Non-Linear Potential Structures in Non-Maxwellian Magneto-Rotating Plasmas

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Certificate

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To my Parents, my Family, Respected Supervisor,my dear Cousin Fareed Ul Haq, Supportive Friends

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Abstract

Shock waves in plasma are very important occurrence, consisting of number of different modes. We have investigated such non-linear wave modes as affected by various non-thermal distribution of electrons, magneto-rotating and relativistic effects. In particular, we have looked into the non-linear ion acoustic shock waves on ion time scale in the presence of collisional rotating magneto-plasma with non-Maxwellian electrons and warm relativistic ions. For this a reductive perturbation method has been employed to derive the respective Zakhrove-Kuznetsov-Burgers (ZKB) equation in the weak limit of non-linearity. We also present a comparison between $(r,\ q)$ and κ distribution for electrons. For that first we review the derivation of ZKB equation by using kappa and Cairns distribution, and then extended for a two indices $(r,\ q)$ distribution for electrons. The study of above solutions reveal that due to non-Maxwellian electrons, it causes the formation of different potential structures. From the numerical analysis of ZKB equation, kappa distribution provides only positive amplitude (compressive) shock potential. Unlike kappa, Cairns distribution exhibits both positive and negative amplitudes (compressive and rarefactive), from further analysis it is noted there is a discontinuity arises that fail our model. It has been observed that linear phase speed depicts both fast and slow ion acoustic waves. In the context of kappa and Cairns distributions it is seen that, the formation of compressive and rarefactive nature in our system is mainly due to the non-linear coefficient A, which changes their sign for the respective structure. The formation of shock waves depend upon the magnitude of kappa and Cairns distribution index. Such plasma system is also investigated by using $(r,\ q)$ distribution function, which exhibits both the compressive and rarefactive bahavior in shock potential. In the proper limits, e.g. ($r = 0$ and $q = \kappa - 1$) we recover the results of kappa cases, and for the case ($r = 0$ and q$\rightarrow \infty$) the finding of Maxwellian potential distribution are recovered. The investigations that are discussed can be useful for plasmas that are magnetized, rotating on the view of non-inertial frame of reference, for example such plasma is observed in different region of space like magnetosphere, and pulsars etc.

Chapter 1

Introduction

In this chapter we shall introduce some of the fundamentals and preliminaries that would be used in the subsequent chapters of the dissertation in hand. For example the plasma system of interest here, various phenomena in such plasmas like relativistic and Coriolis effects, particle distribution functions and non-linear wave modes that are considered here.

1.1 Plasma

Plasma is the four state of matter that is characterized by high energy, electrically charged medium consisting of pasitively charged ions and negative electrons. It is found naturally in different regions like stars, auroras, and sun. Plasmas can also be formed artificially, e.g. in fluorescent bulb, and fusion reactors. Generally we say that, an ionized gas can be a plasma if it satisfy the following condition.

1. Quasi-neutrality
This creteria demands that the densities of ions and electrons of the system are approximately equal, i.e. $n_i = n_e = n$. We can also explain quasi-neutrality by the condition $\lambda_D \ll L$, where λ_D is the Debye length and L denotes the characteristic length of our system. Here Debye length is the spatial scale over which plasma particles can effectively shield the potential applied/induced in the plasma. The Debye length for a simple system can be written as

$$\lambda_D = \left(\frac{k_B T_e}{4\pi n_{eo} e^2} \right)^{\frac{1}{2}}$$

2. Collective behavior

For collective bahavior, we must have large number of particles in the Debye sphere N_D and statistical analysis is valid, i.e. $N_D \gg 1$, where

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$

3. Low collisionality with neutrals

It requires that, $\omega\tau > 1$, where ω is the frequency of plasma oscillation and τ is the mean collision time with neutrals. This condition makes sure that the dynamics of our system is governed by electromagnetic interaction among plasma particles and not by ordinary collisions as in the usual hydrodynamics. The plasma frequency can be estimated from the following expression

$$\omega_{pi} = \left(\frac{4\pi n_{io}e^2}{m_i}\right)^{\frac{1}{2}}$$

1.2 Waves in plasma

A wave is characterized by some disturbance in a medium, and can carry energy and momentum. The properties of waves depend upon medium in which they propagate. Here we focus on various aspects of low frequency waves as supported by the plasma medium, and the effects of various plasma parameters on the wave propagation.

Plasma is a medium, that depending upon various conditions, can support propagation of different types of waves like, electron plasma waves in which we assume an unmagnetized system, where ions are at rest and electrons oscillate. And if thermal effects are included the oscillation couples and give rise to the so-called Langmuir waves. Upon considering ion motion and magnetic field, we can find multiple types of other modes, e.g. ion acoustic, Alfven and magnetosonic waves. In our discussion we focus on non-linear ion acoustic waves in a medium composed of magnetized and rotating plasma, furthermore the relativistic effects have also been taken into account. Finally, the perturbed potential is investigated for non-thermal velocity distributions, namely kappa, Cairns, and $(r,\ q)$ profiles.

1.2.1 Ion acoustic waves

The ion acoustic waves (IAWs) are similar to sound waves, composed of large number of compressions and rarefactions. It is a low frequency waves, due to the larger mass of ions, and have constant frequency in the limit of small Debye length. The linearize dispersion relation for IAWs is given as [1]

$$\frac{\omega}{k} = \left(\frac{\gamma_e k_B T_e + \gamma_i k_B T_i}{M} \right)^{\frac{1}{2}}, \tag{1.1}$$

where γ_i and γ_e are, respectively the ion and electron polytropic coefficients and M denotes the mass of ion. In the limiting case, $T_e \gg T_i$, we find

$$\frac{\omega}{k} = \left(\frac{\gamma_e k_B T_e}{M} \right)^{\frac{1}{2}} \tag{1.2}$$

which shows that the dispersion relation depends on electron temperature and ionic mass.

1.2.2 Non-linear waves

The plasma waves having smallar amplitudes can be analyzed by using the linear analysis, in which second and higher order perturbation are ignored. It can be used effectively to describe the propagation and instabilities (if any). However, if the amplitude of the wave becomes larger, then the linear analysis is no longer valid and one needs to employ methods of non-linear theory. The associated waves are termed as the non-linear modes. The non-linear waves are the one whose interaction can lead to wave steeping, wave breaking, and formation of coherent structures. These are described by special equations, having non-linearity, dispersive, and dissipative terms which make them non-linear and depict multiple structures. Here, we focus on shock like potential structures, described by ZKB and KdVB equations, which will be discussed in the subsequent section. Some of the well-known non-linear wave modes are solitons, shocks and vortices.

1.2.3 Shock waves

In general, shock waves are important in the study of fluid theory, when the speed of the particle in the fluid is greater or comparable with sound speed. The corresponding behavior is sound like whose compression and rarefaction give different non-linear effects. If the speed of fluid greater or less, we called it supersonic or subsonic, respectively. The shock structures can be described by the Zakharov-Kuznetsov-Burgers (ZKB) equation that is given as

$$\frac{\partial f}{\partial t} + Af\frac{\partial f}{\partial x} + B\frac{\partial^3 f}{\partial x^3} + C\frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \right) - D\frac{\partial^2 f}{\partial x^2} = 0, \tag{1.3}$$

where $f = f(x,t)$ is a function of space and time, the coefficients A, B and C describe, respectively the non-linear, dispersive and dissipative coefficients.

1.2.4 Solitons

Solitons are formed when there is a balance between non-linearity and dispersion effects in a medium. These waves can retain their shape over long range distance. The Korteweg-de Vries (KdVB) equation gives solitary and shock waves, and is given by [2, 3]

$$\frac{\partial f}{\partial t} + Af\frac{\partial f}{\partial x} + B\frac{\partial^3 f}{\partial x^2} - C\frac{\partial^2 f}{\partial x^2} = 0 \tag{1.4}$$

1.3 Effect of Coriolis forces in plasma medium

In rotating plasma, as is the case for others systems, the Coriolis force become effective. For example such bahavior is observed in magnetoshpere and pulsurs [4]. The Coriolis force is given as

$$\vec{F} = 2m(\vec{v} \times \vec{\Omega}), \tag{1.5}$$

where $\vec{v}$ is the velocity and $\vec{\Omega}$ denotes the rotating frequency of the particle of mass m. Furthermore, the Coriolis force also produce magnetic field when the plasma fluid rotates, here in this study we consider a magneto-rotating plasmas [5].

1.4 Viscosity and dissipation effects

Viscosity is the measure of internal resistance between different layers in a fluids. It is the internal friction when particles move at different speeds. This effect can cause a transfer of irreversible momentum to those layers on which the speed is low. To account for this effect, we add a term $\eta_i \nabla^2 v_i$ in the force balance equation [6]. Here η_i is the ions kinematic viscosity, which cover the density properties and viscosity [7] and μ_i is also viscosity coefficient known as Bulk viscosity, that appears when there is variation in volume (density) in a fluid, causes losses in energy. Therefore, we also included the term $(\mu_i + \eta_i)\nabla(\nabla \cdot v_i)$ in momentum equation. In our plasma system we take into account both of these two viscosities (kinematic and Bulk), which give rise to the non-linear effects. In some cases the Bulk viscosity is ignored due to incompressible fluid approximation, but in this dessertation we take into account, a compressible plasma system. The formation of shock waves arises due to these viscosities (non-linearity and dissipation) [8]. It is found in different sources that the ion kinematic viscosity give rise to shock strucures [9, 10].

1.5 Relativistic effects in plasmas

When the speed of particles becomes extremely high (closer to speed of light) then one has to take into account the relativistic corrections in fluid equations to describe the plasma system. Such behavior is seen in earth magnetosphere [11], laser-plasma interaction [12] and Van Allen radiation belts [13] by using double probe devices. The relativistic plasmas can also be create artificially by using heating method or strong laser beam. This effect is important when the speed of particle approaches to light speed c. i.e.,

$$v \leq c$$

Let us introduce the Einstein relativistic factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

which for weakly relativistic case can be expanded, via the binomial theorem, to write

$$\gamma = 1 + \frac{v^2}{2c^2} + \frac{4v^2}{8c^2}$$

In our study we consider the weak effects, and upon ignoring the highly order terms that yields

$$\gamma \approx 1 + \frac{v^2}{2c^2}$$

1.6 Reductive perturbation method

There are several methods to solve non-linear partial differential equations (PDEs). Here we use a reductive perturbative method, which is based on small perturbation expansion. In this technique we expand the dependent variables in a small parameter ϵ, whose power is obtained from the dispersion relation [14]. To demonstrate, lets consider a simple system, where ion dynamics is considered. For which the model equations in normalize form can be written as (more detail is provided in the next chapter),

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v_i}{\partial x} = 0, \tag{1.6}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{1.7}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i, \tag{1.8}$$

where every dependent variables is the sum of perturbed and equilibrium part which can be written as $n_i = 1 + \bar{n_i}$, $v_i = \bar{v_i}$, $\phi = \bar{\phi}$.
From equation (1.6) we have

$$\frac{\partial}{\partial t}(1 + \bar{n_i}) + \frac{\partial}{\partial x}(\bar{v_i} + \bar{n_i}\bar{v_i}) = 0,$$

upon neglecting the higher order terms we neglect the higher order terms to write

$$\frac{\partial}{\partial t}\bar{n_i} + \frac{\partial}{\partial x}\bar{v_i} = 0,$$

Similarly equations (1.7) and (1.8) can be written as

$$\frac{\partial \bar{u_i}}{\partial t} + \frac{\partial \bar{\phi}}{\partial x} = 0, \tag{1.9}$$

and orderly,

$$\frac{\partial^2 \bar{\phi}}{\partial x^2} = \bar{\phi} - n_i \tag{1.10}$$

Next, we assume that all the perturbed quantities can be represented by the plane-wave approximation, i.e. proportional to $e^{i(kx - wt)}$, to write above equations, respectively in the form

$$-i\omega n_0 + ikv_{i0} = 0$$

$$-i\omega v_{i0} + ik\phi_0 = 0$$

$$n_0(k^2 + 1)\phi_0$$

which are simplified to write the required dispersion relation.

$$\omega^2 = \frac{k^2}{1 + k^2} \tag{1.11}$$

In the limit of large wavelength (small k) limit, using the binomial expansion and then subtracting (kx) on both side, we find

$$\omega = k(1 + k^2)^{\frac{-1}{2}}$$

$$\omega = k - \frac{1}{2}k^3$$

$$\omega t = kt - \frac{1}{2}k^3 t$$

$$kx - \omega t = k(x - t) + \frac{1}{2}k^3 t \tag{1.12}$$

Following that, let us introduce the so - called stretched coordinates in space and time as

$$\xi = \epsilon^{\alpha}(x - \lambda_0 t), \quad \tau = \epsilon^{\beta} t, \tag{1.13}$$

where α and $\beta = (\alpha + 1)$ are the scaling index and $0 < \epsilon < 1$ is a small parameter. Thus, the power of ϵ and expansion of the dependent variable could be chosen from equation (1.14) [3, 4, 15, 16]. The non-linear PDEs, as introduced in the above discussion, can be solved by using hyperbolic-tangent method, which actually provides an approximate solution of the respective PDE, when there is high non-linear effect as correspondence to dispersive counterpart [17]. In particular, we get a travelling solutions which consist of coherent and localized parts [18, 19]. A detailed derivation is provided in the next chapter.

1.7 Maxwell distribution function

All many body systems present a complicated problems in physics, and if there are complicated interactions - as is the case in plasma charged particles - then the detailed knowledge of each particle trajectory is almost impossible to describe. For such systems laws of statistical mechanics are employed, where the identity of individual particle is not important, and one looks at the average behavior of system. Such theories are quite succesful in describe various observed phenomenon. As we know, a plasma system consists of large numbers of charge species and neutrals particles. All these particles have different velocity that carry energy and momentum, but overall when such plasma system is in thermal equilibrium then we use a called Maxwell distribution function. It turns out - from Boltzman H-theorem - that such distribution is the most probable for an equilibrium condition. The general form of Maxwell distribution function is give as [20]

$$f_M(v) = \frac{n_o}{(2\pi)^{\frac{3}{2}} v_{th}} \exp\left(- \frac{v^2}{2v_{th}} \right) \tag{1.14}$$

where v_{th} is the thermal speed and n_o is the equilibrium number density of system particles (electron and ions). Figure (1.1), shows that the average particle velocity is zero that means the system is in thermal equilibrium.

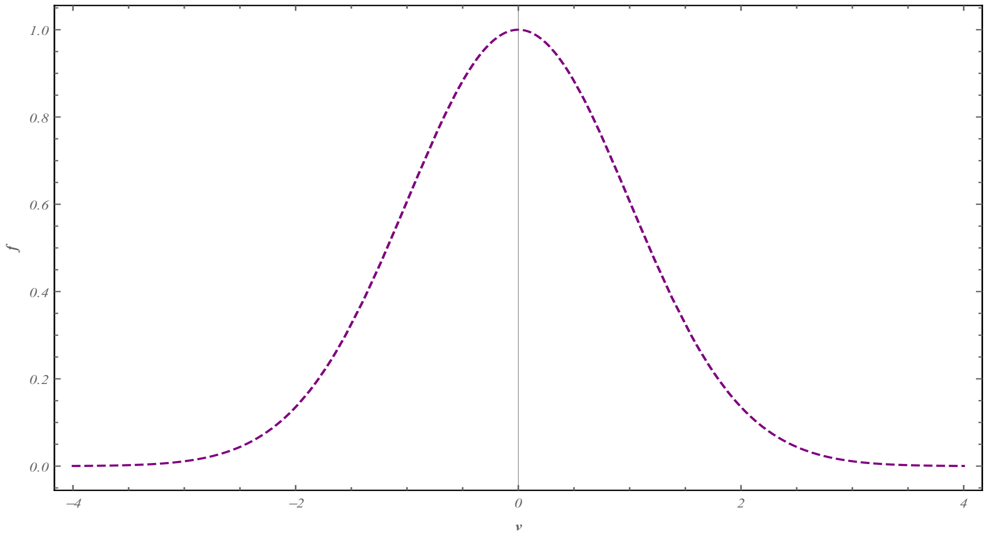

Figure 1.1: *Structure of non-thermal VDF for an equilibrium plasma system.*

1.8 Non-Maxwellian distribution functions

Although, many plasma systems can be described by using the well-known Maxwellian distribution over velocities. And the model works well for highly collisional plasmas, where a thermal equilibrium is achieved and the system attains a Maxwellian profile. However, from the data analysis of various space/astronomical plasmas with the help of satellites it is found, there exist a large number of particle whose distribution functions is different from Maxwellian. Most of such systems consist of highly non-thermal electrons corresponds to collisionless and inhomogenous plasma. These high energy electrons spectra was studied within the plasma sheet and give an empirical velocity distribution formula called **kappa** distribution function, [21, 22] which is given as

$$f_\kappa(v,\phi) = \frac{n_{eo}}{(2\pi)^{\frac{3}{2}}\theta^3}\frac{\Gamma[1+\kappa]}{\Gamma[\kappa-\frac{1}{2}]}\left(1+\frac{v^2-\frac{2e\phi}{m_e}}{\kappa\theta^2}\right)^{-(1+\kappa)}, \tag{1.15}$$

where κ is the spectral index that measures the non-thermal effects and Γ denote the gamma function. Such Lorentzian profile describe the high energy tails in the distribution, and in the limit of very high κ one retrieves the Maxwellian (bell like) profile as depicted in the following figure.

In plasma physics and astrophysics, the kappa distribution in valuable for characterizing the departures from thermal equilibrium that are commonly encountered in these high- energy environments. It provides a more accurate description of particle velocity or energy distribution than the Maxwellian distribution, which assumes thermal equilibrium. Researchers often fit experimental data to the kappa distribution or use it in simulations to better understand and model the behavior of particles in plasmas and other non-equilibrium systems.

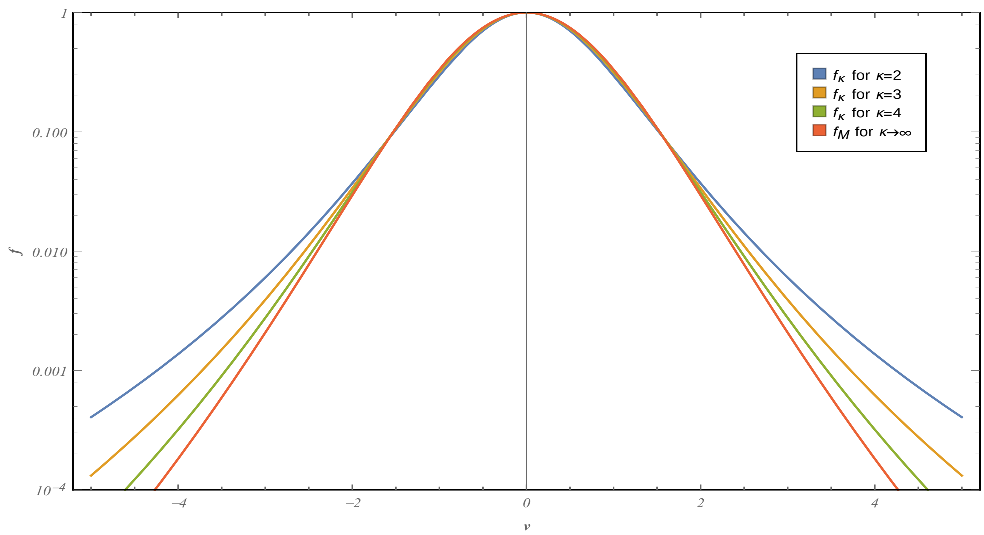

Figure 1.2: *Comparison of Kappa and Maxwellian distribution functions for different values of index (κ).*

In figure (1.1), we note that there is major deviation for small κ, however as the values of κ increases, then the respective distribution approaches to Maxwell counterpart, which corresponds to the dissipation of high energy electrons out from the system. Such system, can be treated by using linear analysis. This means that when there is non-linear effects exist in a system, for that we may use kappa distribution function. [8, 23–26].

Twenty years ago, electrostatic solitary waves were explain by another non-Maxwellian distribution on the ion time scale known as **Cairns** distribution, that was used to model the data as observed by Freja satellite. This type of distribution denotes the existance of rarefactive ion acoustic solitary wave [21, 27]. The Cairns distribution function is given as

$$f_c(v) = \frac{n_{eo}}{(2\pi)^{\frac{1}{2}}(1+3\alpha)v_{th}}\left(1+\frac{\alpha v^4}{v_{th}^4}\right)\exp\left(-\frac{v^2}{2v_{th}^2}\right), \tag{1.16}$$

where α corresponds to the non-thermal electrons papulation, and the distribution approaches to Maxwellian, when $\alpha \to 0$. This scenario is observed in the figure (1.2), means that as the value of non-thermal electron papulation (α) goes to minimun values, the given distribution function approaches to Maxwellian. Here we also note that one may get the shoulders like distribution, although such distributions are generally unstable. In systems undergoing phase transition (e.g., solid to liquid, liquid to gas), you may observe shoulder profiles in energy or density distributions. The shoulder corresponds to particles or molecules that are in a transitional state. This shoulder represents a secondary state or condition in the system that is less probable but still significant. The presence of a shoulder indicates that there is a non-negligible population of particles deviating from the most probable state.

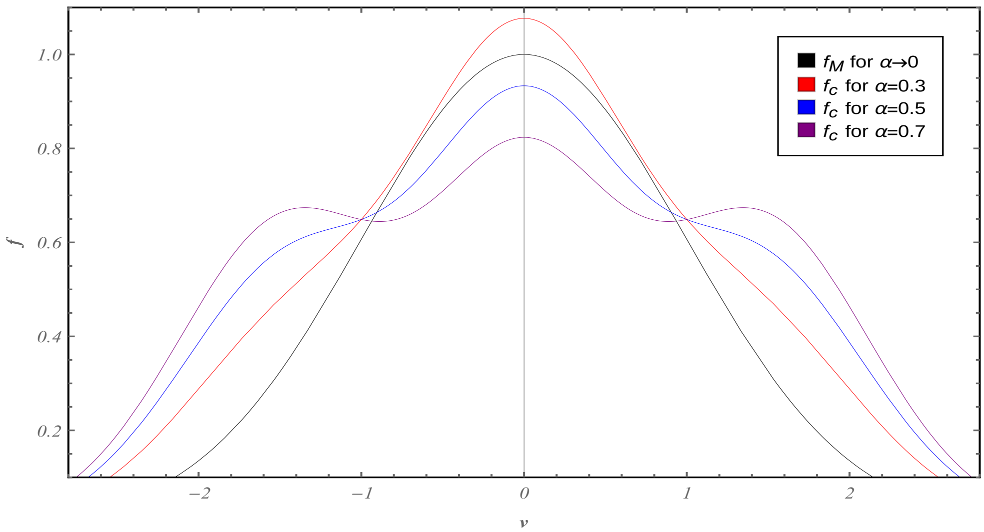

Figure 1.3: *The Cairns VDF for different values of index (α), we recover the Maxwellian counterpart as $\alpha \to 0$.*

There is another type of more general non-thermal electrons distribution known as (r,q) distribution function [28] that can be written as

$$f_{rq}(v) = \frac{3\Gamma[q](q-1)^{\frac{-3}{(2+2r)}}}{4\pi\beta^{\frac{3}{2}} v_{th}^{\frac{3}{2}} \Gamma[q - \frac{3}{2+2r}]\Gamma[1+\frac{3}{2+2r}]} \left(1 + \frac{1}{q-1}\left(\frac{v^2 - \frac{2e\phi}{m_e}}{\beta v_{th}^2}\right)^{r+1}\right)^{-q}, \tag{1.17}$$

where

$$\beta = \frac{3(q-1)^{\frac{-1}{(1+r)}}\Gamma[q - \frac{3}{2+2r}]\Gamma[\frac{3}{2+2r}]}{2\Gamma[q - \frac{5}{2+2r}]\Gamma[\frac{5}{2+2r}]}$$

In above equations v_{th} is the thermal speed of the electrons and r, q are the spectral indices. In the limiting case of small r and large q, we recover the Maxwellian distribution. Figure (1.3) is plotted for different value of q, upon which the ditribution function shows deviation from Maxwell distribution.

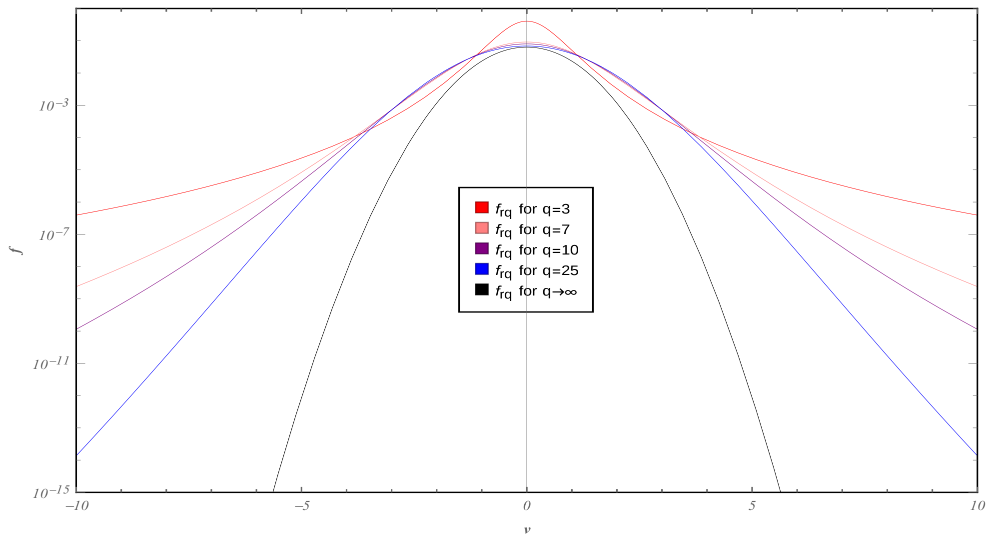

Figure 1.4: *Behavior of normalized (r,q) distribution function against normalized valocity, for $r = 0$.*

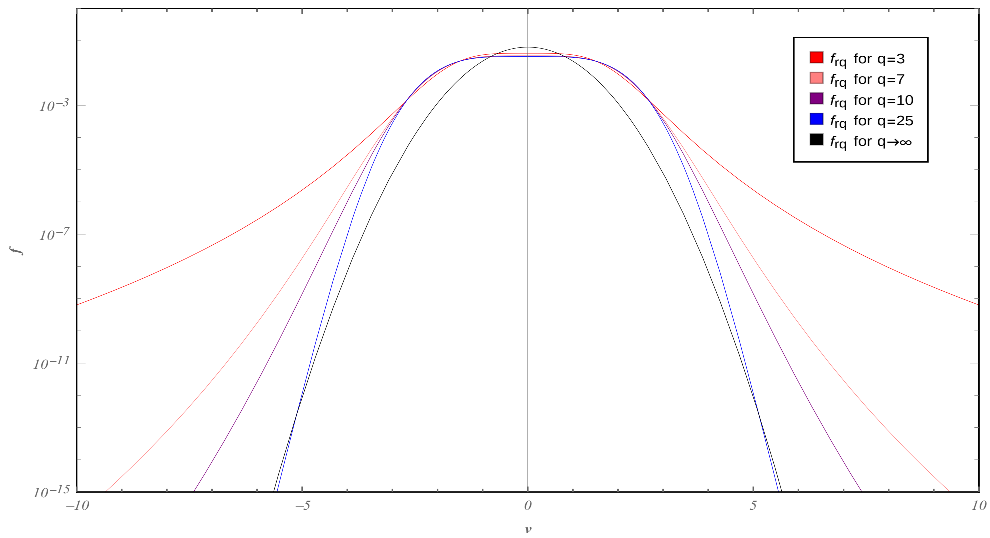

Figure 1.5: *Same as previous figure, but for $r = 1$.*

Figure (1.4), once again plot for different values of q, and fix value of r unlike as we do in above figure. In this figure we observe that, when the value of q increases the the $(r,\ q)$ distribution deviate to Maxwellian counterpart.

The obvious advantage of using two indices distribution function is that once can model huge variety of model data.

Here in this study, we have used a more general (in comparison with kappa and Cairns) velocity distribution function to see the effects of flat-top as well as high energy tails in the VDF. Which is supposed to give a better insight into the PES systems. This type is present in many astrophysical environments, such as the early universe, neutrons stars, active galactic nuclei, Earth's ionosphere, chromosphere, corona, solar winds, and pulsars. Moreover, the laboratory plasmas consisting of pasitive and negative ions with equal masses have been investigated experimentally by Oohara et al. [29–31]. Flate-top particles distributions are commonly observed in space plasmas, e.g., in the downstream of Earth's bow shock, and is caused by various physical processes such as by the interaction of particle with electrostatic potential generated by the bow shock [**?**].

Chapter 2

Shock structures in magneto-rotating relativistic plasmas with non-Maxwellian electrons

In this chapter we shall review the shock like distribution of the perturbed potential in a magnetized non-thermal plasma. For that the corresponding ZKB equation is derived by using a reductive perturbation method in the fluid description of plasmas. Finally, the impact of various parameters on shock profiles are discussed.

2.1 Model equations

Here we consider a plasma system, which is magnetized, weakly relativistic, composed of warm ions and non-Maxwellian electrons. We consider that the magnetic field is along z-direction, i.e., $\vec{B} = B_0\ \hat{z}$, where B_0 is the magnitude of the magnetic field. In equilibrium the quasi-neutrality condition demands that $n_{i0} = n_{e0}$, where n_{i0} and n_{e0} are the unperturbed densities of ions and electrons. The corresponding ion fluid equations, namely continuity, force balanced and Poisson's equation, for our system can be written as

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{2.1}$$

$$\begin{gathered} m_i n_i \left(\frac{\partial}{\partial t} + \vec{v_i} \cdot \vec{\nabla} \right) \gamma \vec{v_i} = e n_i (\vec{E} + \vec{v_i} \times \vec{B}) - \vec{\nabla} p_i + 2 m_i n_i (\vec{v_i} \times \vec{\Omega}) + \\ m_i n_i \eta_i \nabla^2 \vec{v_i} + m_i n_i (\eta_i + \mu_i) \vec{\nabla} (\vec{\nabla} \cdot \vec{v_i}), \end{gathered} \tag{2.2}$$

and, respectively

$$\vec{\nabla} \cdot \vec{E} = 4\pi e (n_i - n_e). \tag{2.3}$$

The left hand side of equation (2.2) is the convective term and the right side contains Lorentz force, pressure gradient, Coriolsis force and viscosity terms. The variables $\vec{v_i}$, n_i, m_i are velocity, density and mass of an ion, Orderly, the symbol η_i is the ion kinematic viscosity and μ_i is the ion bulk viscosity. In our study we focus on bulk viscosity because our plasma is compressible. The Einstein relativistic factor is defined as, $\gamma = \left(1 - \frac{v_{iz}^2}{c^2}\right)^{\frac{-1}{2}}$, and is considered to be weak for our plasma system, hence the binomial expansion can be applied write, $\gamma = 1 + \frac{1}{2}\frac{v_{iz}^2}{c^2}$.
We take v_i in along z-direction because in other two direction the relativistic effects is weak. In equation (2.2), $\vec{\Omega}$ is the frequency of the rotating plasma whose direction is same as magnetic field, i.e., $\vec{\Omega} = \Omega_0 \hat{z}$.
The electron in our system are assumed to have non-Maxwellian distribution, in particular we consider kappa (κ) and Cairns (α) VDF.
The 3D kappa (κ) distribution function is given by

$$f_\kappa(v) = \frac{n_{e0}}{\theta^3}\left(\frac{1}{\pi\kappa}\right)^{\frac{3}{2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\left(1 + \frac{v^2 - 2e\phi/m_e}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{2.4}$$

where θ is the modified thermal speed for electrons and κ is the spectral index which quantifies the deviation from the Maxwellian counterpart. For physically valid VDF we must have which is $\kappa > \frac{3}{2}$. Upon integrating the above distribution over velocity space, the corresponding electron density takes the form [21]

$$n_0 = n_{e0}\left[1 - \frac{e\phi}{(\kappa - \frac{3}{2})T_e}\right]^{-(\kappa-\frac{1}{2})} \tag{2.5}$$

Similarly the Cairns (α) distribution can be written as,

$$f_e(v) = \frac{n_{e0}}{(3\alpha+1)\sqrt{2\pi V^2}}\left(1 + \frac{\alpha v^4}{v_{th}^4}\right)\exp\left(\frac{v^2}{2v_{th}^2}\right), \tag{2.6}$$

where $v_{th} = \sqrt{\frac{T_e}{m_e}}$ is the warm electrons thermal speed and α defines the population of non-thermal electrons. The respective electronic distribution can be written as [35]

$$n_{e0} = n_{e0}\left[1 - \beta\left(\frac{e\phi}{T_e}\right) + \beta\left(\frac{e\phi}{T_e}\right)^2\right]\exp\left(\frac{e\phi}{T_e}\right) \tag{2.7}$$

Here $\beta = \frac{4\alpha}{(1+3\alpha)}$, and in the limit when α (or β) approaches to zero, we recover the Maxwellian counterpart, same is true for kappa distribution when $\kappa \to \infty$.

2.2 Normalizaton of model equations

In mathematical physics, its always easy to deal with dimensionless variables, following that we can write our system of equations in normalized form. For that various normalization factors are defined as follow;

- The electron and ion densities, n_e and n_i are normalized by the equilibrium values, i.e., $\bar{n}_i = \frac{n_i}{n_{io}}$ and $\bar{n}_e = \frac{n_e}{n_{eo}}$
- Space variables (x, y, z) are normalized by electron Debye length which is given by, $\lambda_D = \left(\frac{T_e}{4\pi n_{eo} e^2}\right)^{\frac{1}{2}}$, i.e., $\bar{x} = \frac{x}{\lambda_D}$, $\bar{y} = \frac{y}{\lambda_D}$, $\bar{z} = \frac{z}{\lambda_D}$.
- The velocity of fluid and the light speed are normalized by ion acoustic speed C_s, where $C_s = \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}$.
- The ions cyclotron frequency $\omega_{ci} = \frac{eB_0}{m_i c}$ and the rotation frequency are normalized by ion plasma frequency $\omega_{pi} = \left(\frac{4\pi n_{io} e^2}{m_i}\right)^{\frac{1}{2}}$.
- Electric field is also normalized as, $\bar{E} = \frac{E}{e\lambda_{Di} T_e}$.
- The time variable is normalized by the inverse of ion plasma frequency, i.e., $\bar{t} = t\omega_{pi}$.
- The electrostatic potential is normalized as, $\bar{\phi} = \frac{e\phi}{T_e}$.
- The coefficients of ion fluid viscosity are normalized by $\omega_{pi}\lambda_D^2$.
- Note that we can express ion acoustic speed as, $C_s = \lambda_D \omega_{pi}$.

Upon introducing the diamensionless variables, the continuity equation can be written as,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0$$

$$\frac{\partial(\bar{n}_i n_{eo} \omega pi)}{\partial \bar{t}} + \frac{\partial}{\partial(\bar{x}\lambda_D)}(\bar{n}_i n_{io} C_s \bar{v}_i) = 0$$

$$n_{eo}\omega_{pi}\lambda_D \frac{\partial \bar{n}_i}{\partial \bar{t}} + \frac{\partial}{\partial(\bar{x})}(\bar{n}_i \bar{v}_i n_{io} C_s) = 0$$

$$\frac{n_{e0} C_s}{n_{i0}} \frac{\partial \bar{n}_i}{\partial \bar{t}} + \frac{\partial}{\partial \bar{x}}(\bar{n}_i \bar{v}_i C_s) = 0$$

As $\frac{n_{e0}}{n_{io}} = 1$, hence the continuity equation becomes

$$\frac{\partial \bar{n}_i}{\partial \bar{t}} \frac{\partial}{\partial \bar{x}}(\bar{n}_i \bar{v}_i) = 0 \tag{2.8}$$

The equation of motion can also be written in normalized variables as follows,

$$m_i n_i\left(\frac{\partial}{\partial t}+\vec{v_i}\cdot\vec{\nabla}\right)\gamma\vec{v_i}=en_i(\vec{E}+\vec{v_i}\times\vec{B})-\vec{\nabla}p_i+2m_in_i(\vec{v_i}\times\vec{\Omega})+m_in_i\eta_i\nabla^2\vec{v_i}+m_in_i(\eta_i+\mu_i)\vec{\nabla}(\vec{\nabla}\cdot\vec{v_i})$$

After using the equation of state $p=n_iT_e$ we find

$$\left(\frac{\partial}{\partial t}+\vec{v_i}\cdot\vec{\nabla}\right)\gamma\vec{v_i}=-\frac{e}{m_i}\vec{\nabla}\phi+\omega_{ci}\left(\vec{v_i}\times\hat{z}\right)-\frac{T_i}{m_i}\frac{\vec{\nabla}n_i}{n_i}+2\Omega_0\left(\vec{v_i}\times\hat{z}\right)+\eta_i\vec{\nabla}^2v_i+\left(\eta_i+\mu_i\right)\vec{\nabla}\left(\vec{\nabla}\cdot\vec{v_i}\right)$$

$$\left(\frac{\partial\omega_{pi}}{\partial\bar{t}}+\frac{C_s}{\lambda_D}(\bar{v_i}\cdot\bar{\nabla})\right)C_s\gamma\bar{v_i}+\frac{eT_e}{m_i\lambda_D}\bar{\nabla}\bar{\phi}-\omega_{pi}C_s\bar{\omega_{ci}}\left(\bar{v_i}\times\hat{z}\right)+\frac{T_i\cancel{n_{io}}}{m_i\lambda_D\cancel{n_{io}}}\frac{\bar{\nabla}\bar{n_i}}{\bar{n_i}}-2\bar{\Omega_0}\omega_{pi}C_s\left(\bar{v_i}\times\hat{z}\right)=$$
$$\omega_{pi}C_s\bar{\eta_i}\frac{\cancel{\lambda_D^2}}{\cancel{\lambda_D^2}}\bar{\nabla}^2\bar{v_i}+\omega_{pi}C_s(\bar{\eta}+\bar{\mu})\bar{\nabla}\left(\bar{\nabla}\cdot\bar{v_i}\right)\frac{\cancel{\lambda_D^2}}{\cancel{\lambda_D^2}}$$

$$\omega_{pi}C_s\left(\frac{\partial}{\partial t}+\frac{\cancel{C_s}}{\cancel{C_s}}(\bar{v_i}\cdot\bar{\nabla})\right)\gamma\bar{v_i}+\frac{eT_e}{m_i\lambda_D}\bar{\nabla}\bar{\phi}-\omega_{pi}C_s\bar{\omega_{ci}}\left(\bar{v_i}\times\hat{z}\right)+\frac{T_i}{m_i\lambda_D}\frac{\bar{\nabla}\bar{n_i}}{\bar{n_i}}-2\bar{\Omega_0}\omega_{pi}C_s\left(\bar{v_i}\times\hat{z}\right)=$$
$$\omega_{pi}C_s\bar{\eta_i}\bar{\nabla}^2\bar{v_i}+\omega_{pi}C_s(\bar{\eta}+\bar{\mu})\bar{\nabla}\left(\bar{\nabla}\cdot\bar{v_i}\right)$$

$$\left(\frac{\partial}{\partial t}+(\bar{v_i}\cdot\bar{\nabla}\right)\gamma\bar{v_i}+\frac{\cancel{eT_em_i}}{\cancel{eT_em_i}}\bar{\nabla}p\bar{h}i-\frac{\cancel{\omega_piC_s}}{\cancel{\omega_piC_s}}\bar{\omega_{ci}}\left(\bar{v_i}\times\hat{z}\right)+\frac{T_i\cancel{4\pi n_{io}e^2m_i}}{\cancel{4\pi n_{io}e^2m_i}T_e}\frac{\bar{\nabla}\bar{n_i}}{\bar{n_i}}-\frac{\cancel{\omega_{pi}C_s}}{\cancel{\omega_{pi}C_s}}2\bar{\Omega_0}\left(\bar{v_i}\times\hat{z}\right)=$$
$$\frac{\cancel{\omega_{pi}C_s}}{\cancel{\omega_{pi}C_s}}\bar{\eta}\bar{\nabla}^2\bar{n_i}+\frac{\cancel{\omega_{pi}C_s}}{\cancel{\omega_{pi}C_s}}\left(\bar{\eta_i}+m\bar{u}_i\right)\bar{\nabla}\left(\bar{\nabla}\cdot\bar{v_i}\right)$$

$$\left(\frac{\partial}{\partial t}+\bar{v_i}\cdot\bar{\nabla}\right)\gamma\bar{v_i}+\bar{\nabla}\bar{\phi}-\bar{\omega_{ci}}\left(\bar{v_i}\times\hat{z}\right)+\sigma\frac{\bar{\nabla}\bar{n_i}}{\bar{n_i}}=2\Omega_0\left(\bar{v_i}\times\hat{z}\right)+\bar{\eta_i}\bar{\nabla}^2\bar{v_i}+\left(\bar{\eta_i}+\bar{\mu_i}\right)\bar{\nabla}\left(\bar{\nabla}\cdot\bar{v_i}\right)$$

$$\left(\frac{\partial}{\partial t}+\bar{v_i}\cdot\bar{\nabla}\right)\gamma\bar{v_i}=-\bar{\nabla}\bar{\phi}-\sigma\frac{\bar{\nabla}\bar{n_i}}{\bar{n_i}}+2\Omega_0\left(\bar{v_i}\times\hat{z}\right)+\bar{\omega_{ci}}\left(\bar{v_i}\times\hat{z}\right)+\bar{\eta_i}\bar{\nabla}^2\bar{v_i}+\left(\bar{\eta_i}+\bar{\mu_i}\right)\bar{\nabla}\left(\bar{\nabla}\cdot\bar{v_i}\right),\tag{2.9}$$

where $\Omega_c=\omega_{ci}+2\Omega_0$ and $\sigma=\frac{T_i}{T_e}$, represents the temperature ratio of ions to electrons.

Poisson's equation also transforms in the following manner,

$$\frac{\bar{\nabla}^2\bar{\phi}T_e}{\lambda_D^2e}=4\pi e\left(n_{eo}\bar{n_e}-n_{io}\bar{n_i}\right)$$

$$\frac{\cancel{T_e4\pi n_{io}e^2}}{\cancel{T_e4\pi n_{io}e^2}}\bar{\nabla}^2\bar{\phi}=\left(\bar{n_e}-\bar{n_i}\right)$$

$$\bar{\nabla}^2\bar{\phi}=\bar{n_e}\bar{n_i}$$

The normalized version of kappa density distribution takes the form,

$$n_0 = n_{e0}\left[1 - \frac{e\phi}{(\kappa - \frac{3}{2})T_e}\right]^{-(\kappa - \frac{1}{2})} \tag{2.10}$$

$$\cancel{n_{e0}}\bar{n_e} = \cancel{n_{e0}}\left[1 - \frac{\bar{\phi}}{(\kappa - \frac{3}{2})}\right]^{-(\kappa - \frac{1}{2})}$$

$$\bar{n_e} = \left[1 - \frac{\bar{\phi}}{(\kappa - \frac{3}{2})}\right]^{-(\kappa - \frac{1}{2})}, \tag{2.11}$$

where $\bar{\phi} = e\frac{\phi}{T_e}$. Likewise, for the Cairns VDF we find

$$\cancel{n_{e0}}\bar{n_e} = \cancel{n_{e0}}\left[1 - \beta\left(\frac{\cancel{e}\cancel{T_e}\bar{\phi}}{\cancel{T_e}\cancel{e}}\right) + \beta\left(\frac{\cancel{e}\cancel{T_e}\bar{\phi}}{\cancel{T_e}\cancel{e}}\right)^2\right]\exp\left(\frac{\cancel{e}\cancel{T_e}\bar{\phi}}{\cancel{e}\cancel{T_e}}\right)$$

$$\bar{n_e} = \left[1 - \beta\bar{\phi} + \beta\bar{\phi}^2\right]\exp(-\bar{\phi}). \tag{2.12}$$

From now on, for a mathematical ease we shall omit the overhead bar from our normalized variables.

The continuity, force balanced and Poisson's equations can be written in cartesian form as

$$\frac{\partial n_i}{\partial t} + \left(\frac{\partial n_i}{\partial x}\hat{x} + \frac{\partial n_i}{\partial y}\hat{y} + \frac{\partial n_i}{\partial z}\hat{z}\right)\cdot\left(v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z}\right)$$

$$\frac{\partial}{\partial t}ni + \frac{\partial}{\partial x}n_i v_{ix} + \frac{\partial}{\partial y}n_i v_{iy} + \frac{\partial}{\partial z}n_i v_{iz} = 0, \tag{2.13}$$

$$\frac{\partial}{\partial t}\left(v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z}\right) + \left((v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z})\cdot(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z})\right)\left(v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z}\right) + \left(\frac{\partial}{\partial x}\phi\hat{x} + \frac{\partial}{\partial y}\phi\hat{y} + \frac{\partial}{\partial z}\phi\hat{z}\right) - \Omega_c\left((v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z})\times\hat{z}\right) - \frac{\sigma}{n_i}\left(\frac{\partial}{\partial x}n_i - \frac{\partial}{\partial y}n_i - \frac{\partial}{\partial z}n_i\right) = \eta_i\left((\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})(v_{ix}\hat{x} + v_{iy}\hat{y} + v_{iz}\hat{z})\right) + \left(\eta_i + \mu_i\right)\left[\frac{\partial^2}{\partial x^2}v_{ix}\hat{x} + \frac{\partial^2}{\partial y^2}v_{iy}\hat{y} + \frac{\partial^2}{\partial z^2}v_{iz}\hat{z} + \frac{\partial^2}{\partial x\partial y}v_{iy}\hat{x} + \frac{\partial^2}{\partial x\partial z}v_{iz}\hat{x} + \frac{\partial^2}{\partial y\partial x}v_{ix}\hat{y} + \frac{\partial^2}{\partial y\partial z}v_{iz}\hat{y} + \frac{\partial^2}{\partial z\partial x}v_{ix}\hat{z} + \frac{\partial^2}{\partial z\partial y}v_{iy}\hat{z}\right]$$

The x, y and z-component of above equations, can be separated to write

$$\begin{aligned}&\frac{\partial}{\partial t}v_{ix} + v_{ix}\frac{\partial}{\partial x}v_{ix} + v_{iy}\frac{\partial}{\partial y}v_{ix} + v_{iz}\frac{\partial}{\partial z}v_{ix} + \frac{\partial}{\partial x}\phi - \Omega_c v_{iy} + \frac{\sigma}{n_i}\frac{\partial}{\partial x}n_i = \\ &\eta_i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)v_{ix} + \left(\eta_i + \mu_i\right)\left(\frac{\partial^2}{\partial x^2}v_{ix} + \frac{\partial^2}{\partial x\partial y}v_{iy} + \frac{\partial^2}{\partial x\partial z}v_{iz}\right)\end{aligned} \tag{2.14}$$

$$\frac{\partial}{\partial t}v_{iy}+v_{ix}\frac{\partial}{\partial x}v_{iy}+v_{iy}\frac{\partial}{\partial y}v_{iy}+v_{iz}\frac{\partial}{\partial z}v_{iy}+\frac{\partial}{\partial y}\phi+\Omega_c v_{ix}+\frac{\sigma}{n_i}\frac{\partial}{\partial y}n_i=$$
$$\eta_i\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{iy}+\left(\eta_i+\mu_i\right)\left(\frac{\partial^2}{\partial y^2}v_{iy}+\frac{\partial^2}{\partial x\partial y}v_{ix}+\frac{\partial^2}{\partial z\partial y}v_{iz}\right) \quad (2.15)$$

$$\frac{\partial}{\partial t}\gamma v_{iz}+v_{ix}\frac{\partial}{\partial x}\gamma v_{iz}+v_{iy}\frac{\partial}{\partial y}\gamma v_{iz}+v_{iz}\frac{\partial}{\partial z}\gamma v_{iz}=-\frac{\partial}{\partial z}\phi-\frac{\sigma}{n_i}\frac{\partial}{\partial z}n_i+$$
$$\eta_i\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{iz}+\left(\eta_i+\mu_i\right)\left(\frac{\partial^2}{\partial z^2}v_{iz}+\frac{\partial^2}{\partial x\partial z}v_{ix}+\frac{\partial^2}{\partial z\partial y}v_{iy}\right), \quad (2.16)$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=n_e-n_i. \quad (2.17)$$

Inserting equation (2.11) into (2.17), and then expand it by using binomial expansion yields

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=1+\frac{(\kappa-\frac{1}{2})}{(\kappa-\frac{3}{2})}\phi+\frac{(\kappa-\frac{1}{2})(\kappa+\frac{1}{2})}{2(\kappa-\frac{3}{2})^2}\phi^2-n_i$$
$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=1+c_1\phi+c_2\phi^2-n_i,$$

where c_{12} for kappa VDF is given by

$$c_{1,2}=\begin{cases}\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\\ \\ \frac{\kappa-\frac{1}{2}\kappa+\frac{1}{2}}{2(\kappa-\frac{3}{2})^2}\end{cases} \quad (2.18)$$

Now, using equation (2.5) into (2.17), and following the same steps to write

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\left(1-\beta\phi+\beta\phi^2\right)\exp(\phi)-n_i$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\left[1+(1-\beta)\phi+\frac{\phi^2}{2}\right]-n_i$$

Here c_{12} coefficients of Cairns density function are given by

$$c_{1,2}=\begin{cases}1-\beta\\ \\ \frac{1}{2}\end{cases} \quad (2.19)$$

2.3 The reductive perturbation method

We use a reductive perturbation method to derive ZKB equation. This schemes are helpul in the study of waves which have small but measurable amplitudes. In particular, here we consider small amplitude of ion acoustic shock waves. Now first we transform the coordinates, also called stretching of coordinates, to define space and time variables as following

$$\begin{aligned} \xi &= \epsilon^{\frac{1}{2}} x, & \eta &= \epsilon^{\frac{1}{2}} y, \\ \zeta &= \epsilon^{\frac{1}{2}} (z - \lambda_0 t), & \tau &= \epsilon^{\frac{3}{2}} t \end{aligned} \tag{2.20}$$

where $0 < \epsilon << 1$, and λ_0 is the linear wave speed. The next step, in our scheme, is to expand the dependent variables in the power series of epsilon as following

$$\begin{aligned} n_i &= 1 + \epsilon n_1 + \epsilon^2 n_2 + ... \\ v_{ix} &= \epsilon^{\frac{3}{2}} u_1 + \epsilon^2 u_2 + ... \\ v_{iy} &= \epsilon^{\frac{3}{2}} v_1 + \epsilon^2 v_2 + ... \\ v_{iz} &= v_0 + \epsilon w_1 + \epsilon^2 w_2 + ... \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + ... \end{aligned} \tag{2.21}$$

We assume that the damping is flimsy such that different viscosities can be written as

$$\begin{aligned} \eta_i &= \epsilon^{\frac{1}{2}} \eta_0, \\ \mu_i &= \epsilon^{\frac{1}{2}} \mu_0 \end{aligned}$$

where η_0 and μ_0 have values of very small magnitude. The respective derivatives are then transformed as

$$\frac{\partial f(\xi,\eta,\zeta,\tau)}{\partial x} = \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y} + \frac{\partial f}{\partial \zeta}\frac{\partial \zeta}{\partial z} + \frac{\partial f}{\partial \tau}\frac{\partial \tau}{\partial t}.$$

$$\frac{\partial f(\xi,\eta,\zeta,\tau)}{\partial x} = \epsilon^{\frac{1}{2}} \frac{\partial f}{\partial \xi} + 0 + 0 + 0$$

$$\frac{\partial}{\partial x} = \epsilon^{\frac{1}{2}} \frac{\partial}{\partial \xi} \tag{2.22}$$

$$\frac{\partial}{\partial t} = \epsilon^{\frac{3}{2}} \frac{\partial}{\partial \tau} - \lambda_0 \epsilon^{\frac{1}{2}} \frac{\partial}{\partial \zeta} \tag{2.23}$$

$$
\begin{aligned}
\frac{\partial}{\partial y} &= \epsilon^{\frac{1}{2}}\frac{\partial}{\partial \eta} & \frac{\partial}{\partial z} &= \epsilon^{\frac{1}{2}}\frac{\partial}{\partial \zeta} \\
\frac{\partial^2}{\partial x^2} &= \epsilon\frac{\partial^2}{\partial \xi^2} & \frac{\partial^2}{\partial y^2} &= \epsilon\frac{\partial^2}{\partial \eta^2} \\
\frac{\partial^2}{\partial z^2} &= \epsilon\frac{\partial^2}{\partial \zeta^2} & \frac{\partial^2}{\partial x\partial y} &= \epsilon\frac{\partial^2}{\partial \xi\partial \eta} \\
\frac{\partial^2}{\partial x\partial z} &= \epsilon\frac{\partial^2}{\partial \xi\partial \zeta} & \frac{\partial^2}{\partial y\partial z} &= \epsilon\frac{\partial^2}{\partial \zeta\partial \eta}
\end{aligned}
\tag{2.24}
$$

Next, we have to use all these transform derivatives in our model equations and then campare coefficients of various powers of ϵ. From Poisson's equation the different powers of ϵ can be seperated as

$$
(n_1 - c_1\phi_1)\epsilon + (n_2 - c_2\phi_1^2 - c_1\phi_2 + \phi_1''[\xi] + \phi_1''[\zeta] + \phi_1''[\eta])\epsilon^2 + (-2c_2\phi_1\phi_2 + \phi_2''[\zeta] + \phi_2''[\eta] + \phi_2[\xi])\epsilon^3 - c_2\phi_2^2\epsilon^4 + O\epsilon^8 = 0
\tag{2.25}
$$

Similarly, from continuity equation we have

$$
\begin{aligned}
& v_0'[\zeta]\epsilon^{\frac{1}{2}} - (\lambda_0 n_1'[\zeta] + v_0 n_1'[\zeta] + n_1 v_0'[\zeta] + w_1'[\zeta])\epsilon^{\frac{3}{2}} + (u_1'[\xi] + v_1'[\eta])\epsilon^2 + (w_1[\zeta]n_1'[\zeta] + n_1'[\tau] - \lambda_0 n_2'[\zeta] \\
& + v_0 n_2'[\zeta] + u_2'[\xi] + n_2[\zeta]v_0'[\zeta] + v_2'[\eta] + n_1 w_1'[\zeta] + w_2'[\zeta])\epsilon^{\frac{5}{2}} + (v_1 n_1'[\eta] + u_1 n_1'[\xi] + n_1 u_1'[\xi] + n_1 v_1'\eta)\epsilon^3 \\
& + (w_2 n_1'[\zeta] + v_2 n_1'[\eta] + u_2 n_1'[\xi] + w_1 n_2'[\zeta] + n_2'[\tau] + n_1 u_2[\xi] + n_1 v_2'[\eta] + n_2 w_1[\zeta] + n_1 w_2'[\zeta])\epsilon^{\frac{7}{2}} \\
& + (v_1 n_2'[\eta] + u_1 n_2'[\xi] + n_2 u_1'[\xi] + n_2 v_1'[\eta])\epsilon^4 + (w_2 n_2'[\zeta] + v_2 n_2'[\eta] + u_2 n_2'[\xi] + n_2 u_2'[\xi] + n_2 v_2'[\eta] \\
& + n_2 w_2'[\zeta])\epsilon^{\frac{9}{2}} + O\epsilon^{\frac{15}{2}} = 0
\end{aligned}
\tag{2.26}
$$

From equation of motion the x-component provides the following relation among various epsilon power terms

$$
\begin{aligned}
& (-v_1\Omega_c + \sigma n_1'[\xi] + \phi_1'[\xi])\epsilon^{\frac{3}{2}} + (-v_2\Omega_c + v_0 u_1'[\zeta] - \lambda_0 u_1'[\zeta])\epsilon^2 + (w_1 u_1[\zeta] + u_1'[\tau])\epsilon^3 + (w_2 u_1'[\zeta] \\
& + u_2 u_1'[\xi] + u_1 u_2'[\xi] + v_2 v_1'[\eta] + v_1 v_2'[\eta])\epsilon^4 + (-\sigma n_1 n_1'[\xi] + \sigma n_2'[\xi] + v_0 u_2'[\zeta] - \lambda_0 u_2'[\zeta] + \phi_2[\xi])\epsilon^{\frac{5}{2}} \\
& + (-\sigma n_2 n_1'[\xi] - \sigma n_1 n_2'[\xi] + u_1 u_1'[\xi] + w_1 u_2'[\zeta] + u_2[\tau] + v_1 v_1'[\eta])\epsilon^{\frac{7}{2}} - (\sigma n_2 n_2'[\xi] + w_2 u_2'[\zeta] \\
& + u_2 u_2'[\xi] + v_2 v_2'[\eta])\epsilon^{\frac{9}{2}} + O\epsilon^{\frac{15}{2}} = 0
\end{aligned}
\tag{2.27}
$$

Likewise, the y and z-components yield, respectively

$$(u_1\Omega_c+\sigma n_1'[\eta]+\phi_1'[\eta])\epsilon^{\frac{3}{2}}+(u_2\Omega_c+v_0v_1'[\zeta]-\lambda_0v_1'[\zeta])\epsilon^2+(w_1v_1'[\zeta]+v_1'[\tau])\epsilon^3+(w_2v_1'[\zeta]+v_2v_1'[\eta]$$
$$+u_2v_1'[\xi]+v_1v_2'[\eta]+u_1v_2'[\xi])\epsilon^4+(-\sigma n_1n_1'[\eta]+\sigma n_2'[\eta]+v_0v_2'[\zeta]-\lambda_0v_2'[\zeta]+\phi_2[\eta])\epsilon^{\frac{5}{2}}+(-\sigma n_2n_1'[\eta]$$
$$-\sigma n_1n_2'[\eta]+v_1v_1'[\eta]+u_1v_1'[\xi]+w_1v_2'[\zeta]+v_2[\tau])\epsilon^{\frac{7}{2}}+(-\sigma n_2n_2'[\eta]+w_2v_2'[\zeta]+v_2v_2'[\eta]+u_2v_2'[\xi])\epsilon^{\frac{9}{2}}$$
$$+O\epsilon^{\frac{15}{2}}=0 \tag{2.28}$$

and

$$O\epsilon^0+O\epsilon^1+O\epsilon^2+(v_1w_1'[\eta]+\frac{3v_0^2v_1w_1'[\eta]}{2c^2}+u_1w_1'[\xi]+\frac{3u_1v_0^2w_1'[\xi]}{2c^2})\epsilon^3+(\frac{3v_1v_0w_1w_1'[\eta]}{c^2}+$$
$$\frac{3u_1v_0w_1w_1'[\xi]}{c^2}+v_1w_2'[\eta]+\frac{3v_1v_0^2w_2'[\eta]}{2c^2}+u_1w_2'[\xi]+\frac{3u_1v_0^2w_2'[\xi]}{2c^2})\epsilon^4+(\sigma n_1'[\zeta]+v_0w_1'[\zeta]+\frac{3v_0^3w_1'[\zeta]}{2c^2}-$$
$$\lambda_0w_1'[\zeta]-\frac{3\lambda_0v_0^2w_1'[\zeta]}{2c^2}+\phi_1'[\zeta])\epsilon^{\frac{3}{2}}+.....=0 \tag{2.29}$$

Equating coefficients of ϵ in equation (2.25) yields

$$n_1-c_1\phi_1=0$$

$$n_1=c_1\phi_1 \tag{2.30}$$

Similarly, comparing $\epsilon^{\frac{3}{2}}$ terms in equation (2.26) and (2.28) to write

$$\frac{\partial n_1v_0}{\partial\zeta}-\lambda_0\frac{\partial n_1}{\partial\zeta}+\frac{\partial w_1}{\partial\zeta}=0$$

$$v_0n_1-\lambda_0n_1+w_1=0$$

$$n_1(\lambda_0-v_0)=w_1$$

$$n_1=\frac{w_1}{(\lambda_0-v_0)} \tag{2.31}$$

and, orderly

$$u_1\Omega_c+\sigma\frac{\partial n_i}{\partial\eta}+\frac{\partial\phi_1}{\partial\eta}=0 \tag{2.32}$$

Using equation (2.30) into equation (2.32) we find

$$u_1\Omega_c+\sigma c_1\frac{\partial\phi_1}{\partial\eta}+\frac{\partial\phi_1}{\partial\eta}=0$$

$$u_1=-\Omega_c^{-1}(1+\sigma c_1)\frac{\partial\phi_1}{\partial\eta} \tag{2.33}$$

Coefficients of $\epsilon^{\frac{3}{2}}$ in equation (2.27) give,

$$-v_1\Omega_c + \sigma\frac{\partial n_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \xi} = 0 \tag{2.34}$$

which, upon using equation (2.30) provides

$$-v_1\Omega_c + \sigma c_1\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \xi} = 0$$

$$v_1 = \Omega_c^{-1}(1+\sigma c_1)\frac{\partial \phi_1}{\partial \xi} \tag{2.35}$$

Coefficient of $\epsilon^{\frac{3}{2}}$ in equation (2.29) results in

$$\sigma\frac{\partial n_1}{\partial \zeta} + v_0\frac{\partial w_1}{\partial \zeta} + \frac{3v_0^3}{2c^2}\frac{\partial w_1}{\partial \zeta} - \lambda_0\frac{\partial w_1}{\partial \zeta} - \frac{3v_0^2}{2c^2}\frac{\partial w_1}{\partial \zeta} + \frac{\partial \phi_1}{\partial \zeta} = 0 \tag{2.36}$$

$$\left(v_0 - \lambda_0 + \frac{3v_0^3}{2c^2} - \frac{3v_0^2\lambda_0}{2c^2}\right)\frac{\partial w_1}{\partial \zeta} + \frac{\partial \phi_1}{\partial \zeta} + \sigma\frac{\partial n_1}{\partial \zeta} = 0$$

By using values of n_1 the above equation can be written as,

$$\frac{\partial}{\partial \zeta}\left[\phi_1 + \sigma c_1\phi_1 + \left(v_0 - \lambda_0 + \frac{3v_0^3}{2c^2} - \frac{3v_0^2\lambda_0}{2c^2}\right)w_1\right] = 0$$

$$\left[\phi_1 + \sigma c_1\phi_1 + \left(v_0 - \lambda_0 + \frac{3v_0^3}{2c^2} - \frac{3v_0^2\lambda_0}{2c^2}\right)w_1\right] = 0,$$

$$\phi_1(1+\sigma c_1) + w_1 v_0(1+1.5\gamma_0) - (1.5\gamma_0+1)w_1\lambda_0 = 0$$

$$\phi_1(1+\sigma c_1) + w_1(v_0 - \lambda_0 + 1.5v_0\gamma_0 - 1\cdot 5\gamma_0\lambda_0) = 0$$

$$\phi_1(1+\sigma c_1) + (1+1.5v_0)(v_0-\lambda_0)w_1 = 0 \tag{2.37}$$

$$\phi_1(1+\sigma c_1) = (\lambda_0 - v_0)\gamma_1 w_1$$

where $\gamma_0 = \frac{v_0^2}{c^2}$, $\gamma_1 = 1 + 1.5\gamma_0^2$ and

$$w_1 = \frac{\phi_1(1+\sigma c_1)}{(\lambda_0 - v_0)\gamma_1} \tag{2.38}$$

Now comparing the coefficient of ϵ^2 in equation (2.28) to write

$$u_2\Omega_c + v_0\frac{\partial v_1}{\partial \zeta} - \lambda_0\frac{\partial v_1}{\partial \zeta} = 0$$

$$(v_0 - \lambda_0)\frac{\partial v_1}{\partial \zeta} + u_2\Omega_c = 0$$

$$u_2\Omega_c = (\lambda_0 - v_0)\frac{\partial v_1}{\partial \zeta} \tag{2.39}$$

Similarly, from equation (2.27) we find

$$-v_2\Omega_c + v_0\frac{\partial u_1}{\partial \zeta} - \lambda_0\frac{\partial u_1}{\partial \zeta} = 0$$

$$(v_0 - \lambda_0)\frac{\partial u_1}{\partial \zeta} - v_2\Omega_c = 0$$

$$v_2\Omega_c = -(\lambda_0 - v_0)\frac{\partial u_1}{\partial \zeta} \tag{2.40}$$

And orderly, from equation (2.25) we find

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} - c_2\phi_1^2 = c_1\phi_2 - n_2 \tag{2.41}$$

Upon that solving equations (2.30), (2.31) and (2.38) to obtain phase speed λ_0 as

$$c_1\phi_1 = \frac{w_1}{(\lambda_0 - v_0)}$$

$$\phi_1 = \frac{w_1}{c_1(\lambda_0 - v_0)}$$

$$\cancel{w_1} = \frac{(1+\sigma c_1)}{(\lambda_0 - v_0)\gamma_1}\frac{\cancel{w_1}}{c_1(\lambda_0 - v_0)}$$

$$(\lambda_0^2 + v_0^2 - 2v_0\lambda_0)c_1\gamma_1 = (1+\sigma c_1)$$

$$\lambda_0^2 c_1\gamma_1 + v_0^2 c_1\gamma_1 - 2\lambda_0 c_1\gamma_1 v_0 = 1 + \sigma c_1$$

$$\lambda_0^2 - 2\lambda_0 v_0 + \frac{c_1\gamma_1 v_0^2 - 1 - \sigma c_1}{c_1\gamma_1} = 0$$

$$\lambda_0^2 c_1\gamma_1 - 2c_1\gamma_1\lambda_0 v_0 + c_1\gamma_1 v_0^2 - 1 - \sigma c_1 = 0$$

$$\lambda_0 = 2c_1\gamma_1 v_0 \pm \left((2c_1\gamma_1 v_0)^2 - 4c_1\gamma_1(c_1\gamma_1 v_0^2 - 1 - \sigma c_1)\right)^{\frac{1}{2}}$$

$$\lambda_0 = v_0 \pm \left(\frac{(4c_1\gamma_1 + 4c_1^2\gamma_1\sigma)}{4c_1^2\gamma_1}\right)^{\frac{1}{2}}$$

$$\lambda_0 = v_0 \pm \left(\frac{(1+\sigma c_1}{c_1\gamma_1}\right)^{\frac{1}{2}} \tag{2.42}$$

This equation shows the phase speed for ion acoustic waves, where pasitive sign shows the fast mode of ion acoustic waves and negative sign corresponds to the slower counterpart.

Coefficients of $\epsilon^{\frac{5}{2}}$ in equation (2.26) yield

$$w_1\frac{\partial n_1}{\partial \zeta}+\frac{\partial n_1}{\partial t}+v_0\frac{\partial n_2}{\partial \zeta}-\lambda_0\frac{\partial n_2}{\partial \zeta}+\frac{\partial u_2}{\partial \xi}+\frac{\partial v_2}{\partial \eta}+n_1\frac{\partial w_1}{\partial \zeta}+\frac{\partial w_2}{\partial \zeta}+n_2\frac{\partial v_0}{\partial \zeta}=0$$

$$\frac{\partial n_1}{\partial t}-\lambda_0\frac{\partial n_2}{\partial \zeta}+\frac{\partial u_2}{\partial \xi}+\frac{\partial v_2}{\partial \eta}+\frac{\partial w_2}{\partial \zeta}+\frac{\partial n_1 w_1}{\partial \zeta}+\frac{\partial n_2 v_0}{\partial \zeta}=0$$

$$(v_0-\lambda_0)\frac{\partial n_2}{\partial \zeta}+\frac{\partial n_1}{\partial t}+\frac{\partial u_2}{\partial \xi}+\frac{\partial v_2}{\partial \eta}+\frac{\partial w_2}{\partial \zeta}+\frac{\partial n_1 w_1}{\partial \zeta}=0$$

$$-\frac{\partial n_1}{\partial t}-\frac{\partial n_1 w_1}{\partial \zeta}=-(\lambda_0-v_0)\frac{\partial n_2}{\partial \zeta}+\frac{\partial u_2}{\partial \xi}+\frac{\partial v_2}{\partial \eta}+\frac{\partial w_2}{\partial \zeta}\cdot \tag{2.43}$$

Likewise, same comparison in equation (2.29) provides the following

$$-\sigma n_1\frac{\partial n_1}{\partial \zeta}+\sigma\frac{\partial n_2}{\partial \zeta}+w_1\frac{\partial w_1}{\partial \zeta}+\frac{3v_0^2 w_1}{2c^2}\frac{\partial w_1}{\partial \zeta}+\frac{3v_0^2 w_1}{c^2}\frac{\partial w_1}{\partial \zeta}-\frac{3v_0\lambda_0 w_1}{c^2}\frac{\partial w_1}{\partial \zeta}+\frac{\partial w_1}{\partial \tau}+\frac{3v_0^2}{2c^2}\frac{\partial w_1}{\partial \tau}+$$
$$v_0\frac{\partial w_2}{\partial \zeta}+\frac{3v_0^3}{2c^2}\frac{\partial w_2}{\partial \zeta}-\lambda_0\frac{\partial w_2}{\partial \zeta}-\frac{3v_0^2\lambda_0}{2c^2}\frac{\partial w_2}{\partial \zeta}+\frac{\partial \phi_2}{\partial \zeta}-2\eta_0\frac{\partial^2 w_1}{\partial \zeta^2}-\mu_0\frac{\partial^2 1_1}{\partial \zeta^2}-\eta_0\frac{\partial^2 w_1}{\partial \eta^2}-\eta_0\frac{\partial^2 w_1}{\partial \xi^2}=0 \tag{2.44}$$

First of all we solve $w_1\frac{\partial w_1}{\partial \zeta}$ term in above equation as

$$-\frac{3v_0\lambda_0 w_1}{c^2}\frac{\partial w_1}{\partial \zeta}+-\frac{3v_0^2 w_1}{2c^2}\frac{\partial w_1}{\partial \zeta}+\frac{3v_0^2 w_1}{c^2}\frac{\partial w_1}{\partial \zeta}+w_1\frac{\partial w_1}{\partial \zeta}$$

Multiplying and Dividing by $2v_o$ to write

$$-\frac{3\lambda_0\gamma_0^2}{2v_0}\frac{\partial w_1^2}{\partial \zeta}+\frac{3v_0\gamma_0^2}{2v_0}\frac{\partial w_1^2}{\partial \zeta}+w_1\frac{\partial w_1}{\partial \zeta}+\frac{3\gamma_0^2 w_1}{2}\frac{\partial w_1}{\partial \zeta}$$

$$=-\lambda_0\gamma_2\frac{\partial w_1^2}{\partial \zeta}+v_0\gamma_2\frac{\partial w_1^2}{\partial \zeta}+\gamma_1 w_1\frac{\partial w_1}{\partial \zeta}$$

$$=(-\lambda_0+v_0)\gamma_2\frac{\partial w_1^2}{\partial \zeta}+\gamma_1 w_1\frac{\partial w_1}{\partial \zeta}, \tag{2.45}$$

where $\gamma_2=\frac{3\gamma_0^2}{2}$.
Secondly, we try to solve $\frac{\partial w_1}{\partial \tau}$ term in (2.44) as follows

$$\frac{\partial w_1}{\partial \tau}+\frac{3v_0^2\gamma_0^2}{2c^2}\frac{\partial w_1}{\partial \tau}$$

$$=\left[1+\frac{3\gamma_0^2}{2}\right]\frac{\partial w_1}{\partial \tau}$$

$$= \gamma_1 \frac{\partial w_1}{\partial \tau},$$

next term in equation in (2.44) can be simplified as

$$v_0 + \frac{3v_0^3}{2c^2}\frac{\partial w_2}{\partial \zeta} - \lambda_0 \frac{\partial w_2}{\partial \zeta} - \frac{3v_0^2\lambda_0}{2c^2}\frac{\partial w_2}{\partial \zeta}$$

$$= \frac{\partial w_2}{\partial \zeta} v_0 \left(1 + \frac{3}{2}\gamma_0^2\right) - \lambda_0 \left(1 + \frac{3}{2}\gamma_0^2\right)\frac{\partial w_2}{\partial \zeta}$$

$$= \left(v_0 - \lambda_0\right)\gamma_1 \frac{\partial w_2}{\partial \zeta}$$

After plugging in all of the above three solvable steps in equation (2.44) and rearranging the terms yields

$$-(-\lambda_0 + v_0)\gamma_1 \frac{\partial w_2}{\partial \zeta} + \frac{\partial \phi_2}{\partial \zeta} + \sigma \frac{\partial n_2}{\partial \zeta} = (\lambda_0 - v_0)\gamma_2 \frac{\partial w_1^2}{\partial \zeta} - \gamma_1 \frac{\partial w_1}{\partial \tau} - \gamma_1 w_1 \frac{\partial w_1}{\partial \zeta} + \sigma n_1 \frac{\partial n_1}{\partial \zeta} + \eta_0 (\frac{\partial^2 w_1}{\partial \xi^2} + \frac{\partial^2 w_1}{\partial \eta^2} + \frac{\partial^2 w_1}{\partial \zeta^2}) + (\eta_0 + \mu_0)\frac{\partial^2 w_1}{\partial \zeta^2}. \tag{2.46}$$

2.4 Extraction of ZKB equation

From equation (2.41), we have

$$\frac{\partial}{\partial \zeta}\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right] - c_2 \frac{\partial \phi_1^2}{\partial \zeta} = c_1 \frac{\partial \phi_2}{\partial \zeta} - \frac{\partial n_2}{\partial \zeta}$$

$$\frac{\partial \phi_2}{\partial \zeta} = \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \eta^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \zeta^2}\right) + \frac{1}{c_1}\left(\frac{\partial n_1}{\partial \zeta}\right) - \frac{c_1}{c_2}\left(\frac{\partial \phi_1^2}{\partial \zeta}\right) \tag{2.47}$$

Using equation (2.43) and rearrange it to write

$$\frac{\partial w_2}{\partial \zeta} = -\frac{\partial n_1}{\partial t} - \frac{\partial}{\partial \zeta}(n_1 w_1) - \frac{\partial u_2}{\partial \xi} - \frac{\partial v_2}{\partial \eta} + (\lambda_0 - v_0)\frac{\partial n_2}{\partial \zeta} \tag{2.48}$$

Upon using equations (2.47) and (2.48) into equation (2.46), we find

$$-(\lambda_0 - v_0)\gamma_1 \left[-\frac{\partial n_1}{\partial t} - \frac{\partial}{\partial \zeta}(n_1 w_1) - \frac{\partial u_2}{\partial \xi} - \frac{\partial v_2}{\partial \eta} + (\lambda_0 - v_0)\frac{\partial n_2}{\partial \zeta}\right] + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left[n_2 + \frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} - c_2 \phi_1^2\right] + \sigma \frac{\partial n_2}{\partial \zeta} = 2(\lambda_0 - v_0)^3 \gamma_2 n_1 \frac{\partial n_1}{\partial \zeta} - \gamma_1 (\lambda_0 - v_0)\frac{\partial n_1}{\partial \tau} + \sigma n_1 \frac{\partial n_1}{\partial \zeta} - (\lambda_0 - v_0)^2 n_1 \frac{\partial n_1}{\partial \zeta} + \eta_0 \left[\frac{\partial^2 w_1}{\partial \xi^2} + \frac{\partial^2 w_1}{\partial \eta^2} + \frac{\partial^2 w_1}{\partial \zeta^2}\right] + (\eta_0 + \mu) + \frac{\partial^2 n_1}{\partial \zeta^2}.$$

$$(\lambda_0 - v_0)\gamma_1 \frac{\partial n_1}{\partial \tau} + (\lambda_0 - v_0)\gamma_1 \frac{\partial n_1 w_1}{\partial \zeta} - (\lambda_0 - v_0)^2 \gamma_1 \frac{\partial n_2}{\partial \zeta} + (\lambda_0 - v_0)\gamma_1 \frac{\partial u_2}{\partial \xi} + (\lambda_0 - v_0)\gamma_1 \frac{\partial v_2}{\partial \eta} + \frac{1}{c_1}\frac{\partial}{\partial \zeta}$$
$$\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \eta^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \zeta^2}\right) + \frac{1}{c_1}\frac{\partial n_2}{\partial \zeta} - 2\frac{c_1}{c_2}\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \sigma \frac{\partial n_2}{\partial \zeta} = 2(\lambda_0 - v_0)^3 \gamma_2 n_1 \frac{\partial n_1}{\partial \zeta} -$$
$$\gamma_1(\lambda_0 - v_0)\frac{\partial n_1}{\partial \tau} + \sigma n_1 \frac{\partial n_1}{\partial \zeta} - (\lambda_0 - v_0)^2 n_1 \frac{\partial n_1}{\partial \zeta} + \eta_0 \left[\frac{\partial^2 w_1}{\partial \xi^2} + \frac{\partial^2 w_1}{\partial \eta^2} + \frac{\partial^2 w_1}{\partial \zeta^2}\right] + (\eta_0 + \mu) + \frac{\partial^2 w_1}{\partial \zeta^2}.$$

$$(\lambda_0 - v_0)\gamma_1 \frac{\partial n_1}{\partial \tau} + (\lambda_0 - v_0)\gamma_1 \left[n_1 \frac{\partial w_1}{\partial \zeta} + w_1 \frac{\partial n_1}{\partial \zeta}\right] - (\lambda_0 - v_0)^2 \gamma_1 \frac{\partial n_2}{\partial \zeta} + (\lambda_0 - v_0)\gamma_1 \frac{\partial u_2}{\partial \xi} +$$
$$(\lambda_0 - v_0)\gamma_1 \frac{\partial v_2}{\partial \eta} + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \eta^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \zeta^2}\right) + \frac{1}{c_1}\frac{\partial n_2}{\partial \zeta} - 2\frac{c_2}{c_1}\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \sigma \frac{\partial n_2}{\partial \zeta} =$$
$$2(\lambda_0 - v_0)^3 \gamma_2 n_1 \frac{\partial n_1}{\partial \zeta} - \gamma_1(\lambda_0 - v_0)\frac{\partial n_1}{\partial \tau} + \sigma n_1 \frac{\partial n_1}{\partial \zeta} - (\lambda_0 - v_0)^2 n_1 \frac{\partial n_1}{\partial \zeta} + \eta_0 \left[\frac{\partial^2 w_1}{\partial \xi^2} + \frac{\partial^2 w_1}{\partial \eta^2} + \frac{\partial^2 w_1}{\partial \zeta^2}\right] +$$
$$(\eta_0 + \mu)\frac{\partial^2 w_1}{\partial \zeta^2}. \tag{2.49}$$

Using $w_1 = n_1(\lambda_0 - v_0)$ in above equation, we find

$$(\lambda_0 - v_0)\gamma_1 \frac{\partial n_1}{\partial \tau} + \left[2\gamma_1(\lambda_0 - v_0)^2 n_1 \frac{\partial n_1}{\partial \zeta}\right] - (\lambda_0 - v_0)^2 \gamma_1 \frac{\partial n_2}{\partial \zeta} + (\lambda_0 - v_0)\gamma_1 \frac{\partial u_2}{\partial \xi} + (\lambda_0 - v_0)\gamma_1 \frac{\partial v_2}{\partial \eta} +$$
$$\frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \eta^2}\right) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \zeta^2}\right) + \frac{1}{c_1}\frac{\partial n_2}{\partial \zeta} - 2\frac{c_2}{c_1}\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \sigma \frac{\partial n_2}{\partial \zeta} = 2(\lambda_0 - v_0)^3 \gamma_2$$
$$n_1 \frac{\partial n_1}{\partial \zeta} - \gamma_1(\lambda_0 - v_0)\frac{\partial n_1}{\partial \tau} + \sigma n_1 \frac{\partial n_1}{\partial \zeta} - (\lambda_0 - v_0)^2 n_1 \frac{\partial n_1}{\partial \zeta} + c_1 \eta_0 (\lambda_0 - v_0)\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right] +$$
$$c_1(\eta_0 + \mu_0)(\lambda_0 - v_0)\frac{\partial^2 \phi_1}{\partial \zeta^2}. \tag{2.50}$$

Next, using equations (2.30), (2.39) and (2.40) in equation (2.50) yields

$$(\lambda_0 - v_0)c_1 \gamma_1 \frac{\partial \phi_1}{\partial \tau} + 2(\lambda_0 - v_0)^2 c_1^2 \gamma_1 \phi_1 \frac{\partial \phi_1}{\partial \zeta} - \gamma_1(\lambda_0 - v_0)^2 \frac{\partial n_2}{\partial \zeta} + (\lambda_0 - v_0)\gamma_1$$
$$\left[\frac{(\lambda_0 - v_0)}{\Omega_c}\frac{\partial}{\partial \xi}(\frac{\partial v_1}{\partial \zeta})\right] + (\lambda_0 - v_0)\gamma_1 \left[\frac{-(\lambda_0 - v_0)}{\Omega_c}\frac{\partial}{\partial \eta}(\frac{\partial u_1}{\partial \zeta})\right] + \frac{1}{c_1}\frac{\partial n_2}{\partial \zeta} + \frac{1}{c_1}\frac{\partial}{\partial \zeta}(\frac{\partial^2 \phi_1}{\partial \xi^2}) + \frac{1}{c_1}\frac{\partial}{\partial \zeta}(\frac{\partial^2 \phi_1}{\partial \eta^2}) +$$
$$\frac{1}{c_1}\frac{\partial^3 \phi_1}{\partial \zeta^3} - 2\frac{c_2}{c_1}\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \sigma \frac{\partial n_2}{\partial \zeta} - 2\gamma_2 c_1^2 (\lambda_0 - v_0)^3 \phi_1 \frac{\partial \phi_1}{\partial \zeta} + \gamma_1 c_1 (\lambda_0 - v_0)\frac{\partial \phi_1}{\partial \tau} + \gamma_1 c_1^2 (\lambda_0 - v_0)^2 \phi_1 \frac{\partial \phi_1}{\partial \zeta} -$$
$$\sigma c_1^2 \phi_1 \frac{\partial \phi_1}{\partial \zeta} - c_1 \eta_0 (\lambda_0 - v_0)\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right] + c_1(\eta_0 - \mu_0)(\lambda_0 - v_0)\frac{\partial^2 \phi_1}{\partial \zeta^2} = 0$$

$$(\lambda_0 - v_0)c_1\gamma_1\frac{\partial\phi_1}{\partial\tau} + 2(\lambda_0 - v_0)^2 c_1^2\gamma_1\phi_1\frac{\partial\phi_1}{\partial\zeta} - \gamma_1(\lambda_0 - v_0)^2\frac{\partial n_2}{\partial\zeta} + \frac{(\lambda_0 - v_0)^2\gamma_1}{\Omega_c}\frac{\partial^2 v_1}{\partial\xi\partial\zeta} - \frac{(\lambda_0 - v_0)^2\gamma_1}{\Omega_c}$$
$$\frac{\partial^2 u_1}{\partial\eta\partial\zeta} + \frac{1}{c_1}\frac{\partial n_2}{\partial\zeta} + \frac{1}{c_1}\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\xi^2}) + \frac{1}{c_1}\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\eta^2}) + \frac{1}{c_1}\frac{\partial^3\phi_1}{\partial\zeta^3} - 2\frac{c_2}{c_1}\phi_1\frac{\partial\phi_1}{\partial\zeta} + \sigma\frac{\partial n_2}{\partial\zeta} - 2\gamma_2 c_1^2(\lambda_0 - v_0)^3\phi_1$$
$$\frac{\partial\phi_1}{\partial\zeta} + \gamma_1 c_1(\lambda_0 - v_0)\frac{\partial\phi_1}{\partial\tau} + \gamma_1 c_1^2(\lambda_0 - v_0)^2\phi_1\frac{\partial\phi_1}{\partial\zeta} - \sigma c_1^2\phi_1\frac{\partial\phi_1}{\partial\zeta} - c_1\eta_0(\lambda_0 - v_0)\left[\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2} + \frac{\partial^2\phi_1}{\partial\zeta^2}\right]$$
$$- c_1(\eta_0 + \mu_0)(\lambda_0 - v_0)\frac{\partial^2\phi_1}{\partial\zeta^2} = 0 \tag{2.51}$$

Which upon making use of equations (2.33) and (2.35) transforms to the following form

$$(\lambda_0 - v_0)c_1\gamma_1\frac{\partial\phi_1}{\partial\tau} + 2(\lambda_0 - v_0)^2 c_1^2\gamma_1\phi_1\frac{\partial\phi_1}{\partial\zeta} - \gamma_1(\lambda_0 - v_0)^2\frac{\partial n_2}{\partial\zeta} + \frac{(\lambda_0 - v_0)^2\gamma_1}{\Omega_c^2}(1 + \sigma c_1)\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\xi^2}) +$$
$$\frac{(\lambda_0 - v_0)^2\gamma_1}{\Omega_c^2}(1 + \sigma c_1)\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\eta^2}) + \frac{1}{c_1}\frac{\partial n_2}{\partial\zeta} + \frac{1}{c_1}\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\xi^2}) + \frac{1}{c_1}\frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\eta^2}) + \frac{1}{c_1}\frac{\partial^3\phi_1}{\partial\zeta^3} - 2\frac{c_2}{c_1}\phi_1\frac{\partial\phi_1}{\partial\zeta} +$$
$$\sigma\frac{\partial n_2}{\partial\zeta} - 2\gamma_2 c_1^2(\lambda_0 - v_0)^3\phi_1\frac{\partial\phi_1}{\partial\zeta} + \gamma_1 c_1(\lambda_0 - v_0)\frac{\partial\phi_1}{\partial\tau} + \gamma_1 c_1^2(\lambda_0 - v_0)^2\phi_1\frac{\partial\phi_1}{\partial\zeta} - \sigma c_1^2\phi_1\frac{\partial\phi_1}{\partial\zeta} - c_1\eta_0$$
$$(\lambda_0 - v_0)\left[\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2} + \frac{\partial^2\phi_1}{\partial\zeta^2}\right] - c_1(\eta_0 + \mu_0)(\lambda_0 - v_0)\frac{\partial^2\phi_1}{\partial\zeta^2} = 0$$

$$\left[2\gamma_1 c_1^2(\lambda_0 - v_0)\right]\frac{\partial\phi_1}{\partial\tau} + \left[2\gamma_1 c_1^3(\lambda_0 - v_0)^2\phi_1 - 2c_2 - 2\gamma_2(\lambda_0 - v_0)^3 c_1^3 + \gamma_1(\lambda_0 - v_0)^2 c_1^3 - \sigma c_1^3\right]\phi_1\frac{\partial\phi_1}{\partial\zeta} +$$
$$\frac{\partial^3\phi_1}{\partial\zeta^3} + \frac{c_1\gamma_1(1 + \sigma c_1)(\lambda_0 - v_0)^2}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^2} + \frac{c_1\gamma_1(1 + \sigma c_1)(\lambda_0 - v_0)^2}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\eta^2} + \frac{\partial n_2}{\partial\zeta} + \frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\xi^2}) + \frac{\partial}{\partial\zeta}$$
$$(\frac{\partial^2\phi_1}{\partial\eta^2}) + \sigma c_1\frac{\partial n_2}{\partial\zeta} - c_1\gamma_1(\lambda_0 - v_0)^2\frac{\partial n_2}{\partial\zeta} - c_1^2\eta_0(\lambda_0 - v_0)\left[\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2} + \frac{\partial^2\phi_1}{\partial\zeta^2}\right] - c_1^2(\eta_0 + \mu_0)$$
$$(\lambda_0 - v_0)\frac{\partial^2\phi_1}{\partial\zeta^2} = 0$$

$$\left[2\gamma_1 c_1^2(\lambda_0 - v_0)\right]\frac{\partial\phi_1}{\partial\tau} + \left[3(\lambda_0 - v_0)^2 c_1^3\gamma_1 - 2(\lambda_0 - v_0)^3 c_1^3\gamma_2 - 2c_2 - \sigma c_1^3\right]\phi_1\frac{\partial\phi_1}{\partial\zeta} + \frac{\partial}{\partial\zeta}(\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2})$$
$$\left[1 + \Omega_c^{-2} c_1\gamma_1(\lambda_0 - v_0)^2(1 + \sigma c_1)\right] + \frac{\partial^3\phi_1}{\partial\zeta^3} + \left[1 + \sigma c_1 - \gamma_1 c_1(\lambda_0 - v_0)^2\right]\frac{\partial n_2}{\partial\zeta} - c_1^2\eta_0(\lambda_0 - v_0)\left[\frac{\partial^2\phi_1}{\partial\xi^2} +\right.$$
$$\left.\frac{\partial^2\phi_1}{\partial\eta^2} + \frac{\partial^2\phi_1}{\partial\zeta^2}\right] - c_1^2(\eta_0 + \mu_0)(\lambda_0 - v_0)\frac{\partial^2\phi_1}{\partial\zeta^2} = 0 \tag{2.52}$$

From equation (2.35) we have

$$(\lambda_0 - v_0)^2\gamma_1 c_1 = 1 + \sigma c_1 \tag{2.53}$$

Now inserting equation (2.53) in (2.52) to cancel $\frac{\partial n_2}{\partial \zeta}$ term and yielding

$$\left[2\gamma_1 c_1^2(\lambda_0 - v_0)\right]\frac{\partial \phi_1}{\partial \tau} + \left[3(\lambda_0 - v_0)^2 c_1^3 \gamma_1 - 2(\lambda_0 - v_0)^3 \gamma_2 c_1^3 - 2c_2 - \sigma c_1^3\right]\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \frac{\partial^3 \phi_1}{\partial \zeta^3} + \frac{\partial}{\partial \zeta}\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \Omega_c^{-2} c_1 \gamma_1 (\lambda_0 - v_0)^2 (1 + \sigma c_1)\right] - c_1^2 \eta_0 (\lambda_0 - v_0)\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right] - c_1^2(\eta_0 + \mu_0)(\lambda_0 - v_0)\frac{\partial^2 \phi_1}{\partial \zeta^2} = 0 \tag{2.54}$$

Divide equation (2.54) by $2c_1^2\gamma_1(\lambda_0 - v_0)$ to write

$$\frac{\partial \phi_1}{\partial \tau} + \frac{3(\lambda_0 - v_0)^2 c_1^3 \gamma_1 - 2(\lambda_0 - v_0)^3 \gamma_2 c_1^3 - 2c_2 - \sigma c_1^3}{2c_1^2\gamma_1(\lambda_0 - v_0)}\phi_1 \frac{\partial \phi_1}{\partial \zeta} + \frac{1}{2c_1^2\gamma_1(\lambda_0 - v_0)}\frac{\partial^3 \phi_1}{\partial \zeta^3} + \frac{1 + \Omega_c^{-2}(\lambda_0 - v_0)^2 \gamma_1 c_1 (1 + \sigma c_1)}{2c_1^2\gamma_1(\lambda_0 - v_0)}\frac{\partial}{\partial \zeta}\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right] - \frac{\eta_0}{2\gamma_1}\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2}\right] - \frac{\eta_0 + \mu_0)}{2\gamma_1}\frac{\partial^2 \phi_1}{\partial \zeta^2} = 0$$

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \zeta} + B\frac{\partial^3 \phi_1}{\partial \zeta^3} + C\frac{\partial}{\partial \zeta}\left[\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right] - D\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \zeta^2} - E\frac{\partial^2 \phi_1}{\partial \zeta^2} = 0 \tag{2.55}$$

Which is the desired ZKB equation, where

$$A = \frac{3(\lambda_0 - v_0)^2 c_1^3 \gamma_1 - 2(\lambda_0 - v_0)^3 \gamma_2 c_1^3 - 2c_2 - \sigma c_1^3}{2c_1^2\gamma_1(\lambda_0 - v_0)} \tag{2.56}$$

$$B = \frac{1}{2c_1^2\gamma_1(\lambda_0 - v_0)} \tag{2.57}$$

$$C = \frac{1 + \Omega_c^{-2}(\lambda_0 - v_0)^2 \gamma_1 c_1 (1 + \sigma c_1)}{2c_1^2\gamma_1(\lambda_0 - v_0)} \tag{2.58}$$

$$D = \frac{\eta_0}{2\gamma_1} \tag{2.59}$$

$$E = \frac{\eta_0 + \mu_0)}{2\gamma_1} \tag{2.60}$$

2.5 Solution of ZKB equation

First we transform the ZKB equation by defining the travelling coordinate as following

$$\chi = l_x \zeta + l_y \eta + l_z \zeta - U_0 \tau, \tag{2.61}$$

where l_x, l_y and l_z are direction cosines which shows relation between propagating vector k and ξ, η and ζ axis, such that $l_x^2 + l_y^2 + l_z^2 = 1$.

The derivatives change accordingly, i.e.

$$
\begin{aligned}
\frac{\partial}{\partial\tau} &= \frac{\partial\chi}{\partial\tau}\frac{\partial}{\partial\chi}, & \frac{\partial}{\partial\tau} &= -U_o\frac{\partial}{\partial\chi} \\
\frac{\partial}{\partial\zeta} &= \frac{\partial\chi}{\partial\zeta}\frac{\partial}{\partial\chi}, & \frac{\partial}{\partial\zeta} &= l_z\frac{\partial}{\partial\chi} \\
\frac{\partial}{\partial\xi} &= \frac{\partial\chi}{\partial\xi}\frac{\partial}{\partial\chi}, & \frac{\partial}{\partial\xi} &= l_x\frac{\partial}{\partial\chi} \\
\frac{\partial}{\partial\eta} &= \frac{\partial\chi}{\partial\eta}\frac{\partial}{\partial\chi}, & \frac{\partial}{\partial\xi} &= l_y\frac{\partial}{\partial\chi} \\
\frac{\partial^2}{\partial\xi^2} &= l_x^2\frac{\partial^2}{\partial\chi^2}, & \frac{\partial^2}{\partial\eta^2} &= l_y^2\frac{\partial^2}{\partial\chi^2} \\
\frac{\partial^2}{\partial\zeta^2} &= l_z^2\frac{\partial^2}{\partial\chi^2}, & \frac{\partial^3}{\partial\zeta^3} &= l_z^3\frac{\partial^3}{\partial\chi^3} \\
=> \frac{\partial}{\partial\zeta}\frac{\partial^2}{\partial\xi^2} &= l_x^2 l_z^2\frac{\partial^3}{\partial\chi^3}, & \frac{\partial}{\partial\zeta}\frac{\partial^2}{\partial\eta^2} &= l_y^2 l_z^2\frac{\partial^3}{\partial\chi^3}
\end{aligned}
\tag{2.62}
$$

Using equations (2.62) into (2.55) yields

$$
-U_0\frac{\partial\phi_1}{\partial\chi} + Al_z\phi_1\frac{\partial\phi_1}{\partial\chi} + \left[Bl_z^2 + C(l_x^2 + l_y^2)\right]l_z\frac{\partial^3\phi_1}{\partial\chi^3} - \left[(l_x^2 + l_y^2) + El_z^2\right]\frac{\partial^2\phi_1}{\partial\chi^2} = 0
$$

$$
-U_0\frac{d\phi_1}{d\chi} + Al_z\phi_1\frac{d\phi_1}{d\chi} + Hl_z\frac{d^3\phi_1}{d^3\chi} - G\frac{d^2\phi_1}{d\chi^2} = 0 \tag{2.63}
$$

Here $G = D + El_z^2$ and $H = l_z^2 B + (l_x^2 + l_y^2)C$. Equation (2.63) gives the shock waves solution, because it contains both dispersive and dissipative terms. The amplitude of this shock waves can be found by using tangent hyperbolic method. For this we assume that our potential is bounded at $\chi = \pm\infty$. The complete solution is given by,

$$
\phi_1(\chi) = \frac{3G^2}{25HAl_z^2}\left[2 - 2tanh\left(\frac{G\chi}{10Hl_z}\right) + sech^2\left(\frac{G\chi}{10Hl_z}\right)\right] \tag{2.64}
$$

Which is the shock waves solution of ZKB equation. The value $\phi_m = \frac{9G^2}{25HAl_z^2}$ is the amplitude and $\frac{G}{10Hl_z}$ is the width of the shock waves, χ is the moving coordinate and U_0 is the normalized shock speed. The coefficients D, E denote the dissipative and B, C represent dispersive terms. The pasitive and negative amplitude depends on the sign of coefficient. There will be compression of amplitude when the coefficient of non-linear term is pasitive and rarefaction of amplitude when that is negative.

2.6 Graphical results and discussion

In this section, we present the numerical analysis to describe the shock parameters as affected by different plasma variables like σ, l_z etc.

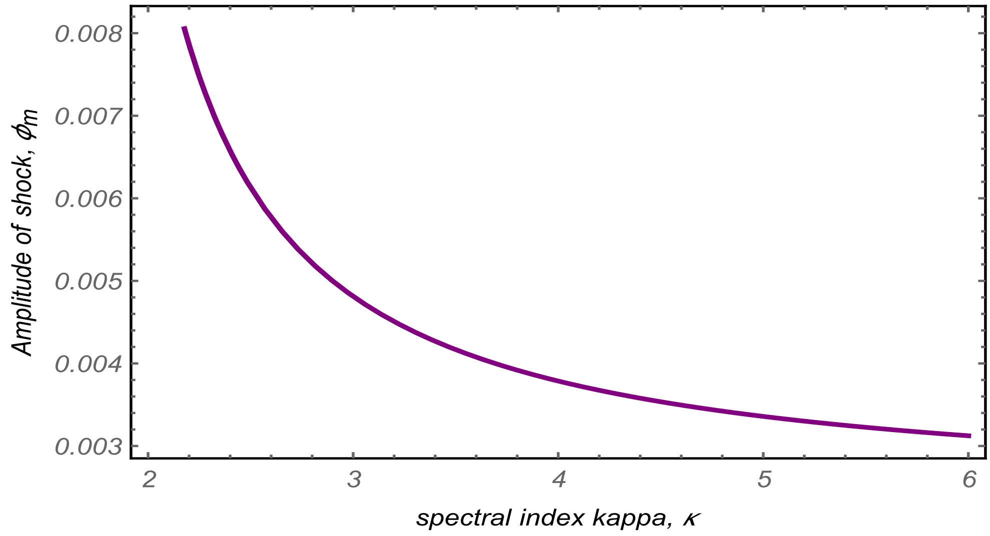

Figure 2.1: *Variation of ϕ_m with the spectral index κ.*

Figure (2.1) indicates the potential amplitude decrease with κ. That implies that for higher values of κ - that is for Maxwellian VDF - the potential amplitude is smallar in comparison with the scenario when superthermal particle are more in numbers (small value of κ).

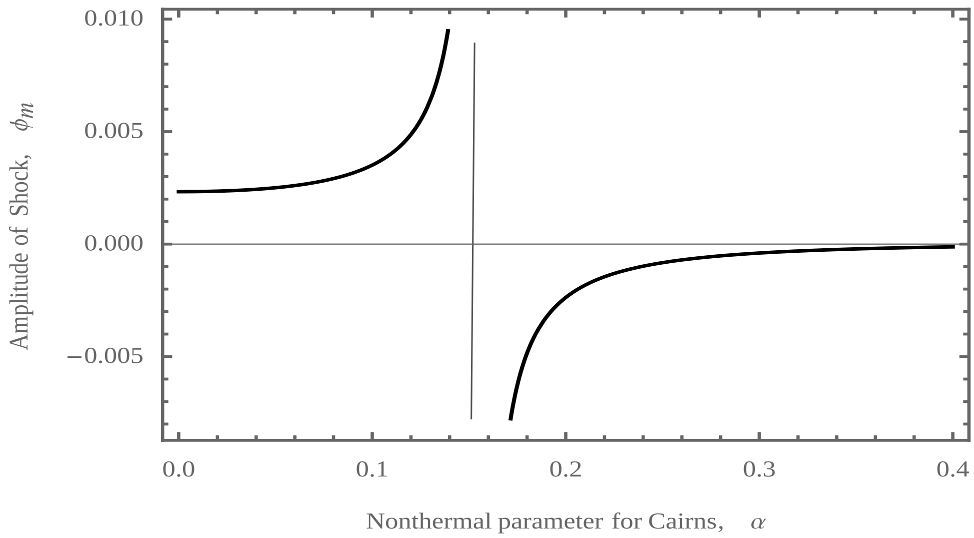

Figure 2.2: *Effect of Cairns parameter α on the ion acoustic shock amplitude ϕ_m.*

Figure (2.2) depicts the effect of Cairns VDF on ϕ_m of ion acoustic shock waves.

Variation for cairns distribution can be seen by a parameter α (non-thermal electrons population). We note that ϕ_m changes with alpha variation. To evaluate the critical value of α we have plotted the distribution of perturbed potential, as a function of Cairns parameter in figure (2.2). Here we see that at $\alpha = 0.15$ the shock amplitude changes its sign and thus a transition between compressive and rarefactive shock takes place.

In figure (2.3), we have plotted A and H against α, we observe that the compressive and rarefactive nature of ion acoustic shock waves corresponds to pasitive and negative sign of non-linear coefficient in equation (2.55). This figure is consistant with previous one.

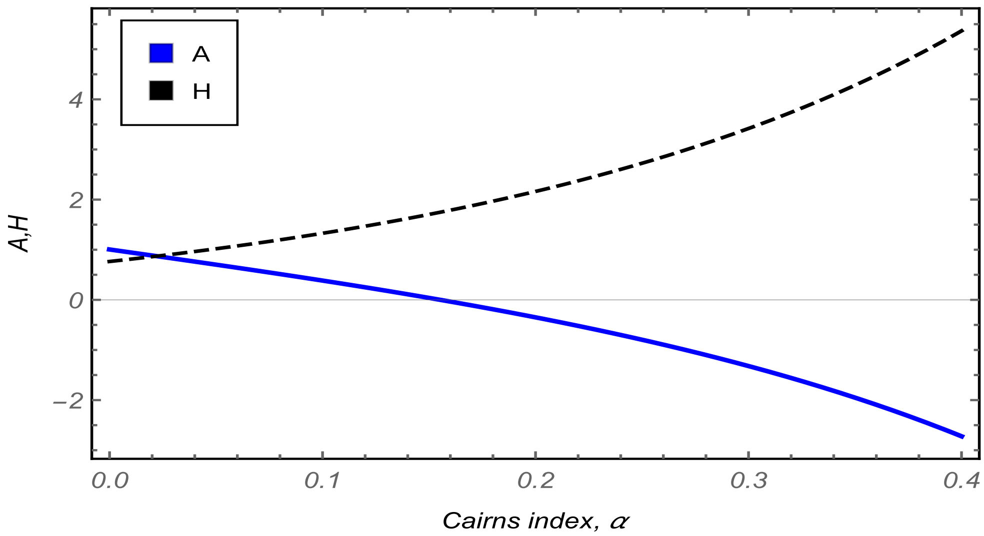

Figure 2.3: *Variations in non-linear and dispersive coefficients with respect to Cairns index α.*

In figure (2.4), we see that as the value of kappa κ increases the structure of shock profile decreases. Its indicate that there is fewer of non-thermal electrons and it is likely that shock profile have high degree of Maxwellian electrons.

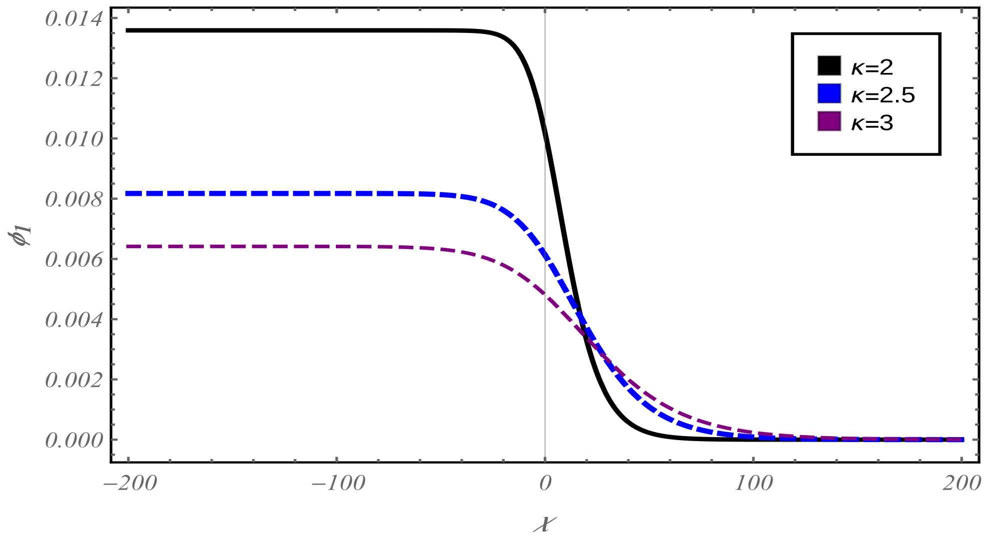

Figure 2.4: *Structure of shock wave profiles for different values of* κ*.*

Figure (2.5) shows that as the value of α (Non-thermal electrons population) increases the amplitude of shock profile decreases. This corresponds to rarefactive nature of shock ionic waves.

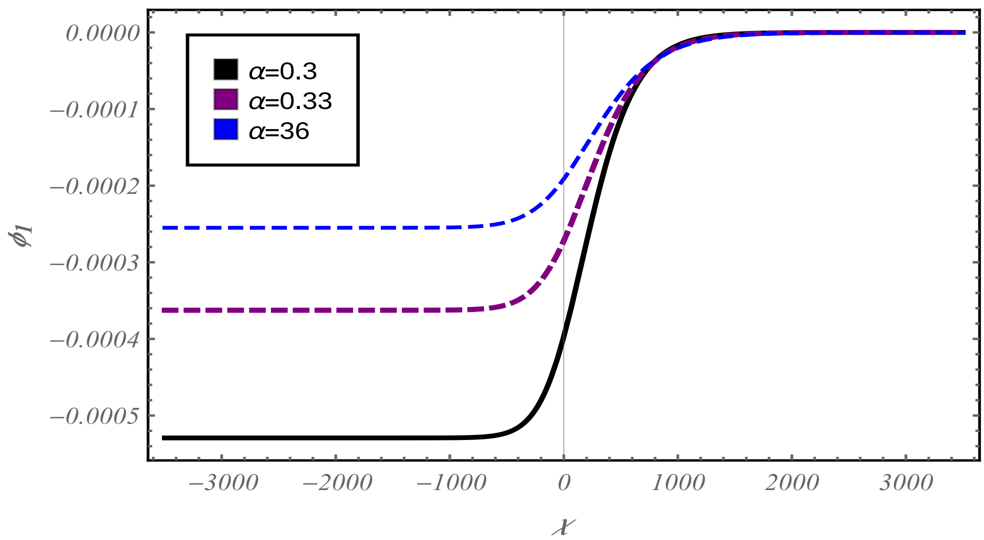

Figure 2.5: *Structure of shock wave profiles for different values of* α*.*

Figure (2.6) shows that as we increase the values of viscosity index eta (η), i.e. The dissipation in the system and it variate the shock potential for kappa electrons. We see in this figure that as dissipative variable increases, the shock potential increase in such manners that is is easily predicted.

In figure (2.7), a similar trend is observed for Cairns VDF. However, the amplitude ϕ_m in case of later is more than kappa VDF.

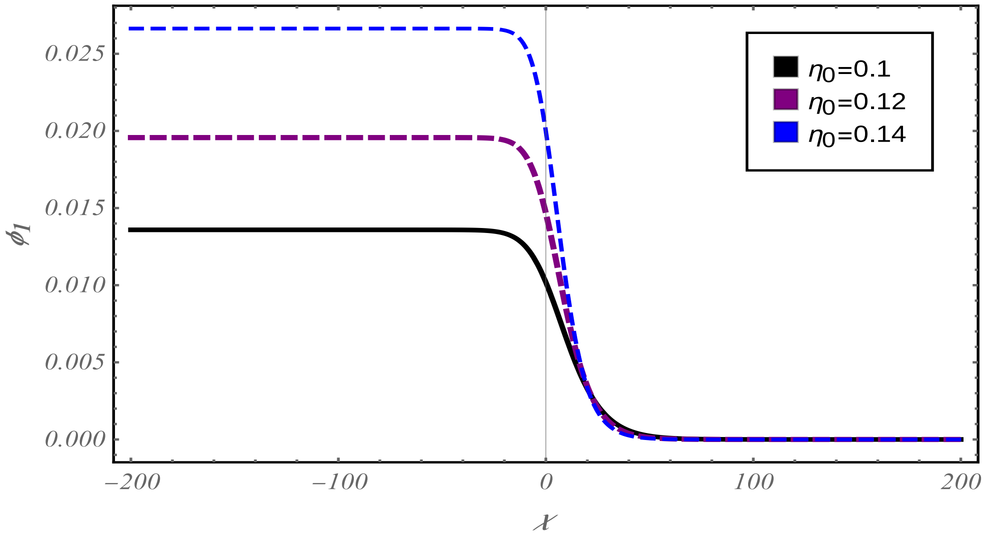

Figure 2.6: *Behavior of shock wave profile ϕ_1 for different value of η (ion kinematic viscosity).*

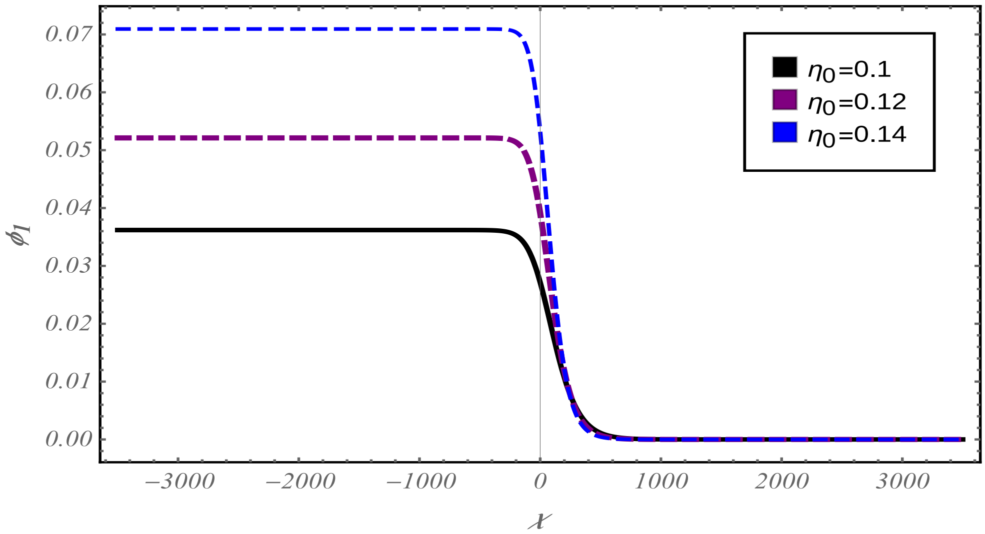

Figure 2.7: *Same as the last figure but for kappa distributed electrons.*

In figure (2.8), we see that ionic shock wave profile varies as we increase the rotational frequency Ω. The high amplitude of the potential corresponds to high values of rotational frequency for kappa electrons.
In fig. (2.9), a similar trend is also observed for Cairns VDF but exhibiting a higher strength

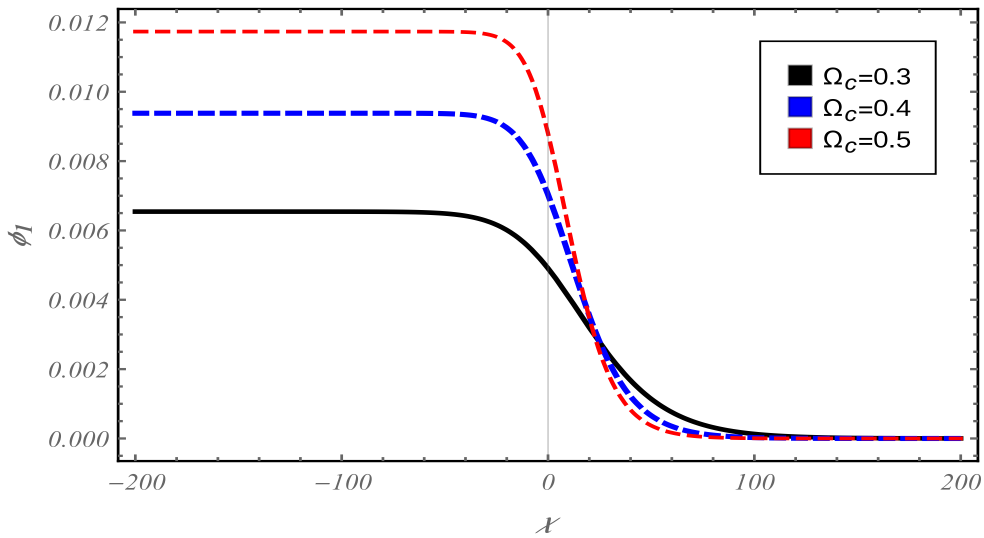

Figure 2.8: *Behavior of shock profile for different value of Ω (rotaional frequency).*

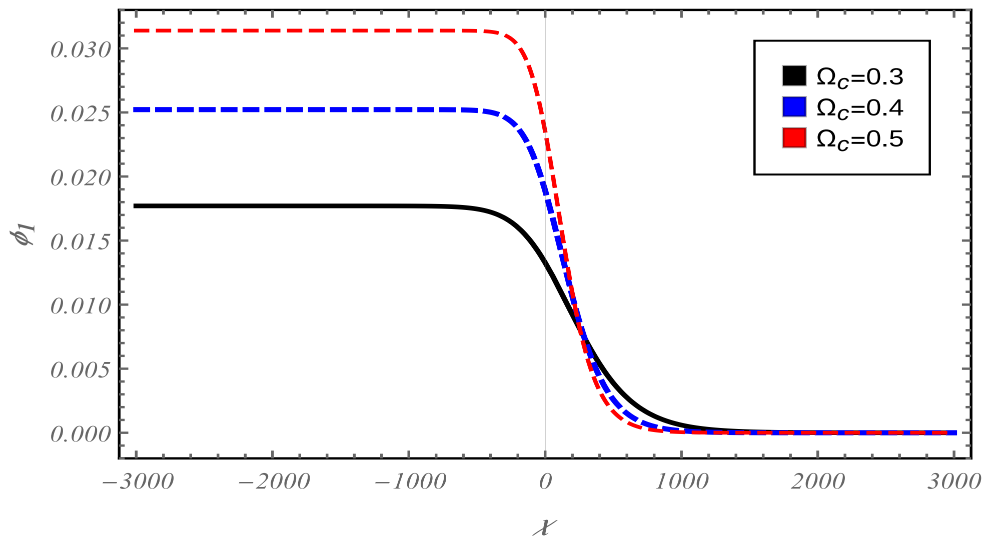

Figure 2.9: *Same as previous figure for Cairns distribution of electrons.*

Figures (2.10) and (2.11) shows how the perturbed potential distribution for both kappa and Cairns electronic distributions, as affected by direction cosine l_z. Here we note that as we enlarge the l_z, the obliqueness angle deminish which shows that there is enlarged behavior of cyclotron character. Thus the magnitude of shock potential is diminish as the l_z gets larger values.

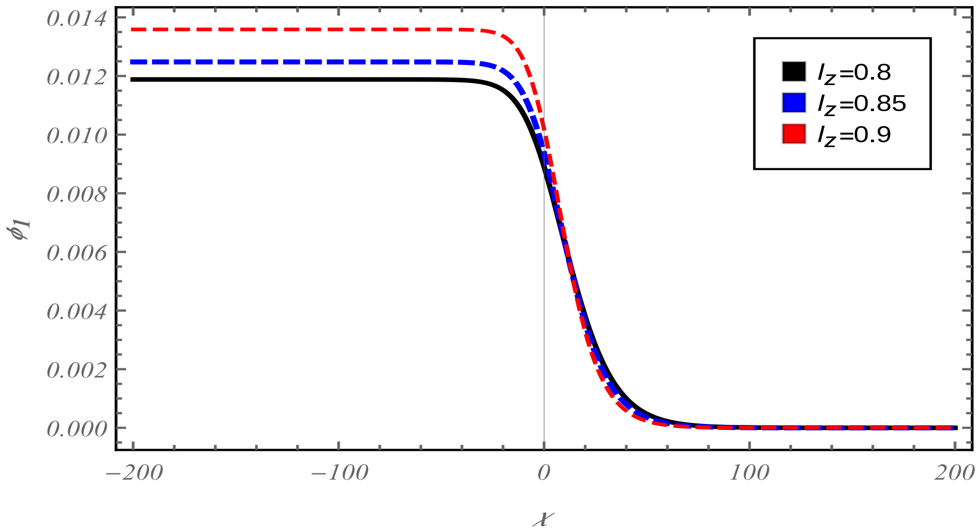

Figure 2.10: *The shock potential distribution as affected by direction cosine l_z and kappa spectral index.*

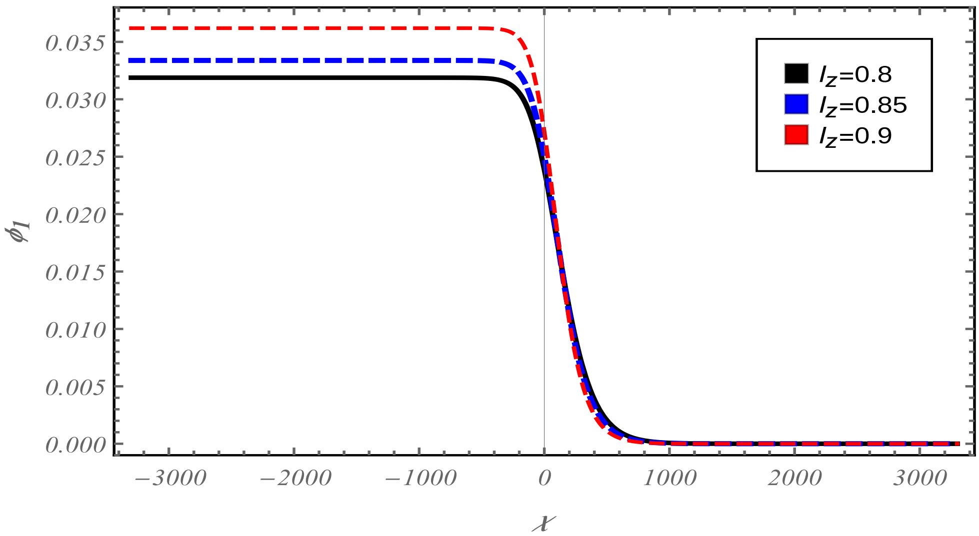

Figure 2.11: *Behavior of shock potential ϕ_1 with respect to direction cosine l_z and Cairns VDF.*

In figure (2.12), we extend our study to see bahavior of ϕ_1 (perturbed potential) for different values of ion to electron temperature ratio σ. When we increase σ, the shock profile diminish both for kappa and Cairns electron distributions.

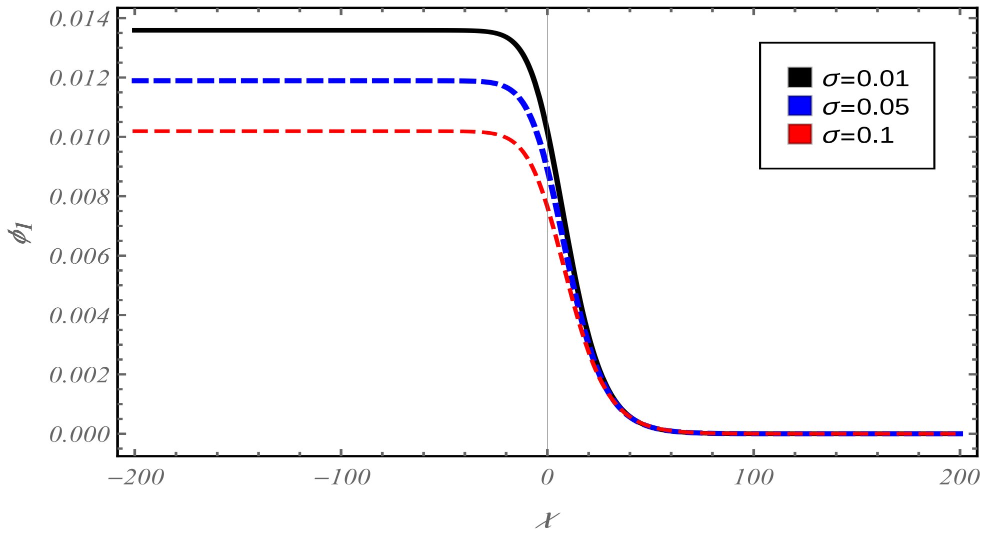

Figure 2.12: *Structures of shock profile ϕ_1 for different values of ion electron temperature ratio σ.*

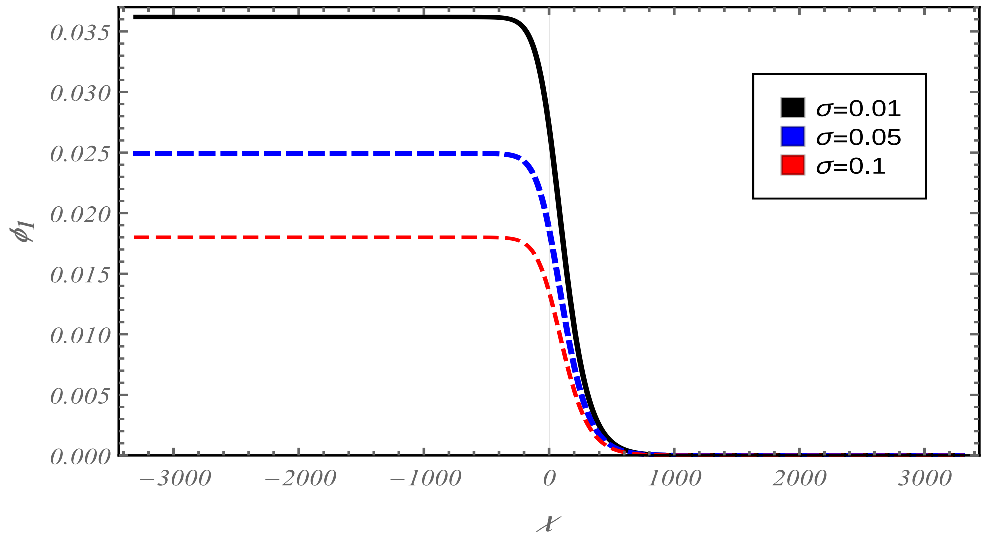

Figure 2.13: *Same as previous figure but for kappa distributed electrons.*

But in figure (2.13), it is seen that as compared to kappa, shock profile have enlarge amplitude for Cairns case.

Chapter 3

IA shock-like potential with (r, q) distributed electrons

3.1 Model equations

Again consider a plasma system, which is magnetized, composed of warm ions and non-Maxwellian electrons. We also consider that magnetic field is along z-direction, i.e., $\vec{B} = B_0\hat{z}$. The equilibrium quasi-neutrality condition demands that $n_{i0} = n_{e0}$, where n_{e0} and n_{i0} are, respectively the unperturbed densities of ions and electrons. The corresponding ion fluid equations, namely continuity, force balanced, and Poisson's equation, for our system can be written as

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{3.1}$$

$$m_i n_i\left(\frac{\partial}{\partial t} + \vec{v_i}\cdot\vec{\nabla}\right)\vec{v_i} = en_i(\vec{E} + \vec{v_i}\times\vec{B}) - \vec{\nabla}p_i + 2m_i n_i(\vec{v_i}\times\vec{\Omega}) + m_i n_i \eta_i \nabla^2\vec{v_i}, \tag{3.2}$$

and, respectively

$$\vec{\nabla}\cdot\vec{E} = 4\pi e(n_i - n_e) \tag{3.3}$$

Here all the symbols have their usual meaning, as defined in previous chapter. The electrons (r, q) distribution function is given by [28]

$$f_{rq}(v) = \frac{3\Gamma[q](q-1)^{\frac{-3}{(2+2r)}}}{4\pi\beta^{\frac{3}{2}}v_{th}^{\frac{3}{2}}\Gamma[q-\frac{3}{2+2r}]\Gamma[1+\frac{3}{2+2r}]}\left(1+\frac{1}{q-1}\left(\frac{v^2-\frac{2e\phi}{m_e}}{\beta v_{th}^2}\right)^{r+1}\right)^{-q}, \tag{3.4}$$

where

$$\beta = \frac{3(q-1)^{\frac{-1}{(1+r)}}\Gamma[q-\frac{3}{2+2r}]\Gamma[\frac{3}{2+2r}]}{2\Gamma[q-\frac{5}{2+2r}]\Gamma[\frac{5}{2+2r}]} \tag{3.5}$$

Here v_{th} is the thermal velocity, r and q are the spectral indices. By choosing different values of these indices, one can model huge number of non-Maxwellian plasmas. Upon integrating the above distribution over velocity space, the corresponding electron density takes the form [36]

$$n_e = n_{e0}(1 + A_1\phi + A_2\phi^2) \tag{3.6}$$

with

$$A_1 = \frac{(q-1)^{\frac{-1}{r+1}}\Gamma[q-\frac{1}{2r+2}]\Gamma[\frac{1}{2r+2}]}{2\beta\Gamma[\frac{3}{2r+2}]\Gamma[q-\frac{3}{2r+2}]}, \tag{3.7}$$

$$A_2 = \frac{-(q-1)^{\frac{-2}{r+1}}\Gamma[q+\frac{1}{2r+2}]\Gamma[\frac{-1}{2r+2}]}{8\beta^2\Gamma[\frac{3}{2r+2}]\Gamma[q-\frac{3}{2r+2}]} \tag{3.8}$$

In the limit when $r = 0$ and $q \to \infty$, we recover the Maxwellian counterpart for which $A_1 = 1$ and $A_2 = \frac{1}{2}$. For a limit when value of $r = 0$ and $q \to \kappa + 1$ we recover the kappa VDF whose coefficients are given as, $A_1 = \frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}$, $A_2 = \frac{(\kappa-\frac{1}{2})(\kappa+\frac{1}{2})}{2(\kappa-\frac{3}{2})^2}$. For physically acceptable results, values of r and q must satisfy the condition, $q > 1$ and $q(1+r) > \frac{5}{2}$.

3.2 Normalization of model equations

Here again we use the same normalized variable as discussed in the last chapter. The continuity equation (2.8), in its normalized form, is given as

$$\frac{\partial \bar{n}_i}{\partial \bar{t}}\frac{\partial}{\partial \bar{x}}(\bar{n}_i\bar{v}_i) = 0, \tag{3.9}$$

Likewise, the force balance expression takes the following form

$$m_i n_i\left(\frac{\partial}{\partial t} + \vec{v_i}\cdot\vec{\nabla}\right)\vec{v_i} = -en_i\frac{\partial\phi}{\partial x} + en_i(\vec{v_i}\times\vec{B}) - k_BT_i\frac{\vec{\nabla}n_i}{n_i} + 2m_in_i(\vec{v_i}\times\vec{\Omega}) + m_in_i\nabla^2\vec{v_i}$$

Divide above equation with m_in_i, and then by $C_s\omega_{pi}$ gives,

$$\left(\frac{\partial}{\partial t} + \vec{v_i}\cdot\vec{\nabla}\right)\vec{v_i} = \frac{-e\cancel{n_i}}{m_i\cancel{n_i}}\frac{\partial\phi}{\partial x} + \frac{e\cancel{n_i}}{m_i\cancel{n_i}}(\vec{v_i}\times\vec{B}) - \frac{k_BT_i}{m_i}\frac{\vec{\nabla}n_i}{n_i} + \frac{2\cancel{m_in_i}}{\cancel{m_in_i}}(\vec{v_i}\times\vec{\Omega}) + \frac{\cancel{m_in_i}}{\cancel{m_in_i}}\eta_i\nabla^2\vec{v_i}$$

$$\left(\omega_{pi}\frac{\partial}{\partial\bar{t}}+\frac{C_s}{\lambda_D}\bar{v}_i\cdot\bar{\nabla}\right)\bar{v}_iC_s=-\frac{k_BT_e}{m_i\lambda_D}\frac{\partial\bar{\phi}}{\partial\bar{x}}+\frac{eB_0C_s}{m_i}(\bar{v}_i\times\hat{z})-\frac{K_BT_i}{m_i\lambda_D}\frac{\bar{\nabla}\bar{n}_i}{\bar{n}_i}+2C_s\omega_{pi}\Omega_0(\bar{v}_i\times\hat{z})+\omega_{pi}\bar{\eta}_iC_s\nabla^2\bar{v}_i$$

$$\omega_{pi}\left(\frac{\partial}{\partial\bar{t}}+\frac{C_s}{\lambda_D\omega_{pi}}\bar{v}_i\cdot\bar{\nabla}\right)\bar{v}_iC_s=\frac{-k_BT_e}{m_i\lambda_D}\frac{\partial\bar{\phi}}{\partial\bar{x}}+\omega_{ci}C_s(\bar{v}_i\times\hat{z})-\frac{k_BT_i}{m_i\lambda_D}\frac{\bar{\nabla}\bar{n}_i}{n_i}+2\omega_{ci}C_s\Omega_0(\bar{v}_i\times\hat{z})+\omega_{pi}C_s\bar{\eta}_i\nabla^2\bar{v}_i$$

$$\left(\frac{\partial}{\partial\bar{t}}+\bar{v}_i\cdot\bar{\nabla}\right)\bar{v}_i=-\frac{\partial\bar{\phi}}{\partial\bar{x}}-\sigma\frac{\bar{\nabla}\bar{n}_i}{\bar{n}_i}+\Omega_c(\bar{v}_i\times\hat{z})+\bar{\eta}_i\bar{\nabla}^2\bar{v}_i, \tag{3.10}$$

where $\Omega_c=\omega_{ci}+2\Omega_0$ and σ represents the temperature ratio of ion to electron.
Poisson's equation in normalized form reads

$$\frac{\partial^2\bar{\phi}}{\partial^2\bar{x}}=n_e-n_i. \tag{3.11}$$

For a mathematical ease we shall omit the overhead bar from our normalized variables.
The continuity, forced balanced, and Poisson's equations can be written in the cartesian coordinates

$$\frac{\partial ni}{\partial t}+\frac{\partial n_iv_{ix}}{\partial x}+\frac{\partial n_iv_{iy}}{\partial y}+\frac{\partial n_iv_{iz}}{\partial z}=0, \tag{3.12}$$

$$\frac{\partial}{\partial t}\left(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z}\right)+\left((v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z})\cdot(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z})\right)\left(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z}\right)+$$
$$\left(\frac{\partial}{\partial x}\phi\hat{x}+\frac{\partial}{\partial y}\phi\hat{y}+\frac{\partial}{\partial z}\phi\hat{z}\right)-\Omega_c\left((v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z})\times\hat{z}\right)-\frac{\sigma}{n_i}\left(\frac{\partial}{\partial x}n_i-\frac{\partial}{\partial y}n_i-\frac{\partial}{\partial z}n_i\right)=$$
$$\eta_i\left((\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z})\right)$$

$$\frac{\partial}{\partial t}\left(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z}\right)+\left((v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z})\cdot(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z})\right)\left(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z}\right)-$$
$$\left(\frac{\partial}{\partial x}\phi\hat{x}+\frac{\partial}{\partial y}\phi\hat{y}+\frac{\partial}{\partial z}\phi\hat{z}\right)+\Omega_c\left((v_{iy}\hat{x}-v_{ix}\hat{y}+0)\right)-\frac{\sigma}{n_i}\left(\frac{\partial}{\partial x}n_i-\frac{\partial}{\partial y}n_i-\frac{\partial}{\partial z}n_i\right)=$$
$$\eta_i\left((\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})(v_{ix}\hat{x}+v_{iy}\hat{y}+v_{iz}\hat{z})\right)$$

Next the x, y, and z-component of the above equation can be seperated to write

$$\begin{aligned}&\frac{\partial}{\partial t}v_{ix}+v_{ix}\frac{\partial}{\partial x}v_{ix}+v_{iy}\frac{\partial}{\partial y}v_{ix}+v_{iz}\frac{\partial}{\partial z}v_{ix}+\frac{\partial}{\partial x}\phi-\Omega_cv_{iy}+\frac{\sigma}{n_i}\frac{\partial}{\partial x}n_i=\\&\eta_i\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{ix}\end{aligned} \tag{3.13}$$

$$\begin{aligned}&\frac{\partial}{\partial t}v_{iy}+v_{ix}\frac{\partial}{\partial x}v_{iy}+v_{iy}\frac{\partial}{\partial y}v_{iy}+v_{iz}\frac{\partial}{\partial z}v_{iy}+\frac{\partial}{\partial y}\phi+\Omega_cv_{ix}+\frac{\sigma}{n_i}\frac{\partial}{\partial y}n_i=\\&\eta_i\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)v_{iy}\end{aligned} \tag{3.14}$$

$$\frac{\partial}{\partial t}v_{iz} + v_{ix}\frac{\partial}{\partial x}v_{iz} + v_{iy}\frac{\partial}{\partial y}v_{iz} + v_{iz}\frac{\partial}{\partial z}v_{iz} = -\frac{\partial}{\partial z}\phi - \frac{\sigma}{n_i}\frac{\partial}{\partial z}n_i + \eta_i\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)v_{iz}, \tag{3.15}$$

and, respectively

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = n_e - n_i \tag{3.16}$$

Inserting equation (3.6) into (3.17) yields

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = (1 + A_1\phi_1 + A_2\phi_2) - n_i \tag{3.17}$$

3.3 Application of RPT

For the transformation of coordinates and expansion of dependent variables, we use the same analysis as in chapter 2.
From Poisson's equation the different powers of ϵ can be seperated as

$$(n_1 - A_1\phi_1)\epsilon + (n_2 - A_2\phi_1^2 - A_1\phi_2 + \phi_1''[\xi] + \phi_1''[\zeta] + \phi_1''[\eta])\epsilon^2 + (-2A_2\phi_1\phi_2 + \phi_2''[\zeta] + \phi_2''[\eta] + \phi_2[\xi])\epsilon^3 - A_2\phi_2^2\epsilon^4 + O\epsilon^8 = 0 \tag{3.18}$$

Similarly, from continuity equation we have

$$-(\lambda_0 n_1'[\zeta] + w_1'[\zeta])\epsilon^{\frac{3}{2}} + (u_1[\xi] + v_1'[\eta])\epsilon^2 + (w_1[\zeta]n_1'[\zeta] + n_1'[\tau] - \lambda_0 n_2'[\zeta] + u_2'[\xi] + v_2'[\eta] + n_1 w_1'[\zeta] + w_2'[\zeta])\epsilon^{\frac{5}{2}} + (v_1 n_1'[\eta] + u_1 n_1'[\xi] + n_1 u_1'[\xi] + n_1 v_1'\eta)\epsilon^3 + (w_2 n_1'[\zeta] + v_2 n_1'[\eta] + u_2 n_1'[\xi] + w_1 n_2'[\zeta] + n_2'[\tau] + n_1 u_2'[\xi] + n_1 v_2'[\eta] + n_2 w_1'[\zeta] + n_1 w_2'[\zeta])\epsilon^{\frac{7}{2}} + (v_1 n_2'[\eta] + u_1 n_2'[\xi] + n_2 u_1'[\xi] + n_2 v_1'[\eta])\epsilon^4 + (w_2 n_2'[\zeta] + v_2 n_2'[\eta] + u_2 n_2'[\xi] + n_2 u_2'[\xi] + n_2 v_2'[\eta] + n_2 w_2'[\zeta])\epsilon^{\frac{9}{2}} + O\epsilon^{\frac{11}{2}} = 0 \tag{3.19}$$

From equation of motion the x-component provides the relation among various powers of ϵ as

$$-(v_1\Omega_c + \sigma n_1'[\xi] + \phi_1'[\xi])\epsilon^{\frac{3}{2}} + (-v_2\Omega_c - \lambda_0 u_1'[\zeta])\epsilon^2 + (w_1 u_1'[\zeta] + u_1'[\tau] - \eta_0 u_1''[\xi] - \eta_0 u_1''[\eta] - \eta_0 u_1''[\zeta])\epsilon^3 + (w_2 u_1'[\zeta] + u_2 u_1'[\xi] + u_1 u_2'[\xi] + v_2 u_1'[\eta] + v_1 u_2'[\eta])\epsilon^4 + (-\sigma n_1 n_1'[\xi] + \sigma n_2'[\xi] - \lambda_0 u_2'[\zeta] + \phi_2[\xi])\epsilon^{\frac{5}{2}} + (-\sigma n_2 n_1'[\xi] - \sigma n_1 n_2'[\xi] + u_1 u_1'[\xi] + w_1 u_2'[\zeta] + u_2'[\tau] + v_1 u_1'[\eta] - \eta_0 u_2''[\xi] - \eta_0 u_2''[\eta] - \eta_0 u_2''[\zeta])\epsilon^{\frac{7}{2}} - (\sigma n_2 n_2'[\xi] + w_2 u_2'[\zeta] + v_2 u_2'[\xi] + u_2 u_2'[\xi])\epsilon^{\frac{9}{2}} + O\epsilon^{\frac{15}{2}} = 0 \tag{3.20}$$

Likewise, the y and z-components provide

$$(u_1\Omega_c + \sigma n_1'[\eta] + \phi_1'[\eta])\epsilon^{\frac{3}{2}} + (u_2\Omega_c - \lambda_0 v_1'[\zeta])\epsilon^2 + (w_1 v_1'[\zeta] + v_1'[\tau] - \eta_0 v_1''[\xi] - \eta_0 v_1''[\eta] - \eta_0 v_1''[\zeta])\epsilon^3$$
$$+ (w_2 v_1'[\zeta] + v_2 v_1'[\eta] + u_2 v_1'[\xi] + v_1 v_2'[\eta] + u_1 v_2'[\xi])\epsilon^4 + (-\sigma n_1 n_1'[\eta] + \sigma n_1 n_2'[\eta] - \lambda_0 v_2'[\zeta] + \phi_2[\eta])\epsilon^{\frac{5}{2}}$$
$$+ (-\sigma n_2 n_1'[\eta] - \sigma n_1 n_2'[\eta] + v_1 v_1'[\eta] + u_1 v_1'[\xi] + w_1 v_2'[\zeta] + v_2'[\tau] - \eta_0 v_2''[\xi] - \eta_0 v_2''[\eta] - \eta_0 v_2''[\zeta])\epsilon^{\frac{7}{2}}$$
$$+ (-n_2 n_2'[\eta] + w_2 v_2'[\zeta] + v_2 v_2'[\eta] + u_2 v_2'[\xi])\epsilon^{\frac{9}{2}} + O\epsilon^{\frac{15}{2}} = 0 \tag{3.21}$$

and, respectively

$$(v_1 w_1'[\eta] u_1 w_1'[\xi])\epsilon^3 + (v_1 w_2'[\eta] + u_1 w_2'[\xi])\epsilon^4 + (\sigma n_1'[\zeta] - \lambda_0 w_1[\zeta] + \phi_1[\zeta])\epsilon^{\frac{3}{2}} + (-\sigma n_1 n_1[\zeta] + \sigma n_2'[\zeta]$$
$$+ w_1 w_1'[\zeta] + w_1'[\tau] - \lambda_0 w_2'[\zeta] + \phi_2'[\zeta] - \eta_0 w_1''[\xi] - \eta_0 w_1''[\eta] - \eta_0 w_1''[\zeta])\epsilon^{\frac{5}{2}} + (-\sigma n_2 n_1'[\zeta] - \sigma n_1 n_2'[\zeta]$$
$$+ w_2 w_1'[\zeta] + v_2 w_1[\eta] + u_2 w_1'[\xi] + w_1 w_2[\zeta] + w_2'[\tau] - \eta_0 w_2''[\xi] - \eta_0 w_2''[\eta] - \eta_0 w_2''[\zeta])\epsilon^{\frac{7}{2}} + (-\sigma n_2 n_2'[\zeta]$$
$$+ w_2 w_2'[\zeta] + v_2 w_2'[eta] u_2 w_2'[\xi])\epsilon^{\frac{9}{2}} + o\epsilon^{\frac{15}{2}} = 0 \tag{3.22}$$

Equating coefficients of ϵ in equation (3.18) yields

$$n_1 - A_1\phi_1 = 0$$

$$n_1 = A_1\phi_1 \tag{3.23}$$

From equation (3.19), the coefficients of $\epsilon^{\frac{3}{2}}$ can be written as

$$-\lambda_0 \frac{\partial n_1}{\partial \zeta} + \frac{\partial w_1}{\partial \zeta} = 0$$

$$n_1 = \frac{w_1}{\lambda_0} \tag{3.24}$$

Similarly, comparing the coefficients of $\epsilon^{\frac{3}{2}}$ for x, y and z component in EOM results in

$$-v_1\Omega_c + \sigma \frac{\partial n_1}{\partial \xi} + \frac{\partial \phi_1}{\partial \xi} = 0$$

which can be simplified as following

$$v_1\Omega_c = (1 + \sigma A_1)\frac{\partial \phi_1}{\partial \xi} \tag{3.25}$$

$$u_1\Omega_c + \sigma \frac{\partial n_1}{\partial \eta} + \frac{\partial \phi_1}{\partial \eta} = 0$$

$$u_1\Omega_c = (1+\sigma A_1)\frac{\partial \phi_1}{\partial \eta} \tag{3.26}$$

and, orderly

$$\sigma\frac{\partial n_1}{\partial \zeta} - \lambda_0\frac{\partial w_1}{\partial \zeta} + \frac{\partial \phi_1}{\partial \zeta} = 0$$

$$\sigma n_1 - \lambda_0 w_1 + \phi_1 = 0$$

$$w_1 = \frac{(1+\sigma A_1)\phi_1}{\lambda_0} \tag{3.27}$$

Upon that solving equations (3.23), (3.24) and (3.27) to obtain the phase speed (λ_0) as

$$\frac{w_1}{\lambda_0} = A_1\phi_1$$

$$\frac{w_1}{\phi_1} = \frac{(1+\sigma A_1)}{\lambda_0}$$

$$A_1\lambda_0^2 = 1+\sigma A_1$$

$$\lambda_0^2 = \frac{1+\sigma A_1}{A_1}$$

$$\lambda_0 = \left(\frac{1+\sigma A_1}{A_1}\right)^{\frac{1}{2}} \tag{3.28}$$

Coefficients of ϵ^2 in equations (3.19), (3.20) and (3.21) can be written as

$$-v_1\Omega_c - \lambda_0\frac{\partial u_1}{\partial \zeta} = 0$$

$$v_2\Omega_c = \lambda_0\frac{\partial u_1}{\partial \zeta} \tag{3.29}$$

$$u_2\Omega_c - \lambda_0\frac{\partial v_1}{\partial \zeta}$$

$$u_2\Omega_c = \lambda_0\frac{\partial v_1}{\partial \zeta} \tag{3.30}$$

and, orderly

$$\frac{\partial^2 \phi_1}{\partial^2 \xi^2} + \frac{\partial^2 \phi_1}{\partial^2 \eta^2} + \frac{\partial^2 \phi_1}{\partial^2 \zeta^2} = A_2\phi_1^2 + A_1\phi_2 - n_2$$

$$\frac{\partial^2 \phi_1}{\partial^2 \xi^2} + \frac{\partial^2 \phi_1}{\partial^2 \eta^2} + \frac{\partial^2 \phi_1}{\partial^2 \zeta^2} - A_2\phi_1^2 = A_1\phi_2 - n_2 \tag{3.31}$$

From equations (3.19) and (3.22), the coefficients of $\epsilon^{\frac{5}{2}}$ can be seperated to write

$$w_1\frac{\partial n_1}{\partial \zeta} + \frac{\partial n_1}{\partial \tau} - \lambda_0\frac{\partial n_2}{\partial \zeta} + \frac{\partial u_2}{\partial \xi} + n_1\frac{\partial w_1}{\partial \zeta} + \frac{\partial v_2}{\partial \eta} + \frac{\partial w_2}{\partial \zeta},$$

$$-\lambda_0\frac{\partial n_2}{\partial\zeta}+\frac{\partial u_2}{\partial\xi}+\frac{\partial v_2}{\partial\eta}+\frac{\partial w_2}{\partial\zeta}=-\frac{\partial n_1}{\partial\tau}-\frac{\partial n_1 w_1}{\partial\zeta}, \tag{3.32}$$

$$-\sigma n_1\frac{\partial n_1}{\partial\zeta}+\sigma\frac{\partial n_2}{\partial\zeta}+w_1\frac{\partial w_1}{\partial\zeta}+\frac{\partial w_1}{\partial\tau}-\lambda_0\frac{\partial w_2}{\partial\zeta}+\frac{\partial\phi_2}{\partial\zeta}-\eta_0\left[\frac{\partial^2 w_1}{\partial\xi^2}+\frac{\partial^2 w_1}{\partial\eta^2}+\frac{\partial^2 w_1}{\partial\zeta^2}\right]=0 \tag{3.33}$$

3.4 Derivation of ZKB equation

Upon taking the derivative of equation (3.30), we obtain the following

$$\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial_1^\phi}{\partial\zeta^2}\right)-A_2\frac{\partial\phi_1^2}{\partial\zeta}=A_1\frac{\partial\phi_2}{\partial\zeta}-\frac{\partial n_2}{\partial\zeta},$$

$$\frac{\partial\phi_2}{\partial\zeta}=\frac{1}{A_1}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right)-\frac{A_2}{A_1}\frac{\partial\phi_1^2}{\partial\zeta}+\frac{1}{A_1}\frac{\partial n_2}{\partial\zeta} \tag{3.34}$$

Inserting equations (3.25), (3.26), (3.29), (3.30), (3.32) and (3.34) into equation (3.33) yields

$$\frac{\partial w_2}{\partial\zeta}=\lambda_0\frac{\partial n_2}{\partial\zeta}-\frac{\partial u_2}{\partial\xi}-\frac{\partial v_2}{\partial\eta}-\frac{\partial n_1}{\partial\tau}-\frac{\partial n_1 w_1}{\partial\zeta}$$

$$-\sigma n_1\frac{\partial n_1}{\partial\zeta}+w_1\frac{\partial w_1}{\partial\zeta}+\frac{\partial w_1}{\partial\tau}-\eta_0\left[\frac{\partial^2 w_1}{\partial\xi^2}+\frac{\partial^2 w_1}{\partial\eta^2}+\frac{\partial^2 w_1}{\partial\zeta^2}\right]+\sigma\frac{\partial n_2}{\partial\zeta}-\lambda_0\left(\lambda_0\frac{\partial n_2}{\partial\zeta}-\frac{\partial u_2}{\partial\xi}-\frac{\partial v_2}{\partial\eta}-\frac{\partial n_1}{\partial\tau}\right.$$
$$\left.-\frac{\partial n_1 w_1}{\partial\zeta}\right)+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right)-\frac{A_2}{A_1}\frac{\partial\phi_1^2}{\partial\zeta}+\frac{1}{A_1}\frac{\partial n_2}{\partial\zeta}=0$$

$$-\sigma n_1\frac{\partial n_1}{\partial\zeta}+w_1\frac{\partial w_1}{\partial\zeta}+\frac{\partial w_1}{\partial\tau}-\eta_0\left[\frac{\partial^2 w_1}{\partial\xi^2}+\frac{\partial^2 w_1}{\partial\eta^2}+\frac{\partial^2 w_1}{\partial\zeta^2}\right]+\sigma\frac{\partial n_2}{\partial\zeta}-\lambda_0\left[\lambda_0\frac{\partial n_2}{\partial\zeta}-\left(\frac{\partial^2}{\partial\xi\partial\zeta}\lambda_0\Omega_c^{-2}\right.\right.$$
$$\left.(1+\sigma A_1)\frac{\partial\phi_1}{\partial\xi}\right)-\left(\frac{\partial^2}{\partial\eta\partial\zeta}\lambda_0\Omega_c^{-2}(1+\sigma A_1)\frac{\partial\phi_1}{\partial\eta}\right)-\left.\frac{\partial n_1}{\partial\tau}-\frac{\partial n_1 w_1}{\partial\zeta}\right]+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right)$$
$$-\frac{A_2}{A_1}\frac{\partial\phi_1^2}{\partial\zeta}+\frac{1}{A_1}\frac{\partial n_2}{\partial\zeta}=0$$

$$-\sigma n_1\frac{\partial n_1}{\partial\zeta}+w_1\frac{\partial w_1}{\partial\zeta}+\frac{\partial w_1}{\partial\tau}-\eta_0\left[\frac{\partial^2 w_1}{\partial\xi^2}+\frac{\partial^2 w_1}{\partial\eta^2}+\frac{\partial^2 w_1}{\partial\zeta^2}\right]+\sigma\frac{\partial n_2}{\partial\zeta}-\lambda_0\left[\lambda_0\frac{\partial n_2}{\partial\zeta}-\Omega_c^{-2}\lambda_0\right.$$
$$\left.(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}-\Omega_c^{-2}\lambda_0(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}-\frac{\partial n_1}{\partial\tau}-\frac{\partial n_1 w_1}{\partial\zeta}\right]+\frac{1}{A_1}\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{1}{A_1}\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}$$
$$+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}-\frac{2A_2}{A_1}\phi_1\frac{\partial\phi_1}{\partial\zeta}+\frac{1}{A_1}\frac{\partial n_2}{\partial\zeta}=0$$

Using equations (3.23), (3.24) and (3.27) in above and multiplying with A_1 to write

$$\sigma\frac{\partial n_2}{\partial\zeta}-\lambda_0^2\frac{\partial n_2}{\partial\zeta}+\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}+A_1\lambda_0\frac{\partial\phi_1}{\partial\tau}+2\lambda_0^2A_1^2\phi_1\frac{\partial\phi_1}{\partial\zeta}$$
$$+\frac{1}{A_1}\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{1}{A_1}\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}-\frac{2A_2}{A_1}\phi_1\frac{\partial\phi_1}{\partial\zeta}+\frac{1}{A_1}\frac{\partial n_2}{\partial\zeta}-A_1^2\sigma\phi_1\frac{\partial\phi_1}{\partial\zeta}+A_1^2\lambda_0^2\phi_1\frac{\partial\phi_1}{\partial\zeta}$$
$$+A_1\lambda_0\frac{\partial\phi_1}{\partial\tau}-A_1\lambda_0\eta_0\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]=0$$

$$A_1\sigma\frac{\partial n_2}{\partial\zeta}-A_1\lambda_0^2\frac{\partial n_2}{\partial\zeta}+A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}+A_1^2\lambda_0\frac{\partial\phi_1}{\partial\tau}$$
$$+2\lambda_0^2A_1^3\phi_1\frac{\partial\phi_1}{\partial\zeta}+\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}-2A_2\phi_1\frac{\partial\phi_1}{\partial\zeta}+\frac{\partial n_2}{\partial\zeta}-A_1^3\sigma\phi_1\frac{\partial\phi_1}{\partial\zeta}+A_1^3\lambda_0^2\phi_1\frac{\partial\phi_1}{\partial\zeta}$$
$$+A_1^2\lambda_0\frac{\partial\phi_1}{\partial\tau}-A_1^2\lambda_0\eta_0\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]=0$$

$$(A_1\sigma-A_1\lambda_0^2+1)\frac{\partial n_2}{\partial\zeta}+\Big(A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\Big)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+\Big(A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}\Big)$$
$$+2A_1^2\lambda_0\frac{\partial\phi_1}{\partial\tau}+\Big(2\lambda_0^2A_1^3-2A_2-A_1^3\sigma+A_1^3\lambda_0\Big)\phi_1\frac{\partial\phi_1}{\partial\zeta}+\Big(-A_1^2\lambda_0\eta_0\Big)\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]$$
$$+\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}=0$$

Now, using $\lambda_0^2A_1=1+\sigma A_1$ to make the above equation in first order terms, and then divide with $(2\lambda_0A_1^2)$ to write

$$\Big(\cancel{A_1\lambda_0^2}-\cancel{A_1\lambda_0^2}\Big)\frac{\partial n_2}{\partial\zeta}+\Big(A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\Big)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+\Big(A_1\lambda_0^2\Omega_c^{-2}(1+\sigma A_1)\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}\Big)$$
$$+2A_1^2\lambda_0\frac{\partial\phi_1}{\partial\tau}+\Big(3\lambda_0^2A_1^3-2A_2-A_1^3\sigma\Big)\phi_1\frac{\partial\phi_1}{\partial\zeta}+\Big(-A_1^2\lambda_0\eta_0\Big)\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]$$
$$+\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial}{\partial\zeta}\frac{\partial^2\phi_1}{\partial\eta^2}=0$$

$$\frac{\partial\phi_1}{\partial\tau}+\frac{3A_1^3\lambda_0^2-A_1^3\sigma-2A_2}{2A_1^2\lambda_0}\phi_1\frac{\partial\phi_1}{\partial\zeta}+\frac{1}{2\lambda_0A_1^2}\frac{\partial^3\phi_1}{\partial\zeta^3}+\frac{1+A_1\lambda_0^2\Omega_c^{-2}(1+A_1\sigma)}{2\lambda_0A_1^2}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial\phi_1}{\partial\eta^2}\right)$$
$$-\frac{\cancel{A_1^2}\cancel{\lambda_0}\eta_0}{\cancel{\lambda_0}\cancel{A_1^2}2}\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]$$
$$\implies\frac{\partial\phi_1}{\partial\tau}+A_{rq}\phi_1\frac{\partial\phi_1}{\partial\zeta}+B_{rq}\frac{\partial^3\phi_1}{\partial\zeta^3}+C_{rq}\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial\phi_1}{\partial\eta^2}\right)-D_{rq}\left[\frac{\partial^2\phi_1}{\partial\xi^2}+\frac{\partial^2\phi_1}{\partial\eta^2}+\frac{\partial^2\phi_1}{\partial\zeta^2}\right]=0 \tag{3.35}$$

which is the desired ZKB equation, where non-linear, dispersive and dissipative coefficients are given by

$$A_{rq} = \frac{3A_1^3\lambda_0^2 - A_1^3\sigma - 2A_2}{2A_1^2\lambda_0}, \tag{3.36}$$

$$B_{rq} = \frac{1}{2\lambda_0 A_1^2}, \tag{3.37}$$

$$C_{rq} = \frac{1 + A_1\lambda_0^2\Omega_c^{-2}(1 + A_1\sigma)}{2\lambda_0 A_1^2}, \tag{3.38}$$

and, respectively

$$D_{rq} = \frac{\eta_0}{2} \tag{3.39}$$

For benchmarking our results with previous chapter, we apply the limits $r = 0$ and $q \to \kappa$+*1*.

3.5 Numerical results and discussion

The transform form of ZKB equation can be written as

$$-U_0\frac{d\phi_1}{d\chi} + A_{rq}l_z\phi_1\frac{\partial\phi_1}{d\chi} + H_{rq}l_z\frac{d^3\phi_1}{d\chi^3} - D_{rq}\frac{d^2\phi_1}{d\chi^2}, \tag{3.40}$$

where $H_{rq} = C_{rq}(l_x^2 + l_y^2) + B_{rq}l_z^2$ and the coefficients of dissipative, non-linear terms remain the same. The complete solution of the above equation takes the form

$$\phi_1(\chi) = \frac{3D_{rq}^2}{25H_{rq}A_{rq}l_z^2}\left[2 - 2\tanh\left(\frac{D_{rq}\chi}{10H_{rq}l_z}\right) + sech^2\left(\frac{D_{rq}\chi}{10H_{rq}l_z}\right)\right] \tag{3.41}$$

where $\phi_m = \frac{9D_{rq}^2}{25H_{rq}A_{rq}l_z^2}$ is the amplitude and $\frac{D_{rq}}{10H_{rq}l_z}$ is the width of shock structure. Here we use the following plasma parameters for our numerical analysis,
$\sigma = 0.01$,
$\eta_0 = 0.1$,
$l_z = 0.9$,
$\omega_{ci} = 0.3$,
$\Omega_0 = 0.01$.

3.6 Potential profiles for (r, q) VDF

Figure (3.1), displays the behavior of non-linear coefficient (A) for kappa and (r, q) distributions. Here we have, for our comparison, r = 0, 1, 2 and q = 3 to 12, for which the value of κ is choosen accordingly ($\kappa = q - 1$). We note that both VDFs give the same value of non-linear coefficient only for $r = 0$ and $q \to \kappa$+*1* case, which benchmark our finding for the non-linear coefficient.

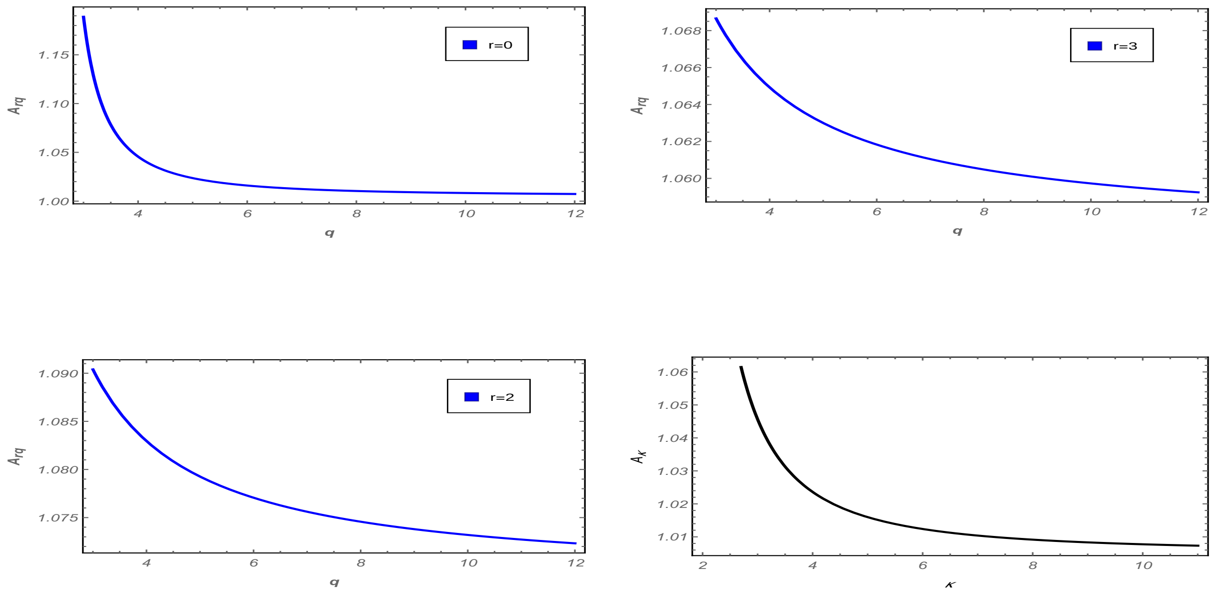

Figure 3.1: *Comparison of the non-linear coefficient for kappa and (r,q) VDFs, for $r = 0, 2, 3$ in our discussion.*

Figures (3.2) and (3.3) also shows non-linear behavior for kappa and (r,q) distributions. Here we use the same values as in the previous figure.

Figure 3.2: *Behavior of the non-linear coefficients B for kappa and (r,q) VDFs, by using $r = 0, 2, 3$.*

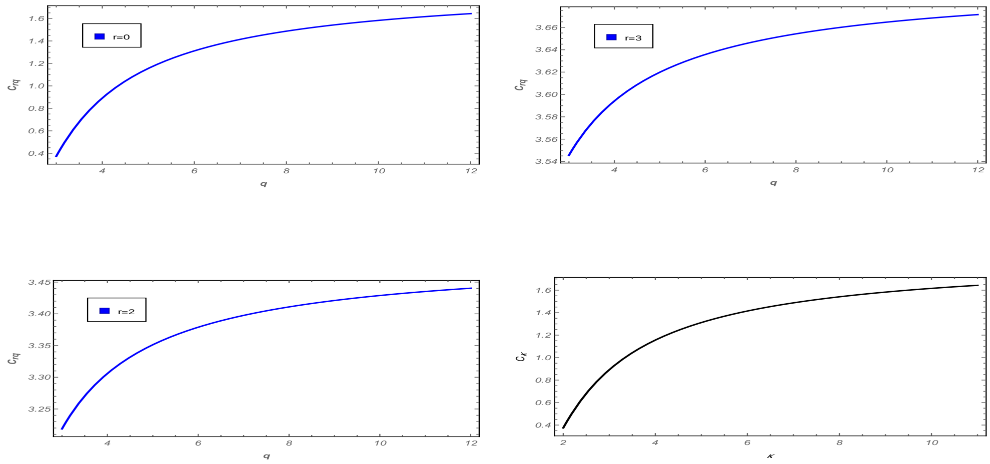

Figure 3.3: *Behavior of the non-linear coefficients C for kappa and (r,q) VDFs, by using* $r = 0, 2, 3$.

Figure (3.4), shows the variation of dispersive coefficient H for kappa and (r,q) distributions. Here we use the same limit and values of r, q just as above. In this figure, we observe that the dispersive coefficient H is mainly responsible for different structures of shock potential.

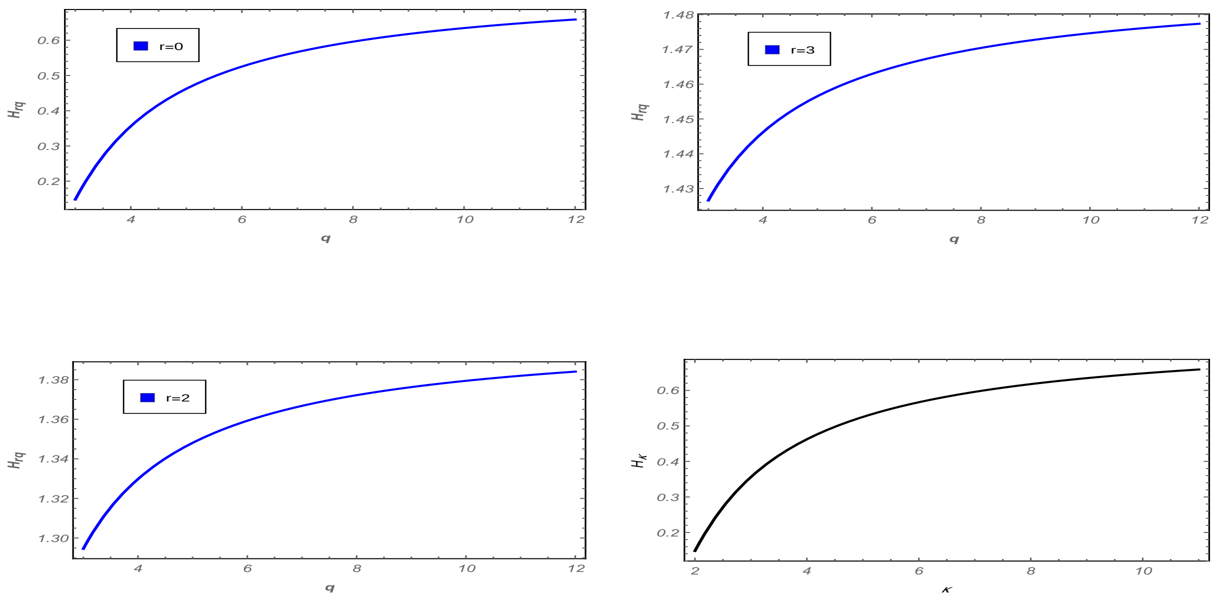

Figure 3.4: *Comparison of dispersive coefficient H for kappa and (r,q) VDFs.*

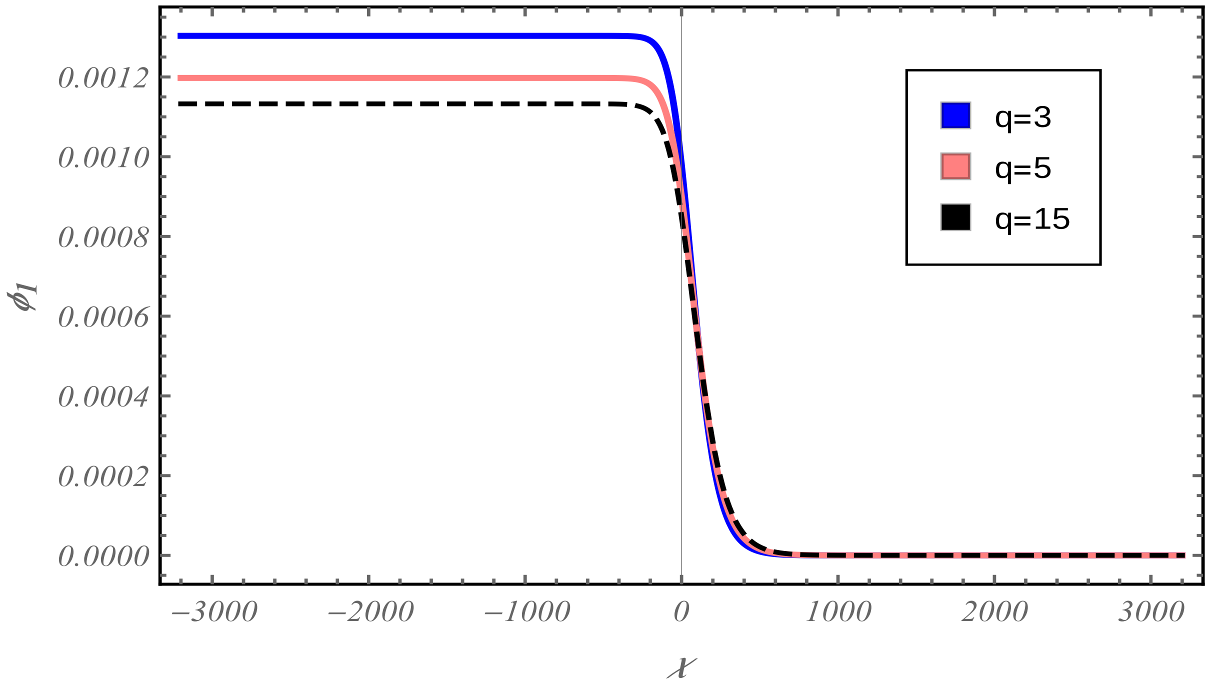

Figure 3.5: *Variation of perturbed potential for different values of q and r=1.*

Figure (3.5), shows the behavior of perturbed potential for different value of q. As the value of q increases, the shock potential strength decrease which correspond to lower energy and high tail electrons. This means that as the values of q decrease, there is enlarging numbers non-thermal electrons which maximize amplitudes of shock potential.

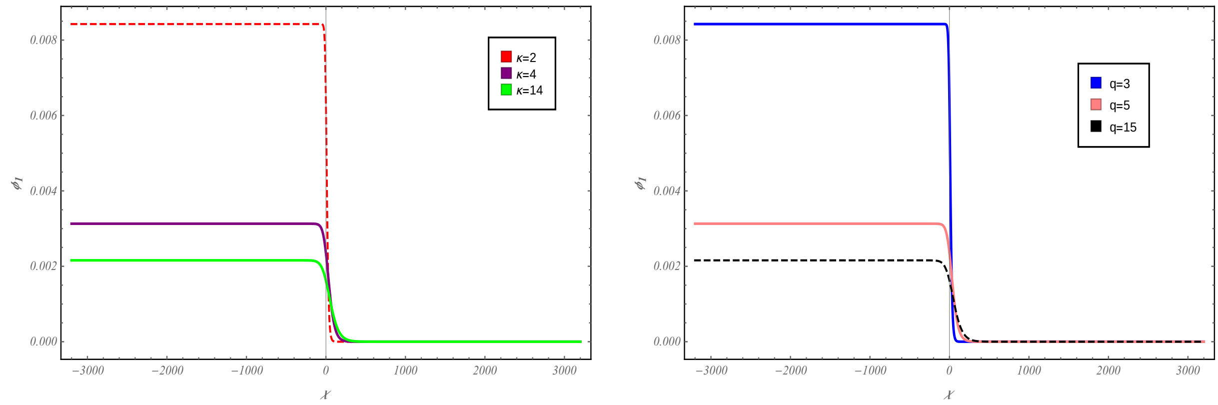

Figure 3.6: *Comparison of perturbed potential for (r,q) with kappa VDF.*

Figure (3.6) displays that, in the limit of r and q our perturbed potential behavior goes to kappa distribution, which tells that as the value of κ increase the strength of shock potential decreases which corresponds to low energy electrons.

Chapter 4

Summary and outlook

The study of ion acoustic shock waves in magneto-rotating plasmas, incorporated in this study is not only a contribution for the understanding of fundamental plasma physics, but it also help in studying non-linear waves generated due to various process like dissipation and dispersion in the important space media like magnetosphere and pulsars. In this thesis, we have studied the ion acoustic shock waves in a medium that is magneto-rotating and relativistic (chapter 2) plasma and well as non relativistic (as studied in chapter 3) systems. For such systems we derived the respective Zakharov Kuznetsov Burgers (ZKB) equation by using a reductive perturbation technique. Due to collisions the two viscosities - ion kinematic and bulk have been included in the force balanced equation. The Coriolis term, accounting for rotating effects also added because the system is viewed in a non-inertal frame of reference.

For huge number of non-thermal electrons we have used kappa and Cairns distributions and extended the study to a more generalized non-thermal profile called $(r,\ q)$ distribution. It is observed that after employing kappa and Cairns distribution, the former depicts only compressive shock profile while the later exhibits both compressive and rarefactive perturbed potentials. However, in a specific limit our model is no more applicable when there is a discontinuity. Moreover it is seen that the non-linear term (coefficient A) is responsible for the formation of compressive and rarefactive shock structures. Further we have plotted shock structure against kappa and Cairns parameter that shows, the strength of ion acoustic shock waves is greater for Cairns as compared to kappa distribution. After reviewing the one index VDFs we have extended our study to a more general non-Maxwellian distribution function, namely the $(r,\ q)$ profields. With the use of such VDF one can model a huge number of observed data exhibiting both high energy particles as well flat-top distributions. Thus, the study can be used to analyze the potential distribution for

a large number of plasma systems. To benchmark our findings proper limits have been applied to recover the kappa and Maxwellian potential distributions. For the said VDF both compressive and rarefactive shock structures have been obtained. The given work and models in this dissertation may also be applied in future studies for different types of complex medium, that arises enormous non-linear effects. The results which were obtained in this thesis have some correspondence to the rotating flows in magnetoplasmas that are infer to exist in planetary magnetosphere of Saturn and Jupiter, and respectively in magnetosphere.
The work has also been extended for electron-positron-ion plasmas which is not discuss here.

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