Standard Model Gauge Coupling Unification and Vacuum Stability with Vector-like Particles

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Certificate

This is to certify that the dissertation entitled *"**Standard Model Gauge Coupling Unification and Vacuum Stability with Vector-like Particles**"* is a record of the bonafide work done by **Mr. Abdul Rehman**, Roll No.02181313021. Thesis is submitted to the Quaid-i-Azam University, Islamabad in partial fulfillment of the requirements for the award of the degree of **Master of Philosophy in Physics**.

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DEDICATON

To my loving Parents

and

my Advisor, Dr. Mansoor Ur Rehman

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ABSTRACT

Unification of the three Standard Model (SM) gauge couplings at the scale from 5.5×10^{15} GeV to 2.4×10^{18} GeV (reduced Planck scale) has been achieved in the models where the matter part of SM is extended by postulating the existence of new vector-like particles. The new vector-like particles carry the same quantum numbers as the SM particles do and so they are called as the standard vector-like particles. Masses of these particles are of the order of 1 TeV. The vacuum stability of the SM Higgs boson up to the Planck scale is also achieved by this simple extension of SM. Two models have been considered in this dissertation to achieve gauge coupling unification and vacuum stability, in the first model only vector-like fermions are added in the SM, where in the second model a potentially successful Dark Matter candidate is also considered along with the vector-like fermions. For describing the tiny neutrino masses, type I seesaw has also been included in the analysis and the impact of type I seesaw physics on the predictions of vacuum stability is also discussed.

Contents

Chapter 1

Introduction

The Standard Model (SM) of particle physics which was developed in 1970s is a remarkably successful theory to describe the fundamental particles and their interactions. SM has survived more than two decades of almost all the precision tests at high energy particle accelerators and is regarded as the "Science's most experimental successful theory". Even though the SM is currently the best description there is for the subatomic world, it does not explain the complete picture and still consider to be incomplete. The reason being that SM does not address many problems and there are reasons to believe in physics beyond the SM, both theoretical and experimental [1]. The unification of the three gauge couplings, and the reason why there is a huge difference between the electroweak and Planck scale or what prevents the mass of Higgs boson to receive the quadratic corrections (Hierarchy problem) [2] are amongst the theoretical reasons. In the SM, neutrinos are assumed to have zero mass. But now there is a strong evidence from the observation of neutrino oscillation experiments that neutrinos do have very tiny masses. We can not even generate these tiny neutrino masses within the mathematical framework of SM. Neutrinos with non zero mass is the only direct experiment evidence to believe that the SM is not complete. There are indirect evidence as well and among them is Dark matter (DM), which makes up most of the universe but can only be detect through gravitational effects. There is no explanation of the nature of DM in the SM. After the discovery of Higgs boson in 2012, the collider experiments have now discovered every particle in the SM. Properties of these particles were gradually revealed, among these properties masses of the Higgs boson and top quark are important to determine the behavior of Higgs quartic coupling. The measurement of the Higgs boson mass has recently found to be 125.3 ± 0.21 GeV [3] , and a combined analysis of collider experiments reported the mass of top quark as 173.34 ± 0.76 GeV [4]. A running of Higgs quartic coupling becomes negative around 10^{10} GeV using the central values of Higgs and top quark masses. This behavior implies that our vacuum is not stable at that moment [5].

To address the above mentioned problems, many extensions of SM were proposed. Among these extensions Supersymmetry (SUSY) is the most popular one, which treats the fermionic and bosonic degree of freedom on equal footing. SUSY offers solution to many problems of SM for example, the hierarchy problem. The quadratic divergences coming from the fermionic loop exactly cancels the loop contribution coming from the bosonic part. The unification of the three gauge couplings is another feature of SUSY and the lightest supersymmetric particle (LSP) can be a good candidate of dark matter.

Another extension of SM is the grand unified theory (GUT) which extends the gauge sector of SM. GUT unifies the three gauge couplings of SM at a scale around 10^{16} GeV, this scale is also known as GUT scale. Above the GUT scale the three gauge groups of SM are unified in a single gauge group, and with just one gauge coupling. Popular choices of GUT groups are SU(5) and SO(10), with both supersymmetric and non supersymmetric versions. However the simplest choice of non supersymmetric SU(5) has already been ruled out by proton decay constraint.

There are various other extensions of SM present for example Extra dimensions, Effective field theory, String theory and so on. All these physics beyond the SM theories aims to fill the deficiencies of SM, but there are still no signs of any of these theories in the experiments.

In this dissertation we will consider the simplest extension of SM in which we extend only the matter part of it by adding vector like particles which are SM like. These particles are called standard vector like because they carry the same quantum numbers as the standard model particle carry. Similar extension have been already proposed before which also includes the non standard vector like particles. We can add as many number of vector like particles as we want and also the scalars because they do not contribute to the gauge anomaly. The only restriction we have on adding new particles is that the gauge couplings should remain perturbative up to the Planck scale ($\sim 10^{19}$ GeV). We will derive the perturbative conditions on the gauge couplings in chapter 4. The main motivation of our work is that we can achieve the unification of the three SM gauge coupling. Another motivation is that we can simultaneously achieve the vacuum stability with this simple extension. Here we will consider two scenarios for unification. In the first scenario we will consider only the vector-like fermions with the SM particles and in the second scenario we will also consider Minimal Dark Matter (MDM) candidate [6] with these vector like fermions to unify the three gauge couplings. The scale we will consider for the gauge coupling unification is between 5×10^{15} GeV to the reduced Planck scale 2.4×10^{18} GeV. The reason to choose the upper limit is obvious that at that scale gravity became important and we can not simply ignore it, while the lower scale is chosen to avoid the proton decay arising from the dimension six effective operator[1] [7].

To describe the tiny neutrino masses we have included type-I seesaw in our analysis. Type-I seesaw includes a right handed sterile neutrino (per generation of fermions) in the Standard model [8]. As the right handed neutrino do not carry any gauge charge so it will not effect the running of gauge couplings, but it will effect the evolution of top Yukawa and Higgs quartic coupling which in turns effect the vacuum stability bound. The seesaw mechanism and its impact on the prediction of vacuum stability will be discussed in detail latter.

[1]Proton has a measured life time of $\sim 10^{33}$ years [9]. It is automatically stable in SM because of the accidental symmetry known as baryon number conservation, but this symmetry explicitly breaks when we includes higher order terms and use the effective description of SM theories, allowing the proton to decay through operators which are suppressed by $1/\Lambda_{GUT}$. To be consistent with experimental value of proton life time Λ_{GUT} should be greater that 5×10^{15}GeV [10].

Chapter 2

Standard Model A Brief Introduction

2.1 Motivation of Standard Model

The objective of Particle Physics is to understand the basic structure and laws of nature all the way from the largest dimensions in the universe that is to say the formation of Galaxies, Stars etc, to all the way down to the smallest dimensions of the microworld. Historically we knew about what are the different elements in nature, we knew about Hydrogen, Helium, Oxygen, Gold, Lead and so on, which are all made of different atoms. But a great simplification was made when we realize that all atoms are just made of three particles the Proton, Neutron and the Electron. In principle we can build a very simple universe from just three particles. But it became much more complicated in the beginning of 20th century when we found many many new particles from cosmic rays. There was not really a system established to organize the zoo of particles, so we started calling these new particles things like Pie(π), Sigma(σ), Delta(Δ) so on, and soon we found ourselves running out of symbols to name these particles, so we started organizing these particles according to there properties, like spin, electric charge, mass of the particle and life time of the particles. To simplify the picture new fundamental particles called Quarks were predicted and the whole zoo of particles could be described by combinations of these quarks, and this was the birth of Standard Model [11][20][21].

2.2 Standard Model

The Standard Model of Particle Physics was developed in early 1970's and has been tested many times through experiments[19]. The Standard Model (SM) is based on the idea that everything in the universe is made up of few basic building blocks, known as the Fundamental Particles, and they interact with each others through three Fundamental Forces (Gravity is treated Classically in SM). The SM at the time of its development has tested many times through experiments and has successfully explained almost all the experimental results. Hence it has been established as a well tested theory. In the beginning of the development of SM, there were only three quarks up(**u**), down(**d**) and strange(**s**). The Charm(**c**), bottom(**b**) and top(**t**) were predicted and were discovered afterwards which gave us great confidence on the model. In addition to these quarks there is another set of fundamental building blocks of matter so called Leptons, these are electron(**e**), muon(μ), tau(τ) and their neutrino partners electron neutrino(ν_e), muon neutrino(ν_μ) and tau neutrino(ν_τ).

The three Fundamental forces include in the SM are the Weak force, explains the energy production of sun and is responsible for the radioactive Beta decay. The mediators of weak force are W's and Z bosons. The Electro Magnetic(EM) Force acts on the charge particles and is responsible for the propagation of light or for the fact that the magnet can pick up paper clips, the corresponding force carrier is Photon. Finally the Strong force acts on Quarks mediated by Gluon's, the gluon literally glues together the quarks in the neutrons and protons and holds the nucleus together.

Last but not the least is the Higgs boson which is responsible for giving mass to the elementary particles, The idea was put forward about 40 years ago simultaneously in three now famous papers; written by Robert Brout and Francois Englert, Peter Higgs and Gerald Guralnik, Richard Hagen and Tom Kibble was that all fundamental particles get there mass by interacting with Higgs field, which spread across everywhere and the fluctuation of this field give rise to Higgs particle, which was later discovered in 2012 and Peter Higgs along with another scientist Francois Englert were awarded Nobel Prize in Physics for the prediction of this particle.

2.3 Mathematical Description of SM

The Standard Model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C, L and Y denotes color, Left and hypercharge respectively. The Matter fields in the SM are given in the table below

Name	Spin	Generations	$(U(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_C)$
ϕ	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
Q_L	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
L_L	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
d_R	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
u_R	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
e_R	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1})$

Table 2.1: Standard Model matter contents.

Where in the Last column the transformations of fields under SM gauge group are shown, the hypercharge of fields are assigned from the Gell-Mann–Nishijima relation $Q = I_3 + Y$, with I_3 being the isospin. In addition to these Matter fields there are the gauge fields corresponding to every gauge group shown in the table below

Name	Gauge Group	Coupling	Name
B	$U(1)_Y$	g_1	Hypercharge
W	$\mathrm{SU}(2)_L$	g_w	Isospin
G	$\mathrm{SU}(3)_C$	g_s	Color

Table 2.2: Gauge fields in Standard model.

In the Standard Model Quantum Chromodynamics (QCD) is the theory describes the strong interactions. It is a non abelian gauge theory which is based on the gauge group $SU(3)_C$, Quarks belongs to the fundamental representation and transforms as triplet under this group, gluons which are the mediator of strong interactions belong to the adjoint representation of $SU(3)_C$. All other fundamental particles transforms as singlet under this group and do not experience any strong interaction.

The Lagrangian of QCD can be written as [22]

$$\mathcal{L}_S = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{\psi}_i (i\gamma^\mu D_\mu - m)\psi_i \ , \tag{2.1}$$

where $G^a_{\mu\nu}$ is the field strength tensor and is given by

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G_{b\mu} G_{c\nu} \ . \tag{2.2}$$

D_μ is the covariant derivative and is defined as

$$D_\mu = \partial_\mu \delta - i g_s T_a G^a_\mu \ , \tag{2.3}$$

where g_s is the strong coupling constant, the indice a runs over color and has values from 1 to 8. T_a are the generator of the gauge group which satisfy the relation $[T_a,T_b] = if_{abc}T_c$. Where f_{abc} are the structure constant of the group. In case of SU(3) these generators T_a are related to 3×3 Gell-Mann matrices [11] by

$$T_a = \frac{\lambda_a}{2} \tag{2.4}$$

the corresponding Lagrangian is invariant under $SU(3)_C$ infinitesimal local gauge transformations.

The Electroweak part of SM describes the weak and electromagnetic interactions. The corresponding Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{EW} &= -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu}_i - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - i\bar{\psi}_j\gamma^\mu D_\mu\psi_j + (D_\mu\phi)(D^\mu\phi)^\dagger \\ &\quad + Y_L\bar{L}_L\phi e_R + Y_u\bar{Q}_L\bar{\phi}U_R + Y_d\bar{Q}_L\phi d_R - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 , \end{aligned} \tag{2.5}$$

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths of the non-abelian gauge fields of $SU(2)_L$ and the only abelian gauge field of $U(1)_Y$ respectively and are defined as

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_w\epsilon^{ijk}W_{j\mu}W_{k\nu} , \tag{2.6}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu . \tag{2.7}$$

The covariant derivative here is defined as

$$D_\mu = \partial_\mu - ig_wT_iB^i_\mu - ig_1\frac{Y}{2}W_\mu . \tag{2.8}$$

The generators of $SU(2)$ obeys the relation $[T_i,T_j] = i\epsilon_{ijk}T_k$ and related to Pauli spin matrices [11] by the relation

$$T_i = \frac{\tau_i}{2} . \tag{2.9}$$

The above Lagrangian is invariant under $SU(2)_L$ and $U(1)_Y$ infinitesimal local gauge transformations. The massless gauge fields belong to the adjoint representation of $SU(2)_L$ and transforms as triplet with the charge fields defined by

$$W^\pm_\mu = \frac{(W_1 \mp iW_2)_\mu}{\sqrt{2}} , \tag{2.10}$$

and neutral component of W_i mixes with the $U(1)$ gauge field to form physical states Z_μ and A_μ

$$Z_\mu = W^3_\mu \cos\theta_w + B_\mu \sin\theta_w , \tag{2.11}$$

$$A_\mu = B_\mu \cos\theta_w - W^3_\mu \sin\theta_w \,, \tag{2.12}$$

here θ_w is the weak mixing angle or Weinberg angle related to g_1 and g_w by

$$\tan\theta_w = \frac{g_1}{g_w} \,. \tag{2.13}$$

L_l and Q_l in eqn.2.5 are the left handed leptons and quark doublet defined as

$$L_L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu \\ e \end{pmatrix} \quad , \quad Q_L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix} u \\ d \end{pmatrix} .$$

e_R , u_R and d_R are the lepton,up-type quark and down-type quark singlets respectively defined as

$$e_R = \frac{1}{2}(1+\gamma_5)e \,, \quad u_R = \frac{1}{2}(1+\gamma_5)u \,, \quad d_R = \frac{1}{2}(1+\gamma_5)d \,.$$

Where L_L , Q_L , e_R , u_R and d_R also have an implicit 3-component generation indices for three generations of quarks and leptons in the SM. Y_l, Y_u and Y_d are the Yukawa couplings of lepton, up-type and down-type quarks respectively.

The masses of fermions and non-abelian gauge fields can be generated by the Higgs mechanism [25], in which the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken. The Higgs field which is a complex scalar iso-spin doublet $(\phi^\dagger, \phi^0)$ gets a non zero vacuum expectation value (vev), in fact only neutral component of ϕ is allowed to acquire vev, and the reason behind this is, if the charged component is allowed to acquire vev, the Electromagnetic (EM) symmetry will break. Which is not realized in nature.

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{2.14}$$

with $v^2 = -\mu^2/Y_i$, the mass terms of physical gauge bosons are then given by

$$M_w = \frac{1}{2}g_w v \,, \quad M_z = \frac{v}{2}\sqrt{g_w^2 + g_1^2} \,, \quad M_A = 0 \,.$$

The photon still remains massless. Higgs field also generates masses of fermions through Yukawa couplings $m_f = Y_f v/\sqrt{2}$. Now if we consider all the three families we need to Diagonalize the mass matrix which give rise to Cabibbo-Maskawa-Kobayashi (CKM) matrix [26]. Note that we can not generate the masses of neutrino through spontaneous symmetry breaking as we have done it for other fermions because of the absence of right handed neutrino in the SM.

2.4 Salient features of SM

The SM is highly successful theory and its beauty as a theory can be listed in the following points

- **Renormalizability**

A model is said to be renormalizable if it entails all infinities may be absorbed by a finite number of counter-terms [14], expressing all quantities in terms of the renormalized physical parameters. SM certainly has this feature embedded in it, because all the terms in the SM Lagrangian are of Mass dimension four or less, which is essential for the theory to be renormalizable.

- **Unitarity**

Unitarity says that the probabilities of an event can be at maximum approach one but not greater then that and this is an obvious and essential requirement for a Quantum field theory(QFT), if the theory is not unitary then it can not describe the Nature completely, and it is necessarily missing some information. SM as a QFT and is consistent with unitarity constraint.

- **Unification of EM and Weak force**

Before the SM there was no gauge theory for weak force and to describe the beta decay which is caused by the weak force, Fermi had given the theory which describe beta decay as a four fermion interaction at a single point [27]. However this theory was immediately in trouble because it was non-renormalizable. The only way to fix this problem is to regulate the theory at high energies and the only consistent way to regulate the contact interaction is to explain it as an exchange of another particle. The right particle that can be exchanged to match basic experimental test is a vector boson, proposed by Glashow, Weinberg and Salam. The discovery of the W and Z particles in 1983 brought experimental verification of particles whose prediction had already contributed to the Nobel Prize in 1979, The Photon, particle involved in the Electromagnetic(EM) interaction, along with W and Z provide the necessary pieces to unify the Weak and EM interactions.

- **Prediction of relationship between W and Z boson Masses**

The Weinberg angle(θ_W) or some time known as weak mixing angle is a parameter of Electroweak interaction. It is the angle by which the unphysical

W^0 and B^0 states are related to the physical vector bosons Z^0 and the photon(γ) [28]

$$m_Z = \frac{m_W}{\cos\theta_w} \tag{2.15}$$

it also gives the relation between W and Z boson masses

$$m_Z = \frac{m_W}{\cos\theta_w} \tag{2.16}$$

and can also be expressed in terms of coupling constants of $SU(2)$ and $U(1)$

$$\sin\theta_w = \frac{g_1}{\sqrt{g_w^2 + g_1^2}} \tag{2.17}$$

the value of Weinberg angle varies with the momentum transfer, at which it is measured. This running of θ_w with momentum was the key prediction of Electroweak theory and is in great agreement with the experiments. Along with this the weak charged and neutral current structure of SM agrees with the experiments as well.

- **Anomaly Cancellation**

Anomaly cancellation within each generation is another key feature of SM [29][30]. Anomaly is the breaking down of symmetry when we include Quantum effects in the theory. The anomaly cancellation is required in any gauge theory for its quantum consistency . And in order to identify the anomaly one need to only worry about the calculation of triangular diagram of the form AVV (A=Axial Vector, V=Vector currents)

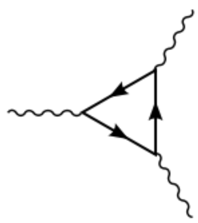


Figure 2.1: Triangular Anomaly.

For a given representation of fermion D of gauge group G the triangular anomaly fig.2.1 can then be written as [31]

$$A(D)d^{abc} \equiv \mathrm{Tr}[T^a\{T^b, T^c\}] \tag{2.18}$$

Where d^{abc} denotes the anomaly coefficient associated with the fundamental representation and T^a denotes the generators of G. The anomaly coefficient for real and pseudo real representations is zero $A(D) = 0$ [14], while the anomaly coefficient for common representations of $SU(N)$ is given in [32]

For the case of SM which is based on the gauge group $SU(3) \times SU(2) \times U(1)$ we encounter cubic anomalies and also mixed anomalies in case when the three generators in the triangle comes from different factors. But we only need to check one generation for anomaly as the generations simply repeats itself. Now the possible anomalies are given by

- $SU(3) \times SU(3) \times SU(3)$ **:** From table.2.3 we can see that there are as many Quarks belongs to 3 as $\bar{3}$ so the theory is vector-like with respect to $SU(3)$ $(A(3) = -A(\overline{3}))$, so the $SU(3)^3$ anomaly cancels.

- $SU(2) \times SU(2) \times SU(2)$ **:** All the representations of $SU(2)$ are only real or pseudo-real, so are automatically anomaly free. We can also check this explicitly
$$d^{abc} \;=\; tr(\tau^a\{\tau^b, \tau^c\})$$
where τ's are the Pauli spin matrices, but
$$\{\tau^b, \tau^c\} \;=\; 2\delta^{bc} I$$
and so
$$d^{abc} \;=\; 2tr(\tau^a \delta^{bc}) \;=\; 0$$
as τ's are traceless.

- $SU(2) \times SU(2) \times SU(3)$ **:** These anomalies vanishes because of the traceless property of generators
$$tr(\lambda^a\{\tau^b, \tau^c\}) \;=\; tr(\lambda^a) tr\{\tau^b, \tau^c\} \;=\; 0$$
similarly $SU(3) \times SU(3) \times SU(2)$ also vanishes.

- $SU(2) \times U(1) \times U(1)$ & $SU(3) \times U(1) \times U(1)$ **:** Also Vanishes because of the same argument as above.

- $SU(2) \times SU(2) \times U(1)$ **:** Only left handed particles contribute to this anomaly because only they carry both $SU(2)_L$ and $U(1)$ charge (right handed particles are singlet under $SU(2)_L$)
$$tr(Y\{\tau^b, \tau^c\}) \;=\; 2\delta^{bc} tr(Y)$$
$$tr(Y) \;=\; 2 \times 3 \times \left(\frac{1}{6}\right) + 2 \times 1 \times \left(-\frac{1}{2}\right) \;=\; 0$$

- $SU(3) \times SU(3) \times U(1)$ **:** Here all particles carrying $SU(3)$ and $U(1)$ charge contributes to this anomaly

$$tr(Y\{\lambda^b, \lambda^c\}) \;=\; 2\delta^{bc} tr(Y)$$
$$tr(Y) \;=\; 3 \times 2 \times \left(\frac{1}{6}\right) + 3 \times 1 \times \left(\frac{1}{3}\right) + 3 \times 1 \times \left(-\frac{2}{3}\right) \;=\; 0$$

- $U(1) \times U(1) \times U(1)$ **:** As the Generators of $U(1)$ are just numbers so for this anomaly we need to just sum cube of hypercharge of all the particles

$$6\left(\frac{1}{6}\right)^3 + 2\left(-\frac{1}{2}\right)^3 + 3\left(\frac{1}{3}\right)^3 + 3\left(-\frac{2}{3}\right)^3 + 1(1)^3 \;=\; 0$$

So we see that all the anomalies are accidentally canceled in the SM and we can conclude from here that SM is anomaly free. Note here that the assignment of hypercharge is essential for the cancellation of anomaly so instead of the Gell-Mann Nishijima relation this is another way to check for the hypercharge assignment and we can find that instead of this assignment there is yet another possibility for the hypercharge assignment if we require the SM to be anomaly free [33].

- **Higgs**

The Discovery of Higgs boson is itself a great triumph of SM, it was the last missing piece of SM and after its discovery on 4^{rth} of July 2012 at ATLAS and CMS experiments at CERN's Large Hadron Collider (LHC) completes the SM [3]. The Higgs boson as proposed within the SM, is the simplest manifestation of the Brout-Englert-Higgs Mechanism [25]. Nobel prize in Physics was awarded to Peter Higgs and Francois Englert jointly for the theoretical discovery of the mechanism that describe the very origin of mass of subatomic particles.

2.5 Unanswered Questions in SM

Despite the fact that Standard Model answers many of the question regarding the structure and stability of matter and its interaction, it is still incomplete. There are still many question which SM do not answer, which can be listed as follows

Hierarchy Problem

Why is there a desert between the Electro-Weak scale ($\sim$ 100 GeV) and Plank scale ($\sim 10^{19}$ GeV), or in other words why is gravity so weak as compared to other three forces ? What prevent quantities (like Higgs boson mass) at the Electro-Weak Scale, from getting loop corrections (quantum correction) on the order of Plank scale [34].

Neutrino Mass

Neutrinos are massless in SM because the SM incorporate only left-handed neutrino (and right-handed anti-neutrino) and so they can not have mass from the Higgs mechanism [25][34]. But there are strong evidence from experiments, that neutrino do have mass. So giving mass to neutrino in SM is also a problem.

Matter-Antimatter Asymmetry

Why do we only observe matter in the universe and almost no antimatter [35]? What is te reason for this asymmetry between matter and antimatter.

Three Generations

Why are there only three generations of leptons and quarks in SM ? Is there any possibility of 4^{rth} generation in the SM, which is yet to be discovered ?

Fundamental particles

Are Leptons and Quarks fundamental or they themselves are made up of even more fundamental particles ?

So many Parameters

Why are there some twenty a-priori parameters (couplings and masses of particles)in the SM for which their values are only determined from experiments without any theoretical understanding of these values ?

Dark Matter

What is Dark matter made up of ? that we can not see and that has visible gravitational effects in the cosmos.

Dark Energy

What is the cause of accelerated expansion of the universe we observed ? if gravity is the only force acting on the large scale in the cosmos.

Gravity

The SM describe only three of the four fundamental forces at the quantum level, gravity is only treated classically, which is also a question mark on the completion of SM [1].

Chapter 3

The Renormalization Group Equations

In this chapter we will introduce some of the terminologies used in the work presented in this dissertation. We will first of all consider a simple theory and calculate its beta function. Then at the end we will derive the beta function of Standard Model gauge couplings. The detailed calculation will not be included here because this is just to give a qualitative understanding of beta function. Detailed calculation can be seen from any text book of quantum field theory for example [13][14].

We will start by considering a theory of a pseudo-scalar field ϕ and a Dirac field ψ. This theory was proposed by Yukawa in 1934 to explain the nature of nuclear force, he proposed that the massive bosons (ϕ in our case) mediate the interaction between nucleons (ψ in our case) [11]. Now to write the Lagrangian of this theory we need to consider all possible terms (no gauge interaction will be considered here) whose coefficient have zero or positive mass dimension (necessary for the theory to be renormalizable [12]) and which respect the symmetries of the original Lagrangian, for example Lorentz symmetry, Parity, time reversal and so on. Therefore the only allowed Yukawa interaction term in the Lagrangian is

$$\mathcal{L}_{\mathcal{Y}} = iy\phi\overline{\psi}\gamma_5\psi \ ,$$

where y is the Yukawa coupling constant and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ are the well know Dirac matrices. Note here that as ϕ is a pseudo-scalar ($P^{-1}\phi(x,t)P = -\phi(-x,t)$) this term conserve parity as it should be because we are talking about strong force which conserves parity. Now the complete Lagrangian of this theory can be written as the sum of free Lagrangian and the interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathcal{I}} \ , \tag{3.1}$$

where

$$\mathcal{L}_0 \ = \ i\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}M^2\phi^2 \ , \tag{3.2}$$

and

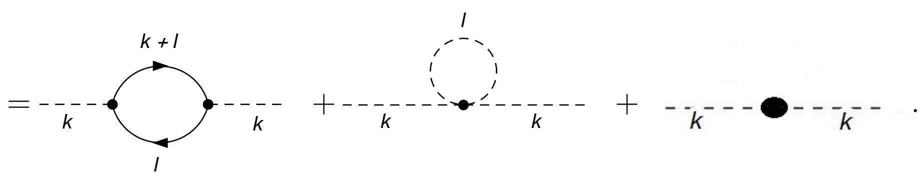
$$\mathcal{L}_{\mathcal{I}} \ = \ iZ_y y\phi\overline{\psi}\gamma_5\psi - \frac{1}{24}Z_\lambda\lambda\phi^4 + \mathcal{L}_{ct} \tag{3.3}$$

where terms with only ϕ and ϕ^3 are forbidden due to parity and λ is the scalar quartic coupling. The field introduced in the above Lagrangian are the renormalized field, $\mathcal{L}_{ct}$ is the counter term Lagrangian which is introduced to remove the infinites coming from quantum corrections.

$$\begin{aligned}\mathcal{L}_{ct} \ &= i(Z_\psi - 1)\overline{\psi}\partial\!\!\!/\psi - (Z_m - 1)m\overline{\psi}\psi - \frac{1}{2}(Z_\phi - 1)\partial^\mu\phi\partial_\mu\phi \\ &- \frac{1}{2}(Z_M - 1)M^2\phi^2\end{aligned} \tag{3.4}$$

3.0.1 Loop correction to Scalar propagator

Lets start our calculation from scalar propagator, the full scalar propagator $\Pi(k^2)$ up to one-loop level and its counter term correction is given below



The amplitude of the diagram with fermion loop using Feynman rules [13] is given by

$$i\Pi_{\psi-loop}(k^2) = (-1)(-y^2)\left(\frac{1}{i}\right)^2 \int \frac{d^4l}{(2\pi)^4} Tr\Big[S(l\!\!\!/ + k\!\!\!/)\gamma_5 S(l\!\!\!/)\gamma_5\Big], \tag{3.5}$$

where

$$S(p\!\!\!/) = \frac{-p\!\!\!/ + m}{p^2 + m^2 + i\epsilon}.$$

After solving the above amplitude using dimensional regularization [13] in d = $4 - \epsilon$ space-time dimensions, we finally get

$$\Pi_{\psi-loop}(k^2) = -\frac{y^2}{4\pi}\left[\frac{1}{\epsilon}(k^2 + 2m^2) + \frac{1}{6}k^2 + m^2 + ...\right] \quad . \tag{3.6}$$

Similarly the other two diagrams gives

$$\Pi_{\phi-loop}(k^2) = \frac{\lambda}{(4\pi)^2}\left[\frac{1}{\epsilon} + \frac{1}{2} - \frac{1}{2}\left(M^2/\mu^2\right)\right]M^2 \quad , \tag{3.7}$$

and

$$\Pi_{ct}(k^2) = -(Z_\phi - 1)k^2 - (Z_M - 1)M^2 \; . \tag{3.8}$$

Note that the amplitude diverges in the limit $\epsilon \to 0$, we will set Z_ϕ and Z_M so that they cancel the divergences. we get

$$Z_\phi = 1 - \frac{y^2}{4\pi^2}\left(\frac{1}{\epsilon} + \text{finite}\right) , \tag{3.9}$$

and

$$Z_M = 1 + \left(\frac{\lambda}{16\pi^2} - \frac{y^2}{2\pi^2}\frac{m^2}{M^2}\right)\left(\frac{1}{\epsilon} + \text{finite}\right) . \tag{3.10}$$

3.0.2 Loop correction to Fermionic propagator

Now turning our attention to fermionic propagator,the only diagram from which ψ receive the one-loop correction with the counter-term is shown in fig. below

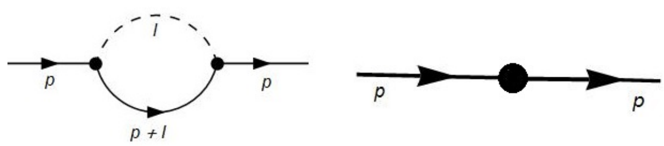


Figure 3.1: One loop correction to scalar propagator and the counter term correction.

we have

$$i\Sigma_{1-loop}(\not{p}) = (-y)^2\left(\frac{1}{i}\right)^2 \int \frac{d^4l}{(2\pi)^4}\left[\gamma_5 S(\not{p} + \not{l})\gamma_5\right]\Delta(l^2) \; , \tag{3.11}$$

where
$$S(\not{p}) = \frac{-\not{p} + m}{p^2 + m^2 + i\epsilon} \quad \text{and} \quad \Delta(l^2) = \frac{1}{l^2 + M^2 - i\epsilon} \; .$$
By solving eqn.3.11 we finally get
$$\Sigma_{1-loop}(\not{p}) = -\frac{y^2}{16\pi^2}\left[\frac{1}{\epsilon}(\not{p} + 2m) + ...\right] \; . \tag{3.12}$$
And the counter-term gives
$$\Sigma_{ct}(\not{p}) = -(Z_\psi - 1)\not{p} - (Z_m - 1)m \; . \tag{3.13}$$
Again the finiteness of $\Sigma(\not{p})$ requires
$$Z_\psi = 1 - \frac{y^2}{16\pi^2}\left(\frac{1}{\epsilon} + \text{finite}\right) \tag{3.14}$$
$$Z_m = 1 - \frac{y^2}{8\pi^2}\left(\frac{1}{\epsilon} + \text{finite}\right) \; . \tag{3.15}$$

3.0.3 Loop correction to Yukawa vertex

Next we turn to the one-loop correction to the Yukawa vertex. The vertex function iV_y is defined as the sum of one-particle irreducible diagrams with one incoming and one outgoing fermion having momenta p and $\prime p$, and one incoming scalar having momentum $k = \prime p - p$. where the original vertex $-Z_y y\gamma_5$ is the first term in the sum and the diagram of fig.3.2 is the second. The total sum can be written as
$$iV_y(p', p) = -Z_y y\gamma_5 + iV_{y-1loop}(p', p) + \mathcal{O}(y^5), \tag{3.16}$$
where the one loop diagram is shown below

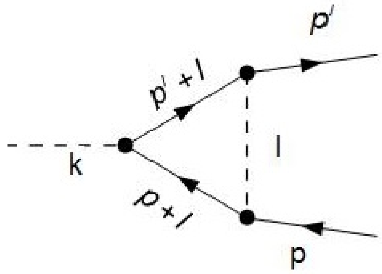


Figure 3.2: One-loop correction to the Yukawa vertex.

And its amplitude is given by

$$iV_{y-1loop}(p', p) = (-y^3)\left(\frac{1}{i}\right)^3 \int \frac{d^4l}{(2\pi)^4}\left[\gamma_5 S(\not{p}'+\not{l})\gamma_5 S(\not{p}+\not{l})\gamma_5\right]\Delta(l^2), \quad (3.17)$$

where

$$S(\not{p}) = \frac{-\not{p}+m}{p^2+m^2+i\epsilon} \quad \text{and} \quad \Delta(l^2) = \frac{1}{l^2+M^2-i\epsilon}\ .$$

We proceed in the usual way and get

$$iV_{y-1loop}(p', p) = \frac{y^3}{8\pi}\left[\frac{1}{\epsilon}+...\right]. \quad (3.18)$$

From eq.3.16 we can see that the finiteness of iV_y requires

$$Z_Y = 1 + \frac{y^2}{8\pi^2}\left(\frac{1}{\epsilon}+\text{finite}\right). \quad (3.19)$$

3.0.4 4-Point Vertex

Finally we will turn to the correction of ϕ^4 vertex $iV_4(k_1, k_2, k_3, k_4)$, the tree level contribution is $-iZ_\lambda\lambda$. Now the Feynman diagram of tree level and the one loop correction is shown in the figure below.

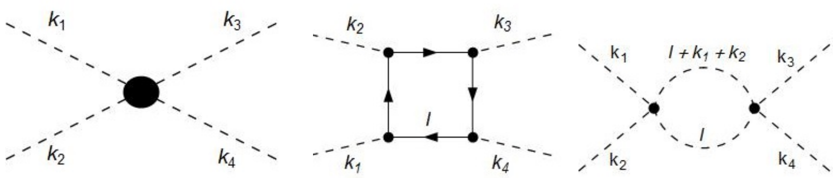


Figure 3.3: Corrections to the 4-point vertex up to one loop, there are other diagrams which can be obtained by the permutation of external momenta in all possible inequivalent ways.

The diagram with closed fermionic loop gives

$$V_{4,\psi\ loop} = -\frac{3y^4}{\pi^2}\left(\frac{1}{\epsilon}+\text{finite}\right), \quad (3.20)$$

and the close scalar loop diagram gives

$$V_{4,\phi\ loop} = \frac{3\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon}+\text{finite}\right). \quad (3.21)$$

Now the complete 4-Point function can be written as

$$V_4 = -Z_\lambda \lambda + V_{4,\psi\, loop} + V_{4,\phi\, loop} + ... \quad . \tag{3.22}$$

From here we can see that the finiteness of V_4 requires

$$Z_\lambda = 1 + \left(\frac{3\lambda^2}{16\pi^2} - \frac{3y^4}{\lambda\pi^2}\right)\left(\frac{1}{\epsilon} + \text{finite}\right), \tag{3.23}$$

and the higher order corrections.

3.0.5 Calculation of Beta Function

Now we have everything in hand to calculate the beta function of the Yukawa theory we have discussed above. The Lagrangian written at the start was in terms of renormalized parameters and it can also be written in terms of bare parameters(y_0, λ_0, ϕ_0, and so on), and we can find a relation between the bare couplings and renormalized couplings [14]

$$y_0 = Z_\phi^{-\frac{1}{2}} Z_\psi^{-1} Z_y \tilde{\mu}^{\frac{\epsilon}{2}}\ y\ , \tag{3.24}$$

and

$$\lambda_0 = Z_\phi^{-2} Z_\lambda \tilde{\mu}^{\epsilon}\ \lambda\ . \tag{3.25}$$

We define

$$\ln(Z_\phi^{-\frac{1}{2}} Z_\psi^{-1} Z_y) = \sum_{n=1}^{\infty} \frac{G_n(y,\lambda)}{\epsilon^n}\ , \tag{3.26}$$

and

$$\ln(Z_\phi^{-2} Z_\lambda) = \sum_{n=1}^{\infty} \frac{L_n(y,\lambda)}{\epsilon^n}\ . \tag{3.27}$$

Now taking the logarithm of eqn.3.24 and eqn.3.25 we get

$$\ln y_0 = \sum_{n=1}^{\infty} \frac{G_n(y,\lambda)}{\epsilon^n} + \ln y + \frac{1}{2}\epsilon \ln \tilde{\mu}\ . \tag{3.28}$$

$$\ln \lambda_0 = \sum_{n=1}^{\infty} \frac{L_n(y,\lambda)}{\epsilon^n} + \ln \lambda + \epsilon \ln \tilde{\mu}\ . \tag{3.29}$$

As we are only interested in one loop corrections here so we need to expand $G_n(y, \lambda)$ and $L_n(y, \lambda)$ up to first order. Now using the results from the last sections we finally obtain

$$G_1(y,\lambda) = \frac{5y^2}{16\pi^2} + \ldots\ , \tag{3.30}$$

$$L_1(y,\lambda) = \frac{3\lambda}{16\pi^2} + \frac{y^2}{2\pi^2} - \frac{3g^4}{\pi^2\lambda} + \ldots\ . \tag{3.31}$$

Now here we use the fact that the bare couplings should be independent of the fake parameter μ,then by differentiating eqn.3.28 and eqn.3.29 with respect to $\ln\mu$ we get

$$0 = \sum_{n=1}^{\infty}\left(y\frac{\partial G_n}{\partial y}\frac{dy}{d\ln\mu} + y\frac{\partial G_n}{\partial\lambda}\frac{d\lambda}{d\ln\mu}\right)\frac{1}{\epsilon^n} + \frac{dy}{d\ln\mu} + \frac{1}{2}\epsilon y\ , \tag{3.32}$$

$$0 = \sum_{n=1}^{\infty}\left(\lambda\frac{\partial L_n}{\partial y}\frac{dy}{d\ln\mu} + \lambda\frac{\partial L_n}{\partial\lambda}\frac{d\lambda}{d\ln\mu}\right)\frac{1}{\epsilon^n} + \frac{d\lambda}{d\ln\mu} + \frac{1}{2}\epsilon\lambda\ . \tag{3.33}$$

In any renormalizable theory,$\frac{dy}{d\ln\mu}$ and $\frac{d\lambda}{d\ln\mu}$ must remain finite in the limit $\epsilon \to 0$.Therefore we can write

$$\frac{dy}{d\ln\mu} = -\frac{1}{2}\epsilon y + \beta_y(y,\lambda)\ , \tag{3.34}$$

$$\frac{d\lambda}{d\ln\mu} = -\epsilon\lambda + \beta_\lambda(y,\lambda)\ . \tag{3.35}$$

The above two equations are known as the renormalization group equations for Yukawa and lambda respectively. Now substituting them in eqns.3.32,3.33 and matching the power of ϵ we find

$$\beta_g(y,\lambda) = y\left(\frac{1}{2}y\frac{\partial}{\partial y} + \lambda\frac{\partial}{\partial\lambda}\right)G_1\ , \tag{3.36}$$

$$\beta_\lambda(y,\lambda) = \lambda\left(\frac{1}{2}y\frac{\partial}{\partial y} + \lambda\frac{\partial}{\partial\lambda}\right)L_1\ . \tag{3.37}$$

Using the values of eqn.3.30 and eqn.3.31 in the above equations we get the final expressions of beta function

$$\beta_y(y,\lambda) = \frac{5y^3}{16\pi^2} + \ldots, \tag{3.38}$$

and

$$\beta_\lambda(y,\lambda) = \frac{1}{16\pi^2}\left(3\lambda^2 + 8\lambda y^2 - 48y^4\right) + \dots \,. \tag{3.39}$$

This is the general way to derive the beta function of any theory. In our previous discussions we have not considered the gauge couplings, which arises when we require local gauge invariance of the theory under certain gauge group. Now in the next example we will also consider the gauge couplings in the theory and derive their beta functions.

3.1 Beta function of SM gauge couplings

In this section we will use the general result of [15][16], to calculate the beta function of SM gauge couplings. where the procedure to derive the result is the same as discussed in the above the sections.

The general result for the two loop beta function of gauge couplings is given by

$$\beta_i = \frac{b_i}{(4\pi^2)} g_i^3 + \sum_{j=1}^{3} \frac{b_{ij}}{(4\pi^4)} g_i^3 g_j^2 \tag{3.40}$$

Now for the case of SM (which is the product group of U(1), SU(2) and SU(3), see Chapter 2) i, j take values 1,2,3 which refer to U(1), SU(2) and SU(3) respectively. In eqn.3.40, g_i are the gauge couplings constant associated to each group. b_i and b_{ij} are defined as follows

$$b_i = \frac{2}{3}T(F_i)d(F_{\bar{i}}) + \frac{1}{3}T(S_i)d(S_{\bar{i}}) - \frac{11}{3}C_2(G_i)\ , \tag{3.41}$$

$$\begin{aligned} b_{ij} \;=&\; \left(\frac{10}{3}C_2(G_i) + 2C_2(F_i)\right)T(F_i)d(F_{\bar{i}}) + \left(\frac{2}{3}C_2(G_i) + 4C_2(S_i)\right)T(S_i)d(S_{\bar{i}}) \\ &- \left(\frac{34}{3}\right)(C_2(G_i))^2\ . \end{aligned} \tag{3.42}$$

Where the meanings of the notation used are as follows. The fermion multiplets are assumed to transform according to the irreducible representation F_i with respect to the group G_i (here G_i are U(1), SU(2) and SU(3) for $i = 1, 2, 3$ respectively). Similarly for bosons F is replaced by S. $\bar{i}$ means other than i (for example $\bar{1} = 2 \times 3$). For the irreducible representation R we have the relation [17]

$$R^a R^a \;=\; C_2(R) I\ , \tag{3.43}$$

and

$$\mathrm{Tr}[R^a R^b] \;=\; T(R)\delta^{ab}\ . \tag{3.44}$$

R^a here is the matrix representation of the generators of the group. $C_2(R)$ and $T(R)$ are related by the identity

$$C_2(R)d(R) \;=\; T(R)d(G)\ . \tag{3.45}$$

Where $d(G)$ is the number of generators of the group and $d(R)$ is defined as the dimension of the representation. $C_2(R)$ is the quadratic Casimir operator (which commutes with every operator of the group) of the representation and $C_2(G)$ is defined as the quadratic Casimir for the adjoint representation. Now for the case of representation of $U(1)$ we have $C_2(R) = T(R) = y^2$, where $T(R)$ is known as dynkin index and y is the hypercharge (normalized appropriately) [18]. The Scalar representation in the SM is complex, and the fermion representation is complex and chiral. Thus for the fundamental representation of $SU(N)$ (for scalar or fermion) we have $T = \frac{1}{2}$ and $d = N$ and $T = 0$ for singlet representation [23][24]. Now to calculate b_i we need the representations of SM particles under the SM gauge group and are given in table 2.3. Now from eqn.3.41

$$\begin{aligned} b_1 = \frac{2}{3}n_G\Big[&T(q_1)d(q_{3\times 2}) + T(u_1^c)d(u^c_{3\times 2}) + T(d_1^c)d(d^c_{3\times 2}) + T(l_1)d(l_{3\times 2}) \\ &+ T(e_1^c)d(e^c_{3\times 2})\Big] + \frac{n_H}{3}T(H_1)d(H_{3\times 2}) - \frac{11}{3}C_2(G_1)\ , \end{aligned}$$

now by using the canonical normalization of hypercharge ($y = \sqrt{\frac{3}{5}}Y$) where Y can be read from table 2.3) we get

$$\begin{aligned} b_1 = \frac{2}{3}n_G\left(\frac{3}{5}\right)\Bigg[&\left(\frac{1}{6}\right)^2(3\times 2) + \left(\frac{-2}{3}\right)^2(3\times 1) + \left(\frac{1}{3}\right)^2(3\times 1) + \left(\frac{1}{2}\right)^2(1\times 2) \\ &+ (-1)^2(1\times 1)\Bigg] + \frac{n_H}{3}\left(\frac{3}{5}\right)\left(\frac{-1}{2}\right)^2(1\times 2) - \frac{11}{3}(0)\ , \end{aligned}$$

where n_G is the no of generations of fermions and n_H is the no of generation for scalars.

$$b_1 = \frac{4}{3}n_G + \frac{1}{10}n_H\ . \tag{3.46}$$

Similarly for b_2 corresponding to $SU(2)$ we get

$$b_2 = \frac{2}{3}n_G\left[T(q_2)d(q_{3\times1}) + T(u_2^c)d(u_{3\times1}^c) + T(d_2^c)d(d_{3\times1}^c) + T(l_2)d(l_{3\times1}) + T(e_2^c)d(e_{3\times1}^c)\right] + \frac{n_H}{3}T(H_2)d(H_{3\times1}) - \frac{11}{3}C_2(G_2)\ ,$$

as u, d and e transform as singlet under $SU(2)$ so $T = 0$ for all of them and we get

$$b_2 = \frac{4}{3}n_G + \frac{1}{6}n_H - \frac{22}{3}\ . \tag{3.47}$$

Finally for b_3 corresponding to $SU(3)$ we have

$$b_3 = \frac{2}{3}n_G\left[T(q_3)d(q_{1\times2}) + T(u_3^c)d(u_{1\times2}^c) + T(d_3^c)d(d_{1\times2}^c) + T(l_3)d(l_{1\times2}) + T(e_3^c)d(e_{1\times2}^c)\right] + \frac{n_H}{3}T(H_3)d(H_{1\times2}) - \frac{11}{3}C_2(G_3)\ ,$$

l and e are singlet under $SU(3)$ so $T = 0$ for them and we get

$$b_3 = \frac{4}{3}n_G - 11\ . \tag{3.48}$$

Now we have three generations of fermions and one generation of scalar(Higgs) in case of SM, so using $n_G = 3$ and $n_H = 1$ in eqns. 3.46, 3.47, 3.48 we get

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right)\ . \tag{3.49}$$

On the same footing we can calculate bij and is given by

$$B_{ij} \ = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}\ . \tag{3.50}$$

Eqns.3.49 and 3.50 are known as one and two loop beta coefficients of SM gauge couplings respectively. We will use these results in Chapter4 for the running of SM gauge couplings.

Chapter 4

Extended Standard Model

After briefly discussing the SM in the 2nd chapter and defining the terminologies is the last chapter, we will discuss in detail some of the problems of SM which are relevant to our work at the start of this chapter and after that we will discuss a particular solution to these problems.

4.1 Status Of Standard Model

4.1.1 Gauge Coupling Evolution

As mentioned in the previous chapter that there are three gauge groups in SM with there corresponding gauge couplings, from now on we will call these couplings g_1 (for $U(1)$) , g_2 (for $SU(2)$) and g_3 (for $SU(3)$). At scale Mz (Mass of Z boson) these couplings are very different to each other in terms of strength, and are independent to each other. Due to higher order correction or loop corrections we see that each of these couplings is actually function of some energy scale (μ). So the three couplings evolves with energy and the evolution of these couplings is described by the Renormalization Group Equations (RGE) [15], which at the two loop are given below.

$$\frac{dg_i}{dln\mu} = \frac{b_i}{16\pi^2}g_i^3 + \frac{g_i^3}{(16\pi^2)^2}\left(\sum_{j=1}^{3} B_{ij}g_j^2 - C_i^t y_t^2\right) , \tag{4.1}$$

where i runs from 1 to 3 for the three couplings, b_i and B_{ij} are the one and two loop beta coefficients of gauge couplings we have derived in Chapter 3.

$$b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) , \quad B_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix} \tag{4.2}$$

C_i^t are the coefficients for the top Yukawa couplings (we are not considering the contribution of other quarks because these contributions are negligibly small) and is given by

$$C_i^t = \left(\frac{17}{10}, \frac{3}{2}, 2\right) . \tag{4.3}$$

The gauge couplings at Mz has been determined very precisely by around 1990, and people use these values as initial conditions for the running of these couplings, but now the values of these couplings are also known at Mt (Top mass Scale) with high precision, and we will use these values as our initial conditions [36].

$$g_1(Mt) = \sqrt{\frac{5}{3}}\left(0.35761 + 0.00011\left(\frac{Mt}{GeV} - 173.10\right)\right), \tag{4.4}$$

$$g_2(Mt) = 0.64822 + 0.00004\left(\frac{Mt}{GeV} - 173.10\right), \tag{4.5}$$

$$g_3(Mt) = 1.1666 - 0.00046\left(\frac{Mt}{GeV} - 173.10\right)\Bigg), \tag{4.6}$$

Now if we Plot these couplings from Mt to Plank scale, the coupling would look like the one shown below.

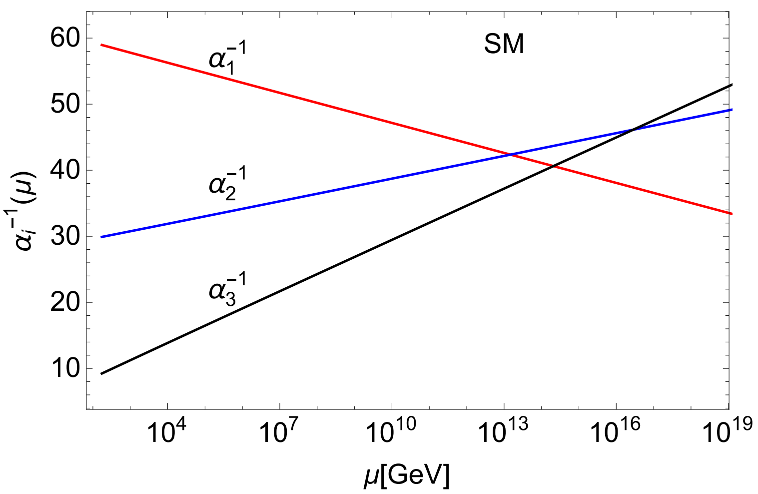


Figure 4.1: Gauge coupling evolution in SM.

As can be seen in Fig above, the three couplings seems to come close to each other when extrapolated using the SM expression, but are not unified at a single point on the energy scale.

4.1.2 Vacuum Stability

The potential for Higgs is given by

$$V = -\mu^2\phi^2 + \lambda\phi^4 \ .$$

The stability of SM-vacuum depends on the sign of Higgs quartic coupling(λ) in the last term of the above equation. The potential will be unbounded from below for negative λ and therefore unstable [34]. For the Higgs mass (125 GeV) we know precisely the value of λ at weak scale from the tree level relation $m_H^2 = 2\lambda v^2$, which is around 0.13 for vacuum expectation value v equals 246 GeV. which is positive definitely. But this is not the compete story, at large values of ϕ the Higgs potential is (to the good approximation) given by $V \approx \lambda(\phi)\phi^4$, where $\lambda(\phi)$ is the running quartic coupling evaluated at scale ϕ. The running of λ is obtained by solving the RGE given by

$$\frac{d\lambda}{dln\mu} = \frac{1}{16\pi^2}\beta_\lambda^1 + \frac{1}{(16\pi^2)^2}\beta_\lambda^2 \ , \tag{4.7}$$

where β_λ^1 and β_λ^2 are the one and two loop beta functions for λ defined as

$$\beta_\lambda^1 = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) + 12y_t^2 - 12y_t^4, \tag{4.8}$$

and

$$\begin{aligned}\beta_\lambda^2 = & -78\lambda^3 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2g_2^2 - \frac{1887}{200}g_1^4\right)\lambda \\ & - 3\lambda y_t^4 + \frac{305}{8}g_2^6 - \frac{867}{120}g_1^2g_2^4 - \frac{3411}{1000}g_1^6 - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 \\ & - \frac{9}{2}g_2^4y_t^2 + 10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)y_t^2 - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 \\ & - 72\lambda^2y_t^2 + 60y_t^6.\end{aligned} \tag{4.9}$$

Where $g's$ are the gauge couplings whose running are define by eq 4.1, y_t is the top Yukawa couplings whose RGE is given by

$$\frac{dy_t}{dln\mu} = y_t\left(\frac{1}{16\pi^2}\beta_t^1 + \frac{1}{(16\pi^2)^2}\beta_t^2\right). \tag{4.10}$$

Here the one-loop beta function is defined as

$$\beta_t^1 = \frac{9}{2}y_t^2 - \left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right) \tag{4.11}$$

While the two-loop is given by

$$\begin{aligned}\beta_t^2 = & -12y_t^4 + \left(\frac{393}{80}g_1^2 + \frac{225}{16}g_2^2 + 36g_3^2\right)y_t^2 + \frac{1187}{600}g_1^4 - \frac{9}{20}g_1^2g_2^2 \\ & + \frac{19}{15}g_1^2g_3^2 - \frac{23}{4}g_2^4 + 9g_2^2g_3^2 - 108g_3^4 + \frac{3}{2}\lambda^2 - 6\lambda y_t^2.\end{aligned} \tag{4.12}$$

Now by looking at the above equations, it can be seen that if Higgs were decoupled from the rest of matter then with the increase in energy λ would grow and would eventually explode into a **Landau Pole** (The Landau pole is the energy scale at which the coupling constant of quantum field theory becomes infinite). However the top Yukawa coupling provides a negative contribution to the evolution equation that works towards decreasing λ at large energies. Within SM the top Yukawa coupling is large as compared to Higgs self coupling, so Yukawa wins. This behavior can also be seen from the plot shown below

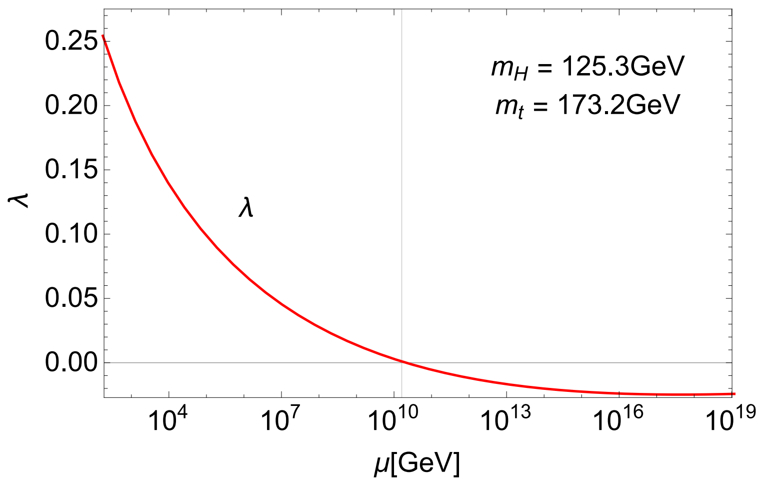


Figure 4.2: Evolution of Higgs quartic coupling in SM.

Where λ is evaluated using the central value of top quark mass M_t = 173.2 GeV and α_s (strong coupling constant) = 0.1184 [36] at M_Z. We have evaluated these couplings from the top pole mass to the Plank scale, where the boundary conditions at top mass scale [36] are given below

$$y_t(Mt) \;=\; 0.93558 + 0.00550\left(\frac{Mt}{GeV} - 173.1\right) \tag{4.13}$$

and for λ

$$\lambda(Mt) \;=\; 0.1271 - 0.00004\left(\frac{Mt}{GeV} - 173.1\right) + 0.00206\left(\frac{Mh}{GeV} - 125.66\right) \tag{4.14}$$

One can see from the graph shown above that λ becomes negative at the scale around 10^{10} GeV which naively means that the Higgs potential is not bounded from below or we can say that the SM vacuum is not stable up to the Planck scale.

4.1.3 Dark Matter

Dark Matter(DM) is called dark because it is not made up of ordinary matter which we see. It is not in the form of planets and stars, nor it emits any type of radiation which we can see. We can say at this point that we are much more certain what DM is not, then we are about what it is. Observation shows that there is much more dark matter than the normal matter in the universe. Scientist believes that there is about $\sim 27\%$ dark matter $\sim 68\%$ dark energy(which is the reason why our universe expands with acceleration rather then deceleration) and only $\sim 5\%$ ordinary matter in the whole universe[35].

There are many theories in the past to explain dark matter, but most of them are ruled out now, still at this point a few dark matter possibilities are viable, one of them is that dark matter may be made up of baryonic matter tied up in brown dwarfs or in dense small chunks of heavy elements, these are known as massive compact halo objects or "MACHOs". But the most profound theory is that dark matter is made up of Non baryonic matter, like exotic particles(Axions) or Weakly Interacting Massive Particles (WIMPs) [35]. But the truth is yet to e revealed about what actually is this Dark matter.

4.1.4 Massless Neutrinos

Neutrino are spin half particle and are electrically neutral cousins of charged lepton (e, μ, τ), due to this property they are drastically different from other spin half particles. They also do not carry color charge like their cousins leptons so they do not take part in strong interactions as well. For a long time it was thought that neutrinos do not have mass at all, but during the past decade, from the experimental observations involving neutrino emission from solar burning, from the cosmic-ray experiments and also the production of neutrino from terrestrial source like accelerators and reactors, it is now a very strong believe of physicist that neutrinos do have a tiny mass like other leptons and quarks, leading to neutrino oscillation phenomenon [35]. But there is no other experimental prove to believe about the nature of neutrinos, whether they are Dirac like or Majorana like, or exactly how much mass do these neutrinos have. But still for the tiny mass of neutrino, it is impossible to generate this mass within the SM, and to generate neutrino mass we need to go beyond the SM.

4.2 Extension Of SM

There are many extension of SM present in the literature like super-symmetry, which consider the symmetry between fermions and bosons and its minimal model (MSSM), which considers only minimum number of new particles with the SM particles, Extra Dimensions, Grand Unified theories (GUT), String theory and so on. But none of these theories are proved experimentally. So there is still open window for theorist to make new models.

The one extension of SM, we will be discussing there is to add Vector-like particles in the SM, the addition of Vector-like particles is motivated by the fact that we can avoid the gauge anomalies here, as the anomaly cancels within themselves. specifically we will consider only Standard Vector-like particles. why are we calling them Standard ? It is because the vector-like particles transforms the same way as that of SM fermions with their hermitian conjugate. All the possible Standard vector-like particles with their quantum numbers under SM gauge group are given in the table below [37].

Standard Model like Vector Particles	Irreducible representation $(SU(3)_3, SU(2)_L, U(1)_y)$
$Q+\overline{Q}$	$(3,2,\frac{1}{6})+(\overline{3},2,-\frac{1}{6})$
$U+\overline{U}$	$(3,1,\frac{2}{3})+(\overline{3},1,-\frac{2}{3})$
$D+\overline{D}$	$(3,1,-\frac{1}{3})+(\overline{3},1,\frac{1}{3})$
$L+\overline{L}$	$(1,2,\frac{1}{2})+(1,2,-\frac{1}{2})$
$E+\overline{E}$	$(1,1,1)+(1,1,-1)$
G	$(8,1,0)$
W	$(1,3,0)$

Table 4.1: Possible Standard Vector-like Particles with their quantum numbers under SM Gauge group.

For extra particles which belong to real representation or adjoint representation do not need to be vector-like because they do not contribute to the anomalies [33], so note from the above table that G and W do not need to be vector-like as they belong to adjoint representation of $SU(3)$ and $SU(2)$ respectively.

The contribution to one loop β-coefficient (b_1, b_2, b_3) by these extra Particles are given in the table below.

Standard Model like Vector Particles	Contribution to (b_1, b_2, b_3) $(\triangle b_1, \triangle b_2, \triangle b_3)$
$Q+\overline{Q}$	$(\frac{1}{5},3,2)$
$U+\overline{U}$	$(\frac{8}{5},0,1)$
$D+\overline{D}$	$(\frac{2}{5},0,1)$
$L+\overline{L}$	$(\frac{3}{5},1,0)$
$E+\overline{E}$	$(\frac{6}{5},0,0)$
G	$(0,0,3)$
W	$(0,2,0)$

Table 4.2: Standard Vector-like Particles and their contributions to one-loop beta functions.

Where these contribution need to multiply with $(\frac{2}{3})$ when considering them as fermions and $(\frac{1}{3})$ when considering as scalars.

4.3 Gauge Coupling Unification Condition

In this section we will derive the condition on the one loop beta function (b_1, b_2, b_3) which is necessary for the gauge coupling unification. we will start with the one loop gauge coupling RGEs

$$\frac{dg_i}{dln\mu} = \frac{b_i}{16\pi^2} g_i^3 \,, \tag{4.15}$$

where $g's$ are related to $\alpha's$ by the relation given by

$$\alpha_i = \frac{g_i^2}{4\pi} \,, \quad \text{or} \quad g_i = \sqrt{4\pi}\alpha^{\frac{1}{2}} \,. \tag{4.16}$$

So we can write eqn. 4.15 in terms of α as

$$\sqrt{4\pi}\frac{d(\alpha_i^{\frac{1}{2}})}{dln\mu} = \frac{b_i}{16\pi^2}(4\pi)^{\frac{3}{2}}\alpha_i^{\frac{3}{2}} \,.$$

Simplifying this we get

$$\frac{1}{\alpha_i^2}\frac{d\alpha_i}{dln\mu} = \frac{b_i}{2\pi} \,. \tag{4.17}$$

Now as we know that

$$\frac{d}{dln\mu}\left(\frac{1}{\alpha}\right) = -\frac{1}{\alpha^2}\frac{d\alpha}{dln\mu} \,,$$

using this in above eq we get

$$\frac{d}{dln\mu}(\alpha_i^{-1}) = -\frac{b_i}{2\pi} \,. \tag{4.18}$$

Integrating the above equation from M_f to M_G

$$\int_{M_f}^{M_G} d\alpha_i^{-1} = -\frac{b_i}{2\pi}\int_{M_f}^{M_G} dln\mu \,,$$

where M_f is the mass scale at which the new particles contribute to the RGEs and M_G is the GUT scale, where all the three couplings unifies.The solution of above equation is

$$\alpha_i^{-1}(M_G) = \alpha_i^{-1}(M_f) - \frac{b_i}{2\pi}ln\left(\frac{M_G}{M_f}\right) \,. \tag{4.19}$$

Now at GUT scale all these couplings have same value (GCU condition) $\alpha_i^{-1}(M_G) = \alpha_j^{-1}(M_G) = \alpha^{-1}(M_G)$ for $i, j = 1, 2$ and 3. Then by using this the above eq can be written as

$$\left(\frac{bi}{2\pi} - \frac{bj}{2\pi}\right)ln\left(\frac{M_G}{M_f}\right) = \alpha_i^{-1}(M_f) - \alpha_j^{-1}(M_f) \,,$$

or

$$b_i - b_j \;=\; \frac{2\pi}{ln\left(\frac{M_G}{M_f}\right)}\left(\alpha_i^{-1}(M_f) - \alpha_j^{-1}(M_f)\right) .$$

Where $b_i = b'_i + b_i^{SM}$, in which b_i^{SM} is the contribution of SM particles and b'_i is the contribution coming from the new particles. So finally we have

$$b'_i - b'_j \;=\; \frac{2\pi}{ln\left(\frac{M_G}{M_f}\right)}\left(\alpha_i^{-1}(M_f) - \alpha_j^{-1}(M_f)\right) - (b_i^{SM} - b_j^{SM}) . \qquad (4.20)$$

Which are the required conditions for realization of gauge coupling unification.

In addition to the unification condition we need to also take care of the perturbative bound on the couplings i.e the coupling should remain perturbative up-to the Planck scale(to avoid Landau Pole), and for this the condition on the $b'_i s$ can be derived as follows. We require $(\alpha_i^{-1}(M_G) > 0)$ so eqn.4.19 gives

$$\alpha_i^{-1}(M_f) - \frac{b_i}{2\pi} ln\left(\frac{M_G}{M_f}\right) \geq 0 ,$$

or

$$b_i \leq \frac{2\pi}{ln\left(\frac{M_G}{M_f}\right)}\alpha_i^{-1}(M_f) .$$

Again $b_i = b'_i + b_i^{SM}$, so finally we get

$$b'_i \leq \frac{2\pi}{ln\left(\frac{M_G}{M_f}\right)}\alpha_i^{-1}(M_f) - b_i^{SM} . \qquad (4.21)$$

Which are the conditions for the perturbativity of gauge couplings.

4.4 Specific Example

In this section we will discuss a specific example to achieve gauge coupling unification and vacuum stability. We add just a pair of vector-like particles amongst those described in section 4.2, with SM particles. The particles we have chosen with there Quantum numbers under SM gauge group are given below [38].

$$
\begin{aligned}
Q + \bar{Q} &= \left(3, 2, \frac{1}{6}\right) + \left(\bar{3}, 2, -\frac{1}{6}\right) , \\
D + \bar{D} &= \left(3, 1, -\frac{1}{3}\right) + \left(\bar{3}, 1, \frac{1}{3}\right) .
\end{aligned} \tag{4.22}
$$

The SM Lagrangian will receive extra contributions from these new particles. The relevant terms in the Lagrangian is given by

$$
\begin{aligned}
\mathcal{L}' &= - y_1^i \phi Q d_i^c - y_2^i \phi D q_i - y_3^i \phi^c Q u_i^c - \kappa_1 \phi^c \bar{Q} \bar{D} - \kappa_2 \phi Q D \\
&\quad - M_f (\bar{D} D + \bar{Q} Q) + h.c .
\end{aligned} \tag{4.23}
$$

Where ϕ is the SM Higgs doublet and ϕ^c is its charge conjugate defined as $\phi^c \equiv i\sigma_2 \phi^*$, u_i, d_i and q_i are the SM quarks where $i = 1, 2, 3$ being the generation index. κ's and y^i's are the dimensionless couplings called Yukawa couplings for new particles. For the simplest case we assume these couplings to be very small and will not contribute significantly in the RGE analysis so we can simply ignore these contributions [38]. M_f is the mass of these vector like particles and is a free parameter and can have any value from few hundred GeV to several TeV. We can constraint the lower limit on their masses from experimental observations, but we have no bound on the upper limit of their masses, and also these masses do not need to be degenerate. But for the easiest case we can take their masses to be degenerate and equal to 1 TeV.

4.4.1 Gauge Coupling Unification

The first advantage of adding new vector like particles is that we can now achieve gauge coupling unification in the SM. For the evolution of gauge couplings we will again use the two loop RGE

$$
\frac{dg_i}{dln\mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \left(\sum_{j=1}^{3} B_{ij} g_j^2 - C_i^t y_t^2 \right) ,
$$

where b_i, B_{ij} and C_i^t are given in eqns.4.2 and 4.3. Now when the renormalization scale reaches the mass scale of new fermions ($\mu \geq M_f$), the beta function of gauge couplings receive extra contribution from them, which to the one loop is given by

$$
\triangle b_i = \left(\frac{2}{5}, 2, 2 \right) ,
$$

and the two loop contribution is given by

$$\triangle B_{ij} = \begin{pmatrix} \frac{3}{50} & \frac{3}{10} & \frac{8}{5} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{1}{5} & 3 & \frac{114}{3} \end{pmatrix} .$$

The new contribution alter the sloop of the gauge couplings in such a way that all the three couplings unifies at a single point, as can also be seen in the graph below.

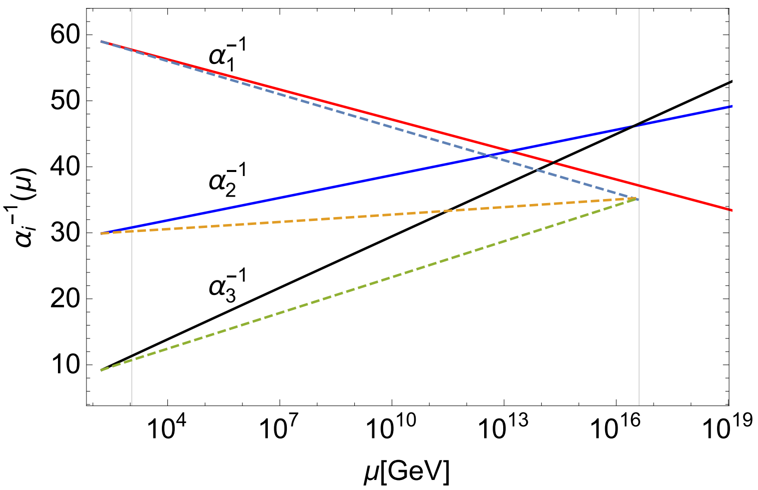


Figure 4.3: Evolution of Gauge coupling in SM(solid lines) and ESM(doted lines).Two vertical lines represent the Scale at which new fermions contribute to the RGE's (1TeV) and where all the three coupling unify($\sim 3 \times 10^{17}$GeV),respectively.

We can see that all the couplings are unified at a single point at the scale around $3 \times 10^{16} GeV$. The unification may not be precise but this is not a big issue for because it depends on where we alter the sloop of couplings or consequently it depends on mass of new fermions (M_f), which is a free parameter in our case. We have taken their mass for simplicity equals 1 TeV, but there is no restriction that it may be around 900 GeV or 1.1 TeV and also it is not necessary that each fermions we added have same (degenerate) mass. For instance if they have different mass then they will change the sloop of the couplings at different point and the story will be different all-together.

4.4.2 Vacuum Stability

As discussed earlier that the SM vacuum stability depends on the positivity of Higgs quartic coupling λ, but in case of only SM particles we have seen that λ goes to negative values from $\sim 10^{10}$ GeV see fig.4.2, which implies that SM vacuum is not stable up to the Planck scale. Now we can achieve the stability of vacuum by adding the particles discusses in the earlier section. How can this be done.? lets look at the RGE of λ

$$\frac{d\lambda}{dln\mu} = \frac{1}{16\pi^2}(12\lambda^2 - 12y_t^4 + ...) . \tag{4.24}$$

Where the complete form of RGEs are given in section 4.1.2. Now from the above eq it can be seen that the strongest contribution is coming from the top Yukawa coupling (y_t) so we need to look for its RGE as well

$$\frac{dy_t}{dln\mu} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2 - 8g_3^2 + ...\right) . \tag{4.25}$$

Where again the complete form of RGEs are given in section 4.1.2. Now if we look at th above equation it can be seen that by adding new fermions, top Yukawa coupling will decrease because of the negative contribution from gauge couplings and consequently the Higgs quartic coupling will increase, making the potential stable up-till Planck scale. This behavior can also be seen from the graph below.

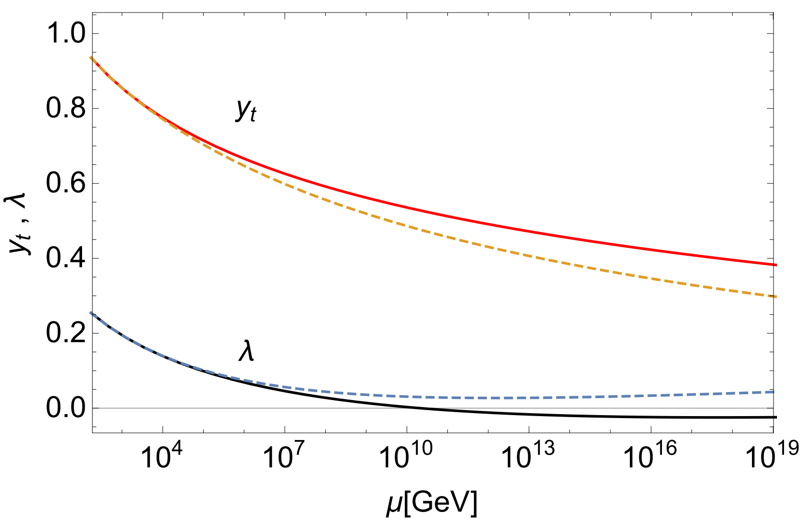


Figure 4.4: Evolution of top Yukawa and Higgs quartic coupling in SM(solid lines) and ESM(doted lines).

Where the solid lines represents the evolution of top Yukawa coupling and Higgs quartic coupling in SM and the dotted lines are the evolution in ESM. Note that λ is now positive up to the Planck scale which implies that our vacuum is now stable up-to the Planck scale.

4.5 SeeSaw Mechanism

Neutrino physics is currently the most interesting topic in physics as neutrino oscillation is the only experimental evidence to believe in the physics beyond the Standard Model [39]. Since a description to describe the tiny masses of neutrino is impossible in the frame work of SM, a possible explanation is so-called seesaw mechanism.

To understand the principle of seesaw mechanism let us assume right handed (RH) neutrino ν_R, beside the usual left handed (LH) neutrino (which are already present in the SM). This addition of only RH neutrino is known as type-I seesaw [40]. There are two other types present in the literature, type-II (which includes a Higgs triplet) and type-III (which includes a triplet fermion) [41]. Now focusing on type-I, we can now construct a Dirac mass term for neutrino

$$\begin{aligned} \mathcal{L}^D_{mass} &= m_D \overline{\nu_R} \nu_L + h.c \,, \\ &= \frac{1}{2}(m_D \overline{\nu_R} \nu_L + m_D \overline{\nu^c_L} \nu^c_R) + h.c \,. \end{aligned} \tag{4.26}$$

Since neutrinos are electrically neutral, we can in general write the Majorana mass term for them as well

$$\mathcal{L}^L_{mass} = \frac{1}{2} m_L \overline{\nu^c_L} \nu_L + h.c \,, \tag{4.27}$$

which is for the LH neutrinos and similarly for the RH neutrinos we can write

$$\mathcal{L}^R_{mass} = \frac{1}{2} m_R \overline{\nu^c_R} \nu_R + h.c \quad = \quad \frac{1}{2} m_R \overline{\nu_R} \nu^c_R + h.c \,. \tag{4.28}$$

Now if we define

$$n_L = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} \quad \text{and} \quad \overline{n^c_L} = \begin{pmatrix} \overline{\nu^c_L} & \overline{\nu_R} \, . \end{pmatrix} \tag{4.29}$$

Then we can write the mass matrix M so that

$$\mathcal{L}_{mass} = \mathcal{L}^D_{mass} + \mathcal{L}^L_{mass} + \mathcal{L}^R_{mass} \; = \; \frac{1}{2} \overline{n^c_L} M n_L \,, \tag{4.30}$$

where

$$M = \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} . \tag{4.31}$$

The positive mass eigenstate for this mass matrix would then be given by

$$m_{1,2} = \left| \frac{1}{2} \left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2} \right) \right| . \tag{4.32}$$

In the seesaw limit the RH neutrino fields $\nu_R = N_R$ are assumed to be much heavier, whereas m_D is of the electroweak scale. Therefore $m_D \ll m_R$. Since ν_L possesses non-zero hypercharge and isospin, the LH Majorana term is forbidden by SM gauge symmetries, hence $m_L = 0$. So in the fundamental theory which respect the SM symmetries we obtain the following mass eigenstates.

$$\begin{aligned} m_1 &\approxeq \frac{m_D^2}{m_R} , \\ m_2 &\approxeq m_R . \end{aligned} \tag{4.33}$$

As a consequence, we have a very heavy neutrino at a mass scale $\Lambda_N = m_R$ of new physics, and a very light neutrino, whose mass is suppressed by m_D/Λ_N. To explain the low experimental upper limit for the neutrino mass, the new scale has to be close to the GUT scale.

Chapter 5

Exploring More Possibilities For GCU

In this chapter we will explore some more possibilities regarding the gauge unification and vacuum stability with vector like particles given in Table.4.1 in the last chapter. We have divided the possibilities of GCU in to two sections. In the first section we will consider only new vector like fermions and in the second section we will also include potentially successful Dark Matter candidate with these new particles and look for their possible combination where gauge unification can be realized.

5.1 Adding Only New Fermions

To achieve gauge coupling unification at two loop level, we first introduce the vector like particles, considering them as fermions. All possible standard vector-like fermions with their contribution to one loop beta functions are given in the table below

SM Vector like Fermions	Contribution to (b_1, b_2, b_3) $(\triangle b_1, \triangle b_2, \triangle b_3)$
$Q+\overline{Q}$	$(\frac{2}{15}, 2, \frac{4}{3})$
$U+\overline{U}$	$(\frac{16}{15}, 0, \frac{2}{3})$
$D+\overline{D}$	$(\frac{4}{15}, 0, \frac{2}{3})$
$L+\overline{L}$	$(\frac{2}{5}, \frac{2}{3}, 0)$
$E+\overline{E}$	$(\frac{4}{5}, 0, 0)$
G	$(0, 0, 2)$
W	$(0, \frac{4}{3}, 0)$

Table 5.1: Vector-like Fermions and their contributions to one-loop beta functions

and the 2-loop contributions are given in the appendix A.

Now if we consider only one family of these new vector-like fermions with the SM particles, we have the following combinations which gives gauge coupling unification.

1. $(Q+\overline{Q})+(D+\overline{D})$,
2. $(L+\overline{L})+G+W$,
3. $(Q+\overline{Q})+(U+\overline{U})+(L+\overline{L})+(E+\overline{E})+G+W$.

Where the first possibility was discussed in last chapter. The masses for all the new fermions are taken to be equal to 1 TeV. The unification scale for each of the above possibility and value of three gauge couplings at the unification scale is given in the table below

Combinations for GCU	Unification Scale	α_i^{-1}
$(Q+\overline{Q})+(D+\overline{D})$	$\sim 3\times 10^{16}$	~ 35.5
$(L+\overline{L})+G+W$	$\sim 5\times 10^{16}$	~ 35.9
$(Q+\overline{Q})+(U+\overline{U})+(L+\overline{L})+(E+\overline{E})+G+W$	$\sim 1\times 10^{17}$	~ 25.1

Table 5.2: Examples of combinations of vector-like fermions which realize GCU, where in the right most column the values of couplings are shown at the unification Scale.

This is the case where we have only allowed one family of these extra fermions, but we have the freedom to allow more then one family of new fermions for the realization of GCU, as there is no restriction in the SM on these vector-like particles, we need to just take care of the perturbative bounds derived in eqn.4.19. And if we allow more that one family we have enormous numbers of combinations which realizes GCU. But in order to restrict the possibilities we have only allowed one or three family of new fermions to have an analogy with the Standard Model (which has either one or 3 families of particles). Now if we apply this restriction, following are the combinations which realizes GCU.

Combinations for GCU	Uni. Scale	α_i^{-1}
$3(Q+\overline{Q})+3(U+\overline{U})+(D+\overline{D})+(L+\overline{L})+(E+\overline{E})$	$\sim 4\times10^{17}$	~ 8.3
$(Q+\overline{Q})+3(U+\overline{U})+(D+\overline{D})+(E+\overline{E})+G+3W$	$\sim 4\times10^{17}$	~ 12.9
$(Q+\overline{Q})+(U+\overline{U})+3(D+\overline{D})+3(E+\overline{E})+G+3W$	$\sim 4\times10^{17}$	~ 12.9
$3(Q+\overline{Q})+(U+\overline{U})+3(D+\overline{D})+(L+\overline{L})+3(E+\overline{E})$	$\sim 4\times10^{17}$	~ 8.3
$(Q+\overline{Q})+(D+\overline{D})+3(L+\overline{L})+(E+\overline{E})+G$	$\sim 1\times10^{17}$	~ 25.1
$3(Q+\overline{Q})+3(U+\overline{U})+(E+\overline{E})$	$\sim 2\times10^{17}$	~ 13.1
$(Q+\overline{Q})+(U+\overline{U})+3(L+\overline{L})+G$	$\sim 1\times10^{17}$	~ 25.1
$(Q+\overline{Q})+(U+\overline{U})+3(D+\overline{D})+(L+\overline{L})+W$	$\sim 7\times10^{16}$	~ 25.1
$(Q+\overline{Q})+3(D+\overline{D})+(E+\overline{E})+W$	$\sim 5\times10^{16}$	~ 28.7

Table 5.3: Examples of combinations of vector-like fermions if we allow only one or three generations of new fermions which realize GCU, where in the right most column the values of three gauge couplings are shown at the unification Scale.

It can be seen from the tables above that values of couplings are positive(within the perturbative bounds) up to the Planck scale, and also the unification scale is around 10^{16} GeV and 10^{17} GeV, which is what we have required from the start.

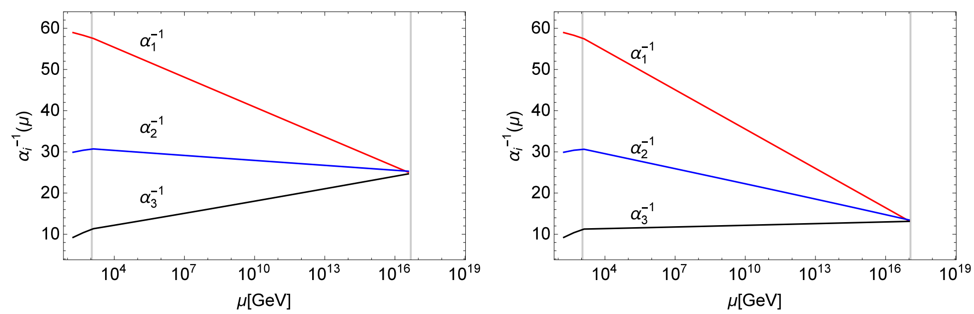


Figure 5.1: GCU with SM + Extra Fermions (one family), corresponds to third one of Table.5.2 (left-panel); SM + Extra Fermions (one or three families), corresponds to sixth one of Table.5.3 (right-panel).

5.2 Minimal Dark Matter

The Dark matter(DM) has been the main focus of physicist for many decades and it calls for the physics beyond the Standard model, because there is no explanation in the Standard model of the nature of DM or what it is made up of [35]. There are many attempts to address this problem which typically includes a rich amount of physics at the electroweak scale including DM candidates. Amongst them supersymmetry is the most promising theory which naturally em-band a DM candidate in it [35]. However no new physics appeared at the collider experiments so far, and the quest for identification of missing mass of the universe is still on.

We will here focus on the Minimal Dark Matter(MDM) approach (it is called MDM because it is described by the minimal gauge-covariant Lagrangian or adding the minimal amount of new physics in the Standard model) focusing on the Dark Matter problem. We will add just one extra ElectroWeak(EW) multiplet χ and assign minimal quantum numbers (spin, isospin and hypercharge) to it that make it a good DM candidate, without ruining the positive features of SM. No ad hoc extra symmetry will be introduced for the stability of DM candidate and this stability will be ensured by the SM gauge symmetry and renormalizibality. Moreover the theory is remarkably predictive because of its minimality [6].

For construction of MDM model we add on top of the Standard Model a single multiplet χ(fermionic or scalar) charged under SM $SU(2)_L \times U(1)_Y$ electroweak interaction, that is a Weakly Interacting Massive Particle (WIMP) not charged under $SU(3)_C$ strong interactions. $\bar{\chi}$ which is the conjugate of χ belongs to the same representation and the theory is vector-like with respect to $SU(2)$ and anomaly free. The minimal Lagrangian for χ would then be written as

$$\mathcal{L}_{DM} = \frac{1}{2}\overline{\chi}(i\not{D} + M)\chi \,, \tag{5.1}$$

for fermionic χ. and for scalar χ the Lagrangian is given by

$$\mathcal{L}_{DM} = \frac{1}{2}|D_\mu\chi|^2 - M^2|\chi|^2 \,. \tag{5.2}$$

Where D_μ is the gauge-covariant derivative and contains the electroweak couplings to the vector bosons ($Z, W^\pm$ and γ) of SM.In the above equations M is the tree level mass of χ and is the only free parameter of the theory. Other terms such as Yukawa couplings with SM fields would in principle come in the Lagrangian, but for the successful DM candidate these terms would be forbidden by Lorentz or gauge invariance as detailed below.

Quantum numbers			DM can	Stable?
$SU(2)_L$	$U(1)_Y$	Spin	decay into	
2	1/2	S	EL	$\times$
2	1/2	F	EH	$\times$
3	0	S	HH^*	$\times$
3	0	F	LH	$\times$
3	1	S	HH, LL	$\times$
3	1	F	LH	$\times$
4	1/2	S	HHH^*	$\times$
4	1/2	F	(LHH^*)	$\times$
4	3/2	S	HHH	$\times$
4	3/2	F	(LHH)	$\times$
5	0	S	(HHH^*H^*)	$\times$
5	0	F	$-$	$\surd$
5	1	S	$(HH^*H^*H^*)$	$\times$
5	1	F	$-$	$\surd$
5	2	S	$(H^*H^*H^*H^*)$	$\times$
5	2	F	$-$	$\surd$

Table 5.4: The possible Minimal DM candidates. Quantum numbers of candidates are listed in the first 3 columns. The 4th column indicates some decay modes of these candidates into SM particles; modes listed in parenthesis correspond to dimension 5 operators.

The assignment of quantum numbers under the gauge group ($SU(2)_L \times U(1)_Y$) will fully determine χ (the number of its $SU(2)_L$ components $n = \{2, 3, 4, 5..\}$ and the hypercharge Y). For a given n (first column of Table. 5.4) there are few possibilities of hypercharge assignment, such that one of the component of χ multiplet has electric charge $Q = I_3 + Y = 0$ (where I_3 is the usual diagonal generator of $SU(2)_L$), which is the requirement for DM candidate. For example for the doublet ($n = 2$) has $I_3 = \pm\frac{1}{2}$ so Y needs to be $= \mp\frac{1}{2}$ so that $Q = 0$. For $n = 3$ one can have $Y = \{0, \pm 1\}$, and so on.

Note from the Table.5.4 that the list of possible candidates stops at $n \leq 5$ for fermions (for scalars the list can be extended up to $n = 7$ but we are not interested in these possibilities here) because larger multiplets would spoil the perturbativity of $SU(2)_L$ coupling g_2 and leads to the Landau pole. In the list of candidates, those having $Y \neq 0$ are excluded by the direct detection searches as discussed in [6], so we will focus only on those candidates having $y = 0$ and amongst those we need to inspect which are stable against decay in to SM particles. Some possible decay operators are shown in the $4th$ column of Table.5.4 for each case. For instance the triplet fermion with $y = 0$ would couple with a SM leptonic doublet L and Higgs field H through a Yukawa operator χLH and decay in a very short time and so the fermionic triplet can not be a viable DM candidate, unless some ad hoc symmetry eliminate this operator which we will discuss in the latter section. For another instance the 5-plet scalar with hypercharge $y = 0$ would couple through dimension 5 operator χHHH^*H^* with four Higgs fields, suppressed by one power of the Planck scale (which is the cut-off scale of the theory), and if we calculate the life time of the operator $\tau \sim M_{PL}^2 TeV^{-3}$ it turns out to be shorter than the age of the universe, so this also is not a viable candidate for DM. Now given the known SM particle contents the larger multiplet (having large value of n) can not couple to SM fields so they are automatically stable and are therefore good candidates of DM. This is the same reason why known massive particles like proton are stable: decay modes do not exist at the renormalizable level. So we can say that the stability of these DM candidates are explained by an accidental symmetry like proton stability. Amongst all the candidates only two possibilities then emerges that survived all the previous constraints and the possibilities are n=5 fermion and n=7 scalar. But for our analysis we will not consider n=7 scalar as it may have non-minimal quartic couplings with the Higgs field [6]. So we will first of all focus on the fermionic 5-plet as the DM candidate, later we will include other possibilities as well.

5.3 Fermionic Dark Matter and GCU

We will consider a 5-plet fermionic Dark Matter (discussed in the last section) with the other Standard Model vector-like particles and look for their possible combinations which realizes gauge coupling unification. The mass of the fermionic DM multiplet will be taken to be 9.6 TeV as mentioned in [6], while the mass of other vector-like fermions will again be takes as 1 TeV. The one and two loop change in beta coefficient of SM because of this 5-plet fermionic DM is given by

$$\triangle b^{DM} = \left(0, \frac{20}{3}, 0\right) \quad ; \quad \triangle B_{ij}^{DM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{560}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

Now it is noticed that the gauge coupling unification can not be realized up-to the reduced Planck scale with only new fermions and 5-plet fermionic DM and can only be realized above the reduced Planck scale as mentioned in the table below.

Combinations	Unification Scale	α_i^{-1}
$2(U+\overline{U})+8(D+\overline{D})+DM$	$\sim 4\times 10^{18}$	~ 7.8
$2(U+\overline{U})+5(D+\overline{D})+(E+\overline{E})+G+DM$	$\sim 5\times 10^{18}$	~ 7.6
$3(U+\overline{U})+4(D+\overline{D})+G+DM$	$\sim 5\times 10^{18}$	~ 7.6
$2(U+\overline{U})+2(D+\overline{D})+2(E+\overline{E})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8
$3(U+\overline{U})+(D+\overline{D})+(E+\overline{E})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8
$4(U+\overline{U})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8
$(U+\overline{U})+3(D+\overline{D})+3(E+\overline{E})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8
$4(D+\overline{D})+4(E+\overline{E})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8
$3(U+\overline{U})+(D+\overline{D})+(E+\overline{E})+2G+DM$	$\sim 1\times 10^{19}$	~ 6.8

Table 5.5: Combinations of vector-like fermions with 5-plet Dark matter where GCU can be realized.

As can be seen from the table the unification scale is above the reduced Planck scale (2.4×10^{18} GeV) for each case, which is not an interesting scenario. However fortunately the GCU can be realize within the Planck scale if we allow one of the SM like new particles (for instance E(1,1,1)) to be a scalar and also varying the mass of vector-like fermions (We can do that as mass is a free parameter in our analysis). Below are some of the examples where GCU can be realized within the Planck scale with the fermionic DM and extra particles.

Combinations for GCU{Masses of Particles}	Uni. Scale	α_i^{-1}
$2(U\overline{U})+8(D\overline{D})+DM+2E(\mathrm{S})$ {2TeV}	$\sim 5\times 10^{17}$	~ 9.8
$3(U\overline{U})+4(D\overline{D})+G+DM+2E(\mathrm{S})${2TeV}	$\sim 9\times 10^{17}$	~ 9.2
$2(U\overline{U})+5(D\overline{D})+(E\overline{E})+G+DM+2E(\mathrm{S})${2TeV}	$\sim 9\times 10^{17}$	~ 9.2
$4(U\overline{U})+2G+DM+2E(\mathrm{S})${3TeV}	$\sim 7\times 10^{17}$	~ 9.6
$4(U\overline{U})+4(E\overline{E})+2G+DM+2E(\mathrm{S})${2TeV}	$\sim 2\times 10^{18}$	~ 8.7

Table 5.6: Combination of vector-like fermions and a scalar(E) with 5-plet Dark matter where GCU can be realized within the Planck scale. The mass of fermionic DM is 9.6 TeV.

Note from the 2nd last column of the table.5.6 that now the gauge coupling unification scale is within the Plank scale. the scalar(E), we have added do not need to be vector like because scalar do not contribute to the anomalies. Also the fermionic 5-plet DM has the same mass as before (9.6 TeV).

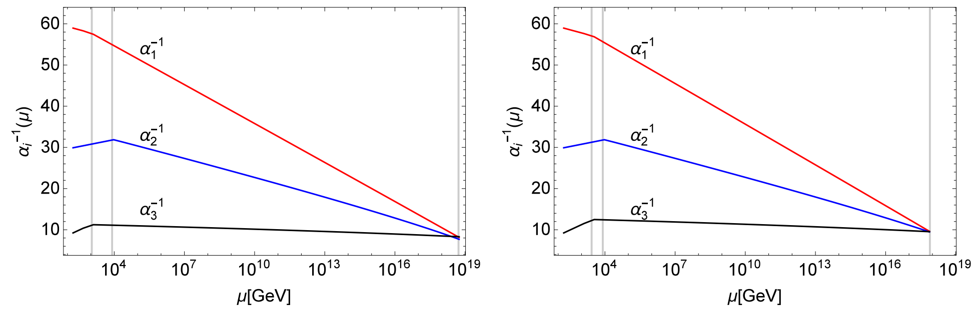


Figure 5.2: GCU with SM + Extra Fermions + fermionic DM, corresponds to first one of Table.5.5 (left-panel); SM + Extra Fermions + Scalar + fermionic DM , corresponds to forth one of Table.5.6 (right-panel).

5.4 Scalar Dark Matter and GCU

Now in the last step we will consider a Scalar $SU(2)_L$ 5-plet Dark Matter and standard vector like particles along with the SM particles and look for there possible combinations for gauge coupling unification. As discussed in sec.5.2 the scalar 5-plet DM is not stable and has non vanishing dimension five coupling with four Higgs with the life time of less then the age of universe, but we can some-how suppress this operator by introducing ad-hoc symmetry for example Z-2 symmetry ($\phi(x) \to -\phi(x)$), and imposing that the Lagrangian is even under this symmetry, in this way we can suppress the couplings of Scalar DM and make it stable.

Now for gauge unification we will consider the mass of scalar DM equals to 9.6 TeV [6] and 1 TeV for standard vector like fermions. The one and two loop contribution to the beta coefficients by the scalar DM is given below

$$\triangle b^{DM} = \left(0, \frac{20}{3}, 0\right) \quad ; \quad \triangle B_{ij}^{DM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{560}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

Again many combination can be found where good gauge coupling unification can be achieved and some of them are shown in the table below.

Combinations for GCU{Masses of Particles}	Uni. Scale	α_i^{-1}
$(Q\overline{Q}) + (U\overline{U}) + 6(D\overline{D}) + (E\overline{E}) + SDM$	$\sim 2 \times 10^{18}$	~ 13.6
$9(D\overline{D}) + 3(L\overline{L}) + SDM$ $\{1.5\ TeV\}$	$\sim 1 \times 10^{18}$	~ 14.1
$(U\overline{U}) + 8(D\overline{D}) + (L\overline{L}) + W + SDM$ $\{1.5\ TeV\}$	$\sim 1 \times 10^{18}$	~ 14.1
$(U\overline{U}) + 9(D\overline{D}) + 2(L\overline{L}) + W + SDM$ $\{1.5\ TeV\}$	$\sim 1 \times 10^{18}$	~ 9.2
$(Q\overline{Q}) + 2(U\overline{U}) + 5(D\overline{D}) + SDM$	$\sim 2 \times 10^{18}$	~ 13.5

Table 5.7: Combination of vector-like fermions and a scalar(E) with 5-plet Dark matter where GCU can be realized within the Planck scale.

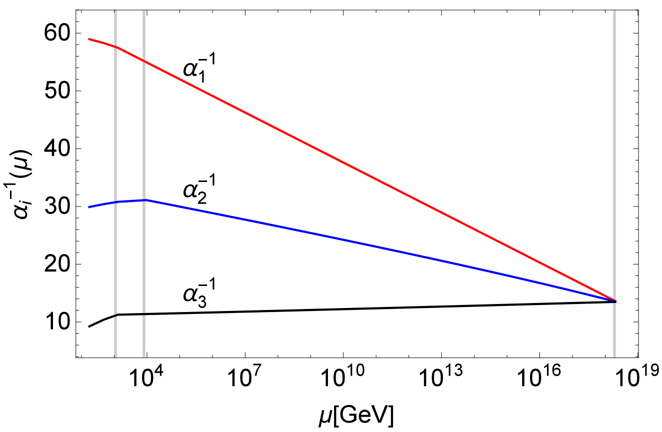


Figure 5.3: GCU with SM + Extra Fermions + Scalar DM, corresponds to last one of Table.5.7.

5.5 Type-I Seesaw and Vacuum Stability

The vacuum stability for all the other cases discussed above is ensured by the fact that by adding the new particles we always enlarge the gauge couplings which in turns make the vacuum stable up to the Planck scale. This behavior is also discussed in detail in the section 4.4.2.

We will here consider the impact of type-I seesaw on the Vacuum stability. The relevant terms in the Lagrangian for neutrino oscillation through type-I seesaw are given by

$$\mathcal{L}_\nu = -y_D^{ij} l_i \nu_j^c \phi^c - \frac{1}{2} M_R^{ij} (\nu^c)^T i \nu_j + h.c \,, \tag{5.3}$$

with $i, j = 1, 2, 3$. Here l_i is the lepton doublet, ν_i^c is the right handed neutrino, y_D^{ij} is the Yukawa coupling of neutrino and M_R^{ij} denotes the mass matrix for the right handed neutrino [38].

Above the scale of the right handed neutrino (M_R) we have the following renormalization group equation for $Y_\nu \equiv y_D^{ij}$,

$$\frac{dY_\nu}{dln\mu} = \frac{Y_\nu}{16\pi^2}\left(3y_t^2 + tr\left[Y_\nu^\dagger Y_\nu\right] + \frac{3}{2}Y_\nu^\dagger Y_\nu - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2\right)\right) . \tag{5.4}$$

As the right handed neutrinos do not carry any gauge charge, so they will not affect the running of gauge couplings. The running of top Yukawa coupling and the Higgs quartic coupling will be modified as follows:

$$\begin{aligned} &\beta_t^{(1)} \rightarrow \beta_t^{(1)} + \mathrm{tr}\left[Y_\nu^\dagger Y_\nu\right] , \\ &\beta_\lambda^{(1)} \rightarrow \beta_\lambda^{(1)} + 4\lambda\ \mathrm{tr}\left[Y_\nu^\dagger Y_\nu\right] - 4\ \mathrm{tr}\left[\left(Y_\nu^\dagger Y_\nu\right)^2\right] . \end{aligned} \tag{5.5}$$

The mixing matrix U_{MNS} is used to diagonalized the light neutrino mass matrix such that

$$M_\nu = \frac{v^2}{2M}Y^T Y \ = \ U_{MNS} D_\nu U_{MNCS}^T , \tag{5.6}$$

where $D_\nu = \mathrm{diag}(m_1, m_2, m_3)$, and for simplicity, we have assumed Y_ν to be real. The mixing matrix has the following tri-bimaximal form [42]

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} . \tag{5.7}$$

The diagonal neutrino mass matrix for the hierarchical case is give by

$$D_\nu \simeq \mathrm{diag}\left(0, \sqrt{\Delta m_{12}^2}, \sqrt{\Delta m_{23}^2}\right) . \tag{5.8}$$

The input values of Δm_{12}^2 and Δm_{23}^2 are fixed from neutrino oscillation data [43] as

$$\begin{aligned} \Delta m_{12}^2 &= 8.2 \times 10^{-5}\mathrm{eV}^2 , \\ \Delta m_{23}^2 &= 2.4 \times 10^{-3}\mathrm{eV}^2 . \end{aligned} \tag{5.9}$$

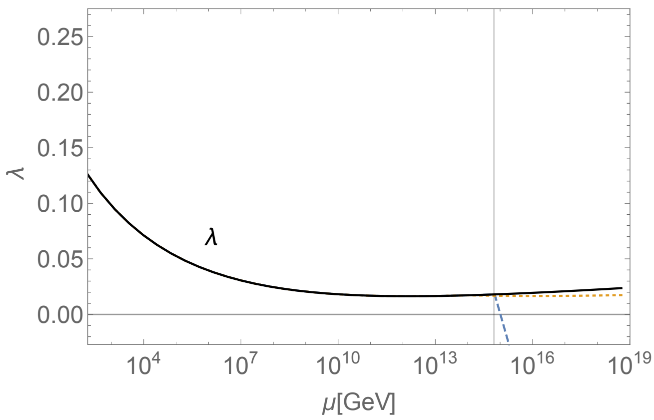


Figure 5.4: Vacuum stability in the extended SM (considered in section.4.4), including type-I seesaw physics. We have considered three different type-I seesaw scales $M_R = 10^{13}$GeV (solid), 10^{14}GeV (dotted), 10^{15}GeV (dashed).

Our findings are presented in fig.5.4 where we have plotted the Higgs quartic coupling for the case discussed in section.4.4, including type-I seesaw physics. Three distinct mass scales for heavy right handed neutrinos are considered here namely, $M_R = 10^{13}$ GeV (solid), 10^{14} GeV (dotted) and 10^{15} GeV (dashed). There is no change in the Higgs quartic coupling (λ) until right handed neutrino mass scale and the behavior is the same as in fig.4.4.2. After the type-I seesaw scale, there is a significant change in λ for the case $M_R = 10^{15}$ only. From eqn.5.5, it can be observed that the Dirac neutrino Yukawa coupling Y_ν gives an additional contribution to the Higgs quartic coupling RGE with the same sign as the top quark contribution. It is natural to expect that λ will decrease with the increase in the Y_ν coupling. For $M_R = 10^{13}$ GeV there is almost no change in the running of λ. For $M_R = 10^{14}$ GeV there is only slight change in λ, since Y_ν is still not large at that scale in comparison to the top Yukawa coupling. For $M_R = 10^{15}$ GeV we see a significant change in λ. In fact λ goes to the negative values for this particular case since now the coupling Y_ν is larger than the top Yukawa coupling, and the two of then together force the Higgs quartic coupling to the negative values.

Chapter 6

Summary and Conclusion

In this dissertation we have explored the possibilities of gauge coupling unification (GCU) at the scale from 5×10^{15} GeV to the reduced Planck scale (2.4×10^{18} GeV) in the extended standard model. These extended models includes extra fermions around the TeV scale and minimal dark matter having mass 9.6 TeV. To avoid gauge anomalies, extra fermions are considered as vector-like. In addition, we restrict ourselves to the standard model like vector fermions (that is, these fermions carry standard model quantum numbers). We have considered various cases for GCU. In the first case we have only considered one family of extra vector-like fermions with SM particles and found that there are only three cases where GCU can be realized. Afterwards we have extended the families of extra fermions and observed that there are enormous number of combinations for GCU. So to restrict the number of possibilities we have allowed only one or three families of extra fermions (to have an analogy with the SM) with the SM particles and the possible combinations where GCU can be realized are listed in Table.5.3.

In the next case we have also considered potentially successful minimal dark matter candidates proposed by Alessandro Strumia in his paper [6], with the extra fermions to achieve GCU. Among the many candidates we have chosen the 5-plet fermion and the 5-plet Scalar for our purpose. The 5-plet fermion is automatically stable against decay in to SM particles and has no free parameter. We have found that if we only consider extra fermions and 5-plet fermionic DM with the SM particle the GCU can only be realized above the reduced Planck scale, which is not a best scenario. Fortunately GCU can be realized within the reduced Planck if we also add a scalar (E(1,1,1)) with these fermions and some of the possibilities are listed in Table.5.6. The scalar 5-plet DM can decay through dimension-5 operator with a life time less the age of universe, but this operator can be suppressed using Z_2 symmetry. Some of the possible combinations where GCU can be realized in this case are listed in Table.5.7

The stability of SM vacuum has also been achieved in these models by ensuring the positivity of Higgs quartic coupling up to the Planck scale. We have assumed the couplings of extra particles to the SM particles are very small and can be neglected. Therefore the only way through which the extra particles affect the Higgs quartic coupling is through the top Yukawa coupling which itself depends strongly on the gauge coupling as can be seen from eqn.4.24 and eqn.4.24, the smaller y_t and the larger g_i makes the Higgs quartic coupling positive up to the Planck scale.

Type-I seesaw (which includes a right-handed neutrino per generation in the SM) has been introduced in the analysis to describe the tiny neutrino masses, and there impact on the predictions of vacuum stability has also been discussed. The right-handed neutrino do not carry any SM gauge charge and therefore do not effect the running the gauge couplings, but they do alter the running of Higgs quartic coupling through Yukawa interaction and therefore effect the vacuum stability bond. The behavior has been discussed in section.5.5.

The vector-like particles, because of their moderate masses, should be accessible at the LHC, and these models will be tested in the ongoing run of LHC.

Appendix A

Two-loop Beta function

In the Appendix, we present the two-loop contribution to the beta function of SM gauge couplings, coming from extra vector-like particles which were introduced in Chapter.4

$$\Delta B^{Q+\overline{Q}} = \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{1}{15} & 3 & \frac{76}{3} \end{pmatrix} \ , \ \Delta B^{U+\overline{U}} = \begin{pmatrix} \frac{64}{75} & 0 & \frac{64}{15} \\ 0 & 0 & 0 \\ \frac{8}{15} & 0 & \frac{38}{3} \end{pmatrix} \tag{A.1}$$

$$\Delta B^{D+\overline{D}} = \begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{38}{3} \end{pmatrix} \ , \ \Delta B^{L+\overline{L}} = \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{49}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

$$\Delta B^{E+\overline{E}} = \begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \ \Delta B^{G} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 48 \end{pmatrix} \tag{A.3}$$

$$\Delta B^{W} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{64}{3} & 0 \\ 0 & 0 & \end{pmatrix} \tag{A.4}$$

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