Effect of Van der Waals Force on Wave Packet Dynamics

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dedicated to my mother(late), father, and my country

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The completion of a thesis may sound pleasant to the ears and calm to the mind, but it has been a long and often arduous road that has been made more bearable by many reputable and dear ones nearby in several ways.

I want to begin by humbly thanking Almighty Allah for giving me the courage to do all of this. Then, I want to officially express my gratitude to my parents for giving me opportunity, resources, and, most importantly, for believing in me. I would especially like to thank my supervisor, Dr. Farhan Saif, for spending so much time reviewing and editing my work from beginning to conclusion.

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Abstract

In light of the fact that nanotechnology is unquestionably a fantastic field of science. It has numerous uses in engineering, science, and defence. It is necessary to gather data on solid surfaces for research and technology in the fields of surface and material sciences. It takes a microscope based on the laws of quantum mechanics to see the nano structure on the surface of a solid.

A tool that can be used to examine solid surfaces is the recurrence tracking microscope. An atomic cloud is bounced off on atomic mirror that is created as a result of the interaction between the force of gravity and optical force, revealing quantum recurrences of material wave packets. A cantilever is attached to the opposite side of this atomic mirror. The atom's initial potential energy determines how long recurrences last when they are bouncing off on atomic mirror. As the structural change manifests itself beneath the cantilever on a solid's surface, the original height of the atoms will shift. When an atomic cloud in a recurrence tracking microscope reaches the insulator's surface, a Van der Waals force exerted on the cloud can stop the phenomenon of quantum recurrences.

In this thesis, by introducing the van der waals force into the system we are attempting to examine its effect on the recurrence tracking microscope.

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Chapter 1

Introduction

It has become increasingly popular over the past ten years to examine the matter waves packet phenomena in quantum mechanical systems using theoretical knowledge, numerical equations, and experimental verification[1]. The interference between quantum states generates the quantum wave packets, which evolve over time. The phenomena of time-dependent interference for bound states with quantized energies is highly fascinating because it has implications for the development of quantum mechanical systems based on matter wave packets, such as the recurrence tracking microscope (RTM).

Additionally, it also realize us about long term time evolution's novel aspects, like quantum wave packet revivals. Researchers have shown a significant deal of interest in the solutions to bound states of quantum mechanical issues in the time domain. By demonstrating wave packet solutions to numerous issues, including the situations of the free particle and the forced particle, the relationship between the quantum mechanical and classical explanations of nature is made clear[1].

The development of the effective potential has enabled us theoretically to create the apparatus of quantum wave packets. This effective potential contains three potentials, gravitational potential, optical potential and van der Waals potential. In this study our focus will be on the interaction of van der waals potential in the atomic mirror. To see the the effect of the van der waals force in atomic mirror we also introduce recurrence tracking microscope (RTM) as it works on the phenomena of quantum wave packet revivals.

Thus, one can find the effect of van der waals force on wave packet dynamics. In this research work, We will investigate the impact of the Van der Waals force on material wave packets by taking recurrence tracking microscope as a reference.

Chapter 1 contains introduction.

Chapter 2 begins with the description of de Broglie's hypothesis, quantum wave packets in triangular potential well, auto correlation function and the phenomena of quantum revivals.

Chapter 3 is completely dedicated to recurrence tracking microscope and it explain the experimental setup, working principle and two modes of operation of recurrence tracking microscope.

Chapter 4 deals with the interaction of van der waals force, the incorporation of the van der waals potential to the effective potential for recurrence tracking microscope, Taylor series of the effective potential. It also explain the wave packets confine in a parabolic potential the tunneling of the material wave packet through the parabolic barrier.

Chapter 5 is about the conclusion of this study.

Chapter 2

General Analysis of Quantum Wave Packets

The inability of classical physics to explain a number of microscopic phenomena, including atomic stability, black body radiation, the photoelectric effect, and atomic spectroscopy, opened the door to the search for novel theories outside of traditional domain.

The quantum of energy was initially conceptualised by Max Planck in 1900, marking the first significant advancement to explain the black body radiation phenomenon. He proposed that the energy exchange between the radiation and its surroundings occurs in discrete or quantized amounts. He claimed that an electromagnetic wave of frequency ω and the matter can only exchange energy in integer multiples of $\hbar\omega$ [2]. The quantization of the electromagnetic field proved to be a thought that has broad ramifications.

In 1905 Einstein realised that Planck's theory of the quantization of electromagnetic waves must also apply to light in his quest to comprehend the photoelectric effect. He adopted Planck's method and proposed that light is constituted of discrete energy units, or particles known as photons, each with an energy of $\hbar\omega$, where ω is the frequency of the light. Another break through was due to Neils Boher in which he introduced the model of Hydrogen atom. In this work he explained that atoms can be found in discrete states of energy and that the interaction of atom with electromagnetic field. These breakthroughs

from Plank, Einstein, Boher and many others lay down the theoretical foundations of experimental confirmation for the Particle aspect of waves.

In 1923, de Broglie introduced a powerful and new concept. He postulated that not only electromagnetic fields exhibit particle like behaviour but conversely, material particles shows wave like behaviour. In 1927 this concepts was confirmed experimentally by Davisson and Germer. They obtained interference patterns from the material particles like atoms and electrons.

2.1 The de Broglie Wave

The energy and momentum of a photon of frequency f is defined as

$$E = hf \tag{2.1}$$

$$P = \frac{hf}{c} \tag{2.2}$$

The relationship between λ and f for electromagnetic fields in free space

$$c = f\lambda \tag{2.3}$$

$$\omega = 2\pi f \tag{2.4}$$

$$k = \frac{2\pi}{\lambda} \tag{2.5}$$

$$\hbar = \frac{h}{2\pi} \tag{2.6}$$

The equation (2.2) appears as

$$P = \hbar k \tag{2.7}$$

$$E = \hbar\omega \tag{2.8}$$

$$\omega = ck \tag{2.9}$$

The equation (2.8) is known as dispersion relation and it reveals that ω is linearly dependant on k. Now a collection of waves of different wave lengths are composed to construct a wave packet propagating with no distortion and its all component waves are moving with same constant velocity c [3]. The photon is characterized by two wave parameters k and ω, which in a sense a particle according to equations (2.7) and (2.8).

A photon has zero rest mass and travel with the velocity of light. With finite rest mass, more familiar particles also have wave properties. For a Particle with mass M and kinetic energy E, the wave length λ of the corresponding matter wave is defined as

$$\lambda = \frac{h}{P} \tag{2.10}$$

The above relation is known as de Broglie wave length and it ascribes wave property to a matter particle [3].

2.2 Introduction to Wave Packets

The term ***wave packet*** refer to a collection of waves of various wave lengths that make up a localized wave function. Phases and amplitudes of these waves are chosen in such a way that they interfere constructively in a small area of space and destructively in other ares.The relationship between classical physics and quantum mechanics may be understood in large part thanks to these wave packets. In order to deal with and unify nature's particle-like behaviour and wave-like activity, the concept of a wave packets aids in the development of mathematical tools [2]. In general, for the discrete case, quantum wave packet in its initial sate is defined as

$$\psi(z,0) = \sum_n a_n \phi_n(z) \tag{2.11}$$

where a_n represents the probability amplitudes and $\phi_n(z)$ represents the

eigen functions of the wave packet. The probability amplitudes a_n can be expressed as

$$a_n = \langle \phi_n(z) \mid \psi(z,0) \rangle \tag{2.12}$$

For the time dependence of the initial wave packet $\psi(z,0)$

$$\psi(z,t) = e^{-\iota \frac{E_n}{\hbar} t} \sum_n a_n \phi_n(z) \tag{2.13}$$

$$\mid \psi(z,t) \rangle = \sum_n \tilde{a}_n(t) \phi_n(z) \tag{2.14}$$

Here $\tilde{a}_n(t)$ represents the time dependent probability amplitudes.

2.3 Particle in a Triangular Potential Well

A triangular potential well is created by a linear potential V(z) such that $V'(z)$ is a constant value and V(z) is bounded by an infinite barrier at z = 0. The potential is created by uniform gravitational field. Since $F = -Mg$ where F is the force, M is mass of the particle and g is gravitational acceleration. We also know the work done $W = Fz$ and to our situation we can say that $V(z) = Fz$. Thus

$$V(z) = Mgz \tag{2.15}$$

The graphical representation of this potential well is triangular in its nature as shown in figure 2.1(a). The triangular potential well has a great importance for the design of semiconductor devices. For example the high electron mobility transistor (HEMT) in which electrons are confined in a triangular potential well produced when different semiconducting materials are used to form a hetero junction. In order to solve the problems related to triangular potential well it is vital to have the solution of the Airy differential equation.

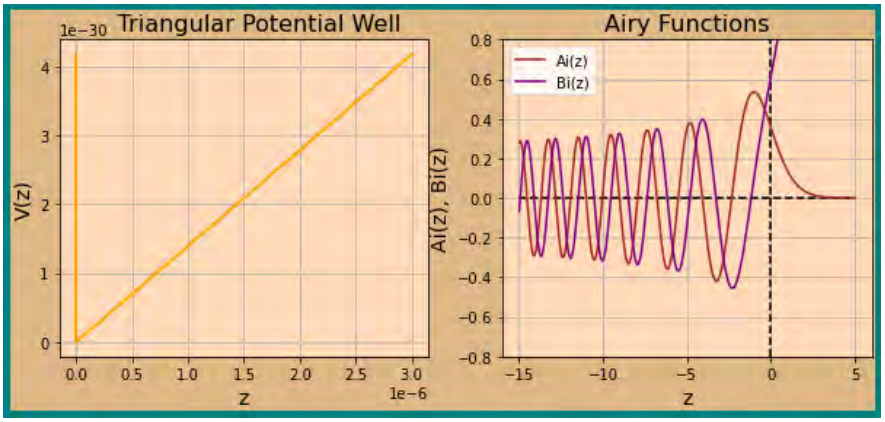

Figure 2.1: (a) on left side: The gravitational triangular potential well for the Rubidium atom .(b) on right side: The behaviour of Airy functions of first kind A_i (z) and second kind B_i (z)

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2.3.1 The Airy Differential Equation

The Airy function is named after the British astronomer George Biddell Airy. The related functions $A_i(z)$ and $B_i(z)$ are linearly independent solutions to the differential equation

$$\frac{d^2y(z)}{dz^2} - zy(z) = 0 \tag{2.16}$$

known as the Airy equation or the Stokes equation. It is the second order linear differential equation with a point where the character of the solution changes from oscillatory to exponential. The solution of the Airy differential equation is written as

$$y = c_1 A_i(z) + c_2 B_i(z) \tag{2.17}$$

$$A_i(z) \approx 0.355\left(1 + \frac{z^3}{6} + \frac{z^6}{180} + \dots\right) - 0.259\left(z + \frac{z^4}{12} + \frac{z^7}{504} + \dots\right)$$

$$B_i(z) \approx 0.615\left(1+\frac{z^3}{6}+\frac{z^6}{180}+\ldots\right)+0.448\left(z+\frac{z^4}{12}+\frac{z^7}{504}+\ldots\right)$$

The Airy functions $A_i(z)$, $B_i(z)$ are shown in figure 2.1(b).

2.3.2 Solution of Time Independent Schrodinger Equation

The time independent Schrodinger equation for one dimensional potential is

$$-\frac{\hbar^2}{2M}\frac{d^2\phi_n(z)}{dz^2}+V(z)\phi_n(z)=E_n\phi_n(z) \tag{2.18}$$

For the uniform gravitational potential well, if we replace V(z) with Mgz the above equation will become

$$-\frac{\hbar^2}{2M}\frac{d^2\phi_n(z)}{dz^2}+Mgz\phi_n(z)=E_n\phi_n(z) \tag{2.19}$$

We can manipulate the above equation to get its solution in the form of airy functions and that's why airy functions are critical for triangular potential well.

$$\phi_n(z)=A_nA_i(\beta z+\gamma_n)+B_nB_i(\beta z+\gamma_n) \tag{2.20}$$

Where A_n and B_n are normalization constants. If we look back to the graph of airy functions in Figure 2.1(b) we can observe the airy function of second kind $B_i(z)$ does not converge which means $\phi_n(z)$ can never be normalized. Thus we can reject $B_i(z)$ as a solution and it leaves only $A_i(z)$ with exponentially decaying behaviour at $z > 0$.

$$\phi_n(z)=A_nA_i(\beta z+\gamma_n) \tag{2.21}$$

Here β is a constant value and γ_n are the negative zeros of the airy functions. We know that there is an infinite barrier at z = 0 which means

$$\phi_n(0)=0 \tag{2.22}$$

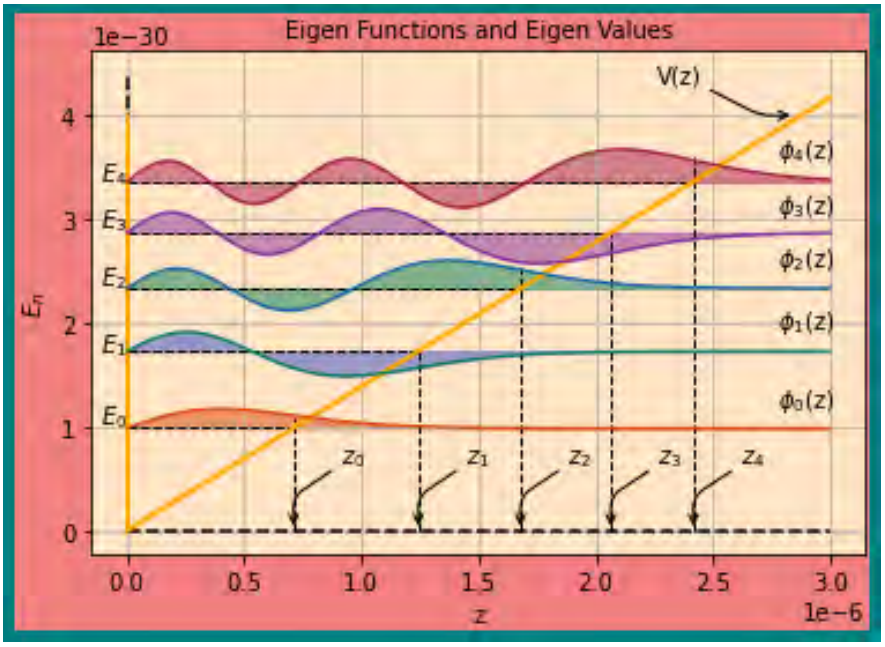

Figure 2.2: This figure shows the first five eigen functions with their classical turning points and the eigen values of a gravitational triangular potential well for Rubidium atom. Here V(z) represents triangular potential well, ϕ_0 to ϕ_4 represent the eigen functions and z_0 to z_4 points on z-axis represent their classical turning points.

Hence the equation for eigen energy values can be obtained as

$$E_n = -\gamma_n Mg \left(\frac{\hbar^2}{2M^2 g} \right)^{1/3}$$

2.3.3 Gaussian Wave Packet in Triangular Potential Well

If we consider the shape of the initial wave packet is gaussian then its mathematical expression can be written as

$$\psi(z,0) = \frac{1}{\sqrt{a\sqrt{2\pi}}} e^{-\frac{z^2}{4a^2}} \tag{2.23}$$

The transformation of initial wave packet $\psi(z,0)$ into a non dimensional space α will provide us

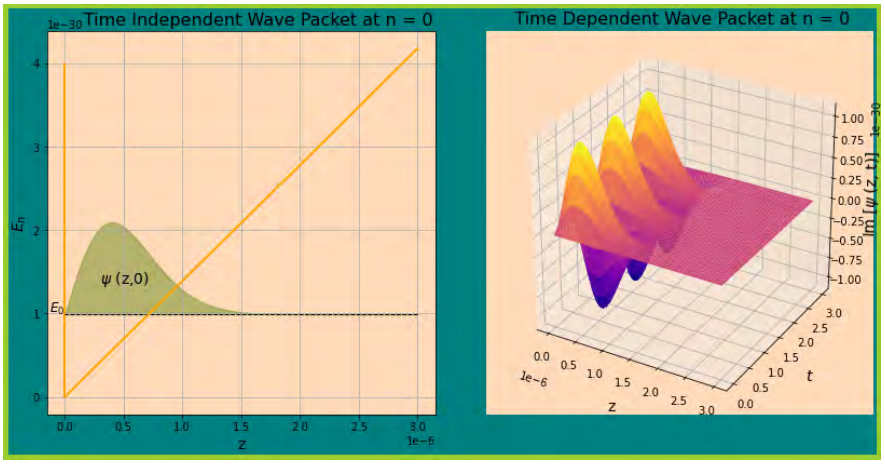

Figure 2.3: (a) on left side: the time independent Gaussian wave packet ψ(z,0) at n = 0 state, decaying along the z-axis. (b) on right side: the imaginary part of the time dependant wave packet ψ(z,t) at n = 0 state. Here it can be observe that wave packet is oscillating along the time axis for three cycles and decaying along the z-axis.

$$\psi(\alpha, 0) = \frac{1}{\sqrt{a\sqrt{2\pi}}} e^{-\frac{(\alpha-\gamma_n)^2}{4a^2\beta^2}} \tag{2.24}$$

Now we confine the gaussian wave packet at E_0 energy level in the triangular potential well as shown in figure 2.3(a). According to the equation (2.12), the probability amplitudes a_n of the wave packet in α space is expressed as

$$a_n = \frac{\sqrt{\chi} A_n}{\beta\sqrt{a\sqrt{2\pi}}} \int_{-\infty}^{+\infty} \phi_n^*(\alpha) e^{-\frac{(\alpha-\gamma_n)^2}{4a^2\beta^2}} \tag{2.25}$$

where χ is a normalization constant.

2.3.4 Solution of Time dependent Schrodinger Equation

The solution of time dependent Schrodinger equation helps us to see the time evolution of the quantum wave packet as figure 2.3(b) shows the time evolution of the imaginary part of the time dependent wave packet.

$$\psi(z, t) = \psi(z, 0) e^{-\iota \frac{E_n}{\hbar} t} \tag{2.26}$$

$$Im[\psi(z,t)] = -\iota\psi(z,0)sin(\omega_n t) \tag{2.27}$$

2.4 The Auto Correlation Function

The one-dimensional temporal development of wave packet solutions, which come from the Schrodinger equation, can be studied using the concept of overlap of the initial state with the time-dependent state. This overlap is referenced as the auto correlation function and is defined as.

$$A(t) = \langle \psi(z,0) \mid \psi(z,t) \rangle \tag{2.28}$$

For n = m

$$A(t) = \sum_m |a_m|^2 e^{-\iota \frac{E_m t}{\hbar}} \tag{2.29}$$

and the probability density of the auto correlation function will be

$$|A(t)|^2 = \sum_n |a_n|^4 + 2\sum_{nm} |a_n|^2 |a_m|^2 cos\left(E_n - E_m\right)\frac{t}{\hbar} \tag{2.30}$$

The above expression is known as auto correlation function.

2.5 Quantum Wave Packet Revivals

The Taylor series expansion for E_m around a mean value $m = r$ provides us

$$E_m = E_r + (m-r)\frac{\partial E_m}{\partial m}\bigg|_{m=r} + \frac{1}{2!}(m-r)^2\frac{\partial^2 E_m}{\partial m^2} + \frac{1}{3!}(m-r)^3\frac{\partial^3 E_m}{\partial m^3}\bigg|_{m=r} + ... \tag{2.31}$$

In this series the first term represents the maximum probable energy, second term represents the first order correction, third term represents the second order correction and so on. Now putting this series in equation (2.29) and auto correlation function will become

$$A(t) = \sum_m |a_m|^2 e^{-\frac{\iota}{\hbar}(E_r+(m-r)\frac{\partial E_m}{\partial m}\Big|_{m=r}+(m-r)^2\frac{\partial^2 E_m}{\partial m^2/2!}\Big|_{m=r}+(m-r)^3\frac{\partial^3 E_m}{\partial m^3/3!}\Big|_{m=r}...)t} \tag{2.32}$$

Since we know that $\omega_r = \frac{E_r}{\hbar}$ thus

$$A(t) = \sum_m |a_m|^2 e^{-\iota(\omega_r+(m-r)\omega^{(1)}+(m-r)^2\omega^{(2)}+(m-r)^3\omega^{(3)}+...)t} \tag{2.33}$$

$$A(t) = e^{-\iota\omega_r t}\sum_m |a_m|^2 e^{-\iota((m-r)\frac{2\pi}{T^{(1)}}+(m-r)^2\frac{2\pi}{T^{(2)}}+(m-r)^3\frac{2\pi}{T^{(3)}}+...)t} \tag{2.34}$$

$$A(t) = e^{-\iota\omega_r t}\sum_m |a_m|^2 e^{-(m-r)\frac{2\pi\iota t}{T^{(1)}}-(m-r)^2\frac{2\pi\iota t}{T^{(2)}}-(m-r)^3\frac{2\pi\iota t}{T^{(3)}}+...} \tag{2.35}$$

Since

$$T^{(i)} = \frac{2\pi}{\omega^{(i)}} \tag{2.36}$$

Here $i = 1, 2, 3, ...$
Thus

$$T^{(1)} = \frac{2\pi\hbar}{|\frac{\partial E_m}{\partial m}\Big|_{m=r}|} \tag{2.37}$$

$$T^{(2)} = \frac{2\pi\hbar}{|\frac{\partial^2 E_m}{\partial m^2}\Big|_{m=r}|/2} \tag{2.38}$$

$$T^{(3)} = \frac{2\pi\hbar}{|\frac{\partial^3 E_m}{\partial m^3}\Big|_{m=r}|/6} \tag{2.39}$$

Here $T^{(1)}$ represents classical period of motion in the bound state, $T^{(2)}$ represents the revival time scale and $T^{(3)}$ represents the super revival time. The ratios of revival time $T^{(2)}$ to the classical period $T^{(1)}$ and super revival time

$T^{(3)}$ to revival time $T^{(2)}$ are given as

$$\frac{T^{(2)}}{T^{(1)}} = \mu \tag{2.40}$$

$$\frac{T^{(3)}}{T^{(2)}} = \zeta \tag{2.41}$$

Here μ and ζ are the scaling constants of their corresponding ratios and they can be effected by the system parameters like depth of the potential well, mass of the particle and the initial state of the matter wave packets.

Chapter 3

Wave Packets on Atomic Mirror

Many of the earliest atom optics researches involved interferometry and there purpose was to focus on phase shift measurements as well as the ability to observe fringes. The results of these experiments provided insight into the significance of enhancing optical components, especially atomic mirrors. The mirror is a commonly used component in optics to avoid chromatic aberration in telescope, Fabry Perot and Michelson interferometers, where performance wise good mirrors are required for partial or total reflection and in the laser cavities as well[7].

In 1982 a first purposed reflection of atoms on the bases of dipole force exerted by an evanescent wave which was created by using laser[8]. In 1987 the first experiment was carried out for the reflection of atoms at grazing incidence[9] and in 1990 for normal incidence[10]. Since then, various groups have been studied evanescent wave atomic mirrors[4].

In order to understand the dynamics of the material wave packets on an atomic mirror, we use an evanescent wave atomic mirror to reflect the matter wave packets of the Rubidium atom.

3.1 The Evanescent Wave Atomic Mirror

In this mirror, the reflection of the matter wave packets is appeared on the bases of dipole force which is generated by using laser beam and exerts on a

neutral atom due to the variation of laser electric field $E(r)$ in space[5].

$$F = -2(\frac{\hbar\Omega_R^2}{\Delta})u(\hat{r})\nabla u(\hat{r}) \tag{3.1}$$

The above equation provides an effective force F while ∇u represents a gradient of u. It reveals us that electromagnetic field exerts a position dependent gradient force on the atom while it interacts with the atom. This gradient force is inversely proportional to the detuning factor Δ and is directly proportional to the square of the Rabi frequency Ω_R.

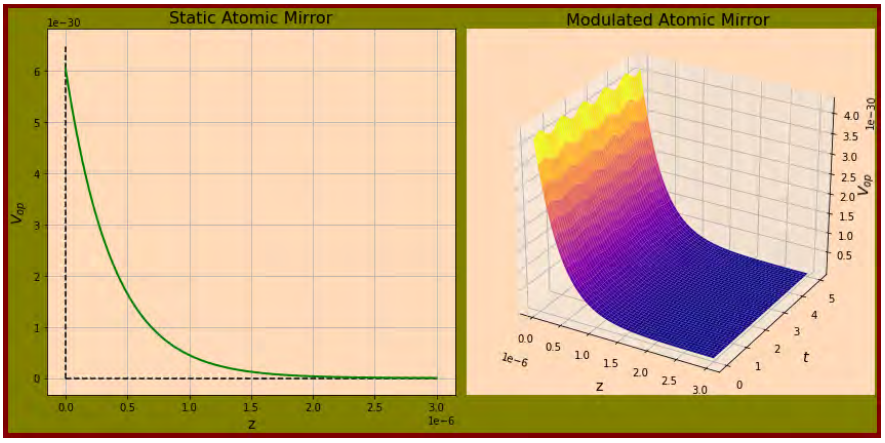

Figure 3.1: (a) left side: the static atomic mirror for the Rubidium atom. (b) right side: five cycles of modulated atomic mirror with modulation height d =10 nm.

The direction of the exerted force on the atom whether it would be made attractive or repulsive depends upon the sign of the detuning factor $\Delta = \nu - \nu_0$. If the field frequency ν is to be set less then the atomic transition frequency ν_0 then detuning factor Δ will be negative, that means force will be attractive and vice versa. The amplitude of the force decreases exponentially with distance from the dielectric surface[6] as shown in figure 3.1(a).

$$F_{opt} = -V_0\kappa e^{-\kappa z} \tag{3.2}$$

3.1.1 Spatial Modulation in Atomic Mirror

When the dielectric surface moves slightly up and down along the z axis, spatial modulation of the atomic mirror can be observed. Hence the optical force F_{opt} will become explicitly time dependent as shown in figure 3.1(b).

$$F_{opt} = V_0 \kappa e^{-\kappa(z - dsin\omega t)} \tag{3.3}$$

3.2 Recurrence Tracking Microscope

This microscope working on the phenomena of quantum recurrence and it can help us for the study of nano structures which appear on the solid surface at the atomic scale resolution and therefore it is named as Recurrence Tracking Microscope (RTM). It can probe surfaces from insulators to conductors, surfaces contain impurities and provides information about the surfaces with periodic structures[7]. Its operation comprises of two modes the Static Mode of operation and the Dynamical Mode of Operation, their details will be given in next sections. The phenomena of quantum recurrence of a material wave packet is already well understood and have been realized experimentally for both oscillating and non oscillating surfaces.

3.2.1 The System

In the Recurrence Tracking Microscope, magneto-optically trapped cold atoms are placed on an atomic mirror. For the matter wave packet, the mirror is created when a monochromatic laser light is reflected from a dielectric film under the condition of total internal reflection. From one end the cantilever is physically attached to the dielectric film while the other end of the cantilever is above the surface under investigation as shown in figure 3.2.

If the setup is in on state, then by switching off the magneto-optical trap, the atoms are allowed to travel towards the atomic mirror under the effect of gravitational force. Here we assume the frequency ν of the optical field that causes the atomic mirror to be tuned such that it is far from the transition fre-

quency ν_0 between any two levels of the atom. The probability of spontaneous emission is [7]

$$P_{sp} = \eta(\frac{\Omega_{max}^2}{4\Delta^2})\tau_{ref} \tag{3.4}$$

Where Δ represents the magnitude of atom field detuning, $\tau_{ref} = \frac{2}{\kappa v_z}$ is the characteristic time of the atom field interaction during the process of reflection of the atomic mirror and η is the decay constant of the higher sate. The velocity of the atom towards the gravitational field along the z direction is represented by the term v_z. Thus the probability of finding the atom in its excited state becomes very minute at the cost of large detuning factor Δ. Due to this reason, we can ignore spontaneous emission in experiments[7].

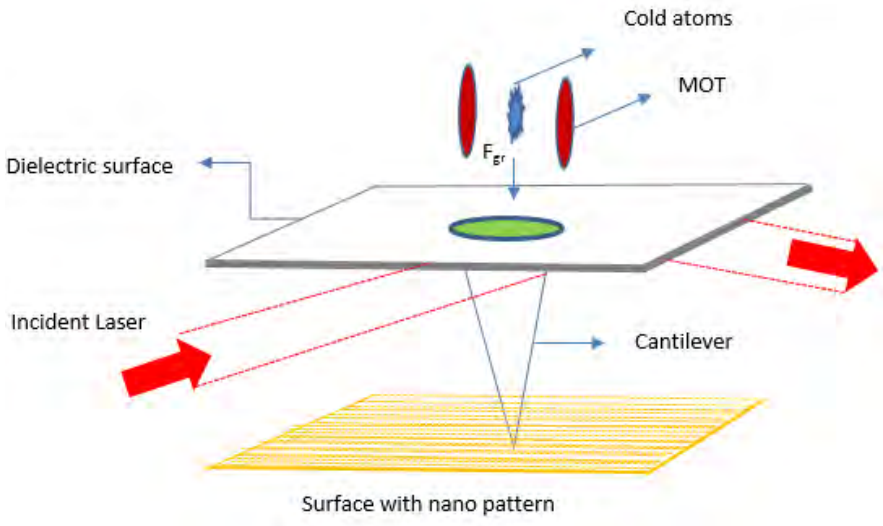

Figure 3.2: The experimental setup for the implementation of recurrence tacking microscope (RTM).

The evanescent wave field $E(z)$ is responsible for the creation of atomic mirror and it varies with position, as a result the Rabi frequency $\Omega = d.E(z)/\hbar$ also becomes position dependent. Thus Ω_{max} represents the maximum Rabi frequency face by the atom at its turning points on the surface of the atomic

mirror. Here the kinetic energy of the atom of mass M is equate as [8]

$$\frac{\hbar\Omega_{max}^2}{4\Delta} = \frac{Mv_z^2}{2} \tag{3.5}$$

The kinetic energy of the atom in the above equation is compensated by the light shift due to the external field.

An exponentially increasing repulsive force, due to the optical potential, is faced by the atoms near the dielectric surface and they are tuned to the blue, which means $\nu > \nu_0$ as defined through equation (3.2). On the other hand, atom faces a constant gravitational force $F = -Mg$ due to gravitational field which push the atom towards the atomic mirror. Thus a bounded motion can be observed from the atom in the presence of gravitational force and optical force together as shown in figure(). We can define the effective Hamiltonian for the bounded atomic dynamics in so generated atomic trampoline or gravitational cavity.

$$H = \frac{p^2}{2M} + Mgz + V_0 e^{-\kappa z} \tag{3.6}$$

Here g represents the constant gravitational acceleration, κ express the decay length of the atomic mirror, M shows the mass of the atom and p represents the center of mass momentum along the z axis.

3.2.2 Static Mode of Operation

A matter wave packet with finite energy passes through destructive and constructive interferences during its temporal evolution. The interference in quantum mechanical evolution of the material wave packet performs an important role and shows itself in the phenomena of quantum recurrences. For a short duration of time the wave packet follows classical evolution and it reemerge itself after a classical period. When few classical periods has passed, it occupies all the available space which follows wave mechanics and collapses. It rebuilds itself after certain time evolution due to its quantum mechanical dynamics. This phenomena of its reappearance is known as quantum revival and the time which passes during its collapse and complete reappearance is

known as quantum revival time. We can also observe the partial appearance of the initially propagated wave packet at the fractions of the quantum revival time and these times are known as fractional revival times.

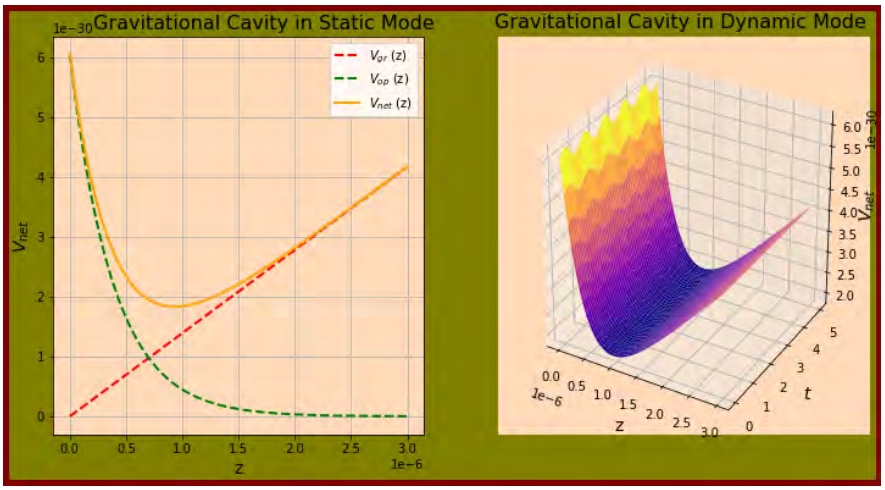

Figure 3.3: (a) on left side: the gravitational cavity when RTM working in static mode. (b) on right side: the gravitational cavity when RTM is working in dynamical mode for the Rubidium atom.

In RTM we approximate the total potential as a triangular well potential and its energy and revival time are defined as

$$E_n = \left(\frac{M\hbar^2 g^2}{2}\right)\alpha_n \tag{3.7}$$

$$T_0^{(2)} = \frac{16E_0^{(2)}}{M\pi\hbar g^2} \tag{3.8}$$

Here E_0 represents the intial mean energy of the wave packet and α_n are the negative zeros of the airy function.

3.2.3 Dynamic Mode of Operation

The slight movement of the surface under investigation, the cantilever changes its position following the surface structures. As a result the initial distance between the bouncing atom and the atomic mirror also change. Due to this rea-

son the atom faces different initial energy E_0 and different revival time $T_0^{(2)}$. If the atomic mirror have spatial modulation as shown in figure(3.4) the bouncing atom shows collapse and revival after a certain period of time. The time of the quantum revival can be calculated as[7]

$$T_{\tilde{\zeta}}^{(2)} = T_0^{(2)} \left[1 - \frac{1}{8} \left(\frac{Mgd}{E_0} \right)^2 \frac{3(1-s)^2 + \tilde{d}^2}{[(1-s)^2 - \tilde{d}^2]^3} \right] \tag{3.9}$$

Here

$$s = \left(\frac{E_n}{E_0} \right)^{\frac{1}{2}} \tag{3.10}$$

$$\tilde{d} = \frac{n^2 \hbar \omega}{4E_0} \tag{3.11}$$

In dynamic mode of operation modulation depends on the frequency ω and the height of the periodic structure d. The relation ship between the height of the periodic structure and the quantum revival time is defined as

$$d = \left(\frac{8}{3} \right)^{\frac{1}{2}} \frac{E_0}{Mg} (1-s)^2 \left[1 - \frac{T_{\tilde{\zeta}}^{(2)}}{T_0^{(2)}} \right] \tag{3.12}$$

Chapter 4

Van der Waals Force Interaction

The spontaneously rising of transient electric and magnetic fields in vacuum or in the material body is the origin of van der Wall's interaction. Generally there are two different ways to observe the fluctuation of charges. Besides the quantum mechanical uncertainties in positions and momenta of the atoms there also exists thermal agitation of the atoms. In the range of zero limit of temperature, thermal agitation of the atoms can be neglected but the Heisenberg's uncertainty principle is unavoidable for the quantum mechanical objects. With the passage of time, the concepts of Van der waals force gradually developed and named after the name of Dutch scientist Van der Waal however other scientists also contribute.

In 1870 Van der waals gave a revolutionary idea of non ideal gas equation for the interaction of particles and Maxwell set the time dependent equations for electric and magnetic fields. Hertz showed the creation and absorption of electromagnetic field waves. By comparing to one atom, it tended to be generalized on interactions between huge bodies when the Van der Waals interaction between two particles were taken into account.

4.1 Van der Waals Force in Atomic Mirror

The potential energy will change from just exponential variation because of the van der waals interaction. The mirror potential is shown in figure 4.1(a)

after the addition of $\frac{1}{z^3}$ attractive potential.

$$V_{vw}^{LJ}(z) \equiv -\frac{\epsilon - 1}{\epsilon + 1}\left(\frac{C^{(3)}}{z^3}\right) \tag{4.1}$$

Here $V_{vw}^{LJ}(z)$ represents the van der waals potential calculated by Lennard-Jones in the range of short distances, ϵ represents the dielectric constant and $C^{(3)}$ represents the van der waals coefficient.

Now at smaller distances, new potential changes significantly from the dipole potential alone. We can see the modification in the atomic mirror such that the height of the total potential is reduced by a factor of 3. The reduction in the height of the total potential can be detected by measuring the reflection threshold as it becomes the function of the kinetic energy of the atom. The experiments shows that it is easy to keep the kinetic energy of the atom constant and change the magnitude of the total potential by changing the magnitude of the dipole potential. This is carried out by varying the detuning or intensity of the laser beam. Thus the attraction of the van der waals force can be balanced by varying dipole force.

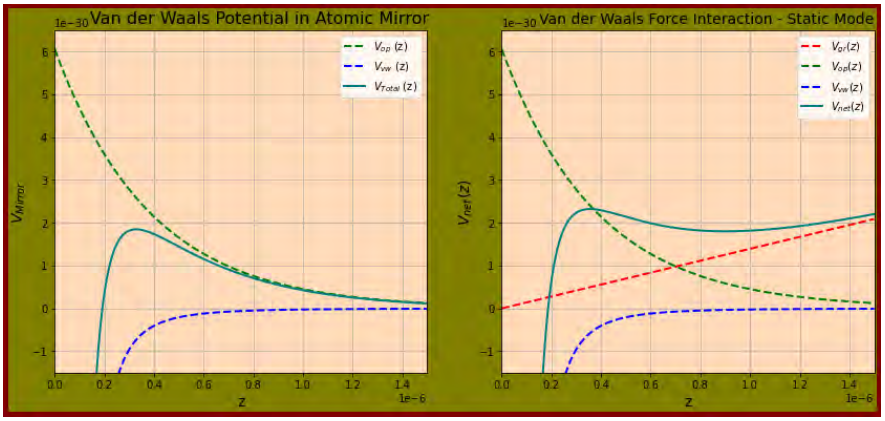

Figure 4.1: (a) on left side: Atomic mirror after the interaction of van der waals force for Rubidium atom. It can be seen the mirror potential is reduced by the factor of 3 after van der waals potential. (b) on right side: van der waals interaction in gravitational cavity in static mode of operation.

4.2 The Effective Potential in Static Mode

For RTM, The effective potential with out van der waals force is already seen in the previous chapter. we can write the Taylor series expansion of the net potential about the point $z = a$

$$V_{net}(z) = V_{net}(a) + (z-a)\frac{\partial V_{net}(a)}{\partial z} + \frac{1}{2}(z-a)^2\frac{\partial^2 V_{net}(a)}{\partial z^2} + ... \tag{4.2}$$

In above equation the first term is a constant value that has no effect on the particle, the second term which contains first derivative and it will disappear at local minima of the net potential and the third term which is parabolic in shape offers an effective potential as shown in figure 4.2(a). Therefore, $V_{net}(z)$ can be approximately expressed as

$$V(z) = \frac{1}{2}\frac{\partial^2 V_{net}(a)}{\partial z^2}(z-a)^2 \tag{4.3}$$

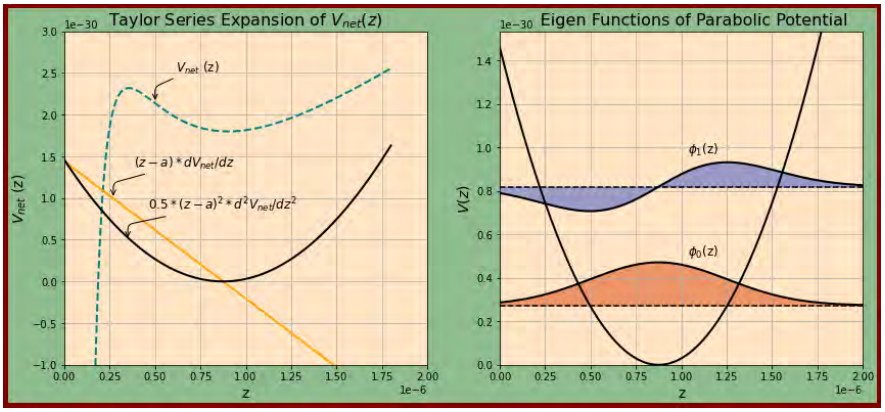

Figure 4.2: (a) on left side: the Taylor series expansion of the effective potential $V_{net}(z)$ about the point $z = a$. The teal colord dotted line represents the effective potential $V_{net}(z)$, the yellow line shows the 2nd term of the series and black parabola shows the 3rd term of the series. (b) on right side: shows the parabolic potential with its first two eigen functions ϕ_0 and ϕ_1 for Rubidium atom respectively.

4.3 The Particle in a Parabolic Potential Well

According to quantum mechanics, when a harmonic oscillator is displaced z distance from its mean position, it comes up against a restoring force F exerting towards its mean position. This force is directly proportional to the displacement z.

According to the Hook's law this force F is expressed as

$$F = -Kz \tag{4.4}$$

Here negative sign indicates that the displacement z and force F are in the opposite directions while K is a proportionality constant. We can calculate the work done by the harmonic oscillator under a constant force F

$$W = -\frac{1}{2}Kz^2 \tag{4.5}$$

The negative work done W in above equation is corresponding to an increase in the potential energy of the particle. Thus

$$V(z) = \frac{1}{2}Kz^2 \tag{4.6}$$

Therefore the potential will increase in a quadratic fashion on both sides of a point $z = a$. The equation of motion for the harmonic oscillator is given as

$$M\frac{d^2z}{dt^2} = -Kz \tag{4.7}$$

$$\frac{d^2z}{dt^2} = -\omega^2 z \tag{4.8}$$

Here $\omega = \sqrt{\frac{K}{M}}$

Now for our case, if a particle with mass M is oscillating with a frequency ω in a parabolic potential V(z) as shown in figure (4.2 - (a)) then the mathematical expressions for the parabolic potential V(z) and ω will be

$$V(z) = \frac{1}{2}\frac{d^2V_{net}(a)}{dz^2}(z-a)^2 \tag{4.9}$$

$$\omega = \sqrt{\frac{\frac{d^2V_{net}(a)}{dz^2}}{M}} \tag{4.10}$$

4.3.1 Solution of the Hermite Differential Equation

To solve the Schrodinger equation for a harmonic oscillator it is mandatory to have the solution of Hermite differential equation. The Hermite differential equation is expressed as

$$\frac{d^2y}{dz^2} - 2z\frac{dy}{dz} + 2\lambda y = 0 \tag{4.11}$$

By using power series we can get the Hermite polynomials as the solutions of the Hermite differential equation. The first few polynomials are

$$H_0(z) = 1 \tag{4.12}$$

$$H_1(z) = 2z \tag{4.13}$$

$$H_2(z) = 4z^2 + b_0 \tag{4.14}$$

4.3.2 Solution of the Time Independent Schrodinger Equation

The time independent Schrodinger equation equation for the harmonic oscillator is

$$-\frac{\hbar^2}{2M}\frac{d^2\phi_n(z)}{dz^2} + \frac{1}{2}\frac{d^2V_{net}(a)}{dz^2}(z-a)^2 = E_n\phi_n(z) \tag{4.15}$$

By using basics quantum mechanics, we can solve this equation and get the eigen energy values and eigen functions for the parabolic potential well.

$$E_n = (n + \frac{1}{2})\hbar\omega \tag{4.16}$$

$$\phi_n(z) = \left(\frac{M\omega}{\hbar\pi}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{M\omega}{\hbar}} z\right) e^{-\frac{M\omega z^2}{2\hbar}} \tag{4.17}$$

The eigen functions and eigen energy values for parabolic potential well are shown in figure 4.2(b). Now the Gaussian wave packet is confined in the parabolic potential well at E_0 level of energy as shown in figure 4.3

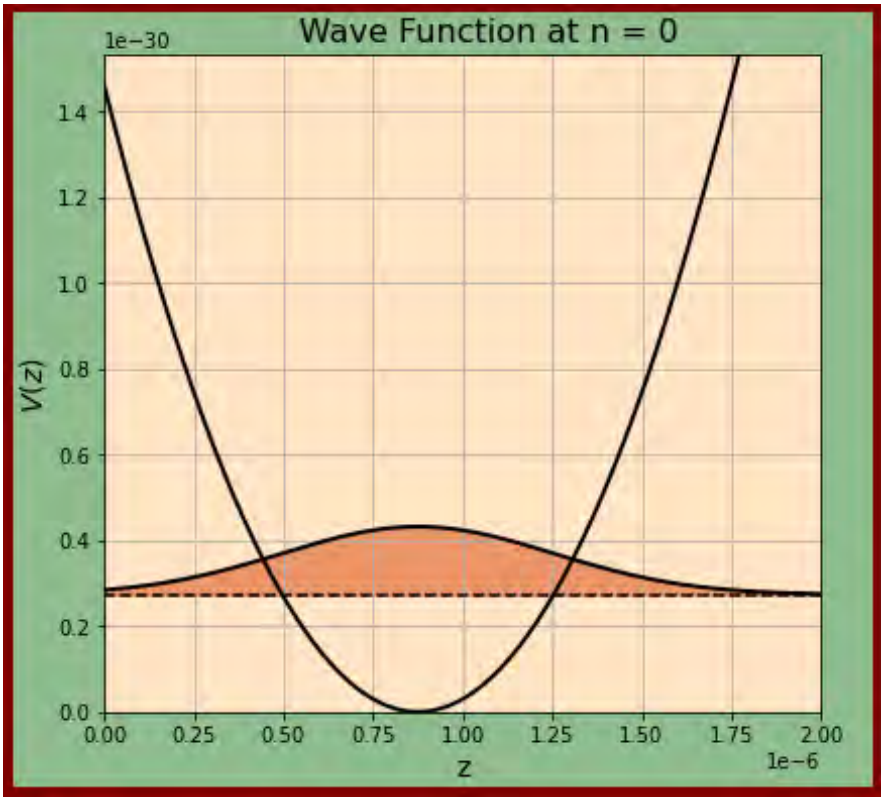

Figure 4.3: The time independent Gaussian wave packet confined in a parabolic potential well at E_0

4.3.3 Solution of the Time dependent Schrodinger Equation

The solution of time dependent Schrodinger equation helps us to see the time evolution of the quantum wave packet as figure 4.4 shows the time evolution of the imaginary part of the time dependent wave packet.

$$\psi(z,t) = \psi(z,0)e^{-\iota\frac{E_n}{\hbar}t} \tag{4.18}$$

$$Im[\psi(z,t)] = -\iota\psi(z,0)sin(\omega_n t) \tag{4.19}$$

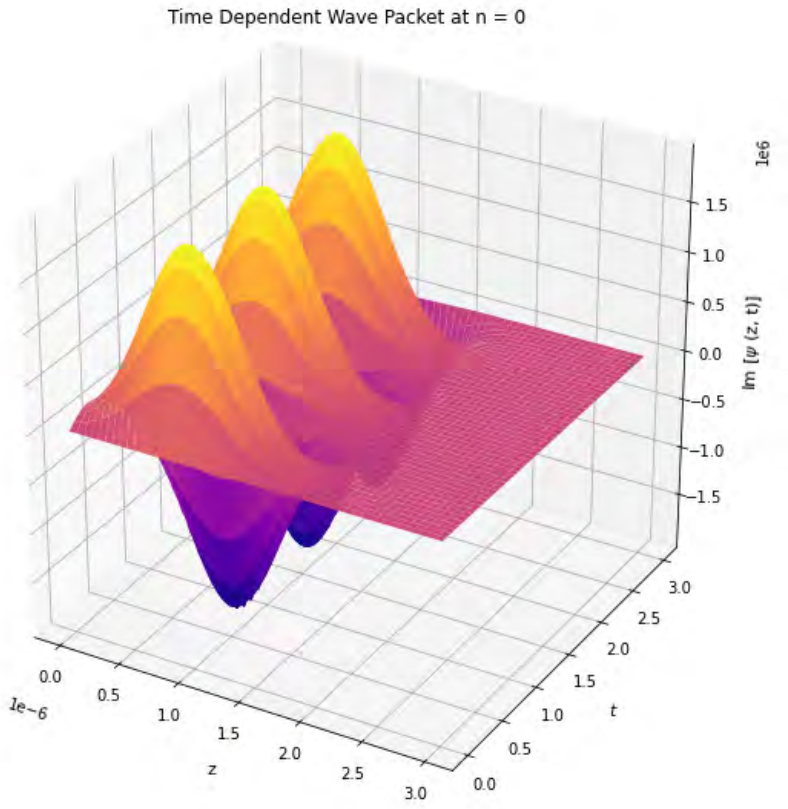

Figure 4.4: The time dependent Gaussian wave packet confined in a parabolic potential well at E_0

4.3.4 Tunneling and Matter Wave Packet Reflection

To observe the phenomena of tunneling through the atomic mirror and for the quantum reflection of matter wave packet with an energy above the barrier height would be an interesting experiment. Due to these effects we can round off the sharp threshold.

To estimate the probability of tunneling or quantum reflection there exist a simple analytical formula through which we can find the transmission probability T for a material wave packet to pass a parabolic height.

$$T = \frac{1}{1 + e^{-\frac{2\pi E}{\hbar\Omega}}} \tag{4.20}$$

Where Ω represents the oscillation frequency of the atom if the harmonic oscillator potential were not inverted and E is the difference between the potential barrier height and the energy of the atom. A negative value of E means a particle whose energy is less then the barrier height would reflect from the barrier with unit probability. At energies within a few percent of reflection threshold, tunneling or quantum reflection occur with non negligible probability.

Chapter 5

Conclusion

In this work we have found an important consequence that the van der waals force performs a key role in the functioning of the evanescent wave atomic mirror. We have seen that in the case of evanescent wave atomic mirror an exponentially decaying potential is obtained at the interface of the dielectric due to the total internal reflection of a laser beam. The height of this decaying optical potential depends on the angle of incidence of the laser beam.

If the matter waves corresponding to the Rubidium atom incident with a kinetic energy lower then the maximum height of the atomic mirror they will be reflected back. We have seen that the van der waals force interaction changes the shape of the total potential and it also reduces reflection threshold of atomic mirror by a factor of 3 which can cause for tunneling of the material wave packet of the Rubidium atom through the atomic mirror. As a result the dynamics of the wave packet vanish and the Rubidium atom stick to the surface of the dielectric due to the van der waals force of attaraction.

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