Effects of slip and heat transfer on the peristaltic motion in an asymmetric channel

By

Fahad Munir Abbasi

Department of Mathematics
Quaid-i-Azam University
Islamabad, Pakistan

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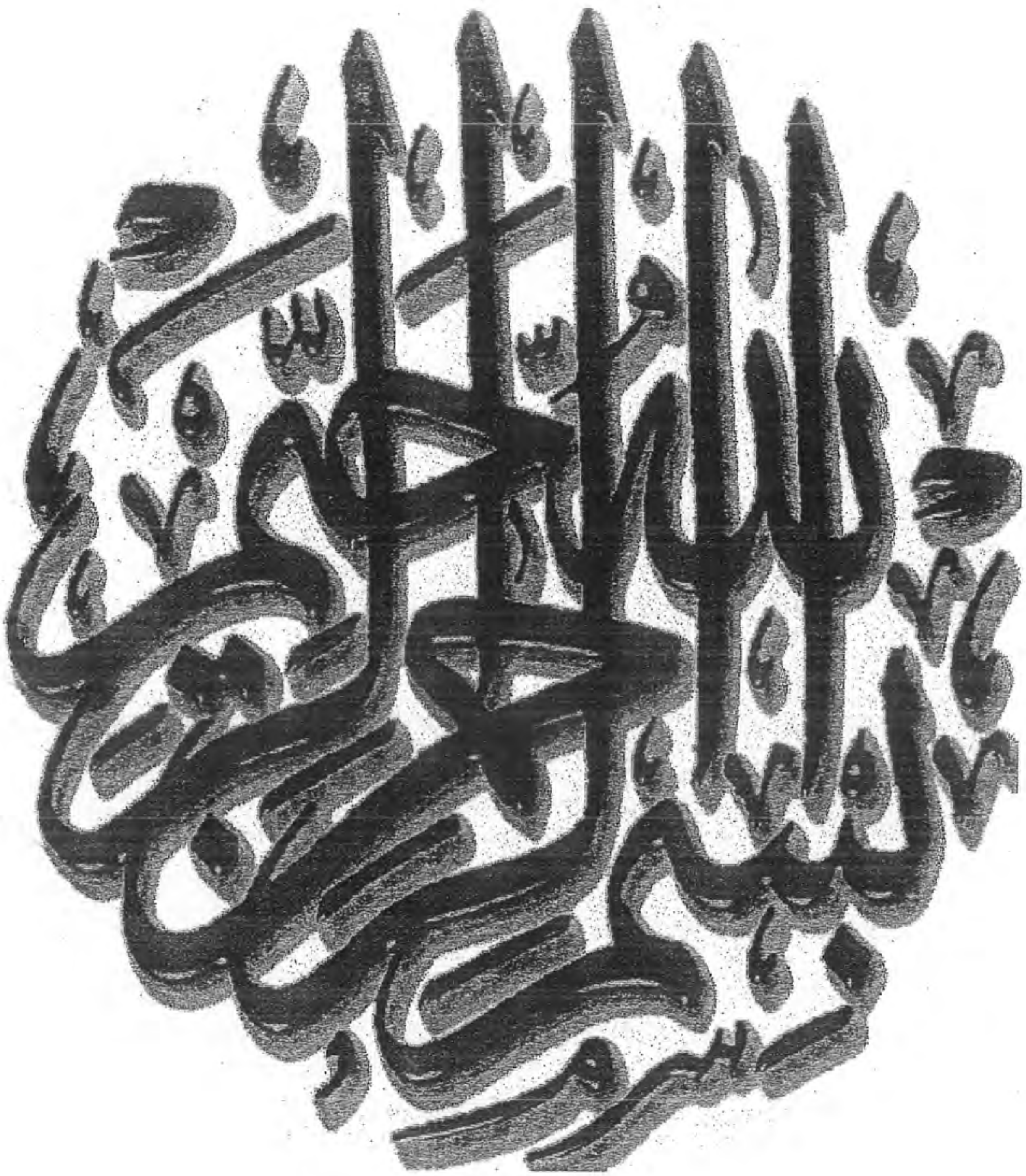

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A Dissertation Submitted in the Partial Fulfillment of the Requirements for the Degree of

MASTER OF PHILOSOPHY

IN

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CERTIFICATE

A DISSERTATION SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF THE MASTER OF PHILOSOPHY

We accept this dissertation as conforming to the required standard

1. Prof. Dr. Muhammad Ayub
(Chairman)

2. Dr. Tasawar Hayat 21/6/10
(Supervisor)

3. Prof. Dr. Muhammad Nawaz Naeem
(External Examiner)

Department of Mathematics
Quaid-i-Azam University
Islamabad, Pakistan
2010

Dedicated to

My Beloved Father,

My Loving Mother,

My Brother & Sisters

And My Friends.

Acknowledgements

In the name of **Allah**, the almighty, the creator, the superb, the centre of all the praise, who gave me courage, guidance and help to overcome all the difficulties. All my respect and love is for the **Holy prophet** (peace be upon him), **Ahhl e bait** and **Sahaba e karam** whose lives and teachings are the ultimate rule of life for us.

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Preface

The peristaltic flow in cannel/tube has attained considerable importance from past few decades and the reason for this is that such flows occur in various physiological and engineering processes. In particular, peristaltic motion occurs in urine transport from kidney to bladder, blood flow in arteries, roller and finger pumps, transportation of certain corrosive fluids etc. Peristaltic motion with heat transfer and magnetic field effects are important in biomagnetic fluid dynamics. In [1-10] are reported some worthy contributions on related topic. In [14] Ali et al. discussed the peristaltic flow of a viscous fluid with variable viscosity in an asymmetric channel. The present dissertation is organized as follows.

Chapter one includes some fundamental definitions related to the topic, that are vital in order to develop the basic understanding of the reader. The material presented in this chapter helps in describing the flow characteristics of the subsequent chapters.

Chapter two is prepared to investigate the peristaltic transport of a viscous fluid in an asymmetric channel. Mathematical problem is formulated under the assumptions of long wavelength and low Reynolds number. Exact solution of the problem is calculated. Moreover, the computations for pressure rise and frictional forces at the upper and lower walls have been carried out and discussed through graphically results. This chapter provides a detailed review of a paper by Ali et al. [14].

In chapter three the effects of heat transfer on a peristaltic flow of a magneto hydrodynamic (MHD) fluid in an asymmetric channel with variable viscosity is explored. Perturbation method is adopted to obtain the results under the assumptions of long wavelength and low Reynolds number. Expressions of stream function, temperature and heat transfer coefficient are constructed and discussed.

Contents

Chapter 1

Relevant definitions and equations

The main purpose of this chapter is to provide some relevant definitions and equations for the subsequent chapters.

1.1 Basic definitions

1.1.1 Fluid

Fluid is defined as a substance that deforms continuously under the action of applied shear stresses of any magnitude.

The basic difference between solids and fluids is that in case of solids, the deformation generated by applied shear stresses is not continuous.

1.1.2 Fluid mechanics

Fluid mechanics is the branch of engineering which is associated with the study of fluids at rest or in motion.

1.1.3 Fluid dynamics

The branch of engineering dealing with the fluids in motion is known as fluid dynamics.

1.1.4 Fluid statics

It is the branch of engineering that deals with the study of fluids at rest.

1.2 Some definitions

1.2.1 Density of a fluid

We know that density of any substance (fluid) is defined as the mass of unit volume of the substance (fluid) at a given temperature and pressure. However (in case of fluids) if the density of the fluid varies throughout the system, then the density at a point is defined as the limiting value in the following way

$$\rho = \lim_{\delta v \to 0} \left(\frac{\delta m}{\delta v} \right). \tag{1.1}$$

In above equation δm denotes the mass element, δv is the volume element enclosing the point under consideration and ρ indicates the fluid density.

1.2.2 Viscosity

It is defined as the ability of a fluid to resist the flow, or it is the internal resistance of a fluid. In a more scientific and more compact way, the ratio of shear stress to the rate of shear strain is known as viscosity. The mathematical relationship for viscosity is

$$\text{Viscosity } (\mu) = \frac{\text{shear stress}}{\text{rate of shear strain}}.$$

Depending upon certain conditions in the several cases, viscosity is also termed as absolute, kinematic or dynamic viscosity. It is an important property of a fluid which plays obvious role in experimental and mathematical analysis regarding flow. Classification of fluids is also made on the basis of viscosity.

1.2.3 Kinematic viscosity

It is defined as the ratio of dynamic viscosity to the density of the fluid. In mathematical form one can write

$$\text{Kinematic viscosity } (v) = \frac{\mu}{\rho}.$$

1.2.4 Pressure

It is known as the magnitude of the applied force to the object (in the perpendicular direction to the surface) per unit area. Mathematically one can write

$$\text{Pressure} = \frac{\text{Magnitude of applied force}}{\text{area}} = \frac{F}{A}$$

1.2.5 Flow

We know that the fluid goes under deformation when different forces act upon it. If the deformation increases continuously or indefinitely then this is known as flow.

1.2.6 Types of flows

(i) Steady flow

The flow in which the physical properties of the fluid (i.e. velocity, pressure, density etc.) at each point of the flow field remain invariant with respect to time is named as "steady flow". For any fluid property ζ we then write

$$\frac{\partial \zeta}{\partial t} = 0. \tag{1.2}$$

(ii) Unsteady flow

The flow in which the fluid property changes with time is called the unsteady flow. In mathematical notation we have

$$\frac{\partial \zeta}{\partial t} \neq 0. \tag{1.3}$$

(iii) Incompressible flow

Flow of constant density fluid is known as incompressible flow. In general all liquids are considered to have an incompressible flow.

(iv) Compressible flow

The flow for which density varies is known as compressible flow. Flow of all the gases have been treated as the compressible flows.

(v) One-, two-, and three-dimensional flows

A flow is classified as one-, two-, or three dimensional depending upon the number of space coordinates appearing in the velocity field.

1.2.7 Stream line

The imaginary line in the fluid drawn in such a way that the tangent to it at any point gives the direction of flow at that point, is called stream line. Thus the stream line shows the direction of motion of a number of particles at the same time.

1.2.8 Stream function

It is a function, which describes the form of pattern of flow, or in other words it is the discharge per unit thickness and it describes flow fields in term of either mass flow rate, for compressible fluids, or volume flow rate, for incompressible fluids. Mathematically for a steady state two dimensional flow field, we may write

$$\mathrm{V} = \nabla \times \boldsymbol{\psi}, \tag{1.4}$$

where $\mathrm{V} = (u, v, 0)$, therefore $\boldsymbol{\psi} = (0, 0, \psi)$. In Cartesian coordinate system, the velocity components in terms of stream function may be defined as

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}. \tag{1.5}$$

The stream function can be used to plot the stream lines (ψ = constant) to analyze the flow behavior graphically.

1.2.9 Ideal and real fluids

Fluids of negligible viscosity are known as ideal fluids. These fluids do not offer any resistance to the shear forces and thus do not practically exist in nature. However, from engineering point of view, gasses are considered as the ideal fluids. On the other hand, fluids of finite viscosity are known as real fluids. These fluids offer considerable resistance against the shear forces. Such fluids are further classified in to two sub classes namely the Newtonian and non-Newtonian fluids.

1.2.10 Newtonian fluid

The fluid for which shear stress is directly proportional to the linear rate of strain is termed as Newtonian fluid. For such fluids, the graph between shear stress and deformation rate is a straight line. Mathematical expression satisfied by such fluid is given below

$$\tau_{yx} \propto \frac{du}{dy},$$

or

$$\tau_{yx} = \mu \frac{du}{dy}, \tag{1.6}$$

where τ_{yx} is the shear stress, μ is the dynamic viscosity (a constant of proportionality) and du/dy is the rate of strain (velocity gradient perpendicular to the direction of shear) for a unidirectional and one-dimensional flow.

For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure, not on the forces acting upon it. In common terms, this means that the fluid continues to flow, regardless of the forces acting on it. If the fluid is incompressible and viscosity is constant across the fluid, the above equation governing the shear stress can be generalized in the Cartesian coordinate system as follows

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial y_j} + \frac{\partial u_j}{\partial y_i} \right). \tag{1.7}$$

In above expression

τ_{ij} is the shear stress on the i^{th} face of a fluid element in the j^{th} direction

u_i is the velocity in the i^{th} direction

y_j is the j^{th} direction coordinate

The most common examples of such fluids are water and gasoline. In this dissertation, the considered flows will be analyzed for the viscous fluid situation.

1.2.11 Types of forces

Surface forces

Such forces act on the surface of any medium through direct contact with the surface. Examples of such forces include pressure and stress.

Body forces

Such forces act throughout the volume of the fluid and are independent of any type of physical contact. Gravity and magnetic forces are examples of two body forces.

1.2.12 Volume flow rate

It is the volume of fluid which passes through a section of pipe or channel in unit time. It is usually represented by the symbol Q. Given an area A, and a fluid flowing through it with uniform velocity V with an angle θ away from the perpendicular to A, then the volume flow rate is

$$Q = A\mathrm{V}\cos\theta.$$

For flow perpendicular to the area A we have $\theta = 0$ and thus the volume flow rate is

$$Q = A\mathrm{V}.$$

1.2.13 No-slip condition

When a fluid flows, the outer most molecules of the fluid near the solid boundary stick with the boundary and the fluid velocity at the boundary is equal to that of the solid boundary. This is known as the no-slip condition.

1.2.14 Slip condition

Although no-slip condition is extensively used in flows of Newtonian and non-Newtonian fluids but in most engineering applications, the no-slip condition does not always hold in reality. For example a large class of polymeric materials slip or stick-slip on the solid boundaries. To counter this situation Navier proposed a general boundary condition that incorporate the possibility of fluid slip at the solid boundary. According to Navier, the relative velocity between the fluid and the solid boundary in the x-direction (at a solid boundary) is directly proportional to the shear stress at that boundary, i.e.

$$u_f - u_w \propto \tau_{xy},$$

or

$$u_f - u_w = \pm\frac{\beta}{\mu}\tau_{xy}, \tag{1.8}$$

where β (constant of proportionality) is the slip parameter having dimension of length, the plus and the minus signs are due to direction of the normal on the wall, u_f is the velocity of the fluid and u_w is the velocity of the wall. This is known as slip condition at solid boundary. For $\beta = 0$ we recover the case of no-slip condition.

1.3 Basics of magnetohydrodynamics

Definition (MHD)

The science that deals with the dynamics of highly conducting fluids in presence of a magnetic field is known as magnetohydrodynamics (MHD).

Maxwell's equations

These equations describe the behavior of electric and magnetic fields in terms of basic laws dealing with the electromagnetism.

Gauss's law of electricity: According to the Gauss's law of electricity, the net electric flux through any closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed in it, mathematically

$$\nabla.\mathbf{E} = \frac{\rho_e}{\epsilon_0} \tag{1.9}$$

where ρ is the charge density, ϵ_0 is the permittivity of the free space and $\mathbf{E}$ is the electric field. For free space i.e. $\rho = 0$, (there is no electric field).

Gauss's law of magnetism: By this law, the total magnetic flux through any closed surface is zero, i.e.

$$\nabla.\mathbf{B} = 0. \tag{1.10}$$

We can further state from this law that the magnetic mono-poles do not exist or magnetic field lines can never end.

Faraday's law of electromagnetic induction: It states that changing the magnetic field induces an electric field. In mathematical notations, we can express that

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.11}$$

where $\mathbf{B}$ is the magnetic field.

Ampere's law: It relates the magnetic field to the changing electric field, i.e.

$$\frac{\nabla \times \mathbf{B}}{\mu_0} = \overline{\mathbf{J}} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{1.12}$$

where μ_0 is the magnetic permeability and $\overline{\mathbf{J}}$ is the current density.

Ohm's law

This states that the total current is proportional to the voltage drop which for a stationary conductor is

$$\overline{\mathbf{J}} = \sigma \mathbf{E} \tag{1.13}$$

and for a moving conductor

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{1.14}$$

where σ is the electrical conductivity.

Lorentz force

Lorentz force for changing magnetic field is given by

$$\mathbf{F} = \rho_c \mathbf{E} + \overline{\mathbf{J}} \times \mathbf{B} \tag{1.15}$$

and for constant magnetic field we have

$$\mathbf{F} = \overline{\mathbf{J}} \times \mathbf{B}$$

1.4 Basics of heat transfer

Heat

We know that the total kinetic energy of the system is known as heat. Heat is one of the most common form of energy that plays a vital role in transfer of energy from one place to another due to difference in temperature (average kinetic energy of the system).

Heat transfer

Heat transfer is the process that deals with the flow of heat within the system. It is different from thermodynamics in the sense that thermodynamics only deals with the flow of heat across the boundary and it is inadequate to explain the flow of heat within the system. As all of the transfer phenomenon are triggered by some gradient, in case of heat transfer the cause is difference in temperature. Heat flows from hotter to cooler side, and it keeps on flowing unless the temperature gradient is zero (or the heat is uniformly distributed throughout the system).

Modes of heat transfer

Following are the modes through which heat can be transferred from one place to another

Conduction The transfer of heat, when it takes place from more energetic particles to the less energetic ones due to particle to particle collusions, is known as conduction. Most of the heat transfer taking place in solids is due to conduction, it also takes place in liquids and gases but not as a major mode of heat transfer. Common example of conduction is rise of temperature of one end of an iron rod, when the other end is heated by any source.

Convection: Heat transfer when it takes place between a solid boundary and the fluid moving adjacent to the boundary, is termed as convection. It involves the combined effects of conduction and fluid motion. Conduction is the mode of heat transfer that is responsible for the transfer of heat in fluids. Example of convection can be taken as heating up of water when it is boiled in any container.

Following are three types of convection

Natural convection

If no external force or agent is involved in the process, or the fluid motion occurs purely due to density difference induced by the temperature difference, then the process is called natural or free convection. The temperature changes in the whole control volume produces a difference in density that in turn induces body forces, these body forces are responsible for generation of flow in case of free convection. These body forces are actually generated by pressure gradients imposed on the whole fluid. Gravity is the most common source of this imposed pressure fields. The body forces in this case are in common termed as buoyancy forces. In general we can say that natural convection would not be possible without thermal expansion and gravity.

Forced convection

Forced convection is the type of convection that involves the fluid flow due to some external agent or source e.g. due to a fan or a pump. Buoyancy forces are negligible is this case.

Radiation Matter in all its forms emit, absorb and transmit radiations. These radiations are in form of electromagnetic waves. The transfer of heat by this mode has the speciality that it does not require any medium of propagation, and radiations can travel through vacuum. Heat transfer by this mode is explained by modified Stefan-Boltzmann law.

Specific heat

The amount of heat energy required to increase the temperature of one kg of any substance by one degree, is know as specific heat of that substance.

Thermal conductivity

The ability to transmit or to conduct heat energy for different materials is different. It is the measure of this ability of a material to conduct heat that is known as thermal conductivity. It is denoted by k. A substance with a large k is a good conductor of heat e.g. iron, whereas a material with low k is a poor conductor but a good insulator e.g. air and wood.

Thermal diffusivity

The ratio of amount of heat conducted to the amount of heat stored per unit volume is known as thermal diffusivity.

Viscous dissipation

It is the transformation of kinetic energy to the internal energy of the fluid due to viscous effects, in other words it is the heating up of fluid.

Note that all the material of heat transfer has been taken for the convenience of the readers from [15].

1.5 Dimensionless numbers

In most of the problems related to fluid mechanics, we use certain dimensionless numbers and parameters. Such non-dimensionalizing is important from two points of view. It simplifies the problem up to a certain extent and it also reduces the cost of the experiment i.e. we can discuss the effects of several parameters in one dimensionless numbers.

We now define some dimensionless numbers for the related material presented in the dissertation.

Reynolds number

It is the ratio of inertial force to the viscous force. It is denoted by Re. By the use of this number, one can predict either the flow is laminar or turbulent. A low Reynolds number means that the fluid is more thicker (viscous) and the flow is laminar where the viscous forces are dominant and vice versa.

Magnetic Reynolds number

In problems involving MHD, magnetic Reynolds number is a more important parameter. It represents the ratio of fluid flux to the magnetic diffusivity. In comparison to ordinary Reynolds number which determines the diffusion of vorticity along the streamlines, magnetic Reynolds number determines the diffusion of magnetic field along the streamlines. A high magnetic Reynolds number means that the magnetic field lines will be frozen in the fluid and will move as the fluid flows. Mathematically it is expressed by the relation

$$\mathrm{Re}_m = \frac{UL}{\eta}$$

where U is the velocity scale of flow, L is the typical length scale of flow and η is the magnetic diffusivity. By considering the small values of Re_m we can neglect the effects of induced magnetic field on the flow.

Hartman number

It is defined as ratio of magnetic body force to the viscous force. It measures the relative importance of drag forces resulting from magnetic induction and viscous forces and determines the velocity profile in Hartman flow. It is denoted by M.

Wave number

The ratio of the channel width to the wavelength is called wave number. Its definition is

$$\delta = \frac{L}{\lambda}$$

in which L is the half width of the channel and λ the wavelength.

Prandtl number

It is the ratio of momentum diffusivity (kinematic viscosity) and the thermal diffusivity. It can be related to the thickness of thermal and velocity boundary layer. In other words it can be defined as the ratio of velocity boundary layer to the thermal boundary layer. When it attains the value 1, the two boundary layers coincide. It is denoted by Pr.

Eckert number

This number expresses the relation between a flow's kinetic energy and enthalpy and is denoted by Ec or E. This number only enters into the problem when the viscous dissipation in the energy equation is taken in to account.

Brinkman number

The product of Prandtl number and Eckert number is called a Brinkman number. Besides this it interprets the ratio of the viscous dissipation to the heat transfer rate.

1.6 Basic governing equations

1.6.1 Continuity equation

The conservation law of mass leads to the fact that mass of a closed system of substances remains constant, regardless of the processes acting inside the system. This principal may be applied to a moving fluid and its mathematical formulation yields the continuity equation. With no source or sink in the control volume, the continuity equation is given by

$$\frac{\partial \rho}{\partial t}+\frac{\partial (\rho u)}{\partial x}+\frac{\partial (\rho v)}{\partial y}+\frac{\partial (\rho \omega)}{\partial z}=0. \tag{1.16}$$

where u, v and ω designate the velocity components in x, y and z directions respectively. The vectorial form of Eq. (1.19) can be presented as follows

$$\frac{\partial \rho}{\partial t} + \nabla . \rho \mathbf{V} = 0, \tag{1.17}$$

which for incompressible fluid becomes

$$\nabla . \mathbf{V} = 0,$$

or

$$\operatorname{div} \mathbf{V} = 0. \tag{1.18}$$

1.6.2 Equation of motion

The vectorial form of equation of motion is

$$\rho \frac{d\mathbf{V}}{dt} = \operatorname{div} \mathbf{T} + \rho \mathbf{b}, \tag{1.19}$$

where $\mathbf{T}$ is the Cauchy stress tensor, $\rho \mathbf{b}$ is the body force and d/dt is the material time derivative. The definition of material derivative is

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + \omega \frac{\partial}{\partial z},$$

or

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V} . \nabla \tag{1.20}$$

and the Cauchy stress tensor $\mathbf{T}$ is

$$\mathbf{T} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}, \tag{1.21}$$

in which $\sigma_{xx}, \sigma_{yy},$ and σ_{zz} denote the normal stresses and $\tau_{ij(i \neq j)}$ are the shear stresses.

1.6.3 Energy equation

The conservation law of energy states that the increase in the internal energy of a thermodynamical system is equal to the amount of heat energy added to the system plus (minus) the amount of energy gained (lost) by the system as a result of the work done on (by) the system by the surroundings. The general form of energy equation is

$$\rho\zeta\frac{dT}{dt} = \overline{\mathbf{T}}.(\nabla\mathbf{V}) + \nabla.(k\nabla T) \tag{1.22}$$

in which ζ is the specific heat at constant volume and k is the thermal conductivity. In case of constant thermal conductivity, Eq. (1.22) becomes

$$\rho\zeta\frac{dT}{dt} = \overline{\mathbf{T}}.(\nabla\mathbf{V}) + k\nabla^2 T).$$

1.7 Peristalsis

The word peristalsis has been originated from a greek word "Peristaltikos", which means compressing. Hence, peristalsis is a mechanism, for mixing and transporting fluids, which is generated by progressive wave of contraction or expansion moving on the wall of a flexible channel/tube.

1.7.1 Peristaltic transport

It is a mechanism of material transport as a result of peristaltic waves induced along the length of the walls of a distensible channel/tube containing the material.

1.7.2 Occurrence of peristalsis in physiology

The mechanism of peristalsis has obvious appearance in moving food through the digestive tract, transport urine from kidneys to the bladder, transport of bile, transport of lymph in the lymphatic vessels, vasomotion of small blood vessels, movement of chyme in the gastrointestinal tract, movement of spermatozoa in the ducts efferents of male reproductive tract and the ovum in the female fallopian tube. It also occurs in small and large intestines. The mechanism of peristaltic motion has been exploited for industrial applications and it plays a vital role in many biomechanical instruments like, blood pumps in heart lung machines and roller pumps.

1.8 Pumping and trapping

A pump moves liquids from lower pressure to higher pressure under certain conditions and this operation of a pump is known as pumping. This may be further elaborated as follows.

1.8.1 Positive and negative pumping

The positive and negative pumping are dependent upon the dimensionless mean flow rate θ. For positive θ we have positive pumping and if θ is negative then it is known as negative pumping.

1.8.2 Adverse and favorable pressure gradient

Pressure gradient generated by peristaltic motion of the wall is adverse if pressure rise per wavelength (ΔP_λ) is positive and favorable if ΔP_λ is negative.

1.8.3 Peristaltic pumping

Peristaltic pumping appears when pumping is positive and pressure gradient is adverse i.e. $\theta > 0$ and $\Delta P_\lambda > 0$.

1.8.4 Free pumping

This possibility holds when pressure gradient becomes zero and pumping remains positive i.e. $\theta > 0$ and $\Delta P_\lambda = 0$.

1.8.5 Co-pumping or augmented pumping

Here pumping is positive and pressure gradient is favorable i.e. $\theta > 0$ and $\Delta P_\lambda < 0$.

1.8.6 Bolus

A volume of fluid bounded by closed stream lines in the wave frame is called bolus.

1.8.7 Trapping

In the wave frame, stream lines under certain conditions split to trap a bolus of fluid which is pushed ahead along with the peristaltic wave with the speed of the wave. This phenomenon is called trapping.

Chapter 2

Peristaltic transport of a viscous fluid in an asymmetric channel with variable viscosity

2.1 Introduction

The purpose of this chapter is to examine the influence of variable viscosity on the peristaltic transport of a viscous fluid in an asymmetric channel. The flow generated is due to waves propagating on the channel walls. Different phase and amplitude of the travelling waves are response to cause an asymmetry. The velocity components and longitudinal pressure gradient are derived after long wavelength and low Reynolds number. The graphs are sketched and discussed. This work provides a detailed review of a paper by T. Hayat and N. Ali [14].

2.2 Definition of the problem

Let us investigate the peristaltic transport of an incompressible Newtonian fluid with variable viscosity in an asymmetric channel of width $d_1 + d_2$. The sinusoidal wave trains propagate with constant speed c on the walls of channel which in turn propel the fluid along the walls.

The wall shapes are described as follows:

$$\begin{aligned}\overline{h_1}\left(\overline{X},\overline{t}\right) &= d_1 + a_1\cos\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\overline{t}\right)\right] \text{....upper wall,} \\ \overline{h_2}\left(\overline{X},\overline{t}\right) &= -d_2 - b_1\cos\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\overline{t}\right)+\phi\right] \text{....lower wall.}\end{aligned} \tag{2.1}$$

In above definitions a_1 and a_2 are the wave amplitudes, λ the wavelength, c the wave speed, $\overline{t}$ the time, and ϕ $(0 \leq \phi \leq \pi)$ is the phase difference. Here $\left(\overline{U},\overline{V}\right)$ denote the velocity components in fixed frame of reference $\left(\overline{X},\overline{Y}\right)$. For $\phi = 0$ this case corresponds to the symmetric channel with waves out of phase and for $\phi = \pi$ the waves are in phase. Further a_1, a_2, d_1, d_2 and ϕ satisfies the condition $a_1^2 + a_2^2 + 2a_1a_2\cos\phi \leq (d_1+d_2)^2$.

The appropriate definition of velocity $\mathbf{V}$ is

$$\mathbf{V}=\left(\overline{U}\left(\overline{X},\overline{Y},\overline{t}\right),\overline{V}\left(\overline{X},\overline{Y},\overline{t}\right),0\right), \tag{2.2}$$

The expression of Cauchy stress tensor $\overline{\mathbf{T}}$ for a viscous fluid is

$$\overline{\mathbf{T}} = -\overline{P}\overline{\mathbf{I}} + \overline{\mu}\overline{\mathbf{A}}_1 \tag{2.3}$$

in which $\overline{\mathbf{I}}$ the identity tensor, $\overline{P}(\overline{X},\overline{Y},\overline{t})$ the pressure, $\overline{\mu}$ the variable dynamic viscosity and first Rivilin Ericksen tensor $\overline{\mathbf{A}}_1$ is

$$\overline{\mathbf{A}}_1 = (\nabla\mathbf{V}) + (\nabla\mathbf{V})^t \tag{2.4}$$

where t denotes the transpose of the matrix. By making use of Eq. (2) $\nabla\mathbf{V}$ and it's transpose are defined as

$$\nabla\mathbf{V} = \begin{bmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{U}}{\partial\overline{Y}} & 0 \\ \frac{\partial\overline{V}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{Y}} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2.5}$$

$$(\nabla\mathbf{V})^t = \begin{bmatrix} \frac{\partial\overline{U}}{\partial\overline{X}} & \frac{\partial\overline{V}}{\partial\overline{X}} & 0 \\ \frac{\partial\overline{U}}{\partial\overline{Y}} & \frac{\partial\overline{V}}{\partial\overline{Y}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.6}$$

From Eqs. (2.6)-(2.7) we have

$$\overline{\mathbf{A}}_1 = \begin{bmatrix} 2\frac{\partial \overline{U}}{\partial \overline{X}} & \frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}} & 0 \\ \frac{\partial \overline{U}}{\partial \overline{Y}} + \frac{\partial \overline{V}}{\partial \overline{X}} & 2\frac{\partial \overline{V}}{\partial \overline{Y}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.7}$$

The relevant equations are

$$\begin{aligned} \operatorname{div} \mathbf{V} &= 0, && (2.8) \\ \rho \frac{d\mathbf{V}}{dt} &= \operatorname{div} \overline{\mathbf{T}}, && (2.9) \end{aligned}$$

where ρ is the fluid density. Eqs. (2.9) and (2.10) give

$$\frac{\partial \overline{U}}{\partial \overline{X}} + \frac{\partial \overline{V}}{\partial \overline{Y}} = 0, \tag{2.10}$$

$$\begin{aligned} \rho \left[\frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right] \overline{U} &= -\frac{\partial \overline{P}}{\partial \overline{X}} + 2\frac{\partial}{\partial \overline{X}} \left[\overline{\mu}(\overline{Y}) \frac{\partial \overline{U}}{\partial \overline{X}} \right] + \frac{\partial}{\partial \overline{Y}} \left[\overline{\mu}(\overline{Y}) \left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right) \right] && (2.11) \\ \rho \left[\frac{\partial}{\partial \overline{t}} + \overline{U} \frac{\partial}{\partial \overline{X}} + \overline{V} \frac{\partial}{\partial \overline{Y}} \right] \overline{V} &= -\frac{\partial \overline{P}}{\partial \overline{Y}} + 2\frac{\partial}{\partial \overline{Y}} \left[\overline{\mu}(\overline{Y}) \frac{\partial \overline{V}}{\partial \overline{Y}} \right] + \frac{\partial}{\partial \overline{X}} \left[\overline{\mu}(\overline{Y}) \left(\frac{\partial \overline{V}}{\partial \overline{X}} + \frac{\partial \overline{U}}{\partial \overline{Y}} \right) \right] && (2.12) \end{aligned}$$

2.3 Governing equations in the wave frame

The frame moving with wave speed c in the positive $\overline{X}$ direction is called wave frame$(\overline{x}, \overline{y})$. Surely, in the laboratory frame the flow is unsteady. However in a coordinate system moving at the wave speed c (wave frame) $(\overline{x}, \overline{y})$ it can be considered as steady. The two coordinate frames are related by the following transformations

$$\begin{aligned} \overline{x} &= \overline{X} - c\overline{t}, \quad \overline{y} = \overline{Y}, \quad \overline{u}(\overline{x}, \overline{y}) = \overline{U}\left(\overline{X}, \overline{Y}, \overline{t}\right) - c, \quad \overline{v}(\overline{x}, \overline{y}) = \overline{V}\left(\overline{X}, \overline{Y}, \overline{t}\right), \\ p(\overline{x}, \overline{y}) &= P(\overline{X}, \overline{Y}, \overline{t}) && (2.13) \end{aligned}$$

where $\overline{u}$ and $\overline{v}$ are the velocity components in the $\overline{x}$ and $\overline{y}$ directions respectively. With the help of these transformations the differential operators in the two frames are related in the following way

$$\frac{\partial}{\partial \overline{X}} = \frac{\partial}{\partial \overline{x}}, \quad \frac{\partial}{\partial \overline{Y}} = \frac{\partial}{\partial \overline{y}} \quad , \frac{\partial}{\partial \overline{t}} = -c\frac{\partial}{\partial \overline{x}} \tag{2.14}$$

In view of Eqs. (2.13) and (2.14), the continuity equation (2.10) and component form of momentum equations (2.11) and (2.12) take the forms

$$\begin{aligned}
\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} &= 0, &(2.15)\\
\rho\left[\overline{u}\frac{\partial}{\partial \overline{x}} + \overline{v}\frac{\partial}{\partial \overline{y}}\right]\overline{u} &= -\frac{\partial \overline{p}}{\partial \overline{x}} + 2\frac{\partial}{\partial \overline{x}}\left[\overline{\mu}(\overline{y})\frac{\partial \overline{u}}{\partial \overline{x}}\right] + \frac{\partial}{\partial \overline{y}}\left[\overline{\mu}(\overline{y})\left(\frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}}\right)\right], &(2.16)\\
\rho\left[\overline{u}\frac{\partial}{\partial \overline{x}} + \overline{v}\frac{\partial}{\partial \overline{y}}\right]\overline{v} &= -\frac{\partial \overline{p}}{\partial \overline{y}} + 2\frac{\partial}{\partial \overline{y}}\left[\overline{\mu}(\overline{y})\frac{\partial \overline{v}}{\partial \overline{y}}\right] + \frac{\partial}{\partial \overline{x}}\left[\overline{\mu}(\overline{y})\left(\frac{\partial \overline{v}}{\partial \overline{x}} + \frac{\partial \overline{u}}{\partial \overline{y}}\right)\right]. &(2.17)
\end{aligned}$$

where $\overline{p}$ is the pressure and $\overline{\mu}(\overline{y})$ is the viscosity function.

2.4 Non-dimensionalization of the problem

In order to non-dimensionalize the above equations we introduce following dimensionless quantities

$$\begin{aligned}
x &= \frac{\overline{x}}{\lambda}, \quad y = \frac{\overline{y}}{d_1}, \quad u = \frac{\overline{u}}{c}, \quad v = \frac{\overline{v}}{c\delta}, \quad t = \frac{c\overline{t}}{\lambda}, \quad a = \frac{a_1}{d_1}, \quad d = \frac{d_2}{d_1}, \quad b = \frac{b_1}{d_1},\\
p &= \frac{d_1^2}{\lambda \mu_0 c}\overline{p}, \quad h_1 = \frac{\overline{h_1}}{d_1}, \quad h_2 = \frac{\overline{h_2}}{d_1}, \quad \mu(y) = \frac{\overline{\mu}(\overline{y})}{\mu_0}, \quad \mathrm{S} = \frac{d_1}{\mu_0 c}\overline{\mathrm{S}}, \qquad (2.18)
\end{aligned}$$

where μ_0 is the constant viscosity. In view of dimensionless quantities introduced in above equation , Eqs. (2.15)–(2.17) can be written as

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, &(2.19)\\
\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u &= -\frac{\partial p}{\partial x} + 2\delta^2\frac{\partial}{\partial x}\left[\mu(y)\frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu(y)\left(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right], &(2.20)\\
\mathrm{Re}\,\delta^3\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)v &= -\frac{\partial p}{\partial y} + 2\delta^2\frac{\partial}{\partial y}\left[\mu(y)\frac{\partial v}{\partial y}\right] + \delta^2\frac{\partial}{\partial x}\left[\mu(y)\left(\delta^2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]. &(2.21)
\end{aligned}$$

where the wave number δ and Reynolds number Re are

$$\delta = \frac{d_1}{\lambda}, \quad \mathrm{Re} = \frac{\rho c d_1}{\mu_0}, \tag{2.22}$$

By employing the assumptions of long wavelength $\delta \ll 1$ and low Reynolds number, the Eqs. (2.20) and (2.21) give

$$-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left[\mu(y)\left(\frac{\partial u}{\partial y}\right)\right]=0, \tag{2.23}$$

and $\partial p/\partial y=0$, which implies that $p\neq p(y)$.

The dimensionless form of the wall shapes becomes

$$h_1=1+a\cos 2\pi x,\ h_2=-d-b\cos\left(2\pi x+\phi\right)$$

2.5 Volume flow rate and boundary conditions

The instantaneous volume flow rate in the laboratory frame can be written as

$$Q=\int_{\overline{h_2}(\overline{X},\overline{t})}^{\overline{h_1}(\overline{X},\overline{t})}\overline{U}\left(\overline{X},\overline{Y},\overline{t}\right)d\overline{Y}, \tag{2.24}$$

where $\overline{h_1}$ and $\overline{h_2}$ are functions of $\overline{X}$ and $\overline{t}$ only. In the wave frame the expression of volume flow rate is

$$q=\int_{\overline{h_2}(\overline{x})}^{\overline{h_1}(\overline{x})}\overline{u}\left(\overline{x},\overline{y}\right)d\overline{y}, \tag{2.25}$$

where $\overline{h_1}$ and $\overline{h_2}$ are functions of x alone. Using Eqs. (2.14), (2.18) and (2.25) we have

$$Q=q+c\overline{h_1}\left(\overline{x}\right)-c\overline{h_2}\left(\overline{x}\right). \tag{2.26}$$

The time mean flow over a period T at a fixed position $\overline{X}$ is

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt. \tag{2.27}$$

Putting Eq. (2.26) into the above expression and then performing the integration one obtain

$$\overline{Q}=q+d_1c+d_2c. \tag{2.28}$$

Defining the non-dimensional time mean flow rates θ and F in the laboratory and wave frames by

$$\theta = \frac{\overline{Q}}{cd_1}, \qquad F = \frac{q}{d_1 c}, \tag{2.29}$$

we have

$$F + d + 1 = \theta. \tag{2.30}$$

The boundary conditions in the wave frame are

$$\begin{aligned}
\overline{u} &= -c \ \text{ at } \overline{y} = \overline{h_1}, \overline{y} = \overline{h_2}, \\
\overline{v} &= -c\frac{dh_1}{dx} \text{ at } y = h_1 \text{ and } \overline{v} = -c\frac{dh_2}{dx} \text{ at } y = h_2.
\end{aligned}$$

Which upon non-dimensionalizing give

$$u = -1 \ \text{ at } y = h_1 \text{ and } y = h_2, \tag{2.31}$$

$$v = -\frac{dh_1}{dx} \text{ at } y = h_1 \text{ and } v = -\frac{dh_2}{dx} \text{ at } y = h_2, \tag{2.32}$$

2.6 Solution expressions

The exact solution of the above problem subject to the boundary conditions is

$$u = \frac{dp}{dx}\left[\left(I_1(y) - I_1(h_1)\right) + \left(\frac{I_1(h_2) - I_1(h_1)}{I_0(h_1) - I_0(h_2)}\right)\left(I_0(y) - I_0(h_1)\right)\right] - 1, \tag{2.33}$$

in which

$$I_0(y) = \int \frac{1}{\mu(y)}dy, \quad I_1(y) = \int \frac{y}{\mu(y)}dy.$$

Using the dimensionless form of the volume flow rate i.e.

$$F = \int_{h_2}^{h_1} u dy.$$

we arrive at

$$F = -\frac{dp}{dx}\left[I_2 - \frac{\{I_1(h_1) - I_1(h_2)\}^2}{I_0(h_1) - I_0(h_2)}\right] - (h_1 - h_2) \tag{2.34}$$

which further gives

$$\frac{dp}{dx} = -\frac{F + h_1 - h_2}{I_2 - \frac{\{I_1(h_1) - I_1(h_2)\}^2}{I_0(h_1) - I_0(h_2)}} \tag{2.35}$$

where

$$I_2(y) = \int \frac{y^2}{\mu(y)} dy$$

Using Eq. (2.33) we get

$$v = L_1' [I_3(h_1) - I_3(y)] + L_2' [I_2(h_1) - I_2(y)] - L_3' (h_1 - y) - h_1'$$

where

$$\begin{aligned} L_1 &= \frac{dp}{dx}, \ L_2 = L_1 \left(\frac{I_1(h_2) - I_1(h_1)}{I_0(h_1) - I_0(h_2)} \right), \ L_3 = -L_2 I_0(h_1) - L_1 I_1(h_1) - 1, \\ h_1' &= \frac{dh_1}{dx}, \ I_3(y) = \int I_1(y)\, dy, \ I_2(y) = \int I_0(y)\, dy. \end{aligned}$$

Note that the differentiation is with respect to x. The non-dimensional form of the pressure rise per wavelength Δp_λ, and frictional forces at the upper and lower walls $F_\lambda^{(u)}$ and $F_\lambda^{(l)}$ respectively given by

$$\begin{aligned} \Delta p_\lambda &= \int_0^1 \left(\frac{dp}{dx} \right) dx, \\ F_\lambda^{(u)} &= \int_0^1 h_1^2 \left(-\frac{dp}{dx} \right) dx, \\ F_\lambda^{(l)} &= \int_0^1 h_2^2 \left(-\frac{dp}{dx} \right) dx. \end{aligned}$$

The influence of variable viscosity on the flow characteristics can be analyzed through I_0, I_1 and I_2 for any given type of viscosity $\mu(y)$. The form of the variable viscosity $\mu(y)$ can be adopted as follows

$$\mu(y) = e^{-\alpha y} \text{ for } \alpha \ll 1 \text{ or } \mu(y) = 1 - \alpha y.$$

in which α is the viscosity parameter. The consideration of such kind of expression for μ is justified physiologically in such a way that a normal person or an animal of the same size takes $(1 - 2L)$ of the fluid every day. The small intestine receives $(6 - 7L)$ of the fluid as secretions from salivary glands, stomach, pancreas, liver and small intestine itself. This points to the dependence of fluid concentration upon y and therefore the selection of μ is appropriate.

Subtituting the above choice of viscosity in the Eq. (2.34) we finally get

$$\begin{aligned}\frac{dp}{dx} &= -\frac{12\left(F+h_1-h_2\right)}{\left(h_1-h_2\right)^3}\left[1-\frac{\alpha}{2}\left\{\frac{6\left(h_1^4-h_2^4\right)-8\left(h_1+h_2\right)\left(h_1^3-h_2^3\right)+3\left(h_1^2-h_2^2\right)\left(h_1-h_2\right)^2}{\left(h_1-h_2\right)^3}\right\}\right] \\ &\quad +O\left(\alpha^2\right) \qquad (2.36)\end{aligned}$$

We note that the results of constant viscosity can be obtained as a special case by taking $\mu(y)=1$. Also if we put $a=b$, $d=1$ and $\phi=0$ the results for symmetric channel can be retained. The expressions for Δp_λ, $F_\lambda^{(u)}$ and $F_\lambda^{(l)}$ involve the integration of dp/dx. As from the expression of dp/dx it is obvious that it can not be integrated analytically. As a result, a numerical integration scheme is adopted for the evaluation of integrals. **Mathematica** is used to evaluate the integrals and later to generate all the plots.

2.7 Discussion

In order to see certain features of Δp_λ, $F_\lambda^{(u)}$ and $F_\lambda^{(l)}$, the integrals involved have been evaluated numerically for different values of a, b, d, and ϕ. The used values of α are 0 and 0.1. Fig. 2.1 is plotted for the effects of upper wave amplitude a on Δp_λ. It is found that with an increase in a pumping increases in the pumping region ($\Delta p_\lambda > 0$), copumping region ($\Delta p_\lambda < 0$) and free pumping region ($\Delta p_\lambda = 0$). It is further noticed that for an appropriately chosen $\Delta p_\lambda < 0$, θ is decreasing function of a. Moreover the pumping curves for $\alpha = 0.1$ lie above the curves of $\alpha = 0$ in pumping region, free pumping and copumping region for all three values of a. The effects of lower wave amplitude b is qualitatively similar to that of a (Fig. 2.2). Fig. 2.3 shows the variation of channel width d on Δp_λ. In pumping region ($\Delta p_\lambda > 0$) pumping decreases with increase in d. For free pumping there is no difference in the pumping curves while in the copumping region ($\Delta p_\lambda < 0$) pumping decreases for large d. For $\alpha = 0.1$ the pumping curves lie above the curves of $\alpha = 0$ in the pumping region. However the situation in copumping region is quite opposite. Fig. 2.4 is prepared for the effects of phase difference ϕ on Δp_λ. Similar effects are observed in the pumping and copumping region while in the free pumping , the pumping decreases with an increase in ϕ.

In order to examine the effects quantitatively, we provide intervals for θ where $\Delta p_\lambda < 0$ and $\Delta p_\lambda > 0$. Observe that as a increases the length of the interval for $\Delta p_\lambda > 0$ increases (Table 1). Similar observation is for the case when viscosity increases from 0 to 0.1 and a fixed (Table 2.1). This means that the interval for θ in which pressure resists the flow increases in length as a and α increase. The lower wave amplitude b has similar effects on the length of interval as that of a (Table 2.2). An increase in the channel width d leads to a decrease in the length of the interval where $\Delta p_\lambda > 0$ for both α $(= 0, 0.1)$ (Table 2.3). The observations regarding the effects of ϕ on Δp_λ are similar to that of d (Table 2.4).

Figs. 2.5 and 2.6 depict the effects of ϕ on the frictional forces $F_\lambda^{(u)}$ and $F_\lambda^{(l)}$ respectively. Interestingly, frictional forces are decreasing functions of ϕ and α. Further, the frictional force at the upper wall is much when compared with the lower wall.

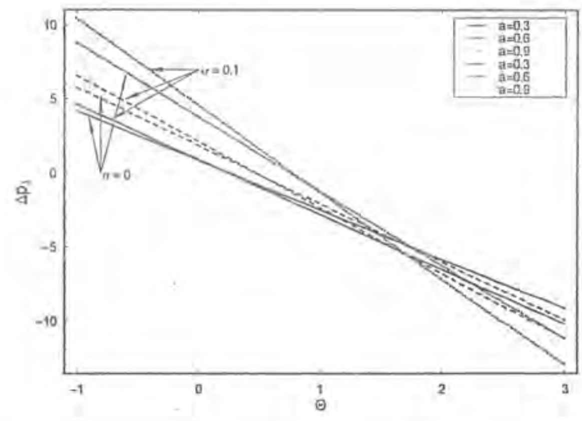


Fig. 2.1: The pressure rise versus flow rate for $b = 0.5$, $d = 2$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$

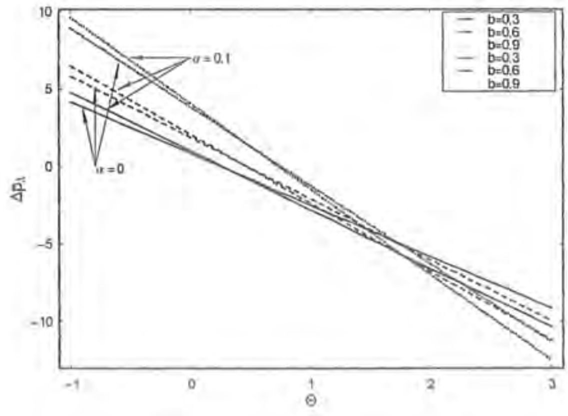


Fig. 2.2: The pressure rise versus flow rate for $a = 0.5$, $d = 2$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$

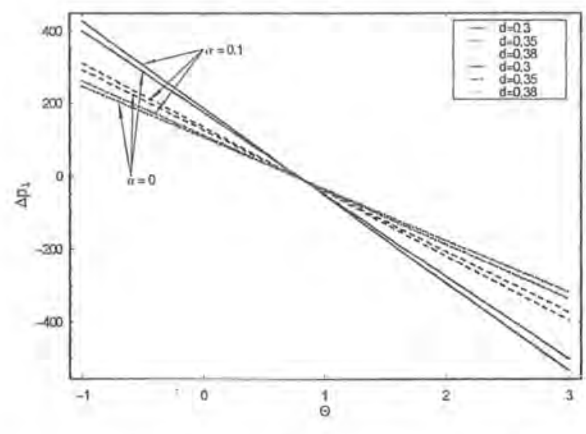


Fig. 2.3: The pressure rise versus flow rate for $b = 0.5$, $a = 0.5$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$

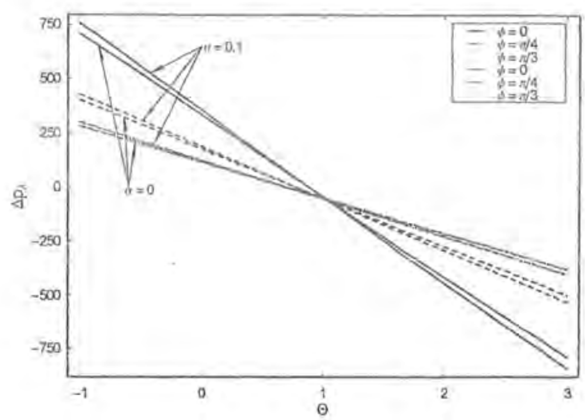


Fig. 2.4: The pressure rise versus flow rate for $a = 0.5$, $b = 0.5$, $d = 0.3$, $\alpha = 0$ and $\alpha = 0.1$.

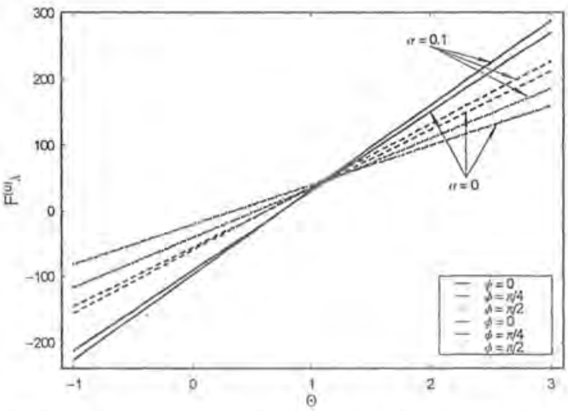


Fig. 2.5: The frictional force at the upper wall versus flow rate for $a = 0.5$, $b = 0.5$, $d = 0.3$, $\alpha = 0$ and $\alpha = 0.1$.

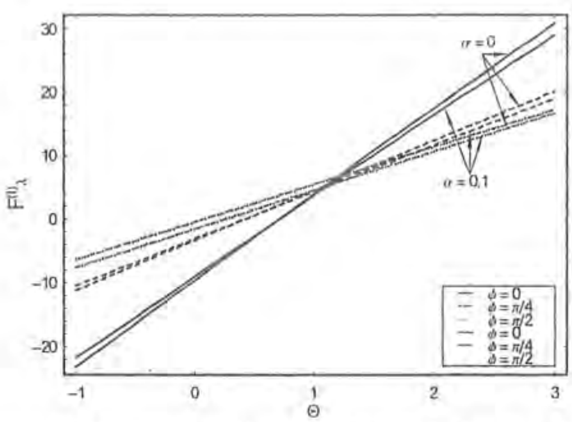


Fig. 2.6: The frictional force at the lower wall versus flow rate for $a = 0.5$, $b = 0.5$, $d = 0.3$, $\alpha = 0$ and $\alpha = 0.1$.

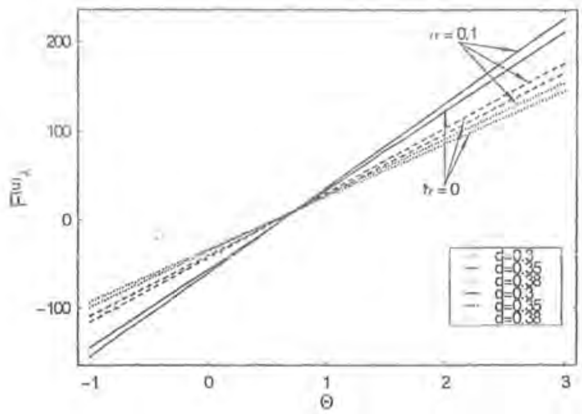


Fig. 2.7: The frictional force at the upper wall versus flow rate for $a = 0.5$, $b = 0.5$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

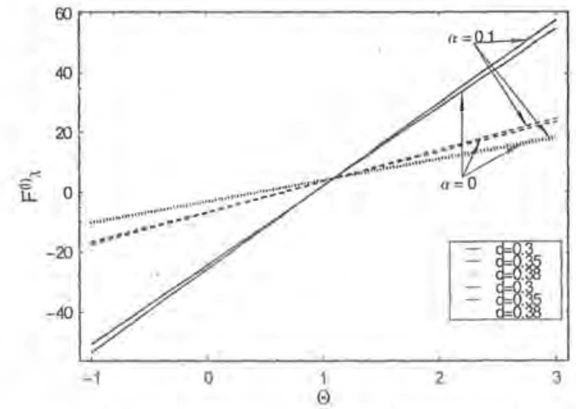


Fig. 2.8: The frictional force at the lower wall versus flow rate for $a = 0.5$, $b = 0.5$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

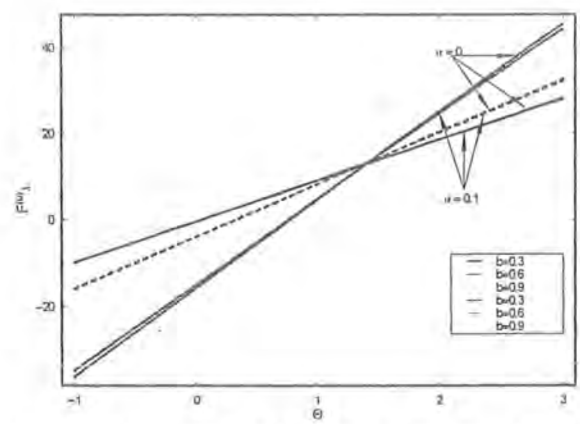


Fig. 2.9: The frictional force at the upper wall versus flow rate for $a = 0.5$, $d = 0.7$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

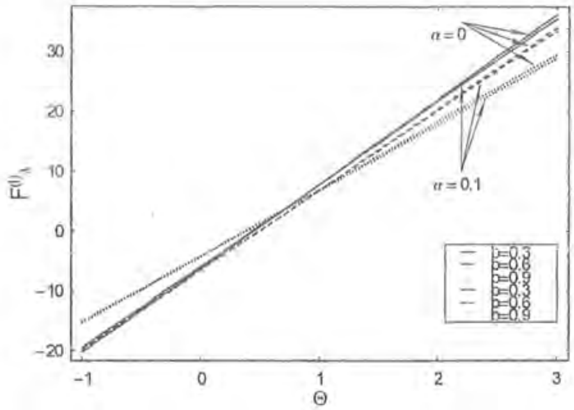


Fig. 2.10: The frictional force at the lower wall versus flow rate for $a = 0.5$, $d = 0.7$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

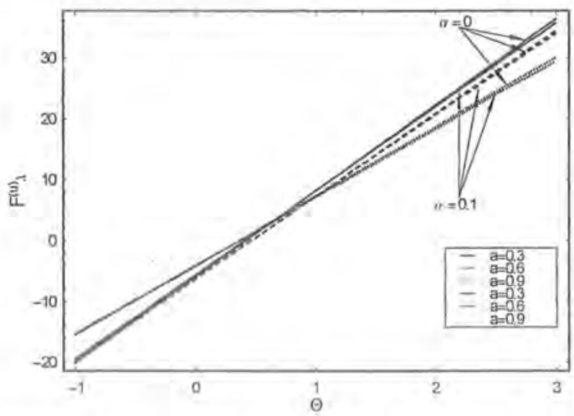


Fig. 2.11: The frictional force at the upper wall versus flow rate for $b = 0.5$, $d = 0.7$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

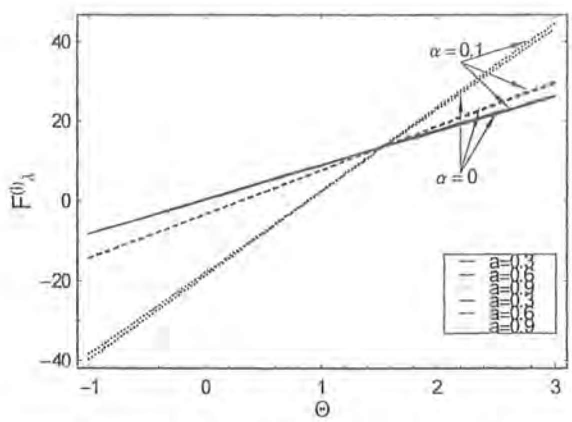


Fig. 2.12: The frictional force at the lower wall versus flow rate for $b = 0.5$, $d = 0.7$, $\phi = \pi/4$, $\alpha = 0$ and $\alpha = 0.1$.

Table 2.1: Intervals for flow rate θ for different values of a

Parameter (a)	Intervals of θ where $\Delta p_\lambda > 0$	Intervals of θ where $\Delta p_\lambda < 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.2001$	$\alpha = 0.0, 0.2001 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2123$	$\alpha = 0.1, 0.2123 < \theta < 3$
0.6	$\alpha = 0.0, -1 < \theta < 0.4890$	$\alpha = 0.0, 0.4890 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.5031$	$\alpha = 0.1, 0.50310 < \theta < 3$
0.9	$\alpha = 0.0, -1 < \theta < 0.7747$	$\alpha = 0.0, 0.7747 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.7938$	$\alpha = 0.1, 0.7938 < \theta < 3$

The other parameters are $b = 0.5$, $d = 2$, and $\phi = \pi/4$.

Table 2.2: Intervals for flow rate θ for different values of b (The other parameters are $a = 0.5$, $d = 2$, and $\phi = \pi/4$).

Parameter (b)	Intervals of θ where $\Delta p_\lambda > 0$	Intervals of θ where $\Delta p_\lambda < 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.2143$	$\alpha = 0.0, 0.2143 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2281$	$\alpha = 0.1, 0.2281 < \theta < 3$
0.5	$\alpha = 0.0, -1 < \theta < 0.4075$	$\alpha = 0.0, 0.4075 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.4188$	$\alpha = 0.1, 0.4188 < \theta < 3$
0.7	$\alpha = 0.0, -1 < \theta < 0.6776$	$\alpha = 0.0, 0.6776 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6889$	$\alpha = 0.1, 0.6889 < \theta < 3$

Table 2.3: Intervals for flow rate θ for different values of d (The other parameters are $a = 0.5$, $b = 0.5$, and $\phi = \pi/4$).

Parameter (d)	Intervals of θ where $\Delta p_\lambda > 0$	Intervals of θ where $\Delta p_\lambda < 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.7838$	$\alpha = 0.0, 0.7838 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8023$	$\alpha = 0.1, 0.8023 < \theta < 3$
0.35	$\alpha = 0.0, -1 < \theta < 0.7811$	$\alpha = 0.0, 0.7811 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.7666$	$\alpha = 0.1, 0.7666 < \theta < 3$
0.38	$\alpha = 0.0, -1 < \theta < 0.7690$	$\alpha = 0.0, 0.7690 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.7564$	$\alpha = 0.1, 0.7564 < \theta < 3$

Table 2.4: Intervals for flow rate θ for different values of ϕ (The other parameters are $a = 0.5$, $d = 2$, and $b = 0.5$).

Parameter (ϕ)	Intervals of θ where $\Delta p_\lambda > 0$	Intervals of θ where $\Delta p_\lambda < 0$
0	$\alpha = 0.0, -1 < \theta < 0.8975$	$\alpha = 0.0, 0.8975 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.9267$	$\alpha = 0.1, 0.9267 < \theta < 3$
$\frac{\pi}{4}$	$\alpha = 0.0, -1 < \theta < 0.7837$	$\alpha = 0.0, 0.7837 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8024$	$\alpha = 0.1, 0.8024 < \theta < 3$
$\frac{\pi}{2}$	$\alpha = 0.0, -1 < \theta < 0.7092$	$\alpha = 0.0, 0.7092 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.7268$	$\alpha = 0.1, 0.7268 < \theta < 3$

Table 2.5: Intervals for flow rate θ for different values of ϕ (The other parameters are $a = 0.5$, $d = 2$, and $b = 0.5$).

Parameter (ϕ)	Intervals of θ where $F_{\lambda}^{(u)} < 0$	Intervals of θ where $F_{\lambda}^{(u)} > 0$
0	$\alpha = 0.0, -1 < \theta < 0.7706$	$\alpha = 0.0, 0.7706 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8336$	$\alpha = 0.1, 0.8336 < \theta < 3$
$\frac{\pi}{4}$	$\alpha = 0.0, -1 < \theta < 0.6006$	$\alpha = 0.0, 0.6006 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6107$	$\alpha = 0.1, 0.6107 < \theta < 3$
$\frac{\pi}{2}$	$\alpha = 0.0, -1 < \theta < 0.4252$	$\alpha = 0.0, 0.4252 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.3126$	$\alpha = 0.1, 0.3126 < \theta < 3$
	Intervals of θ where $F_{\lambda}^{(l)} < 0$	Intervals of θ where $F_{\lambda}^{(l)} > 0$
0	$\alpha = 0.0, -1 < \theta < 0.8125$	$\alpha = 0.0, 0.8125 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8415$	$\alpha = 0.1, 0.8415 < \theta < 3$
$\frac{\pi}{4}$	$\alpha = 0.0, -1 < \theta < 0.2050$	$\alpha = 0.0, 0.2050 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2121$	$\alpha = 0.1, 0.2121 < \theta < 3$
$\frac{\pi}{2}$	$\alpha = 0.0, -1 < \theta < 0.0621$	$\alpha = 0.0, 0.0621 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.0685$	$\alpha = 0.1, 0.0685 < \theta < 3$

Table 2.6: Intervals for flow rate θ for different values of d (The other parameters are $a = 0.5$, $b = 0.5$, and $\phi = \pi/4$).

Parameter (d)	Intervals of θ where $F_\lambda^{(u)} < 0$	Intervals of θ where $F_\lambda^{(u)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.6377$	$\alpha = 0.0, 0.6377 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6807$	$\alpha = 0.1, 0.6807 < \theta < 3$
0.35	$\alpha = 0.0, -1 < \theta < 0.5969$	$\alpha = 0.0, 0.5969 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6308$	$\alpha = 0.1, 0.6308 < \theta < 3$
0.38	$\alpha = 0.0, -1 < \theta < 0.5733$	$\alpha = 0.0, 0.5733 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6026$	$\alpha = 0.1, 0.6026 < \theta < 3$
	Intervals of θ where $F_\lambda^{(l)} < 0$	Intervals of θ where $F_\lambda^{(l)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.8163$	$\alpha = 0.0, 0.8163 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8275$	$\alpha = 0.1, 0.8275 < \theta < 3$
0.35	$\alpha = 0.0, -1 < \theta < 0.5942$	$\alpha = 0.0, 0.5942 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6098$	$\alpha = 0.1, 0.6098 < \theta < 3$
0.38	$\alpha = 0.0, -1 < \theta < 0.3216$	$\alpha = 0.0, 0.3216 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.3365$	$\alpha = 0.1, 0.3365 < \theta < 3$

Table 2.7: Intervals for flow rate θ for different values of b (The other parameters are $a = 0.5$, $d = 2$, and $\phi = \pi/4$).

Parameter (b)	Intervals of θ where $F_\lambda^{(u)} < 0$	Intervals of θ where $F_\lambda^{(u)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.6349$	$\alpha = 0.0, 0.6349 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.6512$	$\alpha = 0.1, 0.6512 < \theta < 3$
0.5	$\alpha = 0.0, -1 < \theta < 0.5643$	$\alpha = 0.0, 0.5643 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.5670$	$\alpha = 0.1, 0.5670 < \theta < 3$
0.7	$\alpha = 0.0, -1 < \theta < 0.5626$	$\alpha = 0.0, 0.5626 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.5871$	$\alpha = 0.1, 0.5871 < \theta < 3$
	Intervals of θ where $F_\lambda^{(l)} < 0$	Intervals of θ where $F_\lambda^{(l)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.3220$	$\alpha = 0.0, 0.3220 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.3364$	$\alpha = 0.1, 0.3364 < \theta < 3$
0.5	$\alpha = 0.0, -1 < \theta < 0.3169$	$\alpha = 0.0, 0.3169 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2900$	$\alpha = 0.1, 0.2900 < \theta < 3$
0.7	$\alpha = 0.0, -1 < \theta < 0.2878$	$\alpha = 0.0, 0.2878 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2762$	$\alpha = 0.1, 0.2762 < \theta < 3$

Table 2.8: Intervals for flow rate θ for different values of a (The other parameters are $b = 0.5$, $d = 2$, and $\phi = \pi/4$).

Parameter (a)	Intervals of θ where $F_\lambda^{(u)} < 0$	Intervals of θ where $F_\lambda^{(u)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.4024$	$\alpha = 0.0, 0.4024 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.4024$	$\alpha = 0.1, 0.4024 < \theta < 3$
0.6	$\alpha = 0.0, -1 < \theta < 0.4126$	$\alpha = 0.0, 0.4126 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.4137$	$\alpha = 0.1, 0.4137 < \theta < 3$
0.9	$\alpha = 0.0, -1 < \theta < 0.3831$	$\alpha = 0.0, 0.3831 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.3967$	$\alpha = 0.1, 0.3967 < \theta < 3$
	Intervals of θ where $F_\lambda^{(l)} < 0$	Intervals of θ where $F_\lambda^{(l)} > 0$
0.3	$\alpha = 0.0, -1 < \theta < 0.0513$	$\alpha = 0.0, 0.0513 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.0513$	$\alpha = 0.1, 0.0513 < \theta < 3$
0.6	$\alpha = 0.0, -1 < \theta < 0.2152$	$\alpha = 0.0, 0.2152 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.2164$	$\alpha = 0.1, 0.2164 < \theta < 3$
0.9	$\alpha = 0.0, -1 < \theta < 0.8125$	$\alpha = 0.0, 0.8125 < \theta < 3$
	$\alpha = 0.1, -1 < \theta < 0.8233$	$\alpha = 0.1, 0.8233 < \theta < 3$

Chapter 3

Heat transfer on the peristaltic transport in an asymmetric channel

3.1 Introduction

This chapter highlights the effects of velocity and thermal slip parameters on the peristaltic motion of variable viscosity and magnetohydrodynamic (MHD) fluid in an asymmetric channel. Heat transfer coefficient and temperature are given due attention with respect to embedded parameters in the problem.

3.2 Mathematical formulation

We consider a variable viscosity viscous fluid in an asymmetric channel of width $d_1 + d_2$.

The fluid is electrically conducting whereas the channel walls are insulating. Fluid is conducting under the effect of constant magnetic field $\mathbf{B}_0$ applied in the $\overline{Y}$ direction. No electric field is applied. Induced magnetic field is also not considered under the assumption that magnetic Reynolds number is very small. Furthermore, it is assumed that the upper and lower channel walls are at temperature T_0 and T_1 respectively. The motion is initiated due to sinusoidal wave train moving with constant speed c. The shape of channel walls surfaces are described by

$$
\begin{aligned}
H_1(\overline{X},\overline{t}) &= d_1 + a_1 \sin\frac{2\pi}{\lambda}(\overline{X}-c\overline{t}), \quad \text{upper wall,}\\
H_2(\overline{X},\overline{t}) &= -d_2 - b_1 \sin\left(\frac{2\pi}{\lambda}(\overline{X}-c\overline{t})+\phi\right), \quad \text{lower wall,}
\end{aligned}
\tag{3.1}
$$

The governing Eqs. are (2.9)-(2.10) and the energy equation. The energy equation can be written as

$$
\rho\zeta\frac{dT}{dt} = \overline{\mathbf{T}}.\,(\nabla\mathbf{V}) + k\nabla^2 T,
$$

where ρ is the fluid density, ζ the specific heat at constant volume, T the temperature. $\overline{\mathbf{T}}$ the Cauchy stress tensor, $\mathbf{V}$ the velocity field and k the thermal conductivity. We know that

$$
\overline{\mathbf{T}}.\,(\nabla\mathbf{V}) = tr\left[\left(\overline{\mathbf{T}}\right)(\nabla\mathbf{V})\right]
$$

in which tr denotes the trace of the matrix. In the wave frame the complete set of governing equations is

2.10 =>

$$
\frac{\partial\overline{U}}{\partial\overline{X}} + \frac{\partial\overline{V}}{\partial\overline{Y}} = 0,
\tag{3.2}
$$

2.11 =>

$$
\begin{aligned}
\rho\left[\frac{\partial}{\partial\overline{t}} + \overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}}\right]\overline{U} &= -\frac{\partial\overline{P}}{\partial\overline{X}} + 2\frac{\partial}{\partial\overline{X}}\left[\overline{\mu}(\overline{Y})\frac{\partial\overline{U}}{\partial\overline{X}}\right] + \frac{\partial}{\partial\overline{Y}}\left[\overline{\mu}(\overline{Y})\left(\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{U}}{\partial\overline{Y}}\right)\right]\\
&\quad -\sigma B_0^2\overline{U},
\end{aligned}
\tag{3.3}
$$

2.12 =>

$$
\rho\left[\frac{\partial}{\partial\overline{t}} + \overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}}\right]\overline{V} = -\frac{\partial\overline{P}}{\partial\overline{Y}} + 2\frac{\partial}{\partial\overline{Y}}\left[\overline{\mu}(\overline{Y})\frac{\partial\overline{V}}{\partial\overline{Y}}\right] + \frac{\partial}{\partial\overline{X}}\left[\overline{\mu}(\overline{Y})\left(\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{U}}{\partial\overline{Y}}\right)\right],
\tag{3.4}
$$

$$
\rho\zeta\left(\overline{U}\frac{\partial}{\partial\overline{X}} + \overline{V}\frac{\partial}{\partial\overline{Y}}\right)T = \overline{\mu}(\overline{Y})\left[2\left\{\left(\frac{\partial\overline{U}}{\partial\overline{X}}\right)^2 + \left(\frac{\partial\overline{V}}{\partial\overline{Y}}\right)^2\right\} + \left(\frac{\partial\overline{V}}{\partial\overline{X}} + \frac{\partial\overline{U}}{\partial\overline{Y}}\right)^2\right] + k\left[\frac{\partial^2 T}{\partial\overline{X}^2} + \frac{\partial^2 T}{\partial\overline{Y}^2}\right],
\tag{3.5}
$$

The above equations in the following nondimensional variables become

$$\begin{aligned}
x &= \frac{2\pi\overline{x}}{\lambda},\ \ y=\frac{\overline{y}}{d_1},\ \ u=\frac{\overline{u}}{c},\ \ v=\frac{\overline{v}}{c\delta},\ \ \delta=\frac{2\pi d_1}{\lambda},\ \ h_1=\frac{H_1}{d_1},\ \ h_2=\frac{H_2}{d_1},\ \ d=\frac{d_2}{d_1}, p=\frac{2\pi d_1^2\overline{p}}{c\lambda\mu_0},\\
\theta &= \frac{T-T_0}{T_1-T_0},\ \ \mu(y)=\frac{\overline{\mu}(\overline{y})}{\mu_0},\ \ M=\left(\frac{\sigma}{\mu_0}\right)^{1/2}B_0 d_1,\ \ Pr=\frac{\mu_0\varsigma}{k},\ \ E=\frac{c^2}{\varsigma(T_1-T_0)},\ \ v_0=\frac{\mu_0}{\rho},\\
a &= \frac{a_1}{d_1},\ \ b=\frac{b_1}{d_1}, Re=\frac{\rho c d_1}{\mu_0},\ \ t=\frac{c\overline{t}}{\lambda}, u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x},
\end{aligned}\tag{3.6}$$

$$\begin{aligned}
\operatorname{Re}\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u &= -\frac{\partial p}{\partial x}+2\delta^2\frac{\partial}{\partial x}\left(\mu(y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]\\
&\quad -M^2(u+1),
\end{aligned}\tag{3.7}$$

$$\operatorname{Re}\delta^3\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v=-\frac{\partial p}{\partial y}+2\delta^2\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial v}{\partial y}\right)+\delta^2\frac{\partial}{\partial x}\left[\mu(y)\left(\delta^2\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right],\tag{3.8}$$

$$\begin{aligned}
\operatorname{Re}\delta\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\theta &= E\mu(y)\left[4\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left\{\left(\frac{\partial u}{\partial y}\right)^2+\delta^4\left(\frac{\partial v}{\partial x}\right)^2-2\delta^2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right\}\right]+\\
&\quad \frac{1}{\Pr}\left[\frac{\partial^2\theta}{\partial y^2}+\delta^2\frac{\partial^2\theta}{\partial x^2}\right],
\end{aligned}\tag{3.9}$$

where ψ is the stream function, v_0 is the kinematic viscosity, M is the Hartman number, Re is the Reynolds number, δ is the wave number, Pr is the Prandtl number, E is the Eckret number, μ_0 is the constant viscosity and θ is the dimensionless temperature. Adopting the long wavelength approximation eqs. $(3.7)-(3.9)$ reduce to

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)-M^2\left(\frac{\partial\psi}{\partial y}+1\right)\tag{3.10}$$

$$0=-\frac{\partial p}{\partial y}\tag{3.11}$$

$$0=\frac{\partial^2\theta}{\partial y^2}+Br\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\tag{3.12}$$

in which the Brinkman number $Br=Pr\ E$ and continuity equation is automatically satisfied.

3.2.1 Derivation of slip condition

In fixed frame of reference the slip conditions at the walls are defined as

$$\overline{U}_{\omega} - \overline{U}\left(\overline{X}, \overline{H_1}, \overline{t}\right) = \frac{\gamma}{\mu}\overline{\tau}_{\overline{XY}}, \tag{3.13}$$

$$\overline{U}\left(\overline{X}, \overline{H_2}, \overline{t}\right) - \overline{U}_{\omega} = \frac{\gamma}{\mu}\overline{\tau}_{\overline{XY}}, \tag{3.14}$$

in which $\overline{U}\left(\overline{X}, \overline{Y}, \overline{t}\right)$ is the longitudinal component of the velocity in the laboratory frame, $\overline{H_1}$ and $\overline{H_2}$ are the shapes of the walls which are given in Eq. (2.1), $\overline{U}_{\omega}$ is the velocity of the walls, $\overline{\tau}_{\overline{XY}}$ is the shear stress and γ is the dimensional slip parameter. Transforming the above expressions in the wave frame through Eq. (2.13) we obtain

$$\overline{u}_{\omega} - \overline{u}\left(\overline{x}, \overline{h}_1\right) = \frac{\gamma}{\mu}\overline{\tau}_{\overline{xy}}, \tag{3.15}$$

$$\overline{u}\left(\overline{x}, \overline{h_2}\right) - \overline{u}_{\omega} = \frac{\gamma}{\mu}\overline{\tau}_{\overline{xy}}. \tag{3.16}$$

Since the wave frame is moving with the speed of the wave travelling along the distensible walls of the channel so the boundaries are stationary in the wave frame and therefore

$$\overline{u}_{\omega} = 0. \tag{3.17}$$

which in view of Eqs. (2.14) become

$$\overline{u}\left(\overline{x}, \overline{h}_1\right) = -\frac{\gamma}{\mu}\overline{\tau}_{\overline{xy}}, \tag{3.18}$$

$$\overline{u}\left(\overline{x}, \overline{h_2}\right) = \frac{\gamma}{\mu}\overline{\tau}_{\overline{xy}}. \tag{3.19}$$

It is convenient to use the non-dimensional variables (2.18) at this stage and obtain

$$u = -\beta\frac{\partial u}{\partial y} - 1 \quad \text{at} \quad y = h_1(x), \tag{3.20}$$

$$u = \beta\frac{\partial u}{\partial y} - 1 \quad \text{at} \quad y = h_2(x), \tag{3.21}$$

where $\beta = \frac{\gamma}{\mu}$.

The non-dimensional boundary conditions in terms of stream function are

$$\psi = \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} + \beta\mu(y)\frac{\partial^2 \psi}{\partial y^2} = -1, \quad \theta + \gamma\frac{\partial \theta}{\partial y} = 0, \quad at \;\; y = h_1, \tag{3.22}$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} - \beta\mu(y)\frac{\partial^2 \psi}{\partial y^2} = -1, \quad \theta - \gamma\frac{\partial \theta}{\partial y} = 1, \quad at \;\; y = h_2, \tag{3.23}$$

$$h_1(x)=1 + a\sin(2\pi x), \quad h_2(x) = -d - b\sin(2\pi x + \phi), \;\; F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} dy, \tag{3.24}$$

$$a^2 + b^2 + 2ab\cos\phi = (1 + d)^2, \tag{3.25}$$

where $\mu(y) = e^{-\alpha y}$ or $\mu(y) = 1 - \alpha y$ for $\alpha \ll 1$; α is the viscosity parameter and β and γ are the non-dimensional velocity and thermal slip parameters respectively.

3.3 Solution of the problem

We solved the system of equations as follows

3.3.1 Case 1 for hydrodynamic fluid ($M = 0$)

For this case the governing system takes the following form

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2 \psi}{\partial y^2}\right), \tag{3.26}$$

$$0 = -\frac{\partial p}{\partial y}, \tag{3.27}$$

$$0 = \frac{\partial^2 \theta}{\partial y^2} + Br\mu(y)\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 \tag{3.28}$$

$$\psi = \frac{F}{2}, \quad \frac{\partial\psi}{\partial y} + \beta\frac{\partial^2\psi}{\partial y^2} = -1, \quad \theta + \gamma\frac{\partial\theta}{\partial y} = 0, \quad at \;\; y = h_1, \tag{3.29}$$

$$\psi = -\frac{F}{2}, \quad \frac{\partial\psi}{\partial y} - \beta\frac{\partial^2\psi}{\partial y^2} = -1, \quad \theta - \gamma\frac{\partial\theta}{\partial y} = 1, \quad at \;\; y = h_2, \tag{3.30}$$

solution for which, are given as

$$\psi = \frac{\left[\begin{array}{c} (2A_1(h_1-y)(h_2-y)(A_1(A_2-2y+(h_1^2+h_1h_2+h_2^2-A_2y-y^2)\alpha) \\ +2(A_2+h_1h_2\alpha-y(2+\alpha y))\beta) - F(A_1(h_2^3-6h_2y^2+h_1^4\alpha+h_2^4\alpha \\ -4h_2^2y^2\alpha+h_1^3(1-2h_2\alpha)-h_1(h_2^2-4h_2y+2y^2)(3+2h_2\alpha) \\ +h_1^2(-3h_2-4(h_2-y)^2\alpha)+2y^3(2+\alpha y))+4(-3h_2^2y-3h_1(h_2^2-4h_2y+y^2) \\ +h_1^4\alpha+h_2^4\alpha-h_1y(-3h_2^2+y^2)\alpha-3h_1^2(h_2+y+h_2(h_2-y)\alpha) \\ +(2-2y\alpha)(h_1^3+h_2^3)+y^3(2+2y\alpha)-h_2y^2(3+\alpha y))\beta+12A_1(A_2-2y)\beta^2)) \end{array}\right]}{(2A_1^2(A_2^2(1+A_2\alpha)+4A_1(2+A_2\alpha)\beta+12\beta^2))}, \tag{3.31}$$

$$\frac{dP}{dx} = -\frac{(6(F+A_1)(A_1(2+\alpha A_2)+4\beta))}{(A_1^2(A_1^2(1+\alpha A_2)+4A_1(2+\alpha A_2)\beta+12\beta^2))}, \tag{3.32}$$

$$\theta = \frac{1}{L_1}(L_2+\gamma(-23h_1^6\alpha+h_2L_3+L_4+L_5+L_6+L_7\gamma)). \tag{3.33}$$

$L_1 = 5A_2^4(A_2+6\beta)(A_2^2(1+2A_1\alpha+8A_2(1+A_1\alpha)\beta+12\beta^2)(A_2+2\gamma))$, $L_2 = (A_2^4(A_2+6\beta)(A_2^2(1+2A_1\alpha)+8A_2(1+A_1\alpha)\beta+12\beta^2)(h_1-y+\gamma)-2(F+A_2)^2Br(A_2(h_1-y)(h_2-y)(A_2(15(h_1^2+h_2^2-2A_1y+2y^2)+(23(h_1^3+h_2^3)+13h_1h_2(A_1)(37(h_1^2+h_2^2)+52h_1h_2)y+18A_1y^2+18y^3)\alpha)+6(h_1^2+h_2^2+2y(-A_1+y)(5+(A_1+3y)\alpha)\beta)$, $L_3 = 120h_2y^3+23h_2^5\alpha+15h_2^4(1+3y\alpha)-12y^4(5+3y\alpha)-30h_2^3y(-1+4y\alpha)+10h_2^2y^2(-9+11y\alpha)$, $L_4 = -6(h_2^4+2h_2^3y-6h_2^2y^2+8h_2y^3-4y^4)(5+h_2\alpha+3y\alpha)\beta+3h_1^5(-5+46h_2\alpha-15\alpha y-2\alpha\beta)-15h_1^4(9h_2^2\alpha+2y\alpha(-4y+\beta)+2(y+\beta)+h_2(7(-1+y\alpha)-2\alpha\beta))+3h_1(-46h_2^5\alpha+10h_2^2y(-3y\alpha)-6\beta+40h_2y^2(3+y\alpha)\beta+52h_2^4(7(-1+y\alpha)+2\alpha\beta))+20h_2^3(y(1-2y\alpha)+3\beta+y\alpha\beta)+4y^3(y(5+3y\alpha)-10(2+y\alpha)\beta))$, $L_5 = 15h_1^2(12h_2^3+9h_2^4\alpha-2h_2y(y(-3y\alpha)+6\beta)-12h_2^2(\beta+y\alpha\beta)+4y^2(3\beta+y(-2+\alpha\beta)))$, $L_6 = -10h_1^3(18h_2^2+y(y(-9+11y\alpha)+6\beta)-6h_2^2(3\beta+y(-1+2\alpha y+\alpha\beta)))$, $L_7 = -15A_2^3(A_2(2+3A_1\alpha)+2(2+A_1\alpha)\beta)$

3.3.2 Case 2 for magnetohydrodynamic (MHD) fluid ($M \neq 0$)

For the case of magnetohydrodynamic fluid the governing equations are

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right) - M^2\frac{\partial^2\psi}{\partial y^2} = 0, \tag{3.34}$$

$$0 = \frac{\partial^2\theta}{\partial y^2} + Br\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{3.35}$$

In order to solve the above equations, we expand flow quantities in terms of small parameter α as follows

$$\psi = \psi_0 + \alpha\psi_1 + o(\alpha^2) \tag{3.36}$$

$$p = p_0 + \alpha p_1 + o(\alpha^2) \tag{3.37}$$

$$F = F_0 + \alpha F_1 + o(\alpha^2) \tag{3.38}$$

$$\theta = \theta_0 + \alpha\theta_0 + o(\alpha^2) \tag{3.39}$$

Zeroth order system

$$\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\psi_0}{\partial y^2}\right) - M^2\frac{\partial^2\psi_0}{\partial y^2} = 0,$$

$$\frac{\partial^2\theta_0}{\partial y^2} + Br\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2 = 0,$$

$$\begin{aligned}
\psi_0 &= \frac{F_0}{2}, \ \ \frac{\partial\psi_0}{\partial y} + \beta\frac{\partial^2\psi_0}{\partial y^2} = -1, \ \ \theta_0 + \gamma\frac{\partial\theta_0}{\partial y} = 0, \ \text{ at } \ y = h_1, \\
\psi_0 &= -\frac{F_0}{2}, \ \ \frac{\partial\psi_0}{\partial y} - \beta\frac{\partial^2\psi_0}{\partial y^2} = -1, \ \ \theta_0 - \gamma\frac{\partial\theta_0}{\partial y} = 1, \ \text{ at } \ y = h_2,
\end{aligned}$$

First order system

$$\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\psi_1}{\partial y^2}\right) - M^2\frac{\partial^2\psi_1}{\partial y^2} = \frac{\partial^2}{\partial y^2}\left[y\frac{\partial^2\psi_0}{\partial y^2}\right],$$

$$\frac{\partial_1^2\theta_1}{\partial y^2} + Br\left[2\left(\frac{\partial^2\psi_1}{\partial y^2}\right)\left(\frac{\partial^2\psi_0}{\partial y^2}\right) - y\left(\frac{\partial^2\psi_1}{\partial y^2}\right)^2\right] = 0,$$

$$\psi_1 = \frac{F_1}{2}, \ \frac{\partial \psi_1}{\partial y} + \beta \frac{\partial^2 \psi_1}{\partial y^2} - \beta y \frac{\partial^2 \psi_0}{\partial y^2} = 0, \ \theta_1 + \gamma \frac{\partial \theta_1}{\partial y} = 0, \ \text{at } y = h_1,$$

$$\psi_1 = -\frac{F_1}{2}, \ \frac{\partial \psi_1}{\partial y} - \beta \frac{\partial^2 \psi_1}{\partial y^2} + \beta y \frac{\partial^2 \psi_0}{\partial y^2} = 0, \ \theta_1 - \gamma \frac{\partial \theta_1}{\partial y} = 0, \ \text{at } y = h_2,$$

The final solution comes out to be

$$\begin{aligned}\psi &= \frac{1}{2A_5}\left[2e^{M(A_1-y)}(F+A_2) - 2e^{My}(F+A_2) + e^{h_2 M}(A_1-2y)(2+FMA_4)\right] - e^{h_1 M} \\ &\quad (A_1-2y)(2+FMA_3) + \alpha\left[m_1+m_2\right] \end{aligned} \tag{3.40}$$

$$m_1 = \begin{bmatrix} -\frac{1}{8A_5^2}(\frac{1}{MA_6}(e^{-M(2A_1+y)}(A_5(A_6(e^{2M(A_1+y)}(F+A_2-8F_1M-2(F+ \\ A_2)My+2(F+A_2)M^2y^2)+e^{3A_1M}(F+A_2+8F_1M+2(F+ \\ A_2)My+2(F+A_2)M^2y^2)+4e^{M(2h_1+3h_2+y)}F_1A_1M^2A_4 \\ -4e^{M(3h_1+2h_2+y)}F_1A_1M^2A_3-8A_4F_1Me^{M(3h_1+4h_2)}+2A_4e^{M(2h_1+3h_2+2y)} \\ -A_1A_4^2Me^{M(2h_1+4h_2+y)}-2A_3e^{M(4h_1+3h_2)}+2A_3e^{M(3h_1+2(h_2+y))}+ \\ A_1A_3^2Me^{M(4h_1+2h_2+y)}))+(F+A_2)(-8A_6e^{M(3A_1+y)}A_2A_1M^3y+ \\ 2e^{4A_1M}(2-M(A_1+A_2^2M)(2+A_1M)+A_2^2M^4(-1+A_1M)\beta^2)) \end{bmatrix}$$

$$m_2 = \begin{bmatrix} -2e^{3A_1M+2My}(-2+M(-2A_1+3M(h_1^2+h_2^2)-2h_1h_2M- \\ M^2(h_1^3+h_2^3)+M^2h_1h_2A_1+A_2^2M^3(1+A_1M)\beta^2)) \\ +e^{M(3h_1+5h_2)}(-2+M(h_2+h_1(-1+2M(A_1+h_1A_2M))-2\beta- \\ 4h_1M(1+h_1M(1+h_1M-h_2M))\beta+A_2M^2(1+2h_1M \\ (1+h_1M))\beta^2))+A_7e^{2M(h_1+2h_2+y)}+2A_8e^{M(2h_1+5h_2+y)}- \\ 2A_9e^{M(5h_1+2h_2+y)}+A_{10}e^{2M(2h_1+h_2+y)}+2A_{11}e^{M(3h_1+4h_2+y)}+ \\ A_{12}e^{M(5h_1+3h_2)}+2A_{13}e^{M(4h_1+3h_2+y)}))))+8e^{A_1M}(F+A_2)A_1My\sinh[A_2M] \end{bmatrix}$$

$$\frac{dP}{dx} = -\frac{(F+A_2)M^3(A_1(-e^{2h_1M}+e^{2h_2M}+2e^{A_1M}A_2M)\alpha - 2A_5e^{h_2M}A_4 + 2A_5e^{h_1M}A_3}{2A_5(e^{h_2M}(2-A_2MA_4)+e^{h_1M}(-2+A_2MA_3))}, \tag{3.41}$$

$$\begin{aligned}\theta &= \frac{1}{24A_5}(N_1 + +\frac{1}{2}Br(F+A_2)M\alpha(N_2 + \frac{1}{A_5A_6}N_3 + N_4 + N_5 + -48e^{MA_1} \\ &\quad F_1M(\cosh[A_2M] - \cosh[(A_2-2y)M] + 2M\gamma\sinh[A_2M]))) \end{aligned} \tag{3.42}$$

$N_1 = \frac{24A_5^2(h_1-y+\gamma)}{A_2+2\gamma} + 12Bre^{MA_1}(F+A_2)^2M^2(\cosh[A_2M] - \cosh[A_2M-2y] + 2M(M(h_1 - y)(h_2-y) - A_2M\gamma + \gamma\sinh[A_2M]))$, $N_2 = -6e^{2My}(F+A_2)(1-My+M^2y^2) + 6e^{2M(A_1-y)}(F + A_2)(1+My(1+My)) + \frac{1}{A_5A_6}(3e^{-2My}(F+A_2)(2e^{3MA_1}A_{14} + e^{2M(h_1+2h_2)}A_{15} - e^{2M(2h_1+h_2)}A_{16} + 2e^{M(A_1+4y)}A_{17} - e^{2M(h_2+2y)}A_{18} + e^{2M(h_1+2y)}A_{19}))$, $N_3 = 6e^{A_1M}M^2(2e^{A_1M}(F-(-A_{14}+A_{17}) + A_2(A_{14}+A_{17})) - e^{2h_2M}((F-A_2)A_{15} + (F+A_2)A_{18}) + e^{2h_1M}((F-A_2)A_{16} + (F+A_2)A_{19}))((h_1 - y)(h_2-y) - A_2\gamma))$, $N_4 = \frac{1}{A_2+2\gamma}(8e^{A_1M}M^3(A_2(h_1-y)(h_2-y)(-12F_1 + (F+A_2)(A_1+y)) - (h_1^2 - 4h_1h_2 + h_2^2 + 2A_1y - 2y^2)(-12F_1 + (F+A_2)(A_1+y))\gamma - 3A_2(-8F_1 + (F+A_2)A_1)\gamma^2)) - \frac{1}{A_6}(24F_1M(e^{h_2M}A_4 + e^{h_1M}A_3)(e^{2M(A_1-y)} - e^{2My} + 4e^{MA_1}M^2((h_1-y)(h_2-y) - A_2\gamma))) - \frac{1}{A_2+2\gamma}(6e^{2h_2M}(F+A_2)(-h_2(1+My-2M\gamma) - 2y(-1+M\gamma) + h_1^2M^2(h_2-y-\gamma)(-1+2M\gamma) - h_2^2M^2(y-\gamma)(-1+2M\gamma) + h_1(-1+My-2M\gamma+h_2^2M^2(1+2M\gamma))))$, $N_5 = -\frac{1}{A_2+2\gamma}(6e^{2h_1M}(F + A_2)(h_1^2M^2(h_2-y-\gamma)(1+2M\gamma) - h_2^2M^2(y-\gamma)(1+2M\gamma) + h_2(1-My+2M\gamma) - 2(y+My\gamma) + h_1(1+My+2M\gamma+h_2^2M^2(1+2M\gamma)))) + \frac{1}{A_5A_6(A_2+2\gamma)}(3(8A_5F_1M(e^{h_2M}A_4 + e^{h_1M}A_4)(A_2 - 2\gamma)(e^{2h_2M}(1-2M\gamma) + e^{2h_1M}(1+2M\gamma) - (F+A_2)(-2e^{M(h_1+3h_2)}(-1+2M\gamma)(-h_2A_{14} + y(A_{14} - A_{17}) + h_1A_{17} + (A_{14}+A_{17})\gamma) + 2e^{M(3h_1+h_2)}(1+2M\gamma)(h_1A_{14} - h_2A_{17} + h_2A_{19} - y(A_{16}+A_{19}) + \gamma A_{16} - \gamma A_{19}) + e^{4h_1M}(-1+2M\gamma)(h_2A_{15} - y(A_{15}+A_{18}) - \gamma A_{15}y(-A_{14}+A_{17}) + (A_{14}+A_{17})\gamma - e^{4h_1M}(1 + 2M\gamma)(h_1A_{16} - + A_{18}(h_1+\gamma)) + e^{2MA_1}(h_2(A_{16}+A_{18}) - y(A_{15}+A_{16}+A_{18}+A_{19}) - 2My(A_{15}-A_{16} + A_{18}-A_{19})\gamma) + \gamma(A_{15} - A_{16} - 2h_2MA_{16} - A_{18} + 2h_2MA_{18} + A_{19} + 2M(A_{15}+A_{16}-A_{18}-A_{19})))))$.

The heat transfer coefficient (Z) at the upper wall is

$$Z_1 = \theta_y\left(h_1\right)_{,x}, \tag{3.43}$$

which upon using the results, gives

$$
\begin{aligned}
Z_1 \;=\;& -\frac{1}{24A_5^2}(a\cos[x](-\frac{24A_5^2}{A_2+2\gamma}+12Bre^{MA_1}(F+A_2)^2M^2(2M(-M(h_1-y)-\\
& M(h_1-y))+2M\sin[M(A_1-2y)])\frac{1}{2}Br(F+A_2)M\alpha(-6e^{2My}(F+A_2)(-M\\
& +2M^2y)-12e^{2My}(F+A_2)M(1-My+M^2y^2)+6e^{2M(A_1-y)}(F+A_2)(M^2y\\
& +M(1+My))-12e^{2M(A_1-y)}(F+A_2)M(1+My(1+My))-\frac{1}{A_5A_6}(6e^{-2My}\\
& (F+A_2)M(2e^{3MA_1}A_{14}+e^{2M(h_1+2h_2)}A_{15}-e^{2M(2h_1+h_2)}A_{16}+2e^{M(A_1+4y)}A_{17}\\
& -e^{2M(h_2+2y)}A_{18}+e^{2M(h_1+2y)}A_{19}))+\frac{1}{A_5A_6}(3e^{-2My}(F+A_2)(8e^{M(A_1+4y)}\\
& MA_{17}-4e^{2M(h_2+2y)}MA_{18}+4e^{2M(h_1+2y)}A_{19}))+\frac{1}{A_5A_6}(6e^{MA_1}M^2(-A_1+\\
& 2y)(2e^{MA_1}(F(-A_{14}+A_{17})+A_2(A_{14}+A_{17}))-e^{2Mh_2}((F-A_1)A_{15}+(F+A_2)\\
& A_{18})+e^{2Mh_1}((F-A_1)A_{16}+(F+A_2)A_{19})))-\frac{1}{A_6}(24F_1M(-2e^{2M(A_1-y)}M-\\
& 2e^{2My}M+4e^{MA_1}M^2(-A_2+2y))(e^{Mh_2}A_4+e^{Mh_1}A_3))+\frac{1}{A_2+2\gamma}(8e^{MA_1}M^3(A_2\\
& (F+A_2)(h_1-y)(h_2-y)-A_2(h_1-y)(-12F_1+(F+A_2)(A_1+y))-\\
& A_2(h_2-y)(-12F_1+(F+A_2)(A_1+y))-(F+A_2)(h_1^2-4h_1h_2+h_2^2+2A_1y\\
& -2y^2)\gamma-2(2A_1-4y)(-12F_1+(F+A_2)(A_1+y))\gamma))-\frac{1}{A_2+2\gamma}(6e^{2Mh_2}\\
& (F+A_2)(h_1M-h_2M-2(-1+M\gamma)-h_1^2M^2(-1+2M\gamma)-h_2^2M^2(-1+2M\gamma)))\\
& -\frac{1}{A_2+2\gamma}(e^{2Mh_1}(F+A_2)(h_1M-h_2M-2(1+M\gamma)-h_1^2M^2(1+2M\gamma)-\\
& h_2^2M^2(1+2M\gamma)))-\frac{1}{A_5A_6(A_2+2\gamma)}(3(F+A_2)(-2e^{(h_1+3h_2)M}(-A_{14}+A_{17})\\
& (-1+2M\gamma)+e^{4h_2M}(-A_{15}-A_{18})(-1+2M\gamma)+2e^{M(3h_1+h_2)}(A_{14}-A_{17})\\
& (1+2M\gamma)-e^{4Mh_1}(-A_{16}-A_{19})(1+2M\gamma)+2e^{MA_1}(-A_{15}-A_{16}-A_{18}-A_{19}-\\
& 2M(A_{15}-A_{16}+A_{18}-A_{19})\gamma)))-96e^{MA_1}F_1M^2\sinh[(A_1-2y)]))), \qquad (3.44)
\end{aligned}
$$

The pressure rise per wave $l$ength (ΔP_{λ_1}) and frictional forces at the upper ($F_{\lambda_{11}}$) and lower ($F_{\lambda_{12}}$) walls are

$$\Delta F_{\lambda_1} = \int_0^{2\pi} \frac{dp_1}{dx} dx, \tag{3.45}$$

$$F_{\lambda_{11}} = \int_0^{2\pi} -h_1^2 \left(\frac{dp_1}{dx} \right) dx,$$

$$F_{\lambda_{12}} = \int_0^{2\pi} -h_2^2 \left(\frac{dp_1}{dx} \right) dx. \tag{3.46}$$

3.4 Discussion of results

The purpose of this section is to see the salient features of temperature θ, heat transfer coefficient Z and stream lines for the velocity slip β, thermal slip γ, flow rate η, viscosity parameter α and Brinkman number Br.

Figs. 3.1 $(a)-(e)$ show the behavior of temperature. Fig 3.1(a) explains that an increase in the velocity slip β decreases the temperature. Fig. 3.1(b) illustrates that the temperature increases with an increase in flow rate η. Temperature increases by increasing in Br and γ see Figs. 3.1(c) and 3.1(d). Fig. 3.1(e) demonstrates the effect of viscosity parameter on the temperature. Obviously there is an increase in the temperature when the value of viscosity parameter increases.

Fig. 3.2 represents the behavior of streamlines for the different values of α and β. Fig. 3.2(b) shows that the size of trapped bolus increases with an increase in the viscosity parameter (α). Figs. 3.2(a) and (b) examine that size of trapped bolus decreases when β increases.

Figs. $3.3-3.5$ represent the behavior of heat transfer coefficient at the upper wall (h_1). Heat transfer coefficient has an oscillatory behavior due to peristalsis. Absolute value of heat transfer coefficient decreases with an increase in β (see Fig. 3.3).

Fig 3.4 shows that absolute value of heat transfer coefficient increases by increasing Br. Fig 3.5 represents that heat transfer coefficient increases with an increase in γ. By comparison of left and right panels, we conclude that the heat transfer coefficient at the upper wall increases with an increase in α.

3.5 Conclusions

Peristalsis of variable viscosity fluid in an asymmetric channel has been studied in the presence of slip condition. The following observations are noted.

- There is a decrease in temperature when β increases.
- The effects of γ, Br and η on temperature are quite opposite to that of β.
- An increases in β reduces the size of trapped bolus.
- The magnitude of the heat transfer coefficient at the upper wall increases when thermal slip parameter increases.
- The no-slip results can be recovered by choosing $\beta = \gamma = 0$.

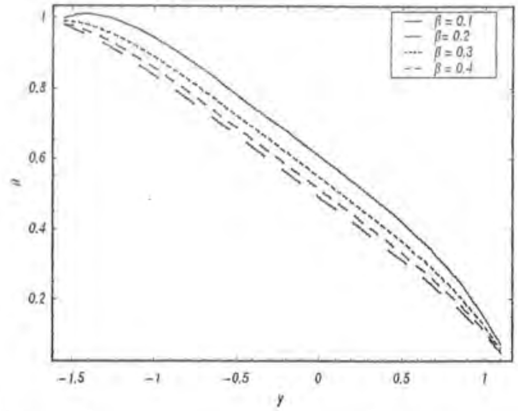


Fig. 3.1(a) : Variation of β on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $M = 1.0$; $\phi = \frac{\pi}{6}$; $x = 0$; $\gamma = 0.2$; $Br = 0.5$ and $\eta = 1.4$.

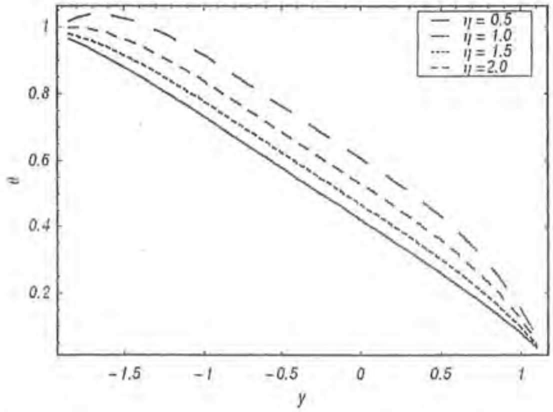


Fig. 3.1(b): Variation of η on the temperature for $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $M = 1$; $Br = 0.5$ and $\gamma = 0.2$.

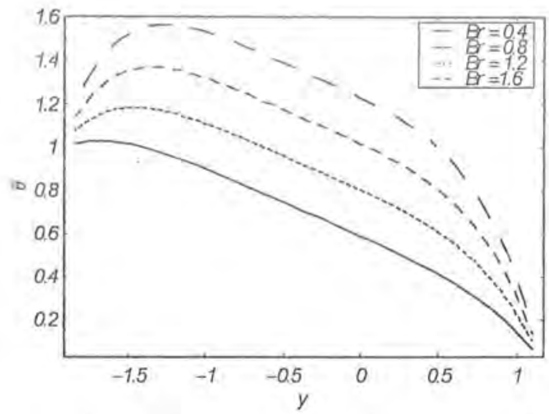


Fig. 3.1(c): Variation of Br on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $M = 1$ and $\gamma = 0.2$.

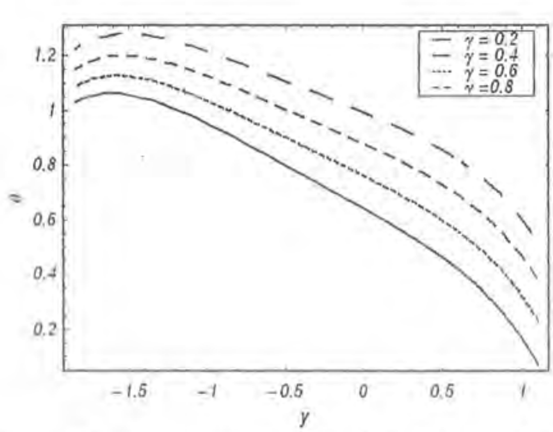


Fig. 3.1(d): Variation of γ on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $Br = 0.5$ and $\eta = 1.4$.

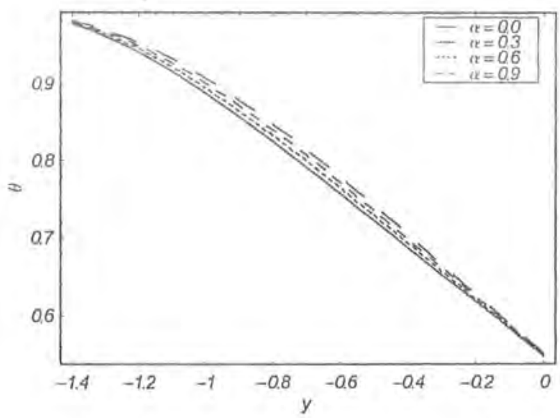


Fig. 3.1(e): Variation of α on the temperature when $d = 1.1$; $\beta = 0.2$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $M = 1$ and $\gamma = 0.2$.

(a)

(b)

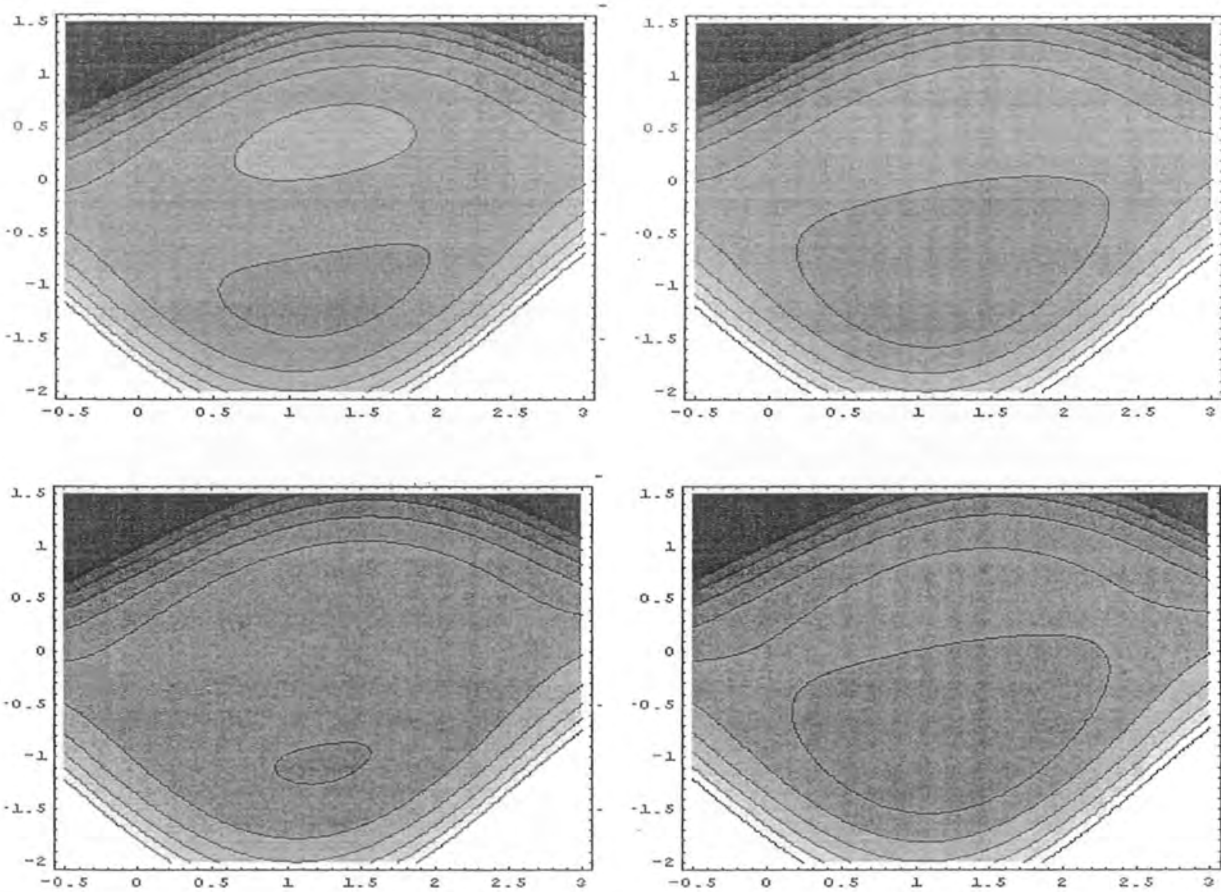
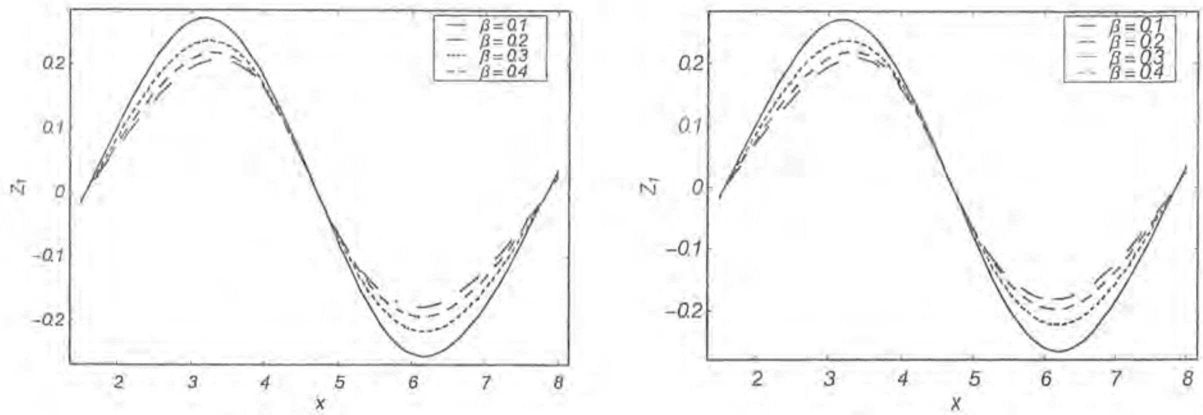
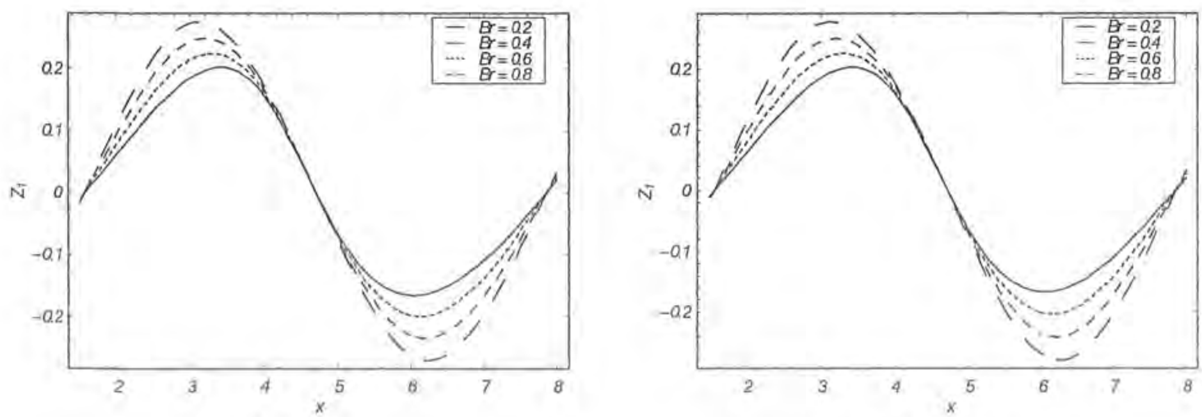


Fig. 3.2: Effect of β on the stream lines (left panels are for $\alpha = 0$,and right panels are for $\alpha = 0.2$), when $a(\beta = 0.4)$, $b(\beta = 0.08)$ and $d = 1.2$; $a = 0.7$; $b = 1.2$; $\phi = \frac{\pi}{6}$; $\eta = 1.4$; $M = 1.0$.



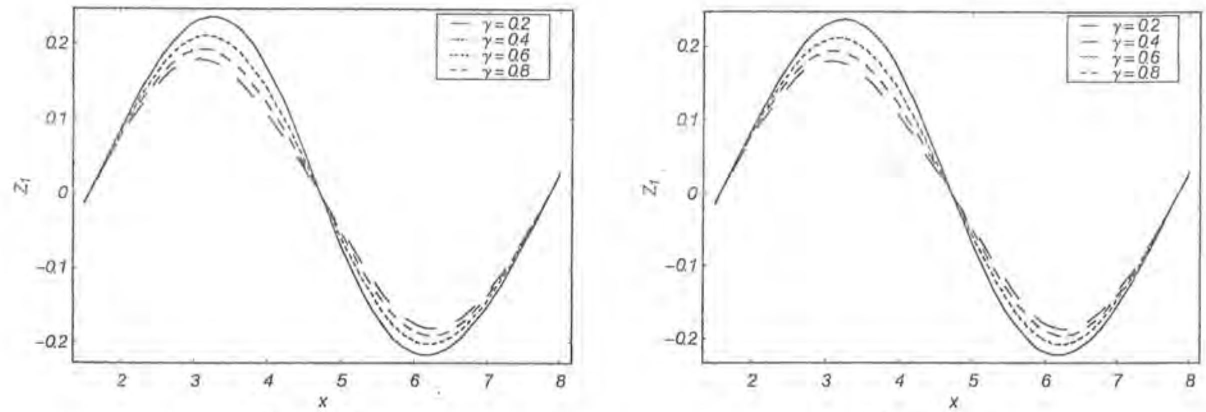
(a) $\alpha = 0$ (b) $\alpha = 0.4$

Fig. 3.3 : Effect of β on the heat transfer coefficient (Z_1) at the upper wall for $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $\gamma = 0.2$; $Br = 0.5$; and $M = 1$.



(a) $\alpha = 0$ (b) $\alpha = 0.4$

Fig. 3.4 : Effect of Br on the heat transfer coefficient (Z_1) at the upper wall for $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $\gamma = 0.2$; $\beta = 0.2$; and $M = 1$.



(a) $\alpha = 0$ (b) $\alpha = 0.4$

Fig. 3.5 : Effect of γ on the heat transfer coefficient (Z_1) at the upper wall when $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $Br = 0.5$; $\beta = 0.2$;and $M = 1$.

Appendix

Here, we present the involved values in solution expressions.

$A_1 = h_1 + h_2$, $A_2 = h_1 - h_2$, $A_3 = 1 + M\beta$, $A_4 = -1 + M\beta$, $A_5 = e^{h_2 M}(2 - MA_2A_4) + e^{h_1 M}(-2 + MA_2A_3)$, $A_6 = e^{h_2 M}A_4 + e^{h_1 M}A_3$, $A_7 = -2 + M(-5h_1 + h_2 + 2Mh_1h_2 + 2Mh_1^2 + 2M^2h_1h_2^2 - 2M^2h_2^3 - 2\beta + 4Mh_2(1 + Mh_2(-1 - Mh_1 + Mh_2))\beta + A_2M^2(1 + Mh_2(-1 + Mh_2))\beta^2)$, $A_8 = 2 + h_2M(-3 + M\beta) + Mh_1(3 - M\beta + 4Mh_2A_4)$, $A_9 = -2 - h_2M(3 + M\beta) + Mh_1(3 + M\beta + 4Mh_2A_3)$, $A_{10} = -2 + M(2\beta - 2M^2h_1^3A_3^2 + 2Mh_1^2A_3^2(1 + Mh_2) + h_2(-5 + M^2\beta^2) + h_1(1 + M(2h_2 - 4\beta - M(1 + 2Mh_2)\beta^2)))$, $A_{11} = -2 + M(h_2 + 2M^2h_1^3A_4 - h_1(-1 + 2M^2h_2^2)A_4 + 2Mh_1^2(A_3 + Mh_2A_4) + Mh_2(-\beta + 2h_2(1 + M(h_2 + \beta - \beta Mh_2))))$, $A_{12} = -2 + M(2\beta + h_2(-1 + M(2h_2A_3^2 + \beta(4 + M\beta) + 2Mh_2^2A_3^2)) - h_1(-5 + M(M\beta^2 + 2Mh_2^2A_3^2 + 2h_2(-1 + M^2\beta^2))))$, $A_{13} = -2 + M(2 + h_1^3M^2A_3 - h_1(-1 + 2M^2h_2^2)A_3 + 2Mh_1^2(1 + h_2M + M(-1 + h_2M)\beta) - h_2(1 + M(\beta 2h_2(-1 + M(h_2 + \beta + h_2M\beta)))))$, $A_{14} = 2 - M(A_1 + A_2^2M)(2 + MA_1) + A_2^2M^4(-1 + A_1M)\beta^2$, $A_{15} = 2 + M(5h_2 + h_1(-1 + 2M(A_1 + h_1A_2M)) - 2\beta - 4Mh_1(1 + h_1M(1 + h_1M - h_2M))\beta + A_2M^2(1 + 2h_1M(1 + h_1M))\beta^2)$, $A_{16} = 2 + M(h_2 - 2\beta + h_2M(-2h_2A_3^2 - \beta(4 + M\beta) - 2Mh_2^2A_3^2) + h_1(-5 + M(M\beta^2 + 2M(h_2 + Mh_2\beta)^2 + 2h_2(-1 + M^2\beta^2))))$, $A_{17} = -2 + M(-2A_1 + 3Mh_1^2 - 2Mh_1h_2 + 3Mh_2^2 - M^2h_1^3 + M^2h_1^2h_2 + M^2h_2^2h_1 - M^2h_2^3 - M^3A_2^2(1 + A_1M)\beta^2)$, $A_{18} = -2 + M(-5h_1 + h_2 + 2Mh_1h_2 + 2Mh_2^2 + 2M^2h_2^2h_1 - 2M^2h_2^3 - 2\beta + 4Mh_2(1 + Mh_2(-1 - Mh_1 + Mh_2))\beta + A_2M^2(1 + 2Mh_2(-1 + Mh_2)\beta^2$, $A_{19} = 2 + M(5h_2 - 2\beta - M^2\beta^2h_2 + 2M^2h_1^3A_3^2 - 2M(1 + Mh_2)h_1^2A_3^2 + h_1(-1 + M(-2h_2 + 4\beta + M(1 + 2h_2M)\beta^2)))$.

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