# Hybrid-ARQ Transceiver for OFDM System


Awais Ahmed

Department of Electronics

Quaid-i-Azam University, IslamabadPakistan


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A thesis entitled "Hybrid-ARQ Transceiver for OFDM System" by Awais Ahmed in partial fulfilment of the requirements for the degree of Master of Philosophy, has been approved and accepted by the following

---

**Advisor**
Dr. Muhammad Zia
Assistant Professor
Department of Electronics
Quaid-i-Azam University, Islamabad
Pakistan

---

**Chairman**
Dr. Farhan Saif
Associate Professor
Department of Electronics
Quaid-i-Azam University, Islamabad
Pakistan

بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

إِنَّ فِى خَلْقِ السَّمَاوَاتِ وَالأَرْضِ وَاخْتِلاَفِ اللَّيْلِ وَالنَّهَارِ وَالْفُلْكِ الَّتِى تَجْرِى فِى الْبَحْرِ بِمَا يَنفَعُ
النَّاسَ وَمَا أَنزَلَ اللهُ مِنَ السَّمَاءِ مِن مَّاءٍ فَأَحْيَا بِهِ الأَرْضَ بَعْدَ مَوْتِهَا وَبَثَّ فِيهَا مِن كُلِّ
دَآبَّةٍ وَتَصْرِيفِ الرِّيَاحِ وَالسَّحَابِ الْمُسَخِّرِ بَيْنَ السَّمَاءِ وَالأَرْضِ لَآيَاتٍ لِّقَوْمٍ يَعْقِلُونَ ○
(البَقَرَة : 164)

Verily, in the creation of the heavens and the earth, and in the alternation of the night and the day, and in the ships (and vessels) which sail through the ocean carrying cargo profitable for the people, and in the (rain) water which Allah pours down from the sky, reviving therewith the earth to life after its death, and (the earth) in which He has scattered animals of all kinds, and in the changing wind directions, and in the clouds (that trail) between the sky and the earth, duty-bound certainly, (in these) are (many) signs for those who put their reason to work.

(Al-Baqarah : 164)

## Acknowledgements

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# Abstract

In this work, we present a novel method which employs joint Automatic Repeat reQuest (ARQ) mechanism for Partial Automatic Repeat reQuest (PARQ) in multiple-input multiple-output (MIMO) systems with orthogonal frequency division multiplexing (OFDM) signaling. The thought behind PARQ is to retransmit only partial copy of information symbols sequence rather than retransmitting the whole sequence. The receiver makes use of joint detection by exploiting observations from multiple transmissions. As a direct consequence of partial retransmission, the bandwidth and power efficiency of the overall system is improved considerably without much decrease in the performance. The channel estimation is pivotal to any communication system and its estimation error directly affects the performance of MIMO-OFDM system. In order to acquire channel knowledge, conventional communication systems rely on pilot assisted approaches at the expense of bandwidth and power resources. In this work, we omit training sequence during retransmission and combine both blind and training based strategies to estimate channels in semi-blind fashion. In order to evaluate efficacy of the proposed PARQ method, we also provide throughput comparison between Chase-Combining hybrid-ARQ (CC-HARQ) approach with proposed partial ARQ method.

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# Nomenclature

## SYMBOLS

| Symbol | Description |
|---|---|
| $R$ | Code Rate |
| $T$ | Symbol Period |
| $N$ | FFT size |
| $\{.\}^H$ | Matrix Hermitian operator |
| $\{.\}^T$ | Matrix Transpose operator |
| $\{.\}^*$ | Matrix Conjugate operator |
| $\{.\}^p$ | Retransmission Symbols |
| $n_r$ | Number of receive antennas |
| $n_t$ | Number of transmit antennas |
| $\mathbf{F}$ | FFT matrix |
| $\mathbf{F}^H$ | IFFT matrix |
| $L$ | Channel order |
| $L_p$ | Length of cyclic prefix |
| $\mathbf{H}$ | Channel matrix |
| $\mathbf{R}_{\{.\}}$ | Covairiance matrix |
| $\mathbf{y}_k$ | Received symbol vector |
| $\mathbf{s}_k$ | Transmitted symbol vector |
| $\mathbf{w}_k$ | Noise symbol vector |

| **Acronym** | **Description** |
|---|---|
| *ACK* | Acknowlegement |
| *ADSL* | Asymmetric Digital Subscriber Line |
| *ARQ* | Automatic Repeat reQuest |
| *AWGN* | Additive White Gaussian Noise |
| *BER* | Bit Error Rate |
| *CC* | Chase Combining |
| *COFDM* | Coded Orthogonal Frequency Division Multiplexing |
| *CP* | Cyclic Prefix |
| *DAB* | Digital Audio Broadcasting |
| *DFT* | Discrete Fourier Transform |
| *DVB* | Digital Video Broadcasting |
| *FEC* | Forward Error Correction |
| *FFT* | Fast Fourier Transform |
| *HARQ* | Hybrid Automatic Repeat reQuest |
| *HOS* | Higher Order Statistics |
| *IDFT* | Inverse Discrete Fourier Transform |
| *IFFT* | Inverse Fast Fourier Transform |
| *I.I.D* | Independent and identically distributed |
| *IR* | Incremental Redundancy |
| *ISI* | Inter Symbol Interference |
| *LOS* | Line Of Sight |
| *LTE* | Long Term Evolution |
| *MIMO* | Multiple Input Multiple Output |
| *MISO* | Multiple Input Single Output |
| *ML* | Maximum Likelihood |
| *MMSE* | Minimum Mean Square Error |
| *MRC* | Maximal Ratio Combining |
| *NACK* | Negative Acknowlegement |
| *NLOS* | Non Line Of Sight |
| *NMMSE* | Normalized Minimum Mean Square Error |
| *OFDM* | Orthogonal Frequency Division Multiplexing |
| *OFDMA* | Orthogonal Frequency Division Multiple Access |
| *PARQ* | Partial Automatic Repeat reQuest |
| *PSK* | Phase Shift Keying |
| *QAM* | Quadrature Amplitude Modulation |
| *QOS* | Quality of Service |
| *SIMO* | Single Input Multiple Output |
| *SISO* | Single Input Single Output |
| *SNR* | Signal to Noise Ratio |
| *SOS* | Second Order Statistics |
| *VOIP* | Voice Over IP |
| *WIMAX* | Worldwide Interoperability for Microwave Access |
| *ZF* | Zero Forcing |

# Chapter 1

# Introduction

## 1.1 Motivation

In recent years, exponential growth in demand for higher data rate in wireless networks has sparked cross-layer receiver design. Diversity techniques in communication system through MIMO are the core of modern communication standards. The use of multiple antennas both at transmitter and receiver can combat many problems including fading and interference. MIMO systems offer advantages like spectral efficiency, capacity [1], beamforming and space time coding [2]. Along with MIMO systems, various techniques are combined to achieve further enhancements like high data rates and power efficiency. Among all the methods, one of these is Orthogonal Frequency Division Multiplexing.

In MIMO OFDM signaling, a frequency-selective wide-band spectrum is divided into several parallel flat-fading equal bandwidth sub-channels in order to combat inter-symbol interference (ISI) [2] . That results into low complexity receiver. This makes channel equalization and receiver design very simple. Due to simple receiver design under OFDM signaling, OFDM is an emerging technology for future wireless standards. In particular, many wireless standards such as WiMAX, Long Term Evolution (LTE), LTE Advanced, DVB (Digital Video Broadcast) have adopted OFDM technology [3].

Automatic Repeat reQuest (ARQ) has been used to ensure system reliability by retransmitting the data packets which are not correctly received during first or subsequent transmissions. Capacity achieving channel codes such as low density parity check (LDPC) and Turbo codes along with ARQ techniques enhance the robustness of the wireless communication systems. In order to maximize throughput for all kinds of channel conditions, hybrid ARQ (HARQ) has been of great interest in research and development community [4]. A HARQ system consists of conventional ARQ method with forward error correcting codes (FEC) for error detection and correction. Function of FEC in HARQ is to reduce the number of transmissions by correcting errors. HARQ method is further classified into type-I and type-II hybrid ARQ methods [5]. In type-I HARQ, receiver does not combine data from multiple transmissions for joint detection. However, in type-II HARQ, receiver keeps data from multiple transmissions for joint detection. Type-II HARQ is further classified into Chase Combining (CC) and Incremental Redundancy (IR). In CC-HARQ [6], if receiver can recover corrupted bits, it saves received data and generates retransmission request for the transmitter. In response to retransmission request, transmitter retransmits full original packet and receiver combines multiple copies of the same packet for joint detection. In IR-HARQ scheme, transmitter sends more parity bits in response to retransmission requests. Type-II HARQ [6, 7] is set to be part of both the present and future wireless standards like long term evolution(LTE) [3], advanced LTE and Worldwide Interoperability for Microwave Access(WiMAX) [8]. Due to efficacy of type-II HARQ, it is adopted in WIMAX, LTE and advanced LTE standards. Most attention has been paid to HARQ methods that involve FEC codes along with ARQ. However, very few work has been done on HARQ techniques that involve modulation layer of the wireless system [9, 10, 10, 11]. In CC-HARQ, after the packet has been received and evaluated whether it is received error free or a retransmission is necessary, CC-HARQ enables transmitter to send a full copy of data packet. In many situations only a few errors occur during transmission, so it would be power and bandwidth efficient to retransmit only a portion of sequence rather than retransmitting the whole sequence. In method proposed for single-input single-output SISO OFDM system [10], a decimated copy of the original OFDM symbol is transmitted against retransmission request. This method is named as

Partial ARQ (PARQ) and improves bandwidth and power efficiency by jointly detecting data from multiple transmissions.

## 1.2 Problem Statement

In our work, we extend work in [10] to MIMO OFDM systems. Previous works ignored the involvement of modulation layer of the wireless systems.In many situations only a few errors occur, so it is desired to transmit only some portion of the data packet. In our simulation results, we demonstrate that PARQ based joint detection scheme for OFDM MIMO systems can significantly reduce the BER of single transmission. We also expect that PARQ is power and bandwidth efficient as compared to full retransmission due to the fact that we omit training during retransmission and retransmit decimated copy of original information sequence.

In traditional ARQ, for distortive channels, channel estimation is carried out for each retransmission. Training sequences are often used for channel estimation for every retransmission which is costly especially for MIMO systems and are prone to inter-symbol interference. So, it is desired to conserve the bandwidth if retransmission uses least number of training sequences. In pursuit of saving the bandwidth, joint channel estimation is introduced but we make use of joint semi-blind channel estimation that promises to be spectrally efficient in partial ARQ.

In this dissertation, we address that joint semi-blind estimation can achieve comparable results to training based estimation. It takes advantage of training symbols during first transmission and blind subspace based channel estimation is used during retransmissions. Particularly, we exploit both training and second order statistics to formulate a semi-blind algorithm achieving better performance.

## 1.3 Dissertation Outline

The organization of remaining dissertation is as follows:
**Chapter** 2 introduces the concepts of MIMO and OFDM. Issues and advantages relating MIMO systems are discussed. A comprehensive overview of OFDM is also presented in this chapter.
**Chapter** 3 summarizes the incentives of using HARQ by evaluating the shortcomings of the methods used previously like ARQ. The protocol of HARQ is given along with different types of HARQ. Puncturing in the HARQ framework is linked with the next chapter by providing its need.
**Chapter** 4 provides the details of proposed algorithm for PARQ Transceiver with MIMO OFDM. New Signal detection algorithm is presented in this chapter. A new modified technique for semi-blind channel estimation is also discussed in this chapter.
**Chapter** 5 consists of the simulation results and performance analysis of the proposed PARQ method. The results demonstrate that the proposed algorithm for signal detection and channel estimation provide a bandwidth and power efficient alternative.
**Chapter** 6 concludes this dissertation by providing a brief summary of major achievements and suggestions for possible extensions to the present work.

# Chapter 2

# MIMO OFDM

Over the past few years, there has been increasing demand for high data rates in wireless communication systems. This demand has attracted research community to provide solutions of exploding demands of users. OFDM has recently evolved as main signaling technique to achieve high throughput at low receiver complexity. In particular, OFDMA signaling is adapted by LTE due to its potential to achieve high throughput. OFDM is basically a type of multi-carrier transmission technique. MIMO OFDM systems offer many advantages such as simple implementation, multi-user system access, robustness to narrow band interference and increase in capacity for slow fading channels. Due to these advantages, MIMO OFDM systems have gained attention in recent standards like LTE, DAB and DVB. The main attraction of OFDM is its property of converting frequency selective channels into multiple and orthogonal frequency flat sub-carriers. Thus, the equalizer at the receiver has a very low complexity. The use of guard band in OFDM further helps to combat against ISI.

## 2.1 Characteristics of Mobile Channel

The quality of communication in mobile radio systems mainly depends upon behavior of the channel. Different obstacles cause reflections to the transmitted signal. Hence, the received signal arrives with different gains and delays.

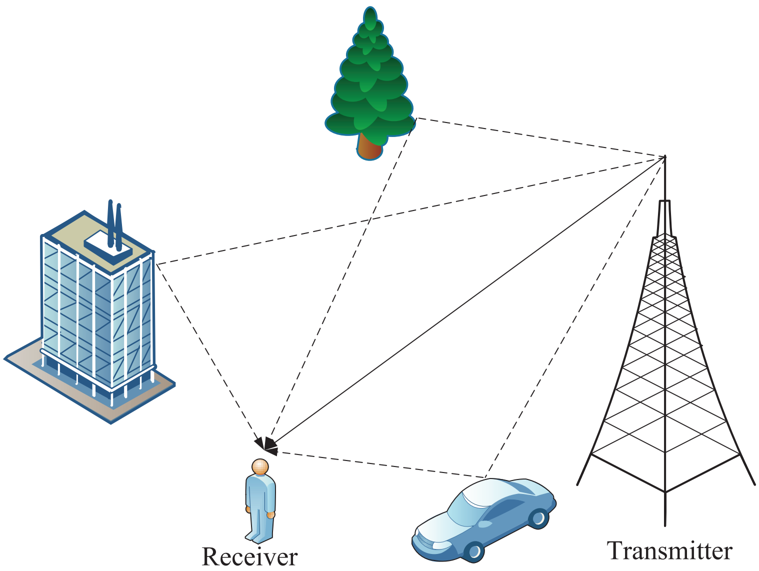


Figure 2.1: Multipath Propagation

Due to these reflections, mobile communication channel or medium is mostly characterized by non-line-of-sight (NLOS). Reliable wireless communication becomes a difficult task when the transmitted signal is degraded by inter symbol interference along with multipath fading. To fully understand the concepts of wireless communication, one must clearly understand the basic propagation characteristics of channel and techniques applied. A description of inherent properties governing wireless communication systems is discussed in the next section.

### 2.1.1 Multipath Propagation Model

Mostly in mobile radios, communication occurs through medium that is rich in scattering. Such mediums are known as NLOS mediums. The presence of various objects like trees, buildings, houses in the environment affect the signals in different ways. These affects include reflection, diffraction and scattering. Due to the reflections from obstacles radio waves travel along different paths and arrive at different times at the receiver as illustrated in Figure 2.1. The interactions between waves introduce multipath fading and when these are combined at any location, their strengths vary depending on the constructive

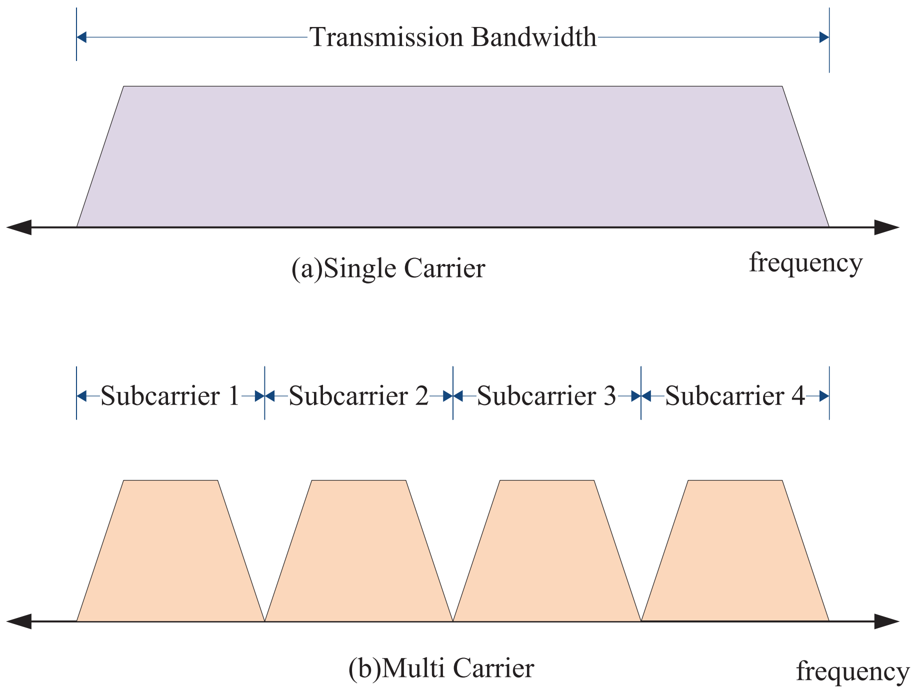


Figure 2.2: Single vs. Multiple Carrier Transmission

and destructive combination. Thus the received signal can vary significantly both in amplitude and phase.

The fading is normally classified into two parts; large-scale fading and small-scale fading [12, 13]. **Large-scale fading** materializes in shadowing and path loss, which attenuates average signal power over large areas. **Small-scale fading** exemplifies itself by the drastic changes in signal amplitude and phase because of the small variations in the environment. To mitigate these fading effects various transmission techniques have been put forward. The analysis of these transmission techniques is presented next.

### 2.1.2 Single Carrier versus Multi-carrier Transmission

In single carrier transmission, transmitted signal is spread over the whole bandwidth and single information symbol is released at a time as is depicted in Figure 2.2. Consider N symbols each having symbol period T, i.e. data rate R=$\frac{1}{T}$, are transmitted through channel having bandwidth B. For reliable communication at data rate R, the minimum bandwidth required is $\frac{R}{2}$. It is evident that due to high data rate requirement, wider bandwidth with its efficient use is needed to meet the aforementioned need. As the signal bandwidth becomes larger than the coherence bandwidth the multipath effects become more significant, resulting in inter-symbol interference (ISI). The complexity of the receiver increases drastically with the increase in ISI. So, single carrier transmission doesn't offer a feasible solution due to high complexity of the receiver.

To resolve the issues of single carrier transmission, multiple carrier transmission was introduced in order to achieve higher data rates with low complexity receiver [14]. The basic idea behind multi-carrier transmission is shown in Figure 2.2, in which a wideband channel is approximated by several narrowband channels, each at a different center frequency and each signal can be modulated independently [14]. This converts frequency selective channels into frequency flat channels which significantly reduces the complexity of the receiver. Multi-carrier transmission schemes are opted with MIMO systems in almost all of the recent standards.

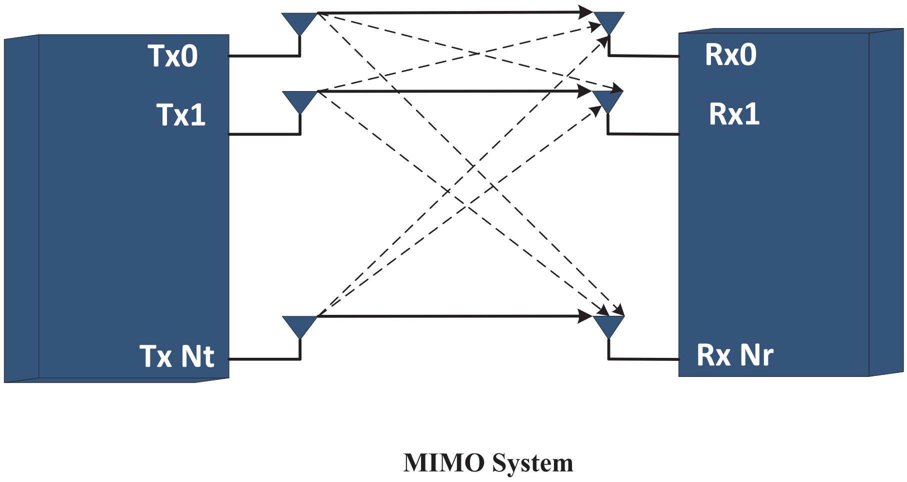


Figure 2.3: MIMO System

### 2.1.3 Multiple Input Multiple Output (MIMO)

The deployment of multiple antennas both at transmitter and receiver, commonly known as Multiple Input Multiple Output (MIMO) has significantly improved the overall throughput of any communication system in a cost effective manner. The data rate can be significantly increased by sending multiple streams through different antennas each of them treated independently. A pictorial description of MIMO system is shown in Figure 2.3.

The concept of MIMO has been under the radar for many years for all kinds of systems such as array signal processing for beamforming. The concept was first introduced in 1984 [15]. In this seminal work, multiple antennas are used to transmit data from multiple users over the same channel. Many works have been published and implemented since then [16, 17]. MIMO systems offer many advantages including diversity, array gain, spectral efficiency and capacity.

**Diversity**

A MIMO system is capable of achieving transmit diversity by transmitting copy of the same information through independent transmit antennas by constructing space-time block code [17, 18]. At the receiver, signals from multiple antennas can be combined to reduce amplitude variations from multipath hence increasing the throughput. On the same lines, multiple antennas can be installed at the receiver to introduce receive diversity. The information from multiple receive antennas can be combined to decrease the probability of error.

**Array Gain**

By exploiting the correlation characteristics between the combined signals from multiple antennas both at transmitter and receiver, Signal to noise ratio(SNR) can be increased. This increase in signal power is termed as array gain and it increases with the number of antennas.

**Spectral Efficiency and Capacity**

Data rates can be increased by transmitting multiple streams through multiple antennas at same time without consuming more bandwidth or power. As the number of transmitters and receivers increase spectral efficiency, system capacity also increases [16].

Precisely due to the above mentioned advantages, MIMO technology is the critical part of present and future wireless standards. A large amount of focus has been laid on the enhancements in MIMO systems like beamforming [19]. A complete overview of MIMO enhancements and their performances can be viewed in [20]. Latest standards like 3GPP release 8 LTE has included MIMO as a critical technology even in its development stage. New releases have also accepted the advantages of MIMO and it is set to be integral part of almost every future standard. Multiple antennas are part of discussion under OFDM signaling in the next section.

## 2.2 OFDM Basics

Orthogonal Frequency Division Multiplexing is specifically a type of multicarrier transmission scheme which offers an attractive solution for high data rate communication systems. OFDM converts the frequency selective channel into multiple frequency non-selective channels. The channels are overlapping but still orthogonal. Orthogonality of sub-carriers overcomes the need for guard bands between sub-channels and makes it highly spectral efficient. In addition, the orthogonality allows the receiver to separate the sub-channels. As a result, the receiver complexity is low in OFDM systems. Due to promising features of OFDM, LTE adapted OFDM together with channel coding. More insight on the background and theoretical model of OFDM is given in the following section.

### 2.2.1 OFDM Background

OFDM has emerged as a strong candidate for recent standards both in wired and wireless environments. OFDM has been adopted by recent and future standards like WiMax, LTE and DVB [3]. It has also been the part of wired standards such as Asymmetric Digital Subscriber Line (ADSL) and High bit rate Digital Subscriber Line (HDSL) [21]. The idea of OFDM was first introduced by Chang in 1966 [22].
In traditional Frequency Domain Multiplexing (FDM) techniques bandwidth

is divided into non overlapping sub-carriers which are independent of other. Thus, each sub-carrier can be modulating separately and interference is reduced. In OFDM, sub-carriers overlap with the neighboring sub-carriers but are still orthogonal to each other. This overlapping conserves valuable bandwidth if the orthogonality remains intact. In the early days of OFDM sinusoidal sub-carriers were used which required signal generators and other hardware, but the complexity and cost of hardware were increased by increase in the number of sub-carriers. It was not until 1971 that a modified OFDM system has been proposed by Weinstein and Ebert [23], which used Discrete Fourier Transform (DFT) to generate orthogonal signals as sub-carriers. This significantly reduced the complexity of OFDM system. Inverse Digital Fourier Transform (IDFT) is used to modulate the signals while demodulation is carried out by taking the DFT. This ensures both the overlapping and orthogonality property. Another notable advancement in OFDM system was made by Peled and Ruiz [24] in the year 1980. They have introduced the notion of cyclic prefix (CP) which is essentially a null or guard band appended cyclically either at the end or beginning of every OFDM symbol. Addition of CP is vital to mitigate inter block interference of OFDM symbols and maintain orthogonality of sub-carriers [2].

OFDM in LTE(3G) and LTE Advanced (4G) allows adaptive modulation and channel coding (AMC) to enhance spectral efficiency. Furthermore, multi-user OFDM in LTE called OFDMA [25] is key to QoS and spectral efficiency.

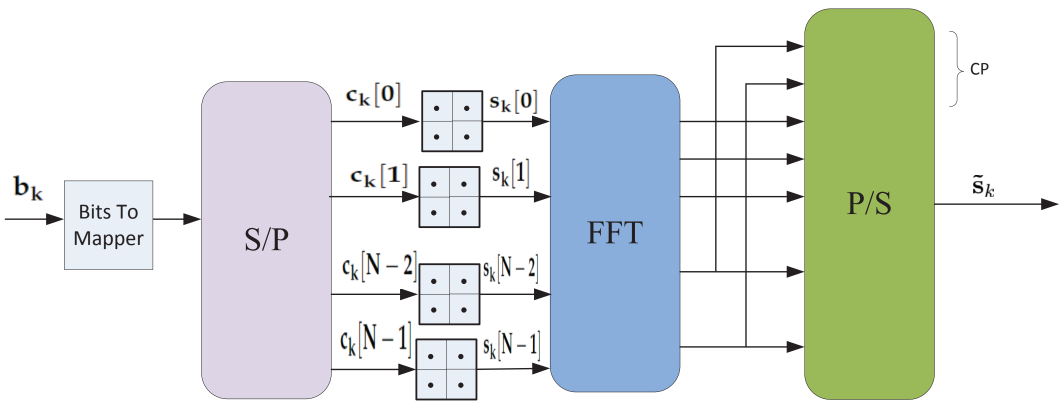


Figure 2.4: OFDM Tranmitter

### 2.2.2 OFDM System Model

In a wireless environment, the receiver may incur ISI due to the multipath where each path has different delay. OFDM is an alternative to counter this ISI. In OFDM, the serial data $b_k$ is first converted into N parallel streams. This conversion transforms the symbol time to be greater than maximum delay spread of channel. This increase in the symbol time reduces the effect of inter-symbol interference. Moreover, the receiver complexity is also reduced significantly, provided the channel response remains constant during the transmission of one OFDM frame. System model of OFDM transmitter is depicted in Figure 2.4. Let us consider MIMO OFDM system with $n_r$ and $n_t$ receive and transmit antennas, respectively. Each transmit antenna will have the information sequence of length $N$. Note that $N$ is number of OFDM sub-carriers. The symbols before the FFT operation are of the form $\mathbf{s}_k \in \mathcal{C}^{n_t(N)\times 1}$, where $k$ is the time index of the OFDM symbol. It is important to note that each sub-carrier among the modulated streams can be modulated independently using Phase Shift Keying (PSK), Quadrature Amplitude Modulation (QAM) or any other modulation technique. But in our case we are restricting to same modulation scheme during whole communication. Some of the ISI is removed due to se-

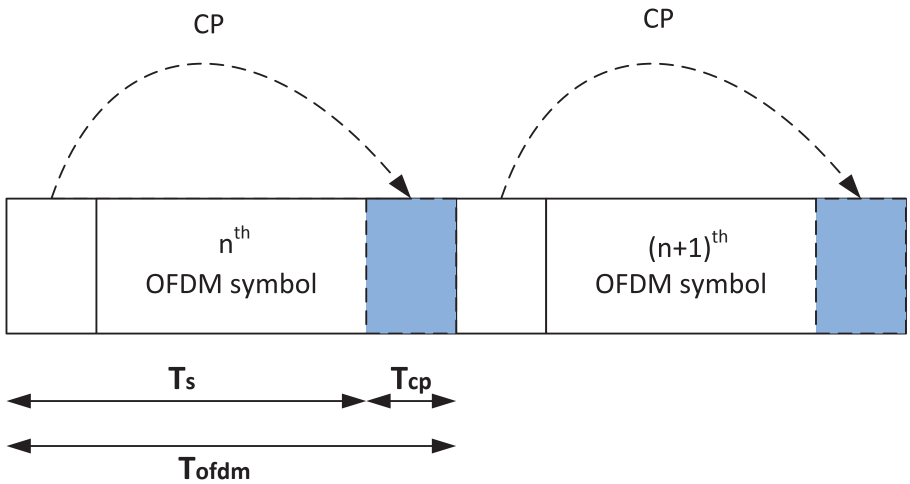


Figure 2.5: Cyclic Prefix

rial to parallel (S/P) conversion, the remaining ISI is removed by inserting a guard band to the time domain signal. The guard band known commonly as

cyclic prefix, is inserted by adding a copy of last $L_{cp}$ values either at the end or beginning of every OFDM symbol. After the insertion of CP, OFDM symbol results in total length of $N + L_{cp}$ symbols. The length $L_{cp}$ of CP is chosen such that it is greater than the maximum delay spread of channel. The process of addition of CP is shown in Figure 2.5. One added advantage of CP is that it turns the linear convolution into circular convolution. This advantage makes the implementation of FFT at the receiver very simple [2].
$N$ point FFT is applied to the modulation symbols to get the symbols $\tilde{\mathbf{s}}_k \in \mathcal{C}^{n_t(N+L_{cp})\times 1}$ as the output of the OFDM transmitter. These symbols are transmitted through the channel.

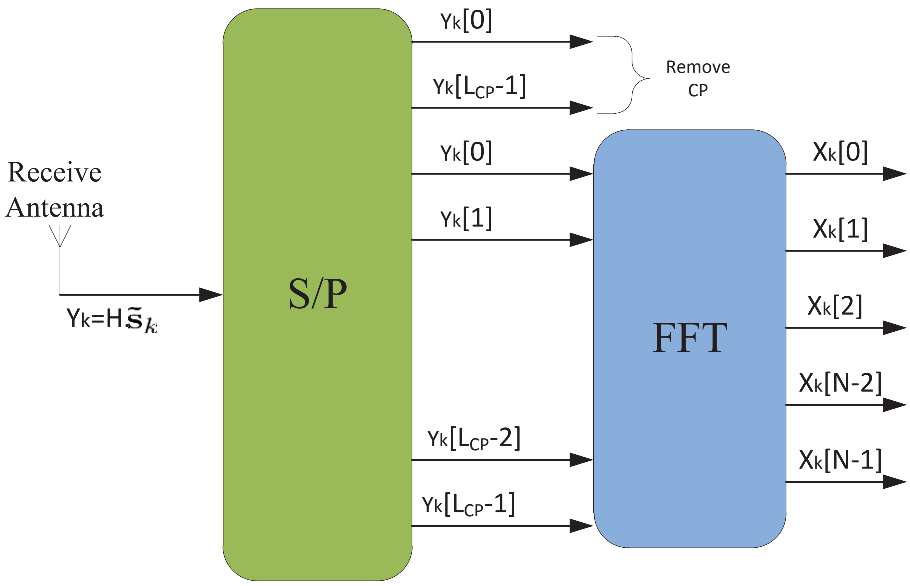


Figure 2.6: OFDM Receiver

Let us denote $\mathbf{h}_{n_r,n_t}$ as the channel response vector from any transmit antenna $n_t$ to the received antenna $n_r$. The length of each vector will be $L+1$. Due to the transformation from linear to circular convolution, the channel matrix of MIMO OFDM will be block circulant. The noise introduced by the channel is considered to be Gaussian. Let $\mathbf{w}_k \in \mathcal{C}^{n_t(N+L)}$ be the additive white Gaussian noise. At the receiver, exactly the inverse operations are performed. System description of OFDM receiver is shown in Figure 2.6. Firstly, $L_{cp}$ many symbols, which were placed at the end of every OFDM symbol at the receiver, are removed from the modulated signal. The remaining samples are converted into frequency domain by the implementation of FFT.

The CP added into the OFDM symbol converts this linear convolution into circular convolution. The circular convolution is efficiently implemented by FFT by simple multiplication. Thus, the received signal can be written in simplified form

$$\mathbf{x}_k = \mathbf{H}\tilde{\mathbf{s}}_k + \mathbf{w}_k, \tag{2.1}$$

where, for the MIMO transmission the channel matrix will be of the form:

$$\mathbf{H} = \begin{bmatrix} H_{(1,1)} & \dots & H_{(1,n_t)} \\ \vdots & \ddots & \vdots \\ H_{(n_r,1)} & \dots & H_{(n_r,n_t)} \end{bmatrix}, \tag{2.2}$$

where

$$H_{(n_r,n_t)} = \begin{bmatrix} h_0 & h_1 & \dots & h_L & 0 & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_L & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & h_0 & h_1 & \dots & h_L \\ h_L & \ddots & \ddots & 0 & h_0 & \dots & h_{L-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \vdots \\ h_1 & \dots & h_L & 0 & \dots & 0 & h_0 \end{bmatrix}. \tag{2.3}$$

The transformation of convolution into multiplication makes equalization very simple. The detailed equalization process will be discussed in Section 2.3.

## 2.3 OFDM Receiver

Transmitted information symbols can be estimated by the receiver in optimal and sub-optimal fashion. The information symbol vector can be estimated using matrix model of $k-th$ sub-carrier in Equation (2.1). There is always trade-off between complexity and performance in the receiver design.

### 2.3.1 Symbol Detection

Consider a MIMO system with $n_r$ and $n_t$ receive and transmit antennas, respectively. The received signal is given in (2.1). Linear receiver design can be implemented as lower complexity ZF and MMSE equalizers. However, optimal ML detection can be carried out at the expense of complexity. Sub-optimal ZF receiver can be written as

$$\widehat{\mathbf{s}}_k = \mathcal{H}^{\dagger}\mathbf{x}_k \tag{2.4}$$

where

$$\mathcal{H}^{\dagger} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \tag{2.5}$$

is pseudo inverse of channel matrix $\mathbf{H}_k$ of the $k-th$ sub-carrier. In some applications, no compromise in the performance can be tolerated. In such applications, optimal detectors which use exhaustive search in their implementation are used. The non-linear optimal ML detector is the most commonly used. Let $\hat{\mathbf{s}}_k^{ML}$ be the ML estimate of the respective symbol vector written as :

$$\hat{\mathbf{s}}_k^{ML} = \arg\min_{\mathbf{s}_k\in\mathcal{A}} \parallel \mathbf{x}_k - \mathbf{H}\cdot\mathbf{s}_k \parallel^2 \tag{2.6}$$

where $\mathcal{A}$ is the set of the constellation symbols. Note that aforementioned optimal and sub-optimal receivers assume full channel knowledge. Receiver can acquire channel knowledge at the expense of bandwidth resource by transmismitting training sequence known to the receiver.

### 2.3.2 Channel Estimation

Non-Coherent signal detection doesn't require any channel knowledge. But in most practical systems, coherent signal detection is the only option. For non-coherent detection, estimation of the channel is required which affects the overall receiver performance. Channel estimation uses training or pilot symbols known to both the transmitter and receiver to estimate the channel.

#### 2.3.2.1 Training based Channel Estimation

To achieve good performance, training symbols are used to estimate the channel. The receiver has complete information about the training symbols. Based on the received signal passed through the channel, the channel is estimated. The tradeoff between better performance is the symbols used in training which reduce the efficiency. The training symbols are embedded in a frame which consists of payload and training symbols. In traditional single antenna systems training symbols constitute only a fraction of the frame. Assuming that $\mathbf{S_t}$ training symbols are transmitted, the received vector $\mathbf{Y_t}$ after passing through the channel $\mathbf{h}$ can be represented by

$$\mathbf{Y}_t = \mathbf{S_t}\mathbf{h} + \mathbf{w}, \tag{2.7}$$

where $\mathbf{w}$ is the additive white Gaussian noise. The least square estimate [26] for training based estimation can be given by

$$\hat{\mathbf{h}} = \min_{\mathbf{h}} \|\mathbf{y}_t - \mathbf{S}_t\mathbf{h}\|^2. \tag{2.8}$$

## 2.4 Summary

In this chapter a brief overview of the OFDM system is presented. The gain of using multi-carrier transmission scheme for transmission is also analyzed. The need and advantages offered by the use of MIMO are presented as well. The receiver design for MIMO OFDM system is the focus of this chapter. Receiver design includes the process of signal detection for coherent systems. Optimal and sub-optimal signal detection is investigated. To conclude, channel estimation based on the training symbols for non-coherent systems is also evaluated.

# Chapter 3

# Hybrid ARQ

Hybrid- ARQ is basically a technique to ensure system reliability and throughput by combining ARQ and FEC, respectively. As opposed to conventional ARQ method,HARQ receiver combines information from multiple transmissions to improve overall performance. A little attention has been paid on FEC in this work. Our focus is solely on the ARQ method.

## 3.1 Retransmission Need

The fundamental objective of any communication system is to transmit data from one location to another without errors at higher data rate. Many approaches exist in literature and implemented in existing communication systems to achieve this objective. In wireless systems, multipath introduces ISI and the noise effects make it very difficult to detect the information sequences at the receiver. When few errors occur, FEC can help to correct the errors if they fall within the error correcting capability of the FEC. In this way the number of retransmissions can be reduced significantly. To further improve the throughput of system many approaches can be considered. One approach is to use more powerful FEC at the transmitter which improves the system reliability. However, such strong coding introduces large amount of redundancy. Also, in case of severe channel, even the powerful coding is not sufficient to guarantee error free communication. Thus, it is necessary to retransmit the whole data if errors occur during communication.

## 3.2 Automatic Repeat reQuest (ARQ)

Automatic Repeat reQuest is basically a feedback method from the receiver to transmitter. It makes use of acknowledgment (ACK) and Negative acknowledgments (NACK) signals for controlling the transmission flow from the transmitter. The process continues until the maximum number of retransmissions are reached which is termed as timeout. An acknowledgment can be defined as a message which is a feedback that assures the successful transmission of packet. If the receiver fails to detect correctly, it feedbacks a negative acknowledgment (NAK) to the transmitter or if the transmitter doesnt receive any ACK within a specific time it retransmits declaring the packet transmitted as erroneous. An ARQ system can be decomposed into three major types depending on the latency and ways in which packets are retransmitted. These three types are Stop and Wait, Go Back N and Selective ARQ. A brief description of all of them is covered in this section.

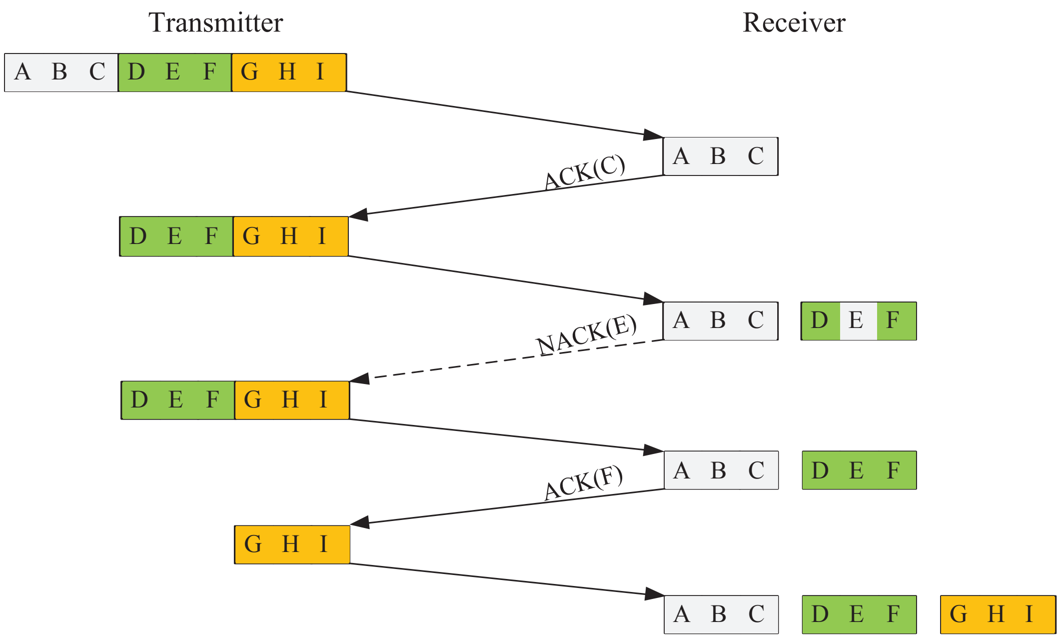


Figure 3.1: Stop and Wait Scheme over Four Transmissions

### 3.2.1 Stop and Wait

In Stop and Wait, the transmitter sends only one packet consisting of codewords at any given time. After sending the packet, transmitter waits for either the ACK or NACK signal. If ACK is received at the transmitter, this implies that the packet has been successfully delivered. The transmitter sends the next packet after successful transmission of previous packet. If NACK is received, the transmitter retransmits the same packet. Figure 3.1 illustrates the procedure of Stop and Wait ARQ. Stop and wait ARQ method suffers from throughput loss as transmitter stays in idle mode while waiting for ACK/NACK. As a result, it introduces large latency in the performance of overall system. Moreover, the efficiency of the overall system using Stop and Wait is considerably lower. The issue of throughput loss can be addressed by Go Back N method discussed next.

### 3.2.2 Go Back N

Unlike the Stop and Wait scheme, N numbers of packets are transmitted on the channel. The transmitter doesnt wait for ACK or NACK. Also, an ACK of $L-th$ packet ensures that all pevious L-1 packets are successfully delivered to the receiver. Figure 3.2 illustrates the Go Back N scheme. The acknowledg-

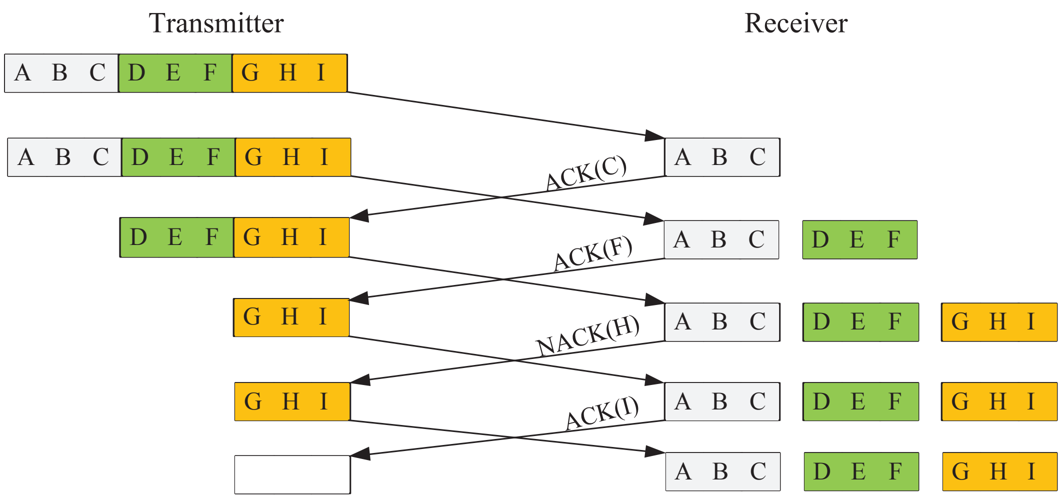


Figure 3.2: Go back N

ment and negative acknowledgments of respective packets in this scheme are differentiated with a number. The main problem with Go Back N design is that whenever an error occurs, the transmitter has to retransmit all the packets since the last acknowledgment. Although among these N retransmitted packets, only a few packets can be in error while the rest are already received successfully. In a communication system, normally the packet size is large so large data sequences will be retransmitted as an overload. This decreases the overall throughput of the system considerably.

### 3.2.3 Selective Repeat

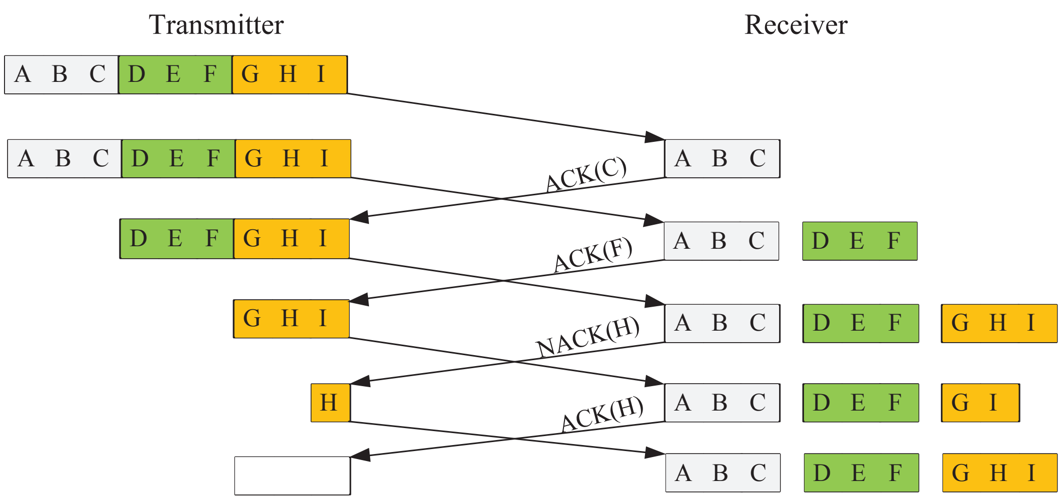


Figure 3.3: Selective Repeat

The issues relating to Go Back N method can be solved by retransmitting only the packets which correspond to negative acknowledgment. This technique is termed as Selective Repeat as only selected erroneous packets are retransmitted. Figure 3.3 illustrates the Selective Repeat scheme. Selective Repeat mechanism needs buffers at both the transmitter and receiver to save data from previous transmissions. The ACKs and NACKs are numbered as was done in Go Back N to represent a particular packet. This scheme increases the throughput of the system substantially. The ARQ schemes are combined with forward error correcting codes to form HARQ which further improves the throughput.

## 3.3 Hybrid ARQ

Although ARQ ensures error free communication but the throughput is variable depending on the channel. In wireless communication, mostly the channels are frequency selective. These channels require large number of retransmissions and throughput of ARQ will be lowered. Moreover, irrespective of the number of errors.For instance, if only a few errors occur during transmission, ARQ will have to retransmit. To minimize such retransmissions, ARQ is combined with FEC to formulate HARQ [27]. In the case of limited errors, FEC can help to decode the information sequences correctly. Thus, decreasing the number of retransmissions. The throughput of the overall system can be enhanced considerably by using HARQ.

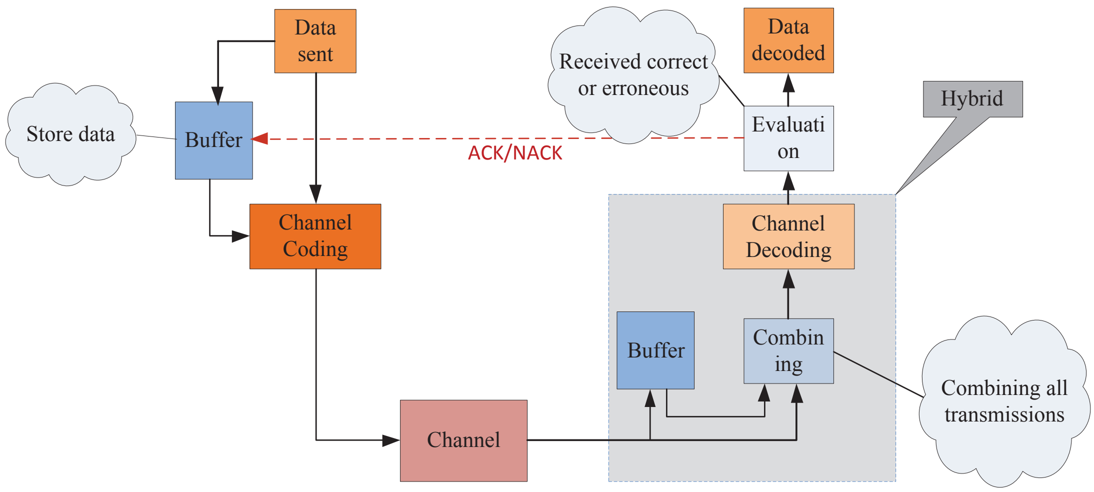


Figure 3.4: HARQ System Diagram

### 3.3.1 Protocol

The main idea behind HARQ is combining information from both the erroneous first transmission and retransmission to decode the particular packet. Prior to HARQ, the erroneous packet was considered to be useless. In HARQ protocol, the erroneous packet is saved in a buffer until the retransmission, and information from both transmissions is used to jointly decode the packet. A graphical view of HARQ protocol is shown in Figure 3.4. When the receiver gets the packet, the first task is to decide whether the packet is transmitted for the first time or retransmission has taken place. This is done by matching the packet number of the current packet with the packets in the buffer. In case packet numbers match, both the packets are combined and forwarded to the decoder. In case of mismatch, the packet is simply forwarded with a copy of packet being stored in the buffer for future use. If the decoding of any packet is successful, the receiver will send ACK to the transmitter and delete that packet from the buffer.

### 3.3.2 Types of HARQ

HARQ is mainly classified into two types Type I (Chase Combining) and Type II (Incremental Redundancy).

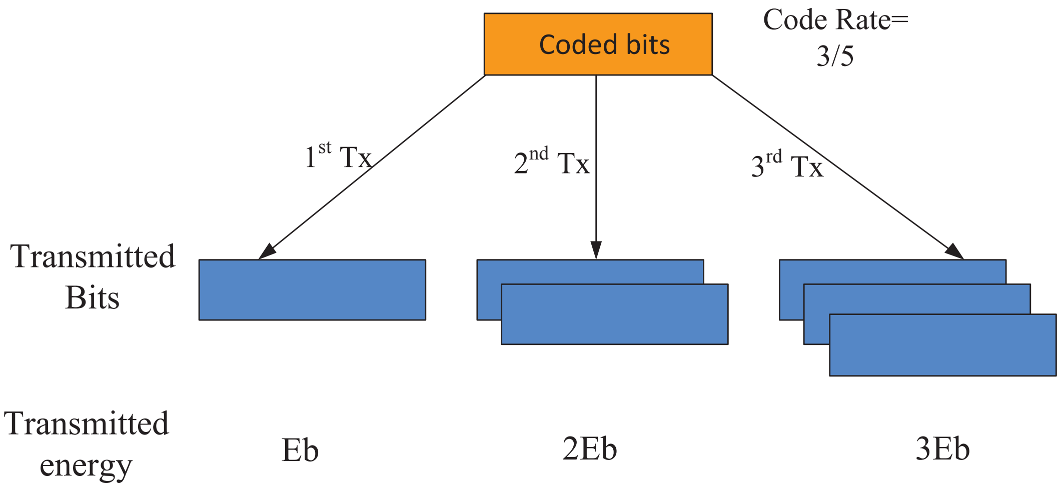


Figure 3.5: Chase Combining

#### 3.3.2.1 Type I (Chase Combining)

In Chase combining (CC), same packet is retransmitted again if NACK is received at the transmitter. This technique was named after D. Chase [6] who introduced it for first time. The data from the first transmission is saved and data from retransmission of same ensemble are combined for decoding at the receiver. The schematic diagram of CC-HARQ is shown in Figure 3.5. The packets are combined using a method commonly known as maximal ratio combining (MRC). Each packet is combined after assigning a weighting factor chosen according to the reliability of estimate. Chase combining uses SNR statistics from all transmissions which makes packets into easily decodable. The code rate during the retransmission is same unlike other approaches of HARQ. The main drawback of Type I HARQ is the inability to provide coding gain.

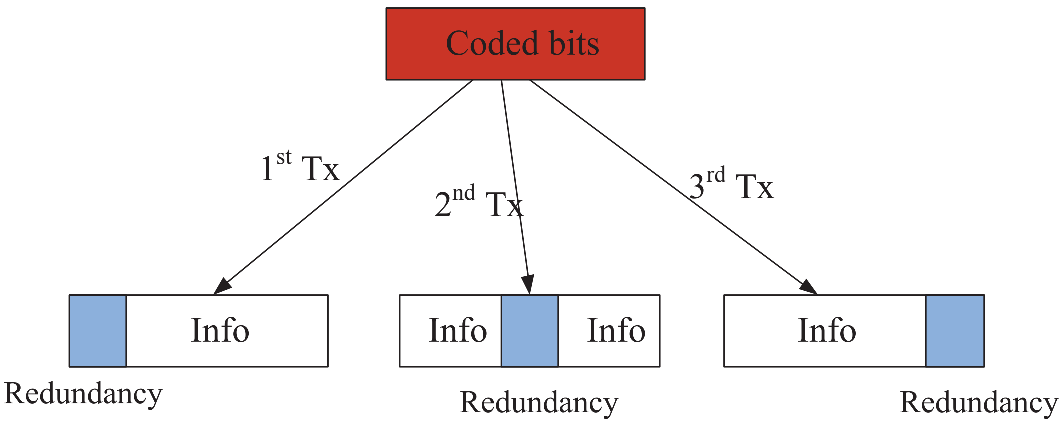


Figure 3.6: Incremental Redundancy

#### 3.3.2.2 Type II (Incremental Redundancy)

In incremental redundancy (IR), only parity bits are retransmitted unlike CC. The data from subsequent transmissions is combined and decoding is done using this combined data. The selection of bits for retransmission is based on different criteria like channel norm or condition number and varies with application need. The code rate during retransmission is lower than the rate during first transmission. Figure 3.6 shows the working of Type II HARQ. At first, data is encoded by any coding scheme to generate a full codeword sometimes referred as mother-code. Some portion of packet after adding redundancy is transmitted during first transmission. If the transmitter gets NACK, a new redundancy is added to same bits. The receiver combines data from both transmissions to form a codeword of lower rate $R_2$ less than $R_1$, where $R_1$, $R_2$ is the rate of first and second transmission, respectively. IR method provides coding gain as opposed to CC, thus its performance is much better than the prior. But this is not true in general. A performance comparison between CC and IR is demonstrated by Dimitris [28]. It is concluded in [28] that CC and IR can perform better based on that application complexity can be kept reasonable. In real HARQ systems, forward error correcting codes (FEC) are also used for reliable communication. The basics of forward error correcting codes is discussed in the next section 3.4.

## 3.4 Low Density Parity Check(LDPC)

Low Density Parity Check codes have been included in many recent wireless communication standards like IEEE 802.16e, IEEE 802.20, IEEE 802.3 and DVB standards. The reason for their inclusion is their ability to approach the channel capacity limits. LDPC codes were first proposed by Robert Gallager in 1962. LDPC codes have many other added advantages like code of any rate and length can be constructed by parity check matrix. The ease of construction as opposed to turbo codes are making LDPC codes a popular entity in present research. The parity check matrix of LDPC codes is sparse i.e it contains only a few number of $1's$ as compared to $0's$. Parity checks are used for the validity of codewords, even if errors occur. Moreover, low density parity check codes use iterative decoding algorithms with the complexity much lower than turbo codes.

## 3.5 Puncturing

Instead of either retransmitting the whole packet or adding different parity bits, HARQ provides the flexibility to transmit only a portion of data (puncturing). Puncturing serves the benefit of saving overhead, bandwidth and increasing the throughput. This problem is identified and explored recently by many researchers [9], [27]. In the next chapter we propose a new HARQ scheme based on puncturing the data packet.

## 3.6 Summary

In this chapter need for the retransmission is discussed at first. Then conventional method of ARQ is presented. Brief overview of techniques used in ARQ are presented. Furthermore, HARQ protocol is reviewed. Two types of HARQ i.e IR-HARQ and CC-HARQ, with their benefits and their demerits are investigated. Within HARQ puncturing or partial retransmission of data is explored. The importance of LDPC codes within the framework of HARQ is also discussed.

# Chapter 4

# HARQ Transceiver Design

The previous chapters gave the basic understanding of MIMO systems, OFDM and HARQ process. In this chapter, a novel method is proposed which integrates Partial Automatic Repeat reQuest (PARQ) with the orthogonal frequency division multiplexing (OFDM) employing multiple input multiple output (MIMO) systems. The motivation behind this work is to retransmit only some fraction of the data instead of retransmitting the whole sequence. At the receiver, joint detection is employed on the observations from multiple transmissions. The bandwidth and power saving of the overall system are the direct benefits we get from the proposed method. It is worth noting that the performance remains still comparable. The channel estimation is an integral part of any communication system and its performance has direct consequence on the overall performance of the system. Conventional systems use pilot based approaches for channel estimation which require resources like bandwidth and power. In the proposed work, we omit training symbols during retransmissions. Semi-blind channel estimation is performed at the receiver. The complete PARQ system description is presented next.

## 4.1 PARQ System Model

### 4.1.1 System Model

We consider MIMO-OFDM system equipped with $n_r$ and $n_t$ receive and transmit antennas, respectively with $N$ sub-carriers depicted in Figure 4.1. There are $n_t$ information symbol streams of length $N$ each transmitted from $n_t$ transmit antennas. Let $\mathbf{s}_{kj} = [s_{kj}(1), \ldots, s_{kj}(\ell) \ \ldots \ \ s_{kj}(N)]^T \in \mathcal{C}^{N\times 1}$ be the symbols vector obtained by mapping information bits onto constellation set $\mathcal{A}$ of cardinality $M$. Note that vector $\mathbf{s}_{kj}$ is transmitted from the $j$-th transmit antenna, where $j = 1, \ldots, n_t$ and $k$ is the time index. After OFDM modulation, signal vector $\tilde{\mathbf{s}}_{kj} \in \mathcal{C}^{(N+L)\times 1}$ is transmitted over frequency selective channel vector $\mathbf{h}_{i,j}$ shown by top branch in Figure 4.1. The channel vector $\mathbf{h}_{i,j}$ is assumed to be of order $L$ from $j$-th transmit antenna to the $i$-th receive antenna.

The stack of symbol vectors $\mathbf{s}_{kj}$ and modulated signals $\tilde{\mathbf{s}}_{kj}$ are

$$\mathbf{s}_k = \left[ \begin{array}{ccc} \mathbf{s}_{k1}^T & \ldots & \mathbf{s}_{kn_t}^T \end{array} \right]^T \quad \text{and} \quad \tilde{\mathbf{s}}_k = \left[ \begin{array}{ccc} \tilde{\mathbf{s}}_{k1}^T & \ldots & \tilde{\mathbf{s}}_{kn_t}^T \end{array} \right]^T. \tag{4.1}$$

In the case of retransmission, instead of retransmitting the same OFDM modulated signal $\tilde{\mathbf{s}}_k$, the signal vector $\mathbf{x}_k$ is decimated to obtain $\mathbf{x}_k^p$ before adding cyclic prefix (CP) as shown by lower branch in Figure 4.1. Thus, in response to retransmission, CP added OFDM modulated signal $\tilde{\mathbf{s}}_k^p \in \mathcal{C}^{n_t(\frac{N}{2}+L)\times 1}$ is transmitted. For the sake of simplicity, we decimate the data symbols by a factor of 2.

The top branch in Figure 4.1 represents the typical MIMO-OFDM transmission. For MIMO-OFDM system, the channel matrix will be block circulant matrix because of the addition of CP to maintain orthogonality of OFDM subcarriers [30]. The bottom branch describes the retransmission procedure. Due to large delay between the first and subsequent transmissions, we assume that channel for each retransmission corresponding to an OFDM symbol is independent. Also, we denote $\mathbf{h}_{i,j}^p$ as the channel response vector for the partial retransmission from $j$-th transmitter to $i$-th receiver. It is eminent that $L$ is the length of CP. We denote additive white Gaussian noise during the first and

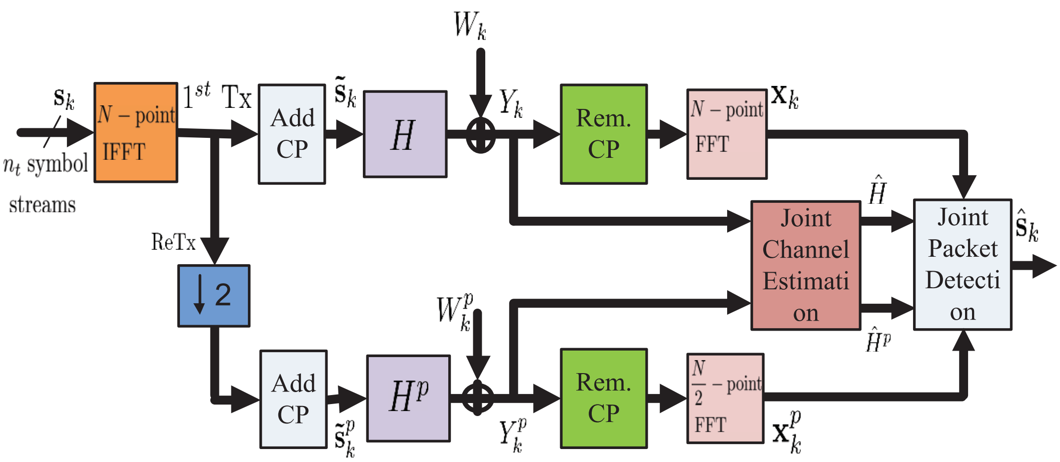


Figure 4.1: System model of PARQ method for MIMO-OFDM signaling

second transmission by $\mathbf{w}_k \in \mathcal{C}^{n_t(N+L)\times 1}$ and $\mathbf{w}_k^p \in \mathcal{C}^{n_t(\frac{N}{2}+L)\times 1}$, respectively.

### 4.1.2 Problem Formulation

A typical erroneous packet has fewer errors. The retransmission of bad packet in full is unnecessary. Thus, most of the times, little more information (partial packet) is sufficient for the receiver to recover from the errors. The proposed method retransmits partial copy of the original data to achieve bandwidth and power efficiency. We generate partial copy of original OFDM frame by decimating output of inverse fast Fourier transformation (IFFT) block. For example, with decimation factor 2, retransmission of partial packet achieves 50% of both bandwidth and energy saving. The receiver preserves observation from the first transmission of a bad packet for joint detection. The joint detector exploits observation from the multiple transmissions at modulation level to enhance reliability of the decoded data. Due to large latency between first transmission and retransmission of a packet, we assume that their channel realizations are independent. The joint detection achieves diversity gain discussed in Section 4.3.

In our problem formulation, for simplicity and without the loss of generality, we consider single retransmission of an erroneous packet. Extension to multiple retransmissions is straightforward. Let FFT and IFFT matrices be $F$ and $F^H$ of order $N \times N$ for conventional SISO-OFDM system with $N$ sub-carriers. For MIMO-OFDM system, we define $\tilde{\mathbf{F}} \in \mathcal{C}^{n_r N \times n_r N}$ and $\hat{\mathbf{F}} \in \mathcal{C}^{n_t N \times n_t N}$ as block FFT and IFFT matrices. The block circulant matrices $\mathbf{H} \in \mathcal{C}^{n_r N \times n_t N}$ and $\mathbf{H}^p \in \mathcal{C}^{n_r \frac{N}{2} \times n_t \frac{N}{2}}$ correspond to the first transmission and subsequent retransmissions, respectively [30]. Each block of the circulant matrix is constructed from the corresponding frequency selective channel of order $L$. We also define block diagonal FFT matrix $\tilde{\mathbf{F}}_p$ of $n_r$ diagonal block, where each block is a FFT matrix $F_p$ of order $\frac{N}{2} \times \frac{N}{2}$ to be applied at the receiver side of the PARQ based MIMO-OFDM system. In order to generate decimated copy of the original data for partial retransmission, we define block IFFT matrix $\hat{\mathbf{F}}_p \in \mathcal{C}^{n_t \frac{N}{2} \times n_t N}$ as follows:

$$\hat{\mathbf{F}}_p = \begin{bmatrix} PF^H & \dots & 0 \\ \vdots & \ddots & 0 \\ 0 & \dots & PF^H \end{bmatrix} \in \mathcal{C}^{n_t \frac{N}{2} \times n_t N},$$

where $PF^H = \begin{bmatrix} F_p^H & F_p^H \end{bmatrix}^T$ is block matrix of $\frac{N}{2}$-point IFFT matrix.

The relationship between the received signal after removing cyclic prefix (CP) and the transmitted vectors for the first transmission and partial retransmission with decimation factor of 2 as shown in Figure 4.1 can be written in matrix form as

$$\mathbf{z}_k = \tilde{\mathbf{F}}\mathbf{H}\hat{\mathbf{F}}\mathbf{s}_k + \tilde{\mathbf{F}}\mathbf{w}_k, \tag{4.2}$$

$$\mathbf{z}_k^p = \tilde{\mathbf{F}}_p\mathbf{H}^p\hat{\mathbf{F}}_p\mathbf{s}_k + \tilde{\mathbf{F}}_p\mathbf{w}_k^p, \tag{4.3}$$

It is clear from (4.2) and (4.3) that partial (punctured) retransmission of original packet transmits fewer symbols that saves bandwidth and fewer symbols translates into less energy. The joint detection by combining observations from multiple transmissions improves detector quality at the expense of moderate increase in complexity. The memory requirement to save data packet for joint detection is same as that of conventional CC-HARQ. Similar to CC-HARQ method, PARQ method also saves full packet observations for joint detection. To achieve further bandwidth saving, we also propose joint semi-blind channel estimation by omitting pilot signal during partial retransmission discussed in Section 4.4. Next, we discuss joint detection by combining observation of multiple transmissions of packet.

## 4.2 JOINT DETECTION

In this section, we develop joint detection model for MIMO-OFDM system with PARQ method. The joint detection model for SISO system is presented in [10]. In order to exploit observation from the first and subsequent transmissions, first, we develop joint detection model for PARQ receiver. Then, we discuss complexity of the maximum likelihood (ML) and zero-forcing (ZF) receivers. Let $\tilde{\mathbf{z}}_k = \left[\mathbf{z}_k^T \;\; (\mathbf{z}_k^p)^T\right]^T \in \mathcal{C}^{\frac{3Nn_r}{2}\times 1}$ be the observation vector constructed from the first transmission and partial retransmission of information symbol vector $\mathbf{s}_k$ for MIMO-OFDM system. The matrix form of system model for joint detection is

$$\tilde{\mathbf{z}}_k = \mathcal{H}\mathbf{s}_k + \tilde{\mathbf{w}}_k,$$

where

$$\mathcal{H} = \begin{bmatrix} \tilde{\mathbf{F}}\mathbf{H}\hat{\mathbf{F}} \\ \tilde{\mathbf{F}}_p\mathbf{H}^p\hat{\mathbf{F}}_p \end{bmatrix} \text{ and } \tilde{\mathbf{w}}_k = \begin{bmatrix} \tilde{\mathbf{F}}\mathbf{w}_k \\ \tilde{\mathbf{F}}_p\mathbf{w}_k^p \end{bmatrix}.$$

Due to the fact that FFT bases diagonalize circulant matrix [30], we have

$$FH_{(i,j)}F^H = D_{(i,j)} = \begin{bmatrix} D^0_{(i,j)} & o \\ o & D^1_{(i,j)} \end{bmatrix} \in \mathcal{C}^{N\times N},$$

where $H_{(i,j)}$ is a circulant matrix corresponding to the channel from transmit antenna $j$ to receive antenna $i$. The diagonal matrix $D_{(i,j)}$ is split into two diagonal blocks $D^0_{(i,j)}$ and $D^1_{(i,j)}$ of size $\frac{N}{2}\times\frac{N}{2}$ each. Note that the elements of diagonal matrix $D_{(i,j)}$ are coefficients of $N$-point FFT of channel vector from transmit antenna $j$ to receive antenna $i$ during the first transmission. Similarly, FFT bases can convert circulant matrix for partial retransmission into block diagonal matrix as follows:

$$\begin{aligned} F_p H^p_{(i,j)} P F^H = F_p H^p_{(i,j)} \begin{bmatrix} F^H_p & F^H_p \end{bmatrix} &= \begin{bmatrix} F_p H^p_{(i,j)} F^H_p & F_p H^p_{(i,j)} F^H_p \end{bmatrix} \\ \hat{D}^p_{(i,j)} &= \begin{bmatrix} D^p_{(i,j)} & D^p_{(i,j)} \end{bmatrix} \in \mathcal{C}^{\frac{N}{2} \times N}, \end{aligned}$$

where diagonal elements of $D^p_{(i,j)}$ are coefficients of $\frac{N}{2}$-point FFT of channel vector from transmit antenna $j$ to receive antenna $i$ during partial retransmission. Thus, joint channel matrix $\mathcal{H}$ can be written as block matrix

$$\mathcal{H} = \begin{bmatrix} D_{(1,1)} & \cdots & D_{(1,n_t)} \\ \vdots & \ddots & \vdots \\ D_{(n_r,1)} & \cdots & D_{(n_r,n_t)} \\ \hat{D}^p_{(1,1)} & \cdots & \hat{D}^p_{(1,n_t)} \\ \vdots & \ddots & \vdots \\ \hat{D}^p_{(n_r,1)} & \cdots & \hat{D}^p_{(n_r,n_t)} \end{bmatrix} \in \mathcal{C}^{\frac{3}{2} n_r N \times n_t N}.$$

Note that problem of joint detection of $n_t N$ symbols can be further divided into $\frac{N}{2}$ smaller problems of joint detection of $2n_t$ symbols each. For example, under MIMO-OFDM system with 2 transmit antennas and 2 receive antennas, joint detection of $2N$ symbols can be decoupled into $\frac{N}{2}$ smaller problems of joint detection of 4 symbols each. As a result, computational complexity of both sub-optimal and optimal detection is significantly lowered. Let $D_{(i,j)}(\ell)$ be the $\ell$-th diagonal element of $N$ point FFT of channel vector $\mathbf{h}_{i,j}$ and represents gain of $\ell$-th sub-carrier, where $\ell = 1, \ldots, N$. The matrix model of joint detection of $m$-th vector of four symbols for $2 \times 2$ MIMO-OFDM system under PARQ scheme with decimation factor of 2 is

$$\tilde{\mathbf{z}}_k(m) = \begin{bmatrix} D_{11}(m) & 0 & D_{12}(m) & 0 \\ 0 & D_{11}(m+\frac{N}{2}) & 0 & D_{12}(m+\frac{N}{2}) \\ D_{11}^p(m) & D_{11}^p(m) & D_{12}^p(m) & D_{12}^p(m) \\ D_{21}(m) & 0 & D_{22}(m) & 0 \\ 0 & D_{21}(m+\frac{N}{2}) & 0 & D_{22}(m+\frac{N}{2}) \\ D_{21}^p(m) & D_{21}^p(m) & D_{22}^p(m) & D_{22}^p(m) \end{bmatrix} \begin{bmatrix} s_{k1}(m) \\ s_{k1}(m+\frac{N}{2}) \\ s_{k2}(m) \\ s_{k2}(m+\frac{N}{2}) \end{bmatrix} + \tilde{\mathbf{w}}_k(m) \tag{4.4}$$

$$\tilde{\mathbf{z}}_k(m) = \mathcal{H}(m)\mathbf{s}_k(m) + \tilde{\mathbf{w}}_k(m), \tag{4.5}$$

where $m = 1, \ldots, \frac{N}{2}$. The matrix model for joint detection in (4.5) represent flat-fading MIMO system of order $3n_r \times 2n_t$. The implementation of $\frac{N}{2}$ joint detectors for flat-fading MIMO systems can be carried out using well-known ML (optimal) or ZF (sub-optimal) detector [30, 31]. The complexity of the ZF receiver is lower than the ML receiver at the expense of performance loss. Next, we discuss complexity of ZF and ML joint receivers for the proposed PARQ method for MIMO-OFDM signaling.

### 4.2.1 Zero Forcing Detection

The ZF detector of the $m$-th joint detection model in (4.5) of MIMO-OFDM for PARQ method is

$$\hat{\mathbf{s}}_k(m) = \mathcal{H}^{\dagger}(m)\tilde{\mathbf{z}}_k(m), \tag{4.6}$$

where $\mathcal{H}^{\dagger}(m)$ is Moore-Penrose pseudo inverse of block matrix $\mathcal{H}(m)$ of the $m$-th joint detector. The joint ZF detector for CC-HARQ method has almost same complexity as that of single transmission. The ZF joint detection of $n_t$ symbols for CC-HARQ with $n_t$ transmit antennas involves pseudo inverse of channel matrix and has complexity $O(n_t^3)$ [32]. Note that ZF joint receiver involves channel matrix $\mathcal{H}(m) \in \mathcal{C}^{3n_r \times 2n_t}$ under decimation factor of 2. Therefore, the complexity of ZF joint detector for proposed PARQ method is $O(8n_t^3)$ [32]. Next, we discuss joint ML detection under PARQ of OFDM-MIMO system.

### 4.2.2 ML Detection

For $n_r \times n_t$ MIMO-OFDM system with proposed PARQ method of $N$ subcarriers, joint detection in (4.5) is modeled as $\frac{N}{2}$ parallel MIMO flat-fading systems of order $3n_r \times 2n_t$. There are $\frac{N}{2}$ smaller joint detectors of $2n_t$ symbols each. Thus, ML estimate of the $m$-th symbol vector as a result of joint detection of $m$-th detector, where $m = 1, \ldots, \frac{N}{2}$, is

$$\hat{\mathbf{s}}_k(m) = \arg \min_{\mathbf{s}_k(m) \in \tilde{\mathcal{A}}} \|\tilde{\mathbf{z}}_k(m) - \mathcal{H}(m)\mathbf{s}_k(m)\|^2, \tag{4.7}$$

where $\tilde{\mathcal{A}}$ is set of $M^{2n_t}$ vectors of length $2n_t$ each. In (4.7), $\mathcal{H}(m) \in \mathcal{C}^{3n_r \times 2n_t}$ is flat-fading channel matrix constructed from FFT coefficients of the channels from the first and second transmissions for joint detection given in (4.5).

The number of multiplications of ML detector of MIMO system with $n_r$ and $n_t$ receiver and transmit antennas, respectively, are $4n_r n_t M^{n_t} + 2n_r n_t M^{n_t}$. For CC-HARQ, the number of multiplications of $N$ joint ML detectors of order $2n_r \times n_t$ are $8Nn_r n_t M^{n_t} + 4Nn_r n_t M^{n_t}$. There are $\frac{N}{2}$ joint ML detector for PARQ method of order $6n_r \times 2n_t$. Thus, the number of multiplications of the joint ML detection of PARQ method of MIMO-OFDM system is $8Nn_r n_t M^{2n_t} + 3Nn_r n_t M^{2n_t}$. The complexity of ML detector for PARQ and CC-HARQ methods is $O(M^{2n_t})$ and $O(M^{n_t})$, respectively.

Next, we provide diversity analysis of proposed PARQ retransmission approach.

## 4.3 Divesity Analysis

In this section, we present diversity analysis of the proposed PARQ method for SISO-OFDM and MIMO-OFDM systems. First, we show that PARQ method for SISO-OFDM system achieves diversity of full retransmission (Chase combining). Note that diversity of joint detector of CC-HARQ method for SISO-OFDM system is 2. The following proposition provides diversity of PARQ joint receiver.

**Proposition 1.** *Joint detection of partial ARQ method for SISO-OFDM signaling* *achieves diversity of full retransmission.*

*Proof.* For SISO-OFDM system, the channel matrix $\mathcal{H}(m)$ and information symbol vector $\mathbf{s}_k(m)$ in (4.5) of joint detector of PARQ method reduce to $3 \times 2$ channel matrix and $2 \times 1$ vector, respectively. The matrix form of model in (4.5) can be rewritten as follows:

$$\tilde{\mathbf{z}}_k(m) = \begin{bmatrix} s_k(m) & 0 & 0 \\ 0 & s_k(m+\frac{N}{2}) & 0 \\ 0 & 0 & s_k(m)+s_k(m+\frac{N}{2}) \end{bmatrix} \begin{bmatrix} D(m) \\ D(m+\frac{N}{2}) \\ D^p(m) \end{bmatrix} + \tilde{\mathbf{w}}_k(m)$$

$$\tilde{\mathbf{z}}_k(m) = \mathcal{S}(m)\vec{\mathcal{H}}(m) + \tilde{\mathbf{w}}_k(m), \tag{4.8}$$

There are $M^{2n_t}$ possible codewords in codebook $\mathcal{U}$. For simplicity and without loss of generality, we omit index $m$ from (4.8). Probability that codeword $\mathcal{S}_q$ is detected when codeword $\mathcal{S}_p$ is transmitted for a given channel vector $\vec{\mathcal{H}}$ is

$$\mathbf{P}_r\left(\mathcal{S}_p \underset{p \neq q}{\longrightarrow} \mathcal{S}_q\right) = \mathbf{Q}\left(\sqrt{\frac{\| \triangle \mathcal{S}_{pq}\vec{\mathcal{H}}\|^2}{N_0}}\right), \tag{4.9}$$

where $\frac{N_0}{2}$ is the noise variance and $\mathcal{S}_p - \mathcal{S}_q = \Delta\mathcal{S}_{pq}$ is an error matrix. The elements of joint channel $\vec{\mathcal{H}}$ are independent identically distributed (i.i.d.) with normal distribution due to the fact that elements of $\vec{\mathcal{H}}$ are obtained by applying orthogonal FFT matrix on frequency-selective channel vector with i.i.d. coefficients of normal distribution. The averaged probability of error in (4.9) over $\vec{\mathcal{H}}$ is

$$\mathbf{E}_{\vec{\mathcal{H}}}\left[\mathbf{P}_r\left(\mathcal{S}_p \underset{p \neq q}{\longrightarrow} \mathcal{S}_q\right)\right] \leq \left(\frac{1}{4\sigma^2}\right)^{-\mathcal{N}} \cdot \prod_{k=1}^{\mathcal{N}} \lambda_k^{-1}, \tag{4.10}$$

where $\lambda_1, \ldots, \lambda_{\mathcal{N}}$ are $\mathcal{N}$ non-zero eigen values of $\Delta\mathcal{S}_{pq}^H \Delta\mathcal{S}_{pq}$. Let $\tilde{\mathcal{M}} =\mid \mathcal{U} \mid$ and $\check{\mathcal{M}} =\mid \check{\mathcal{U}} \mid$ be the number of the codewords and error matrices respectively. Assuming that all codewords are equally likely to be transmitted, then the union bound is

$$\mathbf{P}_u = \frac{1}{\tilde{\mathcal{M}}} \sum_{i=1}^{\check{\mathcal{M}}} \mathbf{Q}\left(\sqrt{\frac{\| (\Delta\mathcal{S}_i)\, \vec{\mathcal{H}}\|^2}{2\sigma^2}}\right), \tag{4.11}$$

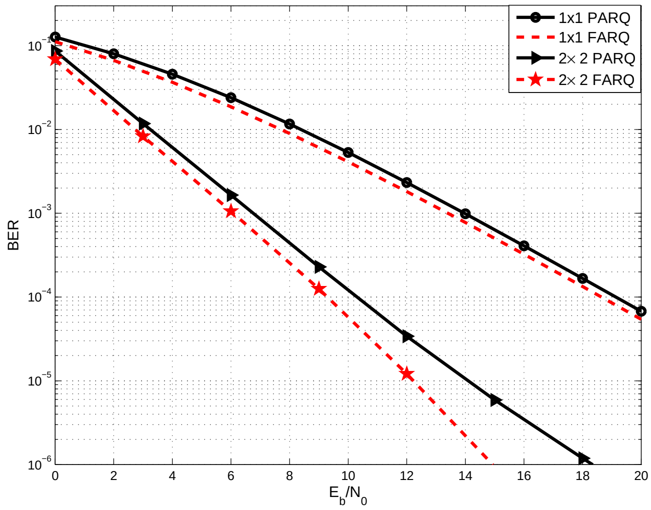


Figure 4.2: Union bound for SISO and MIMO-OFDM systems with PARQ and CC-HARQ methods with 4-QAM constellation

where $\Delta\mathcal{S}_i$ is the $i$-th error matrix of a codeword. Notice that typically $\check{\mathcal{M}} \gg \tilde{\mathcal{M}}$. The union bound averaged over random $\vec{\mathcal{H}}$ is

$$\mathbf{E}_{\tilde{\mathbf{h}}}\left[\mathbf{P}_u\right] \leq \frac{1}{\tilde{\mathcal{M}}}\sum_{i=1}^{\check{\mathcal{M}}}\left(\frac{1}{4\sigma^2}\right)^{-\mathcal{N}} \cdot \prod_{k=1}^{\mathcal{N}}\lambda_{(k,i)}^{-1}, \tag{4.12}$$

where $\lambda_{(k,i)}$ is the $k$-th eigen value of the $i$-th error matrix. Note that $\mathcal{N}$ is the rank of error matrix $\Delta\mathcal{S}_i$. Clearly, the minimum rank of an error matrix $\Delta\mathcal{S}_i$ is 2. That is, when two codeword matrices $\mathcal{S}_p$ and $\mathcal{S}_q$ differ by one information symbol, the resultant error matrix $\Delta\mathcal{S}_{pq}$ has the smallest rank, which is 2. Therefore, PARQ retransmission method achieves diversity of CC-HARQ for SISO-OFDM. $\square$

The rank of error matrix $\Delta\mathcal{S}_i$ for both full and PARQ method for SISO-OFDM signaling is 2. However, PARQ approach suffers from some diversity loss under MIMO-OFDM signaling when compared with CC-HARQ method. For MIMO-OFDM system with $n_r \times n_t = 2 \times 2$ under the proposed PARQ scheme, the matrix model of $m$-th joint detector in (4.5) can be rearranged as

$$\tilde{\mathbf{z}}_k(m) = \left( \begin{bmatrix} \tilde{S}_k(m) & 0 \\ 0 & \tilde{S}_k(m) \end{bmatrix} = \mathcal{S}(m) \right) \begin{bmatrix} D^1(m) \\ D^2(m) \end{bmatrix} + \tilde{\mathbf{w}}_k(m), \tag{4.13}$$

where

$$D^1(m) = \begin{bmatrix} D_{11}(m) & D_{12}(m) & D_{11}(m+\frac{N}{2}) & D_{12}(m+\frac{N}{2}) & D^p_{11}(m) & D^p_{12}(m) \end{bmatrix}^T \tag{4.14}$$

$$D^2(m) = \begin{bmatrix} D_{21}(m) & D_{22}(m) & D_{21}(m+\frac{N}{2}) & D_{22}(m+\frac{N}{2}) & D^p_{21}(m) & D^p_{22}(m) \end{bmatrix}^T \tag{4.15}$$

Note that in $s_{kj}$, $k$ is the time index and $j$ is index of the transmit antenna. It is obvious from (4.13) that if two codewords $\mathcal{S}_p$ and $\mathcal{S}_q$ differ by single symbol, then the rank of error matrix $\Delta\mathcal{S}_{pq}$ is 6, which is smaller than the rank of error matrix of joint detector under CC-HARQ. Note that the rank of joint detector of 4 symbols with CC-HARQ method is 8. Thus, the proposed PARQ method for MIMO-OFDM system achieves diversity gain as compared to single transmission and is lower as compared to CC-HARQ method. Figure 4.2 compares the union bound of PARQ and conventional CC-HARQ retransmission methods for SISO-OFDM and MIMO-OFDM systems. As shown in Figure 4.2, SISO-OFDM system under PARQ achieves full diversity similar to CC-HARQ retransmission method. Although PARQ method suffers from some diversity loss as compared to CC-HARQ method, the impact of diversity on bit error rate (BER) appears at high signal-to-noise ratio (SNR) where BER is low ($10^{-5}$). The effect of BER improvement on throughput at high SNR is marginal. Next, we discuss semi-blind channel estimation to improve bandwidth efficiency.

## 4.4 Joint Channel Estimation

For coherent signal detection, receiver requires estimate of CSI. The accuracy of channel estimation directly affects the overall receiver performance. In channels with long delay spread, specifically for MIMO communication systems, training based channel estimation consumes precious power and bandwidth resources. In some scenarios, channel knowledge is acquired using blind methods which have inherent estimation ambiguity [33–35]. In our proposed PARQ approach for MIMO-OFDM system, pilot assisted method is used for channel estimation during the first transmission. We omit pilot insertion during subsequent retransmissions against ARQ. Our semi-blind channel estimation method combines training based least square (LS) cost with noise subspace based blind cost to uniquely estimate channel. Therefore, we proceed channel estimation in two steps. First, we use observations from the first and second transmissions to estimate noise subspace for blind channel estimation using second order statistics [33]. Secondly, we make use of training data to resolve the ambiguity and improve channel estimate.

### 4.4.1 Data Model

As shown in Figure 4.1, signals $\tilde{\mathbf{s}}_k$ and $\tilde{\mathbf{s}}_k^p$ are transmitted after adding CP during the first and second transmission, respectively, over MIMO frequency selective channels of the corresponding transmissions. Note that even after applying IFFT on i.i.d. input vector $\mathbf{s}_k$, the IFFT vector $\tilde{\mathbf{s}}_k$ remains i.i.d. Let $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{H}}^p$ be the convolution block matrices of $n_r n_t$ blocks for the first and second transmission, respectively. Note that $\hat{H}_{(i,j)}$ and $\hat{H}^p_{(i,j)}$ are the convolution channel matrices between the $j$-th transmit antenna to the $i$-th receive antenna for the first and second transmission, respectively. As shown in Figure 4.1, $\mathbf{y}_k$ and $\mathbf{y}_k^p$ are the observation vectors of the first and second transmissions, respectively. The system for both transmissions can be combined to formulate single system of higher order as

$$\tilde{\mathbf{y}}_k = \tilde{\mathcal{H}}\tilde{\mathbf{s}}_k + \tilde{\mathbf{w}}_k,$$

where $\tilde{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_k^p \end{bmatrix}$, $\tilde{\mathcal{H}} = \begin{bmatrix} \tilde{\mathbf{H}} \\ \tilde{\mathbf{H}}^p \end{bmatrix}$ and $\tilde{\mathbf{w}}_k = \begin{bmatrix} \mathbf{w}_k \\ \mathbf{w}_k^p \end{bmatrix}$. We use $\tilde{\mathbf{y}}_k$ to estimate noise subspace to formulate blind channel estimation problem in the next section.

### 4.4.2 Blind Channel Estimation

The blind channel estimation is an extension of subspace based channel estimation carried out for HARQ systems in [10, 11]. The basic procedure is to generate the second order statistics of the observation vector $\mathbf{y}_k$. The space spanned by eigen vectors of auto-covariance matrix $\mathbf{R}_y$ of joint observation from both transmissions $\tilde{\mathbf{y}}_k$ can be decomposed into two complementary subspaces. They are noise subspace and signal subspace. The generation of orthogonal complement noise and signal subspaces requires joint channel matrix to be full column rank.
Let

$$\mathbf{R}_s = E\left[\tilde{\mathbf{s}}_k \tilde{\mathbf{s}}_k^H\right] = \sigma_s^2.\mathbf{I}_{(L+m_1)n_t} \tag{4.16}$$

$$\mathbf{R}_w = E\left[\tilde{\mathbf{w}}_k \tilde{\mathbf{w}}_k^H\right] = \sigma_w^2.\mathbf{I}_{(m_1+m_2)n_r} \tag{4.17}$$

and

$$\mathbf{R}_y = \tilde{\mathcal{H}} \mathbf{R}_s \tilde{\mathcal{H}}^H + R_w = U \Lambda_y U^H, \tag{4.18}$$

where $\mathbf{R}_y$, $\mathbf{R}_s$ and $\mathbf{R}_w$ are the covariance matrices of observation, source and noise signals, respectively. Since $\tilde{\mathcal{H}}$ and $\mathbf{R}_s$ are full rank matrices, the signal and noise subspace are orthogonal complements of space spanned by $\mathbf{R}_y$. Another underlying assumption is that the noise $\mathbf{w}_k$ is independent of input signal $\mathbf{s}_k$. The null space separated by singular value decomposition (SVD) of observation from multiple transmissions is further used to estimate the channel. That is,

$$\Lambda_y = \mathrm{diag}(\lambda_1, \lambda_1, \ldots, \lambda_{(m_1+L)n_t}, \underbrace{\sigma_n^2, \ldots, \ldots, \ldots, \sigma_n^2}_{(m_1+m_2)n_r-(m_1+L)n_t}), \tag{4.19}$$

with $m_1$ and $m_2$ as smoothing factors for the estimation of noise subspace [11]. The first $(m_1 + L)n_t$ eigen vectors corresponding to the largest eigen values span the signal subspace $\mathbf{U}_s$ while the remaining $(m_1 + m_2)n_r - (m_1 + L)n_t$ vectors represent the noise space $\mathbf{U}_n$. The basic equation to identify the channel using the null space can be given by

$$\mathbf{U}_n^H \tilde{\mathcal{H}} = 0. \tag{4.20}$$

Next we study the procedure for the solution of the above equation.

### 4.4.3 Algorithm Implementation

Now we describe the procedure to solve the linear equation (4.20). After the determination of eigen vectors of the noise subspace $\mathbf{U}_n$, a block toeplitz matrix from each eigen vector of noise subspace is formed. The channel matrices of the first and second transmission is converted into a column vector. The signal and noise subspaces are

$$\mathbf{U} = [\mathbf{U_s} \;\; \mathbf{U}_n],$$

where $\mathbf{U}_s = [\mathcal{U}_1, \ldots, \mathcal{U}_{(m_1+L)n_t}]$ and $\mathbf{U}_n = [\mathcal{U}_{(m_1+L)n_t+1}, \ldots, \mathcal{U}_{(m_1+m_2)n_r}]$. Also, $\mathcal{U}_i$ is $i$-th eigen vector of auto-covariance matrix $\mathbf{R}_y$. The set of homogeneous equations given in (4.20) can be converted into a more manageable form in which channel matrix is transformed into a column vector as follows

$$\tilde{\mathbf{h}}^{\mathbf{T}} \mathcal{U} = 0. \tag{4.21}$$

For simplicity and without loss of generality, we consider $2 \times 2$ MIMO-OFDM system and extension is straightforward. The $i$-th basis vectors $\mathcal{U}_i$ of noise subspace can be partitioned into four sub-vectors $\mathbf{u}_{i,1}$, $\mathbf{u}_{i,2}$, $\mathbf{u}_{i,3}$ and $\mathbf{u}_{i,4}$ having lengths $m_1, m_1, m_2$ and $m_2$, respectively. That is,

$$\mathcal{U}_i = \left[ \begin{array}{cccc} \mathbf{u}_{i,1}^T & \mathbf{u}_{i,2}^T & \mathbf{u}_{i,3}^T & \mathbf{u}_{i,4}^T \end{array} \right]^T, \tag{4.22}$$

where

$$\mathbf{u}_{i,1} = \begin{bmatrix} u_{i,1} \\ \vdots \\ u_{i,m_1} \end{bmatrix}, \ \mathbf{u}_{i,2} = \begin{bmatrix} u_{i,m_1+1} \\ \vdots \\ u_{i,2m_1} \end{bmatrix}, \ \mathbf{u}_{i,3} \ = \begin{bmatrix} u_{i,2m_1+1} \\ \vdots \\ u_{i,2m_1+m_2} \end{bmatrix} \text{ and } \mathbf{u}_{i,4} = \begin{bmatrix} u_{i,2m_1+m_2+1} \\ \vdots \\ u_{i,2m_1+2m_2} \end{bmatrix}.$$

For the $2 \times 2$ MIMO system, (4.20) for the $i$-th eigen vector of noise subspace takes the form

$$\mathcal{U}_i^H \tilde{\mathcal{H}} = \begin{bmatrix} \mathbf{u}_{i,1}^H & \mathbf{u}_{i,2}^H & \mathbf{u}_{i,3}^H & \mathbf{u}_{i,4}^H \end{bmatrix} \begin{bmatrix} \hat{H}_{1,1} & \hat{H}_{1,2} \\ \hat{H}_{2,1} & \hat{H}_{2,2} \\ \hat{H}_{1,1}^p & \hat{H}_{1,2}^p \\ \hat{H}_{2,1}^p & \hat{H}_{2,2}^p \end{bmatrix}. \tag{4.23}$$

Note that (4.23) can be rewritten as

$$\mathcal{U}_i^H \tilde{\mathcal{H}} = \tilde{\mathbf{h}}^H \begin{bmatrix} \mathcal{V}_{i,1}^T & \mathcal{V}_{i,2}^T & 0 & 0 & \mathcal{V}_{i,3}^T & \mathcal{V}_{i,3}^T & 0 & 0 \\ 0 & 0 & \mathcal{V}_{i,1}^T & \mathcal{V}_{i,2}^T & 0 & 0 & \mathcal{V}_{i,3}^T & \mathcal{V}_{i,3}^T \end{bmatrix}^T, \tag{4.24}$$

where $\tilde{\mathbf{h}} = \left[ h_{1,1}^H \ h_{2,1}^H \ h_{1,2}^H \ h_{2,2}^H \ \bar{h}_{1,1}^H \ \bar{h}_{2,1}^H \ \bar{h}_{1,2}^H \ \bar{h}_{2,2}^H \right]^H$, $\bar{h}_{ij} = h_{ij}^p$. Also, $\mathcal{V}_{i,j}$ is a convolution matrix constructed from $\mathbf{u}_{i,j}$. Now we can rewrite (4.24) as

$$\mathcal{U}_i^H \tilde{\mathcal{H}} = \tilde{\mathbf{h}}^H \begin{bmatrix} \mathbf{V}_i \\ \mathbf{V}_{i,p} \end{bmatrix}, \tag{4.25}$$

where

$$\mathbf{V}_i = I_{n_t} \otimes \begin{bmatrix} \mathcal{V}_{i,1} \\ \mathcal{V}_{i,2} \end{bmatrix} \text{ and } \mathbf{V}_{i,p} = I_{n_t} \otimes \begin{bmatrix} \mathcal{V}_{i,3} \\ \mathcal{V}_{i,4} \end{bmatrix}.$$

Here $\otimes$ is kronecker product. We can use all eigen vectors corresponding to noise subspace in order to estimate noise subspace $\mathcal{U}$, which is orthogonal complement of signal subspace, to formulate eigen value problem as

$$\hat{\tilde{\mathbf{h}}} = \min_{\|\tilde{h}\|=1} \tilde{\mathbf{h}}^H \hat{\mathcal{U}} \tilde{\mathbf{h}}, \tag{4.26}$$

where $\hat{\mathcal{U}} = \sum_{i=k_1}^{k_2} \mathbf{V}_i \mathbf{V}_i^H$ and $k_1 = (m_1 + L)n_t$ and $k_2 = (m_1 + m_2)n_r$. Note that the solution of (4.26) is simply the smallest eigen value of noise subspace estimate $\hat{\mathcal{U}}$. Now we are ready to discuss semi-blind channel estimation.

### 4.4.4 Semi-blind Channel Estimation

Training assisted channel estimation using least square (LS) method is the most common practice and has low complexity. However, training based channel estimation consumes precious bandwidth and power resources. The least squares (LS) channel estimate $\hat{\mathbf{h}}$ of channel vector $\mathbf{h}$ of the first transmission can be achieved by minimizing

$$\hat{\mathbf{h}} = \min_{\mathbf{h}} \|\mathbf{y}_t - \mathbf{S}_t \mathbf{h}\|^2, \tag{4.27}$$

where $\mathbf{S}_t \in \mathcal{C}^{T_L \times (L+1)}$ is the block toeplitz convolution matrix constructed from $T_L$ training symbols and $\mathbf{y}_t$ is noisy observation vector corresponding to the training signal.

In practical situations, training based estimates are far superior than blind estimation. The semi-blind approach makes use of both the training and subspace method to estimate channels. In our proposed method, training is used during the first transmission while we omit training sequence for the subsequent transmissions. We combine observations from the PARQ transmission with the first transmission for joint blind channel estimation. Blind channel estimation has inherent ambiguity. This ambiguity can be resolved by combining least squares (LS) cost functions of (4.27) and subspace cost in (4.26) to formulate the semi-blind channel estimation problem as follows:

$$\hat{\hat{\mathbf{h}}} = \min_{\mathbf{h}} \parallel \mathbf{y}_t - \mathbf{S}_t \mathbf{h} \parallel^2 + \tilde{\mathbf{h}}^H \hat{\mathcal{U}} \tilde{\mathbf{h}}. \tag{4.28}$$

The optimal joint channel estimate can be expressed by setting the gradient with respect to $\tilde{\mathbf{h}}$ to zero. Finally, by solving (4.28), we get

$$\hat{\tilde{\mathbf{h}}}_{opt} = \left( \begin{bmatrix} \mathbf{S}_t^H \mathbf{S}_t & 0 \\ 0 & 0 \end{bmatrix} + \hat{\mathcal{U}} \right)^{-1} \begin{bmatrix} \mathbf{S}_t^H \mathbf{y}_t \\ 0 \end{bmatrix}. \tag{4.29}$$

The complexity of semi-blind channel estimation is $O((n_r n_t L)^3)$, where $L$ is order of channel vector.

## 4.5 Summary

In this chapter, we have presented our new transceiver design under the framework of MIMO OFDM. It is shown that *PARQ* is very effective when complexity and bandwidth is limited. A detailed description of *PARQ* system is given in this chapter. Signal detection by zero forcing detection offers less complex solution with performance compromise. Maximum likelihood detection performs optimal but offers a complex solution. Semi-blind method offers a viable solution for channel estimation. It offers many advantages like reduced complexity, bandwidth saving through reduced overhead and comparable performance. The simulation results to verify theses advantages are described in the next chapter.

# Chapter 5

# Results and Discussion

This chapter presents the simulation results and discussion on the findings of the proposed method. The performance of channel estimation and signal detection techniques derived in Chapter 4 is viewed here with the help of MATLAB simulations. The performance of signal detection implemented in the MATLAB environment, is viewed in terms of bit error rate (BER). Three cases of signal detection are considered which are single transmission, full retransmission and proposed partial retransmission technique. PARQ is compared with both single transmission scenario and full retransmission case. The performance of semi-blind channel estimation is also observed. The proposed scheme is compared with training based channel estimation as performance benchmark. Finally, we provide BER comparison with and without PARQ algorithm when LDPC (FEC) is used.

## 5.1 Simulation Setup

For the implementation of HARQ transceiver design, the parameters for simulation are given in this section. QPSK modulation is used to modulate the binary symbols. The channel considered is frequency selective Rayleigh fading channel with length 20. By frequency selective we mean that the symbol period is greater than the maximum excess delay. The number of sub-carriers used in the OFDM are 128. These number of carriers correspond to the size of Fourier transformation. We assumed that the channels don't change during transmission. The effects of bit over loading are considered to be out of the scope of this dissertation. Respective OFDM symbols are assumed to be synchronized. Our simulations is independent of the coding scheme used in HARQ systems for forward error correction.

## 5.2 Performance of PARQ Signal Detection

The performance of signal detection for HARQ Transceiver design proposed in Chapter 4 is analyzed. Perfect Channel State Information (CSI) is assumed for signal detection. For clarity, we proceed in two steps. First, the performance is evaluated for SISO OFDM systems. Secondly, it is extended for systems with multiple antennas. The performance is measured in terms of probability of error or BER versus $\frac{E_b}{N_0}$.

### 5.2.1 Signal Detection for SISO OFDM

The performance for the proposed method in terms of BER is compared with single and full retransmission case under SISO OFDM is shown in Figure 5.1 for SISO OFDM systems. The upper most solid line shows simple SISO OFDM transmission for Rayleigh fading channel. The lower most solid line represents probability of error when full retransmission takes place. Dashed lines represent proposed PARQ technique. The upper dashed line shows PARQ signal detection when zero forcing equalization is used. The lower dashed line stands for maximum likelihood detection in our partial retransmission case. It is clear that, although full retransmission offers the best performance but it suffers from significant use of valuable bandwidth. Figure 5.1 shows that our partial retransmission scheme provides gain of around 4dB over single transmission when linear detector is used. The linear detector provides low complexity solution. If ZF detection is used together with PARQ the minimum performance gain is 5dB. These gains are further improved when multiple antennas are placed which is discussed next.

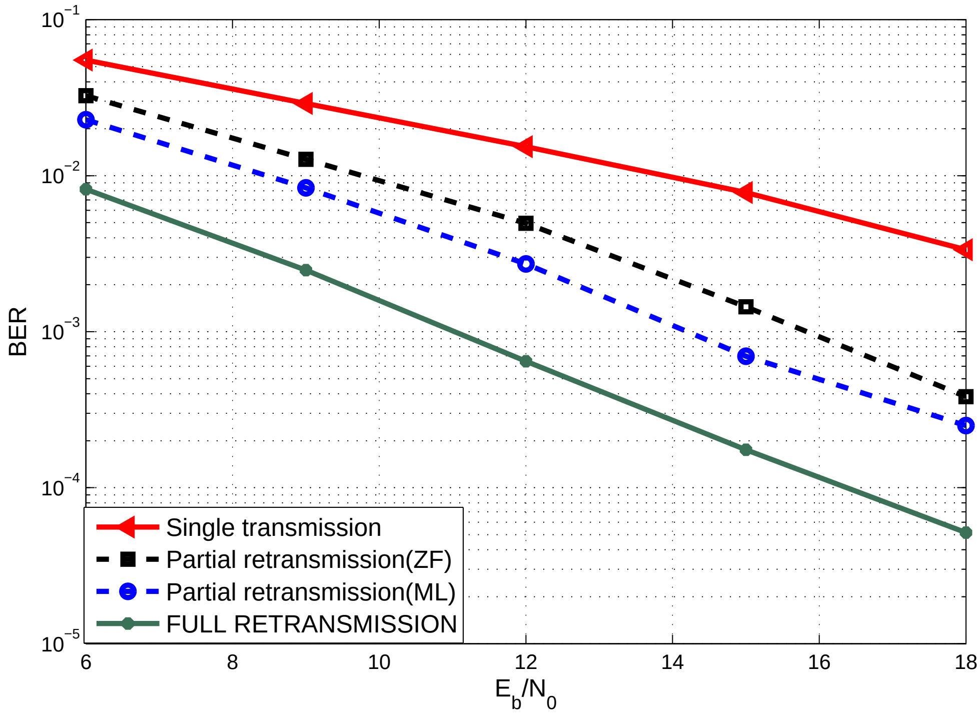


Figure 5.1: BER performance of SISO Partial and Full Retransmission using Perfect Channel State Information.

### 5.2.2 Signal Detection for MIMO OFDM

The performance of signal detection by any scheme becomes prominent when it is measured with multiple antennas. For the sake of simplicity, we have selected the number of antennas at transmitter and receiver as 2. The results follow the same behavior for any general number of antennas either at transmitter or receiver. First we analyze a simple MIMO OFDM system without any retransmission. Zero forcing and maximum likelihood detection is applied to that system. Figure 5.2 shows the performance of traditional MIMO OFDM system. Figure 5.2 depicts that non linear maximum likelihood detection performs overwhelming better than linear zero forcing detection when MIMO systems are installed. The performance gap between ZF and ML detection increases at higher SNR. Next, the performance of our PARQ scheme is evaluated for MIMO OFDM system with 2 antennas at the transmitter and

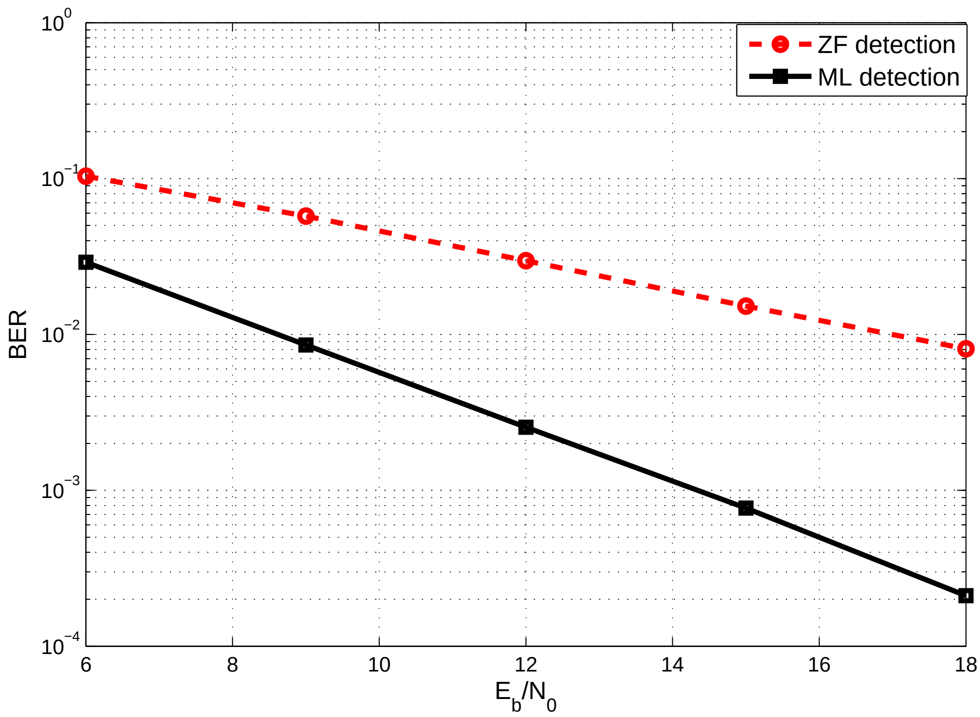


Figure 5.2: BER performance of MIMO system using perfect channel state information for single transmission

receiver. Figure 5.3 shows the comparison of proposed scheme with single transmission scheme. The solid lines represent single transmission MIMO system. The top most line shows zero forcing detection while the bottom solid line expresses the maximum likelihood detection. The dashed lines display the performance of our signal detection mechanism for linear and non linear detection in order from top to bottom. It is clear that our algorithm provides a gain of around 8dB over single transmission when we install two antennas. This performance gain increases significantly at higher SNR. The gain can be increased further if more number of antennas are used at the receiver. Note that for $2 \times 2$ MIMO OFDM system, problem of joint detection under PARQ can be divided into $\frac{N}{2}$ joint detection of four symbols each. There is 50% bandwidth saving because of retransmission of half of the symbols, with marginal performance loss as compared to full retransmission. Perfect channel estima-

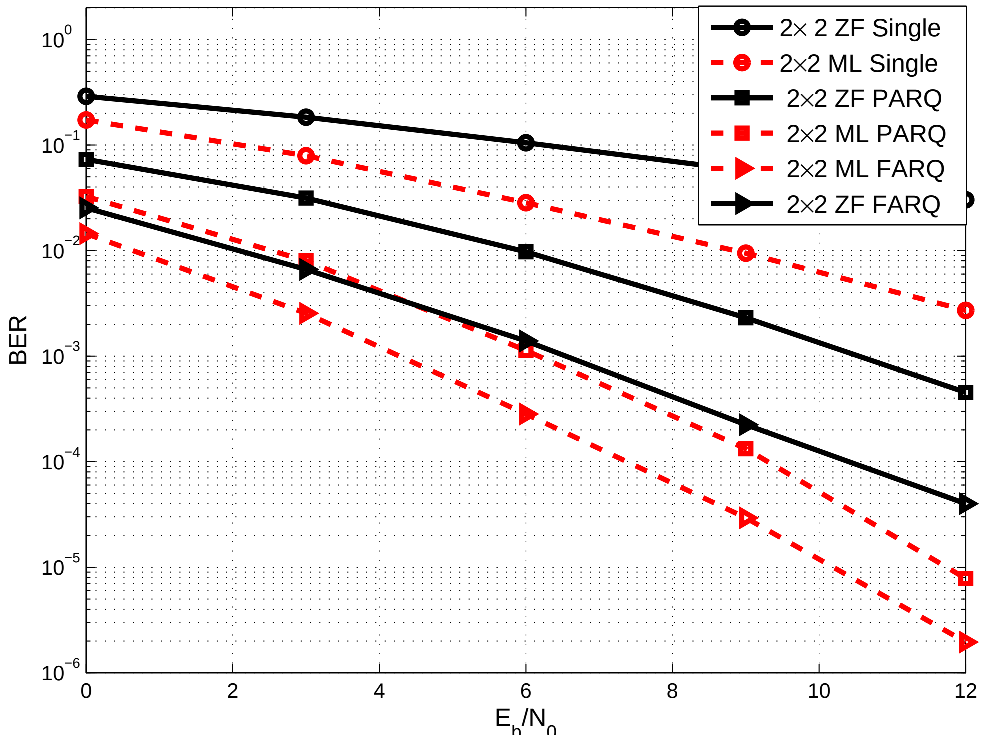


Figure 5.3: BER performance of PARQ MIMO system using perfect channel state information

tion was considered during the signal detection but real systems apply channel estimation techniques. The performance of our technique with the integration of channel estimation is explained in the next section.

### 5.2.3 Channel Estimation

In this section we discuss the performance of our proposed scheme embedded with semi-blind channel estimation technique. During first transmission the training sequence of length 10 is used for the estimation process. Increasing the training length improves the overall estimation of channel during first transmission. Training symbols are only sent during first transmission. The performance metric used for the channel estimation is normalized mean square error (NMSE) given by

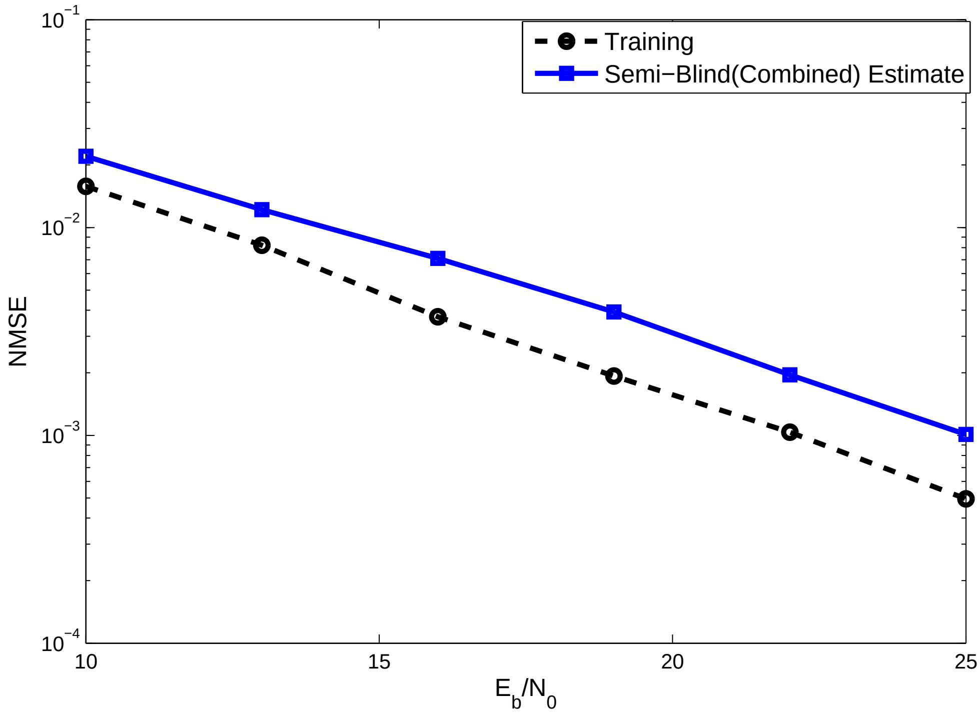


Figure 5.4: MMSE Channel Estimation

$$NMSE = E\left[\frac{\| \hat{\tilde{\mathbf{h}}} - \tilde{\mathbf{h}} \|^2}{\| \tilde{\mathbf{h}} \|^2}\right]. \tag{5.1}$$

Figure 5.4 shows the performance of training and semi-blind channel estimation. The results demonstrate that semi-blind method provides the best performance because it includes both training and most of the data information can be used. The semi-blind estimation is also much stable than the blind estimation for all transmissions because it uses training information as well. The estimation can be made more accurate by increasing the number of training symbols. Next, we compare the effect of various parameters on the performance of channel estimation which include the variations in channel, frame and training lengths.

Figure 5.5 compares normalize mean-square error (NMSE) of semi-blind and training based channel estimation for different channel lengths. In this simu-

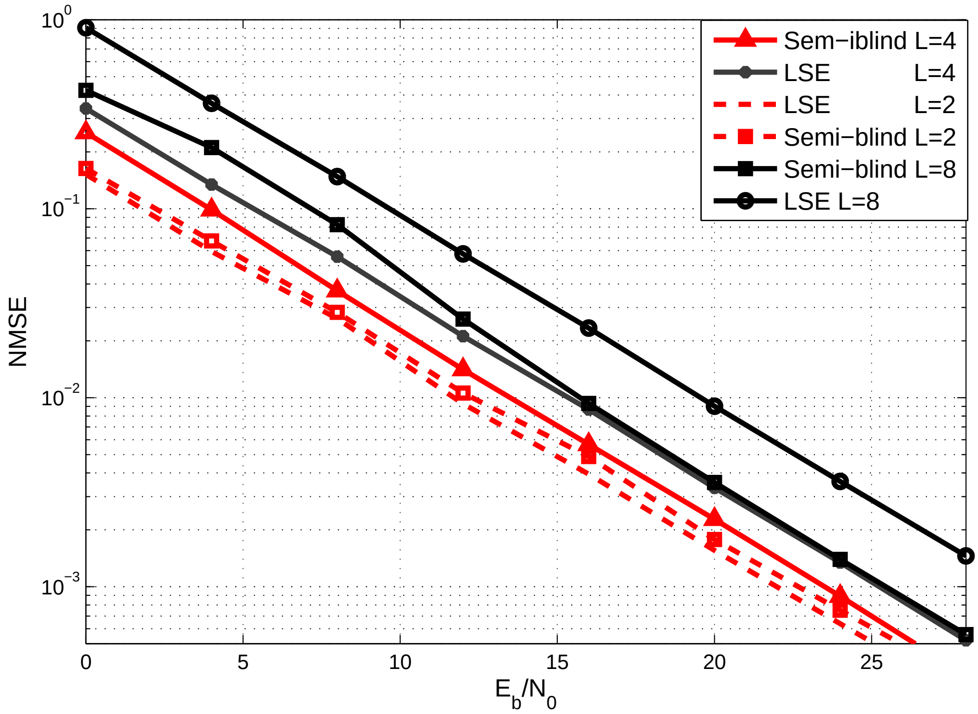


Figure 5.5: Effect of Channel length on semi-blind and training assisted channel estimation under 512-subcarriers OFDM system with training length $T_L$ of 16 symbols

lation, we fix training length $T_L$ and number of sub-carrier of OFDM system to 16 and 512, respectively. It can be noticed from Figure 5.5 that for larger channel length, semi-blind method has lower NMSE as compared to training based least square estimator (LSE). However, for smaller channel length ($L = 2$), LSE performs marginally better than semi-blind channel estimation because of the presence of training based estimation.

Effect of frame (data) length on semi-blind channel estimation is shown in Figure 5.6. NMSE of semi-blind method decreases by increasing number of observations (data length) from 128 data points to 256 point. This gain in performance is not linear in data length. That is, improvement in NMSE when data points are increased from 512 to 1024 is subtle as compared to when data points are increased from 128 to 256. This trend is due to well known behavior of blind channel estimation methods. Figure 5.7 provides comparison be-

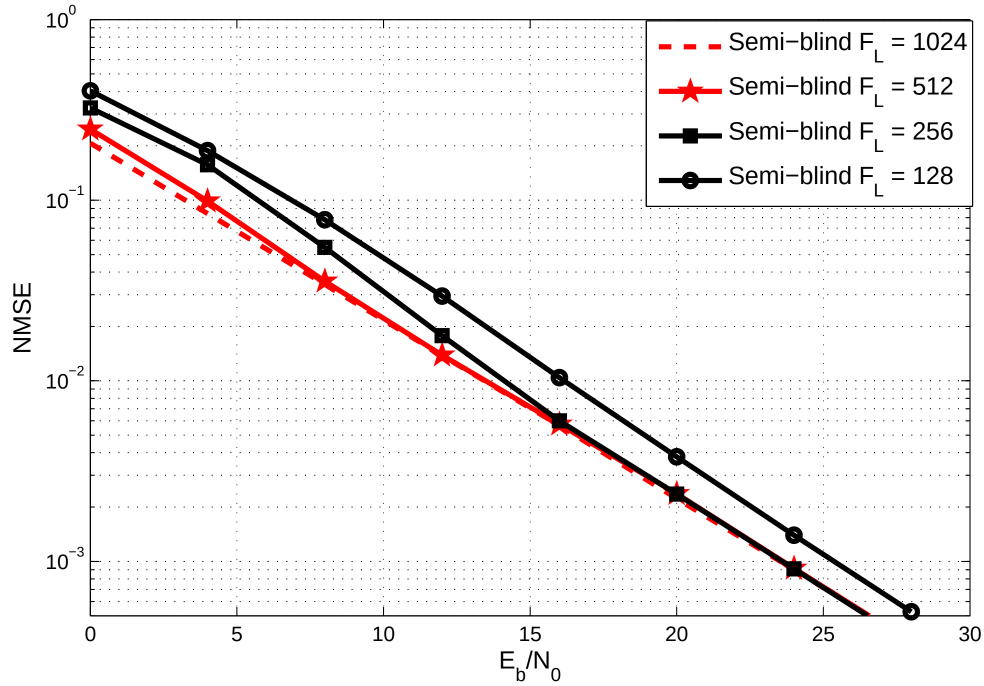


Figure 5.6: Effect of OFDM frame length $F_L$ (observation length of semi-blind method) on NMSE of semi-blind channel estimation with training length $T_L$ and channel length of 16 symbols and 4-taps, respectively

tween joint semi-blind and training assisted channel estimation methods. As shown in Figure 5.7, NMSE gap between joint semi-blind channel estimation and training based LSE reduces by increasing the number of training symbols. For training length of 32 symbols, both methods have similar performance.
Figure 5.8 shows effect of semi-blind channel estimation on the BER performance for ML and ZF algorithm under PARQ transmission. In this simulation, 16 training symbols are used for OFDM system over 4-tap Rayleigh fading channel. Semi-blind channel estimation is better suited for low signal to noise ratio regime.

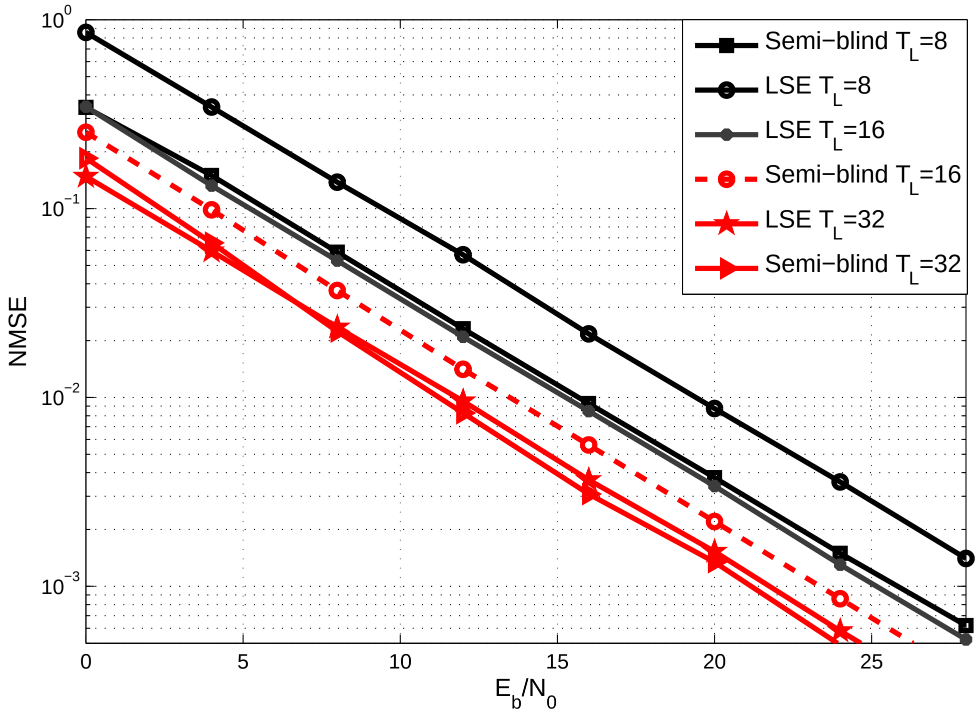


Figure 5.7: Comparison of semi-blind and training based LSE channel estimation for different training length $T_L$ under 512-subcarriers OFDM system with channel length of 4-taps

## 5.3 Performance of PARQ with LDPC

In Figure 5.9, we present performance comparison between partial ARQ and full ARQ with rate R=$\frac{1}{2}$ low density parity check (LDPC) FEC code of codeword length 648 over frequency selective Rayleigh fading under OFDM signaling. As it can be noticed, the performance gap between partial transmission and full transmission is very small as compared to single transmission. The similar trend is seen for frame error rate (FER) comparison between partial and full transmission in Figure 5.10.Consider operating $\frac{E_b}{N_0}$ of 4.5dB of MIMO OFDM system that suffers from FER of approximately 5%. By just partial retransmission, effective frame error rate is almost one frame after 10,000 frame, which saves 50% of bandwidth.

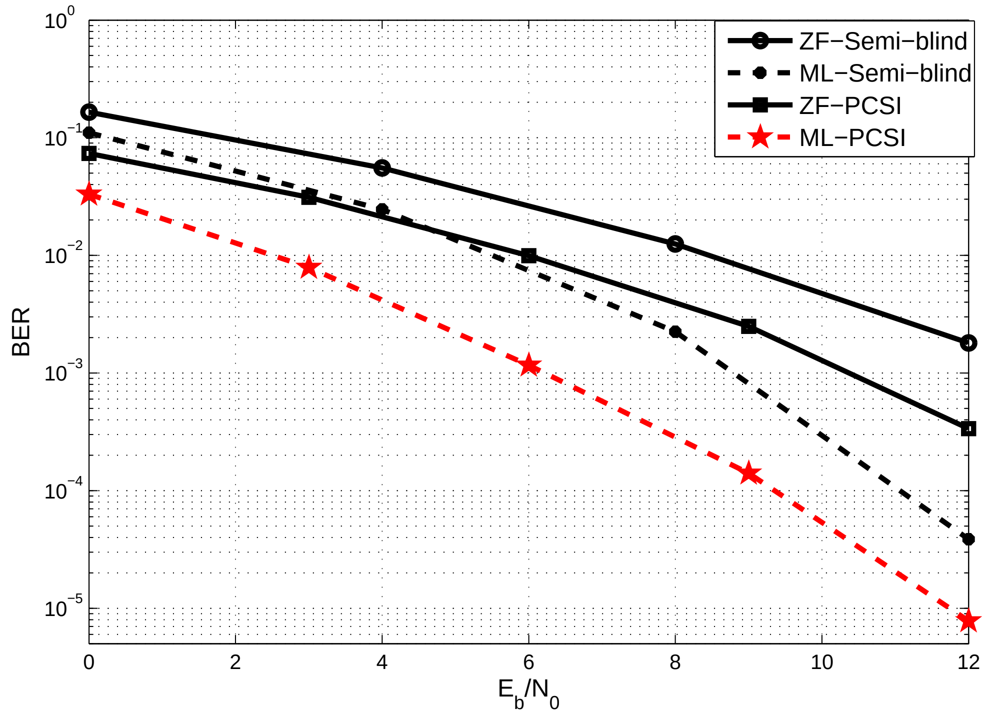


Figure 5.8: BER performance of PARQ with semi-blind channel estimation and PCSI for 128 OFDM subcarriers with channel length of 4 symbols and training length $T_L$ of 16 symbols

## 5.4 Summary

The proposed PARQ retransmission scheme provides a better trade-off between bandwidth saving and performance. Results showed that PARQ method is an efficient method for signal detection and channel estimation. Signal detection through PARQ scheme provides with better performance than single transmission and better throughput than full retransmission. The throughput can further be increased by integrating with the uper layer protocols and adapting to the application requirements.

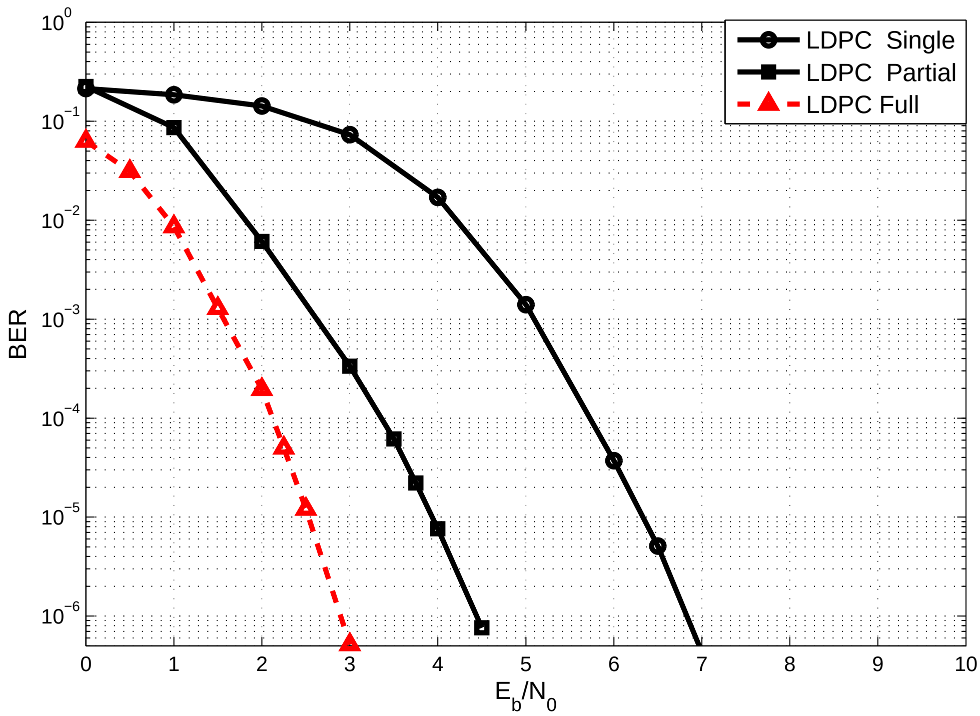


Figure 5.9: BER performance of PARQ and FARQ using FEC ( LDPC) with PCSI.

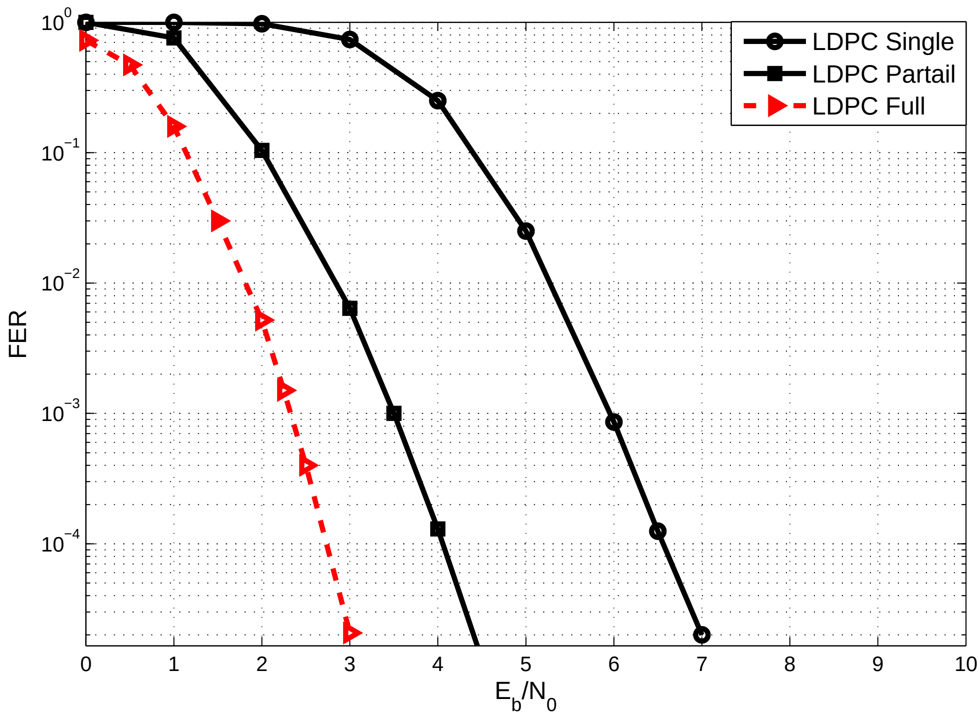


Figure 5.10: FER performance of PARQ and FARQ using FEC ( LDPC) with PCSI.

# Chapter 6

# Conclusion and Future Work

In this thesis, we have presented a PARQ retransmission scheme within the framework of OFDM. The presented scheme gives good tradeoff between bandwidth usage and performance of detection over MIMO channels. Using information from multiple transmissions, we have developed an optimal ML detector. A low complexity ZF equalizer for signal detection is also investigated. Higher layer protocols can adapt to the tradeoff between PARQ signal detection performance and the bandwidth usage according to the requirements of a particular application. This results in improved throughput of the overall system.

The problem of joint channel estimation has also been investigated. First, blind channel estimation algorithm was presented based on only the second order statistics exploiting observations from multiple transmissions. Blind channel estimation suffers from significant performance loss when channel length is long so the channel estimation was extended to include the semi blind case. Semi-blind channel estimation doesn't require training during retransmissions. It offers a comparable performance to training based channel estimation with manageable complexity. It gives better performance for long channels as compared to blind channel estimation.

In certain applications, a fraction of data like parity bits is retransmitted only. The presented PARQ scheme is best suited for these applications. The proposed method can be easily integrated into existing HARQ systems without much alteration. Following section summarizes few of the possible extensions to the presented work in future.

## 6.1 Future Work

The work can be further extended to observing the effect of our proposed PARQ method on overall throughput of the system. We have noticed significant performance gain in BER performance in simulation results. Improvement in BER performance should lower the retransmissions after FEC.
Retransmission can be made more affective based on the condition number of channel matrix. This can be implemented by channel feed-back to the transmitter. The gains symbols can be adaptively adjusted based on channel condition of each sub-carrier. This will help to further improve the bandwidth efficiency. Instead of simply adding cost functions of LS and blind channel estimation, we can also study performance of semi-blind channel estimation by weighted sum of the aforementioned cost functions.

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