Radiative Flow of Powell-Eyring Liquid in Presence of Convective Condition

By

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Department of Mathematics
Quaid-I-Azam University
Islamabad, Pakistan
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A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF

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IN

MATHEMATICS

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2016

CERTIFICATE

On the Prior distribution for the parameter of Gumbel Distribution

By

Mariya Raftab

A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT

OF THE REQUIREMENT FOR THE DEGREE OF

MASTER IN STATISTICS

We accept this thesis as confirming to the required standard.

1. ________ 23/4/2012

Prof. Dr. Javid Shabbir

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3. ________ 16/3/2012

External Examiner

Date: 16-3-2012

DEPARTMENT OF STATISTICS

QUAID-I-AZAM UNIVERSITY ISLAMABAD, PAKISTAN

2012

DECLARATION

I hereby declare that this thesis is result of my individual research and that it has not been submitted concurrently to any other university for any other degree.

Date: Signature

Mariya

بسم الله الرحمن الرحيم

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This thesis submitted in the partial fulfillment of the requirement for the degree of the

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Dedicated to

MY PARENTS AND TEACHERS

WHOSE PRAYERS AND ATTENTIONS

HAVE ALWAYS BEEN

A GREAT SOURCE OF INSIPIRATION AND ENCOURAGMENT

MY LIFE

Acknowledgement

First of all I would like to thanks to Almighty ALLAH, The compassion and the most mercifully, Who bestowed on me his kind blessing to complete this task. All respect to His Prophet (peace be upon him) for enlightening the essence of faith in Allah, converging all his kindness and mercies upon me.

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Abstract

This thesis provides estimation of the parameter of Gumbel type II distribution. We have used two informative priors; Gamma prior and Exponential prior for the unknown parameter of the Gumbel distribution. The graphs of the posterior distribution of the parameter of the Gumble type II distribution using informative and noninformative priors are drawn to check the symmetry of the distribution. The comparison of the prior distribution is made on the basis of coefficient of skewness andposterior risk under three loss functions (Square error, weighted and Quadratic). The Bayesian Hypothesis testing has been done for the testing of the parameter of the Gumble distribution. Censoring scheme (type I and type II) is also used for time to failure data. The posterior distribution, Bayes estimator and Bayes posterior risk using type I and type II censoring is derived. Mixture Prior is used for the parameter of Gumble distributions. The objective of study is to select a suitable prior for parameter of Gumble distribution. Exponential prior for the parameter of the Gumble distribution is recommended for the Bayesian analysis of the Gumble model as the value of the posterior risk using this prior is minimum

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CHAPTER 1

Introduction

The Bayesian approach is more applicable over classical as due to the utilization of prior information. This study provides a Bayesian analysis of unknown parameter of Gumbel distribution. The posterior distribution using informative and noninformative priors are derived. Similarly comparison of the prior has been also made on the basis of Bayes estimator, Bayes posterior risk, coefficient of skewness and Bayes factor. Simulations study is performed using the Bayes estimator and Bayes posterior risk under three loss functions (Quadratic, Square, weighted) using informative (Gamma and Exponential) and non informative (Jeffreys and uniform).This study provide the priority of informative priors on non informative priors.

There are seven chapters in this thesis. Chapter 1 includes a basic introduction about the thesis, scope and objective of study.

Chapter 2 describes the brief description of difference between classical and Bayesian statistics, Prior distributions and its kinds, loss functions and its types, Censoring scheme, type I type II censoring. This chapter includes a brief note on Gumbel distribution and Literature Review about the parameter of Gumbel distribution.

In chapter 3, we derives the posterior distributions of the parameter of Gumbel distribution using informative and noninformative priors.Two informative priors (Gamma and Exponential) noninformative priors (Jefferys and uniform) are also used. The Graphs of posterior distributions using informative and noninformative priors are also derived.

In chapter 4, we have made the comparison of the priors using Bayes estimator, Bayes posterior risk. Bayesian hypotheses testing are also for the parameter of the Gumble type II distributions. Simulation study is also performed using Bayes estimates and Bayes posterior risk under different loss functions (Quadratic, Square, and Weighted).

Chapter 5 describes the posterior distributions using informative priors (Gamma and Exponential) under censoring scheme. Two types of the censoring scheme are used, type I and type II censoring. The Bayes estimates and Bayes posterior risk are also calculated for the posterior distributions using type I and type II censoring. Simulation study is also performed using the Bayes estimators and Bayes Posterior risk under different loss functions (Quadratic, Square, and Weighted) using informative priors (Gamma and Exponentia). The Graphs of posterior distributions using censoring scheme (type I and type II) using informative priors (Gamma and Exponential) is also derived.

In chapter 6, we derived the posterior distributions using mixture of Gamma and Exponential as a priors, and also used Exponential Gamma as a Double prior. The Bayes estimators are calulated for the posterior distributions using mixture priors, Bayes posterior risk and Bayes estimator are also calculated for the posterior distributions using a double prior. The Graphs of posterior distributions using the mixture (Gamma and Exponential) and double (Exponential Gamma) priors is also derived.

Chapter 7 includes the summary of whole work with conclusion and recommendation for the further study.

Objective of the study

The main objectives of this study is as follows:

1. To derive the posterior distribution under suitable priors for the estimation of the parameter of Gumbel distribution with known scale parameter and unknown shape parameter.
2. To Compare the Bayes estimators under different loss functions using informative and noninformative priors.
3. To Compare the Bayes posterior risk under different loss functions using informative and noninformative priors.
4. To Compare the Bayes factor under different loss functions using informative and noninformative priors.
5. To compare the coefficient of skewness under different loss functions using informative and noninformative priors.
6. To Compare the informative priors according to the various graphs of the posterior distribution using informative priors.

1.2 Scope of the study

This study has many benefits in the field of Bayesian inferences:

1. Researchers, students and teachers related to the Bayesian inference can easily utilize the study, especially in working with the parameter estimation of the Gumbel Distribution.

2. Research organization working on the Gumbel distribution can easily use the study to get a good result by taking recommended prior for the parameter estimation of Gumbel Distribution.

CHAPTER 2

Conventional and Bayesian Statistics

2.1 Introduction

Mathematical statistics uses two school of thought, conventional (frequentist), and Bayesian. Thomas Bayes (1763), who first gave the idea of Bayesian statistics by the application of ‘Bayes theorem’. R.A Fisher is the founder of frequentist or classical statistics.

2.2 Classical and Bayesian Approach

The Bayesian philosophy involves a completely different approach to conventional statistics. Bayesian approach is basically the application of Bayes theorem. In Classical statistics we use sample information for making inference about uncertain quantity parameter. Parameters are generally treated as fixed in classical approach and that putting a probability distribution on it doesn't make sense. A statement such as probability between 10.45 and 13.26 or $P(10.45 < \theta < 13.26) = 0.95$ cannot be made because θ is not a random variable. In Bayesian statistics in addition to sample information we utilize the prior information for estimating the unknown quantity. Parameters are treated as random in Bayesian approach. Classical approach basically requires a different procedure. In Bayesian approach we used only the Bayes theorem. Hypothesis testing in classical approach is complicated. Bayesian approach is a straight forward method for Hypothesis testing. In Bayesian approach we used a single sample for making an inference where as in classical many more sample are require for results. In Bayesian approach hypothesis testing are directly related to the probabilities where as in classical approach these probabilities are related to type I and type II error which are not related to the hypothesis testing. Classical approach does not help to solve the Behrens fisher problem. Bayesian approach requires only one credible interval where as classical approach require many confidence interval for interpreting the parameter. It is true also that the Bayesian analysis has certain disadvantages, such as the difficulty of the calculation, but the development of new software like the Win BUGS facilitate the calculation of the posterior distribution. Bayesian analysis can be particularly useful when there is limited data for a given design. A frequentist will design the 95% confidence interval procedure so that out of every 100 values, at least 95 of the resulting confidence intervals will be expected to include the true value of the parameter. The other 5 might be slightly wrong, or they might be complete nonsense, as long as 95 out of 100 inferences are correct. (Of course we would prefer them to be slightly wrong, not total nonsense). Bayesian approaches formulate the problem differently. Instead of saying the parameter has one true value, a Bayesian method says 95% credible interval means out of 100 intervels 95 intervels will contain true parameteric value. In Bayesian Bayes estimate can be only calculated by minimizing the expected loss function where as classical approach requires method of moments, MLE, method of least square.In Bayesian statistics the loss function is the functional value of estimator and parameter,whereas in classical approach the function is simply MSE and some biasness.

2.3 The Prior Distribution

The main contradiction between classicals and Bayesian statistics is the use of prior information. As the parameter is treated as random in Bayesian so we assign a probability to a random variable known specifically as prior. The prior information is the probability distribution that would express uncertainty before the “data”. Parameters of prior distributions are called hyperparameters, to distinguish from the parameter of the model. Prior informations basically depend upon the data which is in use.When we have large amount of data the prior distribution does not yet play role very well, but the use of the prior is still important for making reliable inference by using small amount of data and ensure that any information about it is not wasted. A prior is often the purely subjective assessment of an experienced expert. Some will choose a conjugate prior when they can, to make calculation of the posterior distribution easier.

2.3.1 The Informative Prior

An informative prior expresses specific, definite and complete information about parameters. Therefore some auther called them a “subjective prior”. When prior information is available about parameter, it should be included in the prior distribution of parameter. For example, if the present model form is similar to a previous model form, and the present model is intended to be an updated version based on more current data, then the posterior distribution of parameter from the previous model may be used as the prior distribution of parameter for the present model. Sometimes informative prior information is not simply to be used, such as when it resides in another person, such as an expert. In this case, their personal beliefs about the probability of the event must be elicited into the form of a proper probability density function. This process is called prior elicitation.

2.3.2 The NonInformative Prior

When the information about parameter is not available we use information from hyperparameters distribution hierarchical models (Upton & Cook). These priors are oftenly called noninformative prior. The noninformative prior may be proper and improper. The usual noninformative priors on continous, unbounded variables are improper. A noninformative prior expresses vague or complete lack of information about parameter of population.

2.3.3 Jeffereys Prior

A common noninformative prior is Jeffereys prior which is proportional to the square root of the determinant of the Fisher information matrix. The Jeffreys prior is quite useful for a single parameter but can be serious problem for a nuisense parameter. It can be written as:

$$f(\theta) \propto \mathrm{I}_{\theta}^{1/2}$$

2.3.4 Uniform Prior

One of the most widely use of noninformative prior, due to Laplace (1812), is a uniform (possibly improper prior).The distribution of a uniform prior add no information in Bayesian inference. The form of the uniform prior is:

$$f(\theta) \propto 1$$

2.3.5 Conjugates Prior

A Cojugate prior is define as prior distribution belonging to same parametric family for which the resulting posterior also belongs to the same family. This is an important property, since the Bayes estimator, as well as its statistical properties (variance, confidence interval, etc) can all be derived from the posterior distribution. Conjugate prior are especially useful for sequential estimation, where the posterior of the current measurement is used as the prior in the next measurement.

2.3.6 Proper and Improper Priors

It is necessary for the prior distribution to be proper for making calculation easy. A prior distribution, p(θ) is improper as it does not converge at the given range. An unbounded uniform prior distribution is an improper prior distribution. When the prior distribution is improper posterior distribution is proper, inferences are invalid, it is non-integrable, and Bayes factors cannot be used.

2.4 Loss function

The loss function is used for parameter estimation, the difference between parameter and estimater can be express in the function is known as function. The value of the loss function itself is a randam quantity because it depends on the outcomes of a random variable say (X). Loss function is real valued functions that explicitly provide a loss for decision of given θ. Both Bayesian and frequentist statistical theory involve making a decision based on expected value of loss function however this quantity is defined differently under two approaches. The loss function does not itself completely determine a decision. Therefore the relationship between loss function and the prior probability is determined by comparing the two different loss function which leads to the same decision. In Bayesian theory, Bayes estimator can also be obtained by minimizing the posterior expected value of loss function, it maximizes the posterior expectation of a utility function.

2.4.1 Square error loss function

The square error loss function is also known as MSE. The square error loss function is difference between the value implied by an estimator and true value of quantity being estimated. The square error loss has the disadvantages that it has tendency to be dominated by outlier. The square error loss function is symmetric loss function.

2.4.2 Quadratic loss function

The quadratic loss function is used in linear quadratic optimal control problem. The use of a quadratic loss function is common, for example when using least squares techniques or Taguchi methods. It is often more mathematically tractable than other loss functions because of the properties of variances, as well as being symmetric: an error above the target causes the same loss as the same magnitude of error below the target.

2.5 Bayes risk

Suppose an unknown parameter θ have prior distribution £. Let δ be the estimate of θ, and L(θ, δ) be loss function. The Bayes risk of δ is defined as E£{L(θ, δ)}, the expectation is taken over the probability distribution of estimate with respect to the prior distribution. Bayes Risk is undefined in case of improper prior.

2.6 Risk function

Risk function is also MSE, the most common risk function in use, primially due to its simplicity. Risk function is the Expectation of the Loss function with respect to random variable. The technical result are usually derived in the form which make it very difficult the role played, if any, by different loss function. Loss function the sing of the minimum of the expected risk coincide the Bayes optimal solution.

2.7 Posterior distribution

When applying Bayes' theorem, the prior is multiplied by the likelihood function and then normalized to estimate the posterior probability distribution, which is the conditional distribution of parameter given the data.

2.8 Bayesian Hypothesis testing

In Bayesian approach hypotheses testing procedure is simpler than classical hypotheses testing. The posterior probabilities are calculated and decision are made on the basis of posterior probabilities. Bayesian inference about θ is primarily based on the posterior distribution of θ. For example, we can report our findings through point estimates. We can also use the posterior distribution to construct hypothesis tests or probability statements.

Jeffreys developed a procedure for using data y to test between alternative scientific hypotheses H_o and H_1 , if hypotheses are simple Vssimplethen we compute the probabilities under H_o and H_1. If $p(Ho) > p(H_1)$ then we support H_o and vice versa. If hypothesis are

Composite Vs composite then easy way is to compute the Bayes factor $p(H_o)/p(H_1)$. If the Bayes factor using informative and noninformative prior is less than 1, we will support H_1. If the Bayes factor using informative and noninformative prior is less than 1, we will support Ho.

2.9 Censoring Scheme

Life testing experiment ofhen deal with conserved sanple in order toestimatetheparemter involved in the life distribution. Two type of censoring are generally recognized type I and type II censoring. In type I censoring the experiment continous until a preassign time T, and failure that occurs after T are not observed. In contrast, in type II censoring scheme the experimenter decide to terminate the test after a preassign no of failure observed say $k \leq n$. In either case, the advantage is that it take less time to complete the experiment. In many cases the experimenter may be at liberty to chose between the two censoring scheme. There are many reasons affect in the choice of the two type of censoring scheme that is the time require to complete the experiment etc.

For example, suppose a study is conducted to measure the impact of a drug on mortality. In such a study, it may be known that an individual's age at death is at least 75 years. Such a situation could occur if the individual withdrew from the study at age 75, or if the individual is currently alive at the age of 75.

2.10 Type I Censoring

Type I censoring occurs if an experiment has a set number of subjects or items and stops the experiment at a predetermined time, at which point any subjects remaining are right-censored.

2.11 Type II censoring

Type II censoring occurs if an experiment has a set number of subjects or items and stops the experiment when a predetermined number are observed to have failed; the remaining subjects are then right-censored.

2.12 **P**rogressive Censoring

Progressive cencoring describes how to make exact or approximate inferences for the different statistical models with samples based on progressive censoring schemes. With many concrete examples, the book points out the greater efficiency gained by using this scheme instead of classical right-censoring methods.

2.13 Left Censored data

In a left censored data, the failure time is only known to be before time. For example we may know that a certain unit failed before 10 hours. In other words it could be failed in any interval between 0 to 10 hours.

2.14 Right Censored data

In case of life time data, these data set are composed of unit that did not failed. For example if we have ten units and only six had failed by the end of the test, we would have suspended data for four unfailed units.

2.15 Mixture Models

A mixture model is a probabilistic model for representing the presence of sub-populations within an overall population, without requiring that an observed data-set should identify the sub-population to which an individual observation belongs. Formally a mixture model corresponds to the mixture distribution that represents the probability distribution of observations in the overall population. However, while problems associated with "mixture distributions" relate to deriving the properties of the overall population from those of the sub-populations, "mixture models" are used to make statistical inferences about the properties of the sub-populations given only observations on the pooled population, without sub-population-identity information.

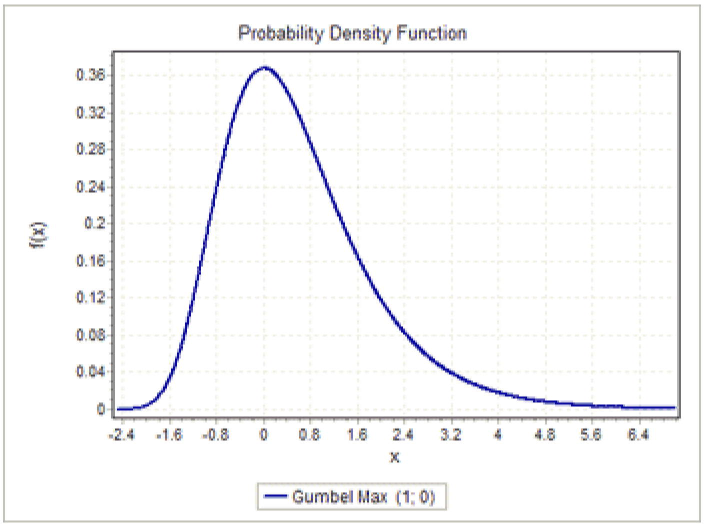
2.16 The Gumbel Distribution

The Gumbel distribution takes its name from Emil J Gumbel (1891-1960) is used to model the distribution of maximum no of sample of various distribution. The Gumbel distribution has two parameters, location and scale; ϕ and θ respectively. It is also known as the Extreme Value Type I distribution. The Gumbel distribution has a thin tailed as compared to Pareto and Cauchy distribution. The Gumbel distribution is continuous distribution define on semi indefinite rang x>*0,* The density of Gumbel type II distribution is;

$$f\left(x;\theta,\phi\right)=\phi\ \theta\ x^{-(\phi+1)}\ e^{-\theta x^{-\phi}}\quad x>0,\theta>0,\phi>0 \tag{2.1}$$

The Gumbel distribution is widely used in reliability and life testing. An attractive feature of Gumbel distribution is that the parameter equations produce an estimate of the mode. Gumbel’s focus was basically on applications of extreme value theory to engineering problems, in particular modeling of meteorological phenomena such as annual flood flows. In environmental sciences it is used to model the extreme associated with the flooding in rainfall. Gumbel distribution is special case of generalized extreme value distribution used in industry for QA/QC. The Gumbel distribution is used to represent the distribution of maximum relate to extreme value that is useful if the distribution of the sample is normal and exponential. Practical use of Bayesian estimation is often associated with difficulties to choose prior information and prior distribution for a Gumbel parameter. The two parameter Gumbel distribution requires a two-dimensional joint prior distribution. The importance of the Gumbel distribution as a model of quantitative phenomena in social and environmental sciences is due to the extreme value. It is oftenly used to see the maximum rang of trading value in financial market produce a satisfactory significant fit. It is also useful in predicting the chance of extreme earth quack, flood or natural disaster. Pieces of Graph paper also incorporate a Gumbel distribution.

Graph of the Gumbel Distribution



2.17 Properties of the Gumble distribution

The mean of the Gumbel type II distribution is always equal to mode $-\gamma$ *scale parameter where γ is Euler constant and its value is -0.5772116. The median of the Gumbel type II distribution is G(0.5). The variance of the Gumbel type II distribution is $\left(\frac{\text{scale} * \pi}{6}\right)$ or $\frac{\beta^2 * \pi}{6}$. The skewness of the Gumbel type II distribution is 1.139547. The Kurtosis of the Gumbel type II distribution is 5.4. The Gumbel type II distribution is always asymmetric distribution. The Gumbel type II distribution iscontinuous distribution define on semi indefinite range *x>0.* The Gumbel type II distribution has a thin tailed as compared to other distribution pareto and Cauchy distribution .The range of the distribution is $0 to \infty$. The Gumbel type II distribution is particular case of Weibull distribution. The c.d.f of Gumbel type II distribution is $1-e^{-bx^{-a}}$.

2.18 Literature Review

A lot of work has been done on the estimation of the parameter of Gumbel type II distribution, some of the related literature review are given below.

Oakes &Manatuga (1992) proceed the work of Lee (1967), give explicit formula for 5x5 dimentional Fisher information matrix for Gumbel's (1960) bivariate type II distribution of extreme values with Weibul marginal. Through numerical evaluation he shows that our parameterization makes the dependent parameter exactly orthogonal on scale parameter.

Battacharyya et.al (1991) developed the inference procedure for the bivariate exponential distribution of Gumbel. The asymmetric properties of maximum liklehood estimates are presented for the case of identical marginals. Optimal test and confidence bound are used for model parameter.

Palutikof et al. (2000) described and reviewed the methods to calculate extreme wind speeds, including 'classical' methods based on the generalized extreme value (GEV) distribution and the generalized Pareto distribution (GPD), and approaches designed specifically to deal with short data sets. The main emphasis was on the needs develop the techniques for calculating the distribution parameters and quantiles. In this regard the techniques applicable to data sets as short as two years, including simulation modelling and methods based on the parameters of the parent distribution, were considered.

Koutsoyiannis and Baloutsos (2000) analyzed the annual series of maximum daily rainfall extending through 1860-1995 in Athens using extreme value distribution. The statistical analysis showed that the conventionally employed Extreme Value Type I (EV1 or Gumbel) distribution is inappropriate for the examined record (especially in its upper tail), whereas this distribution would seem as an appropriate model if fewer years of measurements were available. On the contrary, the General Extreme Value (GEV) distribution appears to be suitable for the examined series and its predictions for large return periods agree with the probable maximum precipitation estimated by the statistical (Hershfield's) method.

Chechile (2001) obtained the posterior distribution assuming that the random sample is taken from the gumbel distribution using the conjugate prior. The normalization constant and the marginal distribution of the scale parameter were obtained. The properties of the posterior distribution were evaluated using an exact Monte Carlo algorithm. Application of the results was discussed using different real life data.

Wu and Lin (2001) derived an exact confidence interval for the shape parameter and an exact joint confidence region for the shape and scale parameters of the Weibull and Gumbel distributions under censored samples. The joint confidence region was used to obtain a conservative lower confidence bound for the reliability function. Further, the optimal criteria to find a best exact confidence interval for the shape parameter and a best exact joint confidence region for the shape and scale parameters were discussed. Two real life examples were used to elaborate the results.

Hirose (2002) derived the confidence intervals for maximum likelihood estimates of percentile points in dielectric breakdown voltage using generalized extreme-value distribution. A Monte Carlo simulation using the generalized extreme-value distribution parameter estimation code reveals the property of the percentile point estimates for extreme-value type distributions. Biases and root mean squared errors of percentile point estimates are evaluated both for the maximum likelihood estimates and for some closed form estimates; maximum likelihood estimates provide reasonable confidence intervals. A real life example was also presented to illustrate the results.

Mousa (2002) obtained the Bayesian estimation for the two parameters of the Gumbel distribution based on record values. Point and interval predictions for the future lower record values were obtained. Based on a recurrence relation of conditional moments of nonadjacent record values, a characterization for the Gumbel distribution is also given. Numerical computations are given to illustrate these procedures.

Clarke (2002) discussed that the widely-used hydrological procedures for calculating events with *T*-year return periods from data that follow a Gumbel distribution assume that the data sequence from which the Gumbel distribution is fitted remains stationary in time. If non-stationarity is suspected, the hypothesis that the data are Gumbel-distributed is temporarily abandoned while testing for trend, but is re-adopted if the trend proves to be not significant. They described an alternative model in which the Gumbel distribution has a (possibly) time-variant mean. Simulated samples from a standard Gumbel distribution were used to calculate the power of each of three trend-testing procedures (Maximum Likelihood, Linear Regression, and the non-parametric Mann-Kendall test) were compared. The ML test was always more powerful than either the Linear Regression or Mann-Kendall tests.

Van Montfort (2003) checks the adequacy of extreme value distribution by taking type-I Gumbel distribution as null hypothesis and taking type-II distribution as alternative hypothesis. Montfort paper give a quick result with high power by proceeding the result of elsewhere dealing with the reduction of power using numerically simplified test.

Balakrishnan et.al (2007) mentioned that the Bayesian estimation and prediction problems for the linear hazard rate distribution using type II censored sample. Markov chain Monte Carlo method are used to generate the Bayesian conditional probabilities.

In practice joint prior for the two parameters Gumble distribution is much difficult. Wanbu (2008) in his paper mentioned that joint prior can be obtained in case of two parameters Gumbel distribution by using a simple Bayesian estimation procedure proposed by Kaminskiy and Vasily (2005). The prior information is presented in the form of intervalassessment of reliability function as the Gumble distribution is widely used in reliability function. Proceeding the idea of Kaminskiy and Vasily we construct the continous joint prior of Gumble parameters as well as the posterior estimate of mean and variance of the parameter of the density function.

Noortwijk et.al (unpublished) mentioned in his paper (Bayesian frequency analysis for extreme river discharge) that Bayesian method has been successfully applied to estimate the design discharge of river Rhine while taking account of statistical uncertainties involved. For this he used seven predictive probability distributions for determining the extreme quantile of discharge.

Kakade et.al (2008) discussed the inferences and MLE of cumulative distribution. He constructs the bootstrap and asymptotic confident interval by using asymptotic distribution. Testing of the reliability based on asymptotic distribution of the maximum likelihood estimator is discussed. Simulation study to investigate performance of the confidence intervals and tests has been carried out.

Miladinovic (2008) describes the application of the Kernel density prior to the Gumbel probability distribution. Bayesian and empirical estimates of the reliability and failure rate function under the Gumble failure model are derived and compared with the kernel density estimates. The comparison of the Bayes estimates of the Gumble reliability function using six different prior including kernel density prior is performed. He use jackknife procedure to improve ML parameter estimates.

Hoppe and Fang (2008) derived the posterior predictive distribution and predictive intervals for gumbel distribution. The data from the outlet side feeder pipes at Ontario nuclear power plants was used to predict the minimum thickness of all remaining uninspectedpipes and to show that with what confidence can it be saidthat the remaining wall thicknesses are above an acceptable minimumto ensure a sufficiently high thickness up to the endof the next operating interval. A hybrid Bayesian method and full Bayesian approach using Markov Chain Monte Carlo was used. It was shown thatthe latter gives larger lower prediction limits and therefore moremargins to fitness for service.

Nakajima et.al (2009) proposed a new approach to model the time dependence in extreme value process. Under a Bayesian approach he used an efficient alogrithem and implementson Markov chain Monte Carlo method and derived a very accurate approximation of the Gumbel distribution by a ten-component mixture of normal distributions.

Al-Aboud (2009) extends the work of Balakrishnan et al.(2004) discussed in classical framework, the point and interval estimation for parameters of extreme value distribution based on censored data. Preceding his work in this paper, Bayes estimates of two (unknown) parameters, the reliability and failure rate functions are obtained by using approximation of Lindley (1980). The estimators are estimated under both symmetric and asymmetric loss functions. A practical example consisting of various types of real data was presented. Finally, in order to investigate the accuracy of estimation, Monte Carlo simulation study was conducted, and the estimated risks of Bayes estimates are computed and compared with the corresponding estimated risks of maximum likelihood estimates.

Thompson et al. (2011) introduced a distributional hypothesis test for left censored Gumbel observations based on the probability plot correlation coefficient (PPCC). Critical values of the PPCC hypothesis test statistic are computed from Monte-Carlo simulations and are a function of sample size, censoring level, and significance level. When applied to a global catalog of earthquake observations, the left-censored Gumbel PPCC tests and likelihood tests are unable to reject the Gumbel hypothesis for 45 of 46 seismic regions.

Evin et al. (2011) studied the mixtures of distributions with normal, Gamma, and Gumbel components. Moving away from the standard normal setting, gamma mixtures are developed in order to model strictly positive hydrological data and Gumbel mixtures for extreme variates. Time dependent and independent cases were considered. Dependent cases were modeled by Markov process. The relevance adequacy of the mixture models was tested by calculating the marginal likelihoods, for a given data, under Bayesian framework.

Mahdavi and Mojtaba (2011) represented a new method for computing the reliability of a system which is arranged in series or parallel model. They obtained the life distribution function of whole structure using the asymptotic Extreme Value (EV) distribution of Type I, or Gumbel theory. All parameters were also estimated by Moments method. Reliability function and failure (hazard) rate and p-th percentile point of each function are determined. Other important indexes such as Mean Time to Failure (MTTF), Mean Time to repair (MTTR), for non-repairable and renewal systems in both of series and parallel structure were computed.

CHAPTER 3

THE POSTERIOR DISTRIBUTION

3.1 Introduction

In this chapters, the Posterior Distribution for the parameter of the Gumbel distribution of the type II is derived using two informative priors (Gamma and Exponential) and noninformative prior (Jeffreys and uniform). The graphs of the posterior distribution are also drawn in this chapter.

3.2 The Posterior Distribution for the Parameter of Gumbel Distribution

The posterior distributions for the parameter of Gumbel type II distribution using informative and noninformative priors are derived below:

3.3 The Posterior Distribution using noninformative prior

We derived the posterior distribution using two noninformative priors the Jeffreys and uniform prior in the following sections.

3.3.1 Using Jefferys Prior

The p.d.f for the parameter of Gumbel Type II distribution for a randam variable **X** having parameters ϕ and θ is:

$$f(x;\theta,\phi) = \phi\ \theta\ x^{-(\phi+1)}\ e^{-\theta x^{-\phi}} \quad x>0, \theta>0, \phi>0 \tag{3.1}$$

The Likelihood function of Gumbel Type II distribution with the parameters ϕ and θ is:

$$L(x;\theta) = \prod_{i=1}^{n} f(x;\theta,\phi)$$

The Fisher's information is:

$$I(\theta) = -E\left[\frac{\partial^2 \ln L(\theta;x)}{\partial\theta^2}\right]$$

The Jeffreys prior for tha parameter θ is:

$$p_J(\theta) \propto \sqrt{\frac{1}{\theta^2}}$$

$$p_J(\theta) \propto \frac{1}{\theta} \quad \theta>0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p(\theta|\mathbf{x}) \propto L(x;\theta)\ f(\theta;\phi)$$

$$p(\theta|\mathbf{x}) \propto \frac{1}{\theta}\ \theta^n\ e^{-\theta\left(\sum_{i=1}^{n} x_i^{-\phi}\right)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{n-1}\ e^{-\theta\left(\sum_{i=1}^{n} x_i^{-\phi}\right)}$$

which is density kernel of gamma distribution with the parameters $\alpha = \text{n}$ and $\beta = \sum_{i=1}^{n} x_i^{-\phi}$

So theposterior distribution of θ given data is $\text{gamma}(\alpha,\beta)$.

Example 3.1 The data regarding failure times of the air conditioning system of an aero plane:

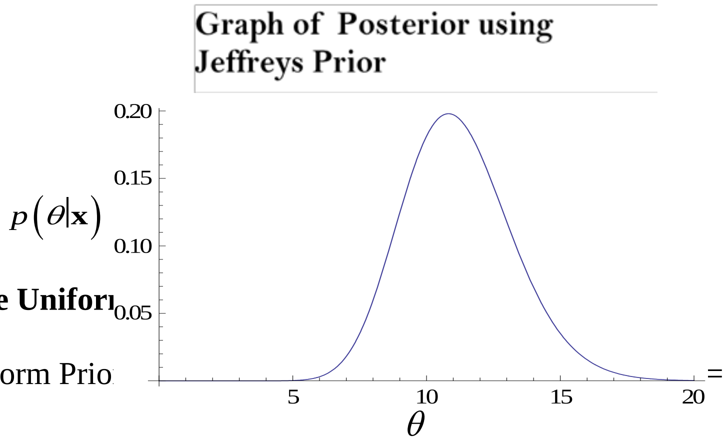
23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12,

120, 11, 14, 71, 11, 14, 11, 16, 3, 90, 1, 16, 52, 95.

where $\phi = 1,\ \text{n} = 30, \sum_{i=1}^{n} x_i^{-\phi} = 2.68$

The graph of the posterior distribution with its parameter is given below.

For the posterior parameters a $\alpha = 30$ $\beta = 2.68$



3.3.2 The Unifor

The Uniform Prio =1)is

$$f(\theta) \propto 1$$

The posterior distribution of θ given data is;

$$p(\theta|\mathbf{x}) \propto \theta^{n} \; e^{-\theta\left(\sum_{i=1}^{n} x_i^{-\phi}\right)}$$

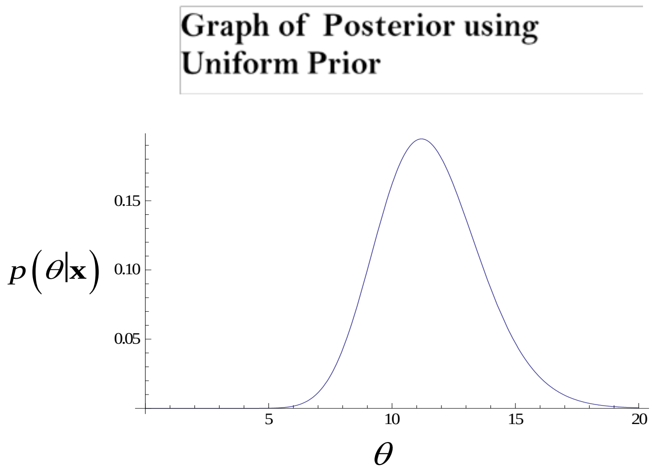
$$p(\theta|\mathbf{x}) \propto \theta^{n+1-1} \; e^{-\theta\left(\sum_{i=1}^{n} x_i^{-\phi}\right)}$$

which is the density kernel of Gamma distribution with the parameters $\alpha = n+1$ and $\beta = \sum_{i=1}^{n} x_i^{-\phi}$. So theposterior distribution of θ given data is $\text{Gamma}(\alpha, \beta)$.

$$\text{where } \phi = 1, \; n = 30, \sum_{i=1}^{n} x_i^{-1} = 2.68$$

The graph of the posterior distribution with its parameters is given below.

For the posterior parameters a $\alpha = 31$, and $\beta = 2.68$.



3.4 The Posterior Distribution using Informative prior

We have used two informative priors Gamma distribution and Exponential distribution as a prior in the following sections:

3.4.1 Using Gamma Distribution as Prior

The informative prior Gamma distribution of θ with hyper parameter '*a*' and 'b' is given below.

$$f(\theta; a, b) = \frac{b^a}{\Gamma a}\, \theta^{a-1}\, e^{-\theta b} \quad \theta > 0,\ a > 0,\ b > 0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p(\theta|\mathbf{x}) \propto \theta^{a-1} e^{-\theta b} \theta^n \prod_{i=1}^{n} x^{-(\phi+1)} e^{-\theta \sum_{i=1}^{n} x^{-\phi}}$$

$$p(\theta|\mathbf{x}) \propto \theta^{a+n-1} e^{-\theta b} e^{-\theta \sum_{i=1}^{n} x^{-\phi}}$$

$$p(\theta|\mathbf{x}) \propto \theta^{a+n-1} e^{-\theta\left(b + \sum_{i=1}^{n} x^{-\phi}\right)}$$

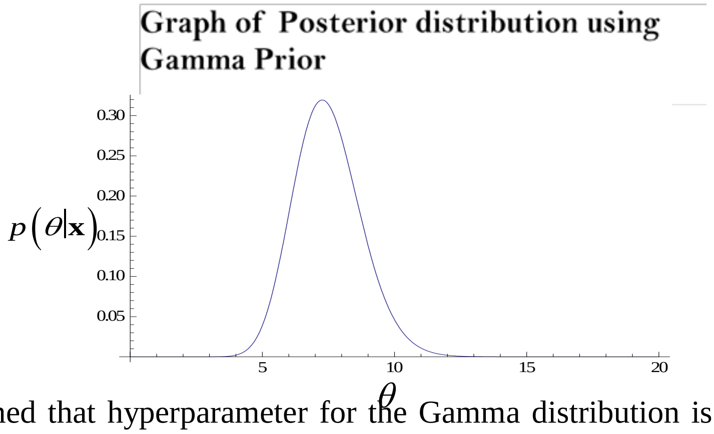
$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1} e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha = a + n$ and $\beta = (b + \sum_{i=1}^{n} x_i^{-\phi})$.So theposterior distribution of θ given data is Gamma distribution with parameters (α, β).

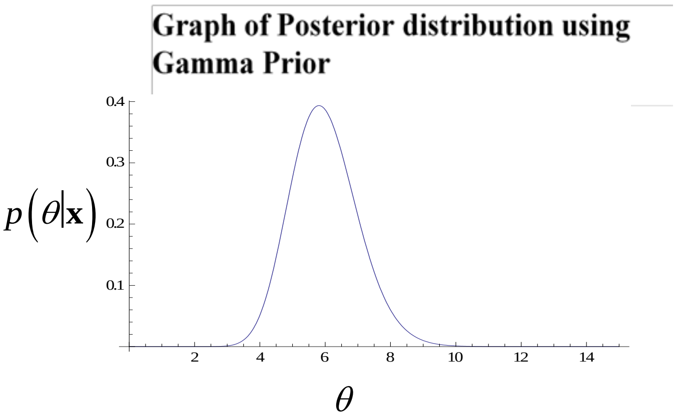
where $\phi = 1,\ n = 30, \sum_{i=1}^{n} x_i^{-1} = 2.68$

The graph of the Gamma distribution (prior) and posterior distribution for the different values of hyperparameter is given below

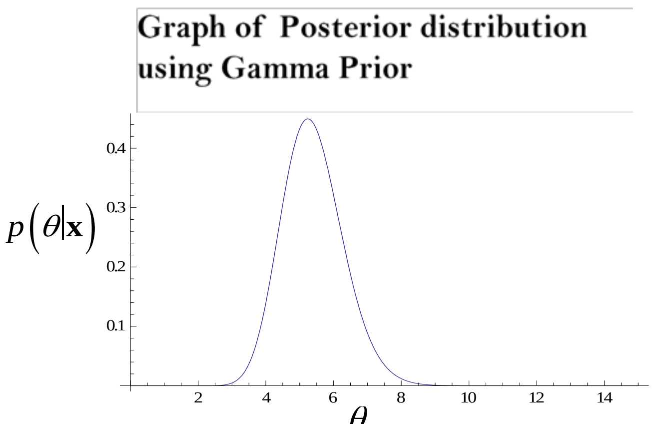
1. It is assumed that hyperparameter for the Gamma distribution is a =5 and b=2 then the posterior parameter will be α = 35, and β = 4.68



2. It is assumed that hyperparameter for the Gamma distribution is a =4 and b = 3 then the posterior parameter will be α = 34 and β = 5.68.



3. It is assumed that hyperparameter for the Gamma distribution is a = 6 and b = 4 then the posterior parameter will be α =36 and β=6.68.



3.4.2 Using Exponential Distribution as Prior

The informative prior Exponential distribution of θ with hyper parameter‘s’ is given below.

$$f\,(\theta;\mathrm{c}) = \mathrm{c}e^{-\theta\mathrm{c}} \quad \theta > 0\,, \quad c > 0$$

The posterior distribution of θ given data is;

$$p(\theta|\mathbf{x}) \propto \theta^{n-1}e^{-\theta c}\theta^{n}\prod_{i=1}^{n} x^{-(\phi+1)}e^{-\theta\sum_{i=1}^{n} x^{-\phi}}$$

$$p(\theta|\mathbf{x}) \propto \theta^{n}\prod_{i=1}^{n} x^{-(\phi+1)}e^{-\theta(\sum_{i=1}^{n} x^{-\phi}+c)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{n}e^{-\theta(\sum_{i=1}^{n} x^{-\phi}+c)}$$

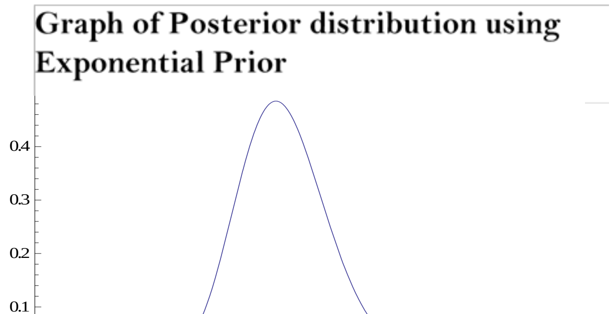
$$p(\theta|\mathbf{x}) \propto \theta^{n+1-1}e^{-\theta(\sum_{i=1}^{n} x^{-\phi}+c)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1}e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha = n+1$ and $\beta = (\mathrm{c}+\sum_{i=1}^{n} x_i^{-\phi})$ .So theposterior distribution of θ given data is gamma distribution with parameters (α,β).

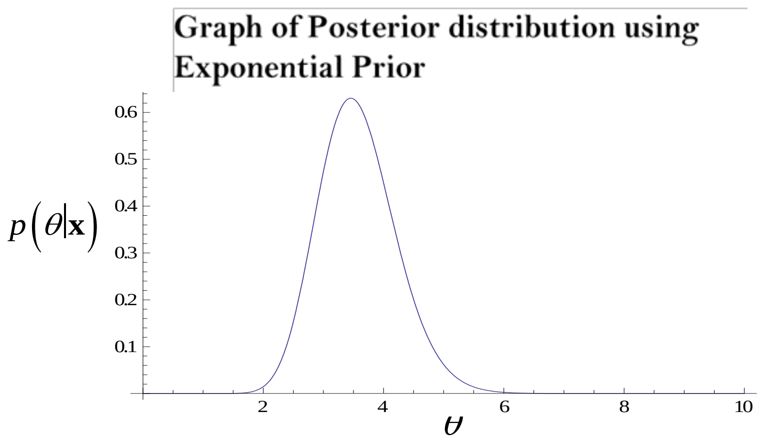
The graph of the posterior distribution for the different values of hyperparameter is given below.

1. It is assumed that hyperparameter for the Exponential distribution is c=2 then the posterior parameter will be α =31and β = 4.68.

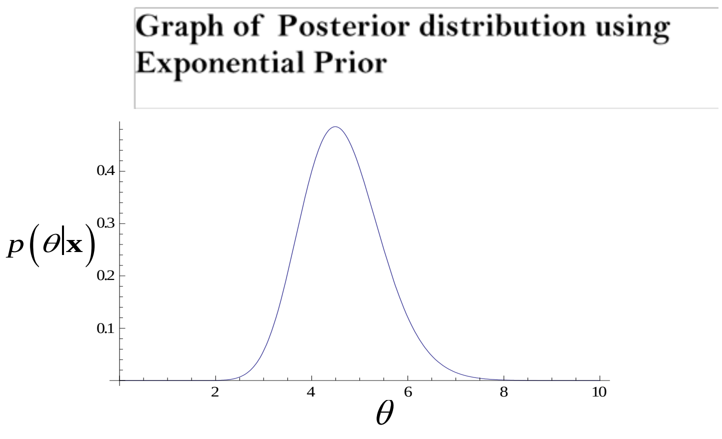


$p(\theta|\mathbf{x})$

2. It is assumed that hyperparameter for the Exponential distribution is c = 3 then the posterior parameter will be α =31 and β= 5.68



3. It is assumed that hyperparameter for the Exponential distribution is c = 4 then the posterior parameters will be α =31 and β= 6.68



CHAPTER 4

Comparisons of Priors

Introduction

In this chapter the informative and noninformative priors are compared on the basis of Bayes estimators, posterior risks, coefficient of skewness. Bayesian Hypotheses testing has been performed in this section. Finally simulation study is performed using Bayes estimator and Bayes Posterior risk under three loss functions (Quadratic, Square, Weighted) using informative (Gamma and Exponential) and noninformative (Jeffreys and Uniform).

4.1 Comparison of Priors using Bayes Estimators

Comparisionof Informative and noninformative Priors using Bayes risk under three loss functionispresentedin the following sections.

(i) The square error loss function

The Bayes estimator is:

The square error loss function for the parameter θ is:

$$L_1 = L(\theta,\hat{\theta}) = (\theta - \hat{\theta})^2$$

The Bayes estimator is:

$$\hat{\theta} = E(\theta)$$

The Bayes estimator for the Jeffreys prior is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{n}{\sum_{i=1}^{n} x_i^{-\phi}}$$

The Bayes estimator Using Uniform prior is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{n+1}{\sum_{i=1}^{n} x_i^{-\phi}}$$

The Bayes estimator Using Gamma distribution as a prior is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{a+n}{b+\sum_{i=1}^{n} x_i^{-\phi}}$$

The Bayes estimator Using Exponential distribution as a prior is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{n+1}{c+\sum_{i=1}^{n} x_i^{-\phi}}$$

(ii) The weighted loss function

The Bayes estimator is:

The weighted loss function is defined as:

$$L_2 = L(\theta, \hat{\theta}) = \frac{(\theta - \hat{\theta})^2}{\theta}$$

The Bayes estimator is:

$$L_2 = \frac{\theta^2 + \hat{\theta}^2 - 2\theta\hat{\theta}}{\theta}$$

$$\frac{\partial E_\theta(L_2)}{\partial \hat{\theta}} = 2\hat{\theta} E_\theta(\theta^{-1}) - 2$$

$$\hat{\theta} = \frac{1}{E(\theta^{-1})}$$

The Bayes estimator using Jeffreys prior is:

$$\hat{\theta} = \frac{n-1}{(\sum_{i=1}^{n} x_i^{-1})}$$

The Bayes estimator Using Uniform as a prior is:

$$\hat{\theta} = \frac{n}{(\sum_{i=1}^{n} x_i^{-1})}$$

The Bayes estimator Using Gamma distribution as a prior is:

$$\hat{\theta} = \frac{a+n-1}{(\sum_{i=1}^{n} x_i^{-1} + b)}$$

The Bayes estimator Using Exponential distribution as a prior

$$\hat{\theta} = \frac{n}{(\sum_{i=1}^{n} x_i^{-1} + c)}$$

(iii) The Quadratic loss function

The Quadratic loss function is defined as:

$$L_3 = L(\theta, \hat{\theta}) = \left(\frac{\theta - \hat{\theta}}{\theta} \right)^2$$

The Bayes estimator is:

$$L_3 = \frac{\theta^2 + \hat{\theta}^2 - 2\theta\hat{\theta}}{\theta^2}$$

$$\frac{\partial E_\theta(L_3)}{\partial \hat{\theta}} = 2\hat{\theta} E_\theta(\theta^{-2}) - 2E_\theta(\theta^{-1})$$

$$\hat{\theta} = \frac{E(\theta^{-1})}{E(\theta^{-2})}$$

The Bayes estimator using Jeffreys prior is:

$$\hat{\theta} = \frac{n-2}{\sum_{i=1}^{n} x_i^{-1}}$$

The Bayes Using Uniform prior is:

$$\hat{\theta} = \frac{n-1}{\sum_{i=1}^{n} x_i^{-1}}$$

The Bayes estimator using Gamma distribution as a prior is:

$$\hat{\theta} = \frac{a+n-2}{(b + \sum_{i=1}^{n} x_i^{-1})}$$

The Bayes estimator Using Exponential distribution as a prior is:

$$\hat{\theta} = \frac{n-1}{(c + \sum_{i=1}^{n} x_i^{-1})}$$

Comparison of priors using Bayes Estimator under different loss functions using informative (Gamma and Exponential) and noninformative (Uniform and Jeffreys) priors are shown in the following table 4.1.

Table 4.1: Bayes Estimators Using Different Loss Functions

Loss function $L(\theta,\hat{\theta})$	Prior Distributions		Posterior parameters (α,β)	Bayes Estimators $\hat{\theta}$
L_1	IP	GP	$(35, 4.68)$	7.4786
		EP	$(31, 4.68)$	6.6239
	NIP	JP	$(30, 2.68)$	11.1940
		UP	$(31, 2.68)$	11.5671
L_2	IP	GP	$(35, 4.68)$	7.2649
		EP	$(31, 4.68)$	6.4102
	NIP	JP	$(30, 2.68)$	10.8209
		UP	$(31, 2.68)$	11.1940
L_3	IP	GP	$(35, 4.68)$	7.0512
		EP	$(31, 4.68)$	6.1965
	NIP	JP	$(30, 2.68)$	10.4477
		UP	$(31, 2.68)$	10.8209

Here GP: Gamma prior, EP: Exponential prior, JP: Jeffreys prior, UP: Uniform prior

The above table describes the Bayes estimators under different loss function using informative (Gamma and Exponential) and noninformative (Jeffreys and uniform) priors.

4.2 Comparision of Priors using Bayes Posterior Risk

The Expectation of a Loss function with respect to the posterior distribution is known as posterior risk.

(i) The square error loss function

The Bayes posterior risk under SELF is variance of posterior distribution:

The posterior risk under square error loss function Using Jeffery prior is:

$$V(\theta|x) = \frac{\alpha}{\beta^2}$$

$$V(\theta|x) = \frac{n}{(\sum_{i=1}^{n} x_i^{-1})^2} \text{ here } \phi = 1$$

The posterior risk undersquare error loss function Using Uniform prior is:

$$V(\theta|x) = \frac{n+1}{(\sum_{i=1}^{n} x_i^{-1})^2}$$

The posterior risk undersquare error loss function Using Gamma as a prior is:

$$V(\theta|x) = \frac{a+n}{(\mathrm{b} + \sum_{i=1}^{n} x_i^{-1})^2}$$

The posterior risk undersquare error loss function Using Exponential as a prior is:

$$V(\theta|x) = \frac{n+1}{(\mathrm{s} + \sum_{i=1}^{n} x_i^{-1})^2}$$

(ii) The Quadratic loss function

The Bayes posterior risk (PR) under QLF is:

The posterior risk Using Jeffery prior is:

$$\mathrm{PR} = E_\theta \, \mathrm{L}(\theta, \hat{\theta}) = E_\theta \left(\frac{\theta - \hat{\theta}}{\theta} \right)^2$$

$$\mathrm{PR} = E_\theta \left(1 - \frac{\hat{\theta}}{\theta} \right)^2$$

$$\text{PR} = \hat{\theta}^2 E_\theta(\theta^{-2}) + 1 - 2\hat{\theta} E_\theta(\theta^{-1})$$

$$\text{PR} = \left(\frac{E_\theta(\theta^{-1})}{E_\theta(\theta^{-2})}\right)^2 E_\theta(\theta^{-2}) + 1 - 2\left(\frac{E_\theta(\theta^{-1})}{E_\theta(\theta^{-2})}\right) E_\theta(\theta^{-1})$$

The posterior risk Using Jefferysprior is:

$$\text{PR} = 1 - \frac{(n-2)}{(n-1)}$$

The posterior risk Using Uniform prior is:

$$\text{PR} = 1 - \frac{(n-1)}{(n)}$$

The posterior risk using a Using Gamma prior is:

$$\text{PR} = 1 - \frac{(a+n-2)}{(a+n-1)}$$

The posterior risk Using Exponential prior is:

$$\text{PR} = 1 - \frac{(n-1)}{(n)}$$

iii) The Weighted loss function

The Bayes posterior risk under WLF is:

$$\text{PR} = \text{E}(\theta) - \text{E}(\theta^{-1})^{-1}$$

The posterior risk Using Jeffery prior is:

$$\text{PR} = \frac{n}{(\sum_{i=1}^{n} x_i^{-1})} - \frac{n-1}{(\sum_{i=1}^{n} x_i^{-1})}$$

The posterior risk Using Uniform as a prior is:

$$\text{PR} = \frac{n+1}{(\sum_{i=1}^{n} x_i^{-1})} - \frac{n}{(\sum_{i=1}^{n} x_i^{-1})}$$

The posterior risk Using Gamma prior is:

$$\text{PR} = \frac{a+n}{(\sum_{i=1}^{n} x_i^{-1} + b)} - \frac{a+n-1}{(\sum_{i=1}^{n} x_i^{-1} + b)}$$

The posterior risk Using Exponential distribution as a prior is:

$$\text{PR} = \frac{n+1}{(\sum_{i=1}^{n} x_i^{-1} + c)} - \frac{n}{(\sum_{i=1}^{n} x_i^{-1} + c)}$$

Comparison of priors using Bayes Posterior risk under different loss functions using informative (Gamma and Exponential) and noninformative (Uniform and Jeffreys) priors is discussed in the following table 4.2

Table 4.2:Bayes Posterior risk Using Different Loss Functions

Loss function $L(\theta,\hat{\theta})$	Prior Distributions		Posterior parameters (α,β)	Bayes Posterior Risk PR
L_1	IP	GP	$(35,4.68)$	1.5979
		EP	$(31,4.68)$	1.4153
	NIP	JP	$(30,2.68)$	4.1768
		UP	$(31,2.68)$	4.3161
L_2	IP	GP	$(35,4.68)$	0.02944
		EP	$(31,4.68)$	0.03333
	NIP	JP	$(30,2.68)$	0.03448
		UP	$(31,2.68)$	0.03333
L_3	IP	GP	$(35,4.68)$	0.2136
		EP	$(31,4.68)$	0.2137
	NIP	JP	$(30,2.68)$	0.3732
		UP	$(31,2.68)$	0.3731

Table 4.2 describes that the comparison of posterior Risk using different loss functions. It is cleared that the posterior risk for noninformative priors (Jeffery and Uniform) is greater than the informative prior (Gamma and Exponential) using Quadratic, Weighted loss Functions and Square error loss functions. On the whole Bayes posterior risk under Gamma distribution is minimum under three loss functions. However, using the noninformative priors the best prior is Uniform as it has minimum posterior risk under all loss function.

4.4 Bayesian Estimation using Simulation

In this section we simulate the Bayes Estimate and Bayes Posterior Risk using different loss (Quadratic, Weighted, Square) functions as well as different sample size by using informative (Gamma and Exponential) and uninformative (Uniform and Jeffreys) priors.

4.4.1 Simulation using Bayes Estimates and Bayes Posterior Risk under Square error loss function

Simulation using Bayes estimator and Bayes Posterior risk under Square error loss function using informative (Gamma and Exponential) and noninformative (Jeffreys and Uniform) priors is given as

Table 4.3 Simulation using Bayes Estimates and Bayes Posterior Risk under Jeffreys Prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	6.10655 (0.77017)	7.19467 (1.02441)	8.19369 (1.36478)	9.20042 (1.72414)	10.1809 (2.13354)
$n_2 = 100$	6.08595 (0.37119)	7.06968 (0.50620)	8.08871 (0.66366)	9.08448 (0.83420)	10.1215 (1.02853)
$n_3 = 200$	6.0439 (0.18203)	7.00418 (0.24955)	8.03708 (0.32382)	9.04363 (0.41454)	10.0153 (0.51504)
$n_4 = 300$	6.0252 (0.12142)	7.02257 (0.16640)	8.02135 (0.21625)	9.02991 (0.27254)	10.0612 (0.33778)
$n_5 = 500$	6.01152 (0.07327)	7.00554 (0.09844)	8.01832 (0.12920)	9.01049 (0.16361)	10.0034 (0.20085)

Table 4.4 Simulation using Bayes Estimates and Bayes Posterior Risk under Uniform Prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	6.22088 (0.78146)	7.30248 (1.06176)	8.35138 (1.38676)	9.41436 (1.73703)	10.4451 (2.17)
$n_2 = 100$	6.10053 (0.37170)	7.17299 (0.51361)	8.18117 (0.66424)	9.12131 (0.84462)	10.1678 (1.03945)
$n_3 = 200$	6.06155 (0.18176)	7.08598 (0.25241)	8.0737 (0.32598)	9.09045 (0.41180)	10.0682 (0.51071)
$n_4 = 300$	6.03901 (0.12229)	7.05038 (0.16562)	8.06155 (0.21545)	9.0586 (0.27342)	10.0343 (0.33702)
$n_5 = 500$	6.02568 (0.07271)	7.02242 (0.09863)	8.02809 (0.12912)	9.04974 (0.16325)	10.0397 (0.20148)

Table 4.5 Simulation using Bayes Estimates and Bayes Posterior Risk using Gamma prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	5.26119 (0.80303)	5.95122 (1.09035)	6.5966 (1.37274)	7.21301 (1.70878)	7.82749 (2.09774)
$n_2 = 100$	5.6543 (0.38210)	6.45408 (0.51944)	7.22733 (0.67863)	7.98545 (0.84967)	8.71244 (1.05183)
$n_3 = 200$	5.80241 (0.18591)	6.70091 (0.25029)	7.5754 (0.33467)	8.45912 (0.41403)	9.29611 (0.51743)
$n_4 = 300$	5.88154 (0.12263)	6.79937 (0.16648)	7.7072 (0.21815)	8.63079 (0.27497)	9.52674 (0.34769)
$n_5 = 500$	5.91824 (0.07281)	6.86896 (0.09919)	7.82052 (0.12913)	8.78708 (0.16437)	9.70859 (0.20207)

Table 4.6 Simulation using Bayes Estimates and Bayes Posterior Risk using Exponential Prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.98538 (0.75398)	5.63923 (1.00126)	6.24389 (1.31)	6.85748 (1.64237)	7.37711 (2.00564)
$n_2 = 100$	5.44871 (0.37263)	6.27038 (0.49624)	6.99786 (0.65158)	7.73629 (0.82214)	8.47489 (1.02094)
$n_3 = 200$	5.70022 (0.18303)	6.57579 (0.24895)	7.47888 (0.32748)	8.33302 (0.40988)	9.18449 (0.50505)
$n_4 = 300$	5.81657 (0.12138)	6.74396 (0.16495)	7.64535 (0.21393)	8.5515 (0.27322)	9.43659 (0.33497)
$n_5 = 500$	5.88167 (0.07262)	6.84429 (0.09891)	7.77781 (0.82214)	8.70299 (0.16364)	9.64704 (0.20157)

For simulation the Bayes estimator and Bayes Posterior risk under square error loss function using informative (Gamma and Exponential) priors and non informative priors is used. The above tables shows the simulation using Bayes estimates and Bayes Posterior risk under Square error loss function for Informative and noninformative priors for the different values of sample size and parameter. After simulation it is observed that as sample size increases the value of the parameter approaches to its true value, and the Posterior risk decrease as the sample sizes increased.

4.4.2 Simulation using Bayes estimates and Bayes Posterior risk

under Weighted loss function

The Simulation using Bayes estimator and Bayes Posterior risk under weighted loss function using informative (Gamma and Exponential) and noninformative (Uniform and Jeffreys) priors is given as

Table 4.7 Simulation using Bayes Estimates and Bayes Posterior risk under Jeffreys Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	6.05733 (0.12266)	6.95253 (0.14301)	7.99101 (0.16524)	8.93386 (0.18508)	9.99331 (0.20309)
$n_2=100$	5.96899 (0.06070)	6.97725 (0.07056)	7.98770 (0.80704)	8.97897 (0.09084)	10.0158 (0.10104)
$n_3=200$	6.0006 (0.03022)	7.02004 (0.03512)	7.99827 (0.04021)	9.0316 (0.04510)	10.0067 (0.05022)
$n_4=300$	6.02434 (0.02003)	7.00807 (0.02339)	8.01139 (0.02609)	9.03238 (0.03009)	9.9994 (0.03335)
$n_5=500$	6.00235 (0.019967)	7.00206 (0.01402)	8.00536 (0.01607)	9.0256 (0.01799)	10.0063 (0.02003)

Table 4.8 Simulation using Bayes Estimates and Bayes Posterior risk under Uniform Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	6.10655 (0.12207)	7.19467 (0.14192)	8.19369 (0.16274)	9.20042 (0.18674)	10.1809 (0.20565)
$n_2=100$	6.08595 (0.06050)	7.06968 (0.07079)	8.08871 (0.08080)	9.08448 (0.09100)	10.1215 (0.10130)
$n_3=200$	6.0439 (0.03021)	7.00418 (0.03516)	8.03708 (0.04011)	9.04363 (0.04513)	10.0153 (0.05031)
$n_4=300$	6.0252 (0.02008)	7.02257 (0.02339)	8.02135 (0.02673)	9.02991 (0.03004)	10.0612 (0.03339)
$n_5=500$	6.01152 (0.01237)	7.00554 (0.01402)	8.01832 (0.01601)	9.01049 (0.01803)	10.0034 (0.02001)

Table 4.9 Simulation using Bayes Estimatesand Bayes Posterior risk under Gamma Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	5.21558 (0.09822)	5.84117 (0.14192)	6.4944 (0.12267)	7.11019 (0.13416)	7.64938 (0.14475)
$n_2=100$	5.57232 (0.0500)	6.37195 (0.07079)	7.20702 (0.06944)	7.94218 (0.07710)	8.62835 (0.08395)
$n_3=200$	5.77524 (0.02896)	6.65009 (0.03516)	7.54008 (0.03724)	8.42761 (0.04151)	9.28736 (0.04546)
$n_4=300$	5.83435 (0.01913)	6.7577 (0.02241)	7.69693 (0.02543)	8.59904 (0.02846)	9.49693 (0.03162)
$n_5=500$	5.90388 (0.01173)	6.86426 (0.01368)	7.80043 (0.01550)	8.79255 (0.01739)	9.68065 (0.01926)

Table 4.10 Simulations using Bayes Estimates and Bayes Posterior risk using Exponential Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	4.87707 (0.09772)	5.52053 (0.11076)	6.12371 (0.12278)	6.70454 (0.13314)	7.21076 (0.14480)
$n_2=100$	5.37798 (0.05413)	6.14306 (0.06178)	6.94554 (0.06947)	7.69965 (0.07653)	8.38107 (0.08360)
$n_3=200$	5.6805 (0.02840)	6.59922 (0.03287)	7.4379 (0.03708)	8.2884 (0.04163)	9.13814 (0.04551)
$n_4=300$	5.78947 (0.01933)	6.69175 (0.02238)	7.63045 (0.02539)	8.50808 (0.02842)	9.37014 (0.03128)
$n_5=500$	5.86467 (0.01176)	6.82504 (0.01358)	7.75019 (0.01554)	8.69572 (0.01739)	9.63613 (0.01921)

For simulation the Bayes estimator and Bayes Posterior risk under weighted loss function using informative (Gamma and Exponential) priors and non informative priors is used. The above tables shows the simulation using Bayes estimates and Bayes Posterior risk under Square error loss function for Informative and noninformative priors for the different values of sample size and parameter. After simulation it is observed that as sample size increases the value of the parameter approaches to its true value, and the Posterior risk decrease as the sample sizes are increased.

4.4.3 Simulation using of the Bayes Estimates and Bayes Posterior risk Under Quadratic loss function

The Simulation using Bayes estimator and Bayes Posterior risk under Quadratic loss function using informative (Gamma and Exponential) and noninformative (Uniform and Jeffreys) priors is given as

Table 4.11 Simulation using Bayes Estimates and Bayes Posterior Risk under Jeffreys Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	5.87893 (0.02040)	6.83896 (0.02040)	7.77992 (0.02040)	8.80499 (0.02040)	9.81102 (0.02040)
$n_2=100$	5.94266 (0.01010)	6.96943 (0.01010)	7.91718 (0.01010)	8.86646 (0.01010)	9.89516 (0.01010)
$n_3=200$	5.97331 (0.005025)	6.96658 (0.005025)	7.91718 (0.005025)	8.98282 (0.005025)	9.95026 (0.005025)
$n_4=300$	5.97099 (0.003344)	6.96892 (0.003344)	7.95405 (0.003344)	8.97897 (0.003344)	9.97974 (0.003344)
$n_5=500$	5.97769 (0.00200)	6.96829 (0.00200)	7.9807 (0.00200)	9.00041 (0.00200)	9.96546 (0.00200)

Table 4.12 Simulation using Bayes Estimates and Bayes Posterior risk using Uniform prior is:

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$
$n_1=50$	6.05733 (0.02)	6.95253 (0.02)	7.99101 (0.02)	8.93386 (0.02)	9.99331 (0.02)
$n_2=100$	5.96899 (0.01)	6.97725 (0.01)	7.98770 (0.01)	8.97897 (0.01)	10.0158 (0.01)
$n_3=200$	6.0006 (0.005)	7.02004 (0.005)	7.99827 (0.005)	9.0316 (0.005)	10.0067 (0.005)
$n_4=300$	6.02434 (0.003)	7.00807 (0.003)	8.01139 (0.003)	9.03238 (0.003)	9.9994 (0.003)
$n_5=500$	6.00235 (0.002)	7.00206 (0.002)	8.00536 (0.002)	9.0256 (0.002)	10.0063 (0.002)

Table 4.13 Simulation using Bayes Estimates and Bayes Posterior risk using Gamma Prior

Sample sizes	$\theta=6$	$\theta=7$	$\theta=8$	$\theta=9$	$\theta=10$

$n_1 = 50$	5.08793 (0.01960)	5.74827 (0.01960)	6.38944 (0.01960)	6.97801 (0.01960)	7.50804 (0.01960)
$n_2 = 100$	5.50191 (0.00990)	6.28679 (0.00990)	7.0981 (0.00990)	7.82837 (0.00990)	8.55254 (0.00990)
$n_3 = 200$	5.73808 (0.00497)	6.61814 (0.00497)	7.50791 (0.00497)	8.39371 (0.00497)	9.23853 (0.00497)
$n_4 = 300$	5.82198 (0.00332)	6.77206 (0.00332)	7.65973 (0.00332)	8.56283 (0.00332)	9.47671 (0.00332)
$n_5 = 500$	5.89856 (0.00199)	6.85553 (0.00199)	7.80434 (0.00199)	8.73983 (0.00199)	9.69897 (0.00199)

Table 4.14 Simulation using Bayes Estimates and Bayes Posterior risk using Exponential prior is:

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.7934 (0.02)	5.4346 (0.02)	5.96745 (0.02)	6.53527 (0.02)	7.08682 (0.02)
$n_2 = 100$	5.37191 (0.01)	6.13752 (0.01)	6.89477 (0.01)	7.63386 (0.01)	8.31 (0.01)
$n_3 = 200$	5.65424 (0.005)	6.55393 (0.005)	7.39888 (0.005)	8.24984 (0.005)	9.08315 (0.005)
$n_4 = 300$	5.78098 (0.003)	6.66969 (0.003)	7.65973 (0.003)	8.50629 (0.003)	9.3527 (0.003)
$n_5 = 500$	5.8708 (0.002)	6.82264 (0.002)	7.75321 (0.002)	8.67104 (0.002)	9.61704 (0.002)

For simulation the Bayes estimator and Bayes Posterior risk under Quadratic loss function using informative (Gamma and Exponential) priors and non informative priors is used. The above tables shows the simulation using Bayes estimates and Bayes Posterior risk under Square error loss function for Informative and noninformative priors for the different values of sample size and parameter. After simulation it is observed that as sample size increases the value of the parameter approaches to its true value, and the Posterior risk decrease as the sample sizes are increased.

4.5 Comparison of Prior Using Coefficient of Skewness

The Coefficient of Skewness using informative prior is calculated by the following formula.

$$\gamma_1 = \sqrt{\frac{{\mu_3}^2}{\mu_2^3}}$$

The Posterior distribution derived for the informative and noninformative priors i.e Uniform, Jeffreys, Gamma and Exponential belongs to the family of Gamma distribution with different hyperparameters. So the Coefficient of Skewness are preaented in Table 4.15.

Table 4.15: Coefficient of skewness using Informative Prior

Name of Prior	Hyperparameter		Moment about mean of the posterior distribution			Coefficient of skewness
			μ_1	μ_2	μ_3	
Gamma Prior	$a = 5$	$b = 2$	0	1.59799	0.70878	.37341
	$a = 4$	$b = 3$	0	1.05385	0.37436	.34603
	$a = 6$	$b = 4$	0	.80676	0.24257	0.3345
	$a = 7$	$b = 5$	0	.62730	0.16266	0.3292
Exponential Prior	c = 2		0	1.41537	0.59751	0.39451
	c = 4		0	0.69471	0.22521	0.38894
	c = 5		0	0.52558	.24030	0.63197
	c = 6		0	0.41145	0.10217	0.38712

The Table 4.15 shows that the coefficient of Skewness of the posterior distribution using informative prior (Gamma and Exponential). From the above table it is clear that coefficient of skewness of each posterior distribution is greater than zero, which suggest that all the posterior distribution are positively skewed, as shown in the graphs in chapter 3, but the coefficient of skewness for the posterior distribution using Gamma prior is less skewed than Exponential distribution. So overall we conclude that Gamma prior is preferable or comparatively suitable than other informative prior (Exponential) as Gamma prior has small value of coefficient of skewness.

4.6 Bayes factor for Hypotheses testing:

The Bayes factor of hypotheses using informative (Gamma and Exponential) and noninformative (Jeffreys and Uniform) priors is given in the following section:

Table 4.16: Bayes factor for hypotheses testing

Null hypothesis	Alternative Hypothesis	Prior Distributions	Posterior Probability	Bayes Factor

H_0	H_1			$P(H_0)$	$P(H_1)$	B
$\theta \geq 6$	$\theta < 6$	IP	GP	0.9794	0.0206	47.5436
			EP	0.97900	0.021	46.6190
		NIP	JP	0.97740	0.0226	44.1327
			UP	0.9736	0.0264	36.8787
$\theta \geq 7$	$\theta < 7$	IP	GP	0.9688	0.0312	31.0512
			EP	0.9682	0.0318	30.4465
		NIP	JP	0.9762	0.0238	41.0168
			UP	0.9761	0.0239	40.8410
$\theta \geq 8$	$\theta < 8$	IP	GP	0.96490	0.03509	27.49571
			EP	0.97617	0.02382	40.36091
		NIP	JP	0.97248	0.02751	35.33992
			UP	0.959588	0.04040	23.75147

The above table describes the comparison of prior using Bayes factor under the following hypotheses.

For $H_0 : \theta \geq 6$ versus $H_1 :\ \theta < 6$, the Bayes factor using informative and noninformative prior is greater than 1. So we support H_0 using informative and noninformativeprior.ForHypothesis $H_0 : \theta \geq 7$ versus $H_1 : \theta < 7$,the Bayes factor using noninformative prior and informative prior is greater than 1.So we support H_0 using informative prior (Gamma). For Hypotheses $H_0 :\ \theta \geq 8$ versus $H_1 : \theta < 8$ the Bayes factor using informative and noninformative prior is greater than 1. So we support H_0 using informative and noninformative prior.

CHAPTER 5

Censoring scheme

5.1 Introduction

In this chapter, the Posterior Distribution for the parameter of the Gumbel distribution of the type II is derived using the censoring scheme. Two censoring schemes are used Type I and Type II censoring using two informative priors (Gamma and Exponential). The Bayes estimators and Posterior risk had been also derived in this sections under different loss functions (Quadratic, Weighted, Square). The graphs for posterior distributions using type I and type II censoring using two informative priors (Gamma and Exponential) had been also drawn in this section.

5.2 Type I Censoring

As discussed in the section 2.11 in Type I censored sample, suppose that a random samples of n units is tested under a predetermine time T at which the test is terminate. Time to failure of 'k' observations is observed at random thus the life times of x_r i=1,2,...n observed only $X_i \le T$ and thus the Likelihood function is given by:

$$L(\theta, x) = \prod_{i=1}^{k} f(x_i;\theta)\{1 - F(t;\theta)\}^{n-k}$$

$$L(\theta, x) = \theta^k \prod_{i=1}^{k} x_i^{-(\phi+1)} e^{-\theta \sum_{i=1}^{k} x_i^{-1}} \left(e^{-\theta(x_{(t)}^{-1})(n-k)}\right)$$

5.2.1 The Posterior Distribution under Type I censoring using Gamma prior.

The informative prior Gamma distribution of θ with hyper parameter 'a' and 'b' is given below.

$$f(\theta;a,b)=\frac{b^a}{\Gamma a}\,\theta^{a-1}\,e^{-\theta b}\quad \theta>0\,,\; a>0\,,\; b>0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p\left(\theta|\mathbf{x}\right)\propto\theta^{a-1}e^{-\theta b}\theta^{k}\prod_{\mathrm{i=1}}^{\mathrm{k}}x_i^{-(\phi+1)}e^{-\theta\sum_{i=1}^{k}x_i^{-1}}\,(e^{-\theta(x_{(t)}^{-1})(n-k)})$$

$$p\left(\theta|\mathbf{x}\right)\propto\theta^{a-1}e^{-\theta b}\theta^{k}e^{-\theta\left(\sum_{i=1}^{k}x_i^{-1}\right)}(e^{-\theta(x_{(t)}^{-1})(n-k)})$$

$$p\left(\theta|\mathbf{x}\right)\propto\theta^{a+k-1}e^{-\theta\left(b+\sum_{i=1}^{k}x_i^{-1}+(x_{(t)}^{-1})(n-k)\right)}$$

$$p(\theta|\mathbf{x})\propto\theta^{\alpha-1}e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha=a+k$ and $\beta=\left(b+\sum_{i=1}^{k}x_i^{-1}+(x_{(t)}^{-1})(n-k)\right)$. So the posterior distribution of θ given data is Gamma distribution with parameters $\left(\alpha,\beta\right)$.

5.2.3 The Posterior Distribution under Type I censoring using Exponential prior.

The informative prior Exponential distribution of θ with hyper parameter'c' is given below.

$$f\left(\theta;\mathrm{c}\right)=\mathrm{c}e^{-\theta\mathrm{c}}\quad\theta>0\,,\quad c>0$$

The posterior distribution of θ given data is:

$$p\left(\theta|\mathbf{x}\right)\propto\mathrm{c}e^{-\theta\mathrm{c}}\theta^{k}\prod_{\mathrm{i=1}}^{\mathrm{k}}x_i^{-(\phi+1)}e^{-\theta\sum_{i=1}^{k}x_i^{-1}}\,(e^{-\theta(x_{(t)}^{-1})(n-k)})$$

$$p\left(\theta|\mathbf{x}\right)\propto\mathrm{ce}^{-\mathrm{c}\theta}\theta^{k}e^{-\theta\left(\sum_{i=1}^{k}x_i^{-1}\right)}(e^{-\theta(x_{(t)}^{-1})(n-k)})$$

$$p\left(\theta|\mathbf{x}\right) \propto \theta^{k+1-1} e^{-\theta\left(c+\sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1} e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha = 1 + k$ and $\beta = (c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)$ .So the posterior distribution of θ given data is Gamma distribution with parameters (α, β).

5.3 Type II Censoring

Let X= $(x_{1,} x_{2,\ldots.} x_r)$ where x_i is the time of ith component to fail since the remaining n-r component have not been yet failed and thus have lifetime greater than x_r .

The likelihood function can be written as:

$$L(\theta, x) = \frac{n!}{(n-r)!} \prod_{i=1}^{r} f(x_{(i);} \theta) \{1 - F(x_{(r)})\}^{n-r}$$

The remaining $1 - F(x_{(r)})$ where x_r is the rth order statistics in $\underline{x} = (x_1, x_2, \ldots. x_r)$.

$$L(\theta, x) = \frac{n!}{(n-r)!} \theta^r \prod_{i=1}^{r} x_{(i)}{}^{-(\theta+1)} e^{-\theta \sum_{i=1}^{r} x_{(i)}} (e^{-\theta(x_{(r)}^{-1})(n-r)})$$

5.3.1 The Posterior Distribution under Type II censoring using Gamma as a prior

The informative prior Gamma distribution of θ with hyper parameter '*a*' and 'b' is given below.

$$f(\theta; a, b) = \frac{b^a}{\Gamma a} \theta^{a-1} e^{-\theta b} \quad \theta > 0,\ a > 0,\ b > 0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p\left(\theta|\mathbf{x}\right) \propto \theta^{a-1}e^{-\theta b}\theta^{r}\prod_{\mathrm{i}=1}^{\mathrm{r}} x_{(i)}{}^{-(\phi+1)}e^{-\theta\sum_{i=1}^{r}x_{(i)}^{-1}}(e^{-\theta(x_{(r)}^{-1})(n-r)})$$

$$p\left(\theta|\mathbf{x}\right) \propto \theta^{a-1}e^{-\theta b}\theta^{r}e^{-\theta\left(\sum_{i=1}^{r}x_{(i)}^{-1}\right)}(e^{-\theta(x_{(r)}^{-1})(n-r)})$$

$$p\left(\theta|\mathbf{x}\right) \propto \theta^{a+r-1}e^{-\theta\left(b+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)\right)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1}e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha = a + r$ and $\beta = \left(b+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)\right)$. So the posterior distribution of θ given data is Gamma distribution with parameters $\left(\alpha,\beta\right)$.

5.3.2 The Posterior Distribution under Type II censoring using Exponential as prior.

The informative prior Exponential distribution of θ with hyper parameter's' is given below.

$$f\left(\theta;\mathrm{c}\right) = \mathrm{c}e^{-\theta\mathrm{c}} \quad \theta > 0, \quad c > 0$$

The posterior distribution of θ given data is:

$$p\left(\theta|\mathbf{x}\right) \propto \mathrm{c}e^{-\theta c}\theta^{r}\prod_{\mathrm{i}=1}^{\mathrm{r}} x_{(i)}{}^{-(\phi+1)}e^{-\theta\sum_{i=1}^{r}x_{(i)}^{-1}}(e^{-\theta(x_{(r)}^{-1})(n-r)})$$

$$p\left(\theta|\mathbf{x}\right) \propto \mathrm{c}e^{-\theta\mathrm{c}}\theta^{r}e^{-\theta\sum_{i=1}^{r}x_{(i)}^{-1}}(e^{-\theta(x_{(r)}^{-1})(n-r)})$$

$$p\left(\theta|\mathbf{x}\right) \propto \theta^{r+1-1}e^{-\theta\left(c+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)\right)}$$

$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1}e^{-\theta\beta}$$

Which is density kernel of Gamma distribution with the parameters $\alpha = r+1$ and $\beta = \left(c + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n-r)\right)$. So the posterior distribution of θ given data is Gamma distribution with parameters (α,β).

5.4 Comparison of a Priors using Bayes Estimator under Type I censoring

(i) The square error loss function

The Bayes estimator Using Gamma distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{a+k}{\left(b + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{1+k}{\left(c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

(ii) The weighted loss function

The Bayes estimator Using Gamma distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha-1}{\beta} = \frac{a+k-1}{\left(b + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha - 1}{\beta} = \frac{k}{\left(c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

(iii) The Quadratic loss function

The Bayes estimator Using Gamma distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha - 2}{\beta} = \frac{a + k - 2}{\left(b + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type I censoring is:

$$\hat{\theta} = \frac{\alpha - 2}{\beta} = \frac{k - 1}{\left(c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)}$$

5.5 Comparison of a Priors using Bayes Estimator under Type II censoring

(i) The square error loss function

The Bayes estimator Using Gamma distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{a + r}{\left(b + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n-r)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha}{\beta} = \frac{1 + k}{\left(c + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n-r)\right)}$$

(ii) The weighted loss function

The Bayes estimator Using Gamma distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha - 1}{\beta} = \frac{a + r - 1}{\left(b + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n - r)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha - 1}{\beta} = \frac{r}{\left(c + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n - r)\right)}$$

(iii) The Quadratic loss function

The Bayes estimator Using Gamma distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha - 2}{\beta} = \frac{a + r - 2}{\left(b + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n - r)\right)}$$

The Bayes estimator Using Exponential distribution as a prior under Type II censoring is:

$$\hat{\theta} = \frac{\alpha - 2}{\beta} = \frac{r - 1}{\left(c + \sum_{i=1}^{r} x_{(i)}^{-1} + x_{(r)}^{-1}(n - r)\right)}$$

Comparison of priors under different loss functions in type I censoring using informative (Gamma and Exponential) noninformative priors (Jeffreys and uniform) is discussed in the following table 5.1.

Table 5.1: Bayes Estimators Using Type I censoring under Different Loss Functions

Loss function	Prior Distributions		Posterior parameters	Bayes Estimator
$L(\theta, \hat{\theta})$		HP	(α, β)	$\hat{\theta}$
L_1	GP	$a = 5$, b=2	(30, 8.68)	3.45622
	EP	$c = 2$	(26, 8.68)	2.99539
	GP	$a = 5$, b=2	(30, 8.68)	3.34101

L_2	EP	$c = 2$	$(26, 8.68)$	2.88012
L_3	GP	$a = 5$, b=2	$(30, 8.68)$	3.22580
	EP	$c = 2$	$(26, 8.68)$	2.76497

The above table describes the Bayes estimator under different loss functions (Quadratic, Weighted, Square) using informative Gamma and Exponential priors in type I censoring.

Comparison of priors under different loss functions in type II censoring using informative (Gamma and Exponential) priors is discussed in the following table 5.2.

Table 5.2: Bayes Estimators Using Type II censoring under Different Loss Functions

Loss function	Prior Distributions	Posterior	Bayes

$L(\theta,\hat{\theta})$				
		HP	(α,β)	$\hat{\theta}$
L_1	GP	$a=5$, b=2	(30,18.58)	1.61463
	EP	$c=2$	$(26,18.58)$	1.39935
L_2	GP	$a=5$, b=2	(30,18.58)	1.50681
	EP	$c=2$	$(26,18.58)$	1.34553
L_3	GP	$a=5$, b=2	(30,18.58)	1.50699
	EP	$c=2$	$(26,18.58)$	1.29171

The above table describes the Bayes estimator under different loss function (Quadratic, Weighted, Square) using informative (Gamma and Exponential) priors in type I censoring.

5.6 Comparison of a Priors using Bayes Posterior risk under Type I censoring

(i) The square error loss function

The Bayes posterior risk Using Gamma distribution prior under Type I censoring is:

$$V(\theta|x)=\frac{\alpha}{\beta^2}=\frac{a+k}{\left(b+\sum_{i=1}^{k}x_i^{-1}+(x_{(t)}^{-1})(n-k)\right)^2}$$

The Bayes posterior risk Using Exponential distribution prior under Type I censoring is:

$$V(\theta|x) = \frac{\alpha}{\beta^2} = \frac{1+k}{\left(c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)\right)^2}$$

(ii) The weighted loss function

The Bayes posterior risk Using Gamma distribution as a prior under Type I censoring is:

$$\text{PR} = \frac{a+k}{b + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)} - \frac{a+k-1}{b + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)}$$

The Bayes posterior risk Using Exponential distribution prior under Type I censoring is:

$$\text{PR} = \frac{1+k}{c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)} - \frac{k}{c + \sum_{i=1}^{k} x_i^{-1} + (x_{(t)}^{-1})(n-k)}$$

(iii) The Quadratic loss function

The Bayes posterior risk Using Gamma distribution as a prior under Type I censoring is:

$$\text{PR} = 1 - \frac{a+k-1}{a+k-2}$$

The Bayes posterior risk Using Exponential distribution prior under Type I censoring is:

$$\text{PR} = 1 - \frac{k}{k-1}$$

5.7 Comparison of a Priors using Bayes Posterior risk under Type II censoring

(i) The square error loss function

The Bayes posterior risk Using Gamma distribution prior under Type II censoring is:

$$V(\theta|x)=\frac{\alpha}{\beta^2}=\frac{a+r}{\left(b+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)\right)^2}$$

The Bayes posterior risk Using Exponential distribution prior under Type I censoring is:

$$V(\theta|x)=\frac{\alpha}{\beta^2}=\frac{1+r}{\left(c+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)\right)^2}$$

(ii) The weighted loss function

The Bayes posterior risk Using Gamma distribution prior under Type I censoring is:

$$\text{PR}=\frac{a+r}{b+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)}-\frac{a+r-1}{b+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)}$$

The Bayes posterior risk Using Exponential distribution prior under Type I censoring is:

$$\text{PR}=\frac{1+r}{c+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)}-\frac{r}{c+\sum_{i=1}^{r}x_{(i)}^{-1}+x_{(r)}^{-1}(n-r)}$$

(iii) The Quadratic loss function

The Bayes posterior risk Using Gamma distribution prior under Type II censoring is:

$$\text{PR}=1-\frac{a+r-2}{a+r-1}$$

The Bayes posterior risk Using Exponential distribution prior under Type II censoring is:

$$\text{PR}=1-\frac{r-1}{r}$$

Comparison of priors under different loss functions (Quadratic, Weighted, Square) in type I censoring using informative (Gamma and Exponential) priors is discussed in the following table 5.3.

Table 5.3: Bayes Posterior risk Using Type I censoring under Different Loss Functions

Loss function $L(\theta,\hat{\theta})$	Prior Distributions	HP	Posterior parameters (α,β)	Bayes Posterior Risk $\hat{\theta}$
L_1	GP	$a=5$, b=2	(30,8.68)	0.39818
	EP	$c=2$	(26,8.68)	0.34509
L_2	GP	$a=5$, b=2	(30,8.68)	0.11520
	EP	$c=2$	(26,8.68)	0.11521
L_3	GP	$a=5$, b=2	(30,8.68)	0.03487
	EP	$c=2$	(26,8.68)	0.04

Table 5.3 describes that the Bayes posterior Risk using different loss functions. It is clear that the posterior risk for informative priors (Gamma and Exponential) under Quadratic loss function is less than the others two loss functions (Weighted and Square) loss functions. On the whole Bayes posterior risk under Gamma distribution is minimum under three loss functions. So the best prior is Gamma as it has minimum posterior risk.

Comparison of priors under different loss functions (Quadratic, Weighted, Square) in type II censoring using informative (Gamma and Exponential) priors is discussed in the following table 5.4.

Table 5.4: Bayes Posterior risk Using Type II censoring under Different Loss Functions

Loss function	Prior Distributions	Posterior	Bayes Posterior

$L(\theta,\hat{\theta})$		HP	(α,β)	$\hat{\theta}$
L_1	GP	$a=5,$ b=2	(30,18.58)	0.08690
	EP	$c=2$	$(26,18.58)$	0.07531
L_2	GP	$a=5,$ b=2	(30,18.58)	0.05382
	EP	$c=2$	$(26,18.58)$	0.05383
L_3	GP	$a=5,$ b=2	(30,18.58)	0.03487
	EP	$c=2$	$(26,18.58)$	0.04

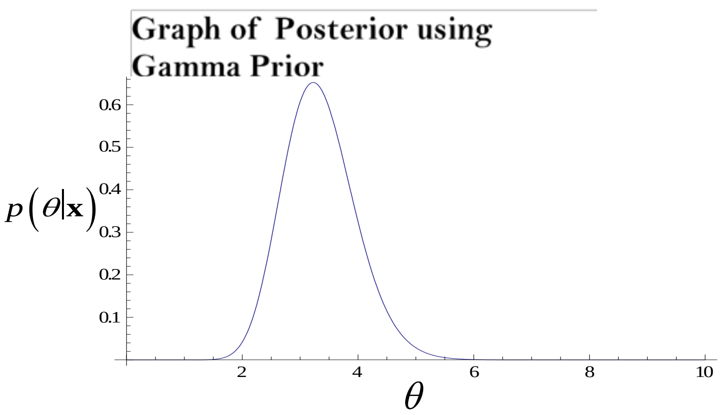
Table 5.4 describes that the Bayes posterior Risk using different loss functions. It is clear that the posterior risk for informative priors (Gamma and Exponential) under Quadratic loss function is less than the others two loss functions (Weighted and Square). On the whole Bayes posterior risk under Gamma distribution is minimum under three loss functions.

5.8 Graphs of the Posterior distribution using type I Censoring

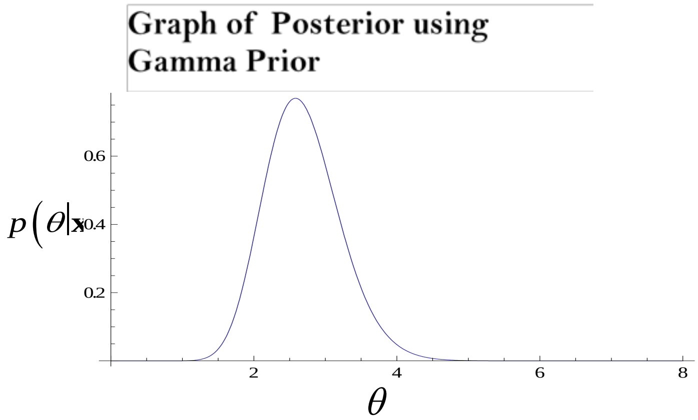
(i) Using Gamma as a Prior

The graph of the posterior distribution with its parameter is given below.

1. It is assumed that the hyperparameters $a=4,\ b=2$ than the posterior parameters will be $\alpha=29,$ and $\beta=8.68$.



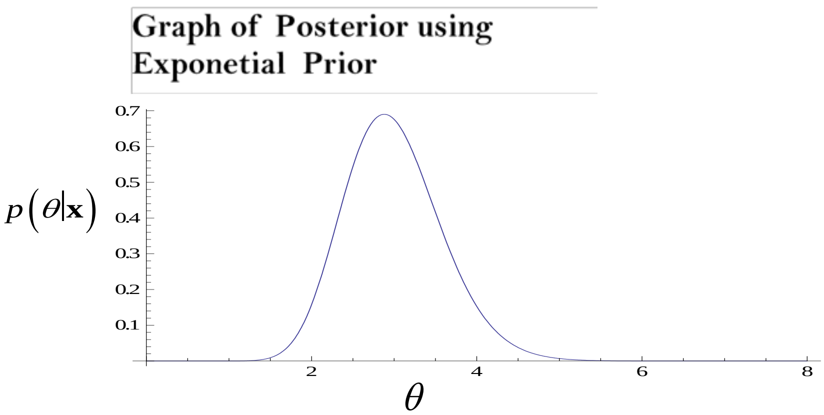
2. It is assumed that the hyperparameters $a = 5, \text{ b} = 3$ than the posterior parameters will be $\alpha = 30,$ and $\beta = 9.68$.



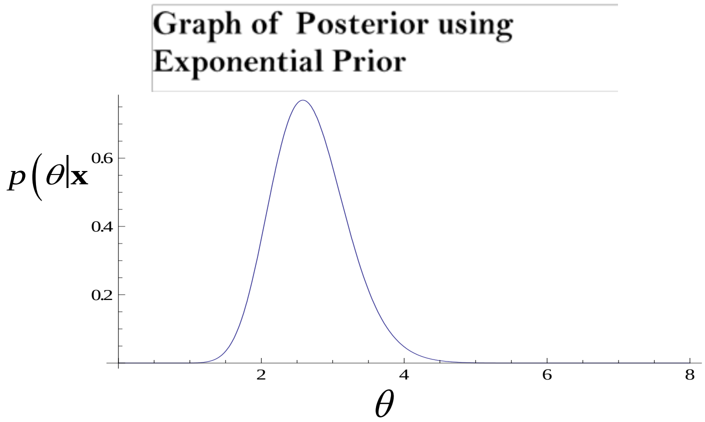
(ii) Using Exponential as a Prior

The graphs of the posterior distribution with its parameter is given below.

1. It is assumed that the hyperparameters $c = 2$ than the posterior parameters will be $\alpha = 26, \ \beta = 8.68$.



2. It is assumed that the hyperparameters $c = 3$ than the posterior parameters will be $\alpha = 26,\ \beta = 9.68$.

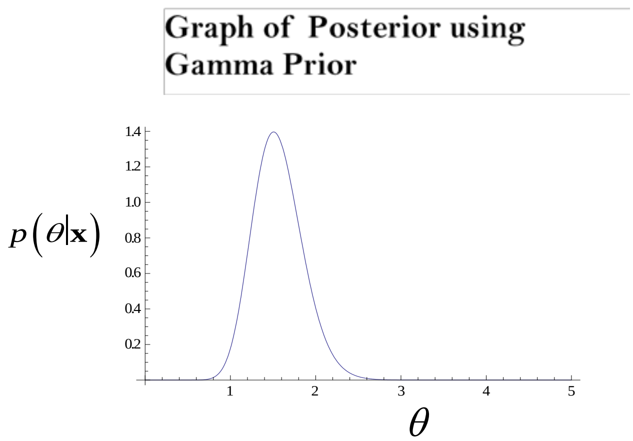


5.9 Graphs of the Posterior distribution using type II Censoring

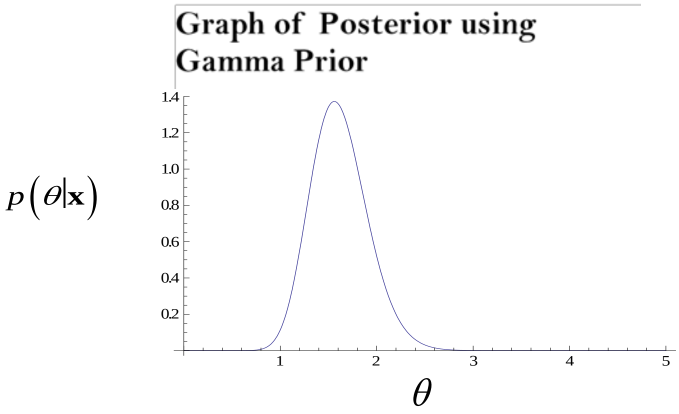
(i) Using Gamma as a Prior

The graph of the posterior distribution with its parameter is given below.

1. It is assumed that the hyperparameters $a = 4,\ \mathrm{b} = 2$ than the posterior parameters will be $\alpha = 29$ and $\beta = 18.58$.



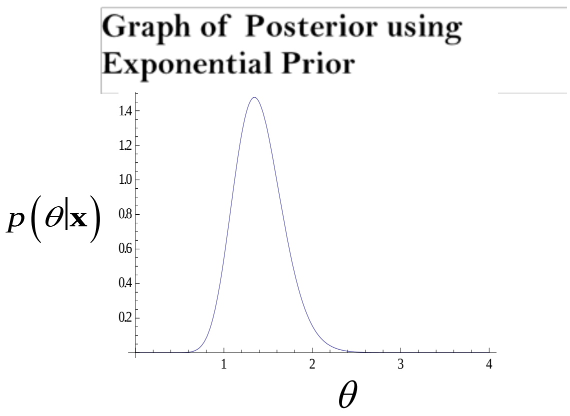
2. It is assumed that the hyperparameters $a = 5,\ b = 2$ than the posterior parameters will be $\alpha = 30,$ and $\beta = 18.58$.



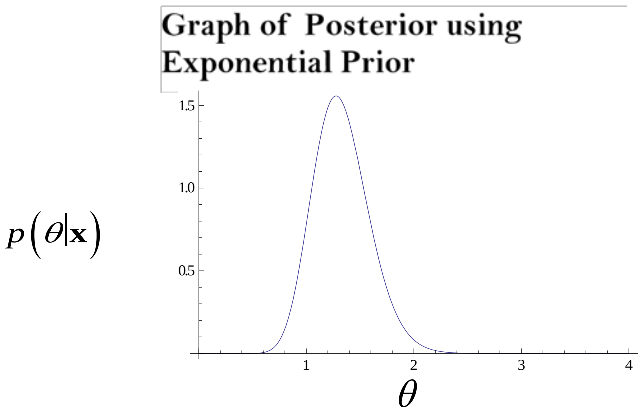
(ii) Using Exponential as a Prior

The graphs of the posterior distribution with its parameter is given below.

1. It is assumed that the hyperparameters $c = 2$ than the posterior parameters will be $\alpha = 26,$ and $\beta = 18.58$



2. It is assumed that the hyperparameters $c = 3$ than the posterior parameters will be $\alpha = 26$, and $\beta = 19.58$.



CHAPTER 6

Posterior Distribution using Mixture and double Priors

6.1 Introduction

In this chapters, the Posterior Distribution for the parameter of the Gumbel type II distribution are derived using two mixture priors (Gamma and Exponential). The posterior distribution using double prior has been also derived in this section. The Bayes estimators under square error loss function using mixture priors has been derived in this section. The Bayes estimators and Bayes posterior risk under different loss functions are calculated using double prior. Simulations using Bayes estimators under Square error loss function using mixture of Gamma and mixture of Exponential as a priors. Finally the Simulation study is performed using Bayes Estimators and Bayes Posterior risk using double prior. The graphs of the posterior distributions using mixtures of Gamma and Exponential priors under Square error loss function and double (Exponential Gamma) prior under three loss functions (Square, Quadratic and Weighted) has been also drawn in the following sections.

6.2 The Posterior Distribution using Mixture of Gamma Distribution as a Prior

The Mixture Prior of Gamma Distribution for the parameter θ (assuming $\phi=1$) is:

$$f\left(\theta_1,\theta_2;a_1,b_1,a_2,b_2\right)=p_1\frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{a_1-1}e^{-\theta_1 b_1}+(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{a_2-1}e^{-\theta_2 b_2},$$

$$\theta_1>0,\theta_2>0,a_1>0,b_1>0,a_2>0,b_2>0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p\left(\theta\middle|\mathbf{x}\right)=\frac{p_1\frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{a_1+n-1}e^{-\theta_1(b_1+\sum_{i=1}^{n}x^{-1})}+(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{a_2+n-1}e^{-\theta_2(b_2+\sum_{i=1}^{n}x^{-1})}}{\int_0^\infty p_1\frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{a_1+n-1}e^{-\theta_1(b_1+\sum_{i=1}^{n}x^{-1})}d\theta_1+\int_0^\infty(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{a_2+n-1}e^{-\theta_2(b_2+\sum_{i=1}^{n}x^{-1})}d\theta_2}$$

$$p\left(\theta|\mathbf{x}\right)=\frac{p_1\frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{\alpha_1-1}e^{-\theta_1\beta_1}+(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{\alpha_2-1}e^{-\theta_2\beta_2}}{\int_0^\infty p_1\frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{\alpha_1-1}e^{-\theta_1\beta_1}d\theta_1+\int_0^\infty(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{\alpha_2-1}e^{-\theta_2\beta_2}d\theta_2}$$

$$p\left(\theta|\mathbf{x}\right)=\frac{p_1\frac{b_1^{a_1}}{\Gamma a_1}\frac{\Gamma(\alpha_1)}{(\beta_1)^{\alpha_1}}p\left(\theta_1|\mathbf{x}\right)+(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\frac{\Gamma(\alpha_2)}{(\beta_2)^{\alpha_2}}p\left(\theta_2|\mathbf{x}\right)}{p_1\frac{b_1^{a_1}}{\Gamma a_1}\frac{\Gamma(\alpha_1)}{(\beta_1)^{\alpha_1}}+(1-p_1)\frac{b_2^{a_2}}{\Gamma a_2}\frac{\Gamma(\alpha_2)}{(\beta_2)^{\alpha_2}}}$$

$$p\left(\theta|\mathbf{x}\right)=w_1p\left(\theta_1|\mathbf{x}\right)+w_2p\left(\theta_2|\mathbf{x}\right)$$

Where $w_1=\dfrac{p_1\frac{b_1^{a_1}}{\Gamma a_1}\frac{\Gamma(\alpha_1)}{(\beta_1)^{\alpha_1}}}{p_1\frac{b_1^{a_1}}{\Gamma a_1}\frac{\Gamma(\alpha_1)}{(\beta_1)^{\alpha_1}}+1-p_1\frac{b_2^{a_2}}{\Gamma a_2}\frac{\Gamma(\alpha_2)}{(\beta_2)^{\alpha_2}}}$ and

$$w_2=\frac{1-p_1\frac{b_2^{a_2}}{\Gamma a_2}\frac{\Gamma(\alpha_2)}{(\beta_2)^{\alpha_2}}}{p_1\frac{b_1^{a_1}}{\Gamma a_1}\frac{\Gamma(\alpha_1)}{(\beta_1)^{\alpha_1}}+1-p_1\frac{b_2^{a_2}}{\Gamma a_2}\frac{\Gamma(\alpha_2)}{(\beta_2)^{\alpha_2}}}$$

which is the complete pdf of mixture of Gamma distribution with parameters $\alpha_1=a_1+n,\ \beta_1=b_1+\sum_{i=1}^{n}x^{-1},\ \alpha_2=a_2+n,\ \beta_2=b_2+\sum_{i=1}^{n}x^{-1}$. So the posterior distribution of θ given data is Gamma distribution with parameters $\left(\alpha_1,\alpha_2,\beta_1,\beta_2\right)$.

6.3 The Posterior Distribution using Mixture of Exponential Distribution as a prior

The Mixture Prior of Exponential Distribution for the parameter θ (assuming $\phi=1$) is:

$$f\left(\theta_1,\theta_2;c_1,c_2\right)=p_1c_1e^{-\theta_1c_1}+(1-p_1)c_2e^{-\theta_2c_2}\quad \theta_1>0,\theta_2>0,c_1>0,c_2>0$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p\left(\theta\middle|\mathbf{x}\right)=\frac{p_1c_1\theta_1^{n+1-1}e^{-\theta_1(c_1+\sum_{i=1}^{n_1}x^{-1})}+(1-p_1)c_2\theta_1^{n+1-1}e^{-\theta_2(c_2+\sum_{i=1}^{n_2}x^{-1})}}{\int_0^\infty p_1c_1\theta_1^{n+1-1}e^{-\theta_1(c_1+\sum_{i=1}^{n_1}x^{-1})}d\theta_1+\int_0^\infty(1-p_1)c_2\theta_1^{n+1-1}e^{-\theta_2(c_2+\sum_{i=1}^{n}x^{-1})}d\theta_2}$$

$$p\left(\theta\middle|\mathbf{x}\right)=\frac{p_1c_1\dfrac{\Gamma(\alpha_1^*)}{(\beta_1^*)^{\alpha_1^*}}\,p\left(\theta_1\middle|\mathbf{x}\right)+(1-p_1)c_2\dfrac{\Gamma(\alpha_2^*)}{(\beta_2^*)^{\alpha_2^*}}\,p\left(\theta_2\middle|\mathbf{x}\right)}{p_1c_1\dfrac{\Gamma(\alpha_1^*)}{(\beta_1^*)^{\alpha_1^*}}+(1-p_1)c_2\dfrac{\Gamma(\alpha_2^*)}{(\beta_2^*)^{\alpha_2^*}}}$$

$$p\left(\theta\middle|\mathbf{x}\right)=w_1^*\,p\left(\theta_1\middle|\mathbf{x}\right)+w_2^*\,p\left(\theta_2\middle|\mathbf{x}\right)$$

where $$w_1^*=\frac{p_1c_1\dfrac{\Gamma(\alpha_1^*)}{(\beta_1^*)^{\alpha_1^*}}}{p_1c_1\dfrac{\Gamma(\alpha_1^*)}{(\beta_1^*)^{\alpha_1^*}}+1-p_1c_2\dfrac{\Gamma(\alpha_2^*)}{(\beta_2^*)^{\alpha_2^*}}}$$ and

$$w_2^*=\frac{(1-p_1)c_2\dfrac{\Gamma(\alpha_2^*)}{(\beta_2^*)^{\alpha_2^*}}}{p_1c_1\dfrac{\Gamma(\alpha_1^*)}{(\beta_1^*)^{\alpha_1^*}}+(1-p_1)c_2\dfrac{\Gamma(\alpha_2^*)}{(\beta_2^*)^{\alpha_2^*}}}$$

which is the complete pdf of mixture of Gamma distribution with parameters $\alpha_1 = 1+n,\ \beta_1 = c_1 + \sum_{i=1}^{n} x^{-1},\ \alpha_2 = 1+n,\ \beta_2 = c_2 + \sum_{i=1}^{n} x^{-1}$. So the posterior distribution of θ given data is the mixture of Gamma distribution with parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2)$.

6.4 The Posterior Distribution using Exponential Gamma Distribution double Prior

The informative prior Exponential distribution of θ with hyperparameter 'c' is given below.

$$f(\theta;\mathrm{c}) = \mathrm{c}e^{-\theta\mathrm{c}} \quad \theta > 0, \quad c > 0$$

The informative prior Gamma distribution of θ with hyperparameters '*a*' and 'b' is given below.

$$f(\theta;a,b) = \frac{b^a}{\Gamma a}\, \theta^{a-1}\, e^{-\theta b} \quad \theta > 0,\ a > 0,\ b > 0$$

now double prior is obtained by combining the above two priors:

$$f^*(\theta) \propto f(\theta;c)\, f(\theta;a,b)$$

$$f^*(\theta) \propto \theta^{a-1} e^{-\theta(b+c)}$$

The posterior distribution of θ given data $\mathbf{x}$ is:

$$p(\theta|\mathbf{x}) \propto \theta^{a-1} e^{-\theta(b+c)} \theta^n \prod_{i=1}^{n} x^{-(\phi+1)} e^{-\theta \sum_{i=1}^{n} x^{-\phi}}$$

$$p(\theta|\mathbf{x}) \propto \theta^{a+n-1} e^{-\theta(b+c+\sum_{i=1}^{n} x^{-\phi})}$$

$$p(\theta|\mathbf{x}) \propto \theta^{\alpha-1} e^{-\theta\beta}$$

which is density kernel of Gamma distribution with the parameters $\alpha = a + n$ and $\beta = (b + c + \sum_{i=1}^{n} x_i^{-\phi})$. So the posterior distribution of θ given data is Gamma distribution with parameters (α, β).

6.4 Bayes Estimator using Square error loss function

The Bayes estimator using Square error loss function is presented in the following section.

(i) The square error loss function

The Bayes estimator is:

The square error loss function for the parameter θ is:

$$\hat{\theta} = w_1 E(\hat{\theta}_1) + w_2 E(\hat{\theta}_2)$$

The Bayes estimator using the mixture of Gamma prior is:

$$\hat{\theta} = w_1 \frac{\alpha_1}{\beta_1} + w_2 \frac{\alpha_2}{\beta_2}$$

$$\hat{\theta} = w_1 \frac{a_1 + n}{b_1 + \sum_{i=1}^{n_1} x_i^{-1}} + w_2 \frac{a_2 + n}{b_2 + \sum_{i=1}^{n_1} x_i^{-1}}$$

The Bayes estimator using the mixture of Exponential prior is:

$$\hat{\theta} = w_1^* \frac{\alpha_1^*}{\beta_1^*} + w_2^* \frac{\alpha_2^*}{\beta_2^*}$$

$$\hat{\theta} = w_1^* \frac{n+1}{c_1 + \sum_{i=1}^{n_1} x_i^{-1}} + w_2^* \frac{n+1}{c_2 + \sum_{i=1}^{n_2} x_i^{-1}}$$

Comparison of priors under Square error loss function using informative prior (mixture of Gamma and Exponential) is discussed in the following table 6.1.

Table 6.1 Bayes Estimator using mixture priors

Loss function $L(\theta, \hat{\theta})$	Prior Distributions		Posterior parameters	Bayes Estimator
	IP	HP	$(\alpha_1, \beta_1, \alpha_2, \beta_2)$	$\hat{\theta}$
L_1	GP	$a_1 = 4,\ b_1 = 2$ $a_2 = 3,\ b_2 = 2$	$(34, 4.68, 33, 4.68)$	7.22115
		$a_1 = 5,\ b_1 = 3$ $a_2 = 5,\ b_2 = 4$	(35, 5.68, 35, 6.68)	6.12569
		$a_1 = 6,\ b_1 = 3$ $a_2 = 5,\ b_2 = 4$	(36, 5.68, 35, 6.68)	6.33280
	EP	$c_1 = 2,\ c_2 = 3$	(31, 4.68, 31, 5.68)	6.05111
		$c_1 = 3,\ c_2 = 3$	(31, 5.68, 31, 5.68)	5.03142
		$c_1 = 3,\ c_2 = 4$	(31, 5.68, 31, 6.68)	5.42771

The above table describes the Bayes estimators under Square error loss function using mixture of Gamma and Exponential priors.

6.5 Simulation using Bayes Estimates under Square error loss function

The Simulation using the Bayes estimator under square error loss function using mixture of Gamma and Exponential prior is given as

Table 6.2 Simulation using Bayes Estimates using mixture of a Gamma as a Prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	5.18132	5.87596	6.5225	7.03863	7.66593
$n_2 = 100$	5.56705	6.35319	7.13468	7.92618	8.64209
$n_3 = 200$	5.7753	6.67563	7.56577	8.42069	9.26427

$n_4 = 300$	5.84283	6.7775	7.70936	8.60311	9.47775
$n_5 = 500$	5.91314	6.8579	7.816	8.75813	9.65463

For simulation the Bayes estimator under square error loss function using mixture of Gamma prior is used. The above tables shows the simulation using Bayes estimates under Square error loss function using the mixture of Gamma prior for the different values of sample sizes and parameters. After simulation it is observed that as sample size increases the value of the parameter approaches to its true value.

Table 6.3 using Bayes Estimates Using mixture of Exponential as a Prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.98607	5.61709	6.25164	6.81912	7.37688
$n_2 = 100$	5.45891	6.24745	6.26106	7.92618	8.48573
$n_3 = 200$	5.71749	6.61125	7.51923	8.32411	9.17655
$n_4 = 300$	5.81293	6.71863	7.63476	8.55296	9.46425
$n_5 = 500$	5.88683	6.83723	7.7921	8.71191	9.65834

For simulation the Bayes estimator under square error loss function using mixture of Exponential prior is used. The above tables shows the simulation using Bayes estimates under Square error loss function using the mixture of Exponential prior for the different values of sample size and parameter. After simulation it is observed that as sample size increases the value of the parameter approaches to its true value.

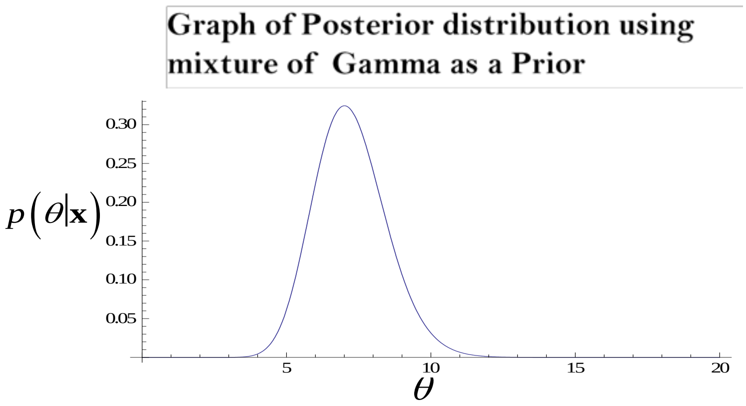
6.6 Graphs of the Posterior distribution using mixture Priors

The graphs of the posterior distribution under Square error loss functions using mixture of Gamma and Exponential as prior are presented in the following sections:

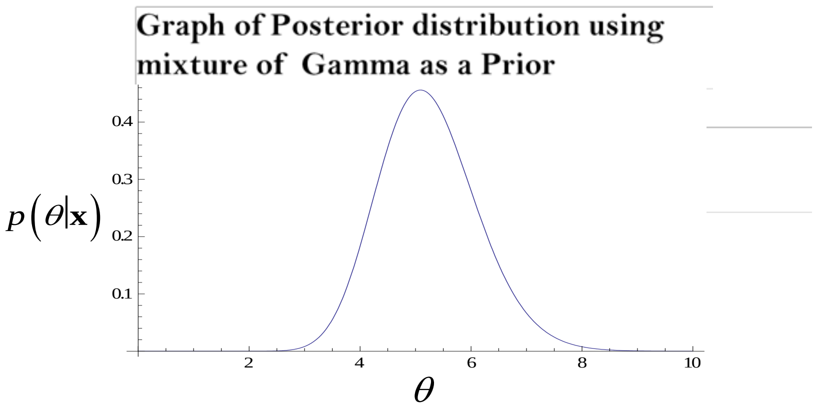
(i) Using Mixture of Gamma as a Prior

The graphs of the posterior distribution with its parameter are given below.

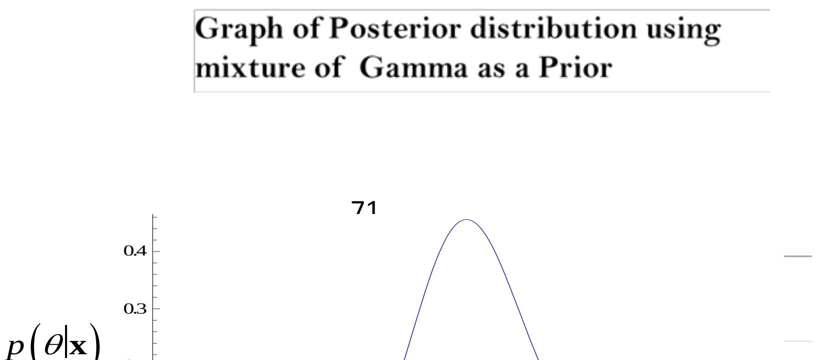
1. It is assumed that the hyperparameters $a_1 = 4,\ b_1 = 2,\ a_2 = 3,\ b_2 = 2$ than the posterior parameters will be $\alpha_1 = 34,\ \beta_1 = 4.68, \alpha_2 = 33$ and $\beta_2 = 4.68$.



2. It is assumed that the hyperparameters are $a_1 = 5,\ b_1 = 3,\ a_2 = 5,\ b_2 = 4$ than the posterior parameters will be $\alpha_1 = 35,\ \beta_1 = 5.68, \alpha_2 = 35$ and $\beta_2 = 6.68$



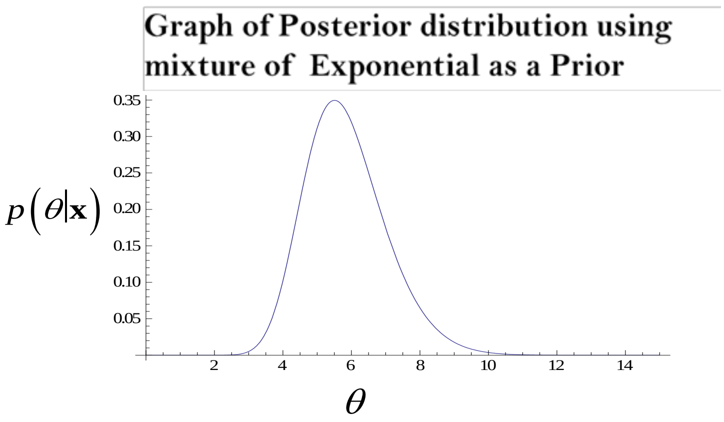
3. It is assumed that the hyperparameters are $a_1 = 6,\ b_1 = 3,\ a_2 = 5,\ b_2 = 4$ than the posterior parameters will be $\alpha_1 = 36,\ \beta_1 = 5.68, \alpha_2 = 35$ and $\beta_2 = 6.68$.



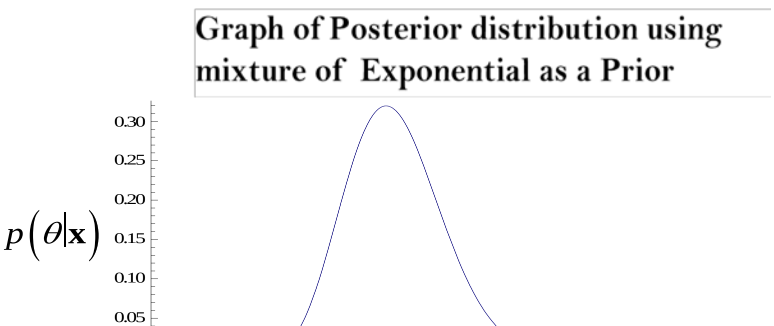
(ii) Using Mixture of Exponential as a Prior

The graphs of the posterior distribution with its parameter is given below.

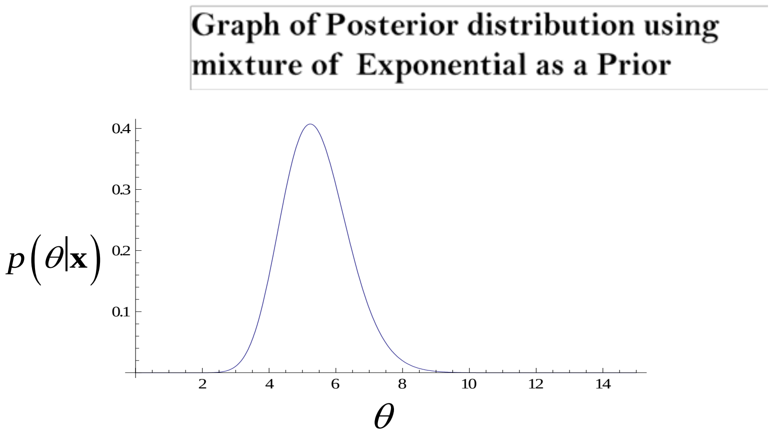
1. It is assumed that the hyperparameters $c_1 = 2,\ c_2 = 3$ than the posterior parameters will be $\alpha_1 = 31,\ \beta_1 = 4.68, \alpha_2 = 31$ and $\beta_2 = 5.68$.



2. It is assumed that the hyperparameters $c_1 = 3,\ c_2 = 3$ than the posterior parameters will be $\alpha_1 = 31,\ \beta_1 = 5.68, \alpha_2 = 31$ and $\beta_2 = 5.68$.



3. It is assumed that the hyperparameters $c_1 = 3,\ c_2 = 4$ than the posterior parameters will be $\alpha_1 = 31,\ \beta_1 = 5.68, \alpha_2 = 31$ and $\beta_2 = 6.68$.



6.7 Bayes Estimator using Exponential Gamma as double prior

(i) The square error loss function

The Bayes estimator using the Exponential Gamma as a double prior is:

$$\hat{\theta} = \frac{a+n}{(b + c + \sum_{i=1}^{n} x_i^{-1})}$$

(ii) The weighted loss function

The Bayes estimator using the Exponential Gamma as a double prior is:

$$\hat{\theta} = \frac{a+n-1}{(b + c + \sum_{i=1}^{n} x_i^{-1})}$$

(iii) The Quadratic loss function

The Bayes estimator using the Exponential Gamma as a double prior is:

$$\hat{\theta} = \frac{a+n-2}{(\mathrm{b}+c+\sum_{i=1}^{n} x_i^{-1})}$$

6.8 Bayes Posterior Risk using Exponential Gamma as double prior

(i) The Square error loss function

The Bayes Posterior risk using the Exponential Gamma as a double prior is:

$$V(\theta|x) = \frac{\alpha}{\beta^2} = \frac{a+n}{(\mathrm{b}+c+\sum_{i=1}^{n} x_i^{-1})^2}$$

(ii) The Weighted loss function

The Bayes Posterior risk using the Exponential Gamma as a double prior is:

$$\mathrm{PR} = \frac{a+n}{(\mathrm{b}+c+\sum_{i=1}^{n} x_i^{-1})} - \frac{a+n-1}{(\mathrm{b}+c+\sum_{i=1}^{n} x_i^{-1})}$$

(iii) The Quadratic loss function

$$\mathrm{PR} = 1 - \frac{a+n-2}{a+n-1}$$

Comparison of priors under different loss functions (Quadratic, Sqtuare, Weighted) using double (Exponential Gamma) is discussed in the following table 6.4.

Table 6.4: Bayes Estimators Using Double Prior under different Loss Functions

Loss functions	Double Prior Distributions	Posterior parameters	Bayes Estimator
$L(\theta, \hat{\theta})$			

			(α,β)	$\hat{\theta}$
L_1	EG	$a=5$, b=2, c=2	$(35, 6.68)$	5.23952
		$a=4$, b=3, c=2	$(34, 7.68)$	4.42708
L_2	EG	$a=5$, b=2, c=2	$(34, 6.68)$	5.08986
		$a=4$, b=3, c=2	$(33, 7.68)$	4.29687
L_3	EG	$a=5$, b=2, c=2	$(33, 6.68)$	4.90411
		$a=4$, b=3, c=2	$(32, 7.68)$	4.16666

The above table describes the Bayes estimator under different loss function (Square, Quadratic, Weighted) using double prior (Exponential Gamma) .

Comparison of priors using posterior risk under different loss functions (Quadratic, Square, Weighted) using double (Exponential Gamma) prior are shown in the following table 6.5.

Table 6.5 : Bayes Posterior risk Using double Prior under Different Loss Functions

Loss functions	Double Prior Distributions		Posterior parameters	Bayes Posterior risk
$L(\theta,\hat{\theta})$			(α,β)	$\hat{\theta}$
L_1	EG	$a=5$, b=2, c=2	$(35, 6.68)$	0.46454
		$a=4$, b=3, c=2	$(34, 7.68)$	0.57644
L_2	EG	$a=5$, b=2, c=2	$(34, 6.68)$	0.10408
		$a=4$, b=3, c=2	$(33, 7.68)$	0.130401
L_3	EG	$a=5$, b=2, c=2	$(33, 6.68)$	0.03125
		$a=4$, b=3, c=2	$(32, 7.68)$	0.03225

Table 6.5 describes that the comparison of posterior Risk using different loss function (Quadratic, Square, Weighted). It is clear that the posterior risk under Quadratic loss function is less than the other two loss functions (Square and Weighted).

6.9 Simulation using Bayes Estimates and Bayes Posterior risk under Square error loss function

The Simulation using Bayes estimator under Square error loss function using Exponential Gamma as double prior is given a

Table 6.6 Simulation using Bayes Estimates and Bayes Posterior risk using double prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.4023 (0.36210)	4.88421 (0.44904)	5.30681 (0.52936)	5.70565 (0.60527)	10.1809 (0.67958)
$n_2 = 100$	5.08316 (0.24810)	5.75511 (0.31688)	6.32708 (0.39083)	6.91358 (0.46169)	10.1215 (0.53743)
$n_3 = 200$	5.49402 (0.18468)	6.30222 (0.19512)	7.05318 (0.24438)	9.04363 (0.30020)	10.0153 (0.36022)
$n_4 = 300$	5.66336 (0.10480)	6.51427 (0.13975)	7.32863 (0.17709)	9.02991 (0.21962)	10.0612 (0.26606)
$n_5 = 500$	5.77856 (0.06657)	6.68609 (0.08902)	7.5923 (0.11433)	8.47617 (0.14235)	10.0034 (0.17415)

For simulation the Bayes estimator and Bayes Posterior risk under square error loss function using double prior is used. The above tables shows the simulation using Bayes estimates under Square error loss function using the double prior for the different values of sample size and parameter. After simulation it is observed that as sample sizes increases the value of the parameter approaches to its true value. And as the sample sizes increases the posterior risk decreases.

6.10 Simulation of the Bayes Estimates under Weighted loss function

The Simulation using Bayes estimator under Weighted loss function using Exponential Gamma as double prior is given as

Table 6.7 Simulation using Bayes Estimates and Bayes Posterior risk using double prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.4023 (0.08226)	4.88421 (0.09031)	5.30681 (0.09825)	5.70565 (0.10587)	10.1809 (0.11204)
$n_2 = 100$	5.08316 (0.04871)	5.75511 (0.05504)	6.32708 (0.060945)	6.91358 (0.06645)	10.1215 (0.07185)
$n_3 = 200$	5.49402 (0.02689)	6.30222 (0.03080)	7.05318 (0.03458)	9.04363 (0.03808)	10.0153 (0.04179)
$n_4 = 300$	5.66336 (0.01855)	6.51427 (0.02138)	7.32863 (0.02418)	9.02991 (0.02685)	10.0612 (0.02962)
$n_5 = 500$	5.77856 (0.01148)	6.68609 (0.01329)	7.5923 (0.01504)	8.47617 (0.01685)	10.0034 (0.018579)

For simulation the Bayes estimator under weighted loss function using double prior is used. The above tables shows the simulation using Bayes estimates under Square error loss function using the double prior for the different values of sample size and parameter. After simulation it is observed that as sample sizes increases the value of the parameter approaches to its true value. And the sample sizes increases the posterior risk decreases.

6.11 Simulations using Bayes Estimates and Bayes Posterior risk under Quadratic loss function

The Simulation using Bayes estimates and Bayes posterior risk under Quadratic loss function using Exponential Gamma as double prior is given as

Table 6.8 Simulation using Bayes Estimates and Bayes Posterior risk using double prior

Sample sizes	$\theta = 6$	$\theta = 7$	$\theta = 8$	$\theta = 9$	$\theta = 10$
$n_1 = 50$	4.4023 (0.98113)	4.88421 (0.98113)	5.30681 (0.98113)	5.70565 (0.98113)	10.1809 (0.98113)
$n_2 = 100$	5.08316 (0.99629)	5.75511 (0.99629)	6.32708 (0.99629)	6.91358 (0.99629)	10.1215 (0.99629)
$n_3 = 200$	5.49402 (0.99501)	6.30222 (0.99501)	7.05318 (0.99501)	9.04363 (0.99501)	10.0153 (0.99501)
$n_4 = 300$	5.66336 (0.9969)	6.51427 (0.9969)	7.32863 (0.9969)	9.02991 (0.9969)	10.0612 (0.9969)
$n_5 = 500$	5.77856 (0.99890)	6.68609 (0.99890)	7.5923 (0.99890)	8.47617 (0.99890)	10.0034 (0.99890)

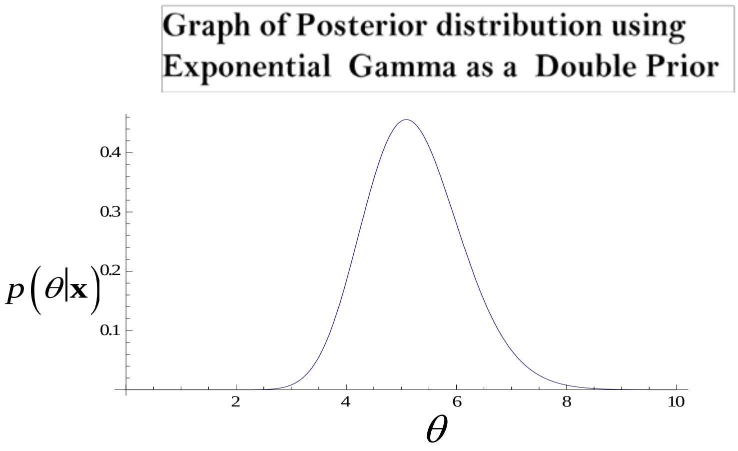
For simulation the Bayes estimator under Quadratic loss function using double prior is used. The above tables show the simulation using Bayes estimates under Quadratic loss function using the double prior for the different values of sample size and parameter. After simulation it is observed that as sample sizes increases the value of the parameter approaches to its true value.

6.12 Graphs of the Posterior distribution using Double Prior

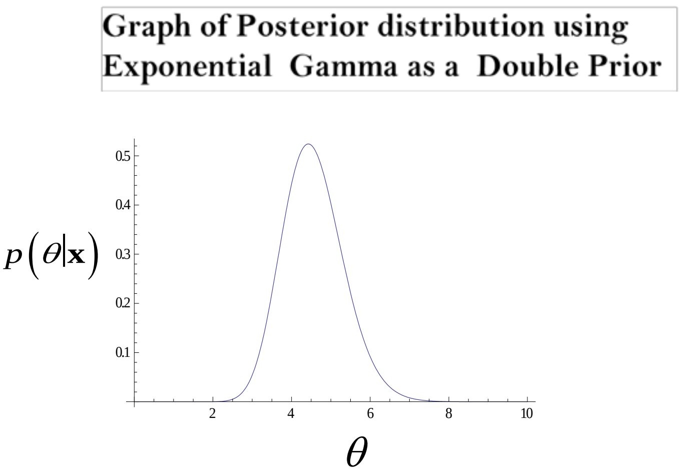
(i) Using Exponential Gamma as a Double Prior

The graphs of the posterior distribution with its parameter is given below.

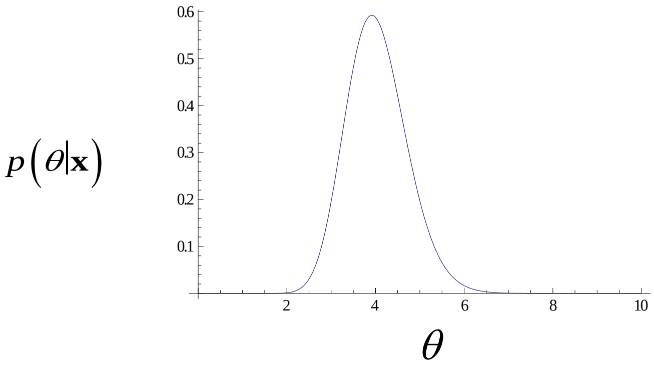
1. It is assumed that the hyperparameters $a = 5,\ b = 2,\ \mathrm{c} = 2$ than the posterior parameters will be $\alpha = 35$ and $\beta = 6.68$.



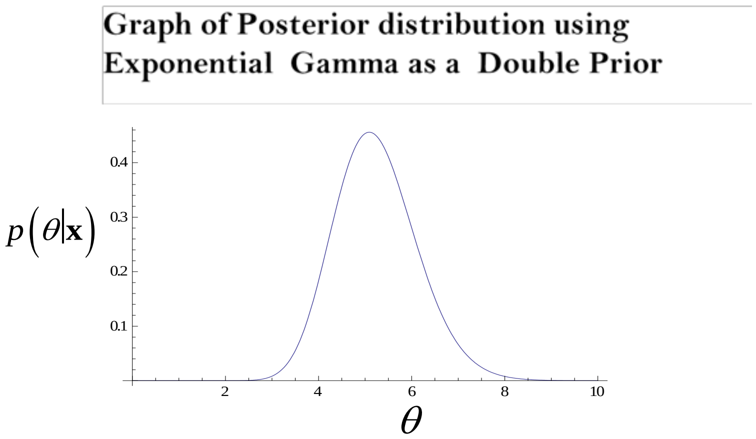
2. It is assumed that the hyperparameters $a = 5,\ b = 3,\ \mathrm{c} = 2$ than the posterior parameters will be $\alpha = 35$ and $\beta = 7.68$.



3. It is assumed that the hyperparameters $a=5,\ b=3,\ \mathrm{c}=3$ than the posterior parameters will be $\alpha=35$ and $\beta=8.68$.



4. It is assumed that the hyperparameters $a=5,\ b=4,\ \mathrm{c}=3$ than the posterior parameters will be $\alpha=35$ and $\beta=9.68$.



CHAPTER 7

Conclusion and Recommendation

7.1 Conclusion

This study provides a Bayesian analysis of unknown parameter of Gumbel type II distribution. Using the informative and non informative priors, the posterior distributions are derived. For checking which of the prior is suitable comparisons are made on the basis of Bayes estimators, Bayes Posterior risk and coefficient of skewness? The posterior distributions are derived using the same work for the noninformative priors.

As explain in detail in chapter 03 about the graphs. The graphs of informative priors are symmetrical, whereas the graphs of the noninformative priors are slightly positively skewed.

As explain in detail chapter 04 about the results. Finally we conclude that the informative prior Gamma distribution is best prior under weighted loss function and Quadratic loss function but in Square error loss function Exponential prior is best for the estimation of Gumble distribution because of minimum posterior risk. Whereas in the noninformative prior uniform prior is suitable under Square error, Quadratic and Weighted loss functions because of minimum posterior risk

Simulation study is performed using the Bayes estimator and Bayes Posterior risk under different loss functions (Quadratic, Square, Weighted), and finally conclude that as sample size increase the value of the parameter approach to its true value, posterior risk decrease as the sample size increase.

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The whole conclusion of the Bayesian analysis of the unknown parameter of the Gumble distribution is that the best informative prior is Gamma. In case of having noninformative prior Uniform is best prior. The Bayes factor for hypotheses testing is also presented for the testing of parameter, and conclude that our hypotheses conclude to H_o.

Chapter 05 describes the posterior distributions using the mixture and double prior is derived and conclude that the posterior distributions are also a mixture of Gamma distribution. Simulation study is performed using the Bayes estimates under Square error loss function using the mixture of Gamma and Exponential as a prior. Simulation study is also performed for the Bayes estimates and Bayes posterior risk under (Square, Quadratic, Weighted) using Exponential Gamma as a prior. The Graph for the posterior distribution using mixture and double priors are slightly positively skewed.

Chapter 06 describes that the posterior distributions are derived using two informative priors (Gamma and Exponential) for type I and type II censoring. The Bayes estimates and Bayes Posterior risk are calculated using type I censoring and conclude that under Square error loss function Exponential prior is best and under Quadratic and Weighted loss function Gamma prior is best as it has minimum posterior risk. Similarly the Bayes estimates and Bayes Posterior risk are calculated using type II censoring and conclude that under Square error loss function Exponential prior is best and under Quadratic and Weighted loss function Gamma prior is best as it has minimum posterior risk. The Graph for the posterior distribution using type I and type II censoring are slightly positively skewed.

7.2 Recommendations

1. The work can be further extended for the mixture of the Gumbel distributions.

2. The work can be further extended to compare the more informative prior.

3. Bayesian analysis can be done for the both unknown parameters of this distribution.

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Appendix-A

Properties of the Gamma distribution

1. The mean of the Gamma distributionis $\frac{\alpha}{\beta}$.

2. The variance of the Gamma distributionis $\frac{\alpha}{\beta^2}$.

3. The median of the Gamma distribution is G(0.5).

4. The mode of the Gamma distribution is $\frac{\alpha-1}{\beta}$.

5. The skewness of the Gamma distributionis $2\sqrt{\frac{1}{\alpha}}$.

6. The Kurtosis of the Gamma distribution is $3+\frac{6}{\alpha}$.

7. The Gamma distribution is always asymmetric distribution.

8. The Gamma distribution iscontinuous distribution define on semi indefinite range $x>0$.

9. The range of the Gammmadistributionis $0 to \infty$.

10. The m.g.f of Gamma distribution is $\left[\frac{\beta}{\beta-t}\right]^{\alpha}$.

11. The c.d.f of Gamma distribution is not in close form.

12. The s.d. of Gamma distribution is $\frac{\sqrt{\alpha}}{\beta}$.

13. The coefficient of variation of Gamma distribution is $\sqrt{\frac{1}{\alpha}}$.

Properties of the Exponential distribution

1. The mean of the Exponential distribution is $\frac{1}{\theta}$.

2. The variance of the Exponential distribution is $\frac{1}{\theta^2}$.

3. The median of the Exponential distribution is $\frac{1}{\theta}\ln 2$.

4. The mode of the Exponential distribution is 0.

5. The skewness of the Exponential distribution is 2.

6. The Kurtosis of the Exponential distribution is 9.

7. The Exponential distribution is always asymmetric distribution.

8. The Exponential distribution is continuous distribution define on semi indefinite range *x>0.*

9. The range of the Exponential distribution is $0 to \infty$.

10. The m.g.f of Exponential distribution is $\frac{\theta}{\theta - \mathrm{t}}$.

11. The c.d.f of Exponential distribution is $1 - e^{-\theta x}$.

12. The s.d. of Exponential distribution is $\frac{1}{\theta}$.

13. The coefficient of variation of Exponential distribution is1.

Programe for Simulated data

```
SIM=1000;BE=RandomReal[UniformDistribution[{0,0}],SIM];
For[i=1,i≤SIM,i++,θ=10;n=500;
u=RandomReal[UniformDistribution[{0,1}],n];
  x=-θ*(Log[(1-u)])^-1;
  total1=Total[(x)^-1];
BE[[i]]=((n-2)/total1);];
Table[BE[i],{BE[i],1,n}]
Mean[BE]
```

```
SIM=1000;BE=RandomReal[UniformDistribution[{0,0}],SIM];
For[i=1,i≤SIM,i++,θ=10;c=3;w=.0000669;n=500;
u=RandomReal[UniformDistribution[{0,1}],n];
  x=-θ*(Log[(1-u)])^-1;
  total1=Total[(x)^-1];
  BE[[i]]=(w*(n+1)/ b+total1+(1-w)*(1+n)/ d+total1);];
Table[BE[i],{BE[i],1,n}]
Mean[BE]
```