Single and dual reference-free cumulative score charts for detecting unknown patterned mean shifts

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In the Name of Allah the Most Merciful and the Most Beneficent

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A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY IN STATISTICS

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Declaration

I "Mehwish Naz" hereby solemnly declare that this thesis entitled "Single and dual reference-free cumulative score charts for detecting unknown patterned mean shifts", submitted by me for the partial fulfillment of Master of Philosophy in Statistics, is the original work and has not been submitted concomitantly or latterly to this or any other university for any other degree.

Dated: ______________ Signature: ______________

Dedication

I am feeling great honor and pleasure to dedicate this research work to

My Beloved Parents

Whose endless affection, prayers and wishes have been a great source of comfort for me during my whole education period.

Acknowledgments

First and foremost I praise and acknowledge The Allah Almighty, The Lord and Creator of the heavens and earth. All my respect and gratitude goes to the Holy Prophet Hazrat Muhammad (SAAW) who enlightens our hearts with the light of Islam and whose way of life has been a continuous guidance for us.

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Mehwish Naz

Abstract

Most of the control charts in the literature focus on detecting constant mean shifts. In practice, dynamic and time-varying process mean shifts are prevalent in the monitoring of feedback-control and autocorrelated processes. The cumulative score (CUSCORE) charts have been proposed in the literature to detect dynamic and time-varying process shifts. One of such control charts is a reference-free CUSCORE (called RFCS-I) chart that may detect unknown patterned mean shifts effectively. In this thesis, single and dual CUSCORE charts are developed to enable an efficient detection of unknown patterned mean shifts. The developed single CUSCORE chart, called the RFCS-II chart, is based on the control charting structure of the Crosier CUSUM (called CU-II) chart. Additionally, two existing single CUSCORE (called RFCS-I) charts are integrated with reflecting boundaries, while an integration of two RFCS-II charts is also made, to develop the dual RFCS-I (DRFCS-I) and dual RFCS-II (DRFCS-II) charts, respectively. Moreover, the RFCS-I and RFCS-II charts are integrated to propose the mixed dual CUSCORE chart, called the MDRFCS chart. In essence, four CUSCORE charts, i.e. the RFCS-II, DRFCS-I, DRFCS-II and MDRFCS charts are suggested in this thesis. The Monte Carlo simulation and a real application illustrate the superiority of these developed CUSCORE charts over their existing counterparts when detecting patterned mean shifts in terms of zero-state and steady-state relative mean indexes.

Contents

List of Tables

Chapter 1

Introduction

In the realm of statistical quality control, the concept of quality pertains to the comparative excellence of a particular item or service. In the context of selecting among competing products and services, quality has demonstrated its pivotal role as a decisive factor. Within every established organization, a comprehensive framework of policies and guidelines exists to uphold the inherent caliber of its deliverables. Quality control, as a technique, emerges as a crucial means of ensuring that goods or services adhere to predetermined quality standards. The fundamental objective of this approach is to provide assurance that the products or services offered meet the specified requirements. Employing various methods to address quality-related issues, statistical process control emerges as one of the most efficacious strategies for attaining and maintaining superior quality standards.

1.1 Statistical Process Control

Statistical process control (SPC) serves as an industrial technique employed for the evaluation and regulation of the quality level within a production process. Its core purpose is to ensure consistency in the manufacturing of products according to their intended design. By closely monitoring the quality of processes and adhering to predetermined standards, SPC aims to attain and maintain optimal process performance. To accomplish this objective, SPC encompasses a comprehensive set of seven fundamental tools, which collectively contribute to enhancing performance throughout every stage of the production process. The following section presents a succinct review of these essential tools, elucidating their significance in the context of SPC.

1. **Cause-and Effect Diagram:** It is a vertical bar graph that shows the most prevalent kinds of problems.
2. **Check Sheet:** A check sheet, also known as tally sheet, is a tool used to collect and analyze data.
3. **Pareto Chart:** A vertical bar graph displaying the relative frequency of flaws in rank-order.
4. **Histogram:** A graph showing frequency distributions or data with vertical bars varying in height.
5. **Defect concentration diagram:** It is a graphic tool used in quality control and process improvement to identify the processes where defects are most concentrated.
6. **Scattar Diagram:** A technique for analyzing a relationship between two continuous variables is a scatter diagram.
7. **Control Chart:** A control chart is a statistical tool used in quality control to monitor a process and detect any changes or variations that may occur.

1.2 Control charts

The concept of control charts, initially introduced in the 1920s by Walter A. Shewhart, plays a crucial role in identifying and addressing potential defects or unsatisfactory outcomes in manufacturing and production processes. By graphically representing variations in processes or non-compliant products, control charts provide a visual tool for monitoring and taking necessary actions to maintain quality standards. The primary objective of a control chart is to detect any unusual variations occurring within the process. In the realm of SPC, two types of variations are recognized: "natural cause variation" and "assignable cause variation". Natural cause variation refers to the inherent fluctuations present in a process or system, even when operating under normal conditions. When such variations exist, it indicates that the process is under control. On the other hand, assignable cause variation, also known as special cause variation, is caused by factors external to the process or system, and can be specifically identified. Its presence indicates a problem within the process that requires investigation and correction. The ultimate objective is to identify the source of assignable cause variation and take appropriate measures to eliminate it, thereby improving the overall process performance.

Within the realm of SPC, two distinct types of charts are employed to monitor and track process variations: memory-less charts and memory-type charts. Memory-less charts, commonly referred to as Shewhart-type charts, solely rely on current data for analysis. These charts excel at detecting significant alterations in process parameters but may be less effective in identifying smaller or moderate shifts. In contrast, memory-type charts are specifically designed to utilize both previous and current data, making them highly proficient in detecting subtle and moderate changes in process parameters. Among the memory-type control charts, the exponentially weighted moving average (EWMA) and Cumulative SUM (CUSUM) are the most widely utilized. While Shewhart charts are typically employed to identify substantial changes or shifts in process parameters, the CUSUM and EWMA charts serve as effective alternatives for detecting smaller to moderate changes in process parameters. By leveraging the strengths of these different chart types, practitioners can enhance their ability to effectively monitor and manage process variations within the context of SPC.

In the realm of process monitoring, $\bar{X}$-type control charts have gained significant popularity for tracking constant shifts in the process mean. However, these conventional charts are inadequate when it comes to detecting dynamic mean shifts, characterized by non-constant, time-varying patterns with unknown characteristics. This limitation arises from the neglect of dynamic information inherent in the process. To address this issue, auto-correlation or feedback-controlled procedures have been introduced to effectively capture dynamic patterns in mean changes. Among the proposed methods, the CUmulative SCORE (CUSCORE) chart has emerged as a promising approach for detecting dynamic process changes over time.

Researchers have made notable contributions to the field by developing various control charting techniques over the years, specifically targeting mean changes with unknown patterns. For instance, Bagshaw and Johnson (1977) presented sequential approaches for monitoring forecast errors in time series models. Building upon the Shewhart-CUSCORE charts, Ncube and Amin (1990) proposed two techniques for simultaneous monitoring of process mean and variation. Further advancements were made by Ncube (1992), who developed CUSUM-CUSCORE charts that assign equal weight to previous sample mean values and past sample score values. The CUSCORE chart has also been utilized for monitoring the rate of occurrences of rare events (Radaelli, 1992) and as an adjunct to the Shewhart chart when specific types of departures are anticipated (Box and Ramírez, 1992).

Additional studies have focused on the economic design of CUSCORE charts (Kim and Jeong, 1993), the combined Shewhart-CUSCORE method for analyzing process variance changes (Ncube, 1994), and the enhancement of CUSCORE charts for detecting process shifts (Cheng, 1995). Moreover, control charts have been proposed for comparing sustained mean shifts and sporadic spikes before and after applying time series modeling (Hu and Roan, 1996), and the impact of serial correlation on CUSCORE chart performance has been examined (Bohm and Hackl, 1996). The CUSCORE technique has also been explored as an interface between Statistical Process Control and Engineering Process Control (Shao, 1998), with applications in monitoring clean room air quality (Ramírez, 1998).

Studies have further contributed algorithms for computing the average run length (ARL) and run length probability distributions of CUSCORE charts for mean monitoring (Luceno, 1999), and suggested modifications that outperform standard CUSCORE charts in terms of ARL performance (Ncube and Ncube, 2000). To reduce mismatch between detectors and fault signatures, a CUSUM-triggered CUSCORE chart has been proposed (Shu et al., 2002). The benefits and drawbacks of the generalised likelihood ratio (GLR) and CUSCORE charts in enhancing sensitivity for fault detection have been discussed (Runger and Testik, 2003). Additionally, a reference-free CUSCORE chart has been recommended to monitor and detect dynamic shifts in the process mean even in the absence of a known reference pattern (Han and Tsung, 2006).

While the CUSCORE chart proves suitable when the reference pattern is known, practical situations often involve unknown reference patterns. Consequently, there is a need for reference-free charting techniques to overcome this limitation and provide effective process monitoring solutions.

By developing four single and dual CUSCORE charts and utilizing Monte Carlo simulations to evaluate their performance characteristics, this thesis provides novel insights into the detection of unknown patterned mean shifts. The outlined structure ensures a logical progression of the research, from the introduction and development of the proposed charts to their evaluation through comprehensive simulations. The conclusions drawn from the results contribute to advancing the field of statistical process control and offer suggestions for future research endeavors.

Chapter 2

Single and dual reference-free cumulative score charts

2.1 Introduction

Control charts are important process monitoring tool in manufacturing and service industries to differentiate assignable causes of variation from common causes of variation. The most commonly used control charts are the $\overline{X}$-type charts for detecting constant shifts in the process mean. These conventional $\overline{X}$-type charts are not suitable in detecting dynamic mean shifts, which include mean shifts that are nonconstant, time-varying and have an unknown pattern because these charts ignore the dynamic information contained in the process. Dynamic patterns in mean shifts are usually prevalent in autocorrelated or feedback-controlled processes. Numerous control charting procedures were developed by researchers over the years for process monitoring involving unknown patterned mean shifts. These procedures are discussed thenceforth.

Bagshaw and Johnson (1977) proposed sequential procedures for monitoring forecast errors in detecting changes in a time series model. Ncube and Amin (1990) suggested two methods for simultaneously monitoring the process mean and process variance by using combined Shewhart-CUSCORE charts. Ncube (1992) proposed the CUSUM-CUSCORE charts that consider equal weight past sample mean values and past sample score values and compared the proposed schemes with the EWMA-CUSCORE charts. Radaelli (1992) adopted the CUSCORE chart in monitoring the rate of occurrence of a rare event. Box and Ramírez

(1992) developed the CUSCORE statistics that can be used as a supplement to the Shewhart chart when a kind of departure specifically feared can be identified in advance. Kim and Jeong (1993) considered an economic design of the CUSCORE chart, while Ncube (1994) proposed the combined Shewhart-CUSCORE procedure for detecting shifts in the process variance. A CUSCORE control approach and its enhancements for detecting process shifts were considered by Cheng (1995). Hu and Roan (1996) suggested control charts for showing the change patterns of two types of additive process shifts, i.e., sustained mean shifts and sporadic spikes, before and after the application of time series modelling. The effect of serial correlation on the in-control average run length (ARL) performance of the CUSCORE chart was studied by Bohm and Hackl (1996). Shao (1998) utilized the CUSCORE technique as an interface between Statistical Process Control and Engineering Process Control. An illustration as to how the CUSCORE charts can be used in detecting changes in the parameters of an integrated moving average model adopted in monitoring clean room air quality was provided by Ramírez (1998). Luceno (1999) gave algorithms to compute the ARL and run length probability distributions of CUSCORE charts for the mean.

An EWMA-CUSCORE procedure for detecting shifts in the process variance was suggested by Ncube and Li (1999). Ncube and Ncube (2000) suggested a CUSCORE chart that considers past history of the values of the quality characteristic's sample mean and past history of the scores, where the suggested scheme surpasses the standard CUSCORE chart, in terms of the ARL performance. Shu et al. (2002) proposed a CUSUM-triggered CUSCORE chart that reduces the mismatch between the detector and fault signature. The advantages and disadvantages of the generalized likelihood ratio (GLR) and CUSCORE charts that were developed for an increase sensitivity in detecting fault signature were presented by Runger and Testik (2003). Luceno (2004) developed CUSCORE charts for detecting level shifts in an autocorrelated process. Han and Tsung (2005) compared the performances of the CUSCORE, GLR test (GLRT) and CUSUM charts in the detection of a dynamic mean shift that eventually approaches a steady-state value. A simple and effective method using the CUSCORE in monitoring the coefficient shifts in an autoregressive moving average process was proposed by Pan (2006). A chapter devoted for discussing the background, development and implementation of the CUSCORE chart in process monitoring was presented by Nembhard

(2006). Han and Tsung (2006) proposed a reference-free CUSCORE chart which can trace and detect dynamic shifts in the process mean quickly even when the reference pattern is unknown. The CUSCORE statistics for detecting spike, step, bump and ramp signals hidden in non-stationary disturbance for feedback-controlled processes were formulated by Nembhard and Valverde-Ventura (2007). Nembhard and Chen (2007) formulated the CUSCORE statistics for monitoring process characteristics which are measured by a generalized minimum variance feedback-control system. Nembhard and Changpetch (2007) derived a CUSCORE statistic and the necessary control limits for detecting a mean shift in a seasonal time series process.

Shu et al. (2008) proposed a weighted CUSUM chart to detect patterned mean shift, while two methods for implementing the CUSCORE chart when the time of the signal is unknown were suggested by Changpetch and Nembhard (2008). The performance of CUSCORE charts in monitoring a nonstationary process subject to the minimum mean-squared error feedback-control was studied by Valverde-Ventura and Nembhard (2008). An integration of the EWMA and GLRT statistics for monitoring processes having patterned mean and variance shifts was made by Zhou et al. (2010). Capizzi and Masarotto (2010) developed two self-starting multivariate CUSCORE charts to monitor the unknown mean of a multivariate normal process. A nonlinear filter chart for monitoring dynamic changes, where the charting statistics comprise a nonlinear combination of the process data was suggested by Han et al. (2010). An analysis of the adaptive EWMA charts' performance in signalling linear drifts was conducted by Su et al. (2011). Chen and Nembhard (2011) developed a multivariate CUSCORE approach based on the sequential likelihood ratio test and fault signature analysis for monitoring the mean vector of an autocorrelated multivariate process. An analysis of the performance of the EWMA chart of the squared deviation under drifts in the process variation was carried out by Huang et al. (2012). Capizzi and Masarotto (2012) proposed an adaptive chart that detects one-sided persistent or time-varying, as well as oscillatory patterned shifts in the process mean when prior information about the direction, magnitude and pattern of the shift is not available. The use of the trigger CUSUM-CUSCORE chart was proposed for solving the drift monitoring problem for two-stage processes by Zhong and Le (2017).

In recent years, little study has been done on CUSCORE and similar control charts for identifying dynamic and pattern mean alterations. In order to close this research gap, four single and dual CUSCORE charts are created in this chapter for identifying unknown systematic mean shifts, respectively. The rest of this chapter is divided into the following sections. The current CUSUM-type and CUSCORE charts are listed in Section 2. Section 3 presents the four CUSCORE charts that were created. The determination of run duration and an assessment of the proposed CUSCORE and current charts are both covered in Section 4. The application of the proposed CUSCORE charts is demonstrated and contrasted with the use of the current charts in Section 5. Concluding remarks are given in Section 6.

2.2 The existing charts

In this section, we briefly review some well-known existing CUSUM-type charts when detecting the patterned shifts in the mean of a process that is assumed to follow a normal distribution.

Let X_t be a measured quality characteristic at the time t, for $t \geq 1$. In addition, suppose that the process, say $\{X_t\}$, is normally distributed with the mean and variance, denoted as μ_0 and σ_0^2, respectively. Symbolically, $X_t \sim \mathcal{N}(\mu_0, \sigma_0^2)$, for $t \geq 1$. Moreover, it is also assumed that after some change point τ, the probability distribution of $\{X_t\}$ changes from $\mathcal{N}(\mu_0, \sigma_0^2)$ to $\mathcal{N}(\mu r_{t-\tau}, \sigma_0^2)$, where $t > \tau$. Clearly, if $r_t = 1$ or $r_t \neq r_{t'}$, the in-control mean μ_0 undergoes either persistent or patterned shifts of sizes $\mu - \mu_0$ or $\mu r_{t-\tau} - \mu_0$, respectively, where $t \neq t' \, (\geq \tau + 1)$. Let $Z_t = (X_t - \mu_0)/\sigma_0$ for $t \geq 1$. Then it can be shown that $Z_t \sim \mathcal{N}(0, 1)$ and $Z_t \sim \mathcal{N}(\mu_1, 1)$ for $t < \tau$ and $t > \tau$, respectively, where $\mu_1 = (\mu r_{t-\tau} - \mu_0)/\sigma_0$.

2.2.1 The CU-I and CU-II charts

Let (A_t^+, A_t^-) be the (upward, downward) CUSUM statistics based on Z_t, for $t \geq 1$, that help in monitoring the irregular changes in the in-control process mean, μ_0, given by

$$A_t^+ = \max\left[0, +Z_t - \delta/2 + A_{t-1}^+\right], \quad A_0^+ = 0, \tag{2.1}$$

$$A_t^- = \max\left[0, -Z_t - \delta/2 + A_{t-1}^-\right], \quad A_0^- = 0, \tag{2.2}$$

where $\max[\cdot]$ is the maximum operator and $\delta \, (> 0)$ is the mean shift size, also called the reference value, which is of particular interest. The CU-I chart with the statistics (A_t^+, A_t^-) is optimal in detecting a step-shift of magnitude δ in μ_0. For a given value of δ, the CU-I chart triggers an out-of-control signal as soon as $A_t^+ > h$ or $A_t^- > h$, where $h\,(> 0)$ is the control limit or decision interval. The value of h is judiciously selected so that the in-control ARL of the CU-I chart reaches a certain threshold. For more details, refer to Montgomery (2009).

Similar to the CU-I chart, another CUSUM statistic was developed by Crosier (1986) for monitoring the mean of a normal process. Let B_t be the CCUSUM statistic based on Z_t, for $t \geq 1$, given by

$$B_t \;=\; (Z_t + B_{t-1}) \times \max\left[0, 1 - \delta/(2C_t)\right], \quad B_0 = 0, \tag{2.3}$$

where $|\cdot|$ is the absolute operator and $C_t = |Z_t + B_{t-1}|$. Here, unlike A_t, the statistic B_t may take positive or negative values. The CU-II chart with the statistic B_t is also approximately optimal in detecting a mean shift of size δ in μ_0. For a given value of δ, the CU-II chart triggers an out-of-control signal as soon as $|B_t| > h$, where $h\,(> 0)$ is the decision interval. The value of h for the CU-II chart is determined similar to that of the CU-I chart. For more details, refer to Crosier (1986).

2.2.2 The CS and RFCS-I charts

In applications where the mean of a process is prone to be perturbed by a 'known' feared signal peculiar to a particular system, it is customary to use the CUSCORE (CS) chart as a signal detector device. For instance, a rotary system may introduce a sinusoidal signal, tool wear may cause a linear-trend signal, etc. The CS chart is based on the concept of an efficient score statistic that aids in an early detection of this feared signal.

Let r_t be the reference pattern of the mean shift that may badly disturb a process. Let (D_t^+, D_t^-) be the (upward, downward) CUSUM statistic based on Z_t and r_t, for $t \geq 1$, given by

$$D_t^+ \;=\; \max\left[0, +r_t\,(+Z_t - \delta r_t/2) + D_{t-1}^+\right], \quad D_0^+ = 0, \tag{2.4}$$

$$D_t^- \;=\; \max\left[0, +r_t\,(-Z_t - \delta r_t/2) + D_{t-1}^-\right], \quad D_0^- = 0, \tag{2.5}$$

Clearly, the CS chart encompasses the CU-I chart with $r_t = 1$. For a given reference pattern

r_t, the CS chart triggers an out-of-control signal as soon as $D_t^+ > h$ or $D_t^- > h$, where $h\,(> 0)$ is the decision interval. When the reference pattern r_t is known in advance, the CS chart may outperform the CU-I chart. For more details, refer to Bagshaw and Johnson (1977) and Luceno (1999).

Although it is known that the efficiency of the CU-I and CS charts depends on a known reference value (δ) and a reference pattern r_t, respectively, an accurate reference pattern (also known as a predetermined fault signature) is usually difficult to obtain in practice. To circumvent this problem, Han and Tsung (2006) suggested a RFCS-I chart for monitoring patterned mean shifts of a normal process. The main idea behind the RFCS-I chart is to estimate the mean shift pattern using Z_t as we rarely know the exact mean change magnitude and pattern. Thus, Han and Tsung (2006) suggested to replace the reference pattern r_t in the CS chart with $|Z_t|$ as it contains real information on the magnitude and pattern of the mean change.

Let (E_t^+, E_t^-) be the (upward, downward) CUSUM statistics of the RFCS-I chart based on Z_t, for $t \geq 1$, given by

$$E_t^+ = \max\left[0, +|Z_t|(+Z_t - |Z_t|/2) + E_{t-1}^+\right], \quad E_0^+ = 0, \tag{2.6}$$

$$E_t^- = \max\left[0, +|Z_t|(-Z_t - |Z_t|/2) + E_{t-1}^-\right], \quad E_0^- = 0. \tag{2.7}$$

Unlike the CU-I and CS charts, as expected, the plotting-statistic of the RFCS-I chart does not depend on the reference value δ, as well as on the reference pattern r_t. The RFCS-I chart triggers an out-of-control signal as soon as $E_t^+ > h$ or $E_t^- > h$, where $h\,(> 0)$ is the decision interval. The RFCS-I chart outperforms the CU-I and CS charts when detecting different patterned shifts in the process mean. For more details, refer to Han and Tsung (2006).

2.3 The proposed charts

In this section, we propose the RFCS-II chart based on the control-charting structure of the CU-II chart, called the RFCS-II chart, for detecting patterned mean shifts. In addition, we also integrate two RFCS-I charts, as well as two RFCS-II charts, to device dual RFCS-I and RFCS-II charts, called DRFCS-I and DRFCS-II charts, respectively, for monitoring dynamic mean shifts of a normal process. On similar lines, a mixed dual chart that integrates the RFCS-I and RFCS-II charts is also suggested.

2.3.1 The RFCS-II chart

The control-charting structure of the RFCS-II chart is similar to that of the CU-II chart. Similar to B_t, let F_t be the CCUSUM statistic of the RFCS-II chart based on Z_t, for $t \geq 1$, given by

$$F_t \;=\; (|Z_t|Z_t + F_{t-1}) \times \max\left[0, 1 - Z_t^2/(2G_t)\right], \quad F_0 = 0, \tag{2.8}$$

where $G_t = ||Z_t|Z_t + F_{t-1}|$. Similar to the RFCS-I chart, the RFCS-II chart does not depend on δ and r_t. The RFCS-II chart triggers an out-of-control signal as soon as $|F_t| > h$, where $h\,(> 0)$ is the decision interval. The values of h is selected so that the in-control ARL of the RFCS-II chart reaches a certain threshold.

2.3.2 The dual and mixed dual charts

In order to increase the sensitivity of the RFCS-I/RFCS-II chart for detecting a range of the mean shift sizes, it is customary to integrate two RFCS-I/RFCS-II charts into a single chart, namely the DRFCS-I and DRFCS-II charts, where the first RFCS-I/RFCS-II chart detects small-to-moderate shifts whilst the second RFCS-I/RFCS-II chart detects moderate-to-large shifts. In addition, for effectively recovering the sensitivity of the DRFCS-I and DRFCS-II charts for several shift sizes, the corresponding CUSUM statistics of these charts are also modified with some reflecting boundaries.

Similar to (E_t^+, E_t^-), let $(H_{1,t}^+, H_{1,t}^-)$ and $(H_{2,t}^+, H_{2,t}^-)$ be the modified CUSUM statistics of the DRFCS-I chart based on Z_t, for $t \geq 1$, given by

$$H_{1,t}^+ \;=\; \max\left[0, +W_{1,t}\,(+Z_t - W_{1,t}/2) + H_{1,t-1}^+\right], \quad H_{1,0}^+ = 0, \tag{2.9}$$

$$H_{1,t}^- \;=\; \max\left[0, +W_{1,t}\,(-Z_t - W_{1,t}/2) + H_{1,t-1}^-\right], \quad H_{1,0}^- = 0, \tag{2.10}$$

and

$$H_{2,t}^{+} = \max\left[0, +W_{2,t}\left(+Z_t - W_{2,t}/2\right) + H_{2,t-1}^{+}\right], \quad H_{2,0}^{+} = 0, \tag{2.11}$$

$$H_{2,t}^{-} = \max\left[0, +W_{2,t}\left(-Z_t - W_{2,t}/2\right) + H_{2,t-1}^{-}\right], \quad H_{2,0}^{-} = 0, \tag{2.12}$$

where

$$W_{1,t} = \min\left(\sqrt{2/\pi}, |Z_t|\right) \quad \text{and} \quad W_{2,t} = \max\left(\sqrt{2/\pi}, |Z_t|\right).$$

It is to be noted that the reflecting boundary of $\sqrt{2/\pi}$ is the mean of $|Z_t|$ for an in-control process. The DRFCS-I chart triggers an out-of-control signal as soon as $\max\left(H_{1,t}^{+}, H_{1,t}^{-}\right) > h_1$ or $\max\left(H_{2,t}^{+}, H_{2,t}^{-}\right) > h_2$, where $h_1\,(> 0)$ and $h_2\,(> 0)$ are the decision intervals. The values of h_1 and h_2 are selected so that the in-control ARL of the DRFCS-I chart reaches a certain threshold.

Similar to F_t, let $I_{1,t}$ and $I_{2,t}$ be the modified CCUSUM statistics of the DRFCS-II chart based on Z_t, for $t \geq 1$, given by

$$I_{1,t} = \left(W_{1,t}\,Z_t + I_{1,t-1}\right) \times \max\left[0, 1 - W_{1,t}^2/(2J_{1,t})\right], \quad I_{1,0} = 0, \tag{2.13}$$

$$I_{2,t} = \left(W_{2,t}\,Z_t + I_{2,t-1}\right) \times \max\left[0, 1 - W_{2,t}^2/(2J_{2,t})\right], \quad I_{2,0} = 0, \tag{2.14}$$

where

$$J_{1,t} = |W_{1,t}\,Z_t + I_{1,t-1}| \quad \text{and} \quad J_{2,t} = |W_{2,t}\,Z_t + I_{2,t-1}|.$$

The DRFCS-II chart triggers an out-of-control signal as soon as $|I_{1,t}| > h_1$ or $|I_{2,t}| > h_2$, where $h_1\,(> 0)$ and $h_2\,(> 0)$ are the decision intervals. The values of h_1 and h_2 are selected similar to that of the DRFCS-I chart.

It is a well-known fact that the CU-II chart is more sensitive than the CU-I chart under both the zero-state (ZS) and steady-state (SS) setups. However, in the SS setup, when detecting moderate-to-large shifts, the CU-I chart prevails over the CU-II chart (Crosier, 1986). Based on this fact, using $I_{1,t}$ and $H_{2,t}$, it is possible to suggest the MDRFCS chart, which integrates the RFCS-II and RFCS-I charts into a single chart. The plotting-statistics of the MDRFCS chart based on Z_t, for $t \geq 1$, is given by

$$K_{1,t} = (W_{1,t}\, Z_t + K_{1,t-1}) \times \max\left[0, 1 - W_{1,t}^2/(2L_t)\right], \quad K_{1,0} = 0, \tag{2.15}$$

$$K_{2,t}^{+} = \max\left[0, +W_{2,t}\,(+Z_t - W_{2,t}/2) + K_{2,t-1}^{+}\right], \quad K_{2,0}^{+} = 0, \tag{2.16}$$

$$K_{2,t}^{-} = \max\left[0, +W_{2,t}\,(-Z_t - W_{2,t}/2) + K_{2,t-1}^{-}\right], \quad K_{2,0}^{-} = 0, \tag{2.17}$$

where $L_t = |W_{1,t}\, Z_t + K_{1,t-1}|$. The MDRFCS chart triggers an out-of-control signal as soon as $|K_{1,t}| > h_1$ or $\max\,(K_{2,t}^{+}, K_{2,t}^{-}) > h_2$, where $h_1\,(> 0)$ and $h_2\,(> 0)$ are the decision intervals. The values of h_1 and h_2 are selected similar to that of the DRFCS-II chart.

2.4 Run length computation and evaluation

In this section, we use the Monte Carlo simulation method to estimate the ZS and SS ARL profiles of the existing (CU-I, CU-II, CS, RFCS-I) and proposed (RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) one-sided charts when sampling from a normal process. Under each simulation run, one hundred-thousand iterations of the run-length are considered for each of the considered control chart. The in-control ZS ARL is set approximately equal to 870. On the lines of Han and Tsung (2006), four different kinds of shift pattern are considered, which include $r_t = 1,\ 3/4 + (1/4)(1/2)^{t-1},\ 5/4 - (1/4)(1/2)^{t-1}$ and $1 + \sin(t\pi/4)$. The values of ARLs for these control charts are displayed in Tables 2.1–2.4. In addition, we also estimate the value of ZS-RMI for each of the control chart with the following formula:

$$\mathrm{RMI}(T) = \frac{1}{m}\sum_{j=1}^{m}\left[\frac{\mathrm{ARL}_{\mu_j}(T) - \mathrm{ARL}_{\mu_j}^{*}(T)}{\mathrm{ARL}_{\mu_j}^{*}(T)}\right], \tag{2.18}$$

where T is the signature of a control chart, $\mathrm{ARL}_{\mu_j}(T)$ denotes the out-of-control ARL of T chart with a shift of μ_j, and $\mathrm{ARL}_{\mu_j}^{*}(T)$ is the minimum of the out-of-control ARLs of the CU-I, CU-II, CS, RFCS-I, RFCS-II, DRFCS-I, DRFCS-II and MDRFCS charts. Similarly, in Table 2.5, the SS-RMI values are reported with the change-points, $\tau = 1$, 6, 11, 26, 51, 76 and 101, under the aforementioned four shift patterns. Note that for a changepoint, say 51, the underlying process remains in the in-control state for $t = 1, 2, \ldots, 50$, and then a patterned shift occurs in the in-control process mean. In case a shift occurs before $t = 50$, then that sequence of observations is ignored when computing the values of the SS-RMI. From Tables 2.1–2.4, the following main points are observed:

Table 2.1: The ZS ARL comparisons of the existing and proposed one-sided charts under constant shift

	Existing				Proposed			
Chart	CU-I	CU-II	CS*	RFCS-I	RFCS-II	DRFCS-I	DRFCS-II	MDRFCS
δ	1.0	1.0	1.0	–	–	–	–	–
h, h_1	4.938	4.641	4.658	9.244	8.394	6.500	5.810	6.500
$\{\mu\}, h_2$	–	–	–	–	–	8.868	8.310	8.150
0.00	873.31	872.84	870.59	872.40	874.43	871.08	869.73	873.59
0.05	579.99	568.01	544.79	530.52	507.98	507.57	484.42	523.32
0.10	394.02	376.33	349.31	329.79	310.87	308.33	286.38	318.35
0.25	135.81	128.83	110.75	105.35	96.74	93.26	85.73	95.60
0.50	37.07	35.07	31.72	33.47	30.82	31.15	28.73	31.02
0.75	16.80	15.87	15.82	18.02	16.60	17.44	16.05	16.82
1.00	10.27	9.72	10.18	12.01	11.06	11.75	10.89	11.18
1.25	7.33	6.93	7.46	8.84	8.14	8.66	8.09	8.14
1.50	5.67	5.41	5.83	6.86	6.34	6.73	6.33	6.29
2.00	3.97	3.77	4.05	4.58	4.24	4.44	4.22	4.15
3.00	2.55	2.43	2.55	2.54	2.37	2.46	2.35	2.32
4.00	2.00	1.92	1.95	1.69	1.59	1.65	1.58	1.56
6.00	1.29	1.20	1.20	1.04	1.03	1.04	1.03	1.02
RMI	0.20	0.14	0.13	0.16	0.09	0.12	0.05	0.09

*The CS chart considers $r_t = (3/4) + (1/4)(1/2)^{t-1}$

Table 2.2: The ZS ARL comparisons of the existing and proposed one sided charts under patterned shift with $r_t = 3/4 + (1/4)(1/2)^{t-1}$

	Existing Charts				Proposed Charts			
chart	CU-I	CU-II	CS^*	RFCS-I	RFCS-II	DRFCS-I	DRFCS-II	MDRFCS
δ	1.0	1.0	1.0	-	-	-	-	-
h_1, h_1	4.938	4.641	4.658	9.244	8.394	6.500	5.810	6.500
$\{\mu\}$, h_2	-	-	-	-	-	8.868	8.310	8.150
0.00	872.36	873.52	869.76	874.46	874.82	870.79	870.28	873.80
$0.05r_t$	643.82	629.90	603.54	594.74	581.67	578.75	559.40	592.78
$0.10r_t$	475.61	460.85	432.27	417.81	394.33	391.50	370.67	404.64
$0.25r_t$	207.74	194.08	171.97	162.50	149.11	143.08	132.32	148.13
$0.50r_t$	65.54	61.38	53.28	53.50	49.02	48.19	44.21	48.01
$0.75r_t$	28.47	26.96	24.83	27.02	24.78	25.49	23.30	24.88
$1.00r_t$	15.91	15.06	14.84	17.16	15.73	16.65	15.27	15.99
$1.25r_t$	10.52	9.87	10.22	12.19	11.17	11.93	11.00	11.34
$1.50r_t$	7.64	7.18	7.68	9.25	8.45	9.10	8.38	8.49
$2.00r_t$	4.86	4.57	4.93	5.88	5.37	5.70	5.32	5.27
$3.00r_t$	2.80	2.65	2.80	2.91	2.67	2.80	2.65	2.60
$4.00r_t$	2.07	1.97	2.01	1.77	1.65	1.71	1.63	1.60
$6.00r_t$	1.29	1.20	1.20	1.04	1.03	1.04	1.03	1.03
RMI	0.22	0.15	0.12	0.17	0.08	0.11	0.04	0.08

Table 2.3: The ZS ARL comparisons of the existing and proposed one sided charts under patterned shift with $r_t = 5/4 - (1/4)(1/2)^{t-1}$

	Existing Charts				Proposed Charts			
chart	CU-I	CU-II	CS^*	RFCS-I	RFCS-II	DRFCS-I	DRFCS-II	MDRFCS
δ	1.0	1.0	1.0	-	-	-	-	-
h, h_1	4.938	4.641	4.658	9.244	8.394	6.500	5.810	6.500
$\{\mu r_t\}$, h_2	-	-	-	-	-	8.868	8.310	8.150
0.00	873.20	872.96	870.40	870.04	872.57	871.97	874.22	873.77
0.05 r_t	524.86	510.50	557.21	468.77	447.30	444.58	424.31	458.44
0.10 r_t	524.86	510.50	557.21	468.77	447.30	444.58	424.31	458.44
0.25r_t	94.01	87.71	113.65	73.85	67.29	65.56	60.38	66.65
0.50r_t	24.19	23.07	27.99	23.98	22.18	22.79	21.11	22.32
0.75r_t	11.83	11.31	12.48	13.54	12.56	13.29	12.36	12.71
1.00r_t	7.79	7.44	7.80	9.32	8.70	9.16	8.61	8.68
1.25r_t	5.88	5.60	5.76	7.04	6.52	6.89	6.50	6.47
1.50r_t	4.75	4.53	4.62	5.57	5.19	5.44	5.18	5.12
2.00r_t	3.50	3.35	3.42	3.87	3.62	3.76	3.59	3.55
3.00r_t	2.39	2.30	2.36	2.33	2.19	2.27	2.18	2.15
4.00r_t	1.96	1.90	1.98	1.65	1.56	1.61	1.55	1.53
6.00r_t	1.29	1.19	1.34	1.04	1.03	1.04	1.03	1.02
RMI	0.19	0.13	0.25	0.16	0.09	0.11	0.05	0.08

Table 2.4: The ZS ARL comparisons of the existing and proposed one sided charts under patterned shift with $r_t = 1 + \sin(t\pi/4)$

	Existing Charts				Proposed Charts			
chart	CU-I	CU-II	CS^*	RFCS-I	RFCS-II	DRFCS-I	DRFCS-II	MDRFCS
delta	1.0	1.0	1.0	-	-	-	-	-
h,h_1	4.938	4.641	4.658	9.244	8.394	6.500	5.810	6.500
$\{\mu r_t\}$, h_2	-	-	-	-	-	8.868	8.310	8.150
0.00	874.31	873.81	869.99	869.78	874.25	872.79	872.89	870.71
0.05 r_t	577.18	566.60	564.66	526.05	506.82	501.33	484.78	518.09
0.10 r_t	387.04	372.08	363.42	327.85	308.66	302.98	284.71	316.24
0.25r_t	129.27	121.06	114.80	102.97	94.25	91.61	84.11	92.54
0.50r_t	32.50	30.77	26.41	31.01	28.27	29.25	26.62	28.37
0.75r_t	13.72	12.79	10.18	15.25	13.80	14.86	13.58	14.03
1.00r_t	7.41	6.81	5.17	8.99	8.07	8.77	8.04	8.04
1.25r_t	4.49	4.13	3.08	5.57	4.92	5.37	4.93	4.84
1.50r_t	3.06	2.85	2.19	3.53	3.15	3.39	3.13	3.07
2.00r_t	2.14	2.07	1.59	1.99	1.87	1.94	1.86	1.84
3.00r_t	1.63	1.51	1.06	1.21	1.15	1.18	1.15	1.14
4.00r_t	1.08	1.05	1.00	1.01	1.00	1.00	1.00	1.00
6.00r_t	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
RMI	0.33	0.26	0.07	0.31	0.21	0.26	0.18	0.21

Table 2.5: The ZS and SS RMI values of the existing and proposed one-sided charts when the ZS in-control ARL is 870

		Existing charts				Proposed charts			
Changepoint	Shift pattern ($\{\mu r_t\}$)	CU-I	CU-II	CS*	RFCS-I	RFCS-II	DRFCS-I	DRFCS-II	MDRFCS
1	$\{\mu\}$	0.1984	0.1370	0.1248	0.1649	0.0871	0.1170	**0.0518**	0.0843
6	$\{\mu\}$	0.1691	0.1549	0.1749	0.1377	0.0998	0.0933	**0.0634**	0.0683
11	$\{\mu\}$	0.1739	0.1661	0.1733	0.1265	0.1065	0.0838	0.0672	**0.0630**
26	$\{\mu\}$	0.1745	0.1711	0.1707	0.1123	0.1096	0.0714	0.0657	**0.0586**
51	$\{\mu\}$	0.1778	0.1729	0.1699	0.1108	0.1111	0.0668	0.0662	**0.0572**
76	$\{\mu\}$	0.1764	0.1741	0.1715	0.1117	0.1130	0.0678	0.0665	**0.0596**
101	$\{\mu\}$	0.1758	0.1732	0.1714	0.1120	0.1136	0.0680	0.0671	**0.0576**
1	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.2174	0.1504	0.1252	0.1657	0.0832	0.1144	**0.0423**	0.0833
6	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1822	0.1588	0.1505	0.1388	0.0989	0.0891	**0.0566**	0.0688
11	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1822	0.1654	0.1454	0.1261	0.1008	0.0785	**0.0579**	0.0637
26	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1852	0.1719	0.1441	0.1127	0.1046	0.0653	0.0576	**0.0575**
51	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1853	0.1731	0.1449	0.1128	0.1086	0.0622	0.0590	**0.0587**
76	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1855	0.1726	0.1444	0.1112	0.1085	0.0616	0.0591	**0.0581**
101	$\{\mu(3/4+(1/4)(1/2)^{t-1})\}$	0.1856	0.1722	0.1436	0.1108	0.1074	0.0605	0.0581	**0.0573**
1	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1896	0.1335	0.2541	0.1604	0.0878	0.1135	**0.0544**	0.0815
6	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1791	0.1679	0.2026	0.1494	0.1152	0.1067	0.0816	**0.0780**
11	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1789	0.1756	0.2072	0.1345	0.1191	0.0967	0.0817	**0.0729**
26	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1785	0.1804	0.2097	0.1200	0.1183	0.0807	0.0794	**0.0654**
51	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1795	0.1807	0.2092	0.1189	0.1214	0.0786	0.0799	**0.0635**
76	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1777	0.1794	0.2071	0.1173	0.1214	0.0766	0.0800	**0.0642**
101	$\{\mu(5/4-(1/4)(1/2)^{t-1})\}$	0.1806	0.1824	0.2106	0.1204	0.1232	0.0807	0.0816	**0.0657**
1	$\{\mu(1+\sin(t\pi/4))\}$	0.3297	0.2563	**0.0663**	0.3091	0.2157	0.2593	0.1843	0.2093
6	$\{\mu(1+\sin(t\pi/4))\}$	0.1678	0.1513	**0.0816**	0.1563	0.1267	0.1194	0.0954	0.1003
11	$\{\mu(1+\sin(t\pi/4))\}$	0.2798	0.2910	**0.0713**	0.2069	0.1894	0.1741	0.1615	0.1423
26	$\{\mu(1+\sin(t\pi/4))\}$	0.3309	0.3564	**0.0712**	0.2732	0.2839	0.2372	0.2464	0.2038
51	$\{\mu(1+\sin(t\pi/4))\}$	0.2827	0.2990	**0.0734**	0.1907	0.1987	0.1548	0.1596	0.1345
76	$\{\mu(1+\sin(t\pi/4))\}$	0.2179	0.2247	0.1767	0.1187	0.1213	0.0856	0.0859	**0.0684**
101	$\{\mu(1+\sin(t\pi/4))\}$	0.1556	0.1434	0.0900	0.1180	0.1104	0.0834	**0.0759**	0.0780

*The CS chart considers $r_t = (3/4) + (1/4)(1/2)^{t-1}$ when $r_t = 1$

1. As expected, the values of out-of-control ARLs of the control charts tend to decrease as the magnitude of the shift increases and vice versa. For example, from Table 2.1, the out-of-control ARLs of the (CU-I, CU-II, CS, RFCS-I, RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) charts at $\mu = 0.05$, 0.50, 2.0 and 6.0 are (579.99, 568.01, 544.79, 530.52, 507.98, 507.57, 484.42, 523.32), (37.07, 35.07, 31.72, 33.47, 30.82, 31.15, 28.73, 31.02), (3.97, 3.77, 4.05, 4.58, 4.24, 4.44, 4.22, 4.15) and (1.29, 1.20, 1.20, 1.04, 1.03, 1.04, 1.03, 1.02), respectively.

2. It can be seen that the RFCS-II chart uniformly outperforms the RFCS-I chart when detecting different patterned mean shifts. For instance, from Table 2.2, the out-of-control ARLs of the (RFCS-I, RFCS-II) charts for $\{\mu r_t\} = 0.05$, 0.50, 2.0 and 6.0 are (594.74, 581.67), (53.50, 49.02), (5.88, 5.37) and (1.04, 1.03), respectively. In addition, from Tables 2.1–2.3, it is observed that the values of ZS-RMI of any of the proposed chart is less than those of the existing charts. For example, the values of ZS-RMI for the (CU-I, CU-II, CS, RFCS-I, RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) charts from Tables 2.1, 2.2 and 2.3 are (0.20, 0.14, 0.13, 0.16, 0.09, 0.12, 0.05, 0.09), (0.22, 0.15, 0.12, 0.17, 0.08, 0.11, 0.04, 0.08) and (0.19, 0.13, 0.25, 0.16, 0.09, 0.11, 0.05, 0.08), respectively. However, from Table 2.4, the CS chart outperforms all charts in terms of ZS-RMI, but when detecting small patterned mean shifts of magnitude less than $0.50(1 + \sin(t\pi/4))$, the proposed charts significantly outperform the CS chart. A shortcoming of CS chart is that its performance is heavily dependent on the actual choice of the reference pattern r_t, which may not be known in practice. In addition, it is worth mentioning that the proposed (RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) charts surpass the existing (CU-I, CU-II, RFCS-I) charts.

3. From Table 2.5, it can be seen that the proposed charts outperform all of the existing charts in terms of ZS-RMI and SS-RMI values when $r_t = 1$, $(3/4) + (1/4)(1/2)^{t-1}$ and $(5/4) - (1/4)(1/2)^{t-1}$, respectively. Note that when the changepoint is equal to one (greater than one), it refers to the ZS (SS) setup. The boldfaced values showcase the minimum values in each row. However, as expected, with a patterned mean shift $r_t = 1 + \sin(t\pi/4)$, when the changepoint is less than or equal to 51, the CS chart outperforms all other charts. In addition, the proposed charts (DRFCS-I, DRFCS-II,

MDRFCS) also surpass the CU-I, CU-II and RFCS-I charts in terms of the ZS-RMI and SS-RMI values.

2.5 Real data application

For illustrating the use of the proposed charts in a real context, a phase-II standardized dataset is adopted, which is related to the hourly chemical process viscosity readings. This complete dataset has been used (and reported) by Han and Tsung (2006) when implementing the two-sided CU-I, CS and RFCS-I charts. For a chemical process control, viscosity measure is taken every hour as a quality characteristic that must be maintained as close as possible in order to meet a fixed target level. In case of any significant deviation from the target level, the process is considered to be out-of-control and chart should signal as early as possible (see Han and Tsung (2006) for more detail and discussion).

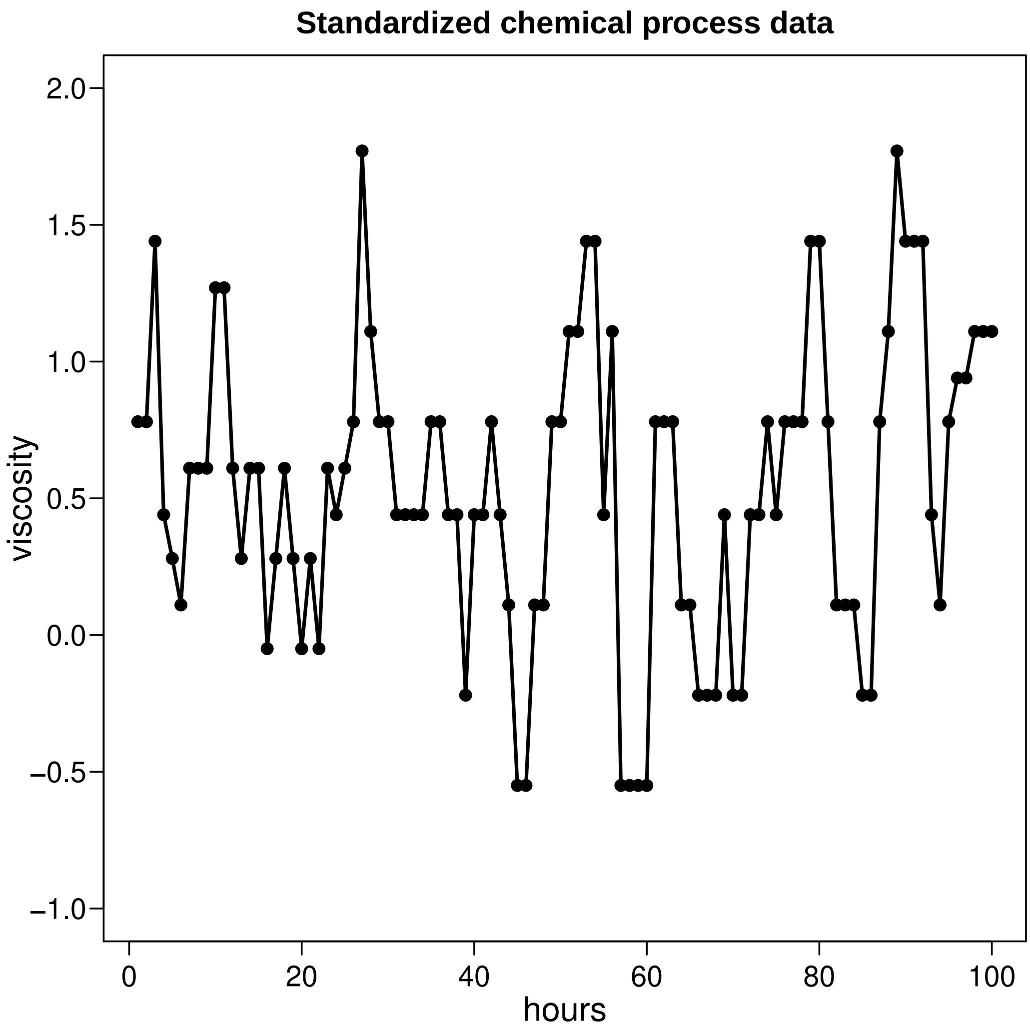

Figure 2.1: A Phase-II standardized dataset related to the chemical process viscosity readings

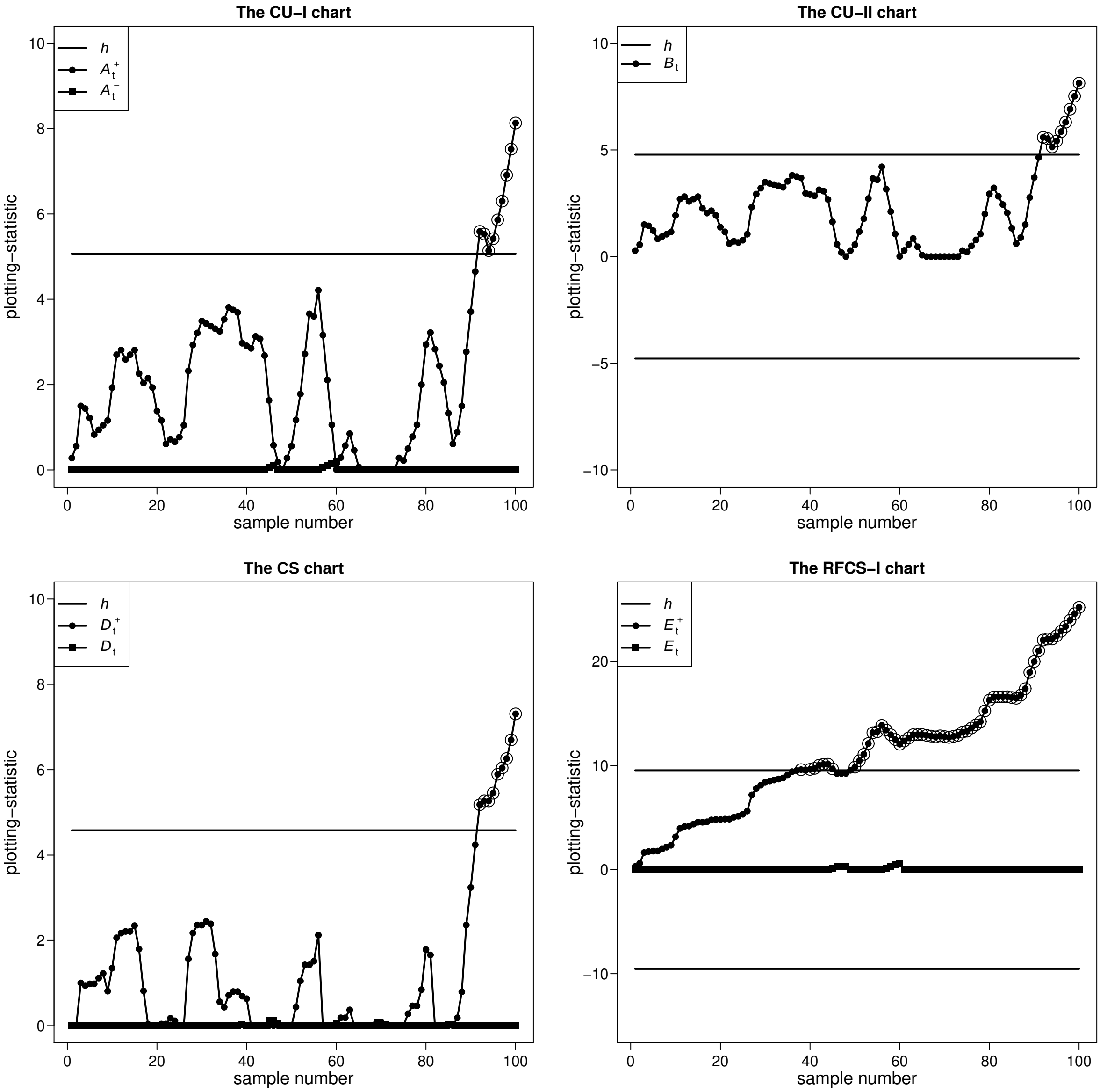

Figure 2.2: The existing two sided charts for the chemical process viscosity dataset

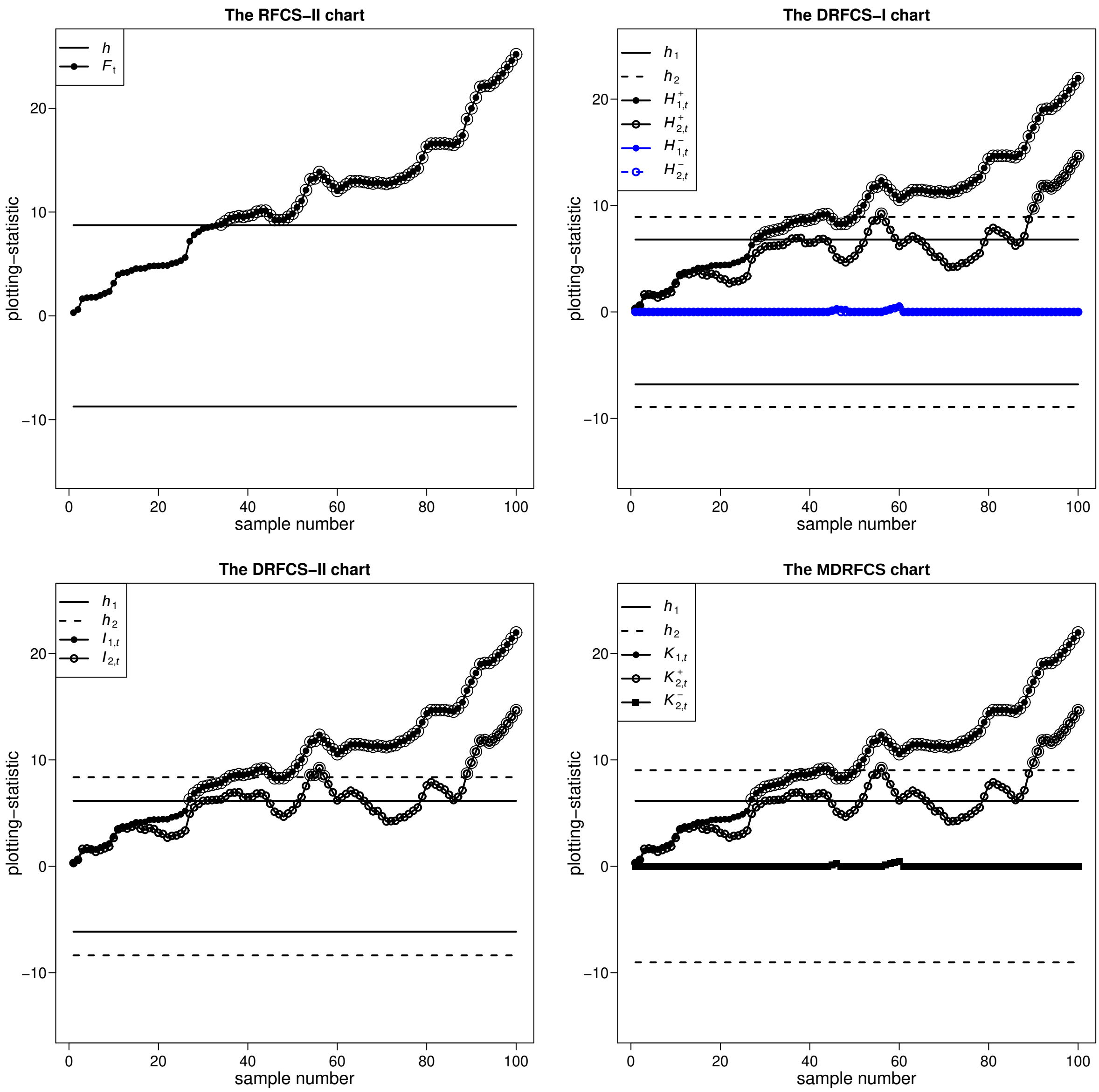

Figure 2.3: The proposed two-sided charts for the chemical process viscosity dataset

From the chemical process, 100 raw readings are collected on-line over 100 hours. These observations are then standardized. A run chart of these standardized observations is displayed in Figure 2.1. The existence of a dynamic or cyclic mean change pattern may be observed from the run chart that is given in Figure 2.1. The existing (CU-I, CU-II, CS, RFCS-I) and proposed (RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) two-sided charts are applied on this dataset, where the in-control ARL for each of the control chart is set to 500. Here, for the CS chart, the reference pattern (r_t) is taken as $\sin(t\pi/4)$. The choices of the parameters for the CU-I, CU-II, CS, RFCS-I, RFCS-II, DRFCS-I, DRFCS-II and MDRFCS charts are $(\delta = 1.0, h = 5.071)$, $(\delta = 1.0, h = 4.780)$, $(\delta = 1.0, h = 4.58)$, $h = 9.542$, $h = 8.736$, $(h_1 = 6.80, h_2 = 8.93)$, $(h_1 = 6.151, h_2 = 8.373)$ and $(h_1 = 6.151, h_2 = 9.030)$, respectively. These control charts are displayed in Figures 2.2 and 2.3.

From Figures 2.2 and 2.3, it can be seen that all of the control charts are triggering out-of-control signals to indicate an upward shift in the in-control process mean. The existing (CU-I, CU-II, CS, RFCS-I) and proposed (RFCS-II, DRFCS-I, DRFCS-II, MDRFCS) two-sided charts have triggered the first out-of-control signals at the (92nd, 92nd, 92nd, 38th) and (34th, 28th, 27th, 27th) samples, respectively. This shows that the proposed charts signal an out-of-control situation earlier than the existing charts.

2.6 Conclusion

This chapter presents four CUSCORE charts for detecting unknown patterned mean shifts in a normal process, namely the single RFCS-II, DRFCS-I, DRFCS-II and MDRFCS charts. In comparing the ARL and RMI performance of the four developed one-sided CUSCORE charts with the four existing one-sided CU-I, CU-II, CS and RFCS-I charts, it is found that the former charts outperform the later. The implementation of the four developed CUSCORE charts is illustrated through real-data viscosity readings, which are compared with the four existing charts mentioned above. According to this dataset, the developed CUSCORE charts issue out-of-control signals faster than the existing charts.

Chapter 3

Conclusion and Future work

3.1 Conclusion

Control charts have traditionally been utilized to detect constant shifts in the mean in various industries. However, in processes that involve feedback-control and autocorrelation, dynamic and time-varying mean shifts are commonly observed, rendering conventional $\bar{X}$-type charts inadequate for their detection. To address this issue, the literature has introduced the CUSCORE chart as a tool capable of identifying dynamic or time-varying process alterations. Over the years, researchers have developed numerous control charting procedures to monitor processes involving unknown patterned mean shifts.

The primary focus of this thesis was the development of four CUSCORE charts designed to detect unknown patterned mean shifts in normal processes. These charts include the single RFCS-II chart, as well as the dual DRFCS-I, DRFCS-II, and MDRFCS charts. To assess their performance, an evaluation was conducted by comparing their ARL and RMI characteristics with four existing one-sided control charts: CU-I, CU-II, CS, and RFCS-I. The results demonstrated that the proposed charts surpassed the performance of the existing ones, exhibiting lower out-of-control ARL and RMI values. Notably, among the proposed charts, the DRFCS-II chart exhibited the highest sensitivity in detecting deviations in the mean shifts.

3.2 Future work

For future research work, the variable sampling interval, variable sample size, and variable sample size and sampling interval approaches can be incorporated into the proposed CUSCORE charts to increase the sensitivity of these charts in detecting patterned mean shifts. As this thesis only considers the ARL criterion, other performance measures, such as the expected ARL, median run length and expected MRL criteria can be utilized in studying the performance of the proposed charts.

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