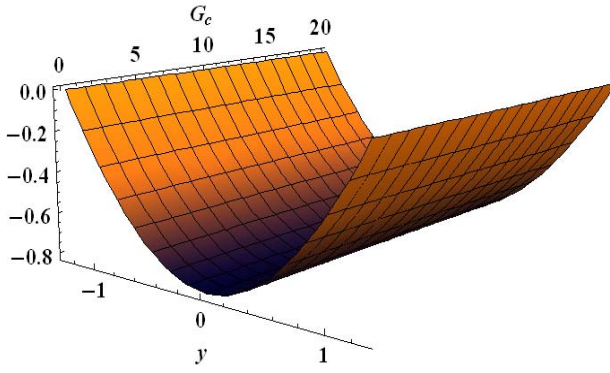
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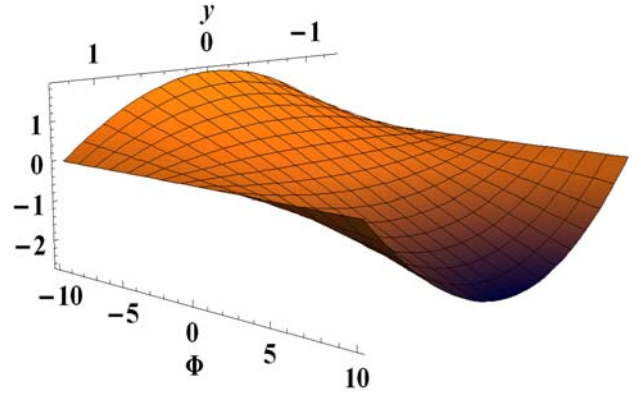
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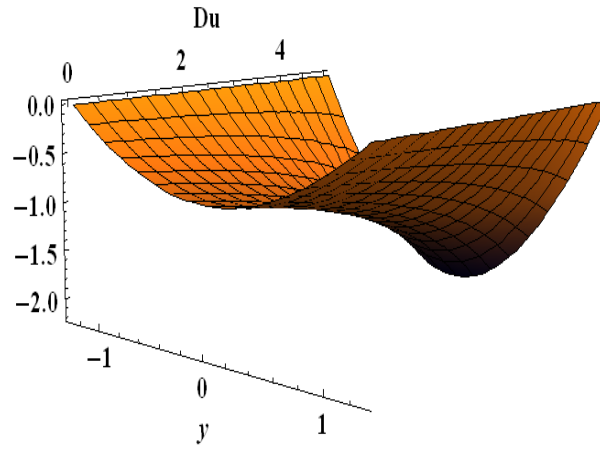
Fig. 8.11 e

Figs. 8.11 (a-e) Diagrams analyzing the development of concentration field when $M$, $G_t$ ,$G_c$, $\Phi$, and $Du$ are varied.

118

# Chapter 9

# Mixed convective heat and mass transfer for peristaltic transport in an asymmetric channel with Soret and Dufour effects

The aim of present chapter is to address the simultaneous effects of heat and mass transfer in the mixed convection peristaltic flow of viscous fluid in an asymmetric channel. The channel walls exhibit the convective boundary conditions. In addition the effects due to Soret and Dufour are taken in to consideration. Resulting problems are solved for the series solutions. Numerical values of heat and mass transfer rates are displayed and studied.

## 9.1    Mathematical analysis

We consider an asymmetric channel of width $d_1 + d_2$ with flexible walls. An incompressible viscous fluid is considered. Cartesian coordinate system is adopted such that the $\overline{X}$-axis is along the length of the channel and the $\overline{Y}$-axis is normal to the $\overline{X}$-axis. The flow is generated due to peristaltic waves propagating on the channel walls with speed c. Moreover, the channel walls possess the characteristics of convective type boundary condition. We intend to study

the flow in the presence of Soret and Dufour effects. The channel walls are described by the following expressions

$$\overline{H}_1(\overline{X},\overline{t}) = d_1 + \xi_1, \quad \text{left wall and } \overline{H}_2(\overline{X},\overline{t}) = -\left(d_2 + \xi_2\right), \quad \text{right wall}$$

where $\xi_1$ and $\xi_2$ are the disturbances produced due to propagation of peristaltic waves at the left and right walls respectively. The values of $\xi_1$ and $\xi_2$ can be written in the forms

$$\begin{aligned}
\xi_1 &= a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right), \\
\xi_2 &= b_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t}) + \gamma\right).
\end{aligned}$$

in which $\overline{t}$ is time, $\gamma$ is the phase difference and $a_1$, $b_1$ are the amplitudes of the waves travelling on the left and right walls with speed c respectively. Temperature and concentration at the walls are assumed to be $T_i$ and $C_i$ ($i = 0$ for left wall and $i = 1$ for right one ) respectively. Velocity field for this problem is $\overline{V} = \left[U(\overline{X},\overline{Y},\overline{t}), V(\overline{X},\overline{Y},\overline{t}), 0\right].$

Fundamental equations governing the flow of an incompressible viscous fluid are

$$\overline{\nabla}.\overline{V} = 0, \tag{9.1}$$

$$\rho\frac{d\overline{V}}{dt} = -\overline{\nabla}P + \mu\left(\overline{\nabla}^2\overline{V}\right) + \rho g\alpha^*\left(T - T_0\right) + \rho g\alpha^{**}\left(C - C_0\right), \tag{9.2}$$

in which $\rho$ is the density of fluid, $\overline{P}$ is the pressure, $\mu$ is the dynamic viscosity, $g$ is the acceleration due to gravity and $\alpha^*$ and $\alpha^{**}$ are the thermal and concentration expansion coefficients respectively. The energy and concentration equations can be expressed as

$$C_p\frac{dT}{dt} = \frac{K}{\rho}\left[T_{\overline{X}\overline{X}} + T_{\overline{Y}\overline{Y}}\right] +$$
$$v\left[2\left(\overline{U}_{\overline{X}}^2 + \overline{V}_{\overline{Y}}^2\right) + \left(\overline{U}_{\overline{Y}} + \overline{V}_{\overline{X}}\right)^2\right] + \frac{DK_T}{\rho C_s}\left[C_{\overline{X}\,\overline{X}} + C_{\overline{Y}\,\overline{Y}}\right], \tag{9.3}$$

$$\frac{dC}{dt} = D\left[C_{\overline{X}\,\overline{X}} + C_{\overline{Y}\,\overline{Y}}\right] + \frac{DK_T}{T_m}\left[T_{\overline{X}\,\overline{X}} + T_{\overline{Y}\,\overline{Y}}\right]. \tag{9.4}$$

Here $C_p$ is the specific heat, $K$ is the thermal conductivity, $D$ is the mass diffusivity, $K_T$ is the thermal diffusion ratio, $C_s$ is the concentration susceptibility, $T_m$ is the mean temperature, $T$

and $C$ are respectively the temperature and concentration of the fluid.

We consider $(\overline{u}, \overline{v})$ and $\overline{p}$ as the velocity components and pressure in the wave frame $(\overline{x}, \overline{y})$. The transformations between laboratory and wave frames are

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}). \tag{9.5}$$

where $(\overline{U}, \overline{V})$ and $\overline{P}$ are the velocity components and pressure in the laboratory frame. The fundamental equations in wave frame are reduced as follows:

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{9.6}$$

$$\rho \left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right) (\overline{u} + c) = -\frac{\partial \overline{p}}{\partial \overline{x}} + \mu \left[ \frac{\partial^2 \overline{u}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} \right] + \rho g \alpha^* (T - T_0)$$
$$+ \rho g \alpha^{**} (C - C_0), \tag{9.7}$$

$$\rho \left( (\overline{u} + c) \frac{\partial}{\partial \overline{x}} + \overline{v} \frac{\partial}{\partial \overline{y}} \right) \overline{v} = -\frac{\partial \overline{p}}{\partial \overline{y}} + \mu \left[ \frac{\partial^2 \overline{v}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{y}^2} \right], \tag{9.8}$$

$$C_p \left( (\overline{u} + c) T_{\overline{x}} + \overline{v} T_y \right) = \frac{K}{\rho} \left[ T_{\overline{x}\overline{x}} + T_{\overline{y}\overline{y}} \right] +$$
$$\upsilon \left[ 2 \left( (\overline{u} + c)_{\overline{x}}^2 + \overline{v}_{\overline{y}}^2 \right) + \left( (\overline{u} + c)_{\overline{y}} + \overline{v}_{\overline{x}} \right)^2 \right] + \frac{DK_T}{\rho C_s} \left[ C_{\overline{x}\overline{x}} + C_{\overline{y}\overline{y}} \right], \tag{9.9}$$

$$(\overline{u} + c) C_{\overline{x}} + \overline{v} C_{\overline{y}} = D \left[ C_{\overline{x}\overline{x}} + C_{\overline{y}\overline{y}} \right] + \frac{DK_T}{T_m} \left[ T_{\overline{x}\overline{x}} + T_{\overline{y}\overline{y}} \right]. \tag{9.10}$$

Letting

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \ \ y = \frac{\overline{y}}{d_1}, \ \ u = \frac{\overline{u}}{c}, \ \ v = \frac{\overline{v}}{c\delta}, \ \ \delta = \frac{d_1}{\lambda}, \ \ h_1 = \frac{\overline{H}_1}{d_1}, \\
h_2 &= \frac{\overline{H}_2}{d_1}, \ \ d = \frac{d_2}{d_1}, \ \ a = \frac{a_1}{d_1}, \ \ b = \frac{b_1}{d_1}, \ \ p = \frac{d_1^2 \overline{p}}{c\lambda\mu}, \ \ \upsilon = \frac{\mu}{\rho}, \\
Re &= \frac{\rho c d_1}{\mu}, \ \ \theta = \frac{T - T_0}{T_1 - T_0}, \ \ \phi = \frac{C - C_0}{C_1 - C_0}, \ \ Du = \frac{D(C_1 - C_0) K_T}{C_s C_p \mu (T_1 - T_0)}, \\
Br &= \Pr E, \ E = \frac{c^2}{C_p(T_1 - T_0)}, \ \ Sr = \frac{\rho D K_T (T_1 - T_0)}{\mu T_m (C_1 - C_0)}, \ \ t = \frac{c\overline{t}}{\lambda}, \\
Sc &= \frac{\mu}{\rho D}, G_t = \frac{\rho g \alpha^* (T_1 - T_0) d_1^2}{\mu c}, \ \ G_c = \frac{\rho g \alpha^{**}(C_1 - C_0) d_1^2}{\mu c}, \\
\Pr &= \frac{\mu C_p}{K}, \ u = \psi_y, \ v = -\psi_x,
\end{aligned}
\tag{9.11}
$$

and adopting long wavelength and low Reynolds number approach we have in terms of stream function $\psi$ the following equations

$$
p_x = \psi_{yyy} + G_t \theta + G_c \phi,
\tag{9.12}
$$

$$
p_y = 0,
\tag{9.13}
$$

$$
\theta_{yy} + Br \left(\psi_{yy}\right)^2 + \Pr Du(\phi_{yy}) = 0,
\tag{9.14}
$$

$$
\frac{1}{Sc}\phi_{yy} + Sr\theta_{yy} = 0.
\tag{9.15}
$$

Here $\upsilon$ denotes the kinematic viscosity, Re the Reynolds number, $Br$ is the Brinkman number, $E$ is the Eckret number, $Pr$ is the Prandtl number, $\delta$ is the wave number, $G_t$ is the temperature Grashoff number, $G_c$ is the concentration Grashoff number, $\phi$ is dimensionless concentration, $\theta$ is the dimensionless temperature and incompressibility condition is automatically satisfied.

Defining $\eta$ and F as the dimensionless mean flows in laboratory and wave frames we have

$$
\eta = F + 1 + d
\tag{9.16}
$$

where

$$
F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} dy.
\tag{9.17}
$$

The convective boundary conditions for the temperature and concentration are given by

$$-K\frac{\partial T}{\partial Y} = l(T - T_w),$$

$$-D\frac{\partial C}{\partial Y} = k(C - C_w).$$

The dimensionless boundary conditions for our problem can be imposed as follows:

$$\psi = \frac{F}{2}, \quad \psi_y = -1, \quad \theta_y + Bi\,(\theta) = 0, \quad \phi_y + Mi\,(\phi) = 0, \ y = h_1,$$

$$\psi = -\frac{F}{2}, \quad \psi_y = -1, \ \theta_y - Bi(\theta - 1) = 0, \ \phi_y - Mi(\phi - 1) = 0, \ y = h_2, \qquad (9.18)$$

where

$$h_1(x) = 1 + a\cos(2\pi x), \ h_2(x) = -d - b\cos(2\pi x + \gamma)\,, \qquad (9.19)$$

$$Bi = \frac{ld_1}{K}, \ Mi = \frac{kd_1}{D}. \qquad (9.20)$$

In above expressions $l$ and $k$ are dimensionless transfer coefficients, $Bi$ is heat transfer Biot-number and $Mi$ is mass transfer Biot-number.

## 9.2   Solution of the problem

The problems consisting of Eqs. (9.12)-(9.15) have been solved for both the perturbation and numerical solutions. Perturbation solution is obtained for small Brinkman number. Numerical solution is presented using Mathematica. For perturbation solutions we have the following zeroth and first order systems:

$$\psi_{0yyyy} + G_t\theta_{0y} + G_c\phi_{0y} = 0, \qquad (9.21)$$

$$\theta_{0yy} + \Pr Du(\phi_{0yy}) = 0, \qquad (9.22)$$

$$\phi_{0yy} + ScSr\theta_{0yy} = 0, \qquad (9.23)$$

$$\psi_0 = \frac{F_0}{2}, \; \psi_{0y} = -1, \; \theta_{0y} + Bi\,(\theta_0) = 0, \;\; \phi_{0y} + Mi\,(\phi_0) = 0, \;\; y = h_1, \tag{9.24}$$

$$\psi_0 = -\frac{F_0}{2}, \; \psi_{0y} = -1, \; \theta_{0y} - Bi(\theta_0 - 1) = 0, \; \phi_{0y} - Mi(\phi_0 - 1) = 0, \; y = h_2,$$

and

$$\psi_{1yyyy} + G_t\theta_{1y} + G_c\phi_{1y} = 0, \tag{9.25}$$

$$\theta_{1yy} + \left(\psi_{0yy}\right)^2 + \Pr Du(\phi_{1yy}) = 0, \tag{9.26}$$

$$\phi_{1yy} + ScSr\,(\theta_{1yy}) = 0, \tag{9.27}$$

$$\psi_1 = \frac{F_1}{2}, \; \psi_{1y} = 0, \; \theta_{1y} + Bi\,(\theta_1) = 0, \;\; \phi_{1y} + Mi\,(\phi_1) = 0, \; y = h_1,$$

$$\psi_1 = -\frac{F_1}{2}, \; \psi_{1y} = 0, \; \theta_{1y} - Bi(\theta_1) = 0, \;\; \phi_{1y} - Mi(\phi_1) = 0, \; y = h_2. \tag{9.28}$$

The perturbation solutions of above problem are given

$$
\begin{aligned}
\psi = {} & -\frac{F\,(h_1 + h_2 - 2y)\left(h_1^2 - 4h_1h_2 + h_2^2 + 2\,(h_1 + h_2)\,y - 2y^2\right)}{2\,(h_1 - h_2)^3} \\
& + \frac{L_0\,(y)}{24\,(2 + Bi\,(h_1 - h_2))\,(h_1 - h_2)^2\,(2 + h_1 M - h_2 M)} \\
& + \frac{Br L_1\,(y)}{241920\,(h_1 - h_2)^6},
\end{aligned}
\tag{9.29}
$$

$$\theta = \frac{1 + Bih_1 - Biy}{2 + Bih_1 - Bh_2} + \frac{Br}{1440\,(-1 + A)}L_2\,(y), \tag{9.30}$$

$$\phi = \frac{1 + Mih_1 - Miy}{2 + Mih_1 - Mih_2} + \frac{BrScSr}{1440\,(-1 + A)}\,[L_3\,(y)], \tag{9.31}$$

where $L_i$s are given below

$$L_1 = \frac{FMy\cosh(h_1) + y\sinh(h_2) - (F+h)\sinh(y)}{(h\cosh(h_1) - \sinh(h_2))^2} + (F+h_1)^2\, G_t^2\left(1 + h_1^2\right) Mi^2$$

$$+ h^2\left(F + h_1\right)^2 Mi^4 + G_t(2\left(F + h_2\right)^2 Mi\left(1 + M^2\right) y\cosh\left(2y\right)\sinh\left(h_2\right)$$

$$+ 16\left(F + h_2\right) Mi^2 y^2 \cosh\left(y\right)) + 62 G_c Bi\left(F + h_1 - 8h_1^2\right) Mi^2 + 8h_2^4 Bi^4)$$

$$\cosh\left(3h_2 Mi\right) + 16 Bi h^2\left(F + h_2\right) M^4 y^2 \cosh\left(h_2 Mi\right)^2 \cosh\left(y\right) G_t(\sinh\left(y\right)$$

$$- 16 Mi\left(F + h_1\right) Biy\sinh\left(h_2\right)^2 \sinh\left(y\right)) - 2F\left(4 + h_2 + h_2 Bi^2\right) - 2y^2$$

$$+ 2y^3 G_c(h_2\left(8 + Bi M i h_1\left(3 + M^2\left(1 + 2h_1^2 - 2y^2\right)\right)\right))) \cosh\left(2h_2\right),$$

$$L_2 = 2y^2 Bi G_t(h_1 + h_2 + y^2)(2F\left(4 + h_1 + 8h Mi^2\right) - G_t 2y^2 + h_2(8 + h_1(3$$

$$+ Bi^2\left(1 + 2h_1^2 - 2y^2\right)))) + Mi^2 Bi^3 G_c(y + h_1 + 2h_2^3) G_c^2 \cosh\left(2y\right)$$

$$+ 2Fh Bi^2 \cosh\left(2y\right) + h^2 Bi^2 \cosh\left(2y\right) + 8\left(F + h_2\right) + e^{(y + 2h_2 + h_1)} G_t(16(F$$

$$+ h_2 + 4h_1)\sinh\left(hM\right)\left(M\left(h - y\right) + \sinh\left(My\right)\right) + \left(F + h_2 + 4h_1\right)\left(1 + Bi^2\right)$$

$$\left(-3 + 2Mi^2 y^2\right) \sinh\left(2My\right) + 16 G_c + (h_1 + 4h_2)^2 y^2 \sinh\left((h2 + y)\right),$$

$$L_3 = \left(1 + Mi^2\right) y - 4h_2^3\left(F + h_1\right) Mi^4\left(1 + Bi^2\right) y^2 - 4\left(F + h_2\right) Bi^2\left(1 + Mi^2\right) y^3$$

$$(h_1 - h_2)(-8F^2 - 2h_2\left(4 + h_1 + 3y\right) - 2Fy + h_2^2\left(F + 2h_1\right) Mi^2 - 2h_1^2\left(F + h_2\right)^2$$

$$Bi Mi^4) + 2(1 + F\left(F + 2h_1\right) Mi^2 + \left(F + h_2 + 4h_1\right)^2 Mi^4) y^2 + \left(F^2\left(1 + h_1^2\right)\right).$$

## 9.3 Graphical analysis

In order to provide a comparative study of the results by the two methods, we have plotted graphs for both solutions. Left panel depicts the perturbation solution whereas right panel for the numerical solution. Graphical analysis is carried out for the dimensionless temperature, concentration and velocity fields. Numerical values for the rates of heat and mass transfer at the left wall are given in the Tables 9.1 and 9.2 respectively.

The values of different parameters in all the graphs are taken as follows:

$$\eta = 1, \gamma = \pi/4, a = 0.5, b = 0.7, d = 0.8, x = 0, G_t = 1, G_c = 2, Bi = 0.5,$$

$$Mi = 0.5, Du = 1, Br = 0.25, Sc = 0.5, Sr = 0.7.$$

The graphs of temperature profile showed that the temperature decreases with an increase in the values of Bi and it increases by increasing Mi. The decrease in the temperature with increase in the heat transfer Biot-number is very rapid for $Bi \leq 1$. It is noted that for $Bi \geq 1$

corresponding decrease in temperature becomes very slow (see Fig. 9.1). This fact is totally opposite to the case of increase in Mi in which temperature not only increases by increasing $Mi$ but also such increase is constant and very slow (Fig. 9.2).

Fig. 9.3 showed that the temperature increases when $G_t$ is increased. Increase in the temperature is smaller for small values of $G_t$ but this increase becomes much for higher values of $G_t$. However this increase in temperature is observed throughout the channel. Unlike the case of $G_t$, small increase in temperature is observed when $G_c$ increases but this increase is not uniform throughout the channel. Near the right wall the temperature tends to decrease very slowly by increasing $G_c$ (see Fig. 9.4).

Figs. 9.5-9.7 are plotted to analyze the concentration profile in detail. It is found that the concentration increases when $Mi$ is increased. This increase is large for $Mi \leq 1$ and very slow for large values of Mi (see Fig. 9.5). It is noticed from Figs. 9.6 and 9.7 that the concentration generally decreases with an increase in $G_t$ and $G_c$. This decrease is much and constant for $G_t$ and $G_c$ respectively. However concentration profile showed different behavior near the right wall. The concentration here increases which is quite different for its behavior at all other parts of the channel. Figs. 9.8 and 9.9 depict that the value of velocity increases by increasing $G_t$ but it remains constant with variation in $G_c$. The velocity in both cases is increased for increasing $G_t$ and $G_c$ in the region where "y" is negative and decreased for positive y.

Tables 9.1 and 9.2 are plotted for the numerical values of rates of heat and mass transfer. It is seen that solution up to first order perturbation gives an agreement with the numerical results up to second decimal place. Table 9.1 shows that heat transfer rate is increasing function of Biot-number. However the Grashoff numbers and mass transfer Biot number reduce the heat transfer rate (see Table 9.1). Table 9.2 illustrates that the mass transfer rate at the wall tends to decrease when $Bi$, $Mi$, $G_c$ and $G_t$ are increased.
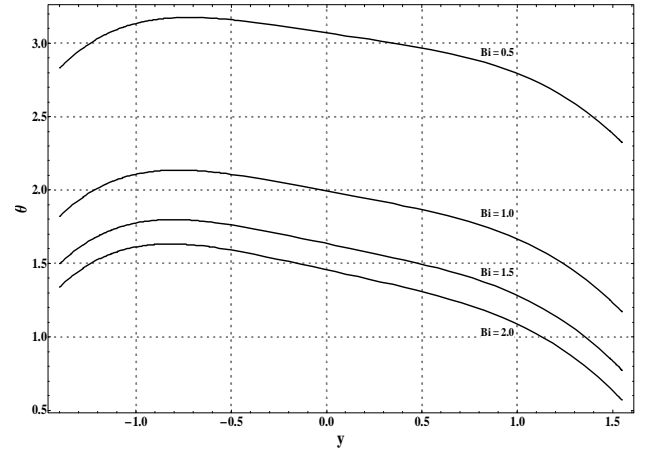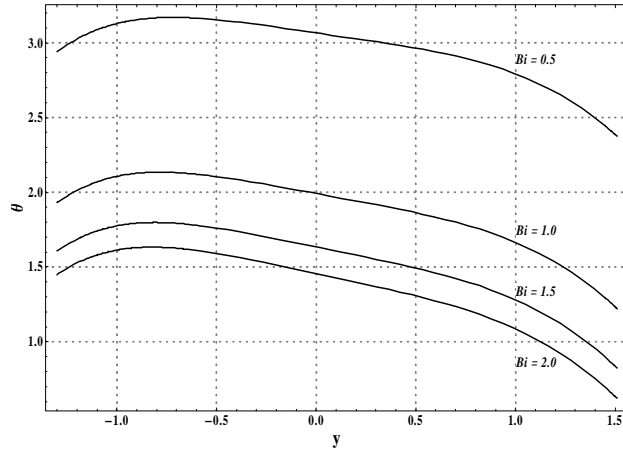
Fig. 9.1 Effect of $Bi$ on the temperature profile.
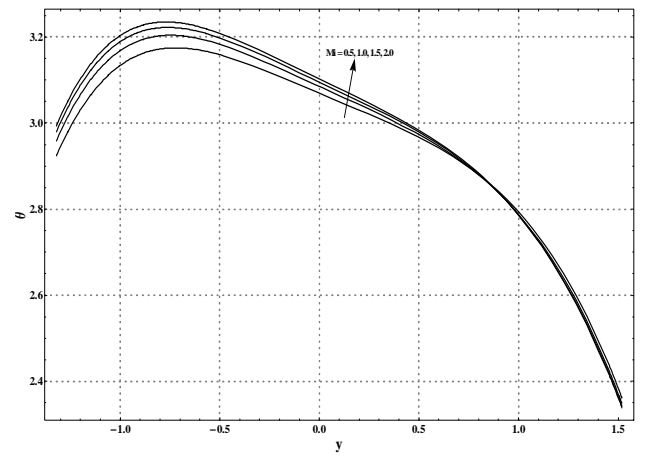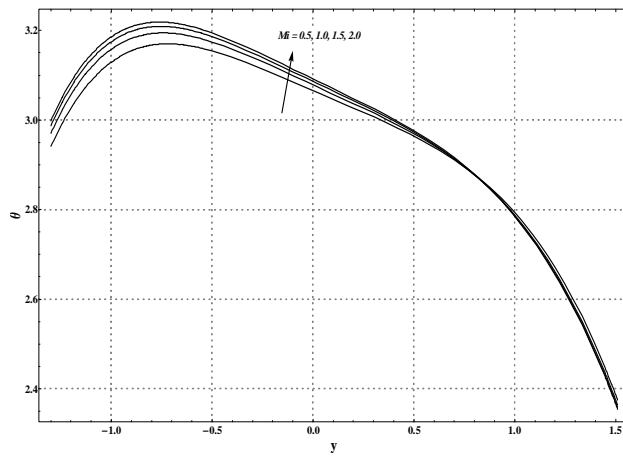


Fig. 9.2 Effect of $Mi$ on the temperature profile.

127
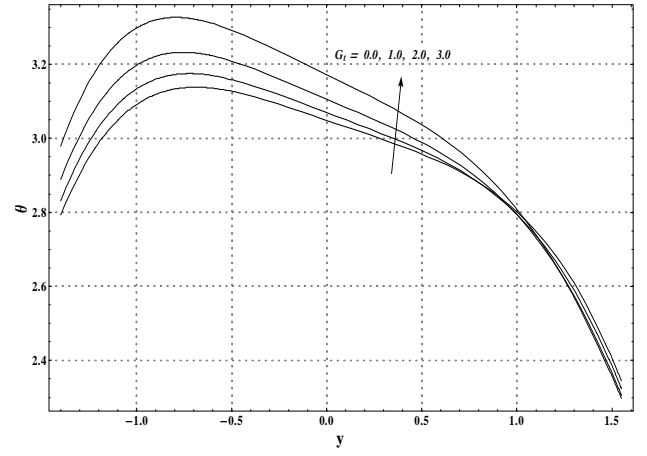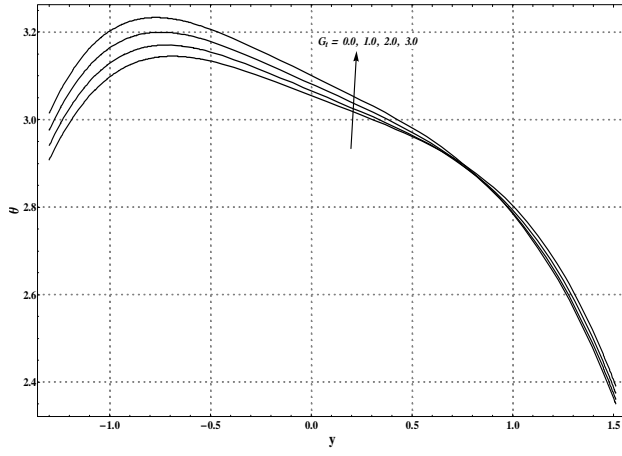
Fig. 9.3 Effect of $G_t$ on the temperature profile.
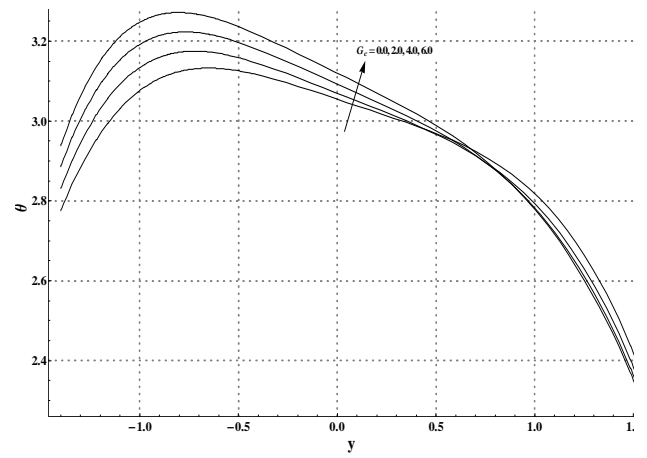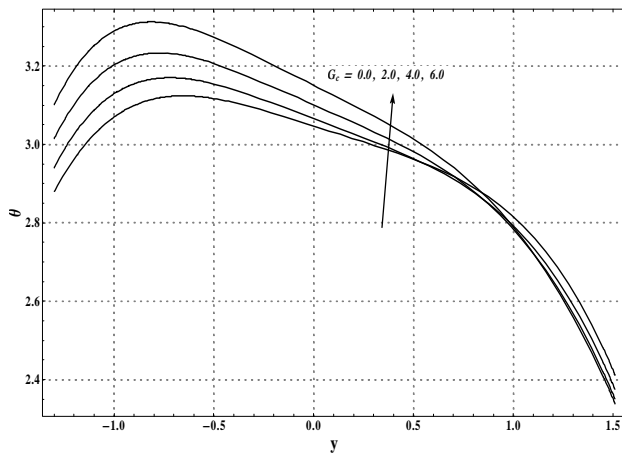


Fig. 9.4 Effect of $G_c$ on the temperature profile.

128
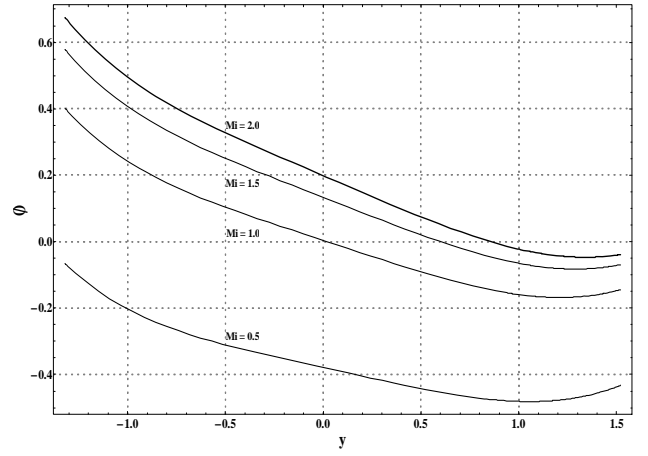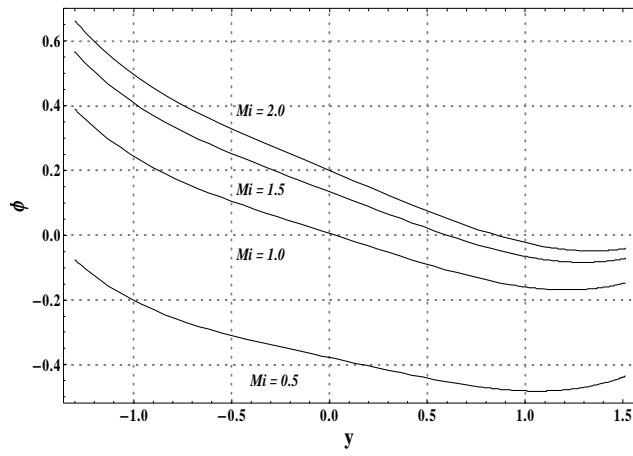
Fig. 9.5 Effect of $Mi$ on the concentration profile.
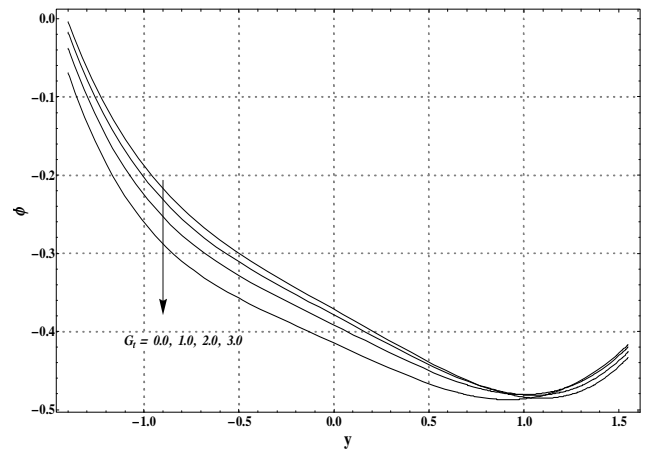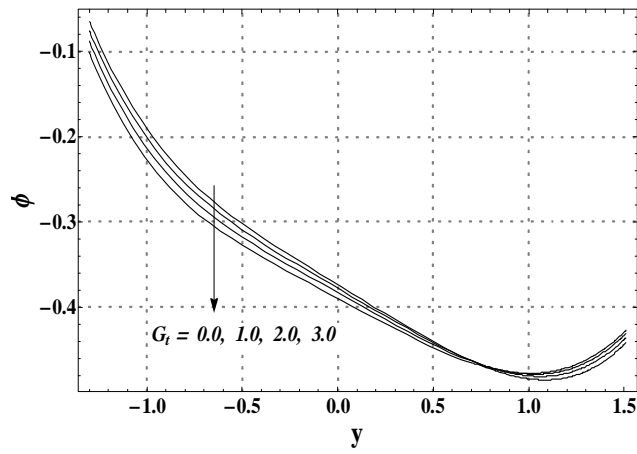


Fig. 9.6 Effect of $G_t$ on the concentration profile.
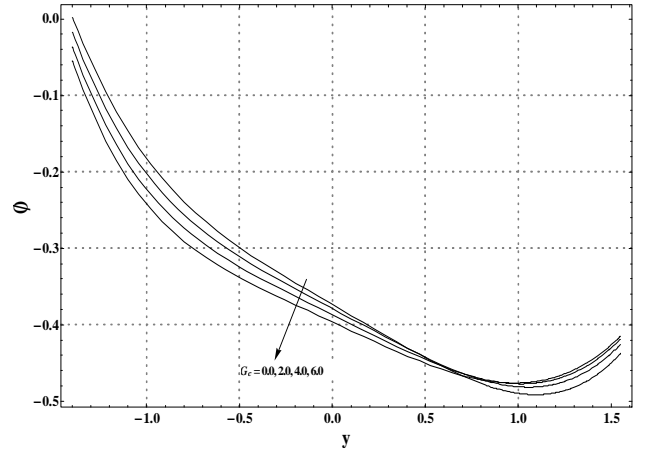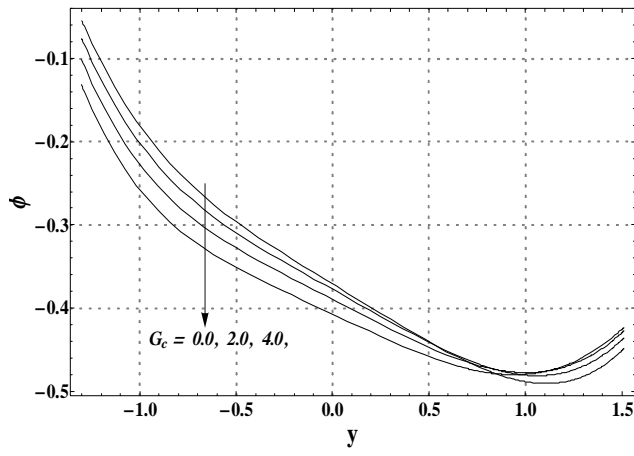
Fig. 9.7 Effect of $G_c$ on the concentration profile.
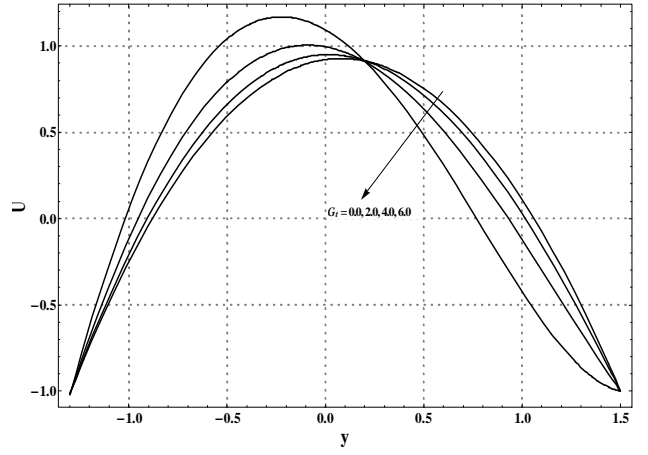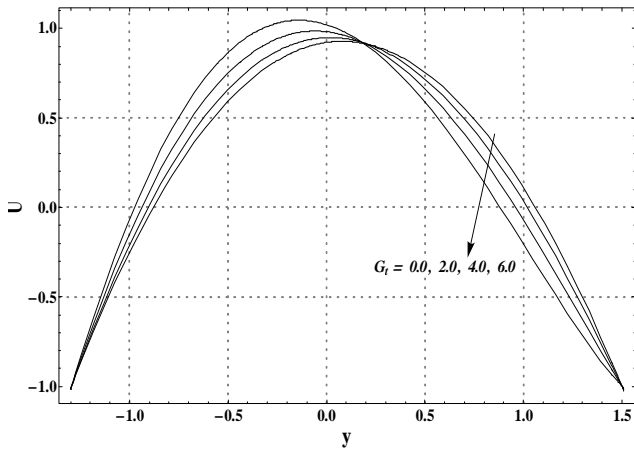



Fig. 9.8 Effect of $G_t$ on the velocity profile.
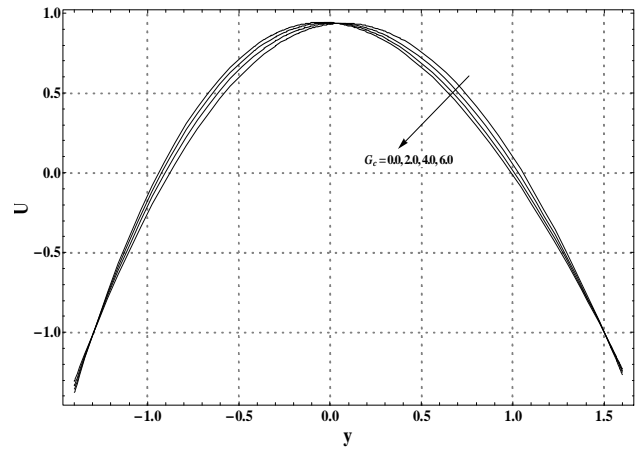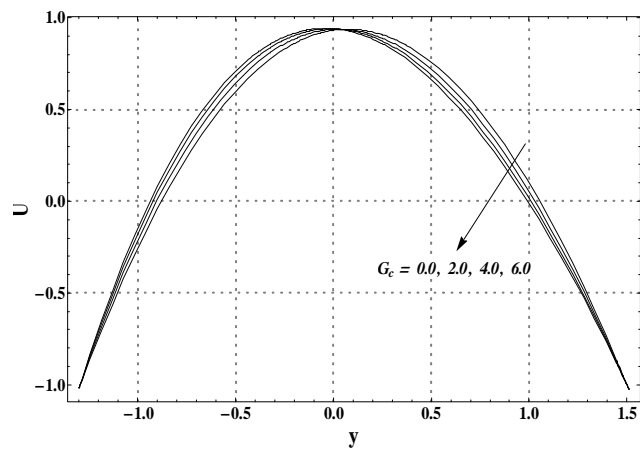
Fig. 9.9 Effect of $G_c$ on the Velocity profile.

| $Bi$ | $G_t$ | $G_c$ | $Mi$ | $-\theta'(h_1)$ | $-\left(\theta'\right)^*(h_1)$ |
|---|---|---|---|---|---|
| 0.5 | 1.0 | 2.0 | 0.5 | 1.1935 | 1.1927 |
| 1.0 | | | | 1.2343 | 1.2343 |
| 1.5 | | | | 1.2550 | 1.2560 |
| 2.0 | | | | 1.2674 | 1.2693 |
| | 0.0 | | | 1.2988 | 1.3018 |
| | 1.0 | | | 1.2674 | 1.2693 |
| | 2.0 | | | 1.2411 | 1.2402 |
| | 3.0 | | | 1.2197 | 1.2176 |
| | | 0.0 | | 1.2439 | 1.2349 |
| | | 1.0 | | 1.2310 | 1.2254 |
| | | 2.0 | | 1.2197 | 1.2176 |
| | | 3.0 | | 1.2100 | 1.2110 |
| | | | 0.5 | 1.2100 | 1.2110 |
| | | | 1.0 | 1.1999 | 1.2006 |
| | | | 1.5 | 1.1955 | 1.1967 |
| | | | 2.0 | 1.1930 | 1.1951 |

Table 9.1. Numerical values of heat transfer rate at $h_1$.

| $Bi$ | $G_t$ | $G_c$ | $Mi$ | $\phi'(h_1)$ | $\left(\phi'\right)^*(h_1)$ |
|---|---|---|---|---|---|
| 0.5 | 1.0 | 2.0 | 0.5 | 0.2190 | 0.2187 |
| 1.0 | | | | 0.2179 | 0.2179 |
| 1.5 | | | | 0.2174 | 0.2176 |
| 2.0 | | | | 0.2171 | 0.2173 |
| | 0.0 | | | 0.2219 | 0.2225 |
| | 1.0 | | | 0.2171 | 0.2173 |
| | 2.0 | | | 0.2139 | 0.2141 |
| | 3.0 | | | 0.2125 | 0.2139 |
| | | 0.0 | | 0.2142 | 0.2146 |
| | | 1.0 | | 0.2131 | 0.2141 |
| | | 2.0 | | 0.2125 | 0.2139 |
| | | 3.0 | | 0.2125 | 0.2140 |
| | | | 0.5 | 0.2125 | 0.2140 |
| | | | 1.0 | 0.1346 | 0.1360 |
| | | | 1.5 | 0.0903 | 0.0914 |
| | | | 2.0 | 0.0618 | 0.0625 |

Table 9.2. Numerical values of mass transfer rate at $h_1$.

# Chapter 10

# Variable viscosity effects on the peristaltic motion of a third-order fluid

The effect of variable viscosity on the peristaltic motion of MHD non-Newtonian fluid in a channel is studied. Constitutive relations of third order fluid are employed in the development of flow. Mathematical analysis is presented when no-slip condition is no longer valid. The series solutions for stream function, longitudinal velocity and pressure gradient are first derived and then discussed in detail. The pressure rise and frictional forces are monitored through numerical integration. Physical interpretation is also made.

## 10.1  Governing problem

We consider an incompressible magnetohydrodynamic (MHD) third order fluid with variable viscosity in a uniform channel of width $2d_1$. A uniform magnetic field $\mathbf{B}_0$ is applied in the $y-$direction. The induced magnetic and electric fields are not considered. Wave shapes are written as

$$H(\overline{X}, \overline{t}) = d_1 + a_1 cos\left[\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right],$$ 

(10.1)

where c is the velocity of propagation of the wave, $a_1$ is the wave amplitude, $\lambda$ is the wavelength, $\bar{t}$ is the time, $\overline{X}$ is the direction of wave propagation and $\overline{Y}$ is transverse to $\overline{X}$.

The constitutive equations for the third order fluid are given through Eqs. (1.17-1.19). Hence we directly write the following governing equations for two dimensional flow of MHD third order fluid as

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{10.2}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XX}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{Y}} - \sigma B_0^2 \overline{U}, \tag{10.3}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \overline{U}\frac{\partial}{\partial \overline{X}} + \overline{V}\frac{\partial}{\partial \overline{Y}}\right)\overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{YY}}}{\partial \overline{Y}}, \tag{10.4}$$

$$
\begin{aligned}
\bar{S}_{\bar{X}\bar{X}} =\ & \mu(2\bar{U}_{\bar{X}}) + \alpha_1(2\bar{U}_{\bar{X}\bar{t}} + 2\bar{U}\bar{U}_{\bar{X}\bar{X}} + 2\bar{V}\bar{U}_{\bar{X}\bar{Y}} + 4\bar{U}_{\bar{X}}^2 + 2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} + 2\bar{V}_{\bar{X}}^2) + \alpha_2 \\
& (4\bar{U}_{\bar{X}}^2 + \bar{U}_{\bar{Y}}^2 + \bar{V}_{\bar{X}}^2 + 2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}) + \beta_1(2\bar{U}_{\bar{X}\bar{t}\bar{t}} + 2\bar{U}_{\bar{t}}\bar{U}_{\bar{X}\bar{X}} + 4\bar{U}\bar{U}_{\bar{X}\bar{X}\bar{t}} + 2\bar{V}_{\bar{t}} \\
& \bar{U}_{\bar{X}\bar{Y}} + 4\bar{V}\bar{U}_{\bar{X}\bar{Y}\bar{t}} + 12\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{t}} + 4\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}} + 14\bar{U}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 10\bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} - \\
& 6\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 6\bar{U}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{X}} + 6\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{t}} + 6\bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} + 4\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}} + 8\bar{U}_{\bar{X}}^3 \\
& + 8\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} + 2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}^2\bar{U}_{\bar{X}\bar{X}\bar{X}} + 4\bar{U}\bar{V}\bar{U}_{\bar{X}\bar{X}\bar{Y}} + 2\bar{U}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{X}} + 2 \\
& \bar{V}^2\bar{U}_{\bar{X}\bar{Y}\bar{Y}}) + \beta_2(8\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{t}} + 8\bar{U}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 8\bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} + 16\bar{U}_{\bar{X}}^3 + 8\bar{U}_{\bar{X}} \\
& \bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} + 4\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}^2 + 2\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{t}} + 2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{X}} - 2\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{X}} + \\
& 2\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{Y}} + 4\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}^2 + 2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}} + 2\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{X}} \\
& - 2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 2\bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}}) + \beta_3(16\bar{U}_{\bar{X}}^3 + 4\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}^2 + \\
& 4\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}^2 + 8\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}),
\end{aligned}
$$

135

$$\bar{S}_{\bar{X}\bar{Y}} = \mu(\bar{U}_{\bar{Y}} + \bar{V}_{\bar{X}}) + \alpha_1(\bar{U}_{\bar{Y}\bar{t}} + \bar{U}\bar{U}_{\bar{X}\bar{Y}} + \bar{V}\bar{U}_{\bar{Y}\bar{Y}} + \bar{V}_{\bar{X}\bar{t}} + \bar{U}\bar{V}_{\bar{X}\bar{X}} - \bar{V}\bar{U}_{\bar{X}\bar{X}} + 2\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}} - 2\bar{V}_{\bar{X}}\bar{U}_{\bar{X}})$$

$$+\beta_1(\bar{U}_{\bar{Y}\bar{t}\bar{t}} + \bar{U}_{\bar{t}}\bar{U}_{\bar{X}\bar{Y}} + \bar{V}_{\bar{t}}\bar{U}_{\bar{Y}\bar{Y}} + \bar{V}_{\bar{X}\bar{t}\bar{t}} + \bar{U}_{\bar{t}}\bar{V}_{\bar{X}\bar{X}} + \bar{V}_{\bar{t}}\bar{U}_{\bar{X}\bar{X}} + 2\bar{U}\bar{V}_{\bar{X}\bar{X}\bar{t}} + 4\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{t}} +$$

$$2\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}} - 2\bar{U}_{\bar{X}}\bar{V}_{\bar{X}\bar{t}} - 4\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{t}} + \bar{U}^2\bar{U}_{\bar{X}\bar{X}\bar{Y}} + \bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}} - \bar{U}\bar{U}_{\bar{X}}\bar{V}_{\bar{X}\bar{X}} + \bar{U}^2\bar{V}_{\bar{X}\bar{X}\bar{X}}$$

$$-5\bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 4\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{X}} + \bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}} + \bar{V}^2\bar{U}_{\bar{Y}\bar{Y}\bar{Y}} + \bar{V}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{X}} + 3\bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}}$$

$$-\bar{V}^2\bar{U}_{\bar{X}\bar{X}\bar{Y}} - 4\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{U}\bar{V}\bar{U}_{\bar{X}\bar{Y}\bar{Y}} + 2\bar{U}\bar{V}\bar{U}_{\bar{X}\bar{X}\bar{X}} + 2\bar{V}\bar{U}_{\bar{Y}\bar{Y}\bar{t}} + 2\bar{U}\bar{U}_{\bar{X}\bar{Y}\bar{t}} + 3\bar{U}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}}$$

$$+5\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}} - 2\bar{V}\bar{U}_{\bar{X}\bar{X}\bar{t}} + 4\bar{U}_{\bar{X}}^2\bar{U}_{\bar{Y}} + 4\bar{V}_{\bar{X}}\bar{U}_{\bar{X}}^2 + 4\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}^2 + 4\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}}^2)$$

$$+\beta_2(8\bar{U}_{\bar{X}}^2\bar{U}_{\bar{Y}} + 6\bar{U}_{\bar{Y}}^2\bar{V}_{\bar{X}} + 2\bar{U}_{\bar{Y}}^3 + 8\bar{V}_{\bar{X}}\bar{U}_{\bar{X}}^2 + 6\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}^2 + 2\bar{V}_{\bar{X}}^3)$$

$$\beta_3(8\bar{U}_{\bar{X}}^2\bar{U}_{\bar{Y}} + 8\bar{U}_{\bar{X}}^2\bar{V}_{\bar{X}} + 2\bar{U}_{\bar{Y}}^3 + 6\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}^2 + 6\bar{U}_{\bar{Y}}^2\bar{V}_{\bar{X}} + 2\bar{V}_{\bar{X}}^3),$$

$$\bar{S}_{\bar{Y}\bar{Y}} = \mu(-2\bar{U}_{\bar{X}}) + \alpha_1(-2\bar{U}_{\bar{X}\bar{t}} - 2\bar{U}\bar{U}_{\bar{X}\bar{X}} - 2\bar{V}\bar{U}_{\bar{X}\bar{Y}} + 4\bar{U}_{\bar{X}}^2 + 2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} + 2\bar{U}_{\bar{Y}}^2)$$

$$+\alpha_2(4\bar{U}_{\bar{X}}^2 + \bar{U}_{\bar{Y}}^2 + \bar{V}_{\bar{X}}^2 + 2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}})$$

$$+\beta_1(-2\bar{U}_{\bar{X}\bar{t}\bar{t}} - 2\bar{U}_{\bar{t}}\bar{U}_{\bar{X}\bar{X}} - 4\bar{U}\bar{U}_{\bar{X}\bar{X}\bar{t}} - 2\bar{V}_{\bar{t}}\bar{U}_{\bar{X}\bar{Y}} - 4\bar{V}\bar{U}_{\bar{X}\bar{Y}\bar{t}}$$

$$+12\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{t}} + 2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}} + 4\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{t}} + 6\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{t}} + 10\bar{U}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} - 2\bar{U}^2\bar{U}_{\bar{X}\bar{X}\bar{X}}$$

$$-4\bar{U}\bar{V}\bar{U}_{\bar{X}\bar{X}\bar{Y}} + 4\bar{U}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{X}} + 6\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}} - 6\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{X}} + 14\bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} - 2\bar{V}^2\bar{U}_{\bar{X}\bar{Y}\bar{Y}}$$

$$+2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}} + 6\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{Y}} - 8\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} - 8\bar{U}_{\bar{X}}^3)$$

$$+\beta_2(2\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{t}} + 2\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}\bar{X}} - 2\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{X}} + 2\bar{U}\bar{U}_{\bar{Y}}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{V}\bar{U}_{\bar{Y}}\bar{U}_{\bar{Y}\bar{Y}}$$

$$-4\bar{U}_{\bar{Y}}^2\bar{U}_{\bar{X}} - 8\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}} + 2\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{t}} + 2\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{t}} + 2\bar{U}\bar{V}_{\bar{X}}\bar{V}_{\bar{X}\bar{X}} - 2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} +$$

$$2\bar{U}\bar{V}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} + 2\bar{V}\bar{V}_{\bar{X}}\bar{U}_{\bar{Y}\bar{Y}} - 4\bar{V}_{\bar{X}}^2\bar{U}_{\bar{X}} + 8\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{t}} + 8\bar{U}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{X}} + 8\bar{V}\bar{U}_{\bar{X}}\bar{U}_{\bar{X}\bar{Y}} - 16\bar{U}_{\bar{X}}^3$$

$$+\beta_3(-16\bar{U}_{\bar{X}}^3 - 4\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}^2 - 4\bar{U}_{\bar{X}}\bar{V}_{\bar{X}}^2 - 8\bar{U}_{\bar{X}}\bar{U}_{\bar{Y}}\bar{V}_{\bar{X}}),$$

where $(\overline{U}, \overline{V})$ and $\overline{P}$ are the velocity components and pressure in the fixed frame $(\overline{X}, \overline{Y})$ and the subscripts denote the partial derivatives

If $(\overline{u}, \overline{v})$ and $\overline{p}$ indicate the velocity components and pressure in the wave frame then defining

$$\overline{x} = \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u} = \overline{U} - c, \quad \overline{v} = \overline{V}, \quad \overline{p}(\overline{x}, \overline{y}) = \overline{P}(\overline{X}, \overline{Y}, \overline{t}) \qquad (10.5)$$

and using

$$x = \frac{\overline{x}}{\lambda},\; y = \frac{\overline{y}}{d_1},\; u = \frac{\overline{u}}{c},\; v = \frac{\overline{v}}{c\delta},\; \delta = \frac{d_1}{\lambda},\; h = \frac{H}{d_1},\; d = \frac{a_1}{d_1},\; p = \frac{a^2\overline{p}}{c\lambda\mu_0},$$

$$S = \frac{d_1}{\mu_0 c}\overline{S}\left(\overline{x}\right),\; \mu(y) = \frac{\overline{\mu}(\overline{y})}{\mu_0},\; M = \left(\frac{\sigma}{\mu_0}\right)^{1/2}B_0 d_1,\; \upsilon_0 = \frac{\mu_0}{\rho},\; Re = \frac{\rho c d_1}{\mu_0},$$

$$t = \frac{c\overline{t}}{\lambda},\; u = \frac{\partial\psi}{\partial y},\; v = -\frac{\partial\psi}{\partial x}, \gamma_2 = \frac{\beta_2 c^2}{\mu_0 d_1^2}, \gamma_3 = \frac{\beta_3 c^2}{\mu_0 d_1^2}, \tag{10.6}$$

we obtain under the long wavelength and low Reynolds number approximation the following expressions

$$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu(y)\frac{\partial^2\psi}{\partial y^2} + 2\Gamma\left(\frac{\partial^2\psi}{\partial y^2}\right)^3\right] - M^2\left(\frac{\partial\psi}{\partial y} + 1\right) = 0, \tag{10.7}$$

$$0 = -\frac{\partial p}{\partial y}, \tag{10.8}$$

in which $\Gamma(=\gamma_2 + \gamma_3)$ is the Deborah number, $\psi$ is the stream function, $\upsilon_0$ is the constant kinematic viscosity, $M$ is the Hartman number, Re is the Reynolds number, $\rho$ is the fluid density, $\sigma$ is the electrical conductivity, $\overline{\mu}(\overline{y})$ is the viscosity function, $\delta$ is the wave number, $\mu_0$ is the constant viscosity and continuity equation is automatically satisfied. Further Eq. (10.8) indicates that $p \neq p(y)$. From Eqs. (10.7) and (10.8) we have

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2} + 2\Gamma\left(\frac{\partial^2\psi}{\partial y^2}\right)^3\right) - M^2\frac{\partial^2\psi}{\partial y^2} = 0. \tag{10.9}$$

The subjected boundary conditions in dimensionless form become

$$\psi = 0,\quad \frac{\partial^2\psi}{\partial y^2} = 0,\quad at\; y = 0,$$

$$\psi = F,\quad \frac{\partial\psi}{\partial y} + \beta_1 S_{xy} = -1,\quad at\; y = h, \tag{10.10}$$

$$h(x)=1 + a\cos(2\pi x),\; F = \int_0^1\frac{\partial\psi}{\partial y}dy. \tag{10.11}$$

In above equations $\mu(y) = e^{-\alpha y}$ or $\mu(y) = 1 - \alpha y$ for $(\alpha \ll 1)$, $\alpha$ is the viscosity parameter, $\beta_1$

is the non-dimensional velocity slip parameter and

$$S_{xy} = \mu(y)\frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left(\frac{\partial^2 \psi}{\partial y^2}\right)^3. \tag{10.12}$$

Pressure rise per wavelength $\Delta p_\lambda$ and frictional force $F_\lambda$ (at the wall) are defined by the following expression

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx} dx. \tag{10.13}$$

$$F_\lambda = \int_0^1 h\left(-\frac{dp}{dx}\right) dx. \tag{10.14}$$

In the next section the problem consisting of Eqs. (10.9) and (10.10) will be solved for small Deborah number $\Gamma$ and small viscosity parameter $\alpha$.

## 10.2   Series expressions

Letting

$$
\begin{aligned}
\psi &= \psi_0 + \Gamma\psi_1 + .... \\
F &= F_0 + \Gamma F_1 + .... \\
p &= p_0 + \Gamma p_1 + ....
\end{aligned}
$$

and

$$
\begin{aligned}
\psi_0 &= \psi_{00} + \alpha\psi_{01} + .... \\
\psi_1 &= \psi_{10} + \alpha\psi_{11} + .... \\
F_0 &= F_{00} + \alpha F_{01} + .... \\
F_1 &= F_{10} + \alpha F_{11} + .... \\
p_0 &= p_{00} + \alpha p_{01} + .... \\
p_1 &= p_{10} + \alpha p_{11} + ....
\end{aligned}
$$

138

we arrive at the following systems:

System for $\psi_{00}$

$$-\frac{\partial p_{00}}{\partial x} + \frac{\partial^3 \psi_{00}}{\partial y^3} - M^2\left(\frac{\partial \psi_{00}}{\partial y} + 1\right) = 0, \tag{10.15}$$

$$\frac{\partial^4 \psi_{00}}{\partial y^4} - M^2\frac{\partial^2 \psi_{00}}{\partial y^2} = 0,$$

$$\psi_{00} = 0 = \frac{\partial^2 \psi_{00}}{\partial y^2} \qquad at \ y = 0,$$

$$\psi_{00} = F_{00} \qquad \frac{\partial \psi_{00}}{\partial y} + \beta_1\frac{\partial^2 \psi_{00}}{\partial y^2} = -1 \qquad at \ y = h.$$

System for $\psi_{01}$

$$-\frac{\partial p_{01}}{\partial x} + \frac{\partial^3 \psi_{01}}{\partial y^3} - M^2\frac{\partial \psi_{01}}{\partial y} = \frac{\partial}{\partial y}\left(y\frac{\partial^2 \psi_{00}}{\partial y^2}\right), \tag{10.16}$$

$$\frac{\partial^4 \psi_{01}}{\partial y^4} - M^2\frac{\partial^2 \psi_{01}}{\partial y^2} = \frac{\partial^2}{\partial y^2}\left(y\frac{\partial^2 \psi_{00}}{\partial y^2}\right),$$

$$\psi_{01} = 0 = \frac{\partial^2 \psi_{01}}{\partial y^2}, \qquad at \ y = 0,$$

$$\psi_{00} = F_{01}, \qquad \frac{\partial \psi_{01}}{\partial y} + \beta_1\frac{\partial^2 \psi_{01}}{\partial y^2} = \beta y\frac{\partial^2 \psi_{00}}{\partial y^2} \qquad at \ y = h.$$

System for $\psi_{10}$

$$-\frac{\partial p_{10}}{\partial x} + \frac{\partial^3 \psi_{10}}{\partial y^3} - M^2\frac{\partial \psi_{10}}{\partial y} + 6\left(\frac{\partial^2 \psi_{00}}{\partial y^2}\right)^2 \frac{\partial^3 \psi_{00}}{\partial y^3} = 0, \tag{10.17}$$

$$\frac{\partial^4 \psi_{10}}{\partial y^4} - M^2\frac{\partial^2 \psi_{10}}{\partial y^2} + 6\frac{\partial}{\partial y}\left(\left(\frac{\partial^2 \psi_{00}}{\partial y^2}\right)^2 \frac{\partial^3 \psi_{00}}{\partial y^3}\right) = 0,$$

$$\psi_{10} = 0 = \frac{\partial^2 \psi_{10}}{\partial y^2}, \qquad at \ y = 0,$$

$$\psi_{10} = F_{10}, \qquad \frac{\partial \psi_{10}}{\partial y} + \beta_1\frac{\partial^2 \psi_{10}}{\partial y^2} + 2\beta_1\left(\frac{\partial^2 \psi_{00}}{\partial y^2}\right)^3 = 0 \qquad at \ y = h.$$

139

The solutions of above systems finally give

$$\psi = \frac{FMy \cosh[hM] + y\left(1 + FM^2\beta_1\right)\sinh[hM] - (F+h)\sinh[My]}{hM\cosh[hM] + \left(-1 + hM\beta_1^2\right)M\beta_1\sinh[hM]} + \alpha m_1 + \Gamma m_2, \quad (10.18)$$

$$u = \frac{1}{32\left(hMCosh[hM] + \left(-1 + hM\beta_1^2\right)M\beta_1\sinh[hM]\right)^4} \times \quad (10.19)$$

$$\left[\begin{array}{c} 72(F+h)^3 M^6 y\Gamma\cosh[My]\left(hM\cosh[hM] + \left(-1 + hM^2\beta_1\right)\sinh[hM]\right) \\ +6u_1 - u_2 + u_3 + u_4 - u_5 + \frac{1}{M}u_6 \end{array}\right],$$

$$\frac{dp}{dx} = -\frac{(F+h)M^3\left(\cosh[hM] + M\beta_1\sinh[hM]\right)}{hM\cosh[hM] + \left(-1 + hM\beta^2\right)M\beta_1\sinh[hM]} + \alpha n_1 + \Gamma n_2, \quad (10.20)$$

with

$$m_1 = \frac{(F+h)\,(m_3 + 2\sinh[hM]\,(m_4) + 2hM\cosh[hM]\,(m_5))}{8M(hM\cosh[hM] + (-1 + hM\beta_1^2)\,M\beta_1\sinh[hM])^2},$$

$$m_2 = \frac{(e^{-3My}e^{-3My}\,(F+h)^3\,M^4)m_9}{32(hM\cosh[hM] + (-1 + hM\beta_1^2)\,M\beta_1\sinh[hM])^4},$$

$$m_3 = -4My\left(h + \beta_1 + hM^2\beta_1\right) + 2My\left(2\beta_1 + h\left(-1 + 2M^2\beta_1\right)\right)\cosh[2hM]$$
$$+ M\left(-2h^2 + y^2 + h\left(-4 + M^2\left((-4 + h)h - y^2\right)\right)\beta_1\right)\sinh[My],$$

$$m_4 = 3y\left(-1 + hM^2\beta_1\right)\cosh[My] + M(-2h^2 + y^2 + h(-4 + M^2((-4 + h)$$
$$h - y^2))\beta_1)\sinh[My],$$

$$m_5 = 3y\cosh[My] + M\left(-y^2 + h\left(h + \beta_1\right)\right)\sinh[My],$$

$$m_6 = -3hM\cosh[3hM] + 12M\left(y + hM\left(h + 2\beta_1 - My\beta_1\right)\right)\sinh[hM]$$
$$+ \left(1 - 9hM^2\beta_1\right)\sinh[3hM],$$

$$m_7 = 3hM\cosh[3hM] - 12M\left(-y + hM\left(h + 2\beta_1 + My\beta_1\right)\right)\sinh[hM]$$
$$+ \left(-1 + 9hM^2\beta_1\right)\sinh[3hM],$$

$$m_8 = 12My\left(h + \beta_1\right) - 16My\beta_1\cosh[2hM] + 4My\beta_1\cosh[4hM]$$
$$-8y\sinh[2hM] + y\sinh[4hM] + h\cosh[hM](12M(-y\cosh[MY]$$
$$+hM\beta_1\sinh[My] + \sinh[3My])) + \sinh[3My],$$

$$m_9 = \left(1 - hM^2\beta_1\right)\sinh[hM] + e^{6My}\,(m_6) + e^{2My}\,(m_7) + 2e^{3My}M\,(m_8).$$

$u_1 = u_7 + u_8,$

$u_2 = 6(F + h)^3 M^6 y \Gamma (u_9),$

$u_3 = 2(F + h)^3 M^5 (1 + 3My) \Gamma (u_{10}),$

$u_4 = 2(F + h)^3 M^5 \Gamma (u_{11}) \, Sinh[My],$

$u_5 = 6(F + h)^3 M^5 \Gamma (Sinh[3hM] + 3hM (u_{12})) Sinh[My],$

$u_6 = 4(F + h)\alpha \left(hM Cosh[hM] + \left(-1 + hM^2 \beta\right) Sinh[hM]\right)^2 (u_{13} + 2Sinh[hM] (u_{14})),$

$u_7 = (F + h)^3 M^5 y \Gamma \cosh [3My] \left(hM \cosh [hM] + \left(-1 + hM^2 \beta_1\right) \sinh [hM]\right)$

$+ 32(hM \cosh [hM] + \left(-1 + hM^2 \beta_1\right) \sinh [hM])^3 ((-(F + h)M Cosh[My]$

$+ Sinh[hM] + FM (Cosh[hM] + M\beta_1 Sinh[hM]))),$

$u_8 = 2(F + h)^3 M^5 \Gamma Cosh[My](12hM \left(-1 - 3My + hM^2 \beta_1\right) Cosh[hM]$

$- 3hM Cosh[3hM]) + 12 \left(1 + M \left(3y + hM \left(h + \beta_1 - 3My\beta_1\right)\right)\right) \sinh [hM]$

$+ \left(1 - 9hM^2 \beta_1\right) \sinh [3hM],$

$u_9 = 12M (h + \beta_1) + 4M\beta(-4Cosh[2hM] + Cosh[4hM]) - 8Sinh[2hM] + Sinh[4hM],$

$u_{10} = 12M(h + \beta_1) + 4M\beta_1(-4Cosh[2hM] + Cosh[4hM]) - 8Sinh[2hM]$

$+ Sinh[4hM], u_{11} = Sinh[3hM] + M(4hM(-y + 3hM\beta_1)Cosh[hM] - 3hCosh[3hM]$

$+ 4(y + hM(3h + 6\beta_1 - My\beta_1))Sinh[hM] - 9hM\beta_1 Sinh[3hM],$

$u_{12} = ((-Cosh[3hM] + M(4hM\beta_1 Cosh[hM] + 4(h + 2\beta_1)Sinh[hM] - 3\beta_1 Sinh[3hM]))),$

$u_{13} = -4M \left(h + \beta_1 + h\beta_1 M^2\right) + 2M \left(2\beta_1 + h \left(-1 + 2M^2 \beta_1\right)\right) Cosh[2hM]$

$+ 3Sinh[2hM] - 4hM^2 \beta_1 Sinh[2hM] + 2hM Cosh[hM]((3 + M^2 \left(-y^2 + h(h + \beta_1)\right))$

$Cosh[My] + My Sinh[My]),$

$u_{14} = \left(-3 + M^2 \left(-2h^2 + y^2 + h \left(-1 + M^2 \left((-4 + h)h - y^2\right)\right) \beta_1\right)\right) Cosh[My]$

$+ My \left(-1 + hM^2 \beta_1\right) Sinh[My].$

$n_1 = \frac{(F+h)M n_3}{8(hM \cosh[hM] + (-1 + hM^2 \beta_1) \sinh[hM])^2},$

$n_2 = \frac{(F+h)^3 M^7 n_4}{16 \left(hM \cosh[hM] + \left(-1 + hM\beta^2\right) M\beta_1 \sinh[hM]\right)^4},$

$n3 = 4M \left(h + \beta_1 + hM^2 \beta_1\right) + 2M \left(h - 2 \left(1 + hM^2\right) \beta_1\right) \cosh [2hM] + \left(-3 + 4hM^2 \beta_1\right)$

$\sinh [2hM],$

$n_4 = 12M (h + \beta_1) + 4M\beta_1 \left(-4 \cosh [2hM] + \cosh [4hM]\right) - 8 \sinh [2hM] + \sinh [4hM].$

## 10.3   Results and discussion

In this section the main objective is to study the influence of various parameters of interest on the flow quantities. In particular the variations of Hartman number $(M)$, Deborah number $(\Gamma)$, amplitude ratio $(a)$, slip parameter $(\beta_1)$ and viscosity parameter $(\alpha)$ on the longitudinal velocity, pressure gradient, pressure rise $(\Delta p_\lambda)$ and frictional force $(F_\lambda)$ per wavelength have been examined.

Figs.10.1-10.3 show the effects of $M$, $\beta_1$ and $\Gamma$ on the longitudinal velocity $u$. The plots show that velocity at the center of the channel and near the walls has opposite behavior. Further the velocity at the center of the channel decreases with an increase in $M$ and $\beta_1$ but it increases by increasing $\Gamma$.

The variation of pressure gradient for certain values of $M$, $\beta_1$ and $\Gamma$ are depicted in the Figs 10.4-10.6 It is anticipated that the absolute value of pressure gradient increases with an increase in $M$ and $\beta_1$ but it decreases when $\Gamma$ is increased.

The pressure rise and frictional force per wavelength have been computed numerically. The pressure rise verses flow rate has been plotted in the Figs 10.7-10.11. It is noticed from Fig. 10.7 that the pressure rise decreases with an increase in $\beta_1$. It is found that magnitude of pressure rise per wavelength increases by increasing $M$ (see Fig. 10.8). Fig. 10.9 depicts an important phenomenon that is the graph for pressure rise for $\Gamma = 0$ is linear whereas in the case of non zero $\Gamma$ it is non linear. Fig 10.11 shows that the pressure rise increases with an increase in the amplitude ratio $a$. The free pumping flux increases by increasing $\alpha$ (see Fig.10.10).

Figs. 10.12-10.16 monitor the features of frictional force $(F_\lambda)$. It is clear that there exists a critical value of $\eta$ below which $F_\lambda$ resists the flow and above which it assists the flow. This critical value of $\eta$ decreases by increasing $\beta_1$ and $M$ (see Figs. 10.12 and 10.13). Contrary to this the critical value of $\eta$ increases with an increase in $a$ and $\alpha$ (see Figs 10.14 and 10.15). It is revealed from Fig. 16 that the effect of $\Gamma$ on frictional force is quiet opposite to that of pressure rise.

Another important fact of interest here is a comparison between the viscous and non-Newtonian fluids (see Figs. 10.3, 10.6, 10.9 and 10.14). In all these Figs. the case of viscous fluid is given by putting $\Gamma = 0$. Fig. 10.3 shows that velocity at the center of the channel in non-Newtonian fluid is larger when compared with a viscous fluid. Fig. 10.6 shows that the

non-Newtonian fluid exhibits larger pressure gradient than the viscous fluid.

Figs. 10.17-10.20 examine the trapping phenomenon. It is observed that the size of the trapped bolus decreases with an increase in $\beta_1$ and $M$. The size of trapped bolus also increases by increasing the viscosity parameter $\alpha$ (see Figs. 10.17 and 10.18). It is noted that by increasing the Deborah number $\Gamma$, the size of the bolus decreases whereas it increases by increasing $a$.
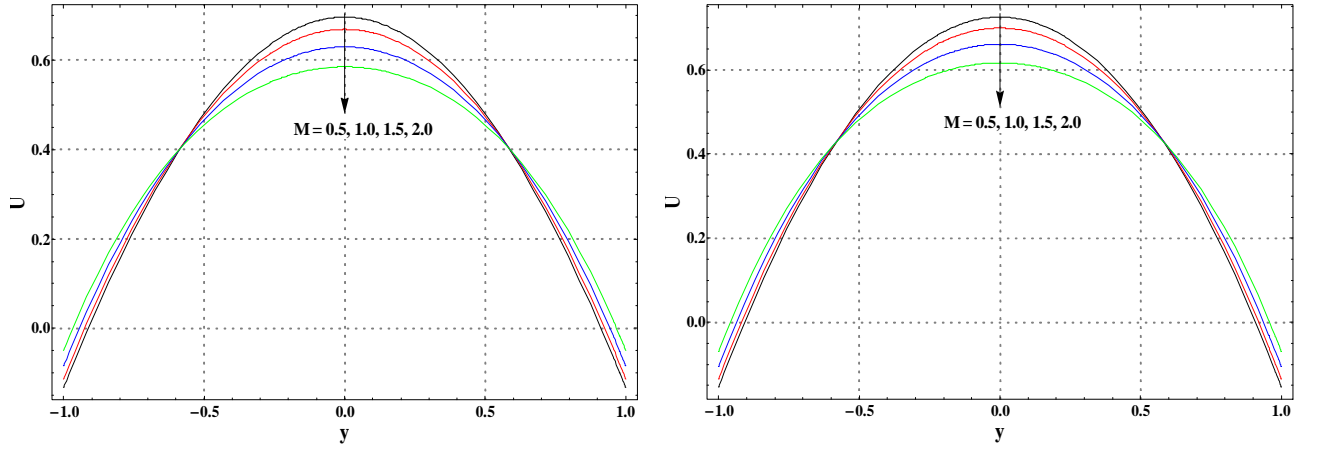
Fig. 10.1. Effect of $M$ on velocity $u$ when $\beta_1 = 0.2, \Gamma = 0.04, x = 1, \eta = 1.3$ and $a = 0.3$. Left panel is for $\alpha = 0$ and right panel is for $\alpha = 0.4$.
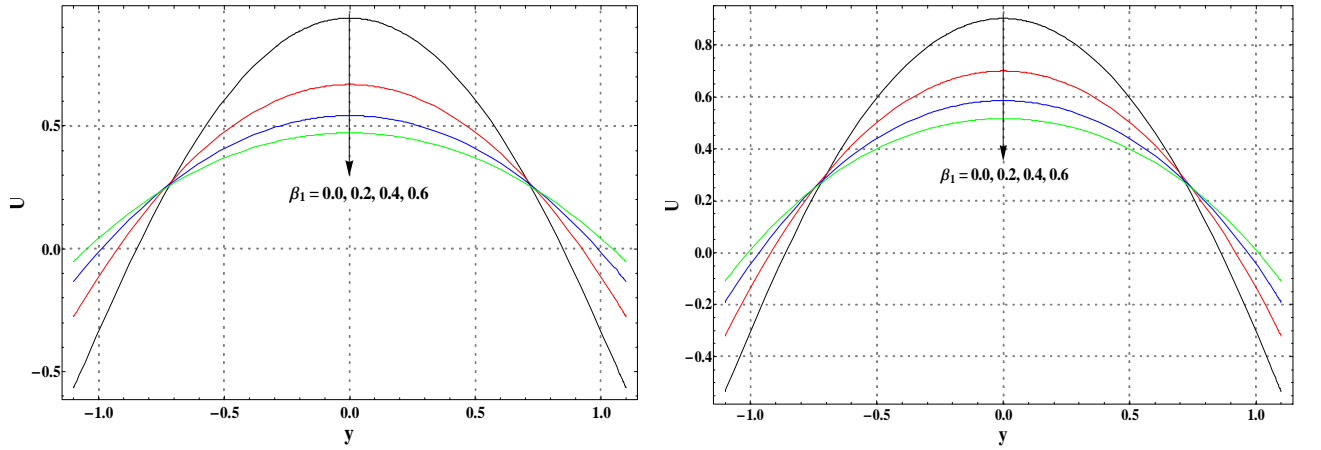


Fig. 10.2. Effect of $\beta_1$ on velocity $u$ when $M = 1, \Gamma = 0.04, x = 1, \eta = 1.3$ and $a = 0.3$. Left panel is for $\alpha = 0$ and right panel is for $\alpha = 0.4$.
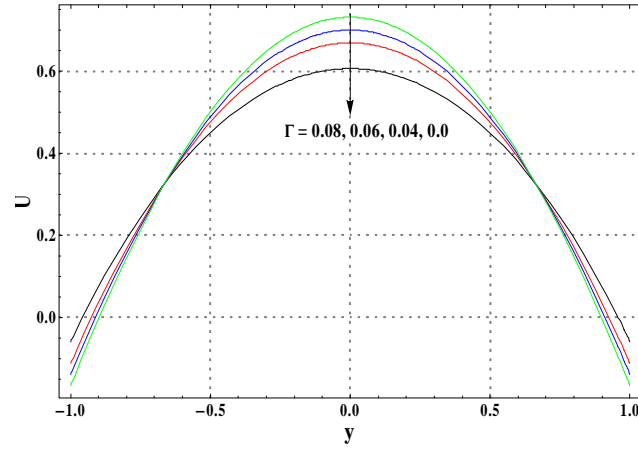
Fig. 10.3. Effect of $\Gamma$ on velocity $u$ when $M = 1$, $\beta_1 = 0.04$, $x = 1$, $\eta = 1.3$ and $a = 0.3$.
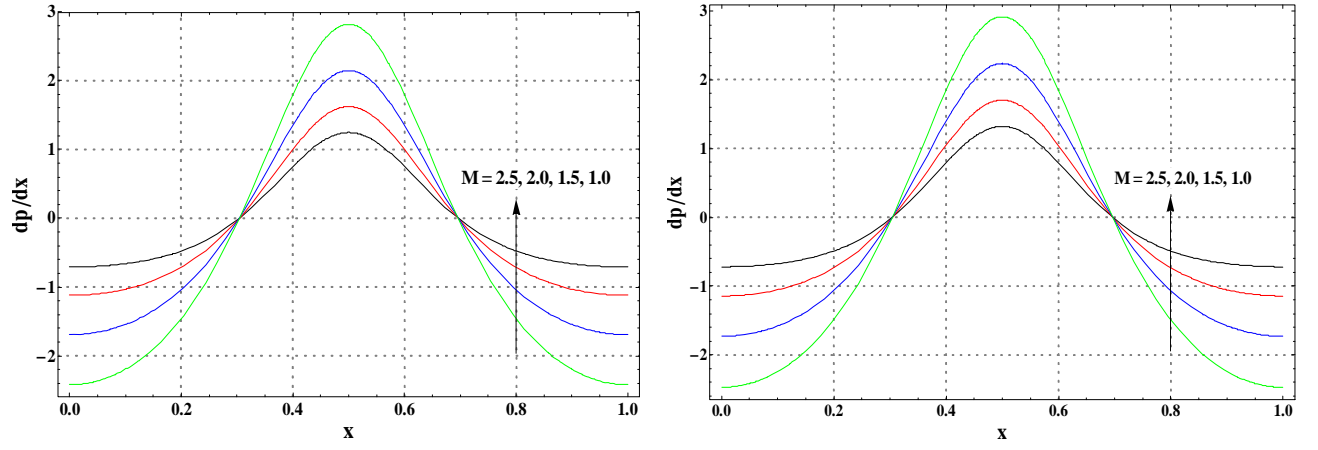


Fig. 10.4. Effect of $M$ on pressure gradient when $\beta_1 = 0.2$, $\Gamma = 0.001$, $\eta = 0.1$ and $a = 0.3$.
Left panel is for $\alpha = 0$ and right panel is for $\alpha = 0.4$.

146
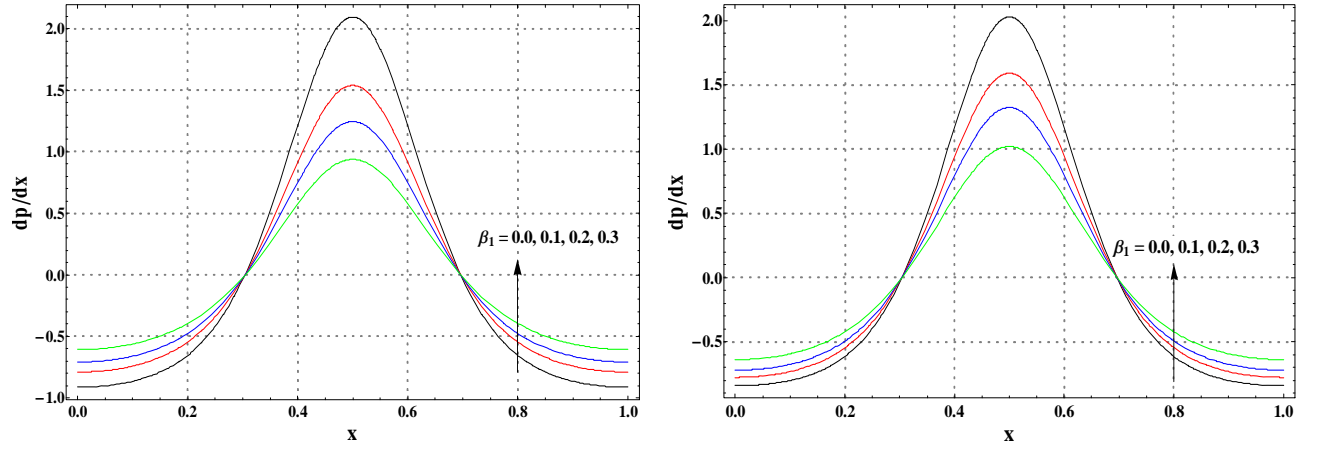
Fig. 10.5. Effect of $\beta_1$ on pressure gradient when $M = 1$, $\Gamma = 0.001$, $\eta = 0.1$ and $a = 0.3$. Left panel is for $\alpha = 0$ and right panel is for $\alpha = 0.4$.
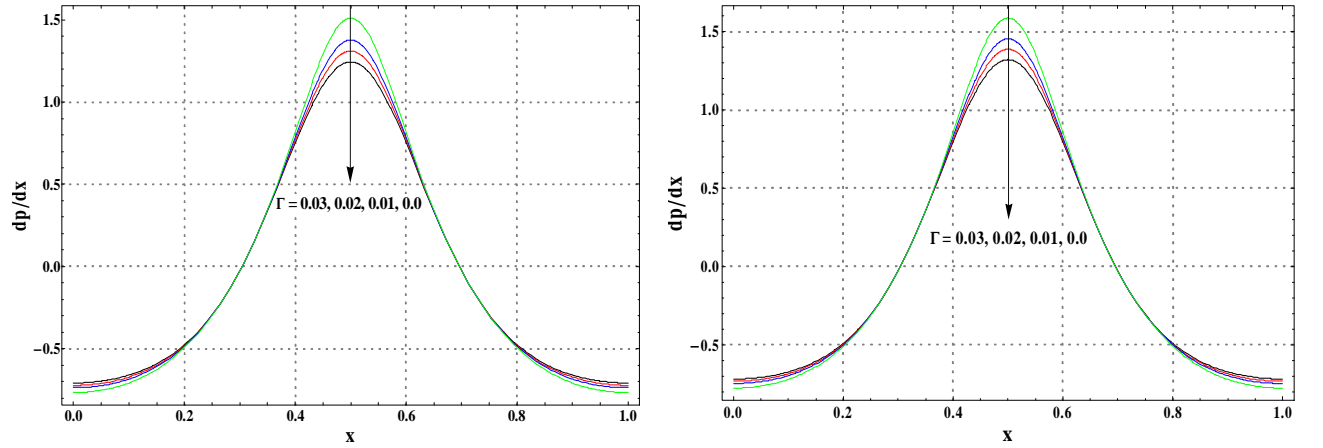


Fig. 10.6. Effect of $\Gamma$ on pressure gradient when $M = 1$, $\beta_1 = 0.2$, $\eta = 0.1$ and $a = 0.3$. Left panel is for $\alpha = 0$ and right panel is for $\alpha = 0.4$.
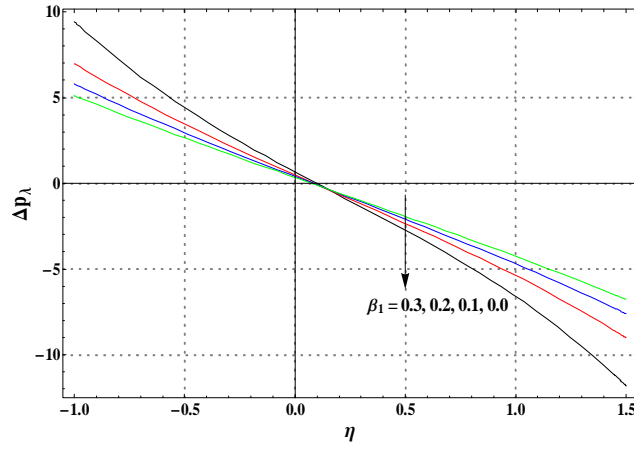
Fig. 10.7. Variation of $\beta_1$ on $\Delta p_\lambda$ when $\Gamma = 0.01$, $M = 1.5$, $a = 0.3$ and $\alpha = 0.2$.
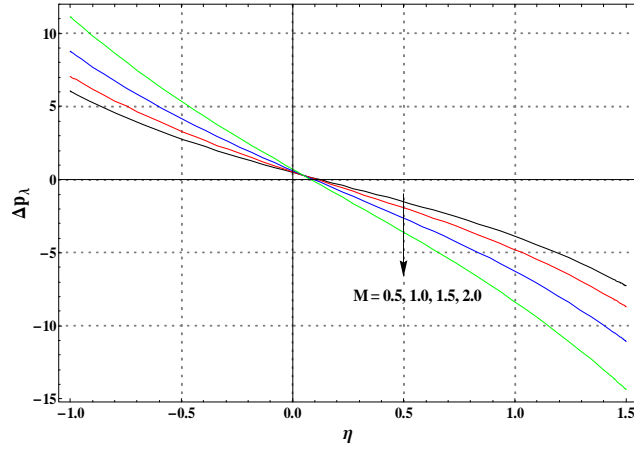


Fig. 10.8. Variation of $M$ on $\Delta p_\lambda$ when $\Gamma = 0.01$, $\beta_1 = 0.02$, $a = 0.3$ and $\alpha = 0.2$.
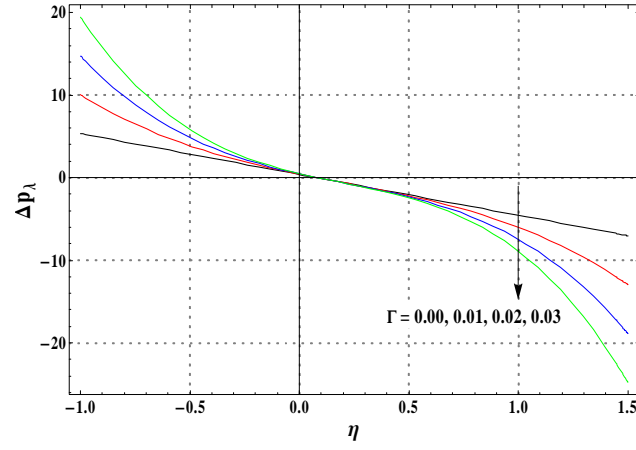
Fig. 10.9. Variation of $\Gamma$ on $\Delta p_\lambda$ when $\beta_1 = 0.02$, $M = 1.5$, $a = 0.3$ and $\alpha = 0.2$.
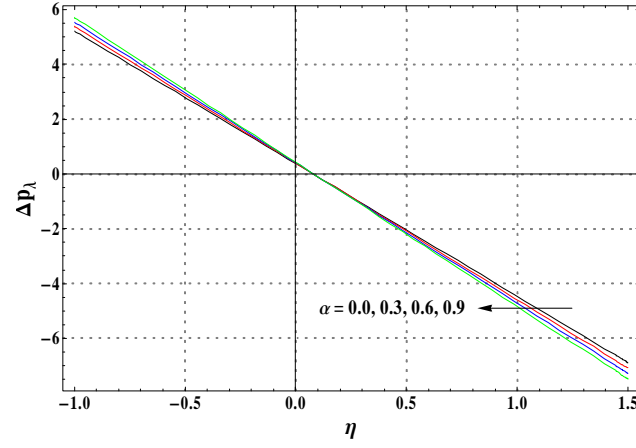


Fig. 10.10. Variation of $\alpha$ on $\Delta p_\lambda$ when $\beta_1 = 0.02$, $M = 1.5$, $a = 0.3$ and $\Gamma = 0.01$.
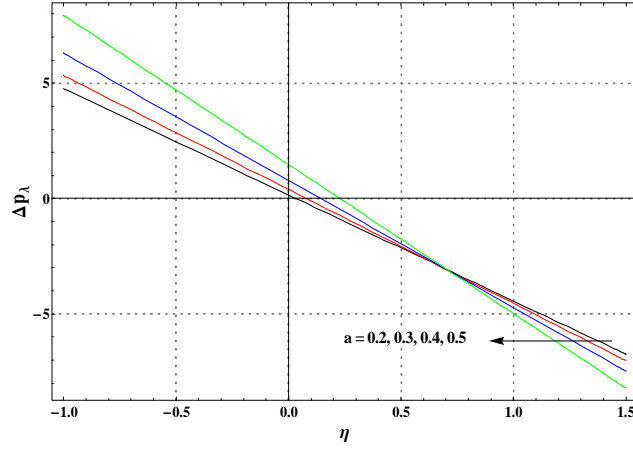
Fig. 10.11. Variation of $a$ on $\Delta p_\lambda$ when $\beta_1 = 0.02$, $M = 1.5$, $\alpha = 0.2$ and $\Gamma = 0.01$.
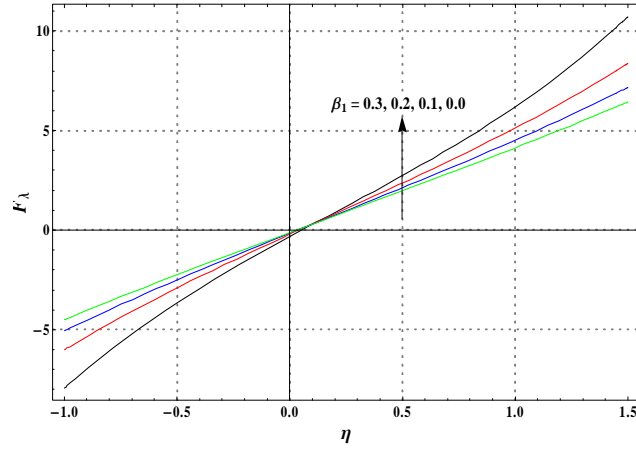


Fig. 10.12. Variation of $\beta_1$ on $F_\lambda$ when $a = 0.3$, $M = 1.5$, $\alpha = 0.2$ and $\Gamma = 0.01$.
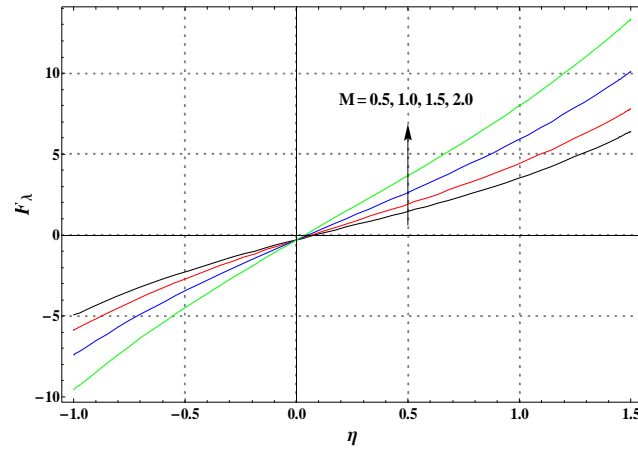
Fig. 10.13Variation of $M$ on $F_\lambda$ when $a = 0.3$, $\beta_1 = 0.02$ $\alpha = 0.2$ and $\Gamma = 0.01$.
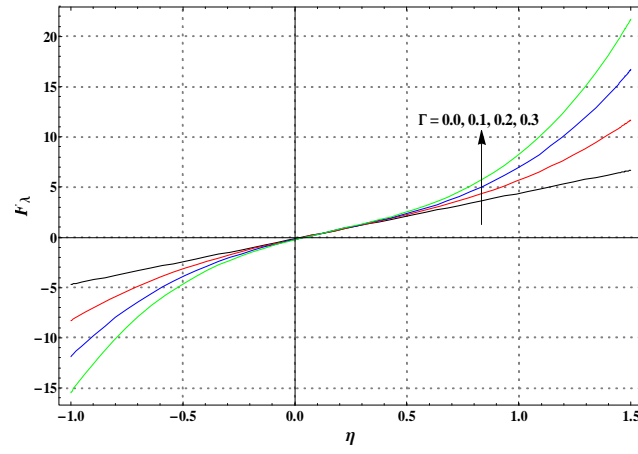


Fig. 10.14. Variation of $\Gamma$ on $F_\lambda$ when $a = 0.3$, $M = 1.5$, $\alpha = 0.2$ and $\beta_1 = 0.02$.
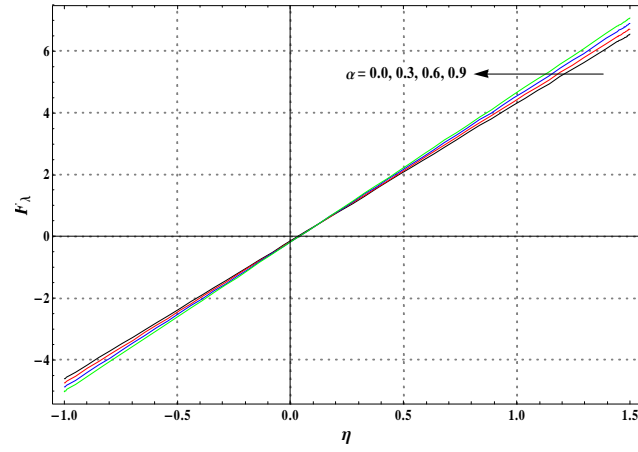
Fig. 10.15. Variation of $\alpha$ on $F_\lambda$ when $a = 0.3$, $M = 1.5$, $\Gamma = 0.01$ and $\beta_1 = 0.02$.
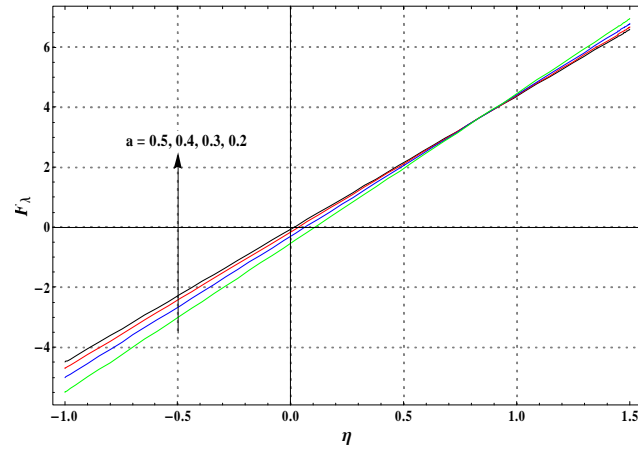


Fig. 10.16. Variation of $a$ on $F_\lambda$ when $\alpha = 0.2$, $M = 1.5$, $\Gamma = 0.01$ and $\beta_1 = 0.02$.
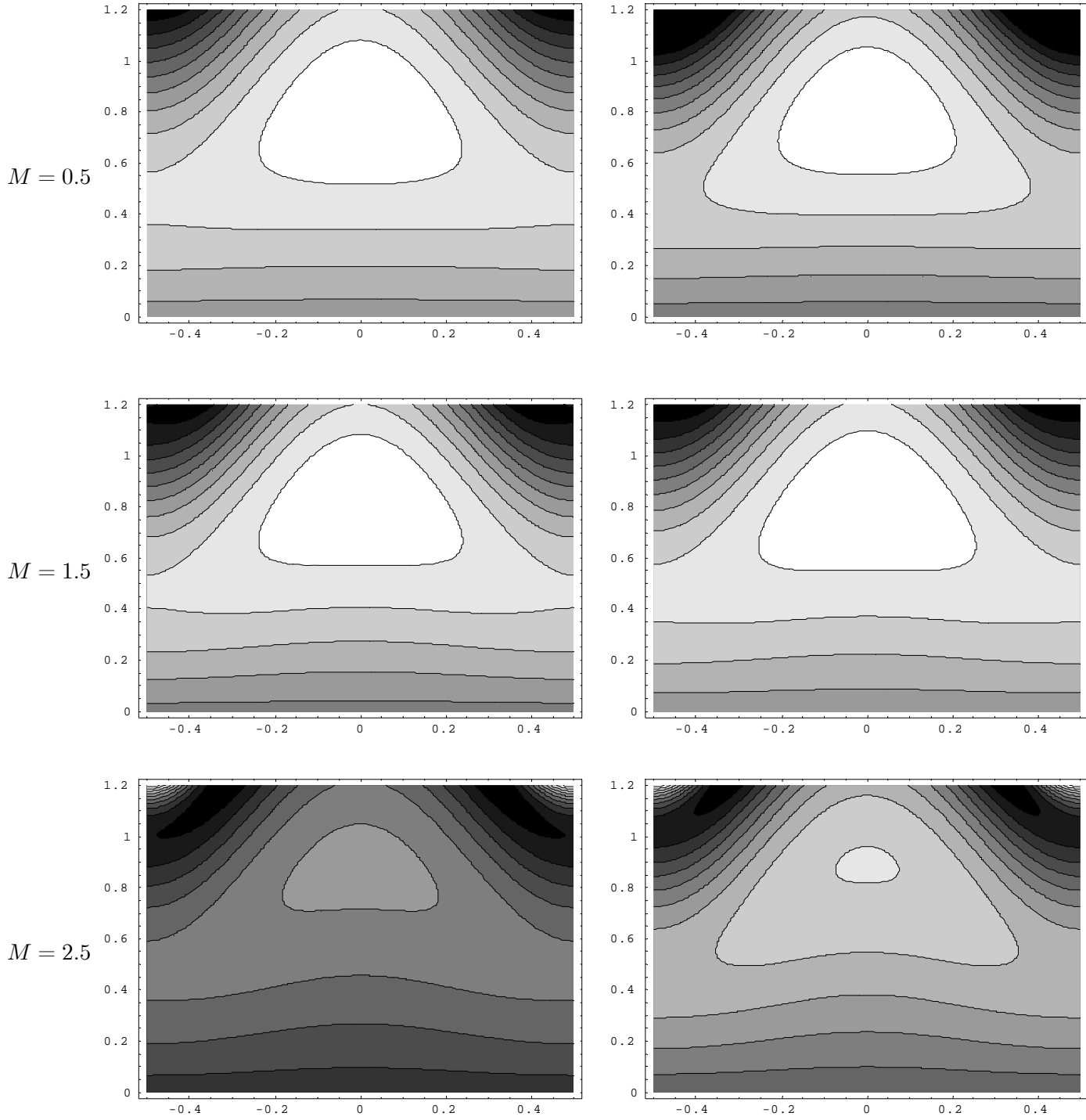
Fig. 10.17. Effect of $M$ on stream lines when $\Gamma = 0.04, \phi = 0.3, \eta = 1.1$, and $\beta = 0.2$ Left panels are for $\alpha = 0$, and right panels are for $\alpha = 0.4$.

153
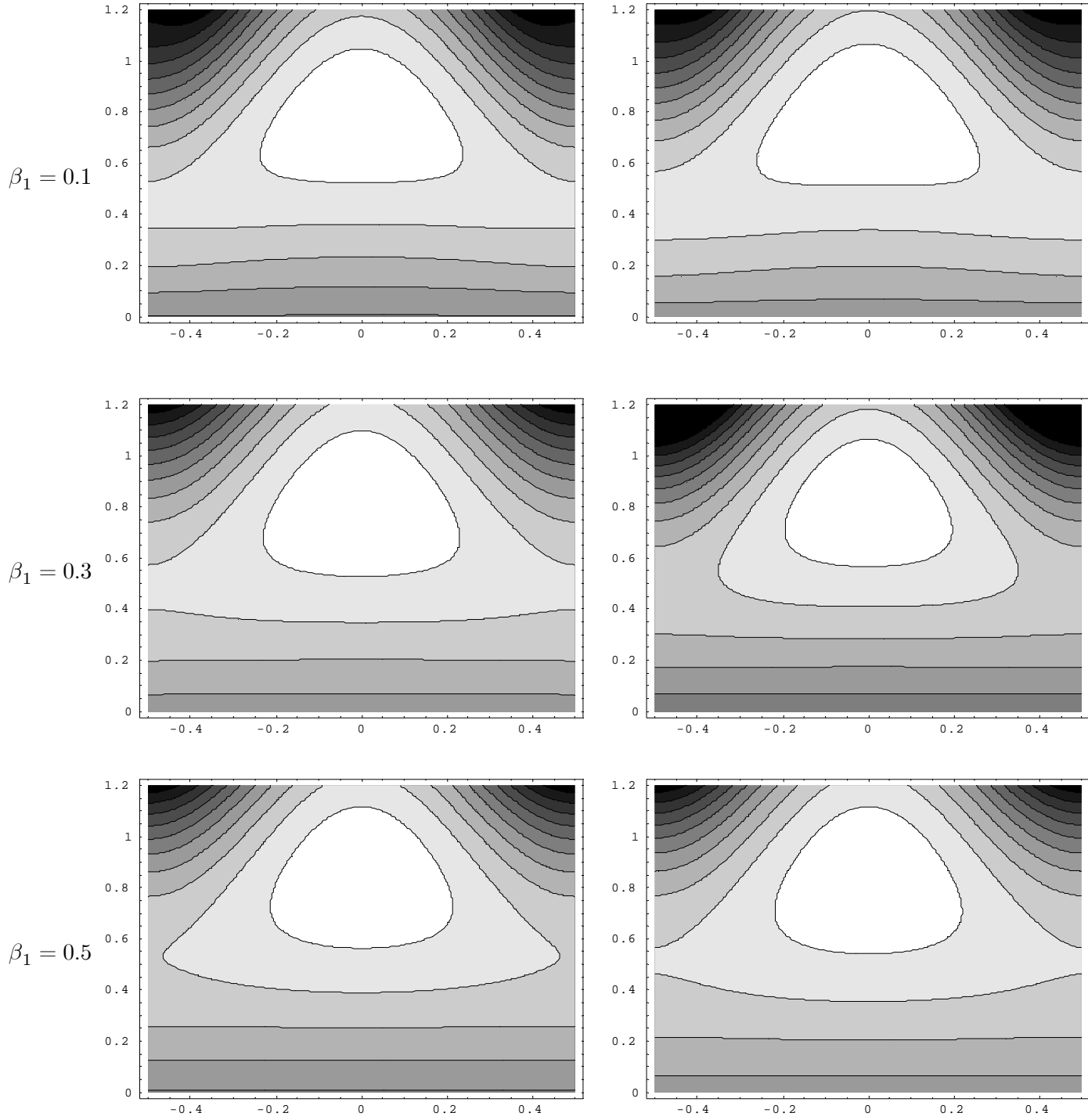
Fig. 10.18. Effect of $\beta_1$ on stream lines when $\Gamma = 0.04, a = 0.3, \eta = 1.1$ and $M = 0.5$. Left panels are for $\alpha = 0$ and right panels are for $\alpha = 0.4$.
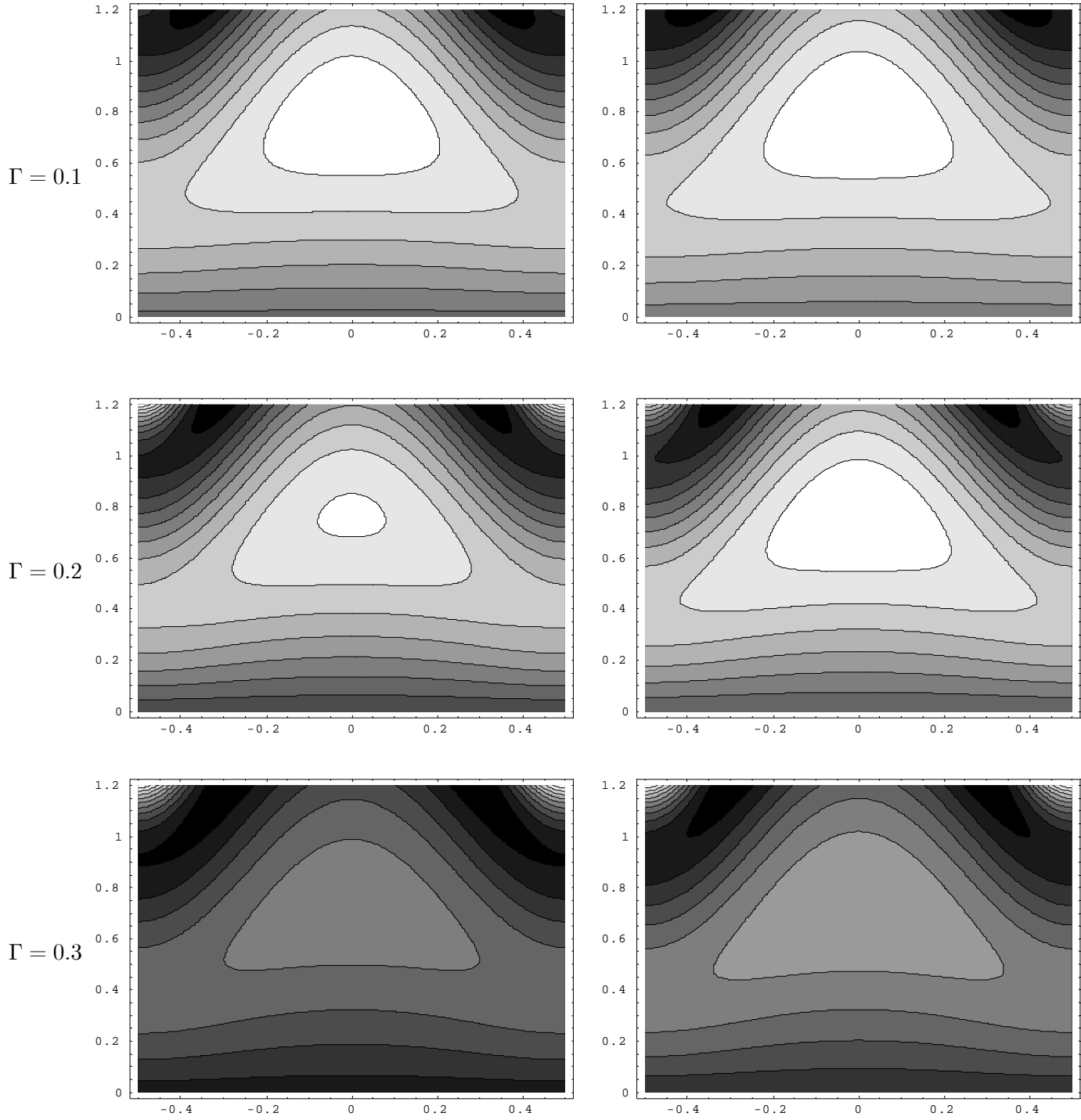
Fig. 10.19. Effect of $\Gamma$ on stream lines when $\beta_1 = 0.2, a = 0.3, \eta = 1.1$ and $M = 0.5$. Left panels are for $\alpha = 0$ and right panels are for $\alpha = 0.4$.

155
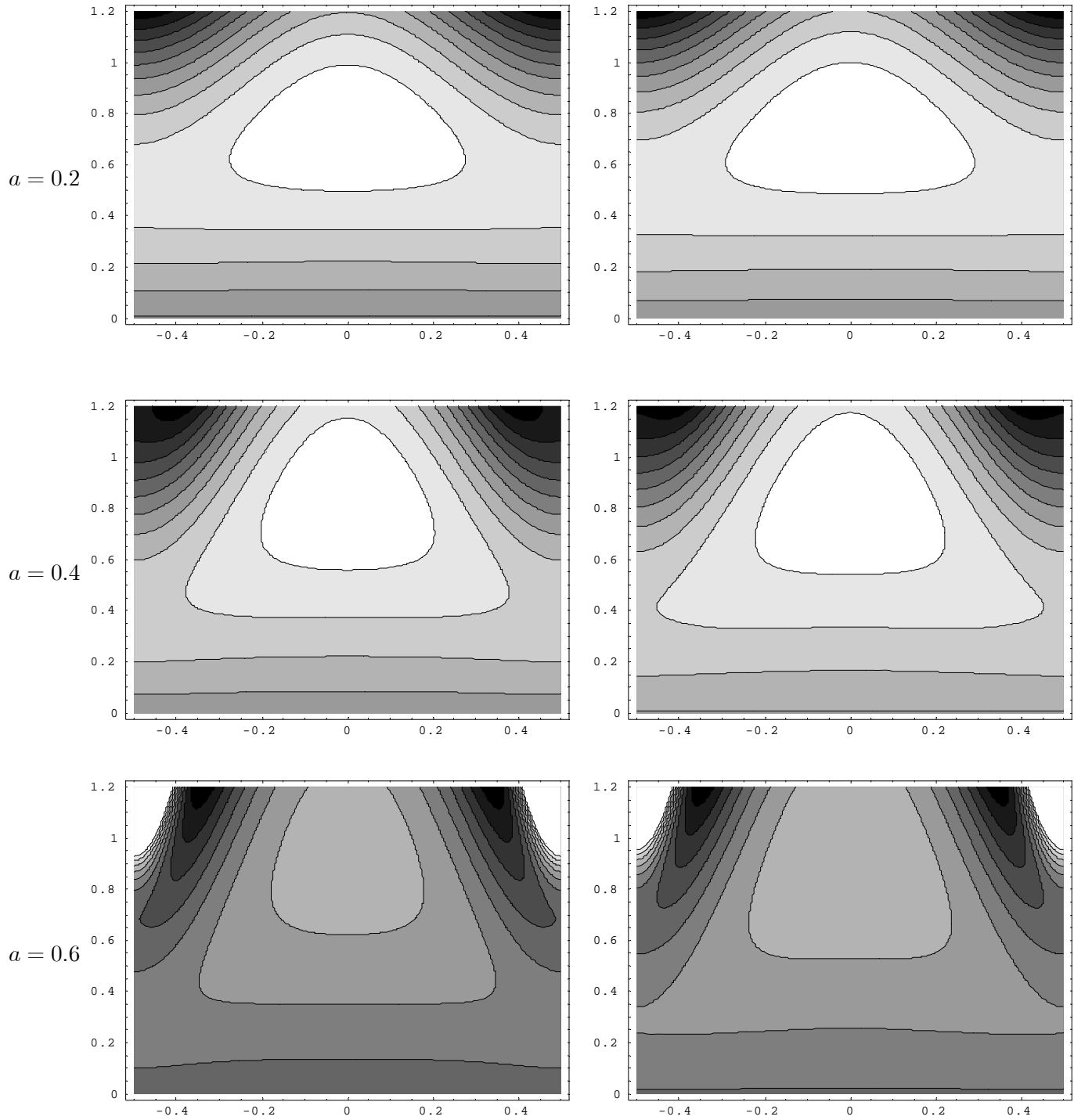
Fig. 10.20. Effect of $a$ on stream lines when $\beta_1 = 0.2, x = 1, \Gamma = 0.04, \eta = 1.1$ and $M = 0.5$.
Left panels are for $\alpha = 0$ and right panels are for $\alpha = 0.4$.

# Chapter 11

# Heat transfer analysis for peristaltic mechanism in variable viscosity fluid

This chapter extends the analysis of previous chapter in the presence of heat transfer. Thermal slip is also considered. Viscous dissipation is present. Perturbation solution is constructed and a comparative study between the cases of constant and variable viscosities is presented and analyzed.

## 11.1   Statement of the problem

Here we study an incompressible magnetohydrodynamic (MHD) third order fluid with variable viscosity in a uniform channel of width $2d_1$. The $\overline{X} - axis$ is selected along the centerline of the channel whereas $\overline{Y}$ is perpendicular to the $\overline{X} - axis$. The fluid is electrically conducting under an applied magnetic field $\mathbf{B}_0$ in the $\overline{Y}-$ direction. The electric and induced magnetic field effects are neglected. Sinusoidal waves propagating on the channel induce the flow. Such waves are described by the following equations

$$H(\overline{X}, \overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right). \tag{11.1}$$

In above expression $c$ is the speed of propagation of the wave, $a_1$ the wave amplitude, $\lambda$ the wavelength and $\overline{t}$ the time. Further the channel walls have a constant temperature $T_0$. The

governing equations are

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{11.2}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XX}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{Y}} - \sigma B_0^2 \overline{U}, \tag{11.3}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{YY}}}{\partial \overline{Y}}, \tag{11.4}$$

$$\rho \zeta \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) T = k \nabla^2 T + \overline{\Phi}, \tag{11.5}$$

where $\overline{\Phi}$ is the dimensional viscous dissipation term. In light of the following dimensionless quantities

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \ y = \frac{\overline{y}}{d_1}, \ u = \frac{\overline{u}}{c}, \ v = \frac{\overline{v}}{c\delta}, \ \delta = \frac{d_1}{\lambda}, \ h = \frac{H}{a}, \ a = \frac{a_1}{d_1}, \ p = \frac{d_1^2 \overline{p}}{c\lambda\mu_0}, \\
S &= \frac{d_1}{\mu_0 c} \overline{S}(\overline{x}), \ \mu(y) = \frac{\overline{\mu}(\overline{y})}{\mu_0}, \ M = \left( \frac{\sigma}{\mu_0} \right)^{1/2} B_0 d_1, \ \upsilon_0 = \frac{\mu_0}{\rho}, \ Re = \frac{\rho c d_1}{\mu_0}, \\
t &= \frac{c\overline{t}}{\lambda}, \ u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x}, \gamma_2 = \frac{\beta_2 c^2}{\mu_0 d_1^2}, \gamma_3 = \frac{\beta_3 c^2}{\mu_0 d_1^2}, \theta = \frac{T - T_0}{T_0}, \\
\mathrm{Pr} &= \frac{\mu_0 C_p}{K}, \ E = \frac{c^2}{C_p T_0}, \ \overline{\zeta} = \frac{\mu_0 c^2}{a^2} \zeta
\end{aligned} \tag{11.6}
$$

equations (11.2)-(11.5) give

$$\delta R_e [(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y}) \frac{\partial \psi}{\partial y}] = -\frac{\partial p}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - M^2 (\frac{\partial \psi}{\partial y} + 1), \tag{11.7}$$

$$-\delta^3 R_e [(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y}) \frac{\partial \psi}{\partial x}] = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y}, \tag{11.8}$$

$$\delta R_e [(\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y})] = E\Phi + \frac{1}{\mathrm{Pr}} (\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}), \tag{11.9}$$

where $C_p$ is the specific heat, $K$ the thermal conductivity, $\Gamma(= \gamma_2 + \gamma_3)$ the Deborah number, $\psi$ the stream function, $\upsilon_0$ the constant kinematic viscosity, $M$ the Hartman number, Re the Reynolds number, $\rho$ the fluid density, $\sigma$ the electrical conductivity, $\overline{\mu}(\overline{y})$ the viscosity function, $\delta$ the wave number, $\mu_0$ the constant viscosity, $Br(= \mathrm{Pr}\, E)$ the Brinkman number, $\Phi$ the dimensionless viscous dissipation term, Pr the Prandtl number and $E$ the Eckert number. The

governing problems subject to long wavelength and low Reynolds number are

$$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}S_{xy} - M^2\left(\frac{\partial \psi}{\partial y} + 1\right) = 0, \quad 0 = -\frac{\partial p}{\partial y}, \tag{11.10}$$

$$\frac{\partial^2 \theta}{\partial y^2} = -Br\zeta, \tag{11.11}$$

$$S_{xy} = \mu(y)\frac{\partial^2 \psi}{\partial y^2} + 2\Gamma\left(\frac{\partial^2 \psi}{\partial y^2}\right)^3, \quad \zeta = \mu(y)(\frac{\partial^2 \Psi}{\partial y^2})^2 + 2\Gamma(\frac{\partial^2 \Psi}{\partial y^2})^4, \tag{11.12}$$

$$\begin{aligned}
\psi &= 0, \quad \frac{\partial^2 \psi}{\partial y^2} = 0, \quad \frac{\partial \theta}{\partial y} = 0 \quad at \ \ y = 0, \\
\psi &= F, \quad \frac{\partial \psi}{\partial y} + \beta_1 S_{xy} = -1, \quad \theta\text{-}\beta_2\frac{\partial \theta}{\partial y}{=}0 \quad at \ \ y = h,
\end{aligned} \tag{11.13}$$

$$h(x){=}1 + \phi\cos(2\pi x), \quad F = \int_0^1 \frac{\partial \psi}{\partial y}dy, \tag{11.14}$$

where $\beta_2$ is the dimensionless thermal slip parameter, $\mu(y) = e^{-\alpha y}$ or $\mu(y) = 1 - \alpha y$ for $(\alpha \ll 1)$; $\alpha$ the viscosity parameter, $\beta_1$ the dimensionless velocity slip parameter, $\psi$ the stream function, $\theta$ the temperature and $F$ the dimensionless flow rate in wave frame. Further Eq. (11.10) indicates that $p \neq p(y)$ and finally we write

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2 \psi}{\partial y^2} + 2\Gamma\left(\frac{\partial^2 \psi}{\partial y^2}\right)^3\right) - M^2\frac{\partial^2 \psi}{\partial y^2} = 0. \tag{11.15}$$

Pressure rise per wavelength $\Delta p_\lambda$ and frictional force $F_\lambda$ (at the wall) are

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx}dx, \quad F_\lambda = \int_0^1 h\left(-\frac{dp}{dx}\right)dx. \tag{11.16}$$

## 11.2 Solution expressions

The solutions for small $\Gamma$ and $\alpha$ are

$$\psi = \frac{FMy\cosh[hM] + y\left(1 + FM^2\beta_1\right)\sinh[hM] - (F + h)\sinh[My]}{hM\cosh[hM] + \left(-1 + hM\beta_1^2\right)M\beta_1\sinh[hM]} + \alpha m_1 + \Gamma m_2, \tag{11.17}$$

$$\frac{dp}{dx} = -\frac{(F+h)\,M^3\,(\cosh{[hM]} + M\beta_1 \sinh{[hM]})}{hM\cosh{[hM]} + (-1 + hM\beta_1^2)\,M\beta_1 \sinh{[hM]}} + \alpha n_1 + \Gamma n_2, \qquad (11.18)$$

$$\begin{aligned}
\theta \;=\;& \frac{Br(F+h)^2 M^2 \left(-Cosh[2hM] + Cosh[2My] + 2M\left(M\left(h^2 - y^2 - 2h\gamma\right) + \beta_2 Sinh[2hM]\right)\right)}{8\,(hMCosh[hM] + (-1 + hM^2\beta_1)\,Sinh[hM])^2} \\
&+ \frac{Br\,(F+h)^2\,M\alpha}{192\,(hMCosh[hM] + (-1 + hM^2\beta_1)\,Sinh[hM])^3} \times \\
&(q_1 + M\,(q_2 - q_3 - q_4 + q_5)) \\
&+ \frac{Br\,(F+h)^4\,M^6\Gamma}{512\,(hMCosh[hM] + (-1 + hM^2\beta_1)\,Sinh[hM])^5} \times \\
&(L_1 + 4\,(L_2 + M\,(L_3 + L_4 + L_5 + L_6))) \qquad\qquad\qquad\qquad\qquad (11.19)
\end{aligned}$$

where the involved $m_i$ $(i = 1 - 8)$, $n_j$ $(j = 1, 2)$, $q_l$ $(l = 1 - 6)$ and $L_k$ $(k = 1 - 8)$ are given as follows:

$$m_1 = \frac{(F+h)(m_3 + 2\sinh[hM](m_4) + 2hM\cosh[hM](m_5))}{8M(hM\cosh[hM] + (-1 + hM\beta^2)M\beta\sinh[hM])^2},$$

$$m_2 = \frac{e^{-3My}e^{-3My}(F+h)^3 M^4 \left(\left(1 - hM^2\beta\right)\sinh[hM] + e^{6My}(m_6) + e^{2My}(m_7) + 2e^{3My}M(m_8)\right)}{32(hM\cosh[hM] + (-1 + hM\beta^2)M\beta\sinh[hM])^4},$$

$$m_3 = -4My\left(h + \beta + hM^2\beta\right) + 2My\left(2\beta + h\left(-1 + 2M^2\beta\right)\right)\cosh[2hM]$$
$$+ y\left(3 - 4hM^2\beta\right)\sinh[2hM], \quad m_4 = 3y\left(-1 + hM^2\beta\right)\cosh[My] + M(-2h^2$$
$$+ y^2 + h\left(-4 + M^2\left((-4 + h)h - y^2\right)\right)\beta)\sinh[My],$$

$$m_4 = 3y\left(-1 + hM^2\beta\right)\cosh[My] + M(-2h^2 + y^2 + h(-4 + M^2((-4 + h)h - y^2))\beta)$$
$$\sinh[My],$$

$$m_5 = 3y\cosh[My] + M\left(-y^2 + h\left(h + \beta\right)\right)\sinh[My],$$

$$m_6 = -3hM\cosh[3hM] + 12M\left(y + hM\left(h + 2\beta - My\beta\right)\right)\sinh[hM]$$
$$+ \left(1 - 9hM^2\beta\right)\sinh[3hM],$$

$$m_7 = 3hM\cosh[3hM] - 12M\left(-y + hM\left(h + 2\beta + My\beta\right)\right)\sinh[hM]$$
$$+ \left(-1 + 9hM^2\beta\right)\sinh[3hM],$$

$$m_8 = 12My\left(h + \beta\right) - 16My\beta\cosh[2hM] + 4My\beta\cosh[4hM] - 8y\sinh[2hM]$$
$$+ y\sinh[4hM] + h\cosh[hM](12M\left(-y\cosh[MY] + hM\beta\sinh[My] + \sinh[3My]\right)$$
$$+ \sinh[3My]).$$

$$n_1 = \frac{(F+h)Mn_3}{8\left(hM\cosh[hM]+(-1+hM^2\beta)\sinh[hM]\right)^2},$$

$$n_2 = \frac{(F+h)^3 M^7 n_4}{16\left(hM\cosh[hM]+\left(-1+hM\beta^2\right)M\beta\sinh[hM]\right)^4},$$

$$n_3 = 4M\left(h+\beta+hM^2\beta\right)+2M\left(h-2\left(1+hM^2\right)\beta\right)\cosh\left[2hM\right]+\left(-3+4hM^2\beta\right)\sinh\left[2hM\right],$$

$$n_4 = 12M\left(h+\beta\right)+4M\beta\left(-4\cosh\left[2hM\right]+\cosh\left[4hM\right]\right)-8\sinh\left[2hM\right]+\sinh\left[4hM\right].$$

$$q_1 = (3+hM^2(-3\beta+4M^2(2y^3+h(h\left(-2h-3\beta+6M^2(h-y)(h+y)\beta\right)-3\left(-1+4h^2M^2\right)$$
$$\beta\gamma))))Cosh[hM]-3\left(1+hM^2\left(-\beta+4h\left(1+\gamma M^2\beta\right)\right)\right)Cosh[3hM]+6Sinh[hM]Sinh[2My],$$

$$q_2 = 3M\left(2(h-y)y+h\left(1+2M^2\left(h^2+y^2\right)\right)\beta\right)Cosh[M(h-2y)]+3M\left(2y(h+y)+h\left(q_6\right)\beta\right)$$
$$Cosh[M(h+2y)],$$

$$q_3 = 3hSinh[hM]+20h^3M^2Sinh[hM]+24h^5M^4Sinh[hM]-24h^3M^4y^2Sinh[hM]$$
$$-8M^2y^3Sinh[hM],$$

$$q_4 = 32h^4M^4\beta Sinh[hM]+24h^2M^4y^2\beta Sinh[hM]+8hM^4y^3\beta Sinh[hM]-36h^2M^2\gamma Sinh[hM]$$
$$-48h^4M^4\gamma Sinh[hM]+96h^3M^4\beta\gamma Sinh[hM]+9hSinh[3hM]+12h^2M^2\gamma Sinh[3hM]$$
$$+3hSinh[M(h-2y)]+6h^3M^2Sinh[M(h-2y)]-6ySinh[M(h-2y)],$$

$$q_5 = 6hM^2y^2Sinh[M(h-2y)]-6h^2M^2\beta Sinh[M(h-2y)]+6hM^2y\beta Sinh[M(h-2y)]$$
$$+3\left(-2y+h\left(-1+2M^2(h-y)(h+y-\beta)\right)\right)Sinh[M(h+2y)],$$

$$q_6 = -1+2M^2(h-y)(h+y).$$

$$L_1 = 16M\left(-6\gamma + h\left(5 + 2M^2\left(L_7\right)\right)\right)Cosh[hM] + 3M\left(36\gamma + h\left(L_8\right)\right)Cosh[3hM] + 16Sinh[hM]$$

$$-25Sinh[3hM] + Sinh[5hM] - 5Sinh[M(h - 4y)],$$

$$L_2 = 5Sinh[M(h - 2y)] + Sinh[M(3h - 2y)] + 5Sinh[M(h + 2y)] + Sinh[M(3h + 2y)])$$

$$-5Sinh[M(h + 4y)],$$

$$L_3 = \left(7h - 4\left(3 + 13hM^2\beta\right)\gamma\right)Cosh[5hM] + 5hCosh[M(h - 4y)] - 20hCosh[M(h - 2y)]$$

$$-48yCosh[M(h - 2y)] + 48h^2M^2\beta Cosh[M(h - 2y)],$$

$$L_4 = 48hM^2y\beta Cosh[M(h - 2y)] - 12hCosh[M(3h - 2y)] - 20hCosh[M(h + 2y)]$$

$$+48yCosh[M(h + 2y)] + 48h^2M^2\beta Cosh[M(h + 2y)] - 48hM^2y\beta Cosh[M(h + 2y)]$$

$$-12hCosh[M(3h + 2y)] + 5hCosh[M(h + 4y)] + 288h^2MSinh[hM],$$

$$L_5 = 192h^4M^3Sinh[hM] - 192My^2Sinh[hM] - 192h^2M^3y^2Sinh[hM] + 112hM\beta Sinh[hM]$$

$$+192h^3M^3\beta Sinh[hM] - 192hM^3y^2\beta Sinh[hM] - 544hM\gamma Sinh[hM] - 384h^3M^3\gamma Sinh[hM]$$

$$-192h^2M^3\beta\gamma\,Sinh[hM] + 16h^2MMSinh[3hM] - 16My^2Sinh[3hM] - 71hM\beta Sinh[3hM]$$

$$-144h^3M^3\beta Sinh[3hM] + 144hM^3y^2\beta Sinh[3hM] - 4hM\gamma Sinh[3hM],$$

$$L_6 = 288h^2M^3\beta\gamma Sinh[3hM] + 31hM\beta Sinh[5hM] - 4hM\gamma Sinh[5hM] + 5hM\beta$$

$$Sinh[M(h - 4y)] + 48h^2MSinh[M(h - 2y)] + 48hMySinh[M(h - 2y)] + 76h$$

$$M\beta Sinh[M(h - 2y)] - 36hM\beta Sinh[M(3h - 2y)] + 48h^2MSinh[M(h + 2y)]$$

$$-48hMySinh[M(h + 2y)] + 76hM\beta Sinh[M(h + 2y)] - 36hM\beta Sinh[M(3h + 2y)]$$

$$+5hM\beta Sinh[M(h + 4y)],$$

$$L_7 = 6y^2 + 3h\left(-2h + \left(-1 + 2M^2(h - y)(h + y)\right)\beta\right) - \left(\beta + 6h\left(-1 + 2hM^2\beta\right)\right)\gamma,$$

$$L_8 = -11 + 4M^2\left(-4h^2 + 4y^2 + 8h\gamma + 7\beta\gamma\right).$$

The heat transfer coefficient is

$$Z = h_x\theta_y. \tag{11.20}$$

## 11.3   Graphical analysis and key points

This section provides a comparative study of temperature for the effects of various parameters in the constant ($\alpha = 0$ left panels) and variable ($\alpha = 0.2$ right panels) fluid viscosities in the presence of heat transfer.

It is observed in Figs. 11.1 (a-d) that $\theta$ decreases when $M$ and $\beta_1$ are increased. Further temperature in variable viscosity fluid is less when compared with constant viscosity fluid. From

Figs. 11.2 (a-d), an increase in $\theta$ is observed when Br increases whereas $\theta$ decreases when $\beta_2$ increases. Although the plots for heat transfer coefficient $Z$ are not included to save space for variation of $M$ and $\Gamma$. It is also seen that impact of $M$ on $Z$ is quite opposite to that of $\beta_1$. $Z$ is an increasing function of $M$. Figs. 11.3 (a-d) show that $Z$ decreases when $\beta_1$ increases whereas it increases by increasing $Br$.

The main points of present chapter can be summarized as follows.

i) Temperature in constant viscosity fluid is greater than variable viscosity situation.

ii) Temperature is decreasing function of thermal slip parameter $\beta_2$.

iii) The qualitative effects of $Br$ and $\beta_1$ on the temperature are opposite.

iv) Behavior of $\beta_1$ on the heat transfer coefficient is opposite to that of $Br$.
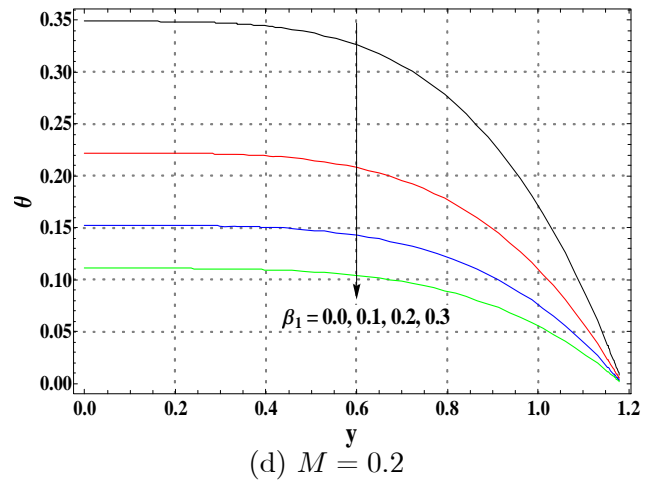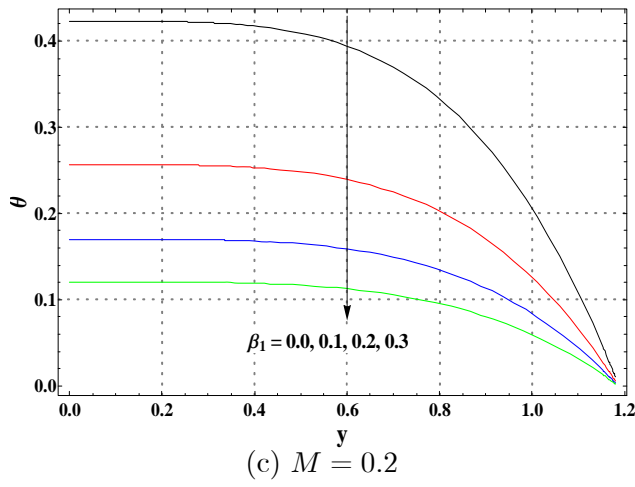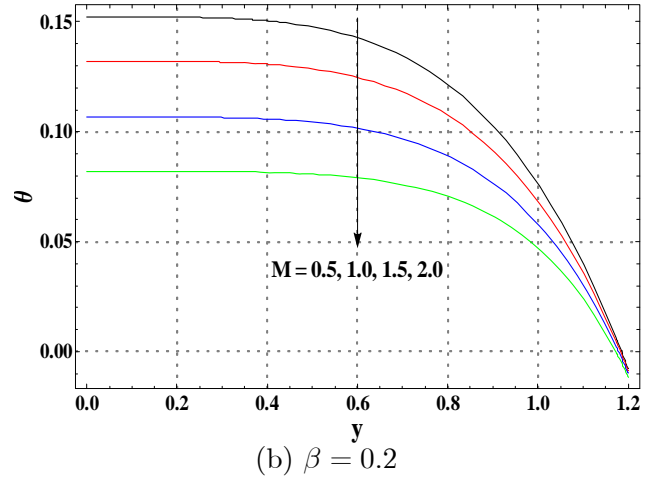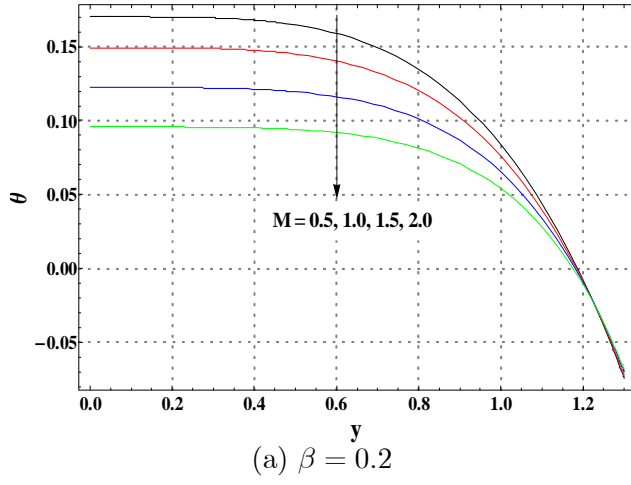
Fig. 11.1 (a-d): Variation of $\eta$ for $\Gamma = 0.04$, $\phi = 0.3$, $\eta = 1.1$, $\gamma = 0.1$ and Br= 0.5.

164

(a) $\beta_2 = 0.2$

(b) $\beta_2 = 0.2$

(c) $Br = 0.5$

(d) $Br = 0.5$
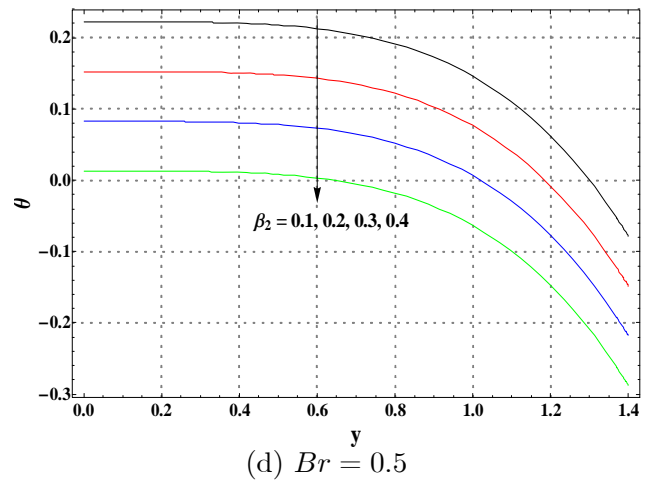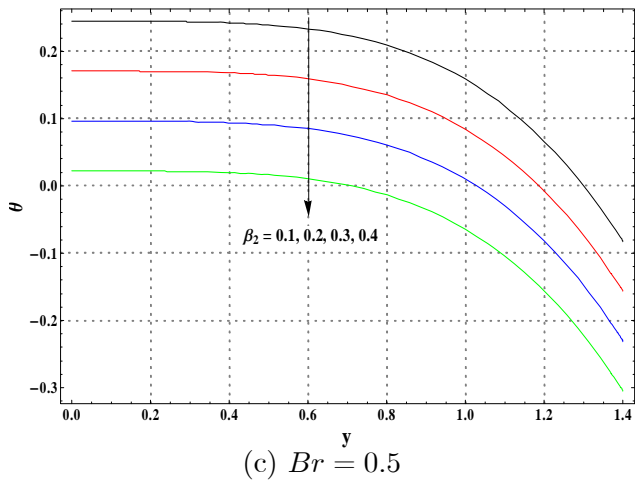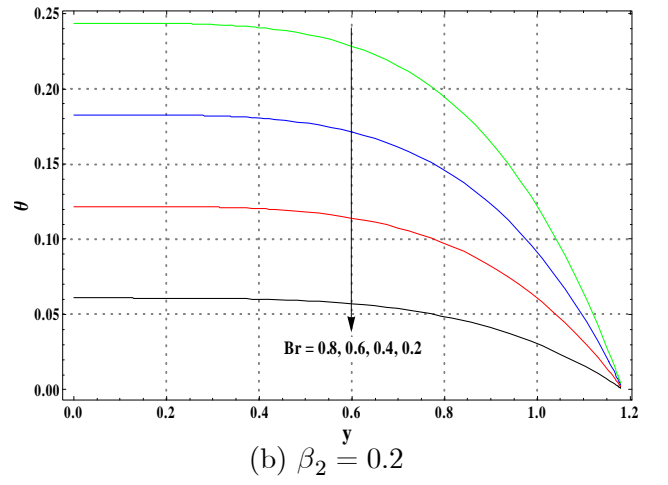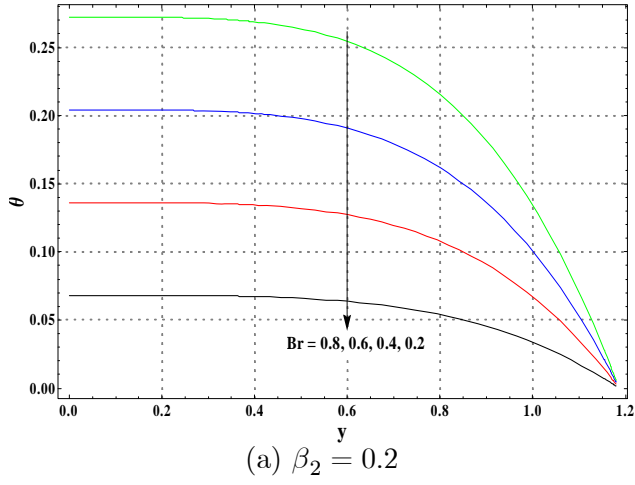
Fig. 11.2 (a-d): Variation of $\eta$ for $M = 1$, $\Gamma = 0.04$, $\phi = 0.3$, $\eta = 1.1$, and $\beta = 0.2$.

(a) $Br = 0.5$

(b) $Br = 0.5$

(c) $\beta_1 = 0.2$

(d) $\beta_1 = 0.2$
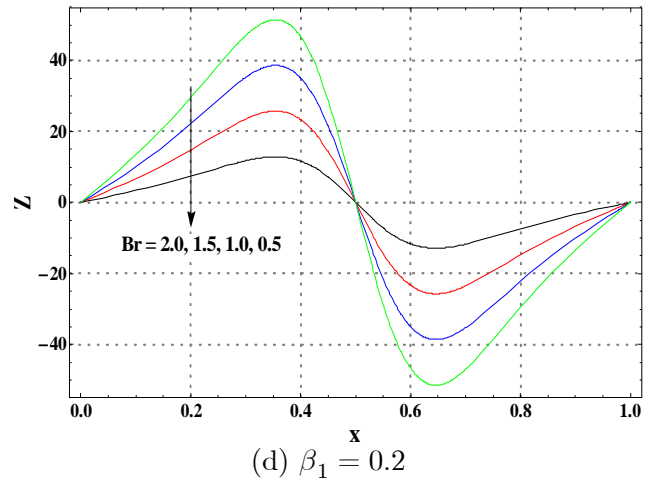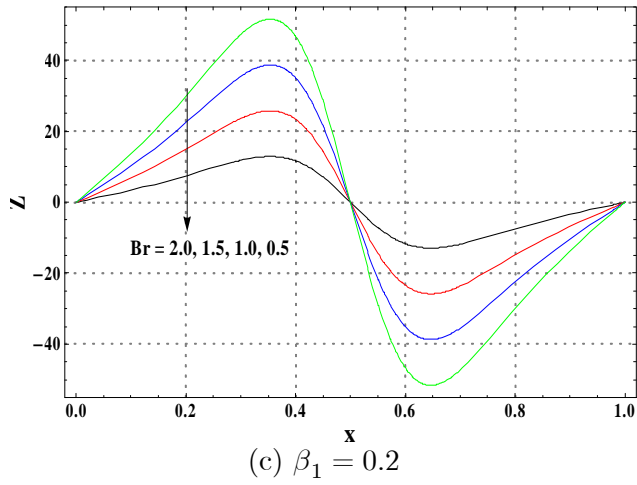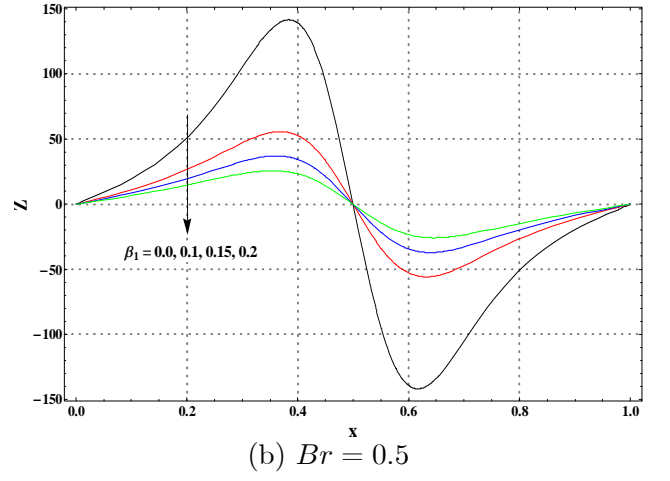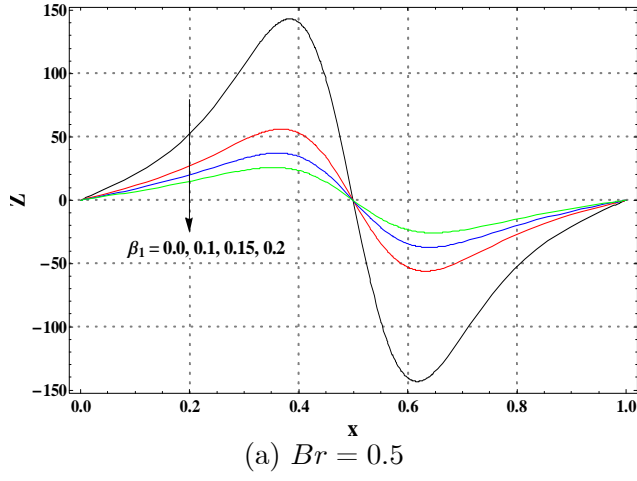
Fig. 11.3 (a-d): Variation of Z when $\Gamma = 0.04$, $\phi = 0.3$, $\eta = 1.1$, and $M = 1$.

166

# Chapter 12

# Soret and Dufour effects on peristaltic transport of a third order fluid

Soret and Dufour effects on peristaltic transport of third order fluid in a symmetric channel have been reported in this chapter. Joule heating effect is also taken in to account. The governing nonlinear problem is solved using perturbation approach. Graphical results are reported and discussed for various parameters of interest entering into the problem

## 12.1   Mathematical formulation

We consider the magnetohydrodynamic (MHD) flow of third order fluid in a channel of width $2d_1$. The $\overline{X}-$ axis is chosen along the walls of channel and $\overline{Y} -$ axis is taken normal to the $\overline{X}-$ axis. A constant magnetic field of strength $\mathbf{B}_0$ is applied in the $\overline{Y} -$ direction. Induced magnetic field is not accounted. Sinusoidal wave propagating on channel walls with constant wave speed $c$ is represented by

$$H(\overline{X},\overline{t}) = d_1 + a_1 cos\left(\frac{2\pi}{\lambda}(\overline{X} - c\overline{t})\right), \tag{12.1}$$

where $a_1$ is the wave amplitude, $\lambda$ is the wavelength and $\bar{t}$ is the time . The continuity, momentum, energy and concentration equations of the problem may be written as follows:

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{12.2}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{U} = -\frac{\partial \overline{P}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XX}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{Y}} - \sigma B_0^2 \overline{U}, \tag{12.3}$$

$$\rho \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) \overline{V} = -\frac{\partial \overline{P}}{\partial \overline{Y}} + \frac{\partial \overline{S}_{\overline{XY}}}{\partial \overline{X}} + \frac{\partial \overline{S}_{\overline{YY}}}{\partial \overline{Y}}, \tag{12.4}$$

$$C_p \left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) T = \frac{K}{\rho} \left( \frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2} \right)$$
$$+ \upsilon \overline{\zeta} + \frac{\sigma B_0^2 \overline{U}^2}{\rho} + \frac{DK_T}{\rho C_s} \left( \frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2} \right), \tag{12.5}$$

$$\left( \frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right) C = D \left( \frac{\partial^2 C}{\partial \overline{X}^2} + \frac{\partial^2 C}{\partial \overline{Y}^2} \right) + \frac{DK_T}{T_m} \left( \frac{\partial^2 T}{\partial \overline{X}^2} + \frac{\partial^2 T}{\partial \overline{Y}^2} \right). \tag{12.6}$$

In the above equations, $\overline{U}$ and $\overline{V}$ are the velocity components along $\overline{X}$ and $\overline{Y}$ directions respectively in fixed (laboratory frame), $\rho$ is the density, $\overline{P}$ is the pressure, $\upsilon$ is the kinematic viscosity, $\sigma$ is the electrical conductivity, $K$ is the thermal conductivity, $B_0$ is the applied magnetic field, $T$ is the temperature field, $C_p$ is the specific heat, $\upsilon \overline{\zeta}$ is the viscous dissipation, $D$ is mass diffusivity, $K_T$ is the thermal diffusion ratio, $C_s$ is the concentration susceptibility, $C$ is the concentration field and $T_m$ is the mean temperature.

Extra stress tensor $\overline{\mathbf{S}}$ in third order fluid is

$$\overline{\mathbf{S}} = \left( \mu + \beta_3 tr \overline{\mathbf{A}}_1^2 \right) \mathbf{A}_1 + \alpha_1 \overline{\mathbf{A}}_2 + \alpha_2 \overline{\mathbf{A}}_1^2 + \beta_1 \overline{\mathbf{A}}_3 + \beta_2 \left( \overline{\mathbf{A}}_1 \overline{\mathbf{A}}_2 + \overline{\mathbf{A}}_2 \overline{\mathbf{A}}_1 \right),$$

in which $\mu$, $\alpha_i$ $(i = 1, 2)$ and $\beta_i$ $(i = 1, 2.3)$ are the material constants. The Rivlin-Ericksen tensors can be represented as follows

$$\overline{\mathbf{A}}_1 = \overline{\mathbf{L}} + \overline{\mathbf{L}}^t,$$
$$\overline{\mathbf{A}}_{n+1} = \frac{d\overline{\mathbf{A}}_n}{dt} + \overline{\mathbf{A}}_n \overline{\mathbf{L}} + \overline{\mathbf{L}}^t \overline{\mathbf{A}}_n \qquad n = 1, 2,$$

where $\overline{\mathbf{L}} = \text{grad}\, \overline{\mathbf{V}}$ and $t$ indicates the matrix transpose.

If $(\overline{U},\overline{V})$ and $(\overline{u},\overline{v})$ are the velocity components in the laboratory $(\overline{X},\overline{Y})$ and wave $(\overline{x},\overline{y})$ frames respectively then defining

$$\overline{x} = \overline{X} - c\overline{t}, \ \ \overline{y} = \overline{Y}, \ \ \overline{u} = \overline{U} - c, \ \ \overline{v} = \overline{V}, \ \ \overline{p}(\overline{x},\overline{y}) = \overline{P}(\overline{X},\overline{Y},\overline{t}), \tag{12.7}$$

and introducing the following dimensionless variables

$$
\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \ \ y = \frac{\overline{y}}{d_1}, \ \ u = \frac{\overline{u}}{c}, \ \ v = \frac{\overline{v}}{c\delta}, \ \ \delta = \frac{d_1}{\lambda}, \ \ h = \frac{H}{d_1}, \ \ a = \frac{a_1}{d_1}, \ \ p = \frac{a^2\overline{p}}{c\lambda\mu}, \\
\theta &= \frac{T - T_0}{T_0}, \phi = \frac{C - C_0}{C_0}, M = \left(\frac{\sigma}{\mu}\right)^{1/2} B_0 d_1, \boldsymbol{v} = \frac{\mu}{\rho}, Re = \frac{\rho c d_1}{\mu}, \\
t &= \frac{c\overline{t}}{\lambda}, u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \ \ Br = \Pr E, \ \ Du = \frac{DC_0 K_T}{C_s C_p \mu T_0}, \ \ Sr = \frac{\rho D K_T T_0}{\mu T_m C_0}, \\
Sc &= \frac{\mu}{\rho D}, \ \ E = \frac{c^2}{C_p T_0}, \ \ \Pr = \frac{\mu C_p}{K}, \ \ S = \frac{d_1}{\mu c}\overline{S}(\overline{x}), \ \ \gamma_2 = \frac{\beta_2 c^2}{\mu a^2}, \gamma_3 = \frac{\beta_3 c^2}{\mu a^2}
\end{aligned}
\tag{12.8}
$$

we have

$$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}S_{xy} - M^2\left(\frac{\partial\psi}{\partial y} + 1\right) = 0, \ \ 0 = -\frac{\partial p}{\partial y}, \tag{12.9}$$

$$0 = \frac{\partial^2\theta}{\partial y^2} + Br\zeta + BrM^2\left(\frac{\partial\psi}{\partial y} + 1\right)^2 + \Pr Du\frac{\partial^2\phi}{\partial y^2}, \tag{12.10}$$

$$0 = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2} + Sr\frac{\partial^2\theta}{\partial y^2}, \tag{12.11}$$

$$S_{xy} = \frac{\partial^2\psi}{\partial y^2} + 2\Gamma\left(\frac{\partial^2\psi}{\partial y^2}\right)^3, \ \ \zeta = (\frac{\partial^2\psi}{\partial y^2})^2 + 2\Gamma(\frac{\partial^2\psi}{\partial y^2})^4, \tag{12.12}$$

where $p$ is the pressure, $\psi$ the stream function, $\Gamma(= \gamma_2 + \gamma_3)$ the Deborah number, $M$ the Hartman number, Re the Reynolds number, $Br$ the Brinkman number, $Du$ the Dufour parameter, $Sr$ the Soret parameter, $Sc$ the Schmidt number, $E$ the Eckret number, $Pr$ the Prandtl number, $\delta$ the wave number, $T_0$ temperature at the wall, $C_0$ concentration at the wall, $\theta$ the dimensionless temperature and $\phi$ the concentration. The boundary conditions are

$$
\begin{aligned}
\psi &= 0, \ \ \frac{\partial^2\psi}{\partial y^2} = 0, \ \ \frac{\partial\theta}{\partial y} = 0, \ \ \frac{\partial\phi}{\partial y} = 0, \ \ \ at \ y = 0, \\
\psi &= F, \ \ \frac{\partial\psi}{\partial y} = -1, \ \ \theta = 0, \ \ \phi = 0, \ \ \ at \ y = h,
\end{aligned}
\tag{12.13}
$$

169

$$h(x)=1+a\cos(2\pi x), \quad F = \int_0^h \frac{\partial\psi}{\partial y}dy, \tag{12.14}$$

where $F$ is the dimensionless volume flow rate and $h$ is the dimensionless wave shape. Pressure rise per wavelength $\Delta p_\lambda$ is

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx}dx. \tag{12.15}$$

We look for solutions in the series form and thus we expand the quantities in terms of small Deborah number as follows:

$$\psi = \psi_0 + \Gamma\psi_1 + ....$$

$$\theta = \theta_0 + \Gamma\theta_1 + ....$$

$$\phi = \phi_0 + \Gamma\phi_1 + ....$$

$$F = F_0 + \Gamma F_1 + ....$$

$$p = p_0 + \Gamma p_1 + ....$$

The resulting zeroth order system is given by

$$\frac{\partial^4\psi_0}{\partial y^4} - M^2\frac{\partial\psi_0^2}{\partial y^2} = 0, \tag{12.16}$$

$$0 = \frac{\partial^2\theta_0}{\partial y^2} + Br\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2 + BrM^2\left(\frac{\partial\psi_0}{\partial y}+1\right)^2 + \Pr Du\frac{\partial^2\phi_0}{\partial y^2}, \tag{12.17}$$

$$0 = \frac{1}{Sc}\frac{\partial^2\phi_0}{\partial y^2} + Sr\frac{\partial^2\theta_0}{\partial y^2}, \tag{12.18}$$

$$\psi_0 = 0, \quad \frac{\partial^2\psi_0}{\partial y^2} = 0, \quad \frac{\partial\theta_0}{\partial y} = 0, \quad \frac{\partial\phi_0}{\partial y} = 0, \quad at \;\; y = 0,$$

$$\psi_0 = F_0, \quad \frac{\partial\psi_0}{\partial y} = -1, \quad \theta_0 = 0, \quad \phi_0 = 0, \quad at \;\; y = h, \tag{12.19}$$

and first order system becomes

$$\frac{\partial^4\psi_1}{\partial y^4} + 2\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi_0^2}{\partial y^2}\right)^3 - M^2\frac{\partial\psi_1^2}{\partial y^2} = 0, \tag{12.20}$$

170

$$0 = \frac{\partial^2 \theta_1}{\partial y^2} + Br\left(2\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^2 \psi_1}{\partial y^2} + \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^4\right)^2 + BrM^2\left(2\frac{\partial \psi_1}{\partial y}\left(\frac{\partial \psi_0}{\partial y} + 1\right)\right) + \Pr Du\frac{\partial^2 \phi_1}{\partial y^2},$$

$$\tag{12.21}$$

$$0 = \frac{1}{Sc}\frac{\partial^2 \phi_1}{\partial y^2} + Sr\frac{\partial^2 \theta_1}{\partial y^2}, \tag{12.22}$$

$$\psi_1 = 0, \quad \frac{\partial^2 \psi_1}{\partial y^2} = 0, \quad \frac{\partial \theta_1}{\partial y} = 0, \quad \frac{\partial \phi_1}{\partial y} = 0, \quad at \ y = 0,$$

$$\psi_1 = F_1, \quad \frac{\partial \psi_1}{\partial y} = 0, \quad \theta_1 = 0, \quad \phi_1 = 0, \quad at \ y = h, \tag{12.23}$$

The heat transfer coefficient at the wall can be computed by

$$Z = h_x \theta_y.$$

## 12.2    Solution expressions

Solving the resulting systems at zeroth and first orders we can write

$$\begin{aligned}
\psi &= -\frac{l_1}{\left(-hM\cosh[hM] + \sinh[hM]\right)^4} \\
&\quad + \frac{e^{-3My}\left(F + h\right)^3 M^4 \Gamma\left(l_2 - l_3 + l_4\right)}{32\left(-hM\cosh[hM] + \sinh[hM]\right)^4}
\end{aligned}$$

$$\begin{aligned}
\theta &= \frac{BrM^2\left(A_1 + 4FA_2 - A_3\cosh[2hM] + A_4 - A_5\right)}{\left(8\left(-1 + A\right)\left(-hM\cosh[hM] + \sinh[hM]\right)^2\right)} \\
&\quad + \frac{-Br\left(F + h\right)^3 M^6\Gamma}{\left(512\left(-1 + A\right)\left(hM\cosh[hM] - \sinh[hM]\right)^5\right)} \\
&\quad \times \left(A_6 - 16hMA_7\cosh[hM] + A_8 + A_9 - A_{10} + A_{11} - A_{12}\right),
\end{aligned}$$

171

$$\phi \;=\; \frac{BrM^2 ScSr\,(B_1 + B_2\cosh[2hM] - B_3)}{\left(8\,(-1+A)\,(-hM\cosh[hM] + \sinh[hM])^2\right)}$$

$$+\frac{Br\,(F+h)^3\,M^6 ScSr\Gamma}{\left(512\,(-1+A)\,(hM\cosh[hM] - \sinh[hM])^5\right)}$$

$$\times\,(-B_4 - 16hM\,(B_5)\cosh[hM] + B_6 - B_7 + B_8 - B_9 - B_{10}),$$

with $l_i s$, $A_i s$ and $B_i s$ are given as follows:

$$l_1 \;=\; (hM\cosh[hM] - \sinh[hM])^3\,(-y\,(FM\cosh[hM] + \sinh[hM]) + (F+h)$$

$$\sinh[My]),$$

$$l_2 \;=\; 24e^{3My}hM^2 y - hM\left(1 - e^{6My} + 12e^{2My}My + 12e^{4My}My\right)\cosh[hM],$$

$$l_3 \;=\; 3e^{2My}\left(-1 + e^{2My}\right)hM\cosh[3hM] + \sinh[hM] - e^{6My}\sinh[hM]$$

$$-12e^{2My}h^2 M^2\sinh[hM],$$

$$l_4 \;=\; 12e^{4My}h^2 M^2\sinh[hM] + 12e^{2My}My\sinh[hM] + 12e^{4My}My\sinh[hM]$$

$$-16e^{3My}My$$

$$\sinh[2hM] - e^{2My}\sinh[3hM] + e^{4My}\sinh[3hM] + 2e^{3My}My\sinh[4hM],$$

$$A = \Pr Du * ScSr,$$

$$A_1 = 2h\left(4 + h + h^3 M^4\right) - 2\left(1 + h^2 M^4\right)y^2 + 2F^2 M^2\left(1 + M^2\right)(h - y)(h + y),$$

$$A_2 = 2 + hM^2\left(1 + M^2\right)(h - y)(h + y),$$

$$A_3 = F^2\left(1 + M^2\right) + 2F\left(4 + h + hM^2\right) - 2y^2 + h\left(8 + h\left(3 + M^2\left(1 + 2h^2 - 2y^2\right)\right)\right),$$

$$A_4 = (F + h)^2\left(1 + M^2\right)\cosh[2My] + 4hM\left(2F + h(2 + h) - y^2\right)\sinh[2hM],$$

$$A_5 = 16h\,(F + h)\,M\cosh[hM]\,(M(h - y) + \sinh[My]) + 16\,(F + h)\sinh[hM](M(h - y)$$
$$+ \sinh[My]),$$

$$A_6 = -288hM - 96h^3 M^3 + 288My + 96h^2 M^3 y,$$

$$A_7 = -16 + 48h^2 M^2 + 12h^3\left(M^2 + 2M^4\right) + F(-5 + 3M^2\left(-3 + 4h^2 - 4y^2\right) + 24M^4$$
$$\left(h^2 - y^2\right)) - h\left(5 + 24M^4 y^2 + 3M^2\left(3 + 4y^2\right)\right),$$

$$A_8 = 256M\left(1 + 3h^2 M^2\right)(h - y)\cosh[2hM] - 288hM\cosh[3hM] - 33FhM\cosh[3hM]$$
$$- 33h^2 M\cosh[3hM] + 27FhM^3\cosh[3hM],$$

$$A_9 = 27h^2 M^3\cosh[3hM] - 48Fh^3 M^3\cosh[3hM] - 48h^4 M^3\cosh[3hM] - 48Fh^3 M^5$$
$$\cosh[3hM],$$

$$A_{10} = 48h^4 M^5\cosh[3hM] + 48FhM^3 y^2\cosh[3hM] + 48h^2 M^3 y^2\cosh[3hM] + 48FhM^5 y^2$$
$$\cosh[3hM] + 48h^2 M^5 y^2\cosh[3hM] + 32hM\cosh[4hM] + 96h^3 M^3\cosh[4hM]$$
$$- 32My\cosh[4hM] - 96h^2 M^3 y\cosh[4hM] + 32My\cosh[5hM] + 7FhM\cosh[5hM],$$

$$A_{11} = 7h^2 M\cosh[5hM] + 3FhM^3\cosh[5hM] + 3h^2 M^3\cosh[5hM] + 5FhM$$
$$\cosh[M(h - 4y)] + 5h^2 M\cosh[M(h - 4y)] + 9FhM^3\cosh[M(h - 4y)] + 9h^2 M^3$$
$$\cosh[M(h - 4y)] - 32hM\cosh[M(2h - 3y)] - 20FhM\cosh[M(h - 2y)] - 20h^2 M$$
$$\cosh[M(h - 2y)],$$

$$A_{12} = 84FhM^3\cosh[M(h - 2y)] - 84h^2 M^3\cosh[M(h - 2y)] - 48My\cosh[M(h - 2y)]$$
$$(F + h) - 48M^3 y\cosh[M(h - 2y)](F + h) - 12hM\cosh[M(3h - 2y)]\left(F + h + M^2\right)$$
$$- 32hM\cosh[M(2h - y)] + 192M\cosh[M(2h - y)]\left(h^3 M^2 + y + h^2 M^2 y\right),$$

$$B_1 \;=\; -8F - 2h\,(4+h) - 2Fh^2\,(F+2h)\,M^2 - 2h^2\,(F+h)^2\,M^4 + 2B_{11}y^2,$$

$$B_2 \;=\; F^2\,(1+M^2) + 2F\,(4+h+hM^2) - 2y^2 + h\,(8+hB_{12}),$$

$$B_3 \;=\; (F+h^2)\,(1+M^2)\cosh[2My] - 4hM\,(2F+h\,(2+h)-y^2)\sinh[2hM] +$$
$$16h\,(F+h)\,M\cosh[hM]\,(M\,(h-y)+\sinh[My]) - 16\,(F+h)\sinh[hM]$$
$$(M\,(h-y)+\sinh[My]),$$

$$B_4 \;=\; 288hM - 96h^3M^3 + 288My + 96h^2M^3y,$$

$$B_5 \;=\; -16 + 48h^2M^2 + 12h^3\,(M^2+2M^4) + FB_{13} - h\,(5+24M^4y^2+3M^2\,(3+4y^2)),$$

$$B_6 \;=\; 256M\,(1+3h^2M^2)\,(h-y)\cosh[2hM] - 288hM\cosh[3hM] - 33hM\cosh[3hM]$$
$$(F+h) + 27hM\cosh[3hM]\,(FM^2+hM^2),$$

$$B_7 \;=\; 48hM\cosh[3hM]B_{14} + 32M\cosh[4hM]\,(h-y) + 96h^2M^2\cosh[4hM]\,(hM-My),$$

$$B_8 \;=\; hM\cosh[5hM]\,(32+7F+7h+3FM^2+3hM^2) + hM\cosh[M\,(h-4y)]$$
$$(5F+5h+9FM^2+9hM^2) - 32hM\cosh[M\,(2h-3y)],$$

$$B_9 \;=\; hM\cosh[M\,(h-2y)]B_{15},$$

$$B_{10} \;=\; hM\cosh[M\,(3h-2y)]\,(12F+12h+12FM^2+12hM^2) - hM\cosh[M\,(2h-y)]$$
$$(32 - 192h^2M^2 - 192\,(y/h) - 192hM^2y),$$

$$B_{11} \;=\; 1 + F\,(F+2h)\,M^2 + (F+h)^2\,M^4, \quad B_{12} = 3 + M^2\,(1+2\,(h^2-y^2)),$$

$$B_{13} \;=\; -5 + 3M^2\,(-3+4h^2-4y^2) + 24M^2\,(h^2-y^2),$$

$$B_{14} \;=\; Fh^2M^2 + h^3M^2 + Fh^2M^4 + h^3M^4 - FM^2y^2 - hM^2y^2 - FM^4y^2 - hM^4y^2,$$

$$B_{15} \;=\; 20F + 20h + 84FM^2 + 84hM^2 + 48Fy + 48hy + 48FM^2y + 48hM^2y.$$

## 12.3   Discussion

The objective of this section is to analyze the plots for temperature, concentration and heat transfer coefficient. Hence Figs. 12.1-12.3 are given in order to achieve such objective. Here Figs. 12.1(a-g) are drawn for temperature $\theta$, Figs. 12.2(a-g) for concentration profile and Figs. 12.3(a-f) for the heat transfer coefficient Z. Explicitly 3-D graphs are plotted to analyze the behavior fully in the domain of interest.

Figs. 12.1 demonstrate that the dimensionless temperature increases with an increase in

M, Br, Du, Sr, Sc, Pr and $\Gamma$. It is also noted from Fig. 12.1a that increase in temperature with respect to M is not much for small M. However the temperature increases abruptly when M increases. Also with the increase in magnitude of applied magnetic field, the temperature increases more due to Joule heating. In Fig. 12.1b it can be seen that there is a linear relationship between Br and $\theta$. Similar observation holds for $\Gamma$ (see Fig. 12.1g). This depicts that rise in temperature is lesser for Newtonian fluid when compared with non-Newtonian fluid.

In Figs. 12.2 (a-g) it is observed that the dimensionless concentration decreases by decreasing M, Br, Du, Sr, Sc, Pr and $\Gamma$. It is found that decrease in temperature is not linear for M, linear and large for Br and $\Gamma$ and smaller bit small for Du, Sr, Sc, Pr.

Figs. 12.3(a-f) explicitly see the behavior of heat transfer coefficient Z when values of different embedded parameters are altered. It is seen in all these Figs. that Z has an oscillatory behavior. It is physically acceptable in view of peristalsis. The magnitude of oscillations in Z increase most rapidly in case of $\Gamma$ when compared with the other cases.
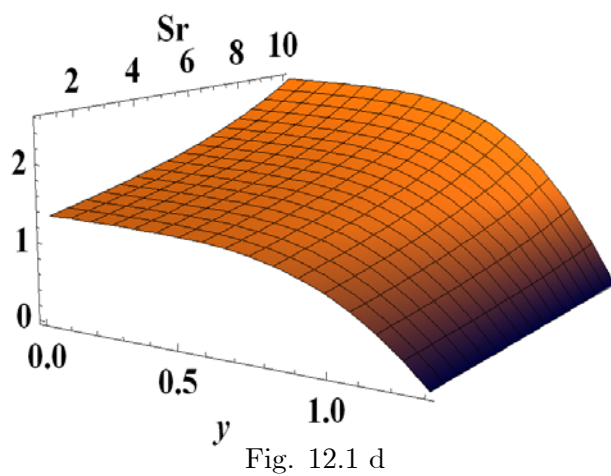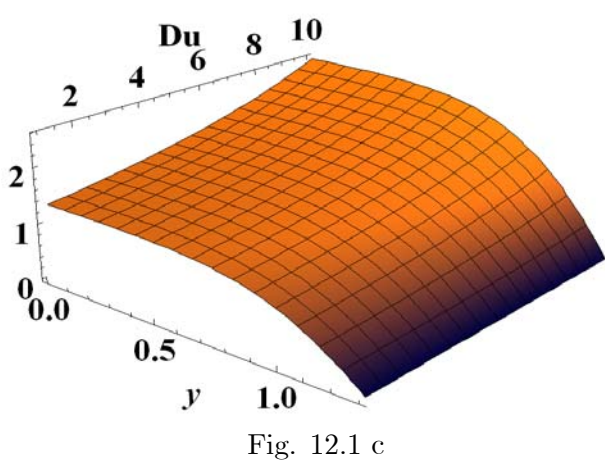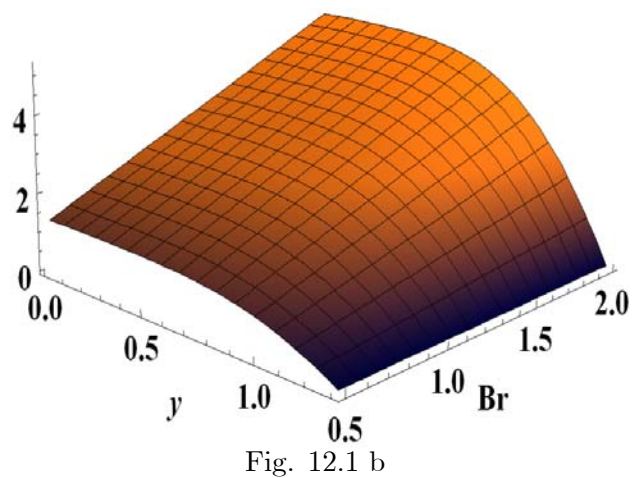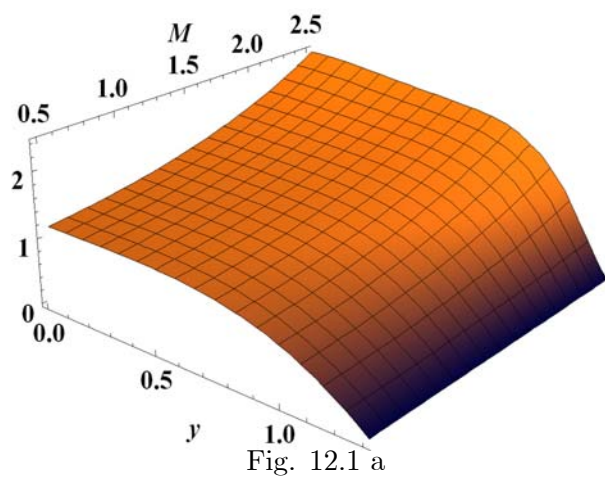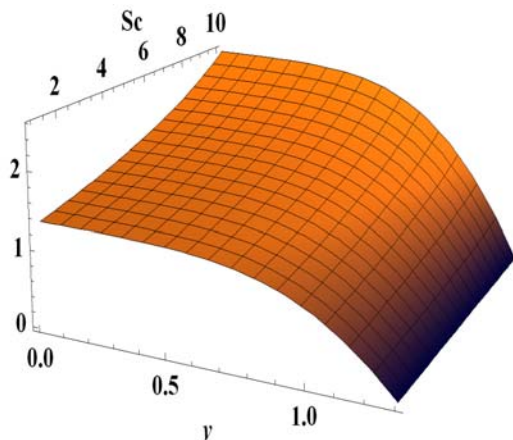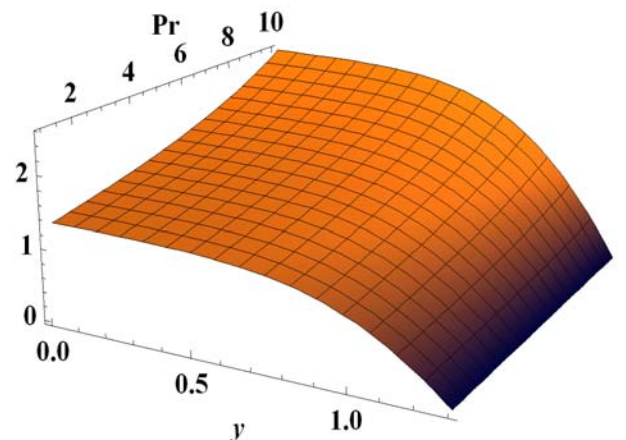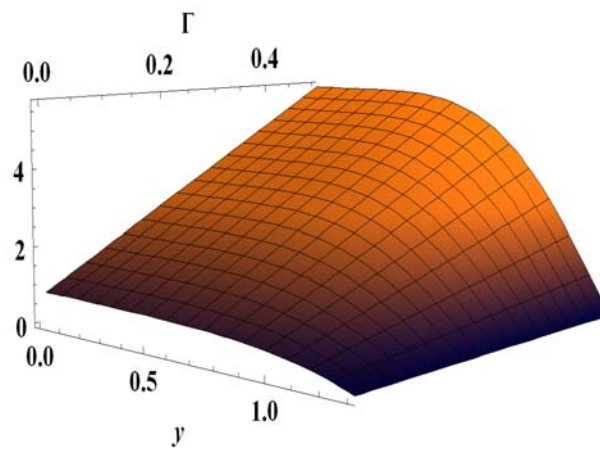
Fig. 12.1 a

Fig. 12.1 b

Fig. 12.1 c

Fig. 12.1 d

176

Fig. 12.1 e



Fig. 12.1 f



Fig. 12.1 g

Figs 12.1(a-g). Behavior of the temperature with change in values of different parameters.
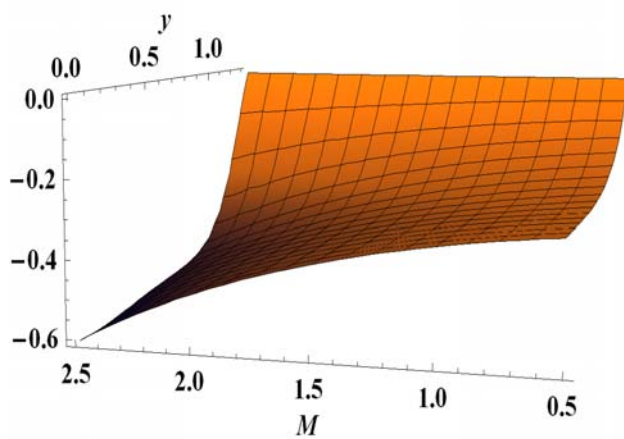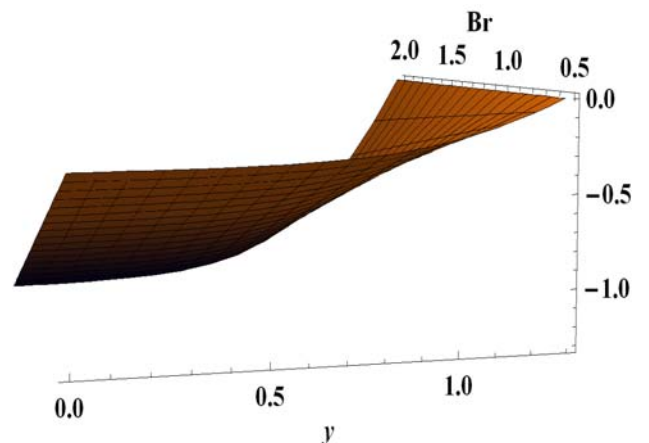
Fig. 12.2 a

Fig. 12.2 b
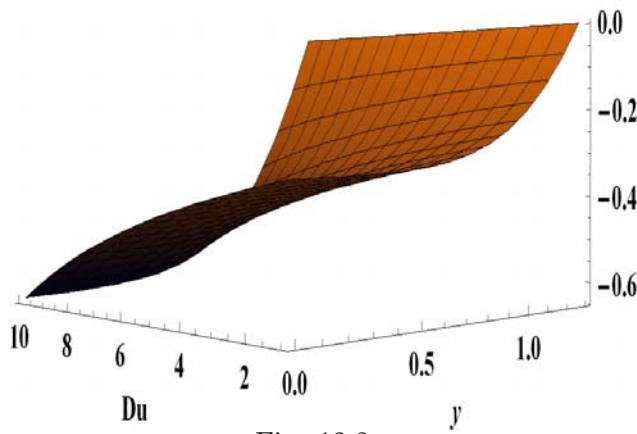
Fig. 12.2 c

Fig. 12.2 d

Fig. 12.2 e



Fig. 12.2 f



Fig. 12.2 g

Figs. 12.2(a-g). Development of concentration field with increase in values of parameters.

Fig. 12.3 a


Fig. 12.3 b


Fig. 12.3 c


Fig. 12.3 d

180

Fig. 12.3 e

Fig. 12.3 f

Figs. 12.3(a-f). Behavior of heat transfer coefficient Z.

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**paper text:**

Contents

> 2**Mixed convective heat and mass transfer** for peristaltic transport **in an** asym-
> metric **channel with**

> 4**Variable viscosity** effects **on the peristaltic motion** of **a third-order fluid**

> 13**Heat transfer analysis for peristaltic mechanism in variable viscosity fluid**

> 4**effects on peristaltic** transport **of a third order fluid**

> 100**1 Introduction The aim of this chapter is to provide** a mterial for **the** better
> understanding **of the**

subse- quent chapters. Here the literature review about peristaltic flows, and some definitions and
fundamental equations are presented. 1.1 Peristaltic transport The peristaltic transport of fluid is caused

> 21**by a progressive wave of area contraction/expansion along the length of a**
> **distensible tube.**

Such wave inturn propells the fluid

> 151**in the direction of propagation of wave.** This type **of**

transport mechanism finds its application in the physiological and industrial processes. These include the
locomotion of worms, urine passage from kidney to bladder, food swallowing by the esophagus,
vasomotion of small blood vessels, sanitary and corrosive fluids transport, movement of bio fluids (chyme
in gastrointestinal tract, bile in the bile duct, ovum in the fallopian tube and spermatozoa in cervical canal)
and in heart lung machine and roller and finger pumps. 1.2 Literature review Despite of the fact that the
mechanism of peristalsis was known to the physiologist since long, mechanical analysis of peristalsis begin
in later half of the twentieth century. Initial investiga- tion of peristaltic pumping was inspired by its
application in human ureter system. Studies of Kiil [1] and Boyarsky [2] are fundamentals in this direction.
Latham [3] in his MS thesis carried out the experimental investigation of peristaltic pump. Shapiro [4]
discussed the inertia free two-dimensional peristaltic transport when

> 137**the wavelength of peristaltic wave is large compared** to **the**

half channel width. Under these assumptions there is a steady flow in a wave frame. Later on these two
considerations (the long wavelength and low Reynolds number approxima- tion) were widely utilized by the
researchers in the analysis of peristaltic flows. The theoretical investigation of Shapiro [4] was found in
good agreement with the experimental work of Latham [3]. These pioneering studies of Latham [3] and
Shapiro [4] were first precise efforts to mechan- ically investigate peristaltic flows and triggered extensive
interest in peristaltic flows. Later on several investigations were made to study peristaltic flows under
different flow configurations. More realistic models for the ureteral waves were proposed by Lykoudis [5]
and Weinberg et al. [6]. Time elapsed during the chyme motion through small intestine was calculated by

Bar- ton and Raynor [7] using the concept of peristaltic motion. Inertia free peristaltic flow in a roller pump was studied by Maginniss [8]. Shapiro et al. [9] examined

> 4the peristaltic flow of Newtonian fluid in a planner channel

and circular

> 80tube. Long wavelength and low Reynolds number approximations were used in

this investigation. In continuation Jaffrin [10]

> 26studied the effects of inertia and streamline curvature on peristaltic

pumping. Yin and Fung [11] and Fung and Yih [12] studied the peristaltic flow for arbitrary Reynolds and wave numbers in axisymmetric tube and planar channel respectively. In these studies the authors performed the analysis for small amplitude ratio. Jaffrin and Shapiro [13] made review for studies available on peristaltic pumping. Mittra and Prasad [14] studied the interaction of peristalsis with poiseuille flow. Wilson and Perel [15] analyzed the interaction of peristalsis with pulsatile flow. Brown and Hung [16] compiled the

> 15computational and experimental investigations on two-dimensional peristaltic flows

up to 1977. Srivastava and Srivastava [17] discussed the interaction of peri- stalsis with pulsatile flow in a cylindrical tube by assuming that the flow is generated by an arbitrary pressure gradient and peristaltic waves. Peristaltic transport of blood was seen by Srivastava and Srivastava [18]. In this study they used the Casson fluid model to describe the non-Newtonian properties of blood. Numerical solutions of two-dimensional peristaltic flow problem were obtained by Takabatake and Ayukawa [19]. Takabatake et al. [20] studied the

> 66peristaltic pumping in circular cylindrical tubes. They obtained the numerical solutions of the problem and

discussed the efficiency of such flow. Fluid mechanics of gastrointestinal tract was studied by Mishra and Rao [21]. They considered the

> 52peristaltic transport in a channel with porous peripheral layer analogous to the flow in gastrointestinal tract. The magnetohydrodynamic character of

fluid has a pivotal role in solidification processes of metals and metal alloys, study of nuclear fuel debris, control of underground spreading of chemical wastes and pollution control, design of MHD power generators, blood pump machines, diagnose and treatment of cancer tumors.

> 15Interaction of peristaltic flow with pulsatile magneto- fluid through a porous medium

was analyzed by Afifi and Gad [22]. Elshehawey et al. [23] examined the

> 5effects of inclined magnetic field on magneto fluid flow

between two inclined wavy plates. In this study they further assumed the boundaries to be porous. Peristaltic flow of blood through a non-uniform channel was inspected by Mekheimer [24]. Blood was assumed to flow

> 81under the influence of a uniform applied magnetic field.

Mekheimer [25] carried out the study of magnetohydrodynamic

> 143nonlinear peristaltic transport in an inclined planar channel. Magnetic fluid model

induced by peristaltic waves was proposed by Siddiqui et al. [26]. Hayat and Ali [27] investigated the

> 13**peristaltically induced motion of MHD third grade fluid in a deformable tube.**

Abd El Naby et al. [28] analyzed the

> 31**effects of magnetic field on trapping** while studying **peristaltic motion** of a
> **generalized Newtonian fluid.** Nonlinear peristaltic transport **in a** planar **channel**

> 15**under the** influence **of** applied **magnetic field was** studied **by** Hayat **et al.**

[29]. Hayat and Ali [30] provided the

> 13**mathematical description of peristaltic hydromagnetic flow in a tube.** Effect of
> **applied**

> 44**magnetic field on** the **peristaltic transport of micropolar fluid**

was discussed by Wang et al. [31]. Hayat et al. [32] studied the

> 13**influence of inclined magnetic field on peristaltic transport of fourth grade
> fluid in an inclined asymmetric channel.**

In several physical situations, the no-slip condition between the fluid and solid boundary is not valid. Such situation arises in fluid mechanics within the body of living beings where the internal linings of the tubular organs are coated with mucus/secretions. Also in the case of artificial heart the no-slip condition is no longer effective. In such situations one may take into account the slip effects while analyzing the flow phenomena. Mandiviwalla and Archer [33] inspected the

> 6**effects of velocity slip on the**

peristaltic pumping in a rectangular

> 10**channel. Hayat et al.** [34] studied **the peristaltic transport** through **a**

porous medium

> 29**under the influence of** partial slip. Hayat **et al.** [35] **studied the peristaltic
> transport** with simultaneous effects **of**

heat transfer and velocity slip.

> 30**Effects of** partial **slip and heat transfer on the peristaltic flow of** MHD
> Newtonian **fluid in an asymmetric channel** were examined **by**

Yildirim and Sezer [36]. Kumari and Radhakrishnamachrya [37] studied the effects

> 99**of slip on heat transfer** for **peristaltic transport in the presence of magnetic
> field.**

Processes that take place at high temperature e.g. in nuclear reactors, turbines, rockets, missile technologies, pumps operated at high temperatures and space vehicles may face variation in fluid viscosity with temperature. Moreover the dependence of fluid viscosity is justified physiologically because normal person or animal of similar size takes 1-2L of the fluid every day. Also 6-7L of the fluid is received by the small intestine as secretions from salivary glands, stomach, pancreas, liver and small intestine itself. Ali et al. [38] carried out the study to examine the

> 1**influence of** velocity **slip on the peristaltic** transport **in a**

planar channel. Fluid viscosity was assumed to be space dependent and a uniform magnetic field was also taken into account. El Naby et al. [39] studied the

> 3**effects of an endoscope and variable viscosity on peristaltic motion.**
> Elshehawey **and**

Gharsseldien [40] examined the

> 52**peristaltic transport of three-layered flow with variable viscosity.** Hayat **and**

Ali [41] examined the

> 45**effect of variable viscosity on the peristaltic transport of Newtonian fluid in an asymmetric channel.**

Ebaid [42] computed

> 55**numerical solution for the MHD peristaltic flow of bio-fluid with variable viscosity in a circular cylindrical tube via Adomian decomposition method.**

> 4**Effects of heat transfer on the** MHD **peristaltic transport of** a variable viscosity **fluid**

were studied by Nadeem and Akbar [43]. The fluid viscosity in this study was considered to vary linearly with temperature and Adomian decomposition Method (ADM) was used

> 88**to obtain the solution of the coupled equations.**

Nadeem et al. [44] discussed the influence

> 4**of heat transfer on the peristaltic transport of fluid with variable viscosity.**

Nadeem and Akbar [45] presented the study for the

> 4**effects of temperature dependent viscosity on peristaltic flow of Jeffrey-six constant fluid in a non- uniform vertical tube.**

Nadeem and Akbar [46] examined the

> 4**influence of temperature dependent viscosity on peristaltic transport of a Newtonian fluid.** The concept **of**

heat transfer has a key role in the analysis of tissues, hemodialysis and oxy- genation. Radhakrishnamachrya and Murty [47] studied influence of

> 25**heat transfer** on **peristaltic transport in a non-uniform channel.**

Vajravelu et al [48] performed the

> 3**heat transfer analysis of** the **peristaltic** transport **in a**

vertical porous annulus under the long wavelength consideration. Simultaneous effects

> 105**of heat transfer and** applied **magnetic field on** the **peristaltic transport** in **a vertical annulus**

were studied by Mekheimer and elmaboud [49]. Srinivas and Kothandapani [50] examined the

10peristaltic transport in an asymmetric channel with heat transfer. Hayat et al. [51] performed the

heat transfer

123analysis for the peristaltic transport of an electrically conduct- ing fluid in

porous space.

26Srinivas and Kothandapani [52] studied the impact of heat and mass transfer on the MHD peristaltic

motion with complaint walls. Hayat and Hina [53] investigated the

3heat and mass transfer effects on the peristaltic motion of Maxwell fluid.

Mekheimer

3et al. [54] analyzed the peristaltic transport under the influence of

heat transfer and space porosity. Nadeem and Akbar [55] analyzed the effect

83of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer.

Ali

64et al. [56] studied the heat transfer effect on peristaltic transport in a

curved channel.

79Hayat and Noreen [57] examined the peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field.

20Effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel

were analyzed by Srinivas and Muthuraj [58].

25Vajravelu et al. [59] discussed the influence of heat transfer on peristaltic transport of Jeffrey fluid in a vertical porous stratum.

Hayat et al. [60] ana- lyzed the

4heat transfer effects on the peristaltic transport in a curved channel with

complaint walls. Mixed convection

2heat and mass transfer in an asymmetric channel with peristalsis

was discussed by Srinivas et al. [61]. Effects

2of heat and mass transfer on the peristaltic flow of Johnson Segalman fluid in a vertical asymmetric channel with induced MHD

was examined by Nadeem and Akbar [62]. Tripathi [63] proposed

29a mathematical model for swallowing of food bolus through the oesophagus under the influence of heat transfer. Akbar et al. [64] computed the effects of

2heat and mass transfer on the peristaltic transport of hyperbolic tangent fluid.

Akbar and Nadeem [65] investigated the

3heat and mass transfer effects in the peristaltic trans- port of Johnson-Segalman fluid.

Effects of

12induced magnetic field were also taken into account

in this study.

2Effects of induced magnetic field and heat transfer on peristaltic transport

were also discussed by Hayat et al. [66]. This study was performed for the Carreau fluid model.

16Peristaltic transport of third order fluid under the effect of applied magnetic field was studied by Hayat et al.

[67]. Hayat et al. [68]also analyzed

4the peristaltic flow of a third order fluid in an asymmetric channel with slip effects. Heat and mass transfer

analysis for

26the peristaltic transport of third order fluid in a diverging tube

was carried out by Nadeem et al. [69]. Akbar et al. [70] studied the

4effects of slip and heat transfer on the peristaltic flow of third order fluid in an inclined asymmetric channel.

1.3 Fundamental laws 1.3.1 Law of conservation of mass Mathematical form of law of conservation of mass is known as continuity equation which for unsteady flow is given by $?? ?? + \nabla \cdot (?V) = 0?$ (1.1) in which $?$ denotes the density of the fluid and V the velocity. Eq. (1?1) for an incompressible fluid is $\nabla \cdot V = 0?$ (1.2) 1.3.2 Law of conservation of linear momentum

38Newton's second law of motion states that time rate of change of linear momentum is equal to the total force acting on the

fluid element. Mathematically it is expressed as $? ?V ?? = \nabla \cdot τ +?b?$ (1.3) for an incompressible flow. Here τ is the Cauchy stress tensor which is different for different fluids, $\nabla?τ$ is the surface force, $?b$ is the body force. The body force $?b$ is different for different flow situations. For instance the expression of body force $?b$ for

113the flow of an electrically conducting fluid in the presence of magnetic field and buoyancy effects is

$?b =?g [?? (? − ?0) + ?? (? − ?0)] + J × B$ (1.4) where g is the gravitational accelration, $?$ and $?0$ the temperatures of fluid and boundary respectively, $?$ and $?0$ the concentrations of fluid and boundary respectively, $??$ and $??$ the volumetric expansion coefficients, J the current density and B the applied magnetic field. Furthermore first two terms

> 139**on the right hand side of Eq.** (1 **?4)** represent **the**

body force due to density differences caused by temperature difference and composition difference. The last term of Eq. (1?4) is the Lorentz force. The Cauchy stress tensor τ is expressed as ??? ? ?? ? ?? ??? ? ?? ? ?? τ = ▢ ? ?? ??? ? ?? ▢ or τ = ▢ ? ?? ??? ? ?? ▢ , (1.5) ▢ ▢ ? ?? ? ?? ??? ▢ ? ?? ? ?? ??? ▢ ▢ ▢ ▢ ▢ where ??? (???) ? ??? (???) and ??? are the normal stresses while all others are shear stresses. In catesian coordinates (?? ?? ?) Eq. (1?3) can be written as ? ?? ?? ?? ?? = µ ?? +??? +??? +??? ? ?? ?? ?? ?? ¶ = µ ?? +??? +??? +??? ? ?? ?? ?? ?? ¶ = µ ?? + ? ?? + ? ?? + ? ?? ¶ 1.3.3 First law of thermodynamics ? (???) ? (???) ?? + ?? + ? (?????) + ???? ? (???) ? (???) ? (???) ?? + ?? + + ???? ?? ? (???) ? (???) ?? + ?? + ? (???) + ???? ?? (1.6) (1.7) (1.8) The law of consrevation of energy is also known as first law of thermodynamics and can be expressed as ? ?(??? ) = τ · ∇V − ∇ · (−? ∇? ) + J · J? 1 ?? ? (1.9) Here ?? is the internal energy, τ · ∇V the viscous dissipation, ?∇? the heat flux, ? the temperature field, ?? the specific heat, ? the electrical conductivity of fluid, ? the thermal conductivity, ? the diffusion coefficient and the last term on RHS of Eq. (1.9) is due to the Joule heating. 1.3.4 Advection diffusion equation The diffusion of chemically reacting species is governed by Fick's second law. It is given by ???? = ?∇2? − ??(? − ?0)?? (1.10) where ? is the concentration field of the diffusing species, ? the oreder of chemical reaction, ? the diffusion coefficient and ?? the reaction rate. 1.3.5 Dufour and Soret effects It has been experimentally verified that the diffusion of energy can be

> 14**caused by a composition gradient.** This fact **is** known as **Dufour** effect **or diffusion-thermo effect. The**

diffusion of diffusing species by temperature gradient is termed as Soret effect

> 14**or thermal diffusion effect. In** most of **the**

studies dealing with the transfer of heat and mass,

> 11**Dufour and Soret** effects **are neglected** under **the** assumptions **that** these effects **are**

> 11**smaller order of magnitude** as **described by Fourier's and Fick's laws.**

However recent developments show that these effects are significant when transfer of heat and mass occurs in the flow of mixture of

> 32**gases with very light molecular weight** (?2, ??) **and** gases with **medium molecular weight** (?2? **air)**

? Eqs. (1.9) and (1.10) can be modified to make them capable of describing Dufour and Soret effects as follows: ?? ??? ?? = ??∇2? + ??? + Φ + ????? ∇2? + 1 ? J · J? (1.11) ?? ?? = ?∇2? + ??? ∇2?? (1.12) ?? where ? ?? is the viscous dissipation term, ? the stress tensor, ? the velocity gradient, ?? the concentration susceptibility, ?? the thermal-diffusion ratio and ?? the mean fluid temperature. 1.3.6 Maxwell's equations Equations which mathematically state a set of laws namely

> 110**Gauss' law** of **electricity, Gauss' law** of **magnetism, Faraday's law** and **Ampere-Maxwell law**

are known as Maxwell equations after the name of James Clerk Maxwell. These equations are ∇ · E = ?? ? (Gauss' law for electricity) ? (1.13) ?0 ∇ · B = 0? (Gauss' law for magnetism) ? (1.14) ?B ∇ × E = − ?? ? (Faraday's law) ? (1.15) ∇ × B = ?0J + ???0 ?? ? (Amapere-Maxwell law) ? ?E (1.16) Here ?? indicates the charge density, ?0 the permittivity of free space, B the magnetic

> 141**field, E the electric** field, J **the current density** and ?? **the electric**

constant. 1.4

> 7**Constitutive equations of a third order fluid The**

Cuachy stress tensor τ in an incompressible homogeneous third order fluid is [67-70]: τ = −?I+ ? + ?3?? A21 A1 + ?1A2 + ?2A21 + ?1A3 + ?2 A1A2 + A2A1 + (1.17) ³ ´ ¡ ¢ in which ?? ?? (? = 1? 2) and ?? (? = 1? 2

7**?3) are the material constants. The Rivlin-Ericksen tensors can be** represented **as** follows A1 = **L+L??** (1.18) **A?+**

1 =

23**?A??? + A?L + L?** A? ? = **1?** 2? **(1.**

19) Chapter 2 MHD mixed convection peristaltic flow with variable viscosity and thermal conductivity This chapter concerns with mixed convection peristaltic flow of an electrically conducting

1**fluid in an** inclined **channel.** Analysis **has been** carried out **in the presence of** Joule heating. **The**

127**fluid viscosity and thermal conductivity are assumed to vary**

with respect to temperature. A nonlinear coupled governing system is computed. Numerical results are presented for the veloc- ity, pressure gradient, temperature and streamlines. Heat transfer rate at the wall is computed and analyzed. Graphs reflecting the contributions of embedded parameters are discussed. 2.1

6**Mathematical formulation We** examine **the peristaltic transport of** viscous **fluid in an** inclined **asymmetric channel of width**

(?1 + ?2) ? The channel is inclined at an angle ?1. The ?−axis is chosen along the length of the channel and ?

20**−axis is taken normal to the** ?−axis. **A uniform magnetic field of** strength **?0** acts parallel to **the** ? −axis. Effects **of**

induced magnetic field are neglected under low magnetic Reynolds number approximation. Electric field effects are also negligible. The viscosity and thermal conductivity are temperature dependent. The following forms of waves propagate along the channel walls ?1(?? ?) = ?1 + ?1, ?2(?? ?) = −?2 − ?2,

87**?1 2? = ?1??? ? (? − ??) ? µ 2? ¶ ?2 = ?1??? (? − ??) + ? ? µ**

? ¶ in which ? is time, ?1 and ?2 represent

171**the upper and lower walls of channel,**

?1 and ?1 are the amplitude of the waves at respective walls, ? the phase difference of two waves and ? and ? are the speed and wavelength of the waves respectively. Appropriate velocity field for this problem is ? = ?(?? ? ? ?)? ? (?? ? ? ?)? 0 ? Laws of conservation of mass and linear momentum give £ ¤ ?? ?? ?? ?? = 0? + (2.1) ? ? +? +? ? ? ? ?? ? ?? = − ?? ? ? (? ) ?? + ? ? (? ) ?? + 2 + µ ?? ?? ?? ¶ ?? ?? −??02? + ???∗ (? − ??) sin ?1 + ?? sin ?1? µ ?? ¶ ?? ?? − ??02? ·

169**µ ?? ?? ¶¸ (2.2) ? ? +? +? ? ? ? ?? ? ?? + ? ?? ?? ? (? ) µ ?? ?? ?? ¶ = −?? +2??** ?(?)?? µ ¶ ?? −???∗ (? − ??) cos **?1**− ?? cos ?1? · **µ ?? ?? ¶¸ + (2.**

3) In above equations ?

122**is the density of fluid, ? is the** electric **conductivity, ? (? ) is the**

temperature dependent dynamic viscosity, ? is the acceleration due to gravity, ?? is the mean value of the temperature of both the channel walls and ?∗ is the thermal expansion coefficient. We consider (?? ?) and ? as the

7**velocity components and pressure in the wave frame (?? ?). The**

? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (2.4) where (? ? ? ) and ?

laboratory frame. The

expressed as ?? ?? ?? ?? = 0? + (2.5) ? (? + ?) ??? + ? ??? (? + ?) = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? ³ −??20´(? + ?) + ???∗ (? − ??¡) sin ?1 +¢ ?? sinh?1? ³ ´i (2.6) ? (? + ?) ??? + ? ??? (?) = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? ³ −´???∗ (? − ??) cos³?1 − ?? c´os ?1? h ³ ´i (2.7) Defining the dimensionless quantities ?= ? ? ? = ? ? = ? ? = ? ? ? ?21? ? ? ? ? = ?1 ? ?1 = ??11 ? ?2 = ?? 12 ? ? = ??21 ? ? = ? ???0 , ?1 ?? ?= ?0 ? ?(?) = ?(? ) ? ? 2 = ? − ?? ? ? 0 µ ?0 ¶ ?02?12? ? = ?? ? ?? = ???∗ (??10−? ?0) ?21 ? 2 ? ? ? = ?1 − ?0 ? ??= ????01? ?? = ???21? ? = ??, ? = −?? (2.8) and adopting long wavelength and low Reynolds number approach we have in terms of stream function ? the following equations ?? = ? ? (?) ?2? − ?2 ?? +1 ?? +??? Si ??1 + Re S?i???1 · ??2 ¸ µ ?? ¶ ? (2.9) ?? = 0? (2.10) and the continuity equation is identically satisfied. Here, ?1?2 are the dimensionless wall shapes,

stream function. Equation (2.10) indicates that ? 6= ?(?) and compatibility equation from Eqs. (2.9) and (2.10) has the form 0= ???22 ?(?)??2??2 −?2 ??2??2 +?????? sin ?? · ¸ µ ¶ The volume flow rate in fixed frame is defined as ?1 ? = ? (? ? ? ? ?)? ? Z?2 and in wave frame we have ?1 ? = ?(?? ?)??? Z?2 in which ?1?2 are functions of ? alone. From Eqs. (2.12), (2.13) and (2.5) one has ? = ? + ??1(?) − ??2(?)? The time averaged flow over a period ?? is ? = 1 ?? ???? (2.11) (2.12) (2.13) ? Z0 which implies that ? = ? + ?? 1 + ??2? Defining ? and ? as the dimensionless mean flows in laboratory and wave frames by ? = , ? = ? ? ??1 ??1 one can write ? = ? +1+?? (2.14) where ?= Z?2 ?? ?? = ?(?1) − ?(?2)? ?1 ?? (2.15) Dimensionless boundary conditions in terms of stream function are ? = ? 2 , ?? = −1 at ? = ?1, ? = − , ?? = −1 at ? = ?2. ? 2 (2.16) 2.2 Heat transfer analysis We have assumed that the walls at ?1 and ?2 have temperature ?0 and ?1 respectively. Energy equation in presence of viscous dissipation and Joule heating effects in laboratory frame is 2 ??? ??? +??? = ∇? ?(?)(∇?) +?(?)"2(µ??¶ ?? 2 + ?? ?? ) + ?? ?? 2 ?? ?? + ¡ ¢ £ ¤ µ ¶ µ ¶ # +??02?2? (2.17) In wave frame above equation takes the form ??? ((? + ?) ?? + ???) = ∇? ?(? ) (∇? ) + ? (? ) "2 (µ ?? ?? 2 ?? 2 + ?? ?? 2 ¶ ) + ¶ µ ?? µ ?? ?? + ¶ ¶ # +?£?20 (? + ?)2 ?¤ (2.18) in which ? is the fluid temperature, ?? is the specific heat, ? is the dimensional tempera- ture dependent thermal conductivity and ? (? ) is the temperature dependent viscosity. The dimensionless energy equation in view of afore mentioned approximations becomes ? {?(?)??}+???(?) ??? 2+???2 ?? 2 + 1 = 0? (2.19) ?? ?? ¡ ¢ µ ¶ where ?? is the Brinkman number, ?(?) the dimensionless temperature dependent thermal conductivity, ? the dimensionless temperature, Pr the Prandtle number and E the Eckert num- ber. These involved quantities are defined as follows: ? = ??1−−???0? ?(?) = ??(?0)? ?? = Pr?? ?? = ??0?0?? ? = ?2 ? (2.20) ?? (?1 − ?0) Dimensionless boundary conditions for the temperature are given by ? = − at ? = ?1 and ? = at ? = ?2? 1 1 2 2 (2.21) Temperature dependent viscosity and thermal conductivity are taken in the form ?(?) = 1 − ??? ?(?) = 1 + ??? (?, ?) ¿ 1? (2.22) Note that ?0 and ?0 are the viscosity and thermal conductivity of the fluid when the temper- ature of the fluid approaches the mean temperature. Also the cases of constant viscosity and thermal conductivity can be recovered by choosing ? and ? equal to zero. The values of ?1 and ?2 are ?1(?) = 1 + ? cos(2??), ?2(?) = −? − ? cos(2?? + ?)? (2.23) We note that Eq. (2.19) is nonlinear in the present analysis. Hence in this study we have nonlinear coupled system. Analytic solution of the resulting nonlinear coupled system seems difficult. Hence the numerical solution through Mathematica is prefered here. The step sizes for x and y are chosen 0.01. The flow quantities of interest will be analyzed in the next section. 2.3 Graphical analysis This section aims to analyze the numerical results for influential parameters. Plots of pressure gradient, axial velocity, temperature and stream function are obtained and examined. Numer- ical values of the heat transfer rate are given in Table 2.1. It is obvious from Figs. 2.1 that pressure gradient for constant viscosity fluid is less when compared with variable viscosity fluid. Temperature dependent thermal conductivity also in- creases the value of pressure gradient. Such results are obvious from the Figs. 2.1 a-b. An increase in ? and ? also increases the pressure gradient. Pressure gradient decreases by increas- ing Hartman and Froud numbers. However this change in pressure gradient due to M and Fr is sufficiently large when compared with that of ? and ? (see Figs. 2.1 d and f). As the value of ?1 is varied from 0 to ??2 the pressure gradient tends to attain significantly large value which highlights the fact that pressure gradient is higher in a vertical channel when compared to horizontal channel (Fig. 2.1 c). For assisting flow (+ve value of acceleration due to gravity g) pressure gradient is large than that of opposing flow (see Fig. 2.1 e). Plots of axial velocity are given in Figs. 2.2 a-d. These Figs. showed that velocity tends to follow parabolic path with maximum value occurring near the centre of the channel. Fig. 2.1a shows that when the viscosity parameter ? is taken zero i.e. the velocity for constant viscosity fluid is seen to have maximum value near the centre of the channel whereas it tends to shift towards the lower wall for non zero values of the viscosity parameter. Moreover a slight increase in the maximum value of velocity is observed when ? increases. Channel inclination angle ? 1 is seen to have almost similar effect on the velocity as that of ? (see Fig. 2.2 b). Maximum value of

velocity in inclined channel is shifted towards the lower wall. Increase in the strength of applied magnetic field decreases the velocity. This change in velocity is maximum when the Hartman number is taken between 1 and 2 (see Fig. 2.2 c). Change in the value of Grashoff number varies the symmetry of the velocity profile relative to the channel i.e. velocity has maximum value near the centre for opposing flow and this change occurs when G? has +ve value. Effect of Grashoff number on the velocity also depends on the channel inclination which is well justified physically (see Fig. 2.2 d). It is further noted that ? has no effect on the axial velocity. Behavior of the dimensionless temperature through ?? ?? ?, ?1? G? and Br are examined in the Figs. 2.3 a-f. Fig. 2.3a depicts that an increase in the viscosity parameter decreases the temperature. This decrease in temperature subject to decrease in ? is uniform throughout the channel. Fig. 2.3 b analyzes the impact of variable thermal conductivity parameter ? on the temperature. Effect of ? on the temperature is noteworthy in the sense that it is not uniform throughout the channel. It means that near the lower wall (?2) an increase in the value of ? decreases the value of temperature of the fluid whereas near the upper wall (?1) it increases the value of ?? Since ? yields the perturbation in the thermal conductivity of the fluid so increase in this parameter gives rise to increase in the ability of the fluid to dissipate or absorb heat. Hence when temperature of the fluid is higher than the temperature of the boundary then increase in ? results in reduction of the temperature of fluid and vice versa. Increase in the Hartman number increases the value of ?? This fact is primarily due to consideration of Joule heating (see Fig. 2.3c). Channel inclination also has non-uniform effect on the temperature. Increase in ?1 near the lower wall increases the temperature but such increase although is not very significant. Similarly the Grashoff number also has very less and non-uniform effect on the temperature (see Figs. 2.3 d and e). Fig. 2.3 f shows that an increase in Br increases the temperature ? significantly. Also when Br is increased then the temperature plots become non-linear. Streamlines for ? and ? are plotted in the Figs. 2.4 and 2.5. It is seen that the size of trapped bolus decreases with an increase in ?? However ? has no significant effect on the size of the bolus. Table 2.1 gives the numerical values of heat transfer rate at the wall ?0(?1) for different values of the embedded parameters. Results showed that an increase in the value of viscosity parameter ? reduces the transfer rate but heat transfer rate increases through increase in ?? The transfer rate decreases via inclined nature of channel. Further an increase in Hartman number increases the transfer rate. 2.4 Main points Effects of variable viscosity and thermal conductivity on the MHD peristaltic transport of fluid in an inclined asymmetric channel are analyzed. Mixed convection and Joule heating effects are present. Main results are reported below. • Variable viscosity and thermal conductivity tend to increase the pressure gradient. • Viscosity parameter, Grashoff number and inclination angle have similar effects on the velocity. However velocity is unperturbed due to change in thermal conductivity parame- ter. • Variable thermal conductivity has no significant change on the bolus size. • Variable viscosity tends to reduce the temperature of fluid. • Viscosity parameter and inclination angle have similar effects on heat transfer rate at the wall. The following values are used in the plots Figs. 2.1 (a-g), ? = 0?1? ?1 = ??6? ? = 0?1? ?? = 3? ?? = 2? ?? = 0?2? ? = 0?5? ?? = 2? ?? = 0?5? ? = 0?4? ? = 0?3? ? = ??4? ? = 0?7 and ? = 0?7? Figs. 2.2 (a-d), ? = 0?1? ?1 = ??6? ? = 0?1? ? = 0? ?? = 3? ?? = 2? ?? = 0?4? ? = 0?5? ?? = 2? ?? = 0?5? ? = 0?4? ? = 0?3? ? = ??4? ? = 0?7 and ? = 0?9? Figs. 2.3 (a-f), ? = 0?1? ?1 = ??6? ? = 0?1? ? = 0? ?? = 5? ?? = 2? ?? = 0?5? ? = 1? ?? = 1? ?? = 0?5? ? = 0?4? ? = 0?3? ? = ??4? ? = 0?7 and ? = 1?5? Figs. 2.4 and 2.5, ? = 0?1? ?1 = ??6? ? = 0?1? ?? = 3? ?? = 2? ?? = 0?2? ? = 0?5? ?? = 2? ?? = 0?5? ? = 0?4? ? = 0?3? ? = ??4? ? = 0?7 and ? = 1?2? 0.2 0.2 0.0 0.0 -0.2 -0.2 dp?dx dp?dx e = 0.0, 0.1, 0.25 -0.4 a = 0.0, 0.2, 0.4 -0.4 -0.6 -0.6 -0.8 -0.8 -1.0 -1.0 -0.4 -0.2 0.0 0.2 0.4 x -0.4 -0.2 0.0 0.2 0.4 Fig. 2.1 a Fig. 2.1 b x 1.0 0.0 0.5 M = 0.0 a1 = p/2 -0.5 0.0 M = 1.0 a1 = p/4 -1.0 dp?dx -0.5 dp?dx -1.5 -1.0 -2.0 a1 = 0 -1.5 -2.5 M = 2.0 -0.4 -0.2 0.0 0.2 0.4 -3.0 -0.4 -0.2 0.0 0.2 0.4 Fig. 2.1 c x Fig. 2.1 d x 0.2 2.5 0.0 -0.2 dp?dx -0.4 -0.6 -0.8 -1.0 Gt = -2, 0, 2 dp?dx 2.0 Fr = 0.5 1.5 1.0 0.5 Fr = 1.0 0.0 Fr = 1.5 -0.5 -0.4 -0.2 0.0 0.2 0.4 x -0.4 -0.2 0.0 0.2 0.4 Fig. 2.1 e Fig. 2.1 f x Figs. 2.1 a-f. Development of pressure gradient in the axial direction. 0.0 0.0 -0.2 -0.2 a = 0.6, 0.4, 0.0 a1 = p/2, p/4, 0 -0.4 -0.4 U U -0.6 -0.6 -0.8 -0.8 -1.0 -1.0 -0.5 0.0 0.5 1.0 y -0.5 0.0 0.5 1.0 Fig. 2.2 a Fig. 2.2 b y 0.0 0.0 -0.2 -0.2 M = 0.0, 0.8, 1.6 Gt = 2, 0, -2 -0.4 -0.4 U U -0.6 -0.6 -0.8 -0.8 -1.0 -1.0 -0.5 0.0 0.5 1.0 -0.5 0.0 0.5 1.0 Fig. 2.2 c y y Fig. 2.2 d Fig. 2.2 a-d Axial velocity for different parameters. 0.8 0.8 0.6 0.6 e = 0.0, 0.1, 0.2 0.4 0.4 a = 0.0, 0.4, 0.8 q 0.2 q 0.2 0.0 0.0 -0.2 -0.2 -0.4 -0.4 -0.5 0.0 0.5 1.0 Fig. 2.3 a y 0.0 -0.5 Fig. 2.3 b y 0.5 1.0 q 0.8 0.6 0.4 0.2 0.0 M = 0.0, 0.5, 1.0 q 0.6 a1 = 0, p/4, p/2 0.4 0.2 0.0 -0.2 -0.2 -0.4 -0.4 -0.5 0.0 0.5 1.0 0.0 0.5 1.0 y Fig. 2.3 c y -0.5 Fig. 2.3 d 0.6 Gt = -2, 0, 2 0.4 Br = 0.1, 0.2, 0.3 0.4 0.2 0.2 q 0.0 0.0 q -0.2 -0.2 -0.4 -0.4 -0.5 0.0 0.5 1.0 0.0 0.5 1.0 y -0.5 Fig. 2.3 e Fig. 2.3 f y Figs. 2.3 a-f. Dimensionless temperature for various parameters. Fig. 2.4 Streamlines for viscosity parameter ?. Fig. 2.5 Streamlines for thermal conductivity parameter ?.

? ? ?1 ? 0?0 0?1 ??6 0?5 0?4 0?6 0?1 0?0 0?1 0?2 0?1 0 ??4 ??2 ??6 0?0 0?4 0?8 −? 0(? 1) 1?0603 1?0257 1?0051 0?9893 1?0524 1?1258 1?1278 1?0247 0?9892 0?9857 1?0284 1?1567

Table 2.1. Numerical values of heat transfer rate at the wall (?1) when ?? = 3? ? ? = 2? ?? = 0?2? ? = 0? ?? = 2? ? ? = 0?5? ? = 0?4? ? = 0?3? ? = ??4? ? = 0?7 and ? = 1?2?

Chapter 3 Hydromagnetic peristaltic transport of variable viscosity fluid with heat transfer and porous medium This chapter examines the peristaltic transport of variable viscosity fluid in a planar channel with heat transfer. The fluid viscosity is taken temperature dependent. The fluid is electrically conducting in the presence of a constant applied magnetic field. Both channel and magnetic field are considered inclined. An incompressible fluid saturates the porous space. Heat transfer analysis is carried out in the presence of viscous dissipation and Joule heating. The resulting problems are solved numerically. A parametric study is performed to predict the impact of embedded variables. Important results have been pointed out in the key findings. 3.1 Mathematical analysis Let us examine the peristaltic transport of an incompressible viscous fluid in a symmetric channel. The channel is taken inclined at an angle ?1 to the vertical. The fluid is electrically conducting in the presence of an inclined magnetic field with constant strength B0. The flow generated is due to peristaltic waves travelling along the channel walls. The wall geometry is chosen in the form ?(?? ?) = ?1 + ?, where ?1 is the half channel width and ? is the disturbance produced due to propagation of peristaltic waves at the walls. This disturbance can be written in the form 2? ? = ?1??? (? − ??) ? µ ? ¶ in which ? is time, ?1 is the amplitude of the peristaltic wave, ? and ? are the speed and wavelength of the waves respectively. Appropriate velocity field for this problem is ? = ?(?????)??(?????)?0 ?

Lawsofconservationofmass,momentumandenergycanberepre- s£ented as follows: ¤ ?? ?? ?? ?? = 0? + (3.1) ? ? ??? + ? ??? ? = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? −??³02???? ?????−? Si?? +??³?∗(? −?0)Si??1+??Si??1− ´ ´ h ³ ?(´?i)? ? ? (3.2) ¡ ¢ ? ? ??? + ? ??? ? = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? −??³02 Si ?? ? ? ??´? − ? Si ?? − ???³∗ (? − ?0´) ? ???1h − ???³???1 − ?´(?i)? ? ? (3.3) ¡ ¢ ?? 2 ?? 2 ?? ?? 2 ??? ??? +??? = ? ??? +??? +?(?) 2 £ " (μ?? ¶ + ) + + ¡ ¢ ¤ μ ?? ¶ μ ?? ?? ¶ # +??02 ?????−? Si?? ?(?)?2 2 + ? ? (3.4) ¡ ¢ Here ? , ? and ?

10**are the velocity components and pressure in the**

laboratory frame (?? ? ? ?), ? the density of fluid, ? is the dynamic viscosity, ? is the acceleration due to gravity, ? is permeability of the porous medium parameter, ?

148**is the** inclination **of applied magnetic field and** ?∗ is **the**

thermal expansion coefficient. We also denote ?? as the specific heat, ? the thermal conductivity and ? the temperature of fluid. We further adopt (?? ?) and ? as the

7**velocity components and pressure in the wave frame (?? ?). The**

7**transformations between laboratory and wave frames are**

? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (3.5) The

10**governing equations in** the **wave frame are given by**

?? ?? ?? ?? = 0? + (3.6) ? (? + ?) ??? + ? ??? (? + ?) = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? −??³02???? ((? + ?) ??´?? − ? Si ??) + ???∗ (?¡ − ?0) Si¢??1 +h?? Si ?³?1 − ?(?)´(?i+?)? (3.7) ? ? (? + ?) ??? + ? ??? (?) = − ???? + 2 ??? ? (? ) ???? + ??? ? (? ) ???? + ???? −??02³Si ?? ((? + ?) ???´? − ? Si ??) − ???³∗ (? − ?0)´????1h− ???³???1 − ?´(?i)(?)? (3.8) ? ?? 2 ?? 2 ?? ?? 2 ??? ((? + ?) ?? + ???) = ? [??? + ???] + ? (? ) "2 (μ?? + μ ?? + ¶ ¶ ) μ ?? ?? + ¶ # +??02 ((? + ?) ???? − ? Si ??)2 + ? (? ) (? + ?)2 ? ? (3.9) Defining the dimensionless quantities ? = ?(?) = ? ? = ? ? ?= ? ?= ? ?= ? ? ? ?1 ? ? ? ? ? = ?1 ?? ? ? ?= ?1 ? ? = ??11 ? ? = ???21??0 , ? = ?0 ? ? ?(? ) ? ? 2 = ? 2 ?0 ? μ ?0 ¶ ?02??21? ?? = ????01? ? = ??? ? = ? −?0?0? ?? = ????∗0??0?21? ? ???21 ? ? = ?21 ? ?? = Pr ?, ? = ???2?0 ? ? ? = ?0?? ? ? ? = ??, ? = −??? (3.10) and adopting

16**long wavelength and low Reynolds number** approach, **the** equations **in** terms **of**

stream function ? are ?? = ?? ?(?) ??2??2 − ?2???2? + ? ? (?) ?? · ¸ ½ ? ? +1 + ?? ¾ μ ¶ ??? Si ??1 + Re Si ??1 ? (3.11) ?? ?? = 0? (3.12) ???+???(?) ??? 2+?? ?2???2?+?(?) ????+1 =0? 2 ? (3.13) ¢ · ¸ μ ¶

3**Note that the incompressibility condition is** automatically **satisfied and**

Eq. (3.11) witnesses ¡ that ? =6 ?(?)? From Eqs. (3.10) and (3.11) we have 0= ???22 ?(?)??2??2 − ?2??? 2?+ ? (?) ?2? ?? · ¸ ½ ? ??2 + ?? ?? Si ??1? (3.14) ¾ μ ¶ Here ? denotes

47**the kinematic viscosity, Re the Reynolds number, ?? the** Brinkman **number, ? the** Eckret **number,** ? ? the Prandtl number, ? **the**

wave number, ?? the Grashoff number, ? ? the Froud number, ? the dimensionless permeability parameter and ? the dimensionless temperature? We consider the temperature dependent viscosity in the form ?(?) = ?−?? which gives ?(?) = 1 − ??? for ? ¿ 1? where ? is the viscosity parameter. Note that for ? = 0 our problem reduces to the constant viscosity case.

16**The dimensionless volume flow rate in** laboratory **frame is** defined **by**

? ?= ? (? ? ? ? ?)? ? (3.15) Z0 and in wave frame we have ? ? = ?(?? ?)?? (3.16) Z0 in which ? is function of ? alone. From Eqs. (3.14), (3.15) and (3.6) one has ? = ? + ??(?)? The time averaged flow over a period ?? is ?? ? = 1 ? Z0 ??? which implies that ? = ? + ??? Defining ? and F as the dimensionless mean flows in the laboratory and wave frames by ? , ? = ? ? (3.17) ?? ?? one can write ? = ? + 1? (3.18) where ? = ? ?? Z0 ?? ??? (3.19) The corresponding boundary conditions for the present flow configuration are ? = 0, ??? = 0, ?? = 0? ? = 0, ? = ? , ?? = −1, ? = 0, ? = ?, (3.20) with ?(?) = 1 + ? cos(2??). (3.21) 3.2 Solution and analysis Numerical solutions of the derived problems are obtained. For that we have taken

0.01 as the step size for the variations of ? and ?. The obtained results are analyzed graphically in this section. The graphs of pressure gradient,

> 30**stream function, velocity and temperature are** examined. Moreover, **the** numerical values **of**

heat transfer rate at the wall are computed and analyzed through Table 3.1. Hence Figs. 3.1(a-h) are plotted to analyze the pressure gradient variation with respect to different embedded parameters. Figs. 3.2-3.8 depict the behavior of streamlines, Figs. 3.9(a-f) for velocity and Figs. 3.10(a-f) for the temperature profile. It is

> 35**seen that the pressure gradient** varies and has **maximum value at the** wider **part of the channel.**

> 178**Minimum value** of **the pressure gradient is**

found near the more occuled part of the channel.

> 27**It is observed that** value of **the** pressure gradient **increases with** an **increase in the**

variable viscosity parameter i.e. the pressure gradient has a higher value for the variable viscosity fluid than that of the fluid with constant viscosity (see Fig. 3.1a). Presence of porous medium decreases the value of pressure gradient (see Fig. 3.1b). Pressure gradient is larger when we move from the horizontal to the vertical channel. Increase in the Hartman number and decrease in the magnetic field inclination tend to decrease the pressure gradient (Figs. 3.1d and f). An increase in Froud number decreases the pressure gradient while the Brinkman number increases its value (Figs. 3.1e and h). Corresponding value of the pressure gradient is higher in the case of resisting (or opposing) flow when compared to that of the assisting flow (Fig. 3.1g). The volume of the fluid trapped within a streamline is usually termed as bolus. This study showed

> 1**that the size of** the **trapped bolus** decreases **with an increase in the** magnetic field **parameter**

M (see Fig. 3.2). Bolus size is

> 14**smaller in** the **case of fluid with**

constant viscosity (see Fig. 3.3).

> 5**Size of the trapped bolus increases with an increase in the** value **of the**

porosity parameter k. It means that the bolus size gets reduced in case of flow through porous medium (Fig. 3.4).

> 64**From Fig. 3.** 5 **it is observed that the**

impact of channel inclination on the size of the trapped bolus is not very significant. However an increase in channel inclination enhances the bolus size by a very small amount. An increase in both magnetic field inclination and Brinkman number give rise to

> 5**an increase in the size of the trapped bolus**

(see Figs. 3.6 and 3.7). Bolus is found to have a comparatively larger size in the case of assisting flow (+ value of g) (see Fig. 31 3.8). Velocity profile is seen to follow a parabolic trajectory with maximum value near the centre of the channel. Such value of the velocity increases when we move from constant viscosity fluid to the fluid with temperature dependent viscosity. It is seen from Fig. 3.9b that the velocity decreases in the case of porous medium. Increase in the Hartrman number decreases the velocity near the centre line whereas the case is opposite for an increase in Brinkman number (see Figs. 3.9d and f). Inclination of the channel results in the increase of the velocity for both assisting and resisting flows (see Fig. 3.9c). Fig. 3.9e revealed that the velocity has a higher value for assisting flow. Dimensionless temperature is analyzed in the Figs. 3.10(a-f). Fig. 3.10 a showed that the temperature is higher for the case of constant

viscosity

> 3**fluid when compared with the** fluid of **variable viscosity.**

Fig. 3.10b

> 3**depicts that an increase in the** value of permeability **parameter** decreases **the**

value of the dimensionless temperature. Dimensionless temperature increases with

> 81**an increase in the value of** the Hartman number **(Fig. 3.10**

c). Similarly temperature is higher for the case of assisting flow (see Fig. 3.10 d). Increase in inclination of channel gives higher value of the temperature whereas the case is opposite for the inclination of magnetic field (see Figs. 3.10 e and f). Numerical values of the heat transfer rate at the wall for different parameters are plotted in Table 3.1.

> 15**It is noted that the heat transfer** rate **decreases** by **increasing**

the viscosity parameter ?? permeability parameter k, Grashoff number G? and channel and magnetic field inclinations (?1 and ? respectively). However it increases due to increase in Hartman number (M) and Brinkman number Br. 3.3 Key findings Numerical solution for the

> 3**peristaltic transport of** the **variable viscosity fluid with** porous medium and **heat transfer**

in an inclined channel is obtained and analyzed. The main points of present attempt are listed below. • Pressure gradient has a higher value for variable viscosity fluid than constant viscosity fluid. • There is a decrease of pressure gradient in presence of porous medium. • Pressure gradient has higher value when channel is vertical and there is resisting flow. •

> 1**Size of** the **trapped bolus increases with an increase in viscosity**

and permeability para- meters. • Trapped bolus has a larger size for assisting flow case. • Fluid velocity with constant viscosity is less when compared to that of variable viscosity. • Effects of porous medium on pressure gradient and velocity are qualitatively similar. • Unlike the velocity, the temperature of the fluid is higher for constant viscosity case. • Temperature is higher for the case of assisting flow. •

> 1**Heat transfer** rate **at the wall** decreases **with** the **increase in**

channel and magnetic field inclinations. The used values of the parameters here are ? = 0?2? ? = 1?0? ? 1? ? = ??4? Re = 5? ? ? = 2? ?? = 0?5? ? = 0?5? ?? = 0?5? ? = 0?3 and ? = 0?3 (unless stated otherwise). 1.6 1.4 1.5 dp?dx 1.2 1.0 0.8 0

> 172**.6 0.4 a = 0, 0.** 3, **0.**

6 dp?dx 1.0 0.5 0.0 k = 5.0 k = 1.5

> 85**k = 0.** 5 **-0.** 4 **-0.2 0.** 0 **0.2 0.** 4 **-0.** 4 **-0.2 0.** 0 **0.2 0.**

4 Fig. 3.1 a x Fig. 3.1 b x 2 1.6 dp?dx 1 0 -1 a1 = p/2 a1 = p/4 a1 = 0 -0.4 -0.2 0.0 Fig. 3.1 c x 0.2 0.4 1.4 dp?dx 1.2 1.0 0.8 0.6 0.4 -0.4 b = 0, p/4, p/6 -0.2 0.0 0.2 Fig. 3.1 d x 0.4 7 6 Fr = 0.5 1.5 5 1.0 dp?dx 4 3 0.5 M = 0.0 dp?dx Fr = 1.0 2 1

> 159**Fr = 0.** 5 **-0.** 4 **-0.2 0.** 0 **0.2**

Fig. 3.1 e x 1.5 dp?dx 1.0 Gt = -2, 0, 2 0.5 -0.4 -0.2 0.0 0.2 Fig. 3.1 g

> 117**x 0.0 -0.** 5 **0.4 -0.** 4 1 **.8 1.** 6 1 **.4**

dp?dx 1.2 1.0 0.8 0.6 0.4 0.4 -0.4 M = 1.0

76**M = 2.0 -0.2 0.0 0.2**

0.4 Fig. 3.1 f x

22**Br = 0, 0.** 5, 1 **.0 -0.** 2 **0.0 0.** 2 **0.**

4 Fig. 3.1 h x Figs. 3.1 (a-h) Pressure gradient for the various parameters. Fig. 3.2 Streamlines for variation of Hartman number. Fig. 3.3 Streamlines for variation of viscosity parameter. Fig. 3.4 Streamlines for variation of porosity parameter. Fig. 3.5 Streamlines for variation of channel inclination. Fig. 3.6 Streamlines for different inclinations of magnetic field. Fig. 3.7 Streamlines for different Brinkman number. Fig. 3.8 Streamlines for different Grashoff number. 0.5 0.5

34**a = 0.0, 0.3,** 0.6 k = **0.** 4, **1,** 10 0 **.0 0.0**

36**U U -0.5 -0.5** -1.0 -1 **.0** -1 **.0 -0.** 5 **0.0**

y Fig. 3.9

60**a 0.** 5 **1.0 -1.0 -0.** 5 **0.0**

Fig. 3.9 b

180**y 0.5 1.0 0.**

5 a1 = 0, p/4, p/2 0.5 M=0,2,4 0

92**.0 0.0 U U -0.** 5 **-0.** 5 -1 **.0 -1.0** -1.0 **-0.** 5 **0.0 0.**

18**5 1.0 y -1.0 -0.5 0.0 0.5 1.0**

Fig. 3.9 c Fig. 3.9 d y 0.5 Gt = -2, 0, 2 0.5

22**Br = 0, 0.** 5, 1 **.0 0.0 0.0**

33**U U -0.** 5 **-0.** 5 -1 **.0** -1 **.0** -1 **.0 -0.** 5 **0.0 0.** 5 1 **.0 -1.0 -0.** 5 0 **.0 0.** 5 **1.0**

Fig. 3.9 e y y Fig. 3.9 f Figs. 3.9 (a-f) Velocity profile for different parameter. 1.4 1.2

175**a = 0.** 0 **1.2** k = **0.** 5 **1.0**

1.0 k = 2.0 a = 0.3 0

23**.8 0.8** q **a = 0.6** q **0.** 6 **0.6**

54**k=10 0.** 4 **0.** 4 **0.** 2 **0.** 2 **0.0** -1 **.0 -0.**

5 0.0 0.5 1.0 y Fig. 3.10

114**a 0.0 0.5** 1 **.0 -1.0 -0.** 5 **0.0**

Fig. 3.10 b y M = 2.0 1.0 Gt = -2, 0, 2 1.5 0.8 M = 1.0 q

22**1.0 0.** 6 q **M = 0.0** 0 **.4 0.** 5 **0.** 2 **0.0 0.0** -1 **.0 -0.** 5 **0.** 0 **0.** 5 1 **.0** -1 **.0 -0.** 5 **0.0 0.**

5 1.0 Fig. 3.10 c y Fig. 3.10 d y 1.4 1.0 a1 = 0, p/4, p/2 b=0 1.2 0.8 b = p/4 1.0 q 0

> 181.**6 0.4** 0.8 **q 0.** 6 **0.**

4 b =

> 48**p/2 0.** 2 **0.** 2 **0.0** -1 **.0 -0.** 5 **0.0 0.** 5 1 **.0 0.0** 1 **.0 -0.** 5 **0.**

5 Fig. 3.10 e y -1.0 Fig. 3.10 f y 0.0 Figs. 3.10 (a-f) Temperature profile for different parameters. . ? 0?0 0?3 0?6 0?2 ? 1?0 0?5 2?0 10?0 1?0 ? 1?0 0?0 1?0 2?0 ?? 2 ?1 ? ?? ?0(?) ??4 ??4 0?5 2?32081 1? 76871 1?44276 2?24094 1?71568 1?52373 1?61917 1?91839 2?8159 1?0 −2?0 0?0 2?0 1?0 0?0 ??4 ??2 ??4 0?0 ??4 ??2 1?95425 1?93242 1?91839 1?93242 1?92427 1?92153 2?22499 1?92427 1? 62345 ??4 0?1 0?401872 0?2 0?795321 0?3 1?18027 Table 3.1. Numerical values of heat transfer rate at the wall for different parameters. Chapter 4

> 102**Peristaltic flow in an asymmetric channel**

with convective boundary conditions and Joule heating This chapter concentrates

> 63**on the peristaltic** transport **of viscous fluid in** an **asymmetric channel. The channel**

walls exhibit convective type boundary conditions. Both cases of hydrodynamic and magnetohydrodynamic (MHD) fluids are considered. Analytic solutions

> 70**for stream function and** temperature **are** constructed. **Numerical integration is** carried out for **pressure rise**

per wavelength. Effects of influential flow parameters are pointed out through graphs. 4.1 Mathematical analysis

> 5**Consider the peristaltic transport of an incompressible** MHD viscous **fluid in** an asymmetric **channel of width**

?1 + ?2. The

> 56**?-axis is taken along** the length **of the** channel whereas the ? **-axis** is **taken normal to the** ?-axis.. **The**

flow is due to the waves travelling on the walls of channel. A constant

> 32**magnetic field** ?0 **is applied in the ? - direction. The**

> 17**fluid is** electrically **conducting while the channel walls are non-conducting. The** upper ?1 **and** lower ?2 **walls are** maintained **at the**

temperature ?0 and ?1 respectively. The geometry of the walls is given as follows 2? ?1(?? ?) = ?1 + ? 1??? (? − ??) , μ ? ¶ ?2(?? ?) = −?2 − ?1??? 2? ? (? − ??) + ? ? (4.1) μ ¶ where ?1 and ?1 are the amplitudes of the waves travelling

> 35**on the upper and lower walls,**

?

> 97**is the wavelength, ? is the wave speed, ? is the** phase difference **and ? is the** time. **The**

laws of mass, momentum and energy yield ?? ?? ?? ?? = 0? + (4.2) ? ? ? =− +? +? ? ? ?? + ? +

> 42**μ ?? ?? ?? ¶ ?? · ????2?** 2 **??2??2**¸ − ??0?2 ? ? ? ? ? =− +? +? ? ? ?? +? ?2? **?2?**μ

?? ?? ?? ¶ ?? ·**??2 ?? 2 ¸ + ? ??? ??? + ? ??? + ? ??? ? = ? ???2?2** + ???2 **?2**

(4.3) (4.4) +? 2 ³ ?? 2 + ???? + ???? + ???? + ??02?2? 2 ´ h 2 i ?? (4.5) · µ³ ´ ³ ´ ¶ ³ ´ ¸ where (?? ? )

> 6**are the velocity components in (?? ? ) directions** respectively, ? **is the**
> **pressure,**

?

> 21**is the dynamic viscosity, ? is the** density **of fluid, ? is the** electrical conductivity,
> ? **is the** thermal conductivity **and**

?? is the specific heat. We have an interest to transform our problem from the fixed frame (?? ? ) to the moving/wave frame (?? ?). For that we define ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (4.6) On setting ?= ? ? ? = ? ? = ? ? = ? ? ? ? ? ?1 ? ?? ? ? = ?1 ? ?1 = 1 ? ? ?1 ?2 = ?2 ? ? = ?2 ? ? = ?1 ? ? = ?1 ? ? = ?21? , ? = ? ? ?1 ?1 ?1 ?1 ??? ? ?? = ??1 ? ? = ?? ? 1?2 ? ? ? ? = µ ¶ ? ?0?1? ? = ? 1 − ?0 ? − ?0 ? ?? = Pr ?, ?= ?2 ?? (?1 − ?0) ? ?? = ???? ? = ??, ? = −?? ? ?? ?? dimensionless form of Eqs. (4?2 − 4?5) take the following form ?? ?? ?? ?? = 0? + ?Re (?+1)?? +??? ?? ?? =− ?? + ?2?2? ?2? −?2(?+1)? + µ ¶ ??

> 146**µ ??2 ??2 ¶ ?3** Re (? + **1)** ???? + ? ???? = − ???? + ?2 ?2 **??2??2 ??2??**

2 + ? µ ¶ µ ¶ ? Pr

> 62**Re (? + 1) ???? + ? ???? = ?2 ???2?** 2 + ??2??2 +?? **2?2 ???? + ???? + ?**
> **2????+???? +???2(?+** 1 **)2? 2 ³ 2 ´ 2**

· µ ³ ´ ¡ ¢ ¶ ³ ´ ¸ (4.7) (4.8) (4.9) (4.10) (4.11) Here ? denotes the kinematic viscosity, ? the Hartman

> 2**number, Re the** Reynolds **number, ?? the** Brinkman number, ? **the** Eckret
> **number, ? ? the Prandtl number, ? the** wave **number** and ? **the**

dimensionless temperature. Making use of the

> 12**long wavelength and low Reynolds number** approximation **we**

have ?? ?3? ?? ??3 = − ?2 ?? + 1 ? (4.12) µ ?? ¶ ?? ?? = 0? (4.13) 0 = ??2??2 + ?? ??2??2 2 + ???2 ???? 2 µ +1 ? (4.14) ¶ µ ¶ where Eq. (4.12) indicates that ? 6= ? (?). Defining ? and F as the dimensionless mean flows in laboratory and wave frames by ? = ? ? = ? ? ? (4.15) ??1 ??1 which after using ? = ? + ? (?1 + ?2) give ? = ? +1+?? (4.16) where ? = ?1 ?? Z?2 ?? ??? (4.17) The convective boundary condition for the temperature can be expressed as follows −? ?? = ?(? − ??)? ?? in which ?

> 5**is the thermal conductivity, ? is the** wall **heat** transfer coefficient and ?? **is the**
> **temperature** of **the**

wall. Physically this condition states that the

> 157**rate of heat transfer from the** wall **to the**

fluid (or vice versa) is proportional to the difference of their temperature. Having in mind the asymmetric nature of considered channel, we choose ?1 and ?2 the respective heat transfer coefficients of upper and lower walls. Hence the dimensionless boundary conditions are ?= ? ?? ?? 2 , ?? = −1, ?? + ??1? = 0? ?? ? = ?1, ?=− ? 2 , ?? ?? ?? = −1, ?? − ??2(? − 1) = 0, ?? ? = ?2, (4.18) ?1(?) = 1 + ? cos(2??), ?2(?) = −? − ? cos(2?? + ?) ? (4.19) ??1 = ?1?1 , ??2 = ?2?1 ? ? ? (4.20) where ??1?2 are the Biot-numbers for the upper and lower walls. Pressure rise per wavelength is given by ∆?? = 1 ?? Z0 ?? ??? (4.21) 4.2 Solution expressions The solutions of Eqs. (4.11) and (4.13) subject to the conditions (4.17) are ? = 2?1 (?2 − ?3) ? −?−?? ?= 1 −?5 + (1 + ??1?1) ?6 + (?1) ¶ ? ? ?4 µ ? ? 1 For ? = 0 one has ? = − (?1 + ?2 − 2?) (−2 (? 21 (−?1?−2)?(?2)13− ?) (?2 − ?) + ? ?8) ? (4.22) (4.23) (4.24) ? = 1 + 6????(2?(?1+ ?−1 ?−2)?32)2 3?? 24????2((??1+−?1?2−)2?2)2+ − ? 1 ??2?2 − 3?? (? + ?1 − ?2)2 (?1 + ?2 − 2?)4 ?3? 4 (?1 − ?2)6 + ?2 ? (4.25) and L?s, A?s and C?s are given as follows: ?1 = ??1 ??2 (?1 − ?2)3 + 6?? (? + ?1 − ?2)2 + 6??2?? (? + ?1 − ?2)2 (−1 + ??2?2) ? ?2 = (³?1 −³?2)3 (??1 + ??2 + ??2??1?1 − ??2??1´?2) ? ´ ?3 = −??1 ??2 (?1 − ?2)3 + 6?? (? + ?1 − ?2)2 + 6??2?? (? + ?1 − ?2)2 ? ?1 = ³??1? (³−2 + ?1? − ?2?) + ???2 (2 + ?1? −´ ?2?) ? ´ ?2 = (³? + ?1 − ?2) 2?(?1+?2)? − 2??2? ? ´ ?3 = ?? ? (?1 + ?2 −³?2?) ?? ?2 (−2 + ? ?´ ) − ?? ?1

(2 + ? ? ) ? ?4 = ??1 + ??2 + ??2??1?1 − ??2??1?2? ³ ´ ?5 = ?? (1 − ??2 + ?2) ? 2 (? ?2 + ?3 − ?4 − ??1 (?5 + ?6)) ? ?6 = ??2 − (?1) × ???2 (−?2 − ?7) + ??(???12)?2 × ?8 − ?9? 1 ? 7 = ??? 2 ?10 − ¡?11?2 ? ¢ µ ¶ ? 8 = ?21 − 4¡?1?2 + ?22 + 2¢ (?1 + ?2) ? 2?2 ? − ? 2 1 = ¡4 ??1? (−2 + ?1? − ?2? ) + ??2? (2¢+ ?1? − ?2? ) ? ?2 = −³2?2?1?(?+?1−?2)2 1+?2 +2?2?2?(?+?1−´?2)2 1+?2 −8??1?(?+?1−?2) ?¡?1?(−2¢+?1? −?2?)+??2?(2¡+?1?−¢?2?) ? ?3 = 8??2?(?+?1−?2)?³ ??1?(−2+?1?−??2)+??2?(2+?1?−?2?´) −4?1 ?2?2? (2 + ?1? −³ ?2?)2 + ?2?1? (2 + ?1? − ?2?) ? ´ ?4 = 2?(?1+³?2)??13? ?5 = ??1? (? + ?1 − ?2)2 1 + ?2 + 2?2?2? (? + ?1 − ?2)2 1 + ?2 −8??1? (? + ?1 − ?2)¡ ??1? (¢−2 + ?1? − ?2?) + ???2?¡(2 + ?1?¢ − ?2?) +8??2? (? + ?1 − ?2) ?³ ´ ?6 = 2?21 ?2?2? (2 + ?1? − ?2? )2 + ?2?1? (2 − ?1? + ?2? )2 − 2?(?1+?2)? ? 13 ? ?7 = 8??2³? (? + ?1 − ?2) ? ??1? (−2 + ?1? − ?2? ) + ??2? (2 + ?1? − ?2? )´ +4?2?12? ³ ´ ?8 = ?2? 1? (? + ?1 − ?2)2 1 + ?2 + 2?2?2? (? + ?1 − ?2)2 1 + ?2 + 8?14? ¡ ¢ ¡ ¢ ?9 ?10 = 8??2? (? + ?1 − ?2) ?? 1? (−2 + ?1? − ?2?) + ??2? (2 + ?1? − ?2?) + 2?2?2? = ?2?(?1+?2−?) (? + ?1 − ?2)2 1 + ?2 + 2?2?? (? + ?1 − ?2) 1 + ?2 ³ ´ +8??(?1+?2−?) (? + ?1 − ?2)¡?14? ¢ ¡ ¢ ?11 ?12 ?13 ?14 = 8??? (? + ?1 − ?2) ? ??1? (−2 + ?1? − ?2? ) + ??2? (2 + ?1? − ?2?³)2 + ?2?1? (2 − ?1? + ? 2?)2 − 2?(?1+?2)??13? ´ = 4 + ?21? 4 − 2?1?2? 4 + ?22? 4 + 2? (?1 − ?2) ? 2 1 + ? 2 + ? 2 ? 2 + ? 4 ? = ??1? (? + ?1 − ?2) ??1? (−2 + ?1? − ?2? ) + ?¡?2? (2 +¢?1? −¡ ?2? ) ? ¢ ³ ´ 4.3 Graphical analysis This section is concerned with the discussion of streamlines, pressure gradient and pressure rise per wavelength. This objective is achieved through Figs. 4.1-4.4. Dimensionless temperature profile is observed in the Figs. 4.5-4.7 for various parameters. In addition

> 1the heat transfer rate at the upper wall is

numerically tabulated. Such numerical values

> 152are presented in Table 4.1. Figs. 4.1

(a and b) examine the streamlines plotted against rate of volume flow ? when ? = 0. It is observed that there is a lower limit of flow rate below which no trapping occurs. This shows that without sufficient volume of the fluid flowing per unit time through the channel, trapping is unlikely to occur. The "threshold" or critical value of flow rate in present problem is observed which approximately is equal to 1.6. It is noticed that when the value of flow rate is increased from the critical value then the

> 17size of the trapped bolus not only increases

but also it gets elongated (see Figs. 4.1a & 1b). Figs. 4.2 showed

> 17that the size of the trapped bolus decreases for

a given value of flow rate and eventually vanishes when Hartman number increases. Fig. 4.3 plots the pressure gradient. It is

> 35seen that the pressure gradient is maximum at the centre of the channel i.e.
> in the wider part

of the channel. Moreover the corresponding value of the pressure gradient is decreased when ? has higher values. Pressure rise per wavelength

> 27decreases with an increase in flow rate and

for a fixed value of

> 27flow rate the pressure rise also decreases by increasing the

value of ?. Figs. 4.5a and 4.5b examine the temperature profile. Fig 4.5a includes the Joule heating effect whereas Fig. 4.5b is without such effect. It is shown that there is no significant effect of ? on the temperature profile when there is no Joule heating. However the temperature in presence of Joule heating effect. Figs. 4.6

> 160(a and b) analyze the effects of ??1 and ??2

separately on the temperature. It is found that variation of ??1 has significant effect near the upper wall and hardly has any effect near the lower wall. Similarly the variation of ??2 is significant near the lower wall. Further the temperature decreases by increasing ??1??2? Fig. 4.7 is for the case when both the walls have same heat transfer coefficient i.e. ??1 = ??2 = ??. Here

> **14the temperature** also **decreases with an increase**

in ??. However such decrease is initially large. Numerical values

> **14of heat transfer** rate **at the** upper **wall is given**

in Table 4.1. Here $?0(?1)$ is the heat transfer rate for ? given in (4.21) and $(?*)0(?1)$ is one when we neglect the Joule heating effect. It is seen that the transfer rate increases when M and ??'s are increased. This increase in heat transfer differs a lot in the two cases when Joule heating is considered and neglected. It is seen that ?? for one wall has very little or no effect (for large value of Bi) on transfer rate at the other wall. 4.4 Concluding remarks Effects of convective boundary conditions and Joule heating

> **12on the peristaltic** transport **of** MHD **viscous fluid** are **examined. The**

key findings of the presented analysis are mentioned below. • There is a critical value of the flow rate below which trapping is not likely to occur. • The applied magnetic field tends to decrease the size of trapped bolus. • Magnetic field decreases the magnitude of pressure gradient and pressure rise. • Temperature rises in presence of Joule heating. • Biot number decreases the dimensionless temperature. • Effects of Hartman and Biot numbers

> **14on the heat transfer** rate **at the wall** are opposite. Fig. **4.**

1 Streamlines for M = 0. Fig. 4.2 Streamlines for M 6= 0. 0 M=0 -20 M=1 dp?dx

> **136-40 M=2 -60 M=3 -80 M=4**

-0

> **115.6 -0.** 4 **-0.** 2 **x 0.0 0.2 0.4**

Fig. 4.3 Pressure gradient ????? for different values of M. 100 80 60 Dpl 40 20 0 -20

> **34M = 0,1,** 2,3 **,4 -0.** 4 **-0.** 2 **0.0**

h 0.2 0.4 Fig. 4.4 Pressure rise $\Delta$?? for different values of M.

> **170Fig. 4.5** (a) Fig. **4.5** (b) **Fig. 4.5**

Dimensionless temperature with variation in M.

> **164Fig. 4.6 (a) Fig. 4.6 (b)**

Fig. 4.6 Dimensionless temperature with variation in ??1?2. Fig. 4.7 Dimensionless temperature with ??1 = ??2 = ??? ? ??1 ??2 ?? −?0(?1) −(?*)0(?1) 0?5 1?0 2?0 2?0 12?1396 10?1291 1?0 18?1183 10?1291 1?5 27?9366 10?1291 2?0 41?4483 10?1291 0?5 10?5343 2?70455 1?0 20?839 5?17941 1?5 31?1436 7?65427 2?0 41?4483 10?1291 0?5 0?5 46?7126 11?3298 1?0 1?0 46?7727 11?3899 1?5 1?5 46?8055 11?4227 2?0 2?0 46?8262 11?4434 Table: 4.1. Effect of various parameters on heat transfer rate at the upper wall. Chapter 5 Peristaltic motion with Soret and Dufour effects This chapter examines the magnetohydrodynamic (MHD) peristaltic flow in a symmetric chan- nel

> **11in the presence of Ohmic heating. Effects of Soret and Dufour**

are retained.

> **30Long wavelength and low Reynolds number assumptions are**

employed. Attention is mainly focused to analyze the flow quantities for various parameters of interest. 5.1 Mathematical analysis

> **3We consider the flow of an incompressible** MHD **fluid in a channel of**

width 2?1. The ?− axis is chosen along the walls of channel and ? − axis is selected normal to it.

> 9**A uniform magnetic field** B0 **is applied in the ? direction** and **induced**
> **magnetic field is neglected. The**

channel walls are represented by 2? ?(?? ?) = ?1 + ?1??? ? (? − ??) , (5.1) μ ¶ where ?1

> 21**is the** wave **amplitude,** ? is **the wavelength, c is the** wave **velocity and ? is the**
> **time. The equations** governing **the**

two-dimensional flow are ?? ?? ?? ?? = 0? + (5.2) ? ? +? +? ? ? + (5.3) μ ?? ?? ?? ¶ ? =−???? +?·????2?2 ??2??2¸ −??02?? ? ? +? +? ? ? ?? ?2? ?2? + ? (5.4)

> 42**μ ?? ?? ?? ¶ ? =−?? +?·??2 ??2¸** ??? ??? + ? ??? + ? ??? ? = ? ???2 **?2** + ??2
> **??2 + ? 2 ???? + ???? + ???? + ??´?? 2** + ??02 **?2 + ????? 2 ³ 2**

> 182**h i ?2?2 + ?2?2**

(5.5) · μ³ ?? ?? ´ ³ ´ ¶ ³ ´ ¸

> 59**h i ? +? +? ? ? + + ?2? ?2? · ?? ?? ?? ¸ ? = ? · ???2?2 ??2??2 ¸ ??? ? ? ·??2**
> **?? 2**

¸ + ? (5.6) where (???) are velocity components in the laboratory frame (???), ? is pressure, ? the concentration field, ? the temperature field, ? mass diffusivity, ?? thermal diffusion ratio, ?? specific heat, ?? concentration susceptibility, ?0 concentration at boundary, ?? and ?0 the fluid mean temperature and temperature at boundary respectively. If (???)

> 24**are the velocity components in the wave frame**

(???) then introducing ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?) (5.7) equations (5.2)-(5.6) in wave frame are reduced as follows: ?? ?? ?? ?? = 0? + (5.8) ? (? + 1)?? + ? ?? = − ?? ??

> 65**μ ¶ ???? + ? ??2??2** ??2 **??2**− ??02(? + **1)?** + (5.9) · ¸ ?? ?? ?? **?2? ?2? ? (? +**
> **1)??** + ??? = −?? + ? ??2 **??2**

+ ? (5.10)

> 131**μ ¶ · ¸ ??? (? + 1) ??? + ? ??? ? = ? ??2??2 + ??2??2 + ? 2 ???? 2 + ?? ³ 2**

´ 2 h i ?? + ???? + ???? + ??02(? + 1)2 + ????? ??2??2 + ??2??2 (5.11) ·μ ¡ ¢ ³ ´ ¶ ³ ´ ¸ h

> 51**i (? + 1) ?? + ? ?? ? = ? ??2??2 + ??2??** 2 ??? ? ? + **?2? ?2?** ? (5.12) · ¸ · ¸ ? ? ·
> ??2 + **??2** ¸ Defining ?= ?= ?= ?? = ? ? ? ? ? = ? ? = ? ? = ? ? ? ? ? = **?1 ? ? = ? ? = ?**
> **1** ? ? = ?21? ? ?1 ? ?? ? **?1**

?1 ??? ? − ?0 ? − ?0 ? ? ? = 0 ? ? ? = 0 ? 1?2 ?0?1? υ = ? ?? = ???1 ? ? ? ? ? ?? μ ¶ ? ?? = Pr ?? ?? = ?????0?????0 ¸ ?? = ???? ?0 ? ????0 ? ? ?2 ??? ? ? = ???? and ? = − ?? ?? ? ? = ???0 ? ? ? = ? ?? ? (5.13) the equations (5?7) − (5?11) in dimensionless form are stated as follows: ?? ?? ?? ?? = 0? + (5.14)

> 112**?Re (?+1)??** +??? ?? ?? = − ???? + ?2 **??2??2 ??2??2**

− ? 2(? + 1)? + (5.15) μ ¶ μ ¶ ?3

> 112**Re (? + 1)** ???? + ? ???? = − ???? + ?2 **?2 ??2??2 ??2??**

2 + ? (5.16) μ ¶ μ ¶ ? Pr

62**Re (? + 1) ???? + ? ???? = ?2 ??2?? 2 ???2?2 + ?? 2?2 + ?? 2 ?? 2**

¶ "Ã µ ?? + + ?2 ?? + ?? 2 µ ¶ µ ?? ¶ ! µ ?? ?? ¶ # +???2(? + 1)2 + ?? Pr ?2 ?2? + ?2? ? (5.17) µ ??2 ?? 2 ¶ ? (?+1)?? +??? ?? ?? = 1 ?2?2? ?2? +?? ?2?2? ?2? + + ? (5.18) µ ¶ ?? µ ??2 ??2 ¶ µ ??2 ??2 ¶ Making use of the long wavelength and low Reynolds number approximations we obtain ?? ?3? ?? ??3 = − ?2 ?? +1 ? µ ?? ¶ ??4??4 − ? 2 ????22 = 0? ?? ?? = 0? (5.19) (5.20) (5.21) 0 = ???2?2 + ?? ??2??2 2 + ???2 ? (5.22) µ ?? 2 ?2? ¶ µ ?? + 1 + Pr ?? ??2 ¶ 0 = ?? ??2 + ?? ??2 1 ?2? ?2? ? (5.23) in which ? 6= ? (?) ? υ denotes the kinematic viscosity, ? the Hartman number, Re the Reynolds number, ?? the Brinkman number, ?? the Dufour number, ?? the Soret number, ?? the Schmidt number, ? the Eckret number, ? ? the Prandtl number, ? the wave number and the dimensionless temperature and concentration are ? and ? respectively. The boundary conditions are ? = 0, ?2? ?? ?? ??2 = 0, ?? = 0? ?? = 0? ?? ? = 0, ? = ? , ?? ?? = −1, ? = 0, ? = 0, ?? ? = ?, (5.24) ?(?) = 1 + ? cos(2??), ? = ? ?? Z0 ?? ??? (5.25) Pressure rise per wavelength ∆?? at this stage is 1 ∆?? = ????? ?? (5.26) Z0 Heat transfer coefficient ? for this problem is defined as ? = ????? 5.2 Solutions expressions Solving the above equations subject to the corresponding boundary conditions one finds that ? = ? ? ? cosh (?? ) + ? sinh (?? ) − (? + ?) sinh (? ?) ?? cosh (?? ) − sinh (?? ) ? (5.27) ?? (? + ?) ? 3 cosh (?? ) ?? = − ?? cosh (?? ) − sinh (?? ) ? (5.28) ? = 8 (−1 +??P?r?2? (?2???1?−) ?(−2?c?oshc[o(?sh?(?)?+ )?+3 +sin?h4()?? ))2 ? ? = 8 (−?1?+?P2r????????(???1)+(−??2? coscho[s(h2?(???))−+?s3in+h (??4?) ))2 ? where ?1=? 4+?+?3?4 − 1+?2?4 ?2+?2?2 1+?2 (?−?)(?+?)+2? 2+??¡2 1+?2 (?¢−?¡)(?+?) ?¢?2=?2 1+¡?2 +2¢? 4+?+??2 −2?2 +¡? 8+?¡3+?2¢1+2?2−2?2 ¢ ?3=(?¡+?)2 1¢+?2¡cosh[2??]+¢4?? 2?¡+?(2¡+?)−?¡2 sinh[2??]−¢¢1¢6?(?+?)?cosh¡[??](?¢(?−?)+sinh[??])? ?¡4=16(? +?)sinh[¢??](?(?−?)+sinh[??]) ?1=−8?−2?(4+?)−2??2(?+2?)?2−2?2(?+?)?4 +2 1+?(?+2?)?2+(?+?)?2?4 ? 2??2=?2 1+?2 + 2? ³4 + ? + ?? 2 − 2?2 + ? 8 + ? 3 +´ ? 2 1 + 2?2¡− 2?2 ¢ ? ?3 ¡= (? + ?)2 1¢+? 2 cosh¡[2? ?] ¡− 4?? 2¡? + ? (2 + ?) ¢−¢¢?2 sinh[2?? ]? ?4 = 16? (? +¡?) ? cos¢h[?? ] (? (? − ?) +¡sinh[? ?]) − 16 (? +¢?) sinh[?? ] (? (? − ?) + sinh[? ?]) ? (5.29) (5.30) 5.3 Discussion Here Figs. 5.1(a-e) are drawn for the temperature, Figs. 5.2(a-e) for concentration and Figs. 5.3(a-e) for the heat transfer coefficient. Tables 5.1 and 5.2 describe the temperature and concentration gradients. It is observed that with an increase in M the temperature increases. An increase in temperature becomes more apparent with an increase in M (see Fig. 5.1a). This fact agrees well to the expected results of Ohmic heating i.e. the temperature increases with an increase in the magnetic field strength. This fact is quite opposite to the observed results of behavior of M on temperature in the previous attempts by several other authors who considered the magnetic field but ignored the effects of Ohmic heating which in turn gives the lesser accurate prediction about the behavior of temperature for different values of M. From Figs. 5.1(b-d), the temperature increases with an increase in Schmidt, Dufour and Soret numbers but contrary to the previous case this increase is not much. Fig. 5.1(e) shows that there is rapid increase in temperature with an increase in the Brinkman number. From Figs. 5.2(a-e), we can clearly see that there is decrease in the concentration with an increase in M, Sr, Sc and Br. This decrease is more in case of Hartman number M and Brinkman number Br when compared with the other parameters. The result that concentration decreases effectively with an increase in ? is of considerable importance. This means that we can use the heating effect of applied magnetic field to develop small thermal gradients which in turn can be used to control the concentration of any particular substance in the fluid. This result has wide range of applications especially for cure of several diseases, for instance clearing several type of blockages in many arteries that exhibit peristaltic mechanism. The thermal gradients can be used more efficiently in place of radiations with lesser side effects. These thermal gradients can also be used for the transport of several minerals and medicine within the human body on the desired location. Heat transfer coefficient has its own physical significance in problems related to heat transfer. It is always helpful to examine the behavior of heat transfer coefficient (? in this case) for a given problem. Keeping this in mind the Figs. 5.3(a-e) are explicitly plotted to examine how ? develops throughout the flow field. In all the Figs. 5.3(a-e), Z has oscillatory behavior which is due to peristalsis. In all these Figs. the absolute value of heat transfer coefficient is positive. Fig. 5.3(a) shows a rapid fluctuation in Z for changing values of M. From Figs. 5.3(b-d), it is quite obvious that the Du, Sc and Sr effect the heat transfer coefficient but this effect is relatively smaller when compared with the effects of other parameters like Br, M and d. Since Soret and Dufour effects have gained reasonable interest in convective heat and mass transfer. One cannot neglect such effects when there is appreciable density difference in the flow regime. The concentration gradient in fact yields the thermal energy flux (referred as Dufour effect) whereas the temperature gradients contribute to the mass fluxes (referred as Soret effect). One can neglect Soret and Dufour effects in heat and mass transfer mechanism when such effects are small in comparison to the effects by Fourier's or Fick's laws. Motivated by this fact we computed the temperature and concentration gradients at ? = ? in the Tables 5.1 and 5.2. It is noticed from Table 1 that magnitude of temperature gradient increases when ?? Du amd Br are increased. However the situation is reverse when there is an increase in a. Table 5.2 depicts that concentration gradient is an increasing function of M, Du, Br and Sr. Further the concentration gradient is decreasing function of a. ?a? Du=0.5,Sc=0.5,Sr=0.5,Br=0.5 ?b? M=1,Sc=0.5,Sr=0.5,Br=0.5 1 1.5 M= 0.5 M= 1.0 Du = 0.2 M= 1.5 Du = 0.4 Du = 0.6 1.25 M= 2.0 0.8 Du = 0.8 1 0.6 q q 0.75 0.4 0.5 0.25 0.2 0 0 0 0.2 0.4 0.6 0.8 1 1.2 y 0 0.2 0.4 0.6 0.8 1 1.2 y ?c? Du=0.5,Sc=0.5,M=1,Br=0.5 1 Sr = 0.2 Sr = 0.4 0.8 Sr = 0.6 Sr = 0.8 0.6 q 0.4 0.2 0 0 0.2 0.4 0.6 y 0.8 1 1.2 ?d? Du=0.5,Sr=0.5,M=1,Br=0.5 ?e? Du=0.5,Sc=0.5,M=1,Sr=0.5 1 Sc = 0.2 Sc = 0.4 1.4 Br = 0.2 Br = 0.4 0.8 SScc == 00..68 1.2 Br = 0.6 Br = 0.8 1 0.6 0.8 q q 0.4 0.6 0.2 0.4 0.2 0 0 0.2 0.4 0.6 0.8 1 1.2 0 0.2 0.4 0.6 0.8 1 1.2 y y Fig. 5.1(a-e) Variation of temperature when ? = 0?3? ? = 1?4? and ? = 0? ?a? Du=0.5,Sc=0.5,Sr=0.5,Br=0.5 ?b? M=1,Sc=0.5,Sr=0.5,Br=0.5 0 0 -0.1 -0.1 f -0.2 f -0.2 -0.3 -0.3 M = 0.5 M = 1.0 Du = 0.2 M = 1.5 Du = 0.4 M = 2.0 Du = 0.6 -0.4 Du = 0.8 -0.4 0 0.2 0.4 0.6 0.8 1 1.2 0 0.2 0.4 0.6 0.8 1 1.2 y y ?c? M=1,Sc=0.5,Du=0.5,Br=0.5 0 -0.25 -0.5 f -0.75 -1 -1.25 Sr = 0.2 Sr = 0.4 Sr = 0.6 -1.5 Sr = 0.8 0 0.2 0.4 0.6 0.8 1 1.2 y 0 ?d? M=1,Sr=0.5,Du=0.5,Br=0.5 ?e? M=1,Sr=0.5,Du=0.5,Sc=0.5 -0.05 -0.1 -0.1 f -0.2 -0.15 0.3 -0.2 -0.3 -0.4 Sc = 0.2 Sc = 0.4 Sc = 0.6 Sc = 0.8 0 0.2 0.4 0.6 0.8 1 1.2 y -0.25 -0.3 -0.35 0 0.2 0.4 0.6 y 0.8 1 Br = 0.2 Br = 0.4 Br = 0.6 Br = 0.8 1.2 Fig. 5.2(a-e) Variation of concentration when ? = 0?

3? ? = 1?4? and ? = 0? ?a? Br=0.5,Sr=0.5,Du=0.5,Sc=0.5 ?b? Br=0.5,Sr=0.5,M=1,Sc=0.5 20 M = 0.5 1.0 10 M = 1.5 = 2.0 10 5 Du = 0.2 Du = 0.4 Du = 0.6 Du = 0.8 Z 0 Z 0 -10 -5 -20 -10 0 0.2 0.4 0.6 0.8 1 x 0 0.2 0.4 0.6 0.8 1 x ?c? Br=0.5,Du=0.5,M=1,Sc=0.5 10 Sr = 0.2 Sr = 0.4 Sr = 0.6 Sr = 0.8 5 Z 0 -5 -10 0 0 0.2 0.4 0.6 0.8 1 x ?d? Br=0.5,Du=0.5,M=1,Sr=0.5 ?e? Sc=0.5,Du=0.5,M=1,Sr=0.5 40 10 Br = 0.2 Br = 0.4 Br = 0.6 5 20 Br = 0.8 Sc = 0.2 Sc = 0.4 Sc = 0.6 Sc = 0.8 Z 0 Z 0 -5 -20 -10 -40 0 0 0.2 0.4 0.6 0.8 1 0 0.2 0.4 0.6 0.8 1 x x Fig. 5.3(a-e) Variation of Z when ? = 0?3? ? = 1?4? and ? = 0? ? ?? ?? ? 0?5 1?0 0?5 0?3 1? 0 1?5 2?0 0?0 0?5 1?0 1?5 ?0(?) −2?8569 −3?6054 −5?0997 −7?7157 −2?7040 −3?0903 −3?6054 −4? 3265 0?0 0?5 1?0 1?5 0?2 0?0 −3?0903 −6?1807 −9?2711 −3?3563 0?4 −2?5868 0?6 −2?11325 0?8 −1? 81531 Table 5.1: Variation of ???? ?=? via different parameters. ? ?? ?? ? ?? ?0(?) 0?5 1?0 0?5 0?3 0?5 0?71424 1?0 0?90136 1?5 1?2749 2?0 1?9289 0?0 0?6760 0?5 0?7725 1?0 0?9013 1?5 1?0816 0? 0 0?0 0?5 0?7725 1?0 1?5452 1?5 2?3178 0?2 0?8390 0?4 0?6467 0?6 0?5283 0?8 0?4538 0?2 0? 2787 0?4 0?5753 0?6 0?8914 0?8 1?2291 Table 5.2: Variation of ???? ?=? via different parameters. Chapter 6 Soret and Dufour effects on peristaltic transport of MHD fluid with variable viscosity This chapter looks at the Soret and Dufour effects on the magnetohydrodynamic (MHD) peri- staltic flow of variable viscosity fluid in a symmetric channel. Modeling is presented in the presence of Ohmic heating. Mathematical analysis is carried out through large wavelength and small Reynolds number. Results for the stream function, temperature and concentration are developed. The variations of sundry parameters on the flow quantities of interest are sketched and examined. 6.1 Mathematical analysis Let us investigate the magnetohydrodynamic flow of an incompressible viscous fluid in a channel with width 2?1. The ?− axis is chosen along the walls of channel and ? − axis is taken normal to the ?− axis. A constant magnetic field of strength B0 is exerted in the ? − direction. Induced magnetic field is not accounted. Sinusoidal wave propagates along the channel walls with constant wave speed ? represented by ?(?? ?) = ?1 + ?1??? 2? ? (? − ??) . (6.1) μ ¶ If (? ? ? ) and (?? ?) are the velocity components in the laboratory (?? ? ) and wave (?? ?) frames then transformations between laboratory and wave frames are ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (6.2) In above expressions ?1 is the wave amplitude, ? is the wavelength, ? is the time and ? and ? are the pressures in the laboratory and wave frames respectively. Introducing the variables in the forms ?= ? ? ? = ? ? = ? ? = ? ? ? ? ? = ?1 ? ? = ? ? ? = ?1 ? ? = ?21? ? ?1 ? ?? ? ? ?1 ???0 ?= ? − ?0 ? ? = ? − ?0 ? ? = ? 1?2 ? ?0?1? υ = ?0 ? ?? = ???1 ? 0 ? 0 μ ?0 ¶ ? ? 0 ?= ?? ? ?? =??????=−????? ??=Pr?? ??=??????0??0??0, ??=???0??????00? ?? = ?0? ?= ?2 ??? ?¯(?¯) ? ?? = ? ? and?(?)= ?0 ? (6.3) ?? ???0 the governing equations after adopting long wavelength and low Reynolds number approxima- tion become ???? ?? ?(?)??2??2 − ?2 = ? μ ?? +1 ? ¶ μ ?? ¶ ??2 ?(?) ??2 − ? ?2 ?2? 2 ??2 = 0? μ ¶ ??2 0 = ??2??2 + ???(?) ??2??2 + ???2 2 ?? 2 ? μ ?2? +1 ¶ μ ?? ¶ + Pr ?? ??2 1 ?2? ?2? 0 = ?? ??2 + ?? ??2 ? (6.4) (6.5) (6.6) (6.7) where ? is the pressure, ? the concentration field, ? the temperature field, ? mass diffusivity, ?? the thermal diffusion ratio, ?? the specific heat, ?? the concentration susceptibility, ?(?) the variable viscosity, ?? the fluid mean temperature, υ the kinematic viscosity, ? the Hartman number, Re the Reynolds number, ?? the Brinkman number, ?? the Dufour parameter, ?? the Soret parameter, ?? the Schmidt number, ? the Eckret number, ? ? the Prandtl number, ? the wave number, ? the stream function, ? the dimensionless temperature and ? the concentration. The boundary conditions are ? = 0, ?2? ?? ?? ??2 = 0, ?? = 0? ?? = 0? ?? ? = 0, ? = ? , ?? ?? = −1, ? = 0, ? = 0, ?? ? = ?, (6.8) ?(?)=1 + ? cos(2??), ? = ? ?? Z0 ?? ??? (6.9) Pressure rise per wavelength ∆?? is ∆?? = ?? ??? 1 ?? (6.10) Z0 The dimensionless expression of space dependent viscosity is ?(?) = ?−?? = 1 − ?? ? ¿ 1? where "?" is the viscosity parameter. We look for solutions in the series form represented below ? = ?0 + ??1 + ???? ? = ?0 + ??1 + ???? ? = ?0 + ??1 + ???? Employing the procedure of perturbation method and retaining the results up to order ?(?) we have ? = ?1 + 8(−?? co(s?h(+???))?+?s2inh(??))2 ? (6.11) ? = − 8 (−1 + Pr ??????) (?????c2o?sh1 (?? ) − sinh (?? ))2 (6.12) − 32 (−?1??+ ?Pr[??2??+??? 3?+)(???4−co?s5h +(???6)−−?si7n+h(??8?] ))3 ? ? = −8(−1+Pr?????????2??)(?????[?co1s−h?(?2?+?) −3]sinh(??))2 + ????????6[4?(−4+1?+5P+r2?(???6?+???)7(+??14?co(?sh8(+???9)−−?si1n0h)−(??? 11)−)3?12+?13)]? (6.13) where the involved A?(? = 1? 2)? B?(? = 1 − 8) and C?(? = 1 − 13) are ?1 = ? ? ? cosh(???? )c+os?hs(i?n?h(?)−?si)n−h((???+?)) sinh(? ?) ? ?2 = ??3 + (?? ? cosh (?? ) + ?3? 2 − ? sinh (?? ))2 sinh (? ?) ? ?3 = −1 + 2??2 2 + cosh (2?? ) + 2? ? cosh (? ?) (¡−?? ¢ cosh (?? ) + sinh (?? )) − 2?? sinh (2?? ) ? ?1 = −8? − 2? (4 + ?) − 2? + ?2 (? + 2?) ? 2 − 2?2 (? + ?)2 ? 4 + 2(1 + ? (? + 2?) ? 2 + (? + ?)2 ? 4)?2 + (? 2 1 + ? 2 +2? 4 + ? + ?? 2 − 2?2 + ? 8 + ? 3 + ? 2 1 + 2?2 − 2?2 ) cosh (2¡?? ) − (¢? + ?)2 1 + ¡? 2 cosh (2?¢?) − 4?? 2¡? + ?¡(2 + ?) −¡ ?2 sinh (2??¢)¢?+ 16? (? + ?) ? c¡osh (??¢) (? (? − ?) + sinh (¡? ?)) − 16 (? + ?) si¢nh (?? ) (? (? − ?) + sinh (? ?))? ?2 = −8?3 (? + ?) (? − ?) sinh (2?? ) ? 4 − 12 (? + ?) ?9 cosh (?? ) − 16?4 (? + ?) ? 4 cosh (?? )3 + 8?3 (? + ?) ? 4 (? − ?) cosh (2?? ) ? ?3 = 12 (? + ?) (3 (? + ?) + 3 ? + ? − 8?2 ? 2 + 8?4? 4) cosh (3?? ) + 16?2 (? + ?) ? 4?2 cosh (?? )2 cosh (? ?) −¡2? (? + ?)2¢? 2 1 + ? 2 ? cosh (?? ) cosh (2? ?) ? ?4 = (? + ?) ? ?10 sinh (?? ) ? ?5 = 64? 1?2? 3 c¡osh (??¢)2 sinh (?? ) + 2?3 (? + ?)2 ? 3 1 + ? 2

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9**cosh (2? ?) sinh (?? ) + 2** (? + ?)2 ? **1** + ? 2 ? **cosh (2? ?) sinh**

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(??¡) + 16 (?¢ + ?) ? 2?2 cosh (? ?) sinh (?? )2 − 8?4 (? + ?) ?¡ 5 (? − ?¢) sinh (2?? ) ? ?6 = ?? ?11 cosh (?? ) sinh (2?? ) ? ?7 = ?? (−4? + ? 2 − 4? + 2? ? + ?2 + (? + ?)2 1 + ?2 ? 2 + ?2 (? + ?)2 ? 4) sinh (3?? ) − 16?2 (? + ?) ? 3? cosh (?? )2 sinh (?¡ ?) + 1¢6?3 (? + ?) ? 3 sinh (?? )2 sinh (? ?) − 16 (? + ?) ? ? sinh (?? )2 sinh (? ?) − 8?4 (? + ?) ? 4 sinh (2?? ) sinh (? ?) ? ?8 = 16? (? + ?) ? 2? sinh (2?? ) sinh (? ?) + (? + ?)2 1 + ? 2 −3 + 2? 2?2 sinh (?? ) sinh (2? ?) − ? (? + ?) ? cosh (?? ) ?12? ?9 =¡? 1 +¢?¡ 2 (3 + 4?? ¢2(−3? +2?3? 2 + 2? − 2? 2?3)) + ?(3 + ? 2(3 + 4?(? −3 + ? 2 −¡3 + 2? ¢1 + ? + ?? 2 +2 (−3 + ?) + 2? 2? − 2? 2 1 + ? 2 ?3)))? ?10¡= −3? (4 +¡? ? + ?) +¡ ?(? −3 + 5¢?¢2¢ +? (−3 + ? (16 + 5)))? 2 +¡?¡3 (? +¢?) 5 + 4?2 ? 4 + 4?5 (? + ?) ? 6 + 4¡ (? + ?) ¢ 1 + ? 2 ? − 4?3 (? + ?) ? 4 1 + ? 2 ?¡2 − 4 (?¢+ ?) ? 2 1 + ? 2 ?3? ?11 = −3 (? + ?)2 +¡ −3? 2¢−6? ? ? 32? 1? + (−¡3 + 2? (¢8 + ?)) ?2 + 4 (4 + ?¡ ) ?3 + 2?¢4 ? 2 + 2?2 (? + ?)2 ?¡4? ?12 = 16? 2?2 sinh (? (? − ?)) + (? + ?) 1 + ? 2 −3 + 2? 2?2¢¡ sinh (2? ?) + 16 ? 2?2 sinh (? (? + ?)) ? ?1 = 2?14 − 16? (? + ?¡) ? 2 (?¢−¡ ?) cosh (?? )¡− (? 2 1 + ? 2 +2? 4 + ? + ?? 2 − 2?2 + ? 8 + ? 3 + ? 2 1 + 2?2 − 2?2 ) cosh (2?? ) ?¡?2 = 8?¢ cosh (¡? (? − ?)) − ¢8? cosh (? (¡? − ?)¡) + ? 2 co¡ish (2? ?) +

2?¢?¢¢cosh (2? ?) + ?2 cosh (2? ?) + ? 2? 2

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9**cosh (2? ?) + 2?** ?? 2 **cosh (2?** ?) + ?2? 2 cosh **(2? ?) + 8 (? + ?) cosh**

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(? (? + ?)) + 16? ?? sinh (?? ) + 16?2? sinh (?? ) − 16? ? ? sinh (?? ) − 16?? ? sinh (?? ) ? ?3 = 8? ?? sinh (2?? ) + 8?2?

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9**sinh (2?? ) + 4?** 3? **sinh (2?? ) − 4??** ?2 **sinh** (2?? ) + 8? ?? **sinh**

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(? (? − ?)) + 8?2? sinh (? (? − ?)) − 8? (? + ?) ? sinh (? (? + ?)) ? ?4 = (3 (? + ?)2 + 3? 2 + 6? (1 − 4?) ? + ? (64? 1 + 3 (1 − 8?) ?) ? 2) cosh (3?? ) ? ?5 = 2? cosh (?? )¡2 ?15? ?6 = −8?3 (? + ?) ? 4 (? − ?) + 2 (? + ¢?)2 ? 1 + ? 2 ?3? 2 + ? cosh(? ?)? ?7 = 2? ? sinh (?? ) + 16 (? + ?) ? 2?2 cosh (? ?) sinh¡ (?? )2 ¢??¡8 = 64? 1?¢? cosh (?? ) + 32?3 (? + ?) ? 3 (? − ?) cosh (2?? ) − 64? 1? ? cosh (3?? ) ? ?9 = ?16 sinh (?? ) ? ?10 = 32?4 (? + ?) ? 4 (? − ?) sinh (2?? ) + (−? (? + ?) −8 + 5? + 5? + (5? + (5 − 16?) ?) ? 2 + 8? 1 4 + ? + ? + ? + ? − 4?2 ? 2 ) sinh (3??¡ ) ? ?11 = 4? ? 1 4 − 4?2? 2 + 4? (?¢+ ?) −¡3? + ?2? 2 (3?¡ + ?) + 4?¢(? 1 ¢ 4 − 4?2? 2 +

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43**4? (? + ?¡) ? + ?2? 2¢(−? + ?) ) cosh¡(2??** ) + 4? **2?** (? + ¢?) ? (?3? **2 −¡2?)**

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sinh (2?¢? ) sinh (? ?) +¡ 4? (? + ?)2 1 + ?¢ 2 −3 + 2? 2?2 sinh (?? ) sinh (2? ?) ? ?12 = cosh (?? ) (? + ?) (? 1 + ? 2 3 +¡4?? 2 −¢3¡? + 2?3? 2 +¢2? − 2? 2?3 + cosh (?? ) ?(3 + ? 2(3 + 4¡?2(−3 +¢?¡ 2 −3 + 2?¡ 4 + ? + ?? 2 + 2 cosh (??¢¢) (−3 + ?) +2? 2? − 2? 2 1 + ? 2 ?3)))? ?13 = 64?¡1?? 2 cos

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119¡**h (2??** ) + 4?¢(¢? + **?)2 ? 2** 1 + ? **2** ? cosh **(2? ?) + ¡2??**−3 **¢2?** 1?? **2** sinh **(2??**

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) + (? + ?) ?18 ? ?14 = ? 4 + ? +¡ ?3? 4 ¢ − 1 + ?2? 4 ?2 + ? ¡2? 2 1 + ? 2 (? − ?) (? + ?) + 2? (2 +¢?? 2 1 +¡? 2 (? − ?) (?¢+ ?)), ?1¡5 = 16?2 (?¢ + ?) ? 3?2 c¡osh (? ?¢) − (? (? + ?) −8 + 5? + 5? +¡(5? + (5¢ − 16?) ?) ? 2 +8? 1 4 + ? + ? + ? + ? + 4?2 ? 2 ) sinh (??¡) − 8((4? 1 −1 + ?2? 2 + (? + ?) (? + ¢?2 2 (−¡? + ?)))) sinh¡(? ?) ? ?16 =¢8? 1 ¢4 + ? + ? + ? + ? −¡ 4?2 ? 2 +¢(? + ?) ?17? ?17 = ?(−56 + ? 1 + ? 2 −13 + 24?¡2? 2 + 16?4?¡ 4 + ?(−13 ¢+ ? 2¢(−13 + 8?(10 + ? 1 + ? 2 3 + 2?2¡? 2 ))))¢+¡ 16 (? + ?) 1 + ? 2 ? − 1¢6?3 (? + ?) ? 4 1 + ? 2 ?2 − 16 (¡? + ?) ?¢¡2 1 + ? 2 ?¢3? ?18 = 16? 2?2 s¡inh (? (¢? − ?)) + (? + ?) 1 +¡ ? 2 −¢3 + 2? 2?2 sinh (2? ?) +¡ 16? 2?2¢2 sinh (? (? + ?)) ? ¡ ¢ ¡ ¢ Heat transfer coefficient is defined as follows: ? = ????? 6.2 Discussion This section describes the impacts of pertient parameters on the temperature and concentration. We recall that theme of this study is to point out the influences of Soret and Dufour. Therefore, the results of various parameters associated with velocity are not included. In order to achieve the desired objective, we present the plots in such a way that the left panels are for constant viscosity (i.e. for ? = 0) and the right ones are for variable viscosity ( ? = 0?4). It is observed in all the graphs related to temperature that the rise in temperature for variable viscosity fluid is relatively slower when compared with fluid having constant viscosity. Here Figs. 6.1-6.5 are drawn for temperature whereas the Figs. 6.6-6.10 show variation of concentration. Temperature increases with an increase in M (Fig. 6.1). Further an increase in temperature is abrupt in view of Ohmic heating. The variations of Du, Sr, Sc and Br on the temperature are displayed in the Figs. 6.2-6.5. These Figs. indicate that there is an increase in temperature by increasing Du, Sr, Sc and Br. It is also found that an increase in temperature is more for Br when compared with the other parameters. Figs. 6.6-6.10 are presented to examine the behavior of embedded parameters on the con- centration. Increase in concentration is observed when M and Du are increased (see Figs. 6.6 & 6.7). Concentration also increases when Sr, Sc and Br are increased (Figs. 6.8-6.10). Behavior of heat transfer coefficient Z for various parameters is shown in the Figs. 6.11-6.15. As expected Z shows an oscillatory behavior which is because of peristalsis. It is also noted that there is no variation in Z for amplitude ratio (?) between 0 and 0.3. It is observed from Fig. 6.11 that Z increases when M is increased. The absolute values of Z in variable viscosity fluid are more than the constant viscosity fluid. Figs 6.12-6.15 show that Z increases with the increase in Du, Sr, Sc and Br. Effects of Du, Sr, Sc and Br on Z are opposite to that of M. Fig. 6.1 Effect of M on ? when ?? = 0?5? ?? = 0?5? ?? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.2 Effect of Du on ? when ? = 0?5? ?? = 0?5? ?? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.3 Effect of Sr on ? when ?? = 0?5? ? = 0?5? ?? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.4 Effect of Sc on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.5 Effect of Br on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.6 Effect of M on ? when Du=0.5, Sr=0.5, Sc=0.5, ? = 0?3? ? = 1?4 , Br=0.5 and x=0. Fig. 6.7 Effect of Du on ? when ? = 0?5? ?? = 0?5? ?? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.8 Effect of Sr on ? when ?? = 0?5? ? = 0?5? ?? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and x=0. Fig. 6.9 Effect of Sc on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4? ?? = 0?5 and ? = 0? Fig. 6.10 Effect of Br on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 , ?? = 0?5 and ? = 0? Fig. 6.11 Effect of M on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 and ?? = 0?5 and ? = 0? Fig. 6.12 Effect of Du on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 and ?? = 0?5? Fig. 6.13 Effect of Sr on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 and ?? = 0?5? Fig. 6.14 Effect of Sc on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 and ?? = 0?5? Fig. 6.15 Effect of Br on ? when ?? = 0?5? ?? = 0?5? ? = 0?5? ? = 0?3? ? = 1?4 and ?? = 0?5? Chapter 7 Soret and Dufour effects

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102**on peristaltic flow in an asymmetric channel**

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This chapter studies the combined

2**effects of heat and mass transfer on** the **peristaltic flow of** magnetohydrodynamic (MHD) **fluid in an asymmetric channel.** Asymmetry in channel **is** induced because **of**

the wave amplitude and phases. Analysis

1**has been** performed **in the presence of slip condition.**

Attention has been given to the Soret and Dufour effects. The obtained results are plotted and discussed. 7.1 Mathematical analysis We consider MHD

63**flow of** incompressible **fluid in an asymmetric channel. The**

?-axis is selected along the channel walls and ? −axis perpendicular to the ?-axis. The flow is due to the sinusoidal wave trains travelling

6**on the walls of the channel with speed**

?? The geometry of the waves is given below

87**2? ?1(?? ?)** = ?1 + **?1** cos (? − ??) , **μ ? ?2(?? ?) = −?2 − ?1** cos 2? ¶ ? (? − ??) + ? ? **μ**

¶ (7.1) (7.2) in which a1 and b1 are the amplitudes of the waves travelling

153**on the upper and lower walls** respectively, ? **is the**

phase difference of two waves and ?1 + ?2 is the

176**width of the channel.** The **phase difference**

? ranges 0 ≤ ? ≤ ?? Here ? = 0

37**corresponds to** the **symmetric channel with waves out of phase and for ? = ? the waves are in phase.** Moreover ?1? ?2? **a1? b1 and ? satisfy the**

condition ?12 + ?21 + 2?1?1 cos (?) ≤ (?1 + ?2)2 ?

16**A uniform magnetic field** B0 **is applied in the transverse direction to the flow.** Effects of induced **magnetic**

field are not considered by choosing low magnetic Reynolds number. However, Joule heating and

104**Soret and Dufour effects are** taken into account. **The**

equations

104**of mass, momentum, energy and concentration** in **the**

laboratory frame (?? ? ) are ?? ?? ?? ?? = 0? + (7.3) ? + ?? + ? ?? + ? ?? ? = −?? ?? 2??? ? ? ? ?? + ? ?? + ? μ ¶ μ ¶ ?? · μ???? ???? ¶¸ − ??02?? (7.4) ? ? ? + ? + ? ? ? ?? + ? ? ?? ?? + + ? 2? ?? μ ?? ?? ?? ¶ ? = −?? ?? · μ?? ?? ¶¸ ? (7.5) ??

42**μ ?? ¶ ??? ??? + ? ??? + ? ??? ? = ? ???2?** 2 + **???2?2 +? 2** ???? + ???? + ???? + ????´ 2 + ??02 **?2 + ????? 2 ³ 2** h i · ½³ ´ ³ ´ ¾ ³ ´ ¸ h **?2?**

109**2 + ?2?2 ?? ?? ? i** (7.6) ? + ? + ? ? ? **?2? ?2? ??? ?2? ?2? ? = ? + + + ? · ?? ?? ?? ¸ ·??2 ?? 2 ¸ ? ? ·??2 ?? 2**

¸ (7.7) where ?? ? are

24**the velocity components in the ? and ? directions respectively, ? the** pressure, ? **the** temperature, ? **the**

concentration, ? the density of fluid, ? the dynamic viscosity, ?? the specific heat, ? thermal conductivity, ?? fluid mean temperature, ? the mass diffusivity, ?? the concentration susceptibility and ?? the thermal diffusion ratio. Using the following transformations in order to transform our problem in to wave frame (???) ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (7.8) We define the following dimensionless quantities ?= ? ? ? = ? ? = ? ? = ? ? ? ?1 ? ??? ? ? = ?1 ? ?1 = ??11 ? ?2 = ??12 ? ? = ??21 ? ? = ??? 21?? ? ? ?= ? − ?0 ? − ?0 ? 1?2 ? ???1 ?1 ?1 ?1 − ?0 ? ? = ?1 − ?0 ? ? = µ ¶ ? ?0?1? υ = ? ?? = ? ? ? ? = ? ? = ? ? 1 ? 1 ?= ?? ? ? ??=Pr?? ??=???(???1?−(??10−)??0?), ??= ???? (?1 − ?0) ??? (?1 − ?0) ? ?? = ? ? ?= ?2 ?? ?? (?1 − ?0) ? ?? = ??? ? ? = ???? and ? = − ???? ? ? (7.9) in which ? is the wave number, ? the non-dimensional temperature, ? the dimensionless con- centration, ? the

1**Hartman number, Re the Reynolds number, ?? the** Brinkman **number,**

?0 and ?0 the temperature and concentration at upper wall, ?1 and ?1 the temperature and con- centration at lower wall,

2**Pr the Prandtl number, ? the** Eckert **number, ?? the Soret number and**

?? the Schmidt number. We find that ? does not depend upon ? i.e. ????? = 0 and ?−component of momentum equation reads to ?? ?3? ?? ??3 = − ?2 ?? + 1 ? (7.10) µ ?? ¶ where (?? ?)

31**are the velocity components in the wave** frame, ? the pressure **in wave frame** and ? **the**

stream function. Now the compatibility, energy and concentration equations are given by ??4??4 − ? 2 ????22 = 0? 0 = ???2?2 + ?? ??2??2 + ???2 2 µ ?? 2 ?2Φ +1 ? ?¶ µ ?? ?¶ + Pr ?? ??2 1 ?2? ?2? 0 = ?? ??2 + ?? ??2 ? (7.11) (7.12) (7.13) The non-dimensional boundary conditions in terms of slip are ?= ? 2 , ?? ?? ?Φ ?? + ?1 ??2 = −1, ? + ?2 ?? = 0? ? + ?3 ?? = 0? ?? ? = ?1, ?2? ?=− , 2 ? ?? ?? − ?1 ??2 = −1, ? − ?2 ?? = 1, ? − ?3 ?? = 1, ?? ? = ?2,(7.14) ?2? ?? ?Φ in which ?1? ?2 and ?3 are the dimensionless velocity, temperature and concentration slip parameters respectively. These are defined as ?1 = ???1∗1 ? ?2 = ??∗21 ? ?3 = ??∗31 ? where ?∗1? ?∗2 and ?∗3

1**are the dimensional velocity,** temperature **and** concentration **slip parameters respectively.** Now the **dimensionless** expressions **of**

wave shapes are ?1(?)=1 + ? cos(2??), ?2(?)= − ? − ? cos(2?? + ?)? (7.15) Flow rate and pressure rise per wavelength ∆?? are defined as follows: ? = ?1 ?? Z?2 ?? ??? (7.16) 1 ∆?? = ?? ?? ??? (7.17) Z0

15**Heat transfer coefficient** ?1 **at the upper wall is defined by**

?1 = ?1???? The solution of the involved problems through Mathematica are analyzed in the next section. 7.2 Discussion This section is especially prepared to examine the effects of pertinent parameters. Here we arrange the variations of such parameters in the Figs. 7.1-7.4. These Figs. depict the plots of stream lines,

14**temperature, concentration and heat transfer coefficient** at **the**

upper wall. Figs. 7.1(a-d) illustrate the streamlines for an increase in velocity slip parameter ?1? The volume of fluid trapped within a streamline is termed as bolus. It is observed that the trapped bolus gets elongated and its size increases when the slip parameter increases. Figs. 7.2(a-i) exhibit the temperature variation. It is seen that the temperature increases due to increase in M, ?2? Du, Sr, Sc, Pr and ?. However it is found that the temperature decreases when d and ?1 are increased? Increase in ? is rapid and much in cases of M and Pr (see Figs. 7.2a and 7.2f) but slower and small for ?2? Du, Sr and Sc (see Figs. 7.2b, 2c,2d and 2e). Figs. 7.3(a-h) demonstrate how concentration field develops when certain parameters of interest are varied. Key observations from these Figs. are

26**that concentration decreases with** the **increase in M,** ?3? Du, **Sr,**

Sc and Pr whereas it increases by increasing ?1 and ?? Here decrease in ? is much for M, ?3? Sr, Sc, Pr and small for Du. Moreover the concentration remains invariant for thermal slip parameter ?2? Figs. 7.4(a-e) are plotted to see

> 1the behavior of heat transfer coefficient Z1 at the upper wall?

Because of peristalsis it is found that, Z1 has an oscillatory behavior. Moreover Z1 has maximum value near the upper wall. It increases with the increase in M and ?? Further Z1 decreases with increase in d and ?1? An interesting and noteworthy fact is that the thermal slip parameter ?2 has no meaningful variation on Z1? 7.3 Main points Simultaneous

> 2effects of heat and mass transfer on the peristaltic transport of viscous fluid in an asymmetric channel with slip effects are

analyzed. They main observations are summarized as follows: • The trapped bolus gets elongated as the value of viscosity parameter is increased. •

> 17Effects of Hartman number M, Dufour number Du and

temperature slip parameter ?2 on the temperature are opposite to that of velocity slip parameter ?1. • Increase in temperature subject to an increase in Hartman number M is rapid when compared to that of other parameters. • Increase in applied magnetic field results in decrease of concentration. • Thermal slip parameter has no effect on the fluid concentration. •

> 29Absolute value of the heat transfer coefficient increases with increase in

the Hartman number M. Fig. 7.1. Effect of velocity slip parameter ?1 on stream lines. 5 20

> 77b2 = 0.1, 0.2, 0.3, 0.4

15

> 19M = 0.5, 1.0, 1.5, 2.0

> 1440 q 10 q -5 5 -10 0 -1.5 -1 .0

-0.5 y 0.0 Fig. 7.2

> 34a 0. 5 1. 0 1.5 -2 -1 0 1

2 Fig. 7.2 b y 4.0 Du = 0.5, 1.0, 1.5, 2.0 4.0 Sr = 0.5, 1.0, 1.5, 2.0 3.5 3.5 3.0 3.0 2.5 2.5

> 49q q 2.0 2. 0 1.5 1.5 1.0 1 .0 0.5 0.5 -1.5

-1.0 -0.5 0.0 0.5 1.0 -1.5 -1.0 0.5 1.0 -0.5 0.0 Fig. 7.2 c y Fig. 7.2 d y 4.0 40 Pr = 2.0 Sc = 0.5, 1.0, 1.5, 2.0 3.5 Pr = 1.5 3.0 30 q 2.5 q 20 Pr = 1.0 2.0 1.5 Pr = 0.5 10 1.0 0.5 -1.5 -1.0 -0.5 0.0 0

> 93.5 1.0 0 y -1.5 -1.0 -0.5 0.0 0.5 1.0

Fig. 7.2 e Fig. 7.2 f y 4 4 g =

> 86p/6, p/4, p/3, p/2

> 69d = 0.3, 0.6, 0.9, 1.2

3 3 q

48q 2 2 1 1 0 -0. 5 0.0 0. 5 1 .0

-1.0 -0.5 0.0 0.5 1.0 Fig. 7.2 g y y Fig. 7.2 h 3 2 q 1

8b1 = 0.1, 0. 2, 0. 3, 0.4 0 -1 -1 .5 -1.0 -0. 5 0. 0 0.

5 1.0 1.5 y Fig. 7.2 i Figs. 7.2(a-i). Effects of various parameters on temperature ?? 1.5 1.0 1.0 0.8 f 0.5 0.0 -0.5

49M = 2.0, 1.5, 1.0, 0.5 f 0. 6 0. 4 0.

2

8b1 = 0.1, 0. 2, 0. 3, 0. 4 -1.0 0.0 -1.5 -0.2

-1.5 -1.0 -0.5 0.0 0.5 1.0 -1.5 -1.0 y Fig. 7.3 a -0.5 0.0 Fig. 7.3 b

18y 0.5 1.0 1.5 1.0 0. 8 1 .0 0.

6

125f 0. 4 0. 2 0.0 b2 = 0 .1, 0. 2, 0. 3, 0.

4

28f 0. 5 0.0 b3 = 0. 4, 0. 3, 0. 2, 0.

1 -1.5 -1.0 -0.5 0.0 Fig. 7.3 c

18y 0.5 1.0 -1.5 -1.0 -0.5 0.0 0.5

y Fig. 7.3

135d 1.0 1.5 1.0 1.0

f 0.8 0.6 0.4 Du = 2.0, 1.5, 1.0, 0.5 0.5 f 0.0 Sr = 2.0, 1.5, 1.0, 0.5 0.2 -0.5 0.0 -1.0 -0.2 -1.5 -1.0 -0.5 0.0 Fig. 7.3 e

84y 0.5 1 .0 -1 .5 -1. 5 -1.0 -0.5 0.0 0.5 1.0

Fig. 7.3 f y 1

162.0 1.0 Pr = 2.0, 1.5,

1.0, 0.5 0.5 0.5 Sc = 2.0, 1.5, 1.0, 0.5 f 0.0

90f 0.0 -0. 5 -0. 5 -1.0 -1.0 -1.5 -1. 5 -1.0 -0. 5 0. 0 0.

5 1.0 -1.5

98y -1.5 -1.0 -0.5 0.0 0.5 1.0

Fig. 7.3 g Fig. 7.3 h y Figs. 7.3(a-h). Effects of various parameters on concentration. 20 15 15 10 10 Z1 5 Z1

19M = 2.0, 1.5, 1.0, 0.5 5 0

69**d = 0.3, 0.6, 0.9, 1.2** 0 **-5**

-5 -10 -3 -2 -1 0

111**1 2 3** 3 **y -3 -2 -1 0 1 2** Fig. 7.4 **a** Fig. 7.4 **b**

y 15 8 6 10 Z1 4 2 0 -2 g =

86**p/6, p/4, p/3, p/2**

Z1 5 0

149**b1 = 0.1, 0.2, 0.3, 0.**

4 -4 -5 -6 -3 -2 -1 0 y Fig. 7.4

150**c 1 2 3 -3 -2 -1** 0 **y**

Fig. 7.4 d 1 2 3 8 6 4 Z1 2 0 -2 b2 = 0.4, 0.3, 0.2, 0.1 -4 -3 -2 -1 0 1 2 3 y Fig. 7.4 e Figs. 7.4(a-e). Behavior of ?1 for different parameters. Chapter 8 Peristaltic transport in a vertical channel subject to Soret and Dufour effects This chapter looks at the simultaneous

2**effects of heat and mass transfer** in **peristaltic transport of** viscous **fluid.**

Mathematical modelling is provided

32**in the presence of** Joule heating **and** Soret and **Dufour effects.**

Perturbation solution is prepared for small Brinkman number. Flow quantities of interest are plotted and analyzed. 8.1 Mathematical formulation Here our aim

3**is to develop a model** for **the peristaltic** transport **of** viscous **fluid in a** vertical **channel.**

156**An incompressible fluid is** considered **in a** vertical **channel.**
Magnetohydrodunamic (MHD) character **of**

fluid is accounted. The flow is generated by sinusoidal waves having con- stant speed ?. The channel width is taken as 2?1. An incompressible and magnetohydrodynamic (MHD) fluid fills the region in the channel. The fluid is conducting through uniform applied magnetic field B0? Induced magnetic field is not taken into account. Electric field is absent. Geometry of flow problem is given below The wall shape is taken as ?(?? ?) = ?1 + ?1??? 2? μ ? (? − ??) , (8.1) ¶ where ?1 is the wave amplitude, ? the wavelength and ? the time. The velocity field in two- dimensional flow is given by ? = ? (? ? ? ? ?)? ? (? ? ? ? ?)? 0 ? £ ¤

95**Conservation laws of mass, momentum and energy**

in the present flow consideration are reduced in the following expressions ?? ?? ?? ?? = 0? + (8.2) ? ? =− +? +? +? ? ? ? ?? + μ ?? ?? ?? ¶ ?? +???∗ (? − ?0) ? ·???2?2 ???2?2¸−??02? +???∗(? −?0) (8.3) ? ? +? +? ? ? ?? ?2? ?2? + ? (8.4)

65μ ?? ?? ?? ? =−?? +?·??2 ??2¸ ¶ ??? ??? + ? ??? + ? ??? ? = ? ???2?2 + ?? 2?? 2 + ? **2** ???? + ???? + ???? + ???? + ??02? + ????? **???2?2** + ???2 **?2**

+ Φ∗ 2 ³ 2 ´2 h 2 i (8.5) · μ³ ´ ³ ´ ¶ ³ ´ ¸

59**h** i ? + ? + ? ? ? **?2? ?2? ??? ?2? ?2?** ? = ? + + + ? (8.6) · ?? ?? ?? ¸ **·??2 ?? 2** ¸ ? ? ·??2 ?? **2**

¸ In above expressions (? ? ? )

> 80**are the velocity components in the** laboratory **frame (? ? ? ), ? the**

pressure, ? the

> 56**acceleration due to gravity,** ?∗ the **coefficient of thermal expansion, ?**∗ ∗ **the** coefficient **of**

expansion due to concentration, ? the thermal conductivity, ? the concentration field, ? the temperature field, ? the mass diffusivity, ?? the thermal diffusion ratio, Φ∗ the constant heat addition/absorption, ?? the specific heat, ?? the concentration susceptibility, ?0 the concentration at boundary, ?? and ?0 the fluid mean temperature and temperature at the boundary respectively. If (?? ?)

> 24**are the velocity components in the wave frame** (?? ?) then using the
> **transformations** ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?) (8.7) **and**

taking ?= ?= ?= ?= Φ= ? ? ? = ? ? = ? ? = ? ? ? ?1 ? ? ? ? = ?1 ? ? = ? ? ? = ?1 ? ? = ?21? ? ?? ? ?1 ?1 ??? ? − ?0 ? ? = ? − ?0 ? ? 2 = ?02?21? υ = ? ?? = ???1 ? ? ? ? 0 ? 0 μ ¶ ? ? ? ?? ? ? ? ?? = Pr ?? ?? = ?????0?????0 , ?? = ?????????00 ? ?? = ?? ? ???2?0 ? ? ? = ??? ? ?? = ????∗??0?12 ? ?? = ???∗? ∗??0?21 ? ? ?2Φ∗ ?? ?? ? (? − ?0)? ? = ?? , ? = − ?? ? (8.8) definition of stream function ? can be written as incompressibility condition (8.2) is clearly satisfied while Eqs. (8.3)-(8.6) through standard ?? ? 3? ?? ??3 = − ?2 ?? + 1 + ??? + ???? (8.9) μ ?? ¶ ??4??4 − ? 2 ????22 ?? ?? + ?? ?? + ?? ?? = 0? (8.10) ?? ?? = 0? (8.11) 0 = ??2??2 + ?? ??2??2 + ??? 2 ???? + 1 + Pr ?? ??2??2 + Φ? 2 2 (8.12) μ ¶ μ ¶ 1 ?2? ?2? 0 = ?? ??2 + ?? ??2 ? (8.13) From Eq (8.11) ? 6= ? (?) ? Here υ denotes the kinematic viscosity, ? the

> 1**Hartman number, Re the Reynolds number, ?? the** local temperature Grashoff **number,**

?? the local concentration Grashoff number, Φ the dimensionless heat source/sink parameter, ?? the Brinkman number, ?? the Dufour number, ?? the Soret number, ?? the Schmidt number, ? the Eckret number, ? ? the Prandtl number, ? the wave number and the dimensionless temperature and concentration are denoted by ? and ? respectively. The boundary conditions are prescribed in the following forms ? = 0, ?2? ?? ?? ??2 = 0, ?? = 0? ?? = 0? ?? ? = 0, ? = ? , ?? ?? = −1, ? = 0, ? = 0, ?? ? = ?, (8.14) ?(?)=1+ ? cos(2??), ? = ? ?? Z0 ?? ??? (8.15) where ? is the dimensionless form of the wall shape. The dimensionless mean flow rates in laboratory (?) and wave (? ) frames are related by ? = ? + 1? Pressure rise per wavelength Δ?? can be put into the following relation 1 Δ?? = ?? ??? ?? (8.16) Z0 8.2 Series expressions We are interested to develop the series solutions of the resulting problems. For that

> 70**we expand the** stream function **in** terms **of small** Brinkman **number and**

write ? = ?0 + ???1 + ???? Similarly the other physical quantities like ?? ?? ? and ? are also expanded. It is worth men- tioning to point out that the condition ?? ¿ 1 is not an unrealistic condition. For example, the most common Newtonian fluid i.e. water, at 25?? (298 Kelvin ≈ room temperature) has dynamic viscosity ? = 0?89 cP (centi poise) and thermal conductivity 0.58 W/mK (watt per meter Kelvin). If we consider the characteristic speed 1m/s (meter per second) and wall tem- perature to be 310 Kelvin (≈ equal to body temperature), then the Brinkman number comes out to be 1?2787 × 10−4 ¿ 1? This provides us a confidence regarding the selection of Brinkman number ?? as a small quantity. Resulting zeroth and first order systems are given by ??0 ?3?0 − ?2 ??0 = ?? ??3 μ + 1 + ???0 + ???0? ?? ¶ 0 = ??2??20 + Pr ?? ??2??20 + Φ? 0 = ?1? ??2??20 + ?? ??2??20 ? ?0 = 0, ??2??20 = 0? ????0 = 0? ?? ? = 0, ? 0 = ?0, ????0 = −1, ?0 = 0, ?0 = 0, ?? ? = ?. First order system ??1 ?3?1 − ?? ??3 2 ????1 + ???1 + ???1? = 0 = ??2??21 + Pr ?? ??2??21 + ?2?0 + ?2 ??0 + 1 2 2 μ ??2 ? ¶ μ ?? ¶ 0 = ?1? ??2??21 + ?? ??2??21 ? ?1 = 0, ?2?1 = 0, ???1 = 0? ??1 = 0? ?? ? = 0, ??2 ?? ?? ?1 = ?1, ????1 = 0, ?1 = 0, ?1 = 0, ?? ? = ?. The solutions of above systems yield the following expressions + ?5? ?+(?)³ ´ ?= ?−??6???2(?+2?) −6 (−1 + ?) ?1 (?) + 2??? ?1? (6?3) ? ?? − ?11 ?4 (?) + ?12 ? " # −?2 + ?2 ? ???? ? = ¡2 (−1 + ?¢) 360? 26 − ? ?= ?8 + ???10 360 (−1 + ?)3 ?29 ? where ?1 = 3(−1+?)(? +?)?2?2(?) ??(?+2?)? ? 2 = ¡(−1 + ?) ? 2 1 + ?? + ?2??¢(−1 + ?? ) ? ?3 = 16?−2??+3?(?³+?)? −1 + ?2?2 (?? − ??????) Φ2? ´ ? 4 = ?6 (?) + ?7 (?) (−1¡+ ?) (? + ?¢) + ?8 (?) + ?9 (?) ? ?5 = (−1 + ?)3 ? 10 1 + ?? + ?2?? (−1 + ?? ) ? ?6 = (−1 + ?)3 1 +³?? + ?2?? (−1 + ??) ? ´ ?7 = 720 (−1 + ?³)2 ??(?−?)?2?2?1 (?) + ?2 (?) + ?3 (?) + ?4 (?) + ?5 (?) ? ´ ?8 = ???? 180 (−1 + ?)2 (? − ?) (? + ?) Φ ? ?9 = 360 (³−1 + ?)3 1 + ?? + ?2?? (−1´+ ?? ) ? ?10 = ?³1 (?) + ?2 (?) ´+³?3 (?) + ?4 (?) + ?5 (?) ? ´ ?11 = 1 + ?? + ?2?? (−1 + ?? ) ? ?12 = ?5 (?) + ??355 ??(?+2?)?4 + ?10 (?) ? ³ ´ n o ? = ? ???????? and ???? ??? and ??? are given below ?1 = ? ?? cosh((????c)o+s?h(s?in? h()?−?sin)−h((???+?)))2sinh(??) ? ?2 = ?1(?2?3 + ??(?? ? cosh (?? ) + ?3? 2 − ? ) cosh (?? ))2 cosh (? ?) ? + sinh (?? )) − 2?? sinh (2?? ) ? ¡ ¢ ?3 = (? + ?)2 1 + ?2 2 + ?2 (? + ?)2 4) + ??(2 (? + ?)2 ? 1 + ? 2 ? cosh (2? ?) sinh¡ (?? ) +¢ 16 (? + ?) ? 2?2 cosh (? ?))? ¡ ¢ ?4 = ? + ? − 8?2 ? 2 + 8?4? 4) cosh (3?? )

+ 16?2 (? + ?) ? 4?2 cosh (?? )2 cosh (? ?) ??(sin¡h (? ?) − 16 (¢? + ?) ? ? sinh (?? )2 sinh (? ?))? ?5 =
2? 4 + ? + ?? 2 − 2?2 + ??(? 8 + ? 3 + ? 2 1 + 2?2 − 2?2 )) cosh (2?? ) ? ?6 = 2?2¡(? + ?)(2? 4 ¢+ ? + ??
2 − ?¡?2?2 +¡ ? 8 + ?¡ 3 + ? 2 1 +¢2¢?¢2 − 2?2 )? ?7 = (? + 2?3)? 2 co¡sh (2? ?) + 2?¢ ?? 2 cosh (2?¡ ?)
+ ?¡2? 2 cos¡h (2? ?) + 8 (?¢¢+¢ ?) ? ?8 = ?()?+2???(16 (? + ?) sinh (?? ) (? (? − ?) + sinh (? ?)))? ?9 =
16? 2?2 sinh (? (? − ?)) + (? + ?) 1 + ? 2 −3 + 2? 2?2 sinh (2? ?) + 16?? +? 2?2 sinh (? (? + ?)) ? ¡ ¢ ¡ ¢?
10 = 1 + ? 2 ? − 4?3 (? + ?) ? 4 1 + ? 2 ?2 − 4 (? + ?) ? 2 1 + ? 2 ?3? ?1 = ?¡1(−8? 2¢− 2? (4 + ? + 3?) −
2¡? ? + ?2¢(? + 2?) ? 2 − 2?2 ¡(? + ?)2¢? 4) +2(1 + ? (? + 2?) ? 2 + (? + ?)2 ? 4)?2 + (? 2 1 + ? 2 + 2? 4 +
? + ?? 2 −2?2 + ? 8 + ? 3 + ? 2 1 + 2?2 − 2?2 ) cosh¡(2?? ) −¢ (? +¡?)2 1 + ? 2 c¢osh (2? ?) ? ?2 = 16?
¡(? + ?¡) ? ???3(¡? + ? 2?) cosh¢¢(?¢? ) (? (? − ?) + sinh (?¡?)) − 16¢(? + ?) ? sinh (?? ) (? (? − ?) + ?
sinh (? ?))? ?3 = 12 (? + ?) ?9 cosh (?? ) − 16?4 (? + ?) ?3 + 8?3 (? + ?) ? 4 (? − ?) cosh (2?? ) ? ?4 =
128? 1?3? 4 cosh (?? )2 sinh (?? ) + 2?3 (? + ?)2 ? 3 1 + ? 2 cosh (2? ?) + 8?3 (? +?)?4(?−?)?5? ¡ ¢ ?5 =
?4? ?(−4? ? + ? 2 − 4? + 2??2? ? + ?2 + (? + ?)2 1 + ?2 ? 2 + ?2 (? + ?)2 ? 4)? ?1 = 16? (? + ?) ? 2 (? − ?)
cosh (?? ) ??? − (? 2 1 + ?¡ 2 + 2¢? 4 + ? + ?? 2 −2?2 + ? 8 + ? 3 + ? 2 1 + 2?2 − 2?2 ) cosh (2¡?? ) ? ¢ ¡
¢ ?2 = ? 2?¡ 2

> 9**cosh¡(2? ?) +¡ 2?** ?? 2 **cosh (¢2¢?¢** ?) + ?2? 2 cosh **(2? ?) + 8** (? + ?) ? 2 **cosh (2?**

?) + 2? ? cosh (2? ?) ? ?3 = 32?2 (? + ?) ? 3?2 cosh (? ?) ?2 − (? (? + ?) −8 + 5? + 5? + (5? + (5 − 16?)
?) ? 2 ? ?4=8? 4+?+?+ ?+?+4?2 ?2 )sinh(??¡)−8((4? −1+?2?2 +(?+?)? ¢ ?5 = ?(−¡64 + ? ? 1 +¡? 2 ?6
−13¢+ 24¢?2? 2 + 16?4? 4 + ¡??3(−13 + ?¢ 2(−13 + 8?(10 + ?)))? ¡ ¢ ¡ ¢ 8.3 Graphical results and
analysis This section emphasizes the impact of embedding flow parameters in the solution expressions.
Here plots are prepared in the two and three dimensions. Arrangement of the graphs is made in such a
way that Figs. 8.1 (a-d) show the pressure gradient, Figs. 8.2 (a-d) the pressure rise per wavelength, Figs.
8.3 (a-d) the 3-D plots for velocity profile, Figs. 8.4-8.9 the stream lines and Figs. 8.10 and 8.11 the 3D
plots for temperature and concentration fields respectively. Figs. 8.1 (a-d) reveal that pressure gradient
decreases by increasing ? and ??? However, the

> 26**pressure gradient increases with an increase in**Φ and Du. **It is** further noticed
> **that**

the change in dp/dx is larger in the cases of Φ and M when compared with Du and Sr. Pressure rise per
wavelength via mean flow rate ? is displayed in the Figs. 8.2 (a-d). These Figs. show that the pressure rise
decreases when flow rate increases. For a given constant value of the mean flow rate, the

> 5**pressure rise increases with an increase in**Φ **and** ?? but **it**

decreases with increase in ?? (see Figs. 8.2 a,c and d). Behavior of pressure rise for M is however
different. The pressure rise increases for negative value of mean flow rate when ? increases. However, the
behavior is opposite for positive values of mean flow rate (Fig. 8.2b). We displayed the 3D plots for
velocity, temperature and concentration. Here

> 5**it is observed from Fig.** 8.3 **that the** velocity **decreases**

when ? is increased. Such decrease largely depends upon the values of ?? Physically the decrease in
velocity is due to the reason that the Lorentz force has a role of retarding force. Consequently it increases
the friction resistance which opposes the motion of fluid. Figs. 8.3 (b-d) depict

> 31**that the velocity is increasing function of**

Φ and ??? However reverse situation is observed for ?? i.e. the velocity decreases when ?? increases.
Trapping is considered interesting feature of the peristaltic transport. The volume of fluid that gets trapped
in a stream line during peristaltic motion is known as bolus. Figs. 8.4-8.9 depict

> 5**that the size of trapped bolus decreases** when ?, ?? **and**

?? are larger. It is also found that the results are opposite for the cases of ??? Φ and ??. The variations of
temperature and concentration fields

> 3**can be seen through Figs. 8.10** and **8.**

11. These Figs show that the temperature increases when ?, ??, Φ and Du are increased. Clearly the rise
in temperature with an increase in M and Du is more than Φ and ??. Effects of ?? on the temperature are
not significant. 99 Figs. 8.11 plot the influences of ?? Φ? ??, ?? and ?? on the concentration field. Here
concentration field decreases when there is an increase in ?, Φ and ??. Further, the variations of
concentration field for ?? and ?? are insignificant. It is also concluded that temperature increases and
concentration decreases when ? increases. 0 0 M = 1.0 -1

129**F = 2.0** -5 **F** = 1 **.0** M = **2.0** -2 -10 -3 **F = 0.**

0 dp?dx M = 3.0 dp?dx -4 -15 -5 F = -2.0 -20 M = 4.0 -6 -7 -0.4 -0.2 Fig.

50**8.1** a **x 0.0 0.2 0.4 -0.** 4 **-0.** 2 **Fig.**

71**8.1** b **x 0.0 0.2 0.4**

-2 0 Du = 6.0 Sr = 0.0 Sr = 1.0 -1 Du = 4.0 -3 Sr = 2.0 Du = -2 -4 Du = 0.0 Sr = 3.0 dp?dx -3 dp?dx -5 -4 -6 -5 -6 -7 -7 -0.4 -0.2 Fig.

50**8.1** c **x 0.0 0.2 0.4 -0.** 4 **-0.** 2 **Fig.**

71**8.1** d **x 0.0 0.2 0.4**

Figs. 8.1 (a-d) Behavior of pressure gradient for variations in M, Φ? Du and Sr. 30 20 10 0 Dpl M =

101**1.0** -10 **M = 2.0 M = 3.0 -20 M = 4.0 -30**

-1 0 1 2 3 Fig. 8.2 a Fig. 8.2 b h 10 10 5 5 0 Gt = 1.0 Dpl -5 Gt = 2.0 Gt = 3.0 -10 Gt = 4.0 -15 0 Gc = 1.0 Dpl -5 Gc = 2.0 Gc = 3.0 -10 Gc = 4.0 -15 -20 -20 -25 -1 0 1 2 3 -1 0 h 1 2 3 h Fig. 8.2 c Fig. 8.2 d Fig. 8.2 (a-d) Pressure rise per wavelength. Fig. 3 a Fig. 8.3 c Fig. 8.3 d Fig. 8.3(a-d) Velocity profile with variations of ?, Φ? ?? and ??. Fig. 8.4 Effect of Hartman number (?) on stream lines when Φ = 1? ?? = 0? 5, ?? = 0?5? ?? = 0?25? ?? = 2 and ?? = 1? Fig. 8.5 Effect of ?? on stream lines for Φ = 1? ?? = 0?5, ?? = 0?5? ?? = 0?25? ?? = 2 and ? = 1? Fig. 8.6 Effect of ?? on stream lines when Φ = 1? ?? = 0?5, ?? = 0?5? ?? = 0?25? ?? = 2 and ? = 1? Fig. 8.7 Effect of Φ on stream lines when ?? = 2? ?? = 0?5, ?? = 0?5? ?? = 0?25? ?? = 2 and ? = 1? Fig. 8.8 Effect of ?? on stream lines for Φ = 1? ?? = 2? ?? = 0?5, ?? = 0?5? ?? = 0?25? ?? = 2 and ? = 1? Fig. 8.9 Effect of ?? on stream lines when Φ = 1? ?? = 2? ?? = 0?5, ?? = 0?5? ?? = 0?25? ?? = 2 and ? = 1?

75**Fig. 8.** 10 **a Fig. 8.** 10 **b**

75**Fig. 8.** 10 **c Fig. 8.** 10 **d**

Fig. 8.10 e Figs. 8.10 (a-e) Variation of ?, ?? ,??? Φ and ?? on temperature.

166**Fig. 8.11 a Fig. 8.11 b**

75**Fig. 8.** 11 **c Fig. 8.** 11 **d**

Fig. 8.11 e Figs. 8.11 (a-e) Diagrams analyzing the development of concentration field when ?, ?? ,??? Φ? and ?? are varied. Chapter 9

2**Mixed convective heat and mass transfer** for peristaltic transport **in an asymmetric channel with**

32**Soret and Dufour effects The** aim **of**

present chapter is to address the

12**simultaneous effects of heat and mass transfer** in **the** mixed convection **peristaltic flow of** viscous **fluid in** an asymmetric **channel. The**

channel walls exhibit the convective boundary conditions. In addition the effects due to Soret and Dufour are taken in to consideration. Resulting problems are solved for the series solutions. Numerical values of heat and mass transfer rates are displayed and studied. 9.1 Mathematical analysis We consider an

asymmetric channel of width ?1 + ?2 with flexible walls. An incompressible viscous fluid is considered. Cartesian coordinate system is adopted such

> 9that the ?-axis is along the length of the channel and the ? -axis is normal to the ?-axis. The flow is

generated due to peristaltic

> 1waves propagating on the channel walls with speed c.

Moreover, the channel walls posses the characteristics of convective type boundary condition. We intend to study the flow

> 32in the presence of Soret and Dufour effects. The

channel walls are described by the following expressions ?1(?? ?) = ?1 + ?1, left wall and ?2(?? ?) = − (?2 + ?2) ? right wall where ?1 and ?2 are the disturbances produced

> 2due to propagation of peristaltic waves at the left and right walls respectively. The

values of ?1 and ?2 can be written in the forms 2? ?1 = ?1??? ? (? − ??) ? μ 2? ¶ ?2 = ?1??? (? − ??) + ? ? μ ? ¶ in which ? is time, ? is the phase difference and ?1? ?1 are the amplitudes of the waves travelling on the left and right walls with speed c respectively. Temperature and concentration at the walls are assumed to be ?? and ?? (? = 0 for left wall and ? = 1 for right one ) respectively. Velocity field for this problem is ? = ?(?? ? ? ?)? ? (?? ? ? ?)? 0 ?

> 10Fundamental equations governing £the flow of an incompress¤ible viscous fluid

are ∇?? = 0? (9.1) ? ?? = −∇? + ? ∇2? + ???∗ (? − ?0) + ???∗∗ (? − ?0) ? (9.2) ?? ³ ´ in which ?

> 66is the density of fluid, ? is the pressure, ? is the dynamic viscosity, ? is the

acceler- ation due to gravity and ?∗ and ?∗∗ are the thermal and concentration expansion coefficients respectively. The energy and concentration equations can be expressed as ?? ???? = ?? ?? + ?? ? ? + ? 2 ?2? + ? 2? + ?? +£? + ????¤?? ? ? + ?? ? 2 ? ? ? (9.3) h ³ ´ ¡ ¢ i £ ¤ ?? ?? = ? ?? ? + ?? ? ????? ? ? + ?? ? + ? ? (9.4) Here ?? is the specific heat, ? is the

> 6thermal conductivity, ? is the mass diffusivity, ?? is the £ ¤ £ ¤ thermal diffusion ratio, ?? is the concentration susceptibility, ?? is the

mean temperature, ? and ? are respectively the

> 14temperature and concentration of the fluid.

We consider (?? ?) and ? as the

> 7velocity components and pressure in the wave frame (?? ?). The

> 7transformations between laboratory and wave frames are

? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (9

> 24.5) where (? ? ? ) and ? are the velocity components and pressure in the laboratory frame.

The fundamental

37**equations in wave frame are** reduced as follows: ?? ?? ?? ?? = **0?**

+ (9.6) ? ? ?? ?2? ? (? + ?) ?? + ? ?? (? + ?) = − ?? + ? ??2 + µ ¶ · ??2??2 + ???∗ (? − ?0) +???∗∗ (? − ?0) ? ¸ (9.7) ? (? + ?) ?? + ? ?? ? = − ?? + ? ??2 ??2 ? ? ?? ?2? ?2? + ? (9.8) µ ¶ · ¸ ?? ((? + ?) ?? + ???) = ?? [??? + ???] + ? 2 (? + ?)2? + ?2? + (? + ?)? + ?? + ?????? [??? + ???] ? 2 (9.9) · ³ ´ ³ ´¸ (? + ?) ?? + ??? = ? [??? + ???] + ??? ?? [??? + ???] ? (9.10) Letting ?= ? ? ? ?= ? ?= ? ?= ? ? ? ?1 ? ?? ? ? = ?1? ? 1 = ?1? ? ?1 ?2 = ?2? ?= ?2? ?= ?1? ?= ?1? ?= ?21? , ? = ? ? ?1 ?1 ?1 ?1 ??? ? ?? = ??1 ? ? = ? − ?0 ? ? = ? − ?0 , ?? = ? (?1 − ?0) ?? , ? ?1 − ?0 ?1 − ?0 ????? (?1 − ?0) ?? = Pr ?, ? = ?2 ? ?? = ???? (?1 − ?0) ?? ?? (?1 − ?0) ??? (?1 − ?0) ? ? = ? ? ?? = ? ?? ? ?? = ???∗ (?1 − ?0) ?21 ? ?? = ???∗∗ (?1 − ?0) ?12 ? ?? ?? ??? ?? = ? ? ?=??,?=−??? (9.11) and adopting long wavelength and low Reynolds number approach we have in terms of stream function ? the following equations ?? = ???? + ??? + ???? ?? = 0? ??? + ?? ??? 2 + Pr ??(???) = 0? ????? + ????? = 0? 1¡ ¢ (9.12) (9.13) (9.14) (9.15) Here ? denotes the

47**kinematic viscosity, Re the Reynolds number,** ?? is **the** Brinkman **number,**

?

1**is the** Eckret **number, ? ?** is the Prandtl **number, ?** is the wave number, **??** is **the** temperature Grashoff **number, ??** is **the** concentration Grashoff **number, ?** is dimensionless concentration, ? is **the**

dimensionless temperature and incompressibility condition is automatically satisfied. Defining ? and F as the dimensionless mean flows in laboratory and wave frames we have ? = ? +1+? (9.16) where ?1 ?? ?= Z?2 ?? ??? (9.17) The convective boundary conditions for the temperature and concentration are given by −? ?? = ?(? − ??)? ?? −??? =?(?−??)? ?? The dimensionless boundary conditions for our problem can be imposed

88**as follows:** ? = ? 2 , **??=−1, ??+??(?)=0? ??+??(?)=0?** ?=?1, ? = − , ??=−1, ?? −??(?−1 **)=0,** ??−??(?−1 **)=0,?=?2,**

? 2 (9.18) where ?1(?) = 1 + ? cos(2??), ?2(?) = −? − ? cos(2?? + ?) ? (9.19) ?? = ?? ? 1, ?? = ??1? ? (9.20) In above expressions ? and ? are dimensionless transfer coefficients, ?? is heat transfer Biot-number and ?? is mass transfer Biot-number. 9.2 Solution of the problem The problems consisting of Eqs. (9.12)-(9.15) have been solved for both the perturbation and numerical solutions. Perturbation solution is obtained for small Brinkman number. Numerical solution is presented using Mathematica. For perturbation solutions we have the following zeroth and first order systems: ?0???? +

78**???0? + ???0? = 0?** ?0?? + **Pr ??(?0??) = 0? ?0?? + ?????0?? = 0?**

(9.21) (9.22) (9.23) ?0 = ?20 , ?0? = −1, ?0? + ?? (?0) = 0? ?0? + ?? (?0) = 0? ? = ?1, ?0 = −?0 , ?0? = −1, ?0? − ??(?0 − 1) = 0, ?0? − ??(?0 − 1) = 0, ? = ?2, 2 and ?1???? + ???1? + ???1? = 0? ?1?? + ?0?? 2 +

74**Pr ??(?1??)** = 0? **?1?? + ???? (?1??)** = 0? ¡ ¢ **?1 = ?1 , ?1?** = 0, **?1? + ?? (?1)** = 0? **?1? + ?? (?1)** = 0? ? = **?1, 2 ?1** = −?21 , **?1?** = 0, **?1? − ??(?1)** = 0, **?1? − ?? (?1)**

= 0, ? = ?2. The perturbation solutions of above problem are given ? = − ? (?1 + ?2 − 2?) ?21 −24(??11? −2+?2?)223 + 2 (?1 + ?2) ? − 2?2 ¡ ?0 (?) ¢ + 24 (2 + ?? (?1 − ?2)) (?1 − ?2)2 (2 + ?1? − ?2?) + ???1 (?) 241920 (?1 − ?2)6 ? ? = 2 + ???1 − ??2 1440 (−1 + ?) ?2 (?) ? 1 + ???1 − ??? + ?? ? = 21++??????11−−??????2 144?0?(−??1?+? ?) [?3 (?)] ? + where ??? are given below (9.24) (9.25) (9.26) (9.27) (9.28) (9.29) (9.30) (9.31) ?1 = ??? cosh((??c1o)s+h?(?si1n)h−(s?in2

108**)h−(?(?2)+)2?)** sinh(?) + (? + ?1 **)2 ?2?** 1 + ?21 **??2 +?2** (? + ?1 **)2** ??4) + **??(2 (? + ?2)2** ?? 1 + **?2**

? co¡sh (2?) s¢inh (?2) +16 (? + ?2) ??2?2 cosh (?)) + 62???? ?¡ + ?1 −¢8?21 ??2 + 8?42??4) cosh (3? 2??) + 16???2 (? + ?2) ?4?2 c¡osh (?2??)2 co¢sh (?) ??(sinh (?) −16?? (? + ?1) ??? sinh (?2)2 sinh (?)) − 2? 4 + ?2 + ?2??2 − 2?2 +2?3??(?2 8 + ?????1 3 + ?2 1 + 2?21 − 2¡?2 ))

43**cosh (2?¢2) ? ?2 = 2?2??¡??(?** 1 + **?2 +¡?2)(**

2? 4¡ + ?1 + 8???2¢¢¢− ??2?2 + ?2(8 + ?1(3 +??2 1 + 2?21 − 2?2 ))) + ??2?¡?3??(? + ?1 + 2?¢32)?2?

cosh (2?) +2??¡??2 cosh (2?) +¢?2??2 cosh (2?) + 8 (? + ?2) + ?(?+2?2+?1)??(16(? +?2 + 4?1) sinh (??) (? (? − ?) + sinh (??))) + (? + ?2 + 4?1) 1 + ??2 −3 + 2? ?2?2 sinh (2? ?) + 16?? + (?1 + 4?2)2?2 sinh ((? 2 + ?))¡? ¢ ?¡3 = 1 + ??2¢? ? − 4?32 (? + ?1) ??4 1 + ??2 ?2 − 4 (? + ?2) ??2 1 + ??2 ?3 (?1 −¡?2)(−8? 2¢ − 2?2 (4 + ?1 + 3?) −¡2?? + ?¢22 (? + 2?1) ??2 − 2?¡21 (? + ?2¢)2 ????4) + 2(1 + ? (? + 2?1) ??2 + (? + ?2 + 4?1)2 ??4)?2 + (?2 1 + ?21 ? 9.3 Graphical analysis ¡ ¢ In order to provide a comparative study of the results by the two methods, we have plotted graphs for both solutions. Left panel depicts the perturbation solution whereas right panel for the numerical solution. Graphical analysis is carried out for the dimensionless temperature, concentration and velocity fields. Numerical values for the rates of

20**heat and mass transfer at the** left wall **are** given **in the Tables**

9.1 and 9.2 respectively. The values of different parameters in all the graphs are taken as follows: ? = 1?? = ??4?? = 0?5?? = 0?7?? = 0?8?? = 0??? = 1??? = 2??? = 0?5? ?? = 0?5??? = 1??? = 0?25??? = 0?5??? = 0?7? The graphs of temperature profile showed

15**that the temperature decreases with an increase in the** values **of**

Bi and it increases by increasing Mi. The decrease in the temperature with increase in the heat transfer Biot-number is very rapid for ?? ≤ 1? It is noted that for ?? ≥ 1 corresponding decrease in temperature becomes very slow (see Fig. 9.1). This fact is totally opposite to the case of increase in Mi in which temperature not only increases by increasing ?? but also such increase is constant and very slow (Fig. 9.2). Fig. 9.3 showed that the temperature increases when ?? is increased? Increase in the tem- perature is smaller for small values of G? but this increase becomes much for higher values of G?. However this increase in temperature is observed throughout the channel. Unlike the case of ??? small increase in temperature is observed when G? increases but this increase is not uniform throughout the channel. Near the right wall the temperature tends to decrease very slowly by increasing ?? (see Fig. 9.4)? Figs. 9.5-9.7 are plotted to analyze the concentration profile in detail. It is found that the concentration increases when ?? is increased? This increase is large for ?? ≤ 1 and very slow for large values of Mi (see Fig. 9.5). It is noticed from Figs. 9.6 and 9.7

5**that the concentration** generally **decreases with an increase in** ?? and ??. This decrease **is**

much and constant for ?? and ?? respectively. However concentration profile showed different behavior near the right wall. The concentration here increases which is quite different for its behavior at all other parts of the channel. Figs. 9.8 and 9.9 depict that the value of velocity increases by increasing ?? but it remains constant with variation in ??? The velocity in both cases is increased for increasing ?? and ?? in the region where "y" is negative and decreased for positive y. Tables 9.1 and 9.2 are plotted for the numerical values of rates of heat and mass transfer. It is seen that solution up to first order perturbation gives an agreement with the numerical results up to second decimal place. Table 9.1 shows that heat transfer rate is increasing function of Biot-number. However the Grashoff numbers and mass transfer Biot number reduce the heat transfer rate (see Table 9.1). Table 9.2 illustrates that the mass transfer rate at the wall tends to decrease when ??, ??, ?? and ?? are increased. 3.0 3.0 Bi = 0.5 Bi = 0.5 2.5 2.5 2.0 q 2.0 q Bi = 1.0 1.5 Bi = 1.5 1.5 1.0 1.0 Bi = 2.0 Bi = 1.0 Bi = 1.5 Bi = 2.0 -1.0 -0.5 0.0 0.5 1.0 1.5 0.5 -1.0 -0.5 0.0 0.5 1.0 y y Fig. 9.1 Effect of ?? on the temperature profile. 3.2 Mi = 0.5, 1.0, 1.5, 2.0 3.2 Mi=05.,1.0,1.5,2.0 3.0 3.0 q 2.8

40**q 2.8 2.6 2.6 2.4 2.**

4 -1.0 -0.5 0.0 0.5 1.0 1.5 -1.0 -0.5 0.0 0.5 1.0 y y Fig. 9.2 Effect of ? ? on the temperature profile. 3.2 Gt = 0.0, 1.0, 2.0, 3.0 3.2

57**Gt = 0.0,** 1 **.0,** 2 **.0,** 3 **.0** 3 **.0**

q 2.8

40**q 2.8 2.6 2.6 2.4 2.**

4 -1.0 -0.5 0.0 0.5 1.0 1.5 -1.0 -0.5 0.0 0.5 1.0 y y Fig. 9.3 Effect of ?? on the temperature profile. 3.2

46**Gc=0.0,2.0,4.0,6.0**

3.2

46**Gc = 0.0, 2.0, 4.0, 6.0**

3.0 3.0 q

40q 2. 8 2. 8 2.6 2.6 2. 4 2.4

-1.0 -0.5 0.0 0.5 1.0 1.5 1.0 -1.0 -0.5 0.0 0.5 y

173y 0. 6 0.6 0.4 Mi = 2.

0 0

155.4 0.2 f 0.0 -0.2

Mi = 2.0

82Mi = 1. 5 0. 2 Mi = 1.5 ∆ 0.0

82Mi = 1.0 Mi = 1. 0 -0. 2 Mi = 0 .5 -0.

4

120Mi = 0.5 -1.0 -0. 5 0. 0 0. 5 1.0 1 .5 -0.

4 y -1.0 -0

118.5 0.0 y 0.5 1.0 1.5

Fig. 9.5

11Effect of ? ? on the concentration profile. 0. 0 -0. 1 -0. 1 -0.

2

133-0.2 f -0.3 f -0.3 -0.4 Gt = 0.

0, 1.0, 2.0, 3.0 -0.4

57Gt = 0.0, 1 .0, 2 .0, 3 .0 -1 .0 -0. 5 0.0 0. 5 1 .0 1.5 -0.

5 y -1.0 -0

128.5 0.0 y 0.5 1.0 Fig.

9.6

11Effect of ?? on the concentration profile. 0. 0 -0. 1 -0. 1 -0.

762 -0.2 f ∆ -0. 3 -0 .3 -0. 4 Gc = 0 .0,

2.0, 4.0, -0.4

46Gc=0.0,2.0,4.0,6.0

-0.5 -1.0 -0.5 0.0 0.5 1.0 1.5 -0.5

39y -1.0 -0.5 0.0 0.5 1.0 y

Fig. 9.7

11**Effect of ?? on the concentration** profile. **1.0** 1 **.0**

0.5 0.5

28**U 0.0** U **0.0** Gt **=0.0,** 2 **.0,** 4 **.0,** 6 **.0** Gt = **0.0,** 2 **.0,** 4 **.0,** 6 **.0** -0.5 **-0.**

5 -1.0 -1.0 -1.0 -0.5 0.0 0

36**.5 1.0** 1.5 **y -1.0 -0.5 0.0 0.5 1.0** y **1.0** 1.0 **0.** 5 **0.5** 0 **.0** Gc=0 **.0,** 2.0,4 **.0,**

6.0

121**U 0.** 0 **U** Gc = **0.0,** 2 **.0,** 4 **.0,** 6 **.0 -0.** 5 **-0.**

5 -1.0 -1.0 -1.0 -0.5 0.0 0

18**.5 1.0** 1.5 **y -1.0 -0.5 0.0 0.5 1.0**

y Fig. 9.9 Effect of ?? on the Velocity profile. ?? ?? ?? ?? −?0(?1) − ?0 ∗ (?1) 0?5 1?0 2?0 0?5 1?1935 ¡1?1¢927 1?0 1?2343 1?2343 1?5 1?2550 1?2560 2?0 1?2674 1?2693 0?0 1?2988 1?3018 1?0 1?2674 1?2693 2?0 1?2411 1?2402 3?0 1?2197 1?2176 0?0 1?2439 1?2349 1?0 1?2310 1?2254 2?0 1?2197 1?2176 3?0 1?2100 1?2110 0?5 1?2100 1?2110 1?0 1?1999 1?2006 1?5 1?1955 1?1967 2?0 1?1930 1?1951 Table 9.1. Numerical values of heat transfer rate

161**at ?1. ?? ?? ?? ?? ?0(?1)** ?0 ∗ **(?1)** 0?5 1 **?0 2?** 0 **0?**

5 0?2190 ¡ 0?¢2187 1?0 0?2179 0?2179 1?5 0?2174 0?2176 2?0 0?2171 0?2173 0?0 0?2219 0?2225 1?0 0?2171 0?2173 2?0 0?2139 0?2141 3?0 0?2125 0?2139 0?0 0?2142 0?2146 1?0 0?2131 0?2141 2?0 0?2125 0?2139 3?0 0?2125 0?2140 0?5 0?2125 0?2140 1?0 0?1346 0?1360 1?5 0?0903 0?0914 2?0 0?0618 0?0625 Table 9.2. Numerical values of mass transfer rate at ?1. Chapter 10

4**Variable viscosity** effects **on the peristaltic motion** of **a third-order fluid**

The

45**effect of variable viscosity on the peristaltic** motion **of** MHD non **-Newtonian fluid in** a **channel**

is studied. Constitutive relations of third order fluid are employed in the development of flow. Mathematical analysis is presented when no-slip condition is no longer valid. The

17**series solutions** for **stream function,** longitudinal **velocity and pressure gradient are**

first derived and then discussed in detail. The

6**pressure rise and frictional forces are** monitored through **numerical integration.**

Physical interpretation is also made. 10.1 Governing problem We

12**consider an incompressible magnetohydrodynamic (MHD)** third order **fluid** with variable viscosity **in a** uniform **channel of**

width 2?1.

16 **A uniform magnetic field** B0 **is applied in the ?−direction. The** induced **magnetic**

and electric fields are not considered. Wave shapes are written as ?(?? ?) = ?1 + ?1??? 2? ? (? − ??) , (10.1) · ¸ where

6 **c is the velocity of propagation** of **the**

wave, ?1

9 **is the** wave **amplitude, ? is the wavelength, ? is the** time, ? is **the** direction of **wave**

propagation and ? is transverse to ?? The constitutive equations for the third order fluid are given through Eqs. (1.17-1.19). Hence we directly write the following governing equations for two dimensional flow of MHD third order fluid as ?? ?? ?? ?? =0? + (10.2) ? ? +? +? ? ? + + (10.3) µ ?? ?? ?? ?=−?? ???? ???? −?? 02?? ¶ ?? ?? ?? ? ? +? +? ? ? + + (10.4)

51 **µ ?? ?? ? =− ?? ???? ????? ?? ¶ ?? ?? ?? ?¯?¯?¯ = ?(2?¯?¯)+?** 1 **(2?¯?¯?** **+2?¯?¯?¯?¯ +** 2?¯?¯?¯?¯ +4 **?¯?2 +2?¯?¯?¯?¯ +2?¯?2)+?2** (4 **?¯?2** +?¯?2 +? **¯?2 +2?¯?¯?¯?¯)+?1(2?¯?¯?¯?+**

2?¯?? ?¯?¯ +4?¯?¯?¯ ?+2?¯ ?¯?¯?¯ +4?¯?¯?¯ ?+12?¯?¯?¯ ?+4?¯?¯?¯ ?+14?¯?¯?¯ ?¯?¯ +10?¯?¯?¯?¯?¯ − 6?¯?¯?¯?¯?¯ +6?¯?¯?¯?¯?¯ +6?¯?¯?¯ ?+6?¯?¯?¯?¯?¯ +4?¯?¯ ?¯?¯ +8?¯?3 +8?¯?¯?¯?¯?¯ +2?¯?¯?¯ ?+2?¯ 2?¯?¯?¯ +4?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ +2 ?¯2?¯?¯?¯)+?2(8?¯?¯?¯?¯ +8?¯?¯?¯?¯?¯ +8?¯?¯?¯?¯?¯ +16?¯?3 +8?¯?¯?¯?¯?¯ +4?¯?¯?2 +2?¯?¯?¯ ?+2?¯?¯?¯ ?+2?¯?¯?¯?¯?¯ −2?¯?¯?¯?¯? + 2?¯?¯?¯?¯? +2? ¯?¯?¯?¯ +4?¯?¯?¯ ?2 +2?¯?¯?¯ ?+2?¯?¯?¯ ?+2?¯?¯?¯?¯? −2?¯?¯?¯?¯? +2?¯?¯?¯ ?¯?¯ +2?¯?¯?¯?¯?¯)+?3(16?¯?3 +4?¯?¯?¯?2 + 4?¯?¯?¯?2 +8?¯?¯?¯?¯?¯)?¯?¯?¯ ?(?¯?¯ +?¯?¯)+?1(?¯?¯?+?¯?¯?¯ ?+?¯?¯?¯ ?+?¯?¯?¯¯+?¯?¯?¯ −?¯?¯?¯?¯ +2?¯?¯?¯?¯ −2?¯?¯?¯?¯) = +?1(?¯?¯ ??+?¯ ??¯?¯?¯ +?¯ ??¯?¯?¯ +?¯?¯?¯?¯?¯ +?¯ ??¯?¯?¯ +2?¯?¯?¯?¯ +4?¯? ¯?¯?¯?¯+ 2?¯?¯?¯?¯ ?−2?¯?¯?¯?¯ −4?¯?¯?¯?¯?¯ +?¯ 2?¯?¯?¯?¯ +?¯?¯?¯?¯?¯ −?¯?¯?¯?¯?¯ +?¯ 2?¯?¯?¯?¯ −5?¯?¯?¯?¯?¯ +4?¯?¯?¯?¯?¯ +?¯?¯?¯?¯?¯ +?¯ 2?¯?¯?¯ +?¯?¯?¯?¯?¯ +3? ¯?¯?¯?¯?¯ −?¯ 2?¯?¯?¯ −4?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯ +2? ¯?¯?¯?¯ +3?¯?¯?¯?¯?¯ +5?¯?¯?¯?¯?¯ −2?¯?¯?¯?¯ +4?¯?2?¯?¯ +4?¯?¯?¯?2 +4?¯?¯?¯?2 +4?¯?¯?¯?2 ) +?2(8?¯?2?¯?¯ +6?¯?2?¯?¯ +2?¯?3 +8?¯?¯?¯?2 +6?¯?¯?¯?2 +2?¯?3 )?3(8?¯? 2¯?¯?¯ +8?¯?2?¯?¯ +2?¯?3 +6?¯?¯?¯?2 +6?¯?2?¯?¯ +2?¯?3 )?¯?¯?¯ = ?(−2?¯?¯)+?1(−2?¯?¯ ¯−2?¯?¯?¯?¯ −2?¯?¯?¯?¯ +4?¯?2 +2?¯?¯?¯?¯ +2?¯?2 ) +?2(4?¯?2 +?¯?2 +?¯?2 +2?¯?¯?¯) +?1(−2?¯?¯ ?−2?¯?¯?¯?¯ −4?¯?¯?¯?¯ −2?¯?¯?¯?¯?¯ −4?¯?¯?¯?¯ +12?¯?¯?¯?¯?¯ +2?¯?¯ ?¯?¯ +4?¯?¯?¯?¯ +6?¯?¯?¯ ?+10?¯?¯?¯?¯?¯ −2?¯ 2?¯?¯?¯ −4?¯?¯?¯?¯?¯ +4?¯?¯?¯?¯ ?¯ +6?¯?¯?¯?¯?¯ −6?¯?¯?¯?¯?¯ +14?¯?¯?¯?¯?¯ −2?¯ 2?¯?¯?¯ +2?¯?¯?¯?¯?¯ +6?¯?¯ ?¯?¯?¯ −8?¯?¯?¯?¯?¯ −8?¯?3 ) +?2(2?¯?¯?¯?¯¯+2?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ −2?¯?¯?¯?¯ ?¯ +2?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ −4?¯?2?¯?¯ −8?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯¯+2?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯?¯ −2?¯?¯?¯?¯?¯ + 2?¯?¯?¯?¯?¯ +2?¯?¯?¯?¯?¯ −4?¯?2?¯?¯ +8?¯?¯?¯?¯ +8?¯?¯?¯?¯?¯ +8?¯?¯?¯?¯?¯ −16?¯?3 +?3(−16?¯?3 −4?¯?¯?¯?2 −4?¯?¯?¯?2 −8?¯?¯?¯ ¯?¯)? where (???) and ?

3 **are the velocity components** and pressure **in the fixed frame (???) and the**

subscripts denote the partial derivatives If (?? ?) and ? indicate the

17 **velocity components and pressure in the wave frame** then **defining**

? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?) (10.5) and using ? = ? = ? = ? ? ? ? = ? ? = ? ? = ? ? ? ?2? ? ? ? ? = ?1 ? ? = ? ? = ?1 ? ? = ?? ? ? ?1 ?1 ?1 ???0 ?(?) µ ?0 ¶1?2 ?0?1? ?0 = ?0 ? ?? = ????01 ? ? ?0? ?0 ? ?1 ? (?) ? ?(?) = ? ? = ?? ? ? ? = ????? ? = −???????2 = ??02??221??3 = ??03?? 221? (10.6) we obtain

3 **under the long wavelength and low Reynolds number** approximation **the**

following expressions − ???? ?? ?(?)??2??2 +2Γ ??2??2¶3#−?2 ???? + ? " + 1 = 0? µ µ ¶ 0 = −?? ?? ? (10.7) (10.8) in which Γ(= ?2 + ?3) is the Deborah number, ?

1 **is the stream function,** ?0 **is the** constant **kinematic viscosity, ? is the Hartman**

**number, Re is the Reynolds number, ? is the**

124**fluid density, ? is the electrical conductivity,** ?(?) is **the** viscosity function, ? **is the**

wave number, ?0 is the constant viscosity and continuity equation is automatically satisfied. Further Eq. (10.8) indicates that ? =6 ?(?)? From Eqs. (10.7) and (10.8) we have ?2 ??? ??? 3 ??2 ?(?) ??2 + 2Γ ??2 Ã − ? 2 ??? = 0? µ ¶ ! ??2 The subjected boundary conditions in dimensionless form become ? = 0, ?2? ??2 = 0, ?? ? = 0, ? = ? , ?? ?? + ?1??? = −1, ?? ? = ?, ?(?)=1 + ? cos(2??), ? = 1 ?? Z0 ?? ??? (10.9) (10.10) (10.11) In above equations ?(?) = ?−?? or ?(?) = 1 − ?? for (? ¿ 1), ? is the viscosity parameter, ? 1 is the non-dimensional velocity slip parameter and ??2 ??2 3 ??? = ?(?) ??2 + 2Γ ??2 ? (10.12) µ ¶

3**Pressure rise per wavelength ∆?? and frictional force ?? (at the wall) are** defined **by**

the following expression 1 ?? ∆?? = ?? ??? (10.13) Z0 ?? = ? − ?? ??? 1 ?? (10.14) Z0 µ ¶ In the next section the problem consisting of Eqs. (10.9) and (10.10) will be solved for small Deborah number Γ and small viscosity parameter ?? 10.2 Series expressions Letting ? = ?0 +

68**Γ?1 + ???? ? = ?0 + Γ?1 + ???? ? = ?0 + Γ?1**

+ ???? and ?0 = ?00 + ??01 + ???? ?1 = ?10 + ??11 + ???? ?0 = ?00 + ??01 + ???? ?1 = ?10 + ??11 + ???? ?0 = ?00 + ??01 + ???? ?1 = ?10 + ??11 + ???? we arrive at the following systems: System for ?00 − ??00 ?? + ??3??300 − ?2(???00 + 1) = 0? (10.15) ??4?400 − ? 2 ??2??200 = 0? ?00 ?2?00 = 0 = ?? 2 ?? ? = 0? ?00 = ?00 ????00 + ?1 ??2??200 = −1 ?? ? = ?? System for ?01 − ??01 + ?3?01 −?2??01 = ? ?2?00 ?? ? ? (10.16) ??3 ?? ?? µ ??2 ¶ ?4?01 −?2?2?01 = ?2 ? ?2?00 ? ??4 ??2 ??2 µ ??2 ¶ ?01 = 0 = ??2??201 ? at ? = 0? ?00 = ?01? ????01 + ?1 ??2??201 = ?? ?2?00 at ? = ?? ??2 System for ?10 − ?? 10 ?? + ?3?10 − ?2 ??10 + 6 ?2?00 2 ?3?00 = 0? (10.17) ??3 ?? µ ??2 ¶ ??3 ?4?10 −?2?2?10 +6 ?2?00 ? ??4 ??2 ?? Ã µ ??2 ¶ 2 ?3?00 = 0? ??3 ! ?10 = 0 = ?2?10 ? at ? = 0? ??2 3 ?10 = ?10? ????10 + ?1 ??2??210 + 2?1 ??2??200 = 0 at ? = ?? µ ¶ The solutions of above systems finally give ? = ??? cosh[??]+? 1+??2?1 sinh[??]−(? +?)sinh[??] ??cosh[??¡]+ −1+?¢??21 ??1sinh[??] + ??1 + Γ?2? (10.18) ¡ ¢ ?= 1 32 ?? ????[?? ] + −1 + ?? ?21 ? ?1 sinh [?? ] 4 × (10.19) □ +6?1 − ?¡2 + ?3 + ?4 − ?5 +¡ ?1 ?6 ¢ ¢ □ ? 7¡2(? +?)3?6?Γc¡osh[??] ??¢cosh[??]+ −¢1+??2?1 sinh[??] □ □ ?? ?? = − ??(?co+sh?[)???3] (+cos −h1[?+??]?+??2 ??1s?in1hsi[n?h?[?])? ] + ??1 + Γ?2? (10.20) ¡ ¢

168**with ?1 ?2 ? 3 ? 4 ?5 ? 6 ? 7**

? 8 ?9 = (? + ?) (?3 + 2 sinh[?? ] (?4) + 2?? cosh[?? ] (?5)) 8? (?? cosh [?? ] + −1 + ?? ?21 ? ?1 sinh [?? ])2 ? (?−3???−3?¡? (? + ?)3 ?¢4)?9 = 32(?? cosh [?? ] + −1 + ?? ?21 ? ?1 sinh [?? ])4 ? = −4?? ? + ?1 + ??¡2?1 + 2??¢ 2?1 + ? −1 + 2?2?1 cosh[2??] +? −¡2?2 + ?2 + ? −4¢+ ? 2 (−¡4 + ?) ?¡− ?2 ? 1 sin¢h¢[? ?]? = 3? −¡1 + ?? 2?1 cos

142**¡h[? ?] + ?¡ (−2?2 + ?2** + ?¢(¢−4 +¢ ? 2((−4 + ?) ? −¡ ?2))?1)

sinh[?¢ ?]? = 3? cosh[? ?] + ? −?2 + ? (? + ?1) sinh[? ?]? = −3?? cosh[3?? ]¡+ 12? (? + ?? (¢? + 2?1 − ? ??1)) sinh[?? ] + 1 − 9?? 2?1 sinh[3?? ]? = 3?¡? cosh[3?? ]¢− 12? (−? + ?? (? + 2?1 + ? ??1)) sinh[?? ] + −1 + 9?? 2?1 sinh[3?? ]? = 12¡? ? (? + ?1) − ¢16? ??1 cosh[2?? ] + 4? ??1 cosh[4?? ] −8? sinh[2?? ] + ? sinh[4?? ] + ? cosh[?? ](12? (−? cosh[? ] +?? ?1 sinh[? ?] + sinh[3? ?])) + sinh[3? ?]? = 1 − ?? 2?1 sinh[?? ] + ?6?? (?6) + ?2?? (?7) + 2?3??? (?8) ? ¡ ¢ ?1 = ?7 + ?8? ?2 = 6(? + ?)3 6?Γ (?9) ? ?3 = 2(? + ?)3? 5(1 + 3? ?)Γ (?10) ? ?4 = 2(? + ?)3? 5Γ (?11) ????[? ?]? ?5 = 6(? + ?)3? 5Γ(????[3?? ] + 3?? (? 12))????[? ?]? ?6 = 4(? + ?)? ?? ????[?? ] + −1 + ?? 2? ????[?? ] 2 (?13 + 2????[?? ] (?14))? ?7 = (? + ?)3 ?¡5?Γ cosh [3? ?] (?¡?

165**cosh [?? ]¢+ −1 + ??¢ 2?1** sinh [?? ]) +32(?? **cosh [?? ] + −1 + ?? 2?** 1 **sinh**

[?? ])3((¡−(? + ?)? ?¢???[? ?] +????[?? ] + ? ? (?¡???[?? ] + ?¢1????[?? ]))? ?8 = 2(? + ?)3? 5Γ????[? ?](12?? −1 − 3? ? + ?? 2?1 ????[?? ] −3?? ????[3?? ]) + 12 (1 + ? (3? +¡?? (? + ?1 − 3? ??1))) sinh [?? ] + 1 − 9?? 2?1 sinh [3?? ] ? ?9¡= 12? (? + ?¢1) + 4? ?(−4????[2?? ] + ????[4?? ]) − 8????[2?? ] + ???? [4?? ]? ?10 = 12? (? + ?1) + 4? ?1(−4????[2?? ] + ????[4?? ]) − 8????[2?? ] +????[4?? ]? ?11 = ???? [3?? ] + ? (4?? (−? + 3?? ?1)????[?? ] − 3?????[3?? ] +4(? + ?? (3? + 6?1 − ? ??1))????[?? ] − 9?? ? 1????[3?? ])? ?12 = ((−????[3?? ] + ? (4?? ?1????[?? ] + 4(? + 2?1)????[?? ] − 3?1????[3?? ])))? ?13 = −4? ? + ?1 + ??1? 2 + 2? 2?1 + ? −1 + 2? 2?1 ????[2?? ] +3????[2?¡? ] − 4?? 2?1????¢¢[2?? ] +¡ 2?? ?? ¡??[?? ]( 3 + ¢?¢? 2 −?2 + ?(? + ?1) ????[??] + ??????[??])? ¡ ¡ ¢¢ ?14 = −3 + ?2 −2?2 + ?2 + ? −1 + ?2 (−4 + ?)? − ?2 ?1 ????[??] +?? ¡−1+??2¡?1 ????[??]? ¡ ¡ ¢¢ ?1 = 8(??cosh[??](+?(+−?1)+????32? 1)sinh[??])2? ¡ ? ¢ ?2 = 16(?? cosh[?? ]+(−1+?? ? )? ?1 sinh[?? ]) (?+?)3?7?24 4? ?3 = 4? ? + ?1 + ??2?1 + 2? ? − 2 1 + ??2 ?1 cosh[2??] + −3 + 4??2?1 sinh [2??¡] ? ¢¡¡ ¢¢¢ ?4 = 12? (? + ?1) + 4? ?1 (−4

cosh [2?? ] + cosh [4?? ]) − 8 sinh [2?? ] + sinh [4?? ] ? 10

> 163.**3 Results and discussion In this section the**

main objective is

> 24**to study the influence of** various **parameters of** interest **on the**

flow quantities. In particular the variations of Hartman number (?)? Deborah number (Γ)? amplitude ratio (?)? slip parameter (?1) and viscosity parameter (?)

> 21**on the** longitudinal **velocity, pressure gradient, pressure rise (∆??) and frictional**

force (??) per wavelength have been examined. Figs.10.1-10.3 show the effects of ?, ?1 and Γ on the longitudinal velocity ?? The plots show that velocity at the center of the channel and near the walls has opposite behavior. Further the velocity at the center of the channel decreases with an increase in ? and ?1 but it increases by increasing Γ?

> 10**The variation of pressure gradient for** certain **values of** ?, ?1 and **Γ**

are depicted in the Figs 10.4-10.6

> 2**It is** anticipated **that the absolute value of** pressure gradient **increases with** an increase in ? **and** ?1 but **it**

decreases when Γ is increased. The

> 6**pressure rise and frictional** force **per wavelength** have been **computed** numerically. **The**

pressure rise verses flow rate has been plotted in the Figs 10.7-10.11.

> 5**It is** noticed **from Fig.** 10.7 **that the** pressure rise **decreases with an increase in** ?1? **It is**

found that magnitude of pressure rise per wavelength increases by increasing ? (see Fig. 10.8). Fig. 10.9 depicts an important phenomenon that is the graph for pressure rise for Γ = 0 is linear whereas in the case of non zero Γ it is non linear. Fig 10.11 shows

> 10**that the pressure** rise **increases with an increase in** the amplitude **ratio**

?. The

> 3**free pumping flux increases by increasing**

? (see Fig?10?10)? Figs. 10.12-10.16 monitor the features of frictional force (??)?

> 3**It is clear that there exists a critical value**

of ? below which ?? resists the flow and above which it assists the flow. This critical value of ? decreases by increasing ?1 and ? (see Figs. 10.12 and 10.13). Contrary to this the critical value of ? increases with an increase in ? and ? (see Figs 10.14 and 10.15). It is revealed from Fig. 16 that the effect of Γ on frictional force is quiet opposite to that of pressure rise. Another important fact of interest here is a comparison between the viscous and non- Newtonian fluids (see Figs. 10.3, 10.6, 10.9 and 10.14). In all these Figs. the case of viscous fluid is given by putting Γ = 0? Fig. 10.3 shows that velocity at the centre of the channel in non-Newtonian fluid is larger when compared with a viscous fluid. Fig. 10.6 shows that the 132 non-Newtonian fluid exhibits larger pressure gradient than the viscous fluid. Figs. 10.17-10.20 examine the trapping phenomenon.

> 27**It is observed that the size of the trapped bolus decreases with** an **increase**

**in** ?1 and ?? **The size of trapped bolus** also **increases** by increasing **the** viscosity **parameter**

? (see Figs. 10.17 and 10.18). It is noted

167**that by increas- ing the** Deborah **number**Γ? the size **of the**

bolus decreases whereas it increases by increasing ?? 0.6 0.6

19**M = 0.5, 1.0, 1.5, 2.**

116**0 M = 0.5, 1.0, 1.5, 2.0**

0.4 0.4

33**U U 0.** 2 **0.** 2 **0.0 0.0** -1 **.0 -0.** 5 **0.0 0.** 5 1 **.0 -1.0 -0.** 5 0 **.0 0.** 5 **1.0**

y y Fig. 10.1. Effect of ? on velocity ? when ?1 = 0?2? Γ = 0?04? ? = 1? ? = 1

8**?3 and ? = 0?3?** Left panel **is for**

? = 0 and right panel is

8**for ? = 0?** 4? **0.** 8 **0.6 0.**

5 0.4 b1 = 0.0, 0.2, 0

130**.4, 0.6 U** b1 = **0.0, 0.** 2, **0.** 4, **0.**

6

61**U 0.** 2 **0.0 0.0 -0.** 2 **-0.** 5 **-0.** 4 -1 **.0 -0.** 5 **0.** 0 **0.** 5 1 **.0** y -1 **.0 -0.** 5 **0.0 0.** 5 1 **.0**

y Fig. 10.2. Effect of ?1 on velocity ? when ? = 1? Γ = 0?04? ? = 1? ? = 1?3 and ? = 0?3? Left panel is for ? = 0 and right panel is for ? = 0?4 0.6

134**G = 0.08, 0.06, 0.04, 0.** 0 **0.**

4

28**U 0.** 2 **0.0** -1 **.0 -0.** 5 **0.0** y **0.** 5 1 **.0**

Fig. 10.3. Effect of Γ on velocity ? when ? = 1? ?1 = 0?04? ? = 1? ? = 1?3 and ? = 0?3? 3 3 2 2 1 1 dp?dx

19**M = 2.5, 2.0, 1.5, 1.0 0**

dp?dx

91**M = 2.** 5, **2.** 0, **1.5, 1.0** 0 -1 **-1** -2 -2 **0.** 0 **0.2 0.** 4 **0.** 6 **0.**

39**8 1.0 x 0.0 0.2 0.4 0.6 0.8 1.0 x Fig.**

10.4. Effect of ? on pressure gradient when ?1 = 0?2? Γ = 0?001? ? = 0?1 and ? = 0?3? Left panel is for ? = 0 and right panel is for ? = 0?4 2.0 1.5 1.0 dp?dx 0.5 0.0 2.0 1.5

67**b1 = 0.0, 0.1, 0.2, 0.3**

dp?dx 1.0 0.5 0.0

8**b1 = 0.** 0, **0.** 1, **0.** 2, **0.** 3 **-0.** 5 **-0.5** -1.0 **0.** 0 **0.2 0.** 4 **0.** 6 **0.** 8 1 **.0 0.** 0 **0.2 0.** 4 **0.** 6 **0.**

8 1.0 x x Fig. 10.5. Effect of ?1 on pressure gradient when ? = 1? Γ = 0?001? ? = 0?1 and ? = 0?3? Left panel is for ? = 0 and right panel is for ? = 0?4? 1.5 1.5 1.0 1.0 dp?dx 0.5

132**G = 0.03, 0.02, 0.01, 0.0**

dp?dx 0.5

53**G = 0.** 03, **0.** 02, **0.** 01, **0.0 0.0 0.0 -0.** 5 **-0.** 5 **0.0 0.** 2 **0.** 4 **0.** 6 **0.** 8 1 **.0 0.0 0.** 2 **0.**

4 0.6 0.8 1.0 x x Fig. 10.6. Effect of Γ on pressure gradient when ? = 1? ?1 = 0?2? ? = 0?1 and ? = 0?3? Left panel is for ? = 0 and right panel is for ? = 0?4? 10 5 Dpl 0 -5

41**b1 = 0.** 3, **0.** 2, **0.1, 0.** 0 -10 **-1.0** -0.5 0 **.0**

0.5 1.0 1.5 h Fig. 10.7. Variation of ?1 on ∆?? when Γ = 0?01? ? = 1?5? ? = 0?3 and ? = 0?2? 10 5 Dpl 0 -5

89**M = 0.5, 1.0, 1.5, 2.0** -10 -15 -1 **.0** -0 **.5** 0 **.0** 0 **.5** 1 **.0** 1 **.5**

h 20 10 Dpl 0 -10

28**G = 0.00, 0.** 01, **0.** 02, **0.** 03 -20 -1 **.0 -0.** 5 **0.** 0 **0.**

5 1.0 1.5 h Fig. 10.9. Variation of Γ on ∆?? when ?1 = 0?02? ? = 1?5? ? = 0?3 and ? = 0?2? 6 4 2 Dpl -2 0 -4 -6

96**a = 0.0, 0.** 3, **0.** 6, **0.** 9 -1 **.0 -0.** 5 **0.0 0.** 5 **1.0**

1.5 h 5 Dpl 0 -5

54**a = 0.** 2, **0.** 3, **0.** 4, **0.** 5 **-1.0 -0.** 5 **0.0 0.** 5 **1.**

0 1.5 h Fig. 10.11. Variation of ? on ∆?? when ?1 = 0?02? ? = 1?5? ? = 0?2 and Γ = 0?01? 10 5 b1 = 0.3, 0.2, 0

78**.1, 0.0 Fl 0** -5 **-1.0 -0.**

5 0.0 0.5 1.0 1.5 h 10

19**M = 0.5, 1.0, 1.5, 2.0**

5 Fl 0 -5 -10 -1.0 -0.5 0.0 0.5 1.0 1.5 h Fig. 10.13Variation of ? on ?? when ? = 0?3? ?1 = 0?02 ? = 0?2 and Γ = 0?01? 20 15 10

154**G = 0.0, 0.** 1, 0 **.2, 0.** 3 **5 Fl 0**

-5 -10 -15 -1.0 -0.5 0.0 0.5 1.0 1.5 h 6

34**a = 0.0, 0.** 3, **0.** 6, **0.** 9 **4 2**

Fl 0 -2 -4 -1.0 -0.5 0.0 0.5 1.0 1.5 h Fig. 10.15. Variation of ? on ?? when ? = 0?3? ? = 1?5? Γ = 0?01 and ?1 = 0?02? 6 4

23**a = 0.** 5, **0.4, 0.3, 0.2** 2 Fl **0** -2 -4 **-1.0** -0.5 **0.0** 0.5 1 **.0**

1.5

58**h 1.** 2 **1.** 2 **1 1 0.** 8 **0.** 8 ? = **0?** 5 **0.** 6 **0.** 6 **0.** 4 0.4 **0.** 2 **0.** 2 **0 0** -**0.** 4 -**0.** 2 **0 0.** 2 0.4
**-0.** 4 -**0.**

2 0 0.2 0.4 1.2 1.2 1 1 0.8 0.8 ? = 1?5 0.6 0.6 0.4 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 0.4 1.2
1.2 1 1 0.8 0.8 ? = 2?5 0.6 0.6 0.4 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 0.4 Fig. 10.17. Effect
of ? on stream lines when Γ = 0?04? ? = 0?3? ? = 1?1? and ? = 0?2 Left 1.2 1.2 1 1 0.8 0.8 ?1 = 0?1 0.6
0.6 0.4 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 0.4 1.2 1.2 1 1 0.8 0.8 ?1 = 0?3 0.6 0.6 0.4 0.4
0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 0.4 1.2 1.2 1 1 0.8 0.8 ?1 = 0?5 0.6 0.6 0.4 0.4 0.2 0.2 0 0
-0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 0.4 Fig. 10.18. Effect of ?1 on stream lines when Γ = 0?04? ? = 0?3? ? =
1?1 and ? = 0?5? Left 1.2 1.2 1 1 0.8 0.8

23**Γ = 0?** 1 **0.** 6 **0.** 6 **0.4 0.** 4 **0.**

2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 1.2 1.2 1 1 0.8 0.8

23**Γ = 0?** 2 **0.** 6 **0.** 6 **0.4 0.** 4 **0.**

2 0 -0.4 -0.2 0 0.2 0.4 0.2 0 -0.4 -0.2 0 0.2 1.2 1.2 1 1 0.8 0.8

23**Γ = 0?** 3 **0.** 6 **0.** 6 **0.4 0.** 4 **0.**

2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 Fig. 10.19. Effect of Γ on stream lines when ?1 = 0?2? ? = 0?
3? ? = 1?1 and ? = 0?5? Left 1.2 1.2 1 1 0.8 0.8 ? = 0?2 0.6 0.6 0.4 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4
-0.2 0 0.2 1.2 1.2 1 1 0.8 0.8 ? = 0?4 0.6 0.6 0.4 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 1.2 1.2
1 1 0.8 0.8 ? = 0?6 0.6 0.4 0.6 0.4 0.2 0.2 0 0 -0.4 -0.2 0 0.2 0.4 -0.4 -0.2 0 0.2 Fig. 10.20. Effect of ? on
stream lines when ?1 = 0?2? ? = 1? Γ = 0?04? ? = 1?1 and ? = 0?5? Chapter 11

13**Heat transfer analysis for peristaltic mechanism in variable viscosity fluid**

This chapter extends the analysis of previous chapter in the presence of heat transfer. Thermal slip is also
considered. Viscous dissipation is present. Perturbation solution is discussed and a comparative study
between the cases of constant and variable viscosities is presented and analyzed. 11.1 Statement of the
problem Here we study

30**an incompressible magnetohydrodynamic (MHD) third order fluid** with
variable viscosity **in** a uniform **channel**

of width 2?1. The ? − ???? is selected along the centerline of the channel whereas ? is perpendicular to
the ? − ????. The

138**fluid is electrically conducting** under an applied **magnetic field** B0 **in the ? −
direction.**

The electric and induced magnetic field effects are neglected. Sinusoidal waves propagating on the
channel induce the flow. Such waves are described by the following equations ?(?? ?) = ?1 + ?1??? 2? μ ?
(? − ??) . (11.1) ¶ In above expression ? is the speed of propagation of the wave, ?1 the wave amplitude, ?
the wavelength and ? the time. Further the channel walls have a constant temperature ?0. The governing
equations are ?? ?? ?? ?? = 0? + ? ? +? +? ? ? ?? ?? ?? ¶ ? = −???? ?????? ?????? − ??02?? + + μ ? ?
+ ? + ? ? ? ? = − ?? ???? ??? ? ? + + μ ?? ?? ?? ¶ ?? ?? ?? ?? ??¯ + ?¯ ??¯ + ?¯ ??¯ ? = ?∇2? + Φ¯ ? ?
? ? μ ¶ (11.2) (11.3) (11.4) (11.5) where Φ¯ is the dimensional viscous dissipation term. In light of the
following dimensionless quantities ?= ? ? ? ? ? ? ? ? = ?1 ? ? = ? ? = ?? ? ? = ?1 ? ? = ? ? = ??11 ? ? =
???21??0 ? ? ? ? 1?2 ? = ?1 ?(?) ? ?0?1? ?0 = ?0 ? ?? = ??1 ?0??(?)? ?(?)= ?0 ? ? =μ?0¶ ? ? ?0 ?=
?? ? ? ? = ????? ? = −??????2 = ??02??221??3 = ??03??221?? = ? −?0?0? Pr= ?0??? ? = ?2 ? ? = ?0?
2 ? ? ???0 ?2 equations (11.2)-(11.5) give ???[( ???? ??? ???? ??? ???? ] = − ???? + ? ?????? +
?????? − ? 2( ???? − ) + 1)? −?3??[( ???? ??? ???? ??? ???? ] = − ???? + ?2 − ) ???? ???? + ? ? ?? ??
???[(???? ???? ???? ????)] = ?Φ + Pr(?2 ????2?2 ??2??2)? − 1 + (11.6) (11.7) (11.8) (11.9) where ?? is
the specific heat, ? the thermal conductivity, Γ(= ?2 + ?3) the Deborah number, ? the stream function, ?0
the constant

47**kinematic viscosity, ? the** Hartman **number,** Re **the** Reynolds **number, ? the**

fluid density, ? the electrical conductivity, ?(?) the viscosity function, ? the wave number, ?0 the constant

viscosity, ??(= Pr ?) the Brinkman number, Φ the di- mensionless viscous dissipation term, Pr the Prandtl number and ? the Eckert number. The governing problems subject to long wavelength and low Reynolds number are − ?? + ? ?? ?? ??? − ?2 ?? + 1 = 0? 0 = − ?? ? (11.10) μ ?? ¶ ?? ??2 = −???? ?2? (11.11) ? 2? ?2? 3 ??? = ?(?) ??2 + 2Γ μ ??2 ¶ ? ? = ?(?)(??2?Ψ2)2 + 2Γ(??2?Ψ2)4? (11.12) ? = 0, ?2? ?? ??2 = 0, ?? = 0 ?? ? = 0, ? = ? , ?? ?? ?? + ?1??? = −1, ?-?2 ?? =0 ?? ? = ?, (11.13) ?(?)=1 + ? cos(2??), ? = 1 ?? Z0 ?? ??? (11.14) where ?2 is the dimensionless thermal slip parameter, ?(?) = ?−?? or ?(?) = 1 − ?? for (? ¿ 1) ; ? the viscosity parameter, ?1 the dimensionless velocity slip parameter, ? the stream function, ? the temperature and ? the dimensionless flow rate in wave frame. Further Eq. (11.10) indicates that ? =6 ?(?) and finally we write ?2 ?2? ?2? 3 ??2 ?(?) ??2 + 2Γ ??2 Ã − ? 2 ?2? = 0? (11.15) μ ¶ ! ??2

1 ?? 1 ?? △?? = ?? ??? ?? = ? − ?? ??? (11.16) Z0 Z0 μ ¶ 11.2 Solution expressions The solutions for small Γ and ? are ?= ? ? ? cosh??[??co]s+h[???¡1]++? ?−12+?1?¢?sin?h21[????]1−si(n?h+[???)]sinh [??] + ??1 + Γ?2? (11.17) ¡ ¢ ?? = − ?? cosh [?? ] + −1 + ?? ?21 ? ?1 sinh [?? ] + ??1 + Γ?2? ?? (? + ?) ? 3 (cosh [?? ] + ? ?1 sinh [?? ]) (11.18) ¡ ¢ ? = ??(? + ?)2?2 −????[2??] + ????[2??] + 2? ? ?2 − ?2 − 2?? + ?2????[2??] ¡ 8 (?? ????[?? ] + (−1 + ??¡2?1)¡????[?? ])2 ¢ ¢¢ + ?? (? + ?)2 ? ? 192 (?? ????[?? ] + (−1 + ?? 2?1) ????[?? ])3 × (?1 + ? (?2 − ?3 − ?4 + ?5)) + ?? (? + ?)4 ? 6Γ 512 (?? ????[?? ] + (−1 + ?? 2?1) ????[?? ])5 × (?1 + 4 (?2 + ? (?3 + ?4 + ?5 + ?6))) (11.19) where the involved ?? (? = 1 − 8) ? ?? (? = 1? 2) ? ?? (? = 1 − 6) and ?? (? = 1 − 8) are given as follows: ?1 = (?8?+?(?)(??c3o+s2h[s?in?h[]?+?(−] (1?+4?)?+2???2?)?co?shsi[n?h?[?]?(?]5)2)) ? ?2 = ?−3???−3??(?+?)33?2(4?(?((1c−os?h?[?2??)]+si(n −h1[?+???]+??62?)??(??s6in)+h[??2??]?)(4?7)+2?3???(?8))) ? ?3 = −4? ? ? + ? + ?? 2? + 2? ? 2? + ? −1 + 2? 2? cosh[2?? ] +? 3 − 4??¡2? sinh[2?? ]? ¢?4 = 3? ¡−1 + ??¡ 2?

¡?) sinh[? ?]?¢ ?4 = 3?¡ −1 + ??¡2? cosh[? ?] + ?¢¢(−2?2 + ?2 +

+ ? (? + ?) sinh[? ?]? ?6 = −3?? cosh[3?? ]¡+ 12? (? + ?? ¢(? + 2? − ? ??)) sinh[?? ] + 1 − 9?? 2? sinh[3?? ]? ?7¡ = 3?? cosh¢[3?? ] − 12? (−? + ?? (? + 2? + ? ??)) sinh[?? ] + −1 + 9?? 2? sinh[3?? ]? ?8¡ = 12? ? (? +¢?) − 16? ?? cosh[2?? ] + 4? ?? cosh[4?? ] − 8? sinh[2?? ] +? sinh[4?? ] + ? cosh[?? ](12? (−? cosh[? ? ] + ?? ? sinh[? ?] + sinh[3? ?]) + sinh[3? ?])? ?1 = 8(?? cosh[?? ](+?(+−?1)+???3 2?) sinh[?? ])2 ? ?2 = (?+?)3?7?42 16(?? cosh[?? ]+(−1+?? ? )? ? sinh[?? ]) 4? ?3 = 4? ? + ? + ?? 2? + 2? ? − 2 1 + ?? 2 ? cosh [2?? ] + −3 + 4?? 2? sinh [2?? ] ? ?4 = 12?¡(? + ?) + 4? ?¢(−4 cos¡h [2??¡] + cosh [¢4??¢ ]) − 8 sinh [2?¡? ] + sinh [4??¢ ] ? ?1 = (3 + ?? 2(−3? + 4? 2(2?3 + ?(? −2? − 3? + 6? 2(? − ?)(? + ?)? − 3 −1 + 4?2? 2 ??))))????[?? ] − 3 1 + ?? 2 −? + 4¡? 1 + ?? 2? ????[3?? ] + 6?¢???[?¡? ]????[2? ?¢]? ?2 = 3? 2(? − ?)? +¡ ? 1 + 2?¡ 2 ?2 + ?¡2 ? ????¢[¢?¢ (? − 2?)] + 3? (2?(? + ?) + ? (?6) ?) ????[?(¡? + 2?)]? ¡ ¡ ¢¢ ¢ ?3 = 3?????[??] + 20?3?2????[??] + 24?5?4????[??] − 24?3?4?2????[??] −8?2?3????[??]? ?4 = 32?4? 4?????[??] + 24?2?4?2?????[??] + 8??4?3????[??] − 36?2?2????[??] −48?4?4?????[??] + 96?3? 4??????[??] + 9?????[3??] + 12?2?2????[3??] +3?????[?(? − 2?)] + 6?3?2????[?(? − 2?)] − 6????[? (? − 2?)]? ?5 = 6??2?2????[?(? − 2?)] − 6?2?2?????[?(? − 2?)] + 6??2??????[?(? − 2?)] +3 −2? + ? −1 + 2?2(? − ?)(? + ? − ?) ????[?(? + 2?)]? ?6 =¡ −1 + 2?¡2(? − ?)(? + ?)? ¢¢ ?1 = 16? −6? + ? 5 + 2? 2 (?7) ????[?? ] + 3? (36? + ? (?8)) ????[3?? ] + 16?????[?? ] −25????[3¡?? ] + ???¡?[5?? ] − 5??¢¢¢?[? (? − 4?)]? ?2 = 5????[? (? − 2?)] + ????[? (3? − 2?)] + 5????[? (? + 2?)] + ????[? (3? + 2?)]) −5????[? (? + 4?)]? ?3 = 7? − 4 3 + 13?? 2? ? ????[5?? ] + 5?????[? (? − 4?)] − 20?????[? (? − 2?)] −48??¡???[?¡(? − 2?)] + 48¢2¢? 2?????[? (? − 2?)]? ?4 = 48?? 2??????[? (? − 2?)] − 12?????[? (3? − 2?)] − 20?????[? (? + 2?)] +48?????[? (? + 2?)] + 48?2? 2?????[? (? + 2?)] − 48?? 2??????[? (? + 2?)] −12?????[? (3? + 2?)] + 5?????[? (? + 4?)] + 288?2? ????[?? ]? ?5 = 192?4? 3????[?? ] − 192? ?2????[?? ] − 192?2? 3? 2????[?? ] + 112?? ?????[?? ] +192?3? 3?????[?? ] − 192?? 3?2?????[?? ] − 544?? ?????[?? ] − 384? 3? 3?????[?? ] −192?2? 3?? Si ??[?? ] + 16?2? ? ????[3?? ] − 16? ?2????[3?? ] − 71?? ?????[3?? ] −144?3? 3?????[3?? ] + 144?? 3?2?????[3??] − 4?? ?????[3?? ]? ?6 = 288?2? 3??????[3?? ] + 31?? ?????[5?? ] − 4?? ?????[5?? ] + 5?? ? ????[? (? − 4?)] + 48?2? ????[? (? − 2?)] + 48?? ?????[? (? − 2?)] + 76? ? ?????[? (? − 2?)] − 36?? ?????[? (3? − 2?)] + 48?2? ????[? (? + 2?)] −48?? ?????[? (? + 2?)] + 76?? ?????[? (? + 2?)] − 36?? ?????[? (3? + 2?)] +5?? ?????[? (? + 4?)]? ?7 = 6?2 + 3? −2? + −1 + 2? 2(? − ?)(? + ?) ? − ? + 6? −1 + 2?? 2? ?? ?8 = −11 + 4?¡2 −4?2¡ + 4?2 + 8?? + 7?? ? ¢ ¢ ¡ ¡ ¢¢ The heat transf¡er coefficient is ¢ ? = ????? (11.20) 11.3 Graphical analysis This section provides a comparative study of temperature for the effects of various parameters in the constant (? = 0 left pannels) and variable (? = 0?2 right pannels) fluid viscosities in the presence of heat transfer. It is observed in Figs. 11.1 (a-d) that ? decreases when ? and ?1 are increased. Further temperature in variable viscosity fluid is less when compared with constant viscosity fluid. From Figs. 11.2 (a-d), an increase in ? is observed when Br increases whereas ? decreases when ?2 increases. Although the plots for heat transfer coefficient ? are not included to save space for variation of ? and Γ? It is also seen that impact of ? on ? is quite opposite to that of ?1? ? is an increasing function function of ?? Figs. 11.3 (a-d) show that ? decreases when ?1 increases whereas it increases by increasing ??? 11.4 Key findings The main points of

i) Temperature in constant viscosity fluid is greater than variable viscosity situation. ii) Temperature is decreasing function of thermal slip parameter ?2. iii) The qualitative effects of ?? and ?1 on the temperature are opposite. iv) Behavior of ?1

> 103**on the heat transfer coefficient is**

opposite to that of ??? 0.15 0.15 0.10 0.10 q 0.05 q

> 19**M = 0.5, 1.0, 1.5, 2.0**

0.05 0.00

> 106**M = 0.5, 1.0,** 1 **.5,** 2.0 **-0.** 05 **0.** 00 **0.** 0 **0.** 2 **0.** 4 0.6 **0.**

8 1.0 1.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2 y y

> 60**(a) ? = 0?2** (b) ? = **0?2 0.** 35 **0.** 4 **0.** 30 **0.** 3 **0.** 25 **0.**

20 q 0.2 q 0.15 0.1

> 67**b1 = 0.0, 0.1, 0.2, 0.3 0.**

10

> 8**b1 = 0.** 0, 0. 1, **0.** 2, **0.** 3 **0.** 05 **0.** 0 **0.** 00 **0.** 0 **0.2 0.** 4 0.6 **0.** 8 1 **.0** 1.2 **0.** 0 **0.** 2 **0.4 0.** 6 **0.**

8 1.0 1.2 y y (c) ? = 0?2 (d) ? = 0?2 Fig. 11.1 (a-d): Variation of ? for Γ = 0?04? ? = 0?3? ? = 1?1? ? = 0?1 and Br= 0?5. 0.25 0.25 0.20 0.20 0.15 q 0.10 0.15 q 0

> 158**.10 0.** 05 **0.** 05 **Br = 0.** 8, **0.** 6, **0.** 4, **0.**

2

> 22**Br = 0.** 8, **0.** 6, **0.** 4, **0.** 2 **0.** 00 **0.** 00 **0.0 0.** 2 **0.** 4 0.6 **0.**

> 73**8 1.0** 1.2 **y 0.0 0.2 0.4 0.6 0.8 1.0** 1.2 **y** (a) **?2**

= 0?2

> 107**(b) ?2 = 0?** 2 **0.** 2 **0.** 2 **0.1 0.** 1 **0.0 0.0**

q q -0.1

> 77**b2 = 0.1, 0.2, 0.3, 0.**

> 41**4 b2 = 0.** 1, **0.2, 0.3, 0.4 -0.** 1 **-0.** 2 **-0.** 2 **-0.** 3 **-0.3 0.** 0 **0.** 2 **0.** 4 **0.** 6 **0.**

8 1.0 1.2 1.4 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 y y (c) ?? = 0?5 (d) ?? = 0?5 Fig. 11.2 (a-d): Variation of ? for ? = 1? Γ = 0?04? ? = 0?3? ? = 1?1? and ? = 0?2?? 150 150 100 100 50 50

> 126**Z 0 Z 0** -50 b1 = **0.0, 0.** 1, **0.** 15, **0.**

2 -50

> 8**b1 = 0.0, 0.** 1, **0.** 15, **0.**

2 -100 -100 -150 0.0 0.2 0.4 0.6 0

> 72.**8 1.0** -150 **x 0.0 0.2 0.4 0.6 0.8 1.0** (a) ?? = **0?** 5 (b) ?? = **0?**

5 x 40 40 20 20 Z 0 Z 0 Br = 2.0, 1.5, 1.0, 0.5 Br = 2.0, 1.5, 1.0, 0.5 -20 -20 -40 -40 0.0 0.2 0.4 0.6 0.8 1.0 0.0 0.2 0.4 0.6 0.8 1.0 (c) ?1 = 0?2 x (d) ?1 = 0?2 x Fig. 11.3 (a-d): Variation of Z when Γ = 0?04? ? = 0? 3? ? = 1?1? and ? = 1? Chapter 12 Soret and Dufour

> 4**effects on peristaltic** transport **of a third order fluid**

Soret and Dufour effects

> 44**on peristaltic transport of third order fluid in** a symmetric **channel** have been reported **in**

this chapter. Joule heating effect is also taken in to account. The

> 37**governing nonlinear** problem **is solved using perturbation**

approach. Graphical

> 177**results are reported** and discussed **for various** parameters **of**

interest entering into the problem 12.1

> 6**Mathematical formulation We consider the** magnetohydrodynamic (MHD) flow **of third order fluid in** a **channel of width**

2?1? The ?− axis is chosen along the walls of channel and ? −

> 20**axis is taken normal to the** ?− axis. **A** constant **magnetic field of** strength B0 **is applied in the**

? − direction. Induced magnetic field is not accounted. Sinusoidal wave propagating on channel walls with constant wave speed ? is represented by 2? ?(?? ?) = ?1 + ?1??? ? (? − ??) , (12.1) µ ¶ where ?1

> 44**is the wave amplitude, ? is the wavelength** and ? **is the**

time . The continuity, momentum, energy and concentration equations of the problem may be written as follows: ?? ?? ?? ?? = 0? + ? ? +? +? ? ? + + µ ?? ?? ?? ¶ ? = −??? ?????? ?????? − ??02?? ? ? +? +? ? ? ? =− ?? ???? ??? ? ? + + µ ?? ?? ?? ¶ ?? ?? ?? ?? ??? + ? ??? + ? ??? ? = ?? ???2?2 + ????2?2 (12.2) (12.3) (12.4) ³ +?? + ???20?2 + ??´???? ???2

> 145**?2³+ ????2? ? ´ 2** (12.5) ³ ´ ? + ? ? ? + ? ?? ? = ? **µ??2** ?? 2¶ **?2? ?2? ??? ?2?**

?2? + + + ? (12.6) µ ?? ?? ¶ ? ? ? µ??2 ?? 2¶ In the above equations, ? and ? are the velocity components along ? and ? directions re- spectively

> 5**in fixed** (laboratory **frame), ? is** the **density, ? is the pressure, ? is the kinematic viscosity, ? is the** electrical **conductivity, ? is the**

thermal conductivity, ?0 is the applied mag- netic field, ? is the temperature field, ??

> 7**is the specific heat,** ?? is **the** viscous **dissipation,**

? is mass diffusivity, ??

> 11**is the thermal diffusion ratio, ?? is the concentration susceptibility, ? is the**

concentration field and ?? is the mean temperature.

> 147**Extra stress tensor S** in **third order fluid is**

S = ? + ?3??A21 A1 + ?1A2 + ?2A21 + ?1A3 + ?2 A1A2 + A2A1 ? ³ ´ ¡ ¢ in which ?? ?? (? = 1? 2) and ?? (? = 1? 2?3)

> 7**are the material constants. The Rivlin-Ericksen tensors can be** represented **as** follows A1 = **L+L?? A?+**

1 = ?A??? +

> 179**A?L + L?** A? ? = **1? 2?** where **L**

= grad V and ? indicates the matrix transpose. If (???) and (???)

> 31**are the velocity components in the** laboratory (???) **and wave (???) frames respectively**

then defining ? = ? − ??? ? = ? ? ? = ? − ?? ? = ? ? ?(?? ?) = ? (? ? ? ? ?)? (12.7) and introducing the following dimensionless variables ?= ? ? ? ?2? ? ? ? = ? ? = ? ? = ? ? ? ? = ?1 ? ? = ? ? = ?1 ? ? = ? ??? ? ? 1 ? ?? ? ?1 ?1 ?= ? − ?0 ? ? = ? − ?0 ? ? = ? 1?2 ?0?1? υ = ? ?? = ??1 ? ? ?0 ?0 µ ¶ ? ? ? ?= ?? ? ?? = ?????? = −????? ?? = Pr?? ?? = ?????0?????0, ?? = ?????????00? ??= ?? ? ? = ???2?0 ? ? ? = ???? ? ? = ??1? ? (?) ? ?2 = ??2??22 ? ?3 = ??3??22 ? (12.8) we have − ?? ?? ?? + ? ???−?2 ?? +1 =0? 0=− ?? ? (12.9) µ ?? ¶ ?? 0 = ???2?2 + ??? + ???2 ?? 2 ?2? ? (12.10) µ ?? +1 +Pr????2 ¶ 1 ?2? ?2? 0 = ?? ??2 + ?? ??2 ? (12.11) ??? = ??2??2 + 2Γ ??2??2 ? ? = ( ??2??2 )2 + 2Γ( ??2??2 )4? 3 (12.12) µ ¶ where ? is the pressure, ? the stream function, Γ(= ?2 + ?3) the Deborah number, ?

> 1**the Hartman number, Re the Reynolds number,** ?? the Brinkman number, ?? **the**

Dufour para- meter, ?? the Soret parameter, ?? the Schmidt number, ? the Eckret number, ? ? the Prandtl number, ? the wave number, ?0 temperature at the wall, ?0 concentration at the wall, ? the dimensionless temperature and ? the concentration. The boundary conditions are ?=0, ?=?, ?2? ?? ?? ??2 = 0, ?? = 0? ?? = 0? ?? ? = 0, ?? ?? = −1, ? = 0, ? = 0, ?? ? = ?, (12.13) ?(?)=1 + ? cos(2??), ? = ? ?? Z0 ?? ??? (12.14) where ? is the dimensionless volume flow rate and ? is the dimensionless wave shape. Pressure rise per wavelength Δ?? is 1 Δ?? = ?? ??? ?? (12.15) Z0 We look for solutions in the series form and thus

> 2**we expand the quantities in** terms **of small** Deborah **number as**

follows: ? = ?0 +

> 68Γ**?1 + ???? ? = ?0 + Γ?1 + ???? ? = ?0 + Γ?1 + ???? ? = ?0** + Γ **?1** + ???? ? = ?0 + Γ **?1**

+ ???? The resulting zeroth order system is given by ??4??40 − ? 2 ????220 = 0? (12.16) 0 = ??2??20 + ?? ??2??20 + ???2 ????0 + 1 2 2 + Pr ?? ?2?0 ? µ ¶ µ ¶ ??2 (12.17) 0 = ?1? ??2??20 + ?? ??2??20 ? (12.18) ?0 = 0, ?2?0 = 0, ??0 = 0? ??0 = 0? ?? ? = 0, ??2 ?? ?? ?0 = ?0, ????0 = −1, ?0 = 0, ?0 = 0, ?? ? = ?, (12.19) and first order system becomes ??4??41 + 2 ???22 ????220 − ? 2 ????221 = 0? 3 (12.20) µ ¶ 0 = ??2??21 + ?? 2??2??20 ??2??21 + ??2??20 + ???2 ????1 ????0 + 1 4 2 Ã ! 2 + Pr ?? ?2?1 ? µ ¶ µ µ ??2 ¶¶ (12.21) 0 = ?1? ??2??21 + ?? ??2??21 ? (12.22) ?1 = 0, ??2??21 = 0, ??1 = 0? ??1 = 0? ?? ? = 0, ?? ?? ?1 = ?1, ????1 = 0, ?1 = 0, ?1 = 0, ?? ? = ?, (12.23) The

> 103**heat transfer coefficient at the** wall can **be** computed **by**

? = ????? 12.2 Solution expressions

> 12**Solving the resulting systems at zeroth and first orders**

we can write ? = − (−?? cosh[?? ] + sinh[?? ])4 ? 1 + ?−3?? (? + ?)3 ? 4Γ (?2 − ?3 + ?4) 32 (−?? cosh[?? ] + sinh[?? ])4 ? = ??? 2 (?1 + 4? ?2 − ?3 cosh[2?? ] + ?4 − ?5) 8 (−1 + ?) (−?? cosh[?? ] + sinh[?? ])2 ³ ´ + −?? (? + ?)3 ? 6Γ 512 (−1 + ?) (?? cosh[?? ] − sinh[?? ])5 × ³(?6 − 16?? ?7 cosh[?? ] + ?8 + ?9 − ?10 ´+ ? 11 − ?12) ? ?= ???2???? (?1 + ?2 cosh[2??] − ?3) 8(−1 + ?)(−?? cosh[??] + sinh[??])2 + ³ ?? (? + ?)3 ? 6???Γ ´ 512 (−1 + ?)(?? cosh[??] − sinh[??])5 × ³(−?4 − 16?? (?5) cosh[?? ] + ?6 − ?7 + ?´8 − ?9 − ?10)

? with ???, ??? and ??? agiven as follows: ?1 = (?? cosh[?? ] − sinh[?? ])3 (−? (? ? cosh[?? ] + sinh[?? ]) + (? + ?) sinh[? ?])? ?2 ? 3 = 24?3? ? ?? 2 ? − ?? 1 − ?6? ? + 12?2? ? ? ? + 12?4? ? ? ? cosh[?? ]? = 3?2? ? −1 + ?2? ? ?¡? cosh[3?? ] + sinh[?? ] − ?6? ? sinh¢[?? ] −12?2?¡ ? ?2 ? 2 sinh¢[?? ]? ?4 = 12?4? ? ?2 ? 2 sinh[?? ] + 12?2? ? ? ? sinh[?? ] + 12?4? ? ? ? sinh[?? ] −16?3? ? ? ? sinh[2?? ] − ?2? ? sinh[3?? ] + ? 4? ? sinh[3?? ] + 2?3? ? ? ? sinh[4?? ]? ? = Pr ?? ∗ ????? ?1 = 2? 4+?+?3?4 −2 1+?2?4 ?2+2?2?2 1+?2 (?−?)(?+?)? ? 2 = 2+¡??2 1+2?¢(?−¡?)(?+?)¢? ¡ ¢ 3 = ?2 1+?¡2 +2?¢ 4+?+??2 2?2+? 8+? 3+?2 1+2?2 2?2 − − ? ? 4 = (?¡+?)2 1¢+?2 ¡cosh[2??]+4¢?? 2?+¡?(2+?¡)−?2 s¡inh[2??]? ¢¢¢ ? 5 = 16?(?+¡?)?cos ¢h[??](?(?−?)+¡sinh[??])+16(?+¢?)sinh[??](?(?−?) + sinh[? ?])? ?6 = −288??−96?3?3+288?+96?2?3?? ?7 = −16+48?2?2+12?3 ?2+2?4 +?(−5+3?2 −3+4?2−4?2 +24?4 ?2−?2 )−? 5+24?¡4?2+3?2¢3+4?2 ? ¡ ¢ ? 8 = ¡256? 1¢+3?2¡?2 (? ?)cosh[2??¡] 288¢?¢?cosh[3??] 33???cosh[3??] − − − −33?2¡? cosh[3??¢] + 27? ?? 3 cosh[3?? ]? ?9 = 27?2? 3 cosh[3?? ] − 48? ?3? 3 cosh[3?? ] − 48?4? 3 cosh[3?? ] − 48? ?3? 5 cosh[3?? ]? ?10 = 48?4? 5 cosh[3?? ] + 48? ?? 3?2 cosh[3?? ] + 48?2? 3?2 cosh[3?? ] + 48? ?? 5?2 cosh[3?? ] + 48?2? 5?2 cosh[3?? ] + 32?? cosh[4?? ] + 96?3? 3 cosh[4?? ] −32? ? cosh[4?? ] − 96?2? 3? cosh[4?? ] + 32? ? cosh[5?? ] + 7? ?? cosh[5?? ]? ?11 = 7?2? cosh[5?? ] + 3? ?? 3 cosh[5?? ] + 3?2? 3 cosh[5?? ] + 5? ?? cosh[? (? − 4?)] + 5?2? cosh[? (? − 4?)] + 9? ?? 3 cosh[? (? − 4?)] + 9?2? 3 cosh[? (? − 4?)] − 32?? cosh[? (2? − 3?)] − 20? ?? cosh[? (? − 2?)] − 20?2? cosh[? (? − 2?)]? ?12 = 84? ?? 3 cosh[? (? − 2?)] − 84?2? 3 cosh[? (? − 2?)] − 48? ? cosh[? (? − 2?)] (? + ?) − 48? 3? cosh[? (? − 2?)] (? + ?) − 12?? cosh[? (3? − 2?)] ? + ? + ? 2 −32?? cosh[? (2? − ?)] + 192? cosh[? (2? − ?)] ?3? 2 + ? + ?2? 2¡? ? ¢ ¡ ¢ ?1 = −8? − 2? (4 + ?) − 2? ?2 (? + 2?) ? 2 − 2?2 (? + ?)2 ? 4 + 2?11?2? ?2 = ? 2 1 + ? 2 + 2? 4 + ? + ?? 2 − 2?2 + ? (8 + ??12) ? ? 3 = ? ¡+ ?2 1¢+ ? 2 ¡cosh[2? ?] − 4¢?? 2? + ? (2 + ?) − ?2 sinh[2?? ] + 1¡6? (? +¢ ¡?) ? cos¢h[?? ] (? (? ?) +¡sinh[? ?]) 16 (? +¢?) sinh [?? ] − − (? (? − ?) + sinh[? ?]) ? ?4 = 288?? − 96?3? 3 + 288? ? + 96?2? 3?? ?5 = −16 + 48?2? 2 + 12?3 ? 2 + 2? 4 + ? ?13 − ? 5 + 24? 4?2 + 3? 2 3 + 4?2 ? ? 6 = 256? 1 + 3?2? 2 (? −¡ ?) cosh[2??¢ ] − 288?? c¡osh[3?? ] − 33?? co¡sh[3??¢]¢ (? + ? ¡) + 27?? co¢sh[3?? ] ? ? 2 + ?? 2 ? ? ? 7 = 48?? cosh[3?? ]?14 + 32?¡cosh[4?? ] (?¢− ?) + 96?2? 2 cosh[4?? ] (?? − ? ?) ? ?8 = ?? cosh[5?? ] 32 + 7? + 7? + 3? ? 2 + 3?? 2 + ?? cosh[? (? − 4?)] 5? + 5? + 9? ?¡ 2 + 9?? 2 − 32?? cosh[? (2¢? − 3?)]? ? 9 = ¡?? cosh[? (? − 2?)]?15? ¢ ?10 = ?? cosh[? (3? − 2?)] 12? + 12? + 12? ? 2 + 12?? 2 − ?? cosh[? (2? − ?)] 32−192?2?2 192(¡???) 192??2? ? − − ¢ ?11 = ¡1+?(? +2?)?2+(?+?)?2?4? ?12¢=3+?2 1+2 ?2−?2 ? ?13 = −5+3?2 −3+4?2−4?2 +24?2 ?2−?2 ? ¡ ¡ ¢¢ ?14 = ??2? 2+?¡3?2+???2?4¢+?3?4−¡??2?2¢−??2?2−??4?2 ??4?2? − ?15 = 20? + 20? + 84? ? 2 + 84?? 2 + 48? ? + 48?? + 48? ? 2? + 48?? 2??? 12.3

> 1**Discussion The** objective of this section is to analyze the plots for **temperature,** concentration and **heat transfer coefficient.**

Hence Figs. 12.1-12.3 are given in order to achieve such objective. Here Figs. 12.1(a-g) are drawn for temperature ?? Figs. 12.2(a-g) for concentration profile and Figs. 12.3(a-f) for the heat transfer coefficient Z. Explicitly 3-D graphs are plotted to analyze the behavior fully in the domain of interest. Figs. 12.1 demonstrate

> 25**that the** dimensionless **temperature increases with an increase in** M, **Br,**

Du, Sr, Sc,

> 2**Pr and**Γ? **It is also noted** from **Fig.** 12.1a **that**

increase in temperature with respect to M is not much for small M. However the temperature increases abruptly when M increases. Also with the increase in magnitude of applied magnetic field, the temperature increases more due to Joule heating. In Fig. 12.1b it can be seen that

> 6**there is a linear relationship between** Br **and** ?? Similar observation holds **for** Γ

(see Fig. 12.1g). This depicts that rise in temperature is lesser for Newtonian fluid when compared with non-Newtonian fluid. In Figs. 12.2 (a-g) it is observed that the dimensionless concentration decreases by decreas- ing M, Br, Du, Sr, Sc, Pr and Γ? It is found that decrease in temperature is not linear for M, linear and large for Br and Γ and smaller bit small for Du, Sr, Sc, Pr. Figs. 12.3(a-f) explicitly see the behavior of heat transfer coefficient Z when values of different embedded parameters are altered. It is seen in all these Figs. that Z has an oscillatory behavior. It is physically acceptable in view of peristalsis. The magnitude of oscillations in Z increase most rapidly in case of Γ when compared with the other cases. Fig. 12.1 a Fig. 12.1 b Fig. 12.1 c Fig. 12.1 d Fig. 12.1 e Fig. 12.1 f Fig. 12.1 g Figs.12.1(a-g). Behavior of the temperature with change in values of different parameters

> 94**Fig. 12.2** a **Fig. 12.** 2 b **Fig. 12.**

2 c

> 94**Fig. 12.2** e **Fig. 12.** 2 d **Fig. 12.**

2 f Fig. 12.2 g Figs. 12.2(a-g). Development of concentration field with increase in values of parameters. Fig. 12.3 a Fig. 12.3 b Fig. 12.3 c Fig. 12.3 d Fig. 12.3 e Fig. 12.3 f Figs. 12.3(a-f). Behavior of heat transfer coefficient Z.

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Hayat, M. U. Qureshi and N. Ali, The influence of slip on the peristaltic motion of a third order fluid in an asymmetric channel, Phys. Lett. A 372 (2008) 2653. [69] S. Nadeem, N. S. Akbar, N. Bibi and S. Ashiq, Influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube, Comm. Nonlinear Sci. Numer. Simulat. 15 (2010) 2916. [70] N. S. Akbar, T. Hayat, S. Nadeem and A. A. Hendi, Effects of slip and heat transfer on the peristaltic flow of a third order fluid in an inclined asymmetric channel, Int. J. Heat and Mass Transfer 54 (2011) 1654. 174 Fig. 9.4 Effect of ?? on the temperature profile. Fig. 9.8 Effect of ?? on the velocity profile. Fig. 10.8. Variation of ? on Δ?? when Γ = 0?01? ?1 = 0?02? ? = 0?3 and ? = 0?2?? Fig. 10.10. Variation of ? on Δ?? when ?1 = 0?02? ? = 1?5? ? = 0?3 and Γ = 0?01? Fig. 10.12. Variation of ?1 on ?? when ? = 0?3? ? = 1?5? ? = 0?2 and Γ = 0?01? Fig. 10.14. Variation of Γ on ?? when ? = 0?3? ? = 1?5? ? = 0?2 and ?1 = 0?02? Fig. 10.16. Variation of ? on ?? when ? = 0?2? ? = 1?5? Γ = 0?01 and ?1 = 0?02? panels are for ? = 0? and right panels are for ? = 0?4?() panels are for ? = 0 and right panels are for ? = 0?4? panels are for ? = 0 and right panels are for ? = 0?4? Left panels are for ? = 0 and right panels are for ? = 0?4? 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 1.5 1.5 1.5 1. 1.5 1.5 1.5 1.5 0.4 0.4 0.4 0.4 0.4 0.4