Effect of Self-Gravity in Dust Contaminated Plasma

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بسم الله الرحمن الرحيم

IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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CERTIFICATE

This is to certify that the theoretical work in this dissertation has been carried out by ***Amna Shahzadi*** under my supervision in Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan.

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To

My Honourable Teachers and Loving Parents

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Amna Shahzadi

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Chapter 1

Introduction

1.1 Plasma, the fourth state of matter

The term plasma was first introduced by Tonk and Langmuir to describe the glowing discharge which is produced in a electrical discharge tube. Plasma is a Greek word that means "moldable material" or "jelly". The plasma is a quasi-neutral gas of charged and neutral particles which exhibit collective behavior. It is believed that plasma makes up more than 99 percent of our universe. Because of this, it is commonly referred to as the fourth state of matter. Its properties differ significantly from those of the other three states of matter.

Plasma is formed by ionizing neutral gases, which contain equal amounts of positive and negative charges (electrons and ions), as well as neutral particles. Because the two fluids are attempting to electrically neutralize each other on macroscopic length scales, plasma is referred to as quasi-neutral (quasi means that there are minor departures from perfect neutrality). When ions and electrons move together, they engage via long-range coulomb force, which is more efficient than short-range electrical interactions. Plasma is dominated by collective movements, as more particles are subjected to the same long-range coulomb force. Rather than uncorrected interactions between nearby particles, it involves linked movements of a large number of particulates.

Plasma exhibits several novel features that are not present in an ordinary gas. It is because a plasma is a dynamical fluid and a good electrical conductor. As a result, plasma is defined as an electrically neutral material. It includes a high number of free electrons and ions that interact and show collective behavior as a result of the long range interactions [1].

1.2 Criteria for Plasma

Unless it meets specific criteria, any ionized gas cannot be considered plasma. The plasma must meet the following requirements:

1.2.1 Macroscopic Neutrality

A plasma system is macroscopically neutral in the absence of exogenous imbalance forces. This means that in an equilibrium plasma system, there is a volume large enough to accommodate a high number of particles yet too tiny in comparison to the typical length scale of plasma parameter changes (temperature and density). The net electrical charge in the above mentioned region is zero, thus the internal space charge field cancel each other, leaving the bulk of plasma neutral. In the event of a charge imbalance, an electrostatic potential energy is generated to restore the plasma's quasi-neutrality across a specific length scale. We named the length of plasma the Debye length [2].

1.2.2 Debye Shielding

In the characterization of plasma, the shielding effect is critical. Due to an electrostatic field is shielded out due to the plasma species collective reaction inside a Debye radius (λ_{Dr}) separation of order that is

$$\lambda_{Dr} = \left(\frac{K_B T_e}{4\pi n_e e^2}\right)^{1/2} \tag{1.1}$$

Where

K_B =Boltzman constant

T_e = Temperature of electrons

n_e = Number density of electron

The Debye sphere is a sphere with a radius of λ_{Dr} within the plasma, in which N is the number of particles,

$$N = \frac{4}{3}\pi n_e \lambda_{Dr}^3 \tag{1.2}$$

Now the first condition of plasma if L shows the length of the plasma system

$$L \gg \lambda_{Dr} \tag{1.3}$$

Plasma particles are Debye shielded as a result of collective activity, therefore the second criterion for plasma is

$$N \gg 1 \tag{1.4}$$

Or

$$n_e \lambda_{Dr}^3 \gg 1 \tag{1.5}$$

This shows that number of particles in Debye sphere should be very large.

1.2.3 Frequency of Plasma

Plasma's macroscopic charge neutrality is an essential property. In the event that a plasma particles are disturbed by a sort of external imbalance force tends to return plasma to its initial charge neutrality. These people as a group the frequency of electrons in a plasma system characterizes its behavior plasma frequency (ω_{pe}) is a natural frequency that is determined by

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \tag{1.6}$$

This demonstrates that the plasma frequency is influenced by electron mass and density. Collisions of electrons and neutrals tend to reduce plasma oscillation by a factor of two. Their amplitude is progressively diminishing. In order for plasma oscillations to be observed, It is required that the electron-plasma frequency (υ_{pe}) be somewhat damped higher than the frequency of electron-neutral collisions υ_{en}

$$\upsilon_{pe} > \upsilon_{en} \tag{1.7}$$

where $\upsilon_{pe} = \frac{\omega_{pe}}{2\pi}$.Otherwise, the collisions will drive the electrons to be in perfect equilibrium with the neutrals, and the plasma will act like a neutral gas; consequently, the plasma's third criteria is

$$\omega\tau > 1 \tag{1.8}$$

where $\tau = 1/\upsilon_{en}$, denotes the average time an electron spends between collisions with neutrals, while ω is angular frequency of typical plasma oscillations [2]. So an ionized gas will be termed as plasma if it satisfy the above three conditions.

1.3 Dusty Plasma

A dusty plasma is a low-temperature multi-species fully or partially ionized gas containing electrons, ions, dust grains (charged, massive, solid particles) and neutrals. The dust grains are generally graphite, magnetite, silicates, and amorphous carbon. Grains may have different size (micrometer or sub-micrometer size), nature and origin. Grains being massive ($10^{10} - 10^{12}$ ions mass) may be positively are negatively charged ($10^3 - 10^5$ elementary charge) depending on the surrounding environments and charging mechanisms. In laboratory condition, thermal velocity of electrons being higher than the ions gives usually negative charge to the dust grain. In astrophysical conditions, dust grain may become positively charged due to photoionization and secondary electron emission. The desirable feature of studying dusty plasmas in the laboratory is that dust particles can be observed visually since dust particles are big enough and their dynamics being slow enough due to their high inertia can be tracked and recorded using fast video camera.

A plasma having dust particles may be termed as dusty plasma or dust in a plasma depending on three fundamental lengths: the dust grain size r, their average separation a and and the Debye length λ_D:[2]

a) $r << \lambda_D < a$ dust in plasma, where dust particles are considered isolated.

b) $r << a < \lambda_D$; dusty plasma, in which the effects of neighboring particles is significant.

Dusty plasmas may be found in many astrophysical environments like Inter-stellar clouds, Circumstellar clouds, Molecular clouds, Asteroid zones, Interplanetary dust, Earth's magnetosphere, Comet tails, nebula and Planetary rings etc. [5, 6, 7]. Dusty plasmas have two distinct characteristics that set them apart from regular plasmas.

1. The plasma frequency, as well as the cyclotron frequency of dust grains and ions, are widely separated due to the vast size and mass of the dust grains, resulting in different frequency modes arising from the dust and ions inertial effects.

2. The grain charges are quite high and can change, resulting in novel phenomena in grain plasma as well as grain interactions, i.e. the charge fluctuation of dust particles dampening waves that would otherwise travel as regular frequency modes [3, 4].

Because of their importance in understanding the space environment, such as cometary tails, planetary rings, asteroid zone, and lower ionosphere, there has been a growing interest in the study of dusty plasmas[8, 9].

1.4 Characteristics of Dusty Plasma

A dusty plasma is formed by a combination of electrons, ions, neutrals, and charged dust grains or macro-particles. The dust particles or grains come in a variety of forms and sizes, but they are classified as point particles based on their distinctive lengths. We can distinguish between "dust in a plasma" and "dusty plasma" based on the characteristic lengths. Dust in a plasma is referred to as a dusty plasma. when $r_D \ll \lambda_D < b_m$(The dust-charged grains are considered as isolated protected particles), when if $\lambda_D > b_m \gg r_D$, This relates to "dusty plasma," which indicates that the charged dust grains participate in collective activity. where λ_D is Debye length of plasma, b_m is mean inter grain distance and r_D represent radius of dust particle[4].

1.4.1 Macroscopic Neutrality in Dust

The dusty plasma is considered to be a quasi-neutral in the absence of an external imbalance force, which implies that the net electrical charge in the dusty plasma disappears. The quasi-neutrality criterion is met when the system is in equilibrium.

$$q_i n_{io} = q_e n_{eo} - q_D n_{D0} \tag{1.9}$$

where n_{eo}, n_{io} and n_{D0} represent the number density of electrons,ions and dust respectively and $q_D = -Z_D e$, $(Z_D e)$ indicate the dust charge, when the particle is positive(negative), where Z$_D$ and Z_D are the charge state of dust and charge state of ion respectively and $q_D = Z_D e\ Ds$ the charge ion.

1.4.2 Dusty Plasma Debye Shielding

Any external disturbance or electric field from any source with a non-zero potential is filtered by the plasma particles, which is one of the plasma system's essential features. This plasma phenomenon is known as Debye shielding, and it results in a distance termed the Debye length, across which other charge particles in a plasma experience the influence of the charge particle's electric field For a dusty plasma, the Debye length is defined as

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{(\lambda_{De} + \lambda_{Di})^{1/2}} \tag{1.10}$$

where λ_D shows the Debye length of dusty plasma, where $\lambda i = (TD/4\pi n_{D0} e^2)^{1/2}$ and $\lambda_{De} = (Te/4\pi n_{e0} e^2)^{1/2}$ indicate the ion and electron Debye length, respectively. Dusty plasma contain negatively charged dust particles have n$_{D0} \gg$ n$_{e0}$ and T$e \geq$ T$_D$ for example $\lambda_{De} \gg \lambda_{De}$, so we have, $\lambda_D \simeq \lambda_{Di}$. while in positive dust particles, we have, $\lambda_D \simeq \lambda_{De}$.

1.4.3 Dust Plasma Frequency

Every plasma system must have some degree of quasi-neutrality. When dusty plasma is temporarily disrupted from equilibrium, the collective movements of charge particles in the plasma attempt to restore plasma quasi neutrality, which may be represented by the plasma frequency, indicated by ω_p. The plasma frequency may be calculated in the following way.

i) Using the continuity equation for c-th species

$$\nabla_t n_c + \nabla \cdot (n_c v_c) \tag{1.11}$$

ii) equation of momentum is

$$\partial_t v_c + (v_c \cdot \nabla)\, v_c = -\frac{q_c}{m_c} \nabla \Phi \tag{1.12}$$

iii) Poisson's equation is given as

$$\nabla^2 \Phi = -4\pi \sum q_c n_c \tag{1.13}$$

we assume small perturbation like we use $n_c = n_{c0} + n_{c1}$ and assume $n_{c1} \ll n_{c0}$,we obtained the following equation by linearize and combine the above equation

$$\partial t^2 \nabla^2 \Phi + 4\pi \sum \frac{q_c^2 n_{c0}}{m_c} \nabla^2 \Phi = 0 \tag{1.14}$$

By integrating the equation (1.14) twice w.r.t the region r(x,y,z) with the boundary condition [$\Phi = 0$ at equilibrium (r=0)]. We get the differential form of equation which is

$$\frac{\partial^2 \Phi}{\partial t^2} + \omega_p^2 \Phi = 0 \tag{1.15}$$

where the term ω_p^2 is equal to

$$\omega_p^2 = \sum_d \frac{4\pi q_D^2 n_{D0}}{m_D} = \sum_d \omega_{pc}^2 \tag{1.16}$$

and $\omega_{pc}^2 = \sqrt{\frac{4\pi q_c^2 n_{c0}}{m_c}}$ shows the frequency of plasma related to plasma species d. It indicates that due to different mass of plasma species i.e. dust, ions and electron. Their frequency of oscillation are different.

1.4.4 Coulomb Coupling Parameter

The electrostatic potential energy (together with the screening effect) is derived by assuming two dust grains, each with same charge q, separated at a distance of b is

$$E_c = \frac{q^2}{b} \exp\left(-\frac{b}{\lambda_D}\right) \tag{1.17}$$

and the thermal energy of dust is $K_B T$. So the Coulomb coupling parameter is given by

$$\Gamma_c = \frac{Z_D^2 e^2}{b k_b T} \exp\left(-\frac{b}{\lambda_D}\right) \tag{1.18}$$

If $\Gamma_c \gg 1$, it shows that dust plasma is a strongly coupled system, when $\Gamma_c \ll 1$, represents weakly coupled dusty plasma. New phenomena, such as the creation of Coulomb crystals, may arise in dusty plasma with high coupling ($\Gamma_c \geqslant 170$) [4].

1.5 Plasma Instabilities

If the disturbance given in the system grows in amplitude exponentially with time, the system never returns to its original position thus we have what is called as instability where the free energy of plasma gets converted into growing mode resulting in generation of different waves. The plasma instabilities has an important role for controlled thermonu- clear fusion and in laboratory experimental situations. classify plasma instabilities in two ways:[8]

Macro-Instabilities

The macro-instabilities are the result of coordinate space non equilibrium. Using ‡fluid theory or MHD theory one can study macro-instabilities. Some familiar examples of macro-instabilies are Jeans self-gravitational instability, Rayleigh-Taylor instability and Kelvin-Helmholtz instability.

Micro-Instabilities

The micro-instabilities, are usually driven by velocity space anisotropy, an inner process in the plasma. The micro-instabilities can be treated theoretically by kinetic equations. Some familiar examples of micro-instabilies are Beam-driven instability, loss-cone instability, whistler instability.

1.6 Self Gravitational Instability

Gravitational instability also known as Jeans instability, who first proposed it, has a great interest in astrophysics as it is the most basic instability in astrophysical plasma. In astrophysical objects, the collapse of an object is assigned to a gravitational force which is responsible for the production of jeans instability. The gas pressure force (hydrostatic) gives a threshold for the instability of jeans. The dynamics of large bodies like stars, planets, and satellites is typically governed by the gravitational force, that is always attractive and cannot be shielded. The possible explanation for formation of solar system lies in this instability. After the work of jeans there has been a lot of interest in the study of this instability in astrophysics because of the interest for the mechanism of the formation of stars. It has been suggested that the star is formed by the condensation of gases. After the work of Jean for the gravitational instability of a homogeneous infinite medium several authors have tries to study this problem under various assumptions and modification. Chandrasekhar [9] has shown that the criterion of jeans remains unchanged in the presence of magnetic field and rotation. The magnetic field's direction has a small Effect on the criterion of jeans. Larson [10] has made of numerical calculations of the dynamics of a spherical collapse of a protostar solar mass with initial condition under its own force of gravity. Krautschneider [11] has discussed the problem of the formation of stars via the gravitational contraction of clouds of grains. Cadez [12] has studied the Jeans criterion of a static self-gravitating cloud. Moreover many authors havestudied the self gravitational instability of dusty plasma with various Effects i.e. Finite Larmor radius (FLR) Effect, Hall Effect, Electron Inertia Effect, Thermal and Radiative Effects, Collisional Effects, Porosity and Rotation Effects.

Moreover from the above discussion, we see that the analysis of the gravitational instability of plasma is a current area of research. Also the study of gravitational instability is important to understand the problems associated with the formation of stars, the formation of molecular clouds and the formation of large-scale structures.

We consider a neutral fluid in which quantities such as the $\rho(x;t)$, $P(x;t)$, $T(x;t)$, *and* $v(x;t)$ can all defined at any point. We also include gravitational Effects with the gravitational potential Ψ. The behavior of the fluid is governed by following fundamental equations.

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho v\right) = 0 \tag{1.19}$$

$$\frac{\partial v}{\partial t} + v.\nabla v = -\nabla\Psi - \frac{\nabla p}{\rho} \tag{1.20}$$

where the gravitational potential Ψ satisfies Poisson's equation.

$$\nabla^2\Psi = 4\pi G\rho \tag{1.21}$$

describe the self-gravity of the fluid (G is constant of gravitation). We assume that initially the fluid has uniform density ρ_0 and pressure P_0, and zero velocity. We introduce a small perturbation such that

$$\rho = \rho_0 + \rho_1$$

$$P = P_0 + P_1$$

$$v = v_1$$

$$\Psi = \Psi_0 + \Psi_1$$

Using these in continuity, momentum and Poisson's equation and upon linearization we get

$$\frac{\partial \rho_1}{\partial t} + \rho_0\nabla.v = 0 \tag{1.22}$$

$$\rho_0 \frac{\partial v_1}{\partial t} = -\nabla\Psi_1 - c_s^2 \nabla\rho_1 = 0 \tag{1.23}$$

We use the relation

$$P_1 = c_s^2 \rho_1 \tag{1.24}$$

$$\nabla^2 \Psi_1 = 4\pi G \rho_1 \tag{1.25}$$

We take the time derivative of Equation (1.19) and the divergence of Equation (1.20) and employing Equation (1.22) to eliminate $\nabla^2\Psi_1$, we arrive at

$$\frac{\partial \rho_1}{\partial t^2} - c_s^2 \nabla^2 \rho_1 = -4\pi G \rho_0 \rho_1 \tag{1.26}$$

We assume the perturbation to be proportional to $expi(kx - \omega t)$ therefore

$$\nabla \rightarrow ik$$

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

and we obtain relation between wave number and frequency given as

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0 \tag{1.27}$$

Assuming a non-vanishing wave number k we can distinguish two different cases:

1. If k is sufficiently large then: $c_s^2 k^2 - 4\pi G \rho_0 > 0$, The ω is real and equilibrium is stable with respect to this perturbation (amplitude does not increase with time).

2. If $c_s^2 k^2 - 4\pi G\rho_0 < 0$ then ω is in the form of $i\alpha$ where α is real. The equilibrium is unstable, therefore there exist perturbations which grow exponentially with time.

3. And for $c_s^2 k^2 - 4\pi G\rho_0 = 0$ we get the critical wave number

$$k_j = \left(\frac{4\pi G\rho_0}{c_s^2}\right)^{\frac{1}{2}} \tag{1.28}$$

or a critical wavelength (jeans wavelength) ($\lambda = \frac{2\pi}{k}$)

$$\lambda_j = \left(\frac{\pi}{G\rho_0}\right)^{1/2} c_s \tag{1.29}$$

Therefore the perturbation with wavelength $\lambda > \lambda_j$ ($i.e. k < k_j$) is unstable. This condition is called the Jeans criterion after James Jeans who derived it in 1902.

1.7 Dust Particles in Space

A complicated system of electrons, numerous types of ions, and negatively or positively charged dust particles can be described as the dusty plasma. The dusty plasma covers the great part of space and has a long history. We'll talk about the important role of dusty plasma in space.

1.7.1 Interplanetary space

It is full of dust called as 'interplanetary dust'. Its existence was known from zodiacal light. Which is a band of light in the night sky that is considered to represent reflected sunlight from cometary dust concentrated in the zodiac plane, or ecliptic. The light is seen in the tropics, where the ecliptic is almost vertical, in the west after dusk and in the east before dawn.

1.7.2 Comets

A brilliant comet serves as an excellent cosmic laboratory for studying dusty plasma interactions and their physical and dynamic consequences. Comets are formed up of grains of non-volatile dust particles and frozen gases and are tiny, fragile, and irregular in shape. It has distinct and dynamic features, such as a coma, which is a cloud of diluting components that expands in size and brighter as the comet approaches the Sun. A dust tail is formed by the sun's radiation pressure and solar wind, resulting in a long white tail of dust particles propelled away from the sun, as shown in Figure (1.1) below.

1.7.3 Planetary Rings

The presence of dust particles in the ring of our solar system's big planets (Jupiter, Saturn, and Neptune) is widely known. Since Galilee discovered the ring of Saturn in 1610, astronomers Fig(1.1). Two different tails,one is thin blue plasma tail and second is broad white dust tail. Fig (1.2); The radial spokes in Ring B is showed by Saturn's ring have struggled to fully comprehend it. A, B, and C are the names of the three major rings as shown in figure (1.2).

Figure 1-1: Figure 1.1: Hale-Bopp comet showing two different tails.

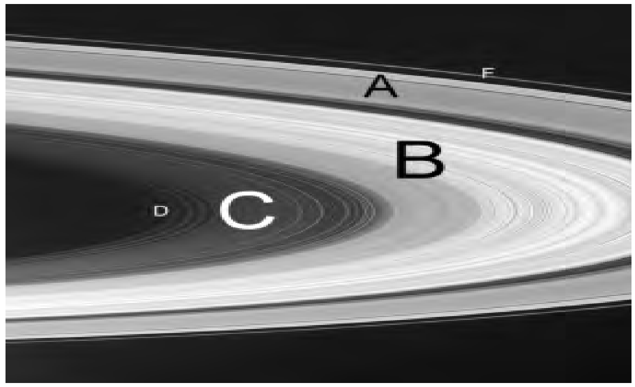

Figure 1-2: Figure 1.2: Saturn's ring showing the radial spokes in ring B.

1.7.4 Earth Atmosphere

Dust particles are prevalent in this region of the earth's atmosphere during the polar summer mesopause, which is detected at altitudes of 80 to 90 km. The presence of dust particles causes noctilucent clouds (NLCs), polar mesospheric summer echoes (PMSE), and substantial radar back scatter, all of which occur during the creation of the polar summer mesopause.

1.8 Fusion Plasma Devices

The dust grains are found in the fusion devices. The dust impurities are produced by arcing, erosion and sputtering of wall of container. In fusion devices the dust particles are ranging in size from 10nm to 100micrometer and some are also in millimeters. The dust particle in fusion device may be radioactive for example tritium, the presence of such a type of dust may also create hazard of explosion. At present time these challenges are overcome by employing different type of technique.

1.9 Aspects of Dusty Plasmas

The physics of the dusty plasmas has gained more and more interest over the last few years from academic point of the view as well as from view of its new aspects in the space and the semiconductor technology, astrophysics, biophysics, crystal physics, plasma chemistry,magnetic fusion devices etc. Due to its vital importance some of the application of dusty plasma are given below.

1. Dusty plasma devices used to produced and study dusty plasma in laboratory.

2. By using technique of PECVD (plasma enhanced chemical vapor deposition) a thin film layer is applied to material to enhanced the surface properties.

3. Plasma used in fabrication of microelectronics for example semiconductor chips, solar cell and flat panel display.

1.10 Motivations

The physics of dusty plasma is rapidly growing subject of science. It is found that collective process in a dusty plasma would have an excellent future perspective, because of its potential applications in different plasma space environment such as in the astrophysical environment.

The main aim of early dusty plasma investigations was obtaining a good control of contamination in plasma processing reactors, either by eliminating dust particles from gas phase or by preventing them from getting in contact with the surface. This task has been accomplished and the knowledge got in the course of these elaborate studies can be utilized in research directions. Applications of macroscopic grains is one of the recent developments in the material science. Now dust particles in the plasma are not considered as unwanted pollutants.The multicomponent dusty plasmas are usually found in many low temperature laboratory devices and industrial processes. Besides the fundamental knowledge and industrial applications, the increasing interest in dusty plasma has inducced the development of numerous experimental diagnostics. All these applications make dusty plasma a rapidly expanding field of research, which in the coming decade will provide a large amount of novel and exciting developments in fundamental studies as well as in plasma technology.

In this dissertation, we have investigated the propagation of electrostatic modes in self-gravitating dusty plasma, which consist of extremely massive, negatively charged dust grains, non-thermally distributions and Boltzmann distributed electrons. It has been observed that self-gravitating field destabilize these low frequency dust electrostatic modes. It is stressed here that these results may be useful for understanding the electrostatic disturbances in astrophysical dusty plasma systems.

1.11 Layout of Dissertation

The present dissertation is related to self gravitational instability in magnetized dusty plasma. The dissertation is organized as follow: In the first chapter, introduction to plasma, dusty plasma, fundamental characteristics of the dusty plasma, occurrence of dusty plasma in space and self-gravitational instability are briefly described. At the end of this chapter, we have also presented the motivations for studying electrostatic waves in self-gravitating dusty magneto-plasma.

In the second chapter, we focus on electrostatic waves in nonuniform dusty magnetoplasma and study the dispersion properties of long frequency electrostatic waves and Shukla-Varma (SV) mode.

Chapter 3 focuses on a physical model, governing equations and the dispersion relation for coupled dust ion-acoustic and Jeans modes are derived and a purely growing instability related to an ion response is studied in self gravitating unmagnetized plasma.

In chapter 4, we have discussed susceptibility relations for electron and ion resulting from kinetic or hydrodynamic theory with the detailed calculation of dust susceptibility relation. Further, using these susceptibilities relations we obtained dispersion relation and the influence of an external magnetic field in a self-gravitating dusty plasma in several interesting cases is discussed.

In chapter 5 we derive the dispersion relation for the low frequency electrostatic modes propagating parallel and perpendicular to the external magnetic field in cold magnetized self gravitational dusty plasma.

Chapter 2

Electrostatic Waves in Non-Uniform Dusty Magnetoplasma

2.1 Introduction

In this chapter, we shall study linear properties of two dimensional convective cell motion in a multi-component magnetoplasma composed of electrons, ions, and charged dust grain in the presence of the equilibrium density gradients [13]. It is a well established fact that all the plasma systems, especially the dusty plasma, always contain some region of the inhomogeneity which cause the drift motions and waves associated with them in the magnetized dusty plasma. Thus, a non-uniform dusty magnetoplasma is considered containing the immobile dust grains and the equilibrium density gradient $\frac{\partial n_{s0}}{\partial x}$ (the unperturbed plasma number densities n_{s0}(x) are assumed as non-uniform along the x-axis) and study the dispersion properties of low-frequency plasma (in comparison with ω_{ci}), long-wavelength (in comparison with ion gyroradius) electrostatic and electromagnetic waves. An external magnetic field is applied along z-axis.

2.2 Basic Model Equations

The following basic equation are used to obtain dispersion relation of convective cell frequency.

2.2.1 Continuity equation

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha v_\alpha) = 0 \tag{2.1}$$

where $n_\alpha = n_{\alpha 0} + n_{\alpha 1}$ is the number density, v_α is the velocity of the $(\alpha = i, e)$ species, represents ions and electrons.

2.2.2 Momentum Equation

$$m_\alpha n_\alpha \frac{\partial v_\alpha}{\partial t} + (v_\alpha . \nabla) \ v_\alpha = q_\alpha E - \nabla P_\alpha \tag{2.2}$$

Where m_α is the mass of particle

2.2.3 Charge Conservation Equation

$$\nabla \cdot (e n_{i0} v_i - e n_{eo} v_e) = 0 \tag{2.3}$$

2.2.4 Quasi-Neutrality Condition

The quasi-neutrality condition at equilibrium is given as.

$$q_i n_{io} = q_e n_{eo} - q_d n_{d0} \tag{2.4}$$

2.3 Electrostatic Waves

In electric field ($E_\perp = -\nabla_\perp \phi$) of low-frequency waves, perpendicular components of the electron and the ion fluid velocities are given as [21].

$$V_{(e)\perp} \approx \frac{c}{B_0}\hat{z} \times \nabla_\perp \phi - \frac{ck_B T_e}{eB_0 n_{e0}}\hat{z} \times \nabla_\perp n_{e1} \tag{2.5}$$

And

$$V_{(i)\perp} \approx \frac{c}{B_0}\hat{z} \times \nabla_\perp \phi - \frac{ck_B Ti}{eB_0 n_{i0}}\hat{z} \times \nabla_\perp n_{i1} - \frac{c}{B_0 \omega ci}(\frac{\partial}{\partial t} + u_{i*}\cdot\ \nabla)\nabla_\perp \phi \tag{2.6}$$

where $u_{i*} = (cT_i/eB_0 n_{i0})\hat{z} \times \nabla n_{i0}(x)$ is the unperturbed ion diamagnetic drift velocity.

2.3.1 Dispersion Relation

We first consider the propagation of the coupled convective cells and the dust drift-acoustic waves. We substitute Eq. (2.5) into Eq. (2.1) for the electron continuity equation and yields.

$$\frac{\partial(n_{e0} + n_{e1})}{\partial t} + \{\nabla_\perp \cdot n_{e0} V_{(e)\perp}\} + n_{e0}\nabla_z V_{ez} = 0 \tag{2.7}$$

$$\nabla_\perp \cdot n_{e0} V_{(e)\perp} = \nabla_\perp \cdot n_{e0} \left\{ (\frac{c}{B_0}\hat{z} \times \nabla_\perp \phi) - \frac{ck_B T_e}{eB_0 n_{e0}}\hat{z} \times \nabla_\perp n_{e1} \right\}$$

Use the relation for this

$$\nabla.(a\ \vec{A}) = \ a\ (\nabla.\vec{A})\ +\ \vec{A}.\nabla a \tag{2.8}$$

$$n_{e0}(\nabla_\perp.(\frac{c}{B_0}\hat{z} \times \nabla_\perp \phi) + (\frac{c}{B_0}\hat{z} \times \nabla_\perp \phi).\nabla_\perp n_{eo}$$

as $\nabla_{\perp}.\frac{c}{B_0}\hat{z} \times \nabla_{\perp}\phi = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{c}{B_0} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \end{bmatrix} = 0$

Finally Eq. (2.7) becomes

$$\frac{\partial n_{e1}}{\partial t} + \frac{c}{B_0}\hat{z} \times \nabla\phi \cdot \nabla n_{e0} + n_{e0}\frac{\partial V_{ez}}{\partial z} = 0 \tag{2.9}$$

Now using Eq. (2.2) momentum equation for electron, where we use parallel to z-component of the velocity V_{ez} as

$$m_e n_e(\frac{\partial V_e z}{\partial t}) = -en_e(E) - \nabla P_e \tag{2.10}$$

where $P_e = K_B T_e n_{e1}$

$$m_e(\frac{\partial V_{ez}}{\partial t}) = -e(E) - \frac{K_B T_e}{n_{e0}}\frac{\partial}{\partial z}n_e$$

$$\frac{\partial V_{ez}}{\partial t} = \frac{e}{m_e}\nabla_z\phi - \frac{K_B T_e}{m_e n_{e0}}\frac{\partial}{\partial z}n_e$$

where $E = -\nabla\phi$

$$\frac{\partial V_{ez}}{\partial t} = \frac{e}{m_e}\nabla_z[\phi - \frac{K_B T_e}{en_{e0}}n_e] \tag{2.11}$$

Similarly equation of motion for ion is

$$\frac{\partial V_{iz}}{\partial t} = -\frac{e}{m_i}\nabla_z\left(\phi - \frac{3K_b T_i}{en_{io}}n_{i1}\right) \tag{2.12}$$

Now we use Eq.(2.3) charge conservation for electrons and ions velocity and use some approximation,

$$\nabla \cdot [en_{i0}(v_{\perp i} + v_{zi}) - en_{eo}(v_{\perp e} + v_{ze})] = 0 \tag{2.13}$$

$$\nabla \cdot (en_{i0}v_{\perp i} + en_{i0}v_{zi}) - \nabla \cdot (en_{eo}v_{\perp e} + en_{eo}v_{ze}) = 0 \tag{2.14}$$

Putting value of $v_{\perp i}$ and $v_{\perp e}$,

$$\nabla \cdot n_{i0}\tfrac{c}{B_0}\hat{z} \times \nabla_\perp \phi - \nabla \cdot n_{i0}\tfrac{ck_B Ti}{eB_0 n_{i0}}\hat{z} \times \nabla_\perp n_{i1} - \nabla \cdot n_{i0}\tfrac{c}{B_0\omega_{ci}}(\tfrac{\partial}{\partial t} + u_i * \cdot \nabla)\nabla_\perp \phi$$

$$+\nabla \cdot (n_{i0}v_{zi}) - \nabla \cdot n_{eo}\frac{c}{B_0}\hat{z} \times \nabla_\perp \phi - \nabla \cdot n_{eo}\frac{ck_B T_e}{eB_0 n_{e0}}\hat{z} \times \nabla_\perp n_{e1} + \nabla \cdot n_{eo}\,(v_{ze}) = 0 \tag{2.15}$$

Now, we solve this equation, we use again the relation

$$\nabla \cdot (a\,\vec{A}) = \;a\,(\nabla \cdot \vec{A})\; + \;\vec{A} \cdot \nabla a$$

$$\nabla \cdot en_{i0}\frac{c}{B_0}(\hat{z} \times \nabla_\perp \phi) = en_{i0}\frac{c}{B_0}(\nabla \cdot \hat{z} \times \nabla_\perp \phi) + (\hat{z} \times \nabla_\perp \phi) \cdot \nabla en_{i0}\frac{c}{B_0} \tag{2.16}$$

Since $\nabla_\perp \cdot \frac{c}{B_0}\hat{z} \times \nabla_\perp \phi = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{c}{B_0} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \end{bmatrix} = 0$

Eq. (2.16) becomes

$$(\hat{z} \times \nabla en_{i0}\frac{c}{B_0}) \cdot \nabla_\perp \phi \tag{2.17}$$

$$\nabla \cdot (\frac{ck_B Ti}{eB_0 n_{i0}}\hat{z} \times \nabla_\perp n_{i1}) = \frac{ck_B Ti}{eB_0 n_{i0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{i1}) \tag{2.18}$$

$$\nabla \cdot \frac{en_{i0}c}{B_0\omega_{ci}}(\frac{\partial}{\partial t})\nabla_\perp \phi = \frac{en_{i0}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2 \phi \tag{2.19}$$

$$\nabla \cdot \frac{en_{i0}c}{B_0\omega_{ci}}(u_i * \cdot \nabla)\nabla_\perp \phi = \frac{en_{i0}c}{B_0\omega_{ci}}(\nabla \cdot u_i * \cdot \nabla)\nabla_\perp \phi \tag{2.20}$$

We neglect Eq. (2.20) by using approximation i.e., $(T_i \ll T_e)$

$$\nabla \cdot en_{eo}\frac{c}{B_0}\hat{z} \times \nabla_\perp \phi = \hat{z} \times (\nabla en_{eo}\frac{c}{B_0}) \cdot \nabla_\perp \phi \tag{2.21}$$

$$\nabla \cdot (\frac{ck_BT_e}{eB_0 n_{e0}}\hat{z} \times \nabla_\perp n_{e1}) = \frac{ck_BT_e}{eB_0 n_{e0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{e1}) \tag{2.22}$$

Substituting the Eqs. (2.17) , (2.18) , (2.19) , (2.21) and (2.22) into Eq. (2.15) yields

$$\begin{aligned}&(\hat{z} \times \nabla en_{i0}\frac{c}{B_0}) \cdot \nabla_\perp \phi - \frac{ck_BTi}{eB_0 n_{i0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{i1}) - \frac{en_{i0}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2 \phi \\ &+\nabla \cdot (en_{i0}v_{zi}) - \hat{z} \times (\nabla en_{eo}\frac{c}{B_0}) \cdot \nabla_\perp \phi + \frac{ck_BT_e}{eB_0 n_{e0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{e1}) - \nabla \cdot en_{eo}v_{ze}\end{aligned} \tag{2.23}$$

re-arranging the equation

$$\begin{aligned}&(\hat{z} \times \nabla en_{i0}\frac{c}{B_0}) \cdot \nabla_\perp \phi - \hat{z} \times (\nabla en_{eo}\frac{c}{B_0}) \cdot \nabla_\perp \phi - \frac{ck_BTi}{eB_0 n_{i0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{i1}) \\ &+\frac{ck_BT_e}{eB_0 n_{e0}}(\nabla \cdot \hat{z} \times \nabla_\perp n_{e1}) - \frac{en_{i0}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2 \phi + \nabla \cdot (en_{i0}v_{zi}) - \nabla \cdot en_{eo}v_{ze}\end{aligned} \tag{2.24}$$

We impose the quasi-neutrality approximation $(n_{e1} \approx n_{i1})$ on the above equation valid for the dense plasma in which i.e. $\omega_{pi} >> \omega_{ci}$, The first two terms become in above equation become

$$(\hat{z} \times \nabla en_{i0}\frac{c}{B_0}) \cdot \nabla_\perp \phi - \hat{z} \times (\nabla en_{eo}\frac{c}{B_0}) = \frac{c}{B_0}\hat{z} \times \nabla q_{d0}n_{d0}) \cdot \nabla_\perp \phi$$

Similarly second and third term vanish by applying $n_{e1} \approx n_{i1}$, Finally we get

$$\frac{c}{B_0}(\hat{z}\times\nabla q_{d0}n_{d0})\cdot\nabla_\perp\phi-\frac{en_{i0}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi+\nabla\cdot(en_{i0}v_{zi})-\nabla\cdot(en_{eo}v_{ze})=0 \tag{2.25}$$

$$\frac{c}{B_0}(\hat{z}\times\nabla q_{d0}n_{d0})\cdot\nabla_\perp\phi-\frac{en_{i0}c}{B_0\omega_{ci}}\frac{\partial}{\partial t}\nabla_\perp^2\phi+en_{i0}\nabla\cdot(v_{zi}-\frac{n_{eo}}{n_{i0}}v_{ze})=0$$

Multiply the above by $\frac{B_0\omega_{ci}}{en_{i0}c}$ and rearrange, we get the final equation

$$\frac{\partial\nabla_\perp^2}{\partial t}\phi+\frac{\omega_{ci}^2}{\omega_{pi}^2}\frac{4\pi c}{B_0}\hat{z}\times\nabla(qd0n_{d0})\cdot\nabla\phi-\frac{B_0\omega_{ci}}{c}\frac{\partial}{\partial z}(v_{iz}-\frac{n_{e0}}{n_{i0}}v_{ez})=0 \tag{2.26}$$

Where

$$\omega_{pi}=\sqrt{\frac{4\pi e^2n_{i0}}{m_i}},\omega_{ci}=\sqrt{\frac{eB_0}{m_ic}}$$

Equations (2.9), (2.11), (2.12) and (2.26) are the equations for the convective cells and dust drift-ion acoustic waves in non-uniform dusty magnetoplasma.

2.4 Limiting case

2.4.1 $|\partial/\partial t|\gg V_{Te}|\partial/\partial z|$

Using $|\partial/\partial t|\gg V_{Te}\partial/\partial z$ and ignoring parallel ion dynamics, we have from equations (2.11) and (2.26)

$$\frac{\partial v_{ez}}{\partial t}=\frac{e}{m_e}\frac{\partial\phi}{\partial z} \tag{2.27}$$

and

$$\frac{\partial\nabla_\perp^2}{\partial t}\phi+\frac{\omega_{ci}^2}{\omega_{pi}^2}\frac{4\pi c}{B_0}\hat{z}\times\nabla(qd0n_{d0})\cdot\nabla\phi-\frac{n_{e0}B_0\omega_{ci}}{n_{i0}c}\frac{\partial v_{ez}}{\partial z})=0 \tag{2.28}$$

Now we linearize the above equation as $\frac{\partial}{\partial t} \longrightarrow i\omega$ and $\nabla \longrightarrow ik$ by assuming that v_{ez} and ϕ are proportional to $exp(-i\omega t + ik.r)$,and obtaining linear dispersion relation for convective cell frequency and shukla varma mode frequency [14, 15].

$$v_{ez1} = \frac{-ek_z}{m_e \omega} \phi_1 \tag{2.29}$$

$$\omega k_\perp^2 i\phi_1 + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} \hat{z} \times \nabla_x (qd0n_{d0}) ik\phi_1 - \frac{n_{e0} B_0 \omega_{ci}}{n_{i0} c} ik_z v_{ez1} = 0 \tag{2.30}$$

Substitute v_{ez1} in the above equation

$$\omega k_\perp^2 i\phi_1 + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} \hat{z} \times \nabla_x (qd0n_{d0}) ik\phi_1 - \frac{n_{e0} B_0 \omega_{ci}}{n_{i0} c} ik_z \left(\frac{-ek_z}{m_e \omega} \phi_1 \right) = 0$$

$$\omega^2 + \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} \frac{k_y}{k_\perp^2} \frac{\partial}{\partial x} (qd0n_{d0}) \omega + \frac{eB_0 \omega_{ci}}{m_e c} \frac{n_{e0}}{n_{i0}} \frac{k_z^2}{k_\perp^2} = 0 \tag{2.31}$$

$$\omega = \frac{-\frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} \frac{k_y}{k_\perp^2} \frac{\partial}{\partial x} (qd0n_{d0}) \pm \sqrt{\left(\frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{4\pi c}{B_0} \frac{k_y}{k_\perp^2} \frac{\partial}{\partial x} (qd0n_{d0}) \right)^2 - 4\,(1)\, \frac{eB_0 \omega_{ci}}{m_e c} \frac{n_{e0}}{n_{i0}} \frac{k_z^2}{k_\perp^2}}}{2}$$

$$\omega = \omega_{sv} \pm \frac{1}{2} \sqrt{(\omega_{sv}^2 + 4\omega_{cc}^2)} \tag{2.32}$$

Where ω_{sv} is the Shukla Varma frequency which is equal to

$$\omega_{sv} = -\frac{4\pi c \omega_{ci}^2 k_y \partial (qd0n_{d0})/\partial x}{B_0 k_\perp^2 \omega_{pi}^2} \tag{2.33}$$

and

$$\omega_{cc} = \sqrt{\frac{n_{e0}}{n_{i0}}} (\omega_{ce} \omega_{ci})^{1/2} \frac{k_z}{k_\perp} \tag{2.34}$$

where ω_{cc} is the modified convective cell frequency [16]. We note that in a homogeneous plasma $\nabla_x \to 0$, the Shukla-Varma (SV) mode is disappeared, and have the following result

$$\omega = \pm\frac{1}{2}\sqrt{4\omega_{cc}^2}$$

$$\omega = \omega_{cc}$$

2.5 Result and Discussion

We have investigated the properties of convective cells in non-uniform multicomponent dusty plasma which is embedded in a homogeneous magnetic field. It is found that presence of static dust grains, produces periodic oscillations having real part of the frequency directly proportional to the gradient of equilibrium dust number density (SV). The result of our investigation would be useful for understanding the properties of convective motion in dusty plasma. Although there is some evidence of finite frequency broadband electrostatic waves in a laboratory experiment, we are not able to correlate the observations with frequency spectrum of the convective cell due to incomplete information about the wave spectrum data. However, the present investigation would stimulate more theoretical, numerical simulation and experimental studies in the area of collective effect in dusty plasma.

Chapter 3

Jeans Criterion and A Purely Growing Instability in Dusty Plasma

3.1 Introduction

Jeans instability a well known instability in self-gravitating systems is the result of an imbalance between the incompressibility of the fluid and self-gravitating force. The charged dust grains experience electrical and gravitational forces. Here, we consider that the self gravitational instability of a dusty plasma accounting for the effect of inertia of the ions as well as the whole dynamic of the dust. Considering the frequency regime with phase velocity smaller (larger) than thermal velocity (of electron,ion and dust), the non-static ion response results in a purely growing instability [17]. The instability has a vital role in the understanding of the phenomenon of the formation of galaxies and also levitation/condensation of charged grain in planetary rings.

3.2 Governing Equations

We consider three component plasma with the electrons, ions and negatively charged dust grain. At equilibrium, the plasma is quasi-neutral as gravitational force balances the pressure gradient. The multifluid theory holds provided that distance between the grains of dust and the dimensional (spatial) scales in the plasmas are larger than the size of the grains. The equations are built using the magnetohydrodynamic model for dusty plasma. The dispersion relation and general criteria for instability of jeans are obtained using plane wave solutions of linearized perturbed equations. We expect the instability of the balance (equilibrium) against electrostatic disturbances including the phase velocity ($\mathbf{v}_{ph} = \omega/k$) satisfying the condition: $k\mathbf{v}_{td}, k\mathbf{v}_{ti} < k\mathbf{v}_{ph} < k\mathbf{v}_{te}$. In the local approximation, for electrons and ions fluids, the perturbation in number density are, respectively,

$$n_{e1} = n_{e0}\frac{e\phi}{T_e} \tag{3.1}$$

and

$$\frac{\partial \mathbf{v}_{i1}}{\partial t} = -\frac{e}{m_i}\boldsymbol{\nabla}\phi$$

along with $n_{i1} = \frac{n_{i0}}{\omega}k\mathbf{v}_{i1}$ gives

$$n_{i1} = n_{i0}\frac{k^2 e\phi}{m_i\omega^2} \tag{3.2}$$

Using fluid theory, dynamics of dust is governed by following equations,

$$\frac{\partial n_{d1}}{\partial t} + \boldsymbol{\nabla}\cdot(\mathbf{n}_{d0}\mathbf{v}_{d1}) = 0 \tag{3.3}$$

$$\frac{\partial \mathbf{v}_{d1}}{\partial t} = -\frac{q_{d0}}{m_d}\boldsymbol{\nabla}\phi - \boldsymbol{\nabla}\boldsymbol{\Psi}_1 \tag{3.4}$$

and

$$\boldsymbol{\nabla}^2\Psi_1 = 4\pi G m_d n_{d1} \tag{3.5}$$

where n_{d1} and v_{d1} are perturbations in dust number density and dust ‡fluid velocity, respectively, and Ψ_1 is the perturbed gravitational potential. With Poisson's equation

$$\boldsymbol{\nabla}^2\phi = 4\pi e(n_{e1} - n_{i1}) + 4\pi q_{d0}n_{d1} \tag{3.6}$$

3.3 Dispersion Relation for Coupled Dust Ion-acoustic and Jeans Modes

Rewriting Eq. (3.3) as

$$\frac{\partial n_{d1}}{\partial t} + n_{d0}(\boldsymbol{\nabla} \cdot \mathbf{v}_{d1}) = 0$$

using $\nabla \sim ik$ and $\frac{\partial}{\partial t} \sim -i\omega$ we have

$$-i\omega n_{d1} + n_{d0} i\mathbf{k} \cdot \mathbf{v}_{d1} = 0$$

$$n_{d1} = \frac{n_{d0}}{\omega} k\mathbf{v}_{d1} \tag{3.7}$$

and Eq. (3.5) gives

$$-\mathbf{k}^2 \Psi_1 = 4\pi G m_d n_{d1}$$

$$\Psi_1 = -\frac{4\pi G m_d n_{d1}}{k^2} \tag{3.8}$$

and Eq. (3.4) gives

$$-i\omega \mathbf{v}_{d1} = \frac{q_{d0}}{m_d} ik\phi - ik\Psi_1$$

using value of Ψ_1 in Eq. (3.8) gives

$$-i\omega \mathbf{v}_{d1} = \frac{q_{d0}}{m_d} ik\phi + ik\frac{4\pi G m_d n_{d1}}{k^2}$$

$$\mathbf{v}_{d1} = -\frac{q_{d0} k\phi}{m_d \omega} - \frac{4\pi G m_d n_{d1}}{\omega k} \tag{3.9}$$

using Eq. (3.7) in Eq. (3.9)

$$\frac{\omega n_{d1}}{k n_{d0}} = -\frac{q_{d0} k \phi}{m_d \omega} - \frac{4\pi G m_d n_{d1}}{\omega k}$$

$$n_{d1} = -\frac{q_{d0} n_{d0} k^2 \phi}{m_d \omega^2} - \frac{4\pi G m_d n_{d1}}{\omega^2}$$

using $\omega_j^2 = 4\pi G m_d n_{d1}$in above equation we get

$$n_{d1} = -\frac{q_{d0} n_{d0} k^2 \phi}{m_d \omega^2} - \frac{\omega_j^2}{\omega^2} n_{d1}$$

$$n_{d1} \frac{\omega^2 + \omega_j^2}{\omega^2} = -\frac{-q_{d0} n_{d0} k^2 \phi}{m_d \omega^2}$$

$$n_{d1} = -\frac{q_{d0} n_{d0} k^2 \phi}{m_d(\omega^2 + \omega_j^2)} \tag{3.10}$$

Using Eq. (3.1), Eq. (3.2) and Eq. (3.10) in Poisson's equation

$$\nabla^2 \phi = 4\pi e(n_{e1} - n_{i1}) + 4\pi q_{d0} n_{d1}$$

We proceed as follows

$$-k^2 \phi = 4\pi e \left[\frac{n_{e0} e \phi}{T_e} - \frac{n_{i0} e k^2 \phi}{m_i \omega^2} \right] - \frac{4\pi q_{d0}^2 n_{d0} k^2 \phi}{m_d(\omega^2 + \omega_j^2)}$$

$$k^2 \phi + \frac{4\pi n_{e0} e^2 \phi}{T_e} - \frac{4\pi n_{i0} e^2 k^2 \phi}{m_i \omega^2} - \frac{4\pi q_{d0}^2 n_{d0} k^2 \phi}{m_d(\omega^2 + \omega_j^2)} = 0$$

$$k^2 \phi + \frac{\phi}{\lambda_{De}^2} - \frac{n_{e0} T_e}{n_{e0} T_e} \frac{4\pi n_{i0} e^2 k^2 \phi}{m_i \omega^2} - \frac{\omega_{pd}^2 k^2 \phi}{(\omega^2 + \omega_j^2)} = 0$$

$$k^2 \phi + \frac{\phi}{\lambda_{De}^2} - \frac{n_{i0} T_e}{n_{e0} m_i} \frac{k^2 \phi}{\lambda_{De}^2 \omega^2} - \frac{\omega_{pd}^2 k^2 \phi}{(\omega^2 + \omega_j^2)} = 0$$

$$k^2 \phi \left[1 + \frac{1}{k^2 \lambda_{De}^2} - \frac{n_{i0} T_e}{n_{e0} m_i} \frac{1}{\lambda_{De}^2 \omega^2} - \frac{\omega_{pd}^2}{(\omega^2 + \omega_j^2)} \right] = 0$$

$$\frac{1+k^2\lambda_{De}^2}{k^2\lambda_{De}^2} - \frac{n_{i0}T_e}{n_{e0}m_i}\frac{1}{\lambda_{De}^2\omega^2} - \frac{\omega_{pd}^2}{(\omega^2+\omega_j^2)} = 0$$

$$\frac{1+k^2\lambda_{De}^2}{k^2\lambda_{De}^2}\left[1 - \frac{n_{i0}T_e}{n_{e0}m_i}\frac{k^2\lambda_{De}^2}{(1+k^2\lambda_{De}^2)\lambda_{De}^2\omega^2} - \frac{\omega_{pd}^2 k^2\lambda_{De}^2}{(1+k^2\lambda_{De}^2)(\omega^2+\omega_j^2)}\right] = 0$$

$$1 - \frac{n_{i0}T_e}{n_{e0}m_i}\frac{k^2\lambda_{De}^2}{(1+k^2\lambda_{De}^2)\omega^2} - \frac{C_{da}^2 k^2}{(1+k^2\lambda_{De}^2)(\omega^2+\omega_j^2)} = 0$$

$$1 - \frac{C_{ss}^2 k^2}{(1+k^2\lambda_{De}^2)\omega^2} - \frac{\omega_{da}^2}{(1+k^2\lambda_{De}^2)(\omega^2+\omega_j^2)} = 0$$

$$1 - \frac{\omega_{ss}^2}{\omega^2} - \frac{\omega_{da}^2}{(\omega^2+\omega_j^2)} = 0 \tag{3.11}$$

where $C_{ss} = (\frac{n_{i0}T_e}{n_{e0}m_i})^{\frac{1}{2}}$,is modified ion acoustic speed,$C_{da} = \omega_{pd}\lambda_{De} = (\frac{T_e}{m_d}\frac{Z_{d0}n_{d0}}{n_{e0}})^{\frac{1}{2}}$ is the modified dust acoustic speed. $\omega_{ss}^2 = \frac{C_{ss}^2 k^2}{(1+k^2\lambda_{De}^2)}$ and $\omega_{da}^2 = \frac{C_{da}^2 k^2}{(1+k^2\lambda_{De}^2)}$are modified ion and dust acoustic frequencies respectively.

3.3.1 Some interesting limiting cases

Case 1:

Firstly for $\boldsymbol{\nabla\Psi}_1 = 0$, $\omega_j = 0$ and from Eq. (3.11)we have

$$1 - \frac{\omega_{ss}^2}{\omega^2} - \frac{\omega_{da}^2}{\omega^2} = 0$$

$$\omega^2 - \omega_{ss}^2 - \omega_{da}^2 = 0$$

$$\omega = (\omega_{ss}^2 + \omega_{da}^2)^{\frac{1}{2}} \tag{3.12}$$

that is the frequency of dust acoustic wave.

Case 2:

From Eq. (3.11) we may proceed as

$$\omega^2 - \omega_{ss}^2 - \frac{\omega^2\omega_{da}^2}{(\omega^2 + \omega_j^2)} = 0$$

$$\omega^2(\omega^2 + \omega_j^2) - \omega_{ss}^2(\omega^2 + \omega_j^2) - \omega^2\omega_{da}^2 = 0$$

$$\omega^4 + \omega^2\omega_j^2 - \omega^2\omega_{ss}^2 - \omega_{ss}^2\omega_j^2 - \omega^2\omega_{da}^2 = 0$$

$$\omega^4 + \omega^2(\omega_j^2 - \omega_{ss}^2 - \omega_{da}^2) - \omega_{ss}^2\omega_j^2 = 0 \tag{3.13}$$

$$\omega^4 + \omega^2 A - B = 0 \tag{3.14}$$

where $A = (\omega_j^2 - \omega_{ss}^2 - \omega_{da}^2)$, and $B = \omega_{ss}^2\omega_j^2$, The solution of Eq. (3.13) is

$$\omega = -\frac{1}{2}A \pm \frac{1}{2}\sqrt{A^2 + B} \tag{3.15}$$

For $\omega_j^2 \ll \omega_{ss}^2$ Eq. (3.13) gives

$$\omega^4 + \omega^2(-\omega_{ss}^2 - \omega_{da}^2) = 0$$

$$\omega^4 = \omega^2(\omega_{ss}^2 + \omega_{da}^2)$$

or

$$\omega = (\omega_{ss}^2 + \omega_{da}^2)^{\frac{1}{2}}$$

The unstable root of Eq. (3.14) gives [15]

$$\gamma = \operatorname{Im}\omega = (\frac{1}{2}A + \frac{1}{2}\sqrt{A^2 + B})^{\frac{1}{2}}$$

and

$$\gamma_{\max\approx}(\omega_j^2 - \omega_{ss}^2)^{\frac{1}{2}} \backsimeq \omega_j \tag{3.16}$$

where $\gamma_{\max}$is the maximum growth rate of instability.

3.4 Conclusion

We considered the Jeans instability of self-gravitating dusty plasma considering the response of the ion dynamics. The possibility of an ion related purely growing instability in self-gravitating unmagnetized dusty plasma is found. Thus, the current instability has a rate of growth much faster than that found in Ref [18].Clearly, the instability examined is quite vigorous and, therefore, might be one of the possible candidates to condensation in astrophysical objects including interplanetary dust, circumstellar dust, planetary rings and galaxies.

Chapter 4

Stability of Self Gravitating Magnetized Dusty Plasma

4.1 Introduction

Most of the space and astrophysical systems(particularly, the ionosphere, protostars, circumstellar disks, suoernova remnants) are partially ionized and contain a significant fraction of dust particulates. The charged dust grains are held under the influence of electromagnetic and gravitational forces, whereas the electrons and ions experience only electric force because their masses are much smaller than that of the dust grains. As most of the astronomical objects have strong magnetic fields and density inhomogeneties, it is of practical interest to consider the electrostatic instability of self-gravitating magnetized dusty plasma.

Here, we have studied the modification of electrostatic waves in self-gravitating magnetized dusty plasma in the presence of self-gravitating force and magnetic fields. For this purpose, the susceptibility relation for electrons, ions and dust grains are derived using hydrodynamic equations. It is found that magnetic fields contribute to the stability of self-gravitating dusty plasma systems.

4.2 General Dispersion Relation

The propagation of electrostatic waves in three-component, self-gravitating, magnetized dusty plasmas whose constituents are electrons, ions and charged dust grains is considered. In the equilibrium, the gravitational force on the grains is balanced by the gradient of the plasma pressure, and also the plasma is assumed to be quas-ineutral, i.e., $\mathrm{n}_{i0}=\mathrm{n}_{e0}+\mathrm{Z}_{d0}\mathrm{n}_{d0}$, where Z_{d0} is the number of charges residing on the negatively charged dust grains. Here, n_{e0} , n_{i0} and n_{d0} are the equilibrium number densities of the electrons, ions and dust grains, respectively. Dust grains are assumed as negatively charged point particles. The grain sizes and inter-grain spacings are also assumed to be much smaller than dusty plasma Debye radius. Before discussing a number of frequency regimes where self gravitation has an important role, we discuss and derive the susceptibility relation for electrons, ions and dust.

4.2.1 Electron and ion susceptibility

The electron and the ion number density perturbations n_{j1} in the presence of wave potential ϕ is given as

$$n_{j1} = -\frac{k^2}{4\pi q_j}\chi_j\phi \tag{4.1}$$

where k is wave number, $q_e = -e$ and $q_i = -e$, e being the magnitude charge on electron and χ_j is the susceptibility of electrons and ions. The susceptibilities χ_j of the electrons and the ions in magnetized plasma are obtained using following equations

$$\frac{\partial n_j}{\partial t} + n_{j0}\boldsymbol{\nabla}\cdot\mathbf{v}_j = 0 \tag{4.2}$$

$$\frac{\partial \mathbf{v}_j}{\partial t} = -\frac{q_j}{m_j}\boldsymbol{\nabla\phi} + \mathbf{v}_j \times \boldsymbol{\omega}_{cj} \tag{4.3}$$

$$\nabla^2\phi = -4\pi\sum q_j n_j \tag{4.4}$$

where $\boldsymbol{\omega}_{cj} = \frac{q_j B_S}{m_j}$ is cyclotron frequency, B is external static magnetic field. Solving momentum equation for parallel and perpendicular velocity components and using $\nabla \sim ik$

and $\frac{\partial}{\partial t} \sim -i\omega$ we have

$$\mathbf{v}_{\parallel} = \frac{q_j k_{\parallel} \phi}{\omega m_j} \tag{4.5}$$

and

$$\mathbf{v}_{\perp} = -\frac{i q_j (k_{\perp} \times \boldsymbol{\omega}_{cj})\phi}{m_j(\boldsymbol{\omega}_{cj}^2 - \omega^2)} - \frac{i q_j k_{\perp} \phi}{m_j} \frac{\omega}{(\boldsymbol{\omega}_{cj}^2 - \omega^2)} \tag{4.6}$$

Now using values of $\mathbf{v}_{\perp}$ and $\mathbf{v}_{\parallel}$in following continuity equation

$$n_{j1} = \frac{n_{j0}}{\omega}(k_{\parallel} \cdot \mathbf{v}_{\parallel} + k_{\perp} \cdot \mathbf{v}_{\perp}) \tag{4.7}$$

we get

$$n_{j1} = \frac{k_{\parallel}^2 q_j n_{j0} \phi}{\omega^2 m_j} - \frac{k_{\perp}^2 q_j n_{j0} \phi}{m_j(\boldsymbol{\omega}_{cj}^2 - \omega^2)} \tag{4.8}$$

Now comparing $n_{j1} = -\frac{k^2}{4\pi q_j}\chi_j \phi$ and Eq. (4.8) we get

$$\chi_j = \frac{k_{\perp}^2}{k^2} \frac{}{(\boldsymbol{\omega}_{cj}^2 - \omega^2)} - \frac{k_{\parallel}^2}{k^2} \frac{\boldsymbol{\omega}_{pj}^2}{\omega^2} \tag{4.9}$$

where $\boldsymbol{\omega}_{pj} = \left(\frac{4\pi q_j^2 n_{j0}}{m_j}\right)^{\frac{1}{2}}$ is the plasma frequency of electrons and ions.

4.2.2 Dust susceptibility

Many processes affect the motion of dust particles. Most importantly, the dust particles provide inertia to the wave. Furthermore, dust-neutral collisions results in wave damping. Here a hydrodynamic approach is used to derive the dust susceptibility.

In the presence of the ßelectrostatic, Lorentz and gravitational force the cold dust grain dynamics is governed by the following equation[19]. Starting with the dust momentum equation

$$\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{q_{d0}}{m_d}\boldsymbol{\nabla}\boldsymbol{\phi} - \boldsymbol{\nabla}\boldsymbol{\Psi}_1 + \mathbf{v}_d \times \boldsymbol{\omega}_{cd} \tag{4.10}$$

where $B = B_0$, $\Psi_1$is perturbed gravitational potential. For motion along z- axis (parallel motion)

$$\frac{\partial \mathbf{v}_{d\|}}{\partial t} = -\frac{q_{d0}}{m_d}\boldsymbol{\nabla}_{\|}\boldsymbol{\phi} - \boldsymbol{\nabla}_{\|}\boldsymbol{\Psi}_1 \tag{4.11}$$

using $\nabla \sim ik$ and $\frac{\partial}{\partial t} \sim -i\omega$

$$-i\omega \mathbf{v}_{d\|} = -\frac{q_{d0}}{m_d} ik_{\|}\phi - ik_{\|}\Psi_1 \tag{4.12}$$

From the Poisson's equation

$$\boldsymbol{\nabla}^2\Psi_1 = 4\pi G m_d n_{d1}$$

$$-\mathbf{k}^2\Psi_1 = 4\pi G m_d n_{d1}$$

$$\Psi_1 = -\frac{4\pi G m_d n_{d1}}{k^2} \tag{4.13}$$

Using value of Ψ_1 in Eq. (4.12) we have

$$-i\omega \mathbf{v}_{d\|} = \frac{q_{d0}}{m_d} ik_{\|}\phi + ik_{\|}\frac{4\pi G m_d n_{d1}}{k^2}$$

$$\mathbf{v}_{d\|} = -\frac{q_{d0}k_{\|}\phi}{m_d\omega} - ik_{\|}\frac{4\pi G m_d n_{d1}}{\omega k^2} \tag{4.14}$$

For perpendicular motion Eq. (4.11) gives

$$\frac{\partial \mathbf{v}_{d\perp}}{\partial t} = -\frac{q_{d0}}{m_d}\boldsymbol{\nabla}_{\perp}\boldsymbol{\phi} - \boldsymbol{\nabla}_{\perp}\boldsymbol{\Psi}_1 + \mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}$$

Fourier transforming above equation, we get

$$-i\omega \mathbf{v}_{\perp} = -\frac{q_{d0}}{m_d} ik_{\perp}\phi - ik_{\perp}\boldsymbol{\Psi}_1 + \mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd} \tag{4.15}$$

Taking right cross product of above Eq. (4.15) with $\boldsymbol{\omega}_{cd}$ we have

$$-i\omega(\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) = -\frac{q_{d0}\phi}{m_d}(k_\perp \times \boldsymbol{\omega}_{cd}) - i(k_\perp \times \boldsymbol{\omega}_{cd})\boldsymbol{\Psi}_\mathbf{1} + (\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) \times \boldsymbol{\omega}_{cd}$$

$$-i\omega(\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) = -\frac{q_{d0}\phi}{m_d}(k_\perp \times \boldsymbol{\omega}_{cd}) - i(k_\perp \times \boldsymbol{\omega}_{cd})\boldsymbol{\Psi}_\mathbf{1} - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp}$$

$$-i\omega(\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) = -\frac{q_{d0}\phi}{m_d}(k_\perp \times \boldsymbol{\omega}_{cd}) + i(k_\perp \times \boldsymbol{\omega}_{cd})\frac{4\pi G m_d n_{d1}}{k^2} - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp}$$

$$-i\omega(\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) = \left[-\frac{iq_{d0}\phi}{m_d} + i\frac{4\pi G m_d n_{d1}}{k^2}\right](k_\perp \times \boldsymbol{\omega}_{cd}) - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp}$$

$$-i\omega(\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd}) = \beta(k_\perp \times \boldsymbol{\omega}_{cd}) - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp} \tag{4.16}$$

where $\beta = (-\frac{iq_{d0}\phi}{m_d} + i\frac{4\pi G m_d n_{d1}}{k^2})$, Eq. (4.15) can be written as

$$-i\omega\mathbf{v}_{d\perp} = \left[-\frac{iq_{d0}\phi}{m_d} + i\frac{4\pi G m_d n_{d1}}{k^2}\right]k_\perp + (\mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd})$$

$$-i\omega\mathbf{v}_{d\perp} - \beta k_\perp = \mathbf{v}_{d\perp} \times \boldsymbol{\omega}_{cd} \tag{4.17}$$

Using Eq. (4.16) in Eq. (4.15)

$$-i\omega(-i\omega\mathbf{v}_{d\perp} - \beta k_\perp) = \beta(k_\perp \times \boldsymbol{\omega}_{cd}) - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp}$$

$$-\omega^2\mathbf{v}_{d\perp} + i\omega\beta k_\perp = \beta(k_\perp \times \boldsymbol{\omega}_{cd}) - \boldsymbol{\omega}_{cd}^2\mathbf{v}_{d\perp}$$

$$(\boldsymbol{\omega}_{cd}^2 - \omega^2)\mathbf{v}_{d\perp} = \beta(k_\perp \times \boldsymbol{\omega}_{cd}) - i\omega\beta k_\perp$$

$$\mathbf{v}_{d\perp} = \frac{\beta(k_\perp \times \boldsymbol{\omega}_{cd}) - i\omega\beta k_\perp}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \tag{4.18}$$

Now from equation of continuity

$$\frac{\partial n_{d1}}{\partial t} + n_{d0}\boldsymbol{\nabla} \cdot \mathbf{v}_d = 0$$

$$-i\omega n_{d1} + n_{d0} i\mathbf{k} \cdot \mathbf{v}_d = 0$$

we get

$$n_{d1} = \frac{n_{d0}}{\omega}(\mathbf{k}_\parallel \cdot \mathbf{v}_{d\parallel} + \mathbf{k}_\perp \cdot \mathbf{v}_{d\perp}) \tag{4.19}$$

Using Eq. (4.14) and Eq. (4.18) in Eq. (4.19) we have

$$n_{d1} = \frac{n_{d0}}{\omega}\left[\mathbf{k}_\parallel \cdot (\frac{q_{d0}}{\omega m_d} k_\parallel \phi - ik_\parallel \frac{4\pi G m_d n_{d1}}{k^2\omega}) + \mathbf{k}_\perp \cdot (\frac{\beta(k_\perp \times \boldsymbol{\omega}_{cd}) - i\omega\beta k_\perp}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)})\right]$$

or

$$n_{d1} = \frac{nq_{d0}}{\omega^2 m_d} k_\parallel^2 \phi - \frac{4\pi G m_d n_{d0}}{k^2\omega^2} k_\parallel^2 n_{d1} - \frac{in_{d0}\beta k_\perp^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \tag{4.20}$$

using value of β in Eq.(4.20)

$$n_{d1} = \frac{nq_{d0}}{\omega^2 m_d} k_\parallel^2 \phi - \frac{4\pi G m_d n_{d0}}{k^2\omega^2} k_\parallel^2 n_{d1} - \frac{in_{d0}(-\frac{iq_{d0}\phi}{m_d} + i\frac{4\pi G m_d n_{d1}}{k^2})k_\perp^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)}$$

$$n_{d1} = \frac{n_{d0}q_{d0}}{\omega^2 m_d} k_{\|}^2 \phi - \frac{4\pi G m_d n_{d0}}{k^2\omega^2} k_{\|}^2 n_{d1} - \frac{\frac{q_{d0}n_{d0}\phi}{m_d} k_{\perp}^2 + \frac{4\pi G m_d n_{d0}}{k^2} n_{d1} k_{\perp}^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)}$$

where $\omega_j^2 = 4\pi G m_d n_{d0}$

$$n_{d1} = \frac{n_{d0}q_{d0}}{\omega^2 m_d} k_{\|}^2 \phi - \frac{4\pi G m_d n_{d0}}{k^2\omega^2} k_{\|}^2 n_{d1} - \frac{\frac{q_{d0}n_{d0}\phi}{m_d} k_{\perp}^2 + \frac{\omega_j^2}{k^2} n_{d1} k_{\perp}^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)}$$

$$n_{d1} = \left[\frac{n_{d0}q_{d0}}{\omega^2 m_d} k_{\|}^2 - \frac{q_{d0}n_{d0}k_{\perp}^2}{m_d(\boldsymbol{\omega}_{cd}^2 - \omega^2)}\right]\phi + \left[\frac{\omega_j^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \frac{k_{\perp}^2}{k^2} - \frac{\omega_j^2}{\omega^2}\frac{k_{\|}^2}{k^2}\right] n_{d1}$$

$$n_{d1} = \frac{\left[\frac{n_{d0}q_{d0}}{\omega^2 m_d} k_{\|}^2 - \frac{q_{d0}n_{d0}k_{\perp}^2}{m_d(\boldsymbol{\omega}_{cd}^2 - \omega^2)}\right]\phi}{\left[1 - \frac{\omega_j^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \frac{k_{\perp}^2}{k^2} + \frac{\omega_j^2}{\omega^2}\frac{k_{\|}^2}{k^2}\right]} \tag{4.21}$$

Now using value of $n_{d1} = -\frac{k^2\phi}{4\pi q_{d0}}\chi_d$ in Eq. (4.21) we have

$$-\frac{\chi_d k^2\phi}{4\pi q_{d0}}\chi_d = \frac{\left[\frac{n_{do}q_{do}}{m_d\omega^2} k_{\|}^2 - \frac{q_{d0}n_{d0}k_{\perp}^2}{m_d(\boldsymbol{\omega}_{cd}^2 - \omega^2)}\right]\phi}{\left[1 - \frac{\omega_j^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \frac{k_{\perp}^2}{k^2} + \frac{\omega_j^2}{\omega^2}\frac{k_{\|}^2}{k^2}\right]}$$

$$\chi_d = \frac{\frac{1}{k^2}\frac{4\pi q_{d0}^2 n_{do}}{m_d}\left[\frac{k_{\|}^2}{\omega^2} - \frac{k_{\perp}^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)}\right]}{\left[1 - \frac{\omega_j^2}{(\boldsymbol{\omega}_{cd}^2 - \omega^2)} \frac{k_{\perp}^2}{k^2} + \frac{\omega_j^2}{\omega^2}\frac{k_{\|}^2}{k^2}\right]}$$

$$\chi_d = \frac{\frac{1}{k^2\omega^2}(-\omega_{pd}^2)\left[\frac{k_{\perp}^2\omega^2}{(\omega^2 - \boldsymbol{\omega}_{cd}^2)} + k_{\|}^2\right]}{\left[1 + \frac{\omega_j^2}{(\omega^2 - \boldsymbol{\omega}_{cd}^2)} \frac{k_{\perp}^2}{k^2} + \frac{\omega_j^2}{\omega^2}\frac{k_{\|}^2}{k^2}\right]}$$

where $\omega_{pd}^2 = \frac{4\pi q_{d0}^2 n_{do}}{m_d}$

or

$$\chi_d = \frac{(-\omega_{pd}^2)\left[\frac{k_\perp^2\omega^2}{(\omega^2-\boldsymbol{\omega}_{cd}^2)} + k_\parallel^2\right]}{\left[k^2\omega^2 + \frac{\omega_j^2\omega^2}{(\omega^2-\boldsymbol{\omega}_{cd}^2)}\frac{k_\perp^2}{k^2} + k_\parallel^2\omega_j^2\right]} \tag{4.22}$$

Above Eq. (4.22) is required dust susceptibility relation [20]. For unmagnetized plasma ($B_0 = 0$) we proceed as follow

$$\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{q_{d0}}{m_d}\boldsymbol{\nabla}\boldsymbol{\phi} - \boldsymbol{\nabla}\boldsymbol{\Psi}_1 \tag{4.23}$$

$$-i\omega\mathbf{v}_d = -\frac{q_{d0}}{m_d}ik\phi - ik\boldsymbol{\Psi}_1 \tag{4.24}$$

with $\Psi_1 = -\frac{4\pi G m_d n_{d1}}{k^2}$ above Eq. (4.24) gives

$$-i\omega\mathbf{v}_d = -\frac{q_{d0}}{m_d}ik\phi + ik\frac{4\pi G m_d n_{d1}}{k^2}$$

$$\mathbf{v}_d = \frac{q_{d0}}{m_d}ik\phi - ik\frac{4\pi G m_d n_{d1}}{k^2\omega} \tag{4.25}$$

$$n_{d1} = \frac{n_{d0}}{\omega}(\mathbf{k}\cdot\mathbf{v}_d) \tag{4.26}$$

Using Eq. (4.25) in Eq. (4.26) we have

$$n_{d1} = \frac{n_{do}q_{do}}{m_d\omega^2}k^2\phi - \frac{4\pi G m_d n_{d0}}{k^2\omega^2}n_{d1}k^2$$

$$n_{d1} = \frac{n_{do}q_{do}}{m_d\omega^2}k^2\phi - \frac{\omega_j^2}{\omega^2}n_{d1}$$

$$\frac{\omega^2+\omega_j^2}{\omega^2}n_{d1} = \frac{n_{do}q_{do}}{m_d\omega^2}k^2\phi$$

$$n_{d1} = \frac{n_{do}q_{do}}{m_d(\omega^2 + \omega_j^2)}k^2\phi \tag{4.27}$$

now using $n_{d1} = -\frac{k^2\phi}{4\pi q_{d0}}\chi_d$ in Eq. (4.27) we get

$$-\frac{k^2\phi}{4\pi q_{d0}}\chi_d = \frac{n_{do}q_{do}}{m_d(\omega^2 + \omega_j^2)}k^2\phi$$

$$\chi_d = \frac{4\pi n_{do}q_{do}^2}{m_d}\frac{1}{(\omega^2 + \omega_j^2)}$$

Finally we get [2]

$$\chi_d = -\frac{\omega_{pd}^2}{(\omega^2 + \omega_j^2)} \tag{4.28}$$

The following dispersion relation

$$\epsilon(\omega, k) = 1 + \chi_e + \chi_i + \chi_d = 0 \tag{4.29}$$

gives frequencies of electrostatic modes in self gravitational dusty plasma.

4.3 Stability analysis

Below are several interesting cases giving affect of external magnetic field on self gravitating unmagnetized dusty plasma.

Case A: $\omega_{cd} \ll \omega \ll \omega_{ci,} k_\perp \rho_{e,i,d} \ll 1$

Here we consider electron and ions to be strongly magnetized, and cold dust grain unmagnetized. Under these approximation

$$\chi_j = \frac{k_\perp^2}{k^2}\frac{\omega_{pj}^2}{(\boldsymbol{\omega}_{cj}^2 - \omega)} - \frac{\omega_{pj}^2}{\omega^2}\frac{k_\parallel^2}{k^2}$$

gives

$$\chi_j = \frac{k_\perp^2}{k^2}\frac{\omega_{pj}^2}{\boldsymbol{\omega}_{cj}^2} - \frac{\omega_{pj}^2}{\omega^2}\frac{k_\parallel^2}{k^2} \tag{4.30}$$

Accordingly Eq. (4.29) gives

$$\epsilon = 1 + \frac{k_\perp^2}{k^2}\left(\frac{\omega_{pi}^2}{\boldsymbol{\omega}_{ci}^2} + \frac{\omega_{pe}^2}{\boldsymbol{\omega}_{ce}^2}\right) - \frac{k_\parallel^2}{k^2\omega^2}(\omega_{pi}^2 + \omega_{pe}^2) - \frac{\omega_{pd}^2}{(\omega^2 + \omega_j^2)} = 0 \tag{4.31}$$

For $k_\parallel^2 = 0, k^2 \sim k$ above equation gives

$$1 + \left(\frac{\omega_{pi}^2}{\boldsymbol{\omega}_{ci}^2} + \frac{\omega_{pe}^2}{\boldsymbol{\omega}_{ce}^2}\right) - \frac{\omega_{pd}^2}{(\omega^2 + \omega_j^2)} = 0 \tag{4.32}$$

In the high density limit ($\omega_{pi}^2 \gg \boldsymbol{\omega}_{ci}^2$) we get

$$\frac{\omega_{pi}^2}{\boldsymbol{\omega}_{ci}^2} - \frac{\omega_{pd}^2}{(\omega^2 + \omega_j^2)} = 0$$

$$\omega^2 + \omega_j^2 = \frac{\omega_{pd}^2\omega_{ci}^2}{\boldsymbol{\omega}_{pi}^2}$$

or

$$\omega^2 = \omega_{DLH}^2 - \omega_j^2 \tag{4.33}$$

Where dust lower-hybrid frequency is

$$\omega_{DLH}^2 = \frac{\omega_{pd}^2\omega_{ci}^2}{\boldsymbol{\omega}_{pi}^2}$$

or

$$\omega_{DLH}^2 = \frac{eB}{m_i}\frac{Z_{do}eB}{m_d}\frac{Z_{do}n_{do}}{n_{i0}}$$

$$\omega_{DLH}^2 = \omega_{ci}\omega_{cd}\frac{Z_{do}n_{do}}{n_{i0}} \tag{4.34}$$

Clearly, the jeans instability is reduced by magnetic field. In the absence of gravitational force, one gets the dust lower-hybrid oscillations which includes the ions and charge dust grains dynamics. The dynamics of electrons is not important. When the magnetic field disappears, we get the standard Jeans instability in dusty plasmas.

Case B : $\omega_{cd} \ll \omega \sim \omega_{ci}, k_{\|}\mathbf{v}_{te} \gg \omega, k_{\perp}\rho_e \ll 1, 1 \gg k_{\perp}\rho_d \gg k_{\perp}\rho_{i,}\omega, |\ \omega - \omega_{ci}| \gg k_{\|}\mathbf{v}_{ti}$

In this case Eq.(4.29) takes form as

$$\epsilon = 1 + \frac{1}{k^2\lambda_{De}^2} - \frac{2\Delta_i\omega_{ci}^2}{k^2\lambda_{Di}^2(\omega^2-\omega_{ci}^2)} - \frac{\omega_{pd}^2}{\omega^2+\omega_{jd}^2} = 0 \tag{4.35}$$

or

$$\frac{1+k^2\lambda_{De}^2}{k^2\lambda_{De}^2} - \frac{2\Delta_i\omega_{ci}^2}{k^2\lambda_{Di}^2(\omega^2-\omega_{ci}^2)} - \frac{\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0$$

$$\frac{1+k^2\lambda_{De}^2}{k^2\lambda_{De}^2}\left[1 - \frac{2\Delta_i\omega_{ci}^2k^2\lambda_{De}^2}{k^2\lambda_{Di}^2(\omega^2-\omega_{ci}^2)(1+k^2\lambda_{De}^2)} - \frac{\omega_{pd}^2k^2\lambda_{De}^2}{(\omega^2+\omega_{jd}^2)(1+k^2\lambda_{De}^2)}\right] = 0$$

$$1 - \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2}{\lambda_{Di}^2(\omega^2-\omega_{ci}^2)(1+k^2\lambda_{De}^2)} - \frac{k^2C_{de}^2}{(\omega^2+\omega_{jd}^2)} = 0 \tag{4.36}$$

where $C_{de}^2 = \frac{\omega_{pd}^2\lambda_{De}^2}{(1+k^2\lambda_{De}^2)}$

$$(\omega^2-\omega_{ci}^2) - \frac{k^2C_{de}^2(\omega^2-\omega_{ci}^2)}{(\omega^2+\omega_{Jd}^2)} = \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}$$

$$(\omega^2-\omega_{ci}^2)(\omega^2+\omega_{jd}^2) - k^2C_{de}^2(\omega^2-\omega_{ci}^2) = \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2(\omega^2+\omega_{jd}^2)}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}$$

$$\omega^4 + \omega^2\omega_{Jd}^2 - \omega^2\omega_{ci}^2 - \omega_{ci}^2\omega_{Jd}^2 - k^2C_{de}^2\omega^2 + k^2C_{de}^2\omega_{ci}^2 = \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2\omega^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)} + \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2\omega_{Jd}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}$$

$$\omega^4 - \omega^2\left[\omega_{ci}^2 + k^2C_{de}^2 - \omega_{Jd}^2 + \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}\right] - \omega_{ci}^2\omega_{Jd}^2\left(1 + \frac{2\Delta_i\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}\right) = 0$$

or

$$\omega^4 - B_3\omega^2 - C_3 = 0 \tag{4.37}$$

where $B_3 = \left[\omega_{ci}^2 + k^2C_{de}^2 - \omega_{Jd}^2 + \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}\right]$, $C_3 = \omega_{ci}^2\omega_{Jd}^2\left(1 + \frac{2\Delta_i\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)}\right)$, For $B_3^2 \gg 4C_3$

$$\omega^2 \simeq \frac{+B_3 \pm \sqrt{B_3^2}}{2A_3}$$

For $A_3 \simeq 1$

$$\omega^2 \simeq B_3$$

$$\omega^2 \simeq \omega_{ci}^2 + k^2C_{de}^2 + \frac{2\Delta_i\omega_{ci}^2\lambda_{De}^2}{\lambda_{Di}^2(1+k^2\lambda_{De}^2)} - \omega_{Jd}^2 \tag{4.38}$$

Above Eq. (4.38) represents the electrostatic ion-cyclotron waves modified by gravitational-effect.

Case C : $\omega_{ci} \ll \omega \ll \omega_{ce}, \omega \ll k_{\parallel}\mathbf{v}_{te}, \frac{\omega}{\omega_{ce}}\frac{k_{\parallel}}{k_{\perp}}, \mathbf{k}_{\perp}\rho_e \ll 1 \ll k_{\perp}\rho_i$

Here Eq. (4.29) gives

$$\epsilon = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{1}{k^2\lambda_{Di}^2} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2} = 0 \tag{4.39}$$

$$1 + \frac{1}{k^2}\left(\frac{\lambda_{De}^2 + \lambda_{Di}^2}{\lambda_{De}^2\lambda_{Di}^2}\right) - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2} = 0$$

$$1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2} = 0$$

$$\frac{1 + k^2\lambda_D^2}{k^2\lambda_D^2} = \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2}$$

$$\omega^2 + \omega_{Jd}^2 = \frac{\omega_{pd}^2 k^2\lambda_D^2}{1 + k^2\lambda_D^2}$$

$$\omega^2 = \frac{\omega_{pd}^2 k^2\lambda_D^2}{1 + k^2\lambda_D^2} - \omega_{Jd}^2 \tag{4.40}$$

Finally we get

$$\omega^2 = k^2 C_d^2 - \omega_{Jd}^2 \tag{4.41}$$

where $C_d^2 = \frac{\omega_{pd}^2\lambda_D^2}{1+k^2\lambda_D^2}$, Eq. (4.41) represents fast dust-acoustic waves in dusty magnetoplasma modified by gravitational force.

Case D:$\omega_{cd}, \omega_{ci} \ll \omega \ll \omega_{ce}, \mathbf{k}_\parallel \mathbf{v}_{tj} \ll \omega, \mathbf{k}_\perp \rho_j \ll 1$

Here Eq. (4.29) gives

$$\epsilon = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{k_\perp^2}{k^2} - \frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2} = 0 \tag{4.42}$$

$$A_L + \frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0 \tag{4.43}$$

where $A_L = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{k_\perp^2}{k^2}$

$$1 + \frac{1}{A_L}\frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2} - \frac{1}{A_L}\frac{\omega_{pi}^2}{\omega^2} - \frac{1}{A_L}\frac{\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0$$

$$1 - \frac{1}{A_L}\frac{\omega_{pi}^2}{\omega^2}(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) - \frac{1}{A_L}\frac{\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0$$

or

$$1 - \frac{\omega_{LH}^2}{\omega^2} - \frac{1}{A_L}\frac{\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0 \tag{4.44}$$

where $\omega_{LH}^2 = \frac{\omega_{pi}^2}{A_L}(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2})$

$$\omega^2 - \omega_{LH}^2 - \frac{1}{A_L}\frac{\omega^2\omega_{pd}^2}{\omega^2+\omega_{Jd}^2} = 0$$

$$\omega^2(\omega^2+\omega_{Jd}^2) - \omega_{LH}^2(\omega^2+\omega_{Jd}^2) - \frac{\omega^2\omega_{pd}^2}{A_L} = 0$$

$$\omega^4 + \omega^2\omega_{Jd}^2 - \omega^2\omega_{LH}^2 - \omega_{LH}^2\omega_{Jd}^2 - \frac{\omega^2\omega_{pd}^2}{A_L} = 0$$

$$\omega^4 + \omega^2(\omega_{Jd}^2 - \omega_{LH}^2 - \frac{\omega_{pd}^2}{A_L}) - \omega_{LH}^2\omega_{Jd}^2 = 0$$

$$\omega^4 + \omega^2 B_3 - \omega_{LH}^2\omega_{Jd}^2 = 0 \tag{4.45}$$

where $B_3 = (\omega_{Jd}^2 - \omega_{LH}^2 - \frac{\omega_{pd}^2}{A_L})$, The solution of Eq. (4.45) gives

$$\omega^2 = -\frac{B_3}{2} \pm \frac{1}{2}\sqrt{B_3^2 + 4\omega_{LH}^2\omega_{Jd}^2} \tag{4.46}$$

However for $k_\parallel^2\omega_{pe}^2 \gg k^2\omega_{ce}^2$ and $\omega^2 \gg \omega_{Jd}^2$ Eq. (4.46) gives

$$1 - \frac{\omega_{LH}^2}{\omega^2} - \frac{1}{A_L}\frac{\omega_{pd}^2}{\omega^2} = 0$$

$$\omega^2 - \omega_{LH}^2 - \frac{\omega_{pd}^2}{A_L} = 0$$

$$\omega^2 = \omega_{LH}^2 + \frac{\omega_{pd}^2}{A_L}$$

$$\omega^2 = \frac{\omega_{pi}^2}{A_L}(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{\omega_{pd}^2}{A_L} \tag{4.47}$$

Putting value of A_L in the above equation we have

$$\omega^2 = \frac{\omega_{pi}^2\omega_{ce}^2k^2}{\omega_{pe}^2k_\perp^2}(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{\omega_{pd}^2\omega_{ce}^2k^2}{\omega_{pe}^2k_\perp^2}$$

$$\omega^2 = \frac{k^2}{k_\perp^2}\frac{\omega_{pi}^2}{\omega_{pe}^2}\omega_{ce}^2(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{k^2}{k_\perp^2}\frac{\omega_{pd}^2\omega_{ce}^2}{\omega_{pe}^2}$$

$$\omega^2 = \frac{k^2}{k_\perp^2}\frac{eB}{m_e}\frac{eB}{m_i}(\frac{n_{i0}}{n_{e0}})(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{k^2}{k_\perp^2}\frac{n_{d0}Z_{d0}^2e^2}{\epsilon_0 m_d}\frac{\epsilon_0 m_e}{n_{e0}e^2}\frac{e^2B^2}{m_e^2}$$

$$\omega^2 = \frac{k^2}{k_\perp^2}\omega_{ce}\omega_{ci}(\frac{n_{i0}}{n_{e0}})(1 + \frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{k^2}{k_\perp^2}\frac{eB}{m_e}\frac{Z_{d0}eB}{m_d}(\frac{Z_{d0}n_{d0}}{n_{e0}})$$

Finally we get

$$\omega^2 = \frac{k^2}{k_\perp^2}\omega_{ce}\omega_{ci}(\frac{n_{i0}}{n_{e0}})(1+\frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{k_\parallel^2}{k^2}) + \frac{k^2}{k_\perp^2}\omega_{ce}\omega_{cd}(\frac{Z_{d0}n_{d0}}{n_{e0}}) \tag{4.48}$$

Eq. (4.48) represents lower hybrid mode being modified by dynamics of dust particles.

4.4 Conclusions

We studied the properties of dispersion relation of the self gravitational dusty plasmas in the presence of an ambient magnetic field. To this end, we got general dispersion relations by deriving new function of dielectric response, also the general dielectric constants for electrons and ions that already exist in the literature. Our general dispersion relation, Eq. (4.29) reproduces the earlier results of unmagnetized gravitating dusty plasmas, when the external magnetic field is set zero. In the frequency regime $\omega_{cd} \ll \omega \ll \omega_{ci}$ and for $k_\parallel = 0$ (exactly perpendicular propagation) we get a mode of constant frequency $\omega \sim \sqrt{\omega_{cd}\omega_{ci}}$(ion-dust-hybrid frequency). Here we also find that external magnetic field stabilizes the self gravitational (Jeans) instability. The stabilization is due to Lorentz force that opposes the gravitational force. To summarize, the present work has an important role to understanding the stability of magnetized dusty clouds, dusty protostars where gravitational and electromagnetic forces are significant. When pressure gradients, Lorentz forces and electrostatic forces are not as much dominant as gravitational forces the self gravitational collapse results fragmentation of dusty objects.

Chapter 5

Electrostatic Modes in a Self-Gravitating Dusty Plasma

5.1 Introduction

The theoretical investigations, where the gravitational force is neglected, are only valid in a plasma regime in which the electrostatic force is much greater than the gravitational force. The low frequency dust acoustic modes for which the dust particles mass provides the inertia and the presence of intertialess ions and electrons provide the restoring force has been studied by number of the authors [22,23]. When the effect of the fast particles, dust temperature, external magnetic field are taken into account, these effect dractically modify the electrostatic modes in self-gravitating dusty plasma. Now we are considering the three components dusty plasma which consist of extremely massive dust grains, ions and electrons in the presence of external magnetic field. It turns out that the self-gravitational force and free electrons drives the electrostatic mode unstable, whereas, the non-thermal ions and magnetic field plays a stabilizing role. In this section, we are going to derive the dispersion relation for the electrostatic mode in cold magnetized dusty plasma [24].

5.2 Cold Magnetized Dusty Plasma

5.2.1 Governing Equations

We are considering three component dusty plasma which consists of extremely massive, negatively charged inertial cold dust grains, non-thermally distributed ions and Boltzmann distributed electrons in the presence of an external static magnetic field $\mathrm{B}_0 = \hat{\mathrm{z}}\mathrm{B}_0$; where $\hat{\mathrm{z}}$ is a unit vector along z-direction. At equilibrium we have $\mathrm{n}_{i0} = \mathrm{Zn}_{d0} + \mathrm{n}_{e0}$; where n_{i0}; n_{e0}; n_{d0} are the unperturbed ions, electrons and dust number densities, respectively and Z_d is the number of elecrons residing on the dust grain surface. The basic linearized governing equations are

$$\frac{\partial n_{d1}}{\partial t} + \nabla.\left(n_{d0} u_{d1}\right) = 0 \tag{5.1}$$

$$\frac{\partial u_{d1}}{\partial t} = \frac{Z_d e}{m_d} \nabla \Phi_1 - \nabla \Psi_G - \omega_{cd}\left(u_{d1} \times \hat{z}\right) \tag{5.2}$$

$$\nabla^2 \Phi_1 = -4\pi e\left(n_{i1} - n_{e1} - Z_d n_{d1}\right) \tag{5.3}$$

$$\nabla^2 \Psi_1 = -4\pi e G m_d n_{d1} \tag{5.4}$$

$$n_{e1} = n_{e0} \exp\left(\frac{e\Phi_1}{T_e}\right) \tag{5.5}$$

$$n_{i1} = n_{i0}\left(1 + \frac{\beta e \Phi_1}{T_i} + \frac{\beta e^2 \Phi_1^2}{T_i^2}\right) \exp\left(-\frac{e\Phi_1}{T_i}\right) \tag{5.6}$$

Where $\omega_{cd} = \frac{Z_d e B_0}{c m_d}$ is dust cyclotron frequency, $\beta = 4\alpha/(1+3\alpha)$ with being a parameter determining the number of non-thermal ions, here β is describing fast particle i.e., non-thermalions. $T_i(T_e)$ is the ion (electron) temperature expressed in the energy units.

5.3 Dispersion Relation

From Eqs. (5.3), (5.5) and (5.6), we get

$$\nabla^2\Phi_1 = -4\pi e Z_d n_{d1} + \frac{1}{\lambda_{De}^2}\Phi_1 + \frac{1}{\lambda_{Di}^2}\left(1-\beta\right)\Phi_1 \tag{5.7}$$

Where $\lambda_{De}{=}\sqrt{T_e/4\pi e^2 n_{e0}}$, $\lambda_{Di}{=}\sqrt{T_i/4\pi e^2 n_{i0}}$ are the electron and ion Debye lengths. Separating Eq. (5.2) inti its components, we have

$$\frac{\partial u_{d1x}}{\partial t} = \frac{Z_d e}{md}\frac{\partial}{\partial x}\Phi_1 - \frac{\partial}{\partial x}\Psi_{G_1} - \omega_{cd}u_{d1y} \tag{5.8}$$

$$\frac{\partial u_{d1y}}{\partial t} = \frac{Z_d e}{md}\frac{\partial}{\partial y}\Phi_1 - \frac{\partial}{\partial y}\Psi_{G_1} - \omega_{cd}u_{d1x} \tag{5.9}$$

$$\frac{\partial u_{d1z}}{\partial t} = \frac{Z_d e}{md}\frac{\partial}{\partial z}\Phi_1 - \frac{\partial}{\partial x}\Psi_{G_1} \tag{5.10}$$

Assuming plane-wave approximation i.e. all the perturbed quantities are behaving sinusoidally, we have Ψ_{G_1} , u_{d1}, $\Phi_1\ \alpha \exp\left(ik.r - i\omega t\right)$, Eqs. (5.8), (5.9) and (5.10) yields

$$-i\omega u_{d1x} = \frac{Z_d e}{m_d} ik_x\Phi_1 - ik_x\Psi_{G_1} - \omega_{cd}u_{d1y} \tag{5.11}$$

$$-i\omega u_{d1y} = \frac{Z_d e}{m_d} ik_y\Phi_1 - ik_y\Psi_{G_1} - \omega_{cd}u_{d1x} \tag{5.12}$$

$$-i\omega u_{d1z} = \frac{Z_d e}{m_d} ik_z\Phi_1 - ik_z\Psi_{G_1} \tag{5.13}$$

Substitute Eq. (5.11) into Eq. (5.12), we get

$$\left(\omega^2 - \omega_{cd}^2\right) u_{d1x} = \left(\omega k_x - i k_y \omega_{cd}\right) \Psi_{G_1} + \frac{Z_d e}{m_d} \left(i k_y \omega_{cd} - \omega k_x\right) \Phi_1 \tag{5.14}$$

Similarly

$$\left(\omega^2 - \omega_{cd}^2\right) u_{d1y} = \left(\omega k_y - i k_x \omega_{cd}\right) \Psi_{G_1} + \frac{Z_d e}{m_d} \left(i k_x \omega_{cd} - \omega k_y\right) \Phi_1 \tag{5.15}$$

From Eq. (5.13), we have

$$\omega u_{d1x} = k_z \Psi_{G_1} - \frac{Z_d e}{m_d} k_z \Phi_1 \tag{5.16}$$

To derive the dispersion relation for an obliquely propagating electrostatic mode in a gravitating magnetized dusty plasma, the Fourier transform of Eqs. (5.1) , (5.4) and (5.7) give

$$\omega n_{d1} = n_{d0} \left(k_x u_{d1x} - k_y u_{d1y} - k_z u_{d1z}\right) \tag{5.17}$$

$$\Psi_1 = -\frac{4\pi e G m_d n_{d1}}{k^2} \tag{5.18}$$

$$\Phi_1 = \frac{4\pi e Z_d n_{d1}}{\left(i^2 k^2 - \lambda_{De}^{-2} - (1-\beta)\, \lambda_{De}^{-2}\right)} \tag{5.19}$$

From Eqs. (5.14) to (5.19), we obtain

$$\omega^2 \left(\omega^2 - \omega_{cd}^2\right) k^2 + \omega_{jd}^2 \left(\omega^2 k^2 - \omega_{cd}^2 k_{\|}^2\right) = \frac{\omega_{pd}^2 \left(\omega^2 k^2 - \omega_{cd}^2 k_{\|}^2\right)}{k^2 + \lambda_{De}^{-2} + (1-\beta)\, \lambda_{Di}^{-2}} \tag{5.20}$$

This is the desired dispersion relation for an obliquely propagating electrostatics mode in a gravitating magnetized dusty plasma with fast ions (β) and free electrons. If the time and space variables are in units of the dust plasma period $\omega_{jd}^{-1} = \left(m_d/4\pi e Z_d^2 n_{d0} e^2\right)^{1/2}$ and the Debye length $\lambda_{Dd} = \left(T_i/4\pi e Z_d^2 n_{d0} e^2\right)^{1/2}$, respectively, $\omega_{cd} = Z_d B_0 e/m_d c\omega_{pd}$ is the dust cyclotron frequency normalized to ω_{pd} in which c is the speed of light in vacuum, $\sigma_i = T_i/T_e$ where T_i and T_e are in energy units. $\beta = 4\alpha/\left(1+3\alpha\right)$ with α being a parameter determining the number of non-thermal ions, $\mu_0 = \mu/\left(1-\mu\right)$ and $\mu_1 = 1/\left(1-\mu\right)$ with $\mu = n_{e0}/n_{i0}$, and $\delta = G\left(md/Z_d e\right)^2$ with G being the universal gravitational constant. In normalized form we can write Eq. (5.20) as

$$\left[\omega^2\left(\omega^2-\omega_{cd}^2\right)k^2+\delta\left(\omega^2 k^2-\omega_{cd}^2 k_{\parallel}^2\right)\right]\left[\left(1-\beta\right)\mu_1+\sigma_i\mu_0 k^2\right] = k^2\left(\omega^2 k^2-\omega_{cd}^2 k_{\parallel}^2\right) \tag{5.21}$$

5.4 Limiting Cases

We shall consider two interesting limiting cases which are

5.4.1 Case 1. Parallel Propagation ($k_{\perp} = 0$)

The dispersion relation for the low-frequency electrostatic mode which propagates along the magnetic field B_0 $\left(k_{\perp} = 0\right)$, can be expressed as

$$\omega^2\left(\omega^2-\omega_{cd}^2\right)k_{\parallel}^2+\omega_{jd}^2\left(\omega^2-\omega_{cd}^2\right)k_{\parallel}^2 = \frac{\omega_{pd}^2 k_{\parallel}^4\left(\omega^2-\omega_{cd}^2\right)}{k_{\parallel}^2+\lambda_{De}^{-2}+\left(1-\beta\right)\lambda_{Di}^{-2}}$$

$$\omega^2 = \frac{\omega_{pd}^2 k_{\parallel}^2}{k_{\parallel}^2+\lambda_{De}^{-2}+\left(1-\beta\right)\lambda_{Di}^{-2}}-\omega_{jd}^2 \tag{5.22}$$

It is evident from Eq. (5.22) that for parallel propagation case, the mode is independent of B_0 and it depends on the relative values of ω_{jd}^2 and $k_\parallel^2/\left(k_\parallel^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}\right)$ terms. Therefore the condition for the mode to be unstable can be written as

$$\omega_{jd}^2 > \frac{\omega_{pd}^2 k_\parallel^2}{k_\parallel^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}$$

and the growth rate γ of this unstable mode is given by

$$\gamma = \operatorname{Im}\omega = \sqrt{\omega_{jd}^2 - \frac{\omega_{pd}^2 k_\parallel^2}{k_\parallel^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}} \tag{5.23}$$

In normalized form we have the above equation

$$\gamma = \operatorname{Im}\omega = \sqrt{\delta - \frac{k_\parallel^2}{k_\parallel^2 + \sigma_i\mu_0 + (1-\beta)\,\mu_1}} \tag{5.24}$$

This equation implies that in the presence of non-thermal ions the mode is stable, whereas the effect of gravitational force (ω_{jd}) and free electrons destabilize the mode.

5.4.2 Case 2. Perpendicular propagation $\left(k_\parallel = 0\right)$

From Eq. (5.20), we can write

$$\omega^2\left(\omega^2 - \omega_{cd}^2\right)k_\perp^2 + \omega_{jd}^2\omega^2 k_\perp^2 = \frac{\omega_{pd}^2 k_\perp^4 \omega^2}{k_\perp^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}$$

$$\omega = \sqrt{\left(\omega_{cd}^2 + \frac{\omega_{pd}^2 k_\perp^2}{k_\perp^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}\right) - \omega_{jd}^2} \tag{5.25}$$

Condition for mode to be unstable will be

$$\omega_{jd}^2 > \left(\omega_{cd}^2 + \frac{\omega_{pd}^2 k_\perp^2}{k_\perp^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}\right) \tag{5.26}$$

The growth rate γ of this unstable mode is given by

$$\gamma = \operatorname{Im}\omega = \sqrt{\omega_{jd}^2 - \left(\omega_{cd}^2 + \frac{\omega_{pd}^2 k_\perp^2}{k_\perp^2 + \lambda_{De}^{-2} + (1-\beta)\,\lambda_{Di}^{-2}}\right)} \tag{5.27}$$

In normalized form we have the above equation

$$\gamma = \operatorname{Im}\omega = \sqrt{\delta - \left(\omega_{cd}^2 + \frac{k_\perp^2}{k_\perp^2 + \sigma_i\mu_0 + (1-\beta)\,\mu_1}\right)} \tag{5.28}$$

It is evident that the growth rate depend on gravitational force term (δ), free electrons $(\mu,)$, cyclotron frequency of dust (ω_{cd}) and the fast particles (β). It is seen that the magnitude of the magnetic field and presence of non-thermal ions decreases the growth rate γ while the gravitational force term enhances the growth rate.

5.5 Result and Discussion

To summarize, we have considered multicomponent magnetized plasma whose constituents are electrons, ions, dust and neutrals. In this chapter, the dispersion relation for the low-frequency electrostatic modes propagating parallel and perpendicular to the external magnetic field in self-gravitating dusty plasma with non-thermal ions are derived in cold dusty plasma. When the gravitational force is taken into account, these modes become unstable. The effect of gravitational force, number of free electrons and ion temperature make these modes unstable. Fast ions, dust temperature and external magnetic field try to stabilize the modes. The growth rate γ for these instabilities decreases with the magnitude of magnetic field, number of non-thermal ions and the dust temperature and increase with the ratio of grain mass to grain charge $(m_d/Z_d e)$ with the ratio of ion temperature to electron temperature and with number of free electrons. These results are useful to explain gravitational condensation of dust grain in planetary system and to understand features of electrostatic disturbances in space plasmas.

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