Study of linear and nonlinear ion acoustic waves in contaminated dusty plasma.

Anbreen Shafiat

Department of Physics

Quaid-i-Azam University

Islamabad, Pakistan

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Study of linear and nonlinear ion acoustic waves in contaminated dusty plasma.

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In

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By

Anbreen Shafiat

Department of Physics

Quaid-i-Azam University

Islamabad, Pakistan

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Certificate

This is certifying that Ms. Anbreen Shafiat D/O Muhammad Shafiat has carried out the theoretical work in this dissertation under my supervision in Plasma, Department of Physics, Quaid-i-Azam University, Islamabad and satisfying the dissertation requirement for the degree of Master of Philosophy in Physics.

Supervisor

Prof. Dr. Arshad Majeed Mirza
Department of Physics
Quaid-i-Azam University
Islamabad, Pakistan

Submitted through

Chairman

Prof. Dr. kashif Sabeeh
Department of Physics
Quaid-i-Azam Universit
Islamabad, Pakistan

DEDICATED

TO

MY BELOVED PARENTS,
SIBLINGS AND NIECE MUNTAHA

Acknowledgment

All the praises to Almighty ALLAH, the most merciful and the sovereign power, who made me able to accomplish this research work. I offer my humble and sincere words of thanks to his Holy Prophet Muhammad (P.B.U.H) who is forever a source of guidance and knowledge for humanity.

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ANBREEN SHAFIAT

Abstract

This theses focus mainly on the theoretical investigation of a new low frequency electrostatic wave in an unmagnetized collisionless dusty plasma, new acoustic waves originating from a balance of dust particle inertia and plasma pressure. It is shown that these waves can propagate linearly as a normal mode in dusty plasma, also discussed linearly the electromagnetic waves that may exist in a nonuniform dusty megnetoplasma by considering the mixed mode. The different limiting cases are examined for the couple drift-Alfven-Shukla-Varma modes in a non-uniform dusty magnetoplasma.

The primary objective of this study was the use of Reductive Perturbation Method (RPM) to examine and discuss the stability of the ion acoustic waves (IAWs) in unmagnetized electron-ion plasmas. The ion acoustic wave (IAWs) propagates with the phase velocity and depends upon ion mass and electron temperature. The nonlinear Schrödinger equation for low amplitude ion acoustic wave (IAWs) packet in plasmas with q-nonextensive electron distribution is obtained using the standard reductive perturbation technique and the stability of the ion acoustic wave (IAWs) is also discussed. The problem of modulational instability (MI) of ion-acoustic waves (IAWs) in a two-component plasma with Cairns--Tsallis distributed electrons is investigated, using the standard multiple scale reductive perturbation method, we derive a nonlinear Schrödinger equation NLSE and the MI of the IAWs is discussed. The nonlinear Schrödinger equation is derived and then the stability of model is explained. The stability depends on q-nonextensive parameter, wave number of ion acoustic wave (IAWs) and the velocity of cold beam. For graphical representation of models plots are made with the help of software mathematic. In this model studied frequency and group velocity of ion acoustic waves (IAWs) depends on nonextensive parameter q.

There is also examined the obliquely propagating nonexrensive dust ion acoustic solitary waves in a dusty plasma. The reductive perturbation method has been employed to derive K-dV equation which admits a solitary wave solution. In this model the amplitude and width of K-dV solitons depends on q-nonextensive parameter, phase shift, and ratio of number densties of dust and ions at equilibrium, and width is also depends on ω_{ci}.

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Chapter 1

Introduction

1.1 Plasma

Plasma is fourth state of matter, it is an ionized gas consist of charge and neutral particle. Plasma is generaly neutral [1]. It is nonconducting liquids, huge number of charged particle in vaporous form can't be allocate by standard hypothesis of gasses. Due to its closest neighbour in plasma the kinetic energy is much larger than its potential energy [2].

Plasma is a dynamic medium which exhibit a vast variety of nonlinear accurance. Researching electromagnetic wave multiplication is a noticable around the most mandatory diagonastic lab and space plasma. In high temperature plasma the collisional scattering smash are feeble, the energized waves can grow an abnormal state and depict a variety of nonlinear behaviour [2].

Charged particles in plasma, shows a firm reaction to electromagnetic fields. The reaction constantly shows an electric current or space charge and fix unique electromagnetic fields. With these lines plasma liquids have to be assign with as electromagnetic liquids which are shown by coupled arrangement of liquid conditions and Maxwell's conditions for electromagnetism.

1.1.1 Existence of plasma

In plasma charged particles tempertaure is high so their collisions wind up in thermonuclear reactions. Plasma consisting the bulk of universe and accured naturally but rare on earth, surround among different phenomena, the star crona, solar wind, nebula and region of earth.

1.1.2 Quasineutral

Quasineutrality means sufficiently neutral so plasma density is n. The confusion of quasi-neutrality can be clear up considering plasma is finite on a cylinder, the plasma seems neutral from outside of chamber, that is positive particles is equivalent to the negative particles, due to small fluctuation in control lack of bias there will be electric and attractive forces inside the cylinder [1].

1.1.3 Debye length

A basic normal for the conduct of plasma is its capacity to shield out electric potential that are connected to it. Considering to set an electric field by place two charged balls linked with a battery as shown in figure.

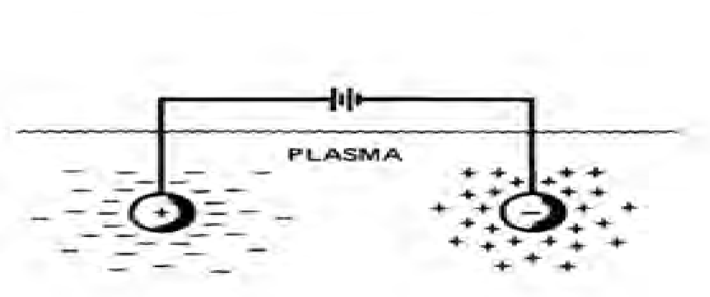

Figure 1.1: Debye Length

The ball linked with positive terminal of bettery is comformed by an electron cloud and rigidly bound with other ball. The temperature at connected potential are shield out by this capacity of plasma.The temperature is measure of thermal motions, the particle at the edge of cloud have enough energy to escape from the elctric potential due to limited temperature. Potential is equivalents to the thermal energy of particles at the radius where edge of cloud accured and the shielding is not ended. Electron debye constanst is given as

$$\lambda_{De} = \left(\frac{\varepsilon_0 k_B T_e}{n_0 e}\right)^{\frac{1}{2}}$$

similarly ion debye constant defined as

$$\lambda_{Di} = \left(\frac{\varepsilon_0 k_B T_i}{n_0 e}\right)^{\frac{1}{2}}$$

where k_B is boltzmann constant, T_e and T_i elctrons and ions temperature, e charge, n is number densties, at equilibrium $n_i = n_e = n_0$.

1.1.4 Plasma frequency

Plasma containing negative and positive ions and neutral atoms. A group of electrons shift from their mean positions, so which will move back the electron by a abandon group pf positively charged particles. Due to inertia electron will move back and exceed their fundamental position without collisions, and keep on moving back and forth. Oscillation frequency is given as

$$\omega_p = \left(\frac{e^2 n_{e0}}{m_e \varepsilon_0}\right)^{\frac{1}{2}}$$

where n is the electron number density at equilibrium e and m electron charge and mass [1].

1.2 Dusty Plasma

It is observed in our universe almost 99% of matter is in the form of plasma [3]. Dusty plasma contains electron ion plasma and with further charged component submicrons sized particles, and in general they are not neutral. The complexity of the system increased by extra component of macro particles. Thats why it is also called complex plasma. These grains differ in shapes and size in case they are artificially made.

1.2.1 Characteristics of Dusty Plasmas

Dusty plasma contains dust particle or dust grains, depending on the radius of dust grains (r_d), the debye raduis of plasma (λ_D) , the everage intergrain distance (a) and the dimensions of dusty plasma. The condition $r_d < \lambda_D < a$ here a collection of isolated screened grains are considered for charged dust grains particles while the condition for $r_d < a < \lambda_D$ in which the collective behaviour of dust charge particles accured.

Differences between electron-ion and dusty plasma

Characteristics	Electron–ion plasma	Dusty plasma
Massive particle charge	$q_i = Z_i e$	$q_d = Z_d e =\gg q_i$
Quasi-neutrality condition	$n_{e0} = Z_i n_{i0}$	$Z_d n_{d0} + n_{e0} = Zi n_{i0}$
Charge dynamics	q_i =constant	$\frac{\partial q_d}{\partial t}$ = net current
Plasma frequency	ω_{pi}	$\omega_{pd} \ll \omega_{pi}$
Debye radius	λ_{De}	$\lambda_{De} \gg \lambda_{Di}$
Particle size	uniform	dust size distribution
linear waves	IAW, LHW ,etc	DIAW, DAW, etc
Non linear waves	IA solitons/shocks	DA/DIA solitons/shocks
$E \times B_0$ particle drift	ion drift at low B_0	dust drift at high B_0
Interaction	repulsive only	attractive between grains
Crystallization	no crystallization	dust crystallization
Phase transition	no Phase transition	Phase transition

1.2.2 Dusty Plasmas in Space

In space dusty plasma found everywhere [4]. There are a large number of systems in space like circumstellar clouds, solar system, interstellar clouds, etc. Where the existence of charged dust particle has been rooted. A huge medium of dust and gass are filled the space between the stars (interstellar space). As new batch of stars are produce in the course of collapsing massive molecules clouds by decreasing constantly gas contents of interstellar medium with time. The development of stellar clusters is risen by the fragmentation of these clouds. The dust grains as a dielectric(silicates,ices etc), and metallic (madnetic, amorphous carbon, etc) are found in interstellar space or circumstellar space.

1.2.3 Interplanetary space

A large amount of dust is filled interplanetary space called as 'interplanetary dust'. The dust particles was known from zodiacal light, are found in interplanetary space. The inner solar system are distributed by throughtout the zodiacal light due to dust grains, with firm contributions from the asteriod belt [5]. For the past two decades in the startosphere NASA has commonly collected interplanetary dust by using high altitude research aircrafts. The dust particle are accumulated by inertial influence onto plastic plates at altitude of 18 to 20 km.These plastic plates are coated with highly viscous selicon oil. The size of dust particles are 5 to 20 mm on the collectors. The appearance dust particles of interoplaner space are too much flimsy and fluffy.The interplanetary dust particles are shown in figure (1.2) .

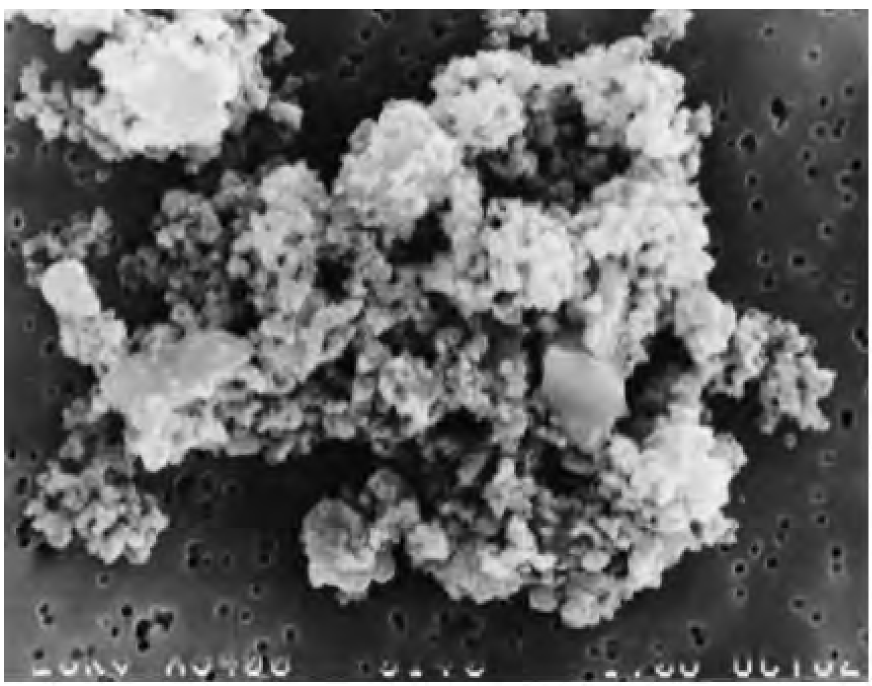

Figure 1.2: Interplanetary dusty plasma in space.

1.3 Low-frequency electrostatic waves in abounded dusty magnetoplasma

The equilibrium quasineutrality condition of an electron ion plasma can change due to presence of static charge dust grains. Hence when stationary dust charges are added the existing plasma wave spectra of an electron are modified [6]. The two normal mode of an unmagnetized dusty plasma DIA and IA waves are accured. Several laboratory dusty plasma devices has been observed dust acoustic and dust ion acoust modes [7].

In an external magnetic field large amount of dusty plasma in laboratory and space environment are confined, the properties of dusty plasma waves is a magnetoplasma is actual interest to analyse [8]. The dust ion-acoustic solitary wave in an unmagnetized collisional dusty plasma, which consists on ions having positive charge, dust fluid with negative charge, q nonextensive electrons and background neutral particles. We formulated nonlinear model by the damped modified Korteweg–de Vries (D-mKdV) equation by applying reductive perturbation technique. We also constructed the new solitary wave solutions for nonlinear D-mKdV equation with the help of two techniques.

1.4 Electron-ion plasmas

In different medium nonlinear analysis of dispersive waves describe that how solitons are create and how they propagate. The characteristics of solution of K-dV equation by using numerical techniques in the mid 60,s of 19th century zubusky and krushal studied extensively. They studied that developing of periodic sinosidal initial perturbation into many solitons. They regained their identity after the interaction of solitons with each other. In electron ion plasma the theoretically small and finite amplitude of ion acoustic wave has been described by washimi and taniuti in 1966. They take the fluid equation in an electron ion plasma and used boltzmann distribution of electrons, while ions are taken cold and dynamics. They consider in their model the lower order nonlinear term and dispersive terms to obtain ion solitary waves. In university of california at Los Angeles with the help of Double plasma device they predicted the formation and propagation of ion acoustic solitons. They observed that the solitons interact nonlinearly which are moving in same directions and those are moving in opposite direction collide and having little effect on each other [2].

1.5 Velocity distributions of electrons

The particles are consider to be in thermodynamic coordinations and maxwellian appropriate capacity in magnetohydrodynamics.This distribution function allow fluid flow and diverse temperature that is parallel and opposite in direction to the magneticfield. Since plasmas are in thermal equilibrium, more complex distribution fuction may be used. The distribution function depict the thermal motion due to speeds and postions of the individual paerticles. On everging over a large number of particles the utilization of statistical discription of plasma, the macroscopic phenomena can accured. In thermal equilibrium maxwellian plasma does not have a free vitality to discover any plasma wave insecurity. In the meantime diverse species in plasma have distinguishing temperature [9, 10, 11].

1.6 Solitons

The superposition of different sinosoidal waves having different frequencies is called arbitray pulse. In dispersive medium these linear waves travels with diffrent velocity and pulse spread out, but in non dispersive medium the velocity of each wave train is same. If nonlinear effects are noteable in a medium then latest traits may be explained. The rest part of wave move slower than the crest or point of large amplitude of wave then wave become steeper and steeper and at the end breaks down. Solitons is known as "solitary wave" for long time, the medium where dispersion and nonlinearity have important roles. It is noted that shape of solitons preserved upon interaction [2].

In a dusty plasma the dynamic of dust ion acoustic solitons is examined. Photoelectric effect raise the both a positive dust grains charge , due to intense electromagnetic radiation and in the absence of electromagnetic radiation negative dust charge exist. The nondissipative soliton can exist are described by mach number and plasma parametres. The negatively charged dust grains in dusty plasma, both compression and rarefraction solitons are propagate, while in plasma only compression solitons can exist due to positively charged dust grains. Numerically solving the hydrodynamic equations for ions and dust grains and dust grain charging studied the compression and rarefraction preturbations of solitons [12].

1.6.1 Properties and Application of Solitons

Important properties of solitons are:

1.These are travel with constant velocity without changing of its permanent shape, these are localized structures.

2.Slitons have particle like propeties because they can attract and repel each other.

3.When they cross each other their shape and velocities remains same.

Applications of solitons are:

1.Waves in plasma physics.

2.Propagation of compressional waves.

J. S. Russel in 1844 and tsunami observed water waves and pressure waves repectively.

1.7 Theoratical methods

In a plasma, it is generally less demanding to portray the movement of charged particles if starting point of the fields (electric and magnetic) was outside. The movement of charged particles in electric and magnetic fields can be depicted by explaining the condition of movement for every individual molecule. In any case, because of the movement of the charged particles, the neighborhood charge focuses are made and subsequently electric fields.

Fluid theory

The circumstance turns out to be extremely mind boggling when E and B fields in a plasma are not recommended. It is confused to take after the direction of every last molecule. The magnetohydrodynamic approach utilized as a part of fluid mechanics works for plasmas too. This methodology ignores the character of the individual molecule in a plasma and they are dealt with as a solitary fluid component. This methodology is helpful to concentrate low frequency wave wonders in exceptionally directing fluids [13].

1.7.1 Kinetic theory

In the fluid aproach maxwellian velocity distribution of every species is to be considered. In case of plasma where the fluid approximation is lacking, considering every species velocity distribution. For every species the relationship between particles, fields and velocity distribution given by dynamic hypothesis treatment. Simple form of kinetic equation is Vlasov condition [13].

1.7.2 Reductive perturbation technique

Nonlinear equations can be find by this technique, it is applicable to nonlinear waves having small amplitude e.g Korteweg-de Vries equation, Nonlinear Schrödinger equation (NLS), etc. If a physical system can be discribed by single dependent variable then nonlinear equation seems very simple.The original equation which are used to determined physical system are not simple, consisting many dependent variables.

For example when we determined plasma as fluid we need equation of fluid velocity, density, and other variable with equation of state to solve the system.Thermodynamic conditions are used to check these conditions are taken or not, while using these equations. All dependent variable are expanded in terms of a small parameter ε (ε is ratio of the amplitude of wave to equilibrium value) For example

$$n = n_0 + n_1 + n_2 + n_3.......$$

$$v = v_0 + v_1 + v_2 + v_3....$$

The boundry conditions inform us that first term will be present or not [14, 15, 16].

1.8 Low-frequency potential structures in a nonuniform dusty magnetoplasma

An unmagnetized dusty plasma asist the dust ion acoustic DIA, dust acoustic DA, and dust lattics DL [17, 18, 19]. In low temperature dusty plasma discharges, dispersion properties of waves experimetally determined [20, 21, 22, 23]. Laboratory and space plasmas are ingraft in an external magnetic field B_0. In a magnetized dusty plasma the plasma have spectra examined by many authors [24, 25, 26]. In the uniform dusty magnetoplasma the existance of plasma wave spectra reformed by presence of stationary charged dust grains.The dust grains dynamics involved by the new possibility of new eigenmods [28], however besides the modified drift waves the appearance of a low frequency with the comparison of ion gyrofrequency in a nonuniform dusty magnetooplasma.

Ion gyrofrequency

$$\Omega_i = \frac{eB_0}{m_i}$$

where B_0 is the strenght of magnetic field m_i is mass of ion and e is the magnitude of the electron charge, fluet like electrostatic mode which is now called as Shukla–Varma (SV) mode [25].

1.9 Layout of Dissertation

The work done in this dissertation are ordered as follows:

In the first chapter, introduction to plasma existance of plasma, quasineutral, debye length, plasma frequency, dusty plasma, solitons and its properties are described briefly and theoratical methods and Low-frequency potential structures in a nonuniform dusty magnetoplasma effects in dusty plasma are also discussed.

In the second chapter, we have discussed acoustic modes and derive a dispersion relation for dust acoustic modes and acoustic modes, and we have presented the linear theory of electromegtic waves in nonuniform dusty magnetoplasma to get the coupled drift Alfven Varma modes. In order to get that first generalized magnetohydrodynamic equations are derived for case of static charged dust particulates. For quasineutral plasma MHD equations consists of the ion continuity equation, the generalized momentum equation and Faraday's law. In order to get a dispersion relation we then use fourier transformed MHD equations. We also examined the dispersion relation for coupled drift-Alfven-shukla-Varma modes in nonuniform megnetoplasma in various limiting cases.

In third chapter, to study the modulational instability of ion acoustic waves in unmagnetized electron ion plasma by using standard multiple scale method. Electron taken as q-nonextensive distributed while ion are assumed to be cold. We use nonlinear method to derive group velocity is a function of wave vector that gives information ion acoustic waves propegate. At the end we derived standard nonlinear schrodinger equation and also described in unmagnetized electron ion plasma, ion acoustic waves modulational instability.

In fourth chapter we determined the basic features of obliquely propegating dust ion acoustic (DIA) solitary waves (SWs) in nonextensive magnetized dusty plasma. To derive Korteweg-de Vries equation, the reductive perturbation method has been used. In a collisionless, three component of magnitized dusty plasma containing negatively charged stationary dust in non-inertial electrons following nonextensive q-distribution and inertial ions.

Chapter 2

Acoustic modes

In uniform collisionless, unmegnitized dusty plasmas there are two types of acoustic modes with a weak Coulomb coupling between the charged dust grains.

Types of acoustic waves

Dust acoustic waves(DA)

Dust ion acoustic waves(DIA)

2.0.1 Dust acoustic waves

In collisionless dusty plasma Dust Acoustic waves pridicted . Collisionless dusty plasma contains electron, ions, and negatively charged dust grains. The thermal speed of elctron and ion much greater than dust acoustic waves phase velocity [26]. In a multicomponent collisionless dusty plasma

Boltzmann electron and ion number density is

$$n_{e1} \approx n_{eo} \frac{e\phi}{k_B Te} \tag{2.1}$$

and

$$n_{i1} \approx n_{io} \frac{e\phi}{k_B Ti} \tag{2.2}$$

Dust continuity equation

$$\frac{\partial n_{d1}}{\partial t} + n_{do}\nabla \cdot v_d = 0 \tag{2.3}$$

Dust momentum equation

$$m_d n_d \frac{\partial v_{1d}}{\partial t} = -q_{do} n_{do} \nabla \phi_1 - 3k_{BT_d} \nabla n_{d1} \tag{2.4}$$

Poisson's equation

$$\nabla^2 \phi = 4\pi \left(e n_{e1} - e n_{i1} - q_{do} n_{d1} \right) \tag{2.5}$$

using Eq.(2.1) and Eq.(2.2) in Eq. (2.5) we get

$$\nabla^2 \phi = 4\pi \frac{e^2 n_{eo} \phi}{k_B T_e} - 4\pi \frac{e^2 n_{io} \phi}{k_B T_I} - 4\pi q_{do} n_{d1}$$

As $k_D^2 = 4\pi \frac{e^2 n_{eo} \phi}{k_B T_e} - 4\pi \frac{e^2 n_{io} \phi}{k_B T_i}$

$$\nabla^2 \phi = k_D^2 - 4\pi q_{do} n_{d1} \tag{2.6}$$

Applying plane waves approximation, so that all perturbed quantities are proportional to $\exp(ik_z - i\omega t)$. (This gives us $\partial/\partial t = -i\omega$ and $\nabla = ik$).Now replacing $\partial/\partial t \to -i\omega$ and $\nabla \to ik$ in Eq. $(2.3 - 2.6)$

$$-i\omega n_{d1} + ikn_{do} v_{d1} = 0 \tag{2.7}$$

$$-i\omega m_d n_d v_{d1} = -q_{do} n_{do} ik\phi_1 - 3ik_B T k n_{d1} \tag{2.8}$$

$$-k^2 \phi = k_D^2 - 4\pi q_{do} n_{d1} \tag{2.9}$$

from Eq. (2.7) we get n_{d1}

$$n_{d1} = \frac{k_D^2 + k^2}{4\pi q_{do}} \phi \tag{2.10}$$

using value of n_{d1} from Eq. (2.10) in Eq. (2.7)

$$-i\omega\frac{k_D^2+k^2}{4\pi q_{do}}\phi+ikn_{do}v_{d1}=0$$

$$v_{d1}=-\frac{\omega}{k}\left(\frac{k_D^2+k^2}{4\pi q_{do}n_{do}}\right)\phi \tag{2.11}$$

putting value of v_{d1} and n_{d1} from Eq. (2.10) and Eq. (2.11) in Eq. (2.8)

$$-\omega n_{do}m_d\frac{\omega}{k}\left(\frac{k_D^2+k^2}{4\pi q_{do}n_{do}}\right)\phi=q_{do}n_{do}k\phi_1-3k_BTk\left(\frac{k_D^2+k^2}{4\pi q_{do}}\right)\phi$$

$$\frac{k_D^2+k^2}{4\pi q_{do}}\left(-m_d\frac{\omega^2}{k}+3k_BTk\right)=q_{do}n_{do}k$$

$$\frac{k_D^2+k^2}{k^2}\left(-\omega^2+\frac{3k_BTk^2}{m_d}\right)=\frac{4\pi q_{do}^2n_{do}}{m_d} \tag{2.12}$$

here $\omega_{pd}^2=\frac{4\pi q_{do}^2n_{do}}{m_d}$

$$\frac{k_D^2+k^2}{k^2}\left(-\omega^2+\frac{3k_BTk^2}{m_d}\right)=\omega_{pd}^2$$

$$\frac{k_D^2+k^2}{k^2}=\frac{\omega_{pd}^2}{\left(-\omega^2+\frac{3k_BTk^2}{m_d}\right)} \tag{2.13}$$

where

$\lambda_{De}=\frac{k_BT_e}{4\pi n_{eo}}$ and $\lambda_{Di}=\frac{k_BTi}{4\pi n_{ei}}$

using value of λ_{De} and λ_{Di} in Eq. (2.13)

$$\frac{k_D^2+k^2}{k^2}=\frac{\omega_{pd}^2}{\left(-\omega^2+\frac{3k_BTk^2}{m_d}\right)}$$

$$1+\frac{k_D^2}{k^2}=\frac{\omega_{pd}^2}{\left(\omega^2-\frac{3k_BTk^2}{m_d}\right)}$$

As $\lambda_D^2 = \frac{k_B T_d}{4\pi n_{do} q_{2_d}}$ and $\lambda_D^2 = \frac{1}{\omega_{pd}^2}$

$$1 + \frac{k_D^2}{k^2} = \frac{\omega_{pd}^2}{\left(\omega^2 - \frac{3k_B T k^2}{m_d}\right)}$$

$$\left(\omega^2 - \frac{3k_B T k^2}{m_d}\right) = \frac{\omega_{pd}^2 k^2}{\frac{1}{\lambda_D^2} + k^2}$$

$$\omega^2 = \frac{3k_B T k^2}{m_d} + \frac{C_D^2 k^2}{1 + k^2 \lambda_D^2} \tag{2.14}$$

$C_D = \lambda_D \omega_{pd}$

Dust acoustic speed condition $\omega \gg \frac{3k_B T k^2}{m_d}$ so Dust acoustic wave frequency [26].

$$\omega = \frac{C_D k}{\left(1 + k^2 \lambda_D^2\right)^{\frac{1}{2}}} \tag{2.15}$$

For longer wavelength $k^2 \lambda_D^2 << 1$

$$\omega = k \omega_{pd} \lambda_D^2$$

$$\omega^2 = k \omega_{pd}^2 \frac{\lambda_{De} \lambda_{Di}}{(\lambda_{De} + \lambda_{Di})^{\frac{1}{2}}}$$

by using values of debye radius λ_{De}, λ_{Di} and dust plasma frequency ω_{pd} we get.

$$\omega^2 = k^2 \left(\frac{4\pi n_{d0} z_{d0}}{m_d}\right) \left(\frac{k_B T_e}{4\pi n_{e0} e^2} \times \frac{k_B T_i}{4\pi n_{i0} e^2}\right) \left(\frac{k_B T_e}{4\pi n_{e0} e^2} + \frac{k_B T_i}{4\pi n_{i0} e^2}\right)^{-1}$$

$$\omega^2 = k^2 \left(\frac{n_{d0} z_{2d0}}{m_d}\right) \left(\frac{k_B T_e}{n_{e0} e^2} \times \frac{k_B T_i}{n_{i0} e^2}\right) \left(\frac{4\pi n_{e0} e^2}{k_B T_e}\right) \left(1 + \frac{T_i n_{e0}}{T_e n_{i0}}\right)^{-1}$$

$$\omega^2 = k^2 \left(\frac{n_{d0} z_{d0}^2}{m_d}\right) \left(\frac{k_B T_i}{n_{i0}}\right) \left(1 + \frac{T_i n_{e0}}{T_e n_{i0}}\right)^{-1}$$

$$\omega = k z_{d0} \left(\frac{n_{d0}}{n_{i0}}\right)^{\frac{1}{2}} \left(\frac{k_B T_i}{m_d}\right)^{\frac{1}{2}} \left(1 + \frac{T_i n_{e0}}{T_e n_{i0}}\right)^{-\frac{1}{2}}$$

so dust acoustic phase velocity is

$$V_p = kz_{d0}^2 \left(\frac{n_{d0}}{n_{i0}}\right)\left(\frac{k_B T_i}{m_d}\right)\left(1+\frac{T_i n_{e0}}{T_e n_{i0}}\right)^{-\frac{1}{2}} \tag{2.16}$$

2.0.2 Dust ion acoustic waves

The restoring force comes from hot electrons that is Boltzmann distributed (inertialess) and ion mass gives the inertia [27]. In an electron-ion-dust plasma due to negatively charge dust grains the DIA waves. Phase speed is smaller than speed of usual ion acoustic speed but for positively charge dust grain DIA waves phase speed is larger than usual acoustic speed, $C_i = \left(\frac{k_B Te}{m_i}\right)^{\frac{1}{2}}$.

where k_B is boltzmann contant, m_i is mass of ion and T_e is electron mass [28].

With the DIA waves the perturbed number density of electron is given by

$$n_{e1} \approx n_{eo}\frac{e\phi}{k_B Te}$$

Continuity equation

$$\frac{\partial n_{d1}}{\partial t} + n_{io}\nabla.v_{d1} = 0$$

Momentum equation

$$m_i n_{do}\frac{\partial v_{d1}}{\partial t} = -en_{do}\nabla\phi_1 - 3k_B T_d \nabla n_{d1} \tag{2.18}$$

Poisson's equation

$$\nabla^2\phi = 4\pi\left(en_{e1} - q_{do}n_{d1} - eni1\right) \tag{2.19}$$

$$\nabla^2\phi = 4\pi e^2 n_{eo}\frac{\phi}{k_B Te} - 4\pi q_{do}n_{d1} - 4\pi e^2 n_{io}\frac{\phi}{k_B Ti}$$

where

$$n_{e1} \approx n_{eo}\frac{e\phi}{k_B Te}$$

$$n_{i1} \approx n_{io}\frac{e\phi}{k_B Ti}$$

for stationary dust grains $n_{d1} \approx 0$

$$\nabla^2\phi = k_{De}^2\phi_1 - 4\pi q_{do}n_{d1} - 4\pi e^2 n_{io}\frac{\phi}{k_B Ti} \tag{2.20}$$

where $k_{D_e}^2 = 4\pi n_{eo}\frac{e^2\phi}{k_B Te}$

Applying plane waves approximation, so that all perturbed quantities are proportional to $\exp(ik_z - i\omega t)$ (This gives us $\partial/\partial t = -i\omega$ and $\nabla = ik$), Now replacing $\partial/\partial t \to -i\omega$ and $\nabla \to ik$ in Eq. $(2.17 - 2.20)$.

$$-i\omega n_{d1} + n_{io}kv_{d1} = 0 \tag{2.21}$$

$$-i\omega m_d n_{do}v_{d1} = -iken_{d0}\phi_1 - ik3k_B T_i n_{d1} \tag{2.22}$$

$$-k^2\phi_1 = k_{De}^2\phi_1 - 4\pi q_{do}n_{d1} - 4\pi e^2 n_{io}\frac{\phi}{k_B Ti} \tag{2.23}$$

Eq. (2.21) can also be expressed as

$$n_{d1} = \left(\frac{k_{De}^2 + k^2}{4\pi q_{do}} - \frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\phi_1 \tag{2.24}$$

where n_{d1} is dust number density perturbation.

using value of dust number density from Eq. (2.24) in dust continuity Eq. (2.21) we get.

1.

$$\omega\left(\frac{k_{De}^2 + k^2}{4\pi q_{do}} - \frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\phi_1 + n_{do}kv_{d1} = 0$$

$$v_{d1} = \frac{-\omega}{kn_{do}}\left(\frac{k_{De}^2 + k^2}{4\pi q_{do}} - \frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\phi_1 \tag{2.25}$$

where v_{d1} is dust fluid velocity.

using Eq. (2.24) and Eq. (2.25) in momentum ion Eq. (2.22)

$$\begin{aligned} & i\omega m_d n_{do}\frac{\omega}{kn_{do}}\left(\frac{k_{De}^2 + k^2}{4\pi q_{do}} - \frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\phi_1 \\ = \; & -ikq_{do}n_{d0}\phi_1 + ik3k_B T_i\frac{\omega}{kn_{do}}\left(\frac{k_{De}^2 + k^2}{4\pi q_{do}} - \frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\phi_1 \end{aligned}$$

$$\omega m_d n_{do}\frac{\omega}{k n_{do}}\left(\frac{k_{De}^2+k^2}{4\pi q_{do}}-\frac{e^2 n_{io}}{k_B T_i q_{do}}\right)=-k q_{do} n_{d0}+k 3 k_B T_i\frac{\omega}{k n_{do}}\left(\frac{k_{De}^2+k^2}{4\pi q_{do}}-\frac{e^2 n_{io}}{k_B T_i q_{do}}\right)$$

$$\left(\frac{k_{De}^2+k^2}{4\pi q_{do}}-\frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\left[-m_d\frac{\omega^2}{k}+3k_B T_d\right]=-k q_{do} n_{d0}$$

Multiplying both side by $4\pi q_{do}$

$$4\pi q_{do}\left(\frac{k_{De}^2+k^2}{4\pi q_{do}}-\frac{e^2 n_{io}}{k_B T_i q_{do}}\right)\left[-m_d\frac{\omega^2}{k}+3k_B T_d\right]=-4\pi k q_{do}^2 n_{d0}$$

$$\frac{1}{m_d}\left[-m_d\frac{\omega^2}{k}+3k_B T_d k\right]\left(k_{De}^2+k^2-\frac{4\pi e^2 n_{io}}{k_B T_i}\right)=\frac{-4\pi k q_{do}^2 n_{d0}}{m_d}$$

$$\left[-\frac{\omega^2}{k^2}+\frac{3k_B T_d}{m_d k}\right]\left(k_{De}^2+k^2-\frac{m_i}{k_B T_i}\omega_{pi}^2\right)=-\omega_{pd}^2 \tag{2.26}$$

Where ω_{pi} is the Langmuir frequency, n_{io} is density of ion, and m_i is mass of ion [29].

$$\omega_{pi}^2=\frac{4\pi e^2 n_{io}}{m_i},\qquad \omega_{pd}^2=\frac{4\pi q_{do}^2 n_{d0}}{m_d},\qquad V_{Ti}^2=\frac{m_i}{k_B T_i}$$

Suppose $\omega \gg kV_{Ti}, kV_{Td}$ so dust ion acoustic wave dispersion relation [30].

$$-\frac{\omega^2}{k^2}\left(k_{De}^2+k^2-\frac{k^2}{\omega^2}\omega_{pi}^2\right)=-\omega_{pd}^2$$

$$-\frac{\omega^2}{k^2}\left(k_{De}^2+k^2\right)+\omega_{pi}^2=-\omega_{pd}^2$$

$$-\frac{\omega^2}{k^2}\left(k_{De}^2+k^2\right)=-\left(\omega_{pd}^2+\omega_{pi}^2\right)$$

so dust ion acoustic(DIA) wave dispersion relation

$$1+\frac{k_{De}^2}{k^2}-\frac{\omega_{pd}^2+\omega_{pi}^2\left(\omega_{pd}^2+\omega_{pi}^2\right)}{\omega^2}=0 \tag{2.27}$$

Dust plasma frequency is much smaller than ion plasma frequecncy due to smaller mass of ion as compared to dust grains mass. As $m_i \ll m_d$ so $\omega_{pd} \ll \omega_{pi}$, debye wavelenghth for dust grains is $k_{De}^2 = \frac{1}{\lambda_{De}^2}$

$$\omega^2 = \frac{\left(\omega_{pd}^2 + \omega_{pi}^2\right)\lambda_{De}^2 k^2}{1 + \lambda_{De}^2 k^2} \tag{2.28}$$

let

$$C_S = \omega_{pi}\lambda_{De} = \left(\frac{n_{io}}{n_{eo}}\right)^{\frac{1}{2}} c_s$$

$$c_s = \left(\frac{k_B T_i}{m_i}\right)^{\frac{1}{2}}$$

Eq. (2.28) gives

$$\omega^2 = \frac{k^2 C_S^2}{1 + \lambda_{De}^2 k^2}$$

Condition

for longer wavelength limit $k^2\lambda_{De}^2 \ll 1$

applying limit $k^2\lambda_{De}^2 \ll 1$ in Eq. (2.72) we get

$$\omega^2 = k^2 C_S^2$$

$$\omega^2 = k^2 \left[\left(\frac{n_{io}}{n_{eo}}\right)^{\frac{1}{2}} c_s\right]^2$$

$$\omega = k\left(\frac{n_{io}}{n_{eo}}\right)^{\frac{1}{2}} c_s$$

DIA wave phase velocity $V_P = \frac{\omega}{k}$

$$V_P = \left(\frac{n_{io}}{n_{eo}}\right)^{\frac{1}{2}} c_s \tag{2.30}$$

For negatively charged dust grains $n_{io} \gg n_{eo}$ Eq. (2.30) expressed that c_s is smaller than dust acoustic waves phase velocity. Debye radius of electron is larger when in the back ground plsama the electron density depletion allocated by increase in the phase velocity. Thus raise the phase velocity of DIA waves due to appearance of stronger space charge electric field.

2.1 Waves in non-uniform megnetoplasma

In an elctron -ion plasma due to an external magnetic field notably modifies the dispersion properties of electromagnetic waves. Drift motions and associated waves in magnetized dusty plasma caused by some region of inhomogeneity in dusty plasma.

Considering nonuniform dusty megnetoplasma consisting static dust grains and unperturbed number densties of plasma $n_{s0}(x)$, suppose that is inhomogenous along x-axis (equilibrium density gradient ∂n_{s0}
and study the dispersion properties of longer wavelength (comparing with the ion gyroradius) and low frequency (comparing with ω_{ci}) electrostatic and electromagnetic waves.

In an electron ion plasma when neutral dust grain are added, dust grains are charged that can modify the wave propegation [31].

At equilibrium the quasi-neutrality condition is

$$en_{i0} - en_{e0} + q_{d0}n_{d0} = 0$$

the electric field of low frequency waves

$$\mathbf{E}_{\perp} = -\nabla_{\perp}\phi$$

The electron and ion fluid velocity prependicular componenets are [32].

$$v_{e\perp} \approx \frac{c}{B_0}\hat{z} \times \nabla_{\perp}\phi - \frac{ck_BT_e}{eB_0n_{e0}}\hat{z} \times \nabla_{\perp}n_{e1} \tag{2.31}$$

$$v_{i\perp} \approx \frac{c}{B_0}\hat{z} \times \nabla_{\perp}\phi - \frac{ck_BT_i}{eB_0n_{i0}}\hat{z} \times \nabla n_{i1} - \frac{c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + ui^{*}\cdot\nabla\right)\nabla_{\perp}\phi \tag{2.32}$$

2.2 Electromagnetic waves

To study electromagnetic waves in a non-uniform dusty magnetoplasma. Consider different types of mixed modes (mixture ofelectrostatic and electromagnetic waves) and a purely electromagnetic mode, namely a non-ducted dust.

2.2.1 Mixed mode (static dust)

Low β

$$\beta = \frac{8\pi n_0 k_B T}{B^2} \ll 1 \tag{2.32}$$

Ampere's law

$$\nabla \times B = \frac{4\pi e}{c}\left(n_i v_i - n_e v_e - Z_{do} n d v_d\right) = \left(\frac{4\pi}{c}\right) J \tag{2.33}$$

Electron continuity equation

$$\frac{\partial n_{e1}}{\partial t} + \nabla.\left(n_e v_e\right) = 0 \tag{2.34}$$

where $n_e = n_{e0} + n_{e1}$

Ion continuity equation

$$\frac{\partial n_{i1}}{\partial t} + \nabla.\left(n_i vi\right) = 0 \tag{2.35}$$

Poisson's equation, for stationary dust grains $(n_d \to 0)$

$$\nabla^2 \phi = 4\pi\left(n_{e1} - n_{i1}\right) \tag{2.36}$$

$$B = \nabla A_Z \times \hat{z}$$

parallel component of vetor potential is A_Z

using value of B in Eq. (2.33) , neglecting parallel component of ion and dust current densitis $n_i, n_{d.}$

$$\nabla \times (\nabla A_Z \times \hat{z}) = \frac{4\pi e}{c}\left(n_e v_{eZ}\right)$$

$$v_{eZ} = \frac{c}{4\pi e n_{eo}}\left(\nabla_\perp \times (\nabla_\perp A_Z \times \hat{z}\ )\right)$$

$$v_{eZ} = \frac{c}{4\pi e n_{eo}}\left[(\nabla_\perp A_Z.\hat{z}\ )\nabla_\perp - (\nabla_\perp.\hat{z}\ )\nabla_\perp A_Z\right]$$

since $(\nabla_\perp.\hat{z}\ )\,\nabla_\perp A_Z = 0$

now

$$v_{eZ} = \frac{c}{4\pi e n_{eo}}\left[\left(\nabla_\perp^2 A_Z\ \right)\right] \tag{2.37}$$

where v_{eZ} is parallele component of electron fluid velocity.

Parallel component of electron continuity equation using Eq. (2.33) we get

$$\frac{\partial n_{e1}}{\partial t} + \nabla_\perp.\left(n_e v_{\perp e}\right) + n_e\frac{\partial}{\partial z}v_{ez}\ = 0$$

here $n_e = n_{e0} + n_{e1}$ and $n_{e1} \ll n_{e0}$

$$\frac{\partial n_{e1}}{\partial t} + \nabla_\perp.\left(n_{e0} v_{\perp e}\right) + n_{e0}\frac{\partial}{\partial z}v_{ez}\ = 0 \tag{2.38}$$

using value $v_{\perp e}$ and v_{ez} from Eq. (2.31) and Eq. (2.37) in Eq. (2.38)

$$\frac{\partial n_{e1}}{\partial t} + \nabla_\perp.\left(n_{e0}\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi - \frac{n_{eo}ck_BT_e}{eB_0n_{e0}}\hat{z}\times\nabla_\perp n_{e1}\right) + n_{e0}\frac{c}{4\pi e n_{eo}}\frac{\partial}{\partial z}\left(\nabla_\perp^2 A_Z\ \right)\ = 0$$

assuming $T_e \to 0$

$$\frac{\partial n_{e1}}{\partial t} + \nabla_\perp\cdot\left(n_{e0}\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi\right) + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z}\ = 0$$

$$\frac{\partial n_{e1}}{\partial t} + \left[n_{e0}\nabla_\perp\cdot\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi - \frac{c}{B_0}\hat{z}\times\nabla_\perp\phi\cdot\nabla_\perp n_{eo}\right] + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z}\ = 0$$

as $\quad \nabla\cdot\hat{z}\times\nabla = \begin{bmatrix}\partial x & \partial y & \partial z\\ 0 & 0 & 1\\ \partial x & \partial y & \partial z\end{bmatrix} = 0$

$$\frac{\partial n_{e1}}{\partial t} + \left[-\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi\cdot\nabla_\perp n_{eo}\right] + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z}\ = 0 \tag{2.39}$$

Prependicular components of Ion continuity equation.

$$\frac{\partial n_{i1}}{\partial t} + \nabla_{\perp} \cdot (n_{i0} \cdot v_{\perp i}) + n_{ei} \frac{\partial v_{\perp i}}{\partial z} = 0 \tag{2.40}$$

using value of $v_{\perp i}$ in Eq. (2.40) .

$$\frac{\partial n_{i1}}{\partial t} + \nabla_{\perp} \cdot \left(n_{i0} \cdot \left(\frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi - \frac{ck_B T_i}{eB_0 n_{i0}} \hat{z} \times \nabla n_{i1} - \frac{c}{B_0 \omega_{ci}} \left(\frac{\partial}{\partial t} + ui^* \cdot \nabla \right) \nabla_{\perp} \phi \right) \right) + n_{ei} \frac{\partial v_{\perp i}}{\partial z} = 0$$

$$\left[\begin{array}{c} \frac{\partial n_{i1}}{\partial t} + \nabla_{\perp} \cdot \left(n_{i0} \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi \right) - \nabla_{\perp} \cdot \left(n_{io} \frac{ck_B T_i}{eB_0 n_{i0}} \hat{z} \times \nabla n_{i1} \right) - \nabla_{\perp} \cdot \left(n_{eo} \frac{c}{B_0 \omega_{ci}} \frac{\partial}{\partial t} \nabla_{\perp} \phi \right) \\ + \nabla_{\perp} \cdot \frac{n_{eo} c}{B_0 \omega_{ci}} \left(ui^* \cdot \nabla \right) \nabla_{\perp} \phi + n_{ei} \frac{\partial v_{\perp i}}{\partial z} \end{array} \right] = 0 \tag{2.41}$$

consider second term from Eq. (2.41)

$$\nabla_{\perp} \cdot \left(n_{i0} \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi \right) = \frac{c}{B_0} \hat{z} \times (\nabla_{\perp} \phi \cdot \nabla_{\perp} n_{io}) + \left(n_{io} \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi \right) \cdot \nabla_{\perp}$$

$$\nabla_{\perp} \cdot \left(n_{i0} \frac{c}{B_0} \hat{z} \times \nabla_{\perp} \phi \right) = \frac{c}{B_0} \hat{z} \times (\nabla_{\perp} \phi \cdot \nabla_{\perp} n_{io})$$

consider 4th and 5th term from Eq. (2.41)

$$\nabla_{\perp} \cdot \left(n_{eo} \frac{c}{B_0 \omega_{ci}} \frac{\partial}{\partial t} \nabla_{\perp} \phi \right) = \frac{n_{io} c}{B_0 \omega_{ci}} \left(\frac{\partial}{\partial t} \right) \nabla_{\perp}^2 \phi$$

$$\nabla_{\perp} \cdot \frac{n_{io} c}{B_0 \omega_{ci}} \left(ui^* \cdot \nabla \right) \nabla_{\perp} \phi = \frac{n_{io} c}{B_0 \omega_{ci}} \left(ui^* \cdot \nabla \right) \nabla_{\perp}^2 \phi$$

using value of theses terms in Eq. (2.41), assume $T_i \to 0$ we get

$$\frac{\partial n_{i1}}{\partial t} = -\frac{c}{B_0} \hat{z} \times (\nabla_{\perp} \phi \cdot \nabla_{\perp} n_{io}) + \frac{n_{io} c}{B_0 \omega_{ci}} \left(\frac{\partial}{\partial t} \right) \nabla_{\perp}^2 \phi + \frac{n_{io} c}{B_0 \omega_{ci}} \left(ui^* \cdot \nabla \right) \nabla_{\perp}^2 \phi = 0 \tag{2.42}$$

subtractiing Eq. (2.39) and Eq. (2.42)

$$\left[\begin{array}{c} \frac{\partial}{\partial t} \left(n_{e1-} n_{i1} \right) - \frac{c}{B_0} \hat{z} \times \left(\nabla_{\perp} n_{eo} - \nabla_{\perp} n_{io} \right) \cdot \nabla_{\perp} \phi + \frac{c}{4\pi e} \frac{\partial \nabla_{\perp}^2 A_Z}{\partial z} + \frac{n_{io} c}{B_0 \omega_{ci}} \frac{\partial}{\partial t} \nabla_{\perp}^2 \phi \\ + \frac{n_{io} c}{B_0 \omega_{ci}} \left(ui^* \cdot \nabla \right) \nabla_{\perp}^2 \phi \end{array} \right] = 0 \tag{2.43}$$

Poisson's equation

$$\nabla^2\phi = 4\pi e\left(n_{e1} - n_{i1}\right)$$

using poisson's equation in Eq. (2.43)

$$\left[\begin{array}{c}\frac{\partial}{\partial t}\left(\frac{1}{4\pi e}\nabla_\perp^2\phi\right) - \frac{c}{B_0}\hat{z}\times\left(\nabla_\perp n_{eo} - \nabla_\perp n_{io}\right)\cdot\nabla_\perp\phi + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z} + \frac{n_{io}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi \\ +\frac{n_{io}c}{B_0\omega_{ci}}\left(ui^*\cdot\nabla\right)\nabla_\perp^2\phi\end{array}\right] = 0$$

$$q_d n_{do} = n_{io} - n_{eo}$$

now we get

$$\left[\begin{array}{c}\frac{\partial}{\partial t}\left(\frac{1}{4\pi e}\nabla_\perp^2\phi\right) - \frac{c}{B_0}\hat{z}\times\nabla_\perp\left(q_d n_{do}\right)\cdot\nabla_\perp\phi + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z} - \frac{n_{io}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi \\ +\frac{n_{io}}{B_0\omega_{ci}}\left(u_{i0}\hat{y}\cdot\nabla\right)\nabla_\perp^2\phi\end{array}\right] = 0$$

$$\left[\begin{array}{c}\frac{\partial}{\partial t}\left(\frac{1}{4\pi e}\nabla_\perp^2\phi\right) - \frac{c}{B_0}\hat{z}\times\nabla_\perp\left(q_d n_{do}\right)\cdot\nabla_\perp\phi + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z} + \frac{n_{io}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi \\ +\frac{n_{io}}{B_0\omega_{ci}}u_{i0}\frac{\partial}{\partial y}\nabla_\perp^2\phi\end{array}\right] = 0$$

$$\left[\begin{array}{c}\frac{\partial}{\partial t}\left(\frac{1}{4\pi e}\nabla_\perp^2\phi\right) - \frac{c}{B_0}\nabla_\perp\left(\frac{q_d n_{do}}{e}\right)\cdot\hat{z}\times\nabla_\perp\phi + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z} \\ +\frac{n_{io}c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + u_{i0}\frac{\partial}{\partial y}\right)\nabla_\perp^2\phi\end{array}\right] = 0 \tag{2.44}$$

as

$$\frac{c}{B_0}\nabla_\perp\left(\frac{q_d n_{do}}{e}\right)\cdot\hat{z}\times\nabla_\perp\phi = \frac{c}{eB_0}\left(q_d n_{do}\right)\frac{\partial}{\partial x}\ln\left(q_d n_{do}\right)\cdot\hat{z}\times\nabla_\perp\phi$$

so now Eq. (2.44) is

$$\left[\begin{array}{c}\frac{\partial}{\partial t}\left(\frac{1}{4\pi e}\nabla_\perp^2\phi\right) + \frac{c}{B_0}\left(q_d n_{do}\right)\frac{\partial}{\partial x}\ln\left(q_d n_{do}\right)\cdot\hat{z}\times\nabla_\perp\phi + \frac{c}{4\pi e}\frac{\partial\nabla_\perp^2 A_Z}{\partial z} \\ +\frac{n_{io}c}{B_0\omega_{ci}}\left(\frac{\partial}{\partial t} + u_{i0}\frac{\partial}{\partial y}\right)\nabla_\perp^2\phi\end{array}\right] = 0$$

multiplying both side by $\frac{B_0\omega_{ci}}{n_{io}c}$, we get

$$\left(\frac{\partial}{\partial t}+u_{i0}\frac{\partial}{\partial y}\right)\nabla_{\perp}^{2}\phi+\frac{\omega_{ci}q_{d0}n_{d0}}{en_{io}}\frac{\partial}{\partial x}\ln\left(q_{d}n_{do}\right)\cdot\hat{z}\times\nabla_{\perp}\phi$$
$$+\frac{B_0\omega_{ci}}{4\pi en_{io}}\frac{\partial\nabla_{\perp}^{2}A_Z}{\partial z}+\frac{B_0\omega_{ci}}{4\pi en_{io}c}\frac{\partial}{\partial t}\nabla_{\perp}^{2}\phi = 0 \tag{2.45}$$

as

$$\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0\\ 0 & 0 & 1\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0\end{bmatrix}=\frac{\partial}{\partial x}\frac{\partial}{\partial y}\text{ so}$$

$$\frac{\omega_{ci}}{en_{io}}q_{d0}n_{d0}\frac{\partial}{\partial x}\ln\left(q_{d}n_{do}\right)\cdot\hat{z}\times\nabla_{\perp}\phi=\frac{\omega_{ci}}{n_{io}}q_{d0}n_{d0}\frac{\partial}{\partial x}\ln\left(q_{d}n_{do}\right)\frac{\partial\phi}{\partial x}$$

Now from Eq. (2.45) we get

$$\left(\frac{\partial}{\partial t}+u_{i0}\frac{\partial}{\partial y}\right)\nabla_{\perp}^{2}\phi+\frac{\omega_{ci}}{en_{io}}q_{d0}n_{d0}\frac{\partial}{\partial x}\ln\left(q_{d0}n_{d0}\right)\frac{\partial\phi}{\partial y}+\frac{B_0\omega_{ci}}{4\pi en_{io}}\frac{\partial\nabla_{\perp}^{2}A_Z}{\partial z}+\frac{B_0\omega_{ci}}{4\pi en_{io}c}\frac{\partial}{\partial t}\nabla_{\perp}^{2}\phi=0 \tag{2.46}$$

sssuming

$$\delta_d=\frac{q_{d0}n_{d0}}{en_{io}}\quad,\quad k_d=\frac{\partial}{\partial x}\ln\left(q_{d0}n_{d0}\right)\quad,\quad\frac{V_A^2}{c}=\frac{B_0\omega_{ci}}{4\pi en_{io}}$$

$$\left(\frac{\partial}{\partial t}+u_{i0}\frac{\partial}{\partial y}\right)\nabla_{\perp}^{2}\phi+\omega_{ci}\delta_d k_d\frac{\partial\phi}{\partial y}+\frac{V_A^2}{c}\frac{\partial\nabla_{\perp}^{2}A_Z}{\partial z}+\frac{V_A^2}{c}\frac{\partial}{\partial t}\nabla_{\perp}^{2}\phi=0 \tag{2.47}$$

Electron momentum equation

$$mn_e\frac{d\nu_e}{dt}=-en_{eo}E-\nabla\rho \tag{2.48}$$

Electron momentum equation (parallel component)

$$mn_{e0}\left[\frac{\partial\nu_{\perp e}}{\partial t}+\frac{\partial v_{ez}}{\partial t}\right]=-en_{eo}E_z-k_BT_e\nabla_{\perp}n_{e1}-k_BT_e\nabla_z n_{e1}-e\left(\nu_{\perp}\times B\right)-e\left(v_{ez}\times B\right)$$

$$mn_{e0}\left[\frac{\partial \nu_{\perp e}}{\partial t}+\frac{\partial v_{ez}}{\partial t}\right]=en_{eo}E_z-k_BT_e\frac{\partial n_{e1}}{\partial z}-e\left(v_{ez}\times B\right)$$

$$\left[\frac{\partial \nu_{\perp e}}{\partial t}+\frac{\partial v_{ez}}{\partial t}\right]=\frac{en_{eo}}{mn_{e0}}E_z-\frac{k_BT_e}{n_{e0}}\frac{\partial n_{e1}}{\partial z}-e\left(v_{ez}\times B\right)$$

where

$$E_z=-\frac{\partial \phi}{\partial z},\, B=B_0\hat{z}$$

so now we get

$$\frac{\partial v_{e\perp}}{\partial t}+\frac{\partial v_{ez}}{\partial t}=\frac{e}{m}\frac{\partial \phi}{\partial z}-\frac{k_BT_e}{n_{e0}}\frac{\partial n_{e1}}{\partial z} \tag{2.49}$$

using Eq. (2.32) and Eq. (2.37) in Eq. (2.49)

$$\left[\frac{\partial}{\partial t}\left(\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi-\frac{ck_BT_e}{eB_0n_{e0}}\hat{z}\times\nabla_\perp n_{e1}\right)+\frac{\partial}{\partial t}\left(\frac{c}{4\pi en_{eo}}\nabla_\perp^2A_Z\right)\right]=\frac{e}{m}\frac{\partial \phi}{\partial z}-\frac{k_BT_e}{n_{e0}}\frac{\partial n_{e1}}{\partial z} \tag{2.50}$$

considering equations

$$\frac{\partial n_{e1}}{\partial t}+\left[-\frac{c}{B_0}\hat{z}\times\nabla_\perp\phi\cdot\nabla_\perp n_{eo}\right]+\frac{c}{4\pi e}\frac{\partial \nabla_\perp^2 A_Z}{\partial z}=0 \tag{2.51}$$

$$\left(\frac{\partial}{\partial t}+v_{e0}\frac{\partial}{\partial y}\right)A_z-\lambda_e^2\frac{\partial}{\partial t}\nabla_\perp^2A_z+c\frac{\partial \phi}{\partial z}-\frac{ck_BT_e}{e}\frac{n_{e1}}{n_{e0}}=0 \tag{2.52}$$

$$\left(\frac{\partial}{\partial t}+u_{i0}\frac{\partial}{\partial y}\right)\nabla_\perp^2\phi+\omega_{ci}\delta_dk_d\frac{\partial \phi}{\partial y}+\frac{V_A^2}{c}\frac{\partial \nabla_\perp^2A_Z}{\partial z}+\frac{V_A^2}{c}\frac{\partial}{\partial t}\nabla_\perp^2\phi=0 \tag{2.53}$$

linearizing above three equations as $\frac{\partial}{\partial t}\rightarrow -i\omega$ and $\nabla\rightarrow ik$

now

$$-i\omega n_{e1}-\left[-\frac{c}{B_0}\hat{z}\times\frac{\partial}{\partial x}n_{eo}\cdot ik_y\phi\right]-\frac{c}{4\pi e}ik_zk_\perp^2A_z=0$$

$$n_{e1}=\frac{c}{\omega B_0}\frac{\partial}{\partial x}n_{eo}k_y\phi-\frac{c}{4\pi\omega e}ik_zk_\perp^2A_z=0 \tag{2.54}$$

and

$$-i\omega A_z+v_{e0}ik_yA_z+\lambda_e^2i\omega\left(ik_\perp\right)^2A_z+cik_z\phi-\frac{ck_BT_e}{e}ik_z\frac{n_{e1}}{n_{e0}}=0$$

as we know that

$$\lambda_e^2 = \frac{k_B T_e}{4\pi n_{e0} e^2}, \qquad \omega_e^* = v_{e0} k_y$$

so

$$-\omega A_z + \omega_e^* A_z - \lambda_e^2 \omega k_\perp^2 A_z + c k_z \phi - \lambda_e^2 e 4\pi k_z n_{e1} = 0 \tag{2.55}$$

$$-i\omega i^2 k_\perp^2 \phi + u_{io} i k_y i^2 k_\perp^2 \phi + \omega_{ci} \frac{q_{d0} n_{d0}}{e n_{io}} \frac{\partial}{\partial x} \ln\left(q_{d0} n_{d0}\right) i k_y + \frac{V_A^2}{c} \left(-i\omega\right) i^2 k_\perp^2 \phi + \frac{V_A^2}{c^2} \left(i k_z\right) i^2 k_\perp^2 A_z = 0$$

$$\omega k_\perp^2 \phi + \omega_i^* k_\perp^2 \phi + \frac{\omega_{ci}}{e n_{io}} \frac{\partial}{\partial x} \left(q_{d0} n_{d0}\right) k_y + \frac{V_A^2}{c^2} \left(-i\omega\right) i^2 k_\perp^2 \phi + \frac{V_A^2}{c^2} \left(i k_z\right) i^2 k_\perp^2 A_z = 0$$

$$\omega k_\perp^2 \phi + \omega_i^* k_\perp^2 \phi + \frac{\omega_{ci}}{e n_{io}} \frac{\partial}{\partial x} \left(q_{d0} n_{d0}\right) k_y + \frac{V_A^2}{c^2} \omega k_\perp^2 \phi + \frac{V_A^2}{c^2} k_z k_\perp^2 A_z = 0 \tag{2.56}$$

where

$$\omega_{sv} = -\frac{4\pi e \omega_{ci} k_y \frac{\partial}{\partial x} \left(q_{d0} n_{d0}\right)}{B_0 k_\perp^2 \omega_{pi}^2}$$

$$\omega_{pi}^2 = -\frac{n_{i0e^2}}{\varepsilon_0 m_i}$$

$$\omega_{ci}^2 = \frac{e^2 B_0^2}{c^2 m_i^2}$$

now ratio of $\frac{\omega_{ci}}{\omega_{pi}^2}$,we get

$$n_{i0} e = \frac{B_0 \omega_{pi}^2}{4\pi c \omega_{ci}}$$

using value of $n_{i0} e$ in Eq. (2.56)

$$\omega k_\perp^2 \phi + \omega_i^* k_\perp^2 \phi + \frac{\omega_{ci}^2 4\pi c}{\omega_{pi}^2 B_0} k_y \frac{\partial}{\partial x} \left(q_{d0} n_{d0}\right) \phi + \frac{V_A^2}{c^2} \omega k_\perp^2 \phi + \frac{V_A^2}{c^2} k_z k_\perp^2 A_z = 0$$

$$\omega k_\perp^2\phi + \omega_i^* k_\perp^2\phi \ + \frac{\omega_{ci}^2 4\pi c k_\perp^2}{\omega_{pi}^2 B_0 k_\perp^2} k_y \frac{\partial}{\partial x}\left(q_{d0} n_{d0}\right)\phi + \ \frac{V_A^2}{c^2}\omega k_\perp^2\phi + \frac{V_A^2}{c^2} k_z k_\perp^2 A_z = 0 \tag{2.57}$$

using value of ω_{sv} in above Eq. (2.57)

$$\omega k_\perp^2\phi + \omega_i^* k_\perp^2\phi \ + \omega_{sv} k_\perp^2\phi + \ \frac{V_A^2}{c^2}\omega k_\perp^2\phi + \frac{V_A^2}{c^2} k_z k_\perp^2 A_z = 0$$

so now we get

$$A_z = \frac{c\omega}{V_A^2 k_z}\phi - \frac{c\omega_i^*}{V_A^2 k_z}\phi + \frac{\omega k^2}{c k_\perp^2 k_z}\phi + \frac{c\omega_{sv}}{V_A^2 k_z}\phi \tag{2.58}$$

using Eq. (2.58) in Eq. (2.54)

$$n_{e1} = \frac{c}{\omega B_0}\frac{\partial}{\partial x} n_{eo} k_y \phi - \frac{c}{4\pi\omega e} i k_z k_\perp^2 \left(\frac{c\omega}{V_A^2 k_z}\phi - \frac{c\omega_i^*}{V_A^2 k_z}\phi + \frac{\omega k_\perp^2}{c k_\perp^2 k_z}\phi + \frac{c\omega_{sv}}{V_A^2 k_z}\phi\right) = 0$$

as

$$-\frac{c k_B T_e}{e B_0 n_{e0}} k_y \frac{\partial}{\partial x}\left(n_{e0}\right) = \omega_e^* \quad , \quad \frac{ck}{eB_0} k_y \frac{\partial}{\partial x}\left(n_{e0}\right) = -\frac{n_{e0} e}{k_B T_e}\omega_e^*$$

so

$$n_{e1} = \frac{e n_{e0}}{\omega k_B T_e}\omega_e^*\phi - \frac{c}{4\pi e V_A^2} k_\perp^2 c\phi + \frac{c}{4\pi\omega e} k_\perp^2 \frac{c\omega_i^*}{V_A^2}\phi + \frac{c}{4\pi\omega e} k_\perp^2 \frac{c\omega_{sv}}{V_A^2}\phi \tag{2.59}$$

using Eq. (2.59) in Eq. (2.55)

$$-\omega A_z - \omega_e^* A_z - \lambda_e^2 \omega k_\perp^2 A_z + c k_z \phi - \lambda_e^2 e 4\pi k_z \left(\begin{array}{c} \frac{e n_{e0}}{\omega k_B T_e}\omega_e^*\phi - \frac{c}{4\pi e V_A^2} k_\perp^2 c\phi + \frac{c}{4\pi\omega e} k_\perp^2 \frac{c\omega_i^*}{V_A^2}\phi \\ + \frac{c}{4\pi\omega e} k_\perp^2 \frac{c\omega_{sv}}{V_A^2} \end{array}\right) = 0 \tag{2.60}$$

using value of A_Z from Eq. (2.58) in Eq. (2.60), so we get

$$\begin{aligned}
&-\omega\left(\frac{c\omega}{V_A^2 k_z}\phi-\frac{c\omega_i^*}{V_A^2 k_z}\phi+\frac{\omega k^2}{ck_\perp^2 k_z}\phi+\frac{c\omega_{sv}}{V_A^2 k_z}\phi\right)-\omega_e^*\left(\frac{c\omega}{V_A^2 k_z}\phi-\frac{c\omega_i^*}{V_A^2 k_z}\phi+\frac{\omega k^2}{ck_\perp^2 k_z}\phi+\frac{c\omega_{sv}}{V_A^2 k_z}\phi\right)\\
&-\lambda_e^2\omega k_\perp^2\left(\frac{c\omega}{V_A^2 k_z}\phi-\frac{c\omega_i^*}{V_A^2 k_z}\phi+\frac{\omega k^2}{ck_\perp^2 k_z}\phi+\frac{c\omega_{sv}}{V_A^2 k_z}\phi\right)\\
=\quad&-ck_z\phi+\lambda_e^2 e4\pi k_z\left(\begin{array}{c}\frac{en_{e0}}{\omega k_B T_e}\omega_e^*\phi-\frac{c}{4\pi e V_A^2}k_\perp^2 c\phi+\frac{c}{4\pi\omega e}k_\perp^2\frac{c\omega_i^*}{V_A^2}\phi\\+\frac{c}{4\pi\omega e}k_\perp^2\frac{c\omega_{sv}}{V_A^2}\end{array}\right)
\end{aligned}$$

$$\begin{aligned}
&\frac{-c\omega^2}{V_A^2 k_z}\phi+\frac{c\omega\omega_i^*}{V_A^2 k_z}\phi+\frac{\omega^2 k^2}{ck_\perp^2 k_z}-\frac{c\omega\omega_{sv}}{V_A^2 k_z}\phi-\frac{c\omega\omega_e^*}{V_A^2 k_z}\phi+\frac{c\omega_e^*\omega_i^*}{V_A^2 k_z}\phi\\
&-\frac{c\omega_e^*\omega_{sv}}{V_A^2 k_z}\phi-\frac{c\lambda_e^2\omega^2 k_\perp^2}{V_A^2 k_z}\phi-\frac{c\lambda_e^2 k_\perp^2\omega\omega_i^*}{V_A^2 k_z}\phi-\frac{\lambda_e^2\omega^2 k^2 k_\perp^2}{ck_\perp^2 k_z}\phi-\frac{c\lambda_e^2\omega k_\perp^2\omega\omega_{sv}}{V_A^2 k_z}\phi\\
=\quad&-ck_z\phi+\lambda_e^2 4\pi k_z\frac{e^2 n_{e0}}{\omega k_B T_e}\omega_e^*\phi-\frac{\lambda_e^2 e4\pi k_z c}{4\pi e V_A^2}k_\perp^2 c\phi+\frac{\lambda_e^2 e4\pi k_z}{4\pi\omega e}k_\perp^2\frac{c^2\omega_i^*}{V_A^2}\phi+\frac{\lambda_e^2 e4\pi k_z c}{4\pi\omega e}k_\perp^2\frac{c\omega_{sv}}{V_A^2}
\end{aligned}$$

$$\begin{aligned}
&\frac{-c\omega^2}{V_A^2 k_z}\phi+\frac{c\omega\omega_i^*}{V_A^2 k_z}\phi+\frac{\omega^2 k^2}{ck_\perp^2 k_z}\phi-\frac{c\omega\omega_{sv}}{V_A^2 k_z}\phi-\frac{c\omega\omega_e^*}{V_A^2 k_z}\phi+\frac{c\omega_e^*\omega_i^*}{V_A^2 k_z}\phi-\frac{c\omega_e^*\omega_{sv}}{V_A^2 k_z}\phi\\
&-\frac{c\lambda_e^2\omega^2 k_\perp^2}{V_A^2 k_z}\phi-\frac{c\lambda_e^2 k_\perp^2\omega\omega_i^*}{V_A^2 k_z}\phi-\frac{\lambda_e^2\omega^2 k^2}{ck_z}\phi-\frac{c\lambda_e^2\omega k_\perp^2\omega\omega_{sv}}{V_A^2 k_z}\phi\\
=\quad&-ck_z\phi+\frac{\lambda_e^2 k_z}{\omega}\left(\frac{4\pi e^2 n_{e0}}{k_B T_e}\right)\omega_e^*\phi-\frac{\lambda_e^2 k_z c}{V_A^2}k_\perp^2 c\phi+\frac{\lambda_e^2 k_z k_\perp^2 c^2\omega_i^*}{\omega V_A^2}\phi+\frac{\lambda_e^2 k_z k_\perp^2 c^2\omega_{sv}}{\omega V_A^2}
\end{aligned}\quad(2.61)$$

where $\left(\frac{4\pi e^2 n_{e0}}{k_B T_e}\right)=\lambda_e^2$

multiplying Eq. (2.61) by $\frac{\omega V_A^2 k_z}{c\phi}$ we get

$$\begin{aligned}
\omega^3\quad&+\omega^2\omega_i^*+\frac{\omega^3 V_A^2 k^2}{ck_\perp^2}-\omega^2\omega_{sv}-\omega^2\omega_e^*+\omega\omega_e^*\omega_i^*-\frac{\omega_e^*\omega^2 k^2 V_A^2}{ck_\perp^2}-\omega\omega_e^*\omega_{sv}\\
&-\lambda_e^2 k_\perp^2\omega^3-\omega^2\omega_i^*\lambda_e^2 k_\perp^2-\lambda_e^2\omega^3 k^2\frac{V_A^2}{c}-\lambda_e^2 k_\perp^2\omega^2\omega_{sv}\\
=\quad&-\omega k_z^2 V_A^2+\frac{\lambda_e^2 k_z^2}{\lambda_e^2 c}\omega_e^* V_A^2-\omega\lambda_e^2 k_z^2 k_\perp^2+\lambda_e^2 k_z^2 k_\perp^2\omega\omega_i^*+\lambda_e^2 k_z^2 k_\perp^2\omega\omega_{sv}
\end{aligned}$$

Ignoring term $\frac{\omega^3 V_A^2 k^2}{ck_\perp^2}$ and $\frac{\omega_e^* \omega^2 k^2 V_A^2}{ck_\perp^2}$

$$\left[\begin{array}{c} \omega^3 - \omega^3 \lambda_e^2 k_\perp^2 + \omega^2 \omega_i^* - \omega^2 \omega_i^* \lambda_e^2 k_\perp^2 - \omega^2 \omega_{sv} - \lambda_e^2 k_\perp^2 \omega^2 \omega_{sv} - \omega^2 \omega_e^* + \omega \omega_e^* \omega_i^* - \omega \omega_e^* \omega_{sv} - \omega \lambda_e^2 k_z^2 k_\perp^2 \\ -\lambda_e^2 k_z^2 k_\perp^2 \omega \omega_i^* - \lambda_e^2 k_z^2 k_\perp^2 \omega \omega_{sv} - \omega k_z^2 V_A^2 - \frac{V_A^2 k_z^2}{c} \omega_e^* \end{array}\right] = 0$$

$$\left[\begin{array}{c} \omega^3 \left(1 + \lambda_e^2 k_\perp^2\right) - \omega^2 \omega_i^* \left(1 + \lambda_e^2 k_\perp^2\right) - \omega^2 \omega_{sv} \left(1 + \lambda_e^2 k_\perp^2\right) - \omega^2 \omega_e^* + \omega \omega_e^* \omega_i^* - \omega \omega_e^* \omega_{sv} - \omega \lambda_e^2 k_z^2 k_\perp^2 \\ -\lambda_e^2 k_z^2 k_\perp^2 \omega \omega_i^* - \lambda_e^2 k_z^2 k_\perp^2 \omega \omega_{sv} - \omega k_z^2 V_A^2 - \frac{V_A^2 k_z^2}{c} \omega_e^* \end{array}\right] = 0$$

$$\left[\begin{array}{c} \left(1 + \lambda_e^2 k_\perp^2\right) \left[\omega^3 - \omega^2 \omega_i^* - \omega^2 \omega_{sv}\right] - \omega^2 \omega_e^* + \omega \omega_e^* \omega_i^* - \omega \omega_e^* \omega_{sv} - \omega \lambda_e^2 k_z^2 k_\perp^2 \\ -\lambda_e^2 k_z^2 k_\perp^2 \omega \omega_i^* - \lambda_e^2 k_z^2 k_\perp^2 \omega \omega_{sv} - \omega k_z^2 V_A^2 + \frac{V_A^2 k_z^2}{c} \omega_e^* \end{array}\right] = 0$$

dividing both side by $\left(1 + \lambda_e^2 k_\perp^2\right)$

$$\left[\begin{array}{c} \left[\omega^3 - \omega^2 \omega_i^* - \omega^2 \omega_{sv}\right] - \frac{\omega^2 \omega_e^*}{\left(1+\lambda_e^2 k_\perp^2\right)} + \frac{\omega \omega_e^* \omega_i^*}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{\omega \omega_e^* \omega_{sv}}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{\omega \lambda_e^2 k_z^2 k_\perp^2}{\left(1+\lambda_e^2 k_\perp^2\right)} \\ -\frac{\lambda_e^2 k_z^2 k_\perp^2 \omega \omega_i^*}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{\lambda_e^2 k_z^2 k_\perp^2 \omega \omega_{sv}}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{\omega k_z^2 V_A^2}{\left(1+\lambda_e^2 k_\perp^2\right)} + \frac{V_A^2 k_z^2}{c\left(1+\lambda_e^2 k_\perp^2\right)} \omega_e^* \end{array}\right] = 0$$

where

$$\frac{\omega_e^*}{\left(1 + \lambda_e^2 k_\perp^2\right)} = \omega_m, \qquad \frac{V_A^2 k_z^2}{\left(1 + \lambda_e^2 k_\perp^2\right)} = \omega_{IA}^2,$$

So we get

$$\left[\begin{array}{c} \left[\omega^3 - \omega^2 \omega_i^* - \omega^2 \omega_{sv}\right] - \omega_m \omega^2 + \omega_m \omega \omega_i^* - \omega_m \omega \omega_{sv} \\ -\frac{V_A^2 k_z^2 k_\perp^2 \rho_S^2 \omega}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{V_A^2 k_z^2 k_\perp^2 \rho_S^2 \omega_i^*}{\left(1+\lambda_e^2 k_\perp^2\right)} - \frac{V_A^2 k_z^2 k_\perp^2 \rho_S^2 \omega_{sv}}{\left(1+\lambda_e^2 k_\perp^2\right)} \end{array}\right] = \omega_{IA}^2 \omega - \omega_{IA}^2 \omega_e^*$$

$$\left[\begin{array}{c} \omega^2 \left[\omega - \omega_i^* - \omega_{sv}\right] - \omega_m \omega \left(\omega + \omega_i^* - \omega_{sv}\right) \\ -\omega_{IA}^2 k_\perp^2 \rho_S^2 \omega - \omega_{IA}^2 k_\perp^2 \rho_S^2 \omega_i^* - \omega_{IA}^2 \rho_S^2 \omega_{sv} \end{array}\right] = \omega_{IA}^2 \omega - \omega_{IA}^2 \omega_e^*$$

$$\omega^2 \left[\omega - \omega_i^* - \omega_{sv}\right] - \omega_m \omega \left(\omega + \omega_i^* - \omega_{sv}\right) - \omega_{IA}^2 k_\perp^2 \rho_S^2 \left(\omega_e - \omega_i^* - \omega_{sv}\right) = \omega_{IA}^2 \left(\omega - \omega_e^*\right)$$

$$\left[\omega^2 - \omega \omega_m - \omega_{IA}^2 k_\perp^2 \rho_S^2\right] \left(\omega + \omega_i^* - \omega_{sv}\right) = \omega_{IA}^2 \left(\omega - \omega_e^*\right) \tag{2.62}$$

2.2.2 Properties of electromagnetic waves in nonuniform dusty magnetoplasma in various limiting case

Case 1

For homogeneous dusty plasma $(\omega_j = 0)$

When the parallel component of phase velocity is lesser than electron thermal plasma speed V_{Te}, the frequency of the dispersive alfven waves is generated . We neglected the parallel component of electron inertial effect $k_y^2\lambda_e^2 << 1$ then we get from Eq. (2.62)

$$\left[\omega^2 - \omega_{IA}^2 k_\perp^2 \rho_S^2\right](\omega) = \omega_{IA}^2(\omega)$$

$$\omega^2 = \omega_{IA}^2\left(1 + k_\perp^2\rho_S^2\right)$$

$$\omega^2 = k_z V_A\left(1 + k_\perp^2\rho_S^2\right)^{\frac{1}{2}} \tag{2.63}$$

Which is dispersive kinetic alfven waves in an intermediate plasma.

for $\frac{\omega}{k_z} >> V_{Te}$ and neglacting the parallel electron pressure gradient term $k_\perp^2\rho_S^2$, so

$$\omega = \frac{k_z V_A}{\left(1 + k_y^2\lambda_e^2\right)^{\frac{1}{2}}}$$

Which is dispersive inertial alfven waves frequency at very low β plasma $\left(\frac{m_e}{m_i} >> \beta\right)$.

Case 2 $\left(\omega >> \omega_m, \omega_j^*\right)$

The dispersive Alfven waves lineraly coupled with SV mode, From Eq. (2.62).

$$\omega = \omega_{sv}$$

In a cold dusty plasma $(T_j = 0)$ with $\omega_i^* << \omega$ and $V_{Te} << \frac{\omega}{k_z}$, we get from Eq. (2.62)

$$\omega^2\left(\omega - \omega_{sv}\right) = \omega_{IA}\omega$$

by using value of $\omega_{IA} = \frac{k_z^2 V_A^2}{1 + k_y^2\lambda_e^2}$

$$\omega^2 - \omega\omega_{sv} - \frac{k_z^2 V_A^2}{1 + k_y^2 \lambda_e^2} = 0 \tag{2.64}$$

which depicts the coupling of inertial Alfven waves and SV modes because of parallel electron motion in the wave electric and magnetic fields. Neglacting the parallel component of pressure gradient force ($k_s \rho_s \to 0$) and $\omega_i^* = 0$.

we get from Eq. (2.63)

$$\left(\omega^2 - \omega\omega_m\right)(\omega - \omega_{sv}) = \omega_{IA}\left(\omega - \omega_e^*\right)$$

where

$$\omega_m = \frac{\omega_e^*}{1 + k_y \lambda_e^2} \quad , \quad \omega_{IA} = \frac{k_z^2 V_A^2}{1 + k_y^2 \lambda_e^2}$$

so we get

$$\left(\omega^2 - \omega \frac{\omega_e^*}{1 + k_y \lambda_e^2}\right)(\omega - \omega_{sv}) = \frac{k_z^2 V_A^2}{1 + k_y^2 \lambda_e^2}\left(\omega - \omega_e^*\right)$$

$$\left(\omega\left(1 + k_y \lambda_e^2\right) - \omega_e^*\right)(\omega - \omega_{sv})\,\omega = k_z^2 V_A^2 \left(\omega - \omega_e^*\right) \tag{2.65}$$

which reveales that linearly coupling of megnatostatic drift modes $\omega = \omega_m$, the inertial Alfven waves $\omega = \omega_{IA}$, the elactron drift mode $\omega = \omega_e$ and SV mode $\omega = \omega_{sv}$.

Case 3 ($k_z v_{ez} = 0$)

Assuming parallel motion of electron vanish completely from Eq. (2.62) we get

$$\left(\omega^2 - \omega\omega_m\right)\left(\omega - \omega_i^* - \omega_{sv}\right) = 0$$

$$\left(\omega^2 - \omega\omega_m\right) = 0 \quad \text{and} \quad \left(\omega - \omega_i^* - \omega_{sv}\right) = 0 \tag{2.66}$$

hence the modified SV mode ($\omega = \omega_i^* + \omega_{sv}$) and flute like magnetostatic mode ($\omega = \omega_m$) arises as independent normal modes of a non uniform dusty magnetoplasma consisting of warm ions.

Case 4 $\left(k_y^2\lambda_e^2 << 1\right)$

When λ_e is much smaller than the prependicular wavelength for $\omega >> \omega_i^*$.

So from Eq. (2.62) we get

$$\left[\omega^2 - \omega\omega_e^* - k_z^2 V_A^2 k_\perp^2 \rho_S^2\right](\omega - \omega_{sv}) = k_z^2 V_z^2 (\omega - \omega_e^*)$$

$$\omega(\omega - \omega_e^*)(\omega - \omega_{sv}) - k_z^2 V_z^2 (\omega - \omega_e^*) = k_z^2 V_A^2 k_y^2 \rho_S^2 (\omega - \omega_{sv})$$

$$(\omega - \omega_e^*)\left(\omega^2 - \omega\omega_{sv}\right) - k_z^2 V_z^2 (\omega - \omega_e^*) = k_z^2 V_A^2 k_y^2 \rho_S^2 (\omega - \omega_{sv})$$

$$\left[\omega^2 - \omega\omega_{sv} - k_z^2 V_z^2\right](\omega - \omega_e^*) = k_z^2 V_A^2 k_y^2 \rho_S^2 (\omega - \omega_{sv}) \tag{2.66}$$

In a dusty plasma due to finite Larmour radius correction of ions at the electron tempera-trure, coupling between SV mode and drift kinetic Alfven waves are

$$\left[\omega^2 - \omega\omega_i^* - k_z^2 V_z^2\right](\omega - \omega_e^*) = k_z^2 V_A^2 k_y^2 \rho_S^2 (\omega - \omega_i^*) \tag{2.67}$$

which is dispertion relation of the coupled drift -kinetic Alfven waves without charged dust grains, in a warm electron ion magnetoplasma.

2.3 Conclusion

In this chapter, in dusty plasmas reveales the linearly propegation of dust acoustic waves and dust ion aouctic waves. In DA waves we study the low frequency and long wavelength cumulative oscillations. We shall consider the modes in which dust particle dynamics is crictical. We describe in the thermodynamical equilibrium the combined motion of negtively charged dust grains in the framework of ions and hot electrons and determined a latest type of sound wave, having low frequency namely the dust-acoustic waves. We have shown that the pressure of the inertialess ion and electron produce restoring force in the DA waves, while inertia due to dust mass assist the waves. The dust plasma frequency is much greater than the frequency of DA waves. We also determined the phase velocity of DIA waves, due to $n_{i0} < n_{e0}$ for negatively dust charge c_s is less than phase velocity of DIA waves, where $c_s = \left(\frac{k_B T_i}{m_i}\right)^{\frac{1}{2}}$. In the back ground of plasma the electron density plasma depletion due to increase in phase velocity, hence electron debye radius become larger. We have also study the electromagnetic waves, present in non uniform dusty magnetoplasma and considering static dust and derive the genral dispersion relation. We observe that when $\omega >> \omega_m, \omega_{j*}$ the dispersive ALfven waves are linearly coupled with the SV mode $\omega = \omega_{sv}$ also various other limiting cases are discussed to obtained magnetostatic mode $(\omega = \omega_m)$, coupling of drift-coupling Alfven waves and SV modes and dispersion relation of the coupled drift- kinetic Alfven waves in a warm electron ion magnetoplasma without charged dust grains.

Chapter 3

Modulational instability of ion acoustic waves in plasma with q-nonextensive electron distribution

3.1 Model

Consider the slow amplitude modulation of linear wave, and one dimentional motion of ion acoustic waves in unmagnetized electron-ion plasma, consisting of q-nonextensive distributed electrons and cold ions.

Continuity equation for ion number density n is given by

$$\frac{\partial v}{\partial t} + \frac{v \partial v}{\partial x} = -\frac{e}{m} \frac{\partial \phi}{\partial x} \tag{3.1}$$

where ϕ is the electrostatic potential and m and e are mass and charge of ions.

The system is closed by poisson's equation, which is given below

$$\varepsilon_o \frac{\partial^2 \phi}{\partial x^2} = e \left[n_e - n \right] \tag{3.2}$$

where n_e and n are number densities of electron and ion respectively.Velocity v and electrostatic potential ϕ are normalized by the ion-acoustic speed $Cs = \left(\frac{T_e}{m}\right)^{\frac{1}{2}}$ and $\frac{T_e}{e}$. The space x and time t are in the units of electron debye length

$$\lambda_D = \left(\frac{T_e}{4\pi e^2 n_{eo}}\right)^{\frac{1}{2}}$$

and reciprocal of ion plasma frequency equation of motion

$$\omega_{pi} = \left(\frac{4\pi e^2 n_o}{m_i}\right)^{\frac{1}{2}}$$

here the q tells us the strength of nonextensivity. Resulting normalized sets of equations are given as

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{3.3}$$

$$\frac{\partial u}{\partial t} + \frac{u\partial (u)}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{3.4}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = [n_e - n] \tag{3.5}$$

where

$$n_e = [1 + (q-1)\phi]^{\frac{q+1}{2(q-i)}} \tag{3.6}$$

after aplying bionomial theorem

$$n_e = 1 + c_1\phi + c_2\phi^2 + c_3\phi^3 ...$$

when we use n_e in the poisson's equation then it is transformed as

$$\frac{\partial^2 \phi}{\partial x^2} = 1 + c_1\phi + c_2\phi^2 + c_3\phi^3 ... - n \tag{3.7}$$

where

$$c_1 = \frac{(q+1)}{2}$$

$$c_1 = \frac{(q+1)(q-3)}{8}$$

$$c_2 = \frac{(q+1)(q-3)(3q-5)}{48}$$

3.2 Outline of Method

Let Ψ be any system variables and describing the State of the system at time t and position x. Consider small deviation from equilibrium state which tells us that and described by

$$\Psi = \Psi^{(0)} + \Sigma_{n-1}^{\infty} \varepsilon^n \Psi^{(n)} \tag{3.8}$$

We use the standard reductive perturbation technique to study the modulation of IAW and obtain the NLSE. The independent variables are

$$\xi = \varepsilon (x - v_g t)$$

and

$$\tau = \varepsilon^2 t$$

where ε is the small parameter and v_g is the group velocity of IAW which strongly depends upon the dispersion relation. The dependent variables are

$$\begin{aligned} n &= 1 + \Sigma_{n-1}^{\infty} \varepsilon^n \Sigma_{n-1}^{\infty} n_i^n (\xi, \tau) e^{\imath l(kx - \omega t)} \\ u &= \Sigma_{n-1}^{\infty} \varepsilon^n \Sigma_{n-1}^{\infty} u_i^n (\xi, \tau) e^{\imath l(kx - \omega t)} \\ \phi &= \Sigma_{n-1}^{\infty} \varepsilon^n \Sigma_{n-1}^{\infty} \phi_i^n (\xi, \tau) e^{\imath l(kx - \omega t)} \end{aligned} \tag{3.9}$$

where n, u and ϕ must satisfy the reality condition

$$\psi_{-\imath}^{(n)} = \left(\psi_{-\imath}^{(n)}\right)^*$$

now dependent variable can be written as

$$n = 1 + \left(\varepsilon n_1^1 + \varepsilon^2 n_1^2 + \varepsilon^3 n_1^3 + ...\right) e^{\grave{\imath} l\theta} \tag{2.10}$$

$$v = \left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta}$$

$$\phi = \left(\varepsilon \phi_1^1 + \varepsilon^2{}^2_1 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta} \tag{3.12}$$

where

$$\theta = kx - \omega t$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \varepsilon v_g \frac{\partial}{\partial \xi} + \varepsilon^2 \frac{\partial}{\partial \tau} \tag{3.13}$$

$$\frac{\partial}{\partial t}\left(\varepsilon\left(x - v_g t\right)\right) = -\varepsilon v_g, \frac{\partial}{\partial t}\left(\varepsilon^2 t\right) = \varepsilon^2$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} + \varepsilon \frac{\partial}{\partial \xi} \tag{3.14}$$

now in case if initially

$$\frac{\partial}{\partial x} = 0$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} = \varepsilon \frac{\partial}{\partial \xi}, \frac{\partial}{\partial \xi} = \varepsilon \tag{3.15}$$

$$\frac{\partial^2}{\partial x^2} = \varepsilon \frac{\partial}{\partial \xi}\frac{\partial}{\partial x} = \varepsilon^2 \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} = \varepsilon^2 \frac{\partial^2}{\partial \xi^2}, \frac{\partial \xi}{\partial x} = \varepsilon \tag{3.16}$$

$$\frac{\partial}{\partial t} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial \tau}\frac{\partial}{\partial \tau} = -\varepsilon v_g \frac{\partial}{\partial \xi} + \varepsilon^2 \frac{\partial}{\partial \tau}, \frac{\partial \xi}{\partial \tau} = -\varepsilon v_g, \frac{\partial \tau}{\partial t} = \varepsilon^2 \tag{3.17}$$

Lets proceeding for continuity equation

First term

$$\frac{\partial n}{\partial t} = \frac{\partial n}{\partial t} - \varepsilon v_g \frac{\partial n}{\partial \xi} + \varepsilon^2 \frac{\partial n}{\partial \tau}$$

$$\begin{aligned}\frac{\partial n}{\partial t} &= \frac{\partial}{\partial t}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}-\varepsilon v_g\frac{\partial}{\partial \xi}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}\\ &+\varepsilon^2\frac{\partial}{\partial \tau}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}\end{aligned}$$

$$\begin{aligned}\frac{\partial n}{\partial t} &= -il\omega\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}-\varepsilon v_g\frac{\partial}{\partial \xi}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}\\ &+\varepsilon^2\frac{\partial}{\partial \tau}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}\end{aligned} \tag{3.18}$$

Second Term

$$n = 1+\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}$$

$$v = \left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}$$

$$\frac{\partial nv}{\partial x} = \frac{\partial}{\partial x}\left[1+\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}\right]\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}$$

$$\begin{aligned}\frac{\partial nv}{\partial x} &= \frac{\partial}{\partial x}\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}+\varepsilon\frac{\partial}{\partial \xi}\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}\\ &+\left(\frac{\partial}{\partial x}+\varepsilon\frac{\partial}{\partial \xi}\right)\left[\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)\left(\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{i2l\theta}\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial nv}{\partial x} &= ilk\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}\\ &+\left(\frac{\partial}{\partial \xi}\varepsilon^2 v_1^1+\frac{\partial}{\partial \xi}\varepsilon^3 v_1^2+\frac{\partial}{\partial \xi}\varepsilon^4 v_1^3+...\right)e^{il\theta}\\ &+\left(\frac{\partial}{\partial x}+\varepsilon\frac{\partial}{\partial \xi}\right)\left[\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{i2l\theta}\right]\end{aligned} \tag{3.19}$$

Combined continuity equation by using equations 3.18 and 3.19

$$\begin{aligned}\frac{\partial n}{\partial t}+\frac{\partial nv}{\partial x} &= -il\omega\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}-\varepsilon v_g\frac{\partial}{\partial\xi}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)\\ &+\varepsilon^2\frac{\partial}{\partial\tau}\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right)e^{il\theta}+ilk\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)\\ &+\left(\frac{\partial}{\partial\xi}\varepsilon^2 v_1^1+\frac{\partial}{\partial\xi}\varepsilon^3 v_1^2+\frac{\partial}{\partial\xi}\varepsilon^4 v_1^3+...\right)e^{il\theta}\\ &+\left(\frac{\partial}{\partial x}+\varepsilon\frac{\partial}{\partial\xi}\right)\left[\begin{pmatrix}\varepsilon v_1^1+\varepsilon^2 v_1^2\\ +\varepsilon^3 v_1^3+...\end{pmatrix}\begin{pmatrix}\varepsilon n_1^1+\varepsilon^2 n_1^2\\ +\varepsilon^3 n_1^3+...\end{pmatrix}e^{i2l\theta}\right] \qquad (3.20)\end{aligned}$$

comparing ε order tern for $n = 1$, $l = 1$

$$(-il\omega)\,\varepsilon n_1^1 e^{il\theta}+ilk\left(\varepsilon v_1^1\right)e^{il\theta}=0$$

$$\omega n_1^1+kv_1^1=0 \qquad (3.21)$$

now equation of motion

$$v=\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}$$

$$\theta=kx-\omega t$$

First term

$$\frac{\partial v}{\partial t}=-il\omega\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}+e^{il\theta}\frac{\partial}{\partial t}\left(\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}\right)$$

$$\frac{\partial v}{\partial t}=-il\omega\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}+e^{il\theta}\left[-\varepsilon v_g\frac{\partial}{\partial\xi}+\varepsilon^2\frac{\partial}{\partial t}\right]\left(\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)e^{il\theta}\right)$$

$$\begin{aligned}\frac{\partial v}{\partial t} &= -il\omega\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+.......\right)e^{il\theta}\\ &+e^{il\theta}v_g\left(\varepsilon^2\frac{\partial}{\partial\xi}v_1^1+\varepsilon^3\frac{\partial}{\partial\xi}v_1^2+\varepsilon^3\frac{\partial}{\partial\xi}v_1^3+.......\right)e^{il\theta}\\ &+e^{il\theta}\left(\varepsilon^3\frac{\partial}{\partial\tau}v_1^1+\varepsilon^4\frac{\partial}{\partial\tau}v_1^2+\varepsilon^5\frac{\partial}{\partial\tau}v_1^3+......\right) \qquad (3.22)\end{aligned}$$

Second term

$$\frac{\partial v}{\partial x} = ilk\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta} + e^{il\theta}\varepsilon\frac{\partial}{\partial \xi}\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta}$$

$$\frac{\partial v}{\partial x} = ilk\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta} + e^{il\theta}\left(\varepsilon^2\frac{\partial}{\partial \xi}v_1^1 + \varepsilon^3\frac{\partial}{\partial \xi}v_1^2 + \varepsilon^4\frac{\partial}{\partial \xi}v_1^3 + ...\right) e^{\grave{\imath} l\theta}$$

$$\begin{aligned} v\frac{\partial v}{\partial x} \quad = \quad & ilk\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right)^2 e^{\grave{\imath} 2l\theta} + \\ & e^{i2l\theta}\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right)\left(\varepsilon^2\frac{\partial}{\partial \xi}v_1^1 + \varepsilon^3\frac{\partial}{\partial \xi}v_1^2 + \varepsilon^4\frac{\partial}{\partial \xi}v_1^3 + ...\right) \end{aligned} \tag{3.23}$$

combining Eq. (3.22) and Eq. (3.23)

$$\begin{aligned} v\frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} \quad = \quad & ilk\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right)^2 e^{\grave{\imath} 2l\theta} \\ & + e^{i2l\theta}\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right)\left(\varepsilon^2\frac{\partial}{\partial \xi}v_1^1 + \varepsilon^3\frac{\partial}{\partial \xi}v_1^2 + \varepsilon^4\frac{\partial}{\partial \xi}v_1^3 + ...\right) \\ & - il\omega\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{\grave{\imath} l\theta} \\ & + e^{il\theta} v_g\left(\varepsilon^2\frac{\partial}{\partial \xi}v_1^1 + \varepsilon^3\frac{\partial}{\partial \xi}v_1^2 + \varepsilon^3\frac{\partial}{\partial \xi}v_1^3 + ...\right) e^{\grave{\imath} l\theta} \\ & + e^{il\theta}\left(\varepsilon^3\frac{\partial}{\partial \tau}v_1^1 + \varepsilon^4\frac{\partial}{\partial \tau}v_1^2 + \varepsilon^5\frac{\partial}{\partial \tau}v_1^3 + ...\right) \end{aligned} \tag{3.24}$$

Third term

$$\phi = 1 + \left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right) e^{\grave{\imath} l\theta}$$

$$\frac{\partial \phi}{\partial x} = ilk\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right) e^{\grave{\imath} l\theta} + e^{il\theta}\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right)$$

$$\frac{\partial \phi}{\partial x} = ilk\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right) e^{\grave{\imath} l\theta} + e^{il\theta}\varepsilon\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right) \tag{3.25}$$

combining all terms from Eq. (3.24) and Eq. (3.25)

$$
\begin{aligned}
& ilk\left(\varepsilon v_1^1 v_1^1+\varepsilon^2 v_1^1 v_1^2+\varepsilon^3 v_1^1 v_1^3+...\right) e^{i2l\theta} \\
& +e^{i2l\theta}\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right)\left(\varepsilon^2 \frac{\partial}{\partial \xi} v_1^1+\varepsilon^3 \frac{\partial}{\partial \xi} v_1^2+\varepsilon^4 \frac{\partial}{\partial \xi} v_1^3+...\right) \\
& -il\omega\left(\varepsilon v_1^1+\varepsilon^2 v_1^2+\varepsilon^3 v_1^3+...\right) e^{il\theta} \\
& -e^{il\theta} v_g\left(\varepsilon^2 \frac{\partial}{\partial \xi} v_1^1+\varepsilon^3 \frac{\partial}{\partial \xi} v_1^2+\varepsilon^4 \frac{\partial}{\partial \xi} v_1^3+...\right) e^{il\theta} \\
& +e^{il\theta}\left(\varepsilon^3 \frac{\partial}{\partial \tau} v_1^1+\varepsilon^4 \frac{\partial}{\partial \tau} v_1^2+\varepsilon^5 \frac{\partial}{\partial \tau} v_1^3+...\right) \\
= \; & ilk\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right) e^{il\theta}+e^{il\theta} \varepsilon \frac{\partial}{\partial x}\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right) \qquad (3.26)
\end{aligned}
$$

comparing ε order term for $n=1, l=1$, hence we get

$$
e^{il\theta}(-il\omega) \varepsilon v_1^1=-ilk\left(\varepsilon \phi_1^1\right) e^{il\theta}
$$

$$
\omega v_1^1=k \phi_1^1 \qquad (3.27)
$$

Finally for poison's equation

$$
\frac{\partial^2 \phi}{\partial x^2}=1+c_1 \phi+c_2 \phi^2+c_3 \phi^3-n
$$

$$
\begin{aligned}
\frac{\partial^2 \phi}{\partial x^2} = \; & 1+c_1\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right) e^{il\theta}+c_2\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right)^2 e^{il2\theta} \\
& +c_3\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right)^3 e^{il3\theta}-1-\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right) e^{il\theta}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \phi}{\partial x^2} = \; & c_1\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right) e^{il\theta}+c_2\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right)^2 e^{il2\theta} \\
& +c_3\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right)^3 e^{il3\theta}-\left(\varepsilon n_1^1+\varepsilon^2 n_1^2+\varepsilon^3 n_1^3+...\right) e^{il\theta} \qquad (3.28)
\end{aligned}
$$

now

$$
\frac{\partial^2 \phi}{\partial x^2}=\frac{\partial}{\partial x}\left[ilk\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right) e^{il\theta}+e^{il\theta} \frac{\partial}{\partial x}\left(\varepsilon \phi_1^1+\varepsilon^2 \phi_1^2+\varepsilon^3 \phi_1^3+...\right)\right]
$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial x^2} &= -l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+e^{il\theta}ilk\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\\ &\quad+\frac{\partial}{\partial x}\left[e^{il\theta}\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial x^2} &= -l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+e^{il\theta}ilk\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\\ &\quad+e^{il\theta}ilk\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)+e^{il\theta}\frac{\partial^2}{\partial x^2}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial x^2} &= -l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+e^{il\theta}2ilk\frac{\partial}{\partial x}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\\ &\quad+e^{il\theta}\frac{\partial^2}{\partial x^2}\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)\end{aligned} \qquad (3.29)$$

as from Eq. (3.15) and Eq. (3.17)

$$\frac{\partial}{\partial x}=\frac{\partial\xi}{\partial x}\frac{\partial}{\partial\xi}=\varepsilon\frac{\partial}{\partial\xi},\frac{\partial}{\partial\xi}=\varepsilon$$

$$\frac{\partial^2}{\partial x^2}=\varepsilon\frac{\partial}{\partial\xi}\frac{\partial}{\partial x}=\varepsilon^2\frac{\partial}{\partial\xi}\frac{\partial\xi}{\partial x}\frac{\partial}{\partial\xi}=\varepsilon^2\frac{\partial^2}{\partial\xi^2},\frac{\partial\xi}{\partial x}=\varepsilon$$

now

$$\begin{aligned}\frac{\partial^2\phi}{\partial x^2} &= -l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+e^{il\theta}2ilk\frac{\partial}{\partial\xi}\left(\varepsilon^2\phi_1^1+\varepsilon^3\phi_1^2+\varepsilon^4\phi_1^3+...\right)\\ &\quad+e^{il\theta}\frac{\partial^2}{\partial\xi^2}\left(\varepsilon^3\phi_1^1+\varepsilon^4\phi_1^2+\varepsilon^5\phi_1^3+...\right)\end{aligned} \qquad (3.30)$$

now combining both sides of poison's equations by using Eq. (3.29) and Eq. (3.30)

$$\begin{aligned}&-l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+e^{il\theta}2ilk\frac{\partial}{\partial\xi}\left(\varepsilon^2\phi_1^1+\varepsilon^3\phi_1^2+\varepsilon^4\phi_1^3+...\right)\\ &+e^{il\theta}\frac{\partial^2}{\partial\xi^2}\left(\varepsilon^3\phi_1^1+\varepsilon^4\phi_1^2+\varepsilon^5\phi_1^3+...\right)\\ =\;& c_1\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{i}l\theta}+c_2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)^2e^{\grave{i}l2\theta}\\ &+c_3\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)^3e^{\grave{i}l3\theta}-\left(\varepsilon n_1^1+\varepsilon^2n_1^2+\varepsilon^3n_1^3+...\right)e^{\grave{i}l\theta}\end{aligned}$$

comparing order of terms $n = 1, l = 1$, we get

$$-l^2 k^2 e^{\imath l\theta} \varepsilon \phi_1^1 = -\varepsilon n_1^1 e^{\imath l\theta} + \varepsilon \phi_1^1 c_1 e^{\imath l\theta}$$

$$-k^2 \phi_1^1 = \phi_1^1 c_1 - n_1^1$$

now considering the Eq. (3.21), Eq. (3.27) and Eq. (3.32)

we get for the the first homonic $n = 1, l = 1$

$$\omega n_1^1 + k v_1^1 = 0$$

$$\omega v_1^1 = k \phi_1^1$$

$$-k^2 \phi_1^1 = \phi_1^1 c_1 - n_1^1$$

by solving above equations we get dispersion relation

$$n_1^1 = \frac{k v_1^1}{\omega}$$

and

$$v_1^1 = \frac{k \phi_1^1}{\omega}$$

$$n_1^1 = \frac{k}{\omega} \frac{k \phi_1^1}{\omega}$$

so

$$-k^2 \phi_1^1 = \phi_1^1 c_1 - \frac{k^2}{\omega^2} \phi_1^1$$

$$\omega^2 = \frac{k^2}{c_1 + k^2}$$

$$\omega = \sqrt{\frac{k^2}{c_1 + k^2}} \tag{3.33}$$

So wave dispersion relation describes that the wave is moving or a simple oscillation, if the frequency IAW frequency ω is the function of wave vector k then the IAW is propagating, else if it is not function of wave vector then there will be simple oscillations.

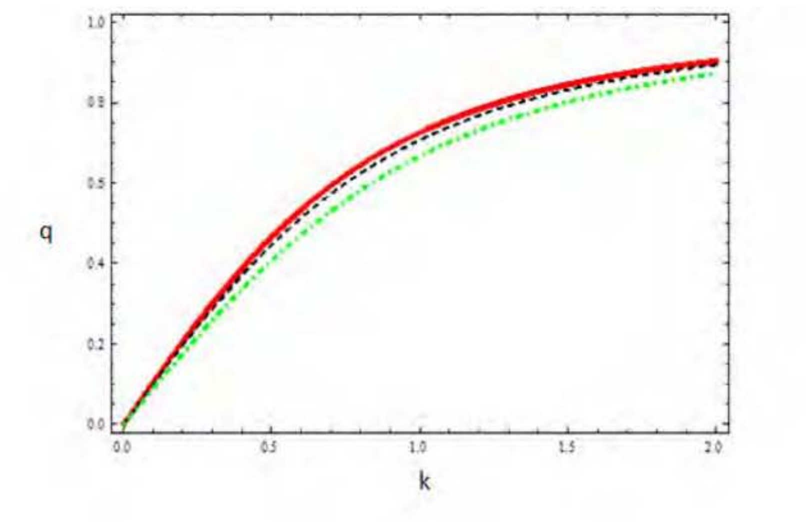

Figure 3.1:Variation of frequency of IAW with the wave vector k for different values of non-extensive parameter q. Solid curve corresponds to q=0.8;Dashed curve to q=1; DotDashed curve to q=1.5

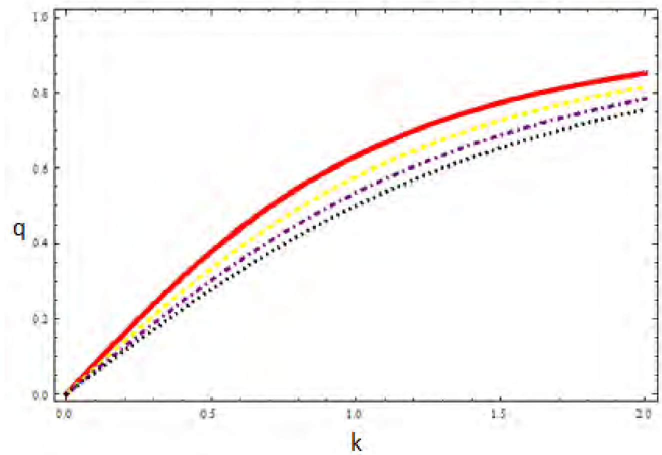

Figure 3.2:Variation of frequency of IAW with the wave vector k for different values of non-extensive parameter q.Solid curve corresponds to q=2;Dashed curve to q=3; DotDashed curve to q=4;Dotted curve to q=5

when the value of non-extensive parameter q is increased then the frequency ω grows slowly with the wave vector. The frequency ω of IAW depends upon the non-extensive parameter q,and the frequency ω of IAW grows fastly for smaller value of non-extensive parameter as compared to the large value of non-extensive parameter.

The first order reduced equations for 1st harmonics in terms of the potential $\phi_1^{(1)}$ are

$$n_1^{(1)} = \left(c_1 + k^2\right) \phi_1^{(1)} \tag{3.34}$$

$$n_1^{(1)} = \frac{k}{\omega} \phi_1^{(1)} \tag{3.35}$$

Eq. (3.34), Eq. (3.35) are the perturbed number density and velocity in terms of first order and first harmonic potential which is usually known to us. These results are very important and used again and again to convert all the equations into known first harmonic and first order potential. It is observed that zeroth order terms as well as higher orders terms are zero.

$$\psi_{l>n}^{(n)} = \frac{k}{\omega}\phi_1^{(1)}, \psi_0^{(n)} = 0$$

3.3 Second order terms:second and zeroth harmonics,group velocity

Considering Eq. (3.20) , comparing ε^2order term for $n = 2, l = 1$ only $e^{\grave{\imath}l\theta}$ term

$$-il\omega\varepsilon^2 n_1^2 e^{il\theta} - e^{\grave{\imath}l\theta}\varepsilon^2 v_g \frac{\partial n_1^1}{\partial\xi} + e^{il\theta}\varepsilon^2 \frac{\partial v_1^1}{\partial\xi} + \varepsilon^2 v_1^2 ilke^{\grave{\imath}l\theta} = 0$$

$$n_1^2 = \frac{i}{\omega}\left(v_g \frac{\partial n_1^1}{\partial\xi} - \frac{\partial v_1^1}{\partial\xi}\right) + \frac{k}{\omega}v_1^2$$

now considering Eq. (3.26)

$$\begin{aligned}
& ilk\left(\varepsilon v_1^1 v_1^1 + \varepsilon^2 v_1^1 v_1^2 + \varepsilon^3 v_1^1 v_1^3 + ...\right) e^{\grave{\imath}2l\theta} \\
& + e^{i2l\theta}\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right)\left(\varepsilon^2 \frac{\partial}{\partial\xi} v_1^1 + \varepsilon^3 \frac{\partial}{\partial\xi} v_1^2 + \varepsilon^4 \frac{\partial}{\partial\xi} v_1^3 + ...\right) \\
& - il\omega\left(\varepsilon v_1^1 + \varepsilon^2 v_1^2 + \varepsilon^3 v_1^3 + ...\right) e^{il\theta} - e^{il\theta} v_g \left(\varepsilon^2 \frac{\partial}{\partial\xi} v_1^1 + \varepsilon^3 \frac{\partial}{\partial\xi} v_1^2 + \varepsilon^4 \frac{\partial}{\partial\xi} v_1^3 + ...\right) e^{\grave{\imath}l\theta} \\
& + e^{il\theta}\left(\varepsilon^3 \frac{\partial}{\partial\tau} v_1^1 + \varepsilon^4 \frac{\partial}{\partial\tau} v_1^2 + \varepsilon^5 \frac{\partial}{\partial\tau} v_1^3 + ...\right) \\
= \; & ilk\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right) e^{il\theta} + e^{il\theta}\varepsilon \frac{\partial}{\partial x}\left(\varepsilon\phi_1^1 + \varepsilon^2\phi_1^2 + \varepsilon^3\phi_1^3 + ...\right)
\end{aligned}$$

comparing ε^2 order term for $n = 2, l = 1$, containing only $e^{\grave{\imath}l\theta}$ term

$$-il\omega\varepsilon^2 v_1^2 e^{\grave{\imath}l\theta} - e^{\grave{\imath}l\theta} v_g \varepsilon^2 \frac{\partial v_1^1}{\partial\xi} = -e^{\grave{\imath}l\theta}\varepsilon^2 \frac{\partial\phi_1^1}{\partial\xi} - \varepsilon^2\phi_1^2 ilke^{il\theta}$$

$$-i\omega v_1^2 - v_g \frac{\partial v_1^1}{\partial\xi} = \frac{\partial\phi_1^1}{\partial\xi} -^2 \phi_1^2 k$$

$$v_1^2 = \frac{i}{\omega}\left(v_g \frac{\partial v_1^1}{\partial\xi} - \frac{\partial\phi_1^1}{\partial\xi}\right) + v_g \frac{k}{\omega} = \frac{\partial\phi_1^1}{\partial\xi}\phi_1^2 \tag{3.37}$$

considering Eq. (3.31)

$$
\begin{aligned}
& -l^2k^2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{\imath}l\theta}+e^{il\theta}2ilk\frac{\partial}{\partial\xi}\left(\varepsilon^2\phi_1^1+\varepsilon^3\phi_1^2+\varepsilon^4\phi_1^3+...\right)\\
& +e^{il\theta}\frac{\partial^2}{\partial\xi^2}\left(\varepsilon^3\phi_1^1+\varepsilon^4\phi_1^2+\varepsilon^5\phi_1^3+...\right)\\
= \; & c_1\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)e^{\grave{\imath}l\theta}+c_2\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)^2e^{\grave{\imath}l2\theta}\\
& +c_3\left(\varepsilon\phi_1^1+\varepsilon^2\phi_1^2+\varepsilon^3\phi_1^3+...\right)^3e^{\grave{\imath}l3\theta}-\left(\varepsilon n_1^1+\varepsilon^2n_1^2+\varepsilon^3n_1^3+...\right)e^{\grave{\imath}l\theta}
\end{aligned}
$$

comparing ε^2 oreder term for $n=2, l=1$ consisting only $e^{il\theta}$ term

so we get

$$-k^2l^2\varepsilon^2\phi_1^2e^{\grave{\imath}l\theta}-2ilk\varepsilon^2e^{il\theta}\frac{\partial\phi_1^1}{\partial\xi}=-c_1\left(\varepsilon^2\phi_1^2\right)e^{\grave{\imath}l\theta}-\varepsilon^2n_1^2e^{\grave{\imath}l\theta}$$

$$-k^2l^2\phi_1^2-2ilk\frac{\partial\phi_1^1}{\partial\xi}=c_1\left(\phi_1^2\right)-n_1^2$$

$$2ilk\frac{\partial\phi_1^1}{\partial\xi}=c_1\left(\phi_1^2\right)-n_1^2+k^2\phi_1^2 \tag{3.38}$$

note now from equation $(3.36-3.38)$ we found $\left(\text{in term of } \phi_1^1\right)$ ϕ_1^2, n_1^1 and v_1^1

$$-i\omega n_1^2+ikv_1^2=\left(v_g\frac{\partial n_1^1}{\partial\xi}-\frac{\partial v_1^1}{\partial\xi}\right)=f_1 \tag{3.39}$$

$$-i\omega v_1^2+ik\phi_1^2=\left(v_g\frac{\partial v_1^1}{\partial\xi}-\frac{\partial\phi_1^1}{\partial\xi}\right)=f_2 \tag{3.40}$$

$$n_1^2+\left(c_1+k^2\right)\phi_1^2=-2ilk\frac{\partial\phi_1^1}{\partial\xi}=f_3 \tag{3.41}$$

for n_1^2, multiplying Eq. (3.39)and Eq. (3.40) by ω and k respectively. we get

$$-i\omega^2n_1^2+ik\omega v_1^2=f_1\omega \tag{3.42}$$

$$-ik\omega v_1^2+ik^2\phi_1^2=f_2k \tag{3.43}$$

adding Eq. (3.42) and Eq. (3.43)

$$-i\omega^2 n_1^2 + ik\omega v_1^2 - ik\omega v_1^2 + ik^2\phi_1^2 = f_1\omega + f_2 k$$

$$-i\omega^2 n_1^2 + ik^2\phi_1^2 = f_1\omega + f_2 k \tag{3.44}$$

multiplying Eq. (3.44) by i on both sides

$$\omega^2 n_1^2 - k^2\phi_1^2 = if_1\omega + if_2 k \tag{3.45}$$

where

$$\left(v_g \frac{\partial n_1^1}{\partial \xi} - \frac{\partial v_1^1}{\partial \xi}\right) = f_1 \tag{3.46}$$

$$\left(v_g \frac{\partial v_1^1}{\partial \xi} - \frac{\partial \phi_1^1}{\partial \xi}\right) = f_2 \tag{3.47}$$

use following equations in Eq. (3.46) and Eq. (3.47)

$$v_g = c_1 \frac{\omega^3}{k^3}$$

$$n_1^1 = \frac{k^2}{\omega^2}\phi_1^1$$

$$v_1^1 = \frac{k}{\omega}\phi_1^1$$

we get

$$f_1 = c_1 \frac{\omega^3}{k^3}\frac{k^2}{\omega^2}\frac{\partial \phi_1^1}{\partial \xi} - \frac{k}{\omega}\frac{\partial \phi_1^1}{\partial \xi}$$

$$i\omega f_1 = ic_1 \frac{\omega^2}{k}\frac{\partial \phi_1^1}{\partial \xi} - ik\frac{\partial \phi_1^1}{\partial \xi} \tag{3.49}$$

$$ikf_2 = ic_1 \frac{\omega^3}{k^3}\frac{k^2}{\omega}\frac{\partial \phi_1^1}{\partial \xi} - ik\frac{\partial \phi_1^1}{\partial \xi}$$

$$ikf_2 = ic_1 \frac{\omega^2}{k}\frac{\partial \phi_1^1}{\partial \xi} - ik\frac{\partial \phi_1^1}{\partial \xi} \tag{3.50}$$

now using value of $i\omega\ f_1$ and ikf_2 from Eq. (3.49) and Eq. (3.50) in Eq. (3.45) , so we get

$$\omega^2 n_1^2 = k^2\phi_1^2 + ic_1\frac{\omega^2}{k}\frac{\partial\phi_1^1}{\partial\xi} - ik\frac{\partial\phi_1^1}{\partial\xi} + ic_1\frac{\omega^2}{k}\frac{\partial\phi_1^1}{\partial\xi} - ik\frac{\partial\phi_1^1}{\partial\xi}$$

$$n_1^2 = \frac{k^2\phi_1^2}{\omega^2} + \frac{2ic_1}{k}\frac{\partial\phi_1^1}{\partial\xi} - \frac{2ik}{\omega^2}\frac{\partial\phi_1^1}{\partial\xi} \tag{3.51}$$

for v_1^2 considering Eq. (3.40) using value of v_g and v_1^1

$$v_g = c_1\frac{\omega^3}{k^3}, v_1^1 = \frac{k}{\omega}\phi_1^1$$

$$i\omega v_1^2 = ik\phi_1^2 - c_1\frac{\omega^3}{k^3}\frac{k}{\omega}\frac{\partial\phi_1^1}{\partial\xi} + \frac{\partial\phi_1^1}{\partial\xi}$$

$$v_1^2 = \frac{k}{\omega}\phi_1^2 + i\frac{\partial\phi_1^1}{\partial\xi}\left(c_1\frac{\omega}{k^2} - \frac{1}{\omega}\right) \tag{3.52}$$

3.3.1 Compatibility condition

Compatibility condition can be calculated as

from Eq. (3.33)

$$\omega = \sqrt{\frac{k^2}{c_1 + k^2}}$$

$$\frac{d\omega}{dk} = \frac{\sqrt{k^2 + c_1} - \frac{k}{\sqrt{k^2+c_1}}k}{k^2 + c_1}$$

$$\frac{d\omega}{dk} = \frac{k^2 + c_1 - k^2}{(k^2 + c_1)^{\frac{3}{2}}}$$

$$\frac{d\omega}{dk} = \frac{c_1}{(k^2 + c_1)^{\frac{3}{2}}} \tag{3.53}$$

as from equation 3.33

$$\frac{\omega}{k} = \frac{1}{\sqrt{(k^2 + c_1)}}$$

$$\sqrt{(k^2 + c_1)} = \frac{k}{\omega} \tag{3.54}$$

using Eq. (3.54) in Eq. (3.53) we get group velocity

$$\frac{d\omega}{dk} = c_1 \frac{\omega^3}{k^3} \tag{3.53}$$

Group velocity gives us information that ion acoustic wave is propagating or it is just a simple oscillation. As group velocity is function of wave vector k so ion acoustic wave is propagating.

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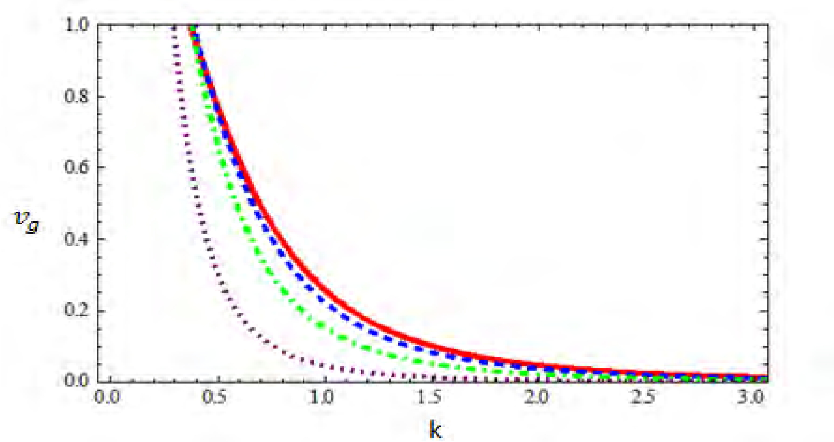

Figure 3.3:Variation of; the Group velocity with the carrier wave number k for different values of q-non-extensive paramer q. Solid curve corresponds to q=-0.1; Dashed to q=-0.3; DotDashed curve to q=-0.6 and Solid curve to q=-0.9.

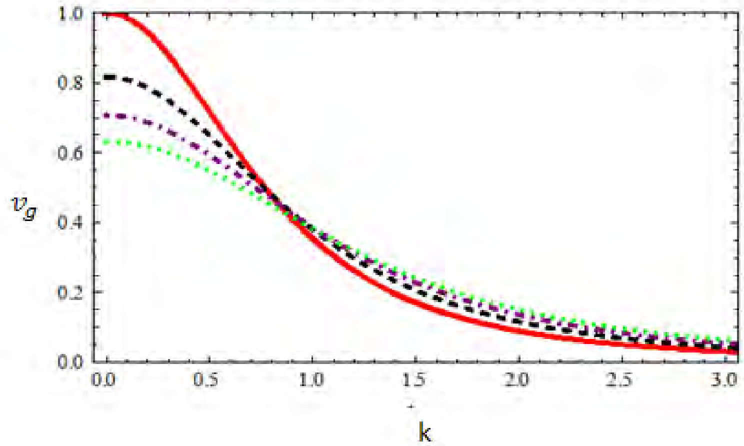

Figure 3.4:: Variation of; the Group velocity with the carrier wave number k for different values of q-non-extensive parameter q. Dotted curve corresponds to q=4; DotDashed to q=3; Dashed curve to q=2 and Solid curve to q=1.

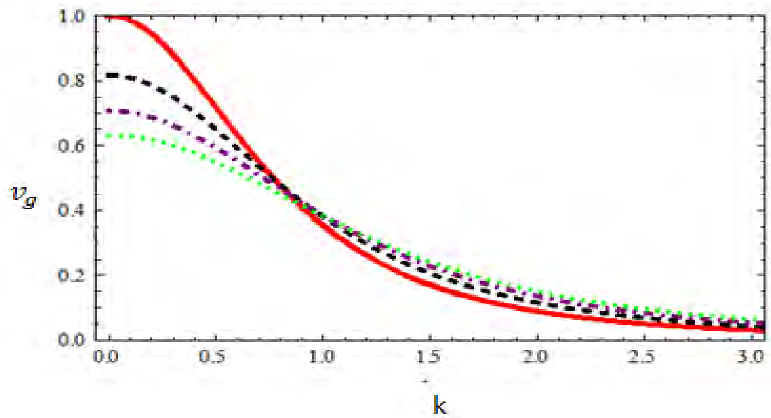

Figure 3.4: Variation of; the Group velocity with the carrier wave number k for different values of q-non-extensive parameter q. Dotted curve corresponds to q=4; DotDashed to q=3; Dashed curve to q=2 and Solid curve to q=1.

comparing ε^2 order terms for $n = 2, l = 2$ (for equation of motion)

$$-il\omega\varepsilon^2 v_1^2 e^{il\theta} - e^{il\theta} v_g \varepsilon^2 \frac{\partial v_1^1}{\partial \xi} + ilk\varepsilon^2 v_1^1 v_1^1 e^{i2l\theta} = -e^{il\theta}\varepsilon^2 \frac{\partial \phi_1^1}{\partial \xi} - \varepsilon^2 \phi_1^2 ilke^{il\theta}$$

now put $l = 2$ in $e^{il\theta}$ term and $l = 1$ in $e^{i2l\theta}$ term, neglecting $\frac{\partial v_2^1}{\partial \xi}, \frac{\partial \phi_2^1}{\partial \xi}$

$$-\omega v_2^2 + kv_1^1 v_1^1 = -k\phi_2^2$$

$$v_2^2 = \frac{kv_1^1}{\omega} + \frac{k}{\omega}\phi_2^2 \tag{3.54}$$

comparing ε^2 order terms for $n = 2, l = 2$ (for poisson's equation)

$$-k^2 l^2 \varepsilon^2 \phi_1^2 e^{il\theta} + 2ilk\varepsilon^2 e^{il\theta} \frac{\partial \phi_1^1}{\partial \xi} = c_1 \left(\varepsilon^2 \phi_1^2\right) e^{il\theta} - c_2 \left(\varepsilon^2 \phi_1^1\right)^2 e^{i2l\theta} - \varepsilon^2 n_1^2 e^{il\theta}$$

now put $l=2$ in $e^{il\theta}$ term and $l=1$ in $e^{i2l\theta}$ term and neglecting $\frac{\partial \phi_2^1}{\partial \xi}$

$$-4k^2\phi_2^2 = c_1\phi_2^2 - c_2\left(\phi_1^1\right)^2 - n_2^2 \tag{3.55}$$

comparing ε^2 order term for $n=2, l=2$ (for continuity equation)

$$-il\omega\varepsilon^2 n_1^2 e^{il\theta} - e^{il\theta} v_g \varepsilon^2 \frac{\partial n_1^1}{\partial \xi} + e^{il\theta}\varepsilon^2 \frac{\partial v_1^1}{\partial \xi} + \varepsilon^2 v_1^2 ilke^{il\theta} + ilke^{i2l\theta}\varepsilon^2 v_1^1 n_1^1 = 0$$

now put $l=2$ in $e^{il\theta}$ term and $l=1$ in $e^{i2l\theta}$ term and neglecting $\frac{\partial v_2^1}{\partial \xi}, \frac{\partial n_2^1}{\partial \xi}$

$$2\omega n_2^2 = 2v_2^2 k + kv_1^1 n_1^1$$

$$n_2^2 = v_2^2\frac{k}{\omega} + \frac{kv_1^1 n_1^1}{2\omega}$$

solving Eq. $(3.54-3.56)$ simultaneously, we get

$$\left(4k^2+c_1\right)\phi_2^2 - c_2\left(\phi_1^1\right)^2 = \left(\frac{kv_1^{1^2}}{\omega} + \frac{k}{\omega}\phi_2^2\right)\frac{k}{\omega} + \frac{kv_1^1 n_1^1}{2\omega}$$

$$\left(4k^2+c_1\right)\phi_2^2 - \frac{k^2}{\omega^2} = c_2\left(\phi_1^1\right)^2 + \frac{k^2}{\omega^2}v_1^{1^2} + + \frac{kv_1^1 n_1^1}{2\omega}$$

now use

$$n_1^1 = \frac{k^2}{\omega^2}\phi_1^1 \qquad , \qquad v_1^1 = \frac{k}{\omega}\phi_1^1$$

$$k^2 + c_1 = \frac{k^2}{\omega^2}$$

hence

$$\left[\left(4k^2+c_1\right) - \left(k^2+c_1\right)\right]\phi_2^2 \;= \left[c_2 + \frac{3k^4}{2\omega^4}\right]\phi_1^{1^2}$$

$$3k^2\phi_2^2 = \left[c_2 + \frac{3\left(k^2+c_1\right)^2}{2}\right]\phi_1^{1^2}$$

$$\phi_2^2 = \left[\frac{c_2}{3k^2} + \frac{\left(k^2+c_1\right)^2}{2}\right]\phi_1^{1^2}$$

$$\phi_2^2 = A_\varphi {\phi_1^1}^2 \tag{3.57}$$

$$A_\varphi = \left[\frac{c_2}{3k^2} + \frac{\left(k^2+c_1\right)^2}{2}\right]$$

use Eq. (3.37) in Eq. (3.35)

$$n_2^2 = \left(4k^2+c_1\right) A_\varphi \left(\phi_1^1\right)^2 - c_2 \left(\phi_1^1\right)^2$$

$$n_2^2 = \left[\left(4k^2+c_1\right) A_\varphi - c_2\right] \left(\phi_1^1\right)^2$$

$$n_2^2 = A_n \left(\phi_1^1\right)^2 \tag{3.58}$$

$$A_n = \left(4k^2+c_1\right) A_\varphi - c_2$$

now considering

$$n_2^2 = v_2^2 \frac{k}{\omega} + \frac{k v_1^1 n_1^1}{2\omega}$$

use

$$n_1^1 = \frac{k^2}{\omega^2}\phi_1^1 \quad , \quad v_1^1 = \frac{k}{\omega}\phi_1^1 \quad , \quad n_2^2 = A_n \left(\phi_1^1\right)^2$$

$$A_n \left(\phi_1^1\right)^2 = v_2^2 \frac{k}{\omega} + \left(\phi_1^1\right)^2 \frac{k^4}{2\omega^4}$$

$$v_2^2 = \frac{\omega}{k}\left[A_n - \frac{\left(c_1+k^2\right)^2}{2}\right]\left(\phi_1^1\right)^2$$

$$v_2^2 = A_u \left(\phi_1^1\right)^2 \tag{3.59}$$

where

$$A_u = \frac{\omega}{k}\left[A_n - \frac{\left(c_1+k^2\right)^2}{2}\right]$$

Similarly B_Φ,B_n,B_u are calculated by comparing $n=3$ and $l=0$ terms for continuity equation, motion equation and comparing $n=2$ and $l=0$ terms for poison's equation

Continuity equation given for $n=3$ and $l=0$

$$-v_g\frac{\partial n_0^2}{\partial \xi}+\frac{\partial v_0^2}{\partial \xi}+2\frac{\partial v_1^1 n_1^1}{\partial \xi}=0$$

$$-v_g\partial n_0^2+\partial v_0^2+2\partial v_1^1 n_1^1=0$$

integrating

$$-v_g n_0^2+v_0^2+2v_1^1 n_1^1=0 \tag{3.60}$$

equation of motion for $n=3$ and $l=0$

$$-v_g\frac{\partial v_0^2}{\partial \xi}+\frac{\partial \phi_0^2}{\partial \xi}+2v_1^1\frac{\partial v_1^1}{\partial \xi}=0$$

$$-v_g\partial v_0^2+\partial \phi_0^2+2v_1^1\partial v_1^1=0$$

integrating

$$-v_g v_0^2+\phi_0^2+2v_1^1 v_1^1=0 \tag{3.61}$$

Poison's equation, comparing $n=2$ and $l=0$

$$-c_1\phi_0^2+n_0^2+2c_2\phi_1^1\phi_1^1 \tag{3.62}$$

solving Eq. $(3.60-3.62)$ simultaneously, we get

$$\phi_2^2=\frac{\left[2c_2v_g^2+3c_1+k^2\right]}{v_g^2c_1-1}\phi_1^{1^2}$$

$$B_\Phi=\frac{\left[2c_2v_g^2+3c_1+k^2\right]}{v_g^2c_1-1}$$

$$B_n=c_1B_\Phi-2c_2$$

$$B_u=-2\frac{\omega}{k}\left[k^2+c_1\right]^2+v_gB_n$$

$$v_0^2 = B_u \left(\phi_1^1\right)^2 \ , \ n_2^2 = B_n \left(\phi_1^1\right)^2 \quad , \quad \phi_2^2 \ = \ B_\Phi \left(\phi_1^1\right)^2$$

now finding the third order term for equation of continuity $n = 3$ and $l = 1$

$$-i\omega n_1^3 + ikv_1^3 - v_g \frac{\partial n_1^2}{\partial \xi} + \frac{\partial n_1^1}{\partial \tau} + \frac{\partial v_1^2}{\partial \xi} + ikv_0^2 v_1^1 - ikv_2^2 v_{-1}^1 + ikn_1^1 v_0^2 + ikn_{-1}^1 v_2^2 = 0$$

using values of $n_1^2, n_1^1, v_1^2, n_0^2, v_1^1, n_2^2, v_{-1}^1, v_0^2, n_{-1}^1, v_2^2$ in terms of ϕ_1^1 the equation can be simplified.

Finding third order terms for equation of motion using $n = 3$ and $l = 1$.Equation obtained as follows

$$-i\omega v_1^3 + ik\phi_1^3 - v_g \frac{\partial v_1^2}{\partial \xi} + \frac{\partial v_1^1}{\partial \tau} + \frac{\partial \phi_1^2}{\partial \xi} + ikv_0^2 v_1^1 - ikv_2^2 v_{-1}^1 + 2ikv_{-1}^1 v_2^2 = 0$$

using values of $n_1^2, n_1^1, v_1^2, n_0^2, v_1^1, n_2^2, v_{-1}^1, v_0^2, n_{-1}^1, v_2^2$, ϕ_1^2 in terms of ϕ_1^1 the equation can be simplified.

Finding third order terms for poissons equation using $n = 2$ and $l = 1$.we get

$$-\left(c_1 + k^2\right) \phi_1^3 + n_1^3 + 2ik \frac{\partial \phi_1^2}{\partial \xi} + \frac{\partial^2 \phi_1^1}{\partial \xi^2} + c_2 \phi_0^2 \phi_1^1 + c_2 \phi_2^2 \phi_{-1}^1 + c_2 \phi_0^2 \phi_1^1 + c_2 \phi_2^2 \phi_{-1}^1 = 0$$

using values of $\phi_2^2, n_1^1, v_1^2, n_0^2, v_1^1, n_{-1}^1, v_2^2$, ϕ_1^2 in terms of ϕ_1^1 the equation can be simplified

Finally solving all above third order equation gives Schrodinger equation .

$$i \frac{\partial \phi_1^1}{\partial \tau} + P \frac{\partial^2 \phi_1^1}{\partial \xi^2} Q \left| \phi_1^{1^2} \right| \phi_1^1 = 0 \tag{3.63}$$

now in nonlinear schrodinger equation the constant P is calculated as

$$\frac{d\omega}{dk} = c_1 \frac{\omega^3}{k^3}$$

$$P = \frac{1}{2} \frac{d^2\omega}{dk^2}$$

$$P = \frac{3}{2}c_1\frac{\omega^3}{k^4}\left[c_1\frac{\omega^2}{k^3}\frac{d\omega}{dk} - \frac{1}{2}c_1 3\frac{\omega^3}{k^4}\right]$$

$$P = \frac{3}{2}c_1\frac{\omega^3}{k^4}\left[c_1\frac{\omega^3}{k^2} - 1\right]$$

using

$$\omega^2 = \frac{k^2}{c_1 + k^2}$$

$$P = \frac{3}{2}c_1\frac{\omega^3}{k^4}\left[\frac{c_1}{c_1 + k^2} - 1\right]$$

$$P = -\frac{3}{2}c_1\frac{\omega^5}{k^4} \tag{3.64}$$

$$Q = \frac{\omega^3}{2k^2}\left[\begin{array}{c} 3c_3 - 2c_2\left(A_\varphi + B_\varphi\right) - 2\frac{k}{\omega}\left(k^2 + c_1\right)\left(A_u + B_u\right) \\ -\left(k^2 + c_1\right)\left(A_n + B_n\right) \end{array}\right]$$

3.4 Stability analysis

In unmagnetized electron ion plasma the modulational instability of ion acoustic waves are examined by splitting amplitude in to two parts.

$$a = \left\{a_0 + \delta a\left(\chi\right)e^{i\Delta\tau}\right\} \tag{3.65}$$

In the above equation a nonlinear frequency shift Δ, real amplitude of IAW is a, the small amplitude perturbation is δa, where $\delta a << a$ and $\chi = K\zeta - \Omega\tau$ is the modulation phase of the wave, where frequency of modulation is $\omega >> \Omega$ and wave number is $k >> K$ respectively.

$$\Delta = -Qa_0^2$$

now when we substitute Eq. (3.66) into Eq. (3.65) and then collecting the same order terms we get

$$i\frac{\partial\delta a}{\partial\tau} + P\frac{\partial^2\delta a}{\partial\zeta^2} + Qa_0^2\left(\delta a + \delta\bar{a}\right) \tag{3.67}$$

where $\delta\bar{a}$ is complex conjugate to δa. Now assuming perturbation of the form

$$\delta a = \{U_0, V_0\} \exp\{i(K\zeta - \Omega\tau) + cc\}$$

where V_0 and U_0 are real constant using above equation in Eq. (3.67), we obtained two coupled equation by separating the real and imaginary part.

$$\frac{\partial V}{\partial \tau} = P\frac{\partial^2 U}{\partial \zeta^2} + 2Qa_0^2 U$$

and for the nontrival solution the following dispersion relation for Ion acoustic waves IAW in e-i plasma is obtained.

$$\frac{\partial V}{\partial \tau} = P\frac{\partial^2 U}{\partial \zeta^2}$$

For the ion acoustic waves the nontrival solution of dispersion relation in electron ion plasma is

$$\Omega^2 = PK^2\left(PK^2 - 2Qa_0^2\right)$$

hence instability growth rate is

$$\Gamma = \mathrm{Im}\left(\Omega\left(K\right)\right)$$

finally instability growth rate is defined as, if $PQ > 0$, the amplitude of IAW a grows and becomes unstable and if $PQ < 0$ the amplitude of IAW a is stable to external perturbation this is called modulational instability. This exist for the critical values is greater than the wave number.

$$K_{cr} = \sqrt{\frac{2Q}{Pa_0}}$$

Where coefficients P and Q are functions of the q-nonextensive parameter so this parameter would alter the conditions of modulational instability, or we can say that the wave remains stable at $K_{cr} << k$ and becomes unstable at $K_{cr} >> k$. Dark solitons ac-cures in the former case, while bright envelope solitons accures in the latter region.

3.5 Graphical representation of analytical result

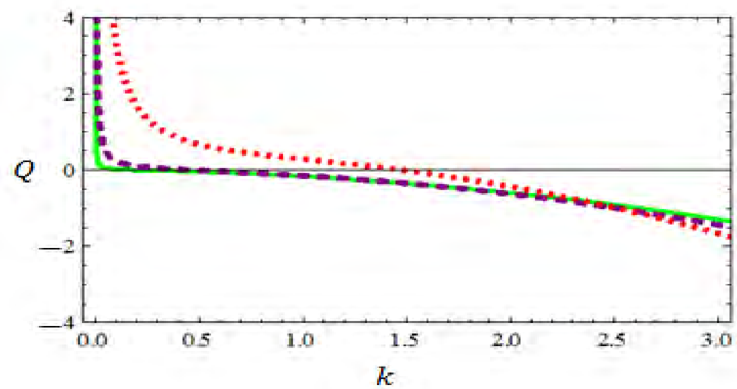

Figure 3.5: Variation of the NLSE coefficient Q with the carrier wave number k for different values of q-non-extensive parameter q. Solid curve corresponds to q=0.1; Dashed curve to q=0.3 and Dotted curve to q=1.

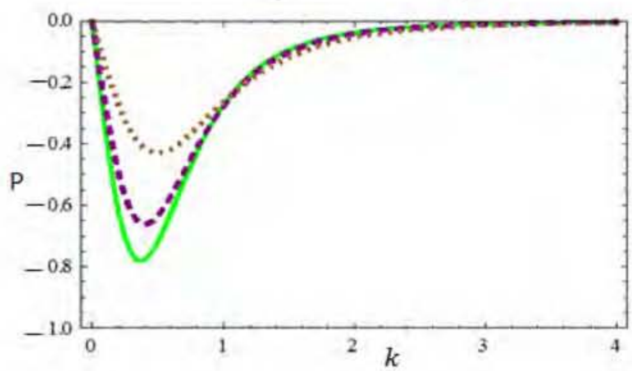

Figure 3.6: Variation of the NLSE coefficient Q with the carrier wave number k for different values of q-non-extensive parameter q. Solid curve corresponds to q=0.1; Dashed curve to q=0.3 and Dotted curve to q=1.

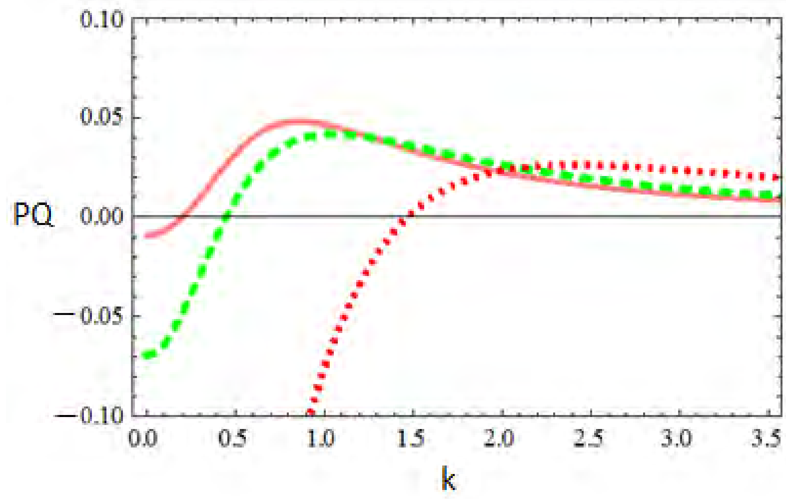

Figure 3.7: Variation of the NLSE coefficients PQ with the carrier wave number k for different values of q-non-extensive parameter q. Solid curve corresponds to q=0.1; Dashed curve to q=0.3 and Dotted curve to q=1.

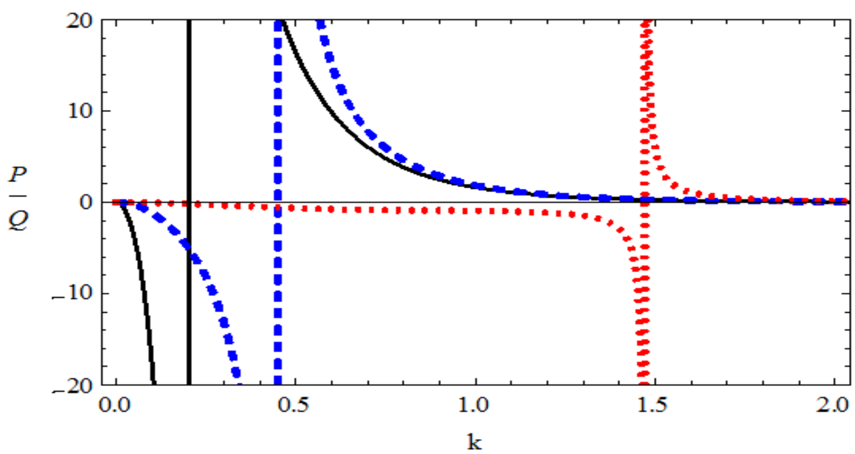

Fig. 3.8 Variation of the NLSE coefficients P/Q with the carrier wave number k for different values of q-non-extensive parameter q. Solid curve corresponds to q=0.1; Dashing curve to q=0.3 and Dotted curve to q=1.

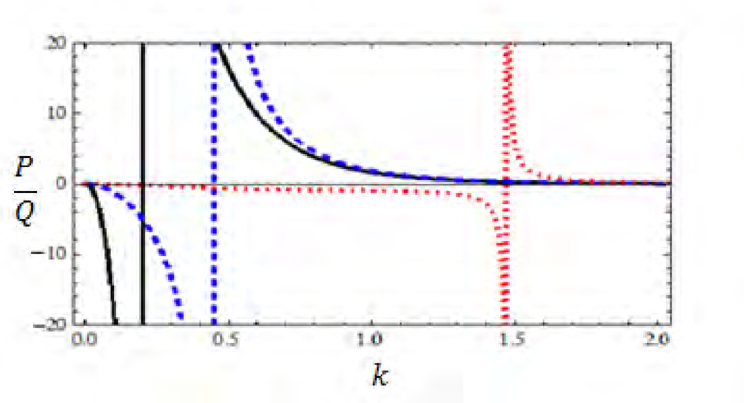

Figure 3.9: Variation of the NLSE coefficients P/Q with the carrier wave number k for different values of q-non-extensive parameter q. Solid curve corresponds to q=0.1; Dashed curve to q=0.3 and Dotted curve to q=1.

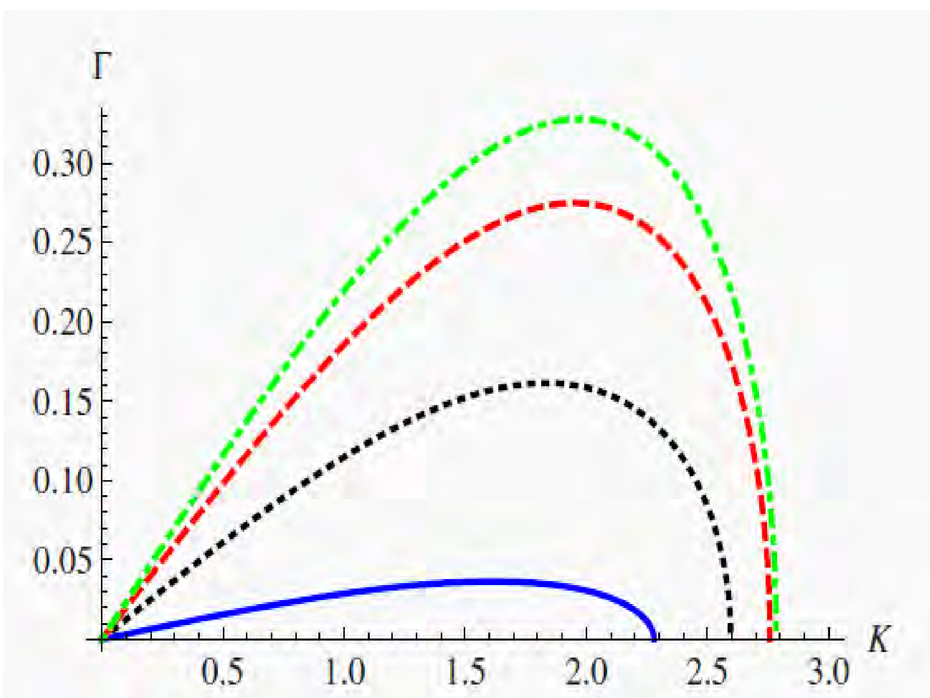

Fig 3.10 Variation of growth rate with wave number for different values of q-non-extensive

parameter q. solid curve q=-0.9 dotted curve q=-0.6 Dashed curve q=-0.3 Dotted dashed curve q=-0.1

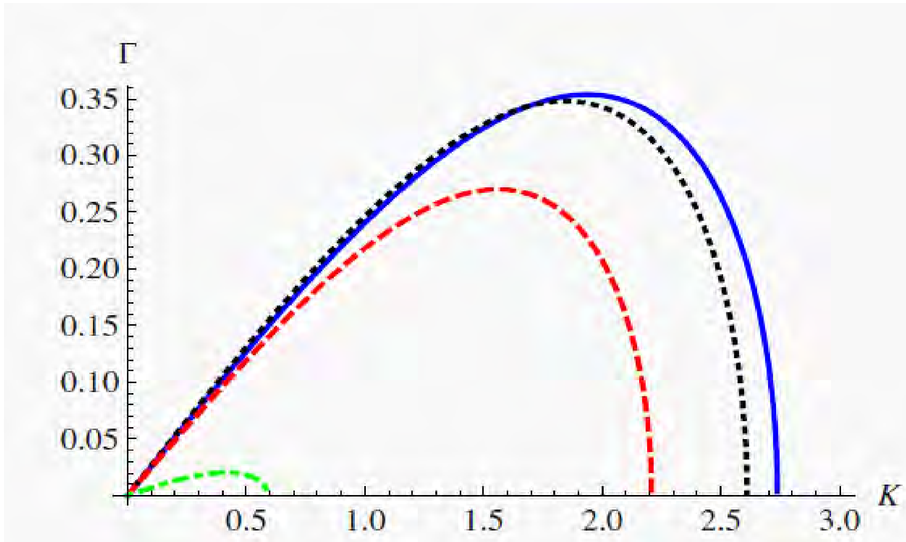

Fig 3.11 Variation of growth rate with wave number k for diffrent value of q non extensive parameter Solid curve q=0.1 dotted curve q=0.3, dashed curve q=0.6, dot-dashed curve q=1.

3.6 Conclusion

We have studied the problem of modulational instability of ion acoustic waves in e-i plasma with electron velocity distribution taken as q-nonextensives. The parameter q justifies the generalized entropy proposed by tsallis. Nonlinear schrodinger equation is derived by using reductive perturbation technique. Frequency of IAW decreases with the increase in the value of parameter q. Three different regions of q-nonextensive parameter on modulational instability are discussed. Bright and dark excitations are formed in each case. The critical value of wave number k at which modulational instability is formed increases for 0<q<1 and decreases for -1<q<0. If we increase the value of k beyond 1 the it gives us large values of critical wave number. One important thing which must be noted that q=0 is not the special value in any distribution but at q=0 our results changes severly.Growth rate increasing with wave number for negative value of q, and decreasing for positive value of q. We have tried our best but fail to find the physical reson for q=0 behaviour. Our theoretical results are applicable in laboratory and space e-i plasmas with q-nonextensive electron velocity distribution.

Standard multiple scale method has been discussed to study the modulational instability of ion acoustic waves (IAWs) in unmagnetized electron ion plasma. Ions are assumed to be cold while electrons taken are q-nonextensive distributed..Group velocity gives us information that ion acoustic wave is propagating or it is just a simple oscillation. As group velocity is function of wave vector k so ion acoustic wave is propagated.

Chapter 4

Obliquely propagation non extensive dust-ion- acoustic solitory wave in dusty magnetoplasma

4.1 Model

We consider a collisionless, three components of magnitized dusty plasma system in which non-linear DIA wave propegated. This system consisting inertial ions, non inertial electron following nonextensive q-distribution, and stationary dust that is negatively charged. At equilibrium

$$n_{i0} = n_{e0} + Z_d n_{d0} 0$$

where n_{e0}, n_{i0}, and n_{d0} are number densities of electron, ion and dust. Z_d is the number of electrons occupy on the surface of dust particle. The range of Z_d is 10^3 to 10^5. For the study of dust ion acoustic waves range of Z_d is used in both experimental and theoratrical observations [33]. In presence of an external magnetic field the phase speed of DIA waves much smaller than the thermal speed of electron and larger than the thermal speed of ion.

External magnetic field

$$B_0 = \acute{z} B_0$$

Equation of motion

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i u_i) = \acute{z} B_0 \tag{4.1}$$

Equation of continuity

$$\frac{\partial u_i}{\partial t} + (u_i \cdot \nabla) u_i = -\nabla \phi + \omega_{ci} (u_i \times \acute{z}) \tag{4.2}$$

Poison's equation

$$\nabla^2 \phi = -n_i + (1 - \mu) n_e + \mu \tag{4.3}$$

The electron density in normalized form

$$n_e = [1 + (q-1)\phi]^{\frac{1+q}{2(q-1)}}$$

$$n_e = [1 + (q-1)\phi]^{\frac{1+q}{2(q-1)}}$$

$$n_e = 1 + c_1\phi + c_2\phi^2 + c_3\phi^3 ... \tag{4.4}$$

where

$$c_1 = \frac{(q+1)}{2}$$

$$c_1 = \frac{(q+1)(q-3)}{8}$$

$$c_2 = \frac{(q+1)(q-3)(3q-5)}{48}$$

ion number density is n_i and it is normalized by equilibrium value n_{i0}.

Fluid speed of ion u_i is normalized by

$$C_i = \left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}$$

Electrostatic wave potential ϕ is normalized by

$$\phi = \frac{k_B T_e}{e} \quad , \quad \mu = \frac{Z_d n_{d0}}{n_{i0}}$$

where Boltzmann constant K_B, nonextensive parametere is q, and e is electron charge's magnitude.

normalization of time variable t is

$$\omega_{pi}^{-1} = \left(\frac{m_i}{4\pi e^2 n_{i0}}\right)^{\frac{1}{2}}$$

and space variable is normalized by Debye length of ion

$$\lambda_{Di} = \left(\frac{k_B T_i}{4\pi e^2 n_{i0}}\right)^{\frac{1}{2}}$$

λ_{Di} is sheath thickness, here we have $n_{i0} >> n_{e0}$ and $T_i \leq T_e$, hence $\lambda_{Di} \simeq \lambda_{De}$ so dust grains of negatively charged in dusty plasma, the temperature and density of ion is determined the thickness of the sheath λ_{Di}.

4.1.1 Outline of method

Dynamical equation for DIA solitary wave is derived by using Eq. $(4.1 - 4.4)$ with small and finite amplitude of nonextensive electrons. We construct for the DIA waves a nonlinear theory and here we follow the reductive perturbation technique. So the independent variable are

$$\xi = \varepsilon^{\frac{1}{2}} \left(l_x x + l_y y + l_z z - V_p t\right)$$

$$\tau = \varepsilon^{\frac{3}{2}} t$$

where the weakness of dispersion is measured by ε that is smaller parameter $(0 < \varepsilon < 1)$, ion acoustic speed (C_i) is normalized the phase speed V_p and $l_{x,} l_y$ and lz along the x, y and z axes, are the direction cosines of the wave vector respectively. So

$$l_x^2 + l_y^2 + l_z^2 = 1$$

here Debye radius (λ_{Di}) normalized the x, y and z, and inverse of ion plasma period $\left(\omega_{pi}^{-1}\right)$ normalized the τ expanding the power series of ε

$$n_i = \left(1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + ...\right) e^{il\theta}$$

$$u_{ix} = \left(0 + \varepsilon^{\frac{3}{2}} u_{ix}^{(1)} + \varepsilon^2 u_{ix}^{(2)} + ...\right) e^{il\theta}$$

$$u_{iy} = \left(0 + \varepsilon^{\frac{3}{2}} u_{iy}^{(1)} + \varepsilon^2 u_{iy}^{(2)} + ...\right) e^{il\theta}$$

$$u_{iz} = \left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta}$$

$$\phi = \left(0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + ...\right) e^{il\theta}$$

where

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \varepsilon^{\frac{1}{2}} V_p \frac{\partial}{\partial \xi} + \varepsilon^{\frac{3}{2}} \frac{\partial}{\partial \tau} \tag{4.5}$$

$$_{p}, \frac{\partial}{\partial t}\left(\varepsilon^{\frac{3}{2}} t\right) = \varepsilon^{\frac{3}{2}}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} + \varepsilon^{\frac{1}{2}} l_z \frac{\partial}{\partial \xi} \tag{4.6}$$

lets proceeding for continuity equation (l_z component), considering Eq. (4.1)

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z}\left(n_i u_{iz}\right) = 0$$

First term

$$\frac{\partial n_i}{\partial t} = \frac{\partial n_i}{\partial t} - \varepsilon^{\frac{1}{2}} V_p \frac{\partial n_i}{\partial \xi} + \varepsilon^{\frac{3}{2}} \frac{\partial n_i}{\partial \tau}$$

$$\begin{aligned}
\frac{\partial}{\partial t}\left(n_i\right) \quad = \quad & \frac{\partial}{\partial t}\left(1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + ...\right) e^{il\theta} \\
& -\varepsilon^{\frac{1}{2}} V_p \frac{\partial}{\partial \xi}\left(1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + ...\right) e^{il\theta} \\
& +\varepsilon^{\frac{1}{2}} \frac{\partial}{\partial \tau}\left(1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + ...\right) e^{il\theta}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial t}(n_i) &= -il\omega\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \\
&\quad -\varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial \xi}\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \\
&\quad +\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \tau}\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \qquad (4.7)
\end{aligned}
$$

Second Term

$$
\begin{aligned}
\frac{\partial(n_i u_{iz})}{\partial z} &= \frac{\partial}{\partial z}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta} \\
&\quad +\left(\frac{\partial}{\partial x}+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}\right)\left[\begin{array}{c}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)\\ \left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)\end{array}\right]e^{i2l\theta}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial(n_i u_{iz})}{\partial z} &= ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta} \\
&\quad +\left(\frac{\partial}{\partial x}+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}\right)\left[\begin{array}{c}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)\\ \left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{i2l\theta}\end{array}\right]
\end{aligned}
$$

now combining both terms of continuity equation

$$
\begin{aligned}
& -il\omega\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \\
& -\varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial \xi}\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \\
& +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau}\left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)e^{il\theta} \\
& +ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)e^{il\theta} \\
& +\left(\frac{\partial}{\partial x}+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}\right)\left[\begin{array}{c}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2 u_{iz}^{(2)}+...\right)\\ \left(1+\varepsilon n_i^{(1)}+\varepsilon^2 n_i^{(2)}+\varepsilon^3 n_i^{(3)}+...\right)\end{array}\right]e^{i2l\theta} \\
= \;& 0
\end{aligned}
$$

comparing equation for order $\varepsilon^{\frac{3}{2}}$, now we get

$$\varepsilon^{\frac{3}{2}} V_p \frac{\partial}{\partial \xi} n_z^{(1)} + l_z \varepsilon^{\frac{3}{2}} \frac{\partial u_{iz}^{(1)}}{\partial \xi} = 0$$

$$V_p \frac{\partial}{\partial \xi} n_{iz}^{(1)} + l_z \frac{\partial u_{iz}^{(1)}}{\partial \xi} = 0$$

$$\frac{\partial}{\partial \xi}\left(V_p n_{iz}^{(1)}\right) = \frac{\partial}{\partial \xi}\left(l_z u_{iz}^{(1)}\right)$$

$$V_p n_{iz}^{(1)} = l_z u_{iz}^{(1)}$$

$$n_{iz}^{(1)} = \frac{l_z}{V_p} u_{iz}^{(1)} \tag{4.10}$$

now considering Eq. (4.2) (z component), so we have

$$\frac{\partial u_{iz}}{\partial t} + \left(u_{iz} \cdot \nabla\right) u_{iz} = -\nabla \phi$$

First term $\frac{\partial u_{iz}}{\partial t}$

$$\frac{\partial u_{iz}}{\partial t} = \frac{\partial u_{iz}}{\partial t} - \varepsilon^{\frac{1}{2}} V_p \frac{\partial u_{iz}}{\partial \xi} + \varepsilon^{\frac{3}{2}} \frac{\partial u_{iz}}{\partial \tau}$$

$$\begin{aligned}\frac{\partial u_{iz}}{\partial t} \quad = \quad & \frac{\partial}{\partial t}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta} - \varepsilon^{\frac{1}{2}} V_p \frac{\partial}{\partial \xi}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta} \\ & + \varepsilon^{\frac{3}{2}} \frac{\partial}{\partial \tau}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta}\end{aligned}$$

$$\begin{aligned}\frac{\partial u_{iz}}{\partial t} \quad = \quad & -il\omega\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta} - \varepsilon^{\frac{1}{2}} V_p \frac{\partial}{\partial \xi}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta} \\ & + \varepsilon^{\frac{3}{2}} \frac{\partial}{\partial \tau}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta}\end{aligned} \tag{4.11}$$

Second term $\left(u_{iz} \cdot \nabla\right) u_{iz}$

$$\frac{\partial u_{iz}}{\partial z} = \frac{\partial}{\partial z}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta} + \varepsilon^{\frac{1}{2}} \frac{\partial}{\partial \xi}\left(0 + \varepsilon^{\frac{3}{2}} u_{iz}^{(1)} + \varepsilon^2 u_{iz}^{(2)} + ...\right) e^{il\theta}$$

$$\frac{\partial u_{iz}}{\partial z} = ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)e^{il\theta}$$

$$\begin{aligned} u_{iz}\frac{\partial u_{iz}}{\partial z} &= ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)^{2}e^{il2\theta} \\ &+\left(0+\varepsilon^{\frac{3}{2}}u+\varepsilon^{2}u_{iz}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)\right)e^{il2\theta} \end{aligned} \quad (4.12)$$

Third term $\nabla\phi$

$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}\phi+\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\phi$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_{z}\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}$$

$$\frac{\partial\phi}{\partial z} = ilk\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_{z}\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta} \quad (4.13)$$

now combining all terms

$$\begin{aligned} & -il\omega\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)e^{il\theta}-\varepsilon^{\frac{1}{2}}V_{p}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)e^{il\theta} \\ & +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)e^{il\theta} \\ & +ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)^{2}e^{il2\theta} \\ & +\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^{2}u_{iz}^{(2)}+...\right)\right)e^{il2\theta} \\ = & -ilkl_{z}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_{z}\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta} \end{aligned}$$

$$\begin{aligned}
& -il\omega\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(2)}+...\right)e^{il\theta}-\varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(2)}+...\right)e^{il\theta} \\
& +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(2)}+...\right)e^{il\theta} \\
& +ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(1)}u_{iz}^{(2)}+...\right)e^{il2\theta} \\
& +\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iz}^{(1)}+\varepsilon^2u_{iz}^{(2)}+...\right)\right)e^{il2\theta} \\
= & -ilkl_z\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+....\right)e^{il\theta} \quad (4.14)
\end{aligned}$$

comparing order terms of $\varepsilon^{\frac{3}{2}}$, we get

$$V_p\varepsilon^{\frac{3}{2}}\frac{\partial u_{iz}^{(1)}}{\partial\xi}=l_z\varepsilon^{\frac{3}{2}}\frac{\partial\phi^{(1)}}{\partial\xi}$$

$$\frac{\partial}{\partial\xi}\left(V_pu_{iz}^{(1)}\right)=\frac{\partial}{\partial\xi}\left(l_z\phi^{(1)}\right)$$

$$u_{iz}^{(1)}=\frac{l_z}{V_p}\phi^{(1)} \quad (4.15)$$

now use Eq. (4.15) in Eq. (4.10), we get

$$n_{iz}^{(1)}=\frac{l_z^2}{V_p^2}\phi^{(1)} \quad (4.16)$$

now considering z component of poisson's Eq. (4.3)

$$\nabla^2\phi=-n_{iz}+(1-\mu)\,n_e+\mu$$

using valuse of n_e from Eq. (4.4)

$$\nabla^2\phi=-n_{iz}+(1-\mu)\left(1+c_1\phi^{(1)}+c_2\phi^{(2)}+c_3\phi^{(3)}...\right)+\mu$$

$$\nabla^2\phi = (1-\mu)\left(\begin{array}{c} 1+c_1\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)e^{il\theta}+c_2\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)^2e^{il2\theta} \\ +c_3\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)^3e^{il3\theta} \end{array}\right)$$
$$-\left(1+\varepsilon n_{iz}^{(1)}+\varepsilon^2 n_{iz}^{(2)}+\varepsilon^3 n_{iz}^{(3)}+...\right)e^{il\theta}+\mu \tag{4.17}$$

$$\frac{\partial^2\phi}{\partial z^2}=\frac{\partial}{\partial z}\left[ilk\left(\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+...\right)e^{il\theta}+e^{il\theta}\frac{\partial}{\partial z}\left(\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+...\right)\right]$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial z^2} &= -l^2k^2\left(\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+...\right)e^{il\theta}+e^{il\theta}ilk\left(\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+\varepsilon^3\phi^{(3)}+...\right) \\ &+\frac{\partial}{\partial z}\left[e^{il\theta}\frac{\partial}{\partial z}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial z^2} &= -l^2k^2\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)e^{il\theta}+e^{il\theta}ilk\frac{\partial}{\partial z}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right) \\ &+e^{il\theta}ilk\frac{\partial}{\partial z}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)+e^{il\theta}\frac{\partial^2}{\partial z^2}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2\phi}{\partial z^2} &= -l^2k^2\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)e^{il\theta}+e^{il\theta}2ilk\frac{\partial}{\partial z}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right) \\ &+e^{il\theta}\frac{\partial^2}{\partial z^2}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)\end{aligned}$$

$$\frac{\partial}{\partial z}=\frac{\partial\xi}{\partial z}\frac{\partial}{\partial\xi}=\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial\xi},\frac{\partial}{\partial\xi}=\varepsilon^{\frac{1}{2}}$$
$$\frac{\partial^2}{\partial z^2}=\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial\xi}\frac{\partial}{\partial z}=\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial\xi}\frac{\partial\xi}{\partial z}\frac{\partial}{\partial\xi}=\varepsilon l_z\frac{\partial^2}{\partial\xi^2},\frac{\partial\xi}{\partial z}=l_z\varepsilon^{\frac{1}{2}}$$

now

$$\frac{\partial^2\phi}{\partial z^2} = -l^2k^2\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)e^{il\theta}+e^{il\theta}2ilk\varepsilon^{\frac{1}{2}}l_z\frac{\partial}{\partial\xi}\left(\varepsilon\phi^1+\varepsilon\phi^2+\varepsilon\phi^3+...\right)$$
$$+e^{il\theta}\frac{\partial^2}{\partial\xi^2}\left(\varepsilon^{\frac{1}{2}}\varepsilon^{\frac{1}{2}}l_z^2\left(\varepsilon\phi^1+\varepsilon\phi^2+\varepsilon\phi^3+...\right)\right) \tag{4.18}$$

using Eq. (4.17) in Eq. (4.18), so we get

$$-l^2k^2\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)e^{il\theta}+e^{il\theta}2ilkl_z\frac{\partial}{\partial\xi}\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)$$
$$+e^{il\theta}\frac{\partial^2}{\partial\xi^2}\varepsilon^{\frac{1}{2}}\varepsilon^{\frac{1}{2}}l_z^2\left(\varepsilon\phi^1+\varepsilon^2\phi^2+\varepsilon^3\phi^3+...\right)$$
$$= (1-\mu)\begin{pmatrix} 1+c_1\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)e^{il\theta}+c_2\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)^2e^{il2\theta} \\ +c_3\left(0+\varepsilon\phi^{(1)}+\varepsilon^2\phi^{(2)}+...\right)^3e^{il3\theta}\end{pmatrix}$$
$$-\left(1+\varepsilon n_{iz}^{(1)}+\varepsilon^2n_{iz}^{(2)}+\varepsilon^3n_{iz}^{(3)}+...\right)e^{il\theta}+\mu \tag{4.19}$$

from Eq. (4.19) comparing ε order of terms

$$(1-\mu)\,c_1\varepsilon\phi^{(1)}-\varepsilon n_{iz}^{(1)}=0$$

as from Eq. (4.16)

$$n_{iz}^{(1)}=\frac{l_z^2}{V_p^2}\phi^{(1)}$$

$$(1-\mu)\,c_1\phi^{(1)}=n_{iz}^{(1)}$$

$$(1-\mu)\,c_1\phi^{(1)}=\frac{l_z^2}{V_p^2}\phi^{(1)}$$

$$(1-\mu)\,c_1=\frac{l_z^2}{V_p^2}$$

where $c_1=\frac{q+1}{2}$

$$(1-\mu)\,\frac{q+1}{2}=\frac{l_z^2}{V_p^2}$$

$$V_p = l_z\left[\frac{2}{(q+1)(1-\mu)}\right]^{\frac{1}{2}} \tag{4.20}$$

The Eq. (4.20) gives linear dispersion relation of phase speed of DIA waves. In the nonextensive plasma system the DIA waves propegate with phase speed V_p .

4.2 First order y and x component of electric field drift.

considering Eq. (4.2)

$$\frac{\partial u_{iy}}{\partial t} + (u_{iy}\cdot\nabla)\,u_{iy} = -\nabla\phi + \omega_{ci}\,(u_{iy}\times \acute{z})$$

First term $\frac{\partial u_{iy}}{\partial t}$

$$\frac{\partial u_{iy}}{\partial t} = \frac{\partial u_{iy}}{\partial t} - \varepsilon^{\frac{1}{2}}V_p\frac{\partial u_{iy}}{\partial \xi} + \varepsilon^{\frac{3}{2}}\frac{\partial u_{iy}}{\partial \tau}$$

$$\begin{aligned}\frac{\partial u_{iy}}{\partial t} &= \frac{\partial}{\partial t}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta} - \varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}\\ &\quad +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}\end{aligned}$$

$$\begin{aligned}\frac{\partial u_{iy}}{\partial t} &= -il\omega\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta} - \varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}\\ &\quad +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}\end{aligned} \tag{4.21}$$

Second term $(u_{iy}\cdot\nabla)\,u_{iy}$

$$\frac{\partial u_{iy}}{\partial y} = \frac{\partial}{\partial y}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta} + \varepsilon^{\frac{1}{2}}l_y\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}$$

$$\begin{aligned}u_{iy}\cdot\frac{\partial u_{iy}}{\partial y} &= \frac{\partial}{\partial y}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)^2 e^{il2\theta}\\ &\quad +\left(0+\varepsilon^{\frac{3}{2}}u_{iy}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\right)e^{il2\theta}\end{aligned}$$

$$
\begin{aligned}
u_{iy}\cdot\frac{\partial u_{iy}}{\partial y} &= ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(1)}u_{iy}^{(2)}+...\right)e^{il2\theta} \\
&+\left(0+\varepsilon^{\frac{3}{2}}u_{iy}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}e^{il2\theta}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\right) \quad (4.22)
\end{aligned}
$$

Third term $\nabla\phi$

$$\frac{\partial\phi}{\partial y}=\frac{\partial}{\partial y}\phi+\varepsilon^{\frac{1}{2}}l_y\frac{\partial}{\partial\xi}\phi$$

$$\frac{\partial\phi}{\partial y}=\frac{\partial}{\partial y}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_y\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}$$

$$\frac{\partial\phi}{\partial y}=ilk\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_y\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta} \quad (4.23)$$

Fourth term $\omega_{ci}\left(u_{iy}\times\acute{z}\right)$

$$\omega_{ci}\left(u_i\hat{y}\times\acute{z}\right)=\omega_{ci}u_{ix}=\omega_{ci}\left(0+\varepsilon^{\frac{3}{2}}u_{ix}+\varepsilon^{2}u_{ix}^{(2)}+...\right)e^{il\theta} \quad (4.24)$$

combining all terms now we have from Eq. $(4.20-4.24)$, we get

$$
\begin{aligned}
& -il\omega\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta}-\varepsilon^{\frac{1}{2}}V_p\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta} \\
& +\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)e^{il\theta} \\
& +ilk\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(1)}u_{iy}^{(2)}+...\right)e^{il2\theta} \\
& +\left(0+\varepsilon^{\frac{3}{2}}u_{iy}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\left(\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(0+\varepsilon^{\frac{3}{2}}u_{iy}^{(1)}+\varepsilon^{2}u_{iy}^{(2)}+...\right)\right)e^{il2\theta} \\
= \; & -ilk\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta}+\varepsilon^{\frac{1}{2}}l_y\frac{\partial}{\partial\xi}\left(0+\varepsilon\phi^{(1)}+\varepsilon^{2}\phi^{(2)}+...\right)e^{il\theta} \\
& -\omega_{ci}\left(0+\varepsilon^{\frac{3}{2}}u_{ix}+\varepsilon^{2}u_{ix}^{(2)}+...\right)e^{il\theta}x \quad (4.25)
\end{aligned}
$$

now comparing order of $\varepsilon^{\frac{3}{2}}$ term

$$\omega_{ci}\varepsilon^{\frac{3}{2}}u_{ix}^{(1)}=-l_y\varepsilon^{\frac{3}{2}}\frac{\partial\phi^{(1)}}{\partial\xi}$$

$$u_{ix}^{(1)} = -\frac{l_y}{\omega_{ci}} \frac{\partial \phi^{(1)}}{\partial \xi}$$

similarly solving for y componenet of momentum equation we get

$$u_{iy}^{(1)} = \frac{l_x}{\omega_{ci}} \frac{\partial \phi^{(1)}}{\partial \xi} \tag{4.27}$$

Eq. (4.26) and Eq. (4.27) are the x, y component of electric field drift.

4.2.1 Comparing higher order term of ε^2

Now again considering the Eq. (4.9) and comparing higher order term of ε^2 ,$l = 1$, we get

$$\varepsilon^{\frac{1}{2}} \varepsilon^{\frac{3}{2}} V_p \frac{\partial u_{ix}^{(1)}}{\partial \xi} - \frac{\partial}{\partial x} \left(\varepsilon^2 \phi^{(2)} \right) = \omega_{ci} \varepsilon^2 u_{iy}^{(2)}$$

$$\varepsilon^2 V_p \frac{\partial u_{ix}^{(1)}}{\partial \xi} - \varepsilon^2 \frac{\partial}{\partial x} \left(\phi^{(2)} \right) = \omega_{ci} \varepsilon^2 u_{iy}^{(2)}$$

$$V_p \frac{\partial u_{ix}^{(1)}}{\partial \xi} = \omega_{ci} u_{iy}^{(2)}$$

using Eq. (4.26)

$$u_{iy}^{(2)} = \frac{l_y V_p}{\omega_{ci}^2} \frac{\partial}{\partial \xi} \left(\frac{\partial}{\partial \xi} \phi^1 \right)$$

$$u_{iy}^{(2)} = \frac{l_y V_p}{\omega_{ci}^2} \frac{\partial^2}{\partial^2 \xi} \phi^1 \tag{4.28}$$

similarly for x component

$$u_{ix}^{(2)} = \frac{l_x V_p}{\omega_{ci}^2} \frac{\partial^2}{\partial^2 \xi} \phi^1 \tag{4.29}$$

here $u_{iy}^{(2)}$, $u_{ix}^{(2)}$ are the second order of momentum equation.

Now for poison's equation, considering Eq. (4.17), comparing ε^2 order of terms

$$\frac{\partial^2}{\partial \xi^2} \left(\varepsilon^{\frac{1}{2}} \varepsilon^{\frac{1}{2}} \varepsilon \phi^1 \right) = (1 - \mu) \left[c_1 \varepsilon^2 \phi^{(2)} + c_2 \left(\varepsilon \phi^{(1)} \right)^2 \right] - n_i^{(2)}$$

$$\frac{\partial^2}{\partial \xi^2}\left(\varepsilon^2 \phi^1\right) = (1-\mu)\left[c_1 \varepsilon^2 \phi^{(2)} + c_2 \left(\varepsilon \phi^{(1)}\right)^2\right] - \varepsilon^2 n_i^{(2)}$$

$$\frac{\partial^2}{\partial \xi^2}\left(\phi^1\right) = (1-\mu)\left[c_1 \phi^{(2)} + c_2 \left(\phi^{(1)}\right)^2\right] - n_i^{(2)}$$

$$\frac{\partial^2}{\partial \xi^2}\left(\phi^1\right) = (1-\mu)\left[c_1 \phi^{(2)} + c_2 \left(\phi^{(1)}\right)^2\right] - n_i^{(2)} \tag{4.30}$$

where

$$c_1 = \frac{q+1}{2} \qquad , \qquad c_2 = -\frac{q+1}{2}\frac{(q-3)}{4}\left(\phi^1\right)^2$$

so from Eq. (4.30)

$$\frac{\partial^2}{\partial \xi^2}\left(\phi^1\right) = -n_i^{(2)} + \frac{l_z^2}{V_p^2}\left(\frac{2}{q+1}\right)\left(\frac{q+1}{2}\right)\phi^{(2)} + \frac{l_z^2}{V_p^2}\left(\frac{2}{q+1}\right)\left(\frac{q+1}{2}\right)\frac{(q-3)}{4}\left(\phi^{(1)}\right)^2$$

$$\frac{\partial^2}{\partial \xi^2}\left(\phi^1\right) = -n_i^{(2)} + \frac{l_z^2}{V_p^2}\phi^{(2)} + \frac{l_z^2}{V_p^2}\frac{(3-q)}{4}\left(\phi^{(1)}\right)^2 \tag{4.31}$$

Eq. (4.31) is second order poisson's equation

comparing $\varepsilon^{\frac{5}{2}}$ terms for $l = 1$, for continuity equation

$$-\varepsilon^{\frac{5}{2}} V_p \frac{\partial n_i^2}{\partial \xi} e^{il\theta} + \varepsilon^{\frac{5}{2}} \frac{\partial n_i^1}{\partial \tau} e^{il\theta} + \varepsilon^{\frac{5}{2}} l_z \frac{\partial u_{iz}^2}{\partial \xi} e^{il\theta} = 0$$

$$-V_p \frac{\partial n_i^2}{\partial \xi} + \frac{\partial n_i^1}{\partial \tau} + l_z \frac{\partial u_{iz}^2}{\partial \xi} = 0 \tag{4.32}$$

comparing $\varepsilon^{\frac{5}{2}}$ terms for $l = 1$,for momentum equation

$$-\varepsilon^{\frac{5}{2}} V_p \frac{\partial u_{iz}^2}{\partial \xi} e^{il\theta} = \varepsilon^{\frac{5}{2}} l_z \frac{\partial u_{iz}^2}{\partial \xi}$$

$$-V_p \frac{\partial u_{iz}^2}{\partial \xi} e^{il\theta} = l_z \frac{\partial u_{iz}^2}{\partial \xi} \tag{4.33}$$

comparing ε^3 terms for $l = 1$, for poisson's equation

$$\varepsilon^3 (1-\mu)\left(c_3 \left(\phi^1\right)^3\right) = \varepsilon^3 l_z^2 \frac{\partial^2 \phi^2}{\partial \xi^2} \tag{4.34}$$

using Eq. $(4.32 - 4.34)$, we eliminate the n_i^2, u_{iz}^2 and ϕ^2 along using n_i^1, u_{iz}^1, in terms of ϕ^1.

finally we get nonlinear propegation of dust ion acoustic waves in a magnetized nonextensive dusty plasma, hense K-dV equation.

$$\frac{\partial \phi^1}{\partial \tau} + A\phi^1 \frac{\partial \phi^1}{\partial \xi} + B\frac{\partial^3 \phi^1}{\partial^3 \xi} = 0 \tag{4.35}$$

where

$$A = \frac{V_P^3}{2l_z^2}\left[\frac{3l_z^4}{V_P^4} + (1-\mu)\frac{(q-3)(q+1)}{4}\right] \tag{4.36}$$

$$B = \frac{V_P^3}{2l_z^2}\left[1 + \left(\frac{1-l_z^2}{\omega_{ci}^2}\right)\right] \tag{4.37}$$

solution of stationary solitary wave of the K-dV equation is obtained by transformation of indepedent variables ξ and τ

$$\eta = \xi - U_0\acute{\tau} \qquad , \qquad \tau = \acute{\tau}$$

where U_0 is constant speed, boundary condition $\phi^1 \to 0$, $\frac{d\phi^1}{d\eta} \to 0$, $\frac{d^2\phi^1}{d\eta^2} \to 0$ at $\eta = \pm\infty$

hence solution of Eq. (3.35) is

$$\phi^1 = \phi_m \sec h^2 \left[\frac{\eta}{\Delta}\right] \tag{4.38}$$

where ϕ_m is amplitude, it is normalized by $\frac{k_B T_e}{e}$ and Δ is width which is normalized by λ_{Dm}, given by

$$\phi_m = \frac{3U_0}{A} \qquad , \qquad \Delta = \sqrt{\frac{4B}{U_0}}$$

4.3 Graphical representation of analytical result

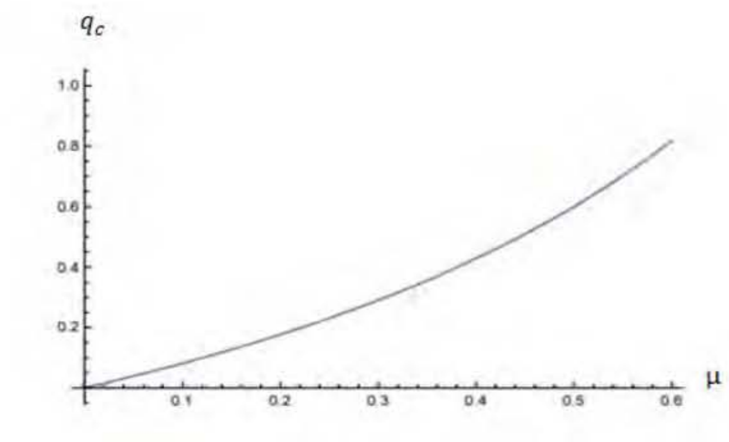

Figure 4.1:Variation of q_c [obtained from $A\,(q = q_c) = 0$]

varies with μ.

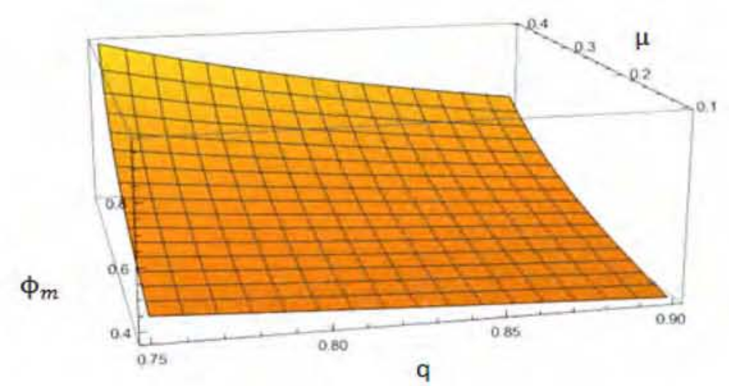

Figure 4.2: Vriation of amplitude of the K-dV Solitons with q

and μ for $\delta = 10°$ and $U_0 = 0.1$.

.

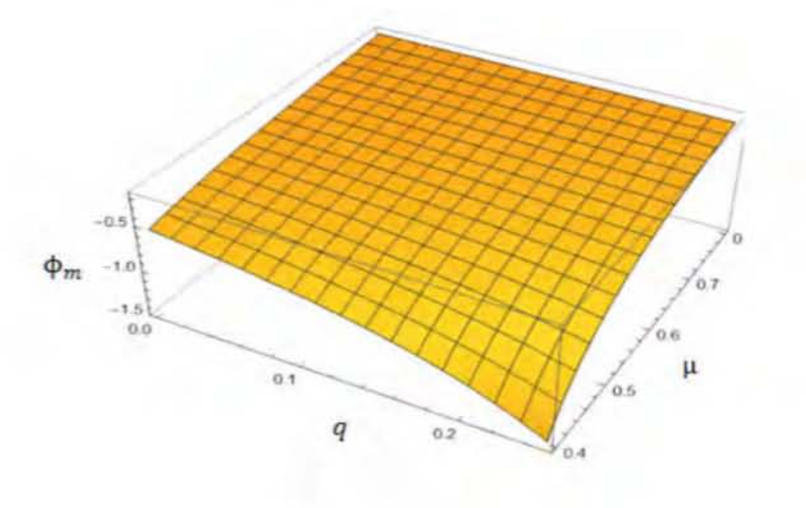

Figure 4.3: Variation of amplitude of the K-dV solitons with q and μ for $\delta = 4^{\circ}$ and $U_0 = 0.1$

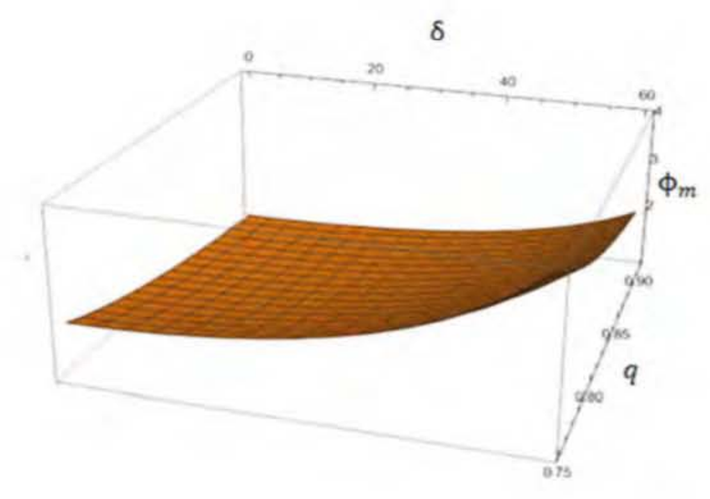

Figure 4.4: Variation of amplitude of K-dV solitons with q and δ for $\mu = 0.5$ and $U_0 = 0.1$.

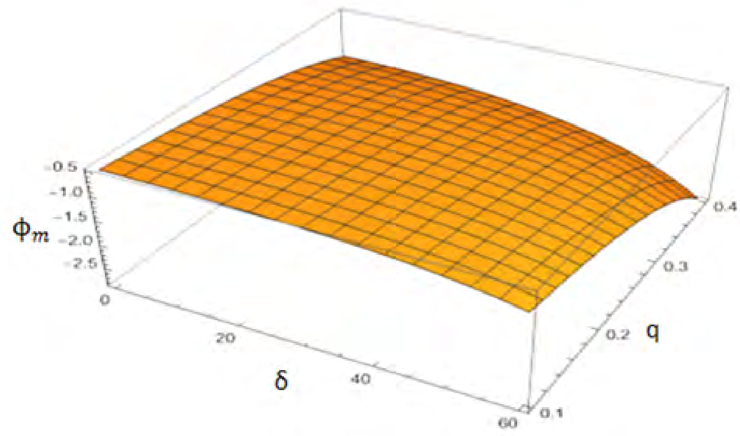

Figure 4.5: Variation of amplitude of the K-dV solitons with q and δ for $\mu = 0.5$ and $U_0 = 0.1$

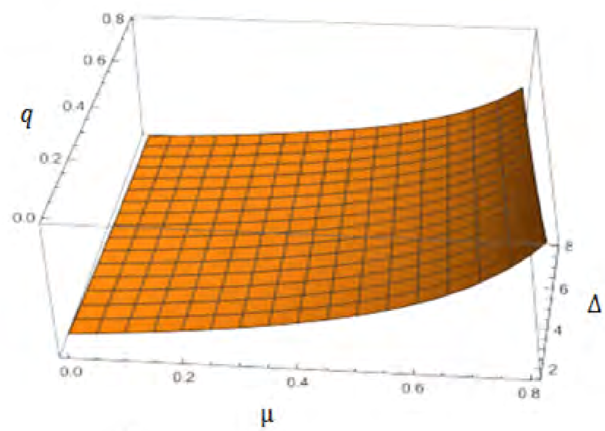

Figure 4.6: Variation of width of the K-dV solitons with q and μ for $\delta = 10^0$,and $U_0 = 1$.

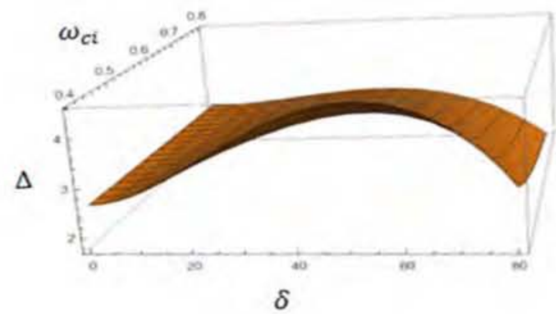

Figure 4.7: Variation of width of the K-dV soliton
with ω_{ci} and δ for $\mu = 0.5$,$q = 0.75$,and $U_0 = 1$.

Polarity the SWs transfer from negative to positive potential at the minimum value of nonextensive prametre q. Amplitude of positive SWs decreases (*increases*) with increasing the value of q, μ. For the lowest range of δ (from $0° to$ $45°$) the width of solitary wave increases, for hiegher range of δ (from $45°$ to $90°$) the width of solitary wave decreases, and at $\delta \rightarrow 90°$ width is goes to zero.The width decreases with increasing (ω_{ci}), which is valid for $\delta < 90°$.

4.4 Conclusion

We have studied consisting of nonextensive electron, negatively charge static dust and inertial ions, in a magnetized dusty plasma system and revealed the presence of obliquely propegating refractive and compressive second order poisson's equation by deriving K-dV equation. KdV equation determined the nonlinear propegation of the DIA waves in magnetized non extensive dusty plasma. As $U_0 > 0$ it depend on the sign of A.The schrodinger waves will be correlated with either negative potential ($\phi_m < 0$) or positive potential ($\phi_m > 0$). When $A > 0$, the schrodinger waves accured with positive potential and $A < 0$,then it exists with a negative potential. If we increased the number density of dust μ, the number density of ions reduced continuously. The parameter q that underpins tsallis generalized entropy is connected to underlying dynamics of the system, the energy of particles of system behaves nonextensively. The amount of its nonextensivity is measured by parametre q.It is also determined that amplitude of schrodinger waves is not effected by magnitude of external magnetic field B_0.It has direct effect on the schrodinger waves width, as width of waves increased by decreasing magnitude of B_0. So the solitary structure become spiky and the system is bounded due to magnetic field. It is found that in the presence of external magneticfield nonextensitivty of electron are modifies the basic features of dust ion acoustic schrodinger waves.The results of this investigation should be helpfull in laboratory plasmas for understanding the nonlinear features of eletrostatic disturbances since the DIA waves are more suitable than the DA waves to examine in laboratory dusty plasma condition. In presence of nonextensive electrons and external magnetic field, we propose to perfom a laboratory experiment in which we can examine the latest features of DIA schrodinger waves propegating in dusty plasma.

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