Impact of Porous Medium in the Flow of Nanofluid over a Variable Thicked Surface

QUAID-I-AZAM UNIVERSITY

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IN

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CERTIFICATE

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

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Chapter 1

Basic definitions and equations

1.1 Introduction

This chapter has been arranged through some definitions and equations which are useful for the next two chapters.

1.2 Some concepts about fluid

1.2.1 Fluid

A material which continuously deforms under the action of applied shear stress is called fluid.

1.2.2 Fluid mechanics

The branch of engineering dealing with the fluids either at rest or in motion. Some common engineering fluids are air (a gas), steam (a vapor) and water (a liquid).

Fluid mechanics can be divided into two categories:

1. The study of fluids at rest is related to hydrostatics.

2. The study of fluids in motion is related to hydrodynamics.

1.2.3 Types of flow system

Internal flow system

Fluid flows surrounded by closed boundaries or surfaces are termed as internal flow systems. i.e flow through pipes, valves, ducts, or open channels etc.

External flow system

Flows having no specific boundaries are termed as external flows. Flow around aeroplane wings, automobiles, buildings or ocean water through which ships, submarines and torpedoes move/sail.

Internal and external flow systems may be laminar, turbulent, compressible or incompressible.

1.2.4 Body force

A force acting throughout the volume of a body without any physical contact. For example gravity, electromagnetic forces etc.

1.2.5 Inertial forces

A force which resists a change in state of an object.

1.2.6 Surface force

A force that acts across an internal or external element in a body through direct physical contact. Pressure and shear forces are examples of surface force.

1.2.7 Shear stress

It is the stress component coplanar with the cross sectional area of a material. The internal forces that adjoining particles of a continuous material exert on each other is called shear stress. It is denoted by a Greek letter τ.

1.2.8 Normal stress

The component of the stress normal to cross section of a material is called normal stress. It is denoted by σ.

1.2.9 Viscosity

Viscosity of fluid is the measure of its internal resistance to deformation caused by tensile or shear stress. A fluid having no resistance to shear stress is called ideal or inviscid. Super fluids have zero viscosity. Viscosity depends on composition and temperature of a fluid. It is also called dynamic/absolute viscosity. It is denoted by μ and is given by

$$\mu = \frac{shear\ stress}{shear\ strain}. \tag{1.1}$$

Viscosity is measured in $kg/m.sec$ and is having dimensions of

$$\left[\frac{M}{LT}\right]. \tag{1.2}$$

1.2.10 Kinematic viscosity

The proportion of absolute viscosity to density of fluid is recognized as kinematic viscosity. It is given by

$$\nu = \frac{\mu}{\rho}. \tag{1.3}$$

It is measured in $m^2/$sec. It is having dimensions of

$$\left[\frac{L^2}{T}\right]. \tag{1.4}$$

1.2.11 Flow types

1) Laminar flow

The flow pattern in which fluid flows in parallel smooth layers with no crossing over between the layers. Smoke rising from a cigarette to the first few centimeters in surrounding shows a clear picture of laminar flow.

2) Turbulent flow

Turbulent flow is a less orderly flow regime, characterizes the pattern of the flow in which the fluid particles have no specific paths or trajectories. Break up of rising smoke into random and haphazard motion represents turbulent flow.

1.2.12 Newton's law of viscosity

It states that

$$\tau_{yx} \propto \frac{du}{dy}, \tag{1.5}$$

$$\tau_{yx} = \mu\frac{du}{dy}, \tag{1.6}$$

in which μ is the absolute viscosity.

1.2.13 Newtonian fluids

Those fluids which obey Newton's law of viscosity are termed as Newtonian fluids. Water, air and gases are Newtonian fluids.

1.2.14 Non-Newtonian fluids

Those fluids for which does not hold Newton's law of viscosity are termed as non-Newtonian or we can say that those fluid in which shear stress is non-linearly proportional to shear strain. Since viscosity depends upon nature of the fluids so for such fluids viscosity remains no more constant and it becomes a function of applied shear stress. Examples of such fluids consist of honey, toothpaste, ketchup, paint etc. Mathematically

$$\tau_{yx} \propto \left(\frac{du}{dy}\right)^{n}, n \neq 1 \tag{1.7}$$

$$\tau_{yx} = \eta\left(\frac{du}{dy}\right), \tag{1.8}$$

$$\eta = \kappa\left(\frac{du}{dy}\right)^{n-1}. \tag{1.9}$$

Here η represents the apparent viscosity, n flow behavior index and κ consistency index. .

Non-Newtonian fluids are divides into three types;

1. Rate type fluids.

2. Integral type fluids.

3. Differential type fluids.

1.3 Mechanisms of heat transfer

Heat transfer occurs from region of higher kinetic energy towards the region of lower kinetic energy or simply from hotter to colder regions. Temperature difference in between the two regions is the main reason for heat transfer. Heat transfer from one place to another by different processes, depending upon the nature of material under consideration.

1.3.1 Conduction

Particles are in constant motion. During motion of the particles, collisions occur, which result in the exchange of kinetic energies. So the process of heat transfer from one place to another due to collision of molecules/particles is called conduction. Such type of heat transfer is observed in solids e.g. heated rod at one end, here no material flow occurs physically.

1.3.2 Convection

It is the process of heat transfer during which particles having more kinetic energies replace the particles with less kinetic energies. The transfer of heat occurs due to random Brownian motion of individual particles from hotter to colder region and this process is known as convection.

1.3.3 Types of convection

Natural convection

The transfer of heat that occurs only due to temperature difference is called natural convection. Dense particles of fluid fall, while lighter particles rise, resulting the bulk motion of fluids. Natural convection can only occurs in the presence of gravitational force.

Forced convection

To increase the rate of heat exchange, fluid motion is generated by external surface forces such as fan, pump etc. Force convection is more efficient than natural convection.

Mixed convection

The process in which heat transfer occurs due to both natural and forced convection is termed as mixed convection.

1.3.4 Radiation

The transmission of heat energy in the form of electromagnetic waves with or without any medium is termed as radiation. Examples are fire, sunlight and very hot objects etc.

1.4 Some concepts about heat

1.4.1 Heat source/sink

Any object which produces and emits heat is termed as a heat source and any object that dissipates heat is termed as a heat sink. Nuclear reactors, sun, fire and electric heaters are heat sources while semiconductors, light emitting diodes, fans and solar cells are examples of heat sink.

1.4.2 Thermal conductivity

Therm means heat and conduction is transfer by collision of molecules. So it is measure of the ability of material to transmit heat from one place to another. Or it can also be characterized as the heat transfer through a unit thickness of a substance in a direction perpendicular to surface area due to unit temperature gradient. It is denoted by k and is given by

$$k = \frac{Ql}{A\Delta T}, \tag{1.10}$$

where Q denotes heat flow in a unit time, A is area of cross section and ΔT denotes change in temperature. In SI system thermal conductivity is measured in $W/m.K$, having dimensions of

$$\left[\frac{ML}{T^3\theta}\right]. \tag{1.11}$$

1.4.3 Thermal diffusivity

It is measure of the capacity of a material to conduct heat energy with respect to its capacity to store heat energy. Thermal diffusivity is measured in $m^2/\sec$, denoted by α and is given by

$$\alpha = \frac{k}{\rho c_p}, \tag{1.12}$$

where k is thermal conductivity, c_p is specific heat and ρ is density.

Thermal diffusivity measures the property of a material for unsteady heat conduction. This illustrate how rapidly a substance responds to change in temperature.

1.4.4 Specific heat

It is an extensive property of a material defined as the amount of heat energy required to enhance temperature of the material by 1^0C. It is measured in $\frac{J}{K}$, having the dimensions of

$$\left[\frac{L^2M}{T^2\theta}\right]. \tag{1.13}$$

1.4.5 Newton's law of heating

The rate of heat loss of a body is directly related to temperature difference between the body and its surrounding.

1.4.6 Fourier's law of heat conduction

It is defined as the rate of heat conduction through a homogeneous material is directly proportional to negative gradient of temperature. Mathematically

$$q \propto -\Delta T, \tag{1.14}$$

$$q = -k\Delta T, \tag{1.15}$$

q is heat flux density, which is the amount of energy flow through a unit area per unit time. q is measured in $\frac{W}{m^2}$, k is material's thermal conductivity and ΔT is temperature gradient.

1.5 Porous medium

A substance which is composed of a solid matrix with interrelated pores is called a porous medium. These pores permit the flow of fluids through the material. The appropriation of pores is unpredictable in nature. The examples of porous medium comprises beach sand, human lungs, rye bread and wood, act.

1.5.1 Porosity

By porosity ϕ of a porous medium we mean the part of the associated void area to the total area of the substance. It means that $1-\phi$ is the fraction that solid area involves the material.

1.6 Dimensionless parameters

1.6.1 Prandtl number

Conduction and convection occur in fluids. For the process of heat transfer, temperature difference is the main cause. Rate of conduction and convection vary in different fluids.

Prandtl number is used to determine the domination of either conduction or convection. It is defined as the ratio of momentum to thermal diffusion rate. It is denoted by Pr and given by

$$\mathrm{Pr} = \frac{\nu}{\alpha}, \tag{1.16}$$

where ν is kinematic viscosity and α is thermal diffusion rate. For many gases over a vast range of temperature and pressure, Pr is approximately treated as constant.

1.6.2 Lewis number

It is defined as the ratio of thermal diffusivity to Brownian diffusivity. Mathematically, it can be written as

$$Le = \frac{\text{thermal diffusivity}}{\text{Brownian diffusivity}} = \frac{\alpha}{D_B} \tag{1.17}$$

Note that in above expression α signifies the thermal diffusivity and D_B represents the Brownian diffusivity.

1.6.3 Hartman number

A dimensionless parameter used to determine relative importance of drag force resulting from magnetic and viscous forces.

It is expressed as the ratio of magnetic to viscous forces given by

$$M = \frac{magnetic\ forces}{viscous\ forces} = B_0 d\sqrt{\frac{\sigma}{\mu}}, \tag{1.18}$$

where B_0 shows characteristic value of magnetic induction, d is characteristic length and μ is dynamic viscosity coefficient.

1.6.4 Brownian motion parameter

The haphazard movement of suspended nanoparticles in a base fluid is called as Brownian motion. Mathematically

$$Nb = \frac{\tau D_B}{\nu}(C_w - C_\infty),$$

where τ the ratio of nanoparticles effective heat capacity and fluid heat capacity, D_B the brownian diffusion coefficient, ν the kinematic viscosity and C_w and C_∞ the surface and ambient concentration respectivelly.

1.6.5 Thermophoresis parameter

Thermophoresis is the phenomenon in which the influence of temperature gradient results in a diffusion of particles is called as thermophoresis. Mathematically

$$Nt = \frac{\tau D_T}{\nu T_\infty}(T_w - T_\infty), \tag{1.19}$$

where D_T is the thermophoretic diffusion coefficient and T_w represents the wall temperature, T_∞ stands for ambient fluid temperature.

1.6.6 Skin friction coefficient

When fluid is passing across a surface then definite amount of drag arises which is termed as skin friction. Mathematically it is stated as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_w^2}, \tag{1.20}$$

where

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=\delta(x+b)^{\frac{1-n}{2}}}, \tag{1.21}$$

in above relation τ_w represents the shear stress at the surface, U_w stands for velocity and ρ denotes the fluid density.

1.6.7 Nusselt number

The nondimensional quantity which denotes the convective to conductive heat transfer ratio is known as Nusselt number (Nu). In mathematical form one can write

$$Nu = \frac{x q_w}{k(T_w - T_\infty)}, \tag{1.22}$$

where

$$q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=\delta(x+b)^{\frac{1-n}{2}}}, \tag{1.23}$$

where q_w denotes the wall heat flux, T_w represents the wall temperature, T_∞ stands for ambient fluid temperature and k represents the thermal conductivity

1.6.8 Sherwood number

It is defined as the ratio of the convective mass transfer to the rate of diffusive mass transport. Mathematically

$$Sh = \frac{(x+b)q_m}{D_B(C_w - C_\infty)}, \tag{1.24}$$

where

$$q_m = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=\delta(x+b)^{\frac{1-n}{2}}}. \tag{1.25}$$

where q_m the mass flux.

1.6.9 Reynolds number

A dimensionless parameter used to predict flow pattern for various fluid flow circumstances. It measures the ratio of inertial to viscous forces and relative importance of the two forces for given flow conditions. It is denoted by Re named after Osborne Reynolds, and is given as

$$\mathrm{Re} = \frac{inertial\ force}{viscous\ force}, \tag{1.26}$$

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}, \tag{1.27}$$

V is maximum velocity, L is length of geometry.

For low Reynolds number, viscous forces are more dominant than inertial forces, laminar flow occurs which characterizes a constant and smooth fluid motion. Similarly, for higher Reynolds number, inertial forces are more dominant than viscous forces, hence turbulent flow occurs.

1.7 Fundamental laws

1.7.1 Mass conservation law

It states that the total mass in any closed system will remain constant. Mathematically

$$\frac{D\rho}{Dt} + \rho \nabla . \mathbf{V} = 0, \tag{1.28}$$

or

$$\frac{\partial\rho}{\partial t}+(\mathbf{V}.\boldsymbol{\nabla})\,\rho+\rho\boldsymbol{\nabla}.\mathbf{V}=0, \tag{1.29}$$

or

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}.\,(\rho\mathbf{V})=0. \tag{1.30}$$

The above expression represents the continuity equation. Here ρ stands for fluid density and $\mathbf{V}$ depicts the velocity profile. Due to steady state flow Eq. (1.30) takes the following form

$$\boldsymbol{\nabla}.\,(\rho\mathbf{V})=0, \tag{1.31}$$

and if the fluid is incompressible then above equation takes the following form

$$\boldsymbol{\nabla}.\mathbf{V}=0. \tag{1.32}$$

1.7.2 Momentum conservation law

This law states that the total momentum of a closed system is conserved. General form of this law is given below

$$\rho\frac{D\mathbf{V}}{Dt}=\boldsymbol{\nabla}.\boldsymbol{\tau}+\rho\mathbf{b}, \tag{1.33}$$

Here inertial, surface and body forces are indicated by the terms $\rho\frac{D\mathbf{V}}{Dt}$, $\nabla.\boldsymbol{\tau}$ and $\rho\mathbf{b}$ respectively.

1.7.3 Energy conservation law

The energy equation for nanofluid can be written as

$$\rho_f c_f\frac{DT}{Dt}=\boldsymbol{\tau}.\mathbf{L}+k\boldsymbol{\nabla}^2T+\rho_p c_p\left(D_B\boldsymbol{\nabla}C.\boldsymbol{\nabla}T+\frac{D_T}{T_\infty}\boldsymbol{\nabla}T.\boldsymbol{\nabla}T\right), \tag{1.34}$$

where ρ_f stands for base fluid density, c_f represents the specific heat of base fluid, T stands for temperature, $\boldsymbol{\tau}$ represents the stress tensor, $\mathbf{L}$ stands for rate of strain tensor, k denotes the thermal conductivity, ρ_p depicts the density of nanoparticles, D_T denotes the thermophoretic diffusion coefficient and D_B stands for Brownian diffusion coefficient.

1.7.4 Concentration law

The concentration equation for nanofluid can be written as

$$\frac{DC}{Dt} = D_B \nabla^2 C + \frac{D_T}{T_\infty} \nabla^2 T \tag{1.35}$$

where C stands for nanoparticles concentration, D_B represents the Brownian diffusion coefficient, D_T stands for thermophoretic coefficient and T depicts the temperature.

1.8 Solution method

1.8.1 Homotopic technique

HAM technique is one of the best and simplest techniques for obtaining convergent series solution for weakly as well as strongly non-linear equations. This method uses the concept of homotopy from topology. Two functions are homotopic if one function continuously deforms into other function, e.g. $f_1(x)$ and $f_2(x)$ are two functions and F^* is a continuous mapping, then

$$F^* : X \times [0, 1] \to Y \tag{1.36}$$

such that

$$F^*(x, 0) = f_1(x) \tag{1.37}$$

$$F^*(x, 1) = f_2(x) \tag{1.38}$$

Liao in 2012 [36] used the homotopic technique for obtaining convergent series solution.

HAM distinguishes itself from other techniques in the following ways:

1. It is independent of small/large parameter.

2. Convergent solution is guaranteed.

3. Freedom for the choice of base functions and linear operators.

Consider a non-linear differential equation

$$\mathcal{N}[u(x)] = 0, \tag{1.39}$$

where $\mathcal{N}$ represents non-linear operator, and $u(x)$ is the unknown function.

Using parameter $p \in [0, 1]$, a system of equations is constructed.

$$(1-p)\,\mathcal{L}\left[\widehat{u}\,(x;p) - u_0\,(x;p)\right] = p\hbar\mathcal{N}\widehat{u}\,(x;p)\,, \tag{1.40}$$

where $\mathcal{L}$ is linear operator, p is embedding parameter and $\hbar$ is convergence parameter. By putting value of p from 0 to 1, unknown function gets the value from $u_0(x)$ to $u(x)$.

$$\widehat{u}\,(x;p) = u_0\,(x) + \sum_{k=0}^{m-1} u_m\,(x)\,q^m, \qquad u_m\,(x) = \frac{1}{m!}\frac{\partial^m \widehat{u}\,(x;p)}{\partial x^m}|_{q=0} \tag{1.41}$$

for $q = 1$ we get

$$u\,(x) = u_0\,(x) + \sum_{m=1}^{\infty} u_m\,(x)\,. \tag{1.42}$$

For mth order problems we have

$$\mathcal{L}_u\left[u_m\,(\eta) - \chi_m u_{m-1}\,(\eta)\right] = \hbar_u \mathcal{R}_m^u\,(\eta)\,, \tag{1.43}$$

$$\mathcal{R}_m^u\,(\eta) = \frac{1}{(m-1)!}\frac{\partial^m \mathcal{N}_u \widehat{u}\,(x;p)}{\partial x^m}|_{p=0} \tag{1.44}$$

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}. \tag{1.45}$$

Chapter 2

Flow due to moving surface with non-linear velocity and variable thickness

2.1 Introduction

This chapter reports the development of homotopy solution for steady flow of an incompressible viscous nanofluid. Flow due to non-linear stretching surface subject to variable thickness analyzed. This paper is detailed review study of work by wahed et al. [32] Nonuniform heat generation $Q(x)$ and nonuniform transverse magnetic field $B(x)$ are considered. Appropriate transformations reduce the partial differential systems into the ordinary differential systems. The resulting systems are solved by homotopy technique. The results for velocity, temperature and concentration are established and discussed

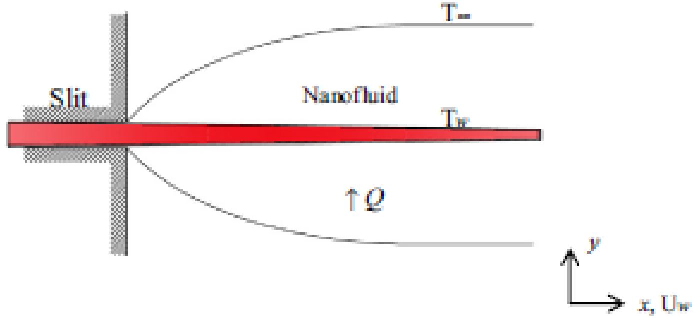

Fig. 1. Physical model and coordinate system.

2.2 Statement

Consider steady laminar two-dimensional incompressible viscous material over a an impermeable stretched sheet with variable thickness. Sheet is stretched with a velocity $U_w = a\,(x+b)^n$. Thickness of the sheet is taken variable, i.e $y = \delta\,(x+b)^{\frac{1-n}{2}}$. We assume coefficient δ so small that the surface is sufficiently thin. The problem is considered for $n \neq 1$. The velocity field of the present flow problem is given by

$$\mathbf{V} = (\mathbf{u}(\mathbf{x},\mathbf{y}), \mathbf{v}(\mathbf{x},\mathbf{y}), \mathbf{0}). \tag{2.1}$$

Here $\mathbf{u}$ and $\mathbf{v}$ are the velocities components in $\mathbf{x}$ and $\mathbf{y}$ directions.

The Cauchy stress tensor for viscous incompressible fluid is defined by

$$\boldsymbol{\tau} = -P\mathbf{I} + \mu\mathbf{A}_1, \tag{2.2}$$

In Eq. (2.2), P denotes the pressure, $\mathbf{I}$ represents the identity tensor and $\mathbf{A}_1$ the Rivlin Ericksen tensor which is expressed as

$$\mathbf{A}_1 = \text{grad}\,\mathbf{V} + (\text{grad}\,\mathbf{V})^T, \tag{2.3}$$

in which

$$\text{grad}\,\mathbf{V} = \begin{bmatrix} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} & \frac{\partial \mathbf{u}}{\partial \mathbf{y}} & 0 \\ \frac{\partial \mathbf{v}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{y}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2.4}$$

$$(\text{grad}\,\mathbf{V})^T = \begin{bmatrix} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{x}} & 0 \\ \frac{\partial \mathbf{u}}{\partial \mathbf{y}} & \frac{\partial \mathbf{v}}{\partial \mathbf{y}} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2.5}$$

Thus we have from $Eq.$(2.3),

$$\mathbf{A}_1 = \begin{bmatrix} 2\frac{\partial \mathbf{u}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{y}} & 0 \\ \frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{y}} & 2\frac{\partial \mathbf{v}}{\partial \mathbf{y}} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2.6}$$

The governing equations for an incompressible nanofluid are given by

$$\nabla.\mathbf{V} = \mathbf{0}, \tag{2.7}$$

$$\rho\frac{DV}{D\bar{t}} = \text{div}\,\boldsymbol{\tau} + J \times B, \tag{2.8}$$

$$(\rho c)_f\frac{DT}{Dt} = k\nabla^2 T + (\rho c)_p\left[D_B \nabla C \nabla T + D_T \frac{\nabla T.\nabla T}{T_\infty}\right] + Q(x)(T - T_\infty), \tag{2.9}$$

$$\frac{DC}{Dt} = D_B \nabla^2 C + \frac{D_T}{T_\infty}\nabla^2 T.$$

Using the boundary layer approximation the above equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2(x)}{\rho}u, \tag{2.11}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2\right] + \frac{Q(x)}{\rho C_p}(T - T_\infty), \tag{2.12}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}(\frac{\partial^2 T}{\partial y^2}), \tag{2.13}$$

with boundary conditions

$$\begin{aligned} u &= U_w,\ v=0,\ T=T_w,\ C=C_w,\ \text{at } y=\delta(x+b)^{\frac{1-n}{2}} \\ u &= 0,\ T=T_\infty,\ C=C_\infty \text{ as } y\to\infty. \end{aligned} \tag{2.14}$$

In the above expressions u and v are velocity components in x and y directions respectively, ν the kinematic viscosity, σ the electrical conductivity, $B(x) = B_0(x+b)^{\frac{1-n}{2}}$ the strength of magnetic field, ρ_f the base fluid density, ρ_p the density of nanoparticles, $Q(x) = Q_0(x+b)^{n-1}$ the heat generation, T the temperature of fluid, T_∞ the temperature above the boundary layer, T_w temperature of surface, τ the effective heat capacity and heat capacity of nanoparticle and base fluid respectivelly, α the thermal diffusion, C the nanoparticles concentration, D_B the Brownian motion coefficient and D_T the thermophoretic diffusion coefficient.

Using the following similarity transformations

$$\begin{aligned} \eta &= \sqrt{\frac{(n+1)a\,(x+b)^{n-1}}{2\nu}}y,\ \psi(x,y)=\sqrt{\frac{2\nu a\,(x+b)^{n+1}}{n+1}}F(\eta), \\ U_w(x) &= a(x+b)^n,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \end{aligned} \tag{2.15}$$

and expressing u and v in terms of stream function $\psi(x,y)$ we have:

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}. \tag{2.16}$$

We adopt

$$u=a(x+b)^nF'(\eta),\ v=-\sqrt{\frac{(n+1)\nu a\,(x+b)^{n-1}}{2}}\left(F(\eta)+\eta\frac{n-1}{n+1}F'(\eta)\right). \tag{2.17}$$

where a and b are constant, η the similarity variable and ψ the stream function. Using the above equations, conservation law of mass is satisfied and the resulting equations (2.11) – (2.14) are as follows

$$F'''+FF''-(\frac{2n}{n+1})F'^2-(\frac{2}{n+1})MF'=0, \tag{2.18}$$

$$\frac{1}{\Pr}\Theta''+F\Theta'+(\frac{2}{n+1})\lambda\Theta+Nb\Theta'\Phi'+Nt\Theta'^2=0, \tag{2.19}$$

$$\Phi'' + \Pr Le F\Phi' + (\frac{Nt}{Nb})\Theta'' = 0, \tag{2.20}$$

with associated boundary conditions by

$$F(\alpha) = \alpha(\frac{1-n}{1+n}),\ F'(\alpha) = 1,\ \Theta(\alpha) = 1,\ \Phi(\alpha) = 1,\ F'(\infty) = 0,\ \Theta(\infty) = 0,\ \Phi(\infty) = 0. \tag{2.21}$$

Here prime denote differentiation with respect to (η). For converting the domain from $[\alpha, \infty)$ to $[0, \infty)$, we use another transformation. Where $\alpha = \delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$ is the surface thickness parameter and $\eta = \alpha = \delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$ indicates the plate surface. We defined $F(\eta) = f(\eta - \alpha) = f(\zeta)$, $\Theta(\eta) = \theta(\eta - \alpha) = \theta(\zeta)$ and $\Phi(\eta) = \phi(\eta - \alpha) = \phi(\zeta)$, therefore the similarity Eqs. $(2.18)-(2.20)$ and the associated boundary condition (2.21) becomes

$$f''' + ff'' - (\frac{2n}{n+1})f'^2 - (\frac{2}{n+1})Mf' = 0, \tag{2.22}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + (\frac{2}{n+1})\lambda\theta + Nb\theta'\phi' + Nt\theta'^2 = 0, \tag{2.23}$$

$$\phi'' + \Pr Le f\phi' + (\frac{Nt}{Nb})\theta'' = 0, \tag{2.24}$$

$$f(0) = \alpha(\frac{1-n}{1+n}),\ f'(0) = 1,\ \theta(0) = 1,\ \phi(0) = 1,\ f'(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0. \tag{2.25}$$

Here n the shape parameter, $\Pr$ the prandtl number, Le the lewis number, M the magnetic field parameter, λ the heat source parameter, Nb Brownian motion parameter and Nt the thermophoresis parameter. The definitions of these parameters are

$$\begin{aligned} \Pr &= \frac{\nu}{\alpha},\ Le = \frac{\alpha}{D_B},\ M = \frac{\beta_0\sigma}{a\rho},\ \lambda = \frac{Q_0}{a\rho C_p},\ Nb = \frac{\tau D_B}{\nu}(C_w - C_\infty), \\ Nt &= \frac{\tau D_T}{\nu T_\infty}(T_w - T_\infty),\ \tau = \frac{(\rho C_p)_p}{(\rho C_p)_f}. \end{aligned} \tag{2.26}$$

The Skin friction (C_{fx}), Nusselt number (Nu) and Sherwood number (Sh) are

$$
\begin{aligned}
C_{fx} &= \frac{\tau_w}{\rho U_w^2}, && (2.27)\\
Nu &= \frac{(x+b)q_w}{k(T_w - T_\infty)}, && (2.28)\\
Sh &= \frac{(x+b)q_m}{D_B(C_w - C_\infty)}. && (2.29)
\end{aligned}
$$

Here ($\tau_w = \mu(\frac{\partial u}{\partial y})_{y=\delta(x+b)^{\frac{1-n}{2}}}$) the surface shear stress, ($q_w = -k(\frac{\partial T}{\partial y})_{y=\delta(x+b)^{\frac{1-n}{2}}}$) the rate of heat transfer and ($q_m = -D_B(\frac{\partial C}{\partial y})_{y=\delta(x+b)^{\frac{1-n}{2}}}$) the rate of mass transfer.

Finally the non-dimensional form of Skin friction, Sherwood number and Nusselt number are given below

$$
\begin{aligned}
\sqrt{R_e} C_{fx} &= 2\sqrt{\frac{n+1}{2}} f''(0), && (2.30)\\
\frac{Sh}{\sqrt{R_e}} &= -\sqrt{\frac{n+1}{2}} \phi'(0), && (2.31)\\
\frac{Nu}{\sqrt{R_e}} &= -\sqrt{\frac{n+1}{2}} \theta'(0). && (2.32)
\end{aligned}
$$

2.3 Solution methodology

The approximate series solutions through the homotopy analysis technique (HAM) require the appropriate initial approximations and auxiliary linear operators for the governing problems which are given in the following forms

$$f_0(\eta) = \alpha(\frac{n+1}{n-1})1 - \exp(-\zeta), \quad \theta_0(\zeta) = \exp(-\zeta), \quad \phi_0(\zeta) = \exp(-\zeta), \tag{2.33}$$

$$\mathcal{L}_f^* = \frac{d^3 f}{d\zeta^3} - \frac{df}{d\zeta}, \quad \mathcal{L}_\theta^* = \frac{d^2\theta}{d\zeta^2} - \theta, \quad \mathcal{L}_\phi^* = \frac{d^2\phi}{d\zeta^2} - \phi. \tag{2.34}$$

The above linear operators have the following properties

$$
\left.
\begin{aligned}
\mathcal{L}_f^*\left[C_1 + C_2\exp(\zeta) + C_3\exp(-\zeta)\right] = 0,\\
\mathcal{L}_\theta^*\left[C_4\exp(\zeta) + C_5\exp(-\zeta)\right] = 0,\\
\mathcal{L}_\phi^*\left[C_6\exp(\zeta) + C_7\exp(-\zeta)\right] = 0,
\end{aligned}
\right\} \tag{2.35}
$$

in which C_j $(j = 1-7)$ stands for arbitrary constants.

2.3.1 Zeroth-order statement

$$(1-Þ^*)\mathcal{L}_f^*\left[\breve{f}(\zeta,Þ^*)-f_0(\zeta)\right] = Þ^*\hbar_f\mathcal{N}_f^*[\breve{f}(\zeta,Þ^*)], \tag{2.36}$$

$$(1-Þ^*)\mathcal{L}_\theta^*\left[\breve{\theta}(\zeta,Þ^*)-\theta_0(\zeta)\right] = Þ^*\hbar_\theta\mathcal{N}_\theta^*[\breve{f}(\zeta,Þ^*),\breve{\theta}(\zeta,Þ^*),\breve{\phi}(\zeta,Þ^*)], \tag{2.37}$$

$$(1-Þ^*)\mathcal{L}_\phi^*[\breve{\phi}(\zeta,Þ^*)-\phi_0(\zeta)] = Þ^*\hbar_\phi\mathcal{N}_\phi^*[\breve{f}(\zeta,Þ^*),\breve{\theta}(\zeta,Þ^*),\breve{\phi}(\zeta,Þ^*)], \tag{2.38}$$

$$\left.\begin{array}{c}\breve{f}(0,Þ^*)=\alpha(\frac{n+1}{n-1}),\ \breve{f}'(0,Þ^*)=1,\ \breve{f}'(\infty,Þ^*)=0,\ \breve{\theta}(0,Þ^*)=1,\\ \breve{\theta}(\infty,Þ^*)=0,\ \breve{\phi}(0,Þ^*)=1,\quad \breve{\phi}(\infty,Þ^*)=0,\end{array}\right\} \tag{2.39}$$

$$\mathcal{N}_f^*\left[\breve{f}(\zeta;Þ^*)\right] = \frac{\partial^3\breve{f}}{\partial\zeta^3}+\breve{f}\frac{\partial^2\breve{f}}{\partial\zeta^2}-\frac{2n}{n+1}\left(\frac{\partial\breve{f}}{\partial\zeta}\right)^2-\frac{2}{n+1}M\frac{\partial\breve{f}'}{\partial\zeta}, \tag{2.40}$$

$$\mathcal{N}_\theta^*\left[\breve{f}(\zeta;Þ^*),\breve{\theta}(\zeta,Þ^*),\breve{\phi}(\zeta,Þ^*)\right] = \frac{\partial^2\breve{\theta}}{\partial\zeta^2}+\Pr\breve{f}\frac{\partial\breve{\theta}}{\partial\zeta}+\Pr Nb\frac{\partial\breve{\theta}}{\partial\zeta}\frac{\partial\breve{\phi}}{\partial\zeta}+\Pr Nt\left(\frac{\partial\breve{\theta}}{\partial\zeta}\right)^2+\left(\frac{2}{n+1}\right)\Pr\lambda\frac{\partial\breve{\theta}}{\partial\zeta}, \tag{2.41}$$

$$\mathcal{N}_\phi^*\left[\breve{f}(\zeta;Þ^*),\breve{\theta}(\zeta,Þ^*),\breve{\phi}(\zeta,Þ^*)\right] = \frac{\partial^2\breve{\phi}}{\partial\zeta^2}+Le\Pr\breve{f}\frac{\partial\breve{\phi}}{\partial\zeta}+\frac{Nt}{Nb}\frac{\partial^2\breve{\theta}}{\partial\zeta^2}. \tag{2.42}$$

Here $Þ^* \in [0,1]$ denotes the embedding parameter, $(\hbar_f,\ \hbar_\theta,\hbar_\phi)$ stand for nonzero auxiliary variables and $(\mathcal{N}_f^*,\ \mathcal{N}_\theta^*,\mathcal{N}_\phi^*)$ represent the nonlinear operators.

2.3.2 $\hat{m}$th-order statement

$$\mathcal{L}_f^*\left[f_{\hat{m}}(\zeta)-\chi_{\hat{m}}f_{\hat{m}-1}(\zeta)\right] = \hbar_f\tilde{\mathcal{R}}_{\hat{m}}^{*f}(\zeta), \tag{2.43}$$

$$\mathcal{L}_\theta^*\left[\theta_{\hat{m}}(\zeta)-\chi_{\hat{m}}\theta_{\hat{m}-1}(\zeta)\right] = \hbar_\theta\tilde{\mathcal{R}}_{\hat{m}}^{*\theta}(\zeta), \tag{2.44}$$

$$\mathcal{L}_\phi^*\left[\phi_{\hat{m}}(\zeta)-\chi_{\hat{m}}\phi_{\hat{m}-1}(\zeta)\right] = \hbar_\phi\tilde{\mathcal{R}}_{\hat{m}}^{*\phi}(\zeta), \tag{2.45}$$

$$\left.\begin{array}{c}f_{\hat{m}}(0)=f'_{\hat{m}}(0)=f'_{\hat{m}}(\infty)=0,\ \theta'_{\hat{m}}(0)+\theta_{\hat{m}}(\infty)=0,\\ \phi'_{\hat{m}}(0)+\phi_{\hat{m}}(\infty)=0,\end{array}\right\} \tag{2.46}$$

$$\tilde{\mathcal{R}}_{\hat{m}}^{*f}(\zeta) = f'''_{\hat{m}-1}+\sum_{k=0}^{\hat{m}-1}(f_{\hat{m}-1-k}f''_k)-\left(\frac{2n}{n+1}\right)\sum_{k=0}^{\hat{m}-1}(f'_{\hat{m}-1-k}f'_k)-\left(\frac{2}{n+1}\right)M(f'_{\hat{m}-1}), \tag{2.47}$$

$$\tilde{\mathcal{R}}^{*\theta}_{\hat{m}}(\zeta) = \theta''_{\hat{m}-1} + \Pr\sum_{k=0}^{\hat{m}-1}(f_{\hat{m}-1-k}\theta'_k) + \Pr Nb\sum_{k=0}^{\hat{m}-1}(\theta'_{\hat{m}-1-k}\phi'_k) + \Pr Nt\sum_{k=0}^{\hat{m}-1}(\theta'_{\hat{m}-1-k}\theta'_k) + \left(\frac{2}{n+1}\right)\Pr\lambda(\theta_{\hat{m}-1}), \tag{2.48}$$

$$\tilde{\mathcal{R}}^{*\phi}_{\hat{m}}(\zeta) = \phi''_{\hat{m}-1}(\zeta) + Le\Pr\sum_{k=0}^{\hat{m}-1}(f_{\hat{m}-1-k}\phi'_k) + \frac{Nt}{Nb}\theta''_{\hat{m}-1}, \tag{2.49}$$

$$\boldsymbol{\chi}_{\hat{m}} = \begin{cases} 0, & \hat{m} \leq 1, \\ 1, & \hat{m} > 1, \end{cases} \tag{2.50}$$

Putting $Þ^* = 0$ and $Þ^* = 1$ we have

$$\breve{f}(\zeta;0) = f_0(\zeta),\ \breve{f}(\zeta;1) = f(\zeta), \tag{2.51}$$

$$\breve{\theta}(\zeta,0) = \theta_0(\zeta),\ \breve{\theta}(\zeta,1) = \theta(\zeta), \tag{2.52}$$

$$\breve{\phi}(\zeta,0) = \phi_0(\zeta),\ \breve{\phi}(\zeta,1) = \phi(\zeta). \tag{2.53}$$

When $Þ^*$ varies from 0 to 1 then $\breve{f}(\zeta;Þ^*)$, $\breve{\theta}(\zeta,Þ^*)$ and $\breve{\phi}(\zeta,Þ^*)$ illustrate variation from the initial approximations $f_0(\zeta)$, $\theta_0(\zeta)$ and $\phi_0(\zeta)$ to the desired final solutions $f(\zeta)$, $\theta(\zeta)$ and $\phi(\zeta)$ respectively. By using Taylor's series expansion we get the following expressions

$$\breve{f}(\zeta;Þ^*) = f_0(\zeta) + \sum_{\hat{m}=1}^{\infty} f_{\hat{m}}(\zeta){Þ^*}^{\hat{m}},\ f_{\hat{m}}(\zeta) = \frac{1}{\hat{m}!}\frac{\partial^{\hat{m}}\breve{f}(\zeta,Þ^*)}{\partial {Þ^*}^{\hat{m}}}\bigg|_{Þ^*=0}, \tag{2.54}$$

$$\breve{\theta}(\zeta,Þ^*) = \theta_0(\zeta) + \sum_{\hat{m}=1}^{\infty} \theta_{\hat{m}}(\zeta){Þ^*}^{\hat{m}},\ \theta_{\hat{m}}(\zeta) = \frac{1}{\hat{m}!}\frac{\partial^{\hat{m}}\breve{\theta}(\zeta,Þ^*)}{\partial {Þ^*}^{\hat{m}}}\bigg|_{Þ^*=0}, \tag{2.55}$$

$$\breve{\phi}(\zeta,Þ^*) = \phi_0(\zeta) + \sum_{\hat{m}=1}^{\infty} \phi_{\hat{m}}(\zeta){Þ^*}^{\hat{m}},\ \phi_{\hat{m}}(\zeta) = \frac{1}{\hat{m}!}\frac{\partial^{\hat{m}}\breve{\phi}(\zeta,Þ^*)}{\partial {Þ^*}^{\hat{m}}}\bigg|_{Þ^*=0}. \tag{2.56}$$

The convergence of Eqs. (2.54) – (2.56) strongly depends upon them ($\hbar_f$, $\hbar_\theta$, $\hbar_\phi$). Considering that ($\hbar_f$, $\hbar_\theta$, $\hbar_\phi$) are chosen in such a way that Eqs. (2.54) – (2.56) converge at $Þ^* = 1$, then

$$f(\zeta) = f_0(\zeta) + \sum_{\hat{m}=1}^{\infty} f_{\hat{m}}(\zeta), \tag{2.57}$$

$$\theta(\eta) = \theta_0(\zeta) + \sum_{\hat{m}=1}^{\infty} \theta_{\hat{m}}(\zeta), \tag{2.58}$$

$$\phi(\eta) = \phi_0(\zeta) + \sum_{\hat{m}=1}^{\infty} \phi_{\hat{m}}(\zeta). \tag{2.59}$$

The general solutions ($f_{\hat{m}}$, $\theta_{\hat{m}}$, $\phi_{\hat{m}}$) of the Eqs. (2.43) – (2.45) in terms of special solutions ($f^*_{\hat{m}}$, $\theta^*_{\hat{m}}$, $\phi^*_{\hat{m}}$) are presented by the following expressions

$$f_{\hat{m}}(\zeta) = f^*_{\hat{m}}(\zeta) + C_1 + C_2 \exp(\zeta) + C_3 \exp(-\zeta), \tag{2.60}$$

$$\theta_{\hat{m}}(\zeta) = \theta^*_{\hat{m}}(\zeta) + C_4 \exp(\zeta) + C_5 \exp(-\zeta), \tag{2.61}$$

$$\phi_{\hat{m}}(\zeta) = \phi^*_{\hat{m}}(\zeta) + C_6 \exp(\zeta) + C_7 \exp(-\zeta). \tag{2.62}$$

in which the constants C_j $(j = 1 - 7)$ through the boundary conditions (2.46) are given by

$$C_2 = C_4 = C_6 = 0,\ C_3 = \left.\frac{\partial f^*_{\hat{m}}(\zeta)}{\partial \zeta}\right|_{\eta=0},\ C_1 = -C_3 - f^*_{\hat{m}}(0), \tag{2.63}$$

$$C_5 = -\theta^*_{\hat{m}}(0)_{\eta=0}, \tag{2.64}$$

$$C_7 = -\phi^*_{\hat{m}}(0)_{\eta=0}. \tag{2.65}$$

2.4 HAM solution

2.4.1 Convergence analysis

Here the series solutions (2.57) – (2.59) involve the auxiliary parameters ($\hbar_f$, $\hbar_\theta$, $\hbar_\phi$). Surely the auxiliary variables ($\hbar_f$, $\hbar_\theta$, $\hbar_\phi$) in series solutions accelerate the convergence. To choose the appropriate values of ($\hbar_f$, $\hbar_\theta$, $\hbar_\phi$) the $\hbar-$curves have been displayed at 25^{th} order of approximations. It is noticed from Figs. $(2-4)$ that the admissible range of $\hbar_f$, $\hbar_\theta$ and $\hbar_\phi$ are $-2.2 \leq \hbar_f \leq -0.3$, $-1.8 \leq \hbar_\theta \leq -0.4$ and $-1.5 \leq \hbar_\phi \leq -0.2$. Moreover the series solutions are convergent in the entire zone of η when $\hbar_f = -1.0 = \hbar_\theta = \hbar_\phi$. Table 2.1 reveals that the 6^{th} order of deformations is enough for the convergent solutions of velocity and 24^{th} order of deformations is enough for both temperature and nanoparticles concentration respectively.

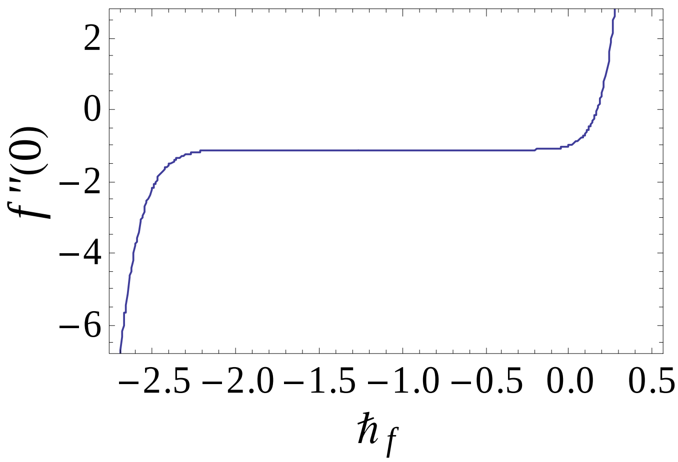

Fig. 2. h-curve for the function $f(\zeta)$.

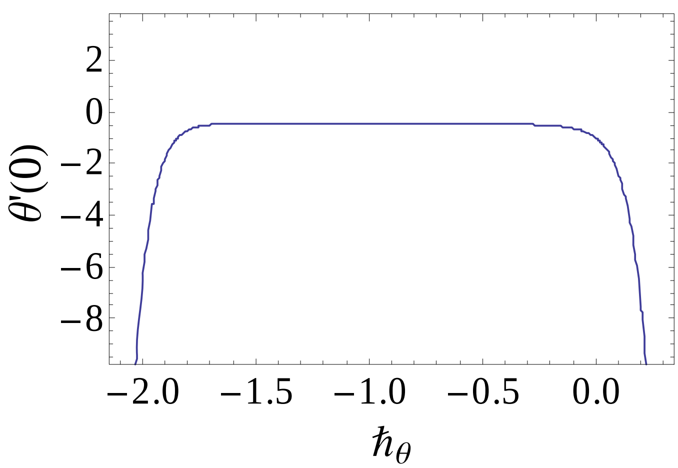

Fig. 3. h-curve for the function $\theta(\zeta)$.

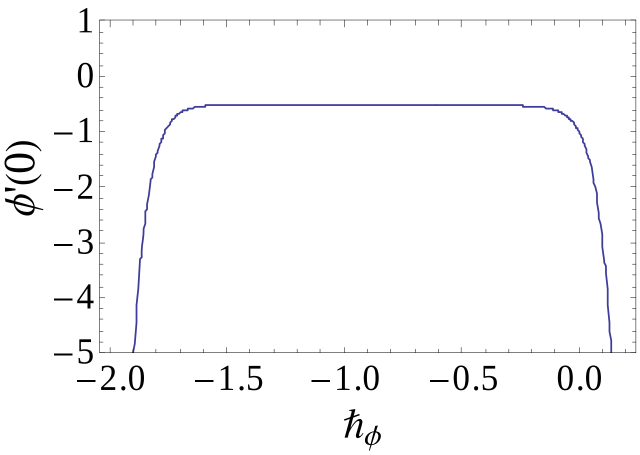

Fig. 4. h-curve for the function $\phi(\zeta)$.

Table 2.1. Convergence of homotopic solutions when $M = 0.5$, $\Pr = 1.0$, $Le = 1.0$, $Nb = 0.3$, $Nt = 0.2$, $\lambda = 0.4$, $n = 0.5$ and $\alpha = 0.2$.

Order of approximation	$-f''(0)$	$\theta'(0)$	$-\phi'(0)$
1	1.10556	0.58333	0.50000
2	1.11628	0.58940	0.54884
3	1.11628	0.56210	0.50861
6	1.11602	0.55031	0.48739
10	1.11602	0.54744	0.48055
15	1.11602	0.54695	0.47918
18	1.11602	0.54690	0.47902
21	1.11602	0.54688	0.47897
24	1.11602	0.54687	0.47896
25	1.11602	0.54687	0.47896
26	1.11602	0.54687	0.47896

2.5 Results and discussion

Our intention here to analyze the characteristics of shape parameter (n), Prandtl number (Pr), Lewis number (Le), magnetic field parameter (M), surface thickness parameter (α), heat source parameter (λ), Brownian motion parameter (Nb) and thermophoresis parameter (Nt) on velocity $f'(\zeta)$, temperature $\theta(\zeta)$ and nanoparticles concentration $\phi(\zeta)$. Moreover, Tables $(2.2-2.4)$ are demonstrated to analyze the behaviors of arising variables on Skin friction (C_{fx}), Nusselt number $(Nu)$and Sherwood number (Sh) respectivelly. Fig. 5 displays the variation of shape parameter (n) on velocity $f'(\zeta)$. From Fig. we observe that velocity is increasing function of (n). Impact of wall thickness parameter α on the velocity profile is shown in Fig. 6. It is observed that increasing values of α lead to decrease the velocity. Fig. 7 shows the effects of magnetic parameter on velocity distribution. Here we seen that velocity decreases for higher values of magnetic parameter (M). Fig. 8 shows the effect of (n) on $\theta(\zeta)$. Temperature is increasing function of (n). Fig. 9 depicts that variation of thickness parameter α yields reduction in temperature. Influence of heat source parameter on temperature profile is plotted in Fig. 10. There is an increase in temperature via source parameter (λ). Brownian motion is a arbitrary movement of suspended particles due to collection of atoms or molecules in the fluid. The increment of (Nb) leads to enhancement of temperature and reduction in nanoparticles concentration as shown in Figs. $(11-12)$. Thermophoresis variable corresponds to enhancement in temperature and nanoparticles concentration as shown in Figs. $(13-14)$. Fig. 15 depicts the variation of shape variable on concentration profile. The concentration enhances when n is increased. Fig. 16 represents the concentration profile for different values of thickness parameter (α). Clearly $\phi(\zeta)$ decreases for larger α. Fig. 17 describes the effect of magnetic parameter on concentration distribution. It is observed that concentration profile enhances for larger M.

Tables (2.2) represents the Skin friction (C_{fx}) for various physical variables appearing in velocity profile. Clearly surface force enhances via α, M and n. Table (2.3) provides the numerical values of dimensionless Nusselt number (Nu) for various physical parameters involved in temperature distribution. Here Nusselt number increases for larger α and reverse trend is noted for higher values of λ, Le, Nb and Nt. Table (2.4) shows that the Sherwood number enhances for larger α, λ, Le and Nt. On the other hand the Sherwood number decays for larger Nt.

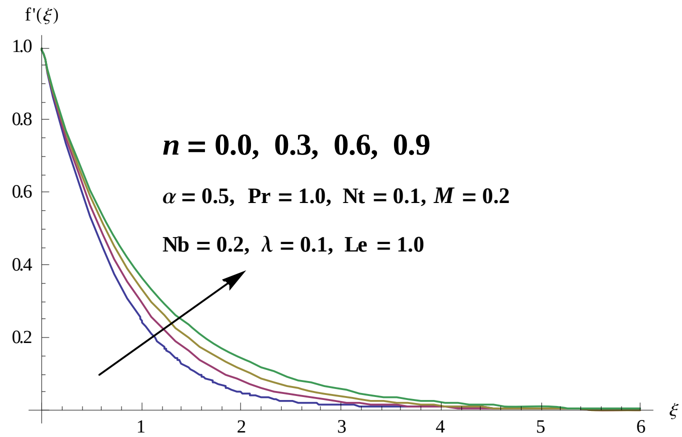

Fig. 5. Effects of n on $f'(\xi)$.

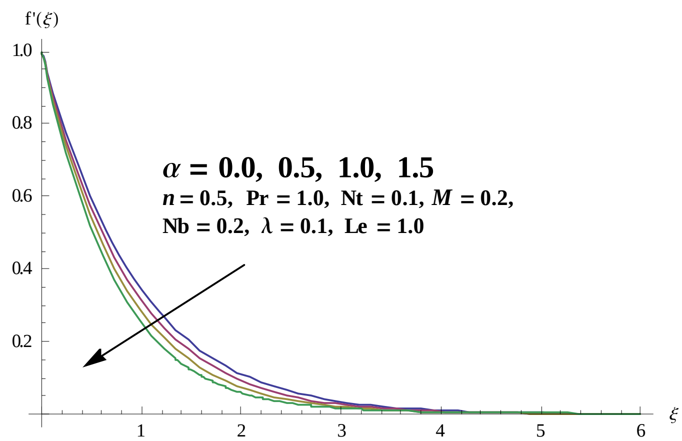

Fig. 6. Effects of α on $f'(\xi)$.

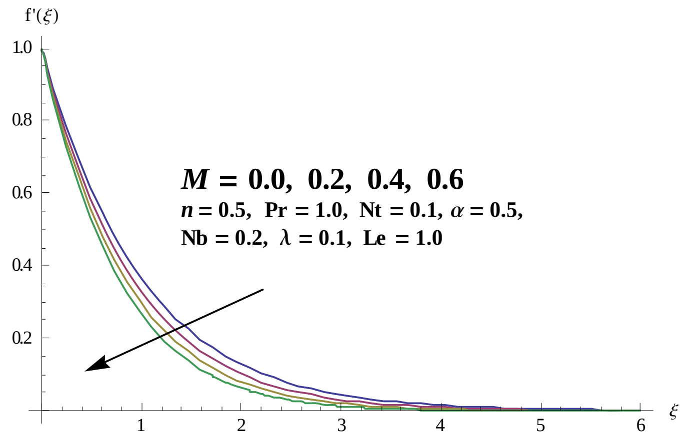

Fig. 7. Effects of M on $f'(\xi)$.

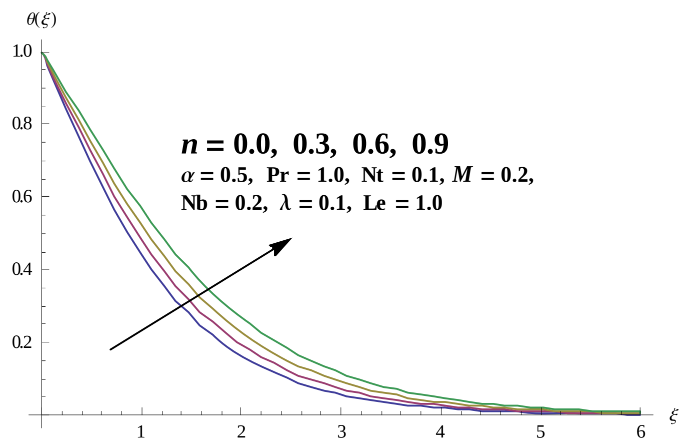

Fig. 8. Effects of n on $\theta(\xi)$.

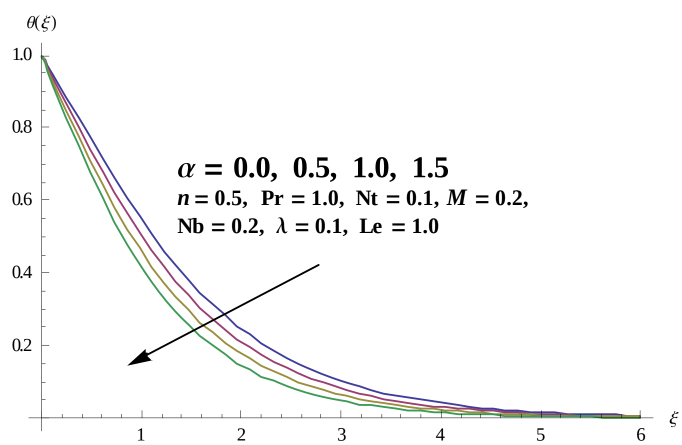

Fig. 9. Effects of α on $\theta(\xi)$.

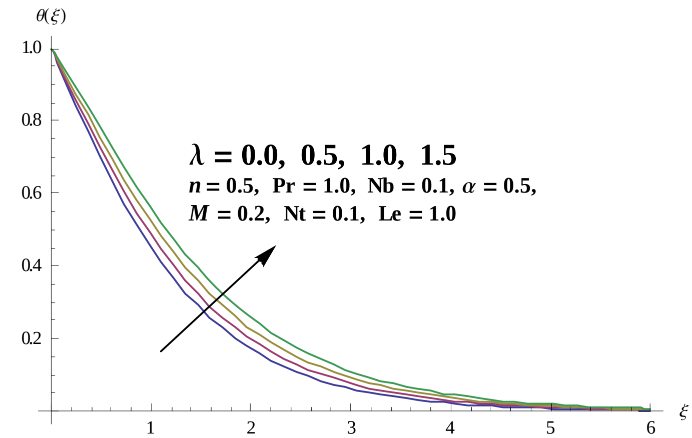

Fig. 10. Effects of λ on $\theta(\xi)$.

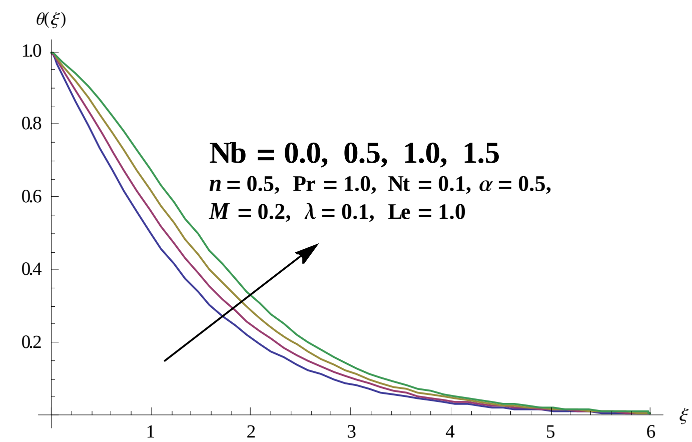

Fig. 11. Effects of Nb on $\theta(\xi)$.

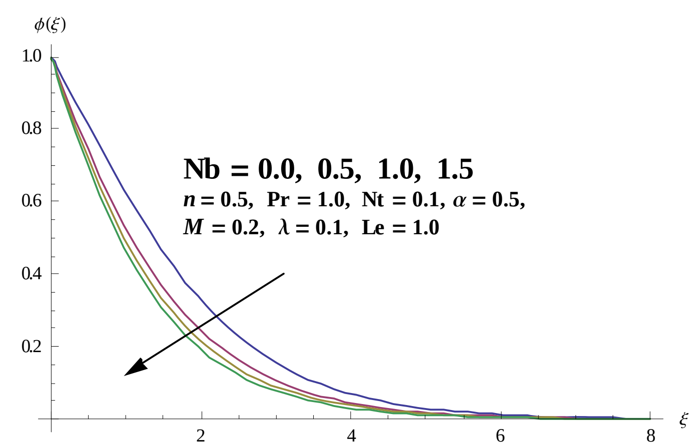

Fig. 12. Effects of Nb on $\phi(\xi)$.

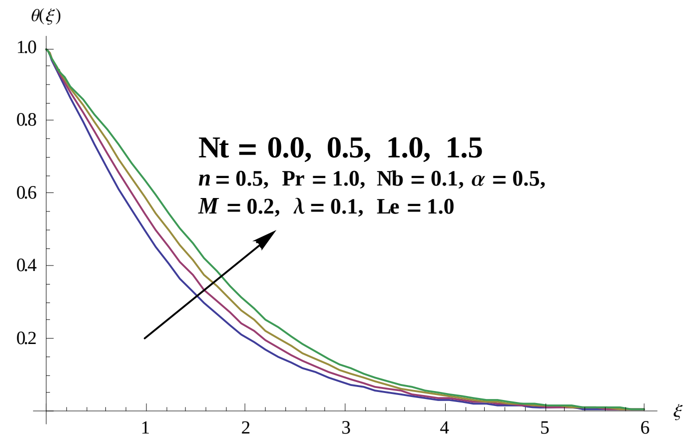

Fig. 13. Effects of Nt on $\theta(\xi)$.

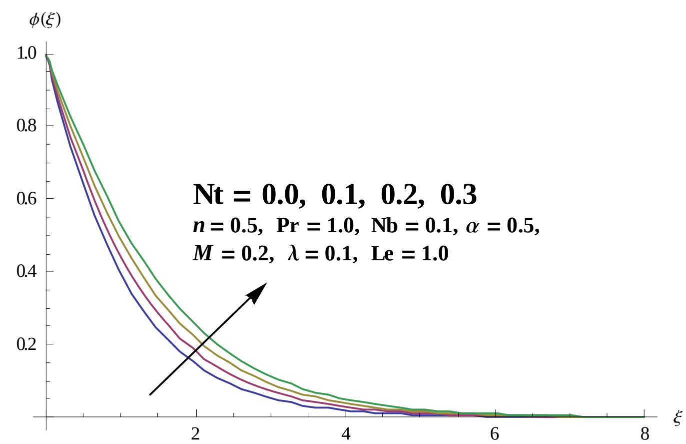

Fig. 14. Effects of Nt on $\phi(\xi)$.

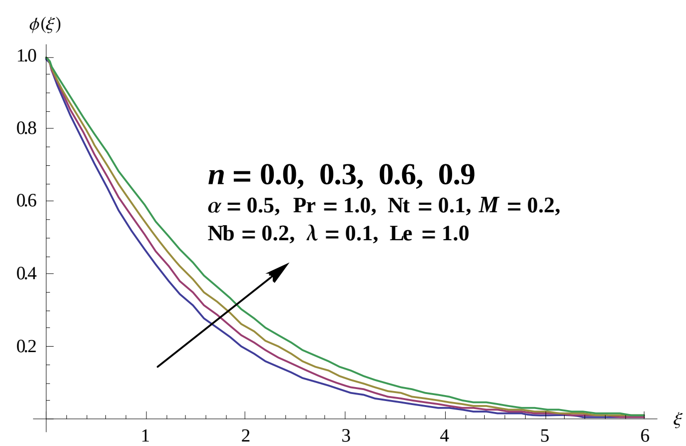

Fig. 15. Effects of n on $\phi(\xi)$.

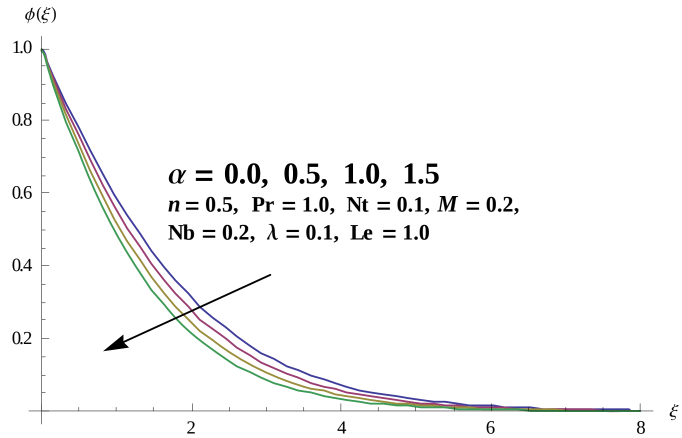

Fig. 16. Effects of α on $\phi(\xi)$.

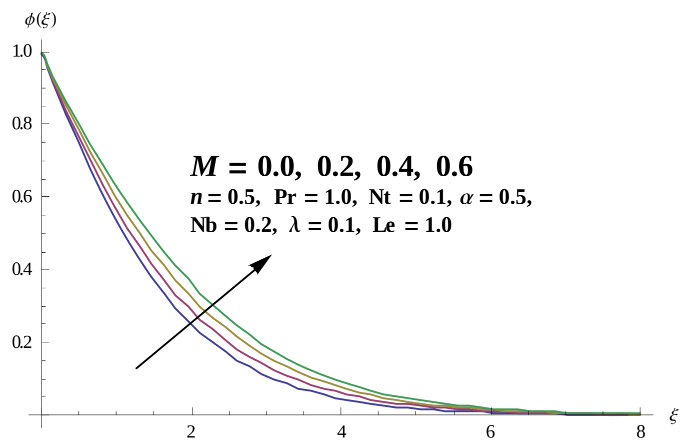

Fig. 17. Effects of M on $\phi(\xi)$.

Table: 2.2. Effects of α, M and n on Skin friction coefficients when $Le = 1.0 = \Pr$,

$Nt = 0.1$, $Nb = 0.2 = \lambda$.

α	n	M	$-(R_e)^{0.5}C_{fx}$
0.0	0.5	0.2	0.8892
0.2			0.9194
0.4			0.9826
	0.0		0.8377
	0.4		0.9400
	0.6		0.9929
		0.0	0.8487
		0.4	1.0711
		0.6	1.1666

Table: 2.3. Impacts of Le, α, Nb, λ and Nt on local Nusselt number (Nu) when $n = 0.5$, $M = 0.4$, $\Pr = 1.0$.

α	λ	Le	Nb	Nt	$-\frac{Nu}{\sqrt{R_e}}$
0.0	0.4	1.0	0.2	0.3	0.2898
0.2					0.3299
0.4					0.3704
	0.0				0.5156
	0.2				0.5049
	0.4				0.4823
		0.5			0.4074
		1.2			0.3868
		1.6			0.3824
			0.1		0.4177
			0.3		0.3648
			0.4		0.3400
				0.1	0.4051
				0.2	0.3751
				0.6	0.3175

Table: 4. Impacts of α, Le, λ, Nb and Nt on local Sherwood number when $n = 0.5$, $M = 0.4$, Pr = 1.0.

α	λ	Le	Nb	Nt	$-\frac{Sh}{\sqrt{R_e}}$
0.0	0.4	1.0	0.2	0.3	0.4023
0.1					0.4025
0.2					0.4210
	0.0				0.3981
	0.2				0.4027
	04				0.4173
		0.6			0.2390
		1.2			0.5435
		1.6			0.6715
			0.1		0.2904
			0.3		0.5036
			0.5		0.5454
				0.2	0.3264
				0.4	0.1133
				0.6	0.0568

2.6 Final remarks

Main points of present study are as follows:

- Flatness of surface directly relates with mechanical properties of surface.
- Enhancement of Brownian motion parameter, shape parameter, thermophoresis parameter and heat source parameter correspond to increase the temperature distribution while opposite behaviour is seen for the variation of thickness parameter.

Chapter 3

Simultaneous effects of melting heat and non-Darcy porous medium in MHD flow of nanofluid towards variable thicked stretched surface

3.1 Introduction

This chapter discuses melting heat transfer and non-Darcy porous medium in MHD stagnation point flow towards a nonlinear stretching sheet with variable thickness. Brownian motion and thermophoresis effects are also accounted. The nonlinear dimensionless ordinary differential equations are solved numerically with builtin shooting technique. Graphically results of various physical parameters on velocity, temperature and concentration are studied. Skin friction coefficient, local Nusselt number and Sherwood number are addressed through tabulated values.

3.2 Formulation

We consider steady two-dimensional MHD stagnation point flow of an incompressible viscous nanofluid in a non-Darcy porous medium. The flow is induced due to stretching sheet at $y = \delta(x+b)^{\frac{1-n}{2}}$. The stretching sheet along x-axis has velocity $u_w = a(x+b)^n$ However the velocity of external flow is $u_e = d(x+b)^n$. Here a, b and c are the positive constants. Heat transfer is studied through melting heat. $T_m$the temperature of melting surface, while T_∞ and C_∞ correspond to the uniform temperature and concentration far away from the surface of the sheet respectively. It is assumed that the effects of viscous dissipation and heat generation/absorption are neglected. Under these assumptions and usual boundary layer approximation the governing equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}\beta^2(u-u_e) - \frac{\nu\epsilon}{k}(u-u_e) - \frac{c_b\epsilon}{\sqrt{k}}(u^2-u_e^2), \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2\right), \tag{3.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{3.4}$$

subjected boundary conditions are presented as follows:

$$u(x,y) = u_w(x) = a(x+b)^n, \quad D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right) = 0 \text{ and } T = T_m \text{ at } y = \delta(x+b)^{\frac{1-n}{2}}, \tag{3.5}$$

$$u\,(x,y) = u_e(x) = d(x+b)^n, \quad T = T_\infty, \quad C = C_\infty, \text{as } y \to \infty, \tag{3.6}$$

$$k_1(\frac{\partial T}{\partial y}) = \rho[\lambda^* + C_S(T_m - T_0)]v \text{ at } y = \delta(x+b)^{\frac{1-n}{2}}. \tag{3.7}$$

In the above equations u and v are the velocity component in the x and y-directions. Here ρ the fluid density, λ^* the latent heat of fluid, ν the kinematic viscosity, k the permeability of porous medium, ε the porosity, c_b the drag coefficient, D_T the thermophoretic diffusion coefficient, D_B the Brownian diffusion coefficient, τ the ratio between effective heat capacity of nanoparticle and base fluid respectively, C_S the heat capacity of surface and T_m the melting temperature.

3.3 Statement

Here we employ transformation procedure to reduce the above partial differential equations and governing boundary conditions into the set of ordinary differential system;

$$\left.\begin{array}{c} \eta = y\sqrt{(\frac{n+1}{2})(\frac{a(x+b)^{n-1}}{\nu})},\ \psi = \sqrt{(\frac{2}{n+1})(x+b)^{n+1}a\nu}F(\eta), \\ \theta(\eta) = \frac{T-T_m}{T_\infty - T_m},\ \phi(\eta) = \frac{C-C_\infty}{C\infty} \end{array}\right\} \tag{3.8}$$

In above equation ψ reperesents the stream function, define by ($u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$) as,

$$u = a(x+b)^n F^{'}(\eta),\ \ v = -\sqrt{(\frac{n+1}{2})a\nu(x+b)^{n-1}}[F(\eta) + \eta\frac{n-1}{n+1}F^{'}(\eta)]. \tag{3.9}$$

Now Eq. (1) is trivially satisfied while Eqs. (3.2) – (3.4) yield

$$\left.\begin{array}{c} F''' - (\frac{2n}{n+1})\left(F^{'}\right)^2 + FF^{''} - (\frac{2}{n+1})(Ha)^2\left(F' - \lambda\right) - (\frac{2}{n+1})(Da)\left(F' - \lambda\right) \\ -(\frac{2}{n+1})\beta\left(F'^2 - \lambda^2\right) + (\frac{2n}{n+1})\lambda^2 = 0, \end{array}\right\} \tag{3.10}$$

$$\Theta'' + \Pr\left(F\Theta' + Nb\Phi^{'}\Theta^{'} + Nt\Theta^{'2}\right) = 0, \tag{3.11}$$

$$\Phi^{''} + \Pr LeF\Phi^{'} + \frac{Nt}{Nb}\Theta^{''} = 0, \tag{3.12}$$

$$\left.\begin{array}{c} F'(\alpha) = 1,\ \ \Theta(\alpha) = 0,\ \ \ M\Theta^{'}(\alpha)\ + \Pr F(\alpha) + \Pr\eta(\frac{n-1}{n+1}) = 0,\ \ \ Nt\Theta^{'}(\alpha) +\ Nb\Phi^{'}(\alpha) = 0, \\ F^{'}(\infty) = \lambda,\ \Theta(\infty) = 1,\ \Phi(\infty) = 0. \end{array}\right\} \tag{3.13}$$

Here $\alpha(= \delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$) represents the surface thickness parameter and $\eta = \alpha = (\delta\sqrt{\frac{1+n}{2}\frac{a}{\nu}}$) the plate surface. We define $F(\eta) = f(\eta-\alpha) = f(\zeta)$, $\Theta(\eta) = \theta(\eta-\alpha) = \theta(\zeta)$ and $\Phi(\eta) = \phi(\eta-\alpha) = \phi(\zeta)$ then Eqs. (3.10) – (3.13) give;

$$\left.\begin{array}{c} f''' - (\frac{2n}{n+1})\left(f^{'}\right)^2 + ff^{''} - (\frac{2}{n+1})(Ha)^2\left(f' - \lambda\right) - (\frac{2}{n+1})(Da)\left(f' - \lambda\right) \\ -(\frac{2}{n+1})\beta\left(f'^2 - \lambda^2\right) + (\frac{2n}{n+1})\lambda^2 = 0, \end{array}\right\} \tag{3.14}$$

$$\theta'' + \Pr\left(f\theta' + Nb\phi^{'}\theta^{'} + Nt\theta^{'2}\right) = 0, \tag{3.15}$$

$$\phi^{''} + \Pr Lef\phi^{'} + \frac{Nt}{Nb}\theta^{''} = 0, \tag{3.16}$$

$$\left.\begin{array}{c} f'(0) = 1, \quad \theta(0) = 0, \quad M\theta^{'}(0) \ + \Pr f(0) + \Pr\alpha(\frac{n-1}{n+1}) = 0, \quad Nt\theta^{'}(0) + \ Nb\phi^{'}(0) = 0, \\ f^{'}(\infty) = \lambda, \theta(\infty) = 1, \phi(\infty) = 0. \end{array}\right\} \tag{3.17}$$

With

$$\begin{array}{lll} M = \frac{C_p(T_\infty - T_m)}{\lambda^* + C_S(T_m - T_0)}, & Da = \frac{\varepsilon\nu}{ka(x+b)^{n-1}}, & Ha = \sqrt{\frac{\sigma}{\rho a}}B_0, \\ \beta = \frac{C_b\varepsilon(x+b)}{\sqrt{k}}, & \lambda = \frac{d}{a}, & \Pr = \frac{\nu}{\alpha}, \\ Nt = \frac{\tau D_T(T_\infty - T_m)}{\nu T_\infty}, & Nb = \frac{\tau D_B C_\infty}{\nu}, & Le = \frac{\alpha}{D_B}, \end{array}$$

in which n, the shape parameter, Da the inverse Darcy number, Ha the Hartman number, β the local inertia coefficient parameter, λ the stretching sheet parameter, Pr the Prandtl number, Nb the Brownian motion parameter, Nt thermophoresis parameter, Le the Lewis number, M the melting parameter, and α the surface thickness parameter.

The skin friction coefficient (C_f) and Nusselt number (Nu) are

$$C_f = \frac{\tau_w}{\rho u_w^2/2}, \ N_u = \frac{(x+b)q_w}{k(T_\infty - T_m)}, \tag{3.18}$$

where the surface shear stress (τ_w) and rate of heat transfer (q_w) define at the surface are

$$\tau_w = \mu(\frac{\partial u}{\partial y})_{y=\delta(x+b)^{\frac{1-n}{2}}}, \ q_w = -k(\frac{\partial T}{\partial y})_{y=\delta(x+b)^{\frac{1-n}{2}}}. \tag{3.19}$$

Now we have

$$C_f\sqrt{\mathrm{Re}} = 2\sqrt{\frac{n+1}{2}}f''(0), \ \frac{Nu}{\sqrt{\mathrm{Re}}} = -\sqrt{\frac{n+1}{2}}\theta'(0). \tag{3.20}$$

with $\mathrm{Re} = \frac{a(x+b)^{n+1}}{\nu}$ as the Reynolds number.

3.4 Numerical procedure

The set of non-linear dimensionless ordinary differential system are solved numerically builtin shooting technique.

3.5 Discussion

Here our intention is to predict the effects of shape parameter (n), inverse Darcy number (Da), Hartman number (Ha), local inertia coefficient parameter (β), stretching sheet parameter (λ), Prandtl number (Pr), thermophoresis parameter (Nt), Brownian motion parameter (Nb), Lewis number (Le), melting parameter (M) and surface thickness parameter (λ) on the velocity $f'(\zeta)$, temperature $\theta(\zeta)$ and concentration $\phi(\zeta)$.

3.6 Dimensionless velocity profile

Mostly study depend on shape parameter n due to its significance because this parameter can controls the surface shape, type of motion and deportment of the boundary layer. Here we notice that the outer shape of surface depends upon the value of shape parameter (n). For $n = 1$ the study is reduced to flat surface. For $n > 1$ the study transformed to surface with decreasing thickness and concave outer shape. However for $n < 1$ the study transformed to surface with increasing thickness and convex outer shape. In addition this parameter controls the type of motion such that for $n = 0$ the motion is reduced to linear with constant velocity. If $n < 1$ then motion is reduced to deceleration case whereas $n > 1$ corresponds to the accelerated motion. Characteristics of shape parameter (n) on velocity distribution $f'(\zeta)$ is shown in Fig. 1. The velocity increases for larger shape parameter(n). Figs. 2 and 3 describe the influence of thickness parameter α on $f'(\zeta)$ when $n < 1$ and $n > 1$. For $n < 1$ the velocity profile decrease for larger α. Physically it shows that stretching velocity decreases. It yields reduction in velocity distribution. However for $n > 1$ the velocity profile increases for larger α. Physically stretching velocity enhances for larger n. Here more deformation is produced in the fluid and thus velocity distribution increases. Fig. 4 illustrates the velocity distribution for various values of stretching sheet parameter. It is seen that velocity distribution enhances for larger stretching sheet parameter. Fig. 5 displays the velocity profile for larger values of melting parameter. Here we noticed that the $f'(\zeta)$ and boundary layer thickness increase through melting parameter (M). Fig. 6 explains the effect of inverse Darcy number (Da) on velocity profile $f'(\zeta)$. It is observed that an increase in value of (Da) decays the velocity $f'(\zeta)$. Fig. 7 gives the effect of velocity profile for different values of Hartman number (Ha). We observed that velocity profile reduces with an enhancement in Hartman number. Fig. 8 indicates the impact of inertial coefficient parameter(β) on $f'(\zeta)$. Clearly it is seen that the velocity profile $f'(\zeta)$ reduces for larger value of inertial coefficient parameter (β).

3.7 Dimensionless temperature profile

The variation of Hartman number on temperature profile is shown in Fig. 9. We observe that temperature is decreasing function of Hartman number (Ha). Fig. 10 depicts that variation of melting parameter (M) yield reduction in temperature. Fig. 11 gives the impacts of (Da) on temperature distribution. Here we noticed that temperature profile reduces when Da increased. Fig. 12 corresponds to the effect of thermophoresis parameter (Nt) on $\theta(\zeta)$. Here temperature is increasing function of Nt. In fact an increase in Nt causes an enhancement in the thermophoresis force which tends to move the nanoparticle from hot to cold area and thus as a result temperature increases. Fig. 13 represents the temperature for higher values of Prandtl number. Clearly $\theta(\zeta)$ increases for larger Prandtl number (Pr). Fig. 14 shows the temperature profile for larger values of (β). There is decrease in $\theta(\zeta)$ for larger β.

3.8 Dimensionless concentration profile

Fig. 15 gives the effect of Lewis number on nanoparticle concentrations $\phi(\zeta)$. An increase in the value of Lewis number shows low nanoparticle concentration. Lewis number depends on Brownian diffusion coefficient. Increasing Lewis number leads to lower Brownian diffusion coefficient which causes weaker nanoparticle concentration profile $\phi(\zeta)$. Fig. 16 represents the effect of Prandtl number on nanoparticle concentrations $\phi(\zeta)$. Nanoparticle concentrations $\phi(\zeta)$ is noted an increasing function of Prandtl number. The Brownian motion is random motion of nanoparticles suspended in the fluid resulting due to fast moving atom or molecules in the fluid.

Figs.17 and 18 represent the effects of Brownian diffusion coefficient (Nb) and thermophoresis parameter (Nt) on nanoparticle concentration $\phi(\zeta)$. Clearly $\phi(\zeta)$ near the surface decays when Nb increases. However the variation of Nb on $\phi(\zeta)$ away from the surface is opposite. Further the impact of Nt on $\phi(\zeta)$ is quite reverse to that of Nb. Fig. 19 describes the effect of inverse Darcy number (Da) on concentration distribution. It is observed that concentration profile enhances for larger Da. Fig. 20 shows the effect of (n) on concentration distribution. It is observed that concentration profile increases via n.

3.9 Dimensionless Skin friction coefficient (C_f) and Nusselt number (Nu)

Table (3.1) provides the numerical values of dimensionless skin friction coefficient for distinct values of physical parameter involved in velocity distribution. Here surface force increases for larger values of Ha, β, Da and α while skin friction coefficient decrease for larger M. Table (3.2) shows that the Nusselt number (Nu) enhances for larger Pr, Nt, α, λ and Le. On the other hand the local Nusselt number decays for larger values of Da.

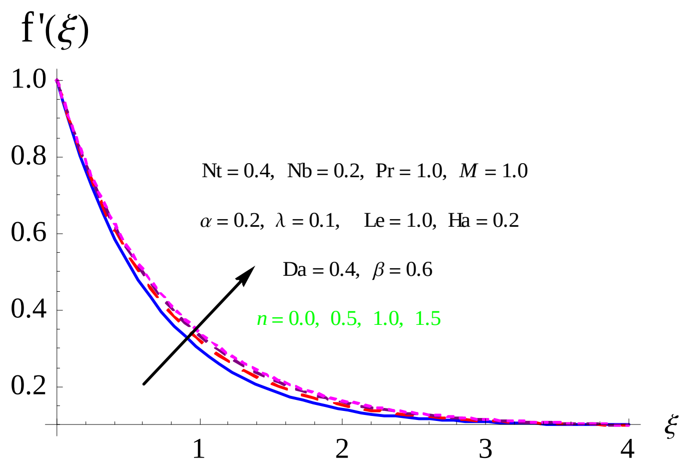

Fig. 1. Effect of n on $f'(\xi)$

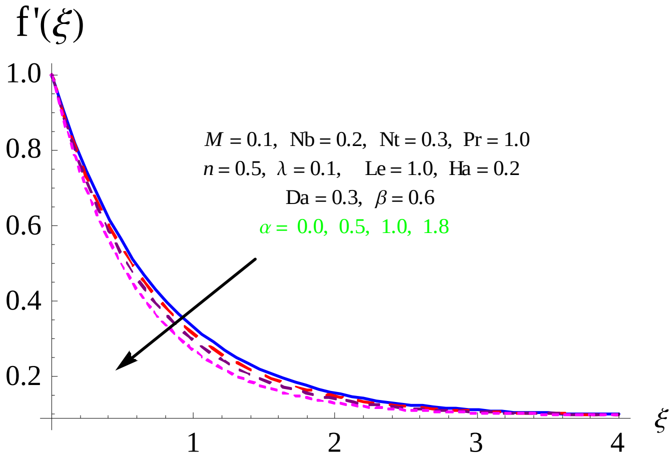

Fig. 2. Effect of α on $f'(\xi)$ for $n \prec 1$

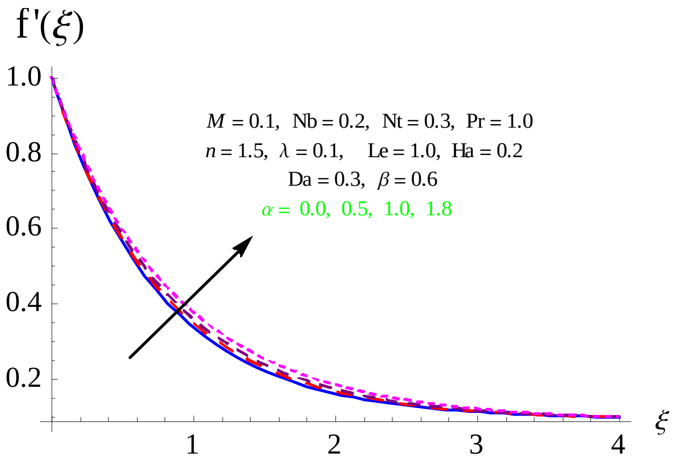

Fig. 3. Effect of α on $f'(\xi)$ for $n \succ 1$

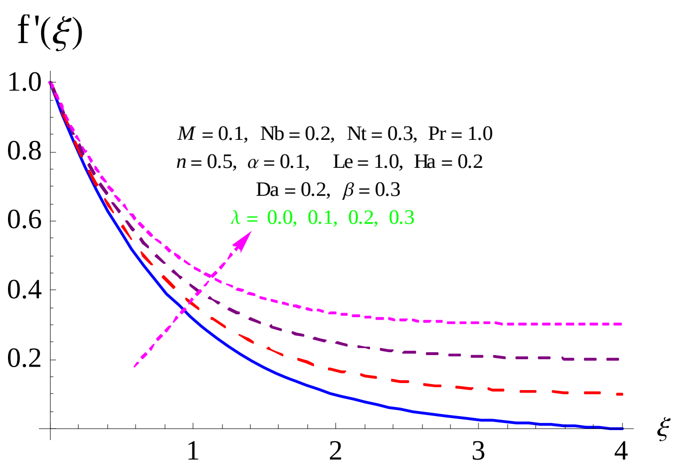

Fig. 4. Effect of λ on $f'(\xi)$

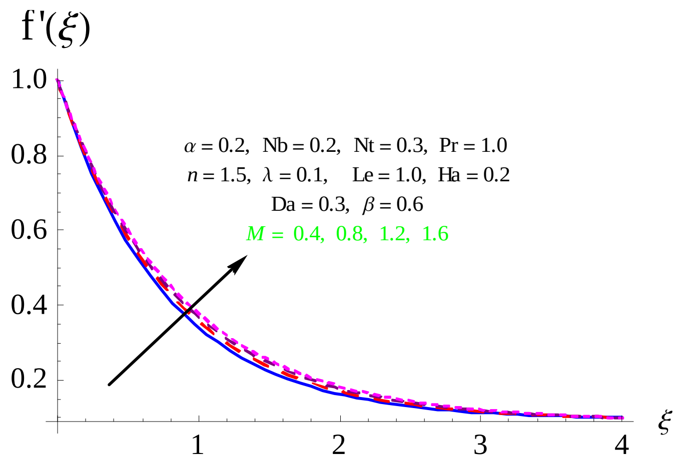

Fig. 5. Effect of M on $f'(\xi)$

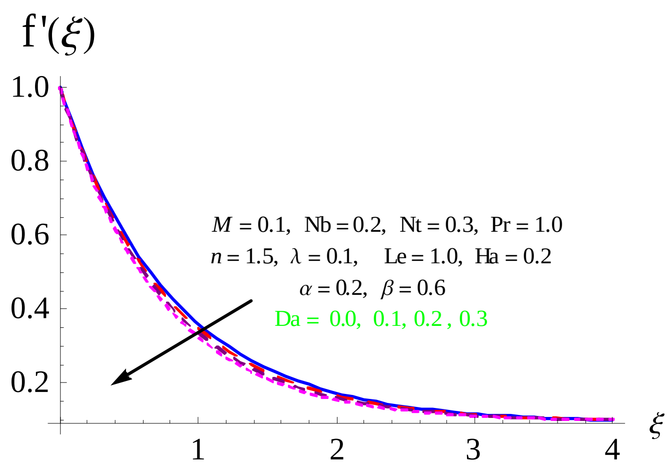

Fig. 6. Effect of Da on $f'(\xi)$

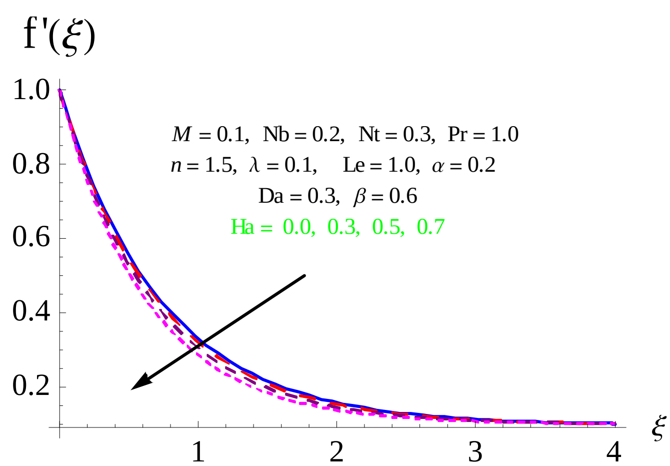

Fig. 7. Effect of Ha on $f'(\xi)$

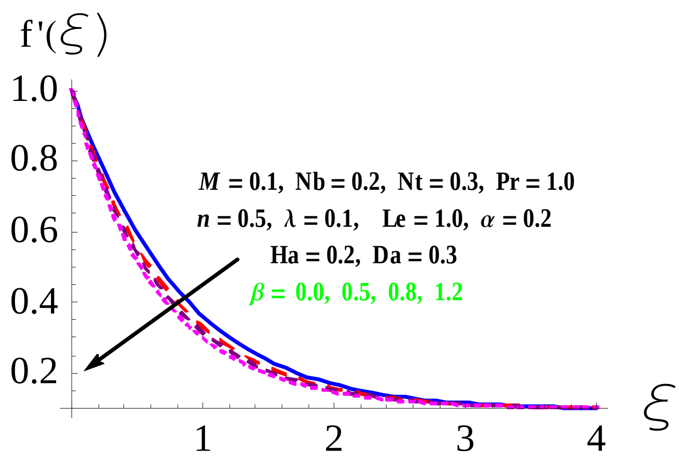

Fig. 8. Effect of β on $f'(\xi)$

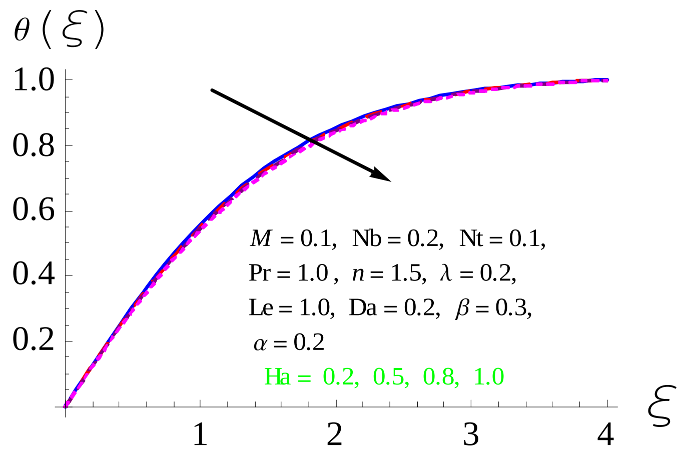

Fig. 9. Effect of Ha on $\theta(\xi)$

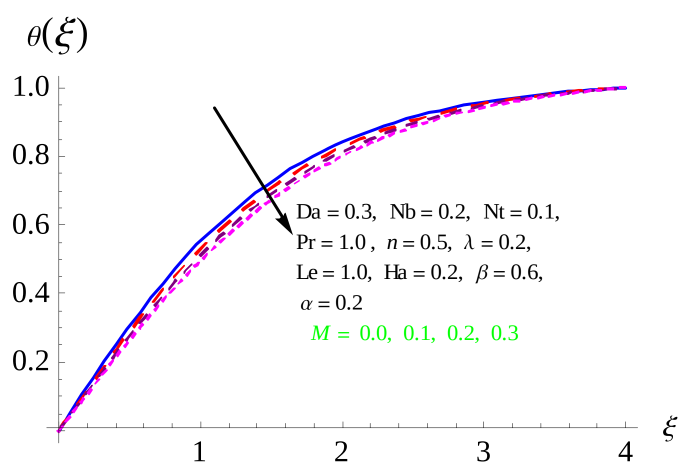

Fig. 10. Effect of M on $\theta(\xi)$

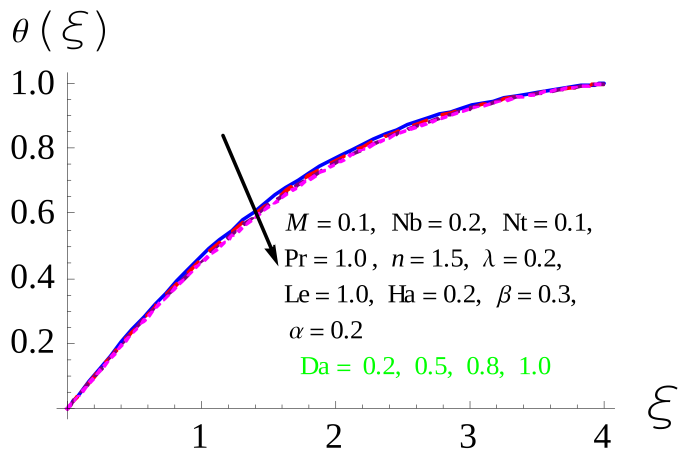

Fig. 11. Effect of Da on $\theta(\xi)$

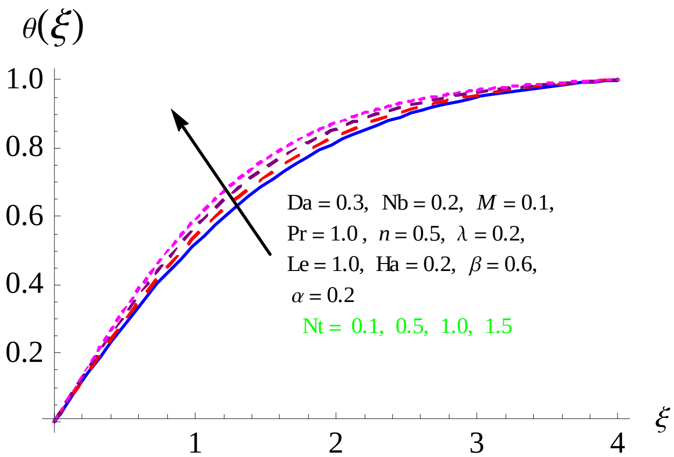

Fig. 12. Effect of Nt on $\theta(\xi)$

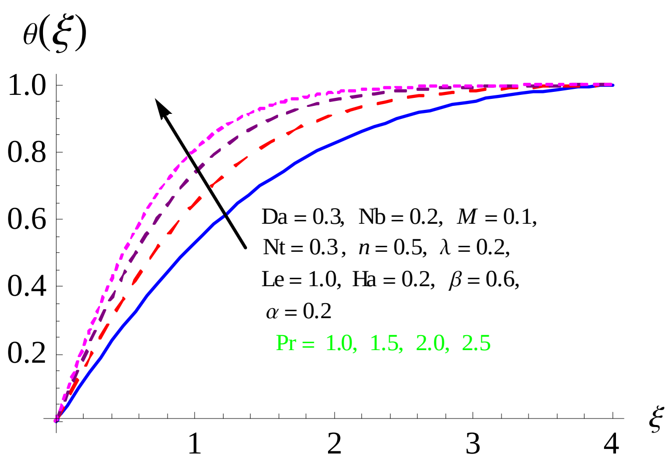

Fig. 13. Effect of Pr on $\theta(\xi)$

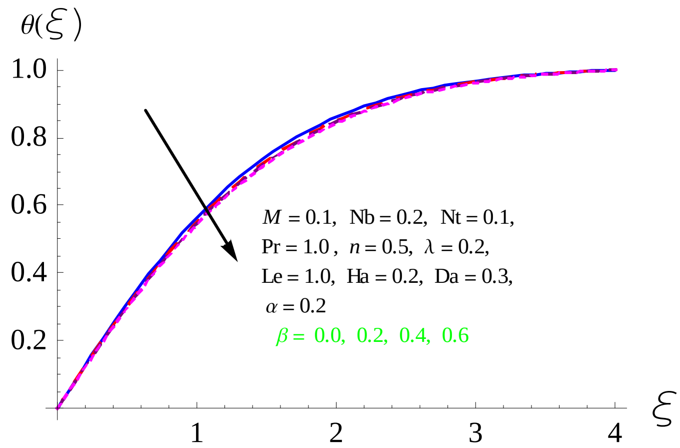

Fig. 14. Effect of β on $\theta(\xi)$

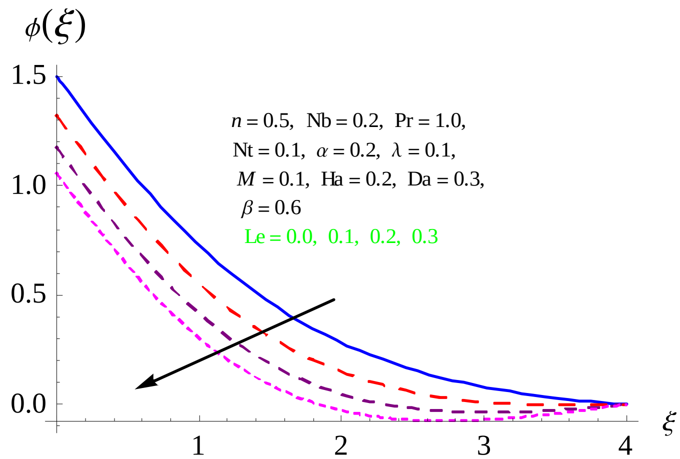

Fig. 15. Effect of Le on $\phi(\xi)$

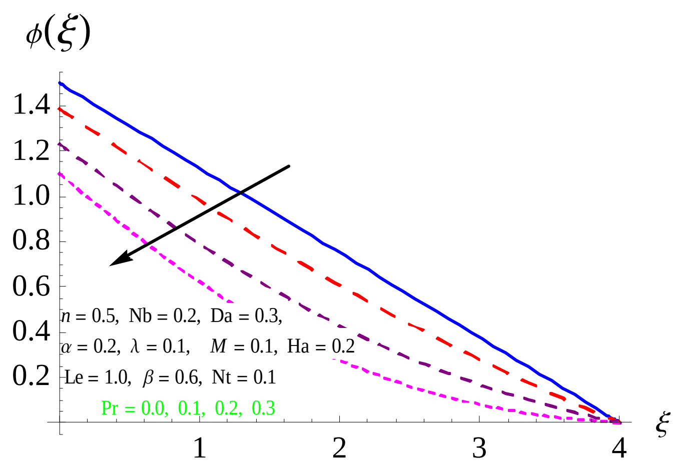

Fig. 16. Effect of Pr on $\phi(\xi)$

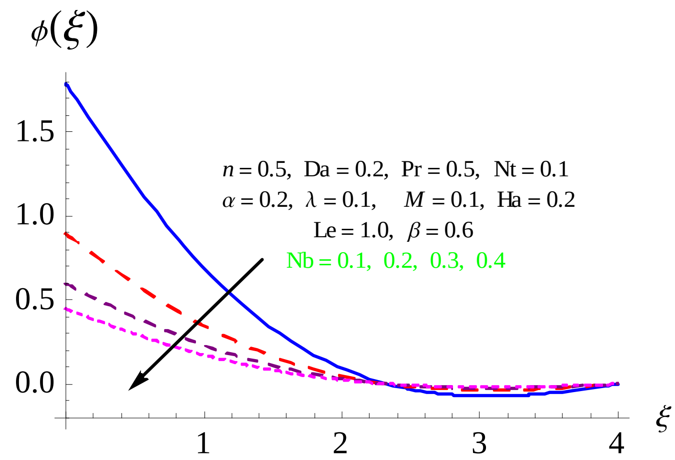

Fig. 17. Effect of Nb on $\phi(\xi)$

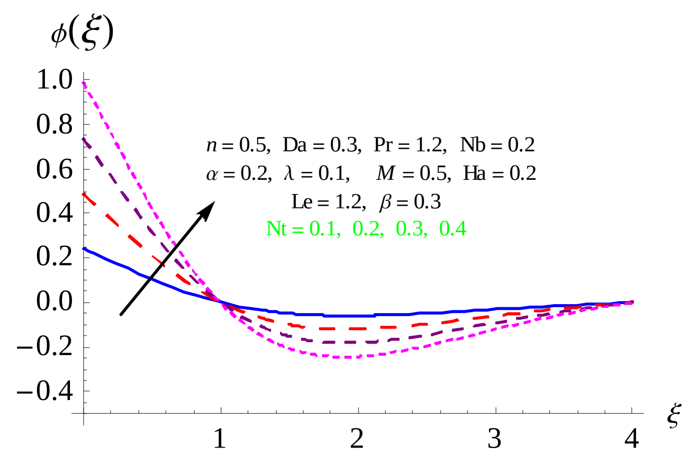

Fig. 18. Effect of Nt on $\phi(\xi)$

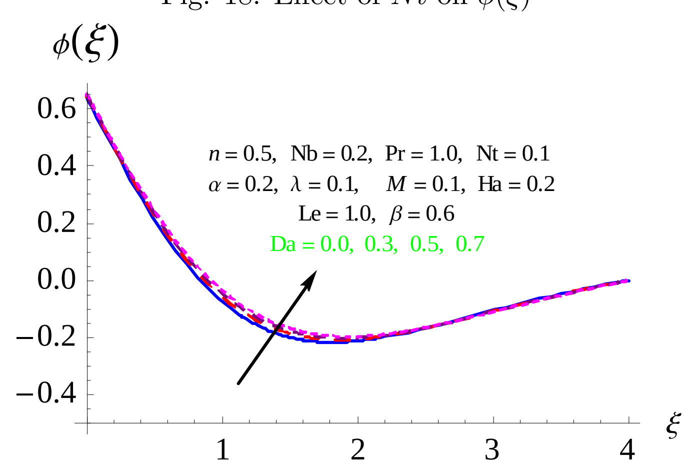

Fig. 19. Effect of Da on $\phi(\xi)$

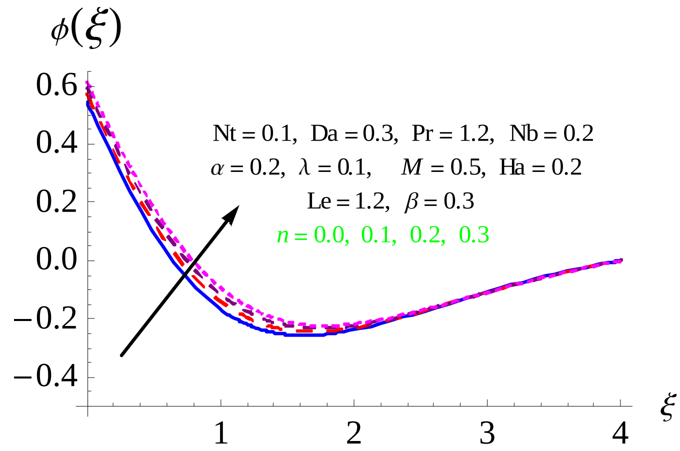

Fig. 20. Effect of n on $\phi(\xi)$

Table 3.1. Impacts of Ha, Da, β, α and M on skin friction coefficient, when Pr =1.0, n=0.5, Nb=0.2, Nt=0.3, λ =0.1, Le=1.0.

Ha	Da	β	α	M	$-C_f\sqrt{\text{Re}}$
0.0	0.3	0.6	0.2	0.1	1.8314
0.1					1.0868
0.2					1.0980
0.3					1.1164
	1.0				1.6088
	1.5				1.6877
	2.0				1.7600
		1.0			1.2092
		2.0			1.5992
		3.0			1.7615
			0.0		1.0742
			0.1		1.0860
			0.2		1.0980
				0.0	1.1217
				0.1	1.0980
				0.2	1.0077

Table 3.2:Impacts of Da, Pr, Nt, α, Le and λ on local Nusselt number when $M = 0.1$, $n = 0.5$, $\alpha = 0.2$, $\lambda = 0.2$, $Ha = 0.2$.

Da	Pr	Nt	α	Le	λ	$-\frac{Nu}{\sqrt{\text{Re}}}$
0.5						0.5322
1.0						0.4231
1.5						0.0479
	1.0					0.5424
	1.5					0.6992
	2.0					0.8416
		0.3				0.5542
		0.5				0.5660
		0.7				0.5777
			0.0			0.5077
			0.1			0.5250
			0.2			0.5424
				1.0		0.5424
				2.0		0.5454
				3.0		0.5472
					0.0	0.5068
					0.1	0.5239
					0.2	0.5424

3.10 Conclusions

MHD stagnation point flow of nanofluid in a non-Darcy porous medium is studied numerically. Melting heat transfer and variable thickness of surface are considered. Numerical simulation witnesses the following findings.

- Velocity and temperature profiles reduce for larger values of inverse Darcy number.
- Concentration has opposite results for inverse Darcy number when compared with velocity and temperature.

- There is enhancement of velocity for melting parameter.
- Decay in temperature is noted for melting parameter M, Hartman number Ha and local inertia coefficient parameter β.
- Concentration reduces for Prandtl number.
- Impact of Da on skin friction is opposite to that of Nusselt numbers.

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 Chapter 1 Basic definitions and equations 1.1 Introduction This chapter has been arranged through some definitions and equations which are useful for the next two chapters. 1.2 Some concepts about fluid 1.2.1 Fluid A material which continuously deforms

15**under the** action **of** applied **shear stress is** called **fluid. 1.** 2 **.2 Fluid mechanics The branch of** engineering dealing with **the fluids** either at rest **or**

in motion. Some common engineering fluids are air (a gas), steam (a vapor) and water (a liquid).

27**Fluid mechanics can be divided into** two categories: 1. **The study of fluids at rest**

is related to hydrostatics. 2. The study of fluids in motion is related to hydrodynamics. 1.2.3 Types of flow system Internal flow system Fluid flows surrounded by closed boundaries or surfaces are termed as internal flow systems. i.e flow through pipes, valves, ducts, or open channels etc. External flow system Flows having no specific boundaries are termed as external flows. Flow around aeroplane wings, automobiles, buildings or ocean water through which ships, submarines and torpedoes move/sail. Internal and external flow systems may be laminar, turbulent, compressible or incompress- ible. 1.2.4 Body force A force acting throughout the volume of a body without any physical contact. For example gravity, electromagnetic forces etc. 1.2.5 Inertial forces A force which resists a change in state of an object. 1.2.6 Surface force A force that acts across an internal or external element in a body through direct physical contact. Pressure and shear forces are examples of surface force. 1.2.7 Shear stress It is the stress component coplanar with the cross sectional area of a material. The internal forces that adjoining particles of a continuous material exert on each other is called shear stress. It is denoted by a Greek letter ?. 1.2.8 Normal stress The component of the stress normal to cross section of a material is called normal stress. It is denoted by ?. 1.2.9 Viscosity Viscosity of fluid is the measure of its internal resistance to deformation caused by tensile or shear stress. A fluid having no resistance to shear stress is called ideal or inviscid. Super fluids have zero viscosity. Viscosity depends on composition and temperature of a fluid. It is also called dynamic/absolute viscosity. It is denoted by ? and is given by ? = ????? ?????? ????? ?????? ? (1.1) Viscosity is measured in ???????? and is having dimensions of ? ? (1.2) · ?? ¸ 1.2.10

12**Kinematic viscosity The** proportion **of** absolute **viscosity to density of fluid is** recognized as **kinematic viscosity. It is** given **by**

? = ? ? ? It is measured in ?2? sec. It is having dimensions of (1.3) ?2 ? · ? ¸ (1.4) 1.2.11 Flow types 1) Laminar flow The flow pattern in which fluid flows in parallel smooth layers with no crossing over between the layers. Smoke rising from a cigarette to the first few centimeters in surrounding shows a clear picture of laminar flow. 2) Turbulent flow Turbulent flow is a less orderly flow regime, characterizes the pattern of the flow in which the fluid particles have no specific paths or trajectories. Break up of rising smoke into random and haphazard motion represents turbulent flow. 1.2.12 Newton's law of viscosity It states that ? ?? ∝ ?? ?? ? (1.5) ?? ? ?? = ? ?? ? (1.6) in which ? is the absolute viscosity. 1.2.13

24**Newtonian fluids** Those **fluids** which **obey Newton's law of viscosity are** termed as **Newtonian fluids. Water,** air **and** gases **are Newtonian fluids. 1.** 2.14 **Non-Newtonian fluids** Those fluids for **which** does **not**

hold

25**Newton's law of viscosity are termed as non-Newtonian**

or we can say that those fluid in which shear stress is non-linearly proportional to shear strain. Since viscosity depends upon nature of the fluids so for such fluids viscosity remains no more constant and it becomes a function of applied shear stress. Examples of such fluids consist of honey, toothpaste, ketchup, paint etc. Mathematically ? ?? ∝ ?? ? ?? ? ? =6 1 (1.7) µ ?? ¶ ? ?? = ? ?? ? (1.8) µ ?? ?−1 ¶ ? = ? ?? ? (1.9) µ ¶ Here ? represents the

15**apparent viscosity? ? flow behavior index and ? consistency index. . Non-Newtonian fluids are**

divides into three types; 1. Rate type fluids. 2. Integral type fluids. 3. Differential type fluids. 1.3 Mechanisms of heat transfer Heat transfer occurs from region of higher kinetic energy towards the region of lower kinetic energy or simply from hotter to colder regions. Temperature difference in between the two regions is the main reason for heat transfer. Heat transfer from one place to another by different processes, depending upon the nature of material under consideration. 1.3.1 Conduction Particles are in constant motion. During motion of the particles, collisions occur, which result in the exchange of kinetic energies. So the process of heat transfer from one place to another due to collision of molecules/particles is called conduction. Such type of heat transfer is observed in solids e.g. heated rod at one end, here no material flow occurs physically. 1.3.2 Convection It is the process of heat transfer during which particles having more kinetic energies replace the particles with less kinetic energies. The transfer of heat occurs due to random Brownian motion of individual particles from hotter to colder region and this process is known as convection. 1.3.3 Types of convection Natural convection The transfer of heat that occurs only due to temperature difference is called natural convection. Dense particles of fluid fall, while lighter particles rise, resulting the bulk motion of fluids. Natural convection can only occurs in the presence of gravitational force. 8 Forced convection To increase the rate of heat exchange, fluid motion is generated by external surface forces such as fan, pump etc. Force convection is more efficient than natural convection. Mixed convection The process in which heat transfer occurs due to both natural and forced convection is termed as mixed convection. 1.3.4 Radiation The transmission of heat energy in the form of electromagnetic waves with or without any medium is termed as radiation. Examples are fire, sunlight and very hot objects etc. 1.4 Some concepts about heat 1.4.1 Heat source/sink Any object which produces and emits heat is termed as a heat source and any object that dissipates heat is termed as a heat sink. Nuclear reactors, sun, fire and electric heaters are heat sources while semiconductors, light emitting diodes, fans and solar cells are examples of heat sink. 1.4.2 Thermal conductivity Therm means heat and conduction is transfer by collision of molecules. So it is measure of the ability of material to transmit heat from one place to another. Or

> 8**it can also be** characterized **as the** heat **transfer through a unit thickness of a**

substance in a direction perpendicular to surface area due to unit temperature gradient. It is denoted by ? and is given by ? = ?∆? ?? ? (1.10) where ? denotes heat flow in a unit time, ? is area of cross section and ∆? denotes change in temperature. In SI system thermal conductivity is measured in ?????, having dimensions of ? ? ? 3? ? (1.11) · ¸ 1.4.3 Thermal diffusivity

> 49**It is measure of the** capacity **of a material to conduct heat** energy **with**

respect to its capacity to store heat energy. Thermal diffusivity is measured in ?2? sec? denoted by ? and is given by ? = ? ? (1.12) ??? where ? is

> 73**thermal conductivity,** ?? is **specific heat and** ? is **density. Thermal** diffusivity measures **the**

property of a material for unsteady heat conduction. This illustrate how rapidly a substance responds to change in temperature. 1.4.4 Specific heat It is an extensive property of a material defined as the amount of heat energy required to enhance temperature of the material by 10?? It is measured in ?? ? having the dimensions of ?2? ? (1.13) · ? 2? ¸ 1.4.5 Newton's law of heating The

> 44**rate of heat loss of a body is directly** related **to** temperature **difference between the body and**

its surrounding. 1.4.6 Fourier's law of

> 78**heat conduction It is** defined as **the** rate **of heat**

conduction through a homogeneous material is directly propor- tional to negative gradient of temperature. Mathematically ? ∝ −∆? ? ? = −?∆? ? (1.14) (1.15) ? is heat flux density, which is the amount of energy flow through a unit area per unit time. ? is measured in ??2 , ? is material's thermal conductivity and ∆? is temperature gradient. 1.5

> 31**Porous medium A** substance **which** is composed **of a solid matrix with** interrelated **pores is called a porous medium.** These pores permit **the** flow **of** fluids through **the material.**

The appropriation of pores is unpredictable in nature. The examples of porous medium comprises beach sand, human lungs, rye bread and wood, act. 1.5.1 Porosity By

> 21**porosity ? of a porous medium we mean the** part **of the** associated **void area to the total area of the**

substance. It means that 1 − ?

> 21**is the fraction that solid area** involves **the material.**

1.6 Dimensionless parameters 1.6.1 Prandtl number Conduction and convection occur in fluids. For the process of heat transfer, temperature difference is the main cause. Rate of conduction and convection vary in different fluids. Prandtl number is used to determine the domination of either conduction or convection.

> 22**It is defined as the ratio of** momentum **to** thermal diffusion rate. **It is denoted by**

Pr and given by Pr = ? ? ? (1.16) where ? is kinematic viscosity and ? is thermal diffusion rate. For many gases over a vast range of temperature and pressure, Pr is approximately treated as constant. 1.6.2

12**Lewis number** It **is** defined **as the** ratio **of thermal diffusivity to** Brownian diffusivity. **Mathematically, it can be** written **as** ?? = Brownian diffusivity ?? **thermal diffusivity**

= ? (1.17) Note that in above expression ? signifies the thermal diffusivity and ?? represents the Brownian diffusivity. 1.6.3 Hartman number A dimensionless parameter used to determine relative importance of drag force resulting from magnetic and viscous forces. It is expressed as the ratio of magnetic to viscous forces given by ???????? ?????? ? ? = ??????? ?????? = ?0? ? ? (1.18) r where ?0 shows characteristic value of magnetic induction, ? is characteristic length and ? is dynamic viscosity coefficient. 1.6.4 Brownian motion parameter The haphazard movement of suspended nanoparticles in a base fluid is called as Brownian motion. Mathematically ??? ?? = ? (?? − ?∞)? where ? the

43**ratio of nanoparticles** effective **heat capacity and fluid heat capacity,** ?? the **brownian diffusion coefficient,** ? the kinematic viscosity **and** ?? and ?∞ **the** surface and ambient **concentration**

respectivelly. 1.6.5 Thermophoresis parameter Thermophoresis is the phenomenon in which the influence of temperature gradient results in a diffusion of particles is called as thermophoresis. Mathematically ??= ????∞?(??−?∞)? (1.19) where ?? is the thermophoretic diffusion coefficient and ?? represents the wall temperature, ?∞ stands for ambient fluid temperature. 1.6.6

11**Skin friction coefficient When fluid is passing across a surface** then definite **amount of drag** arises which **is**

termed as skin friction. Mathematically it is stated as ?? = 12 ????2 ? ? ? (1.20) where ?? ?? = ? μ ??¶?=?(?+?)1−2? ? (1.21) in above relation ? ? represents the shear stress at the surface, ?? stands for velocity and ? denotes the fluid density. 1.6.7

41**Nusselt number The** nondimensional **quantity which** denotes **the convective to conductive heat transfer** ratio **is** known **as Nusselt number**

(? ?). In mathematical form one can write ? ? = ??? ? (1.22) ?(?? − ?∞) where ?? ?? = −? μ ?? ¶?=?(?+?)1−2? ? (1.23) where ?? denotes the wall heat flux, ?? represents the wall temperature, ?∞ stands for ambient fluid temperature and ? represents the thermal conductivity 1.6.8 Sherwood

22**number It is defined as the ratio of the** convective mass transfer **to the** rate **of**

diffusive mass transport. Mathematically ?? = (? + ?)?? ??(?? − ?∞) ? (1.24) where ?? = −?? ?? ?? ¶?=?(?+?)1−2? ? (1.25) μ where ?? the mass flux. 1.6.9 Reynolds number A dimensionless parameter used to predict flow pattern for various fluid flow circumstances. It measures the ratio of inertial to viscous forces and

63**relative importance of** the **two forces for given flow conditions.**

It is denoted by Re named after Osborne Reynolds, and is given as Re = ???????? ????? ??????? ????? ? (1.26) Re = ??? ?? ? = ? ? (1.27) ? is maximum velocity, ? is length of geometry. For low Reynolds number, viscous

10**forces are more dominant than** inertial **forces,** laminar **flow**

occurs which characterizes a constant and smooth fluid motion. Similarly, for higher Reynolds number, inertial

10**forces are more dominant than viscous forces,** hence turbulent **flow**

occurs. 1.7 Fundamental laws 1.7.1 Mass conservation law It states that the total mass in any closed system will remain constant. Mathematically ?? ?? + ?∇?V = 0? (1.28) or ?? ?? + (V?∇) ? + ?∇?V = 0? or ?? ?? + ∇? (?V) = 0? (1.29) (1.30) The above expression represents the continuity equation. Here ? stands for fluid density and V depicts the velocity profile. Due to steady state flow Eq. (1?30) takes the following form ∇? (?V) = 0? (1.31) and if the fluid is incompressible then above equation takes the following form ∇?V = 0? (1.32) 1.7.2 Momentum conservation law This law states that the total momentum of a closed system is conserved. General form of this law is given below ? ?V ?? = ∇?τ +?b? (1.33) Here inertial, surface and body forces are indicated by the terms ? ??V? , ∇?τ and ?b respectively. 1.7.3 Energy conservation law The energy equation for nanofluid can be written as ???????? =τ?L+?∇2?+???? ??∇?? ∇?+?? ? ∇? ?∇? ? (1.34) μ ∞ ¶ where ?? stands for base fluid density, ?? represents the specific heat of base fluid, ? stands for temperature, τ represents the stress tensor, L stands for rate of strain tensor, ? denotes the thermal conductivity, ?? depicts the density of nanoparticles, ?? denotes the thermophoretic diffusion coefficient and ?? stands for Brownian diffusion coefficient. 1.7.4 Concentration law The concentration equation for nanofluid can be written as ???? = ??∇2? + ??∞? ∇2? (1.35) where ? stands for nanoparticles concentration, ?? represents the Brownian diffusion co- efficient, ?? stands for thermophoretic coefficient and ? depicts the temperature. 1.8 Solution method 1.8.1 Homotopic technique HAM technique is one of the best and simplest techniques for obtaining convergent series solution for weakly as well as strongly non-linear equations. This method uses the concept of homotopy from

45**topology. Two functions are homotopic if one function continuously** deforms **into other function,** e.g. **?1(?) and**

?2(?) are two functions and ? ∗ is a continuous mapping, then ? ∗ : ? × [0? 1] → ? (1.36) such that ? ∗(?? 0) = ?1(?) (1.37) ? ∗(?? 1) = ?2(?) (1.38) Liao in 2012 [36] used the homotopic technique for obtaining convergent series solution. HAM distinguishes itself from other techniques in the following ways: 1. It is independent of small/large parameter. 2. Convergent solution is guaranteed. 3. Freedom for the choice of base functions and linear operators.

11**Consider a non-linear differential equation N [?(?)] = 0? where N** represents **non-linear operator, and ?(?) is** the **unknown function.**

Using parameter ? ∈ [0? 1]? a system of equations is constructed. (1 − ?) L [? (?; ?) − ?0 (?; ?)] = ?~N ? (?; ?) ? (1.39) (1.40) where L is linear operator, ? is embedding parameter and ~ is convergence parameter. By b b putting value of ? from 0 to 1, unknown function gets the value from ?0(?) to ?(?). ?−1 ? (?; ?) = ?0 (?) + ?? (?) ??? 1 ??? (?; ?) X?=0 ?? (?) = ?! ??? |?=0 b for ? = 1 we get b ∞ ? (?) = ?0 (?) + ?? (?) ? ?X=1 For mth order problems we have L? [?? (?) − ????−1 (?)] = ~?R?? (?) ? R?? (?) = (? − 1)! 1 ??N?? (?; ?) ??? |?=0 ?? = 0? ⎫ ⎧ 1? ⎩ b ?≤1 ? ??1 (1.41) (1.42) (1.43) (1.44) (1.45) Chapter 2 Flow due to

20**moving surface with non-linear velocity and variable thickness**

2.1 Introduction This chapter reports the development of homotopy solution for steady flow of an incompress- ible viscous nanofluid. Flow due to non-linear stretching surface subject to variable thickness analyzed. This paper is detailed review study of work by wahed et al. [32] Nonuniform heat generation ?(?) and nonuniform transverse magnetic field ?(?) are considered.

29**Appropriate transformations reduce the partial differential systems** into **the ordinary differential systems.** The resulting systems **are**

solved by homotopy technique. The results for

26**velocity, temperature and concentration are** established **and discussed 2.** 2 Statement **Consider steady** laminar **two-dimensional**

incompressible viscous material over a an imperme- able stretched sheet with variable thickness. Sheet is stretched with a velocity ?? = ? (? + ?)?. Thickness of the sheet is taken variable, i.e ? = ? (? + ?) 2 . We assume coefficient ? so small 1−? that the surface is sufficiently thin. The problem is considered for ? 6= 1. The velocity field of the present flow problem is given by

4**V = (u(x? y)? v(x? y)? 0)? (2.** 1) Here **u and v are the** velocities components **in x and y directions. The Cauchy stress tensor for viscous** incompressible **fluid is defined** by **τ = −? I+?** A1? **(2.**

2) In Eq. (2?2), ? denotes

19**the pressure, I** represents **the identity tensor and A1 the Rivlin** Ericksen **tensor** which is expressed **as A1 = grad V + (grad V)?**

? (2.3) in which

19**?u ?x grad V = ⎡ ??vx ⎢** 0 **⎣ ?u ?x (grad V)? = ⎡ ??uy**

⎢ 0 Thus we have from ???(2

40**?3), ⎣ ?u** ?y **?v ?y** 0 **?v ?x ?v**

4?y 0 0 0 ⌉? 0 ⌊⌋ 0 0 ⌉? 0 ⌊⌋ 2 ??ux ??vx + ??uy 0 A1=⌈ ??vx + ?u ?y 2??vy 0 ⌉? ⌊ 0 0 0

⌊ The governing equations for an incompressible nanofluid are given by ⌊⌋ (2.4) (2.5) (2.6) ∇?V = 0? ? ?? ??¯ = div τ +? × ?? (??)? ???? = ?∇2? +(??)? ∇? ?∇? · ??∇?∇?+?? ?∞ +?(?)(? −?∞)? ¸ ???? =??∇2?+?? ∞?∇2?? Using the boundary layer approximation the above equations become ?? ?? ?? ?? = 0? + ? ?? ?? ?2? ??2(?) ?? + ? ?? = ? ??2 + ? ?? ? ?? ?? ?2? ?? ?? ?? ?? ?? + ? ?? = ? ?? + ? ?? ?? ?? ?∞ ?? + (+ ? (?) · ??? (? − ?∞)? ? ???? + ? ???? = ?? ??2??2 ??∞? (??2??2 +)? (2.7) (2.8) (2.9) (2.10) (2.11) (2.12) (2.13) with boundary conditions ? = ??? ? = 0? ? = ??? ? = ??? at ? = ?(? + ?)1−2? (2.14) ? = 0? ? = ?∞? ? = ?∞ as ? → ∞?

42In the above expressions ? and ? are velocity components in ? and ? directions respectively, ? the kinematic viscosity, ? the

electrical conductivity, ?(?) = ?0(? + ?) 1−2?

2the strength of magnetic field, ?? the base fluid density, ?? the

density of nanoparticles, ?(?) = ?0(? + ?)?−1 the heat generation, ?

38the temperature of fluid, ?∞ the temperature above the boundary layer, ?? temperature of surface, ? the

60effective heat capacity and heat capacity of nanoparticle and base fluid respectivelly, ? the

thermal diffusion, ? the nanoparticles concentration, ?? the

37Brownian motion coefficient and ?? the thermophoretic diffusion coefficient. Using the

following similarity transformations ?= (? + 1)? (? + ?)?−1 s 2?? (? + ?)?+1 2? ?? ?(?? ?) = s ? + 1 ? (?) ? ??(?) = ?(? + ?)?? ?(?) = ???−−??∞∞ ? ?(?) = ???−−??∞∞ ? and expressing ? and ? in terms of stream function ?(?? ?) we have: ?? ?? ? = ?? ? ? = − ?? ? We adopt ?=?(?+?)??0(?)? ?=− (?+1)??(?+?)?−1 s 2 ?(?)+? ?−1?0(?) ? μ ?+1 ¶ (2.15) (2.16) (2.17) where ? and ? are constant, ? the similarity variable and ? the stream function. Using the above equations, conservation law of mass is satisfied and the resulting equations (2?11) − (2?14) are as follows ? 000 + ? ? 00 − (2?)? 02 − (2)? ? 0 = 0? ? + 1 ? + 1 (2.18) 1 Θ00 + ? Θ0 + (2)?Θ + ? ?Θ0Φ0 + ? ?Θ02 = 0? Pr ? + 1 (2.19) Φ00 + Pr ??? Φ0 + (? ?)Θ00 = 0? (2.20) ?? with associated boundary conditions by ?(?)=?(11−+??)? ?0(?)=1? Θ(?)=1? Φ(?)=1? ?0(∞)=0? Θ(∞)=0? Φ(∞)=0? (2.21) Here prime denote differentiation with respect to (?)? For converting the domain from [?? ∞) to [0? ∞), we use another transformation. Where ? = ? 1+2? ?? is the surface thickness parameter and ? = ? = ? 1+2??? indicates the plate surfaceq.We defined ?(?) = ?(?−?) = ?(?)? Θ(?)=?(?−?)=q?(?)andΦ(?)=?(?−?)=?(?)thereforethesimilarityEqs. (2?18)−(2?20) and the associated boundary condition (2?21) becomes ?000 + ??00 − (2?)?02 − (2 ? + 1 ? + 1)??0 = 0? 1 ?00+??0+(2)?? + ???0? 0 + ???02 = 0? Pr ? + 1 ?00 + Pr ??? ?0 + (? ?)?00 = 0? ?? ?(0)=?(11−+??)? ?0(0)=1? ?(0)=1? ?(0)=1? ?0(∞)=0? ?(∞)=0? ?(∞)=0? (2.22) (2.23) (2.24) (2.25) Here ? the shape parameter, Pr the

5prandtl number, ?? the lewis number, ? the magnetic field parameter, ? the heat source parameter, ? ? Brownian motion parameter and ? ? the thermophoresis parameter. The

definitions of these parameters are Pr = ? ??= ? ?= ? ? ? ?? ?0?? ?= ?0 ? ??=???(??−?∞)? ?? ???? ? ?? = ??∞(??−?∞)? ?=(???)? ??? (???)? ? (2.26)

62The Skin friction (???), Nusselt number (??) and Sherwood number (??) are

?? ? = ? ? ? (2.27) ???2 ?? = (? + ?)?? ?(?? − ?∞) ? (2.28) ?? = (? + ?)?? ??(?? − ?∞) ? (2.29) Here (?? = ?(????)?=?(?+?)1−2?) the surface shear stress, (?? = −?(????) ?=?(?+?) 2 1−?)

47the rate of heat transfer and (?? = −?? (????)?=?(?+?) 1−?) the rate of mass transfer. 2 Finally the non-dimensional form of

75Skin friction, Sherwood number and Nusselt number are

given below ????? = ? ? + 1?0(0)? (2.30) n ?? r ? √ ? = − ? + 1?0(0)? ? (2.31) ? r √ ?? ? = − ? + 1?0(0)?

given below ? ? ? ? ? − 2 ? ? 1 ? ?0(0)? (2.30) p ? ? ? 2 ? ? − ? ? 1 ? ?0(0)? 2 (2.31) ? ? ? ? ? − ? ? 1 ? 0(0)? (2.32) ? r 2 2.3 Solution methodology The approximate

1**series solutions through the homotopy analysis** technique **(HAM)** require **the appropriate initial** approximations **and auxiliary linear operators for**

the governing problems which are given in the following forms ?0(?) = ?(? − 1

7)**1 − exp(−?)? ?0(?) = exp(−?)? ?0(?) = exp(−?)?** ? + 1 (2.33) **L?∗ = ??3??3 − ???? L∗? = ??2??** 2 − ?? **L∗? = ??2??2 − ?? ? The** above **linear operators have the following properties L?∗ [?1 + ?2 exp(?) + ?3 exp(−?)] = 0? L?∗ [?4 exp(?) + ?5 exp(−?)] = 0?** ⎞ **L?∗ [?6 exp(?) + ?7 exp(−?)] = 0?**

⎨ ⎪ ⎪ ⎭ (2.34) (2.35) in which ?? (? = 1 − 7) stands for arbitrary constants. 2.3.1 Zeroth-order statement

14**(1 − Þ∗)L∗? ?˘(?? Þ∗) − ?0(?) = Þ∗~?N?∗[?˘(?? Þ∗)]?**

(2.36)

14**(1 − Þ∗)L∗? ˘?(?? Þ∗) − ?0(?) = Þ∗~?N?∗[?˘(?? Þ∗)? ˘?(?? Þ∗)?** ˘?(?? Þ∗)]? **h i**

(2.37)

14**(1 − Þ∗)L∗?[?˘(?? Þ∗) − ?0(?)] = Þ∗~?N?∗[?˘(?? Þ∗)? ˘?(?? Þ∗)?** ?˘(?? Þ∗)]? **h i**

(2.38) ?˘(0?Þ∗) = ?(??+−11)? ?˘0(0?Þ∗) = 1? ?˘0

48**(∞?Þ∗) = 0?** ˘?(0 **?Þ∗)** = 1? **?˘(∞?Þ∗) = 0?** ?˘(0 **?Þ∗)** = 1? **?˘(∞?Þ∗) = 0?** ⎞ **(2. 39)** ⎩ N?∗ ?˘(?; **Þ∗)** = ??3??3˘ + ?˘??2 **??2˘ − ?2+?**

1 Ã ????˘!2 ? + 1 ? ????˘0 ⎠ − 2 ? (2.40) h i N?∗ ?˘(?;Þ∗)??˘(??Þ∗)??˘(??Þ∗) = ??2?˘?2+Pr?˘??˘?? +Pr????˘?????˘?+Pr?? ?˘? 2 h i Ã??! +(2) Pr ? ?˘? ?+1 ? ? ? (2.41) N?∗ ?˘(?;Þ∗)?˘?(??Þ∗)??˘(??Þ∗) = ??2??˘2 +??Pr?˘??˘?? ??????2?˘?2 + ? (2.42) h i Here Þ∗ ∈ [0?1] denotes the embedding parameter, (~?? ~??~?) stand for nonzero auxiliary variables and (N?∗? N?∗?N?∗) represent the nonlinear operators. 2.3.2 ?ˆth-order statement L?∗ [??ˆ(?) − ??ˆ??ˆ−1(?)] = ~? R˘∗?ˆ? (?)? L?∗ [??ˆ(?) − ??ˆ ??ˆ−1(?)] = ~? R˘∗?ˆ? (?)? L?∗ ??ˆ(?) − ??ˆ??ˆ−1(?) = ~?R˘∗?ˆ?(?)? ??ˆ

16**(0) = ??0ˆ(0) = ??0ˆ(∞) = 0? ?0? ˆ (0)** + ??ˆ(∞) = **0? £ ¤ ?0? ˆ (0)** + ??ˆ(∞) = **0?** ⎧ ⎞ ?ˆ **−1** ?ˆ **−1** R˘∗?ˆ?(?) = ??0ˆ00 **−1** ⎠ + (??ˆ−1−???00) − **2?** X **?=0** µ ? + **1** ¶ ?X **=0 (??0ˆ−** 1 **−???0) − ? + 1 ?(??0ˆ−1)? 2** µ ¶ **(2.** 43) **(2.** 44) **(2.** 45) **(2.**

46) (2.47)

17**?ˆ−1 ?ˆ−1** R **˘?∗ˆ?(?)** = ??0ˆ0 **−1+** Pr **(??ˆ−1−???** 0)+Pr?? **(?0?ˆ−** 1 **−???0)+** Pr?? X **?=0** X **?=0 ?ˆ−** 1 **(?0?ˆ−** 1 **−??0?)+** ? +2 1 Pr **?(??ˆ−** 1)? X **?=0**

µ (2.48) ¶ R

55˘∗**?ˆ?(?)=?0?ˆ0−** 1(?)+??Pr **?ˆ−1(??ˆ−1−??0?)+** ?????0 **?ˆ0−1?**

(2.49) X?=0

17**0? ?ˆ ≤ 1? χ ?ˆ=** ⎧ (2.50) ⎫ **1? ?ˆ ? 1?** Putting Þ∗ = **0**

and Þ∗ = 1 we have ⎩ ?˘(?;0) = ?0(?)? ?˘(?;1) = ?(?)? (2.51) ?˘(??0) = ?0(?)? ˘?(??1) = ?(?)? (2.52) ? ˘(??0) = ?0(?)? ?˘(??1) = ?(?)? (2.53) When Þ∗ varies from 0 to 1 then ?˘(?;Þ∗)? ˘?(??Þ∗) and ?˘(??Þ∗) illustrate variation

6**from the initial approximations ?0(?)?** ?0(?) **and ?0(?) to the** desired **final solutions ?(?)? ?(?) and ?(?) respectively.** By using **Taylor's** series expansion **we**

get the following expressions ∞ ?˘(?; Þ∗) = ?0(?) + ??ˆ(?)Þ∗?ˆ? ??ˆ(?) = ?ˆ1! ??ˆ??˘Þ(?∗??ˆÞ∗) ¯¯Þ∗=0 ? (2.54) ?Xˆ=1 ¯ ∞ ??ˆ(?)Þ∗?ˆ? ??ˆ(?) = ?ˆ1! ??ˆ?˘?Þ(?∗??ˆÞ∗)¯¯ ∗ ¯ ?˘(?? Þ∗) = ?0(?) + ? (2.55) ?Xˆ=1 ¯Þ =0 ?˘(?? Þ∗) = ?0(?) + ?Xˆ

68=1 ??ˆ(?) Þ ∗?ˆ? ??ˆ(?) = ?ˆ1! ??ˆ??˘ Þ(?∗??ˆÞ∗) Þ ∗=0 ∞ ¯ ? (2.

56) The convergence of Eqs. (2?54) − (2?56) strongly depends upon them (~??¯¯~?? ~? ¯). Considering that (~?? ~?? ~?) are chosen in such a way that Eqs. (2?54) − (2?56) converge at Þ∗ = 1, then ∞ ?(?) = ?0(?) + ??ˆ(?)? ?ˆX=1 ∞ ?(?) = ?0(?) + ??ˆ(?)? ?Xˆ=1 (2.57) (2.58) ∞ ?(?) = ?0(?) + ??ˆ(?)? (2.59) ?Xˆ=1 The general solutions (??ˆ? ??ˆ? ??ˆ) of the Eqs. (2?43) − (2?45) in terms of special solutions (??∗ˆ? ??∗ˆ? ?∗?ˆ) are presented by the following expressions

66??ˆ(?) = ??∗ˆ(?) + ?1 + ?2 exp(?) + ?3 exp(−?)? (2.60) ??ˆ(?) = ?∗?ˆ(?) + ?4 exp(?) + ?5

exp(−?)? (2.61) ??ˆ(?) = ?∗?ˆ(?) + ?6 exp(?) + ?7 exp(−?)? (2.62) in which the constants ?? (? = 1 − 7) through the boundary conditions (2?46) are given by ?2 = ?4 =

56?6 = 0? ?3 = ????∗ˆ?(?) ?=0 ? ?1 = −?3 − ??∗ˆ(0)? ?5 = −??∗ˆ(0)¯?=0? ¯ ?7 = −?∗?ˆ(0)?=

0? (2.63) (2.64) (2.65) 2.4 HAM solution 2.4.1 Convergence analysis Here the series solutions (2?57) − (2?59) involve the auxiliary parameters (~?? ~?? ~?)? Surely the auxiliary variables (~?? ~?? ~?) in series solutions accelerate the convergence. To choose the appropriate values of (~?? ~?? ~?) the ~−curves have been displayed at 25?? order of ap- proximations. It is noticed from Figs. (2 − 4) that the admissible range

9of ~?? ~? and ~? are −2?2 ≤ ~? ≤ −0? 3? −1? 8 ≤ ~? ≤ −0? 4 and −1? 5 ≤ ~? ≤ −0? 2. Moreover the series solutions are convergent in the entire zone of ? when ~? = −1?0 = ~? = ~?. Table

2?1 reveals that the 6??

23order of deformations is enough for the convergent solutions of velocity and

24?? order of deformations is enough

23for both temperature and nanoparticles concentration

respectively. 2 f ''?0? 0 -2 -4 -6 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 Ñ f ???. 2. h-curve for the function ?(?)? 2 0 q'?0? -2 -4 -6 -8 - 2 .0 - 1 .5 - 1 .0 - 0 .5 0 .0 Ñ q ???? 3. h-curve for the function ?(?)? 1 0 f'?0? -1 -2 -3 -4 - 5 - 2 .0 - 1 .5 - 1 .0 - 0 .5 0 .0 Ñf ???? 4. h-curve for the function ?(?)?

29Table 2. 1. Convergence of homotopic solutions when ? = 0? 5? Pr = 1?0?

?? = 1?0? ?? = 0?3? ?? = 0?2? ? = 0?4? ? = 0?5 and ? = 0?2? ????? ?? ????????????? −? 00(0) ?0(0) −?0(0) 1 1.10556 0.58333 0.50000 2 1.11628 0.58940 0.54884 3 1.11628 0.56210 0.50861 6 1.11602 0.55031 0.48739 10 1.11602 0.54744 0.48055 15 1.11602 0.54695 0.47918 18 1.11602 0.54690 0.47902 21 1.11602 0.54688 0.47897 24 1.11602 0.54687 0.47896 25 1.11602 0.54687 0.47896 26 1.11602 0.54687 0.47896 2.5 Results and discussion Our intention here to analyze the characteristics of shape parameter (?)?

34Prandtl number (Pr)? Lewis number (??), magnetic field parameter (?)? surface thickness parameter (?)? heat source parameter (?)? Brownian motion parameter (? ?) and thermophoresis parameter

(? ?) on veloc- ity ?0(?)? temperature ?(?) and nanoparticles concentration ?(?). Moreover, Tables (2?2 − 2?4) are demonstrated to analyze the behaviors of arising variables

65on Skin friction (???), Nusselt number (??)and Sherwood number

(??) respectivelly. Fig. 5 displays the variation of shape parameter (?) on velocity ?0(?). From Fig. we observe that velocity is increasing function of (?). Impact of wall thickness

18parameter ? on the velocity profile is shown in Fig. 6. It is observed that increasing values of

? lead

26to decrease the velocity. Fig. 7 shows the effects of magnetic parameter on velocity distribution. Here we seen that velocity decreases for higher values of magnetic parameter (?). Fig. 8 shows the effect of (?) on ?(?). Temperature is increasing function of (?)? Fig. 9 depicts that variation of thickness parameter ? yields reduc- tion in temperature. Influence of heat source parameter on temperature profile is plotted in Fig. 10. There is an increase in temperature via source parameter (?). Brownian motion is a arbitrary movement of suspended particles due to collection of atoms or molecules in the fluid. The increment of (??) leads to enhancement of temperature and reduction in nanoparticles concentration as shown in Figs. (11 − 12). Thermophoresis variable corresponds to enhance- ment in

2**temperature and nanoparticles concentration as shown** in Figs. **(13** − 14). **Fig.**

15 depicts the variation of shape variable on concentration profile. The concentration enhances when ? is increased. Fig. 16 represents the concentration profile for different values of thickness parameter (?). Clearly ?(?) decreases for larger ?. Fig. 17 describes the effect of magnetic parameter on concentration distribution. It is observed that concentration profile enhances for larger ?. Tables (2?2) represents the Skin friction (???) for various physical variables appearing in velocity profile. Clearly surface force enhances via ?? ? and ?? Table (2?3) provides the numer- ical values of dimensionless

80**Nusselt number (??) for various** physical **parameters** involved **in**

temperature distribution. Here Nusselt number increases for larger ? and reverse trend is noted for higher values of ?? ??? ?? and ??? Table (2?4) shows that the Sherwood number enhances for larger ?? ?? ?? and ??? On the other hand the Sherwood number decays for larger ??? f'?x? 1.0 0.8 0.6 n = 0.0, 0.3, 0.6, 0.9 a = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2 0.4 Nb = 0.2, l = 0.1, Le = 1.0 0.2 1 2 3 4 5 6 x ???? 5? Effects of ? on ?0(?)? f'?x? 1.0 0.8 a = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ???? 6? Effects of ? on ?0(?)? f'?x? 1.0 0.8 0.6 M = 0.0, 0.2, 0.4, 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, a = 0.5, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ???? 7? Effects of ? on ?0(?)? q?x? 1.0 0.8 n = 0.0, 0.3, 0.6, 0.9 0.6 a = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ? ??? 8? Effects of ? on ?(?)? q?x? 1.0 0.8 a = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ? ??? 9? Effects of ? on ?(?)? q?x? 1.0 0.8 l = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nb = 0.1, a = 0.5, M = 0.2, Nt = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ? ??? 10? Effects of ? on ?(?)? q?x? 1.0 0.8 Nb = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, a = 0.5, M = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ???? 11? Effects of ?? on ?(?)? f?x? 1.0 0.8 Nb = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, a = 0.5, M = 0.2, l = 0.1, Le = 1.0 0.4 0.2 2 4 6 8 x ???? 12? Effects of ?? on ?(?)? q?x? 1.0 0.8 Nt = 0.0, 0.5, 1.0, 1.5 0.6 n = 0.5, Pr = 1.0, Nb = 0.1, a = 0.5, M = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ???? 13? Effects of ?? on ?(?)? f?x? 1.0 0.8 0.6 Nt = 0.0, 0.1, 0.2, 0.3 n = 0.5, Pr = 1.0, Nb = 0.1, a = 0.5, 0.4 M = 0.2, l = 0.1, Le = 1.0 0.2 f?x? 1.0 2 4 6 8 ???? 14? Effects of ?? on ?(?)? x 0.8 n = 0.0, 0.3, 0.6, 0.9 0.6 a = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 1 2 3 4 5 6 x ? ??? 15? Effects of ? on ?(?)? f?x? 1.0 0.8 0.6 a = 0.0, 0.5, 1.0, 1.5 n = 0.5, Pr = 1.0, Nt = 0.1, M = 0.2, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 2 4 6 8 x ? ??? 16? Effects of ? on ?(?)? f?x? 1.0 0.8 M = 0.0, 0.2, 0.4, 0.6 0.6 n = 0.5, Pr = 1.0, Nt = 0.1, a = 0.5, Nb = 0.2, l = 0.1, Le = 1.0 0.4 0.2 2 4 6 8 x ? ??? 17? Effects of ? on ?(?)? Table: 2.2. Effects of ?, ? and ? on Skin friction coefficients when ?? = 1?0 = Pr? ? ? = 0?1? ? ? = 0?2 = ?? ? ? ? −(??)0?5??? 0.0 0.2 0.4 0.5 0.0 0.4 0.6 0.2 0.0 0.4 0.6 0.8892 0.9194 0.9826 0.8377 0.9400 0.9929 0.8487 1.0711 1.1666 Table: 2.3. Impacts of ??? ?? ? ?? ? and ? ? on local Nusselt number (? ?) when ? = 0?5? ? = 0?4? Pr = 1?0? ? ? ?? ? ? ? ? − √???? 0.0 0.4 1.0 0.2 0.3 0.2898 0.2 0.3299 0.4 0.3704 0.0 0.5156 0.2 0.5049 0.4 0.4823 0.5 0.4074 1.2 0.3868 1.6 0.3824 0.1 0.4177 0.3 0.3648 0.4 0.3400 0.1 0.4051 0.2 0.3751 0.6 0.3175 Table: 4. Impacts of ?? ??? ?? ? ? and ? ? on local Sherwood number when ? = 0?5? ? = 0?4? Pr = 1?0? ? ? ?? ?? ?? − √???? 0.0 0.1 0.2 0.4 0.0 0.2 04 1.0 0.6 1.2 1.6 0.2 0.1 0.3 0.5 0.3 0.2 0.4 0.6 0.4023 0.4025 0.4210 0.3981 0.4027 0.4173 0.2390 0.5435 0.6715 0.2904 0.5036 0.5454 0.3264 0.1133 0.0568 2.6 Final remarks Main points of present study are as follows: • Flatness of surface directly relates with mechanical properties of surface. • Enhancement of Brownian motion parameter, shape parameter, thermophoresis parame- ter and heat source parameter correspond to increase the temperature distribution while opposite behaviour is seen for the variation of thickness parameter. Chapter 3 Simultaneous effects of melting heat and non-Darcy porous medium

76**in MHD flow of nanofluid** towards variable thicked **stretched surface**

3.1 Introduction This chapter discuses melting heat transfer and non-Darcy porous medium in

61**MHD stagnation point flow towards a** nonlinear **stretching sheet with**

variable thickness.

58**Brownian motion and thermophoresis effects are also** accounted. The **nonlinear** dimensionless ordinary differential **equations are**

solved numerically with builtin shooting technique. Graphically results of vari- ous physical

35**parameters on velocity, temperature and concentration** are studied. **Skin** friction coefficient, local Nusselt number and Sherwood number are addressed through tabulated

values. 3.2

25Formulation We consider steady two-dimensional MHD stagnation point flow of an incompressible viscous nanofluid in a

non-Darcy porous medium. The

40flow is induced due to stretching sheet at

? = ?(?+?)1−2?. The stretching sheet along x-axis has velocity ?? = ?(?+?)? However

37the velocity of external flow is ?? = ?(? + ?)?. Here ?? ? and ? are the

positive constants. Heat transfer is studied through melting heat.

67??the temperature of melting surface, while ?∞ and ?∞ correspond to the uniform temperature

and concentration

1far away from the surface of the

sheet respectively.

32It is assumed that the effects of viscous dissipation and heat generation/absorption are neglected. Under these assumptions and usual boundary layer approximation the governing equations

are, ?? ?? ?? ?? = 0? + ? ???? + ? ???? = ?? ????? + ? ??2??2 ?2(? − ??) − ??(? − ??) − √??? (?2 − ? 2?)? − ? ? ? ? ? ?? ?? + ? ?? = ? ??2 + ? ?? ?? ?? ?? ?2? ?? ?? + ?? (??)2 ? μ ?∞ ?? ¶ ? ?? ?? ?2? ?? ?? + ? ?? = ?? ??2 ?∞ + ?2? ??2 ? μ ¶ subjected boundary conditions are presented as follows: (3.1) (3.2) (3.3) (3.4) ?(?? ?) = ??(?) = ?(? + ?)?? ?? ?? + ?? ?? = 0 and ? = ?? at ? = ?(? + ?)1−2? ? ?? ?∞ ?? μ ¶ (3.5) ? (?? ?) = ??(?) = ?(? + ?)?? ? = ?∞? ? = ?∞? as ? → ∞? (3.6) ?1(????) = ?[?∗ + ??(?? − ?0)]? at ? = ?(? + ?)1−2? ? (3.7)

36In the above equations ? and ? are the velocity component in the x and y-directions. Here ? the fluid density, ?∗ the

latent heat of fluid, ? the kinematic viscosity, ?

77the permeability of porous medium, ? the porosity, ?? the drag coefficient, ?? the

thermophoretic

3diffusion coefficient, ?? the Brownian diffusion coefficient, ? the ratio between effective heat capacity of nanoparticle and base fluid respectively, ?? the

heat capacity of surface and ?? the melting temperature. 3.3 Statement Here we employ transformation procedure to reduce the above

74partial differential equations and governing boundary conditions into the set of

ordinary differential system; ? = ? (?+21)(?(?+??)?−1), ? = (?+21)(? + ?)?+1??? (?), q ?(?) = ??∞−−???? , ?q(?) = ??−∞?∞ (3.8) In above equation ? reperesents the stream function, define by (? = ???? and ? = − ????) as, ? = ?(? + ?)?? 0 (?), ? = − ? + 1)??(? + ?)?−1[? (?) + ? ? − 1 ? 0 (?)]? ((3.9) r 2 ? + 1 Now Eq. (1) is trivially satisfied while Eqs. (3?2) − (3?4) yield ? 000 − (?2+?1) ? 0 2 + ?? 00 − (?+21)(? ?)2 (? 0 − ?) − (?+21)(??) (? 0 − ?) ³ ´ −(?+21)? ? 02 − ?2 + (?2+?1)?2 = 0? (3.10) ¡ ¢ Θ00 + Pr ? Θ0 + ? ?Φ0 Θ0 + ? ?Θ02 = 0? (3.11) Φ00 + Pr ??? Φ0 + ? ? Θ00 = 0? ³ ´ (3.12) ? ? ?0(?)=1? Θ(?)=0? ?Θ0(?) +Pr?(?)+Pr?(??−+11)=0? ??Θ0(?)+ ??Φ0(?)=0? ?0(∞)=?? Θ(∞)=1? Φ(∞)=0? (3.13) Here ?(= ? 1+2? ??) represents the surface thickness parameter and ? = ? = (? 1+? ? 2 ?) the plate surfaqce. We define ?(?) = ?(? − ?) = ?(?), Θ(?) = ?(? − ?) = ?(?) and qΦ(?) = ?(? − ?) = ?(?) then Eqs. (3?10) − (3?13) give; ? 000 − (?2+?1) ? 0 + ? ? 00 − (?+21)(? ?)2 (? 0 − ?) − (?+21)(??) (? 0 − ?) 2 ³ ´ −(?+21)? ? 02 − ?2 + (?2+?1)?2 = 0? (3.14) ⎧ ⎫ ¡ ¢ ⎭ ?00 + Pr ??0 + ???0?0 + ???02 = 0? (3.15) ?00 + Pr ??? ?0 + ? ? ?00 = 0? ³ ´ (3.16) ?? ? 0 (0) = 1? ? (0) = 0? ? ?0 (0) + Pr ? (0) + Pr ?(??−+11) = 0? ? ??0 (0) + ? ??0 (0) = 0? ? 0 (∞) = ?? ?(∞) = 1? ?(∞) = 0? ⎫ (3.17) ⎧ With ⎭ ? = ?∗?+??(??∞(?−??−??)0) ? ?? = ??(?+???)?−1? ?? = ????0? ? = ???√(?+?)? ? ?= ??? Pr= ??q? ?? = ???(???∞∞−??)? ??= ????? ∞? ??= ???? in which ?? the shape parameter, ?? the inverse Darcy number, ?? the Hartman number, ? the local inertia coefficient parameter, ? the stretching sheet parameter, Pr

> 9**the Prandtl number, ?? the Brownian motion** parameter, ?? thermophoresis parameter, ?? **the Lewis number, ? the** melting **parameter, and**

? the surface thickness parameter. The

> 3**skin friction coefficient (??) and Nusselt number (??) are ?? = ??2??** 2? ?? = ? (?∞ − ??) ?? (? + ?)?? ? (3.18) **where the surface shear stress (??) and** rate of **heat**

transfer (??) define at the surface are ?? ?? ?? = ?(?? ?=?(?+?)1−2? ? ?? = −?(?? ?=?(?+?)1−2? ?)) (3.19) Now we have ?? Re = 2 √ ? + 1?00(0)? √?? =− ? + 1?0(0)? (3.20) r 2 Re r 2 with Re = ?(?+??) ?+1 as the Reynolds number. 3.4 Numerical procedure The

> 70**set of non-linear** dimensionless **ordinary differential** system **are solved**

numerically builtin shooting technique. 3.5 Discussion Here our intention is to predict the effects of shape parameter (?)? inverse Darcy number (??), Hartman number (??), local inertia coefficient parameter (?)? stretching sheet parameter (?), Prandtl number (Pr), thermophoresis parameter (??),

> 23**Brownian motion parameter (??), Lewis number** (??), melting parameter (?) **and**

surface

> 2**thickness parameter (?) on the velocity** ?0(?)? **temperature ?(?) and concentration**

?(?) . 3.6 Dimensionless velocity profile Mostly study depend on shape parameter ? due to its significance because this parameter can controls

> 5**the surface shape, type of motion and** deportment of **the boundary layer.** Here **we** notice **that the outer shape of surface depends** upon **the value of** shape parameter (?). **For** ? = 1 **the study** is **reduced to flat surface.**

For ? ?

> 2**1 the study transformed to surface with** decreasing **thickness and** concave **outer shape. However** for ? ? **1 the study transformed to surface with** increasing **thickness and** convex **outer shape.**

In addition this parameter

> 2**controls the type of motion such that** for ? = **0 the motion** is **reduced to linear with constant velocity.** If ? ? 1 then **motion** is **reduced to deceleration** case whereas ? ? **1**

corresponds to the accelerated motion. Characteristics of shape parameter (?) on velocity distribution ? 0(?) is shown in Fig. 1. The velocity increases for larger shape parameter(?). Figs. 2 and 3 describe the influence of thickness parameter ? on ?0(?) when ? ? 1 and ? ? 1. For ? ? 1 the velocity profile decrease for larger ?. Physically it shows that stretching velocity decreases. It yields reduction in velocity distribution. However for ? ? 1 the velocity profile increases for larger ?. Physically stretching velocity enhances for larger ?. Here more deformation is produced in the fluid and thus velocity distribution increases. Fig. 4 illustrates

> 53**the velocity distribution for** various **values of** stretching sheet **parameter. It is** seen **that velocity**

distribution enhances for larger stretching sheet parameter. Fig.

715 displays the velocity profile for larger values of melting parameter.

10Here we noticed that the ?0(?) and boundary layer thickness

increase through melting parameter (?). Fig. 6 explains

18the effect of inverse Darcy number (??) on velocity

profile ?0(?).

18It is observed that an increase in value of (??) decays the velocity

?0(?). Fig. 7 gives the effect of

54velocity profile for different values of Hartman number (??). We observed that velocity profile

reduces with an enhancement in Hartman number. Fig. 8 indicates the impact of inertial coefficient parameter(?) on ?0(?). Clearly it is seen that the velocity profile ?0(?) reduces for larger value of inertial coefficient parameter (?). 3.7 Dimensionless temperature profile The variation of Hartman number on temperature profile

57is shown in Fig. 9. We observe that temperature is decreasing function of

Hartman number (??). Fig. 10 depicts that variation of melting parameter (?) yield reduction in temperature. Fig. 11 gives the impacts of (??) on temperature distribution. Here we noticed that temperature profile reduces when ?? in- creased. Fig. 12 corresponds to

20the effect of thermophoresis parameter (??) on ?(?). Here temperature is increasing function of ??. In

fact an increase in ??

30causes an enhancement in the thermophoresis force which tends to move the nanoparticle from hot to cold area and thus as a result

temperature increases. Fig. 13 represents the temperature for higher values of Prandtl number. Clearly ?(?) increases for larger Prandtl number (Pr). Fig. 14 shows the temperature profile for larger values of (?). There is decrease in ?(?) for larger ?. 3.8 Dimensionless concentration profile Fig. 15 gives the effect of Lewis number on nanoparticle concentrations ?(?).

1An increase in the value of Lewis number

shows low nanoparticle concentration.

1Lewis number depends on Brownian diffusion coefficient. Increasing Lewis number leads to lower Brownian diffusion coefficient which causes weaker nanoparticle concentration profile ?(?). Fig.

16 represents the effect of Prandtl number on nanoparticle concentrations ?(?). Nanoparticle concentrations ?(?) is noted an increasing function of Prandtl number. The

52Brownian motion is random motion of nanoparticles suspended in the fluid resulting due to

2fast moving atom or molecules in the fluid.

Figs.17 and 18 represent the effects of Brownian diffusion coefficient (??) and thermophoresis parameter (??) on nanoparticle concentration ?(?). Clearly ?(?) near the surface decays when ?? increases. However the variation of ?? on ?(?) away from the surface is opposite. Further the impact of ?? on ?(?) is quite reverse to that of ??. Fig. 19 describes the effect of inverse Darcy number (??) on concentration distribution. It is observed that concentration profile enhances for larger ??. Fig. 20 shows the effect of (?)

on concentration distribution. It is observed that concentration profile increases via ?. 3.9

79**Dimensionless Skin friction coefficient (??) and Nusselt**

num- ber (??)

50**Table (3.1)** provides the **numerical values of** dimensionless **skin friction coefficient for** distinct **values of**

physical parameter involved in velocity distribution. Here surface force increases for larger values of ??????? and ? while skin friction coefficient decrease for larger ?. Table (3.2) shows that the Nusselt number (??) enhances for larger Pr???, ?, ? and ??.

69**On the other hand the** local **Nusselt number** decays **for**

larger values of ??. f '?x? 1.0 0.8 Nt = 0.4, Nb = 0.2, Pr = 1.0, M = 1.0 0.6 a = 0.2, l = 0.1, Le = 1.0, Ha = 0.2 Da = 0.4, b = 0.6 0.4 n = 0.0, 0.5, 1.0, 1.5 0.2 1 2 3 4 x Fig. 1. Effect of ? on ?0(?) f '?x? 1.0 0.8 M =

33**0.1, Nb = 0.** 2, **Nt = 0.** 3, Pr = **1.0 n**

= 0.5, l = 0.1, Le = 1.0, Ha = 0.2 Da = 0.3, b = 0.6 0.6 a = 0.0, 0.5, 1.0, 1.8 0.4 0.2 1 2 3 4 x Fig. 2. Effect of ? on ?0(?) for ? < 1 f '?x? 1.0 M = 0.1,

3**Nb = 0.** 2, **Nt = 0.3,** Pr = **1.0 0.8 n**

= 1.5, l = 0.1, Le = 1.0, Ha = 0.2 Da = 0.3, b = 0.6 0.6 a = 0.0, 0.5, 1.0, 1.8 0.4 0.2 1 2 3 4 x Fig. 3. Effect of ? on ?0(?) for ? Â 1 f '?x? 1.0 0.8 M = 0.1,

1**Nb = 0.** 2, **Nt = 0.** 3, **Pr = 1.0** n = **0.**

5, a = 0.1, Le = 1.0, Ha = 0.2 Da = 0.2, b = 0.3 0

13.**6 l = 0.0, 0.1, 0.2, 0.3 0.4 0.2 1 2 3 4** x **Fig.** 4. Effect **of ? on ?0(?) f** '?x? **1.0 0.8 a = 0.** 2, Nb = **0.**

28**2, Nt = 0.** 3, **Pr = 1.** 0 n = 1 **.5,** l = 0.1, **Le = 1.0,**

Ha = 0.2 0.6 Da = 0.3, b = 0.6 M = 0.4, 0.8, 1.2, 1.6 0.4 0.2 1 2 3 4 x Fig. 5. Effect of ? on ?0(?) f '?x? 1.0 0.8 0.6 M = 0.1,

64**Nb = 0.** 2, **Nt = 0.** 3, **Pr = 1.** 0 n = **1.** 5, l = **0.**

1, Le = 1.0, Ha = 0.2 0.4 a = 0.2, b = 0.6 Da = 0.0, 0.1, 0.2 , 0.3 0.2 1 2 3 4 x Fig.

6**6. Effect of ?? on ?0(?) f** '?x? **1.0** M = **0.1,** Nb = **0.2,** Nt = **0.3, Pr = 1.**

0 0.8 n = 1.5, l = 0.1, Le = 1.0, a = 0.2 Da = 0.3, b = 0.6 0.6 Ha = 0.0, 0.3, 0.5, 0.7 0.4 0.2 1 2 3 4 x Fig. 7. Effect of ?? on ?0(?) f '??? 1.0 0.8 0.6 0.4 M = 0.1,

1**Nb = 0.** 2, **Nt = 0.** 3, **Pr = 1.0** n = **0.**

5, l = 0.1, Le = 1.0, a = 0.2 Ha = 0.2, Da = 0.3 b = 0.0, 0.5, 0.8, 1.2 0.2 1 2 3 4 ? Fig. 8. Effect of ? on ?0(?) q??? 1.0 0.8 0.6 M = 0.1, Nb = 0.2, Nt = 0.1, 0.4 Pr = 1.0 , n = 1.5, l = 0.2, Le = 1.0, Da = 0.2, b = 0.3, 0.2 a = 0.2 Ha = 0.2, 0.5, 0.8, 1.0 1 2 3 4 ? Fig. 9. Effect of ?? on ?(?) q?x? 1.0 0.8 0.6 0.4 0.2 Da = 0.3,

1**Nb = 0.** 2, **Nt = 0.** 1, **Pr = 1.0** , n = **0.**

5, l = 0.2, Le = 1.0, Ha = 0.2, b = 0.6, a = 0.2 M = 0.0, 0.1, 0.2, 0.3 1 2 3 4 x Fig. 10. Effect of ? on ?(?) q??? 1.0 0.8 0.6 0.4 0.2 M = 0.1,

28**Nb = 0.2, Nt = 0.1, Pr = 1.** 0 , n = 1 **.5,** l = 0.2, **Le = 1.0,**

Ha = 0.2, b = 0.3, a = 0.2 Da = 0.2, 0.5, 0.8, 1.0 1 2 3 4 ? Fig. 11. Effect of ?? on ?(?) q?x? 1.0 0.8 0.6 0.4

0.2 Da = 0.3, Nb = 0.2, M = 0.1, Pr = 1.0 , n = 0.5, l = 0.2, Le = 1.0, Ha = 0.2, b = 0.6, a = 0.2 Nt = 0.1, 0.5, 1.0, 1.5 1 2 3 4 x Fig. 12. Effect of ?? on ?(?) q?x? 1.0 0.8 0.6 Da = 0.3, Nb = 0.2, M = 0.1, Nt = 0.3 , n = 0.5, l = 0.2, 0.4 Le = 1.0, Ha = 0.2, b = 0.6, a = 0.2 0.2 Pr = 1.0, 1.5, 2.0, 2.5 1 2 3 4 x Fig. 13. Effect of Pr on ?(?) q??? 1.0 0.8 0.6 M = 0

33**.1, Nb = 0.** 2, **Nt = 0.1,** 0.4 Pr = **1.0 , n**

= 0.5, l = 0.2, Le = 1.0, Ha = 0.2, Da = 0.3, a = 0.2 0.2 b = 0.0, 0.2, 0.4, 0.6 1 2 3 4 Fig. 14. Effect of ? on ?(?) ? f?x? 1.5 n = 0.5, Nb = 0.2, Pr = 1.0, Nt = 0.1, a = 0.2, l = 0.1, 1.0 M = 0.1, Ha = 0.2, Da = 0.3, b = 0.6 Le = 0.0, 0.1, 0.2, 0.3 0.5 0.0 1 2 3 4 x Fig. 15. Effect of ?? on ?(?) f?x? 1.4 1.2 1.0 0.8 0.6 0.4 na == 00..52,, NIb==0.01.,2, DMa==00..13,, Ha = 0.2 0.2

1**Le = 1.0,** b = **0.** 6, **Nt = 0.** 1 **Pr** = 0 **.0,**

0.1, 0.2, 0.3 1 2 3 4 x Fig. 16. Effect of Pr on ?(?) f?x? 1.5 n = 0.5, Da = 0.2, Pr = 0.5, Nt = 0.1 1.0 a = 0.2, l = 0.1, M = 0.1, Ha = 0.2 Le = 1.0, b = 0.6

3**Nb = 0.** 1, **0.** 2, **0.** 3, **0.** 4 **0.** 5 **0.0 1 2 3 4** x **Fig.** 17. **Effect of ?? on**

?(?) f?x? 1.0 0.8 0.6 n = 0.5, Da = 0.3, Pr = 1.2, Nb = 0.2 a = 0.2, l = 0.1, M = 0.5, Ha = 0.2 0.4 Le = 1.2, b = 0.3 Nt = 0.1, 0.2, 0.3, 0.4 0.2 0.0 -0.2 -0.4 1 2 3 4 x Fig. 18. Effect of ?? on ?(?) f?x? 0.6 0.4 n = 0.5, Nb = 0.2, Pr = 1.0, Nt = 0.1 a = 0.2, l = 0.1, M = 0.1, Ha = 0.2 0.2 Le = 1.0, b = 0.6 Da = 0.0, 0.3, 0.5, 0.7 0.0 -0.2 -0.4 1 2 3 4 x Fig. 19. Effect of ?? on ?(?) f?x? 0.6 0.4 Nt = 0.1, Da = 0.3, Pr = 1.2, Nb = 0.2 a = 0.2, l = 0.1, M = 0.5, Ha = 0.2 0.2 Le = 1.2, b = 0.3 0.0 n = 0.0, 0.1, 0.2, 0.3 - 0.2 - 0.4 1 2 3 4 x Fig. 20. Effect of ? on ?(?) Table 3.1. Impacts of ????????? and ?

59**on skin friction coefficient,** when Pr =1 **.0, ?=0.** 5, **??=0.2, ??=0.** 3, ? **=0.**

1, ??=1.0. ?? ?? ? ? ? −?? Re 0.0 0.1 0.2 0.3 0.3 1.0 1.5 2.0 0.6 1.0 2.0 3.0 0.2 0.0 0.1 0.2 0.1 0.0 0.1 0.2 1.8314 1.0868 1.0980 1.1164 1.6088 1.6877 1.7600 1.2092 1.5992 1.7615 1.0742 1.0860 1.0980 1.1217 1.0980 1.0077 √ Table 3.2:Impacts

10**of ??, Pr, ??, ?, ?? and ? on local Nusselt number** when ? = **0?**

1, ? = 0?5, ? = 0?2, ? = 0?2, ?? = 0?2. ?? Pr ?? ? ?? ? - √? ? Re 0.5 0.5322 1.0 0.4231 1.5 0.0479 1.0 0.5424 1.5 0.6992 2.0 0.8416 0.3 0.5542 0.5 0.5660 0.7 0.5777 0.0 0.5077 0.1 0.5250 0.2 0.5424 1.0 0.5424 2.0 0.5454 3.0 0.5472 0.0 0.5068 0.1 0.5239 0.2 0.5424 3.10 Conclusions

72**MHD stagnation point flow of nanofluid** in **a** non-Darcy **porous**

medium is studied numerically. Melting heat transfer and variable thickness of surface are considered. Numerical simulation witnesses the following findings. •

38**Velocity and temperature profiles** reduce **for** larger **values of** inverse Darcy **number.**

• Concentration has opposite results for inverse Darcy number when compared with velocity and temperature. • There is enhancement of velocity for melting parameter. • Decay in temperature is noted for melting parameter ?, Hartman number ?? and local inertia coefficient parameter ?. • Concentration reduces for Prandtl number. • Impact of ?? on skin friction is opposite to that of Nusselt numbers.

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