A Comparison of Exponentially Weighted Moving Average Charts for Time and Magnitude Monitoring

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A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY IN STATISTICS

Supervised By

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CERTIFICATE

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STATISTICS

We accept this thesis as conforming to the required standards

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ISLAMABAD, PAKISTAN
2023

Declaration

I "Saman Riaz" hereby solemnly declare that this thesis entitled "A Comparison of Exponentially Weighted Moving Average Charts for Time and Magnitude Monitoring", submitted by me for the partial fulfillment of Master of Philosophy in Statistics, is the original work and has not been submitted concomitantly or latterly to this or any other university for any other Degree.

Dated: 11th July, 2023 Signature: Saman

Dedication

I am feeling great honour and pleasure to dedicate this research work to

My Beloved Parents

Whose endless affection, prayers and wishes have been a great source of comfort during my whole education period
and to

My Supervisor

who inspired hope, ignited the imagination, and instilled a love of learning.

Acknowledgment

First and foremost I praise and acknowledge Allah Almighty, the Lord and Creator of the Universe. All respect and gratitude goes to the Holy Prophet Hazrat Muhammad (Peace Be Upon Him) who enlightens our hearts with the light of Islam and whose way of life has been a continuous guidance for us.

My sincerest thanks go out to my prestigious supervisor, Dr. Sajid Ali, for his valuable guidance, inspiration, and encouragement during the whole study process. I consider myself really fortunate and blessed for his advice, insightful feedback, and compassion throughout my research work.

I extend my sincere gratitude and respect to the Chairman, Prof. Dr. Ijaz Hussain, and all the esteemed Statistics Department faculty members who instructed and mentored me during my academic career.

I am grateful to my parents for their unending love and support throughout my life. I am also appreciative of siblings and entire family for their amazing love and support. Without their encouragement and support, I never could have finished this project. I want to thank all of my friends, classmates and seniors for their support and collaboration, especially Ayesha Talib, Nadeela Bibi, Maria Asghar, Marium khalil and Nabeel Ahmed.

Saman Riaz

Abstract

Science and technology advancements have transformed a wide range of working contexts, including manufacturing and healthcare. For processes to be accurate in relation to the specifications, an assignable cause must be quickly and precisely found. The aim of the study is to compare the EWMA control charts. Traditionally, EWMA charting methodology has been used to track processes with moderate to large shifts. We have modified the EWMA methodology to develop the memory-type rate based EWMA control chart for simultaneous monitoring of time and magnitude of an event. Specifically, the exponential distribution is considered combining with the gamma distribution to develop the suggested chart. In this comparison study, we have used existing Max-EWMA control chart with the suggested chart to claim best performer in the charting scenario. Numerous Monte Carlo simulations are used to access the performance of these charts utilising the ARL criterion. In addition, the charting methodologies are scrutinised on real data sets. The results show that the Max-EWMA control charts is more efficient in detecting simultaneous small to medium-sized shifts with smaller smoothing parameter than the EWMA-Rate control chart. However, the EWMA-Rate chart can detect larger shifts with larger smoothing parameter more quickly. The real data sets have shown the fact precisely.

List of Abbreviations

SQC	Statistical Quality Control
SPC	Statistical Process Control
EWMA	Exponentially Weighted Moving Average
ARL	Average Run Length
TBE	Time-Between-Event
CUSUM	Cumulative Sum
ARL_0	In-Control Average Run Length
ARL_1	Out-of-Control Average Run Length
SDRL	Standard Deviation of Run Length
ATS	Average Time to Signal
IC	In-Control
OOC	Out-of-Control
PDF	Probability Density Function
UCL	Upper Control Limit
UW	Upward
DW	Downward

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Chapter 1

Introduction

The degree to which a set of characteristics satisfies standards is a way to define quality. The nine fundamental quality attributes of statistical output that the European statistical systems utilizes are: rationality and comparability, receptiveness and clarity, punctuality and timeliness, relevance, correctness and reliability.

Statistical quality control (SQC) is a method for evaluating and maintaing the quality of the goods in a wide range of manufacturing sectors, including food and pharmaceutical industries. Generally, there are three foundations for SQC: descriptive statistics, acceptance sampling, and the statistical process control (SPC). The SPC can be used in almost any process, despite the fact that it is typically associated with industrial applications. Any action taken at work is a part of a process, that needs to be scrutinised. Instead correcting issues after they have already happened places an emphasis on problem prevention and early detection. Walter A. Shewhart is the father of statistical process control. Shewhart reached to the conclusion that while all processes exhibit some degree of variation, some processes exhibit variations that are common to all processes known as "common" variability and referred these processes as under control. Other processes show variations that Shewhart referred to as not in-control and that may not present in the system known as the "special" variability.

Three basic stages of activity are involved in the application of SPC: being aware of the specification constraints and the process, removing certain (assignable) sources of variance to ensure process stability. Control charts are used to monitor the ongoing manufacturing process and help finding significant means as well as variation changes. There are several quality control tools to be implemented and one of them is control charts, that are used to track the information from measurements of variations at specific locations on the process modeling.

The need to build a charting technique accounting for rates or proportional data derives from the fact that many real-life and economics contexts involve such data. This thesis intends to offer a method for monitoring time and magnitude. To achieve this, the proposed charts are created using a modified version of the exponentially weighted moving average (EWMA) methodology. The speed at which the out-of-control signal is measured, and different shifts are introduced. This speed is known as the average run length (ARL), which is most popular and commonly used metric for assessing the effectiveness of a control chart.

1.1 Background

Traditional attribute control charts cannot be used in processes where the defect rate is exceptionally low. The term "high quality process" refers to a procedure with an exceedingly low defect rate, such as when hazardous chemicals in a pharmaceutical that can focus on saving your life are being closely examined. A time-between-events (TBE) control chart is typically created to monitor the time interval in order to detect the presence of an assignable reason in high quality processes. The frequency of events is crucial when working with a production process. The magnitude of those occurrences also significantly contributes to sustaining a stability of the process. Magnitude and frequency work together to describe the quality of an event in a comprehensive way. There are numerous control charts, such as cumulative sums (CUSUM), EWMA, or Shewhart charts (such as X, T, R, X&T) which can be used to track the amplitude of an event individually. A memory-type control that considers the process history and is used to identify tiny shifts is the EWMA control chart. The Max-EWMA control chart is modification of the EWMA chart that are described in the literature. This chart is especially helpful for keeping track of two different statistical information concurrently.

Wu et al. (2009b) demonstrated the rate chart which is the ratio of X and T, where X (magnitude) follows gamma distribution having limits 0 to ∞ and T (time) follows exponential distribution, having rate pararmeter $\lambda > 0$. They used average time to signal (ATS) to analyse the quality of a control chart which is equals to ARL_0/λ_0. Thus, it gives quick alert when we have out-of-control process with large shifts.

1.2 Objectives of the Study

The major goal of the thesis is to provide a comprehensive study of monitoring the processes. Specifically,

- To propose memory-type rate based (EWMA) control chart using gamma distribution and exponential distributions.
- To compare rate based EWMA and Max-EWMA control chart.
- To access the performance of control charts by using run length profiles including average run length (ARL), standard deviation run length (SDRL) and quantiles (Q_1, Q_2, Q_3).

The remaining thesis is arranged in the following manner: Literature review is discussed in Chapter 2. The EWMA chart for time and magnitude using exponential and gamma distributions is disussed in Chapter 3. Chapter 4 contains conclusion and future recommendations.

Chapter 2

Literature Review

SQC which uses statistical techniques to monitor a product's or service's quality. To decide whether to accept or reject a set of parts or products depending on the quality of a sample, one technique, called the acceptance sampling is utilised. To decide whether a process should be continued or changed in order to attain the target quality, a second approach known as the statistical process control (SPC) makes use of visual displays called control charts.

The term SPC refers to the application of statistical methods to regulate a process or manufacturing method. You may praeposter the process behavior, identify the problems with internal structures, and resolve the problems associated to productions with the use of SPC tools and methods. The control charts, established by Shewhart (1931), are widely used in industries. A control chart makes it easier to keep track of data and identifies when an unusual occurrence, such as a very high or low observation compared to standard process performance, has occurred.

Control charts make an effort to discern between two categories of process variation. A common cause variation is a part of any process and will always exist. The process is out of statistical control when there is special cause variation (Montgomery, 2020), which results from outside influences. The presence of an out-of-control event can be detected using a variety of techniques. But when more tests are used, the likelihood of a false alert also arises.

There are two types of control charts. The memory-type control charts and memory-less control charts. The two memory-type control charts that most frequently employed in the literature are, EWMA control charts which indigenously presented by Roberts (1959) and CUSUM control charts which is initiated by Page (1954). To quickly alert the user if the process is even slightly off-target, these control methods not only use the most recent observation but also gather data from the past. Due to this hallmark, these charts are able to identify subtle and moderate shifts. Concurrently, they are used to invigilate the process mean and standard deviation. Discoursing the control charts, the CUSUM chart, which is used to track the process location, is available in two versions: Tabular CUSUM and V-Mask CUSUM proposed by Barnard (1959) and further modification done on respective chart by Edwards (1980). Appraise for both procedures are, piling up the deviations up and down from target value and normalization of deviations from the mean, respectively. To further improve the effectiveness of these charts, numerous adjustments have been proposed. While EWMA uses both the past and present information, similar to the CUSUM system, the weights assigned to the data decrease exponentially as observations become older.

The Shewhart-type control charts mainly are memory-less control charts, are mostly used to spot significant alterations. A Shewhart control chart has the drawback of basing its conclusion solely on the current sample, which ignores the past data and leads to a poor performance for minor process perturbations. Shewhart control charts includes mean ($\bar{X}$) for location process, standard deviation (S) and variance (S^2) for disperse process, range (R), T chart, X chart , Rate (X/T) chart and so on.

The run rules schemes are utilised to improve the performance of Shewhart control charts in terms of ARL_0 (in-control) and ARL_1 (out-of-control). However, Steiner (1999) apportioned a comprehensive study on the performance of the ARL of the EWMA control chart with such time-varyig limits applied to mean. Later on, the author noticed that when weights are very small in EWMA, then it would be more sensitive for early shifts of mean. Then, an fast initial response (FIR) methodology has been introduced which narrowed the control chart for first twenty observations by fixing standard deviation, the parameter of normal distribution, to get better detection of shift for location parameter. Many scholars have experimented with the notion of combining various sensitising rules and runs rules systems with the Shewhart-type control charts to improve the executiom of control charts, e.g, Klein (2000), Khoo (2003), Koutras et al. (2007), and Antzoulakos and Rakitzis (2008). For Shewhart-type control charts, the authors proposed various run-rules schemes: including r out of m and modified r out of m. These running rules schemes perform better than the typical Shewhart-type control charts. These techniques had been done due to insensitivity of Shewhart control charts for pointing small average process shifts.

Abbas et al. (2013b) proposed a mixed EWMA-CUSUM control chart to assess the ARL and detect a shift in the process mean, its coequal named CS-EWMA by Abbas et al. (2013a), due to the fact that its plotting statistic is based on an accumulation of the EWMA. The proposed control chart was compared to a few representative control charts, such as the weighted CUSUM designated by Yashchin (1989), the runs rules-based CUSUM and EWMA by Khoo (2003), the fast initial response CUSUM given by Lucas and Crosier (2000), the fast initial response EWMA by Steiner (1999), the adaptive CUSUM with EWMA-based shift estimator recently proposed by Jiang et al. (2008) claimed that ACUSUM could offer a satisfactory detection throughout a multitude of mean shift sizes. The proposed method's effectiveness is evaluated in comparison to other control charts of the CUSUM- and EWMA-type, which are designed to monitor slight and moderate process changes. In contrast to the other schemes, comparisons showed that the proposed system is quite effective at identifying the small variations in the process.

Xie (1999) proposed several interesting single EWMA control chats. The Max-EWMA control chart is the phenomenal contribution which plays enormous role for monitoring the changes in mean and dispersion process. As a rule, Chen et al. (2001) recommended combining the plotting data from the two EWMA control charts—one to identify changes in the process mean and the other to respond to changes in the process dispersion to create Max-EWMA control chart. Sanusi et al. (2020) studied Max-EWMA chart using the maximum of the absolute values of two EWMA statistics - one for controlling the magnitude and the other for the frequency of an event. In addition, the magnitude is assumed to follow a gamma distribution while the frequency is assumed to follow an exponential distribution. The Max-EWMA statistic is initially a non-negative quantity hence it requires only upper control limit (UCL). Sheu et al. (2012) proposed an extended maximum generally weighted moving average (Max-GWMA) control chart to detect both increases and decreases in the process mean and/or variability of a process. There are numerous instances in the field where data are provided on various correlating auxiliary criteria in addition to the quality feature itself. For better understanding of several techniques, see Santos-Fernández (2012).

All the performances of proposed tecchniques based on run-lenth-profile (RLP) beacuse it provides a straight way for comparison. A control chart's performance is typically assessed in terms of its run length characteristics, specifically its ARL and standard deviation run length (SDRL). The in-control ARL ought to be substantial when the process is in an in-control state to prevent numerous false alarms. When the process is out-of-control, it must be small enough to quickly identify any changes to the process mean and/or process dispersion. Additionally, a control chart will perform better with smaller SDRL. Haq (2017) studied the AIB-Max-EWMA chart, a new Max-EWMA chart that uses auxiliary data on a single correlated auxiliary variable to simultaneously monitor both the mean and dispersion of a normally distributed process. The RLP of the proposed control chart, including the ARL and SDRL are calculated using Monte Carlo simulations. The author demonstrated that the existing Max-EWMA chart is covered by the AIB-Max-EWMA chart. Additionally, it turns out that the AIB-Max-EWMA chart is efficient as the Max-EWMA chart.

Chen et al. (2008) revealed that SPC technique used in many areas. The collected data from an events (positive and negative) can be continually monitored by a SPC system, which can then determine if the situation is under or out-of-control and whether any urgent and strengthened action needs to be taken. These monitoring events may involves time interval T between two consecutive occurances and the magnitude X of each occurance.

In high-quality processes with a low incidence rate, control charts based on the TBE are widely employed. The TBE control charts, use the inter-arrival times between the occurrences of events, which follow exponentially distributed (Gan, 1998), as opposed to traditional Shewhart control charts for attributes namely; c chart, u chart, p chart and np chart which are used to monitor the number or proportion of nonconformities or defective products.

The research of TBE control charts has drawn the attention of numerous academicians. The observations distributed exponentially for CUSUM control charts, are easy to implement and compare (Lucas, 1985). Gan (1998) suggested the EWMA control chart for exponential data and estimated its ARL using differential equations and discovered that it was marginally less sensitive than the exponential CUSUM chart. Borror et al. (2003) investigated the exponential CUSUM chart's resilience under the assumption that the TBE observations following a Weibull or just a lognormal distribution, and they demonstrated that it is extremely robust for both moderate and large shifts of parameters.

Aslam et al. (2017) proposed an efficient alternative control charting method using the double moving average and EWMA statistic for the monitoring of exponentially distributed quality characteristics. Various combinations of the shift factors, the EWMA smoothing values, the moving-average-spans, and the desired in-control average run durations are investigated to see how efficiently the suggested control chart performs. The authors claimed that, the proposed chart is more effective than the existing control chart. Zhang et al. (2007) suggested a gamma distribution-based TBE control chart utilising the "random-shifted" model as opposed to the more conventional "fixed-shift model".

Wu et al. (2009a), Liu et al. (2009) and Qu et al. (2014) introduced different charts to jointly monitor time and magnitude. Talib et al. (2022a) suggested a new Max-EWMA chart for time and magnitude monitoring where these are assumed to follow an exponentially modified Gaussian (EMG) distribution to handle the schedule time, positive and negative magnitude shifts. The ARL, SDRL, and quantiles of the run length distribution are used to assess the effectiveness of the chart. The proposed chart is best to monitor the scale parameter of the EMG.

Ali et al. (2022) proposed a Max-EWMA control chart in order to monitor unit interval time and magnitude together using simplex distribution for magnitude while beta distribution for time. The same criteria utilized for checking up the perfomance by adopting different shift sizes and smoothing parameters of the chart as by evaluating run-lenth-profile. The Max-EWMA chart is effective in spotting small to moderate shifts.

By assuming a Weibull distribution, Talib et al. (2022b) proposed a new Max-EWMA control chart which is efficient at identifying various sizes of small shifts. Using beta and unit gamma distribution Akram et al. (2022) suggested Max-EWMA control chart which can monitor an event's time and amplitude simultaneously. They discovered that the suggested chart is more effective at detecting small to medium-sized shifts and as compared to a pure shift, simultaneous shifts can be detected rapidly.

Wu et al. (2009b) used the average time to signal, or steady-state ATS, to assess how well a control chart function. When the shift happens at a random time point, the process enters the steady-state mode and reaches its stationary distribution. Additionally, it is believed that the magnitude X and the time period T are unrelated to one another. Performance evaluations reveal that when compared to an isolated T and X chart, this

T&X chart is significantly more efficient and acts more consistently in a wide shift domain (Wu et al., 2009a). Different parallel changes had been observed for X and T and this proposed approach used a single rate chart to track a single statistic, R, which is the ratio of X to T. Joint X&T chart could not instantly give indication about the changes in the chart signals. Due to this drawback, a new Rate (X/T) chart has been used, which is one of a Shewhart control chart based by utilizing ATS which is mainly equals to ARL_0/λ_0. This gives a quick alert when we have out-of-control process and large shift.

Chapter 3

Time and Magnitude Chart Using Exponential and Gamma Distributions

3.1 Introduction

In terms of statistics, the desired quality of a product is achieved by SQC. The SQC refers to the use of currently available statistical techniques and methodologies to monitor the degree of quality of a good or service. The control chart is a primary tool utilized in SQC. A control chart is a graphical tool shows how the process is performing in relation to the pre-defined specification. Additionally, the SQC can be divided into three different categories: descriptives (data distribution and its features, such as mean, standard deviation, range, etc.), acceptance sampling (method of accepting a substantial amount based on the results of random sampling), and SPC. The SPC makes assumptions about the regular functionality of process using the control chart methodology. The process is said to be in-control (IC) if it works as expected or according to the pre-defined specifications, otherwise it is out-of-control (OOC).

The Shewhart approach is the basic tool developed in SPC for process monitoring, in which we maintain process monitoring decisions on the value of a statistic taken from the most recent sample. This method works well with processes that have large shifts because it disregards historical knowledge about the process. The main drawback of this scheme is its lack of memory-based functionality when handling small shifts. The adoption of memory-type control charts addresses this weakness. Among the most common examples of these are CUSUM chart and EWMA chart.

High-quality processes, often known as processes with exceptionally low defect rates, cannot be observed using traditional attribute control charts. A chart based on inter-arrival time or TBE control chart is used for high-quality processes, like monitoring the necessary amount of chemicals in a life saving treatment. The frequency of an event and its long term effect on the process performance are both taken into account while monitoring a manufacturing process. Therefore, it is necessary to simultaneously monitor two parameters (magnitude and frequency). In addition to making the monitoring process simpler, this will also increase its effectiveness and uniqueness.

The remaining chapter is structured as follows: Section 2 presents the basic concept of the proposed (rate based) EWMA control chart as well as its charting process. Section 3 contains overview of Max-EWMA control chart. Out-of-control features are detailed in Section 4 along with the ARL, SDRL and quantiles. Section 5 provides a real-life data set application on charts and Section 6 provide conclusions.

3.2 Memory-type Rate Based EWMA Control Chart

In this research, we have modified a rate chart to propose an EWMA control chart (Roberts, 1959; Wu et al., 2009b). The plotting statsistic is made up of single EWMA statistic with an event ratio R for simultaneous moitoring of magnitude (X) and frequency (T).

3.2.1 Design of a Chart using Gamma and Exponential Distribution

- A random variable X follows gamma distribution with the shape parameter a>0 and scale parameter b>0, i.e., X $\sim$ Gamma(a,b) with the probability density function (pdf)

$$f(x;a,b) = \frac{1}{b^a\ \Gamma(a)} x^{(a-1)} e^{(-x/b)} \quad , \quad x \geq 0, \quad a,b > 0 \tag{3.1}$$

  where $\Gamma(a)$=(a-1)! is called the gamma function.

  The mean and variance of X are:

$$E(X) = ab \quad and \quad Var(X) = ab^2 \tag{3.2}$$

- A random variable T follows exponential distribution with rate parameter λ>0, i.e., T $\sim$ Exp(λ) with pdf

$$f(t;\lambda) = \lambda e^{-\lambda x} \qquad x \geq 0, \quad \lambda > 0 \tag{3.3}$$

  where the mean and variance of T are:

$$E(T) = \frac{1}{\lambda} \quad and \quad Var(T) = \frac{1}{\lambda^2} \tag{3.4}$$

- The event ratio R:

$$R = X/T \tag{3.5}$$

- The EWMA statistic based on R is defined as:

$$Z_i = \theta R_i \ + \ (1-\theta)Z_{(i-1)} \tag{3.6}$$

where $0<\theta\leq 1$ is one of the two components of the EWMA control chart, and it is known as the smoothing constant. The other one is the L, which is the constant multiplier. The initial value of EWMA statistic is Z_0=μ_0=E(R). A small value of is typically seen to be ideal for detecting small shifts, while a large number is assumed to be suitable to detect significant shifts. Moreover, the proposed chart is considered the upper control limit (UCL) due to the fact that the plotting statistic is to detect the upward (UW) shifts. The UCL is determined by:

$$UCL = \mu_0 + (L\sigma)\sqrt{\frac{\theta}{2-\theta}} \tag{3.7}$$

where the μ_0 and σ are the mean and variance of the event ratio R and the constant multiplier L regulates the control limit's width for IC process. If μ_0 and σ are not known, we can substitute the sample mean and variance of R, which can be computed from historical IC data. The transformed EWMA chart can detect the upward (UW) shift in the X and/or T (and downward (DW) shift as well for a case).

3.2.2 Charting Procedure of a Chart

The construction of EWMA control chart for simultaneous monitoring of an event's magnitude and frequency is stated as:

- Generated 100000 R values and compute mean and variance of R, μ_0=mean(R) and σ=sd(R) and calculate UCL.

- Set Z_0=μ_0 as the initial value, then select the appropriate ARL_0 value. To get the required ARL_0, choose an appropriate value for the smoothing parameter θ as well as the constant multiplier L. The value of the constant multiplier required to generate the 370 and 500 values of standard ARL_0. The performance also includes the standard deviation of average run length along with different quantiles to access the skewness of the run length (RL) distribution.

- Start monitoring the process by plotting the Z_i assigning to the counter number i. The process referred to be IC if Z_i<UCL, otherwise the process is notified as the OOC.

- Investigate the underlying reason(s) for each of the out-of-control points. If Z_i goes above UCL, the process will said to be OOC due to increase in event ratio R. We will label the OOC point by "R^+" or in the context of time and magnitude of an event we can label it "XT".

3.3 Overview of Max-EWMA Control Chart

The research adopted the taditional Max-EWMA charting method (Xie, 1999; Chen et al., 2001) for a comparative study of proposed chart. The maximum value of two independent absolute EWMA statistics for simultaneous monitoring made the plotting statistic of the Max-EWMA control chart. Plotting the maximum absolute values of TBE and magnitude enables joint monitoring of the event's TBE (T) and magnitude (X). The chart can determine whether a shift has taken place in T, X or X&T when an out-of-control (OOC) signal is detected.

For TBE, it is assumed to follow exponential distribution with λ_E as rate parameter, T $\sim$ Exp(λ_E) and magnitude of an event follows gamma distribution where a is shape and b is scale parameter, X $\sim$ Gamma(a,b). Additionally, it is believed that both the magnitude and the interval in time between two consecutive disruptions are independent. The max-EWMA chart is created by defining two independent statistics

$$\mathbf{U_i} = \frac{\mathbf{X_i} - (ab)}{\sqrt{ab^2}}$$

$$\mathbf{V_i} = \lambda_E \mathbf{T_i} - 1$$

Based on $\mathbf{U_i}$ and $\mathbf{V_i}$, the EWMA statistics are expressed as

$$\mathbf{Y_i} = (1-\omega)\mathbf{Y_{(i-1)}} + \omega \mathbf{U_i}\ ,\quad 0 \leq \omega < 1\ ,\ i = 1, 2, ...$$

$$\mathbf{Z_i} = (1-\omega)\mathbf{Z_{(i-1)}} + \omega \mathbf{V_i}\ ,\quad 0 \leq \omega < 1\ ,\ i = 1, 2, ...$$

Assuming the process begins at $\mathbf{Y_0}$=E($\mathbf{U_i}$)=0 and $\mathbf{Z_0}$=E($\mathbf{V_i}$)=0 with smoothing constant $\omega \in$ (0,1] for quick detection of (large and small) shifts (Lucas and Saccucci, 1990) and in-control process parameters are (a_0,b_0) and (λ_0).

The maximum absolute of $\mathbf{Y_i}$ and $\mathbf{Z_i}$ values constitute the monitoring statistic for the Max-EWMA control chart.

$$\mathbf{M_i} = max\{|\mathbf{Y_i}|, |\mathbf{Z_i}|\}$$

As the plotting statistic is a real and non-negative value. The Max-EWMA chart only has the upper control limit (UCL). The UCL can be computed as

$$UCL = E(\mathbf{M_i}) + L\sqrt{V(\mathbf{M_i})}$$

Here, the IC values for the mean and variance of the plotting statistic are E($\mathbf{M_i}$) and V($\mathbf{M_i}$). The constant multiplier $\mathbf{M_i}$ and L regulates the width of the control limit. When E($\mathbf{M_i}$) and Var($\mathbf{M_i}$) are not known, sample means and variance of $\mathbf{M_i}$ derived from previous IC data might be used in their place. The Max-EWMA chart can detect UW and DW shifts in magnitude and time simultaneously.

3.3.1 Shifts in Parameters for both Charts

The shifts in the parameters of T and X are regarded as

$\lambda = \delta_{\lambda_E}\lambda_0$ and $a=\delta_a a_0$ where δ_{λ_E} and δ_a indicated that the shifts in parameter of T and X, respectively. Regarding the occurrence of shifts, we have also taken into account two cases.

3.3.2 Cases

- In the first case, we have considered shifts in the rate parameter of exponential distribution (T $\sim$ Exp(0.01)) along with the shift in first shape parameter of gamma distribution (X $\sim$ Gamma(4,1)).
- In the second case, we have considered shifts in the rate parameter of exponential distribution (T $\sim$ Exp(0.01)) along with the shift in first shape parameter of gamma distribution (X $\sim$ Gamma(7.5,1)).
- In the third case, we have considered shifts in the rate parameter of exponential distribution (T $\sim$ Exp(0.01)) along with the shift in second scale parameter of gamma distribution (X $\sim$ Gamma(1,1)).
- In the final case, we have considered shifts in all parameters, the rate parameter (λ_E) of exponential distribution (T $\sim$ Exp(0.01)) along with the shifts in first shape parameter (a) and scale parameter (b) of gamma distribution (X $\sim$ Gamma(4,1)).

The process is in-control (IC) when δ_{λ_E} and δ_a are equal to one.

3.4 Out-of-Control Characteristics of the Chart

The most used criterion for assessing a control chart's performance is the ARL. It is measured as the average number of subgroups (samples) shown up until an OOC signal is observed. The IC ARL (ARL_0) and the OOC ARL (ARL_1) are the two varieties of ARL that are described in the literature. ARL_0 should ideally be as large as feasible to prevent unwanted delays in the production line. On the other side, the control chart performs better when the value of ARL_1 is lowest. In reality, comparison research shows that the chart with a lower value of ARL_1 beat its rival. We must examine SDRL and run length quantiles. As a result, a complete study of simultaneous monitoring of frequency and magnitude, the ARL together with its SDRL and various quantiles of run length distribution are provided in Tables 3.1 to Table 3.10 of case 1, Tables 3.11 to 3.20 corresponded to case 2, Tables 3.21 to 3.30 of case 3 and Tables 3.31 to 3.35 belongs to the final case 4. For example, for case 1, simultaneous shifts in the shape parameter of the gamma distribution and the rate parameter of the exponential distribution for both charts. For case 2, we considered the same criterion as case 1 but changed the shape parameter of the gamma distribution. In case 3, we introduced shifting process in scale parameter of gamma distribution and rate parameter of exponentail distribution and finally for case 4 the simulation study has done by applying shifts in all three parameters. Furthermore, we assumed different shift sizes for parameters $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00) and δ_b $\in$ (1.000,1.063,1.125,1.188,1.250,1.313). The shifts are considered in terms of percentage, where the value δ=1.16 means there is 16% increase in the parameter value. δ=2.20 means the parameter value is increased by 120%. The value δ=1 conforms no shift in the parameter. However, the ARL_0 value is fixed at approximately 370 and 500. The results are accomplished using R software, (R version 4.2.0, 2022-04-22).

3.4.1 Results and Discussion

Tables 3.1 to 3.20 illustrate the Max-EWMA and EWMA-Rate charts with smoothing parameters $\omega \in$ (0.05,0.10,0.15,0.20,0.25) and $\theta \in$ (0.05,0.10,0.15,0.20,0.25) by taking the various levels of shifts for shape parameter of magnitude (a) and rate parameter of time (λ_E). The parameters of in-control chart presumes for case 1, X $\sim$ Gamma(4,1) and T $\sim$ Exp(0.01) for ARL_0=370 and ARL_0=500. Several levels of upward and downward shifts are taken for a and λ_E, e.g., $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00). Additionally, δ_a=1.00 and δ_{λ_E}=1.00 presents no shift in shape parameter of magnitude and rate parameter of time, whereas δ=1.16 indicates 16% increase in the parameter value and δ=2.60 indicates 160% increase in the parameter value and so on.

To ascertain various run length characteristic features, we do Monte Carlo simulations. Given that the increasing UW and DW shifts result in a comparable degenerating pattern of ARL, simultaneous shifts in the two parameters of X and T are encountered. For example, in Table 3.1, we have simultaneous set of shifts for case 1, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=370 and smoothing parameter 0.05 for both charts, the ARL (SDRL) reduced by 68.38% (67.70%) (for the EWMA-Rate) and 94.78% (98.54%) (for the Max-EWMA) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and ARL (SDRL) increased by 11.16% (5.47%) (for the EWMA-Rate) and reduced by 41.74% (45.52%) (for the Max-EWMA) when 5% increase in shape parameter of X and no change in rate parameter of T, respectively. In Table 3.2, we again have simultaneous set of shifts for case 1, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=500 and alike smoothing parameter 0.05, the ARL (SDRL) reduced by 76.40% (75.91%) (for the EWMA-Rate) and 95.76% (98.80%) (for the Max-EWMA) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and ARL (SDRL) increased by 5.12% (4.15%) (for the EWMA-Rate) and decreased by 45.03% (48.00%) (for the Max-EWMA) when 5% increase in the shape parameter of X and no change in the rate parameter of T, respectively. The results of case 1 with smoothing parameter 0.05 shows that the performance of Max-EWMA chart is better than the EWMA-Rate because the shifts in the parameters are detected more quickly. Table 3.3 under case 1, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=370 and smoothing parameter 0.10 for both charts, the ARL (SDRL) reduced by 79.80% (79.55%) (for the EWMA-Rate) and 91.98% (95.32%) (for the Max-EWMA) when 26% increase in the shape parameter of X and 200% increase in rate parameter of T and ARL (SDRL) decreased by 5.14% (3.89%) (for the EWMA-Rate) and 42.05% (36.55%) (for the Max-EWMA) when 5% increase in shape parameter of X and no change in rate parameter of T, respectively. In Table 3.4, we again have simultaneous set of shifts for case 1, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=500 and alike smoothing parameter 0.10, the ARL (SDRL) reduced by 78.69% (78.53%) (for the EWMA-Rate) and 92.40% (94.83%)

Table 3.1: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.05 and ω=0.05, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		$\boldsymbol{ARL_0}$**=370.45**	$\boldsymbol{\theta}$**=0.05**	**L=0.03622**			$\boldsymbol{ARL_0}$**=370.21**	$\boldsymbol{\omega}$**=0.05**	**L=2.791**		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	162.89	166.75	43	111	227	22.37	4.33	19	22	25
1.05	3.00	151.92	155.66	41	103	211	22.24	4.41	19	22	25
1.10	3.00	142.01	145.39	38	97	197.25	21.95	4.56	19	21	24
1.16	3.00	131.51	134.34	35	90	183	21.31	4.88	19	21	24
1.21	3.00	124.14	126.66	34	85	172	20.45	5.12	18	20	23
1.26	3.00	116.99	119.48	32	80	162	19.29	5.38	16	20	23
1.00	2.60	198.05	202.71	53	135	275	25.72	6.19	21	25	29
1.05	2.60	184.91	189.26	50	126	257	25.49	6.25	21	25	29
1.10	2.60	173.43	177.63	46	118	242	25.02	6.37	21	24	28
1.16	2.60	161.19	165.11	43	110	225	23.97	6.58	20	23	28
1.21	2.60	151.61	155.06	40	103	211	22.67	6.73	19	22	27
1.26	2.60	143.04	146.28	38	98	199	20.97	6.77	17	21	25
1.00	2.20	247.71	253.15	67	169	346	32.27	10.35	25	30	37
1.05	2.20	231.99	236.85	63	158	324	31.76	10.28	25	30	37
1.10	2.20	217.55	222.11	59	148	304	30.75	10.22	24	29	36
1.16	2.20	202.56	206.56	54	138	282	28.64	10.03	22	27	34
1.21	2.20	190.79	195.02	51	130	265	26.21	9.65	20	25	32
1.26	2.20	180.60	185.01	48	123	251	23.42	9.17	17	23	29
1.00	1.80	320.04	327.64	86	219	446	49.47	23.77	33	44	60
1.05	1.80	300.70	306.96	81	206	419	48.14	23.09	32	43	58
1.10	1.80	282.98	288.24	76	194	394	44.36	21.37	30	40	54
1.16	1.80	264.77	269.43	71	182	370	37.99	18.28	26	35	47
1.21	1.80	250.38	255.33	68	171	350	32.11	15.62	21	30	40
1.26	1.80	236.81	241.78	64	162	331	26.80	13.07	17	25	34
1.00	1.40	429.91	437.11	119	296	597	152.54	121.73	67	116	200
1.05	1.40	408.46	416.41	112	281	568	123.99	96.69	56	96	163
1.10	1.40	386.68	394.34	106	266	538	87.77	65.21	42	70	114
1.16	1.40	362.62	370.01	98	249	504	55.79	38.29	29	46	72
1.21	1.40	343.90	350.87	93	236	478	39.76	25.28	22	34	57
1.26	1.40	326.99	333.54	88	225	455	29.88	17.63	17	26	38
1.00	1.00	370.45	339.61	133	270	504	370.21	359.77	115	259.50	509
1.05	1.00	411.30	391.27	138	293	562	215.53	201.55	72	154	295
1.10	1.00	443.26	430.14	141	313	610	113.70	98.96	44	54	152
1.16	1.00	465.63	461.78	141	325	642	60.55	46.38	28	48	80
1.21	1.00	471.16	471.83	138	327	651	40.76	27.90	21	34	52
1.26	1.00	467.44	470.90	134	324	697	29.93	18.42	17	26	38

Table 3.2: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.05 and ω=0.05, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.92	θ=0.05	L=0.03622			ARL_0=499.99	ω=0.05	L=3.037		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	164.08	167.96	44	112	229	24.61	4.83	21	24	27
1.05	3.00	153.15	156.83	41	104	212.25	24.45	4.89	21	24	27
1.10	3.00	143.18	146.48	38	98	199	24.19	5.05	21	24	27
1.16	3.00	132.59	135.35	36	90	184	23.47	5.34	20	23	26
1.21	3.00	125.15	127.57	34	85	174	22.49	5.67	19	22	26
1.26	3.00	117.98	120.41	32	81	164	21.19	5.96	18	21	25
1.00	2.60	199.51	204.11	54	136	277	28.56	7.01	24	27	32
1.05	2.60	186.34	190.73	50	127	259	28.32	7.08	23	27	32
1.10	2.60	174.87	179.20	47	119	244	27.82	7.19	23	27	32
1.16	2.60	162.48	166.32	43	111	227	26.64	7.40	22	26	31
1.21	2.60	152.94	156.37	41	104	213	25.11	7.58	21	25	29
1.26	2.60	144.37	147.53	39	98	201	23.14	7.59	18	23	28
1.00	2.20	249.60	254.90	67	171	349	36.60	12.23	28	34	42
1.05	2.20	233.76	238.67	63	160	327	36.05	12.16	28	34	42
1.10	2.20	219.21	223.69	59	150	306	34.86	12.01	27	33	41
1.16	2.20	204.21	208.66	55	139	285	32.32	11.69	25	31	38
1.21	2.20	192.25	196.44	52	131	267	29.33	11.10	22	28	35
1.26	2.20	182.04	186.39	49	124	253	25.99	10.42	19	25	32
1.00	1.80	322.52	329.77	87	221	449	59.83	30.48	39	52	73
1.05	1.80	302.91	309.34	82	208	421	57.40	29.23	37	50	70
1.10	1.80	285.00	290.32	77	196	397	52.58	26.73	34	47	64
1.16	1.80	266.82	271.37	72	183	373	44.05	22.40	29	40	55
1.21	1.80	252.45	257.11	68	173	353	36.49	18.40	24	34	46
1.26	1.80	238.69	243.59	65	163	334	29.81	15.00	19	28	38
1.00	1.40	438.36	446.50	121	301	610	221.94	187.18	89	166	295
1.05	1.40	413.45	422.05	114	284	575	170.21	140.26	72	129	225
1.10	1.40	390.36	398.14	107	268	543	111.55	87.22	51	87	146
1.16	1.40	365.46	372.92	99	251	508	65.63	46.99	32	53	83
1.21	1.40	346.45	353.18	94	238	482	45.00	29.46	24	38	58
1.26	1.40	329.16	335.69	89	226	458	33.02	19.83	19	28	42
1.00	1.00	500.92	485.21	157	354	689	499.99	489.49	151	349	691
1.05	1.00	525.60	520.79	158	366	724.25	274.83	259.99	90	195	376
1.10	1.00	533.06	533.94	155	369	737	136.85	120.64	52	101	184
1.16	1.00	526.42	531.01	149	364	731	69.59	54.11	32	54	91
1.21	1.00	510.73	517.55	143	352	709	45.64	31.59	23	37	59
1.26	1.00	492.46	500.13	137	339	683	32.96	20.37	19	28	42

Table 3.3: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.10 and ω=0.10, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		$\boldsymbol{ARL_0}$**=370.17**	$\boldsymbol{\theta}$**=0.10**	**L=0.03**			$\boldsymbol{ARL_0}$**=370.29**	$\boldsymbol{\omega}$**=0.10**	**L=3.381**		
$\boldsymbol{\delta_a}$	$\boldsymbol{\delta_{\lambda_E}}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{Q_1}$	$\boldsymbol{Q_2}$	$\boldsymbol{Q_3}$	$\boldsymbol{ARL}$	$\boldsymbol{SDRL}$	$\boldsymbol{Q_1}$	$\boldsymbol{Q_2}$	$\boldsymbol{Q_3}$
1.00	3.00	100.61	102.66	27	69	139	55.59	31.35	34	47	68
1.05	3.00	94.47	96.07	26	65	131	53.31	30.22	33	45	65
1.10	3.00	88.97	90.59	24	61	123	49.34	28.15	31	42	61
1.16	3.00	83.08	84.42	23	57	115	42.45	24.51	26	37	53
1.21	3.00	78.73	79.90	22	54	109	35.96	20.85	22	32	46
1.26	3.00	74.71	75.64	21	51	104	29.64	17.31	17	27	38
1.00	2.60	120.49	122.98	33	83	167	96.08	70.19	47	75	124
1.05	2.60	113.19	115.35	31	77	157	87.46	63.49	43	69	113
1.10	2.60	106.80	108.86	29	73	148	74.42	53.59	38	59	96
1.16	2.60	99.73	101.63	27	68	138	56.47	39.90	29	46	73
1.21	2.60	94.31	96.00	26	65	131	43.33	29.95	22	36	56
1.26	2.60	89.61	91.23	25	61	124	33.19	22.31	17	28	43
1.00	2.20	147.71	150.20	40	101	205	241.58	215.47	88	176	325
1.05	2.20	138.98	141.47	38	95	193	183.54	161.92	69	134	247
1.10	2.20	130.93	133.45	36	90	181	125.19	108.76	49	93	167
1.16	2.20	122.90	125.32	34	84	170	75.44	62.24	32	58	101
1.21	2.20	116.56	118.99	32	80	162	50.93	40.02	23	40	67
1.26	2.20	110.81	112.81	30	76	154	36.03	26.67	17	29	47
1.00	1.80	188.02	191.00	52	129	261	714.16	702.13	218	980	500
1.05	1.80	177.49	180.63	49	121	247	351.13	337.92	110	246	483
1.10	1.80	167.94	170.72	46	115	234	176.08	165.12	59	126	241
1.16	1.80	157.10	159.67	43	108	219	87.25	76.97	33	64	117
1.21	1.80	149.11	151.60	41	102	207	54.61	45.20	23	41	72
1.26	1.80	141.74	144.19	38	97	197	37.15	28.50	17	29	49
1.00	1.40	254.02	256.62	71	175	354	1002.13	996.32	293	698	1384
1.05	1.40	239.41	242.27	67	165	334	405.13	393.94	123	283	564
1.10	1.40	226.41	229.16	63	156	315	186.64	176.82	60	132	256
1.16	1.40	212.34	214.94	59	146	296	89.13	79.42	33	65	120
1.21	1.40	201.98	204.13	56	139	281	55.14	46.02	23	42	73
1.26	1.40	192.21	195.05	53	131	267	37.28	20.77	17	29	49
1.00	1.00	370.17	374.41	103	256	513	370.29	365.53	111	259	510
1.05	1.00	350.96	355.59	98	242	487	241.38	234.73	74	169	332
1.10	1.00	332.73	336.28	93	230	462	144.16	135.61	48	103	197
1.16	1.00	313.53	316.78	88	216	436	78.86	70.02	29	58	106
1.21	1.00	298.79	301.20	84	207	415	51.29	45.57	21	39	68
1.26	1.00	284.96	287.21	80	197	397	35.71	27.55	16	28	47

Table 3.4: EWMA-Rate and Max-EWMA chart when X $\sim$ Gamma(4,1), T $\sim$ Exp(0.01), ARL_0=500 with θ=0.10 and ω=0.10, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.54	θ=0.10	L=0.0479			ARL_0=500.25	ω=0.10	L=3.657		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	141.81	142.77	40	98	197	108.88	80.21	53	85	140
1.05	3.00	133.40	134.52	37	92	185	100.5	73.68	49	79	130
1.10	3.00	126.10	127.24	35	87	174	86.26	62.59	43	68	111
1.16	3.00	118.22	119.35	33	82	163	65.81	46.78	34	54	85
1.21	3.00	112.13	112.93	32	77	155	50.03	34.92	26	42	65
1.26	3.00	106.54	107.35	30	73	147	37.99	25.83	19	32	49
1.00	2.60	168.82	170.16	47	116	235	249.03	219.65	93	182	336
1.05	2.60	159.01	160.30	45	110	221	200.22	174.70	76	148	269
1.10	2.60	150.28	151.45	42	104	208	142.93	122.97	56	107	191
1.16	2.60	140.58	141.70	39	97	195	87.84	72.62	37	67	117
1.21	2.60	133.22	131.24	37	92	184	58.97	46.58	26	46	78
1.26	2.60	126.88	127.95	36	88	175	41.28	30.75	19	33	54
1.00	2.20	205.80	207.33	58	142	285	734.26	713.94	227.75	516	1009
1.05	2.20	193.99	195.43	55	134	269	406.84	388.74	130	287	559
1.10	2.20	183.59	185.21	52	126	255	212.51	198.29	71	152	290
1.16	2.20	172.28	173.88	48	119	240	104.92	92.98	39	77	142
1.21	2.20	163.68	164.95	46	113	228	64.17	53.67	26	48	86
1.26	2.20	155.63	156.70	44	108	216	42.91	33.39	19	34	56
1.00	1.80	260.51	261.65	74	181	363	1379.38	1367.84	403	961	1909
1.05	1.80	246.21	247.57	70	170	343	539.88	527.08	162	378	747
1.10	1.80	232.76	234.10	66	160	324	240.07	228.71	77	170	330
1.16	1.80	218.47	219.59	62	151	304	109.63	98.88	39	80	148
1.21	1.80	207.71	208.57	59	143	289	65.38	55.41	26	49	87
1.26	1.80	197.81	199.00	56	136	274	43.25	33.97	19	34	57
1.00	1.40	344.98	347.93	97	239	478	1469.65	1460.99	427	1021	2036
1.05	1.40	326.78	329.14	92	226	454	551.85	538.31	165	386.5	766
1.10	1.40	309.95	311.72	88	215	430	241.99	230.94	77	171	332
1.16	1.40	291.84	292.71	83	203	406	109.89	99.24	39	80	149
1.21	1.40	278.50	279.81	79	193	387.25	65.45	55.54	26	49	88
1.26	1.40	265.86	267.05	75	184	370	43.27	34.03	19	34	57
1.00	1.00	500.54	504.11	143	346	693	500.25	494.04	148	349	693
1.05	1.00	474.97	478.84	136	328	658	322.33	314.13	99	225	444
1.10	1.00	450.81	454.59	128	311	624	185.85	176.77	60	132	255
1.16	1.00	425.26	428.17	121	295	589	97.58	87.80	35	71	132
1.21	1.00	405.39	408.05	116	281	561	61.21	51.83	25	46	82
1.26	1.00	387.29	389.03	110	269	536	41.55	32.58	19	32	54

Table 3.5: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.15 and ω=0.15, for different levels of shifts in a and λ_E, $\delta_a \in (1.00,1.05,1.10,1.16,1.21,1.26)$ and $\delta_{\lambda E} \in (3.00,2.60,2.20,1.80,1.40,1.00)$

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=370.14	θ=0.15	L=0.0438			ARL_0=370.22	ω=0.15	L=3.788		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	108.95	109.79	31	75	150	1199.23	1192.43	347	834	1662
1.05	3.00	102.94	103.63	29	71	142	526.50	519.80	154	367	730
1.10	3.00	97.40	97.74	28	67	135	259.77	252.62	80	182	358
1.16	3.00	91.37	91.47	26	63	126	126.55	119.32	42	90	173
1.21	3.00	86.83	87.13	25	60	120	76.25	68.90	27	55	103
1.26	3.00	82.80	82.80	24	57	115	50.03	43.36	19	37	67
1.00	2.60	128.30	129.48	36	88	177	1199.23	1192.43	347	834	1662
1.05	2.60	121.47	122.31	34	84	168	526.50	519.80	154	367	730
1.10	2.60	114.96	115.76	33	79	159	259.77	252.62	80	182	358
1.16	2.60	108.12	108.80	31	75	149	126.55	119.32	42	90	173
1.21	2.60	102.94	103.66	29	71	142	76.25	68.90	27	55	103
1.26	2.60	98.02	98.72	28	68	135	50.03	43.36	19	37	67
1.00	2.20	155.29	156.26	44	107	215	1199.23	1192.43	347	834	1662
1.05	2.20	146.93	147.83	41	101	204	526.50	519.80	154	367	730
1.10	2.20	139.31	140.28	39	96	193	259.77	252.62	80	182	358
1.16	2.20	130.75	131.78	37	90	181	126.55	119.32	42	90	173
1.21	2.20	124.66	125.60	35	86	172	76.25	68.90	27	55	103
1.26	2.20	118.92	119.92	34	82	164	50.03	43.36	19	37	67
1.00	1.80	194.66	195.79	56	134	269	1199.15	1192.23	347	834	1662
1.05	1.80	184.44	185.56	52	127	256	526.49	519.80	154	367	730
1.10	1.80	175.25	176.53	49	121	244	259.76	252.61	80	182	358
1.16	1.80	164.68	165.47	46	114	229	126.55	119.32	42	90	173
1.21	1.80	156.79	157.71	44	108	218	76.25	68.89	27	55	103
1.26	1.80	149.80	150.78	42	104	208	50.03	43.36	19	37	67
1.00	1.40	258.25	258.82	74	179	359	1171.00	1165.38	339	814	1620
1.05	1.40	244.54	245.26	70	169	340	520.87	514.59	152	363	722.25
1.10	1.40	231.94	232.44	66	160	323	258.31	251.20	79	181	357
1.16	1.40	218.44	219.01	62	151	304	126.23	119.07	41	90	172
1.21	1.40	208.24	208.66	60	144	289	76.15	68.83	27	55	103
1.26	1.40	198.86	199.63	57	137	276	49.98	43.33	19	37	67
1.00	1.00	370.14	371.84	105	257	512	370.22	367.10	109	258	510
1.05	1.00	351.69	353.75	100	244	487	267.13	263.69	80	186	369
1.10	1.00	334.35	335.75	95	232	464	176.80	171.17	55	124	245
1.16	1.00	315.77	316.71	90	219	438	103.85	97.54	34	74	142
1.21	1.00	301.41	301.78	86	209	418	67.89	61.25	24	49	92
1.26	1.00	288.31	288.53	82	201	401	46.46	40.13	18	34	62

Table 3.6: EWMA-Rate and Max-EWMA chart when X ~ Gamma(4,1), T ~ Exp(0.01), ARL_0=500 with θ=0.15 and ω=0.15, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.10	θ=0.15	L=0.0653			ARL_0=500.20	ω=0.15	L=4.078		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	150.81	151.10	43	104	209	1747.94	1749.63	508	1207	2416
1.05	3.00	142.61	142.84	41	99	197	744.54	736.64	218	518	1031
1.10	3.00	135.23	135.64	39	94	187	351.48	342.53	107	247	485
1.16	3.00	126.97	127.58	36	88	175	164.04	156.82	53	116	224
1.21	3.00	121.12	121.57	35	84	167	95.69	87.71	33	69	131
1.26	3.00	115.60	115.91	33	80	160	60.89	53.60	23	45	82
1.00	2.60	177.43	178.16	51	122	246	1747.94	1749.63	508	1207	2416
1.05	2.60	167.86	167.89	48	116	233.25	744.54	736.64	218	518	1031
1.10	2.60	159.25	159.62	45	110	221	351.48	342.53	107	247	485
1.16	2.60	149.69	150.05	43	104	207	164.04	156.82	53	116	224
1.21	2.60	142.57	142.81	41	99	198	95.69	87.71	33	69	131
1.26	2.60	136.01	136.24	39	94	188	60.89	53.60	23	45	82
1.00	2.20	213.85	214.30	61	147	297	1747.94	1749.63	508	1207	2416
1.05	2.20	202.51	202.85	58	140	281	744.54	736.64	218	518	1031
1.10	2.20	191.79	192.11	55	132	266	351.48	342.53	107	247	485
1.16	2.20	181.00	181.57	52	125	251	164.04	156.82	53	116	224
1.21	2.20	172.45	173.00	49	119	240	95.69	87.71	33	69	131
1.26	2.20	164.74	165.14	47	114	229	60.89	53.60	23	45	82
1.00	1.80	266.90	267.41	76	185	370	1747.76	1749.46	508	1207	2415.25
1.05	1.80	253.60	253.72	73	176	353	744.51	736.62	218	518	1031
1.10	1.80	240.30	240.32	69	166	334	351.47	342.52	107	247	485
1.16	1.80	226.43	226.43	65	156	315	164.04	156.82	53	116	224
1.21	1.80	215.84	215.61	62	149	300	95.69	87.70	33	69	131
1.26	1.80	206.08	206.17	59	142	286	60.89	53.59	23	45	82
1.00	1.40	349.73	351.23	100	242	484	1710.41	1709.74	496	1182	2365
1.05	1.40	331.87	333.08	95	230	460	737.00	729.39	216	513	1021
1.10	1.40	315.76	315.88	90	220	437	349.67	340.71	106	245	483
1.16	1.40	297.48	296.81	85	207	413	163.66	156.46	52	116	224
1.21	1.40	284.45	284.02	82	198	394	95.54	87.61	33	69	130
1.26	1.40	272.30	272.14	78	189	379	60.84	53.55	23	45	82
1.00	1.00	500.10	501.80	144	346	692	500.20	495.81	147	348	693
1.05	1.00	475.12	477.52	137	329	657	362.02	356.73	107	253	499
1.10	1.00	451.90	454.11	130	313	624	235.94	230.39	72	165	325
1.16	1.00	426.93	428.51	123	296	591	134.41	127.71	44	95	184
1.21	1.00	408.03	409.40	117	283	564	85.25	78.01	30	62	116
1.26	1.00	390.41	390.53	112	272	540	56.71	49.74	22	42	76

Table 3.7: EWMA-Rate and Max-EWMA chart when X ~ Gamma(4,1), T ~ Exp(0.01), ARL_0=370 with θ=0.20 and ω=0.20, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.30	θ=0.20	L=0.0543			ARL_0=370.33	ω=0.20	L=4.093		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	113.25	113.76	33	78	156	1353.80	1348.35	391	935	1874
1.05	3.00	107.27	107.35	31	74	148	651.16	647.10	188	453	899
1.10	3.00	101.89	102.26	29	71	140	341.10	334.09	102	238	473
1.16	3.00	95.85	96.00	28	67	132	172.19	166.76	53	121	237
1.21	3.00	91.31	91.19	26	63	126	104.57	98.70	34	75	143
1.26	3.00	87.24	87.29	25	60	120	67.64	62.11	23	49	92
1.00	2.60	132.44	132.89	38	92	183	1353.80	1348.35	391	935	1874
1.05	2.60	125.77	126.27	36	87	174	651.16	647.10	188	453	899
1.10	2.60	119.40	119.98	34	83	165	341.10	334.09	102	238	473
1.16	2.60	112.38	112.91	32	78	155	172.19	166.76	53	121	237
1.21	2.60	107.23	107.43	31	74	148	104.57	98.70	34	75	143
1.26	2.60	102.59	103.05	29	71	141	67.64	62.11	23	49	92
1.00	2.20	159.40	159.89	45	110	221	1353.80	1348.35	391	935	1874
1.05	2.20	151.03	151.49	43	105	209	651.16	647.10	188	453	899
1.10	2.20	143.40	143.97	41	99	199	341.10	334.09	102	238	473
1.16	2.20	135.11	135.51	39	93	186	172.19	166.76	53	121	237
1.21	2.20	128.85	129.36	37	89	178	104.57	98.70	34	75	143
1.26	2.20	123.34	123.74	35	85	170	67.64	62.11	23	49	92
1.00	1.80	198.27	198.64	57	137	274	1353.04	1347.43	391	935	1873
1.05	1.80	187.94	188.46	54	129	261	650.96	646.85	188	452	899
1.10	1.80	178.84	179.50	51	123	248	341.04	334.04	102	238	473
1.16	1.80	168.58	168.91	48	117	235	172.18	166.75	53	121	237
1.21	1.80	160.94	161.27	46	111	224	104.57	98.70	34	75	143
1.26	1.80	153.86	154.24	44	107	214	67.64	62.10	23	49	92
1.00	1.40	260.49	260.49	75	181	362	1293.29	1287.18	373	894	1790
1.05	1.40	247.09	247.13	71	171	343	636.43	632.99	184	442	880
1.10	1.40	234.64	234.47	67	162	327	336.68	329.78	101	235	467
1.16	1.40	221.80	221.53	64	153	308	171.18	165.80	53	120	236
1.21	1.40	211.48	211.10	61	146	294	104.18	89.40	34	74	143
1.26	1.40	202.28	202.29	58	140	280	67.49	61.97	23	92	49
1.00	1.00	370.30	371.12	106	257	512	370.33	366.54	108	258	572
1.05	1.00	352.31	353.55	101	244	487.25	286.82	284.16	84	200	396
1.10	1.00	335.19	336.22	96	233	464	205.76	202.43	62	143	285
1.16	1.00	317.15	317.21	91	220	439	130.04	125.51	41	91	179
1.21	1.00	302.84	302.40	87	211	420	87.80	82.65	29	63	121
1.26	1.00	290.12	289.66	83	202	403	60.46	55.31	21	44	82

Table 3.8: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.20 and ω=0.20, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=500.27	θ=0.20	L=0.0787			ARL_0=500.08	ω=0.20	L=4.396		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	155.51	155.71	45	108	215	2003.82	2010.98	577	1380	2775
1.05	3.00	147.48	147.71	42	102	204	944.53	938.09	272	656	1311
1.10	3.00	140.04	140.24	40	97	194	476.72	470.95	140	334	660
1.16	3.00	131.79	131.99	38	91	182	231.55	226.16	71	161	318.25
1.21	3.00	125.85	126.00	36	87	174	136.81	130.33	44	97	188
1.26	3.00	120.44	120.53	35	84	166	85.86	79.87	29	62	117
1.00	2.60	181.94	182.23	52	126	252	2003.82	2010.98	577	1380	2775
1.05	2.60	172.45	172.38	49	119	239.25	944.53	938.09	272	656	1311
1.10	2.60	163.84	164.07	47	113	228	476.72	470.95	140	334	660
1.16	2.60	154.43	154.57	44	107	214	231.55	226.16	71	161	318.25
1.21	2.60	147.42	147.76	42	102	204	136.81	130.33	44	97	188
1.26	2.60	140.89	141.07	40	98	195	85.86	79.87	29	62	117
1.00	2.20	218.16	218.28	63	151	303	2003.82	2010.98	577	1380	2775
1.05	2.20	206.75	206.86	60	143	286	944.53	938.09	272	656	1311
1.10	2.20	196.28	196.15	57	136	272	476.72	470.95	140	334	660
1.16	2.20	185.28	185.09	53	128	257	231.55	226.16	71	161	318.25
1.21	2.20	177.15	177.14	51	122	246	136.81	130.33	44	97	188
1.26	2.20	169.38	169.55	48	117	236	85.86	79.87	29	62	117
1.00	1.80	352.20	353.06	101	244	487	2002.70	2009.16	577	1380	2774
1.05	1.80	334.61	335.28	96	232	464	944.28	937.72	272	655	1311
1.10	1.80	318.58	318.86	91	222	441	476.68	470.88	140	334	660
1.16	1.80	300.77	300.26	86	209	417	231.54	226.16	71	161	318
1.21	1.80	287.64	287.06	83	200	399	136.80	130.32	44	97	188
1.26	1.80	275.80	275.15	79	192	383	85.86	79.86	29	62	117
1.00	1.40	352.20	353.06	101	244	487	1914.68	1917.56	552	1322	2644.25
1.05	1.40	334.61	335.28	96	232	464	924.48	918.08	266	641	1286
1.10	1.40	318.58	318.86	91	222	441	471.47	465.63	139	330	653
1.16	1.40	300.77	300.26	86	209	417	230.33	224.91	70	161	317
1.21	1.40	287.64	287.06	83	200	399	136.35	129.88	44	96	187
1.26	1.40	275.80	275.15	79	192	383	85.69	79.73	29	61	117
1.00	1.00	500.27	501.33	145	346	692	500.08	496.99	146	348	693
1.05	1.00	475.33	477.19	137	330	657	391.55	386.50	116	272	541
1.10	1.00	452.68	454.01	131	314	625	279.08	275.34	83	195	386
1.16	1.00	428.13	429.69	123	297	592	173.21	168.53	53	122	239
1.21	1.00	409.31	409.76	118	284	566	114.54	109.24	37	81	157
1.26	1.00	391.85	391.63	113	273	542	76.82	71.25	26	55	105

Table 3.9: EWMA-Rate and Max-EWMA chart when X ~ Gamma(4,1), T ~ Exp(0.01), ARL_0=370 with θ=0.25 and ω=0.25, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.29	θ=0.25	L=0.0628			ARL_0=370.33	ω=0.25	L=4.339		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	115.98	116.55	33	80	160	1499.16	1500.80	432	1036	2075
1.05	3.00	109.93	110.05	32	76	151	776.73	773.51	223	538	1081
1.10	3.00	104.46	104.47	30	73	144	427.96	422.83	126	299	595
1.16	3.00	98.66	98.83	28	69	136	224.57	219.50	68	157	310
1.21	3.00	94.02	93.95	27	65	130	138.69	133.77	43	98	191
1.26	3.00	89.99	89.95	26	62	124	89.97	85.42	29	64	124
1.00	2.60	134.93	135.18	39	93	186	1499.16	1500.80	432	1036	2075
1.05	2.60	128.24	128.50	37	89	177	776.73	773.51	223	538	1081
1.10	2.60	122.00	122.33	35	84	168	427.96	422.83	126	299	595
1.16	2.60	115.12	115.50	33	80	158.25	224.57	219.50	68	157	310
1.21	2.60	109.86	110.06	32	76	151	138.69	133.77	43	98	191
1.26	2.60	105.21	105.38	30	73	145	89.97	85.42	29	64	124
1.00	2.20	161.73	162.10	46	112	224	1499.16	1500.80	432	1036	2075
1.05	2.20	153.65	154.05	44	106	213	776.73	773.51	223	538	1081
1.10	2.20	145.98	146.31	42	101	202	427.96	422.83	126	299	595
1.16	2.20	137.63	138.02	39	95	190	224.57	219.50	68	157	310
1.21	2.20	131.40	131.60	38	91	181	138.69	133.77	43	98	191
1.26	2.20	125.88	126.14	36	87	174	89.97	85.42	29	64	124
1.00	1.80	200.39	200.38	58	138	278	1497.00	1498.16	431	1034	2072
1.05	1.80	190.06	190.11	55	131	263	776.02	772.76	223	537	1080
1.10	1.80	181.01	181.38	52	125	251	427.70	422.55	126	298	594
1.16	1.80	170.90	170.98	49	118	238	224.54	219.46	68	157	310
1.21	1.80	163.30	163.51	47	113	227	138.68	133.76	43	98	191
1.26	1.80	156.22	156.42	45	108	217	89.96	85.76	29	64	124
1.00	1.40	261.81	261.71	75	182	363	1392.34	1393.58	401	959	1931
1.05	1.40	248.65	248.48	72	172	346	764.58	744.74	214	516	1037
1.10	1.40	236.26	235.81	68	163	329	418.50	413.82	123	292	581
1.16	1.40	223.36	222.78	64	154	310	222.06	216.98	67	155	307
1.21	1.40	213.28	212.69	62	147	296	137.74	132.81	43	97	190
1.26	1.40	204.36	204.01	59	141	283	89.56	85.02	29	64	123
1.00	1.00	370.29	370.59	106	257	512	370.33	367.83	108	257	513
1.05	1.00	352.49	353.20	101	244.50	488	302.18	300.23	88	210	417
1.10	1.00	335.58	336.11	96	233	465	230.25	227.7	68	160	318
1.16	1.00	317.80	317.61	91	221	440	155.56	152.12	47	109	215
1.21	1.00	303.81	303.46	87	211	422	109.19	105.68	34	77	150
1.26	1.00	291.03	290.37	84	202	404	76.86	72.77	25	55	105

Table 3.10: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.25 and ω=0.25, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.34	θ=0.25	L=0.0897			ARL_0=500.34	ω=0.25	L=4.651		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	158.53	158.77	46	110	219	2249.65	2253.42	647	1549	3124
1.05	3.00	150.51	150.84	43	104	208	1143.93	1138.12	330	794	1584
1.10	3.00	142.97	143.15	41	99	198	610.48	607.49	176	423	845
1.16	3.00	134.92	134.99	39	93	186	310.76	306.65	93	217	430
1.21	3.00	128.80	128.89	37	89	178	187.41	182.16	57	132	258
1.26	3.00	123.55	123.54	35	86	171	118.38	113.27	38	84	162
1.00	2.60	184.68	184.56	53	128	256	2249.65	2253.42	647	1549	3124
1.05	2.60	175.47	175.28	50	121	244	1143.93	1138.12	330	794	1584
1.10	2.60	166.84	166.80	48	115	232	610.48	607.49	176	423	845
1.16	2.60	157.43	157.58	45	109	218	310.76	306.65	93	217	430
1.21	2.60	150.49	150.76	43	104	208	187.41	182.16	57	132	258
1.26	2.60	143.94	143.95	41	100	199	118.38	113.27	38	84	162
1.00	2.20	220.84	220.65	64	153	306	2249.60	2253.39	647	1549	3124
1.05	2.20	209.39	209.48	61	145	290	1143.93	1138.12	330	794	1584
1.10	2.20	199.32	199.00	58	138	276	610.48	607.49	176	423	845
1.16	2.20	188.16	187.98	54	130	261	310.76	306.65	93	217	430
1.21	2.20	179.93	179.76	52	124	250	187.41	182.16	57	132	258
1.26	2.20	172.28	172.23	49	119	240	118.38	113.27	38	84	162
1.00	1.80	272.60	272.23	79	189	379	2246.76	2250.18	646	1547	3120.25
1.05	1.80	259.58	259.42	75	180	361	1143.00	1136.99	330	794	1583
1.10	1.80	246.54	246.00	71	171	343	610.23	607.11	176	423	845
1.16	1.80	233.04	232.08	67	161	324	310.68	306.59	93	217	430
1.21	1.80	222.63	221.71	64	154	309	187.39	182.14	57	132	258
1.26	1.80	213.05	212.78	61.75	147	296	118.37	113.26	38	84	162
1.00	1.40	353.81	354.40	102	245	489	2089.58	2086.19	601	1446	2898
1.05	1.40	336.18	336.80	96	233	466	1101.01	1096.42	317	762	1528
1.10	1.40	320.57	320.53	92	223	444	597.66	595.11	172	414	425
1.16	1.40	302.96	302.53	87	211	420	307.25	303.36	92	214	425
1.21	1.40	289.80	289.08	84	202	402	186.23	180.92	57	131	257
1.26	1.40	277.87	277.17	80	193	386	117.88	112.80	37	83	162
1.00	1.00	500.34	501.06	145	346	692	500.34	498.02	145	347	694
1.05	1.00	475.94	477.46	138	330	658	412.28	409.17	121	286	572
1.10	1.00	453.27	454.96	131	314	626	314.76	311.96	93	219	435
1.16	1.00	428.84	430.26	124	298	593	210.62	207.53	63	147	291
1.21	1.00	410.35	410.52	119	285	568	146.37	142.17	45	103	202
1.26	1.00	393.03	392.75	113	273	544	100.88	96.52	32	71	139

Table 3.11: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.05 and ω=0.05, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.02	θ=0.05	L=0.03412			ARL_0=370.01	ω=0.05	L=2.771		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	180.28	184.14	48	123	252	22.31	4.27	19	22	25
1.05	3.00	168.75	172.43	45	115	235	22.11	4.39	19	22	24
1.10	3.00	157.99	161.47	42	108	220	21.62	4.61	19	21	24
1.16	3.00	146.58	149.78	39	100	203	20.32	5.04	18	20	23
1.21	3.00	137.99	141.16	37	94	191	18.62	5.35	15	19	22
1.26	3.00	130.25	133.26	35	89	180	16.54	5.40	12	17	20
1.00	2.60	218.88	222.89	59	150	360	25.63	6.13	21	25	29
1.05	2.60	204.89	209.31	55	140	285	25.32	6.20	21	24	29
1.10	2.60	192.03	196.18	51	131	268	24.49	6.36	20	24	28
1.16	2.60	178.96	182.68	48	122	250	22.46	6.60	19	22	26
1.21	2.60	168.95	172.26	45	116	236	20.01	6.61	16	20	24
1.26	2.60	159.56	163.10	43	109	222	17.31	6.29	13	17	22
1.00	2.20	272.97	278.84	74	187	381	32.12	10.21	25	30	37
1.05	2.20	256.61	261.86	70	176	359	31.42	10.20	24	30	36
1.10	2.20	240.71	244.80	66	165	337	29.61	9.98	23	28	35
1.16	2.20	224.23	228.41	61	154	313	25.80	9.43	20	25	31
1.21	2.20	211.81	216.21	57	145	295	21.90	8.59	16	21	27
1.26	2.20	200.74	204.93	54	137	279	18.19	7.48	13	17	23
1.00	1.80	351.83	359.01	96	241.50	489	49.76	23.59	33	44	60
1.05	1.80	331.06	338.73	90	227	460	46.99	22.33	32	42	57
1.10	1.80	312.56	319.37	85	214	435	40.78	19.37	28	37	50
1.16	1.80	292.17	297.94	79	200	407	31.19	14.91	21	29	39
1.21	1.80	276.76	282.44	75	190	386	24.23	11.51	16	22	31
1.26	1.80	262.60	268.17	71	180	367	19.01	8.79	12	18	24
1.00	1.40	471.31	477.63	132	325	654	151.30	119.95	67	116	198
1.05	1.40	448.85	455.37	125	308	624	112.07	85.75	52	88	146
1.10	1.40	425.09	433.00	117	292	591	66.97	47.01	34	55	87
1.16	1.40	399.37	407.13	110	274	555	37.66	23.16	21	32	48
1.21	1.40	379.48	387.28	104	261	526	26.01	14.35	16	23	33
1.26	1.40	361.11	368.77	98	248	500	19.45	9.69	12	17	24
1.00	1.00	370.02	336.73	136	271	502	370.01	359.77	115	259	508
1.05	1.00	417.70	392.38	143	300	571	175.30	160.82	61	126	237
1.10	1.00	458.43	442.06	148	324	629	76.51	61.82	33	59	101
1.16	1.00	491.04	482.57	151	344	677	38.35	25.21	21	32	49
1.21	1.00	504.43	502.35	150	350.50	696	25.95	14.81	15	23	33
1.26	1.00	505.38	507.27	146	350	700	19.34	9.78	12	17	24

Table 3.12: EWMA-Rate and Max-EWMA chart when X ~ Gamma(7.5,1), T ~ Exp(0.01), ARL_0=500 with θ=0.05 and ω=0.05, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=500.00	θ=0.05	L=0.034454			ARL_0=500.11	ω=0.05	L=3.011		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	181.40	185.22	49	124	253	24.49	4.77	21	24	27
1.05	3.00	169.88	173.47	46	116	237	24.30	4.86	21	24	27
1.10	3.00	159.03	162.35	42	109	222	23.77	5.09	21	23	27
1.16	3.00	147.69	150.83	39	101	205	22.30	5.57	19	22	25
1.21	3.00	139.06	142.12	37	95	193	20.35	5.90	17	21	24
1.26	3.00	131.26	134.18	35	89	182	17.95	5.93	13	18	22
1.00	2.60	220.22	224.38	60	150	308	28.39	6.92	24	27	32
1.05	2.60	206.25	210.57	56	141	287	28.07	7.03	23	27	32
1.10	2.60	193.35	197.44	52	132	269	27.16	7.19	23	26	31
1.16	2.60	180.16	183.84	48	123	251	24.82	7.39	20	24	29
1.21	2.60	170.11	173.44	46	116	238	21.94	7.34	17	22	27
1.26	2.60	160.71	164.20	43	110	224	18.82	6.94	13	19	23
1.00	2.20	274.86	280.67	74	188	383	36.36	12.05	28	34	42
1.05	2.20	258.15	263.35	70	177	361	35.57	11.96	27	33	41
1.10	2.20	242.20	246.63	66	166	339	33.42	11.62	26	32	39
1.16	2.20	225.58	229.73	62	154	315	28.81	10.80	22	28	35
1.21	2.20	213.27	217.44	58	146	297	24.12	9.67	17	23	30
1.26	2.20	202.16	206.26	55	138	282	19.75	8.24	13	19	25
1.00	1.80	354.02	361.26	96	243	491	59.32	29.96	38	52	72
1.05	1.80	333.09	340.57	90	229	463	55.82	28.09	36	49	68
1.10	1.80	314.40	321.33	85	216	438	47.59	23.76	32	43	58
1.16	1.80	294.06	299.68	80	202	410	35.26	17.48	23	33	44
1.21	1.80	278.43	284.03	75	191	389	26.65	12.95	17	25	34
1.26	1.80	264.37	269.82	71	181	370	20.59	9.66	13	19	26
1.00	1.40	480.31	488.41	133	330	668	217.95	181.79	89	164	290
1.05	1.40	454.16	461.84	126	312	631	149.24	120.65	65	114	196
1.10	1.40	428.83	437.30	118	294	596	80.67	59.45	39	65	105
1.16	1.40	402.12	410.14	110	276	559	42.39	26.88	23	36	54
1.21	1.40	381.88	389.69	104	263	529	28.42	15.88	17	25	36
1.26	1.40	363.44	371.04	99	249	504	20.99	10.46	13	19	26
1.00	1.00	500.00	480.87	162	354	686	500.11	490.78	153	351	686
1.05	1.00	538.96	528.31	167	379	741	219.84	203.27	76	158	299
1.10	1.00	559.85	554.87	166	389.50	775	89.54	72.96	38	68	118
1.16	1.00	563.28	566.06	161	389	779	42.78	28.49	23	35	55
1.21	1.00	553.61	558.81	156	382	769	28.33	16.23	17	25	36
1.26	1.00	536.78	543.79	150	370	745	20.87	10.51	13	19	26

Table 3.13: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.10 and ω=0.10, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.21	θ=0.10	L=0.02505			ARL_0=370.06	ω=0.10	L=3.306		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	102.05	104.69	28	69	141	50.96	27.39	32	43	62
1.05	3.00	95.94	98.20	26	65	133	48.30	26.09	31	42	59
1.10	3.00	90.34	92.52	24	61	125	42.81	23.28	27	37	53
1.16	3.00	84.32	86.33	23	57	117	33.61	18.49	21	30	42
1.21	3.00	79.90	81.48	22	55	111	26.01	14.48	15	24	33
1.26	3.00	75.96	77.32	21	52	106	19.98	11.06	12	18	26
1.00	2.60	122.14	124.96	33	83	169	84.85	59.35	43	67	108
1.05	2.60	114.84	117.62	31	78	159	75.95	52.94	39	81	97
1.10	2.60	108.39	110.86	29	74	150	59.98	41.12	32	49	77
1.16	2.60	101.42	103.93	27	69	141	40.25	26.52	22	34	52
1.21	2.60	96.01	98.33	26	65	133	28.40	17.96	15	25	37
1.26	2.60	91.15	93.28	25	62	126	20.71	12.40	12	18	27
1.00	2.20	149.90	152.55	40	103	209	206.31	178.88	78	152	278
1.05	2.20	141.06	143.78	38	97	196	149.78	129.02	59	111	200
1.10	2.20	133.20	136.23	36	91	184	89.50	74.02	38	68	119
1.16	2.20	125.15	128.16	34	86	173	47.25	35.92	22	38	62
1.21	2.20	118.59	121.57	32	81	164	30.20	20.91	15	25	39
1.26	2.20	112.71	115.47	30	77	156	21.16	13.33	12	18	27
1.00	1.80	191.25	194.29	52	131	266	672.37	650.87	208	473	924
1.05	1.80	180.66	184.00	49	124	251	279.58	265.24	90	199	382
1.10	1.80	170.63	173.81	46	117	238	115.26	103.85	42	84	156
1.16	1.80	159.91	162.90	43	110	223	50.80	41.03	22	39	67
1.21	1.80	151.63	154.56	41	104	211	30.94	22.22	15	25	40
1.26	1.80	144.22	147.23	39	99	200	21.31	13.66	12	18	27
1.00	1.40	257.96	261.44	72	178	360	1088.08	1073.35	322	760	1507
1.05	1.40	243.15	246.11	68	167	340	326.00	313.94	101	230	449
1.10	1.40	230.31	232.92	64	158	321	120.67	110.28	42	87	164
1.16	1.40	216.24	218.74	60	149	302	51.39	41.92	22	39	68
1.21	1.40	205.80	208.52	57	141	286	31.03	22.42	15	25	40
1.26	1.40	196.12	199.04	54	135	273	21.33	13.71	12	18	27
1.00	1.00	370.21	373.53	105	257	512	370.06	364.91	111	259	509
1.05	1.00	353.99	357.88	99	245	490	208.37	200.86	66	147	286
1.10	1.00	337.45	341.78	94	233	468	101.17	91.86	36	73	137
1.16	1.00	318.72	322.55	89	220	443	48.01	39.05	21	37	63
1.21	1.00	304.34	308.00	85	210	424	29.95	21.63	15	24	39
1.26	1.00	290.91	294.88	81	201	405	20.89	13.42	11	18	27

Table 3.14: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.10 and ω=0.10, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.82	θ=0.10	L=0.0397			ARL_0=500.26	ω=0.10	L=3.571		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	140.85	142.29	39	97	195	91.14	63.10	47	72	116
1.05	3.00	132.75	134.26	37	91	183	83.03	57.24	43	66	106
1.10	3.00	125.63	127.24	35	86	174	67.40	45.93	36	55	86
1.16	3.00	117.57	119.08	33	81	163	45.64	30.17	35	39	59
1.21	3.00	111.51	112.84	31	76	154	31.94	20.25	17	28	41
1.26	3.00	106.06	107.15	30	73	147	23.02	13.86	13	20	30
1.00	2.60	167.81	169.65	47	116	233	197.13	166.89	78	147	264
1.05	2.60	158.05	159.50	44	109	220	155.47	130.02	63	117	206
1.10	2.60	149.22	150.58	41	103	207	100.04	81.23	43	77	133
1.16	2.60	140.00	141.58	39	96	194	53.90	40.67	25	43	71
1.21	2.60	132.88	134.54	37	91	183	34.13	23.71	17	28	44
1.26	2.60	126.67	128.53	35	87	175	23.55	14.95	13	20	30
1.00	2.20	204.90	206.11	58	141	285	610.76	584.49	195	432	837
1.05	2.20	193.21	194.53	55	133	268	311.12	292.69	103	222	426
1.10	2.20	182.98	184.80	51	126	254	136.42	123.18	49	99	184
1.16	2.20	171.58	173.35	48	118	239	59.39	48.31	25	46	79
1.21	2.20	163.21	164.84	45	113	228	35.25	25.69	17	28	46
1.26	2.20	155.10	156.54	43	107	216	23.77	15.41	13	20	30
1.00	1.80	259.45	261.47	73	179	361	1468.52	1447.00	432	1030	2033
1.05	1.80	244.81	246.28	70	169	341	428.65	417.19	132	300	590
1.10	1.80	231.76	232.88	66	160	323	151.42	140.31	52	109	206
1.16	1.80	217.81	218.73	62	150	303	60.93	50.57	25	46	81
1.21	1.80	207.25	208.46	59	143	288	35.50	26.15	17	28	46
1.26	1.80	197.68	198.99	56	136	274	23.82	15.53	13	20	30
1.00	1.40	344.24	347.26	97	238	476	1668.93	1655.26	488	1165	2311
1.05	1.40	326.26	328.26	92	226	453	443.44	433.64	134	310	611
1.10	1.40	309.83	311.76	87	214	430	152.72	141.75	52	109	208
1.16	1.40	291.74	293.50	82	202	406	61.05	50.81	25	46	81
1.21	1.40	277.96	279.44	79	192	388	35.51	26.19	17	28	46
1.26	1.40	265.49	267.20	75	184	370	23.81	15.54	13	20	30
1.00	1.00	500.82	504.98	143	346	693	500.26	494.65	148	348	691
1.05	1.00	474.63	478.40	136	328	656	276.51	267.65	86	195	381
1.10	1.00	450.65	454.62	128	310	623	128.03	118.01	44	92	174
1.16	1.00	425.12	428.07	121	294	589	57.19	47.38	24	43	76
1.21	1.00	405.78	408.86	115	281	562	34.37	25.30	17	27	45
1.26	1.00	387.73	390.87	110	268	536	23.39	15.26	13	19	30

Table 3.15: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.15 and ω=0.15, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.54	θ=0.15	L=0.0366			ARL_0=370.12	ω=0.15	L=3.679		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	108.71	109.81	31	75	150	1455.40	1437.14	425	1012	2022.25
1.05	3.00	102.62	103.81	29	71	142	461.17	453.82	137	323	638
1.10	3.00	97.03	98.02	27.75	67	134	177.21	170.04	56	125	243
1.16	3.00	91.14	91.92	26	63	126	72.75	65.26	27	53	98
1.21	3.00	86.77	87.54	25	60	120	41.11	34.11	17	31	54
1.26	3.00	82.71	83.26	23	57	115	26.44	19.93	12	21	34
1.00	2.60	127.97	129.23	36	88	177	1455.40	1437.14	425	1012	2022.25
1.05	2.60	121.15	122.35	34	84	168	461.17	453.82	137	323	638
1.10	2.60	114.73	115.89	32	79	159	177.21	170.04	56	125	243
1.16	2.60	108.01	109.14	30	74	149	72.75	65.26	27	53	98
1.21	2.60	102.81	103.99	29	71	142	41.11	34.11	17	31	54
1.26	2.60	97.86	98.85	28	67	135	26.44	19.93	12	21	34
1.00	2.20	154.84	156.05	43	107	215	1455.40	1437.14	425	1012	2022.25
1.05	2.20	146.36	147.25	41	101	203	461.17	453.82	137	323	638
1.10	2.20	138.79	139.78	39	96	192	177.21	170.04	56	125	243
1.16	2.20	130.80	132.28	37	90	181	72.75	65.26	27	53	98
1.21	2.20	124.81	126.30	35	86	172	41.11	34.11	17	31	54
1.26	2.20	119.00	120.23	33	82	164	26.44	19.93	12	21	34
1.00	1.80	194.78	195.38	55	134	271	1455.17	1436.82	425	1012	2022
1.05	1.80	184.28	185.20	52	127	256	461.12	453.79	137	323	638
1.10	1.80	175.12	176.49	49	121	243	177.20	170.04	56	125	243
1.16	1.80	164.53	165.67	46	114	229	72.75	65.26	27	53	98
1.21	1.80	156.65	157.61	44	108	218	41.11	34.11	17	31	54
1.26	1.80	149.60	150.73	42	103	208	26.44	19.93	12	21	34
1.00	1.40	257.91	258.65	74	179	359	1409.69	1391.38	410	981	1962
1.05	1.40	243.93	244.34	70	169	340	456.38	449.15	136	319	631
1.10	1.40	231.85	232.15	66	160	323	176.52	169.46	56	125	242
1.16	1.40	218.57	218.81	63	151	304	72.65	65.19	26	53	98
1.21	1.40	208.36	208.71	60	144	290	41.06	34.08	17	31	54
1.26	1.40	199.19	199.75	57	138	277	26.42	19.93	12	21	34
1.00	1.00	370.54	372.38	106	258	511	370.12	366.48	109	258	511
1.05	1.00	351.45	355.41	100	243	486	240.86	235.71	73	168	332
1.10	1.00	334.34	335.78	95	232	463	132.46	126.11	43	94	182
1.16	1.00	315.88	317.19	90	219	438	64.52	57.52	24	47	87
1.21	1.00	301.92	302.94	86	209	420	38.52	31.97	16	29	51
1.26	1.00	288.83	289.82	82	200	402	25.47	19.16	12	20	33

Table 3.16: EWMA-Rate and Max-EWMA chart when X ~ Gamma(7.5,1), T ~ Exp(0.01), ARL_0=500 with θ=0.15 and ω=0.15, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.86	θ=0.15	L=0.0552			ARL_0=500.10	ω=0.15	L=3.965		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	150.15	150.55	43	104	208	2230.41	2220.05	652	1553	3089
1.05	3.00	141.94	142.14	40	98	196	662.58	653.20	195	464	920
1.10	3.00	134.50	135.28	38	93	185	239.59	231.78	74	169	328
1.16	3.00	126.94	127.87	36	88	175	92.04	83.88	32	67	125
1.21	3.00	121.01	121.61	34	84	167	49.76	42.32	20	37	66
1.26	3.00	115.34	115.96	33	79	159	30.96	23.93	14	24	41
1.00	2.60	177.13	177.74	50	122	246	2230.41	2220.05	652	1553	3089
1.05	2.60	167.39	167.95	48	116	233	662.58	653.20	195	464	920
1.10	2.60	158.53	159.12	45	110	220	239.59	231.78	74	169	328
1.16	2.60	149.11	149.56	42	103	207	92.04	83.88	32	67	125
1.21	2.60	142.11	142.51	40	98	197	49.76	42.32	20	37	66
1.26	2.60	135.65	136.51	38	94	187	30.96	23.93	14	24	41
1.00	2.20	213.15	212.61	62	148	297	2230.41	2220.05	652	1553	3089
1.05	2.20	201.93	201.52	58	140	280	662.58	653.20	195	464	920
1.10	2.20	191.79	192.00	55	133	266	239.59	231.78	74	169	328
1.16	2.20	180.81	181.30	51	125	251	92.04	83.88	32	67	125
1.21	2.20	172.45	173.11	49	119	240	49.76	42.32	20	37	66
1.26	2.20	164.36	164.91	47	114	228	30.96	23.93	14	24	41
1.00	1.80	266.48	266.41	76	185	370	2230.13	2219.49	652	1552	3088
1.05	1.80	252.51	252.46	73	175	352	662.56	653.16	195	464	920
1.10	1.80	239.65	239.14	69	166	334	239.59	231.78	74	168	328
1.16	1.80	225.97	225.69	65	156	314	92.04	83.87	32	67	125
1.21	1.80	215.54	215.21	62	149	299	49.76	42.32	20	37	66
1.26	1.80	206.26	206.07	59	143	286	30.95	23.93	14	24	41
1.00	1.40	348.88	350.35	100	242	481	2159.56	2148.78	629	1505	2985
1.05	1.40	330.96	331.33	94	230	459	656.07	647.05	193	459	912
1.10	1.40	315.18	315.71	90	219	437	239.79	231.04	74	168	327
1.16	1.40	297.44	297.93	85	206	413	91.91	83.77	32	67	125
1.21	1.40	284.56	284.48	82	198	396	49.70	42.28	20	37	66
1.26	1.40	272.40	272.38	78	189	379	30.94	23.92	14	24	41
1.00	1.00	500.86	502.65	144	347	693	500.10	497.36	146	348	690
1.05	1.00	475.33	477.73	137	330	656	328.96	323.31	99	230	453
1.10	1.00	452.01	453.92	131	312	625	176.41	169.95	55	124	242
1.16	1.00	427.01	429.11	123	296	591	81.72	74.16	29	59	111
1.21	1.00	408.84	410.43	117	283	565	46.76	39.64	19	35	62
1.26	1.00	391.29	392.72	112	271	541	29.93	23.11	14	23	39

Table 3.17: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.20 and ω=0.20, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.35	θ=0.20	L=0.0459			ARL_0=370.48	ω=0.20	L=3.973		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	112.99	113.62	32	78	156	1773.99	1761.65	518	1233	2463.25
1.05	3.00	107.00	107.71	30	74	147	621.46	614.54	183	435	861
1.10	3.00	101.72	102.57	29	70	140	252.71	246.37	77	177	348
1.16	3.00	95.58	96.04	27	66	132	103.68	97.42	34	74	142
1.21	3.00	91.21	91.56	26	63	126	56.70	51.01	21	41	77
1.26	3.00	87.08	87.47	25	60	120	34.74	29.31	14	26	46
1.00	2.60	132.01	132.58	37	91	182	1773.99	1761.65	518	1233	2463.25
1.05	2.60	125.44	126.05	36	87	173	621.46	614.54	183	435	861
1.10	2.60	119.09	119.93	34	82	164	252.71	246.37	77	177	348
1.16	2.60	112.48	113.17	32	77	155	103.68	97.42	34	74	142
1.21	2.60	107.19	107.88	31	74	147	56.70	51.01	21	41	77
1.26	2.60	102.51	103.32	29	71	141	34.74	29.31	14	26	46
1.00	2.20	158.82	159.51	45	110	220	1773.99	1761.65	518	1233	2463.25
1.05	2.20	150.48	151.04	43	104	209	621.46	614.54	183	435	861
1.10	2.20	143.02	143.36	41	99	198	252.71	246.37	77	177	348
1.16	2.20	134.71	135.59	38	93	186	103.68	97.42	34	74	142
1.21	2.20	128.98	129.95	37	89	178	56.70	51.01	21	41	77
1.26	2.20	123.32	123.94	35	85	170	34.74	29.31	14	26	46
1.00	1.80	198.09	197.54	57	137	275	1772.26	1759.80	518	1232	2461
1.05	1.80	187.87	187.61	54	130	261	621.21	614.25	183	861	435
1.10	1.80	178.68	179.20	51	123	248	252.68	246.36	77	177	348
1.16	1.80	168.61	169.03	48	117	234	103.67	97.40	34	74	142
1.21	1.80	160.67	161.24	46	111	224	56.69	51.01	21	41	77
1.26	1.80	153.57	154.31	44	106	213	34.74	29.31	14	26	46
1.00	1.40	260.19	259.84	75	181	362	1658.40	1642.16	483	1153	2306
1.05	1.40	246.40	245.98	71	170	343	606.43	599.00	179	425	841
1.10	1.40	234.64	234.21	68	162	327	250.06	243.84	76	175	344
1.16	1.40	221.44	220.65	64	154	308	103.25	97.08	34	73	141
1.21	1.40	211.49	211.22	61	146	294	56.56	50.88	21	41	76
1.26	1.40	202.45	202.07	58	140	281	34.70	29.28	14	26	46
1.00	1.00	370.35	371.60	107	258	511	370.48	368.05	108	258	512
1.05	1.00	351.97	353.26	101	244	486	268.53	265.43	80	187	371
1.10	1.00	334.91	335.47	96	232	464	165.58	161.10	51	116	228
1.16	1.00	317.14	317.67	91	220	439	86.06	80.72	29	61	118
1.21	1.00	303.27	303.46	87	210	421	51.20	45.92	19	37	69
1.26	1.00	290.36	290.34	83	202	404	32.78	27.58	13	25	44

Table 3.18: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.20 and ω=0.20, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=500.34	θ=0.20	L=0.0669			ARL_0=500.21	ω=0.20	L=4.274		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	154.68	154.84	44	107	214	2787.80	2783.58	802	1933	3873
1.05	3.00	146.49	146.66	42	102	203	933.51	924.83	272	653	1295
1.10	3.00	139.19	139.49	40	96	192	358.91	351.60	107	251	497
1.16	3.00	131.40	132.11	37	91	181	138.68	132.65	44	98	190
1.21	3.00	125.71	126.16	36	87	173	72.73	66.46	26	53	98
1.26	3.00	120.04	120.48	34	83	166	42.69	36.84	17	32	57
1.00	2.60	181.26	181.47	52	125	251	2787.80	2783.58	802	1933	3873
1.05	2.60	171.86	172.10	49	119	239	933.51	924.83	272	653	1295
1.10	2.60	163.09	163.45	47	113	227	358.91	351.60	107	251	497
1.16	2.60	153.72	153.97	44	107	213	138.68	132.65	44	98	190
1.21	2.60	146.68	146.76	42	102	203	72.73	66.46	26	53	98
1.26	2.60	140.37	140.82	40	97	194	42.69	36.84	17	32	57
1.00	2.20	217.26	216.43	63	151	302	2787.80	2783.58	802	1933	3873
1.05	2.20	205.83	204.96	60	143	285	933.51	924.83	272	653	1295
1.10	2.20	195.93	195.45	57	136	272	358.91	351.60	107	251	497
1.16	2.20	184.97	184.83	53	128	257	138.68	132.65	44	98	190
1.21	2.20	176.73	176.88	51	122	246	72.73	66.46	26	53	98
1.26	2.20	168.98	169.17	48	117	235	42.69	36.84	17	32	57
1.00	1.80	269.45	269.06	78	187	374	2785.6	2780.73	801	1932	3869.25
1.05	1.80	256.04	255.15	74	178	356	933.23	924.44	272	653	1295
1.10	1.80	243.12	242.27	70	169	338	358.83	351.52	107	497	250.50
1.16	1.80	229.76	228.74	67	159	319	138.68	132.64	44	98	190
1.21	1.80	219.51	218.74	64	152	305	72.73	66.46	26	53	98
1.26	1.80	209.94	209.24	61	146	291	42.69	36.84	17	32	57
1.00	1.40	350.75	351.94	101	243	484	2593.28	2584.26	752	1803	3595
1.05	1.40	333.02	333.17	95	231	462	911.26	902.22	265	636	1265
1.10	1.40	317.82	318.01	91	220	440	355.34	348.04	106	248	492.25
1.16	1.40	300.02	299.30	86	208	417	138.18	132.19	44	98	189
1.21	1.40	287.01	286.72	83	199	399	72.60	66.39	25	53	98
1.26	1.40	275.45	275.30	79	191	382	42.64	36.81	17	32	57
1.00	1.00	500.34	501.58	145	347	692	500.21	500.36	145	691	347
1.05	1.00	474.95	476.27	138	330	655	369.85	367.50	109	257	510
1.10	1.00	452.09	453.05	131	313	625	227.67	223.20	69	159	314
1.16	1.00	427.21	428.34	123	296	590	114.20	108.36	37	81	157
1.21	1.00	409.43	410.38	118	284	565	65.73	59.89	23	48	89
1.26	1.00	392.20	393.08	113	272	542	40.31	34.70	16	30	54

Table 3.19: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(7.5,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.25 and ω=0.25, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.08	θ=0.25	L=0.0534			ARL_0=370.29	ω=0.25	L=4.214		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	115.52	115.97	33	80	159	2104.64	2089.11	613	1468	2928.25
1.05	3.00	109.71	110.22	31	76	151	808.12	802.03	235	562	1121
1.10	3.00	104.38	104.90	30	72	144	346.56	340.05	103	242	481
1.16	3.00	98.44	98.77	28	68	136	144.80	140.05	45	102	199
1.21	3.00	93.88	94.24	27	65	129	78.48	73.53	26	56	107
1.26	3.00	89.89	90.18	26	62	124	46.67	42.11	17	34	63
1.00	2.60	134.52	134.93	38	93	185	2104.64	2089.11	613	1468	2928.25
1.05	2.60	127.88	128.22	37	89	177	808.12	802.03	235	562	1121
1.10	2.60	121.80	122.38	35	84	168	346.56	340.05	103	242	481
1.16	2.60	115.00	115.71	33	79	158	144.80	140.05	45	102	199
1.21	2.60	109.99	110.52	31	76	151	78.48	73.53	26	56	107
1.26	2.60	105.10	105.72	30	72	145	46.67	42.11	17	34	63
1.00	2.20	161.25	161.70	46	112	224	2104.62	2089.09	613	1468	2928.25
1.05	2.20	152.84	153.11	44	106	212	808.12	802.03	235	562	1121
1.10	2.20	145.41	145.66	42	101	201	346.56	340.05	103	242	481
1.16	2.20	137.32	137.92	39	95	189	144.80	140.05	45	102	199
1.21	2.20	131.40	132.24	37	91	181	78.48	73.53	26	56	107
1.26	2.20	125.99	126.56	36	87	174	46.67	42.11	17	34	63
1.00	1.80	199.98	199.31	58	138	278	2100.17	2083.32	611.75	1466	2923
1.05	1.80	189.89	189.41	55	132	263	807.30	801.09	235	562	1120
1.10	1.80	180.77	180.79	52	125	251	346.38	339.90	103	242	480
1.16	1.80	170.70	170.75	49	118	238	144.77	140.03	45	102	199
1.21	1.80	163.14	163.41	47	113	227	78.47	73.53	26	56	107
1.26	1.80	155.95	156.39	45	108	216	46.67	42.11	17	34	63
1.00	1.40	261.30	260.54	76	181	363	1868.73	1853.07	542	1303	2600
1.05	1.40	247.76	246.99	72	172	345	770.55	765.99	223	535	1068
1.10	1.40	236.03	235.16	68	163	328	339.42	333.41	100	237	471
1.16	1.40	223.33	222.13	65	155	310	143.56	138.83	45	101	197
1.21	1.40	213.38	212.57	62	148	296	78.15	73.23	26	56	107
1.26	1.40	204.32	203.45	59	142	284	46.54	42.00	17	34	63
1.00	1.00	370.08	370.71	107	258	510	370.29	368.26	108	257	513
1.05	1.00	351.97	353.12	101	244	486	289.93	287.11	85	202	400
1.10	1.00	335.27	335.83	96	233	464	196.92	193.65	59	138	271
1.16	1.00	317.79	317.78	91	221	440	110.67	106.39	35	78	152
1.21	1.00	304.00	303.61	87	211	422.25	67.64	63.11	23	48	92
1.26	1.00	291.26	290.70	84	202	405	42.79	38.32	16	31	58

Table 3.20: EWMA-Rate and Max-EWMA chart when X ~ Gamma(7.5,1), T ~ Exp(0.01), ARL_0=500 with θ=0.25 and ω=0.25, for different levels of shifts in a and λ_E, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=500.12	θ=0.25	L=0.0765			ARL_0=500.05	ω=0.25	L=4.525		
δ_a	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	3.00	157.50	157.71	45	109	218	3360.37	3371.33	963	2324	4662
1.05	3.00	149.44	149.70	43	104	207	1245.69	1235.31	363	866	1729
1.10	3.00	142.08	142.28	41	99	196	508.20	503.82	150	355	702
1.16	3.00	134.18	134.80	38	93	185	202.35	197.06	61	142	278
1.21	3.00	128.52	129.01	37	89	177	105.18	100.03	34	75	144
1.26	3.00	123.06	123.36	35	85	170	60.17	55.17	21	43	82
1.00	2.60	183.95	183.72	53	127	255	3360.37	3371.33	963	2324	4662
1.05	2.60	174.65	174.47	50	121	243	1245.69	1235.31	363	866	1729
1.10	2.60	166.09	166.07	48	115	230	508.20	503.82	150	355	702
1.16	2.60	156.55	156.75	45	109	217	202.35	197.06	61	142	278
1.21	2.60	149.64	149.78	43	104	207	105.18	100.03	34	75	144
1.26	2.60	143.29	143.61	41	100	198	60.17	55.17	21	43	82
1.00	2.20	219.61	218.60	64	152	305	3360.33	3371.18	963	2324	4662
1.05	2.20	208.45	207.63	61	144.50	289	1245.69	1235.31	363	866	1729
1.10	2.20	198.40	197.51	58	138	275	508.20	503.82	150	355	702
1.16	2.20	187.61	187.16	54	130	260	202.35	197.06	61	142	278
1.21	2.20	179.49	179.24	51	124	249	105.18	100.03	34	75	144
1.26	2.20	171.70	171.62	49	119	239	60.17	55.17	21	43	82
1.00	1.80	271.32	270.83	78	188	376	3352.67	3362.28	961	2320	4684.25
1.05	1.80	258.24	257.07	75	179	359	1244.65	1233.83	363	865	1728
1.10	1.80	245.35	244.23	71	170	341	507.97	503.54	150	355	702
1.16	1.80	232.11	231.04	67	161	323	202.33	197.05	61	142	278
1.21	1.80	221.95	220.63	65	154	308	105.17	100.02	34	75	144
1.26	1.80	212.57	211.79	82	147	295	60.16	55.17	21	43	82
1.00	1.40	352.15	353.11	101	244	485	2965.41	2970.30	848	2050	4119
1.05	1.40	334.53	334.84	96	232	463	1187.29	1176.45	345	824	1648
1.10	1.40	319.34	319.47	92	222	442	498.46	494.26	146	348	689
1.16	1.40	301.95	301.00	87	210	419	200.76	195.54	61	141	276
1.21	1.40	288.79	288.16	83	201	401	104.74	99.61	34	74	143
1.26	1.40	277.38	276.86	80	193	385	60.01	55.06	21	43	82
1.00	1.00	500.12	501.26	145	346	692	500.05	500.25	145	346	692
1.05	1.00	474.75	475.78	138	330	655	399.25	397.30	117	278	550
1.10	1.00	452.29	453.40	132	313	624	274.54	270.91	82	192	380
1.16	1.00	427.88	428.80	124	297	591	151.68	147.61	46	107	209
1.21	1.00	410.30	410.99	119	285	567	89.97	85.35	29	64	123
1.26	1.00	393.10	393.85	114	273	543	55.11	50.49	19	40	75

Table 3.21: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.05 and ω=0.05, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.35	θ=0.05	L=0.06744			ARL_0=370.15	ω=0.05	L=2.851		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	98.23	99.79	27	67	136	22.89	4.56	20	22	25
1.063	3.00	107.32	109.55	29	73	149	22.02	4.46	20	22	25
1.125	3.00	115.88	118.71	31	79	161	23.04	4.42	20	22	25
1.188	3.00	125.41	128.60	34	85	174	23.07	4.39	20	22	25
1.250	3.00	134.57	138.11	36	92	187	23.08	4.38	20	22	25
1.313	3.00	144.12	147.81	39	98	201	23.10	4.37	20	22	25
1.000	2.60	120.12	123.14	32	82	167	26.38	6.51	22	25	30
1.063	2.60	131.28	134.72	35	89	183	26.53	6.42	22	25	30
1.125	2.60	141.91	145.52	38	96	198	26.59	6.36	22	25	30
1.188	2.60	152.80	156.52	41	104	213	26.62	6.33	22	25	30
1.250	2.60	163.65	167.61	44	111	228	26.64	6.31	22	25	30
1.313	2.60	174.78	178.24	47	119	244	26.63	6.29	22	25	30
1.000	2.20	151.77	155.54	41	103	212	33.23	10.87	26	31	39
1.063	2.20	164.75	168.68	44	112	230	33.51	10.83	26	31	39
1.125	2.20	177.83	181.35	48	121	249	33.64	10.79	26	31	39
1.188	2.20	191.20	194.61	52	131	267	33.67	10.74	26	31	39
1.250	2.20	204.74	208.79	55	140	286	33.66	10.68	26	31	39
1.313	2.20	218.46	222.93	59	149	305	33.61	10.59	26	31	39
1.000	1.80	198.76	202.55	54	136	277	51.96	25.51	34	46	63
1.063	1.80	215.47	220.06	58	147	301	52.92	25.76	35	47	64
1.125	1.80	231.73	236.39	63	159	323	53.31	25.77	35	47	64
1.188	1.80	248.09	253.13	67	170	346	53.28	25.57	35	47	64
1.250	1.80	264.92	270.06	72	181	370	52.95	25.14	35	47	64
1.313	1.80	281.91	286.80	76	193	394	52.34	24.44	35	46	63
1.000	1.40	274.59	279.59	74	188	384	162.90	133.09	70	123	214
1.063	1.40	296.05	300.41	80	203	413	180.08	146.98	77	136	237
1.125	1.40	316.37	320.92	86	218	441	183.96	148.98	79	139	242
1.188	1.40	335.26	339.29	93	231	467	177.19	140.77	78	135	233
1.250	1.40	351.25	353.91	99	243	489	162.82	126.42	74	125	212
1.313	1.40	362.92	363.27	104	252	506	144.11	107.83	68	112	186
1.000	1.00	370.35	365.57	111	260	514	370.15	360.59	113	260	511
1.063	1.00	366.24	356.65	115	259	506	503.42	495.70	151	352	693
1.125	1.00	351.49	335.25	116	251	483	534.51	524.22	163	374	737
1.188	1.00	329.36	305.81	116	240	450	456.45	438.39	146	322	624
1.250	1.00	304.20	275.59	114	224	411	345.38	319.87	119	248	470
1.313	1.00	277.64	243.53	110	208	374	294.21	220.27	93	183	335

Table 3.22: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.05 and ω=0.05, for different levels of shifts in b and λ_E, $\delta_b \in (1.000,1.063,1.125,1.188,1.250,1.313)$ and $\delta_{\lambda E} \in (3.00,2.60,2.20,1.80,1.40,1.00)$

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.11	θ=0.05	L=0.0896			ARL_0=500.05	ω=0.05	L=3.134		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	122.40	123.71	34	84	170	25.59	5.19	22	25	28
1.063	3.00	133.37	135.11	37	92	185	25.68	5.10	22	25	28
1.125	3.00	144.15	146.11	40	99	200	25.73	5.05	22	25	28
1.188	3.00	155.11	157.09	43	106	216	25.75	5.03	22	25	28
1.250	3.00	165.99	167.91	46	114	231	25.75	5.02	22	25	28
1.313	3.00	177.15	178.70	49	122	247	25.75	5.01	22	25	28
1.000	2.60	149.15	151.12	42	102	208	29.83	7.58	25	29	34
1.063	2.60	162.03	164.06	45	111	226	29.98	7.45	25	29	34
1.125	2.60	174.58	176.14	49	120	244	30.05	7.45	25	29	34
1.188	2.60	187.78	189.23	52	129	262	30.07	7.43	25	29	34
1.250	2.60	200.76	202.72	56	138	280	30.09	7.41	25	29	34
1.313	2.60	214.15	216.94	59	147	298	30.08	7.40	25	29	34
1.000	2.20	186.55	188.08	52	128	260	38.53	13.26	29	36	45
1.063	2.20	201.97	204.01	56	139	281	38.87	13.24	30	36	45
1.125	2.20	217.77	220.29	61	149	304	39.00	13.19	30	36	45
1.188	2.20	233.44	236.15	65	160	325	39.03	13.15	30	36	45
1.250	2.20	248.70	251.74	69	171	346	39.02	13.10	30	36	45
1.313	2.20	265.09	268.29	74	182	370	38.96	13.02	30	36	45
1.000	1.80	241.97	244.85	67	166	337	64.36	34.10	41	56	79
1.063	1.80	261.30	264.58	73	179	364	65.67	34.55	42	57	80
1.125	1.80	281.04	284.25	78	193	392	66.14	34.64	42	57	81
1.188	1.80	301.29	304.35	83	207	420	66.15	34.45	42	57	80
1.250	1.80	320.99	324.18	89	222	447	65.83	34.04	42	57	80
1.313	1.80	341.33	345.34	95	235	476	65.17	33.25	42	57	79
1.000	1.40	332.40	335.71	92	229	463	250.65	217.76	97	185	336
1.063	1.40	358.90	363.15	100	247	500	288.26	251.16	110	211	386
1.125	1.40	384.84	388.42	107	265	536	299.80	260.63	115	220	400
1.188	1.40	411.81	415.63	115	284	573	291.29	251.41	114	215	388
1.250	1.40	437.22	441.78	121	301	609	267.50	225.90	108	199	356
1.313	1.40	463.79	468.90	129	320	646	232.86	191.33	97	175	308
1.000	1.00	500.11	504.21	139	346	696	500.05	488.92	151	350.50	691
1.063	1.00	537.02	541.89	150	371	747	711.53	704.51	211	495	981
1.125	1.00	574.17	579.50	161	396	798	794.72	788.04	235	554	1093
1.188	1.00	610.39	617.14	171	421	847	719.04	705.87	219	506	984
1.250	1.00	647.12	653.52	181	446	897	563.14	539.82	181	396	772
1.313	1.00	685.61	692.89	192	473	951	403.92	373.44	139	290	552

Table 3.23: EWMA-Rate and Max-EWMA chart when X $\sim$ Gamma(1,1), T $\sim$ Exp(0.01), ARL_0=370 with θ=0.10 and ω=0.10, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.26	θ=0.10	L=0.1042			ARL_0=370.05	ω=0.10	L=3.545		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	102.79	103.16	30	71	142	96.13	69.79	48	76	123
1.063	3.00	110.55	111.01	32	76	153	101.26	73.19	50	79	130
1.125	3.00	118.53	119.12	34	82	164	103.80	74.78	51	81	133
1.188	3.00	126.86	127.66	36	88	176	105.13	75.67	52	82	135
1.250	3.00	135.10	136.16	38	93	187	105.72	75.96	52	83	135
1.313	3.00	143.21	143.96	41	99	199	105.97	76.07	52	83	136
1.000	2.60	122.31	122.90	35	85	169	198.22	174.22	76	146	266
1.063	2.60	132.01	133.10	37	91	183	226.58	198.84	86	166	304
1.125	2.60	141.31	142.20	40	97	196	243.12	213.28	92	178	325
1.188	2.60	150.92	151.78	43	104	210	251.65	220.59	95	184	337
1.250	2.60	160.36	161.05	46	111	223	255.67	223.93	96	187	343
1.313	2.60	169.79	170.41	48	117	236	257.41	225.32	97	188	346
1.000	2.20	150.00	150.88	43	103	209	463.41	447.08	144	327	635.25
1.063	2.20	161.33	162.00	46	111	224	694.74	671.93	214	491	955
1.125	2.20	172.35	172.67	49	119	240	907.12	879.43	277	641	1252
1.188	2.20	183.66	183.16	53	127	256	1056.37	1025.93	324	745	1459
1.250	2.20	194.46	194.58	56	270	135	1137.10	1104.94	349	801	1569
1.313	2.20	205.80	205.78	59	142	287	1173.82	1140.41	361	827	1618
1.000	1.80	189.52	189.50	54	131	263	699.91	691.71	206	487	968
1.063	1.80	203.41	203.20	58	141	283	1457.18	1452.22	425	1010	2010
1.125	1.80	217.56	218.00	62	150	303	2954.92	2957.04	850	2045	4091
1.188	1.80	231.73	232.37	66	160	322	5557.98	5570.69	1601	3859.5	7685
1.250	1.80	245.50	246.15	70	170	340	**	**	**	**	**
1.313	1.80	259.37	259.91	74	179	360	**	**	**	**	**
1.000	1.40	253.49	253.91	73	175	353	728.49	723.44	213	506	1006
1.063	1.40	271.55	271.58	78	188	378	1593.90	1592.52	462	1103	2203
1.125	1.40	289.21	289.19	82	200	403	3583.52	3589.71	1027	2475	4976
1.188	1.40	307.69	307.67	87	213	428	8336.83	8313.73	2385.25	5805	11552
1.250	1.40	325.83	325.77	93	227	453	**	**	**	**	**
1.313	1.40	344.30	344.27	98	239	479	**	**	**	**	**
1.000	1.00	370.26	369.77	106	257	515.25	370.05	365.59	110	258	511
1.063	1.00	396.56	396.29	113	275	551	507.02	505.41	149	352	700
1.125	1.00	422.94	422.96	121	294	589	611.42	609.91	178	423	844
1.188	1.00	448.45	448.72	128	311	624	678.02	676.79	198	471	933
1.250	1.00	473.82	474.44	135	330	658	710.47	709.77	208	493	978
1.313	1.00	499.49	499.30	143	347	694	725.11	724.74	212	503	999

Table 3.24: EWMA-Rate and Max-EWMA chart when X ~ Gamma(1,1), T ~ Exp(0.01), ARL_0=500 with θ=0.10 and ω=0.10, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.16	θ=0.10	L=0.1531			ARL_0=500.43	ω=0.10	L=3.848		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	144.03	143.96	42	99	200	324.94	297.93	113	234	441
1.063	3.00	155.14	155.25	45	107	215	390.31	359.61	134	280	530
1.125	3.00	165.61	165.45	48	115	230	429.37	395.25	147	309	583
1.188	3.00	176.51	175.78	51	122	245	449.17	413.53	154	323	609
1.250	3.00	187.16	186.29	54	130	260	457.97	421.52	157	329	621
1.313	3.00	197.85	197.24	57	137	275	461.40	424.50	159	332	626
1.000	2.60	170.70	170.42	49	118	237	675.51	657.01	205	474	935
1.063	2.60	182.99	181.91	53	127	255	1080.62	1054.71	328	760	1498
1.125	2.60	195.20	194.48	56	135	271	1483.50	1452.58	447	1041	2049
1.188	2.60	207.89	207.03	60	144	289	1768.65	1730.83	537.75	1238	2438.25
1.250	2.60	220.83	220.36	64	153	307	1920.88	1881.66	585	1343	2655
1.313	2.60	233.81	233.83	67	162	324	1989.15	1943.86	606	1395	2750.25
1.000	2.20	206.57	205.63	60	143	288	940.34	931.09	277	656	1303
1.063	2.20	222.11	221.57	64	154	309	2022.52	2019.01	585	1402	2800
1.125	2.20	237.07	237.50	68	164	328	4157.49	4156.15	1200	2873	5744
1.188	2.20	252.07	251.57	73	175	350	7769.70	7740.35	2239	5404	10805
1.250	2.20	266.97	266.23	77	185	371	**	**	**	**	**
1.313	2.20	281.92	281.04	81	195	392	**	**	**	**	**
1.000	1.80	260.18	259.82	75	180	361	987.85	983.38	290	687	1366
1.063	1.80	278.72	277.71	80	193	388	2256.53	2254.14	651	1560	3134
1.125	1.80	297.00	296.09	85	206	413	5298.19	5287.76	1520	3673	7361
1.188	1.80	315.89	314.88	91	220	439	**	**	**	**	**
1.250	1.80	334.23	333.10	96	232	464	**	**	**	**	**
1.313	1.80	353.34	351.91	102	246	492	**	**	**	**	**
1.000	1.40	345.01	344.15	99	240	480	987.20	982.83	290	686	1366
1.063	1.40	369.62	368.29	106	257	514	2254.14	2252.43	650	1558	3132
1.125	1.40	394.00	392.87	113	274	547	5280.77	5262.95	1518	3660.50	7338
1.188	1.40	418.99	417.98	120	291	583	**	**	**	**	**
1.250	1.40	442.83	442.20	127	308	615	**	**	**	**	**
1.313	1.40	466.91	466.45	134	325	649	**	**	**	**	**
1.000	1.00	500.16	499.22	144	348	694	500.43	494.59	147	349	692
1.063	1.00	534.42	533.90	154	371	740	691.96	686.20	201	481	960
1.125	1.00	567.84	568.17	163	394	786	840.08	836.95	245	583	1164
1.188	1.00	600.92	600.94	172	415	833	927.11	923.70	270	644	1285
1.250	1.00	634.20	634.85	182	439	878	968.67	965.51	283	672	1345
1.313	1.00	669.09	668.52	192	463	926	986.60	981.49	288	685	1371

Table 3.25: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.15 and ω=0.15, for different levels of shifts in b and λ_E, $\delta_b \in (1.000,1.063,1.125,1.188,1.250,1.313)$ and $\delta_{\lambda E} \in (3.00,2.60,2.20,1.80,1.40,1.00)$

Shifts		**EWMA-Rate**					**Max-EWMA**				
		ARL_0=370.41	θ=0.15	L=0.138			ARL_0=370.26	ω=0.15	L=4.003		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	110.08	110.15	32	76	152	732.70	728.60	213	508	1013
1.063	3.00	117.99	118.02	34	82	163	1461.00	1460.99	424	1011	2018
1.125	3.00	126.02	125.95	36	88	175	2966.22	2965.26	858	2045	4120
1.188	3.00	134.20	134.49	39	93	186	6189.86	6199.54	1768	4286	8573
1.250	3.00	142.03	142.25	41	98	197	**	**	**	**	**
1.313	3.00	150.35	150.31	43	104	209	**	**	**	**	**
1.000	2.60	129.91	130.18	37	90	180	732.70	728.60	213	508	1013
1.063	2.60	139.16	139.38	40	96	193	1461.00	1460.99	424	1011	2018
1.125	2.60	148.35	148.36	43	103	206	2966.22	2965.26	858	2045	4120
1.188	2.60	157.87	157.93	46	109	219	6189.86	6199.54	1768	4286	8573
1.250	2.60	167.06	166.87	48	116	232	**	**	**	**	**
1.313	2.60	176.29	175.59	51	122	245	**	**	**	**	**
1.000	2.20	156.94	157.12	45	109	218	732.70	728.60	213	508	1013
1.063	2.20	167.93	167.75	49	116	233	1461	1460.99	424	1011	2018
1.125	2.20	178.79	178.18	52	124	249	2966.22	2965.26	858	2045	4120
1.188	2.20	189.54	188.79	55	131	263	6189.86	6199.54	1768	4286	8573
1.250	2.20	200.30	199.46	58	139	278	**	**	**	**	**
1.313	2.20	211.66	210.78	61	147	294	**	**	**	**	**
1.000	1.80	195.36	194.72	56	135	271	732.69	728.60	213	508	1013
1.063	1.80	209.10	208.19	60	145	291	1460.85	1460.89	424	1011	2018
1.125	1.80	222.95	222.17	64	155	310	2965.69	2964.49	858	2045	4119
1.188	1.80	236.80	236.01	68	164	328	6187.04	6196.76	1767	4283	8568.25
1.250	1.80	250.08	249.82	72	173	347	**	**	**	**	**
1.313	1.80	263.47	262.60	76	183	366	**	**	**	**	**
1.000	1.40	257.58	256.54	75	179	358	726.58	723.30	212	503	1004
1.063	1.40	275.15	273.83	80	191	383	1437.35	1436.24	417	994	1989
1.125	1.40	292.58	291.38	84	203	406.25	2865.81	2871.09	828	1974	3975
1.188	1.40	310.12	309.28	89	215	430	5768.45	5776.49	1641.75	3988	8006
1.250	1.40	327.75	326.76	94	228	455	**	**	**	**	**
1.313	1.40	345.44	343.90	100	240	480	**	**	**	**	**
1.000	1.00	370.41	368.03	107	258	516	370.26	366.41	109	258	512
1.063	1.00	395.81	394.32	114	275	549	492.01	488.21	143	342	680
1.125	1.00	421.03	419.35	121	293	585	590.61	586.87	171	411	818
1.188	1.00	445.96	444.31	129	311	620	658.66	653.24	191	460	911
1.250	1.00	470.41	469.71	135	327.5	652	697.99	693.24	203	487	964
1.313	1.00	494.79	494.07	142	344	686	718.90	713.34	209	501	992

Table 3.26: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.15 and ω=0.15, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.73	θ=0.15	L=0.197			ARL_0=500.06	ω=0.15	L=4.317		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	152.57	152.35	44	106	212	993.82	993.12	291	688	1373
1.063	3.00	163.45	163.16	47	113	227	2048.46	2050.98	587	1420.50	2841
1.125	3.00	173.87	173.03	51	120	242	4277.39	4281.27	1231	2963	5922.25
1.188	3.00	184.55	183.41	54	128	257	**	**	**	**	**
1.250	3.00	194.73	193.73	57	135	270	**	**	**	**	**
1.313	3.00	205.65	204.32	60	143	286	**	**	**	**	**
1.000	2.60	178.81	177.89	52	124	248	993.82	993.12	291	688	1373
1.063	2.60	190.95	189.81	55	133	265	2048.46	2050.98	587	1420.50	2841
1.125	2.60	203.13	201.73	59	141	283	4277.39	4281.27	1231	2963	5922.25
1.188	2.60	215.94	214.75	63	150	300	**	**	**	**	**
1.250	2.60	228.26	227.26	66	159	317	**	**	**	**	**
1.313	2.60	241.33	240.58	70	167	335	**	**	**	**	**
1.000	2.20	214.72	213.45	62	149	299	993.82	993.12	291	688	1373
1.063	2.20	229.64	228.58	66	160	319	2048.46	2050.98	587	1420.50	2841
1.125	2.20	244.53	243.96	71	170	339	4277.39	4281.27	1231	2963	5922.25
1.188	2.20	258.92	257.68	75	180	359	**	**	**	**	**
1.250	2.20	273.49	272.00	79	190	380	**	**	**	**	**
1.313	2.20	288.27	286.51	83	200	401	**	**	**	**	**
1.000	1.80	266.95	265.91	78	185	370	993.80	993.10	290.75	688	1373
1.063	1.80	285.10	283.31	83	198	396	2048.42	2050.96	587	1420	2841
1.125	1.80	303.07	301.79	87	210	421	4276.51	4280.77	1231	2962	5920
1.188	1.80	321.32	320.09	93	224	446	**	**	**	**	**
1.250	1.80	339.20	337.62	98	236	471	**	**	**	**	**
1.313	1.80	358.25	356.21	104	250	498	**	**	**	**	**
1.000	1.40	350.03	347.86	101	244	487	986.97	985.99	288	683	1366
1.063	1.40	373.99	371.53	108	261	521	2019.89	2026.15	580	1398	2795
1.125	1.40	397.61	395.86	115	276	551	4151.25	4168.81	1192	2866	5745
1.188	1.40	421.98	419.98	122	294	586	**	**	**	**	**
1.250	1.40	445.08	442.88	129	310	618	**	**	**	**	**
1.313	1.40	468.68	467.51	135	327	650	**	**	**	**	**
1.000	1.00	500.73	499.22	144	349	695	500.06	495.83	147	348	690
1.063	1.00	534.18	532.73	154	372	740	670.68	606.91	195	467	929
1.125	1.00	566.26	564.98	164	394	784	808.93	806.60	236	562	1118
1.188	1.00	598.84	597.41	172	414	330	901.84	898.32	264	627	1249
1.250	1.00	630.68	629.37	182	437	874	952.52	948.84	279	662	1318
1.313	1.00	664.76	662.74	192	461	920	977.70	971.77	287	679	1354

Table 3.27: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.20 and ω=0.20, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.22	θ=0.20	L=0.164			ARL_0=370.28	ω=0.20	L=4.347		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	113.88	113.84	33	79	157	735.31	731.10	213	511	1019
1.063	3.00	121.76	121.46	35	85	169	1378.88	1379.65	399	958.50	1905
1.125	3.00	129.86	129.86	37	90	180	2603.58	2598.78	751	1801	3615
1.188	3.00	137.80	137.94	40	95	191	5051.17	5047.68	1450	3494	7036
1.250	3.00	145.71	145.60	42	101	203	**	**	**	**	**
1.313	3.00	153.72	153.66	44	106	214	**	**	**	**	**
1.000	2.60	133.45	133.47	39	93	185	735.31	731.10	213	511	1019
1.063	2.60	142.61	142.46	41	99	198	1378.88	1379.65	399	958.50	1905
1.125	2.60	151.83	151.78	44	105	211	2603.58	2598.78	751	1801	3615
1.188	2.60	161.24	160.96	47	112	224	5051.17	5047.68	1450	3494	7036
1.250	2.60	170.26	169.80	49	118	236	**	**	**	**	**
1.313	2.60	179.43	178.70	52	124	249	**	**	**	**	**
1.000	2.20	160.36	160.09	46	111	223	735.31	731.10	213	511	1019
1.063	2.20	171.25	170.84	50	119	238	1378.88	1379.65	399	958.50	1905
1.125	2.20	181.84	181.17	53	126	253	2603.58	2598.78	751	1801	3615
1.188	2.20	192.50	191.24	56	134	268	5051.17	5047.68	1450	3494	7036
1.250	2.20	203.22	201.89	59	141	282	**	**	**	**	**
1.313	2.20	214.49	213.38	62	149	298	**	**	**	**	**
1.000	1.80	198.42	197.34	58	138	276	735.19	731.00	213	511	1019
1.063	1.80	211.92	210.81	61	147	295	1378.47	1379.35	399	958	1904
1.125	1.80	225.30	224.13	65	157	313	2602.30	2597.85	751	1800	3613
1.188	1.80	239.06	237.81	69	166	332	5046.36	5043	1448	3490.50	7029
1.250	1.80	252.21	251.59	73	175	350	**	**	**	**	**
1.313	1.80	265.45	264.19	77	184	369	**	**	**	**	**
1.000	1.40	259.44	258.48	76	180	360	724.62	721.29	210	502	1004
1.063	1.40	276.68	275.13	80	192	385	1342.72	1343.43	390	933	1853
1.125	1.40	293.71	292.15	85	204	408	2476.37	2475.58	715	1711.5	3436
1.188	1.40	311.09	309.87	90	216	432	4590.32	4587.36	1313	3172	6379
1.250	1.40	328.26	326.61	95	229	456	8208.58	8183.65	2362	5693	11406.25
1.313	1.40	345.52	343.28	100	241	480	**	**	**	**	**
1.000	1.00	370.22	367.40	107	258	515	370.28	368.05	109	257	510
1.063	1.00	395.09	392.83	114	275	548	481.98	479.52	141	336	665
1.125	1.00	419.64	417.28	121	292	583	575.69	573.09	168	402	795
1.188	1.00	444.11	441.90	128	310	617	644.85	640.82	187	450	892
1.250	1.00	467.92	466.79	135	326	650	688.08	685.13	200	480	949
1.313	1.00	491.82	490.99	141	312	682	712.00	707.71	207	496	984

Table 3.28: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=500 with θ=0.20 and ω=0.20, for different levels of shifts in b and λ_E, $\delta_b \in (1.000,1.063,1.125,1.188,1.250,1.313)$ and $\delta_{\lambda E} \in (3.00,2.60,2.20,1.80,1.40,1.00)$

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.60	θ=0.20	L=0.231			ARL_0=500.30	ω=0.20	L=4.669		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	156.91	156.55	45	109	218	994.91	990.81	291	692	1379
1.063	3.00	167.51	166.79	49	116	233	1921.96	1923.59	555	1332	2656
1.125	3.00	177.94	177.08	52	123	247	3736.58	3735.79	1073	2585	5175.25
1.188	3.00	188.42	187.27	55	131	262	7420.01	7444.98	2110.75	5132	10264
1.250	3.00	198.88	197.54	58	138	276	**	**	**	**	**
1.313	3.00	209.66	208.47	61	146	291	**	**	**	**	**
1.000	2.60	182.80	181.93	53	127	254	994.91	990.81	291	692	1379
1.063	2.60	194.91	193.53	57	135	271	1921.96	1923.59	555	1332	2656
1.125	2.60	206.96	205.42	60	144	288	3736.58	3735.79	1073	2585	5175.25
1.188	2.60	219.74	218.33	64	153	306	7420.01	7444.98	2110.75	5132	10264
1.250	2.60	231.90	230.54	67	161	322	**	**	**	**	**
1.313	2.60	244.77	243.51	71	170	340	**	**	**	**	**
1.000	2.20	218.56	217.14	63	152	304	994.91	990.81	291	692	1379
1.063	2.20	233.31	231.96	68	162	324	1921.96	1923.59	555	1332	2656
1.125	2.20	247.90	246.88	72	172	344	3736.58	3735.79	1073	2585	5175.25
1.188	2.20	262.08	260.67	76	182	364	7420.01	7444.98	2110.75	5132	10264
1.250	2.20	276.48	274.79	80	192	384	**	**	**	**	**
1.313	2.20	291.20	289.47	84	202	404	**	**	**	**	**
1.000	1.80	270.36	269.00	79	187	376	994.85	990.78	291	692	1379
1.063	1.80	288.06	286.47	83	200	400	1921.68	1923.48	555	1332	2656
1.125	1.80	305.82	304.53	88	212	424	3735.32	3734.75	1073	2584	5174
1.188	1.80	323.87	322.43	93	225	449	7414.53	7439.82	2108	5126.50	10256
1.250	1.80	341.40	339.33	99	238	474	**	**	**	**	**
1.313	1.80	360.31	357.78	104	251	501	**	**	**	**	**
1.000	1.40	351.98	349.39	102	245	489	982.40	978.22	287	683	1363
1.063	1.40	375.57	372.34	109	262	523	1878.53	1882.38	543	1300	2592
1.125	1.40	398.84	396.59	116	277	552	3578.09	3585.21	1027	2471	4963
1.188	1.40	423.16	420.50	122	295	588	6798.47	6818.75	1930	4700	9418
1.250	1.40	445.84	443.29	129	311	619	**	**	**	**	**
1.313	1.40	468.85	467.60	135	327	651	**	**	**	**	**
1.000	1.00	500.60	499.09	145	349	694	500.30	496.65	146	349	691
1.063	1.00	533.05	531.47	154	371	738.25	657.20	653.81	190	456	910
1.125	1.00	565.03	563.08	163.75	393	783	789.62	785.72	229	548	1092
1.188	1.00	597.30	595.78	172	413	827	881.35	876.27	256	613	1221
1.250	1.00	628.24	625.78	182	436	871	938.01	932.84	272	651	1298
1.313	1.00	661.42	659.27	191	458	916	969.70	962.93	283	674	1344

Table 3.29: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(1,1), T ∼ Exp(0.01), ARL_0=370 with θ=0.25 and ω=0.25, for different levels of shifts in b and λ_E, $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=370.00	θ=0.25	L=0.1853			ARL_0=370.16	ω=0.25	L=4.616		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	116.20	116.13	34	81	161	736.10	733.79	213	512	1019
1.063	3.00	124.20	123.83	36	86	172	1322.14	1321.83	383	921	1824.25
1.125	3.00	132.12	132.10	38	92	183	2385.35	2391.22	686	1645	3302
1.188	3.00	139.99	139.88	40	97	194	4376.77	4374.41	1252	3032	6095
1.250	3.00	147.83	147.58	43	102	205	**	**	**	**	**
1.313	3.00	155.94	155.75	45	108	217	**	**	**	**	**
1,000	2.60	135.68	135.56	39	94	188	736.10	733.79	213	512	1019
1.063	2.60	144.77	144.46	42	100	201	1322.14	1321.83	383	921	1824.25
1.125	2.60	153.96	153.76	45	106	214	2385.35	2391.22	686	1645	3302
1.188	2.60	163.24	162.79	47	113	227	4376.77	4374.41	1252	3032	6095
1.250	2.60	172.22	171.65	50	119	239	**	**	**	**	**
1.313	2.60	181.21	180.43	53	126	252	**	**	**	**	**
1.000	2.20	162.33	161.94	47	113	226	736.10	733.79	213	512	1019
1.063	2.20	173.11	172.44	50	120	240	1322.14	1321.83	383	921	1824.25
1.125	2.20	183.64	182.64	53	127	255	2385.35	2391.22	686	1645	3302
1.188	2.20	194.33	192.96	57	135	270	4376.77	4374.41	1252	3032	6095
1.250	2.20	204.90	203.45	60	143	285	**	**	**	**	**
1.313	2.20	215.97	214.86	63	150	300	**	**	**	**	**
1.000	1.80	200.20	199.03	58	139	278	735.84	733.47	213	512	1019
1.063	1.80	213.59	212.57	62	148	297	1321.19	1321.01	382	920	1823
1.125	1.80	226.90	225.19	66	158	315	2382.64	2388.54	685	1643	3297
1.188	1.80	240.28	238.91	70	167	334	4367.25	4364.34	1251	3026	6083
1.250	1.80	253.34	252.43	74	176	352	**	**	**	**	**
1.313	1.80	266.38	264.87	78	185	370	**	**	**	**	**
1.000	1.40	260.63	259.43	76	181	362	720.92	718.81	209	500	998
1.063	1.40	277.63	275.91	81	193	386	1273.81	1273.30	370	887	1756
1.125	1.40	294.38	292.61	85	205	409	2236.80	2245.09	643	1545	3093
1.188	1.40	311.57	310.05	90	216	432	3898.17	3896.09	1121	2693	5426
1.250	1.40	328.62	326.90	95	229	456	6568.34	6563.34	1874	4592	9107
1.313	1.40	345.51	343.27	100	241	480	**	**	**	**	**
1.000	1.00	370.00	366.96	107	258	515	370.16	368.63	109	257	511
1.063	1.00	394.48	391.71	114	275	548	475.11	473.95	139	331	655
1.125	1.00	418.92	416.50	121	292	581	564.65	563.64	164	393	780
1.188	1.00	442.88	440.17	128	308	616	632.41	629.31	184	441	876
1.250	1.00	466.28	465.00	134	325	647	677.92	675.39	197	472	938
1.313	1.00	490.32	489.06	141	342	681	704.72	702.24	204	490	975

Table 3.30: EWMA-Rate and Max-EWMA chart when X ~ Gamma(1,1), T ~ Exp(0.01), ARL_0=500 with θ=0.25 and ω=0.25, for different levels of shifts in b and λ_E, $\delta_b \in (1.000,1.063,1.125,1.188,1.250,1.313)$ and $\delta_{\lambda E} \in (3.00,2.60,2.20,1.80,1.40,1.00)$

Shifts		EWMA-Rate					Max-EWMA				
		ARL_0=500.40	θ=0.25	L=0.259			ARL_0=500.53	ω=0.25	L=4.95		
δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.000	3.00	159.43	158.95	46	110	222	996.73	993.20	291	694	1379
1.063	3.00	169.86	169.28	50	118	236	1841.74	1839.56	534	1277	2547
1.125	3.00	180.30	179.40	52	125	250	3403.54	3406.83	974	2362	4712.25
1.188	3.00	190.87	189.62	55	132	265	6398.45	6415.85	1826	4432	8871
1.250	3.00	201.27	199.91	59	140	280	**	**	**	**	**
1.313	3.00	212.00	210.67	62	147	295	**	**	**	**	**
1.000	2.60	185.22	184.11	54	129	258	996.73	993.20	291	694	1379
1.063	2.60	197.41	196.06	57	137	274	1841.74	1839.56	534	1277	2547
1.125	2.60	209.41	207.99	61	146	291	3403.54	3406.83	974	2362	4712.25
1.188	2.60	222.04	220.33	65	155	309	6398.45	6415.85	1826	4432	8871
1.250	2.60	234.17	232.70	68	163	325	**	**	**	**	**
1.313	2.60	246.63	245.35	72	171	343	**	**	**	**	**
1.000	2.20	220.70	219.17	64	154	307	996.73	993.20	291	694	1379
1.063	2.20	235.50	234.15	68	164	327	1841.74	1839.56	534	1277	2547
1.125	2.20	249.86	248.60	73	173	347	3403.54	3406.83	974	2362	4712.25
1.188	2.20	263.97	262.46	77	183	366	6398.45	6415.85	1826	4432	8871
1.250	2.20	278.36	276.46	81	193	387	**	**	**	**	**
1.313	2.20	292.94	291.00	85	204	406	**	**	**	**	**
1.000	1.80	272.24	270.98	79	189	378	996.45	992.98	291	693	1378
1.063	1.80	289.49	287.58	84	201	402	1840.97	1838.91	534	1277	2545
1.125	1.80	307.46	306.17	89	213	427	3400.35	3403.45	973	2360	4708
1.188	1.80	325.16	323.38	94	226	452	6387.93	6404.58	1823	4418	8859
1.250	1.80	342.59	340.05	99	239	476	**	**	**	**	**
1.313	1.80	361.47	358.51	105	252	503	**	**	**	**	**
1.000	1.40	353.20	350.25	103	246	491	979.01	976.10	285	680	1356
1.063	1.40	376.49	373.18	110	263	524	1783.9	1783.19	518	1236	2460
1.125	1.40	399.88	397.28	116	278	554	3214.99	3219.14	923	2232	4456
1.188	1.40	423.69	420.53	122	296	589	5751.54	5749.24	1646.75	3985	8000.25
1.250	1.40	446.07	443.24	129	311	620	**	**	**	**	**
1.313	1.40	468.73	467.46	135	327	651	**	**	**	**	**
1.000	1.00	500.40	498.73	145	348	694	500.53	497.99	146	349	691
1.063	1.00	532.42	530.36	154	370	738	648.48	646.65	188	451	899
1.125	1.00	564.09	561.98	163	392	782	774.03	771.89	226	539	1071
1.188	1.00	595.79	593.73	172	413	825	865.66	863.56	252	602	1197
1.250	1.00	626.41	623.66	181	434	869	925.90	923.61	268	642	1278
1.313	1.00	659.90	657.23	190	457	913	960.98	958.20	279	666	1327

Table 3.31: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1), T ∼ Exp(0.01), ARL_0=370 and 500 with θ=0.05 and ω=0.05, for different levels of shifts in shape parameter (a), scale parameter (b) of magnitude and rate parameter (λ_E) of time distribution, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts			EWMA-Rate					Max-EWMA				
			ARL_0=370.45	θ=0.05	L=0.03622			ARL_0=370.21	ω=0.05	L=2.791		
δ_a	δ_b	δ_{λ_E}	ARL	SDRL	Q_1	Q_2	Q_3	ARL	SDRL	Q_1	Q_2	Q_3
1.00	1.000	1.00	370.45	339.61	133	270	504	370.21	359.77	115	259.50	509
1.05	1.063	1.40	435.01	441.69	121	300	604	159.72	127.77	70	121	210
1.10	1.125	1.80	328.29	335.44	89	225	456	50.67	24.03	34	45	61
1.16	1.188	2.20	255.37	260.41	69	175	358	32.49	10.31	25	30	37
1.21	1.250	2.60	206.06	210.52	55	140.50	287	23.83	6.13	21	25	29
1.26	1.313	3.00	171.49	175.34	46	117	239	22.46	4.26	19	22	25
			ARL_0=500.92	θ=0.05	L=0.03664			ARL_0=499.99	ω=0.05	L=3.037		
δ_a	δ_b	δ_{λ_E}	ARL	SDRL	Q_1	Q_2	Q_3	ARL	SDRL	Q_1	Q_2	Q_3
1.00	1.000	1.00	500.92	485.21	157	354	689	499.99	489.49	151	349	691
1.05	1.063	1.40	444.80	452.88	122	305	618	235.03	198.41	95	176	312
1.10	1.125	1.80	330.69	337.58	89	227	460	60.61	30.72	39	53	74
1.16	1.188	2.20	257.34	262.14	70	177	360	36.81	12.17	28	34	43
1.21	1.250	2.60	207.73	211.97	56	142	290	28.67	6.97	24	27	32
1.26	1.313	3.00	172.85	176.62	46	118	241	24.69	4.77	21	24	27

Table 3.32: EWMA-Rate and Max-EWMA chart when X $\sim$ Gamma(4,1) , T $\sim$ Exp(0.01) , ARL_0=370 and 500 with θ=0.10 and ω=0.10, for different levels of shifts in shape parameter (a), scale parameter (b) of magnitude and rate parameter (λ_E) of time distribution, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts			**EWMA-Rate**					**Max-EWMA**				
			ARL_0=370.17	θ=0.10	L=0.03			ARL_0=370.29	ω=0.10	L=3.381		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	370.17	374.41	103	256	513	370.29	365.53	111	259	510
1.05	1.063	1.40	257.71	260.43	72	178	360	1603.03	1596.25	466	1116	2213
1.10	1.125	1.80	193.09	195.99	53	132	268	1226.38	1206.44	366	856	1695
1.16	1.188	2.20	151.78	154.29	41	104	211	283.77	254.68	103	206	382
1.21	1.250	2.60	124.85	127.09	34	86	173	101.47	74.27	49	79	130
1.26	1.313	3.00	105.68	107.65	29	72	146	57.13	32.11	35	48	70
			ARL_0=500.54	θ=0.10	L=0.0479			ARL_0=500.25	ω=0.10	L=3.657		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	500.54	504.11	143	346	693	500.25	494.04	148	349	693
1.05	1.063	1.40	349.97	353.04	98	242	485	2481.11	2492.89	721	1719	3428
1.10	1.125	1.80	266.88	268.24	75	185	371	3453.75	3456.00	996	2397	4770
1.16	1.188	2.20	211.24	212.43	60	145	294	1147.01	1114.65	354	808	1579.25
1.21	1.250	2.60	175.19	176.65	49	120	244	284.39	250.80	105	208	382
1.26	1.313	3.00	148.86	150.02	42	103	207	114.29	84.03	55	89	147

Table 3.33: EWMA-Rate and Max-EWMA chart when X ∼ Gamma(4,1) , T ∼ Exp(0.01) , ARL_0=370 and 500 with θ=0.15 and ω=0.15, for different levels of shifts in shape parameter (a), scale parameter (b) of magnitude and rate parameter (λ_E) of time distribution, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts			EWMA-Rate					Max-EWMA				
			ARL_0=370.14	θ=0.15	L=0.0438			ARL_0=370.22	ω=0.15	L=3.788		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	370.14	371.84	105	257	512	370.22	367.10	109	258	510
1.05	1.063	1.40	261.55	262.05	75	181	364	1878.93	1879.47	545	1303	2598
1.10	1.125	1.80	199.71	200.76	57	138	277	3137.75	3146.76	905	2174	4337
1.16	1.188	2.20	159.60	160.49	45	110	222	4311.13	4317.95	1241	2977	5996
1.21	1.250	2.60	132.72	133.54	38	92	183	7026.75	7002.41	2040.75	4881	9719.25
1.26	1.313	3.00	113.89	114.68	32	79	157	**	**	**	**	**
			ARL_0=500.10	θ=0.15	L=0.0653			ARL_0=500.20	ω=0.15	L=4.078		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	500.10	501.80	144	346	692	500.20	495.81	147	348	693
1.05	1.063	1.40	354.45	355.73	101	246	490	2847.80	2856.31	821	1968	3926
1.10	1.125	1.80	273.18	273.18	78	190	380	4913.30	4931.54	1416	3394	6793
1.16	1.188	2.20	219.29	219.29	63	152	305	6956.79	6920.83	2014	4828	9653
1.21	1.250	2.60	183.59	183.92	52	127	255	**	**	**	**	**
1.26	1.313	3.00	157.68	158.08	45	109	219	**	**	**	**	**

Table 3.34: EWMA-Rate and Max-EWMA chart when X $\sim$ Gamma(4,1) , T $\sim$ Exp(0.01) , ARL_0=370 and 500 with θ=0.20 and ω=0.20, for different levels of shifts in shape parameter (a), scale parameter (b) of magnitude and rate parameter (λ_E) of time distribution, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts			**EWMA-Rate**					**Max-EWMA**				
			ARL_0=370.30	θ=0.20	L=0.0543			ARL_0=370.33	ω=0.20	L=4.093		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	370.30	371.12	106	257	512	370.33	366.54	108	258	512
1.05	1.063	1.40	263.69	263.62	76	183	367	2022.95	2022.07	584	1405	2802
1.10	1.125	1.80	203.15	203.34	59	140	282	3470.45	3472.52	999	2412	4795
1.16	1.188	2.20	163.56	163.88	47	113	228	4836.65	4834.22	1394	3351	6703
1.21	1.250	2.60	137.07	137.44	39	95	189	7788.14	7767.04	2242	5407	10830
1.26	1.313	3.00	118.30	118.88	34	82	163	**	**	**	**	**
			ARL_0=500.07	θ=0.20	L=0.0787			ARL_0=500.08	ω=0.20	L=4.396		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	500.07	501.33	145	346	692	500.08	496.99	146	348	693
1.05	1.063	1.40	356.63	357.37	102	247	493	3111.94	3104.16	895	2166	4304.25
1.10	1.125	1.80	276.45	275.88	80	192	384	5496.32	5486.77	1589	3810.50	7599
1.16	1.188	2.20	223.53	223.15	64	155	310	7912.41	7892.95	2286	5494	11007
1.21	1.250	2.60	187.99	187.82	54	130.00	261	**	**	**	**	**
1.26	1.313	3.00	162.47	162.71	46	113	226	**	**	**	**	**

Table 3.35: EWMA-Rate and Max-EWMA chart when X ~ Gamma(4,1) , T ~ Exp(0.01) , ARL_0=370 and 500 with θ=0.25 and ω=0.25, for different levels of shifts in shape parameter (a), scale parameter (b) of magnitude and rate parameter (λ_E) of time distribution, $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.20,1.80,1.40,1.00)

Shifts			**EWMA-Rate**					**Max-EWMA**				
			ARL_0=370.29	θ=0.25	L=0.0628			ARL_0=370.33	ω=0.25	L=4.339		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	370.29	370.59	106	257	512	370.33	367.83	108	257	513
1.05	1.063	1.40	264.94	264.76	76	184	368	2139.30	2137.79	617	1485	2975
1.10	1.125	1.80	205.04	204.92	59	142	284	3790.45	3778.83	1090	2628.50	5264
1.16	1.188	2.20	165.76	165.90	47	115	231	5335.44	5330.76	1533	3701	7382
1.21	1.250	2.60	139.69	139.91	40	97	193	**	**	**	**	**
1.26	1.313	3.00	121.01	121.13	35	84	167	**	**	**	**	**
			ARL_0=500.34	θ=0.25	L=0.0897			ARL_0=500.34	ω=0.25	L=4.651		
δ_a	δ_b	δ_{λ_E}	ARL	$SDRL$	Q_1	Q_2	Q_3	ARL	$SDRL$	Q_1	Q_2	Q_3
1.00	1.000	1.00	500.34	501.06	145	346	692	500.34	498.02	145	347	694
1.05	1.063	1.40	358.24	358.85	103	248	495	3303.35	3298.25	948	2302	4576
1.10	1.125	1.80	278.68	278.06	80	194	387	6046.72	6056.47	1732	4178	8362.25
1.16	1.188	2.20	226.08	225.39	65	156	314	**	**	**	**	**
1.21	1.250	2.60	190.78	190.31	55	132	265	**	**	**	**	**
1.26	1.313	3.00	165.46	165.50	47	115	230	**	**	**	**	**

(for the Max-EWMA) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and ARL (SDRL) decreased by 5.00% (4.23%) (for the EWMA-Rate) and decreased by 35.53% (37.17%) (for the Max-EWMA) when 5% increase in the shape parameter of X and no change in the rate parameter of T, respectively. The results of case 1 with smoothing parameter 0.10 shows that the performance of Max-EWMA chart is better than the EWMA-Rate because the shifts in the parameters are detected more quickly. Tables 3.5 and 3.6 for case 1, when ω and θ is equal to 0.15 noted for ARL for both 370 and 500 nominals values, respectively. Same standards for evaluating shifts have used, the reduction in ARL (SDRL) by 77.62% (77.62%), 76.88% (76.81%) and 86.47% (88.28%), 87.82% (89.28%) for both EWMA-Rate and Max-EWMA charts for ARL_0=370 and 500 respectively, for 26% and 200% UW shifts; and by 4.94% (4.39%), 4.97% (4.49%) and 27.80% (28.73%), 27.59% (28.65%) for DW shifts. Tables 3.7 and 3.8 under case 1, when ω,θ=0.20 and ARL_0=370 taken into account, additional similar degenerating pattern of ARL is observed for increasing UW and DW shifts, (δ_a,δ_{λ_E})= (1.00,3.00), (1.26,3.00) and (1.05,1.00) for both charts. The ARL (SDRL) decreased by 68.89% (68.85%) when no change in shape parameter of X and 200% increase in rate parameter of T for EWMA-Rate chart, for Max-EWMA chart ARL strongly raised by 265.89% (264.41%); the reduction in ARL (SDRL) by 76.42% (76.40%), 81.71% (83.21%) for 26% and 200% UW shifts and 4.78% (4.44%), 22.48% (23.2%) for DW shifts, respectively. The behavior of ARL can also be viewed in the same way for case 1 of ARL_0=500. Tables 3.9 and 3.10 can be interpreted in similar manner.

It is worth noting that, when we have larger smoothing parameter with large shifts the simulated results of Max-EWMA gives substantial results. But if the simultaneous shift is larger, the EWMA-Rate will detect the shift sooner than the Max-EWMA chart.

From Tables 3.1 to 3.10, the quantiles inspection shows that in the most cases, the ARL is greater than the median (Q_2) but less than Q_3 meaning that the run length (RL) distribution is positively skewed. For example, when (δ_a,δ_{λ_E})=(1.26,3.00) and ARL_0 = 370 in case 1 of Table 3.1, the ARL for the EWMA-Rate chart is 116.99, the median is 80 and Q_3 is 162. Interestingly, in few cases the ARL is less (slightly) than the median which means that the distribution is negatively skewed (slightly), for example, in Table 3.1 for ARL_0=370, when (δ_a,δ_{λ_E})=(1.26,3.00) the ARL for Max-EWMA chart is 19.29, the median or Q_2=20 and Q_3=23. Traditionally, the high reduction indicates the best performance. The behavior of quantiles shows that in the most cases RL distribution tends to be positively skewed otherwise slightly negatively skewed. Although the behavior of the ARL does not always follow the conventional rule that smaller values of the smoothing parameter are appropriate for smaller shifts and bigger values are appropriate for larger shifts, for the majority of cases it does.

Tables 3.11 to 3.20 shows the analysis for case 2, The parameters of in-control chart presumes for case 2, X $\sim$ Gamma(7.5,1) and T $\sim$ Exp(0.01) for ARL_0=370 and ARL_0=500. Table 3.11 showing the results for Max-EWMA and EWMA-Rate chart for ARL_0=370.

The simultaneous set of shifts for case 2, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=370 and smoothing parameters 0.05, the ARL (SDRL) reduced by 64.79% (63.98%) and 95.52% (98.54%) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and 12.89% (6.04%) and 52.62% (56.53%) when 5% increase in shape parameter of X and no change in rate parameter of T, respectively. Similarly, in Table 3.12 the ARL_0=500, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) the EWMA-Rate and Max-EWMA chart results reduced in 73.74% (73.16%) and 96.41% (98.81%) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and 7.79% (5.66%) and 56.03% (59.34%) when 5% increase in shape parameter of X and no change in rate parameter of T, respectively.

Table 3.13 shows the results for Max-EWMA and EWMA-Rate chart for ARL_0=370. The simultaneous set of shifts for case 2, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=370 and smoothing parameters 0.10, the ARL (SDRL) reduced by 79.47% (79.10%) and 94.60% (97.01%) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and 4.32% (3.27%) and 43.68% (45.71%)when 5% increase in shape parameter of X and no change in rate parameter of T, respectively. Table 3.14 for ARL_0=500 can be elucidated in a same way. Table 3.15 shows the results for Max-EWMA and EWMA-Rate chart for ARL_0=370. The simultaneous set of shifts for case 2, (δ_a,δ_{λ_E})=(1.26,3.00) and (1.05,1.00) for the EWMA-Rate and Max-EWMA chart, ARL_0=370 and smoothing parameters 0.15, the ARL (SDRL) reduced by 77.64% (77.49%) and 92.85% (94.61%) when 26% increase in shape parameter of X and 200% increase in rate parameter of T and 5.01% (3.94%) and 34.90% (36.29%)when 5% increase in shape parameter of X and no change in rate parameter of T, respectively. Table 3.16 for ARL_0=500 can be expounded in a same way. Tables 3.17 to 3.20 illustrate the ARL performance of EWMA-Rate and Max-EWMA using smoothing constant 0.20 and 0.25 for ARL_0=370 and 500. When we have the simultaneous UW shifts in the parameter of time and no shift in magnitude in Max-EWMA chart, it is interesting to see that the excessive increase in ARL (SDRL) for (δ_a,δ_{λ_E})=(1.00,3.00) is by 379.45% (376.12%) and 457.56% (456.71%) but at the same time when we have simultaneous increasing shifts δ_a,δ_{λ_E})=(1.26,3.00) the ARL (SDRL) is reduced by 96.61% (92.07%) and 91.46% (92.63%). Which indicates the Max-EWMA performs better as it gives smallest ARL value. Meanwhile, the EWMA-Rate chart performs standard in that particular scenario.

Tables 3.21 to 3.30 shows analysis for case 3 for EWMA-Rates and Max-EWMA, X $\sim$ Gamma(1,1) and T $\sim$ Exp(0.01) for ARL_0=370 and 500. For this case, upward and downward shifts are taken for b and λ_E, e.g $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.40,2.20,1.80,1.40,1.00). The simultaneous shifts taken for both charts for ARL_0=370, (δ_b, $\delta_{\lambda E}$)=(1.250,1.40) and (1.125,1.00) for smoothing parameter 0.05, the ARL (SDRL) reduced by 5.06% (4.34%) and 55.99% (65.83%) when 25% increase in scale parameter of X and 40% increase in rate parameter of T and ARL (SDRL) reduced by 5.00% (4.39%) and increased by 44.46% (41.68%) when 12.5% increase in the scale parameter of X and no change in the rate parameter of T, respectively. similarly for other smoothing parameters. The simultaneous shifts in the parameters with small smoothing constant are detected more quickly in the case of the Max-EWMA control chart. However, the EWMA-Rate control chart performs efficiently with large shifts and smoothing parameters. It is absorbing to kmow that when we have large smoothing constants the Max-EWMA and EWMA-Rate (≥ 0.15) gives beyond satisfying results which are represented with the **.

Tables 3.31 to 3.35 represents the case 4 for EWMA-Rate and Max-EWMA, X $\sim$ Gamma(4,1) and T $\sim$ Exp(0.01) for ARL_0=370 and 500. For this case, upward and downward shifts are taken for all three parameters a, b and λ_E, e.g $\delta_a \in$ (1.00,1.05,1.10,1.16,1.21,1.26), $\delta_b \in$ (1.000,1.063,1.125,1.188,1.250,1.313) and $\delta_{\lambda E} \in$ (3.00,2.60,2.40,2.20,1.80,1.40,1.00). The simultaneous shifts taken for both charts for ARL_0=370 with smoothing parameter 0.05, (δ_a, δ_b, $\delta_{\lambda E}$)= (1.21, 1.250, 2.60) the ARL (SDRL) is reduced by 44.30% (43.10%) and 167.87% (63.93%) when 21% increase in shape parameter of X, 25% increase in scale parameter of X and 60% increase in rate parameter of T, respectively. Similarly for ARL_0=500. The simultaneous shifts taken for both charts for ARL_0=370 with smoothing parameter 0.05, (δ_a, δ_b, $\delta_{\lambda E}$)= (1.21, 1.250, 2.60) the ARL (SDRL) is reduced by 62.95% (62.85%) and highly increased by 2004.90% (1992.2%) (in the case of Max-EWMA control chart) when 21% increase in shape parameter of X, 25% increase in scale parameter of X and 60% increase in rate parameter of T, respectively. Similarly for ARL_0=500 and other smoothing parameters. In the case of applying shifts in all parameters with large smoothing constants the EWMA-Rate control chart is highly preferred.

3.5 Real Data Application

The suggested methodology is used on two real data set in this section. An electronic corporation intends to track the sales of one of its key products, a silicon chip is taken from Wu et al. (2009b). A customer's purchase order is referred to as the positive occurrence in this scenario. Here, X is the number of lots in an order, and T is the amount of time (in working hours) among each two orders. The business possesses the t_i and x_i for 50 orders, that were placed during a period when the product was ordinarily sold. While T is fitted to the exponential distribution, the variable X is adapted to the gamma distribution. R software is used to determine the parameters of both data sets using the maximum likelihood criterion. A t_i value of zero in the data set indicates that two orders arrive almost simultaneously.

Table 3.36: Parametric estimates and goodness-of-fit statistics for sales and time between successive orders

Distribution		**ML Estimates**	**AIC**	**BIC**
Gamma	shape	3.134814 (0.596617)	159.6724	163.4962
	scale	0.296307 (0.283774)		
Exponential	rate	0.057142 (0.008078)	388.2201	390.1321

For the implementation of the both charting methodologies, following parameters were considered:

ω=0.05, UCL=12.39738, L=1.13 and ARL_0=50 (for the Max-EWMA chart (a))

θ=0.05, UCL=12.13593, L=0.03 and ARL_0=50 (for the EWMA-Rate chart (b))

ω=0.20, UCL=22.39483, L=1.13 and ARL_0=50 (for the Max-EWMA chart (c))

θ=0.20, UCL=12.00432, L=0.03 and ARL_0=50 (for the EWMA-Rate chart (d))

Figure 3.1 is the representation of the combined Phase I and Phase II data, for the Max-EWMA control chart and EWMA-Rate chart. Since there is no OOC signal in Phase I data, Phase II data analysis reveals that the chart initiates the first OOC signal for Max-EWMA chart at sample 28^{th} and for EWMA-Rate chart at sample 29^{th}. This signal is caused by a simultaneous change in the parameters. Furthermore, in (c) we can see that the OOC signal is triggered at sample point 27^{th} with the larger smoothing constant and shift and (d) triggered OOC point in Phase I. When a drop in sales like this one is seen, the business could use its advertising strategy to boost interest in its goods. The management team can also make timely decisions to bring innovation into the product so they can gain an advantage over their rivals.

The labeling process of the charts followed the criteria in which the Max-EWMA the out-of-control point arise when $|\mathbf{U_i}|$ exceeds UCL, $|\mathbf{V_i}|$ exceeds UCL and iff both exceeds UCL we labeled it by X, T and XT, respectively. In the above figure the OOC points in Max-EWMA chart are because $|\mathbf{U_i}|$ exceeds UCL, so consider the label point X. In the case of EWMA-Rate, the OOC point shall be due to simultaneous increase in X and T so that we labeled it by R+.

Rahali et al. (2021) noted the data set of a company for monitoring the bottleneck machine from 08/01/12 to 27/12/18 (formated DD/MM/YY). All the breakdown dates (in days) and the expected costs (X_i, in euros), which comprise all the expenses of repair and restart (manpower, spare components), as well as the price of manufacturing disruption. It is simple to determine the intervals between two successive breakdowns using these dates (T_i in days). The machine started working on August 1, 2012, and experienced its first failure on October 3, 2012, or $T_1 = 62$ days later. Its anticipated cost to repair and restart the unit is $X_1 = 4890$ euros. The authors mentioned that the data have a minor correlation between TBE and X.

Table 3.37: Parametric estimates and goodness-of-fit statistics for cost and time between two consecutive breakdowns

Distribution		**ML Estimates**	**AIC**	**BIC**
Gamma	shape	18.685066 (1.7173829)	750.8357	754.4041
	scale	276.43027 (0.00030847)		
Exponential	rate	0.017288 (0.00259763)	447.0773	488.8615

For the implementation of the both charting methodologies, following parameters were considered:

ω=0.05, UCL=0.301284, L=1.13 and ARL_0=50 (for the Max-EWMA chart (a))

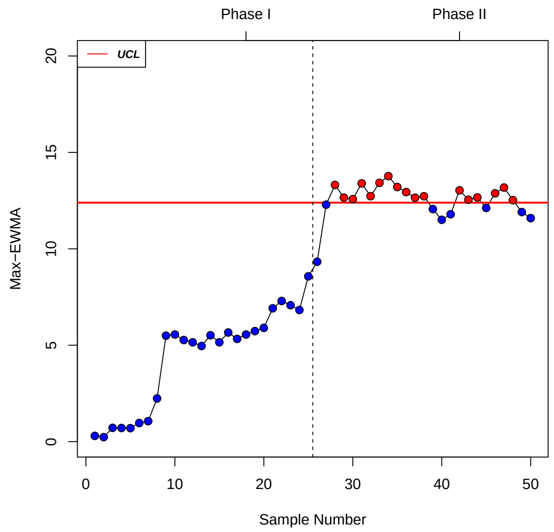
θ=0.05, UCL=92.188, L=0.03 and ARL_0=50 (for the EWMA-Rate chart (b))

ω=0.20, UCL=0.787847, L=1.13 and ARL_0=50 (for the Max-EWMA chart (c))

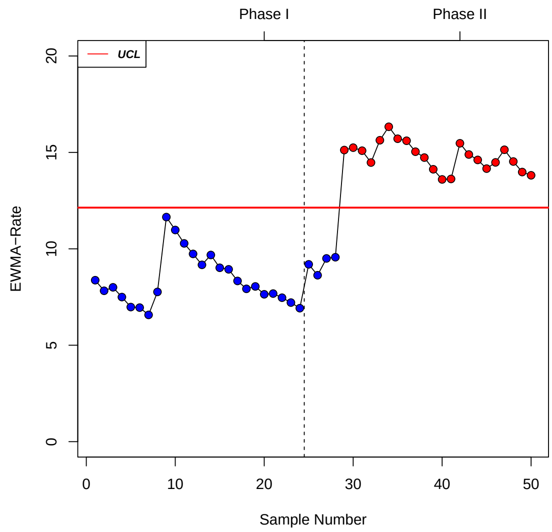
θ=0.20, UCL=93.13087, L=0.03 and ARL_0=50 (for the EWMA-Rate chart (d))

Figure 3.2 is the representation of the combined Phase I and Phase II data, for the Max-EWMA control chart and EWMA-Rate chart. Since there is no OOC signal in any of the Phase I data, Phase II data analysis reveals that the chart initiates the first OOC signal for Max-EWMA chart at sample 25^{th} and for EWMA-Rate chart at sample 25^{th}. This signal is caused by a simultaneous change in the parameters. Furthermore, in (c) we can see that the OOC poin triggered at sample point 38^{th} with the larger smoothing constant and shift and (d) triggered OOC point in Phase II at sample 24^{th}, the grounds of this action is special cause of variation that could be happened in any electronic or sensitive production or manufacturing manners. The early detection can be helpful while competiting with the other industries to perform better in a way and take necessary actions on time for lessen waste.

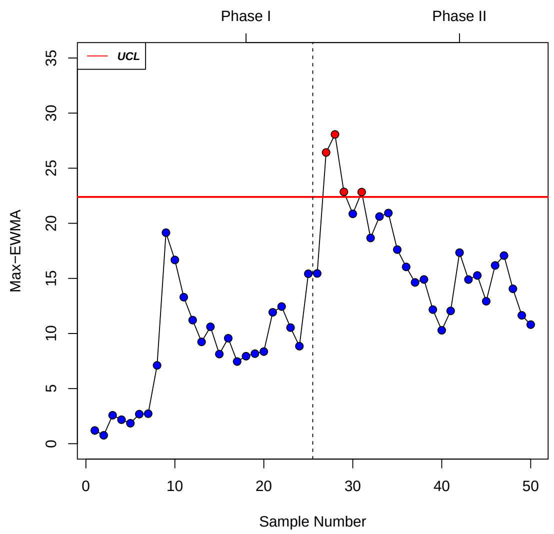
These graphs show that the simultaneous shift is picked up faster than the pure shift. For the real data sets the Max-EWMA chart shows its best performance as the OOC points triggered frequently. Also the figures show that in the case of the EWMA-Rate for both data sets it triggered the OOC point at first with carrying larger smoothing constant and shift, respectively. So, we can easily conclude that in the sensitive industrial or corporated scenarios Max-EWMA chart performs better, as it gives OOC point utimely.



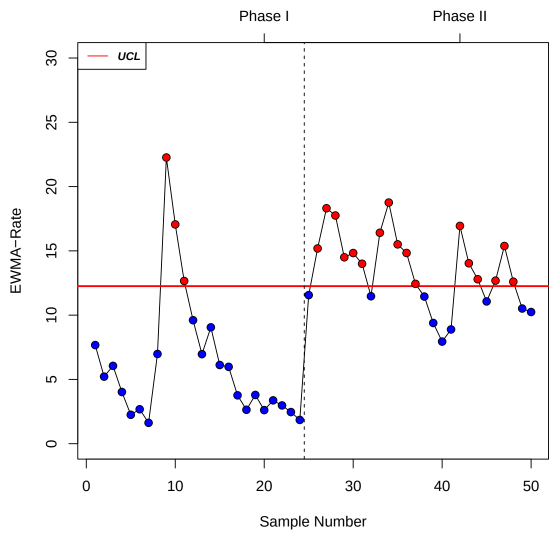
(a) Max-EWMA chart with L=1.13 and UCL=12.39738



(b) EWMA-Rate chart with L=0.03 and UCL=12.13593

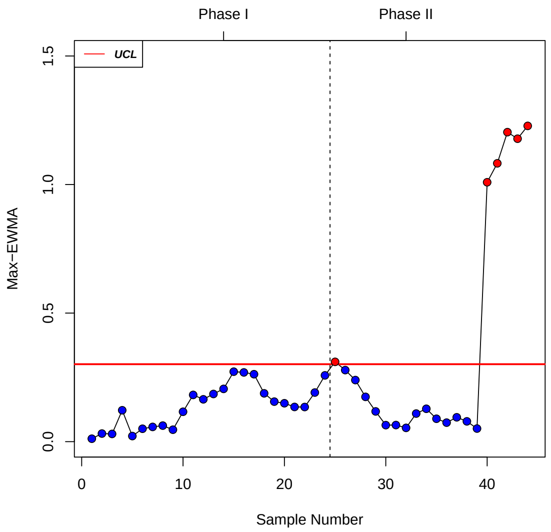


(c) Max-EWMA chart with L=2.01 and UCL=22.39483

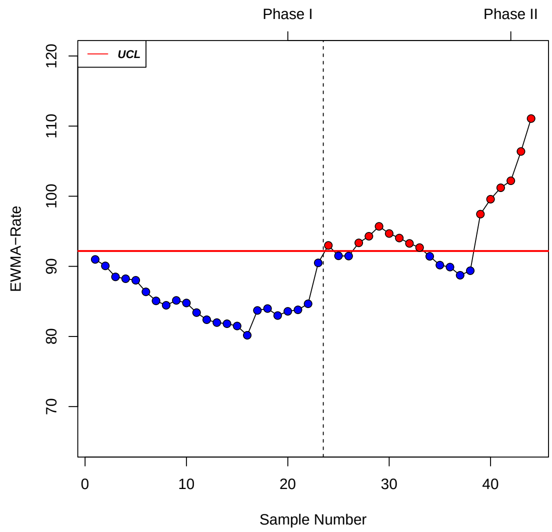


(d) EWMA-Rate chart with L=0.03 and UCL=12.00432

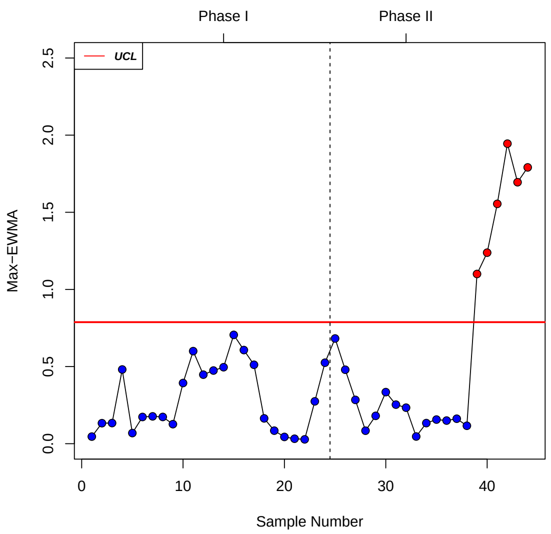
Figure 3.1: Max-EWMA and EWMA-Rate control charts for monitoring the electronic products



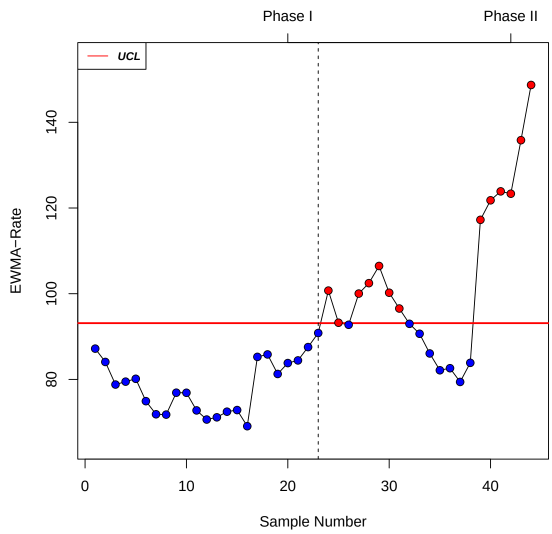
(a) Max-EWMA chart with L=1.13 and UCL=0.3012849



(b) EWMA-Rate chart with L=0.03 and UCL=92.188



(c) Max-EWMA chart with L=2.01 and UCL=0.787847



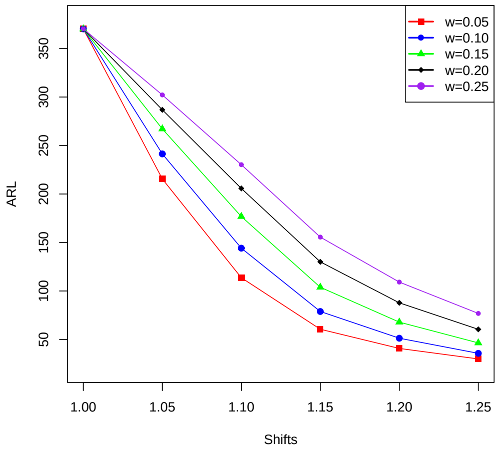
(d) EWMA-Rate chart with L=0.03 and UCL=93.13087

Figure 3.2: Max-EWMA and EWMA-Rate control charts for monitoring the bottleneck machine

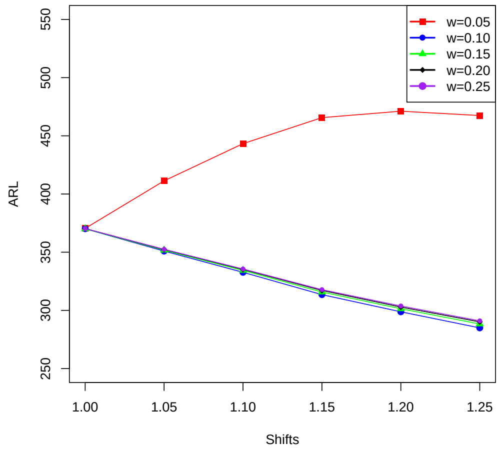
3.6 ARL Graphical Comparison

The graphical representation of whole simulation study is provided to observe the void behavior of both charts and their comparison. Figure 3.3 shows that the individual graphical comparison of ARL which is done by the simulation method. In part a of Max-EWMA control chart the shifts are introduced in shape parameter of magnitude distribution showing smooth exponentially decreasing behavior whereas part b of EWMA-Rate exhibits the less exponentially decreasing behavior. Furthermore part c and d menifests the shifting process in scale parameter of magnitude distribution, unexponentially decreasing behavior has arisen because of small to moderate to large shifting scheme and the smoothing parameters. Charting procedure can easily be observed to conclude that the larger the smoothing constants the larger the unexponentially decreasing bahavior gotten in the case of EWMA-rate chart. Similarly the part e and f. With small smoothing parameter Max-EWMA control chart behaves efficiently on the other hand EWMA-Rate control chart competence the large shifts with smoothing constants.

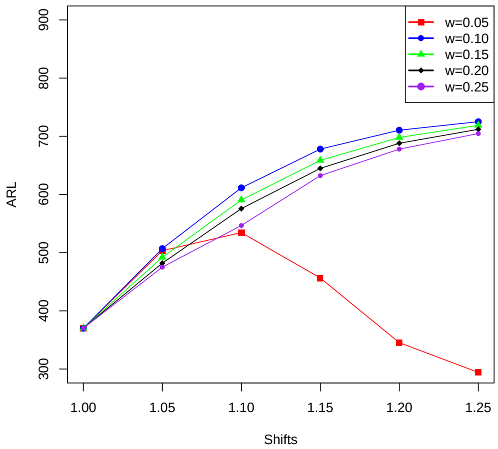
Figure 3.4 illustrates the combine structure of ARL for both charts. Pure shifts in shape and scale parameter of gamma distribution and rate parameter of exponential distribution with smoothing constant 0.20 and 0.10, respectively. When we have shifts in shape parameter with different smoothing constants Max-EWMA acquits effectively. However, the EWMA-Rate charts acts the trend analysis, as it gives sustained behavior after a short period of time.



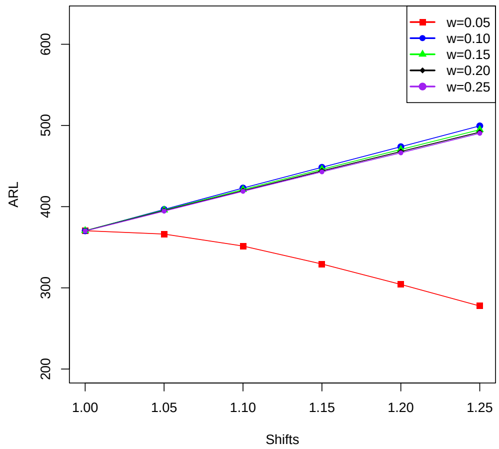
(a) Shift a (Max-EWMA)



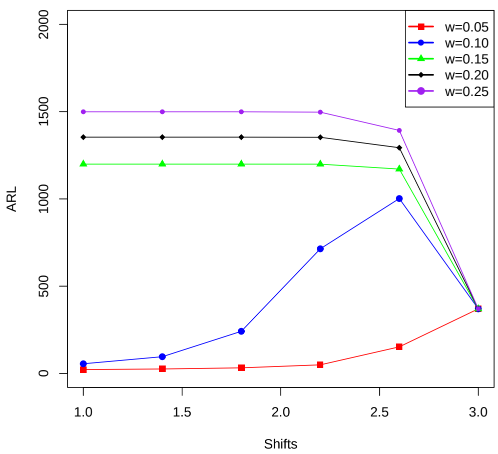
(b) Shift a (EWMA-Rate)



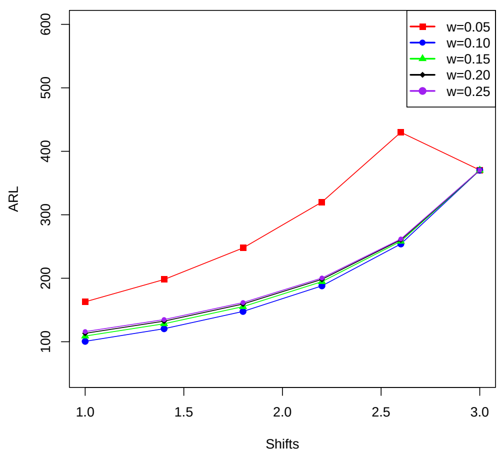
(c) Shift b (Max-EWMA)



(d) Shift b (EWMA-Rate)

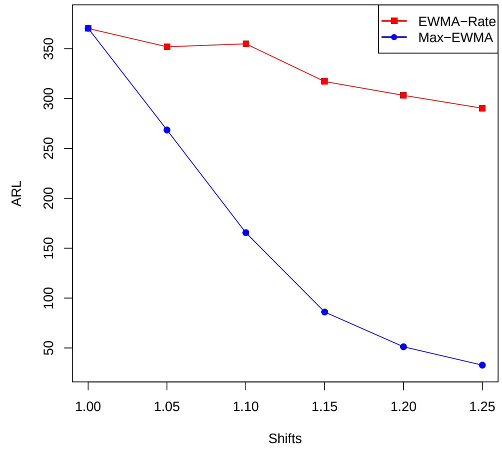


(e) Shift in λ_E (Max-EWMA)

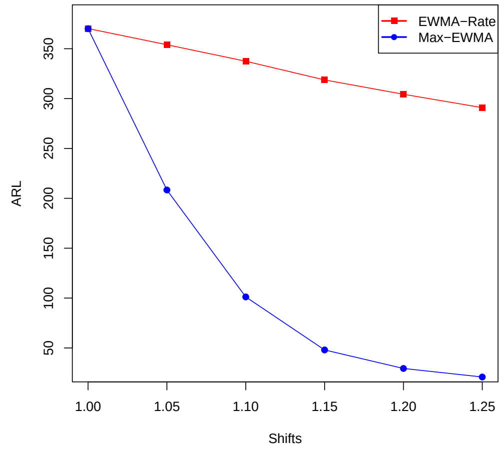


(f) Shift in λ_E (EWMA-Rate)

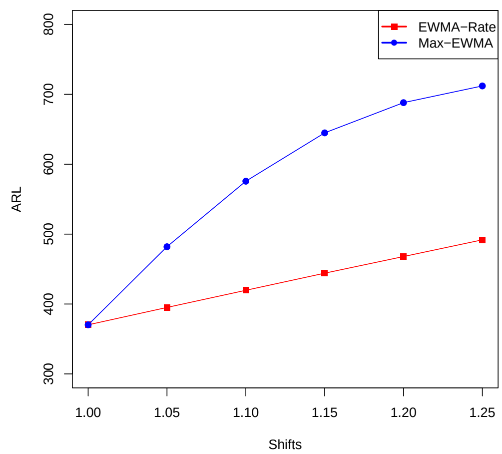
Figure 3.3: Distinctive graphical comparison of ARL of both charts using different smoothing parameters



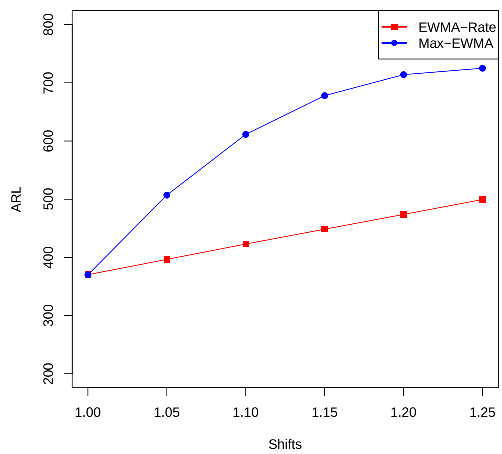
(a) Shift a (Max-EWMA and EWMA-Rate with smooyhing parameter 0.20)



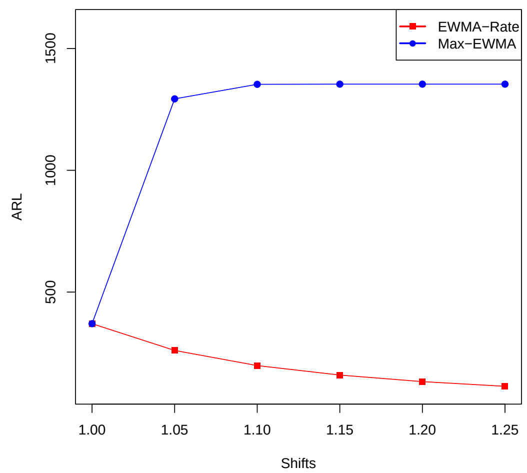
(b) Shift a (Max-EWMA and EWMA-Rate with smoothing parameter 0.10)



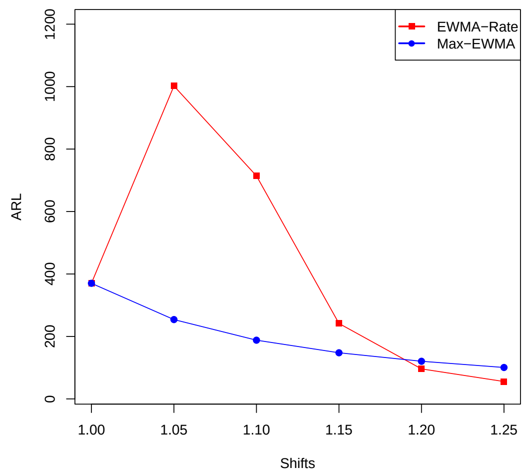
(c) Shift b (Max-EWMA and EWMA-Rate with smoothing parameter 0.20)



(d) Shift b (Max-EWMA and EWMA-Rate with smoothing parameter 0.10)



(e) Shift in λ_E (Max-EWMA and EWMA-Rate with smoothing parameter 0.20)



(f) Shift in λ_E (Max-EWMA and EWMA-Rate with smoothing parameter 0.10)

Figure 3.4: Combine graphical comparison of ARL of both charts using two different smoothing parameters

3.7 Conclusion

Many industries, such as the business or industrial sectors, have been transformed by machines as a result of advances in science and technology. It is crucial to keep track of how well these machines are performing. Several methods for monitoring processes have already been discussed. These methods require diverse distributional forms, assumptions, and ranges of data. Therefore, we have constructed a EWMA chart for simultaneous monitoring the magnitude and TBE of an event under the assumption that TBE follows exponential distribution and magnitude of the event follows gamma distribution. The most widely used criterion, the ARL criterion, has been applied in order to access and analyze the performance of charts. The outcomes show that the suggested chart performs significantly well overall for small to medium-sized (and large) shifts (simultaneous) with larger smoothing parameters (≥ 0.20). Importantly, the Max-EWMA control chart performs better for small to medium-sized shifts. Finally the charting methodology is applied to real data set.

Chapter 4

Conclusion and Recommendations

The charting structures have been successfully applied in several real-world situations. We always want to make sure that system performance is proceeding as planned from the outcomes. Numerous control charts utilizing various distributions, but little attention is devoted for monitoring rates and proportional data. To address this void, we designed a memory-type control chart to track the timing and size of the events while taking distributions with support ranging from 0 to ∞ into account.

In order to simultaneously monitor TBE and magnitude, we compared a rate based EWMA control chart with the Max-EWMA control chart. The effectiveness of the suggested chart is assessed in terms of the ARL criterion based on the gamma and exponential distributions. Various quantiles are also produced to examine the skewness of RL distribution. To understand the behaviour of the control charts, we considered various shift sizes (including upward and downward shifts) and smoothing parameters. The Max-EWMA control chart is capable of accurately detecting small to moderate both upward downward shifts.

Real data sets used for analysing the performances of the charts. Which shows when data has minor correlation between TBE and X it slightly perform better. The performace of suggested rate based EWMA control chart is best performer only when we have larger smoothing parameter and shifts than the Max-EWMA control chart in the real data cases. The Max-EWMA control chart brings out the best results in all the manners. Due to its precisely and anticipatively detecting the OOC points with small to moderate shift sizes and smoothing parameter.

According to the simulation results and the fact that the ARL values fall between the median and 3^{rd} quantile, the run length distribution is positively skewed. This suggests that the majority of the ARL values are clustered on the lower side of the distribution of the run lengths.

In addition to being effective at detecting an OOC signal, the Max-EWMA chart is also easier to use. There are a lot of open sides for enhancing the resreach, other distributions can be applied to the different charts and examinig the comparative performances. Rate based CUSUM chart with the excel combination of distributions can be done subsequently.

Bibliography

Abbas, N., Riaz, M., and Does, R. J. (2013a). CS-EWMA chart for monitoring process dispersion. *Quality and Reliability Engineering International*, 29(5):653–663.

Abbas, N., Riaz, M., and Does, R. J. (2013b). Mixed exponentially weighted moving average–cumulative sum charts for process monitoring. *Quality and Reliability Engineering International*, 29(3):345–356.

Akram, M. F., Ali, S., Shah, I., and Marcon, G. (2022). Unit Interval Time and Magnitude Monitoring Using Beta and Unit Gamma Distributions. *Journal of Mathematics*, 2022.

Ali, S., Akram, M. F., and Shah, I. (2022). Max-EWMA chart using Beta and Simplex distributions for time and magnitude monitoring. *Mathematical Problems in Engineering*, 2022.

Antzoulakos, D. L. and Rakitzis, A. C. (2008). The modified r-out-of m control chart. *Communications in Statistics—Simulation and Computation*, 37(2):396–408.

Aslam, M., Gui, W., Khan, N., and Jun, C.-H. (2017). Double moving average–EWMA control chart for exponentially distributed quality. *Communications in Statistics-Simulation and Computation*, 46(9):7351–7364.

Barnard, G. A. (1959). Control charts and stochastic processes. *Journal of the Royal Statistical Society: Series B (Methodological)*, 21(2):239–257.

Borror, C. M., Keats, J. B., and Montgomery, D. C. (2003). Robustness of the time between events CUSUM. *International Journal of Production Research*, 41(15):3435–3444.

Chen, G., Cheng, S. W., and Xie, H. (2001). Monitoring process mean and variability with one EWMA chart. *Journal of Quality Technology*, 33(2):223–233.

Chen, S.-H., Yang, C.-C., Lin, W.-T., and Yeh, T.-M. (2008). Performance evaluation for introducing statistical process control to the liquid crystal display industry. *International Journal of Production Economics*, 111(1):80–92.

Edwards, R. (1980). Internal analytical quality control using the CUSUM chart and truncated V-mask procedure. *Annals of Clinical Biochemistry*, 17(4):205–211.

Gan, F. (1998). Designs of one-and two-sided exponential EWMA charts. *Journal of Quality Technology*, 30(1):55–69.

Haq, A. (2017). A new maximum EWMA control chart for simultaneously monitoring process mean and dispersion using auxiliary information. *Quality and Reliability Engineering International*, 33(7):1577–1587.

Jiang, W., Shu, L., and Apley, D. W. (2008). Adaptive CUSUM procedures with EWMA-based shift estimators. *IIE Transactions*, 40(10):992–1003.

Khoo, M. B. (2003). Design of runs rules schemes. *Quality Engineering*, 16(1):27–43.

Klein, M. (2000). Two alternatives to the Shewhart X control chart. *Journal of Quality Technology*, 32(4):427–431.

Koutras, M., Bersimis, S., and Maravelakis, P. (2007). Statistical process control using shewhart control charts with supplementary runs rules. *Methodology and Computing in Applied Probability*, 9(2):207–224.

Liu, Y., He, Z., Shu, L., and Wu, Z. (2009). Statistical computation and analyses for attribute events. *Computational statistics & Data Analysis*, 53(9):3412–3425.

Lucas, J. M. (1985). Counted data CUSUM. *Technometrics*, 27(2):129–144.

Lucas, J. M. and Crosier, R. B. (2000). Fast initial response for CUSUM quality-control schemes: give your CUSUM a head start. *Technometrics*, 42(1):102–107.

Lucas, J. M. and Saccucci, M. S. (1990). Exponentially weighted moving average control schemes: properties and enhancements. *Technometrics*, 32(1):1–12.

Montgomery, D. C. (2020). *Introduction to statistical quality control*. John Wiley & Sons.

Page, E. S. (1954). Continuous inspection schemes. *Biometrika*, 41(1/2):100–115.

Qu, L., Wu, Z., Khoo, M., and Shu, L. (2014). A new control chart for monitoring the event frequency and magnitude. *European Journal of Industrial Engineering*, 8(6):789–813.

Rahali, D., Castagliola, P., Taleb, H., and Khoo, M. B. C. (2021). Evaluation of Shewhart time-between-events-and-amplitude control charts for correlated data. *Quality and Reliability Engineering International*, 37(1):219–241.

Roberts, S. (1959). Control chart tests based on geometric moving averages. *Technometrics*, 42(1):97–101.

Santos-Fernández, E. (2012). *Multivariate statistical quality control using R*, volume 14. Springer Science & Business Media.

Sanusi, R. A., Teh, S. Y., and Khoo, M. B. (2020). Simultaneous monitoring of magnitude and time-between-events data with a Max-EWMA control chart. *Computers & Industrial Engineering*, 142:106378.

Sheu, S.-H., Huang, C.-J., and Hsu, T.-S. (2012). Extended maximum generally weighted moving average control chart for monitoring process mean and variability. *Computers & Industrial Engineering*, 62(1):216–225.

Shewhart, W. A. (1931). *Economic control of quality of manufactured product*. Macmillan And Co Ltd, London.

Steiner, S. H. (1999). EWMA control charts with time-varying control limits and fast initial response. *Journal of Quality Technology*, 31(1):75–86.

Talib, A., Ali, S., and Shah, I. (2022a). Max-EWMA chart for time and magnitude monitoring using exponentially modified Gaussian distribution. *Quality and Reliability Engineering International*, 38(2):1092–1111.

Talib, A., Ali, S., Shah, I., and Gul, F. (2022b). Time and magnitude monitoring using Weibull based Max-EWMA chart. *Communications in Statistics-Simulation and Computation*, pages 1–17.

Wu, Z., Jiao, J., and He, Z. (2009a). A control scheme for monitoring the frequency and magnitude of an event. *International Journal of Production Research*, 47(11):2887–2902.

Wu, Z., Jiao, J., and He, Z. (2009b). A single control chart for monitoring the frequency and magnitude of an event. *International Journal of Production Economics*, 119(1):24–33.

Xie, H. (1999). Contributions to qualimetry [phd thesis]. *Winnipeg, Canada: University of Manitoba*.

Yashchin, E. (1989). Weighted cumulative sum technique. *Technometrics*, 31(3):321–338.

Zhang, C., Xie, M., Liu, J., and Goh, T. (2007). A control chart for the gamma distribution as a model of time between events. *International Journal of Production Research*, 45(23):5649–5666.