Scalar Mesons in weak Semi-leptonic decay of B$_{(s)}$

by

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RESEARCH COMPLETION CERTIFICATE

This is to certify that Miss. Hira Waseem bearing Registeration No. 02182113006 has succesfully completed the research work entitled "Scalar Mesons in weak Semi-leptonic decay of $B_{(s)}$" under the supervision of Muhammad Jamil Aslam in the fullfilment of the MPhil degree.

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Declaration

I, Ms. Hira Waseem Roll No. 02182113006, student of M.Phil in the subject of Physics session 2021-2023, hereby declare that the matter printed in the thesis titled "Scalar Mesons in weak Semi-leptonic decay of $B_{(s)}$" is my review work and has not been printed, published or submitted as research work, thesis or publication in any form in any University, Research instititition etc, in Pakistan.

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All praises to Him who is most the gracious and the most merciful, Who created every particle of this Universe and taught man how to hold a pen, Who gave me enough strength, patience and consistency to pursue my dreams. All the achievements, from my very first to until now are possible because of His constant guidance. I am highly thankful for His countless blessings and kindness that helped me to overcome challenges of life and strive for excellence. He has been my constant companion in all the ups and downs of life to face with grace and resilience. I am humbled by His infinite love and compassion, and I pray that He continues to keep me on the path of righteousness.

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Abstract

The decays governed by the flavor-changing-neutral-current-transitions (FCNC), such as $b \to sl^+l^-$, provide an important tool to test the physics in and beyond the Standard Model (SM). In this dissertation, we study the $B \to Sl^+l^-$, where $S = a^0, f^0$ represent the scalar mesons. Being an exclusive process, the matrix elements of inital and final state meson involve the form factors (a non-perturbtive quantity). Using the Light-cone-sum-rules (LCSRs) approach, we calculate the corresponding form factors (FFs). In comparison to other models, i.e., Light cone quark model and Shifman-Vainshtein-Zakharov (SVZ) sum rules approach, the calculated value of FFs is approximately twice in the magnitude to that of the corresponding B meson decaying to pseudoscalar. Using these FFs, we calculate the Branching ratios, lepton's forward-backward asymmetry and the various lepton polarization asymmetries in the SM. Recently, a Bayesian analysis of the lepton-flavor universalities R_K and R_{K^*} is done in a model independent approach and it puts several constraints on the different Wilson coefficients. Using these constraints, we explore their imprints on the above mentioned observables and see if they show any deviations from the corresponding SM predictions.

Chapter 1

Introduction

There are five fundamental forces of nature; electric, magnetic, weak nuclear, strong nuclear, and gravitational. Among these, the gravitational force is the weakest one. It was Scottish Physicist James Clark Maxwell and Dutch Physicist Lorentz who proposed the concept of unification of forces and light is the result of this marvelous idea. So after the unification of the two forces, the number of forces was reduced to four. If a comparison is made between them, then gravitational force is the weakest one and its range is infinite. The range of the weak force is 10^{-18} m which is about 0.1% of the diameter of the proton. Force carrier particles of the electromagnetic, weak and strong forces are the bosons with $(S = 1)$ and the mediator of the gravitational force that is a graviton $(S = 2)$, is not yet found but exists hypothetically.

1.1 Standard Model

Interaction of fundamental particles is governed by the gauge theory which is known as the Standard Model (SM). It has enough strength to predict the new particles that are not even found. Based on the spin, particles can be classified into two classes:

- Bosons, which are integer spin particles $(S = 0, \pm1, \pm2, ...)$.
- Fermions, which carry half integer spins $\left(S = \pm\frac{1}{2}, \pm\frac{3}{2}, ...\right)$.

In SM, the classification of the particles is as follows:

- There are total six quarks and six anti-quarks and each quark can carry only one of three colors red, green and blue. They are grouped in three generations. In first generation up and down quarks are placed, charm and strange are placed in the second generation and top and bottom are placed in third generation. Each quark carries a fractional electric and a color charge. Up, charm and top quarks carries $+2e/3$, whereas down, strange and bottom quarks carries $-e/3$ electric charge. Under $SU(2)$ gauge symmetry, they are left-handed (L) doublet and right-handed (R) singlet, i.e.,

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \tag{1.1.1}$$

$$u_R, d_{R,} c_R, s_R, t_{R,} b_R.$$

- There are total six leptons; electron, muon and tau with their related neutrinos. Just like quarks, leptons also have three generations. Neutrinos are charge-less and massless in the SM. Again, under $SU(2)$, they are classified as left-handed doublets and right-handed singlets, i.e.,

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L, \tag{1.1.2}$$

$$e_R, \mu_R, \tau_R,$$

where right-handed neutrinos are forbidden in the SM.

- Each force carrier is a boson. Photon is the mediator of the electromagnetic force and carries spin $S = 1$.

- Strong force is mediated by the eight gluons that have non-zero color charge and zero electric charge.

- Weak force carries three mediators $W^{\pm}, Z$. W^+ and W^- were discovered at the CERN in 1983 and their mass is 80.379 ± 0.012 GeV$/c^2$ and Z-boson was discovered in the same year with a mass 91.1876 ± 0.0021 GeV$/c^2$.

- As Higgs mechanism is well known because each particle acquire mass by this mechanism. After the discovery of Higgs boson in 2012 at the LHC, with mass 125.18 ± 0.16 GeV/c^2 , the SM is complete.

Free quarks cannot exist in nature due to "**Quark confinemen**t" so they can only exist in bound states called Hadrons. Based on the spin configurations, there are two classes of hadrons; **mesons** and **baryons**. Quark and anti-quark constitutes mesons whereas baryons are constituted by only three quarks or three anti-quarks. Due to color confinement, hadrons exist only in colorless forms. Short range of strong force is the consequence of the quark confinement. Dissimilar to photons, gluons can interact with each other as they carry color charge that results into the "**asymptotic freedom**". Transparency of leptons to strong nuclear force is due to colorless nature. The Gauge symmetry of the Standard model is

$$SU(3)_C \times SU(2)_L \times U(1)_Y,$$

where $SU(3)_C$ represents the three color group of QCD whereas $SU(2)_L \times U(1)_Y$ represents the weak isospin and weak hypercharge groups of the electroweak theory. Symmetry breaking gives:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}.$$

In $SU(2)_L$ representation, the Dirac fields is given in Eq. (1.1.1). In the same representation, the Higgs doublet is

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(\nu + h) \end{pmatrix}, \tag{1.1.3}$$

where $c = v$ is the vacuum expectation value. It has total four degrees of freedom and three of them are responsible for the masses of $W^\pm$ and Z bosons and forth one is responsible for the physical Higgs boson. Electroweak force is the unification of electromagnetic and the weak forces.

1.1.1 SM Lagrangian

In SM, Lagrangian equations are used to describe the fundamental interactions. This Lagrangian has local gauge invariance, and contains all the information of the theory and it is function of fields and their derivatives. The SM Lagrangian is

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW}, \tag{1.1.4}$$

where

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{free} + \mathcal{L}_{QCD}^{int} + \mathcal{L}_{QCD}^{gauge}, \tag{1.1.5}$$

$$\mathcal{L}_{QCD}^{free} = \sum_{f=q} \bar{q}_f(\iota\gamma^\mu\partial_\mu - m)q_f, \tag{1.1.6}$$

$$\mathcal{L}_{QCD}^{int} = \sum_{f=q} -g_S\bar{q}_f\gamma^\mu\frac{\lambda_a}{2}q_f G_\mu^a, \tag{1.1.7}$$

$$\mathcal{L}_{QCD}^{gauge} = -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu}, \tag{1.1.8}$$

with

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_S f_{abc}G_b^\mu G_c^\nu, \tag{1.1.9}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a. \tag{1.1.10}$$

The electroweak part of the Lagrangian is

$$\mathcal{L}_{EW} = \mathcal{L}_{EW}^{free} + \mathcal{L}_{EW}^{int} + \mathcal{L}_{EW}^{gauge} + \mathcal{L}_{EW}^{\phi} + \mathcal{L}_{EW}^{Yuk}, \tag{1.1.11}$$

whereas,

$$\mathcal{L}_{EW}^{free} = \sum_f \bar{L}\iota\partial\!\!\!/ L + \bar{\psi}_R\iota\partial\!\!\!/\psi_R + \bar{\psi}'_R\iota\partial\!\!\!/\psi'_R, \tag{1.1.12}$$

$$\mathcal{L}_{EW}^{int} = -\sum_f \frac{g_W}{\sqrt{2}}\overline{\psi}_L\gamma^\mu V\psi'_L W_\mu^+ + \frac{g_W}{\sqrt{2}}\overline{\psi'}_L\gamma^\mu V^\dagger\psi_L W_\mu^- + e\left(\overline{\psi}\gamma^\mu Q\psi + \overline{\psi'}\gamma^\mu Q\psi'\right)A_\mu +$$

$$\frac{g_W}{\cos\theta_W}\left[\overline{\psi}\gamma^\mu\frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right)\psi + \overline{\psi'}\gamma^\mu\frac{1}{2}\left(c_V^{f'} - c_A^{f'}\gamma^5\right)\psi'\right]Z_\mu, \tag{1.1.13}$$

where $V = V_{CKM}$ is for the quarks and it is one for leptons.

$$\mathcal{L}_{EW}^{gauge} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{1.1.14}$$

$$\mathcal{L}_{EW}^{\phi} = (D_\alpha\phi)^\dagger D^\alpha\phi - \left(\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2\right), \left(\mu^2 < 0, \lambda > 0\right) \tag{1.1.15}$$

$$\mathcal{L}_{EW}^{Yuk} = \sum_f -m_f\overline{\psi}\psi\left(1 + \frac{h}{\nu}\right) + \sum_{f'} -m_{f'}\overline{\psi'}\psi'\left(1 + \frac{h}{\nu}\right). \tag{1.1.16}$$

with $D = \partial_\mu + \iota g_S G_\mu^a \frac{T^a}{2}$.

1.1.2 Strength of the Gauge theory

The SM has successfully predicted the masses of $W^\pm, Z$ bosons and they were found to be in the same mass range. Also, the top quark mass was found to be $m_t = 178 \pm 8^{+17}_{-20}$ GeV/c^2. Systematic related to the mass of Higgs was the main uncertainty [1] gives

$$m_t = 172.9 \pm 0.6 \pm 0.9 \text{ GeV}.$$

This proved to be a marvelous success. Theory of Standard model is renormalized which implies that calculations can be performed at any scale. Top quark is the only quark which decays before the process of **"Hadronization"**. There are total 19 free parameters in Standard model that are worth mentioning here

- Six masses of quarks
- Three masses of charged leptons
- Three coupling constants g_W, g, g_S
- Weak mixing angle θ_W
- Higgs potential parameters λ,μ

- Four independent V_{CKM} elements (three angles and a phase).

Due to neutrinos, there are three masses and four parameters of V_{PMNS} leading the total number of free parameters to 26.

1.1.3 Problems in Gauge theory

Any dynamical mechanism does not lead to the Higgs potential. Vacuum potential energy density due to Higgs potential is

$$\rho_H = V(\phi) = \mu^2 \left(0, \frac{\nu}{\sqrt{2}}\right) \begin{pmatrix} 0 \\ \frac{\nu}{\sqrt{2}} \end{pmatrix} + \lambda \left[\left(0, \frac{\nu}{\sqrt{2}}\right) \begin{pmatrix} 0 \\ \frac{\nu}{\sqrt{2}} \end{pmatrix} \right] = \lambda \left(\frac{\mu^2}{\lambda} \frac{\nu^2}{2} + \frac{\nu^4}{4} \right), \tag{1.1.17}$$

$$= -\lambda \frac{\nu^4}{4} = -\frac{m_H^2 \nu^2}{8}, \tag{1.1.18}$$

as $m_H = 126$ GeV and $\nu = 246$ GeV so $\rho_H \simeq -1.2 \times 10^8$ GeV^4. In general relativity, Vacuum energy density couples to gravity. ρ_H relation with cosmological constant is

$$\Lambda_H = \kappa \rho_H, \text{ with } \kappa = \frac{8\pi G_N}{c^2} \simeq 1.86 \times 10^{-27} cmgr^{-1}, \tag{1.1.19}$$

G_N being the Gravitational constant. In unit of $grcm^{-3}$

$$\Lambda_H = \left(1.86 \times 10^{-27} cmgr^{-1}\right) \underbrace{\frac{-1.2 \times 10^8 GeV^4}{(1.97 \times 10^{-14} GeV.cm)^3}}_{\hbar c} \times \underbrace{1.7827 \times 10^{-24}}_{\text{grGe}V^{-1}} = -5.2 \times 10^{-2} cm^{-2}.$$

Whereas, the measured value of the cosmological constant is

$$\Lambda_{meas} = \kappa \rho_{critical} \Omega_{tot}, \tag{1.1.20}$$

$$= \left(1.86 \times 10^{-27} cmgr^{-1}\right) \left(1.88 \times 10^{-29} \times 0.673^2 gcm^{-3}\right) \times 1,$$

$$= 1.6 \times 10^{-56} cm^{-2}. \tag{1.1.21}$$

So this value exceeds the measured value by the order of 54 in magnitude. That is a very challenging problem in theory. There are some other problems too due to which Physicists are not satisfied with the theory as “why the electric charges are quantized?” Why quarks and leptons are spin half particles?. Why the neutrinos have very small mass? Why top quark is so heavy compared to other family members?.

1.1.4 Tests of the Electroweak theory

Electroweak model has been tested numerously and there are (almost) no significant deviations found between experimental and theoretical results. Using Z^0 production, very large precisions has been achieved. In order to measure Z^0-line shape, data were taken around Z^0 peak. At LEP collider, following cross-section is obtained as in Fig. 1.1.1

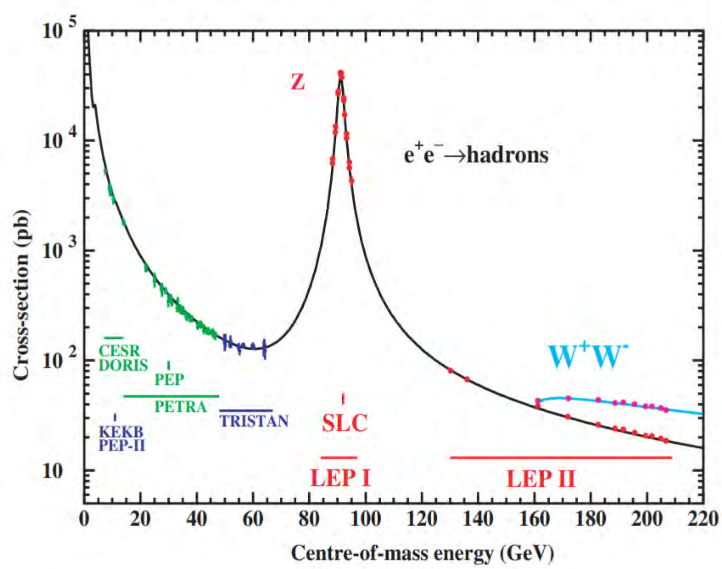

Figure 1.1.1: At various colliders, Hadronic cross-section's measurements [2].

Resonance Z^0 is clearly visible at 91 GeV. Constraints on number of light neutrinos are the consequence of the Z^0 line shape and this number is $N_\nu = 2.9840 \pm 0.0082$. This is represented in the Fig. 1.1.3. Invisible partial decay width is used to find this number. It can be measured by subtracting several quantities

$$\Gamma_{inv} = \Gamma_{total} - \Gamma_{ee} - \Gamma_{\mu\mu} - \Gamma_{\tau\tau} - \Gamma_{had}, \tag{1.1.22}$$

as, it is assumed that only number of neutrinos contribute so

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu\nu}} = \frac{\Gamma_{inv}}{\Gamma_{ll}}(\frac{\Gamma_{ll}}{\Gamma_{\nu\nu}})_{SM}, \tag{1.1.23}$$

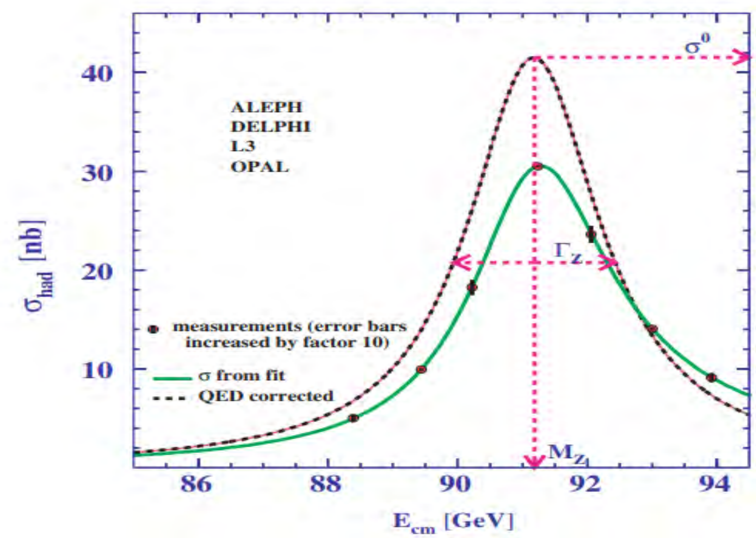

Figure 1.1.2: Around Z^0mass, Hadronic cross-section measurement [2].

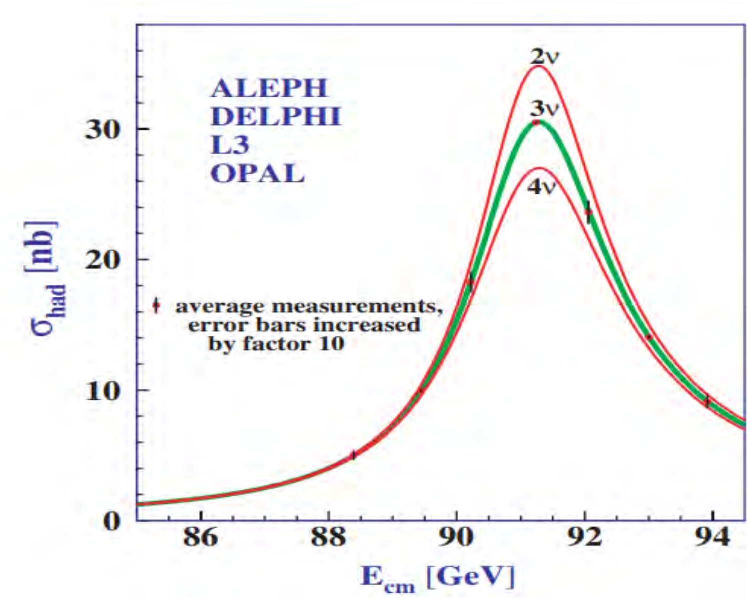

Figure 1.1.3: Constraints on number of light neutrinos [2].

by assuming Lepton universality, $\frac{\Gamma_{inv}}{\Gamma_{ll}}$ is taken and $\left(\frac{\Gamma_{ll}}{\Gamma_{\nu\nu}}\right)_{SM}$ is taken from the predictive power of SM.

As it is well known that

$$\frac{c_V^f}{c_A^f} = 1 - 4\sin^2\theta_W|Q|. \tag{1.1.24}$$

Several channels $f = \mu, \tau, c, b$ give estimation of $\sin^2\theta_W$. As masses of virtual particles start to contribute so indirect sensitivity to Higgs mass is shown in following Fig. 1.1.4.

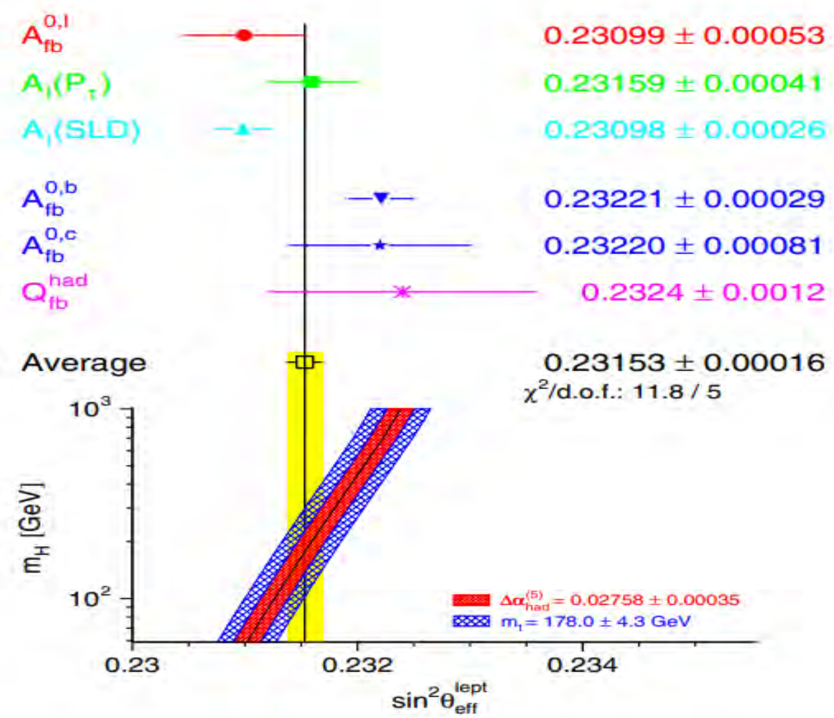

Figure 1.1.4: Indirect sensitivity to Higgs mass and estimation of $\sin^2\theta_W$ [2].

1.2 B-meson Physics

In the last generation of quarks, bottom quark is the lighter one and top-quark, its doublet partner is the heaviest one. In 1973, Kobayashi and Maskawa proposed top and bottom quark [3] and their existence was proved in 1977 [4]. Through generation changing process, light particles can decay and same applies for bottom quark. CKM matrix (explained below) govern flavor changing decays and shows some important characteristics such as loop and box Feynman diagrams, CP asymmetries and flavor oscillations. Bound states in which b quark participate are $\bar{b}u$ called B^+,$\bar{b}d$ called B^0, $\bar{b}s$ called B_s^0, $\bar{b}c$ called B_c^+. Among previously described bound states, B_c^+ is the heaviest one. It was first time produced by CDF collaboration in 1998 and its mass was discovered in 2007 from the decay $B_c^+ \longrightarrow J/\psi\pi^+$ [5, 6]. LHCb made the accurate determination of mass of B_c^+ by using the following decay $B_c^+ \rightarrow J/\psi D^0 K^+$ and its mass came to be $m_{B_c^+} = 6274.28 \pm 1.40 \pm 0.32$ MeV/c^2. Top quarks before "**Hadronization**" decays so $b-$ baryons are the only heaviest bound observed states. The Tevatron accelerator has measured a cross-section of 30 µb for the process where protons collide to produce particles containing bottom quarks ($pp \rightarrow bX$) in the collision energy regime of $\sqrt{s} = 1.96$ TeV, limited to particles with pseudo-rapidity values less than 1. On the other hand, the LHCb experiment at the Large Hadron Collider has observed a cross-section of approximately 72 µb in the collision energy regime of 7 TeV and around 144 µb at 13 TeV for the same process, but restricted to particles with pseudo-rapidity values between 2 and 5.

To measure CKM matrix element, both inclusive and exclusive processes can be used but in both processes, uncertainties have to be faced. From the experimental perspective, Exclusive processes are simpler because Branching fractions can be used to calculate CKM matrix element whereas the form factors calculations can be done by Lattice QCD or QCDSR (QCD sum rules). The averages for inclusive and exclusive B-decays branching fractions are provided by Semi-leptonic B-decays subgroup (subgroup of Heavy Flavor Averaging Group (HFAG)) and so, the methods to calculate the $|V_{cb}|$ and $|V_{ub}|$.

In $\Upsilon(4S)$ decays , there is pair of two least massive lightest B-mesons. Branching fractions are the following:

$$f_1 = {}^{\Gamma\left(\Upsilon(4S)\rightarrow B^+B^-\right)}/_{\Gamma_{tot}(\Upsilon)}, \tag{1.2.1}$$

$$f_2 = {}^{\Gamma\left(\Upsilon(4S)\rightarrow B^+B^-\right)}/_{\Gamma_{tot}(\Upsilon)}, \tag{1.2.2}$$

$$f^{00} = {}^{\Gamma\left(\Upsilon(4S)\rightarrow B^0\bar{B}^0\right)}/_{\Gamma_{tot}(\Upsilon)}, \tag{1.2.3}$$

Their ratio measured in most experimental analysis is

$$R^{1/00} = \frac{f_1}{f^{00}} = \frac{\Gamma\left(\Upsilon\left(4S\right) \to B^+B^-\right)}{\Gamma\left(\Upsilon\left(4S\right) \to B^0\bar{B}^0\right)},$$

$$R^{2/00} = \frac{f_2}{f^{00}} = \frac{\Gamma\left(\Upsilon\left(4S\right) \to B^+B^-\right)}{\Gamma\left(\Upsilon\left(4S\right) \to B^0\bar{B}^0\right)} \tag{1.2.4}$$

and their experimental measurements by various groups are given in Table 1.1.

Table 1.2.1: B^+/B^0production ratios in $\Upsilon\left(4S\right)$.

Exp.	Decay modes	Published value of $R^{1,2/00}$	Assumed value ofτ^+/τ^0
CLEO-Collab.[7]	$J/\psi K^*$	$1.04 \pm 0.07 \pm 0.04$	1.066 ± 0.024
BABAR-Collab.[8]	$(c\bar{c})K^*$	$1.10 \pm 0.06 \pm 0.05$	1.062 ± 0.029
CLEO-Collab.[9]	$D^*l\nu$	$1.058 \pm 0.084 \pm 0.136$	1.074 ± 0.028
Belle-Collab.[10]	dilepton events	$1.01 \pm 0.03 \pm 0.09$	1.083 ± 0.017
BABAR-Collab.[11]	$J/\psi K$	$1.006 \pm 0.036 \pm 0.031$	1.083 ± 0.017
BABAR-Collab.[12]	$(c\bar{c})K^*$	$1.06 \pm 0.02 \pm 0.03$	1.086 ± 0.017
Exp. Ave.		$1.059 \pm 0.027(tot)$	1.076 ± 0.004

From this we can extract

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{B\left(\Lambda_b^0 \to p\mu^-\bar{\nu}_\mu\right)}{B\left(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu}_\mu\right)} R_{FF}. \tag{1.2.5}$$

where R_{FF} can be calculated by using sum rules in the light-front framework. Using R_{FF} $= 0.68 \pm 0.07$ the above ratio results to be $0.083 \pm 0.004 \pm 0.004$. Radiative $b \to s\gamma$ decay mode have photon that is highly polarized in SM. To maximum polarization, small mass of s quark produces small corrections, but there would be clues pointing towards New Physics if there is induced CP violation.

1.3 B-factories

The Standard Model describes fermions in three distinct generations, and the investigation of interactions that differentiate between these generations is referred as flavor physics. Fermions can undergo two types of interactions: coupling with a scalar particle results in Yukawa interaction, while coupling with a gauge boson results in gauge interaction. Gauge couplings are determined by a single coupling constant, and there is no gauge coupling between interaction eigenstates of different generations. Flavor physics is of crucial importance because it can predict **New Physics**. In recent years, several of its predictions were true such as

- Ratio of decay width of $K_L \to \mu^+\mu^-$ to $K^+ \to \mu^+\nu$ predicted charm quark.
- In neutral kaon mixing, third generation of SM was predicted.
- Mass of the charm quark was predicted by the $\Delta m_K = m_{K_L} - m_{K_S}$ and mass of the heaviest quark, top quark was predicted by Δm_B.
- Masses of the neutrinos.

These processes occur very rarely in nature and do not occur at tree level. The principle behind these rare processes is **GIM mechanism** [13].

The small branching ratio of $K^0 \to \mu^+\mu^-$ caused perplexion in the physics community in 1970. This decay is represented by the box diagram in the following Figure 1.5.2. At that time, only two families were known, Glashow, Maiani and Iliopoulos gave a postulate about the existence of charm quark so instead of u quark in the box diagram, another diagram with c quark can be considered too. Therefore, to define s' state

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} c_{\theta_c} & s_{\theta_c} \\ -s_{\theta_c} & c_{\theta_c} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}, \tag{1.3.1}$$

$c \to s'$ and $u \to d'$ quarks are coupled by W boson.

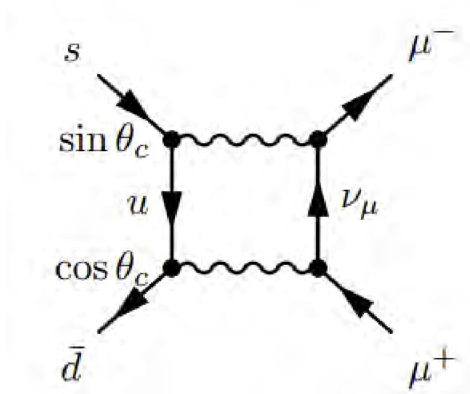

Figure 1.3.1: $K^0 \to \mu^+\mu^-$ box diagram.

$$\begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} u \\ d' \end{pmatrix},$$

So the $c-$quark participation is shown in Fig. 1.5.3. In first diagram, the amplitude is proportional to $g_w^4 \cos\theta \sin\theta$ and in the second diagram to $-g_w^4 \cos\theta \sin\theta$. These two diagrams can be canceled out if both quarks had the same mass, but $m_c >> m_u$ so the second diagram is strongly suppressed and this is called GIM (Glashow, Iliopoulos and Maiani) mechanism. After four years, $c-$quark was discovered in the state J/ψ in 1974. Mass of the charm quark is 1.3 GeV.

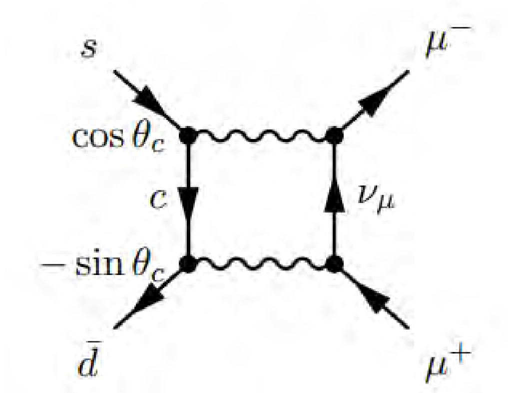

Figure 1.3.2: New box diagram for $K^0 \to \mu^+\mu^-$.

The CKM matrix is further responsible for the suppression of these processes. This is an extended form of the GIM matrix to the three generations, i.e.,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1.3.2}$$

where d', s', b' corresponds to the weak eigenstates and d, s, b represent the mass eigenstates. Following relations hold for CKM matrix

$$\sum_k |V_{ik}|^2 = \sum_i |V_{ik}|^2 = 1,$$

$$\sum_k V_{ik}V_{jk}^* = \sum_k V_{ki}V_{kj}^* = 0(i \neq j),$$

with three families, charge rising current is

$$j^{\mu}_{cc+} = \frac{g_w}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma^{\mu}\frac{1}{2}(1-\gamma_5)V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{1.3.3}$$

The Hermition conjugate of j^{μ}_{cc+} gives charge lowering weak current, i.e.,

$$j^{\mu}_{cc-} = \left(j^{\mu}_{cc+}\right)^{\dagger} = \frac{g_w}{\sqrt{2}}(\bar{d}, \bar{s}, \bar{b})\gamma^{\mu}\frac{1}{2}(1-\gamma_5)V^{\dagger}_{CKM}\begin{pmatrix} u \\ c \\ t \end{pmatrix}, \tag{1.3.4}$$

where

$$V^{\dagger}_{CKM} = \begin{pmatrix} V^*_{ud} & V^*_{cd} & V^*_{td} \\ V^*_{us} & V^*_{cs} & V^*_{ts} \\ V^*_{ub} & V^*_{cb} & V^*_{tb} \end{pmatrix}. \tag{1.3.5}$$

How many free parameters are there in CKM matrix? Decomposition of unitary matrix can be made as product of three rotations plus one parameter that is complex can be termed as phase δ. Therefore,

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{(23)} & s_{(23)} \\ 0 & -s_{(23)} & c_{(23)} \end{pmatrix}\begin{pmatrix} c_{(13)} & 0 & s_{(13)}e^{-\iota\delta} \\ 0 & 1 & 0 \\ -s_{(13)}e^{\iota\delta} & 0 & c_{(13)} \end{pmatrix}\begin{pmatrix} c_{(12)} & s_{(12)} & 0 \\ -s_{(12)} & c_{(12)} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.3.6}$$

$$V_{CKM} = \begin{pmatrix} c_{(12)}c_{(13)} & s_{(12)}c_{(13)} & s_{(13)}e^{-\iota\delta} \\ -s_{(12)}c_{(23)} - c_{(12)}s_{(23)}s_{(13)}e^{\iota\delta} & c_{(12)}c_{(23)} - s_{(12)}s_{(23)}s_{(13)}e^{\iota\delta} & s_{(23)}c_{(13)} \\ s_{(12)}c_{(23)} - c_{(12)}c_{(23)}s_{(13)}e^{\iota\delta} & -c_{(12)}c_{(23)} - s_{(12)}c_{(23)}s_{(13)}e^{\iota\delta} & c_{(23)}c_{(13)} \end{pmatrix}, \tag{1.3.7}$$

where $c_{(ij)}$ represents $\cos\theta_{(ij)}$ and $s_{(ij)}$ represents $\sin\theta_{(ij)}$. Angle $\theta_{(ij)}$ depend on the mixing of the quarks as $\theta_{(12)}$ would be exactly θ_c if there is no mixing between the third and the first generation of quarks. V_{CKM} matrix can be computed only experimentally by constraining and using the Unitarity principle $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$. The current measurements stand at

$$|V_{CKM}^{exp}| = \begin{pmatrix} 0.97428(15) & 0.2253(7) & 3.47(16) \times 10^{-3} \\ 0.2252(7) & 0.97345(16) & 41(1) \times 10^{-3} \\ 8.62(26) \times 10^{-3} & 40.3(1.1) \times 10^{-3} & 0.999152(45) \end{pmatrix}. \tag{1.3.8}$$

For the decays within the family, $u \leftrightarrow d, c \leftrightarrow s, t \leftrightarrow b$, matrix becomes diagonal. Helicity suppression is large for dielectron and dimuon pairs as they have small masses. Due to their suppression in the SM, weak and Higgs-mediated processes are unlikely to exhibit flavor-changing-neutral-current (FCNC) processes. However, since FCNC can occur at higher levels only in Electroweak interactions, these interactions are promising candidates in the search of New Physics.

Experimentally, CLEO in 1994 took the initiative and studied the rare radioactive decay $b \to s\gamma$ [14]. In subsequent years, BaBar and Belle were able to significantly increase their dataset of $B_0\bar{B}_0$ pairs by collecting an impressive 467 million and 772 million pairs in 2008 and 2010, respectively. When these data sets were combined, they produced an astounding integrated luminosity of $1ab^{-1}$ operating at $\Gamma(4S)$, providing an unprecedented amount of data for further analysis and investigation. $b\bar{b}$ cross section measured at the most efficient B- factory, large Hadron collider is $300\mu b$ at the center of mass energy $\sqrt{s}$= 7 TeV [15]. At $\sqrt{s}$= 14 TeV, the corresponding cross-section is $500\mu b$. About 10^{11} hadrons are provided that are produced in a dataset of $1fb^{-1}$. ATLAS, CMS and LHCb are the major experiments of the LHC. Among these, LHCb is specifically focused on the study of production and decay of $c-$ and $b-$hadrons, while all three experiments contribute to the investigation of rare $b-$Hadron decays. While the CMS and ATLAS experiments can generate a final state containing a dimuon pair, the LHCb experiment is capable of producing final states consisting of a photon, a dielectron pair, or only hadrons. In Semi-leptonic decays, B-factories average over neutral and charged B-mesons. CDF and LHCb observed many $b-$Hadron decays such as $B \to K^*l^+l^-, B \to Kl^+l^-, \Lambda_b^0 \to \Lambda\mu^+\mu^-, \Lambda_b^0 \to pK^-\mu^+\mu^-$ and $B_s^0 \to \phi\mu^+\phi^-$ [16, 17, 18, 19, 20].

Branching fractions of some familiar $B-$meson decays are

$$\begin{aligned}
\mathcal{B}\big(B \to K l^+ l^-\big) &= (5.5 \pm 0.7) \times 10^{-7},\\
\mathcal{B}\big(B \to K \mu^+ \mu^-\big) &= (4.43 \pm 0.24) \times 10^{-7},\\
\mathcal{B}\big(B^0 \to K^{*0}(892) l^+ l^-\big) &= \big(1.03^{+0.19}_{-0.17}\big) \times 10^{-6},\\
\mathcal{B}\big(B^0 \to K^{*0} \mu^- \mu^+\big) &= (1.03 \pm 0.06) \times 10^{-6},\\
\mathcal{B}\big(B_s^0 \to \phi \mu^+ \mu^-\big) &= (8.3 \pm 1.2) \times 10^{-7},\\
\mathcal{B}\big(\Lambda_b^0 \to \Lambda \mu^+ \mu^-\big) &= (1.08 \pm 0.28) \times 10^{-6}.
\end{aligned} \tag{1.3.9}$$

The operators of Semi-leptonic decays are more sensitive towards New Physics as compared to Radiative decays. In the case of Semi-leptonic decays, Hadronic uncertainties tend to be larger. This is because the lepton partners in the end state can sometimes originate from a photon that is produced at a flavor-conserving QED vertex.

With this elaborated introduction, the remaining dissertation is organized as follows. In second chapter, we present the Light Cone Sum Rules and their importance in flavor physics. The technique is used to calculate the form factor for the Semi-leptonic $B \to K^{0*}$ decay. Using the recent constrains on various new physics (NP) Wilson coefficients, we compare the results of SM in Chapter 3. Finally, in Chapter 4, we present our conclusions.

Chapter 2

Basic Concepts

This chapter will provide the introduction of the prime concerned terms that are essential for the understanding of the partial leptonic decays of B-mesons into scalar mesons by using the Light front QCD sum Rule approach. Different terms that are involved here are QCD sum rules, Light front sum rules, light cone singularity, light cone limit, the underlying theory of light cone, correlation function, dispersion relation, Operator product expansion, and general technique involved to calculate Wilson coefficients, form factors would be introduced with great and useful details.

2.1 QCD sum rules approach

2.1.1 Introduction

It was first developed in 1970s and is based on the intuition of relating properties of the quark content to the properties of hadrons comprised of the respective quarks and gluons by taking into account correlation functions of the currents in the Euclidean space (i.e., at space-like distances). To extract the properties of hadrons, i.e., mass calculations and resonance couplings, immense work has been done. Basic factors that provide the grounds for this approach are [21]

- Spontaneous symmetry breaking,
- Quark-Hadron duality,

- Asymptotic freedom.

Formalism of QCD approach can be used to calculate masses of light quarks too and the method has been described by Gasser and Leutwyler [22]. Underlying idea of this approach is to tackle the bound state problem by utilizing the property of asymptotic freedom. In more clear words, at first solving by considering short distance and then moved to larger distances where the effects of confinement become considerably important and asymptotic freedom no more exists. The resonances appear implying that gluons and quarks comprise hadrons and are not free at this scale. So, here actually an idea of separation between long distance and short distance physics is introduced. This separation allows us to use perturbative QCD for short distance physics and the properties of hadrons such as mass and decay constants are parameterized in terms of non-perturbative long distance physics. Idea of asymptotic freedom's breakdown appears due to emergence of non-zero expectation values of condensates i.e., quark and gluon condensate operators such as $\langle 0|\bar{q}q|0\rangle$ and $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$, where $q(x)$ is representing quark field and $G^a_{\mu\nu}$ is representing gluon field tensor. These condensates disappear when studied in basic perturbation theory. Hypothesis behind QCD sum rules approach is the existence of gluon and quarks in vacuum field. Valence quarks and gluons injected by external current sources has to develop in vacuum medium rather than empty space.

2.1.1.1 Spontaneous Symmetry breaking

Spontaneous symmetry breaking occurs when there is symmetry in the dynamics of the system that is not manifested in its ground state or equilibrium state. It occurs when the Lagrangian of the system has certain symmetries but the ground state does not preserve it. Spontaneous breaking of a continuous symmetry leads to the appearance of massless scalar (spin-0) bosons. Goldstone model is the best model for it. In mass generating mechanism, a new complex scalar field Φ is introduced. Specifically, it is characterized by the Lagrangian density $\mathcal{L}$.

$$\mathcal{L} = D^*_\mu \phi^* D_\mu \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \mathcal{V}(\phi), \tag{2.1.1}$$

where

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi,$$

and $F_{\mu\nu}$ is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and $\mathcal{V}$ is some potential in Goldstone model. We can define a new field after transformation as

$$B_\mu = A_\mu + \frac{1}{e\eta}\partial_\mu \psi_2,$$

as $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and expansion in ϕ field is taken in as $\frac{1}{\sqrt{2}}\big(\eta + \phi_1 + \phi_2\big)$. From the curvature of potential ϕ_1 mode takes the mass as it is the Higgs boson and we find

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\lambda\eta^2\phi_1^2 + \frac{1}{2}e^2\eta^2 B_\mu B^\mu + \text{cubic terms and quartic terms.} \quad (2.1.2)$$

So, miraculously we are left with heavy scalar (Higgs boson) and heavy gauge vector boson.

2.1.1.2 Asymptotic freedom

Coupling constant g characterizes the strength of interaction between quarks and gluons. g is characterized by a scale μ based on a normalization condition. One can define $g(\mu)$ as a $\gamma_\sigma-$terms of the quark gluon vertex function that is defined as $\Gamma_\sigma(q, p_1, p_2)$, which is manifested by the normalization group equations. Most vital application of RG equation is the dependence of the g on μ scale. In QCD, it can be described by the following equation

$$\frac{dG}{dL} = \beta\big(G\big) = -\sum_{k=0}^{\infty} b_k G^k, \quad (2.1.3)$$

where $G = \frac{g^2}{\left(4\pi\right)^2}$,$L = \ln\big(\mu\big)^2$,$\frac{d}{dL} = \mu^2\frac{d}{d\mu^2}$. Integrating above equation one obtains

$$\ln\left(\frac{\mu^2}{\mu'^2}\right) = \int_{G(\mu')}^{G(\mu)} \frac{dx}{\beta(x)} \equiv \Phi\big(G(\mu)\big) - \Phi\big(G(\mu')\big). \quad (2.1.4)$$

Note that

$$\ln\left(\frac{\mu^2}{\mu'^2}\right) + \Phi\big(G(\mu')\big) = \Phi\big(G(\mu)\big) \text{ does not depend on } \mu' \text{ so,}$$

$$\Phi\big(G(\mu')\big) = \ln\left(\frac{\mu'^2}{\Lambda^2}\right), \tag{2.1.5}$$

where the Λ corresponds to the mass dimension.

$$G(\mu) = \Phi^{-1}\ln\left(\frac{\mu^2}{\Lambda^2}\right), \tag{2.1.6}$$

however, $\Phi \rightarrow \Phi_\phi + 2\phi'$ and $\Lambda \rightarrow \Lambda_{\phi'} \equiv \Lambda e^{-\phi'}$. Under this transformation $G(\mu)$ becomes as

$$G(\mu) = \Phi_{\phi'}^{-1}\left(ln\left(\frac{\mu^2}{\Lambda_{\phi'}^2}\right)\right). \tag{2.1.7}$$

Let's consider for $\phi' = \phi'_0, G(\mu)$ has the following form

$$G(\mu) = \frac{1}{b_0 \ln\left(\frac{\mu^2}{\Lambda_{\phi'}^2}\right)}, \tag{2.1.8}$$

If ϕ' is chosen as $\phi' = \phi'_0 + \delta\phi'$ then $G(\mu)$ would be written as

$$G(\mu) = \frac{1}{b_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)}\left[1 + \sum_{n=1}^{\infty}\left(\frac{\delta\phi'}{ln\left(\frac{\mu^2}{\Lambda_{\phi'}^2}\right)}\right)^n\right]. \tag{2.1.9}$$

As beta function has been calculated up to 4 terms so

$$b_0 = 11 - \frac{2}{3}N_f, \tag{2.1.10}$$

$$b_1 = 102 - \frac{38}{3}N_f, \tag{2.1.11}$$

and

$$b_2 = \frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2, \tag{2.1.12}$$

where N_f is number of quark flavors. The value of b_2 is known only in the MS-bar scheme.

In lowest approximation we get

$$\alpha(Q^2) = \frac{g^2}{4\pi} = 4\pi G(Q) = \frac{4\pi}{\left(11 - \frac{2}{3}N_f\right)\ln\left(\frac{Q^2}{\Lambda^2}\right)}. \tag{2.1.13}$$

Equation (2.1.13) states that coupling constant disappears, i.e., $g \to 0$ when $Q^2 \to \infty$ and this is the property of QCD asymptotic freedom that is the cornerstone of all the perturbative QCD theories.

2.1.1.3 Quark-Hadron duality approximation

Quark-Hadron duality occurs in those processes where one can consider two stage processes that could occur at two distinct scales. These two stage processes are interaction between quarks at short distances as well as at the long distances. The scale for the short distance interaction of quarks is of the order of $\frac{1}{Q}$ and for long distance interaction is of the order of $\frac{Q}{\Lambda^2}$. The reason behind long distance interaction of quarks is the "passage of time". Quark Hadron duality violations are due to the following reasons

1. Quark transitions instead of shorter distances occur at longer distances.
2. Quarks produced at shorter distances but residual interactions are occurring at longer distances.

In the first point, singularities are developed by the point functions at finite value of x^2 and in the second point $x^2 \to \infty$.

2.1.2 History of QCD sum rules approach

Purpose of introducing the history is to mention about the historic time when theory of hadrons was about to be introduced. It would be fair enough to say that QCD was introduced after the talk of Gell-mann and Fritszch as the color octet gluons are introduced. Asymptotic freedom was discovered in 1973 and that time, hard processes were the epicenter of everyone's attention in which short distance physics contribution plays vital role. In the very end of 1960s, the very non-serious attitude of physicists towards field theory made M.Shifman sick, Founder of this approach, M.Shifman was not able to publish his work due to long procedure of publishing according to GLAVILT. Upto 1974, Shifman got the intuition that due to Landau work on zero charge property in QED, everyone has a deep rooted non-serious attitude towards field theory so he started working on himself to solve the problem involving short and long distance physics.

He started working on his idea as to begin from short distance physics where dynamics of quarks and gluons is perturbative and then by considering some approximations, results would be extrapolated to long distance physics to take into account non perturbative effects. M. Shifman says it spectacular effort when he combined his work with V. Novikov, L. Okun and M. Voloshin. It was the result of great amount of hardwork that in 1977, it turned out that all parameters that can give all the information about hadrons can be obtained by sum rules. The gluon condensate was introduced in 1977 and the underlying idea was that " **vacuum is considered as a gluon medium and basic properties of the particles come due to interaction of quarks with this medium so it can be parameterized by condensates**". Shifman and his team were done with this work in 1978 with accurate results[23]. QCD sum rules approach has been very vital for determination of parameters of low-lying Hadronic states. For the problems associated with phenomenology of hadrons, Shifman proposed method of QCD sum rules became a vital tool.

2.2 Operator product expansion (OPE)

2.2.1 Introduction

In this section, we discuss the general procedure involved to calculate all the Hadronic parameters by using sum rules. Correlation functions, dispersion relations and spectral density would be introduced. Correlation function and its relation with all parameters of Hadron would be discussed. Borel transform's role is also explained to take into account the dominance of low lying resonance states. In order to calculate expansion coefficients several techniques developed but most convenient techniques are:

- Formalism utilizing abstract operators.

- A formalism build on the Fock-Schwinger gauge for the gluon field in the absence of particles.

2.2.1.1 Correlation function

Correlation functions are actually $n-$point functions that describe the correlation between colorless external quark currents injected by electromagnetic sources. Correlation functions are calculable by the simplest expansion $(G^a_{\mu\nu}\frac{1}{q^2})$, where $G^a_{\mu\nu}$ is the gluon field strength tensor whereas every current has specific quantum numbers i.e., J, P and C numbers. Here, $G^a_{\mu\nu}$ is treated as an external field. Coefficients in front of these field strength operators are calculated by several techniques involving Feynman diagrams but soon it was realized that methods were inconvenient in order to get non invariant gauge results. QCD sum rules approach ensures gauge invariant gluon field operators appearing in the expansions. The $n-$point Green functions are determined by the asymptotic freedom formula. Power corrections are manifested by the non-perturbative vacuum fluctuations. In vacuum fields everything is fixed by the vacuum condensates. Vacuum condensates are $\langle 0|\bar{\psi}\psi|0\rangle$ and $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$. Some basic steps are following:

- Some colorless currents are given.
- $n-$point Green functions are written in terms of these currents expressing the correlations between different currents in vacuum fields where vacuum fields are actually external fields induced by external quarks and gluons.
- Field considered in this procedure is weak. The weak field means average intensity of field is weaker than the value of momentum.

Consider vector current of quark $q\left(x\right)$. Then the ordered product of two currents is

$$\Pi_{\mu\nu}(q) = \iota\int e^{\iota qx}d^4x\langle T\Big\{j_\mu\big(x\big)j_\nu\big(0\big)\Big\}\rangle, \tag{2.2.1}$$

where $j_\mu\big(x\big)$ is defined as

$$j_\mu\big(x\big) = \bar{q}\gamma_\mu q,$$

this vector current is written for a single massless quark. For large momenta, the time ordered (T-ordered) product of current can be written in the form of expansion of the type

$\left(x^2\right)^k \ln\left(x^2\right)$ so the 2-point function takes the following form

$$\Pi_{\mu\nu}(q) = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) \sum_n C_n\left(Q^2\right)\langle O_n\rangle, \tag{2.2.2}$$

where $Q^2 \equiv -q^2$ and q is the large exchange momenta. $\langle O_n\rangle$ are the condensates and are local gauge invariant operators. C's are the Wilson parameters. Local gauge invariant operators can be written as $1, \bar{\psi}\psi, G^a_{\mu\nu}G^a_{\mu\nu}, \ldots$. The vacuum matrix elements $\langle O_n\rangle$ are of the order of the magnitude of μ^{d_n}, where d_n corresponds the dimensions and n denotes normal. μ is of the order of MeV as it relates to the Hadronic mass scale. For $Q^2 >> \mu^2$, only first few terms can be kept. Wilson coefficient expansion is proven in perturbation theory and beyond perturbative regime effects can be included in it by extending it.

2.2.1.2 Status of OPE in QCD

Infrared renormalon problem arises due to the logarithmic growth of coupling constant $\alpha_s(R)$ at larger distances. These renormalons appear as singularities and these singularities are responsible for the failure of an attempt to sum up series of perturbation theory expansion (PTE). In the last decade, non-perturbative effects were considered crucial at larger distances. NP effects are actually non-vanishing vacuum expectation values and these NP fields are very large in vacuum i.e., $\langle G^a G^a\rangle \approx (600MeV)^4 >> \Lambda^4_{QCD}$. When larger distances are approached or in other words when asymptotic region is ended then NP effects that are coded as power corrections in OPE causes the major changing in the behavior of OPE series. A new method has been introduced to solve these ambiguities in which gluon field is splitted into the background field and perturbative gluon field. Background field is denoted by B_μ and the perturbative gluon field is denoted by a_μ and now the perturbative theory expansion occurs in powers of ga_μ. Propagators are Green functions denoted as $G_{\mu\nu}$ in the background field B_μ. Based on the Feynman Schwinger representation, new closed form of $G_{\mu\nu}$ is introduced. Explicit dependence on B_μ results through the parallel transporters $P\exp\left(\iota g\int_x^y B_\mu dz_\mu\right)$. Now each term in perturbative expansion series contain dependence on B_μ through the Wilson loops $W(C,B)$. Over largest values of N_C, the averaging over background fields are represented by the products of averaged Wilson loops $\langle W(C,B)\rangle$. This product obeys the area law at larger distances and hence it results in calculations of NP-dynamical parameters [24].

2.2.1.3 Calculation of C_1 by using Operator Schwinger method

In external non-abelian gauge field A_μ^a, the eigenvalue product of the Dirac operator is defined as $\mathrm{Det}\,|\iota D_\mu \gamma_\mu - m|$. This determinant in the vacuum to vacuum transition enters as a pre-exponential factor. This $|0\rangle$ to $|0\rangle$ transition can be represented as

$$\langle O|O'\rangle = \int DA_\mu \bar{D}\bar{\psi} D\psi \exp\left[\iota\left\{d^4x\Big(\frac{-1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \bar{\psi}\big(\gamma_\mu \varrho_\mu - m\big)\psi\Big)\right\}\right], \tag{2.2.3}$$

with $\varrho = iD_\mu$. To incorporate all fermions degree of freedom, treating the $A_\mu^a(x)$ as given external field. Then

$$\langle O|O'\rangle = \int DA_\mu \mathrm{Det}\,|\gamma_\mu \varrho_\mu - m| \exp\left\{\frac{-1}{4}\int d^4x G^a_{\mu\nu}G^a_{\mu\nu}\right\}, \tag{2.2.4}$$

$$= \int dA_\mu \exp\left\{\frac{-1}{4}\int d^4x \left[G^a_{\mu\nu}G^a_{\mu\nu} - \iota \ln \mathrm{Det}\,|\not{\varrho} - m|\right]\right\}, \tag{2.2.5}$$

where

$$\langle x|\big(\not{\varrho}^{ab}_{\alpha\beta}\big)|y\rangle = (\gamma_\mu)_{\alpha\beta}\left[\iota\delta^{ab}\frac{\partial}{\partial x_\mu} + g\big(t_c\big)_{ab}A^c_\mu\big(x\big)\right]\delta\big(x-y\big), \tag{2.2.6}$$

and $(t_c)_{ab}$ stands for the generation of color group. A divergence is associated with the determinant of free Dirac operator. To get rid of these divergences, one must take the ratio

$$\frac{\mathrm{Det}\,|\not{\varrho} - m|_A}{\mathrm{Det}\,|\not{\varrho} - m|_{A=0}}.$$

In order to regularize this ratio, an auxiliary fermions Pauli Villars field is introduced. So, final expression is written as

$$D_{regularized} = \mathrm{Det}\left|\frac{\big(\not{\varrho} - m\big)_A}{(\not{\varrho} - m)_{A=0}}\frac{\big(\not{\varrho} - m_R\big)_{A=0}}{(\not{\varrho} - m_R)_A}\right|. \tag{2.2.7}$$

where m_R is called regulator mass. After integration over fermions field, a spare term has to be added in the action of vector field. So

$$
\begin{aligned}
S_{eff} &= S_{cl} + \Delta S, \\
\Delta S &= -\imath \ln D_{regularized} = -\imath \mathrm{Tr} \ln \left| \frac{(\not{\partial} - m)_A}{(\not{\partial} - m)_{A=0}} \frac{(\not{\partial} - m_R)_{A=0}}{(\not{\partial} - m_R)_A} \right|,
\end{aligned} \tag{2.2.8}
$$

here a well known relation is used $\ln Det\,|A| = \mathrm{Tr} \ln |A|$.

Expanding ΔS in powers of $\left(\frac{G}{m^2}\right)^n$ assuming field strength tensor $G^a_{\mu\nu}$ is less than m^2, where m is the mass of quark. As fermions loop for large masses can be found by short distances $\mathcal{O}\left(m^{-1}\right)$, so the effective action can be written in the form of series of local operators

$$
\Delta S = \int d^4x \left[C_1 g^2 \left(\ln \frac{m_R^2}{m^2} \right) G^a_{\mu\nu} G^\alpha_{\mu\nu} + C_2 \frac{g^3}{m^2} f^{abc} G^a_{\mu\alpha} G^b_{\mu\beta} G^c_{\beta\mu} + ... \right]. \tag{2.2.9}
$$

The fundamental role of the gluon field A^a_μ and the $\ln\left(\frac{m_R^2}{m^2}\right)$ has identical structure, so the sum of these two terms would result in the following form

$$
-\frac{g^2}{4} G^a_{\mu\nu} G^a_{\mu\nu} \left[\frac{1}{g^2} - 4C_1 \ln \frac{m_R^2}{m^2} \right].
$$

From the theory of renormalization, we know that the term in the bracket corresponds to the renormalized charge $\frac{1}{g^2_{renormalized}}$, so the value of C_1 is fixed by renormalization. Writing

$$
\frac{1}{g^2_{REN}} = \frac{1}{g^2} - \frac{1}{16\pi^2} \left(\frac{11}{3} N_c - \frac{2}{3} N_f \right) \ln \frac{m_R^2}{m^2}, \tag{2.2.10}
$$

$$
C_1 = -\frac{1}{96\pi^2}. \tag{2.2.11}
$$

Here, N_f represents the number of flavors, which is equal to one in our case. Upto the fourth order, the expansion parameters can be determined [25], and the effective action takes the following form[26]

$$
\Delta S_{effective} = -\frac{1}{32\pi^2} \int d^4x Tr_C \left\{ \frac{2}{3} g^2 G^2_{\mu\nu} \ln \frac{m_R^2}{m^2} - \frac{2}{45} i g^3 G_{\mu\nu} G_{\nu\gamma} G_{\gamma\mu} \frac{1}{m^2} + \frac{g^4}{18} \left(G_{\mu\nu} G_{\mu\nu} \right)^2 \right.
$$

$$-\frac{7}{10}\left\{G_{\mu\alpha},G_{\alpha\nu}\right\}_{+}^{2}+-\frac{29}{70}\left[G_{\mu\alpha},G_{\alpha\nu}\right]_{-}^{2}+\frac{8}{35}\left[G_{\mu\nu},G_{\alpha\beta}\right]_{-}^{2}\frac{1}{m^{4}}\right\}. \tag{2.2.12}$$

Here

$$G_{\mu\nu}=G^{a}_{\mu\nu}t^{a},$$
$$Tr\left(t^{a}t^{b}\right)=\frac{1}{2}\delta^{ab}.$$

The subscripts $-/+$ are marked to avoid the confusion between commutators and anticommutators. Tr_C is representing that the results are valid for any gauge group.

2.2.1.4 Two-point function for massive quarks

When the mass of quark is large, then the quark condensate does not correspond to an independent parameter and problem is restricted to the motion in external field. The 2-point function of vector current is written as

$$\Pi_{\mu\nu}(q)=\iota\int d^{4}x\exp\left(\iota qx\right)\langle T\left[J_{\mu}\left(x\right),J_{\nu}\left(0\right)\right]\rangle, \tag{2.2.13}$$

where

$$J_{\mu}\left(x\right)=\bar{Q}\left(x\right)\gamma_{\mu}Q\left(x\right),$$
$$\Pi_{\mu\nu}\left(q\right)=\iota\int d^{4}x\exp\left(iqx\right)Tr\left\{\gamma_{\mu}S\left(x,0\right)\gamma_{\nu}S\left(0,x\right)\right\}. \tag{2.2.14}$$

Fourier transform of $S\left(x,0\right)$ and $S\left(0,x\right)$ is defined as

$$S\left(p\right)=\int S\left(x,0\right)\exp\left(ipx\right)d^{4}x,$$
$$\tilde{S}\left(p\right)=\int S\left(0,x\right)\exp\left(-ipx\right)d^{4}x,$$

so the above polarization operator takes the following form in momentum space

$$\Pi_{\mu\nu}\left(q\right)=\iota\int\mathrm{Tr}\left\{\gamma_{\mu}S\left(p\right)\gamma_{\nu}\tilde{S}\left(p-q\right)\right\}\frac{d^{4}p}{\left(2\pi\right)^{4}}. \tag{2.2.15}$$

Fourier transform of the vacuum field is written as

$$\begin{aligned}A_{\mu}\left(k\right) &= \int A_{\mu}\left(z\right)\exp\left(\iota kz\right)d^{4}z, \\ &= \frac{-\iota\left(2\pi\right)^{4}}{2} - G_{\rho\mu}\left(0\right)\frac{\partial}{\partial k_{\rho}}\delta^{(4)}\left(k\right) + \frac{\left(-\iota\right)^{2}\left(2\pi\right)^{4}}{3}\left(D_{\alpha}G_{\rho\mu}\left(0\right)\right)\frac{\partial^{2}}{\partial k_{\rho}\partial k_{\alpha}}\delta^{(4)}\left(k\right) + ...,\end{aligned} \tag{2.2.16}$$

where G^2 correction in 2-point function can be written for only those terms in which the external field appeared twice. Therefore, writing the expansion for external field and considering only those terms that contribute in G^2 correction, gives

$$\begin{aligned}A_{\alpha}\left(k_{1}\right) &= -\frac{\iota\left(2\pi\right)^{4}}{2}G_{\rho\alpha}\left(0\right)\frac{\partial}{\partial k_{1\rho}}\delta^{(4)}\left(k_{1}\right), \\ A_{\beta}\left(k_{2}\right) &= -\frac{\iota\left(2\pi\right)^{4}}{2}G_{\sigma\beta}\left(0\right)\frac{\partial}{\partial k_{2\sigma}}\delta^{(4)}\left(k_{2}\right).\end{aligned}$$

As we know that

$$\langle G^{a}_{\mu\nu}\left(0\right)G^{b}_{\alpha\beta}\left(0\right)\rangle = \frac{1}{96}\delta^{ab}\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)\langle G^{c}_{\rho\sigma}G^{\rho\sigma,c}\rangle, \tag{2.2.17}$$

therefore, taking the color trace we get the following form

$$\begin{aligned}\left[\Pi_{\mu\nu}\left(q\right)\right]_{a} &= -\frac{1}{96}\delta^{ab}\langle g^{2}G^{a}_{\mu'\nu'}G^{\mu'\nu',a}\rangle(g_{\rho\sigma}g_{\alpha\beta} - g_{\rho\beta}g_{\sigma\alpha}) \times \int\frac{d^{4}p}{\left(2\pi\right)^{4}}\frac{\partial}{\partial k_{1\rho}}\frac{\partial}{\partial k_{2\sigma}} \\ & Tr_{L}\{\gamma_{\mu}\frac{1}{\not{p} - m}\gamma_{\alpha}\frac{1}{\not{p} - \not{k}_{1} - m}\gamma_{\nu} \times \frac{1}{\not{p}' + \not{k}_{2} - m}\gamma_{\beta}\frac{1}{\not{p}' - m}\}_{k_{1}=0=k_{2}},\end{aligned} \tag{2.2.18}$$

$$\begin{aligned}\left[\Pi_{\mu\nu}\left(q\right)\right]_{b} &= -\frac{1}{48}\langle g^{2}G^{a}_{\mu'\nu'}G^{\mu'\nu',a}\rangle\left(g_{\rho\sigma}g_{\alpha\beta} - g_{\rho\beta}g_{\sigma\alpha}\right) \times \int\frac{d^{4}p}{\left(2\pi\right)^{4}}\frac{\partial}{\partial k_{1\rho}}\frac{\partial}{\partial k_{2\sigma}} \\ & Tr_{L}\left\{\gamma_{\mu}\frac{1}{\not{p} - m}\gamma_{\alpha}\frac{1}{\not{p} - \not{k}_{1} - m}\gamma_{\beta} \times \frac{1}{\not{p}' + \not{k}_{2} - m}\gamma_{\nu}\frac{1}{\not{p}' - m}\right\}_{k_{1}=0=k_{2}}.\end{aligned} \tag{2.2.19}$$

The 2-point function is transverse due to current conservation. So

$$\Pi_{\mu\nu}\left(q\right) = \left(q_{\mu}q_{\nu} - q^{2}g_{\mu\nu}\right)\Pi\left(q^{2}\right). \tag{2.2.20}$$

The indices can be contracted that results into the following form

$$[\Pi_{\mu\mu}(q)]_a = -\iota\langle g^2G^2\rangle \int \frac{d^4p}{(2\pi)^4}\frac{pp'}{(p^2-m^2)^2(p'^2-m^2)^2}, \tag{2.2.21}$$

$$[\Pi_{\mu\mu}(q)]_b = -\iota 4m^2\langle g^2G^2\rangle \int \frac{d^4p}{(2\pi)^4}\frac{pp'-2p^2}{(p^2-m^2)^4(p'^2-m^2)}, \tag{2.2.22}$$

by using Standard methods, the above integrals are calculated, and we get

$$\Pi^G_{\mu\nu}|_{h_Q,1^-} = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\frac{1}{48}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{Q^4}\times\left\{\frac{3(a+1)(a-1)^2}{a^2}\frac{1}{2\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{3a^2-2a+3}{a^2}\right\}, \tag{2.2.23}$$

where $a = 1+\frac{4m_Q^2}{Q^2}$. This is the G^2 correction in case of vector current.

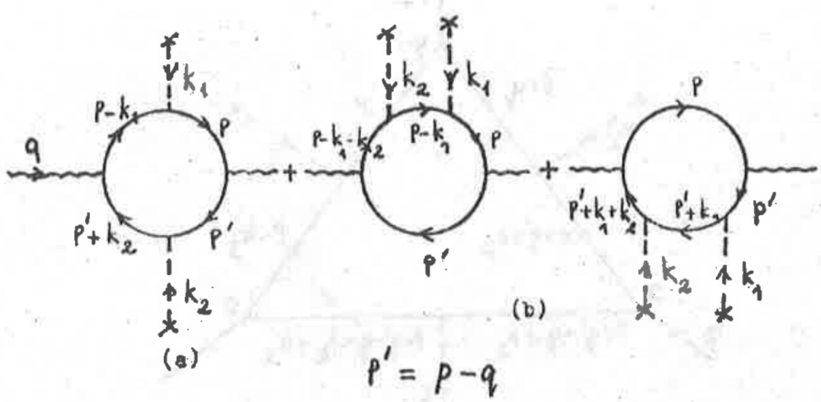

Figure 2.2.1: G^2 correction in case of vector current [27].

Now, the G^2 correction in the case of two point function for pseudo-current can be written as

$$\Pi^{(G)}|_{o^-} = \frac{1}{48}\langle\frac{\alpha_s G^2}{\pi}\rangle\frac{1}{4m_Q^2}\times\left\{\frac{3(3a+1)(a-1)^2}{a^2}\frac{1}{2\sqrt{a}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{9a^2+4a+3}{a^2}\right\}. \tag{2.2.24}$$

Similarly, the two point function for light quarks can be derived. For light quarks, the form of 2-point function is

$$\Pi(q)|_{1^-} = \frac{3}{4\pi^2}q^2\ln Q^2 - \frac{1}{16\pi^2}\frac{1}{q^2}\langle g^2G^2\rangle. \tag{2.2.25}$$

The correlation function for gluon current has the form

$$\iota\int \exp\left(\iota qx\right) d^4x\langle T\left\{j_5\left(x\right)j_5\left(0\right)\right\}\rangle = -8\alpha_s^2\frac{g}{q^2}\langle G^a_{\mu\nu}G^b_{\nu\beta}G^c_{\beta\mu}f^{abc}\rangle + \text{other operators.} \quad (2.2.26)$$

2.2.2 Dispersion relation

It is well known that the correlation function has a dual nature. It represents short distance quark-antiquark fluctuations at large negative q^2 and can be solved in the standard perturbation theory, while at large positive q^2, the correlation function represents the long distance effects that are represented in the non-perturbation theory and are known as non-perturbative effects. Unitarity relation lets to insert the Hadronic intermediate states in the $\Pi_{\mu\nu}(q^2)$, and it results in the following expression

$$2\mathrm{Im}\Pi_{\mu\nu}\left(q\right) = \sum_n \langle vac|j_\mu|b\rangle\langle b|j_\nu|vac\rangle d\upsilon_n \left(2\pi\right)^4\delta^{(4)}\left(q-p_n\right), \quad (2.2.27)$$

where $\sum_n$ means sum over all the possible Hadronic states and $d\upsilon_n$ denotes that the whole phase space region is taken into the account. Above Eq. (2.2.27) is called Hadronic sum. In the Hadronic sum Eq. (2.2.27), vector meson contribution can be taken as

$$\frac{1}{\pi}\mathrm{Im}\Pi^V_{\mu\nu}\left(q^2\right) = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) f_V^2 \delta\left(q^2 - m_V^2\right), \quad (2.2.28)$$

where the decay constant f_V can be defined as

$$\langle V\left(q\right)|j_\mu|0\rangle = f_V m_V \epsilon_\mu^{(V)*}, \quad (2.2.29)$$

ϵ^V_μ stands for the Helicity of vector mesons. m_n corresponds to the sum of Hadronic masses. At $q^2 > m_n^2$, each states contribution is represented by the continuous imaginary part. However, the Hadronic matrix elements corresponding to $\langle n|j|0\rangle$ depends on q^2. Just for the sake of convenience, we separate the ground state vector participation on the right side of Hadronic sum, and a condensed notation is introduced for rest of the excited vector mesons.

The continuum states represented as

$$\mathrm{Im}\Pi\left(q^2\right) = \pi\left(f^2_{Vector}\delta\left(q^2 - m^2_{Vector}\right) + \rho^h_{conti.}\left(q^2\right)\theta_{conti.}\left(q^2 - s^h_{conti.}\right)\right), \tag{2.2.30}$$

where s^h_{conti} represents the threshold of the lowest continuum state. In the case of least massive quark interactions, this threshold is lower than m_V. For analytic function, Cauchy formula is implied and the polarization operator takes the form

$$\Pi\left(q^2\right) = \frac{1}{2\pi\iota}\oint\limits_C dz\frac{\Pi\left(z\right)}{z-q^2}, \tag{2.2.31}$$

$$= \frac{1}{2\pi\iota}\oint\limits_{|z|=R} dz\frac{\Pi\left(z\right)}{z-q^2} + \frac{1}{2\pi\iota}\int_0^R dz\frac{\Pi\left(z+\iota\epsilon\right)-\Pi\left(z-\iota\epsilon\right)}{z-q^2}. \tag{2.2.32}$$

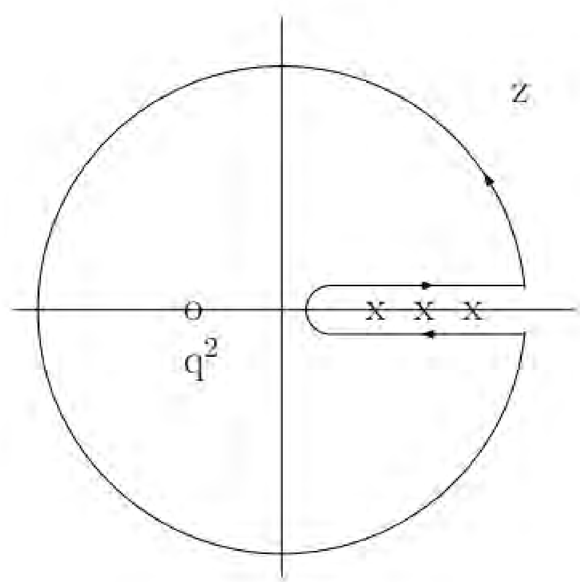

Figure 2.2.2: Contour is shown in complex plane where complex variable is $q^2 = z$. $q^2 < 0$ is shown by open points and crosses show the Hadronic thresholds [27].

Here we have used the fact that $\Pi\left(q^2\right)$ is real valued at $q^2 < t_{min} = min\left\{m^2_V, s^h_0\right\}$. Now, invoking Schwartz reflection principle $\Pi\left(q^2+\iota\epsilon\right) - \Pi\left(q^2-\iota\epsilon\right) = 2\iota\mathrm{Im}\Pi\left(q\right)^2$ at $q^2 > t_{min}$, the dispersion relation takes the following form

$$\Pi\left(q^2\right) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\Pi\left(s\right)}{s - q^2 - \iota\epsilon}, \tag{2.2.33}$$

$\mathrm{Im}\Pi\left(s\right)$ does not disappear at $s \to \infty$, so the dispersion integral does not converge. This problem can be cured by the Taylor expansion of Π at $q^2 = 0$ and then subtracting it from $\Pi\left(s\right)$. Mathematical form of this is

$$\bar{\Pi}\left(q\right)^2 = \Pi\left(q\right)^2 - \Pi\left(0\right). \tag{2.2.34}$$

Eq. (2.2.33) takes the following form

$$\bar{\Pi}\left(q\right)^2 = \frac{q^2}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\Pi\left(s\right)}{s\left(s - q^2\right)}, \tag{2.2.35}$$

from Eq. (2.2.30), we get

$$\Pi\left(q\right)^2 = \frac{q^2 f_{Vector}^2}{m_{Vector}^2\left(m_{Vector}^2 - q^2\right)} + q^2\int_{s_{conti}^h}^{\infty} ds \frac{\rho_{conti}^h\left(s\right)}{s\left(s - q^2\right)} + \Pi\left(0\right). \tag{2.2.36}$$

As electromagnetic interactions used to be gauge invariant, so $\Pi\left(0\right) = 0$. Such relation establishes the sum rules, i.e., significant limitations on the total values of various Hadronic parameters. This suggests that there are specific conditions or restrictions on how the parameters related to the behavior of subatomic particles called hadrons can be combined or added together.

2.2.3 Borel Transformation

Dispersion relations connect physically observed quantities with $n-$point functions. After applying the Borel transformation, expression converts into the sum rule with a weight function that corresponds to the low lying Hadronic states. Borel transformation of an arbitrary function $f(x)$ is written as

$$\tilde{f}\left(\lambda\right) = \frac{1}{2\pi\iota}\int_{c-\iota\infty}^{c+\iota\infty} \exp\left(\frac{\lambda}{x}\right) f\left(x\right) x d\left(\frac{1}{x}\right), \tag{2.2.37}$$

whereas the inverse transformation can be defined as

$$f(x) = \int_0^\infty \tilde{f}(\lambda) \exp\left(-\frac{\lambda}{x}\right) d\lambda / x, \tag{2.2.38}$$

For analytic functions, the Borel transformation has the following form

$$\hat{B}\Pi\left(Q^2\right) = \lim_{Q^2, n\to\infty} \frac{1}{(n-1)!}\left(Q^2\right)^n \left(-\frac{d}{dQ^2}\right)^n \Pi\left(Q^2\right), \tag{2.2.39}$$

where $\Pi\left(Q^2\right)$ is the two point function. Two essential instances are

$$B^{M^2}\left(q^2\right)^k = 0, \tag{2.2.40}$$

$$B^{M^2}\left(\frac{1}{\left(m^2-q^2\right)^k}\right) = \frac{1}{(k-1)!}\frac{\exp\left(-\frac{m^2}{M^2}\right)}{M^{2(k-1)}} \text{ where } k > 0. \tag{2.2.41}$$

Borel transformation factorially suppresses the participation of excited resonances and continuum configuration. In case of heavy currents, following form is more convenient to apply rather than applying Borel transformation

$$M_n\left(q_0^2\right) \equiv \frac{1}{n!}\frac{d^n}{dq^{2n}}\Pi\left(q^2\right)|_{q^2=q_0^2} = \frac{f_V^2}{\left(m_V^2-q_0^2\right)^{n+1}} + \int_0^\infty ds \frac{\rho^h(s)}{\left(s-q_0^2\right)^{n+1}}. \tag{2.2.42}$$

2.2.4 Scale invariance applied to OPE

For an ordinary product of two operators $M(x_1)$ and $N(x_2)$, there exists an expansion in which the four-vector x_1 approaches x_2. This expansion has a specific form that can be described mathematically

$$M(x_1)\,N(x_2) = \sum_n Z_n(x_1-x_2)\,O_n(x_1), \tag{2.2.43}$$

$Z_n(x_1)$ involves powers of the (x_1-x_2) rather than being a delta function. Z_n can have singularities of the following type $\left[(x_1-x_2)^2 - \iota\epsilon\,(x_{1,0}-x_{2,0})\right]^{-p}, \forall p\epsilon R$. Above expansion is valid for any two points much close to each other, i.e., on the light cone. Intermediate states have much higher energy, even larger than the energy regime of starting and ending state and to damp this energy, imaginary part is assigned to the temporal part of four vectors. Two local fields, say $\phi(x_1)$ and $\xi(x_2)$, has the following representation when x_1 is near to x_2

$$[\phi(x_1), \xi(x_2)] = \sum_n E_n(x_1 - x_2) O_n(x_1), \tag{2.2.44}$$

where $E_{n=0}(x_1 - x_2)$ has the following form

$$E_0(z) = E\left[\left(-z^2 + \iota\epsilon z_0\right)^{-p} - \left(-z^2 - \iota\epsilon z_0\right)^{-p}\right]. \tag{2.2.45}$$

The nature of singularities in $C_n(z)$ is determined by using different types of theories having different symmetries. If masses are non-zero then free scalar and spinor field theories are non-invariant. Behavior of singular functions are governed by the scale invariance. Performing a scale transformation on Eq. (2.2.43), we will get

$$s^{d_\phi + d_\xi} \phi(sx_1) \xi(sx_2) = \sum_n Z_n(x_1 - x_2) s^{d(n)} O_n(sx_1). \tag{2.2.46}$$

Considering the left hand side and expanding it gives

$$s^{d_\phi + d_\xi} \sum_n Z_n(sx_1 - sx_2) O_n(sx_1) = \sum_n Z_n(x_1 - x_2) s^{d(n)} O_n(sx_1), \tag{2.2.47}$$

where the local fields are linearly independent so

$$Z_n(sx_1 - sx_2) = s^{-d_\phi - d_\xi + d(n)} Z_n(x_1 - x_2). \tag{2.2.48}$$

If $Z_n(x_1 - x_2)$ is a scalar then behavior of $Z_n(x_1 - x_2)$ can be determined by the Lorentz transformation. Dimension $d_\phi + d_\xi - d(n)$ determines the light cone singularity's strength. If $\psi(x_1)$ is a free scalar field then by invoking Wicks theorem $O_n(x_1)$, we can write

$$\psi(x_1), \nabla_\nu \nabla_\mu \nabla_\pi \psi(x_1), : \psi(x_1) :, \nabla_\nu \nabla_\mu \psi(x_1)) \nabla \psi(x_1), \text{...etc.}$$

This ordering is very crucial while arranging fields because with the increase in dimensions, the singularity of the coefficient decreases. $\nabla_\mu \psi(x_1)$ and $: \psi^2(x_1) :$ carries 2 dimensions.

For zero dimension of the free fields, the corresponding coefficient would be least singular. Lets consider a product of two fields $\psi(x_1)$ and $\psi(x_2)$. By using Wicks theorem, we get the following result

$$:\psi^2(x_1)::\psi^2(x_2) = 2[D(x_1-x_2)]^2 I + 4D(x_1-x_2):\psi(x_1)\psi(x_2): + :\psi^2(x_1)\psi^2(x_2):, \tag{2.2.49}$$

where D is denoting free field singular functions. The normal ordered field $:\psi(x_1)\psi(x_2):$ can be expanded using Wick's product expansion as

$$:\psi(x_1)\psi(x_2):=:\psi^2(x_1): + (x_2-x_1)_\mu :\psi(x_1)\nabla_\mu\psi(x_1): + ... \tag{2.2.50}$$

Similarly, the other products can be expanded too. Pure symmetry considerations help to calculate the singularities in $Z_n(x_1-x_2)$. If the fields $\phi(x_1)$ and $\xi(x_2)$ are of high dimensions, their commutator may contain singular functions. These singular functions can arise due to the fact that the fields are defined at the same point in space-time, which leads to a divergence when calculating the commutator. In particular, if the commutator is evaluated at equal times $(x_0 = y_0)$, then the commutator may contain many derivatives of Delta functions and many divergent constants. This is because the equal-time commutator involves the difference of two operators evaluated at the same point in space and time, which can lead to singularities. These singularities can be difficult to handle and require careful regularization and renormalization techniques in order to make sense of the commutator. In particular, one may need to introduce a regulator to remove the singularities and then take the regulator to infinity to obtain a well-defined result. For a generator Q, there would be a field say $\zeta(x)$, which satisfies

$$[\zeta(x), Q] = q\zeta(x), \tag{2.2.51}$$

where q is some scalar. Above equation would be invariant only if

$$U^\dagger(s)\, QU(s) = Q.$$

For the current $J_0(x)$

$$U^{\dagger}(s)\, J_0(x)\, U(s) = s^3 J_0(sx)\,, \tag{2.2.52}$$

and for x near y

$$[\zeta(x)\,, J_0(y)] = \sum_n k_{n\mu}(x < y)\, O_n(x)\,, \tag{2.2.53}$$

at equal times one has

$$[\zeta(x)\,, J_0(y)] = \sum_n k_n \delta^3 (\boldsymbol{x} < \boldsymbol{y})\, O_n(x) + ST,$$

where, ST means Schwinger term that involves derivatives of the Delta functions. So

$$q\zeta(x) = \sum_n k_n O_n(x)\,. \tag{2.2.54}$$

This implies that $O_n(x)$ has the same dimensions as the $\zeta(x)$.

2.3 Effective field theory:

2.3.1 Introduction

Effective theories allow to perform the calculations even when the exact details of the theory is unknown or not fully understood. Effective field theories (EFTs) can be seen as approximations of more fundamental theories, which themselves may also be EFTs. EFTs are useful because they enable calculations of experimentally observable quantities with a limited amount of error. They rely on a small expansion parameter δ, which determines the order of the expansion to a given degree of precision. The precision of the calculation is directly related to the order of the expansion, with the error scaling as δ to the power of $n+1$. A **Power counting formula** is used to determine the order of the expansion for each diagram. EFTs are characterized by a systematic expansion approach, which permits a well-defined methodology to determine higher order corrections in expansion parameter. This means that it is possible to perform calculations to an arbitrary level of precision by choosing the appropriate order of expansion. As a result, the theoretical error can be made arbitrarily small by selecting a sufficiently large value of n in the power counting formula. The EFTs are not limited to perturbative dynamics and can be applied even in cases where the dynamics are not perturbative. One well-known example of this is chiral perturbation theory (χPT) , which uses an expansion in the ratio $p/\Lambda\chi$, where $\Lambda\chi$ is the scale at which symmetry corresponding to chirality breaks and is roughly equal to 1 GeV. Despite its non-perturbative nature, systematic computations using this expansion in powers of $p/\Lambda\chi$ have shown excellent agreement with experimental results.

Locality is a fundamental principle in the formulation of effective field theories (EFTs), which permits the separation of scales into short- and beyond short-distance contributions. This separation leads to a division of the field theory amplitudes into beyond long-distance Lagrangian parameters and long-distance transition probabilities. The short-distance factors are universal, meaning that they do not depend on the specific long-distance interaction strengths being computed. Experimentally measurable quantities, denoted as O_i , are then expressed as the product of these short-distance coefficients, denoted as C, and the long-distance matrix elements.

In many cases, there are multiple Lagrangian coefficients and transition amplitudes involved in the computation of experimentally measurable quantities, resulting in a expression of the form $O_i = \sum_{ij} C_{ij} M_j$. In certain situations, such as in deep-inelastic scattering, the coefficients C and matrix elements M can depend on a parameter y instead of an index j , which leads to the use of convolution instead of a simple sum in the expression

$$O = \int_0^1 \frac{dy}{y} C\left(y\right) M\left(y\right). \tag{2.3.1}$$

In the case of deep-inelastic scattering, the short-distance coefficient $C\left(y\right)$ is referred to as the large momentum transfer scattering cross section and can be calculated using perturbation theory in Quantum Chromodynamics (QCD). The long-distance transition probabilities are known as parton distribution functions, and are found from experimental data. While the large momenta exchange-scattering cross-section is universal, the parton distribution functions are specific to the particular Hadronic target being studied.

2.3.2 Building EFTs: A Step-by-Step Guide

- **Identify the relevant degrees of freedom:**

  To study the low energy properties, we only consider those parameters that are relevant for the problem available right now. This means that we neglect the heavy particles and only focus on the lighter ones. The heavy and light particles have distinct degrees of freedom, but we only keep the relevant degrees of freedom that are necessary for describing the low energy physics of the system. We integrate out the irrelevant degrees of freedom, which allows us to simplify the equations and make the problem more manageable. The goal is to build an effective field theory that captures the essential features of the low energy physics, while neglecting the details that are not important for the problem being studied. In particle physics, the relevant degrees of freedom might include quarks, gluons, and other elementary particles. The goal is to find the degrees of freedom that are important for describing the physics at the energy scale of interest.

- **Imposing Symmetries:**

  Symmetries are an essential aspect of field theory, which can simplify the equations and help to identify the relevant degrees of freedom. In this step, we identify the symmetries of the system being studied, such as translational invariance, rotational invariance, gauge invariance, and so on. These symmetries can then be used to constrain the possible forms of the EFT and can also help to identify the relevant degrees of freedom. Symmetries are a fundamental aspect of constructing an EFT. By imposing symmetries on the Lagrangian of the underlying problem, we can make predictions about the behavior of the system, without having to solve the underlying equations of motion. This is what makes EFT a powerful tool for studying complex physical systems. There are different types of symmetries, such as continuous, discrete, and global symmetries. In some cases, a problem may not have an inherent or apparent symmetry, but by expanding the problem, we may discover an emerging symmetry. This new symmetry can then be used to simplify the problem and make it more tractable. It is worth noting that an EFT often has more symmetry than the original underlying theory. This is because the EFT is designed to describe the low energy behavior of the system, which is typically more symmetric than the high energy behavior. By neglecting the high energy modes and focusing on the low energy modes, we effectively "integrate out" the complexity of the high energy physics, resulting in a simpler, more symmetric description of the system.

- **Perform power counting:**

  Power counting is a systematic way to organize the terms in the EFT, based on their scaling behavior at different energy scales. In this step, we assign a power to each coupling constant in the theory based on its dimension. This allows us to determine which terms are relevant or irrelevant for a particular process, based on their scaling behavior at different energy scales. The goal is to identify the terms that are most important for describing the physics at the energy scale of interest and to neglect the terms that are less important. In EFT, power counting is a helping tool for determining the relative importance of different terms in the effective Lagrangian. This allows us to identify the terms that are most relevant to the low energy regime physics of the system and neglect the terms that have less impact. The power of a term in the effective Lagrangian is determined by the number of fields and derivatives that it contains. Terms with a higher power are considered to be more important, as they are expected to have a greater influence on the behavior of the system. Conversely, terms with a lower power are typically less important, and they can be neglected in calculations. By performing power counting, we can systematically organize the different terms in the effective Lagrangian and determine which terms should be included in our calculations.

These three steps are crucial for developing an effective field theory that accurately describes the system being studied. By identifying the relevant degrees of freedom, imposing symmetries, and performing power counting, we can construct an EFT that captures the essential features of the system at the energy scale of interest [28].

2.3.3 Examples of EFT

Followings are the some examples of Effective field theory

- The theory of Weak Interactions: Insights from Fermi's Model
- Heavy quark effective theory (HQET)
- Chiral perturbation theory
- Soft collinear effective field theory (SCET)

2.3.3.1 The Theory of Weak Interactions: Insights from Fermi's Model

The Fermi theory of weak interactions, as described in [28], is an EFT that pertains to weak interactions taking place at energies lower than the masses of the W and Z particles. It is essentially a low-energy regime EFT that has been constructed based on the Standard Model "SM". The power counting parameter in EFT is $\delta = \frac{p}{M_W}$, where p represents the momenta of the particles involved in weak decay and is typically of the same size as the mass of the muon in μ decay, or of the hadron or quark masses, or of size of the scale of QCD in Hadronic weak decays. Additionally, the theory also has the customary perturbative expansions in $\alpha_s/(4\pi)$ and $\alpha_e/(4\pi)$. Historically, Fermi's theory has been used to calculate weak decay even when the values of masses of weak bosons were not yet known [29].

2.3.3.2 Heavy Quark Effective Theory /Non-relativistic QCD

Heavy quark effective theory (HQET) and non-relativistic QCD (NRQCD) illustrates the behavior of hadrons in low energy regime that contain a heavy quark, such as those with bottom and charm quarks. In HQET, the expansion variables is $\Lambda_{(QCD)}/m_Q$, where $m_Q = m_b$, m_c represents the mass of the massive quark. Additionally, the theory has a power expansion in strong coupling constant. The matching process from QCD to HQET can be carried out in perturbation theory, since range of strong interaction is small, with values such as $\alpha_s\,(m_b) \approx 0.22$ and $\alpha_s\,(m_b) \approx 0.22 \times (4\pi)$. HQET calculations involve non-perturbative corrections, which can be systematically incorporated into an expansion in $\Lambda_{(QCD)}/m_Q$. NRQCD, on the other hand, is same as HQET but is applied to QQ bound states such as the Υ meson. The heavy quarks move at very low speed in this case, and the expansion parameter is the velocity v of the heavy quarks, which is typically of the order of the strong coupling constant.

2.3.3.3 Chiral Lagrangian Approach

Chiral (χ) Lagrangian approach is an EFT used to illustrate the interactions between pions and nucleons at low momentum exchanged regime. It was formed in the 1960s, and the most modern method for calculating it was formed by Weinberg. The theory illustrates dynamics of QCD in low energy domain, where the full theory is well-formed, but the matching onto the EFT cannot be analytically computed due to its non-perturbative nature. However, latest efforts are still in progress to compute the numerical data.

It should be noted that QCD and χ Lagrangian approach are not expressed in terms of the identical fields. The QCD Lagrangian contains quark and gluon fields, while χPT has meson and baryon fields. The factors of the χPT are typically fitted to experimental data.

Interestingly, investigations in χ Lagrangian theory, such as Weinberg's calculation of pion-pion scattering, were carried out using χ symmetry restoration theory before QCD was even given the ground. This demonstrates that it is possible to perform computations in an EFT without knowledge of its UV origin. The expansion parameter of χPT is $p/\Lambda_{(\chi)}$, where $\Lambda_{(\chi)}$ 1GeV is the scale of chiral symmetry violation. Even though masses of baryons are comparable to scale of chirality $\left(\lambda_{(\chi)}\right)$, χPT can still be applied to baryons because baryon number is conserved. This allows baryons to be treated as massive particles, similar to massive quarks in HQET, as long as the exchanged momentum is smaller than $\Lambda_{(\chi)}$. Additionally, there is an intriguing association between the large-N_c expansion of QCD and baryon chiral dynamics [30].

2.3.3.4 Soft Collinear Effective Field Theory

Soft-collinear effective theory is an EFT that describes QCD processes lying in the domain of high energy. In close association with the collision energy regime, the final states have small invariant mass, such as when two highly energetic protons collide, then jets are produced. SCET is built upon the underlying theory of QCD. The expansion parameters related to the scale of SCET are

- $\xi_1 (Q) \times Q = \Lambda_{(QCD)}$,
- $\xi_2 (Q) \times Q = M_J$,

- $\alpha_s(Q) = \xi_3(Q) \times (4\pi)$,

where Q is the center-of-mass energy of the process representing to high momentum exchange, and M_J is the invariant mass of the jet. Fundamentally developed for the decay of B mesons to least massive particles, for instance $B \to Xs\gamma$, SCET provides a powerful tool for understanding and calculating a wide range of high-energy domain's QCD phenomena.(SCET [31, 32, 33, 34])

2.3.3.5 Standard Model Effective Field Theory (SMEFT)

ESM, which stands for effective Standard model, is an EFT that utilizes SM fields to study potential deviations from the SM and explore Beyond the Standard Model (BSM) physics. The higher dimensional operators in SMEFT are produced at an unknown new physics scale, Λ. Despite the lack of knowledge about Λ, systematic computations can still be carried out within SMEFT.

2.3.4 Effective Hamiltonian for heavy meson decays

Three energy scales can be used to characterize weak decays of heavy mesons and that are

- Λ_{QCD}, scale of QCD strong interactions.
- M_W gives the scale of weak interactions.
- Characteristic energy of the process that is most commonly found by quark mass m_Q .

These scales have the following magnitudes: $O(\Lambda_{QCD}) \approx 0.2$GeV, $O(m_Q) \approx 5$GeV, $O(M_W) \approx$90 GeV, whereas $\Lambda_{QCD} << m_Q << M_W$ so in order to study B-meson decay, effective theory is considered rather than full theory that permits to calculate all physical phenomenon.

2.3.4.1 Effective Field Theory Approach to Weak Interactions in the Standard Model

Weak meson decays occur only at short distances of the order of $O\left(\frac{1}{m_W}\right)$ because of the massive weak bosons. For the given decay to define weak effective Hamiltonian, the dynamical degree of freedom that mediate interactions are removed out of the Standard model

Lagrangian. Formally this leads to operator product expansion

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_n \mathcal{C}_n(\mu)\,\mathcal{O}_n, \tag{2.3.2}$$

Where μ denotes renormalization scale and $\mathcal{C}_i$ are the Wilson coefficient functions. Amplitude for any transition $(X_1 \to X_2)$ can be written as

$$\mathcal{A}(X_1 \to X_2) = \langle X_2 |\mathcal{H}_{eff}| X_1\rangle = \frac{G_F}{\sqrt{2}} \sum_n \mathcal{C}_n(\mu) \langle X_2 |O_n(\mu)| X_1\rangle \equiv \frac{G_F}{\sqrt{2}} \sum_n \mathcal{C}_n(\mu) \langle O_n(\mu)\rangle. \tag{2.3.3}$$

The contributions from the physical scales that are higher than the renormalization scale are contained in Wilson coefficients. Fluctuations that are integrated out are found in Wilson coefficient $\mathcal{C}_n$. Any heavy degrees of freedom would be integrated out. These $\mathcal{C}_n$ depends on m_t, m_W, m_H and these dependencies can be calculated by taking into account box and penguin diagrams where heavy mediators are taking part.

Any scale lower than μ will be dealt by the matrix elements $\langle O_n(\mu)\rangle$ since these matrix elements involve long distances so they can not be treated in perturbation theory, therefore we have to consider non-perturbative effects.

Examples of non-perturbative tools are

- Lattice gauge theory
- Light cone sum rules
- QCD sum rules
- Chiral perturbation theory

Construction of effective Hamiltonian involves calculation of Wilson coefficients and local operators. So the advantage of this process is the calculation of two parts separately (1) Short distance part (2) Long distance part.

2.3.4.2 Effective Hamiltonian beyond Particle Physics Framework

In the effective Hamiltonian, the new physics operators are added that causes shifting in the values of the Wilson coefficients from the SM values. It is essential since it is possible to determine numerical values for the coefficients associated with the Wilson operators from the experiments and presence of NP can be tested. Infinite set of operators is present in this effective Hamiltonian $\mathcal{H}_{eff}^{NP}$.

Relevant SM fields are the constructing elements of the operators required for the construction of the Effective Hamiltonian in New Physics. These all operators are invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$. As there are infinite orders so to drop these operators, high dimensional operators can be dropped. To the NP effective Hamiltonian, a suppression factor Λ_{NP}^{D-6} is added so when dimension gets larger, coupling constant becomes smaller $\frac{\mathcal{C}_i^D}{\Lambda_{NP}^{D-6}}$ so this suppression factor provide a constraint. So the most suitable form of the effective Hamiltonian can be casted as

$$\mathcal{H}_{eff}^{NP} = \sum_{D=7}^{\infty} \sum_{i} \frac{\mathcal{C}_i^{[D]}}{\Lambda_{NP}^{D-6}} \mathcal{O}_i^{[D]}. \tag{2.3.4}$$

Still the large number of operators are involved here.

2.4 Light Cone Sum Rules: Introduction

The formalism of light-front sum rules (LCSR) is a powerful tool in QCD that combines the Operator product expansion (OPE) with the theory of hard exclusive processes. The fundamental idea behind LCSR is to calculate the correlation functions of hadrons using a series expansion of currents near the light-cone.

The LCSR method is based on the operator product expansion (OPE), which allows us to write down a series expansion of local operators in terms of their vacuum expectation values and their derivatives. This expansion is valid in the limit of large spacetime separations, where the participation of higher-dimensional operators become increasingly suppressed. However, in the case of three-point sum rules, there are certain irregularities in the truncated OPE, which can lead to uncertainties in the predictions.

To avoid these irregularities, the LCSR method expands the currents near the light-cone, where the participation of higher-dimensional operators are more strongly suppressed. This expansion involves a partial resummation of local operators, which allows us to include higher-order contributions that are not captured by the truncated OPE.

By combining the light-cone expansion with the OPE, the LCSR method provides a systematic way to calculate Hadronic parameters such as decay widths, form factors, and momentum distributions of partons. These calculations are particularly useful in studying the properties of massive mesons and baryons, where the traditional methods of lattice QCD are not yet fully developed [35, 36, 37]. By using these sum rules, one can calculate hard scatterings and soft contributions.

Vacuum to vacuum correlation functions are employed in SVZ sum rules while starting point of light cone sum rules is different. Here, correlation function is defined as a T-product of currents wrapped between vacuum and on-shell state.

Here we take an example of the following interaction to understand the formalism of light cone sum rules to calculate the required observable

$$e^{+}e^{-} \rightarrow \pi^{0}e^{+}e^{-}$$

The polarization operator for this would be written as

$$W_{\mu\nu}\left(p_1, q\right) = \iota \int d^4x \exp\left(-\iota q.x\right) \langle \pi_0\left(p_1\right) \left| T\left\{ j_{\mu}^{em}\left(x\right) j_{0}^{em}\left(x\right)\right\} \right| 0 \rangle, \tag{2.4.1}$$

$$= \epsilon_{\mu\nu\alpha\beta} p_1^{\alpha} q^{\beta} W\left(Q^2, \left(p_1 - q\right)^2\right), \tag{2.4.2}$$

where p represents the momentum of pion and q represents exchanged momenta, $Q^2 = -q^2$. $j_{\mu}^{em}\left(x\right)$ denotes the electromagnetic current and the dynamics of the process is corresponded by $W\left(Q^2, \left(p_1 - q\right)^2\right)$ [38]. Graphically Eq. (2.4.1) can be represented by the following Figure (2.4.1).

By putting the complete set of Hadronic states we get the following expression

$$W_{\mu\nu}\left(p_1, q\right) = 2\frac{\langle \pi_0\left(p_1\right) \left| j_{\mu}^{em} \right| \rho_h^0\left(p_1 - q\right)\rangle \langle \rho^0\left(p_1 - q\right) \left| j_0^{em} \right| 0\rangle}{m_{\rho}^2 - \left(p_1 - q\right)^2} + \frac{1}{\pi}\int_{s_0^h}^{\infty} ds \frac{\mathrm{Im}W\left(Q^2, s\right)}{s - \left(p - q\right)^2}, \tag{2.4.3}$$

whereas

$$\langle \rho^0\left(p_1 - q\right) \left| j_0^{em} \right| 0\rangle = \frac{f_{\rho}}{\sqrt{2}} m_{\rho} \epsilon_{\nu}^{(\rho)^*}.$$

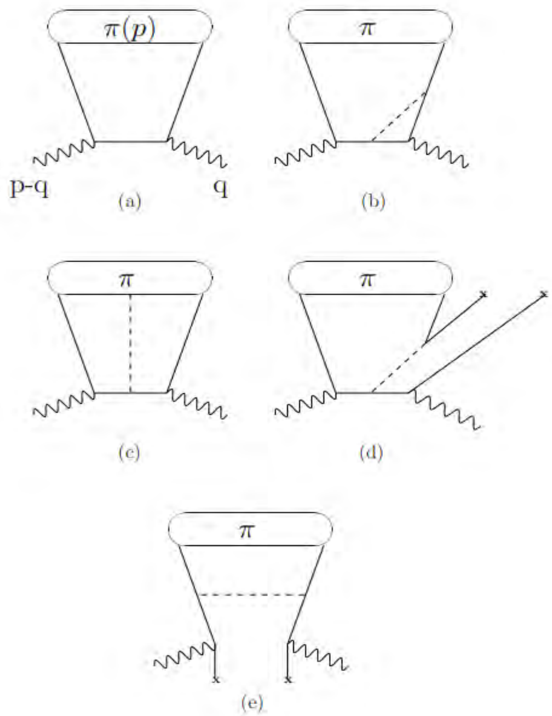

Figure 2.4.1: Expansion of the Correlation function in the light cone limit where $p_1 = p$ is taken here [38].

The contribution of excited and continuum states is included by dispersion integral for the limit $s > s_0^h$. By applying SVZ technique and Borel transformation one can calculate the amplitude in QCD.

2.4.1 Light cone variables

Before going into the detail of light cone expansion, we have to be convenient with the statement that dominant part of the integral in Equation (2.4.3) originates from the light cone expansion near $x^2 = 0$. Therefore, defining a variable $v = |q.p_1| = \left(q^2 - (p_1 - q)^2\right)\frac{1}{2}$, that would be large enough

$$|v| \sim \left|(p-q)^2\right| \sim Q^2 >> \Lambda_{QCD}^2 \tag{2.4.4}$$

defining a ratio

$$\xi = \frac{2v}{Q^2}, \tag{2.4.5}$$

considering a reference frame where momentum of pion is small and finite that is $|\overrightarrow{p}| \sim \mu$, $|p_0| \sim \mu$, $\mu^2 << Q^2, v$. Whereas in this frame, $q_0 \sim \frac{Q^2\xi}{4\mu} + O(\mu)$ and exponential function's argument can be expanded as

$$q.x = q_0 x_0 - q_3 x_3 \simeq \frac{Q^2\xi}{4\mu} x_0 - \left(\sqrt{\frac{Q^4\xi^2}{16\mu^2} + Q^2}\right) x_3 \simeq \frac{Q^2\xi}{4\mu}(x_0 - x_3) - \frac{2\mu x_3}{\xi}, \qquad (2.4.6)$$

moreover, in order to avoid strong oscillations $(x_0 - x_3) \sim \frac{4\mu}{Q^2\xi}$ and at the same time, $x_3 \sim \frac{\xi}{2\mu}$. These two conditions give the following expression

$$x_0^2 \simeq \left(x^3 + \frac{4\mu}{Q^2\xi}\right)^2 \simeq x_3^2 + \frac{4}{Q^2} + O\left(\frac{\mu^2}{Q^4}\right) \qquad (2.4.7)$$

and that is why, $x^2 \sim \frac{1}{Q^2} \longrightarrow 0$ in region Eq. (2.4.4).

2.4.2 Light Cone OPE

Considering only u quark part of the current as in Eq. (2.4.3). Propagator of free massless quark is defined as

$$iS_0(y,0) = \langle 0|T\{u(y)\bar{u}(0)\}|0\rangle = \frac{\iota \not{y}}{2\pi^2 y^4}, \qquad (2.4.8)$$

and after taking the transformation $\gamma_\mu\gamma_\alpha\gamma_\nu \longrightarrow -\iota\epsilon_{\mu\alpha\nu\rho}\gamma^\rho\gamma_5 + ...$, we get the following expression

$$W_{\mu\nu}(p_1,q) = -\iota\epsilon_{\mu\alpha\nu\rho}\int d^4y \frac{y^\alpha \exp(-\iota q.y)}{\pi^2 y^4}\langle\pi_0(p_1)|\bar{u}(y)\gamma^\rho\gamma_5 u(0)|0\rangle. \qquad (2.4.9)$$

To check out the non locality of quark-antiquark operator, expand it around $y = 0$. This gives

$$\bar{u}(y)\gamma^\rho\gamma_5 u(0) = \sum_k \frac{1}{k!}\bar{u}(0)\left(\overleftarrow{D}.y\right)^k \gamma^\rho\gamma_5 d(0), \qquad (2.4.10)$$

and the general splitting of the matrix elements of such operators is

$$\langle\pi_0(p_1)|\bar{u}\overleftarrow{D}_{\alpha_1}\overleftarrow{D}_{\alpha_2}...\overleftarrow{D}_{\alpha_k}\gamma^\rho\gamma_5 u|0\rangle = (-\iota)^k p_{\alpha_1}p_{\alpha_2}...p_{\alpha_k}p_\rho M_k + (-\iota)^k g_{\alpha_1\alpha_2}p_{\alpha_3}...p_{\alpha_k}p_\rho M_k' + ... \qquad (2.4.11)$$

Using Eq. (2.4.5)and Eq. (2.4.11), we get the following expression

$$Q^2 \times W\left(Q^2, (p_1-q)^2\right) = \sum_{k=0}^{\infty} \xi^k M_k + \frac{4}{Q^2} \sum_{k=2}^{\infty} \frac{\xi^{k-2}}{k\,(k-1)} M_k' + ... \tag{2.4.12}$$

Upon closer examination, it becomes apparent that the discrepancy between the regional agents involved in the initial and subsequent phrases of Equation (2.4.12) lies in their level of twist. The concept of "twist" pertains to the contrast between the dimension and spin of a graceless and totally symmetric local operator. In the case of the operators that enter into Eq. (2.4.11), their lowest level of twist is two. This is due to the fact that the operator devoid of derivatives has a dimension of 3 and a Lorentz spin of 1. Additionally, once the matrix elements are considered, the components with a twist of 2 for the operators only play a role in the initial, symmetrical, and traceless term of Eq. (2.4.11), which includes the variable M_r.

In the leading order the matrix element of Eq. (2.4.9) has the following parametric form

$$\langle \pi_0\,(p_1)\,|\bar{u}\,(y)\,\gamma^\rho \gamma_5 u\,(0)\,|0\rangle|_{y^2=0} = -\iota p_\mu \frac{f_\pi}{\sqrt{2}} \int_0^1 du \exp\left(\iota u p.y\right) \varphi_\pi\left(u,\mu\right), \tag{2.4.13}$$

where $\varphi_\pi\left(u,\mu\right)$ represents the distribution amplitude of pion of twist 2 and it is normalized, $\int_0^1 \varphi\left(u,\mu\right) du = 1$. After the expansion of Eq. (2.4.13) and comparing with Eq. (2.4.8) and Eq. (2.4.9), we get relationship of moment of $\varphi_\pi(\mu)$ with the distribution amplitude of twist 2

$$M_k = -\iota \frac{f_\pi}{\sqrt{2}} \int_0^1 du u^k \varphi\left(u,\mu\right). \tag{2.4.14}$$

The behavior of the pion at large distances is encoded in the function $\varphi\left(u,\mu\right)$ multiplied by f_π. Same procedure goes for $d-$quark part. Therefore

$$W^{(tw2)}\left(Q^2, \left((p_1-q)^2\right)\right) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 \frac{du \varphi\left(u,\mu\right)}{\bar{u}^2 Q^2 - u\,(p_1-q)^2}, \tag{2.4.15}$$

where $\bar{u} = 1-u$. In the convolution form

$$W^{(tw2)}\left(Q^2, (p_1-q)^2\right) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 du \varphi\left(u,\mu\right) W\left(Q^2, (p_1-q)^2, \mu, u\right). \tag{2.4.16}$$

whereas at zeroth order in α_s

$$W^{(0)}\left(Q^2,(p_1-q)^2,u\right)=\frac{1}{\bar{u}^2Q^2-u\left(p_1-q\right)^2}, \tag{2.4.17}$$

by utilizing the dispersion relation (2.4.1) and equating it with the outcome of the light-cone expansion, we can derive a summation principle. This involves defining the matrix element

$$\langle\pi_0\left(p_1\right)|j_\mu^{em}|\rho^0\left(p_1-q\right)\rangle=W^{\rho\pi}\left(Q^2\right)m_\rho^{-1}\epsilon_{\alpha\beta\mu\nu}\epsilon^{(\rho)\nu}q^\alpha p_1^\beta, \tag{2.4.18}$$

$$\frac{\sqrt{2}f_\rho W^{\rho\pi}\left(Q^2\right)}{m_\rho^2-\left(p_1-q\right)}+\int_{s_0^h}^{\infty}ds\frac{\frac{1}{\pi}\mathrm{Im}W\left(Q^2,s\right)}{s-\left(p_1-q\right)^2}=\frac{\sqrt{2}f_\pi}{3}\int_0^1\frac{du\varphi_\pi\left(u\right)}{\bar{u}Q^2-u\left(p_1-q\right)^2}. \tag{2.4.19}$$

Eq. (2.4.16) can be represented as

$$W^{(tw2)}\left(Q^2,(p_1-q)^2\right)=\frac{1}{\pi}\int_0^\infty ds\frac{\mathrm{Im}W^{(tw2)}\left(Q^2,s\right)}{s-\left(p_1-q\right)^2}, \tag{2.4.20}$$

where $\mathrm{Im}W^{(tw2)}\left(Q^2,s\right)$ is defined as

$$\mathrm{Im}W^{(tw2)}\left(Q^2,s\right)=\frac{\sqrt{2}f_\pi}{3}\int_0^1 du\varphi_\pi\left(u\right)\delta\left(\bar{u}Q^2-us\right). \tag{2.4.21}$$

By invoking duality approximation

$$\int_{s_0^h}^{\infty}ds\frac{\frac{1}{\pi}\mathrm{Im}W\left(Q^2,s\right)}{s-\left(p_1-q\right)^2}=\int_{s_0^\rho}^{\infty}ds\frac{\mathrm{Im}W\left(Q^2,s\right)}{s-\left(p_1-q\right)^2}=\frac{\sqrt{2}f_\pi}{3}\int_0^{u_0^\rho}\frac{du\varphi_\pi\left(u\right)}{\bar{u}Q^2-u\left(p_1-q\right)^2}, \tag{2.4.22}$$

with $u_0^\rho=\frac{Q^2}{\left(s_0^\rho+Q^2\right)}$. Performing the Borel transformation we get the form factor

$$W^{\rho\pi}\left(Q^2\right)=\frac{f_\pi}{3f_\rho}\int_{u_0^\rho}^1\frac{du}{u}\varphi_\pi\left(u,\mu\right)\exp\left(-\frac{\bar{u}Q^2}{um^2}+\frac{m_\rho^2}{m^2}\right). \tag{2.4.23}$$

2.4.3 Light Front QCD Sum Rules: An Overview

The goal of this section is to determine form factors for the Radiative $B-$meson transitions $B \to V'\gamma$, where V' denotes the vector meson. The correlation function is

$$\iota \int dx \exp\left(\iota q.x\right) \langle V'\left(p,\alpha\right) | T \left\{ \bar{\xi}\left(x\right) \sigma_{\mu\nu} q^{\nu} b\left(x\right) \bar{b}\left(0\right) \iota\gamma_5 \xi\left(0\right) \right\} |vac\rangle = \iota\epsilon_{\mu\nu\rho\sigma}\epsilon^{*(\alpha)\nu} q^{\rho} p^{\sigma} T\left(p+q\right)^2 . \tag{2.4.24}$$

By expressing the relation in $(p+q)^2$, it is possible to isolate the impact of the $B-$meson, which manifests as the singularity contribution within the invariant function $V\left((p+q)^2\right)$, i.e.,

$$V\left((p+q)^2\right) = \frac{f_B m_B^2}{m_b + m_q} \frac{2F_1\left(0\right)}{m_B^2 - \left(p+q\right)^2} + ... \tag{2.4.25}$$

$B-$meson decay constant can be defined as

$$\langle vac|\bar{\xi}\gamma_{\mu}\gamma_5 b|B\left(p\right)\rangle = \iota p_{\mu} f_B, \tag{2.4.26}$$

where m_B, m_b, m_q represents the mass of $B-$meson, light meson and quark mass. After the perturbative expansion of propagator

$$\int dx \exp\left(\iota qx\right) \int \frac{dk}{\left(2\pi\right)^4} \exp\left(-\iota kx\right) \frac{q_{\nu}}{m_b^2 - k^2} \langle V'\left(p,\alpha\right) | T \left\{ \bar{\xi}\left(x\right) \sigma_{\mu\nu} \left(m_b + \not{k}\right) \iota\gamma_5 \xi\left(0\right) \right\} |vac\rangle, \tag{2.4.27}$$

the light cone meson wave function is defined by the above matrix element as

$$\langle vac|\bar{\xi}\left(0\right) \sigma_{\mu\nu} \xi\left(x\right) |V'\left(p,\alpha\right)\rangle = \iota\left(e_{\mu}^{(\alpha)} p_{\nu} - e_{\nu}^{(\alpha)} p_{\mu}\right) f_V^{\perp} \int_0^1 \exp\left(-\iota upx\right) \phi_{\perp}\left(u,\mu^2\right). \tag{2.4.28}$$

Similarly

$$\begin{aligned}\langle 0|\bar{\xi}\left(0\right) \gamma_{\mu} \xi\left(x\right) |V'\left(p,\alpha\right)\rangle &= p_{\mu} \frac{e^{(\alpha)}x}{(px)} f_V m_V \int_0^1 du \exp\left(-\iota upx\right) \phi_{\parallel}\left(u,\mu^2\right) + \\ &\left(e_{\mu}^{(\alpha)} - p_{\mu}\frac{e^{(\alpha)}x}{(px)}\right) f_V m_V \int_0^1 du \exp\left(-\iota upx\right) g_{\nu}^{\perp}\left(u,\mu^2\right),\end{aligned} \tag{2.4.29}$$

$$\langle 0|\bar{\xi}\,(0)\,\gamma_5\xi\,(x)\,|V'\,(p,\alpha)\rangle = \frac{-1}{4}\epsilon_{\mu\nu\rho\sigma}e^{(\alpha)\nu}p^{\rho}x^{\sigma}f_V m_V\int_0^1 du\exp\left(-\iota upx\right)g_{(a)}^{\perp}\left(u,\mu^2\right)/. \tag{2.4.30}$$

The dominant contributions to the fraction of overall momentum carried by quarks in mesons with transverse and longitudinal polarization are described by the leading-twist distributions represented by the functions $\varphi_{\perp}\left(u,\mu^2\right)$ and $\varphi_{\parallel}\left(u,\mu^2\right)$, respectively. By combining Eq. (2.4.26) and Eq. (2.4.29), we get

$$V\left((p+q)^2\right) = \int_0^1 du\frac{1}{m_b^2+\bar{u}um_K^2-u\left(p+q\right)^2}\Big[m_b f_K^{\perp}\phi_{\perp}\left(u\right)+um_K f_K g_{\perp}^{(\nu)}\left(u\right)+ \\ \frac{1}{4}m_K f_K g_{\perp}^{(a)}\Big]+\frac{1}{4}\int_0^1 du\frac{m_b^2-u^2m_K^2}{\left(m_b^2+\bar{u}um_K^2-u\left(p+q\right)^2\right)^2}m_K f_K g_{\perp}^{(a)}\left(u\right). \tag{2.4.31}$$

After applying the Borel transformation, we get

$$\frac{f_B m_B^2}{m_b+m_q}2F_1((0))e^{-\frac{\left(m_B^2-m_b^2\right)}{t}} = \int_0^1 du\frac{1}{u}\exp\left[\frac{\bar{u}}{t}\left(\frac{m_b^2}{u}+m_K^2\right)\right]\theta\left[s_0-\frac{m_b^2}{u}-\bar{u}m_K^2\right]\Big\{m_b \\ f_K^{\perp}\phi_{\perp}\left(u,\mu^2=t\right)+um_K f_K g_{\perp}^{(\nu)}\left(u,\mu^2=t\right)+\frac{m_b^2-u^2m_K^2+ut}{4ut} \\ m_K f_K g_{\perp}^{(a)}\left(u,\mu^2=t\right)\Big\}, \tag{2.4.32}$$

where the vector meson wave functions play a crucial role in this sum rule as they provide important information about the dynamics at large distances. These wave functions contain complex information about how the vector mesons behave over long distances [39].

2.4.4 Difference between QCD sum rules and LC sum rules

QCD sum rules and LC sum rules are two different approaches to analyze the properties of hadrons in QCD, which is known as the "theory of strong interactions". QCD sum rules are a non-perturbative method for studying hadron properties, such as masses and decay constants, based on the Operator product expansion (OPE) and the assumption of quark-hadron duality. QCD sum rules use correlation functions of Hadronic currents, which can be related to perturbative and non-perturbative contributions. The OPE is used to separate perturbative contributions from non-perturbative ones, which are then modeled using Hadronic spectral functions. This allows for the extraction of hadron properties from the spectral functions using sum rules.

LC sum rules, on the other hand, are based on the light-cone quantization of QCD. They use a different framework to study hadron properties, specifically those related to the hadrons light-cone wave function. LC sum rules are based on the Operator product expansion in the light-cone limit, where one of the light-cone coordinates becomes large. This expansion is used to separate the perturbative and non-perturbative contributions to the hadrons light-cone wave function.

In summary, QCD sum rules and LC sum rules are different methods for studying hadron properties in QCD. QCD sum rules are based on the OPE and the assumption of quark-hadron duality, while LC sum rules use light-cone quantization of QCD. Both methods have their advantages and limitations, and can provide complementary insights into the non-perturbative structure of hadrons.

2.5 Form Factors

Matrix elements for the operator $\mathcal{O}_{i\geq 6}^{eff}$ can be described in terms of the form factors in $B \to Sl^+l^-$ interaction whereas these form factors are the function of the squared exchange momentum q^2. Here we discuss them in the context of the scalar meson which is the topic of this dissertation.

2.5.1 Calculation of form factors by using LCSR approach

Form factors are the Hadronic entities that gives us information about the internal structure of partons. Hadronic matrix elements are defined in terms of form factors as

$$\langle S\left(p_1\right)|\bar{q}\gamma_\mu\gamma_5 b|B\left(p_1+q\right)\rangle = -i\left[f_+\left(q^2\right)p_\mu^1 + f_-\left(q\right)^2 q_\mu\right], \tag{2.5.1}$$

$$\langle S\left(p_1\right)|\bar{q}\sigma_{\mu\nu}\gamma_5 q^\nu b|B\left(p_1+q\right)\rangle = -\frac{1}{m_B+m_S}\left[\left(2p_1+q\right)_\mu q^2 - q_\mu\left(m_B^2-m_s^2\right)\right] f_T\left(q^2\right). \tag{2.5.2}$$

Distribution amplitudes that are the crucial for the derivation of form factors for the $B-$meson transition are:

$$\langle S\left(p_1\right)|\bar{q}_{2\beta}\left(z_2\right)q_{1\alpha}|0\rangle = \frac{1}{4}\int_0^1 \exp\left[i\left(yp_1.z_2+\bar{y}p_1.z_1\right)\right] \times\left[\not{p}_1\phi_s\left(y\right)+m_s\left(\phi_S^s\left(y\right)-\sigma_{\mu\nu}p_1^\mu z^\nu\frac{\phi_s^\sigma\left(y\right)}{6}\right)\right], \tag{2.5.3}$$

$$\phi_s\left(y,\mu\right) = f_s\left(\mu\right)6y\left(1-y\right)\left[g_0\left(\mu\right)+\sum_{Gm=1}g_1\left(\mu\right)G_{GP}^{3/2}\left(2y-1\right)\right], \tag{2.5.4}$$

$$\phi_s^s\left(y,\mu\right) = f_s\left(\mu\right)\left[1+\sum_{Gm=1}a_{Gm}\left(\mu\right)G_{GP}^{\frac{1}{2}}\left(2y-1\right)\right], \tag{2.5.5}$$

$$\phi_s^\sigma\left(y,\mu\right) = f_s\left(\mu\right)6y\left(1-y\right)\left[1+\sum_{Gm=1}b_{Gm}\left(\mu\right)G_{GP}^{3/2}\left(2y-1\right)\right], \tag{2.5.6}$$

where $\phi_s\left(y,\mu\right)$ is a function of twist 2, $\phi_s^s\left(y,\mu\right)$ and $\phi_s^\sigma\left(y,\mu\right)$ are functions of twist 3. Due to conservation of G-parity in $SU(3)$ limit $\phi_s\left(y,\mu\right)$ and $\phi_s^s\left(y,\mu\right),\phi_s^\sigma\left(y,\mu\right)$ are symmetric and anti-symmetric that can be tested by replacing $u\longrightarrow 1-u$. Normalization of these distribution amplitudes can be written as

$$\int_0^1 dy\phi\left(y,\mu\right) = f_S, \tag{2.5.7}$$

$$\int_0^1 dy\phi_S^s\left(y,\mu\right) = \int_0^1 dy\phi_S^\sigma\left(y,\mu\right) = \bar{f}_S, \tag{2.5.8}$$

where f_S denotes vector current decay constant and $\bar{f}_S$ denotes scalar density decay constant. In terms of Hadronic matrix elements they can be defined as

$$\langle S\left(p_1\right)|\bar{q}_2\gamma_\mu q_1|0\rangle = p_\mu^1 f_S, \tag{2.5.9}$$

$$\langle S\left(p_1\right)|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S, \tag{2.5.10}$$

both densities can be related to each other through equation of motion

$$\bar{f}_S = \mu_S f_S, \tag{2.5.11}$$

$$B^0(\mu) = \mu_s^{-1} = \frac{m_2(\mu) - m_1(\mu)}{m_s}, \tag{2.5.12}$$

where Eq. (2.5.12) has been solved by using normalization conditions given in Eq. (2.5.7) and 2.5.8. In Eq. (2.5.3, 2.5.4, 2.5.5, 2.5.6) where g_{Gm} denotes Gegenbauer moments and $G_{GP}^{3/2}$ denotes the Gegenbauer polynomials. As g_0 term is either zero or small of order of difference of down and up quark masses or difference of strange and down quark masses or mass of up quark, so are other even Gegenbaur moments. As we know that $\mu_S >> 1$, and Gegenbauer coefficients are suppressed, so that odd Gegenabuer moments results in the domination of LCDA of scalar mesons. Whereas, for the π and ρ mesons the odd Gegenbauer moments disappear [40, 41].

When the three-particle participation are avoided, with the help of equation of motion twist 3 two particle distribution amplitudes can be determined, that leads to

$$(1-2y)\,\phi_S^s(y) = \frac{\left(\phi_S^\sigma(y)'\right)}{6}. \tag{2.5.13}$$

Asymptotic form of distribution amplitudes is

$$\phi_S^s(y) = \bar{f}_s, \tag{2.5.14}$$

$$\phi_S^\sigma(y) = \bar{f}_s 6y(1-y). \tag{2.5.15}$$

The pseudo-scalar meson's distribution amplitudes also have the same asymptotic form. Therefore, to derive form factors, we first of all define a correlation function as the currents sandwiched between a vacuum and final scalar meson state. This can be expressed mathematically as

Table 2.5.1: At the scale of $\mu = 1$ GeV,twist-2 distribution amplitude's decay constant and Gegenbauer moments.

State	$\bar{f}$(MeV)	g_1	g_3
$a_0[1450]$	460±50	-0.58 ± 0.12	-0.49 ± 0.15
$K_0^*[1430]$	445 ± 50	-0.57 ± 0.13	-0.42 ± 0.22
$f_0[1500]$	490 ± 50	-0.48 ± 0.11	-0.37 ± 0.20

Table 2.5.2: At the scale of $\mu = 1$ GeV twist-3 distribution amplitude's decay constant and Gegenbauer moments.

State	$a_1(\times 10^{-2})$	a_2	a_4	$b_1(\times 10^{-2})$	b_2	b_4
a_0	0	$-0.33 \sim -0.18$	$-0.11 \sim 0.39$	0	$0 \sim 0.058$	$0.070 \sim 0.20$
K_0^*	$1.8 \sim 4.2$	$-0.33 \sim -0.025$	$-$	$3.7 \sim 5.5$	$0 \sim 0.15$	$-$
f_0	0	$-0.33 \sim -0.18$	$0.28 \sim 0.79$	0	$-0.15 \sim -0.088$	$0.044 \sim 0.16$

$$\Pi_\mu(p_1, q) = -\int d^4y \exp(\iota q y) \langle S(p_1)|T\{j_{2\mu}(y), j_1(0)\}|0\rangle, \tag{2.5.16}$$

$$\Pi_\mu(p_1, q) = -\int d^4y \exp(iq.y) \langle S(p_1)|T\{\bar{q}_2(y)\gamma_\mu\gamma_5 b(y), b(0) i\gamma_5 q_1(0)\}|0\rangle, \tag{2.5.17}$$

where the currents $j_{2\mu}(y)$ and $j_1(0)$ are defined as $\bar{q}_2(y)\gamma_\mu\gamma_5 b(y)$,$b_1(0)\iota\gamma_5 q_1(0)$. $j_{2\mu}(y)$ represents weak transition of b to q_2 and $j_1(0)$ represents B_{q_1} channel. For interpolating current, vacuum to meson matrix element has the following form

$$\langle B(p_1+q)|\bar{b}i\gamma_5 q|0\rangle = m_B^2 \frac{f_B}{m_b + m_q}. \tag{2.5.18}$$

By using Unitarity relation or in easy words, inserting all intermediate and continuum states we get

$$\Pi_\mu(p_1, q) = -\int d^4y \exp(iq.y) \sum_\lambda \int \frac{d^4k}{(2\pi)^4} \frac{i\exp[-i(k-p).y]}{k^2 - m_\lambda^2 + i\epsilon} \langle S(p)\bar{q}_2(0)\gamma_\mu\gamma_5 b(0)|\lambda\rangle \langle\lambda|\bar{b}(0) i\gamma_5 q_1(0)|0\rangle, \tag{2.5.19}$$

$$\begin{aligned} &= -\int d^4y \sum_\lambda \int \frac{d^4k}{(2\pi)^4} \frac{\exp[-i(k-p-q).y]}{k^2 - m_\lambda^2 + i\epsilon} \langle S(p_1)|\bar{q}_2(0)\gamma_\mu\gamma_5 b(0)|\lambda\rangle\langle\lambda|\bar{b}(0) i\gamma_5 q_1(0)|0\rangle, \\ &= -\int d^4k \frac{\delta^4(k-p_1-q)}{k^2 - m_\lambda^2} \langle S(p_1)|\bar{q}_2(0)\gamma_\mu\gamma_5 b(0)|\lambda\rangle\langle\lambda|\bar{b}(0) i\gamma_5 q_1(0)|0\rangle, \\ &= \frac{-i}{(p_1+q)^2 - m_B^2} \langle S(p_1)|\bar{q}_2(0)\gamma_\mu\gamma_5 b(0)|B(p_1+q)\rangle\langle B(p_1+q)|\bar{b}(0) i\gamma_5 q_1(0)|0\rangle \\ &+ \sum_h \frac{\langle S(p_1)|\bar{q}_2(0)\gamma_\mu\gamma_5 b(0)|h(p_1+q)\rangle\langle h(p_1+q)|\bar{b}(0) i\gamma_5 q_1(0)|0\rangle}{m_h^2 - (p_1+q)^2}. \end{aligned} \tag{2.5.20}$$

Now, using definitions of the matrix elements given in Eq. (2.5.1) and Eq. (2.5.2), we have

$$\Pi_\mu\left(p_1,q\right)=\frac{i}{m_B^2-\left(p_1+q\right)^2}\frac{m_B^2 f_B}{m_b+m_q}\left(-i\right)\left[f_+\left(q^2\right)p_\mu^1+f_-\left(q^2\right)q_\mu\right]+...,\tag{2.5.21}$$

$$\Pi_\mu\left(p_1,q\right)=\frac{m_B^2 f_B}{\left(m_b+m_q\right)\left(m_B^2-\left(p+q\right)^2\right)}\left[f_+\left(q^2\right)p_\mu^1+f_-\left(q^2\right)q_\mu\right]+\int_{s_o^h}^{\infty}\frac{p_+\left(q^2\right)p_\mu^1+p_-\left(q^2\right)q_\mu}{s-\left(p_1+q\right)^2}ds.\tag{2.5.22}$$

The Correlation function can also be calculated if we insert a free quark propagator. Now calculating to the primary order of strong coupling constant α_s, the correlation function by inserting free quark propagator and contracting b field

$$\Pi_\mu\left(p_1,q\right)=-\int d^4y\exp\left(iq.y\right)\langle S\left(p_1\right)\Big|T\left\{\bar{q}_2\left(y\right)\gamma_\mu\gamma_5 iS_F\left(y\right)i\gamma_5 q_1\left(0\right)\right\}\Big|0\rangle.\tag{2.5.23}$$

By transforming the above expression into $k-$space and inserting the value of free quark propagator, we get the following expression

$$\Pi_\mu\left(p_1,q\right)=-\int d^4y\int\frac{d^4k}{\left(2\pi\right)^4}\exp\left(iq.y\right)\exp\left(-ik.y\right)\frac{-i}{m_b^2-k^2}\Bigg[m_b\langle S\left(p_1\right)\Big|\bar{q}_2\left(y\right)\gamma_\mu\gamma_5\left(\not{k}+m_b\right)i\gamma_5\tag{2.5.24}$$

$$q_1\left(0\right)\Big|0\rangle\Bigg],$$

$$=-\int d^4y\int\frac{d^4k}{\left(2\pi\right)^4}\exp i\left\{\left(q-k\right).y\right\}\frac{-i}{m_b^2-k^2}\Bigg[m_b\langle S(p_1)\Big|\bar{q}_2(y)\gamma_\mu q_1(0)|0\rangle-k^\nu\langle S(p_1)\Big|\bar{q}_2(y)$$

$$\gamma_\mu\gamma_\nu q_1\left(0\right)\Big|0\rangle\Bigg],$$

$$=\int d^4y\int\frac{d^4k}{\left(2\pi\right)^4}\exp\left\{\left(i\left(q-k\right).y\right)\right\}\frac{1}{m_b^2-k^2}\Bigg[m_b\langle S\left(p_1\right)\Big|\bar{q}_2\left(y\right)\gamma_\mu q_1\left(0\right)\Big|0\rangle$$

$$-k^\nu g_{\mu\nu}\langle S\left(p_1\right)\Big|\bar{q}_2\left(y\right)q_1\left(0\right)|0\rangle+ik^\nu m_b\langle S\left(p_1\right)\Big|\bar{q}_2\left(y\right)\sigma_{\mu\nu}q_1\left(0\right)\Big|0\rangle\Bigg],$$

$$=-\int d^4y\int\frac{d^4k}{\left(2\pi\right)^4}\exp\left\{i\left(q-k\right).y\right\}\frac{1}{m_b^2-k^2}\Bigg\{m_b p_\mu^1\int_0^1 du\exp i\left(up_1.y\right)\phi_s\left(u\right)-k_\mu m_s$$

$$\int_0^1 du\exp\left(iup_1.y\right)\phi_s^s\left(u\right)-ik^\nu m_s\left(p_\mu^1 X_\nu-p_\nu X_\mu\right)\int_0^1 du\exp\left(iup_1.y\right)\phi_s^\sigma\left(u\right)\frac{1}{6}\Bigg\},$$

$$=-m_b p_\mu^1\int_0^1 du\frac{1}{m_b^2-\left(q+up_1\right)^2}\phi_s\left(u\right)+m_s\int_0^1 du\left(q+up_1\right)_\mu\frac{1}{m_b^2-\left(q+up_1\right)^2}\phi_s^s\left(u\right)$$

$$+ m_s\left(p^1_\mu\frac{\partial}{\partial q^\nu} - p^1_\nu\frac{\partial}{\partial q^\mu}\right)\int_0^1 du\frac{(q+up_1)^\nu}{m_b^2-(q+up_1)^2}\frac{\phi_s^\sigma(u)}{6}. \tag{2.5.25}$$

Solving the last term of the above expression we get

$$\begin{aligned}
m_s\left(p^1_\mu\frac{\partial}{\partial q^\nu} - p^1_\nu\frac{\partial}{\partial q^\mu}\right)\int_0^1 du\frac{(q+up_1)^\nu}{m_b^2-(q+up_1)^2}\frac{\phi_s^\sigma(u)}{6} &= m_s p^1_\mu\int_0^1 du\frac{\phi_s^\sigma(u)}{6}\Bigg[\frac{4}{m_b^2-(q+up_1)^2}+ \\
&\frac{2(q+up_1)^2}{\left(m_b^2-(p_1+q)^2\right)}\Bigg] - m_s p^1_\nu\int_0^1 du\frac{\phi_s^\sigma(u)}{6} \\
&\left(\frac{g^\nu_\mu}{m_b^2-(q+up_1)^2}+\frac{2(q+up_1)_\mu(q+up_1)^\nu}{\left[m_b^2-(q+up_1)^2\right]^2}\right), \\
&= m_s p^1_\mu\int du\phi_s^\sigma(u)\frac{1}{m_b^2-(q+up_1)^2}\Bigg[2+ \\
&\frac{m_b^2-u^2p_1^2+q^2}{m_b^2-(q+up_1)^2}\Bigg] + m_s q_\mu\int_0^1 du\frac{\phi_s^\sigma(u)}{6} \\
&\frac{1}{m_b^2-(q+up_1)^2}\left[1-\frac{m_b^2-u^2p_1^2+q^2}{m_b^2-(q+up_1)^2}\right].
\end{aligned} \tag{2.5.26}$$

So by putting Eq. (2.5.26) in Eq. (2.5.25), we get

$$\begin{aligned}
\Pi_\mu(p_1,q) &= p^1_\mu\int_0^1 du\frac{1}{m_b^2-(q+up_1)^2}\left\{-m_b\phi_s(u)+um_s\phi_s^s(u)+\frac{1}{6}m_s\phi_s^\sigma(u)\left[2+\frac{m_b^2-u^2p_1^2+q^2}{m_b^2-(q+up_1)^2}\right]\right\} \\
&+ q_\mu\int_0^1 du\frac{1}{m_b^2-(q+up_1)^2}\left\{m_s\phi_s^s(u)+\frac{m_s\phi_s^\sigma(u)}{6u}\left[1-\frac{m_b^2-u^2p_1^2+q^2}{m_b^2-(q+up_1)^2}\right]\right\},
\end{aligned} \tag{2.5.27}$$

the imaginary part of the correlation function has the following form

$$\int_0^1 duP(u)\frac{1}{m_b^2-(q+up_1)^2} = \int_0^1 duP(u)\int\frac{ds}{s-(p_1+q)^2}\delta\left(m_b^2-(q+up_1)^2\right)|_{(p_1+q)^2}, \tag{2.5.28}$$

$$= \int_0^1 duP(u)\int\frac{ds}{s-(p_1+q)^2}\delta\left(m_b^2-us+u\bar{u}p_1^2-\bar{u}q^2\right),$$

$$= \int_{u_0}^{1} duP(u) \frac{1}{s-(p_1+q)^2},$$

$$\int_0^1 duP(u) \frac{1}{m_b^2-(q+up_1)^2} = \int_{u_0}^{1} duP(u) \frac{1}{s-(p_1+q)^2}. \tag{2.5.29}$$

Where the positive solution of the equation

$$m_b^2 - us_0 + u(1-u)p^2 - \bar{u}q^2 = 0, \tag{2.5.30}$$

$$m_b^2 - u\left(s_0 - p^2 - q^2\right) - u^2p^2 - q^2 = 0,$$

$$-\left(s_0 - p^2 - q^2\right) + \sqrt{\left(s_0 - p_1^2 - q^2\right) + 4p^2\left(m_b^2 - q^2\right)}\frac{1}{2p_1^2} = u_0.$$

$$\frac{m_b^2 + u\bar{u}p_1^2 - \bar{u}q^2}{u} = s. \tag{2.5.31}$$

Now solving another term

$$\int_0^1 duP(u) \frac{1}{\left[m_b^2-(q+up_1)^2\right]^2} = \int_0^1 duP(u) \frac{1}{\pi} \int \frac{ds}{s-(p_1+q)^2} \mathrm{Im} \frac{1}{\left(m_b^2 - us + u\bar{u}p_1^2 - \bar{u}q^2\right)^2}, \tag{2.5.32}$$

$$= \int_0^1 duP(u) \frac{1}{\pi} \int \frac{ds}{s-(p_1+q)^2} \frac{1}{u}\frac{d}{ds} \mathrm{Im} \frac{1}{\left(m_b^2 - us + u\bar{u}p_1^2 - \bar{u}q^2\right)},$$

$$= \int_0^1 duP(u) \frac{1}{u} \int \frac{ds}{s-(p_1+q)^2} \frac{d}{ds} \delta\left(m_b^2 - us + u\bar{u}p_1^2 - \bar{u}q^2\right),$$

$$= \int_0^1 duP(u) \frac{1}{u} \frac{1}{s-(p_1+q)^2} \delta\left(m_b^2 - us + u\bar{u}p_1^2 - \bar{u}q^2\right)|^{s=s_\circ} +$$

$$\int \frac{ds}{\left[s-(p_1+q)^2\right]^2} \delta\left(m_b^2 - us + u\bar{u}p_1^2 - \bar{u}q^2\right),$$

$$= P(u_0) \frac{1}{u_0} \frac{1}{s_0-(p_1+q)^2} \frac{1}{s_0-(1-2u)p_1^2-q^2} + \int_{u_0}^{1} du \frac{1}{u^2} \frac{P(u)}{\left[s-((p_1+q)^2\right]^2},$$

$$= P(u_0) \frac{1}{u_0} \frac{1}{s_0-(p_1+q)^2} \frac{u_\circ}{m_b^2+u_0^2p_1^2-q^2} + \int_{u_0}^{1} du \frac{1}{u^2} \frac{P(u)}{\left[s-(p_1+q)^2\right]^2},$$

$$= P(u_0) \frac{1}{s_0-(p_1+q)^2} \frac{1}{m_b^2+u_0^2p_1^2-q^2} + \int_{u_0}^{1} du \frac{1}{u^2} \frac{P(u)}{\left[s-(p_1+q)^2\right]^2}. \tag{2.5.33}$$

Finally, we get

$$\frac{m_B^2 f_B f_+ (q)^2}{(m_b + m_q)\left[m_B^2 - (p_1+q)^2\right]} = \int_{u_0}^{1} du \frac{1}{u}\frac{1}{s-(p_1+q)^2}\left[m_b\phi_s(u) + m_s\left\{u\phi_s^s(u) + \frac{1}{3}\phi_s^\sigma(u)\right\}\right]$$
$$+ \int_{u_0}^{1} du \frac{1}{u^2}\frac{1}{\left[s-(p_1+q)^2\right]^2}\frac{1}{6}m_s\phi_s^\sigma(u)\left(m_b^2 - u^2p_1^2 + q^2\right)$$
$$+ \frac{m_s}{6}\frac{\phi_s^\sigma(u_0)}{s_0-(p_1+q)^2}\frac{m_b^2 - u_0^2p_1^2+q^2}{m_b^2+u_0^2p_1^2-q^2}. \tag{2.5.34}$$

$$\frac{m_B^2 f_B f_- (q)^2}{(m_b + m_q)\left[\left[m_B^2 - ((p_1+q)^2)\right]\right]} = \int_{u_0}^{1} du \frac{1}{u}\frac{1}{s-(p_1+q)^2} m_s\left[\phi_s^s(u) + \frac{\phi_s^\sigma(u)}{6u}\right] - \frac{m_s}{6u_0}$$
$$\frac{\phi_s^\sigma(u_0)}{s_0-(p_1+q)^2} - \int_{u_0}^{1} du \frac{1}{u^2}\frac{1}{\left[s-(p_1+q)^2\right]^2}\frac{1}{6}m_s\phi_s^\sigma(u)$$
$$\left(m_b^2 - u^2p_1^2 + q^2\right) \tag{2.5.35}$$

Now applying the Borel transformation

$$f_i\left(q^2\right) = (m_b + m_{q_1})\frac{1}{\pi f_{B_{q_1}} m_{B_{q_1}}^2}\int_{(m_b+m_{B_q})^2}^{s_0} \mathrm{Im}\Pi_i^{QCD}\left(s,q^2\right)\exp\left(\frac{m_{B_{q_1}}^2 - s}{M^2}\right) ds, \tag{2.5.36}$$

and putting $i = +,\ f_{B_{q_1}} = f_B,\ f_i(q^2) = f_+\left(q^2\right)$ and $\exp\left(\frac{m_B^2}{M^2}\right)$ is taken on other side of the equation, we get

$$\frac{\pi f_B m_B^2 f_+\left(q^2\right)}{(m_b+m_q)}\exp\left(\frac{-m_B^2}{M^2}\right) = \int_{u_0}^{1} du \frac{1}{u}\exp\left(\frac{-m_b^2 + u\bar{u}p_1^2 - \bar{u}q^2}{uM^2}\right)\left\{-m_b\phi_s(u) + m_s\left(u\phi_s^s(u)\right)\right.$$
$$\left. + \frac{1}{3}\phi_s^\sigma(u) + \frac{1}{uM^2}\frac{m_s}{6}\phi_s^\sigma(u)\left(m_b^2 - u^2p_1^2 + q^2\right)\right\} + \frac{m_s}{6}\phi_s^\sigma(u_0)$$
$$\exp\left(\frac{-s_0}{M^2}\right)\frac{m_b^2 - u_0^2p_1^2 + q^2}{m_b^2 + u_0^2p_1^2 - q^2}. \tag{2.5.37}$$

Here $\frac{1}{s-(p_1+q)^2} = \exp\left\{-(s-(p_1+q)^2\right\}$. Similarly for other factor (f_-), we can check

$$\frac{\pi f_B m_B^2 f_-\left(q^2\right)}{(m_b+m_q)}\exp\left(\frac{-m_B^2}{M^2}\right) = \int_{u_\circ}^{1} du \frac{1}{u}\exp\left(\frac{-m_b^2 + u\bar{u}p_1^2 - \bar{u}q^2}{uM^2}\right)\left\{m_s\phi_s^s(u) + \frac{\phi_s^\sigma(u)}{6u}\right.$$
$$\left. - \frac{1}{u^2M^2}\frac{m_s}{6}\phi_s^\sigma(u)\left(m_b^2 + u^2p_1^2 - q^2\right)\right\} - \frac{m_s}{6}\phi_s^\sigma(u)\exp\left(\frac{-s_\circ}{M^2}\right). \tag{2.5.38}$$

Same method will be used to derive $f_T(q^2)$

$$\Pi_\mu(p_1,q) = -i\int d^4y\langle S(p_1)|T\left\{\bar{q}_2(y)\sigma_{\mu\nu}\gamma_5 q^\nu b(y), \bar{b}(0)\,i\gamma_5 q_1(0)\right\}|0\rangle. \tag{2.5.39}$$

Again by using Unitarity relation in above expression, we get

$$\begin{aligned}\Pi_\mu(p_1,q) &= -i\int d^4y\exp(iq.y)\sum_\lambda\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_\lambda^2+i\epsilon}\exp\{-i(k-p).y\}\langle S(p_1)|\bar{q}_2(0)\sigma_{\mu\nu}\\ &\quad\gamma_5 q^\nu b(0)|\lambda\rangle\langle\lambda|\bar{b}(0)\,i\gamma_5 q_1(0)|0\rangle,\\ &= \frac{1}{(p_1+q)^2-m_B^2}\langle S(p_1)|\bar{q}_2(0)\sigma_{\mu\nu}\gamma_5 q^\nu b(0)|B(p_1+q)\rangle\langle B(p_1+q)|\bar{b}(0)\,i\gamma_5 q_1(0)|0\rangle + ...\end{aligned} \tag{2.5.40}$$

Just using the definitions of Hadronic matrix elements written in Eq. (2.5.1) and Eq. (2.5.2), implies

$$\Pi_\mu(p_1,q) = \frac{1}{m_B^2-(p_1+q)^2}\frac{m_B^2}{m_b+m_q}f_B\frac{1}{m_B+m_s}\left[(2p_1+q)_\mu q^2 - q_\mu\left(m_B^2-m_s^2\right)\right]f_T((q^2)+..., \tag{2.5.41}$$

$$= \frac{m_B^2 f_B}{m_b+m_q}\frac{1}{m_B+m_s}\left[(2p_1+q)_\mu q^2 - q_\mu\left(m_B^2-m_s^2\right)\right]f_T\left(q^2\right) + ... \tag{2.5.42}$$

Correlation function can also be derived by inserting free quark propagator in the Eq. (2.5.34)

$$\begin{aligned}\Pi_\mu(p_1,q) &= -i\int d^4y\exp(iq.y)\int\frac{d^4k}{(2\pi)^4}\exp(-ik.y)\frac{-i}{m_b^2-k^2}\langle S(p_1)|\bar{q}_2(y)\sigma_{\mu\nu}\gamma_5 q^\nu(\not{k}+m_b)\\ &\quad i\gamma_5 q_1(0)|0\rangle, \qquad (2.5.43)\\ &= -i\int d^4y\exp\{i(q-k).y\}\frac{1}{m_b^2-k^2}\Big[m_b\langle S(p_1)|\bar{q}_2(y)\sigma_{\mu\nu}q^\nu q_1(0)|0\rangle\\ &\quad - k^\rho\langle S(p_1)|\bar{q}_2(y)\sigma_{\mu\nu}q^\nu\gamma_\rho q_1(0)|0\rangle\Big]\\ &= -i\int d^4y\exp\{i(q-k).y\}\frac{1}{m_b^2-k^2}\Big[q^\nu m_b\langle S(p_1)|\bar{q}_2(y)\sigma_{\mu\nu}q_1(0)|0\rangle - k^\rho q^\nu i g_{\nu\rho}\langle S(p_1)|\\ &\quad \bar{q}_2(y)\gamma_\mu q_1(0)|0\rangle + k^\rho q^\nu i g_{\mu\rho}\langle S(p_1)|\bar{q}_2(y)\gamma_\nu q_1(0)|0\rangle - k^\rho q^\nu\epsilon_{\mu\nu\rho\tau}\langle S(p_1)|\bar{q}_2(y)\gamma_\tau q_1(0)|0\rangle\Big].\end{aligned}$$

In deriving Eq. (2.5.35) following relation is used

$$\sigma_{\mu\nu}\gamma_{\rho} = i\left(g_{\nu\rho}\gamma_{\mu} - g_{\mu\rho}\gamma_{nu}\right) + \epsilon_{\mu\nu\rho\tau}\gamma^{\tau}\gamma_{5}, \tag{2.5.44}$$

with $\epsilon_{0123} = 1$. Now

$$\begin{aligned}
\Pi_{\mu}\left(p_1, q\right) = -i\int d^4y\int \frac{d^4k}{\left(2\pi\right)^4}\exp\left\{i\left(q-k\right).y\right\}\frac{1}{m_b^2-k^2}\Big[q^{\nu}m_b\left(-m_s\right)\left(p_{\mu}^1\chi_{\nu}-p_{\nu}^1\chi_{\mu}\right)\int_0^1 du \\
\left(\exp\left(iup.y\right)\right)\frac{\phi_s^{\sigma}\left(y\right)}{6} - i\left(k.q\right)p_{\mu}^1\int_0^1 du\exp\left(iup_1.y\right)\phi_s\left(u\right) + ik_{\mu}q^{\nu}p_{\nu}^1\int_0^1 du \\
\exp\left(iup_1.y\right)\phi_s\left(u\right), \qquad (2.5.45) \\
= \int_0^1 du\frac{1}{m_b^2-\left(q+up_1\right)^2}\phi_s\left(u\right)\left[\left(p_1.q\right)\left(up_1+q\right)_{\mu} - \left(up_1+q\right)qp_{\mu}^1\right] + \\
\underbrace{m_b m_s q^{\nu}\left(p_{\mu}^1\frac{\partial}{\partial q^{\nu}} - p_{\nu}^1\frac{\partial}{\partial q^{\mu}}\right)\int_0^1 du\frac{1}{m_b^2-\left(q+up_1\right)^2}\frac{\phi_s^{\sigma}\left(y\right)}{6}}_{G},
\end{aligned}$$

$$\begin{aligned}
G = m_b m_s q^{\nu}\left(p_{\mu}^1\frac{\partial}{\partial q^{\nu}} - p_{\nu}^1\frac{\partial}{\partial q^{\mu}}\right)\int_0^1 du\frac{1}{m_b^2-\left(q+up_1\right)^2}\frac{\phi_s^{\sigma}\left(y\right)}{6}, \qquad (2.5.46) \\
= m_b m_s q^{\nu}p_{\mu}^1\int_0^1 du\frac{\phi_s^{\sigma}\left(y\right)}{6}\frac{2\left(q+up_1\right)_{\nu}}{\left[m_b^2-\left(q+up_1\right)^2\right]^2} - m_b m_s\left(q.p_1\right)\int_0^1 du\frac{\phi_s^{\sigma}\left(y\right)}{6} \\
\frac{2\left(q+up_1\right)_{\mu}}{\left[m_b^2-\left(\left(q+up_1\right)\right]^2}, \\
= m_b m_s\int_0^1 du\frac{\phi_s^{\sigma}\left(y\right)}{6}\frac{2q\left(q+up_1\right)p_{\mu}^1 - 2\left(p_1.q\right)\left(q+up_1\right)_{\mu}}{\left[m_b^2-\left(q+up_1\right)^2\right]^2}, \\
G = m_b m_s\int_0^1 du\frac{\phi_s^{\sigma}\left(y\right)}{6}\frac{1}{\left[m_b^2-\left(q+up_1\right)^2\right]^2}\left[2q^2p_{\mu}^1 - 2p_1.qq_{\mu}\right]. \qquad (2.5.47)
\end{aligned}$$

Finally combining all terms, we get

$$\begin{aligned}
f_T\left(q^2\right) = p_{\mu}^1\int_0^1 du\frac{1}{\left[m_b^2-\left(q+up_1\right)^2\right]}\Bigg\{-q^2\phi_s\left(u\right) + \frac{m_b m_s}{3}\phi_s^{\sigma}\left(u\right) \\
\frac{q^2}{\left[m_b^2-\left(q+up_1\right)^2\right]}\Bigg\} + q_{\mu}\int_0^1 du\frac{1}{\left[m_b^2-\left(q+up_1\right)^2\right]}\left(p_1.q\right)\phi_s\left(u\right) - \\
\frac{m_b m_s}{3}\phi_s^{\sigma}\left(u\right)\frac{p_1.q}{\left[m_b^2-\left(q+up_1\right)^2\right]},
\end{aligned}$$

$$= \left(-p^1_\mu . q^2 + q_\mu p_1 . q\right) \int_0^1 du \frac{1}{\left[m_b^2 - (q+up_1)^2\right]} \left\{ \phi_s(u) - \frac{m_b m_s}{3} \right.$$

$$\left. \phi_s^\sigma(u) \frac{1}{\left[m_b^2 - (q+up_1)^2\right]} \right\},$$

$$= \frac{-1}{2}\left[(2p_1+q)_\mu q^2 - q_\mu \left(m_B^2 - m_s^2\right)\right] \int_0^1 du \frac{1}{\left[m_b^2 - (q+up_1)^2\right]}$$

$$\left\{ \phi_s(u) - \frac{m_b m_s}{3} \phi_s^\sigma(u) \frac{1}{\left[m_b^2 - (q+up_1)^2\right]} \right\},$$

$$\frac{m_B^2 f_B f_T(q^2)}{(m_b+m_q)\left(m_B^2 - (p_1+q)^2\right)(m_B+m_s)} = \frac{-1}{2} \int_0^1 du \frac{1}{\left[m_b^2 - (q+up_1)^2\right]} \left\{ \phi_s(u) - \frac{m_b m_s}{3} \phi_s^\sigma(u) \right.$$

$$\left. \frac{1}{\left[m_b^2 - (q+up_1)^2\right]} \right\}, \tag{2.5.48}$$

$$\frac{m_B^2 f_B f_T(q^2)}{(m_b+m_q)\left(m_B^2 - (p_1+q)^2\right)(m_B+m_s)} = \frac{-1}{2} \int_{u_0}^1 du \frac{1}{u\left(s-(p_1+q)^2\right)} \phi_s(u) + m_b m_s \frac{1}{6} \int_{u_0}^1 du$$

$$\frac{1}{u^2\left(s-(p_1+q)^2\right)} \phi_s^\sigma(u) + m_b m_s \frac{1}{6} \phi_s^\sigma(u_\circ) \frac{1}{\left(s_0 - (p_1+q)^2\right)}$$

$$\frac{1}{(m_b^2 + u_0^2 p_1^2 - q^2)}. \tag{2.5.49}$$

Now applying the Borel transformation, we get

$$\frac{m_B^2 f_B f_T(q^2)}{(m_b+m_q)(m_B+m_s)} \exp\left(\frac{-m_B^2}{M^2}\right) = \frac{-1}{2} \int_{u_0}^1 du \frac{1}{u} \exp\left(\frac{-m_b^2 + u\bar{u}p_1^2 - \bar{u}q^2}{uM^2}\right)$$

$$\left[\phi_s(u) - \frac{m_b m_s}{3uM^2} \phi_s^\sigma(u)\right] + \frac{m_b m_s}{6} \phi_s^\sigma(u_0) \exp\left(-\frac{s_0}{M^2}\right)$$

$$\frac{1}{(m_b^2 + u_0^2 p_1^2 - q^2)}. \tag{2.5.50}$$

Finally, the relation between f_+ and f_T can be obtained by comparing Eq. (2.5.34) and Eq. (2.5.50), i.e.,

$$\frac{2m_B^2 f_B f_T(q^2) \exp\left({}^{-m_B^2}/{}_{M^2}\right)}{(m_B+m_q)(m_B+m_s)} = \frac{m_B f_+(q^2) f_B \exp\left({}^{-m_B^2}/{}_{M^2}\right)}{(m_B+m_q)}. \tag{2.5.51}$$

$$\frac{2m_B f_T(q^2)}{(m_B+m_s)} = f_+\left(q^2\right). \tag{2.5.52}$$

Similarly comparing Eq. (2.5.35) and Eq. (2.5.50), we get $f_{-}\left(q^2\right)=0$. So sum rules for the form factors $f_{+}\left(q^2\right)$, $f_{-}\left(q^2\right)$ and $f_T\left(q^2\right)$ have been achieved.

Chapter 3

Introduction

In this chapter, we will derive the amplitude and calculate the numerical value by using QCD light cone sum rules for the B-meson decays involving both leptons and hadrons. $B \longrightarrow Sl^+l^-$, where l denotes lepton ($l = e, \mu, \tau$) and S denotes scalar mesons, respectively. We define the approximate range of squared momentum transfer, q^2, within which the Operator product expansion (OPE) for correlators remains valid. Within the effective range, we analyze the behaviors of form factors and differential decay widths. Due to the increasing number of experimentally discovered scalar meson states, significant efforts have been dedicated to investigate their inner structures and classification. Regarding the classification of scalar meson states, there are two well defined scenarios presented from the experimental data. In scenario 1, two scalar nonets are formed by two-quark bound states. One contains the lowest lying scalar states, such as the isoscalars, isodoublets and isovector. The other nonet comprises the corresponding first excited states, such as the isoscalars, isodoublets , and isovector. In scenario 2, the scalar states below 1 GeV are considered members of a four-quark nonet, whereas $f(1370), f_0(1500), a_0(1450)$, and $K_0^*(1430)$ are considered the lowest lying two-quark resonances, and are arranged into another nonet with their corresponding first excited states between 2.0 and 2.3 GeV. These two scenarios are intriguing because they provide us grounds for the study of scalar meson decays [42]. Impact of NP on observables would be discussed in this chapter. We will derive the precise mathematical formulas for the amplitudes using the Wilson coefficients and Hadronic form factors obtained through the application of the LCSR formalism and they are in good match with other models. Configuration of kinematic variables and the adopted convention for polarization for $B-$decay channel would be highlighted. Numerical analysis would be presented with taking into account effects of New Physics.

3.1 Effective Hamiltonian

The generic structure of weak effective Hamiltonian is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} \mathcal{C}_i(\mu) \mathcal{O}_i(\mu), \tag{3.1.1}$$

where G_F represents the Fermi coupling constant, $\mathcal{O}_i(\mu)$ represents to 4 quark propagator and $\mathcal{C}_i(\mu)$ are the Wilson coefficients. Amplitudes are written for an interaction of X_1 and X_2 as following [43]

$$\begin{aligned} \mathcal{M}(X_1 \to X_2) &= \langle X_1 | \mathcal{H}_{eff} | X_2 \rangle, \\ &= \frac{G_F}{\sqrt{2}} \sum_n V^n_{(CKM)} \mathcal{C}_n(\mu) \langle X_1 | O^i(\mu) | X_2 \rangle. \end{aligned}$$

The explicit form of operators can be written as

$$\begin{aligned} \mathcal{O}^1 &= (\bar{c}_\mu b_\nu)_{V-A} (\bar{s}_\nu c_\mu)_{V-A}, \\ \mathcal{O}^2 &= (\bar{c}b)_{V-A} (\bar{s}c)_{V-A}, \\ \mathcal{O}^3 &= (\bar{s}b)_{V-A} \sum_{q=f} (\bar{q}q)_{V-A}, \\ \mathcal{O}^4 &= (\bar{s}_\mu b_\nu)_{V-A} \sum_{q=f} (\bar{q}_\nu q_\mu)_{V-A}, \\ \mathcal{O}^5 &= ((\bar{s}b))_{V-A} \sum_{q=f} ((\bar{q}q))_{V+A}, \\ \mathcal{O}^6 &= ((\bar{s}_\mu b_\nu))_{V-A} \sum_{q=f} ((\bar{q}_\nu q_\mu))_{V+A}, \end{aligned}$$

$$\begin{aligned}
\mathcal{O}^{8} &= \frac{3}{2}\left(\bar{s}_{\mu}b_{\nu}\right)_{V-A}\sum_{q=f}\left(\bar{q}_{\nu}q_{\mu}\right)_{V+A}, \\
\mathcal{O}^{7} &= \frac{3}{2}\left(\bar{s}b\right)_{V-A}\sum_{q=f}e_{q}\left(\bar{q}q\right)_{V+A}, \\
\mathcal{O}^{9} &= \frac{3}{2}\left(\bar{s}b\right)_{V-A}\sum_{q=f}e_{q}\left(\bar{q}q\right)_{V-A}, \\
\mathcal{O}^{10} &= \frac{3}{2}\left(\bar{s}_{\mu}b_{\nu}\right)_{V-A}\sum_{q=f}\left(\bar{q}_{\nu}q_{\mu}\right)_{V-A}, \\
\mathcal{O}^{7\gamma} &= \frac{e}{8\pi^{2}}m_{b}\bar{s}_{\mu}\sigma^{\alpha\beta}\left(1+\gamma^{5}\right)b_{\mu}F_{\mu\nu}, \\
\mathcal{O}^{8g} &= \frac{g}{8\pi^{2}}m_{b}\bar{s}_{\mu}\sigma^{\alpha\beta}\left(1+\gamma^{5}\right)T^{a}_{\mu\nu}b_{\nu}G^{a}_{\alpha\beta}, \\
\mathcal{O}^{9} &= \left(\bar{s}b\right)_{V-A}\left(\bar{l}l\right)_{V}, O^{10} = \left(\bar{s}b\right)_{V-A}\left(\bar{l}l\right)_{A}, \\
\mathcal{O}^{\nu\bar{\nu}} &= \left(\bar{s}b\right)_{V-A}\left(\nu\bar{\nu}\right)_{V-A}, \mathcal{O}^{l\bar{l}} = \left(\bar{s}b\right)_{V-A}\left(\bar{l}l\right)_{V-A}.
\end{aligned} \tag{3.1.2}$$

We can also include the chirality flipped and the scalar type new operators as

$$\begin{aligned}
\mathcal{O}^{7} &= \frac{e}{g^{2}}m_{b}\left(\bar{s}\sigma_{\alpha\beta}P_{R}b\right)I^{\alpha\beta}, \mathcal{O}^{7\prime} = \frac{e}{g^{2}}m_{b}\left(\bar{s}\sigma_{\alpha\beta}P_{L}b\right)I^{\alpha\beta}, \\
\mathcal{O}^{8} &= \frac{1}{g}m_{b}\left(\bar{s}\sigma_{\alpha\beta}T^{a}P_{R}b\right)G^{\alpha\beta a}, \mathcal{O}^{8\prime} = \frac{1}{g}m_{b}\left(\bar{s}\sigma_{\alpha\beta}T^{a}P_{L}b\right)G^{\alpha\beta a}, \\
\mathcal{O}^{9} &= \frac{e^{2}}{g^{2}}\left(\bar{s}\gamma_{\mu}P_{L}b\right)\left(\bar{\mu}\gamma^{\mu}\mu\right), \mathcal{O}^{9\prime} = \frac{e^{2}}{g^{2}}\left(\bar{s}\gamma_{\mu}P_{R}b\right)\left(\bar{\mu}\gamma^{\mu}\mu\right), \\
\mathcal{O}^{10} &= \frac{e^{2}}{g^{2}}\left(\bar{s}\gamma_{\mu}P_{L}b\right)\left(\bar{\mu}\gamma^{\mu}\gamma_{5}\mu\right), \mathcal{O}^{10\prime} = \frac{e^{2}}{g^{2}}\left(\bar{s}\gamma_{\mu}P_{R}b\right)\left(\bar{\mu}\gamma^{\mu}\gamma_{5}\mu\right), \\
\mathcal{O}^{S} &= \frac{e^{2}}{16\pi^{2}}m_{b}\left(\bar{s}P_{R}b\right)\left(\bar{\mu}\mu\right), \mathcal{O}^{S\prime} = \frac{e^{2}}{16\pi^{2}}m_{b}\left(\bar{s}P_{L}b\right)\left(\bar{\mu}\mu\right), \\
\mathcal{O}^{P} &= \frac{e^{2}}{16\pi^{2}}m_{b}\left(\bar{s}P_{R}b\right)\left(\bar{\mu}\gamma_{5}\mu\right), \mathcal{O}^{P\prime} = \frac{e^{2}}{16\pi^{2}}m_{b}\left(\bar{s}P_{L}b\right)\left(\bar{\mu}\gamma_{5}\mu\right),
\end{aligned} \tag{3.1.3}$$

where $P_{L,R} = \frac{1\mp\gamma_5}{2}$ and m_b represents running mass in $\overline{MS}$ scheme.

As we are dealing with the $b \to s$ meson decays with hadrons in the final state, so our required operators are $\mathcal{O}^{7\gamma}, \mathcal{O}^{9}, \mathcal{O}^{10}$ and the last two mentioned in Eq. (3.1.2). For $b \to u$ the effective Hamiltonian has following form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}V_{ub}\bar{u}\gamma_{\mu}\left(1-\gamma_5\right)b\bar{l}\gamma^{\mu}\left(1-\gamma_5\right)\nu_l,$$

Semi-leptonic decays $B \to Sl^+l^-$ in the Standard Model are induced by the following effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F \alpha V_{tb}^* V_{ts}}{\sqrt{2}\pi} \Bigg[C_9^{eff} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{l}\gamma^\mu l + C_{10} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{l}\gamma^\mu \gamma_5 l - \frac{2m_b C_7^{eff}(m_b)}{q^2} \bar{s}\iota\sigma_{\mu\nu} q^\nu (1+\gamma_5) b \bar{l}\gamma^\mu l \Bigg], \tag{3.1.4}$$

where Cabibbo-Kobayashi-Maskawa matrix element is represented by V_{ij}, and C_i^{eff} represents the Wilson coefficients. $C^{9,eff}$ and $C^{7,eff}$ are represented by

$$C^{7,eff}(\mu) = C^7(\mu) + C^{b\to s\gamma}(\mu), \tag{3.1.5}$$

$$C^{9,eff} = C^9(\mu) + Y^{pert}(s') + Y^{LD}(s'), \tag{3.1.6}$$

where $C^{b\to s\gamma}(\mu)$ results from the following interaction $b \to sc\bar{c} \to s\gamma$. $Y^{pert}(s')$ and $Y^{LD}(s')$ represents the short and long distance contributions due to four quark propagators [44]

$$Y^{pert}(s') = h(z', s') - \frac{4}{2}h(1, s') C_3 - \frac{4}{2}h(z', s') C_4 - \frac{3}{2}h(z', s') C_5 - \frac{1}{2}h(z', s') C_6 - \frac{1}{2}h(0, s') C_3 - \frac{3}{2}h(0, s') C_4 + \frac{2}{3}C_3 + \frac{2}{9}C_4 + \frac{2}{3}C_5 + \frac{2}{9}C_6, \tag{3.1.7}$$

moreover, for $y \equiv \frac{4z'^2}{s'} < 1$

$$h(z', s') = -\frac{8}{9}\ln z' + \frac{8}{27} + \frac{4}{9}y - \frac{2}{9}(2+y)|1-y|^{1/2} \left\{ \ln\left|\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\right| - \iota\pi \right\}, \tag{3.1.8}$$

and for $y \equiv \frac{4z'^2}{s'} > 1$

$$h(z', s') = -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}y - \frac{2}{9}(2+y)|1-y|^{1/2} \left\{ 2\arctan\frac{1}{\sqrt{y-1}} \right\}, \tag{3.1.9}$$

$$h(0, s') = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}\iota\pi, \tag{3.1.10}$$

where $z' \equiv \frac{m_c}{m_b}$ and $s' \equiv \frac{q^2}{m_b^2}$. Wilson coefficients values in Standard model are written in the

Table 3.1. The long-distance part is given as

$$Y^{LD}\left(z',s'\right)=\frac{3\pi}{\alpha^2}C^{(0)}\sum_{V_i=\psi_i}\kappa_i\frac{m_{V_i}\Gamma(V_i\to l^+l^-)}{m_{V_i}^2-s'm_b^2-im_{V_i}\Gamma_{V_i}}, \tag{3.1.11}$$

Table 3.1.1: Wilson Coefficients values in SM

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_9	C_{10}
1.119	−0.270	0.013	−0.027	0.009	−0.033	−0.322	4.344	−4.669

Moreover,

$$\mathcal{C}^{b\to s\gamma}\left(\mu\right)=\iota\alpha_s\left[\frac{2}{9}\eta^{14/23}\left(G_1\left(y_t\right)-0.1687\right)-0.03C_2\left(\mu\right)\right], \tag{3.1.12}$$

$$G_1\left(y\right)=\frac{y\left(y^2-5y-2\right)}{8\left(y-1\right)^3}+\frac{3y^2\ln^2 y}{4\left(y-1\right)^4}, \tag{3.1.13}$$

$$C_0=3\left(C_1+C_3+C_5\right)+1\left(C_2+C_4+C_6\right),$$

where $\eta=\frac{\alpha_s(m_W)}{\alpha_s(\mu)}$, $y=\frac{m_t^2}{m_W^2}$. Short distance physics is encoded in Wilson coefficients and any new physics effects would be included in these coefficients. In the definition of operators $\mathcal{O}_{i\geq 7}$, a factor $\frac{16\pi^2}{g^2}$ is taken that allows a plain organization of expansion of the corresponding Wilson coefficients in the perturbation theory. As $C_{7,9}$ always appear in the form of other Wilson coefficients so it would be more convenient to define effective Wilsonian coefficients as [45]

$$C_7^{eff}=\frac{4\pi}{\alpha_s}C_7-\frac{1}{3}C_3-\frac{4}{9}C_4-\frac{20}{3}C_5-\frac{80}{9}C_6, \tag{3.1.14}$$

$$C_8^{eff}=\frac{4\pi}{\alpha_s}C_8+C_3-\frac{1}{6}C_4+20C_5-\frac{10}{3}C_7, \tag{3.1.15}$$

$$C_9^{eff}=\frac{4\pi}{\alpha_s}C_9+Y\left(q^2\right), \tag{3.1.16}$$

$$C_{10}^{eff}=\frac{4\pi}{\alpha_s}C_{10}, \tag{3.1.17}$$

$$C_{7,8,9,10}^{\prime,eff}=\frac{4\pi}{\alpha_s}C'_{7,8,9,10}, \tag{3.1.18}$$

3.2 Kinematics

Decay $B \to Sl^+l^-$can be prescribed the name as quasi-two body decay with $B \to SJ_{eff}$, whereas J_{eff} denotes the di-leptons. Now defining the momenta of each particle in rest frame of $B-$meson that is parent particle

$$p_1 = p_2 + p_3 + p_4, \tag{3.2.1}$$

where p_1 denotes the 4-momentum of $B-$meson, whereas p_2, p_3, p_4 denotes the corresponding momentum of final state scalar meson, electron and positron. Writing

$$p_1 - p_2 = p_3 + p_4, \tag{3.2.2}$$

where

$$p_1 = (m_B, 0, 0, 0)\,, \tag{3.2.3}$$

$$p_1 - p_2 = p_3 + p_4 = q^2 \equiv s, \tag{3.2.4}$$

with q defining the exchanged momenta or the momenta of di-leptons, and m_B is the mass of $B-$mesons. By definition of Mandelstam variable

$$t = (p_3 + p_4)^2\,, \tag{3.2.5}$$

$$= p_3^2 + p_4^2 + 2p_3.p_4$$

$$t = m_e^2 + m_e^2 + 2p_3.p_4, \tag{3.2.6}$$

$$p_3.p_4 = \frac{t - 2m_e^2}{2}. \tag{3.2.7}$$

Similarly

$$(p_1 - p_2)^2 = (p_3 + p_4)^2 = t, \tag{3.2.8}$$

$$t = p_1^2 + p_2^2 - 2p_1.p_2,$$

$$p_1.p_2 = \frac{m_B^2 + m_S^2 - t}{2}. \tag{3.2.9}$$

Defining $s-$Mandelstam variable

$$s \equiv q^2 = (p_1 - p_2)^2 \,,$$

or

$$p_1 - q = p_2,$$

squaring both sides

$$\begin{aligned} p_2^2 &= p_1^2 + q^2 - 2p_1.q, \\ m_S^2 &= m_B^2 + s - 2p_1.q, \\ p_1.q &= \frac{m_B^2 + s - m_S^2}{2}, \end{aligned} \tag{3.2.10}$$

while $s/2 = p_3.q$ and $p_4.q = p_4/s$. Now defining last Mandelstam variable u as

$$u = (p_2 + p_3)^2 \,, \tag{3.2.11}$$

$$\begin{aligned} &= p_2^2 + p_3^2 + 2p_2.p_3, \\ u &= m_S^2 + m_e^2 + 2p_2.p_3, \\ p_2.p_3 &= \frac{u - m_S^2 - m_e^2}{2}. \end{aligned} \tag{3.2.12}$$

The different scalar products can be found as

$$p_2.p_4 = p_2.\left(p_1 - p_2 - p_3\right), \tag{3.2.13}$$

$$\begin{aligned} &= p_1.p_2 - p_2^2 - p_2.p_3, \\ &= \frac{m_B^2 + m_S^2 - t}{2} - m_S^2 - \frac{u - m_S^2 - m_e^2}{2}, \\ p_2.p_4 &= \frac{m_B^2 + m_e^2 - t - u}{2}. \end{aligned} \tag{3.2.14}$$

$$p.q = \left(p_1 + p_2\right)\left(p_1 - p_2\right), \tag{3.2.15}$$

$$= p_1^2 - p_2^2,$$
$$p.q = m_B^2 - m_S^2. \tag{3.2.16}$$

So any combination of momentum can be evaluated with the help of Mandelstam variables, i.e.,

$$t = (p_3 + p_4)^2\,, \tag{3.2.17}$$
$$s = (p_1 - p_2)^2\,, \tag{3.2.18}$$
$$u = (p_2 + p_3)^2\,, \tag{3.2.19}$$
$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2. \tag{3.2.20}$$

Following visual is the rest frame of di-leptons where l^+ can be seen making an angle θ with$z-$axis.

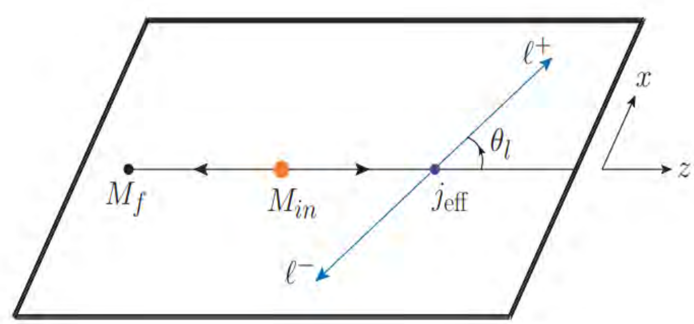

Figure 3.2.1: Kinematics of B-decay channel [46].

3.3 Amplitude calculation within Standard model

Effective Hamiltonian responsible for $b \to s$ transition has the following structure

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u}\gamma_\mu (1-\gamma_5) b \bar{l}\gamma^\mu (1-\gamma_5)\nu_l + h.c., \tag{3.3.1}$$

where l stands for electron, muon, and tau. V_{ub} stands for the CKM matrix element. Similarly for FCNC transition $b \to s$, Hamiltonian has the following representation

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \Bigg[C_9^{eff}(\mu)\left(\bar{s}\gamma_\mu (1-\gamma_5) b\right)\left(\bar{l}\gamma^\mu l\right) + C_{10}\left(\bar{s}\gamma_\mu (1-\gamma_5)\right)b)\left(\bar{l}\gamma^\mu\gamma_5 l\right) - \frac{2m_b C_7^{eff}(\mu)}{q^2}\left(\bar{s}\sigma_{\mu\nu}((1+\gamma_5))q^\nu b\right)\left(\bar{l}\gamma^\mu l\right)\Bigg] + h.c., \tag{3.3.2}$$

as $|\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}| < 0.02$ so the term proportional to $V_{ub}V_{ts}^*$ is neglected. Wilson coefficients expressions are given in the Appendix. As it is well known that

$$\begin{aligned}
\mathcal{M}(b \to s l\bar{l}) &= \langle S|\mathcal{H}_{eff}|B\rangle, \\
&= \langle S|\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \Bigg[C_9^{eff}(\mu)\,\bar{s}\gamma_\mu (1-\gamma_5) b\left(\bar{l}\gamma^\mu l\right) + C_{10}\bar{s}\gamma_\mu (1-\gamma_5) b\left(\bar{l}\gamma^\mu\gamma_5 l\right) - \\
&\quad \frac{2m_b C_7^{eff}(\mu)}{q^2}\bar{s}\sigma_{\mu\nu}(1-\gamma_5) q^\nu b\left(\bar{l}\gamma^\mu l\right)\Bigg]|B\rangle, && (3.3.3) \\
&= \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \Bigg\{ \langle S|C_9^{eff}(\mu)\left[\bar{s}\gamma_\mu L b\right]\left[\bar{l}\gamma^\mu l\right] + C_{10}\left[\bar{s}\gamma_\mu L b\right]\left[\bar{l}\gamma^\mu\gamma_5 l\right] - && (3.3.4) \\
&\quad \frac{2m_b C_7^{eff}(\mu)}{q^2}\left[\bar{s}\iota\sigma_{\mu\nu}q^\nu R\right]\left[\bar{l}\gamma^\mu l\right]|B\rangle\Bigg\}, && (3.3.5)
\end{aligned}$$

where C_{Q_1} and C_{Q_2} are zero in the expression of effective Hamiltonian. Now

$$\hat{s} = \frac{s}{m_B^2}, s = q^2, \tag{3.3.6}$$

$$C_7^{eff} = C_7 - C_{5/3} - C_6, \tag{3.3.7}$$

$$C_9^{eff} = C_9(\mu) + Y(u,\hat{s}) + \frac{3\pi}{\alpha^2} C(\mu) \sum_{V_i=\psi(1s)\ldots\psi(6s)} \frac{K_i \Gamma(V_i \longrightarrow l^+l^-) m_{V_i}}{m_{V_i}^2 - \hat{s}m_B^2 - \iota m_{V_i}\Gamma_{V_i}}, \tag{3.3.8}$$

where the long distance effects due to $c\bar{c}$ resonance are encoded in C_9^{eff} and short distance contributions too. While intermediate states are represented by the last term in above expression of C_9^{eff}. One loop contribution of 4-quark operators represented by the $Y(u,\hat{s})$ and its explicit expression can be found in [47]. By using following definitions of Hadronic transition probabilities, amplitude can be found in terms of Hadronic entities that are form factors so

$$\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B(p+q)\rangle = -\iota\left[f_+\left(q^2\right)p_\mu + f_-\left(q^2\right)q_\mu\right], \tag{3.3.9}$$

$$\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{-1}{m_B+m_S}\left[(2p+q)_\mu q^2 - q_\mu\left(m_B^2-m_S^2\right)\right]f_T\left(q^2\right). \tag{3.3.10}$$

These can also be written as

$$\langle S(p)|\bar{s}\gamma_\mu L b|B(p+q)\rangle = f_+\left(q^2\right)p_\mu + f_-\left(q^2\right)q_\mu, \tag{3.3.11}$$

$$\langle S(p)|\bar{s}\sigma_{\mu\nu}Lq^\nu b|B(p+q)\rangle = \iota\left\{p_\mu q^2 - q_\mu\left(m_B^2-m_S^2\right)\right\}\frac{f_T\left(q^2\right)}{m_B+m_S}, \tag{3.3.12}$$

$$\langle S(p)|\bar{s}b|B(p+q)\rangle = \frac{m_B^2-m_S^2}{m_s-m_b}f_T\left(q^2\right). \tag{3.3.13}$$

By using these definitions, we get the following form of amplitude

$$\mathcal{M}(B\longrightarrow Sl\bar{l}) = \frac{-G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^*\left\{T_\mu^1\left[\bar{l}\gamma_\mu l\right]\right] + T_\mu^2\left[\bar{l}\gamma_\mu\gamma_5 l\right] + T^3\left[\bar{l}l\right]\right\}, \tag{3.3.14}$$

where T_μ^1, T_μ^2 and T^3 are the auxiliary functions defined as

$$T_\mu^1 = \iota\left(C_9^{eff} - C_9'^{eff}\right)f_+\left(q^2\right)p_\mu + \frac{4\iota m_b}{m_B+m_S}\left(C_7^{eff} - C_7'^{eff}\right), \tag{3.3.15}$$

$$T_\mu^2 = \iota\left(C_{10}-C_{10}'\right)\left\{f_+\left(q^2\right)p_\mu + f_-\left(q^2\right)q_\mu\right\} - \frac{\iota}{2m_l\left(m_b+m_s\right)}\left(C_{Q_2}-C_{Q_2}'\right) \tag{3.3.16}$$

$$\left\{f_+\left(q^2\right)p.q + f_-\left(q^2\right)q^2\right\}q_\mu,$$

$$T^3 = \iota\left(C_{Q_1}-C_{Q_1}'\right)\frac{1}{m_b+m_s}\left(f_+\left(q^2\right)p.q + f_-\left(q^2\right)q^2\right), \tag{3.3.17}$$

whereas $C_i' = 0$ and $T^3 = 0$ because $f_-\left(q^2\right)$ doesn't contribute at large recoils. Now taking complex conjugate of Eq. (3.3.14)

$$\mathcal{M}^{\dagger}(B \longrightarrow Sl\bar{l}) = \left[\frac{-G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{ts}^{*}\left\{T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] + T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] + T^{3}\left[\bar{l}l\right]\right\}\right]^{\dagger}, \tag{3.3.18}$$

$$\mathcal{M}^{\dagger} = \frac{-G_F\alpha V_{tb}^{*}V_{ts}}{2\sqrt{2}\pi}\left\{T_{\mu}^{1*}\left[\bar{l}\gamma_{\mu}l\right]^{\dagger} + T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger} + T^{3*}\left[\bar{l}l\right]^{\dagger}\right\}. \tag{3.3.19}$$

Now to calculate the mod square of amplitude

$$\left|\mathcal{M}^2\right| = \left|\mathcal{M}\mathcal{M}^{\dagger}\right|,$$

$$\left|\mathcal{M}^2\right| = \left|\left\{\frac{-G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^{*}\left[T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] + T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] + T^{3}\left[\bar{l}l\right]\right]\right\} \times \left\{\frac{-G_F\alpha V_{tb}^{*}V_{ts}}{2\sqrt{2}\pi}\right.\right. \tag{3.3.20}$$

$$\left.\left.\left(T_{\mu}^{1*}\left[\bar{l}\gamma_{\mu}l\right]^{\dagger} + T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger} + T^{3*}\left[\bar{l}l\right]^{\dagger}\right)\right\}\right|,$$

$$= \left(\frac{-G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^{*}\right)\left(\frac{-G_F\alpha V_{tb}^{*}V_{ts}}{2\sqrt{2}\pi}\right)\left[T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] \times T_{\mu}^{1*}\left[\bar{l}\gamma_{\mu}l\right]^{\dagger} + T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] \times T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger}\right.$$

$$+ T^{3}\left[\bar{l}l\right] \times T^{3*}\left[\bar{l}l\right]^{\dagger} + T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] \times T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger} + T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] \times T^{3*}\left[\bar{l}l\right]^{\dagger} + T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] \times T_{\mu}^{1*}$$

$$\left.\left[\bar{l}\gamma_{\mu}l\right]^{\dagger} + T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] \times T^{3*}\left[\bar{l}l\right]^{\dagger} + T^{3}\left[\bar{l}l\right] \times T_{\mu}^{1*}\left[\bar{l}\gamma_{\mu}l\right]^{\dagger} + T^{3}\left[\bar{l}l\right] \times T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger}\right], \tag{3.3.21}$$

giving each term a name and solving one by one by using Casimir trick here for leptonic part

$$\left|\mathcal{M}^2\right| = \left(\frac{-G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^{*}\frac{-G_F\alpha V_{tb}^{*}V_{ts}}{2\sqrt{2}\pi}\right)\left[\mathcal{M}_1\mathcal{M}_1^{\dagger} + \mathcal{M}_2\mathcal{M}_2^{\dagger} + \mathcal{M}_3\mathcal{M}_3^{\dagger} + \mathcal{M}_1\mathcal{M}_2^{\dagger} + \mathcal{M}_1\mathcal{M}_3^{\dagger} + ...\right], \tag{3.3.22}$$

$$\mathcal{M}_1\mathcal{M}_1^{\dagger} = T_{\mu}^{1}\left[\bar{l}\gamma_{\mu}l\right] \times T_{\mu}^{1*}\left[\bar{l}\gamma_{\mu}l\right]^{\dagger}, \tag{3.3.23}$$

$$= T_{\mu}^{1} \times T_{\mu}^{1*}\left\{Tr\left[\left(\not{p}_3 + m_3\right)\gamma^{\mu}\left(\not{p}_4 - m_4\right)\gamma^{\mu\prime}\right]\right\},$$

$$\mathcal{M}_2\mathcal{M}_2^{\dagger} = T_{\mu}^{2}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right] \times T_{\mu}^{2*}\left[\bar{l}\gamma_{\mu}\gamma_5 l\right]^{\dagger}, \tag{3.3.24}$$

$$= T_{\mu}^{2} \times T_{\mu}^{2*}\left\{Tr\left[\left(\not{p}_3 + m_3\right)\gamma^{\mu}\gamma_5\left(\not{p}_4 - m_4\right)\gamma^{\mu\prime}\gamma_5\right]\right\},$$

$$\mathcal{M}_3\mathcal{M}_3^{\dagger} = 0, \tag{3.3.25}$$

Now solving for cross terms

$$\mathcal{M}_1\mathcal{M}_2^\dagger = T_\mu^1\left[\bar{l}\gamma_\mu l\right]\times T_\mu^{2*}\left[\bar{l}\gamma_\mu\gamma_5 l\right]^\dagger, \tag{3.3.26}$$
$$= T_\mu^1\times T_\mu^{2*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right]\right\},$$
$$\mathcal{M}_1\mathcal{M}_3^\dagger = 0, \tag{3.3.27}$$
$$\mathcal{M}_2\mathcal{M}_1^\dagger = T_\mu^2\left[\bar{l}\gamma_\mu\gamma_5 l\right]\times T_\mu^{1*}\left[\bar{l}\gamma_\mu l\right]^\dagger, \tag{3.3.28}$$

$$= T_\mu^2\times T_\mu^{1*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right]\right\},$$
$$\mathcal{M}_2\mathcal{M}_3^\dagger = 0. \tag{3.3.29}$$

Now summing up all the terms, we get the total amplitude square as

$$\left|\mathcal{M}^2\right| = \frac{G_F^2\alpha^2}{8\pi}\left|V_{tb}V_{ts}\right|^2\Bigg[T_\mu^1\times T_\mu^{1*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right]\right\}+T_\mu^2\times$$
$$T_\mu^{2*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right]\right\}+T_\mu^1\times T_\mu^{2*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right]\right\}$$
$$+T_\mu^2\times T_\mu^{1*}\left\{Tr\left[\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right]\right\}\Bigg], \tag{3.3.30}$$

3.4 Numerical analysis of form factors

Form factors are usually parameterized in single pole or double pole forms

$$f_i\left(q^2\right) = \frac{f_i\left(0\right)}{1-a_i q^2/m_{B_{q_1}}^2}, \tag{3.4.1}$$
$$f_i\left(q^2\right) = \frac{f_i\left(0\right)}{1-a_i q^2/m_{B_{q_1}}^2+\frac{b_i q^4}{m_{B_{q_1}}^4}}, \tag{3.4.2}$$

where the kinematic region is defined as $0<q^2<\left(m_{B_{q_1}}-m_S\right)^2$. Here, a_i and b_i correspond to intrinsic properties beyond perturbation and can be designed by changing the values of momentum transfer that would change the values of form factors.

Table 3.4.1: Numerical values of $f_j(0), a_j$ and b_j involved in calculations of B decay channels[48].

	$f_j(0)$	a_j	b_j
f^+	$0.97^{+0.20}_{-0.20}$	$0.86^{+0.19}_{-0.18}$	
	0.52	1.36	0.86
	0.62 ± 0.16	0.81	-0.21
f^-	$0.073^{+0.02}_{-0.02}$	$2.50^{+0.44}_{-0.47}$	$1.82^{+0.69}_{-0.76}$
f^T	$0.60^{+0.14}_{-0.13}$	$0.69^{+0.26}_{-0.27}$	-
	0.34	1.64	1.72
	0.26 ± 0.07	0.41	-0.32

Here, the coefficients a_i and b_i can be calculated by the first and second derivative of $F(q^2)$ at $q^2 = 0$.

3.5 Decay rate

In the rest frame of the parent particle i.e., $B-$meson, the differential decay width for the semi leptonic $B \to S$ transition can be written as

$$\frac{d\Gamma(B \longrightarrow Sl\bar{l})}{dq^2} = \frac{1}{(2\pi)^3}\frac{1}{32m_B}\int_{u_{min}}^{u_{max}} |\mathcal{M}|^2 \, du, \tag{3.5.1}$$

where $u \equiv (p_S + p_l)^2$ and $q^2 = (p_3 + p_4)^2$. The limits of integration are

$$u_{max} = (E_S^* + E_l^*)^2 - \left(\sqrt{E_S^{*2} - m_S^2} - \sqrt{E_l^{*2} - m_l^2}\right)^2, \tag{3.5.2}$$

$$u_{min} = (E_S^* + E_l^*)^2 - \left(\sqrt{E_S^{*2} - m_S^2} + \sqrt{E_l^{*2} - m_l^2}\right)^2, \tag{3.5.3}$$

where E_S^* and E_l^* are the energies of the scalar particle (S) and the lepton (l), respectively, in the rest frame of leptons [49]. These can be calculated as

$$E_S^* = \frac{m_B^2 - m_S^2 - q^2}{2\sqrt{q^2}}, \tag{3.5.4}$$

$$E_l^* = \frac{\sqrt{q^2}}{2}. \tag{3.5.5}$$

Assembling everything, in Eq. (3.5.1), the final result will read as

$$\frac{d\Gamma(B \longrightarrow Sl\bar{l})}{dq^2} = \frac{G_F^2 \left|V_{tb}V_{ts}\right|^2 m_B^5 \alpha^2}{1536\pi^5}\left(1-\frac{4r_l}{s'}\right)^{1/2}\varphi_S^{1/2}\left[\left(1+\frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S\right], \tag{3.5.6}$$

where

$$s' = {q^2}/{m_B^2}, \qquad r = {m_l^2}/{m_B^2}, \qquad r_s = {m_S^2}/{m_B^2}$$
$$\varphi_S = \left(1-r_S\right)^2 - 2s\left(1+r_S\right) + s^2$$

$$\alpha_S = \varphi_S\left(\left|C_9^{eff}\frac{f_+\left(q^2\right)}{2} - 2\frac{C_7 f_T\left(q^2\right)}{1+\sqrt{r_S}}\right|^2 + \left|C_{10}\frac{f_+\left(q^2\right)}{2}\right|^2\right)$$
$$\delta_S = 6\left|C_{10}\right|^2\left\{\left[2\left\{1+r_S\right\}-s\right]\left|\frac{f_+\left(q^2\right)}{2}\right|^2 + \left(1-r_S\right)\mathrm{Re}\left[f_+\left(q^2\right)\left(f_-\left(q^2\right)-\frac{f_+\left(q^2\right)}{2}\right)\right]\right.$$
$$\left. + s\left|f_-\left(q^2\right)-\frac{f_+\left(q^2\right)}{2}\right|^2\right\}.$$

The numerical values of various input parameters are given in Table 3.3. Using these values, the decay rate becomes

Table 3.5.1: Numerical inputs

G_F	1.166×10^{-2}GeV^{-2}	$\lvert V_{tb}\rvert$	0.9991
m_b	(4.68 ± 0.03)GeV	$m_s(1GeV)$	142MeV
m_{B_0}	5.279GeV	m_{B_s}	142MeV
f_{B_0}	(0.19 ± 0.02)GeV	f_{B_s}	(0.23 ± 0.02)GeV
α_{em}	$\frac{1}{137}$	m_t	174GeV
m_W	80.42GeV	Λ_{QCD}	225MeV

$$\Gamma(B \longrightarrow Sl\bar{l}) = 4.00573\times10^{-7}\text{GeV}. \tag{3.5.7}$$

This value is well in the range of some ongoing and future $B-$factories. Graphically, the profile of the decay rate with square of momentum transfer q^2 is shown in Fig. 3.5.1.

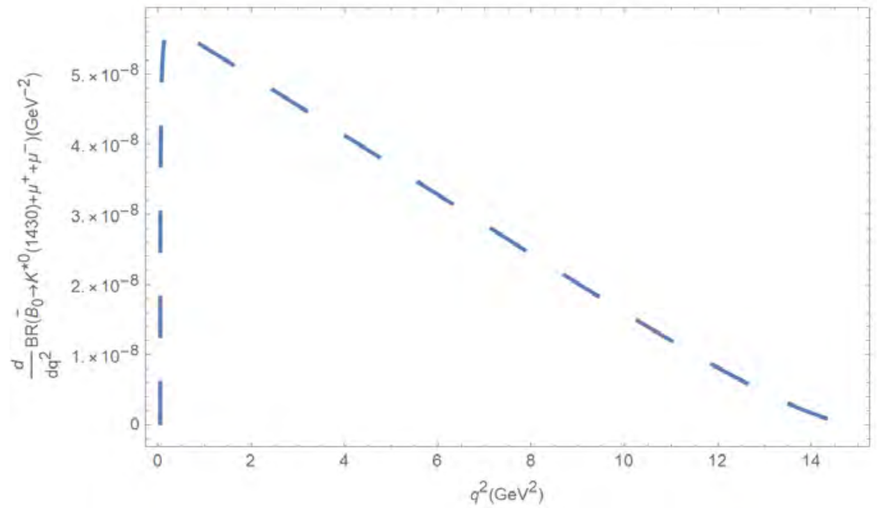

Figure 3.5.1: Branching ratio calculated within SM

3.6 New Physics imprints in the semi leptonic $B \to S$ decays

The exclusive processes involving quark level transitions $b \to sl\bar{l}$ have been measured by several experiments, showing deviations from the SM in the branching ratios of $B \to K^*\mu^+\mu^-$ [50, 51, 52], $B_s \to \phi\mu^+\mu^-$ [53] and in the optimized observables of $B \to K^*\mu^+\mu^-$ [54, 55]. the measurements of the Lepton Flavor Universality (LFU) R_K and R_{K^*}, which is the ratio of the $B \to K^{(*)}\mu^+\mu^-$ to $B \to K^{(*)}e^+e^-$, in different bins of dilepton invariant mass (q^2) provide the signatures of the NP[56, 57, 58].

To accommodate the various discrepancies, let us consider the following weak effective Hamiltonian [59]

$$\mathcal{H}_{eff} = \frac{-4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i O_i + \sum_{i=7}^{8} \left(C_i O_i + C_i' O_i'\right) + \sum_{i=9,10} \left[\left(C_i + C_{il}^{NP}\right) O_i + C_{il}'^{NP} O_i' \right] \right], \tag{3.6.1}$$

$$= \frac{-4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \Big[\left(C_7^{eff} O_7 + C_7'^{eff} O_7' \right) + \left(C_9 + C_{9l}^{NP}\right) O_9 + C_{9l}'^{NP} O_9' + \left(C_{10} + C_{10l}^{NP}\right) O_{10} \tag{3.6.2}$$

$$+C_{10l}^{\prime NP}O_{10}^{\prime}\Bigg], \tag{3.6.3}$$

where the different operators are defined as

$$\begin{aligned}
O_7 &= \frac{e}{6\pi^2}m_b\left(\bar{s}\sigma_{\mu\nu}q^{\nu}P_R b\right)F^{\mu\nu}, O_7^{\prime} = \frac{e}{16\pi^2}m_b\left(\bar{s}\sigma_{\mu\nu}P_L b\right)F^{\mu\nu},\\
O_8 &= \frac{g_s}{16\pi^2}m_b\left(\bar{s}\sigma_{\mu\nu}T^a P_R b\right)G^{\mu\nu a}, O_8^{\prime} = \frac{g_s}{16\pi^2}m_b\left(\bar{s}\sigma_{\mu\nu}T^a P_L b\right)G^{\mu\nu a},\\
O_9 &= \frac{e}{16\pi^2}\left(\bar{s}\gamma_{\mu}P_L b\right)\left(\bar{l}\gamma^{\mu}l\right), O_9^{\prime} = \frac{e}{16\pi^2}\left(\bar{s}\gamma_{\mu}P_R b\right)\left(\bar{l}\gamma^{\mu}l\right),\\
O_{10} &= \frac{e}{16\pi^2}\left(\bar{s}\gamma_{\mu}P_L b\right)\left(\bar{l}\gamma^{\mu}\gamma_5 l\right), O_{10}^{\prime} = \frac{e}{16\pi^2}\left(\bar{s}\gamma_{\mu}P_R b\right)\left(\bar{l}\gamma^{\mu}\gamma_5 l\right).
\end{aligned} \tag{3.6.4}$$

Here, g_s stands for the strong coupling constant and m_b represents the running mass of b-quark in the $\overline{MS}$ technique. By using the same definitions of the matrix elements as mentioned above we get the following expression

$$\mathcal{M} = \langle S(p)|\mathcal{H}_{eff}|B(p+q)\rangle, \tag{3.6.5}$$

$$\begin{aligned}
&= \langle S(p)|\frac{-4G_F}{\sqrt{2}}V_{tb}V_{ts}^{*}\Bigg[\left(C_7^{eff}O_7 + C_7^{\prime eff}O_7^{\prime}\right) + \left(C_9^{eff} + C_{9l}^{NP}\right)O_9 + C_{9l}^{\prime NP}O_9^{\prime} +\\
&\left(C_{10} + C_{10l}^{NP}\right)O_{10} + C_{10l}^{\prime NP}O_{10}^{\prime}\Bigg]|B(p+q)\rangle,\\
&= \frac{-G_F}{2\pi\sqrt{2}}V_{tb}V_{ts}^{*}\Bigg\{\langle S(p)\, C_7^{eff}\left[-\iota\bar{s}\sigma_{\mu\nu}m_b P_R b\frac{1}{s}\right]\left(\bar{l}\gamma^{\mu}l\right) + C_7^{\prime eff}\left[-\iota\bar{s}\sigma_{\mu\nu}q^{\nu}P_L b\frac{1}{s}\right]\left(\bar{l}\gamma^{\mu}l\right) +\\
&\left(C_9^{eff} + C_{9l}^{NP}\right)\left(\bar{s}\gamma_{\mu}P_L b\right)\left(\bar{l}\gamma^{\mu}l\right) + C_{9l}^{\prime NP}\left(\bar{s}\gamma_{\mu}P_R b\right)\left(\bar{l}\gamma^{\mu}l\right) + \left(C_{10} + C_{10l}^{NP}\right)\left(\bar{s}\gamma_{\mu}P_L b\right)\left(\bar{l}\gamma^{\mu}\gamma_5 l\right)\\
&+ C_{10l}^{\prime NP}\left(\bar{s}\gamma_{\mu}P_R b\right)\left(\bar{l}\gamma^{\mu}\gamma_5 l\right)\Bigg\},\\
&= \frac{G_F}{2\pi\sqrt{2}}V_{tb}V_{ts}^{*}\Bigg\{\left(C_9^{eff} + C_{9l}^{NP}\right)\langle S(p)|\bar{s}\gamma_{\mu}\left(1-\gamma_5\right)b\left(\bar{l}\gamma^{\mu}l\right)|B(p+q)\rangle + C_{9^{\prime}l}^{NP}\langle S(p)|\bar{s}\gamma_{\mu}\\
&\left(1+\gamma_5\right)b\left(\bar{l}\gamma^{\mu}l\right)|B(p+q)\rangle - \frac{2m_b}{q^2}C_7^{eff}\langle S(p)|\bar{s}\iota\sigma_{\mu\nu}q^{\nu}\left(1+\gamma_5\right)b|B(p+q)\rangle\left(\bar{l}\gamma^{\mu}l\right) +\\
&\left(C_{10} + C_{10l}^{NP}\right)\langle S(p)|\bar{s}\gamma_{\mu}\left(1-\gamma_5\right)b\left(\bar{l}\gamma^{\mu}\gamma_5 l\right)|B(p+q)\rangle + C_{10^{\prime}l}^{NP}\langle S(p)|\bar{s}\gamma_{\mu}\left(1+\gamma_5\right)b\left(\bar{l}\gamma^{\mu}\gamma_5 l\right)\\
&|B(p+q)\rangle\Bigg\}.
\end{aligned} \tag{3.6.6}$$

By using the Hadronic matrix elements definitions in terms of form factors as mentioned in previous Eq. (3.3.9) and Eq. (3.3.10)

$$\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B(p+q)\rangle = -\iota\left[f_+\left(q^2\right)p_\mu + f_-\left(q^2\right)q_\mu\right], \tag{3.6.7}$$

and contracting above equation with q_μ gives

$$q_\mu\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B(p+q)\rangle = -\iota\left[f_+\left(q^2\right)p_\mu q_\mu + f_-\left(q^2\right)q^2\right]. \tag{3.6.8}$$

At large recoil, there is no contribution of $f_-(q^2)$ term so above expression reduces into the following form

$$q_\mu\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B(p+q)\rangle = -\iota\left[f_+\left(q^2\right)p_\mu q_\mu\right], \tag{3.6.9}$$

$$\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B(p+q)\rangle = -\iota f_+\left(q^2\right)p_\mu, \tag{3.6.10}$$

$$\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{-1}{m_B+m_S}\left[(2p+q)_\mu q^2 - q_\mu\left(m_B^2-m_S^2\right)\right]f_T\left(q^2\right), \tag{3.6.11}$$

giving

$$q^\mu\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{-q^\mu}{m_B+m_S}\left[(2p+q)_\mu q^2 - q_\mu\left(m_B^2-m_S^2\right)\right]f_T\left(q^2\right), \tag{3.6.12}$$

$$q^\mu\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{-1}{m_B+m_S}\left[\left(2p.q+q^2\right)q^2 - q^2\left(m_B^2-m_S^2\right)\right]f_T\left(q^2\right).$$

This could be further reduced to

$$q^\mu\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{-1}{m_B+m_S}2p.qf_T\left(q^2\right), \tag{3.6.13}$$

and by equation of motion

$$f_T\left(q^2\right) = -\frac{m_b-m_s}{m_B-m_S}f_+\left(q^2\right), \tag{3.6.14}$$

we get

$$\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B(p+q)\rangle = \frac{2p_\mu f_+(q^2)}{m_B+m_S}. \tag{3.6.15}$$

By putting all these equations in Eq. (3.6.6), we get the following form of amplitude

$$\mathcal{M} = \frac{G_F}{2\pi\sqrt{2}}V_{tb}V_{ts}^*\Bigg\{\left[\iota\left(C_9^{eff}+C_{9l}^{NP}-C_{9'l}^{NP}\right)f_+\left(q^2\right)p_\mu+\frac{4\iota m_b}{m_B+m_S}\left(C_7^{eff}-C_{7'}^{eff}\right)p_\mu\right]\left[\bar{l}\gamma^\mu l\right]+$$

$$\iota\left(C_{10}+C_{10l}^{NP}-C_{10'l}^{NP}\right)\left(f_+\left(q^2\right)p_\mu+f_-\left(q^2\right)q_\mu\right)\left[\bar{l}\gamma^\mu\gamma_5 l\right]\Bigg\}, \tag{3.6.16}$$

$$= \frac{G_F}{2\pi\sqrt{2}}V_{tb}V_{ts}^*\left\{T_\mu^{1'}\left[\bar{l}\gamma^\mu l\right]+T_\mu^{2'}\left[\bar{l}\gamma^\mu\gamma_5 l\right]\right\}, \tag{3.6.17}$$

$$|\mathcal{M}|^2 = \frac{G_F^2\alpha^2}{8\pi}|V_{tb}V_{ts}|^2\Bigg[T_\mu^{1'}\times T_\mu^{1'*}\left\{Tr\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right\}+T_\mu^{2'}\times$$

$$T_\mu^{2'*}\left\{Tr\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right\}+T_\mu^{1'}\times T_\mu^{2'*}\left\{Tr\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right\}$$

$$+T_\mu^{2'}\times T_\mu^{1'*}\left\{Tr\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right\}\Bigg], \tag{3.6.18}$$

where

$$T_\mu^{1'} = \iota\left[\left(C_9^{eff}+C_{9l}^{NP}-C_{9'l}^{NP}\right)f_+\left(q^2\right)p_\mu+\frac{4\iota m_b}{m_B+m_S}\left(C_7^{eff}-C_{7'}^{eff}\right)p_\mu\right], \tag{3.6.19}$$

$$T_\mu^{2'} = \iota\left[\left(C_{10}+C_{10l}^{NP}-C_{10'l}^{NP}\right)\left(f_+\left(q^2\right)p_\mu+f_-\left(q^2\right)q_\mu\right)\right], \tag{3.6.20}$$

$$T_\mu^{1'*} = -\iota\left[\left(C_9^{eff}+C_{9l}^{NP}-C_{9'l}^{NP}\right)f_+\left(q^2\right)p_\mu+\frac{4\iota m_b}{m_B+m_S}\left(C_7^{eff}-C_{7'}^{eff}\right)p_\mu\right]^*, \tag{3.6.21}$$

$$T_\mu^{2'*} = -\iota\left[\left(C_{10}+C_{10l}^{NP}-C_{10'l}^{NP}\right)\left(f_+\left(q^2\right)p_\mu+f_-\left(q^2\right)q_\mu\right)\right]^*. \tag{3.6.22}$$

This equation is quite involved and to solve it we used the Mathematica 12.1. In order to see if everything is correct, by considering all the NP Wilson coefficients to be zero, the result is

$$\Gamma = 4.00573\times10^{-7}\text{GeV}. \tag{3.6.23}$$

i.e., reproducing correctly the SM result. The Plot for above result is represented in the Figure 3.6.1. To fit the data of different B mesons decays, the LHCb collaboration released an unprecedented accuracy of the Branching ratios using the whole Run 1 and 2 dataset. After performing a global Bayesian analysis of New Physics in Semi-leptonic B decays the highest probability density intervals [HPDI] are used to calculate the values of Wilson coefficients of weak Hamiltonian at low energies that are in great fit to the observed dataset [60] and these are summarized in Table 3.4. By taking different values of Wilson coefficients from the HPDI, decay widths are calculated and graphs are plotted as shown in Figure 3.6.2 and Figure 3.6.3. While to compare results, combine plots are shown in Figure 3.6.4.

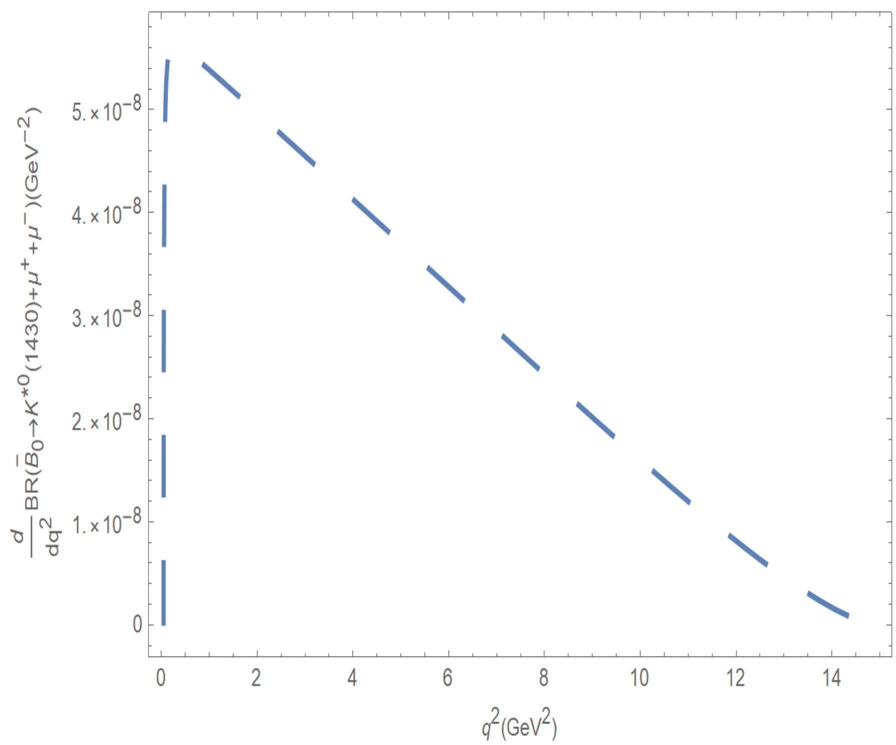

Figure 3.6.1: Branching ratio of $B \to Sl^+l^-$ when all WC's are taken zero

Table 3.6.1: Highest probability density intervals [HPDI] used to calculate the values of Wilson coefficients of weak Hamiltonian at low energies that are in great fit to the observed dataset [60].

	95%HDPI	ΔIC
$C^{NP}_{9,\mu}$	$[-1.10,1.05][-1.25,-0.72]$	$-1.1, 65$
$\{C^{NP}_{9,\mu}, C^{NP}_{10,\mu}\}$	$\{[-0.88,1.14],[-0.08,0.44]\}$	$0.3, 59$
	$\{[-1.24,-0.74],[-0.32,0.03]\}$	
$\{C^{NP}_{9,\mu}, C^{NP}_{9',\mu}\}$	$\{[-1.22,1.41],[-2.77,1.46]\}$,	$-2.3, 64$
	$\{[-1.34,-0.80],[-0.04,0.82]\}$	
$\{C^{NP}_{9,\mu}, C^{NP}_{10',\mu}\}$	$\{[-1.12,1.34],[-0.28,0.22]$,	$-2.2, 62$
	$[-1.38,-0.79][-0.36,0.06]\}$	
$\{C^{NP}_{9,\mu}, C^{NP}_{10,\mu}, C^{NP}_{9',\mu}, C^{NP}_{10',\mu}\}$	$\{[-1.10,1.40],[-0.18,0.60],[-2.66,1.32]$,	$-1.5, 62$
	$[-0.40,0.05],[-0.51,0.77],[-0.43,0.19]\}$,	
	$[-0.33,0.47]\},\{[-1.39,-0.81]$,	

Using these constraints, the values of the decay rate is given in Table 3.5. We can see that the results are close to the SM values leaving very small space for the NP.

Table 3.6.2: New Wilson coefficient values.

C^{NP}_{9}	$C^{NP}_{9'}$	C^{NP}_{10}	$C^{NP}_{10'}$	Γ(GeV)
0	0	0	0	4.00573×10^{-7}
1.40	1.32	0.60	0.47	3.93788×10^{-7}
1.32	1.32	0.47	0.47	4.00573×10^{-7}
-1.10	-1.10	0.47	0.46	4.00573×10^{-7}
-1.10	-0.18	-2.66	-0.33	5.47519×10^{-7}

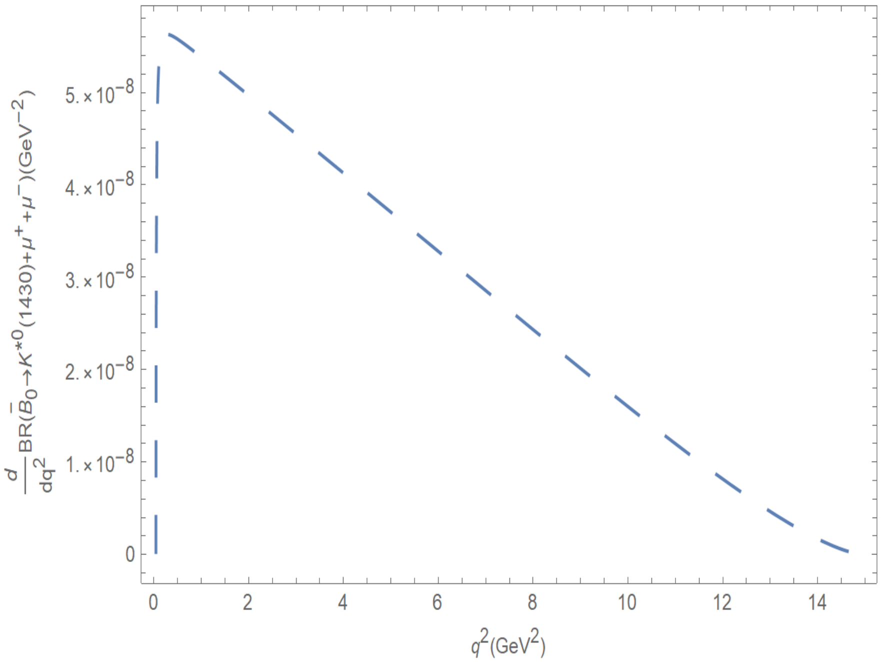

Figure 3.6.2: Differential Branching ratio calculated by taking WC values $C^{NP}_{9,\mu} = 1.40, C^{NP}_{10,\mu}, = 0.60, C^{NP}_{9',\mu} = 1.32, C^{NP}_{10',\mu} = 0.47$.

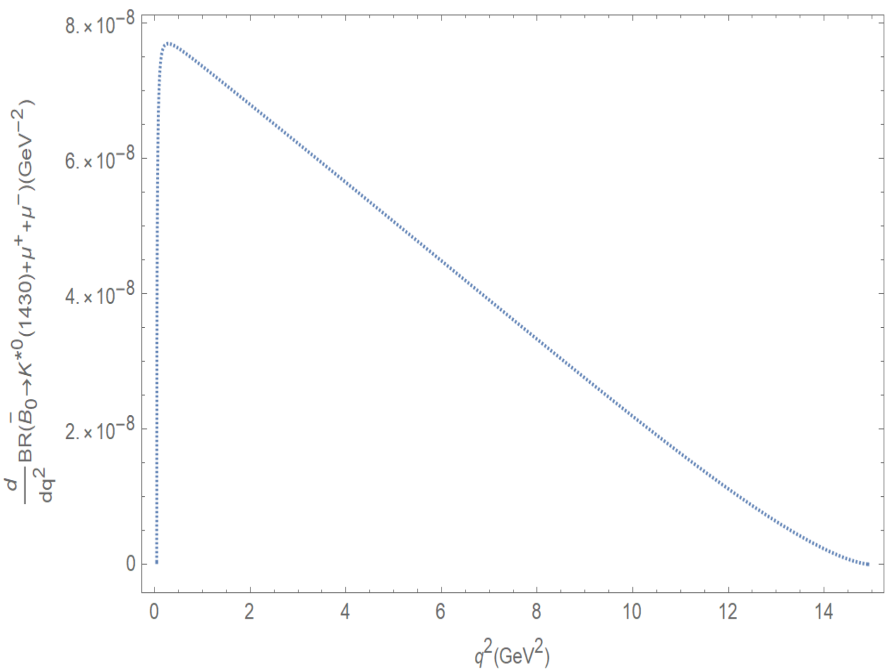

Figure 3.6.3: Differential Branching ratio by taking WC values $C^{NP}_{9,\mu} = -1.10, C^{NP}_{10,\mu}, = -2.66, C^{NP}_{9',\mu} = -0.18, C^{NP}_{10',\mu} = -0.33$.

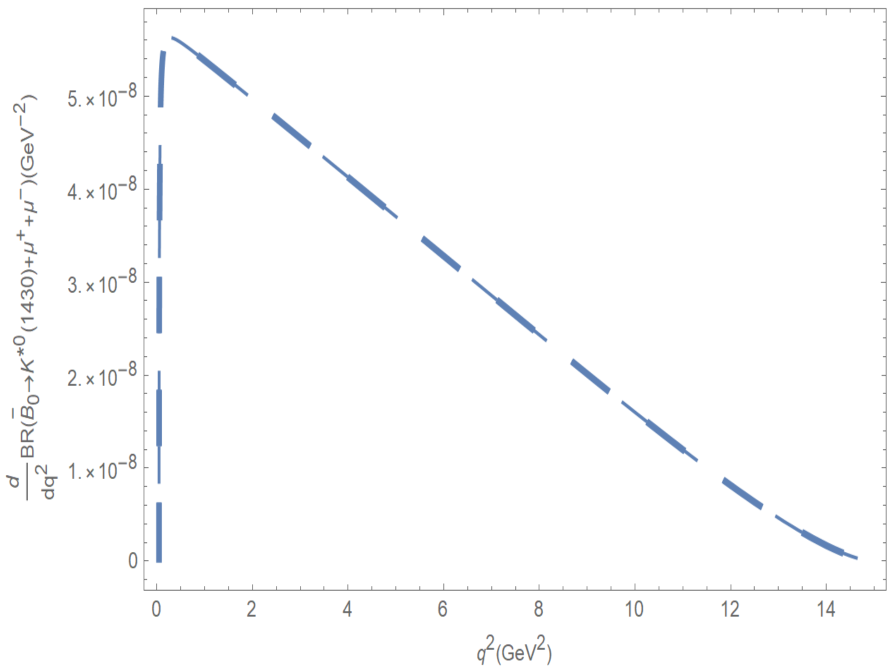

Figure 3.6.4: Combine graphs are plotted to compare SM and New Physics results.

3.7 Polarization Asymmetry

Another interesting parity violating observable that can be used to find the short distance contributions dominated by top quark loops is the lepton polarization asymmetry. In recent times, there has been a notable emphasis by Hewet about the importance of these asymmetries [61]. In finding the expression of the lepton polarization asymmetry, we choose the center of mass of dileptons as our reference frame. After taking into account the polarization of the lepton, the amplitude becomes

$$\mathcal{M}(B \longrightarrow Sl\bar{l}) = \frac{-G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \epsilon_\beta \epsilon_\alpha \left\{ T_\mu^1 \left[\bar{l}\gamma_\mu l\right] + T_\mu^2 \left[\bar{l}\gamma_\mu \gamma_5 l\right] + T^3 \left[\bar{l}l\right] \right\}, \tag{3.7.1}$$

$$\begin{aligned} \left|\mathcal{M}^2\right| = \frac{G_F^2 \alpha^2}{8\pi} \left|V_{tb} V_{ts}\right|^2 \Bigg[& T_\mu^1 \times T_\mu^{1*} \left\{ \frac{1+\gamma_5 \not{s}}{2} Tr\left(\not{p}_3 + m_3\right) \gamma^\mu \left(\not{p}_4 - m_4\right) \gamma^{\mu\prime} \right\} + T_\mu^2 \times T_\mu^{2*} \\ & \left\{ \frac{1+\gamma_5 \not{s}}{2} Tr\left(\not{p}_3 + m_3\right) \gamma^\mu \gamma_5 \left(\not{p}_4 - m_4\right) \gamma^{\mu\prime} \gamma_5 \right\} + T_\mu^1 \times T_\mu^{2*} \Bigg\{ \frac{1+\gamma_5 \not{s}}{2} Tr\left(\not{p}_3 + m_3\right) \gamma^\mu \\ & \left(\not{p}_4 - m_4\right) \gamma^{\mu\prime} \gamma_5 \Bigg\} + T_\mu^2 \times T_\mu^{1*} \left\{ \frac{1+\gamma_5 \not{s}}{2} Tr\left(\not{p}_3 + m_3\right) \gamma^\mu \gamma_5 \left(\not{p}_4 - m_4\right) \gamma^{\mu\prime} \right\} \Bigg], \end{aligned} \tag{3.7.2}$$

where $T^3 = 0$. Polarization asymmetry is a very interesting observable, because it helps to calculate the momentum, spin and other factors of the colliding particle. In the rest frame of a lepton, the four spin vector can be defined as

$$\left(s^{\mu}\right)_{r.s} = \left(0, \hat{\xi}\right). \tag{3.7.3}$$

Along the longitudinal direction of polarization of lepton, the unit vector is defined as

$$\hat{e}_L = \frac{\overrightarrow{p}_l}{|\overrightarrow{p}_l|}, \tag{3.7.4}$$

and the remaining two components that are normal and traverse to the plane are

$$\hat{e}_N = \frac{p_s \times p_l}{|p_s \times p_l|}, \tag{3.7.5}$$

$$\hat{e}_T = \hat{e}_N \times \hat{e}_L. \tag{3.7.6}$$

In this case, the two Mandelstam variables of the interest are s and u, i.e.,

$$s = \left(p_3 + p_4\right)^2, \tag{3.7.7}$$

$$u = \left(p_1 - p_4\right)^2 - \left(p_1 - p_3\right)^2, \left[4m_l^2 \leq s \leq \left(m_b - m_s\right)^2, -u\left(s\right) \leq u \leq u\left(s\right)\right], \tag{3.7.8}$$

$$u\left(s\right) = \sqrt{\left[s - \left(m_b \pm m_s\right)^2\right]^2}. \tag{3.7.9}$$

In the dilepton frame, the variable u is related to the angle between momentum of $B-$meson and l^+, i.e.,

$$z \equiv \cos\theta = u/u_{max}. \tag{3.7.10}$$

We can calculate all the three polarization components - but here the emphasis is on longitudinal polarization asymmetry, which is defined as

$$P_L\left(s'\right) = \frac{\frac{d\Gamma}{ds'}\left(\hat{e}_L\hat{\xi} = 1\right) - \frac{d\Gamma}{ds'}\left(\hat{e}_L\hat{\xi} = -1\right)}{\frac{d\Gamma}{ds'}\left(\hat{e}_L\hat{\xi} = 1\right) + \frac{d\Gamma}{ds'}\left(\hat{e}_L\hat{\xi} = -1\right)}, \tag{3.7.11}$$

where $\hat{\xi} = \pm 1$ denotes the right or left handed lepton in the end state and $\frac{d\Gamma}{ds'}$ means differential decay rate of $B-$meson. In SM, these symmetries arises due to interference of vector or magnetic moment and axial vector operators. The expression of the longitudinal lepton polarization asymmetry (P_L) is found to be

$$P_L\left(s'\right) = \frac{2\left(1-\frac{4r_l}{s'}\right)^{1/2}}{\left(1+\frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S}\mathrm{Re}\left[\varphi_S\left(C_9^{eff}f_+\left(q^2\right)\frac{1}{2} - 2\frac{C_7 f_T\left(q^2\right)}{1+\sqrt{r_S}}\right)\left(C_{10}\frac{f_+\left(q^2\right)}{2}\right)^*\right]. \tag{3.7.12}$$

As the kinematic variable has the form $s = \left(1-\frac{m_S}{m_B}\right)^2$ and $u_{max} = \sqrt{\lambda_B}\sqrt{1-\frac{4m_e^2}{t}}$, where λ is called Kallen function:

$$\lambda_B = m_B^4 + m_S^4 + t^2 - 2m_B^2 m_S^2 - 2m_B^2 t - 2m_S^2 t, \tag{3.7.13}$$

and $t_{max} = \left(\frac{m_B - m_S}{2}\right)^2$ and $t_{min} = 4m_e^2$. Writing

$$u = u_{max} z, \tag{3.7.14}$$
$$du = u_{max} dz,$$

Finally

$$\frac{d\Gamma(B \longrightarrow Sl\bar{l})}{dq^2} = \frac{1}{(2\pi)^3}\frac{1}{32m_B}\int_{u_{min}}^{u_{max}} |\mathcal{M}|^2\, du, \tag{3.7.15}$$
$$= \frac{1}{(2\pi)^3}\frac{1}{32m_B}\int_{-1}^{+1} u_{max}\, |\mathcal{M}|^2\, dz, \tag{3.7.16}$$

by using the Mathematica version 12.1, we plot the $P_L(s')$ that is given in Eq. (3.7.12) and the plot is shown in Fig. 3.7.1. This result is calculated in SM, where $P_L(s')$ of dileptons is around -0.92 for $2 \leq q^2 \leq 14$. Even if $m_l = 0$, then still we get non-zero value of polarization asymmetry at $\hat{s} = \left(1-\frac{m_S}{m_B}\right)^2$. Away from the end points, we till have $P_L(s') = -0.92$. Now taking into account new Wilson coefficient values we get

$$\mathcal{M} = \frac{G_F}{2\pi\sqrt{2}}V_{tb}V_{ts}^*\epsilon_\alpha\epsilon_\beta\left\{T_\mu^{1'}\left[\bar{l}\gamma^\mu l\right] + T_\mu^{2'}\left[\bar{l}\gamma^\mu\gamma_5 l\right]\right\}, \tag{3.7.17}$$

$$\left|\mathcal{M}^2\right| = \frac{G_F^2\alpha^2}{8\pi}\left|V_{tb}V_{ts}\right|^2\left[T_\mu^1 \times T_\mu^{1*}\left\{\frac{1+\gamma_5\not{s}}{2}Tr\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right\}+\right.$$

$$T_\mu^2 \times T_\mu^{2*}\left\{\frac{1+\gamma_5\not{s}}{2}Tr\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right\}+T_\mu^1 \times T_\mu^{2*}\left\{\frac{1+\gamma_5\not{s}}{2}\right.$$

$$\left.\left.\mathrm{Tr}\left(\not{p}_3+m_3\right)\gamma^\mu\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\gamma_5\right\}+T_\mu^2 \times T_\mu^{1*}\left\{\frac{1+\gamma_5\not{s}}{2}\mathrm{Tr}\left(\not{p}_3+m_3\right)\gamma^\mu\gamma_5\left(\not{p}_4-m_4\right)\gamma^{\mu\prime}\right\}\right]. \tag{3.7.18}$$

By putting above amplitude into Eq. (3.7.16) and doing the standard way of calculation we have

$$\frac{d\Gamma\left(B \longrightarrow Sl\bar{l}\right)}{dq^2} = \frac{G_F^2\left|V_{tb}V_{ts}\right|^2 m_B^5\alpha^2}{1536\pi^5}\left(1-\frac{4r_l}{s'}\right)^{1/2}\varphi_S^{1/2}\left[\left(1+\frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S\right], \tag{3.7.19}$$

where

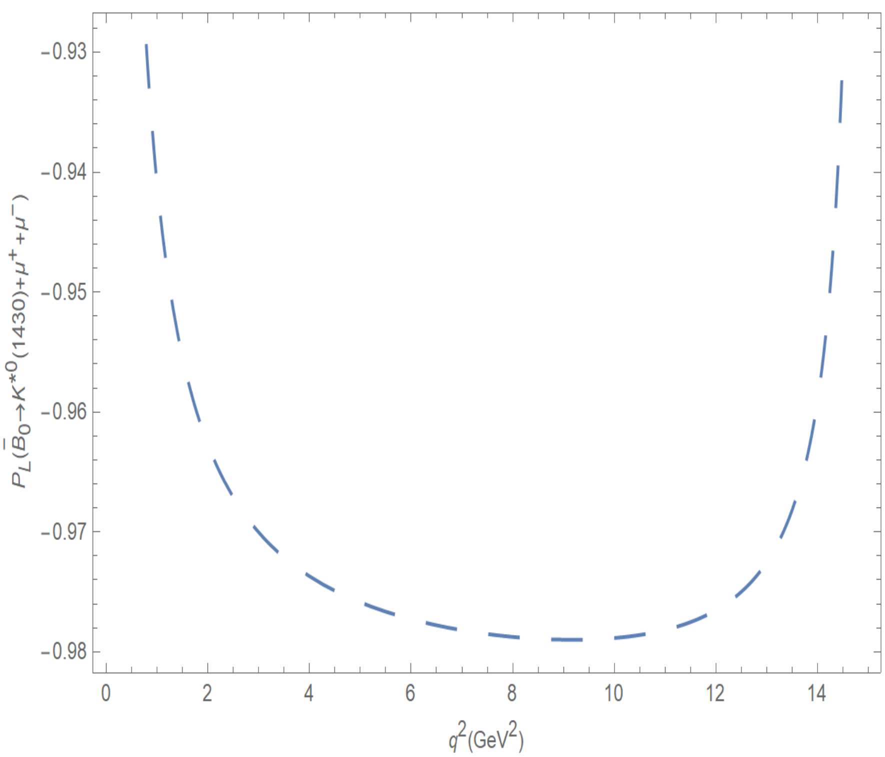

Figure 3.7.1: Longitudinal Polarization asymmetry in Standard Model

$$\alpha_S = \varphi_S \left| \left(C_9^{eff} + C_9^{NP} + C_{9'}^{NP} \right) \frac{f_+ (q^2)}{2} - 2\frac{C_7 f_T (q^2)}{1+\sqrt{r_S}} \right|^2 + \left| \left(C_{10} + C_{10'}^{NP} + C_{10}^{NP} \right) \frac{f_+ (q^2)}{2} \right|^2, \tag{3.7.20}$$

$$\delta_S = 6 \left| \left(C_{10} + C_{10'}^{NP} + C_{10}^{NP} \right) \right|^2 \{ (2\{1 + r_S\} - s) \} \left| \frac{f_+(q^2)}{2} \right|^2 + (1 - r_S) \operatorname{Re} \left[f_+ \left(q^2 \right) f_- \left(q^2 \right) - \frac{f_+ (q^2)}{2} \right] + s \left| f_- \left(q^2 \right) - \frac{f_+ (q^2)}{2} \right| \Bigg\}. \tag{3.7.21}$$

Using Eq. (3.7.11) to calculate the polarization asymmetry, we have

$$P_L(s') = \frac{2(\left(1 - \frac{4r_l}{s'}\right)^{1/2}}{\left(1 + \frac{2r_l}{s'}\right) \alpha_S + r_l \delta_S} \operatorname{Re} \Bigg[\varphi_S \left(\left(C_9^{eff} + C_9^{NP} + C_{9'}^{NP} \right) + f_+ \left(q^2 \right) \right) \frac{1}{2} - 2\frac{C_7 f_T (q^2)}{1+\sqrt{r_S}} \Bigg) \left(C_{10} + C_{10'}^{NP} + C_{10}^{NP} \right) \frac{f_+ (q^2)}{2} \Bigg)^* \Bigg]. \tag{3.7.22}$$

Using the value of the Wilson coefficients corresponding to NP, i.e., $C_9^{NP} = 1.40, C_{9'}^{NP} = 1.32, C_{10}^{NP} = 0.60, C_{10'}^{NP} = 0.47$, the result of the P_L is displayed in Figure 3.7.2.

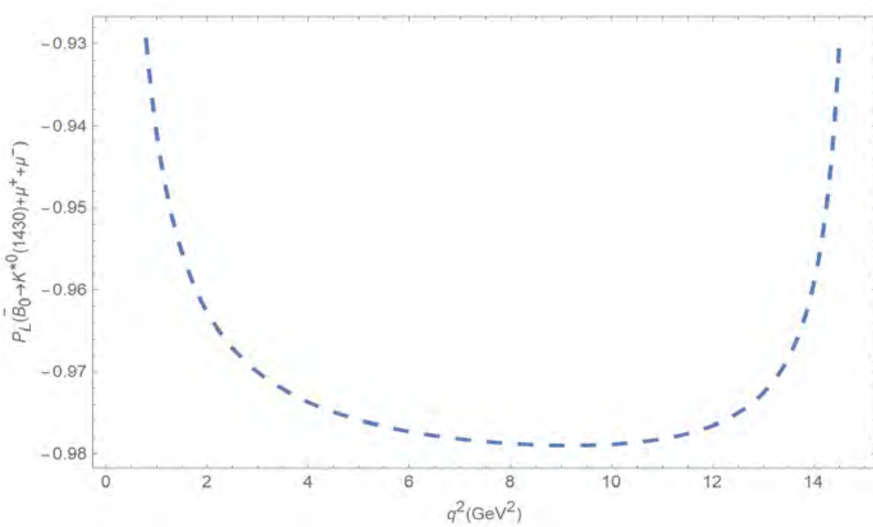

Figure 3.7.2: Longitudinal Polarization asymmetry by taking new values of WC as $C_9^{NP} = 1.40, C_{9'}^{NP} = 1.32, C_{10}^{NP} = 0.60, C_{10'}^{NP} = 0.47$

Here, $P_L(s')$ for dileptons is still-0.92 for 2 Ge$V^2 \leq q^2 \leq 14$ GeV^2, and hence the NP does not give any encouraging results.

In Fig. 3.7.3, we plotted the P_L to compare the difference between the SM and NP results. We can see that the current parametric space does not give any marked deviations from the SM predictions.

Same lines go for the normal and transverse components. Normal component, however, carries a novelty that it is $T-$odd observable because of the non-hermiticity of effective Hamiltonian. Effective Hamiltonian is non-hermition due to intermediate $c\bar{c}$ states. For normal polarization asymmetry, results in SM and then results by taking into account non-zero new Wilson coefficient values are plotted. Now by taking $C_9^{NP} = 1.40, C_{9'}^{NP} = 1.32, C_{10}^{NP} = 0.60, C_{10'}^{NP} = 0.47$, values we plot the graph and acquire value of Normal polarization asymmetry which is 0.30. Different plots are representing the behavior of Polarization asymmetry with respect to Wilsonian values. Its clear from the graphs that no new physics has been observed here if Wilson values are taken from the highest probability density intervals. Outside of these Intervals, we can't be sure about the results until observed. Wilson coefficients calculated at low energy effective Hamiltonian makes all observables in great match with the results of observables of Standard model "SM".

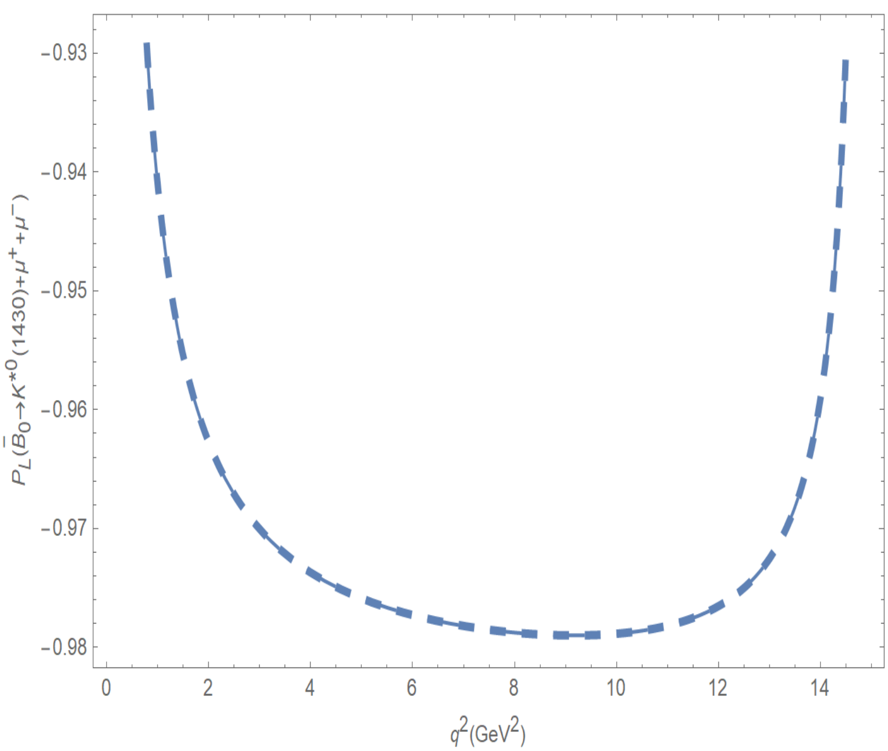

Figure 3.7.3: Combine plot of SM and NP longitudinal asymmetry.

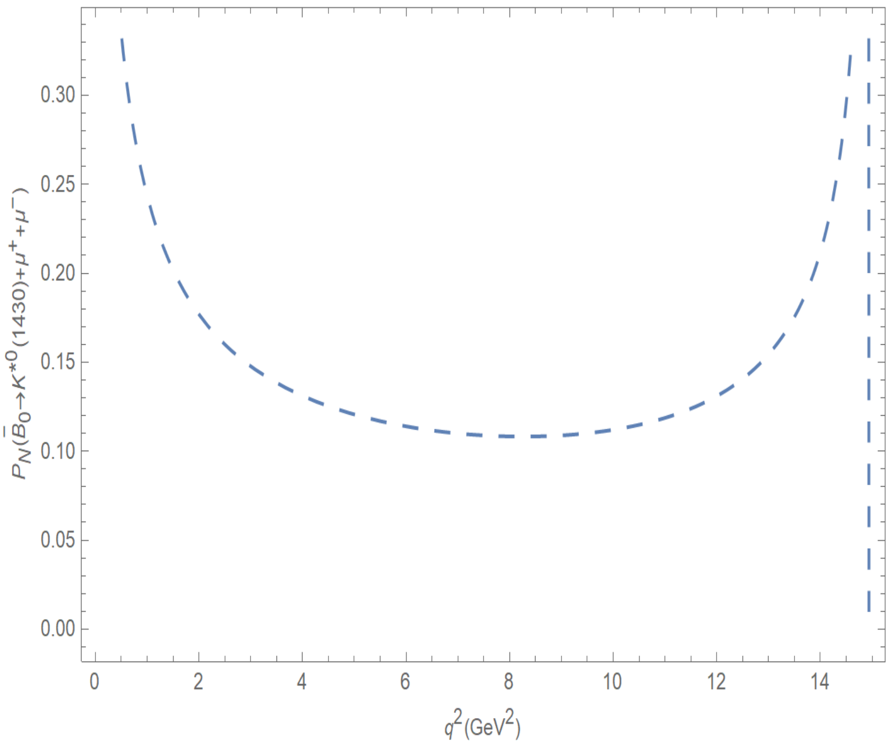

Figure 3.7.4: Normal Polarization in SM.

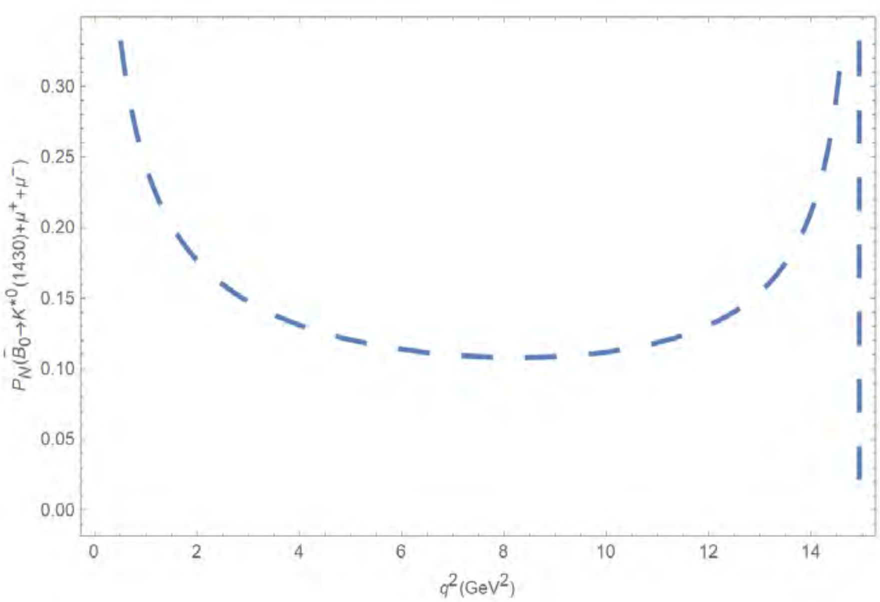

Figure 3.7.5: Normal Polarization by taking values $C_9^{NP} = 1.40, C_{9'}^{NP} = 1.32, C_{10}^{NP} = 0.60, C_{10'}^{NP} = 0.47$.

Now plotting all graphs combined to see if there is any NP.

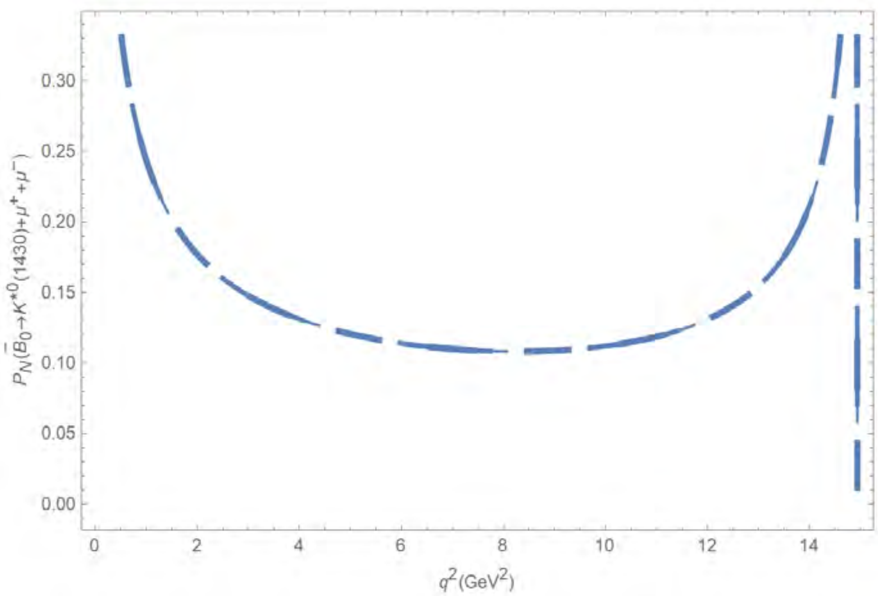

Figure 3.7.6: Combine plots of SM and NP are plotted to compare results for Normal polarization.

These all plots are drawn by taking values of new Wilson coefficients from the Table 3.4. Transverse polarization asymmetry does not contribute here. Average value of Longitudinal polarization asymmetry is -0.92 and of the Normal polarization asymmetry is 0.30, for almost all the values of the WCs.

Chapter 4

Summary and Conclusion

Several results from the last few decades have some $(1-3)\,\sigma$ disagreement with the Standard Model results and the FCNC decays involving $b \to s$ transitions are the pertinent ones. As for the decay $\bar{B}_0 \to K_0^* l^+ l^-$decay, there were discrepancies in different physics observables, where the lepton flavor universality $R_K \equiv (B \to K\mu^+\mu^-)\,/\,(B \to Ke^+e^-)$ is very important. For SM predictions of different physical observables, the form factors, which are non-perturbative quantity and the main source of uncertainties, are the model dependent quantity. In this dissertation, first we reviewed the results of the form factors presented in [49]. Using the framework of light-cone sum rules (LCSR), we calculated the form factors for $\bar{B}_0 \to K_0^*\,(1430)\,l^+l^-$ decay using the leading Fock states up-to twist 3. Interesting, the form factors associated with the scalar currents in $B \to S$ transitions are twice as large as that for the B to pseudo-scalar (P) case. The form factors $f_{+,-,T}\,(q^2 \equiv s)$ we calculated here satisfy the heavy quark symmetry relations. Using these form factors, we verified that the corresponding Branching ratio for $\bar{B}_0 \to K_0^* l^+ l^-$ is of the order of 10^{-7} and the average lepton polarization asymmetry is approximately 1 in the SM.

It is already mentioned that the SM predicts the lepton universality violation ratio to be 1 - but according to LHCb measured data in the bin q^2 ranged from $1 \leq q^2$ (GeV^2) ≤ 6 , this ratio was about $0.745^{0.090}_{-0.074}$ that is 2.6σ deviating from the SM results. It was speculated that NP- amplitudes must have destructive interference with the SM amplitude, that is why this ratio is less than 1. Several research teams have conducted multiple analyses, yielding various solutions to identify the Lorentz structure of the New Physics operators and their relevant

Wilson coefficients that can be used to explore New Physics effects [60]. Here, LUV NP is not found because of no contribution of Wilsonian coefficients C_9. However, the NP can be found if charm loop effects are taken into account. If long distance effects are considered too, may be there would be physics beyond the SM as C_9 would be giving information about NP.

In the last part of the dissertation, we studied the $\bar{B}_0 \to K_0^* l^+ l^-$ decay using the model-independent approach using the NP WCs calculated in [60]. The impact of these WCs on the branching ratio, forward-backward asymmetry and the different lepton polarization asymmetries is calculated. It was found that the results of these observables are consistent with their corresponding Standard Model predictions [49], leaving very small space for the NP in these decays.

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