774

Mixed Convection flow of a non-Newtonian fluid on a rotating cone

By

Salman Saleem

Department of Mathematics
Quaid-i-Azam University,
Islamabad PAKISTAN

2010

بسم الله الرحمن الرحیم

Mixed Convection flow of a non-Newtonian fluid on a rotating cone

By

Salman Saleem

Supervised

By

Dr. Sohail Nadeem

Department of Mathematics
Quaid-i-Azam University, Islamabad
PAKISTAN

2010

Mixed Convection flow of a non-Newtonian fluid on a rotating cone

By

Salman Saleem

A Dissertation Submitted in the partial fulfillment of the requirements for the degree of

MASTER OF PHILOSOPHY

IN

MATHEMATICS

Supervised By

Dr. Sohail Nadeem

Department of Mathematics
Quaid-i-Azam University, Islamabad
PAKISTAN
2010

CERTIFICATE

Mixed Convection flow of a non-Newtonian fluid on a rotating cone

By

SALMAN SALEEM

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

1. Prof Dr. Muhammad Ayub
(Chairman)

2. Dr. Sohail Nadeem
(Supervisor)

3. (External Examiner) 30/6/10

Department of Mathematics
Quaid-i-Azam University, Islamabad
PAKISTAN
2010

Dedicated

To

Babul ilm

Hazrat Ali a.s

Acknowledgement

*All praises to almighty **ALLAH**, the most Beneficent and the most Merciful, who created this universe and gave us the idea to discover. I am highly grateful to Almighty **ALLAH** for His blessing, guidance and help in each and every step of my life. He blessed us with the **Holy Prophet MUHAMMAD** (Sal-Allah-Hu-Alai-Hi Wa-aali-hi Wa-sallam), who is forever source of guidance and knowledge for humanity.*

*I express my deepest and heart-felt gratitude to my supervisor, **Dr. Sohail Nadeem** for his knowledgeable discussions, valuable guidance and inexhaustible inspiration throughout my research. His sympathetic attitude and encouragement prop up me to work harder with keen interest. What ever I had learned and achieved is all because of his superb support.*

*I am also thankful to the Chairman, Department of Mathematics, Prof. **Dr. Muhammad Ayub** and **Dr. Tassawar Hayat** for providing necessary facilities to complete my thesis.*

*My love and gratitude from the core of heart to my loving **Mummy** and **Dady**, caring **brothers**, **sisters** and my friends, **Qasim bhai, Awais bhai, Zeeshan, Waqas, Waqar, Tayyab, Sajjad, Rashid,*** my all **class fellows**, and my **sweet juniors** *for their prayers, support and encouragement, who have always given me love, care and cheer and whose sustained hope in me led me to where I stand today.*

May Almighty Allah shower His choicest blessing and prosperity on all those who assisted me in any way during completion of my thesis.

Preface

Flow over a cone-shaped bodies are often encountered in many engineering applications. In the presence of heat transfer analysis, the boundary layer flow over a rotating cone occures in rotating heat exchangers, design of canisters for nuclear waste disposal, nuclear reactor cooling system and geothermal reservoirs etc. Only a limited attention has been focused to this kind of study. Mention may be made to the interesting works of [1-5]. The study of magnetohydrodynamic [MHD] flows in the presence of heat transfer in the form of either mixed convection or natural convection is important number of technological and industrial applications. Such applications include the production of steel, aluminium, high performance super-alloys or crystals [6-10]. In crystal growth, the magnetic fields are used to suppress the convective motion induced by the arising strong fluxes in order to control the flow in the melt and consequently the crystal quality. Recently Kaharantzas et al [11] have examined the MHD natural convection in a vertical cylinder cavity with sinusoidal upper wall temperature. A number of analytical, numerical and experimental studies have been performed on this topic [12-15]. Motivating the above survey, the purpose of the present dessertation is to discuss the mixed convection MHD flow on a rotating cone in a rotating fluid. Two types of fluid models are taken into account (i) Viscous fluid (ii) Micropolar fluid. The thesis is arranged as follow: In chapter one, we have discussed the unsteady mixed convection flow on a rotating cone in a rotating frame. Basically this paper was solved numerically by [16] and we have done the problem analytically by homotopy analysis method. Chapter two is devoted to the study of mixed convection MHD micropolar fluid on a rotating cone in a rotating fluid. The governing nonlinear partial differential equations of micropolar fluid are first transformed into nonlinear ordinary differential equations with the help of suitable similarity transformations and then solved analytically with the help of homotopy analysis method.

Contents

Chapter 1

Unsteady mixed convection flow on a rotating cone in a rotating fluid

1.1 Introduction

In this chapter, we have presented the unsteady mixed convection flow on a rotating cone in a rotating fluid. The governing equations of viscous fluid along with heat and mass equations are simplified under the assumptions of boundary layer and similarity transformation. The reduced coupled nonlinear ordinary differential equations are then solved analytically with the help of homotopy analysis method. The expressions for local skin friction coefficients and Sherwood number are also computed. A parametric study is also reported through graphs.

1.2 Mathematical Formulation

Consider unsteady laminar incompressible viscous boundary layer flow in the presence of heat and mass transfer analysis over a heated vertical cone rotating in an ambient fluid with time dependent angular velocity $\Omega(t) = (\Omega_1 + \Omega_2)$ around the axis of cone. The curvilinear rectangular coordinate system are considered in such a way that x-axis is taken along a meridional section, y-axis along the circular section and z-axis normal to the cone surface. Both the fluid and the cone are in a state of rigid body rotation about the axis of cone. Let u, v and w are velocity components along x, y and z-axis respectively. The wall temperature T_w and the wall concentration C_w are assumed to vary linearly with the distance x. The ambient temperature T_∞ and concentration C_∞ are assumed to be constants. The governing boundary layer equations of unsteady mixed convection flow on a rotating cone in a rotating fluid in the presence of heat and mass transfer take the following form [16]

$$(xu)_x + (xw)_z = 0, \tag{1.1}$$

$$u_t + uu_x + wu_z - \frac{v^2}{x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon u_{zz} + g\beta\cos\alpha^*\left(T - T_\infty\right) + g\beta^*\cos\alpha^*\left(C - C_\infty\right), \tag{1.2}$$

$$v_t + uv_x + wv_z + \frac{uv}{x} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon v_{zz}, \tag{1.3}$$

$$T_t + uT_x + wT_z = \alpha T_{zz}, \tag{1.4}$$

$$C_t + uC_x + wC_z = DC_{zz}, \tag{1.5}$$

where T is the temperature, C is the concentration, $g\beta\cos\alpha^*$ comes due to effects of gravity, α is the thermal diffusivity and D represents mass diffusivity. The corresponding boundary conditions of rotating cone take the following form.

$$\begin{aligned}
u\left(0,x,z\right) &= u_i(x,z), v(0,x,z) = v_i(x,z)\\
w(0,x,z) &= w_i(x,z), T(0,x,z) = T_i(x,z)\\
C(0,x,z) &= C_i(x,z)\\
u\left(t,x,0\right) &= w\left(t,x,0\right) = 0, v\left(t,x,0\right) = \Omega_1 x\sin\alpha^*(1-st^*)^{-1}\\
T\left(t,x,0\right) &= T_w, C\left(t,x,0\right) = C_w,
\end{aligned} \tag{1.6}$$

for prescribed wall temperature (PWT case) the boundary conditions are defined as

$$-kT_z(t,x,0) = q_w \text{ and } -\rho DC_z(t,x,0) = \dot{m}_w, \tag{1.7}$$

for prescribed heat flux (PHF case) the boundary conditions are defined as

$$\begin{aligned} u(t,x,\infty) &= 0,\ v(t,x,\infty) = v_e = \Omega_2 x \sin\alpha^*(1-st^*)^{-1} \\ T(t,x,\infty) &= T_\infty,\ C(t,x,\infty) = C_\infty, \end{aligned} \tag{1.8}$$

Using the boundary conditions at infinity, Eqs. (1.2) and (1.3) take the form

$$u_t + uu_x + wu_z - \frac{v^2}{x} = -\frac{v_e^2}{x} + \upsilon u_{zz} + g\beta\cos\alpha^*\left(T - T_\infty\right) + g\beta^*\cos\alpha^*\left(C - C_\infty\right), \tag{1.9}$$

$$v_t + uv_x + wv_z + \frac{uv}{x} = \left(v_e\right)_t + \upsilon v_{zz}, \tag{1.10}$$

here α^* is the semi-vertical angle of the cone; υ is the kinematic viscosity; ρ is the density ; σ is the electrical conductivity; t and $t^* (= \Omega\sin\alpha^* t)$ are the dimensional and dimensionless times, respectively; $\Omega_1$and Ω_2 are the angular velocities of the cone and the fluid far away from the surface, respectively; $\Omega\,(= \Omega_1 + \Omega_2)$ is the composite angular velocity; g is the acceleration due to gravity; β is the volumetric coefficient of expansion for concentration; subscripts t, x and z denote partial derivatives with respect to the corresponding variables and the subscripts e, i, w and ∞ denote the conditions at the edge of the boundary layer, initial conditions, conditions at the wall and free stream conditions, respectively; C_w, T_w , C_∞ and T_∞ are the constants; q_w and $\dot{m}_w$ are, respectively, the heat and mass flux at the wall.

Eqs. $(1.1)-(1.5)$ are a system of partial differential equations with three independent variables x, z and t. It has been found that these partial differential equations can be reduced to a system of ordinary differential equations, if we take the velocity at the edge of the boundry layer v_e and the angular velocity of the cone to vary inversely as a linear functions of time. Introducing the following nondimensional quantities for PWT case:

$$
\begin{aligned}
v_e &= \Omega_2 x \sin\alpha^* (1-st^*)^{-1}, \; \eta = \left(\frac{\Omega \sin\alpha^*}{v}\right)^{\frac{1}{2}} (1-st^*)^{-\frac{1}{2}} z \\
t^* &= (\Omega \sin\alpha^*)t, \; u(t,x,z) = -2^{-1}\Omega x \sin\alpha^* (1-st^*)^{-1} f'(\eta) \\
v(t,x,z) &= \Omega x \sin\alpha^* (1-st^*)^{-1} g(\eta), \; \lambda_1 = \frac{Gr_1}{\mathrm{Re}_L^2}, \; \lambda_2 = \frac{Gr_2}{\mathrm{Re}_L^2} \\
w(t,x,z) &= (v\Omega \sin\alpha^*)^{\frac{1}{2}} (1-st^*)^{-\frac{1}{2}} f(\eta), \; \mathrm{Re}_L = \Omega \sin\alpha^* \frac{L^2}{v} \\
T(t,x,z) - T_\infty &= (T_w - T_\infty)\theta(\eta), \; T_w - T_\infty = (T_0 - T_\infty)\left(\frac{x}{L}\right)(1-st^*)^{-2} \\
C(t,x,z) - C_\infty &= (C_w - C_\infty)\phi(\eta), \; (C_w - C_\infty) = (C_0 - C_\infty)\left(\frac{x}{L}\right)(1-st^*)^{-2} \\
Gr_1 &= g\beta \cos\alpha^* (T_0 - T_\infty)\frac{L^3}{v^2}, \; Gr_2 = g\beta\cos\alpha^* (C_0 - C_\infty)\frac{L^3}{v^2} \\
\alpha_1 &= \frac{\Omega_1}{\Omega}, \; N_1 = \frac{\lambda_2}{\lambda_1}, \; \mathrm{Pr} = \frac{v}{\alpha}, \; Sc = \frac{v}{D},
\end{aligned} \tag{1.11}
$$

and for the prescribed heat flux (PHF) case:

$$
\begin{aligned}
v_e &= \Omega_2 x \sin\alpha^* (1-st^*)^{-1}, \; \eta = \left(\frac{\Omega \sin\alpha^*}{v}\right)^{\frac{1}{2}} (1-st^*)^{-\frac{1}{2}} z \\
t^* &= (\Omega \sin\alpha^*)t, \; u(t,x,z) = -2^{-1}\Omega x \sin\alpha^* (1-st^*)^{-1} F'(\eta) \\
v(t,x,z) &= \Omega x \sin\alpha^* (1-st^*)^{-1} G(\eta), \; w(t,x,z) = (v\Omega\sin\alpha^*)^{\frac{1}{2}} (1-st^*)^{-\frac{1}{2}} F(\eta) \\
\lambda_1^* &= \frac{Gr_1^*}{\mathrm{Re}_L^{\frac{5}{2}}}, \; \lambda_2^* = \frac{G^* r_2}{\mathrm{Re}_L^{\frac{5}{2}}}, \; \mathrm{Re}_L = \Omega\sin\alpha^* \frac{L^2}{v}, \; \alpha_1 = \frac{\Omega_1}{\Omega} \\
T(t,x,z) - T_\infty &= \left(\frac{\Omega\sin\alpha^*}{v}\right)^{-\frac{1}{2}} (1-st^*)^{\frac{1}{2}} \left(\frac{q_w}{k}\right)\Theta(\eta), \; \mathrm{Pr} = \frac{v}{\alpha}, \; Sc = \frac{v}{D} \\
C(t,x,z) - C_\infty &= \left(\frac{\Omega\sin\alpha^*}{v}\right)^{-\frac{1}{2}} (1-st^*)^{\frac{1}{2}} \left(\frac{\dot m_w}{\rho D}\right)\Phi(\eta), \; \dot m_w = \dot m_0 \left(\frac{x}{L}\right)(1-st^*)^{-\frac{5}{2}} \\
Gr_1^* &= \frac{g\beta\cos\alpha^* (q_0) L^4}{kv^2}, \; N_1^* = \frac{\lambda_2^*}{\lambda_1^*}, \; Gr_2^* = \frac{g\beta\cos\alpha^*(m_0)L^4}{\rho D v^2}.
\end{aligned} \tag{1.12}
$$

Making use of Eqs. (1.9) and (1.10), Eqs. (1.1) to (1.5) for PWT and PHF cases reduce to,

$$f''' - ff'' + 2^{-1}f'^2 - 2\left(g^2 - (1-\alpha_1)^2\right) - 2\lambda_1(\theta + N_1\phi) - s\left(f' + 2^{-1}\eta f''\right) = 0, \quad (1.13)$$

$$g'' - \left(fg' - gf'\right) + s\left(1 - \alpha_1 - g - 2^{-1}\eta g'\right) = 0, \quad (1.14)$$

$$(\mathrm{Pr})^{-1}\theta'' - \left(f\theta' - f'\frac{\theta}{2}\right) - s\left(2\theta + 2^{-1}\eta\theta'\right) = 0, \quad (1.15)$$

$$(Sc)^{-1}\phi'' - \left(f\phi' - f'\frac{\phi}{2}\right) - s\left(2\phi + 2^{-1}\eta\phi'\right) = 0, \quad (1.16)$$

$$\begin{aligned} f(0) &= 0 = f'(0),\ g(0) = \alpha_1,\ \theta(0) = \phi(0) = 1, \\ f'(\infty) &= 0, g(\infty) = 1 - \alpha_1,\ \theta(\infty) = \phi(\infty) = 0, \end{aligned} \quad (1.17)$$

$$F''' - FF'' + 2^{-1}F'^2 - 2\left(G^2 - (1-\alpha_1)^2\right) - 2\lambda_1^*(\Theta + N_1^*\Phi) - s\left(F' + 2^{-1}\eta F''\right) = 0, \quad (1.18)$$

$$G'' - \left(FG' - GF'\right) + S\left(1 - \alpha_1 - G - 2^{-1}\eta G'\right) = 0, \quad (1.19)$$

$$(\mathrm{Pr})^{-1}\Theta'' - \left(F\Theta' - F'\frac{\Theta}{2}\right) - s\left(2\Theta + 2^{-1}\eta\Theta'\right) = 0, \quad (1.20)$$

$$(Sc)^{-1}\Phi'' - \left(F\Phi' - F'\frac{\Phi}{2}\right) - s\left(2\Phi + 2^{-1}\eta\Phi'\right) = 0, \quad (1.21)$$

$$\begin{aligned} F(0) &= 0 = F'(0), G(0) = \alpha_1,\ \Theta'(0) = \Phi'(0) = -1, \\ F'(\infty) &= 0, G(\infty) = 1 - \alpha_1,\ \Theta(\infty) = \Phi(\infty) = 0, \end{aligned} \quad (1.22)$$

here η is the similarity variable; f, F are the dimensionless stream functions for the PWT and PHF cases respectively; f' and g are the respectively dimensionless velocity along x- and y-directions for PWT case; F' and G are the corresponding velocities for the PHF case, respectively; θ and ϕ are the dimensionless temperature and concentration for the PWT case; Θ and Φ are the the dimensionless temperature and concentration for the PHF case; Re_L is the Reynolds number; Gr_1, Gr_2, Gr_1^*, Gr_2^* are the Grashof numbers; λ_1, λ_2, λ_1^*, λ_2^* are the buoyancy parameters; N_1, N_1^* are the ratio of Grashof numbers for PWT and PHF case, respectively; α_1 is the ratio of angular velocity of the cone to the composite angular velocity; Pr and Sc are the Prandtl and Schmidt numbers, respectively; s is the parameter characterizing the unsteadiness in the free stream velocity. The definitions of surface skin friction coefficient in x- and y-direction for the PWT case are

$$C_{fx} = \frac{[2\mu\left(\frac{\partial u}{\partial z}\right)]_{z=0}}{\rho[\Omega x \sin\alpha^*\left(1 - st^*\right)^{-1}]^2} = -\,\mathrm{Re}_x^{-\frac{1}{2}} f''(0),$$

$$C_{fy} = \frac{[2\mu\left(\frac{\partial v}{\partial z}\right)]_{z=0}}{\rho[\Omega x \sin\alpha^*\left(1 - st^*\right)^{-1}]^2} = -\,\mathrm{Re}_x^{-\frac{1}{2}} g'(0),$$

thus,

$$\begin{aligned} C_{fx}\,\mathrm{Re}_x^{\frac{1}{2}} &= -f''(0), \\ C_{fy}\,\mathrm{Re}_x^{\frac{1}{2}} &= -g'(0), \end{aligned} \tag{1.23}$$

where $\mathrm{Re}_x = \frac{\Omega x^2 \sin\alpha^*(1-st^*)^{-1}}{v}$ is the Reynolds number. The surface skin friction coefficients in x- and y-directions for PHF case are respectively, given by

$$\begin{aligned} \bar{C}_{fx}\,\mathrm{Re}_x^{\frac{1}{2}} &= -F''(0), \\ \bar{C}_{fy}\,\mathrm{Re}_x^{\frac{1}{2}} &= -G'(0). \end{aligned} \tag{1.24}$$

The Nusselt and Sherwood numbers for the PWT case are defined as

$$\begin{aligned} Nu\,\mathrm{Re}_x^{-\frac{1}{2}} &= -\theta'(0), \\ Sh\,\mathrm{Re}_x^{-\frac{1}{2}} &= -\phi'(0). \end{aligned} \tag{1.25}$$

Similarly, the Nusselt and Sherwood numbers for the PHF case are

$$\begin{aligned} \overline{Nu}\,\mathrm{Re}_x^{-\frac{1}{2}} &= \frac{1}{\Theta(0)}, \\ \overline{Sh}\,\mathrm{Re}_x^{-\frac{1}{2}} &= \frac{1}{\Phi(0)}. \end{aligned} \tag{1.26}$$

The equations (1.13) to (1.16) are coupled nonlinear differential equations, to find the analytic solutions we use Homotopy analysis method (HAM).

1.3 Homotopy analysis solution

We express $f(\eta)$, $g(\eta)$, $\theta(\eta)$, $\phi(\eta)$ by a set of base functions

$$\left\{ \eta^k \exp(-n\eta) \,|k \geq 0, n \geq 0 \right\}, \tag{1.27}$$

in the form

$$f(\eta) = a_{0,0}^0 + \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta), \tag{1.28}$$

$$g(\eta) = b_{0,0}^0 + \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta), \tag{1.29}$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} c_{m,n}^k \eta^k \exp(-n\eta), \tag{1.30}$$

$$\phi(\eta) = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} d_{m,n}^k \eta^k \exp(-n\eta), \tag{1.31}$$

in which $a_{m,n}^k$, $b_{m,n}^k$, $c_{m,n}^k$, $d_{m,n}^k$ are the coefficients. According to HAM procedure one can choose the initial guess of the form

$$f_0(\eta) = 0, \tag{1.32}$$

$$g_0(\eta) = (1 - \alpha_1) + (2\alpha_1 - 1)\exp(-\eta), \tag{1.33}$$

$$\theta_0(\eta) = \exp(-\eta), \tag{1.34}$$

$$\phi_0(\eta) = \exp(-\eta). \tag{1.35}$$

The auxiliary linear operators are

$$\mathcal{L}_f = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}, \tag{1.36}$$

$$\mathcal{L}_g = \frac{d^2 g}{d\eta^2} + \frac{dg}{d\eta}, \tag{1.37}$$

$$\mathcal{L}_\theta = \frac{d^2 \theta}{d\eta^2} - \theta, \tag{1.38}$$

$$\mathcal{L}_\phi = \frac{d^2 \phi}{d\eta^2} - \phi, \tag{1.39}$$

which have the following property.

$$\mathcal{L}_f[C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta)] = 0, \tag{1.40}$$

$$\mathcal{L}_g[C_4 + C_5 \exp(-\eta)] = 0, \tag{1.41}$$

$$\mathcal{L}_\theta[C_6 \exp(\eta) + C_7 \exp(-\eta)] = 0, \tag{1.42}$$

$$\mathcal{L}_\phi[C_8 \exp(\eta) + C_9 \exp(-\eta)] = 0, \tag{1.43}$$

where C_i $(i = 1 - 9)$ are arbitrary constants.

1.4 Zeroth-order deformation equation

If $p \in [0, 1]$ is an embedding parameter and $\hbar_f$, $\hbar_g$, $\hbar_\theta$, $\hbar_\phi$ indicate the non zero auxiliary parameters respectively then the zeroth order deformation problems are

$$(1-p)\,\mathcal{L}_f[\hat{f}(\eta;p)-\hat{f}_0\,(\eta)] = p\hbar_f N_f[\hat{f}(\eta;p),\hat{g}(\eta;p),\hat{\theta}(\eta;p),\hat{\phi}(\eta;p)], \tag{1.44}$$

$$(1-p)\,\mathcal{L}_g[\hat{g}(\eta;p)-\hat{g}_0\,(\eta)] = p\hbar_g N_g[\hat{f}(\eta;p),\hat{g}(\eta;p)], \tag{1.45}$$

$$(1-p)\,\mathcal{L}_\theta[\hat{\theta}(\eta;p)-\hat{\theta}_0\,(\eta)] = p\hbar_\theta N_\theta[\hat{f}(\eta;p),\hat{\theta}(\eta;p)], \tag{1.46}$$

$$(1-p)\,\mathcal{L}_\phi[\hat{\phi}(\eta;p)-\hat{\phi}_0\,(\eta)] = p\hbar_\phi N_\phi[\hat{f}(\eta;p),\hat{\phi}(\eta;p)], \tag{1.47}$$

$$\hat{f}(0;p) = 0 = \hat{f}'(0;p),\ \hat{g}(0;p) = \alpha_1,\ \hat{\theta}(0;p) = \hat{\phi}(0;p) = 1, \tag{1.48}$$

$$\hat{f}'(\infty;p) = 0,\ \hat{g}(\infty;p) = 1-\alpha_1,\ \hat{\theta}(\infty;p) = \hat{\phi}(\infty;p) = 0, \tag{1.49}$$

in which the non linear operators $N_f, N_g, N_\theta, N_\phi$ are

$$\begin{aligned} N_f[\hat{f}(\eta;p),\hat{g}(\eta;p),\hat{\theta}(\eta;p),\hat{\phi}(\eta;p)] &= \frac{\partial^3\hat{f}(\eta;p)}{\partial\eta^3} - 2[(\hat{g}(\eta;p))^2-(1-\alpha_1)^2] + \frac{1}{2}\left(\frac{\partial\hat{f}(\eta;p)}{\partial\eta}\right)^2 \\ &\quad -\hat{f}(\eta;p)\frac{\partial^2\hat{f}(\eta;p)}{\partial\eta^2} - 2\lambda_1(\hat{\theta}(\eta;p)+N_1\hat{\phi}(\eta;p)) \\ &\quad -s\left(\frac{\partial\hat{f}(\eta;p)}{\partial\eta}+\frac{1}{2}\eta\frac{\partial^2\hat{f}(\eta;p)}{\partial\eta^2}\right), \end{aligned} \tag{1.50}$$

$$\begin{aligned} N_g[\hat{g}(\eta;p),\hat{f}(\eta;p)] &= \frac{\partial^2\hat{g}(\eta;p)}{\partial\eta^2} - [\hat{f}(\eta;p)\frac{\partial\hat{g}(\eta;p)}{\partial\eta} - \hat{g}(\eta;p)\frac{\partial\hat{f}(\eta;p)}{\partial\eta}] \\ &\quad +s\left(1-\alpha_1-\hat{g}(\eta;p)-\frac{1}{2}\eta\frac{\partial\hat{g}(\eta;p)}{\partial\eta}\right), \end{aligned} \tag{1.51}$$

$$\begin{aligned} N_\theta[\hat{\theta}(\eta;p),\hat{f}(\eta;p)] &= \frac{1}{\Pr}\frac{\partial^2\hat{\theta}(\eta;p)}{\partial\eta^2} - \left(\hat{f}(\eta;p)\frac{\partial\hat{\theta}(\eta;p)}{\partial\eta} - \frac{1}{2}\frac{\partial\hat{f}(\eta;p)}{\partial\eta}\hat{\theta}(\eta;p)\right) \\ &\quad -s\left(2\hat{\theta}(\eta;p)+\frac{1}{2}\eta\frac{\partial\hat{\theta}(\eta;p)}{\partial\eta}\right), \end{aligned} \tag{1.52}$$

$$N_\phi[\hat{\phi}(\eta;p),\hat{f}(\eta;p)] = \frac{1}{Sc}\frac{\partial^2\hat{\phi}(\eta;p)}{\partial\eta^2} - s\left(2\hat{\phi}(\eta;p)+\frac{1}{2}\eta\frac{\partial\hat{\phi}(\eta;p)}{\partial\eta}\right) - \left(\hat{f}(\eta;p)\frac{\partial\hat{\phi}(\eta;p)}{\partial\eta} - \frac{1}{2}\frac{\partial\hat{f}(\eta;p)}{\partial\eta}\hat{\phi}(\eta;p)\right). \quad (1.53)$$

Obviously

$$\hat{f}(\eta;0) = f_0(\eta),\ \hat{f}(\eta;1) = f(\eta), \quad (1.54)$$

$$\hat{g}(\eta;0) = g_0(\eta),\ \hat{g}(\eta;1) = g(\eta), \quad (1.55)$$

$$\hat{\theta}(\eta;0) = \theta_0(\eta),\ \hat{\theta}(\eta;1) = \theta(\eta), \quad (1.56)$$

$$\hat{\phi}(\eta;0) = \phi_0(\eta),\ \hat{\phi}(\eta;1) = \phi(\eta). \quad (1.57)$$

As p goes from 0 to 1, $\hat{f}(\eta;p)$, $\hat{g}(\eta;p)$, $\hat{\theta}(\eta;p)$ and $\hat{\phi}(\eta;p)$ vary from initial guesses $f_0(\eta)$, $g_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$ to final solutions $f(\eta)$, $g(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ respectively. Making the assumption that the auxiliary parameters $\hbar_f$, $\hbar_g$ $\hbar_\theta$ and $\hbar_\phi$ are so properly chosen that the Taylor series of $f(\eta;p)$, $g(\eta;p)$, $\theta(\eta;p)$ and $\phi(\eta;p)$ expanded with respect to embedding parameters converges at $p=1$. Thus we can write

$$\hat{f}(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \quad (1.58)$$

$$\hat{g}(\eta;p) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta)p^m, \quad (1.59)$$

$$\hat{\theta}(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)p^m, \quad (1.60)$$

$$\hat{\phi}(\eta;p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)p^m, \quad (1.61)$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial \eta^m}\bigg|_{p=0}, g_m(\eta) = \frac{1}{m!}\frac{\partial^m g(\eta;p)}{\partial \eta^m}\bigg|_{p=0},$$
$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \theta(\eta;p)}{\partial \eta^m}\bigg|_{p=0}, \phi_m(\eta) = \frac{1}{m!}\frac{\partial^m \phi(\eta;p)}{\partial \eta^m}\bigg|_{p=0}. \quad (1.62)$$

With the help of Eq. (1.62), Eqs. (1.58) to (1.61) can be written as

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \quad (1.63)$$

$$g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta), \quad (1.64)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta), \quad (1.65)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta). \quad (1.66)$$

mth-order deformation equations are defined as

$$\mathcal{L}_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f R_m^f(\eta), \quad (1.67)$$

$$\mathcal{L}_g[g_m(\eta) - \chi_m g_{m-1}(\eta)] = \hbar_g R_m^g(\eta), \quad (1.68)$$

$$\mathcal{L}_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_\theta R_m^\theta(\eta), \quad (1.69)$$

$$\mathcal{L}_\phi[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = \hbar_\phi R_m^\phi(\eta), \quad (1.70)$$

the corresponding boundary conditions for mth deformation problems are

$$f_m(0) = f_m'(0) = g_m(0) = \theta_m(0) = \phi_m(0) = 0, \quad (1.71)$$

$$f_m'(\infty) = g_m(\infty) = \theta_m(\infty) = \phi_m(\infty) = 0, \quad (1.72)$$

where

$$\begin{aligned} R_m^f(\eta) &= f_{m-1}^{'''} - \sum_{k=0}^{m-1} f_k f_{m-1-k}^{'''} + \frac{1}{2}\sum_{k=0}^{m-1} f_k^{'} f_{m-1-k}^{'} - 2[g_k g_{m-1-k} - (1-\alpha_1)^2] \\ & -2\lambda_1(\theta_{m-1} + N_1\phi_{m-1}) - s(f_{m-1}^{'} + \frac{1}{2}\eta f_{m-1}^{'''}), \end{aligned} \quad (1.73)$$

$$R_m^g(\eta) = g_{m-1}^{''} - \sum_{k=0}^{m-1}[f_k g_{m-1-k}^{'} - g_k f_{m-1-k}^{'}] + s(1 - \alpha_1 - g_{m-1} - \frac{1}{2}\eta g_{m-1}^{'}), \quad (1.74)$$

$$R_m^\theta(\eta) = \frac{1}{\Pr}\theta_{m-1}^{''} - \sum_{k=0}^{m-1}[f_k\theta_{m-1-k}^{'} - \frac{1}{2}\theta_k f_{m-1-k}^{'}] - s(2\theta_{m-1} + \frac{1}{2}\eta\theta_{m-1}^{'}), \quad (1.75)$$

$$R_m^\phi(\eta) = \frac{1}{Sc}\phi_{m-1}^{''} - \sum_{k=0}^{m-1}[f_k\phi_{m-1-k}^{'} - \frac{1}{2}\phi_k f_{m-1-k}^{'}] - s(2\phi_{m-1} + \frac{1}{2}\eta\phi_{m-1}^{'}) \quad (1.76)$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1. \end{cases} \quad (1.77)$$

The general solutions of Eqs. (1.67) – (1.72) can be written as

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2\exp(\eta) + C_3\exp(-\eta), \quad (1.78)$$

$$g_m(\eta) = g_m^*(\eta) + C_4 + C_5\exp(-\eta), \quad (1.79)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_6\exp(\eta) + C_7\exp(-\eta), \quad (1.80)$$

$$\phi_m(\eta) = \phi_m^*(\eta) + C_8\exp(\eta) + C_9\exp(-\eta), \quad (1.81)$$

where C_1 to C_9 are constants.

1.5 Convergence of the HAM Solutions

Obviously the series solutions depend upon the non-zero auxiliary parameters $\hbar_f$, $\hbar_g$, $\hbar_\theta$ and $\hbar_\phi$ which can adjust and control the convergence of the HAM solutions. In order to see the range of admissible values of $\hbar_f$, $\hbar_g$, $\hbar_\theta$ and $\hbar_\phi$, the $\hbar$ − *curve* of the functions $f^{''}(0)$, $g^{'}(0)$, $\theta^{'}(0)$ and $\phi^{'}(0)$ are sketched for 15-order of approximations in Figs 1.1 to 1.2. It is found that the range of admissible values of $\hbar_f, \hbar_g, \hbar_\theta$ and $\hbar_\phi$ are $-1.1 \le \hbar_f \le -0.5$, $-1.1 \le \hbar_g \le -0.6$, $-1.3 \le \hbar_\theta \le -0.5$, $-1.0 \le \hbar_\phi \le -0.2$.

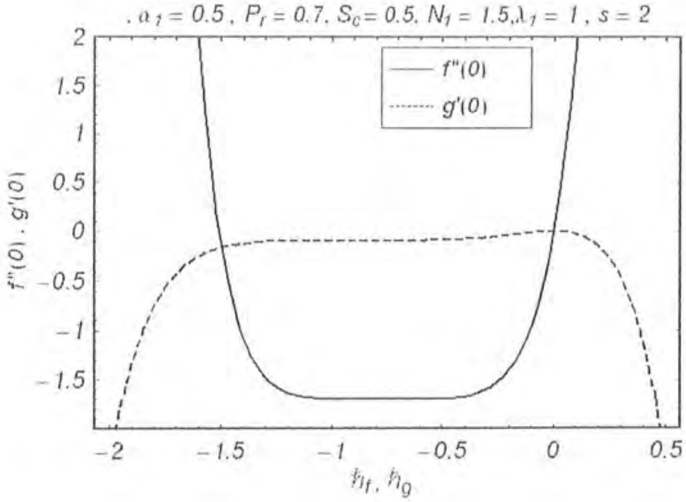

Fig.1.1. $\hbar$−curve of $f^{''}(0)$ and $g^{'}(0)$ at 10th approximation.

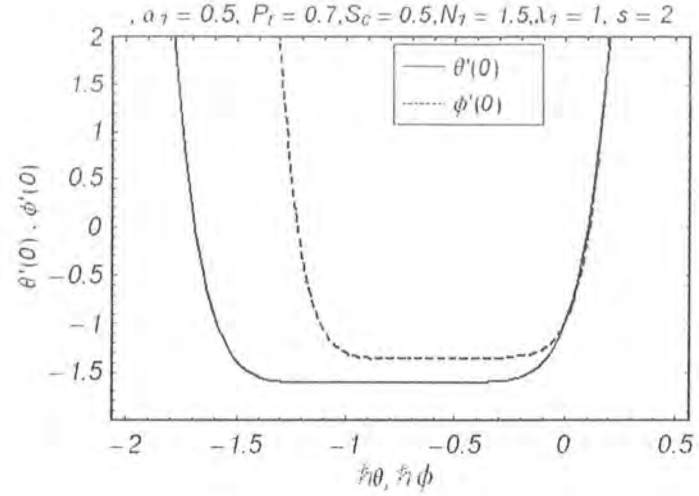

Fig.1.2. $\hbar-$curve for $\theta^{'}(0)$ and $\phi^{'}(0)$ at 10th approximation.

Table 1.1 Convergence of HAM solution for different order of approximations

order of convergence	$-f^{''}(0)$	$-g^{'}(0)$	$-\theta^{'}(0)$	$-\phi^{'}(0)$
1	0.8	0.0	1.4476	1.3333
5	1.0644	0.48753	1.6004	1.3549
10	1.07752	0.55758	1.60334	1.3564
15	1.07757	0.55761	1.60332	1.35646

1.6 Results and discussion

We have applied Homotopy analysis method to solve equations (1.13) – (1.16) subject to boundary conditions (1.17). Homotopy analysis method is applied for the convergence of the series solutions which are shown in the table 1.1.The residual errors for velocity, temperature and concentration are also plotted in Figs. 1.3 – 1.6. The graphical results for velocity, temperature, concentration and skin friction coefficient have been discussed against different physical parameters. Figs. 1.7 and 1.8 shows the effect of the parameter α_1(which is the ratio of the angular velocity of the cone to the composite angular velocity) on the velocity profiles in the tangential and azimuthal directions $(f'(\eta), g(\eta))$. It is clear from the figure the tangential velocity $f'(\eta)$ increases with the increase of α_1 while the azimuthal velocity $g(\eta)$ decreases. Figs.1.9 – 1.10 explains the effects of Pr on θ for $Sc = 2.57$ and $Sc = 0.22$. respectively. It is found that in both cases the temperature field decreases. Figs. 1.11 – 1.12 discuss the effects of Sc on ϕ when $Pr = 0.7$ and $Pr = 7.0$ respectively. It is observed that concentration field decreases for both the cases. In Figs. 1.13 – 1.14 we observe the Effects of $N1$ on $-\theta'(0)$ and $-\phi'(0)$. It is observed that by increasing N_1 the nusselt and the sherwood number decreases. Figs. 1.15 – 1.16 show the effect of buoyancy parameter λ_1 on temperature profile Θ and concentration profile Φ for $Pr = 0.7$. It is seen that both the temperature and concentration profiles decreases by increasing λ_1.

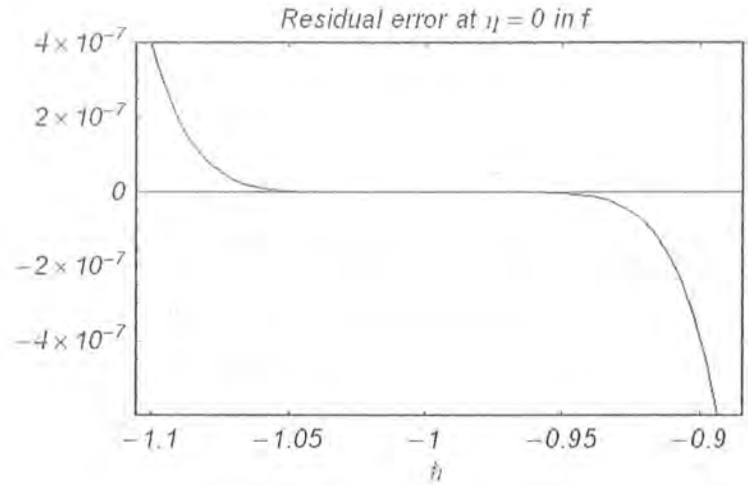

Fig.1.3. Residual error in f

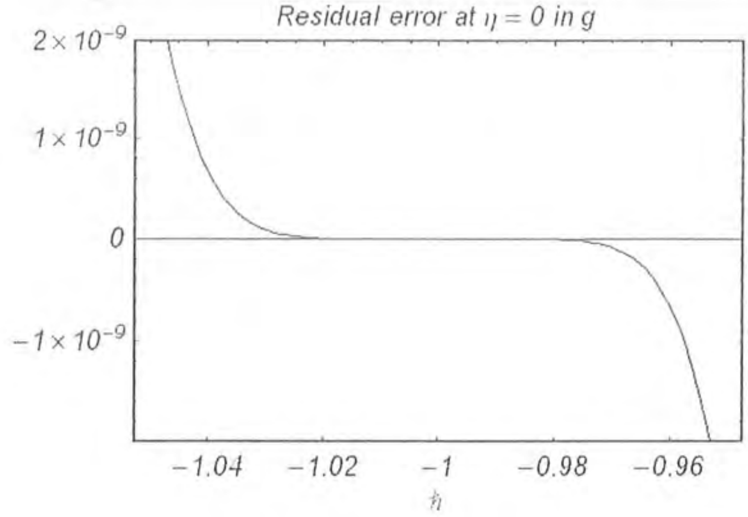

Fig.1.4. Residual error in g

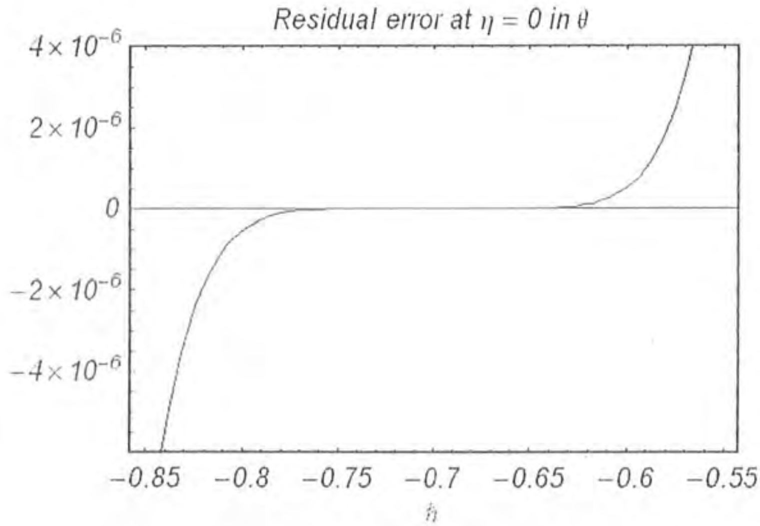

Fig.1.5. Residual error in θ

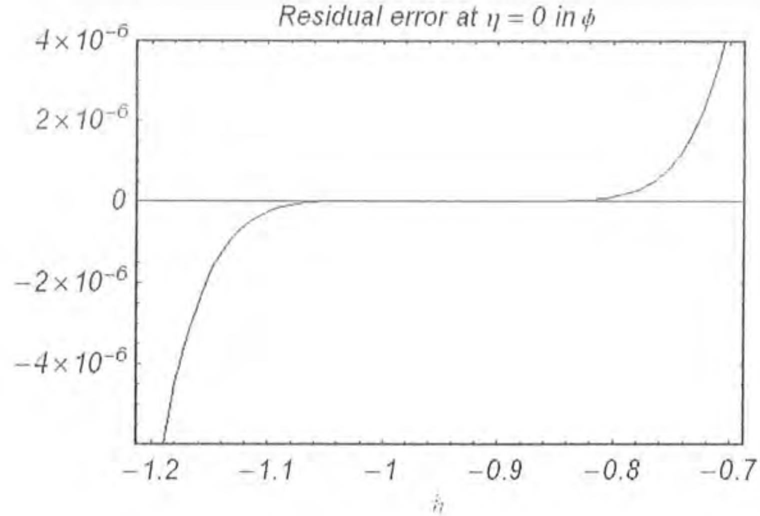

Fig.1.6. Residual error in ϕ

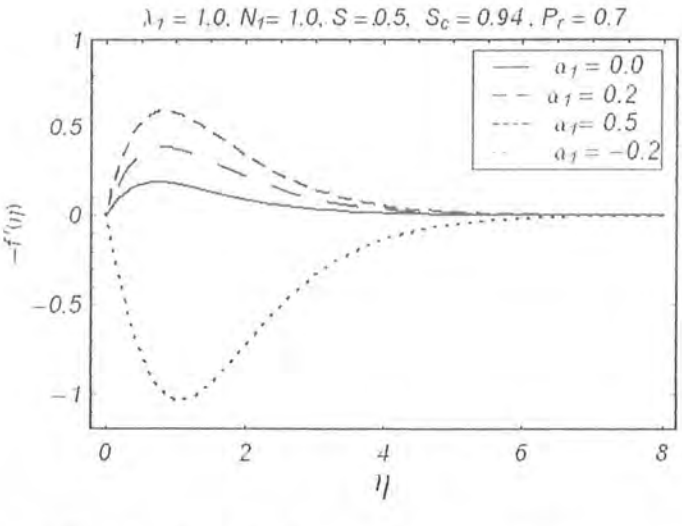

Fig.1.7. Effects of α_1 on $-f'$ when $\lambda_1 = 1.0$

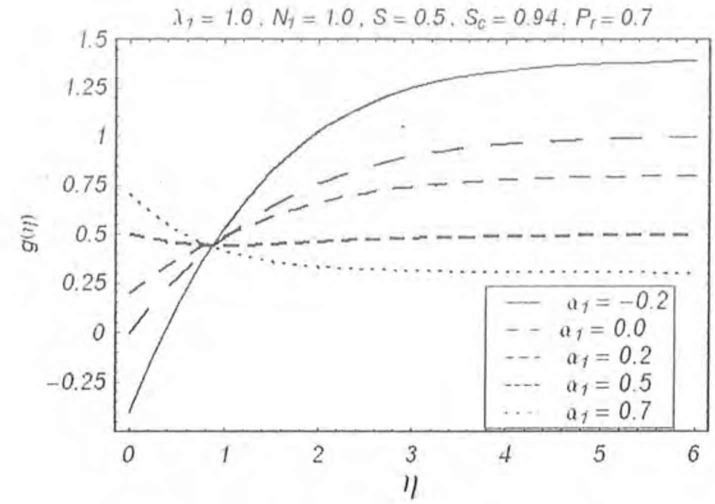

Fig.1.8. Effects of α_1 on g when $\lambda_1 = 1.0$

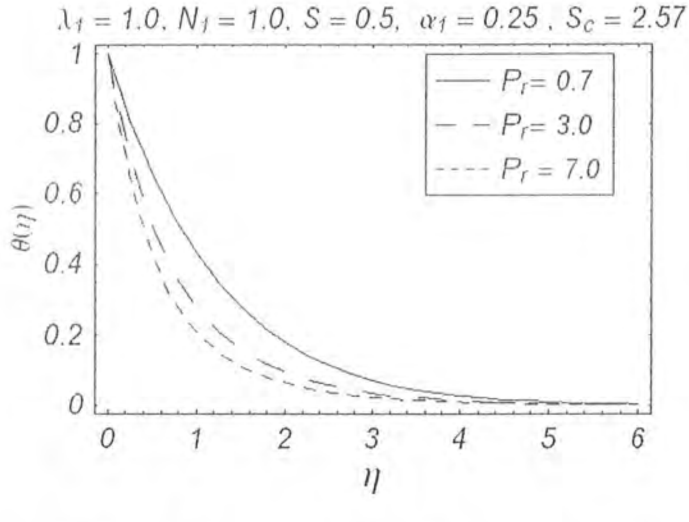

Fig.1.9. Effects of P_r on θ when $S_c = 2.57$

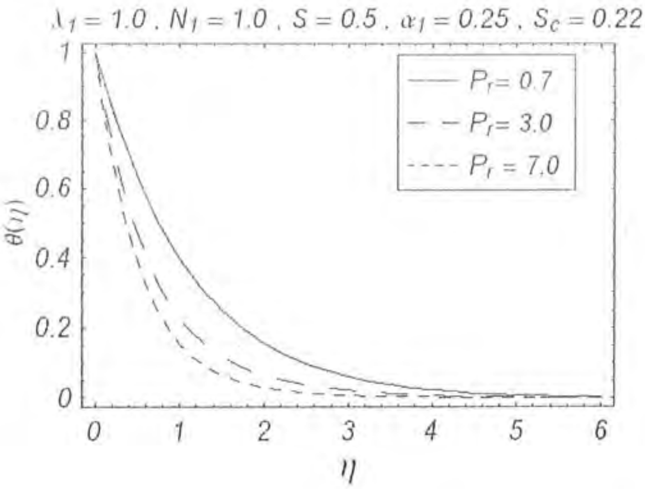

Fig.1.10. Effects of P_r on θ when $S_c = 0.22$

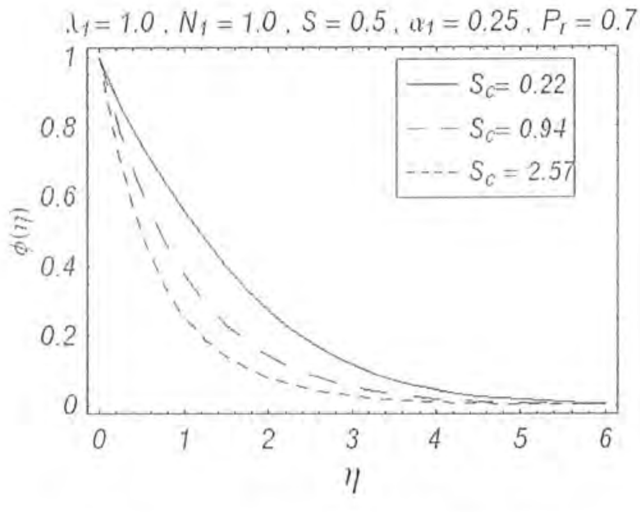

Fig.1.11. Effects of S_c on ϕ when $P_r = 0.7$

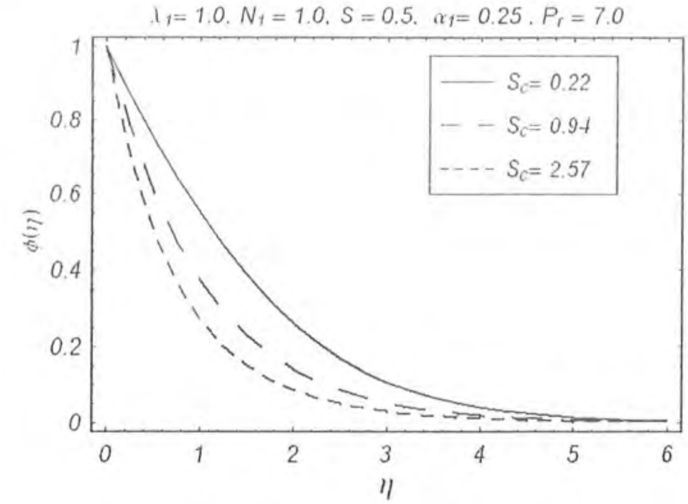

Fig.1.12. Effects of S_c on ϕ when $P_r = 7.0$

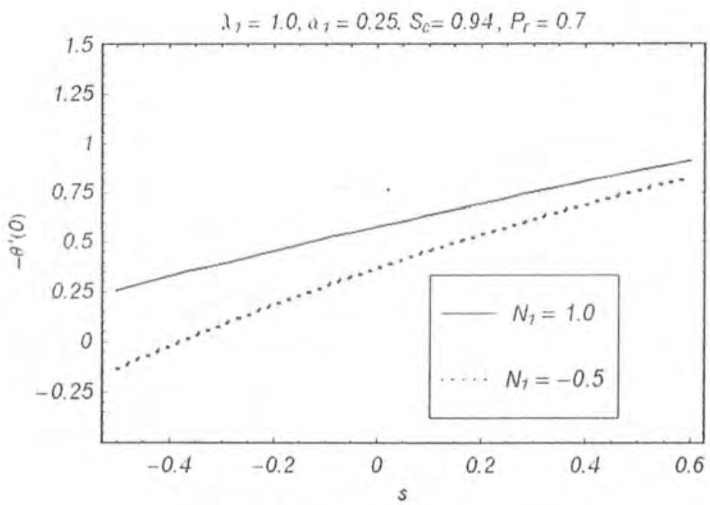

Fig.1.13. Effects of N_1 on $-\theta'(0)$

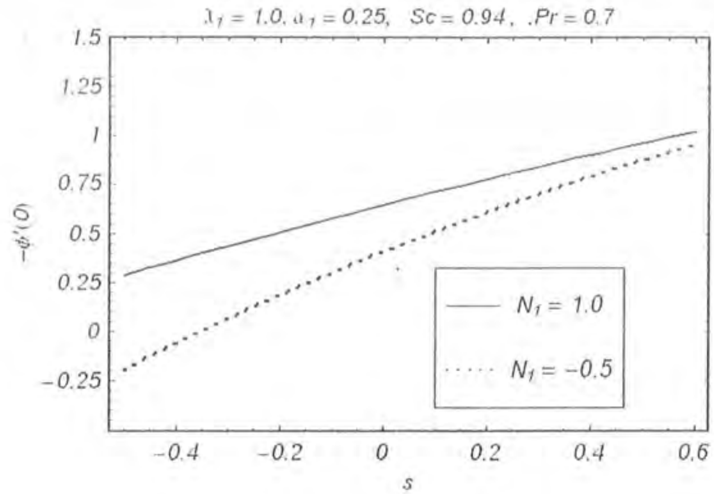

Fig.1.14. Effects of N_1 on $-\phi'(0)$

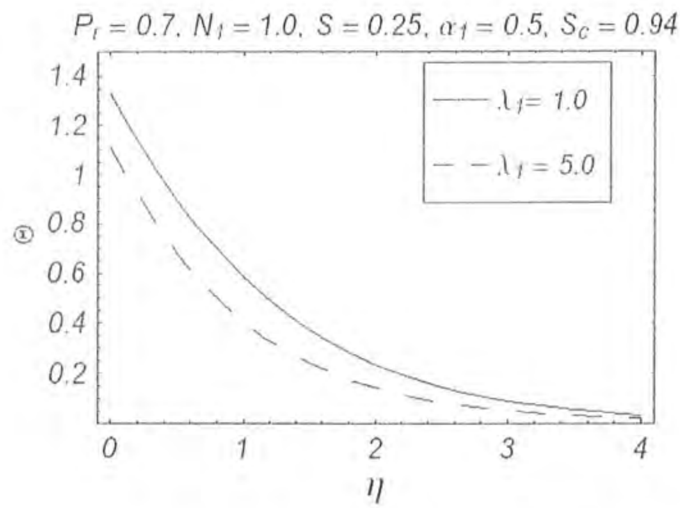

Fig.1.15. Effects of λ_1 on Θ when $P_r = 0.7$

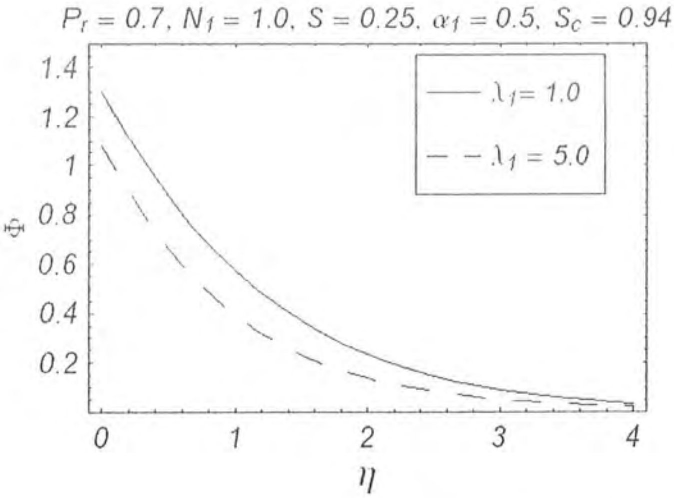

Fig.1.16. Effects of λ_1 on Φ when $P_r = 0.7$

Chapter 2

Mixed convection MHD flow of a micropolar fluid on a rotating cone in a rotating fluid

2.1 Introduction

In this chapter, we have discussed the mixed convection MHD flow of a micropolar fluid on a rotating cone in a rotating fluid. The governing equations of micropolar fluid along with heat equation are firstly simplified by using boundary layer and similarity transformations and then solved by an analytic method, Homotopy analysis method. At the end the graphical results for velocity, microrotation and temperature are plotted.

2.2 Mathematical formulation

Consider boundary layer flow of an incompressible MHD micropolar fluid over a vertical cone rotating in an ambient fluid around the axis of cone. Both the fluid and the cone are in a rigid body rotation about the axis of cone. A constant magnetic field B_0 is applied along the flow direction such that the effect of induced magnetic field are negligible.

The equations for micropolar fluid in the presence of MHD can be written as

$$\operatorname{div}\mathbf{V} = 0, \tag{2.1}$$

$$\rho\frac{\mathrm{DV}}{\mathrm{Dt}} = \operatorname{div}\sigma - k\nabla \times (\nabla \times \mathbf{V}) + \kappa(\nabla \times \mathbf{N}) + \mathbf{J} \times \mathbf{B}, \tag{2.2}$$

$$\rho j\frac{d\mathbf{N}}{dt} = -2\kappa\mathbf{N} + \kappa\nabla \times \mathbf{V} - \gamma(\nabla \times \nabla \times \mathbf{N}) + (\alpha + \beta + \gamma)\nabla(\nabla \times \mathbf{N}), \tag{2.3}$$

$$\rho c_p\frac{d\mathrm{T}}{dt} = \boldsymbol{\sigma}.\mathrm{L} - \operatorname{div}\mathrm{q}, \tag{2.4}$$

in which ρ is the density, $\frac{\mathrm{D}}{\mathrm{Dt}}$ is the total derivative, κ is the vortex viscosity, $\mathbf{N}$ is the microrotation vector, c_p is the specific heat, $\mathbf{T}$ is the temperature, $\mathrm{q}\,(= -k\operatorname{div}\sigma)$ is the heat flux and $\boldsymbol{\sigma}$ is the Cauchy stress tensor. The last term on the right hand side of Eq. (2.2) represents the pondermotive force on the conducting fluid due to interaction of $\mathbf{J}$ (current density) and B (magnetic induction), known as lorentz force. We assume that the induced magnetic field E is negligible in comparison to the applied magnetic field B_0. We seek the velocity field and microinertia normal to xy-plane of the form

$$\mathbf{V}(x, y) = (u(x, y), v(x, y), w(x, y)), \tag{2.5}$$

$$\mathbf{N} = (0, 0, N(x, y)). \tag{2.6}$$

Using the Ohm,s law [11], the last term on the right hand side of Eq. 2.2 can be written as

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2\mathbf{V}. \tag{2.7}$$

Making use of Eqs. (2.5) to (2.7) into Eqs. (2.1) to (2.4), the governing equations in component form can be written as

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{2.8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{w^2}{x}=(\nu+\frac{\kappa}{\rho})\frac{\partial^2 u}{\partial y^2}+g\beta\cos\phi\,(T-T_\infty)+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma}{\rho}B_0^2u, \tag{2.9}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+\frac{uw}{x}=(\nu+\frac{\kappa}{\rho})\frac{\partial^2 w}{\partial y^2}-\frac{\sigma}{\rho}B_0^2w, \tag{2.10}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=-\frac{\kappa}{\rho j}(2N+\frac{\partial u}{\partial y})+\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}, \tag{2.11}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{2.12}$$

The corresponding boundary conditions for the problem are

$$\begin{aligned} u &= 0,\ v=0,\ N=-n\frac{\partial u}{\partial y},\ w=r\Omega,\ T=T_w(x)\ \text{at}\ y=0 \\ u &\to 0,\ v\to 0,\ w\to 0,\ N\to 0,\ T\to T_\infty,\ \text{as}\ y\longrightarrow\infty. \end{aligned} \tag{2.13}$$

In the above equations, u, v and w are the velocity components along x, y and z-axis, respectively, N is the microrotation velocity, ν is the viscosity, ρ is the density, $g\beta\cos\phi$ comes due to effects of gravity, j, γ and k are the microinertia per unit mass, spin gradient viscosity and vortex viscosity, respectively, which are assumed to be constant. n is a constant and $0\le n\le 1$. The case $n=0$, which indicates $N=0$ at the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate. This case is also known as the strong concentration of microelements. The case $n=\frac{1}{2}$ indicates the vanishing of antisymmetric part of the stress tensor and denotes the weak concentration of microelements. The case $n=1$ is used for the modeling of turbulent boundary layer flows. The continuity equation may be satisfied by introducing the stream function Ψ defined as

$$ru=\frac{\partial\psi}{\partial y}\ \text{and}\ rv=-\frac{\partial\psi}{\partial x}.$$

Introducing the nondimensional quantities

$$\begin{aligned}
f(\xi,\eta) &= \frac{\psi}{r\alpha\chi},\ g(\xi,\eta)=\frac{w}{r\Omega},\ h(\xi,\eta)=(\frac{\nu x^2}{\alpha^2\chi^3})N,\\
\theta(\xi,\eta) &= \frac{(T-T_\infty)}{(T_w-T_\infty)},\ \eta=(\frac{y}{x})\chi,\ \xi=\frac{\zeta}{(1+\zeta)},\\
\chi &= \frac{R_e^{\frac{1}{2}}}{\xi},\ \zeta=\frac{R_e^{\frac{1}{2}}}{R_a^{\frac{1}{4}}},\ \Delta=\frac{\kappa}{\mu},\ \lambda=\frac{\gamma}{\mu j},\\
R_e &= x^2\Omega\sin\frac{\phi}{\nu},\ R_a=\frac{g\beta(T_w-T_\infty)x^2\cos\phi}{\alpha\nu},\\
\upsilon &= (\frac{\upsilon}{j\Omega\sin\phi}),\ (T_w-T_\infty)\cdot x^m
\end{aligned} \tag{2.14}$$

With the help of Eq. (2.14), the non-dimensional form of Eqs. (2.8) to (2.13) take the form

$$(1+\Delta)\Pr f''' + \frac{8-(1-\xi)(1-m)}{4}ff'' - \frac{2-(1-\xi)(1-m)}{2}\left(f'\right)^2 + (\Pr)^2\xi^4g^2$$

$$+\Pr(1-\xi)^4\theta + \Delta h' - \Pr M\xi^2 f' - \frac{\xi(1-\xi)(1-m)}{4}\left[f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right]=0, \tag{2.15}$$

$$(1+\Delta)\Pr g'' + \frac{8-(1-\xi)(1-m)}{4}fg' - 2f'g - \Pr M\xi^2 g$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[f'\frac{\partial g}{\partial\xi} - g'\frac{\partial f}{\partial\xi}\right]=0, \tag{2.16}$$

$$\lambda\Pr h'' + \frac{8-(1-\xi)(1-m)}{4}fh' + 2f'h - \Pr\upsilon\Delta(2h+\Pr f'')\xi^2$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[f'\frac{\partial h}{\partial\xi} - h'\frac{\partial f}{\partial\xi}\right]=0, \tag{2.17}$$

$$\theta'' + \frac{8-(1-\xi)(1-m)}{4}f\theta' - mf'\theta - \frac{\xi(1-\xi)(1-m)}{4}\left[f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right]=0, \tag{2.18}$$

$$
\begin{aligned}
f'(\xi,0) &= 0, f(\xi,0)=0, g(\xi,0)=1, \theta(\xi,0)=1, h(\xi,0)=-nf''(\xi,0), \\
f'(\xi,\infty) &\to 0, g(\xi,\infty)\to 0, \theta(\xi,\infty)\to 0, h(\xi,\infty)\to 0,
\end{aligned} \tag{2.19}
$$

where the prime denotes partial differentiation with respect to η, M is the magnetic parameter, Pr is the prandtl number, Δ is the couple parameter and λ and v are the dimensionless material parameters. The quantity m in the above equation comes from the temperature distribution on the cone surface. The temperature is constant, linear and parabolic for $m = 0, 1$ and 2 respectively. For micropolar boundary layer flow, the wall shear stress τ_w is given by

$$
\tau_w = [(\mu + k)\frac{\partial u}{\partial y} + kN]_{y=0}. \tag{2.20}
$$

The local friction factor is defined as follows:

$$
C_{fx} = \frac{\tau_w}{\rho \frac{U^2}{2}},
$$

where

$$
U = r\Omega + [g\beta(T_w - T_\infty)x]^{\frac{1}{2}}.
$$

The couple stress at the wall is given by

$$
m_w = \gamma \left(\frac{\partial N}{\partial y}\right)_{y=0} = \gamma \left(\frac{\alpha^2 \chi^4}{\nu x^3}\right) H'(\xi, 0).
$$

The Nusselt number can be expressed as

$$
\frac{Nu_x}{\chi} = -\theta'(\xi, 0). \tag{2.21}
$$

The Eqs. (2.15) and (2.18) are coupled nonlinear differential equations, to find the analytic solutions we use Homotopy analysis method (HAM).

2.3 Homotopy analysis solution

The highly nonlinear coupled differential equations will be solved analytically by Homotopy analysis method. According to HAM procedure, we express $f(\xi,\eta)$, $g(\xi,\eta)$, $h(\xi,\eta)$ and $\theta(\xi,\eta)$ by a set of base functions

$$\left\{\xi^k \eta^j \exp(-n\eta)\,|k \geq 0, n \geq 0, j \geq 0\right\}, \tag{2.22}$$

in the form

$$f(\xi,\eta) = a_{0,0}^0 + \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=1}^{\infty} a_{j,n}^k \xi^k \eta^j \exp(-n\eta), \tag{2.23}$$

$$g(\xi,\eta) = \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=1}^{\infty} b_{j,n}^k \xi^k \eta^j \exp(-n\eta), \tag{2.24}$$

$$h(\xi,\eta) = \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=1}^{\infty} c_{j,n}^k \xi^k \eta^j \exp(-n\eta), \tag{2.25}$$

$$\theta(\xi,\eta) = \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{n=1}^{\infty} d_{j,n}^k \xi^k \eta^j \exp(-n\eta), \tag{2.26}$$

in which $a_{j,n}^k$, $b_{j,n}^k$, $c_{j,n}^k$ and $d_{j,n}^k$ are the coefficients . Based on the rule of solution expressions and the boundary conditions one can choose the initial guesses f_0, g_0, h_0 and θ_0 as follow:

$$f_0(\xi,\eta) = 0, \tag{2.27}$$

$$g_0(\xi,\eta) = \exp(-\eta), \tag{2.28}$$

$$h_0(\xi,\eta) = -nf_0''(0)\exp(-\eta), \tag{2.29}$$

$$\theta_0(\xi,\eta) = \exp(-\eta). \tag{2.30}$$

The auxiliary linear operators are

$$\mathcal{L}_f = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}, \tag{2.31}$$

$$\mathcal{L}_g = \frac{d^2 g}{d\eta^2} - g, \tag{2.32}$$

$$\mathcal{L}_h = \frac{d^2 h}{d\eta^2} - h, \tag{2.33}$$

$$\mathcal{L}_\theta = \frac{d^2 \theta}{d\eta^2} - \theta, \tag{2.34}$$

which satisfy

$$\mathcal{L}_f[C_1 + C_2 \exp(\eta) + C_3 \exp(-\eta)] = 0, \tag{2.35}$$

$$\mathcal{L}_g[C_4 \exp(\eta) + C_5 \exp(-\eta)] = 0, \tag{2.36}$$

$$\mathcal{L}_h[C_6 \exp(\eta) + C_7 \exp(-\eta)] = 0, \tag{2.37}$$

$$\mathcal{L}_\theta[C_8 \exp(\eta) + C_9 \exp(-\eta)] = 0, \tag{2.38}$$

where $C_i\,(i = 1 - 9)$ are arbitrary constants.

2.3.1 Zeroth-order deformation equation

If $p \in [0, 1]$ is an embedding parameter and $\hbar_f$, $\hbar_g$, $\hbar_h$ and $\hbar_\theta$ indicate the non zero auxiliary parameters respectively then the zeroth order deformation problems are

$$(1-p)\,\mathcal{L}_f[\hat{f}(\xi,\eta;p) - \hat{f}_0\,(\xi,\eta)] = p\hbar_f N_f[\hat{f}(\xi,\eta;p), \hat{g}(\xi,\eta;p), \hat{h}(\xi,\eta;p), \hat{\theta}(\xi,\eta;p)], \tag{2.39}$$

$$(1-p)\,\mathcal{L}_g[\hat{g}(\xi,\eta;p) - \hat{g}_0\,(\xi,\eta)] = p\hbar_g N_g[\hat{f}(\xi,\eta;p), \hat{g}(\xi,\eta;p)], \tag{2.40}$$

$$(1-p)\,\mathcal{L}_h[\hat{h}(\xi,\eta;p) - \hat{h}_0\,(\xi,\eta)] = p\hbar_h N_h[\hat{f}(\xi,\eta;p), \hat{h}(\xi,\eta;p)], \tag{2.41}$$

$$(1-p)\,\mathcal{L}_\theta[\hat{\theta}(\xi,\eta;p) - \hat{\theta}_0\,(\xi,\eta)] = p\hbar_\theta N_\theta[\hat{f}(\xi,\eta;p), \hat{\theta}(\xi,\eta;p)], \tag{2.42}$$

$$\hat{f}(\eta;\xi)\Big|_{\eta=0} = 0 = \left.\frac{\partial \hat{f}(\eta;\xi)}{\partial \eta}\right|_{\eta=0}, \quad \hat{g}(\eta;\xi)|_{\eta=0} = 1 \tag{2.43}$$

$$\hat{\theta}(\eta;\xi)\Big|_{\eta=0}, = 1, \quad \hat{h}(\eta;\xi)\Big|_{\eta=0} = -n\frac{\partial^2 \hat{f}(\eta;\xi)}{\partial \eta^2},$$

$$\left.\frac{\partial \hat{f}(\eta;\xi)}{\partial \eta}\right|_{\eta=\infty} = \hat{g}(\eta;\xi)|_{\eta=\infty} = \hat{\theta}(\eta;\xi)\Big|_{\eta=\infty} = \hat{h}(\eta;\xi)\Big|_{\eta=\infty} = 0, \tag{2.44}$$

in which the non-linear operators N_f, N_g, N_h and N_θ are

$$N_f[\hat{f}(\xi,\eta;p),\hat{g}(\xi,\eta;p),\hat{h}(\xi,\eta;p),\hat{\theta}(\xi,\eta;p)] = (1+\Delta)\,\mathrm{Pr}\,\frac{\partial^3 \hat{f}(\xi,\eta;p)}{\partial \eta^3}^2$$

$$+\frac{8-(1-\xi)(1-m)}{4}\hat{f}(\xi,\eta;p)\frac{\partial^2 \hat{f}(\xi,\eta;p)}{\partial \eta^2} - \frac{2-(1-\xi)(1-m)}{2}\left(\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\right)^2$$

$$+\Delta\frac{\partial \hat{h}(\xi,\eta;p)}{\partial \eta} - \mathrm{Pr}\,M\xi^2\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta} + \mathrm{Pr}(1-\xi)^4\hat{\theta}(\xi,\eta;p) + (\mathrm{Pr})^2\xi^4\hat{g}^2(\xi,\eta;p)$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\frac{\partial^2 \hat{f}(\xi,\eta;p)}{\partial \eta \partial \xi} - \frac{\partial^2 \hat{f}(\xi,\eta;p)}{\partial \eta^2}\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \xi}\right], \tag{2.45}$$

$$N_g[\hat{g}(\xi,\eta;p),\hat{f}(\xi,\eta;p)] = (1+\Delta)\,\mathrm{Pr}\,\frac{\partial^2 \hat{g}(\xi,\eta;p)}{\partial \eta^2} - \mathrm{Pr}\,M\xi^2\hat{g}(\xi,\eta;p)$$

$$+\frac{8-(1-\xi)(1-m)}{4}\hat{f}(\xi,\eta;p)\frac{\partial \hat{g}(\xi,\eta;p)}{\partial \eta} - 2\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\hat{g}(\xi,\eta;p)$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{g}(\xi,\eta;p)}{\partial \xi} - \frac{\partial \hat{g}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \xi}\right], \tag{2.46}$$

$$N_h[\hat{h}(\xi,\eta;p),\hat{f}(\xi,\eta;p)] = \lambda\,\mathrm{Pr}\,\frac{\partial^2 \hat{h}(\xi,\eta;p)}{\partial \eta^2} + \frac{8-(1-\xi)(1-m)}{4}\hat{f}(\xi,\eta;p)\frac{\partial \hat{h}(\xi,\eta;p)}{\partial \eta}$$

$$+2\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\hat{h}(\xi,\eta;p) - \mathrm{Pr}\,v\Delta(2\hat{h}(\xi,\eta;p) + \mathrm{Pr}\,\frac{\partial^2 \hat{f}(\xi,\eta;p)}{\partial \eta^2})\xi^2$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{h}(\xi,\eta;p)}{\partial \xi} - \frac{\partial \hat{h}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \xi}\right], \tag{2.47}$$

$$N_\theta[\hat{\theta}(\xi,\eta;p),\hat{f}(\xi,\eta;p)] = \frac{\partial^2 \hat{\theta}(\xi,\eta;p)}{\partial \eta^2} + \frac{8-(1-\xi)(1-m)}{4}\hat{f}(\xi,\eta;p)\frac{\partial \hat{\theta}(\xi,\eta;p)}{\partial \eta}$$

$$-\frac{\xi(1-\xi)(1-m)}{4}\left[\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{\theta}(\xi,\eta;p)}{\partial \xi} - \frac{\partial \hat{\theta}(\xi,\eta;p)}{\partial \eta}\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \xi}\right]$$

$$-m\frac{\partial \hat{f}(\xi,\eta;p)}{\partial \eta}\hat{\theta}(\xi,\eta;p), \tag{2.48}$$

for $p=0$ and $p=1$, we have

$$\hat{f}(\xi,\eta;0)=f_0\left(\xi,\eta\right),\ \hat{f}(\xi,\eta;1)=f\left(\xi,\eta\right), \tag{2.49}$$

$$\hat{g}(\xi,\eta;0)=g_0\left(\xi,\eta\right),\ \hat{g}(\xi,\eta;1)=g\left(\xi,\eta\right), \tag{2.50}$$

$$\hat{h}(\xi,\eta;0)=\theta_0\left(\xi,\eta\right),\ \hat{h}(\xi,\eta;1)=h\left(\xi,\eta\right), \tag{2.51}$$

$$\hat{\theta}(\xi,\eta;0)=\phi_0\left(\xi,\eta\right),\ \hat{\theta}(\xi,\eta;1)=\theta\left(\xi,\eta\right). \tag{2.52}$$

By Taylor theorem

$$\hat{f}(\xi,\eta;p)=f_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}f_m\left(\xi,\eta\right)p^m, \tag{2.53}$$

$$\hat{g}(\xi,\eta;p)=g_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}g_m\left(\xi,\eta\right)p^m, \tag{2.54}$$

$$\hat{h}(\xi,\eta;p)=h_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}h_m\left(\xi,\eta\right)p^m, \tag{2.55}$$

$$\hat{\theta}(\xi,\eta;p)=\theta_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}\theta_m\left(\xi,\eta\right)p^m, \tag{2.56}$$

$$\begin{aligned}
f_m(\eta) &= \frac{1}{m!}\frac{\partial^m f(\xi,\eta;p)}{\partial \eta^m}\bigg|_{p=0}, g_m(\eta)=\frac{1}{m!}\frac{\partial^m g(\xi,\eta;p)}{\partial \eta^m}\bigg|_{p=0}, \\
h_m(\eta) &= \frac{1}{m!}\frac{\partial^m h(\xi,\eta;p)}{\partial \eta^m}\bigg|_{p=0}, \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m \theta(\xi,\eta;p)}{\partial \eta^m}\bigg|_{p=0},
\end{aligned} \tag{2.57}$$

and

$$f\left(\xi,\eta\right)=f_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}f_m\left(\xi,\eta\right). \tag{2.58}$$

$$g\left(\xi,\eta\right)=g_0\left(\xi,\eta\right)+\sum_{m=1}^{\infty}g_m\left(\xi,\eta\right). \tag{2.59}$$

$$h(\xi,\eta) = h_0(\xi,\eta) + \sum_{m=1}^{\infty} h_m(\xi,\eta). \tag{2.60}$$

$$\theta(\xi,\eta) = \theta_0(\xi,\eta) + \sum_{m=1}^{\infty} \theta_m(\xi,\eta). \tag{2.61}$$

2.3.2 mth-order deformation equation

Differentiating the zeroth order deformation equation (2.39) to (2.42) with respect to p, then setting $p = 0$, and finally dividing them by $m!$, we obtain the mth-order deformation equations

$$\mathcal{L}_f[f_m(\xi,\eta) - \chi_m f_{m-1}(\xi,\eta)] = \hbar_f R_m^f(\xi,\eta), \tag{2.62}$$

$$\mathcal{L}_g[g_m(\xi,\eta) - \chi_m g_{m-1}(\xi,\eta)] = \hbar_g R_m^g(\xi,\eta), \tag{2.63}$$

$$\mathcal{L}_h[h_m(\xi,\eta) - \chi_m h_{m-1}(\xi,\eta)] = \hbar_h R_m^h(\xi,\eta), \tag{2.64}$$

$$\mathcal{L}\theta[\theta_m(\xi,\eta) - \chi_m \theta_{m-1}(\xi,\eta)] = \hbar_\theta R_m^\theta(\xi,\eta), \tag{2.65}$$

$$f_m(\xi,0) = f_m'(\xi,0) = g_m(\xi,0) = h_m(\xi,0) = \theta_m(\xi,0) = 0, \tag{2.66}$$

$$f_m'(\xi,\infty) = g_m(\xi,\infty) = h_m(\xi,\infty) = \theta_m(\xi,\infty) = 0, \tag{2.67}$$

where

$$\begin{aligned} R_m^f(\eta) &= (1+\Delta)\Pr f_{m-1}''' + \frac{8-(1-\xi)(1-m)}{4}\sum_{k=0}^{m-1} f_k f_{m-1-k}'' - \Pr M\xi^2 f_{m-1}' \\ &-\frac{2-(1-\xi)(1-m)}{2}\sum_{k=0}^{m-1} f_k' f_{m-1-k}' + (\Pr)^2\xi^4 g_k g_{m-1-k} + \Pr(1-\xi)^4\theta_{m-1} \\ &+\Delta h_{m-1}' - \frac{\xi(1-\xi)(1-m)}{4}\left[\left(\sum_{k=0}^{m-1} f_k' \frac{\partial f_{m-1-k}'}{\partial\xi} - f_k''\frac{\partial f_{m-1-k}}{\partial\xi}\right)\right], \end{aligned} \tag{2.68}$$

$$
\begin{aligned}
R_m^g(\eta) &= (1+\Delta)\Pr g_{m-1}^{''} + \frac{8-(1-\xi)(1-m)}{4}\sum_{k=0}^{m-1} f_k g_{m-1-k}^{'} \\
&- \Pr M\xi^2 g_{m-1} - 2\sum_{k=0}^{m-1} g_k f_{m-1-k}^{'} \\
&- \frac{\xi(1-\xi)(1-m)}{4}\left[\left(\sum_{k=0}^{m-1} f_k^{'}\frac{\partial g_{m-1-k}}{\partial \xi} - g_k^{'}\frac{\partial f_{m-1-k}}{\partial \xi}\right)\right], \qquad (2.69)
\end{aligned}
$$

$$
\begin{aligned}
R_m^h(\eta) &= \lambda \Pr h_{m-1}^{''} + \frac{8-(1-\xi)(1-m)}{4}\sum_{k=0}^{m-1} f_k h_{m-1-k}^{'} \\
&+2\sum_{k=0}^{m-1} h_k f_{m-1-k}^{'} - \Pr v\Delta(2h_{m-1} + \Pr f_{m-1}^{''})\xi^2 \\
&- \frac{\xi(1-\xi)(1-m)}{4}\left[\left(\sum_{k=0}^{m-1} f_k^{'}\frac{\partial h_{m-1-k}}{\partial \xi} - h_k^{'}\frac{\partial f_{m-1-k}}{\partial \xi}\right)\right], \qquad (2.70)
\end{aligned}
$$

$$
\begin{aligned}
R_m^\theta(\eta) &= \theta_{m-1}^{''} + \frac{8-(1-\xi)(1-m)}{4}\sum_{k=0}^{m-1} f_k \theta_{m-1-k}^{'} - m\sum_{k=0}^{m-1}\theta_k f_{m-1-k}^{'} \\
&- \frac{\xi(1-\xi)(1-m)}{4}\left[\left(\sum_{k=0}^{m-1} f_k^{'}\frac{\partial \theta_{m-1-k}}{\partial \xi} - \theta_k^{'}\frac{\partial f_{m-1-k}}{\partial \xi}\right)\right], \qquad (2.71)
\end{aligned}
$$

$$
\chi_m = \begin{cases} 0 & m \le 1, \\ 1 & m > 1. \end{cases} \qquad (2.72)
$$

The general solutions of Eqs. (2.61) – (2.66) can be written as

$$f_m(\xi,\eta) = f_m^*(\xi,\eta) + C_1 + C_2\exp(\eta) + C_3\exp(-\eta), \qquad (2.73)$$

$$g_m(\xi,\eta) = g_m^*(\xi,\eta) + C_4\exp(\eta) + C_5\exp(-\eta), \qquad (2.74)$$

$$h_m(\xi,\eta) = h_m^*(\xi,\eta) + C_6\exp(\eta) + C_7\exp(-\eta), \qquad (2.75)$$

$$\theta_m(\xi,\eta) = \theta_m^*(\xi,\eta) + C_8\exp(\eta) + C_9\exp(-\eta), \qquad (2.76)$$

where $f_m^*(\xi,\eta)$, $g_m^*(\xi,\eta)$, $h_m^*(\xi,\eta)$ and $\theta_m^*(\xi,\eta)$ are the special solutions. Now it is easy to solve Eqs. (2.61) – (2.66) by using Mathematica one after the other in order m=1,2,3...

2.4 Results and discussion

Equations (2.15)–(2.18) with boundary conditions (2.19) are solved analytically by HAM. The h-curves are plotted for f and θ to show the convergence region which are shown in Figs. 2.1 – 2.2. The analytical results for the velocity, microrotation temperature have been obtained for several values of the mixed convection parameter ξ, magnetic parameter M, coupled parameter Δ and Prandtl number Pr. Figs. 2.3 – 2.14 displays the profiles of the axial velocity f', the tangential velocity g, the microrotation h and the temperature θ for various values of magnetic parameter M and the mixed convection parameter ξ. From Figs. 2.3 – 2.8 we note that, for $\xi = 0$, the axial and tangential velocity increases as the magnetic parameter M increases from 0 to 1. However, for $\xi > 0$ both the axial and tangential velocities decreases as M increases. Also from Figs. 2.9 – 2.11 we conclude that, the magnetic field has only very slight influence on the temperature θ when $\xi = 0$, but the magnetic field is more pronounced for higher values of ξ. From Figs. 2.12 – 2.14. It is obvious that, near the cone surface, the microrotation velocity h decreases for $\xi = 0$ and increases for $\xi > 0$. In Fig. 2.15, the effects of Δ are discussed, it is seen from figure that the axial velocity f increases by increasing Δ. From Fig. 2.16 the temperature profile decreases by increasing Pr.

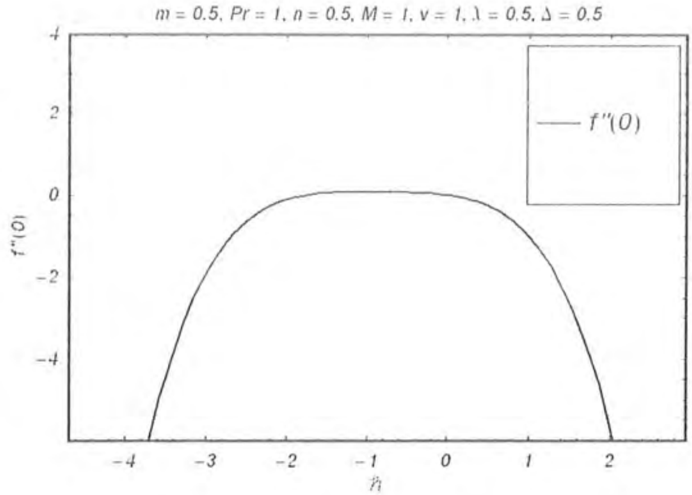

Fig.2.1. $h - curve$ for $f"(0)$

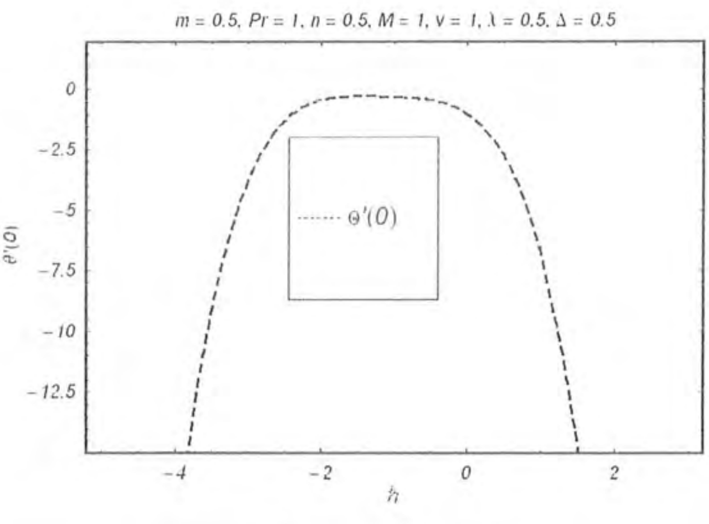

Fig. 2.2. $h - curve$ for $\theta'(0)$

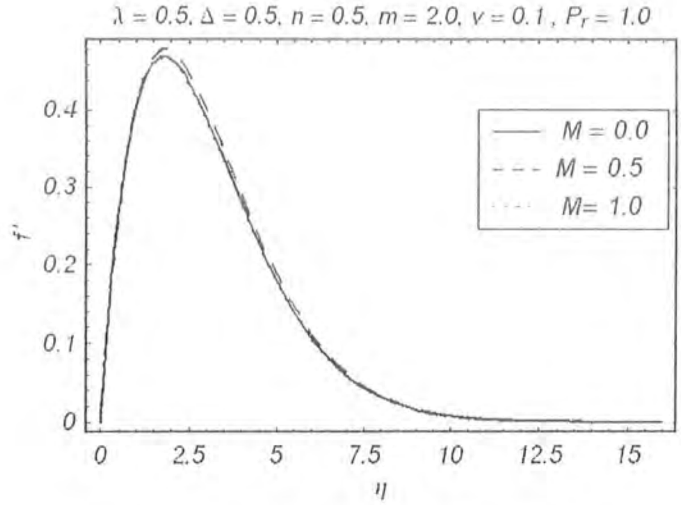

Fig.2.3. Influence of M on Axial velocity f' when $\xi = 0$

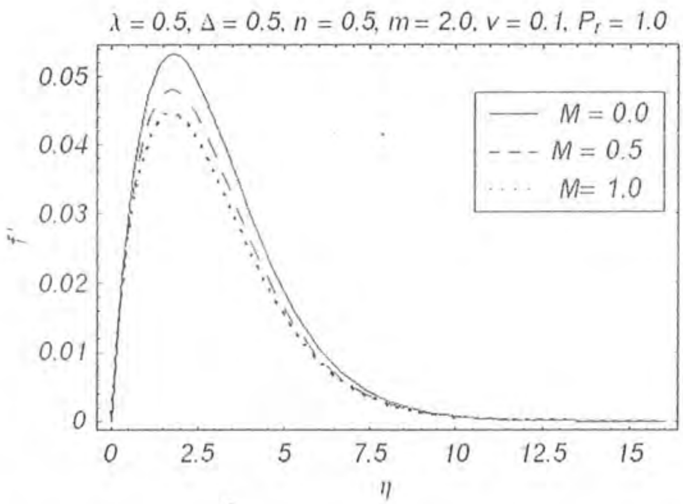

Fig.2.4. Influence of M on Axial velocity f' when $\xi = 0.5$

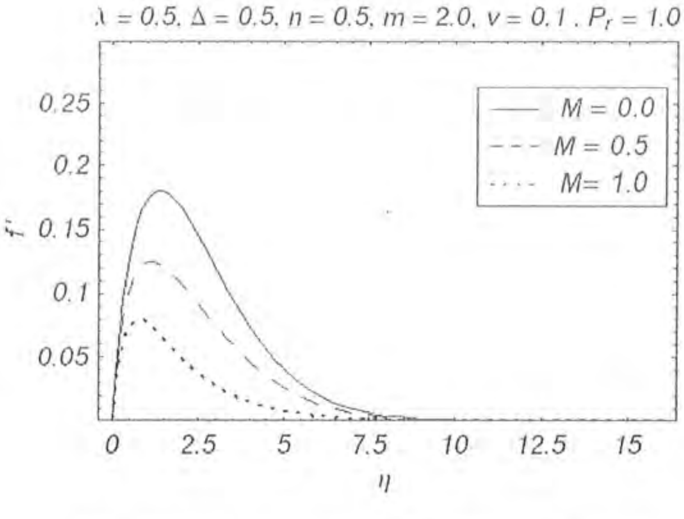

Fig.2.5. Influence of M on Axial velocity f' when $\xi = 1.0$

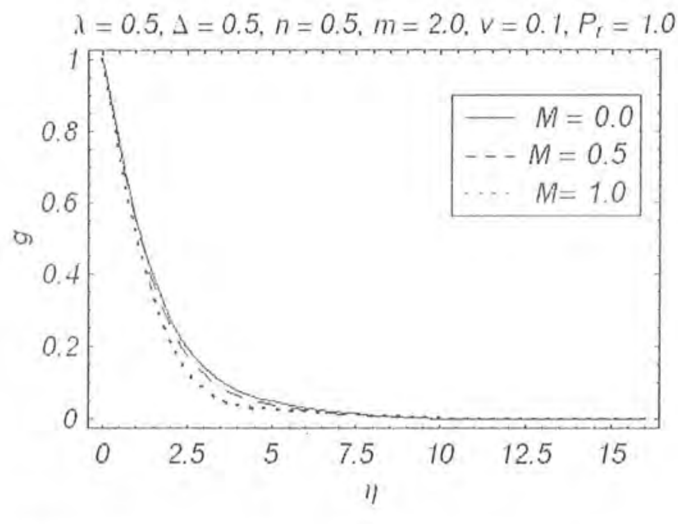

Fig.2.6. Influence of M on Radial velocity g when $\xi = 0$

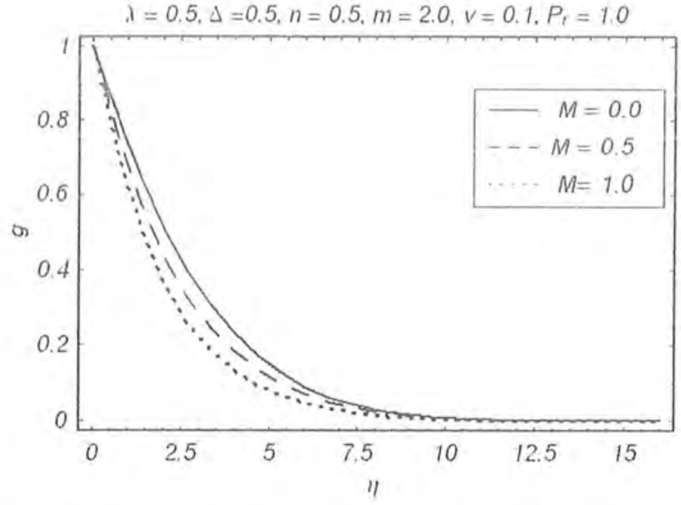

Fig.2.7. Influence of M on Radial velocity g when $\xi = 0.5$

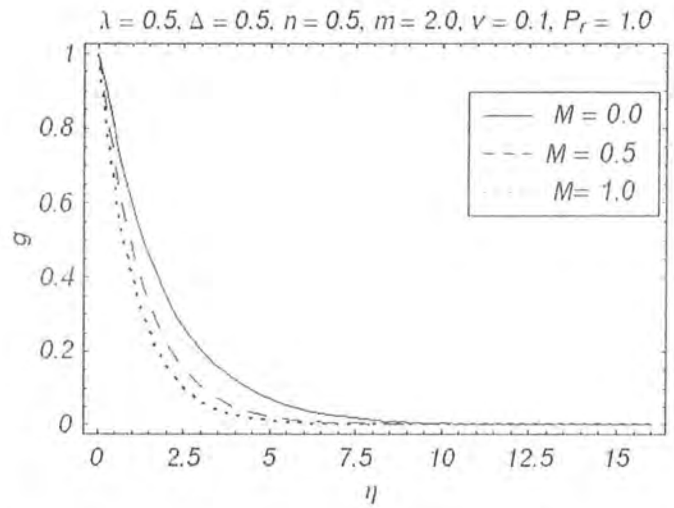

Fig.2.8. Influence of M on Radial velocity g when $\xi = 1.0$

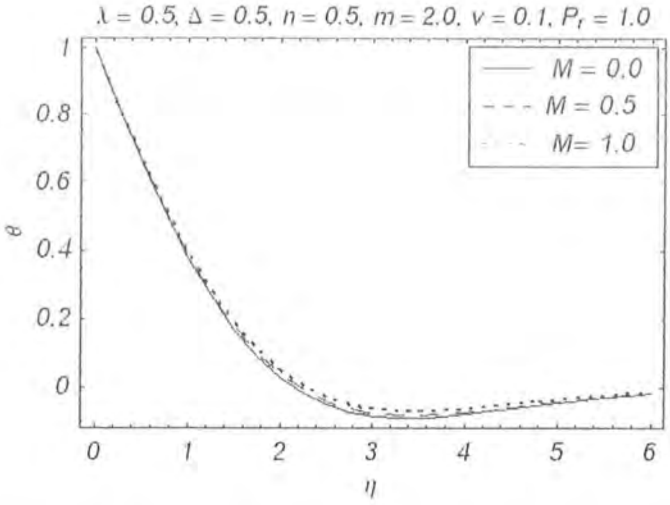

Fig.2.9. Influence of M on Temperature profile θ when

$\xi = 0$

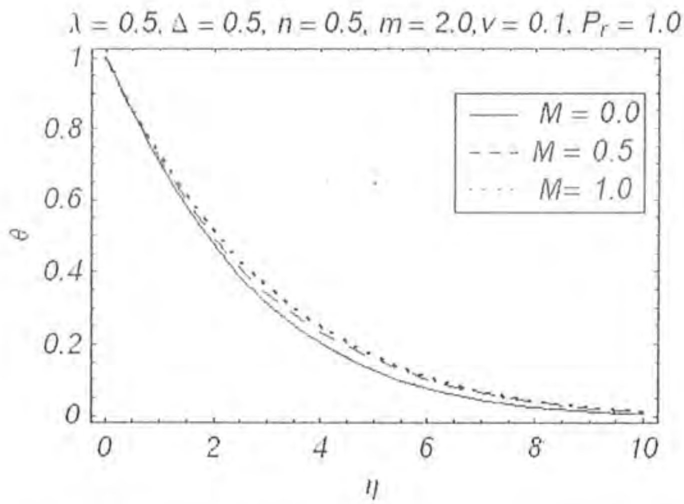

Fig.2.10. Influence of M on Temperature profile θ when

$\xi = 0.5$

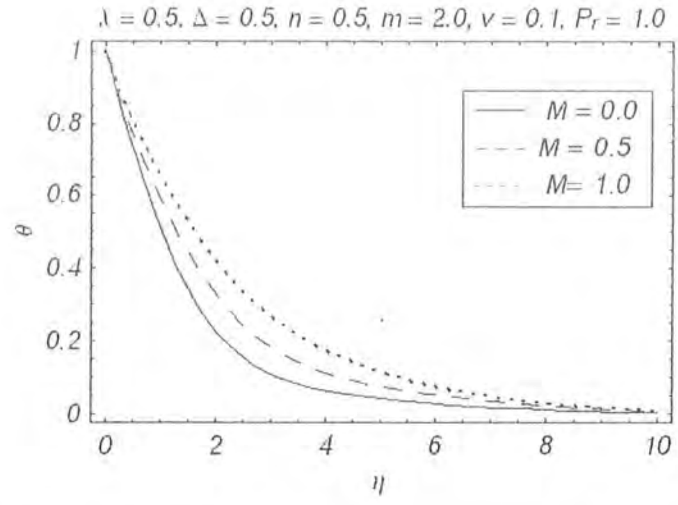

Fig.2.11. Influence of M on Temperature profile θ when

$\xi = 1.0$

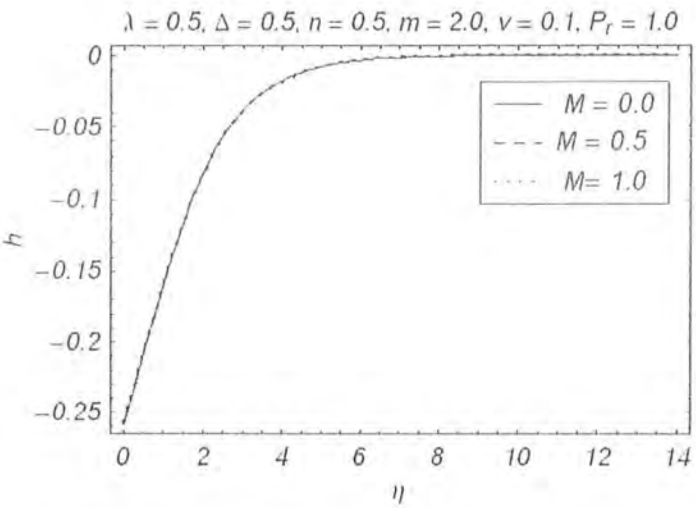

Fig.2.12. Influence of M on Microrotation profile h when

$\xi = 0$

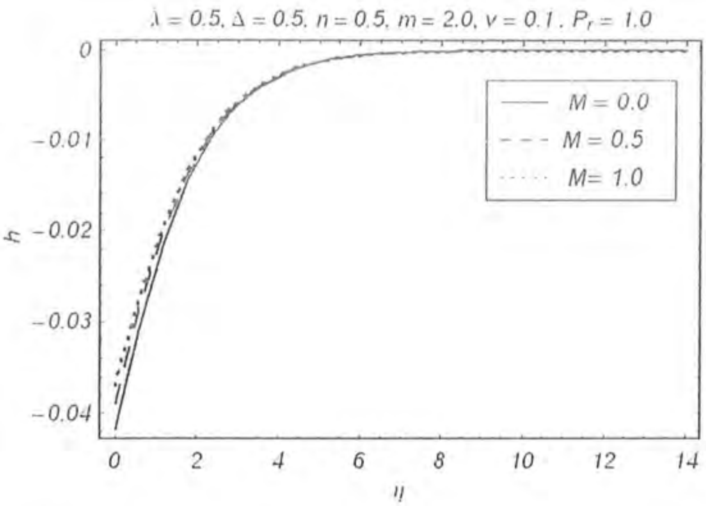

Fig.2.13. Influence of M on Microrotation profile h when $\xi = 0.5$

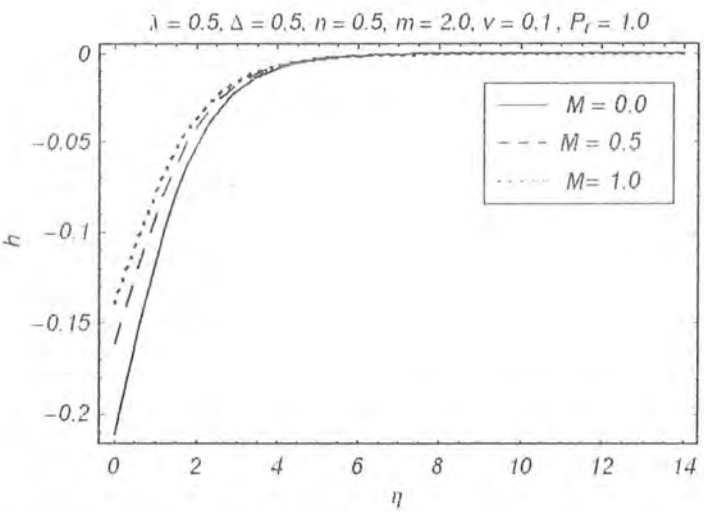

Fig.2.14. Influence of M on Microrotation profile h when $\xi = 1.0$

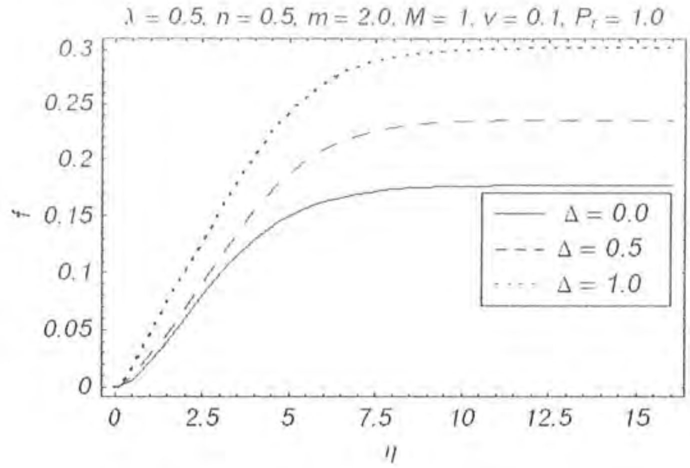

Fig.2.15. Influence of Δ on velocity profile f

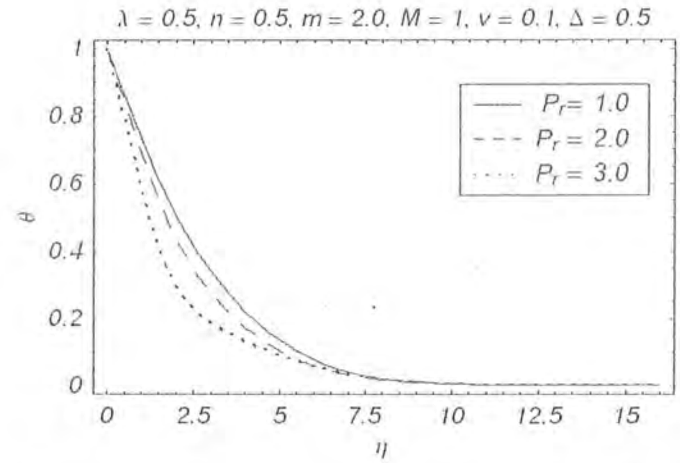

Fig.2.16. Influence of Pr on Temperature θ

Bibliography

[1] R.G.Hering, R.J.Grosh, Laminar combined convection from a rotating cone, ASME Journal of Heat Transfer 85 (1963) 29-34.

[2] C. L. Tien, Heat transfer by laminar flow from a rotating cone,ASME Journal of Heat Transfer 82 (1960) 252-253.

[3] T. Y. Wang, C.kleinstreur, Similarity solutions of combined convection heat transfer from a rotating cone or disk to non-newtonain fluids, ASME Journal of Heat Transfer 112 (1990) 939-944.

[4] K. Himasekhar, P. K. Sarma, K. Janardhan, Laminar mixed convection from a vertical rotating cone, International Communications in Heat and Mass Transfer 16 (1989) 99-106.

[5] H. T. Lin, L. K. Lin, Heat transfer from a rotating cone or disk to fluids of any prandtl number,International Communications in Heat and Mass Transfer 14 (1987) 323-332.

[6] H. Bararnia, Abdoul R. Ghotbi, G. Domairry, On the analytical solution for MHD natural convection flow and heat generation fluid in porous medium, Communications in Nonlinear Science and Numerical Simulation, 14 (2009) Pages 2689-2701.

[7] P. Ganesan, G. Palani, Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux, International Journal of Heat and Mass Transfer, 47 (2004) 4449-4457.

[8] T. K. Aldoss, MHD mixed convection from a vertical cylinder embedded in a porous medium, International Communications in Heat and Mass Transfer, 23 (1996) 517-530e.

[9] Anuar Ishak, Roslinda Nazar, Norfifah Bachok, Ioan Pop, Physica A: Statistical Mechanics and its Applications, 389 (2010) 40-46.

[10] T. K. Aldoss, MHD mixed convection from a vertical cylinder embedded in a porous medium, International Communications in Heat and Mass Transfer, 23 (1996) 517-530.

[11] S.C. Kakarantzas, Magnetohydrodynamic natural convection in a vertical cylindrical cavity with sinusoidal upper wall temperature, International Journal of Heat and Mass Transfer,52 (2009) 250-259.

[12] T. Hayat, Z. Abbas, T. Javed, Mixed convection flow of a micropolar fluid over a non-linearly stretching sheet, Physics Letters A, 372, Issue 5, 28 (2008) 637-647.

[13] Z. Abbas, Y. Wang, T. Hayat, M. Oberlack, Mixed convection in the stagnation-point flow of a Maxwell fluid towards a vertical stretching surface, Nonlinear Analysis: Real World Applications 11, Issue 4 (2010) 3218-3228.

[14] R. G. Hering, R. J. Grosh, Laminar free convection from a non-isothermal cone International Journal of Heat and Mass Transfer,5 (1962) 1059-1068.

[15] Orhan Aydın, Ahmet Kaya, Mixed convection of a viscous dissipating fluid about a vertical flat plate, Applied Mathematical Modelling, 31, Issue 5, (2007) 843-853.

[16] D. Anilkumar, S. Roy, Unsteady mixed convection flow on a rotating cone in a rotating fluid, Applied Mathematics and Computation 115 (2004) 545-561.

[17] R. Nazara, N. Amina, D. Filipb and I. Pop, Stagnation point flow of a micropolar fluid towards a stretching sheet, Int J Non-Linear Mech, 39 (2004) 1227-1235.

[18] Y. Y. Loka, N. Amin, I. Pop, Unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface, Int J Thermal Sci, 42 (2003) 995-1001.

[19] A. C. Eringen, Theory of micropolar fluids, J. Math. Mech. 16 (1966) 1-18.

[20] Chun-I Chen, Non-linear stability characterization of the thin micropolar liquid film flowing down the inner surface of a rotating vertical cylinder, Commun Nonlinear Sci Numer Simulat. 12 (2007) 760–775.

[21] S. Nadeem, Noreen Sher Akbar, M. Y. Malik, Exact and numerical solutions of a micropolar fluid in a vertical annulus, Wiley InterScience, DOI 10.1002/num.20517.

[22] D. Srinivasacharya, J. V. R. Murthy, D. Venugopalam, Unsteady stokes flow of micropolar fluid between two parallel porous plates, Int. J. Engag. Sci. 39 (2001) 1557-1563.

[23] G. Lukaszewicz, Micropolar fluids-theory and applications, Birkhauer, Basel, (1999)

[24] R. Nazara, N. Amina, T. Gro anb and I. Popb,Free convection boundary layer on a sphere with constant surface heat flux in a micropolar fluid, Int Comm Heat Mass Tran, 29 (2002) 1129-1138.

[25] S. J. Liao, Beyond perturbation: introduction to the homotopy analysis method, Chapman & Hall/CRC Press, Boca Raton (2003).

[26] S. J. Liao, On the homotopy analysis method for nonlinear problems, Appl Math Comput 147/2 (2004) 499–513.

[27] S. J. Liao, Notes on the homotopy analysis method: some definitions and theorems. commun Nonlinear Sci. Numer. Simulat 14 (2009) 983-997.