Peristaltic transport of nanofluid in a channel

QUAID-I-AZAM UNIVERSITY

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CERTIFICATE

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We accept this dissertation as conforming to the required standard

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2013

Dedicated

To

My Beloved Parents

And

My Younger Brother

Ashfaq Ahmed

Acknowledgement

*In the name of **Allah** the most beneficent and the most merciful, who granted me ability and potential to fulfill the requirements of my dissertation. I offer my humblest gratitude to the **Holy Prophet Muhammad (S. A. W)** who is forever a torch of guidance and knowledge for humanity.*

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Preface

There is continuous interest of the recent researchers in the peristaltic flows. Such interest is because of wide occurrence in urine transport from kidney to bladder, swallowing food through esophagus, ovum movement in the female fallopian tube, blood pumps in the heart lung machine etc. Latham [1] initially reported experimental work on peristalsis. Afterwards extensive research has been made by different researchers on the topic. For example, peristaltic motion of an electrically conducting fluid in an inclined asymmetric channel has been analyzed by Reddy [2]. Vasudev et al. [3] analyzed the peristaltic flow of magnetohydrodynamic (MHD) Newtonian fluid through porous medium in a vertical tube. Nadeem et al. [4] investigated the effects of heat and mass transfer in peristaltic flow of Williamson fluid. The peristaltic flow of an Eyring-Powel fluid in a vertical annulus has been also examined by Nadeem et al. [5]. Akbar et al. [6] explored heat transfer analysis in peristaltic transport of Williamson fluid. Endoscopic effects in peristaltic flow of an Eyring-Powel fluid are presented by Nadeem et al. [7].

In all the above mentioned studies the no-slip conditions are taken into account. However, there are situations where partial slip conditions becomes significant for example in polishing the valves of artificial heart. Thus Chu and Fang [8] introduced the slip flow in peristalsis. Ali et al. [9] discussed the slip effects on the peristaltic transport of MHD fluid with temperature dependent viscosity. Hayat and Hina [10] discussed the combined effects of slip and heat and mass transfer on peristaltic flow of Williamson fluid. Chaube et al. [11] analyzed the peristaltic transport of micropolar fluid in the presence of slip condition. Tripathi et al. [12] presented the effects of slip condition on peristaltic motion of fractional Burgers' fluid. Hayat et al. [13] investigated the slip effects on peristaltic transport of Maxwell fluid with heat and mass transfer. Mitra and Prasad [14] firstly discussed the effect of wall properties in peristalsis. Srinivas et al. [15] studied the effects of slip conditions, wall properties and heat transfer in peristaltic transport of MHD viscous fluid. Srinivas and Kothandapani [16] analyzed the influence of heat and mass transfer in MHD peristaltic flow through a porous space with compliant walls. Hayat et al. [17] discussed the effects of wall properties on the peristaltic flow of third grade fluid in a curved channel with heat and mass transfer. Hina et al. [18] analyzed the influence of compliant walls on peristaltic motion with heat/mass transfer and chemical reaction

Nanofluid means a liquid suspension that contains tiny particles having diameter less than 100nm. The nanoparticles used in nanofluids are typically made of metals (Al, Cu), oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN) or nanometals (Graphite, carbon nanotubes). Choi [19] discussed that addition of small amount of these nanoparticles increases the thermal conductivity of the base fluid. Buongiorno [20] discussed the nonhomogeneous equilibrium model which suggest that an increase in the thermal conductivity occurs due the presence of two main effects which are Brownian motion and thermophoretic diffusion of nanoparticles. Khan and Pop [21] discussed the boundary layer flow of nanofluid past a stretching sheet. Akbar et al. [22] discussed the slip effects on the peristaltic transport of nanofluid in an asymmetric channel. Akbar and Nadeem [23] analyzed the endoscopic effects on peristaltic flow of nanofluid. With this background, the present dissertation is arranged in following fashion.

First chapter is concerned with the basic definitions and equations. Chapter two describes a model to study the peristaltic transport of nanofluid in a channel with compliant walls. Transport equations involve the combined effects of Brownian motion and thermophoretic diffusion of nanoparticles. Chapter three deals with simultaneous effects of slip and Joule heating in MHD peristaltic transport of nanofluid. The series solution is obtained by homotopy analysis method (HAM), [24-29]. Numerical solution is obtained by the built in routine for solving nonlinear boundary value problems via shooting method through the command *NDSolve* of the software **Mathematica**. Influence of various parameters of interest is analyzed through graphical representations.

Contents

Chapter 1

Introduction

This chapter includes some standard definitions and basic equations for the better understanding of the research presented in the subsequent chapters.

1.1 Basic definition

1.1.1 Fluid

A fluid is defined as a material that will undergo sustained motion when shearing forces are applied, the motion continuing as long as the shearing forces are maintained.

1.1.2 Fluid mechanics

Fluid mechanics is the study of movement of fluids and the effects of forces on them. Fluid includes liquids and gases. Fluid mechanics can be divided into fluid statics and fluid dynamics.

Fluid static

The study of stationary fluid is called fluid statics.

Fluid dynamics

Fluid dynamics is the study of fluid in motion.

1.2 Physical properties of fluid

1.2.1 Pressure

The magnitude of force per unit area that one region of gas, liquid, or solid exerts on another. Pressure is usually measured in Pascal units, atmospheres, or pounds per square inch. Mathematically, one can write.

$$Pressure = \frac{magnitude\ of\ force}{Area} = \frac{F}{A}. \tag{1.1}$$

1.2.2 Density

The density of fluid is the mass per unit volume of a fluid. The density of the fluid is defined as the limiting value.

$$\rho = \lim_{\delta v \longrightarrow 0} \frac{\delta m}{\delta v}. \tag{1.2}$$

In the above equation δm is the mass of small volume δv surrounding the point consideration and ρ is the density.

1.2.3 Viscosity

It is the physical property which measures the resistance of a fluid when different forces act upon it. Mathematically, the ratio of shear stress to rate of shear strain is known as viscosity or dynamic viscosity.

$$dynamic\ viscosity = \mu = \frac{Shear\ stress}{Rate\ of\ shear\ strain}. \tag{1.3}$$

1.2.4 Kinematic viscosity

The ratio of dynamic viscosity to density of a fluid is known as kinematic viscosity. i.e.

$$\nu = \frac{\mu}{\rho} \tag{1.4}$$

in which ν indicates the kinematic viscosity.

1.3 Some concepts about peristaltic transport

1.3.1 Peristalsis

The word peristalsis is taken from a Greek word "Peristaltikos" which means clasping and compressing. Therefore, it is defined as a periodic wall oscillations generated by involuntary movement of muscle fibers.

1.3.2 Peristaltic transport

It is mechanism, for mixing and transporting fluids, which is generated by progressive waves of area contraction or expansion moving on the wall of the channel/tube.

1.3.3 Pumping

Pumping phenomenon is a specific feature of peristaltic transport. The operation of a pump of moving liquids from a lower pressure to higher pressure under certain conditions is called pumping.

Positive and negative pumping

The positive and negative pumping are dependent upon the mean flow rate θ. If θ is positive then it is called positive pumping and it corresponds to negative pumping for $(\theta < 0)$.

Adverse and favorable pressure gradient

Pressure gradient generated by peristaltic motion of the wall is adverse if pressure rise per wavelength (ΔP_λ) is positive and favorable when (ΔP_λ) is negative.

Peristaltic pumping

Here the flow rate is positive $(\theta > 0)$ and pressure rise is adverse $(\Delta P_\lambda > 0)$.

Augmented pumping.

It occurs when flow rate is positive $(\theta > 0)$ and pressure rise is favorable $(\Delta P_\lambda < 0)$.

Retrograde pumping

In this case flow rate is negative ($\theta < 0$) and the pressure rise is adverse ($\Delta P_\lambda > 0$).

Free pumping

Here the flow rate is positive ($\theta > 0$) but pressure rise is neither adverse nor favorable. In other words $\Delta P_\lambda = 0$.

Free pumping flux

The critical value of mean flow rate θ corresponding to $\Delta P_\lambda = 0$ is called free pumping flux.

1.4 No slip and slip conditions

When the fluid in contact with the solid surface takes the velocity of that surface then such type of condition is known as no slip condition. This is due to the viscous property of fluids. However, no slip condition is not adequate for fluid past a permeable walls, slotted plates, rough and coated surfaces, foam, polymer solution etc. In such fluids slip condition is appropriate.

1.5 Nanofluids

Nanofluids are new class of advanced heat-transfer fluids engineered by dispersing nanoparticles smaller than 100nm (nanometer) in diameter in conventional heat transfer fluids. These particles can be found in the metals such as (Al,Cu), oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN) or nonmetals (Graphite, carbon, nanotubes).

1.6 Dimensionless numbers

1.6.1 Brownian motion parameter

The random motion of nanoparticles within the base fluid is called Brownian motion, and results from continuous collisions between the nanoparticles and molecules of the base fluid. In mathematical form one can write

$$Nb = \frac{\tau D_B(C_1 - C_0)}{\nu}, \tag{1.5}$$

in which τ denotes the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid, D_B the Brownian motion coefficient, ν the kinematic viscosity, and C_1 and C_0 the concentration at the upper and lower wall respectively.

1.6.2 Thermophoresis parameter

Particles can diffuse under the effect of temperature gradient. This phenomenon is called thermophoresis, and is the "particle" equivalent of the well-known Soret effect for gaseous or liquid mixtures. Mathematically, it can be defined as

$$Nt = \frac{\tau D_T(T_1 - T_0)}{T_m \nu}, \tag{1.6}$$

in above expression D_T is thermophoretic diffusion coefficient, T_m the mean temperature, and T_1 and T_0 the temperature at the upper and lower wall respectively.

1.6.3 Reynolds number

The ratio of inertial forces to viscous forces is known as Reynolds number. Mathematically, it can be written as

$$\mathrm{Re} = \frac{inertial\ forces}{viscous\ forces} = \frac{c\rho d}{\mu}. \tag{1.7}$$

In above expression c is the velocity, d is the length scale, ρ is the density, and μ is the dynamic viscosity.

1.6.4 Prandtl number

It is defined as the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. Mathematically, one can express it as

$$\Pr = \frac{momentum\ diffusivity}{thermal\ diffusivity} = \frac{\mu c_p}{\kappa}, \tag{1.8}$$

where, c_p is specific heat and κ is thermal conductivity.

1.6.5 Eckert number

The ratio of kinetic energy to enthalpy of the flow is expressed by Eckert number, In mathematical form we can write

$$Ec = \frac{kinetic\ energy}{enthalpy} = \frac{c^2}{c_p(T_1 - T_0)}, \tag{1.9}$$

where c shows the velocity.

1.6.6 Hartman number

It is the ratio of magnetic body force to the viscous force. It is defined by

$$M = B_0 d\sqrt{\frac{\sigma}{\mu}}. \tag{1.10}$$

Here B_0 is applied magnetic field, d is the length scale and σ is the electrical conductivity.

1.6.7 Schmidt number

It is the ratio of diffusion of momentum to the diffusion of mass in a fluid

$$S_c = \frac{\nu}{D_B}, \tag{1.11}$$

where D_B is the diffusivity.

1.6.8 Amplitude ratio

Ratio of wave amplitude to the channel width is called amplitude ratio. i.e

$$\epsilon = \frac{a}{d}, \tag{1.12}$$

where a is wave amplitude.

1.6.9 Wave number

The ratio of channel width to the wavelength. Mathematically, we have

$$\delta = \frac{d}{\lambda}, \tag{1.13}$$

where λ is the wavelength.

1.7 Mechanisms of heat transfer

Heat transfer mechanism can be grouped into following categories:

1.7.1 Conduction

Regions with greater molecular kinetic energy will pass their thermal energy to regions with less molecular energy through direct collisions of the molecules, such a process is known as conduction.

1.7.2 Convection

This type of heat transfer takes place between bodies due to actual movement of the fluid molecules. Generally, transfer of heat taking place between a moving fluid and adjacent boundaries is via convection.

Natural convection

Convection is called natural convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to variation of temperature in the fluid.

Forced convection

Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump or the wind.

1.7.3 Radiation

A process in which heat is transferred by the electromagnetic waves is called radiation. This process plays vital role when heat is transferred through vacuum.

1.7.4 Joule heating

When current passes through a conductor, electrical energy is dissipated due to resistance of the conductor. Energy is dissipated in the form of heat. This phenomenon is called Joule heating. Also it is known as Ohmic heating.

1.8 Some basic equations

1.8.1 Continuity equation

This equation is derived from the law of conservation of mass. In mathematical form the continuity equation is expressed as follows

$$\frac{\partial \rho}{\partial t}+\nabla.(\rho \mathbf{V})=0, \tag{1.14}$$

where ρ is the fluid density, t the time and $\mathbf{V}$ the velocity field. For incompressible fluids, the continuity Eq. becomes

$$\nabla.\mathbf{V}=\mathbf{0}. \tag{1.15}$$

1.8.2 Equation of motion

We will consider the magnetohydrodynamic (MHD) case in chapter three. Thus law of conservation of momentum is expressed as

$$\rho\frac{d\mathbf{V}}{dt}=-\nabla p+\operatorname{div}\mathbf{T}+\mathbf{J}\times\mathbf{B}, \tag{1.16}$$

where $\mathbf{T}$ is the Cauchy stress tensor , p is the pressure, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic field and $\frac{d}{dt}$ is the material derivative defined by

$$\frac{d}{dt}=\frac{\partial}{\partial t}+(\mathbf{V}.\nabla). \tag{1.17}$$

1.8.3 Energy equation

The thermal energy equation for a nanofluid with viscous dissipation and Joule heating effects can be written as

$$\rho_f C_f\left(\frac{\partial}{\partial t}+\mathbf{V}.\nabla\right)T=\kappa\nabla^2T+\mathbf{T}.\mathbf{L}+\rho_pC_p\left[D_B\nabla C.\nabla T+\frac{D_T}{T_m}\nabla T.\nabla T\right]+\frac{1}{\sigma}\mathbf{J}.\mathbf{J}, \tag{1.18}$$

in which C_f the specific heat of basefluid, ρ_f the density of basefluid, ρ_p the density of nanoparticles, κ is the thermal conductivity, L the rate of strain tensor and T the temperature of fluid.

D_B the Brownian motion coefficient, D_T the thermophoretic diffusion coefficient, T_m the mean temperature and σ the electric conductivity of the fluid.

The expression of Cauchy stress tensor $\mathbf{T}$ in viscous incompressible fluid is written as

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1, \tag{1.19}$$

in which p is the pressure, $\mathbf{I}$ the identity tensor, μ the dynamic viscosity and $\mathbf{A}_1$ represents the first Rivlin-Ericksen tensor given by

$$\mathbf{A}_1 = (\mathrm{grad}\,\mathbf{V}) + (\mathrm{grad}\,\mathbf{V})^*, \tag{1.20}$$

where superscript $*$ indicates the matrix transpose, where

$$\mathrm{grad}\,\mathbf{V} = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.21}$$

using Eqs. (1.21) and (1.20), Eq. (1.19) becomes

$$\mathbf{T} = \begin{bmatrix} -p + 2\mu u_x & \mu(u_y + v_x) & 0 \\ \mu(u_y + v_x) & -p + 2\mu v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{1.22}$$

Further we define

$$\mathbf{T.L} = tr\,(\mathbf{TL}) = \mathbf{T}_{xx} + \mathbf{T}_{xy}\,(u_y + v_x) + \mathbf{T}_{yy}. \tag{1.23}$$

1.8.4 Concentration equation

The concentration equation for nanofluids is defined by

$$\left(\frac{\partial}{\partial t} + \mathbf{V}.\boldsymbol{\nabla}\right) C = D_B \boldsymbol{\nabla}^2 C + \frac{D_T}{T_m} \boldsymbol{\nabla}^2 T, \tag{1.24}$$

in which C is the concentration of nanoparticles, D_B is the Brownian motion parameter, D_T is the coefficient of diffusivity and T_m the mean temperature.

1.9 Magnetohydrodynamics

It is the branch in which we study the dynamics of a high conducting fluid where the magnetic field is present. Blood, liquid metals, salt water are some examples of MHD fluids.

1.9.1 Maxwell's equations for MHD

The Maxwell equations connect electric field $\mathbf{E}$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ the magnetic field. Here $\mathbf{B}_0$ and $\mathbf{b}$ are the applied and induced magnetic fields respectively. The Maxwell's equations are given by

$$\nabla.\mathbf{E} = \frac{\rho_e}{\epsilon_o} \quad \text{(Gauss's law for electricity)}, \tag{1.25}$$

$$\nabla.\mathbf{B} = 0 \quad \text{(Gauss's law for magnetism)}, \tag{1.26}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday's law for induction)}, \tag{1.27}$$

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}) \quad \text{(Ampere's law with Maxwell equation)}. \tag{1.28}$$

In the above equations ρ_e is the electric charge density, ϵ_0 is the permitivity of free space, μ_0 is the permeability of free space, and the current density $\mathbf{J}$ is given by

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad \text{(Ohm's law)}, \tag{1.29}$$

where σ is the electrical conductivity and $\mathbf{V}$ is the velocity. In the absence of electrical field $\mathbf{E}$ Eq. (1.29) yields

$$\mathbf{J} = \sigma(\mathbf{V} \times \mathbf{B}), \tag{1.30}$$

the magnetic Reynolds niumber is chosen small enough so that induced magnetic fields effects are ignored. Hence Lorentz force yields

$$\mathbf{J} \times \mathbf{B} = \sigma B_0^2 \mathbf{V}. \tag{1.31}$$

where $\mathbf{B}_0$ is the applied magnetic field.

1.10 Solutions methods

1.10.1 Homotopy analysis method (HAM)

The homotopy analysis method (HAM) is based on the concept of homotopy. A homotopy describes a continuous variation in mathematics. "A homotopy between two continuous functions $f(x)$ and $g(x)$ from a topological space X to topological space Y is formally defined to be continuous function H : $X \times [0, 1] \to Y$ from the product of the space X with unit interval [0 1] to Y such that, if $x \in X$ then $H(x, 0) = f(x)$ and $H(x, 1) = g(x)$". consider a non-linear differential equation

$$\mathcal{N}[u(x)] = 0, \tag{1.32}$$

where $\mathcal{N}$ is non-linear operator and $u(x)$ is an unknown function, first of all we construct a zeroth-order deformation equation

$$(1-q)\mathcal{L}[\phi(x;q) - u_0(x)] = q\hbar H(x)\mathcal{N}[\phi(x;q)], \tag{1.33}$$

where $u_0(x)$ is an initial guess, $\mathcal{L}$ is an auxiliary linear operator , $\hbar$ is the non-zero auxiliary parameter, $H(x)$ is auxiliary function and $\phi(x;q)$ is an unknown function of x and q. We have $\phi(x;0) = u_0(x)$ and $\phi(x;1) = u(x)$ when $q = 0$ and $q = 1$ respectively. Now by Lio [24] we expand the function $\phi(x;q)$ in a taylor series with respect to embedding parameter q gives as

$$\phi(x;q) = u_0(x) + \sum_{m=1}^{\infty} u_m(x)q^m, \tag{1.34}$$

where

$$u_m(x) = \frac{1}{m!}\frac{\partial^m \phi(y;q)}{\partial q^m}\bigg|_{q=0}. \tag{1.35}$$

If the series solution (1.34) converges at $q = 1$, then HAM series solution is

$$u(x) = u_0(x) + \sum_{m=1}^{\infty} u_m(x). \tag{1.36}$$

Differentiating equation (1.33) m-times with respect to q, dividing them by $m!$ and finally setting $q = 0$, we get a following mth-order deformation equation

$$\mathcal{L}\left[u_m(x) - \chi_m u_{m-1}(x)\right] = \hbar H(x)\mathcal{R}_m(x), \tag{1.37}$$

where

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \tag{1.38}$$

$$\mathcal{R}_m(x) = \frac{1}{(m-1)!}\frac{\partial^{m-1}\mathcal{N}[\phi(x;q)]}{\partial q^{m-1}}\bigg|_{q=0}. \tag{1.39}$$

HAM contains an auxiliary parameter $\hbar$ and auxiliary function $H(x)$ helps us to control the convergence of the series solution.

1.10.2 Numerical method

The numerical solution is computed by built in routine for solving nonlinear boundary value problems via shooting method through the command *NDSolve* of the software **Mathematica**.

Chapter 2

Peristaltic transport of nanofluid in a channel with wall properties

2.1 Introduction

This article discusses the peristaltic transport of nanofluid in a symmetric channel. The channel walls are compliant. Brownian motion and thermophoresis effects are taken into account. Mathematical model is carried out using long wavelength and low Reynolds number assumptions. The coupled boundary value problem is solved by homotopy analysis method (HAM) and shooting technique. The graphs are sketched for different parameters appearing in the solution expressions. The contents of this chapter provide a detailed review of a paper by Mustafa et al. [29].

2.2 Physical model

We consider the two-dimensional flow of an incompressible viscous nanofluid in a symmetric channel of width $2d_1$. The flow is caused by sinusoidal waves propagating along the compliant walls of channel. Here, u and v denote the components of velocity in the axial x and transverse y directions respectively.

The channel walls can be expressed as follows:

$$y = \eta(x,t) = +\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right] \qquad \text{at} \qquad \text{upper wall,} \tag{2.1}$$

$$y = \eta(x,t) = -\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right] \qquad \text{at} \qquad \text{lower wall.} \tag{2.2}$$

Here η is the wall displacement, c is the wave speed, λ and a are the wavelength and amplitude respectively.

2.3 Problem formulation

The velocity profile **V** for the two-dimensional flow is given by

$$\mathbf{V} = (u(x,y,t), v(x,y,t), 0). \tag{2.3}$$

The continuity (1.15), momentum (1.16), energy (1.18) and concentration (1.24) equations can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.4}$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]u = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right], \tag{2.5}$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]v = -\frac{1}{\rho_f}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right], \tag{2.6}$$

$$\begin{aligned}\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]T &= \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \frac{\nu}{c_f}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \\ &\quad + \tau D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \tau\frac{D_T}{T_m}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right],\end{aligned} \tag{2.7}$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]C = D_B\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right] + \frac{D_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{2.8}$$

The corresponding boundary conditions for the flow are given by

$$u=0,\quad T=\begin{Bmatrix}T_1\\T_0\end{Bmatrix},\quad C=\begin{Bmatrix}C_1\\C_0\end{Bmatrix}\quad \text{at}\quad (y=\pm\eta)\,, \tag{2.9}$$

$$\frac{\partial}{\partial x}L(\eta)=\frac{\partial p}{\partial x}=\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\rho_f\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right\}\quad \text{at}\quad (y=\pm\eta)\,, \tag{2.10}$$

where an operator L is used to represent the motion of compliant walls with viscous damping forces as follows:

$$L=-\tau_1\frac{\partial^2}{\partial x^2}+m_1\frac{\partial^2}{\partial t^2}+d\frac{\partial}{\partial t}.$$

In the above equation ν is the kinematic viscosity, p is the pressure, α is the thermal diffusivity, D_B and D_T are the Brownian motion parameter and thermophoretic diffusion parameter respectively, ρ_f the density of nanofluid, τ_1 the elastic tension, d the coefficient of viscous damping, m_1 the mass per unit area, T_m the mean temperature, T_1 and C_1 and T_0 and C_0 are the temperature and mass concentration at the upper and lower walls of channel respectively.

Defining

$$\begin{aligned} u^* &= \frac{u}{c},\quad v^*=\frac{v}{c}\quad ,x^*=\frac{x}{\lambda}\quad ,y^*=\frac{y}{d_1},\quad t^*=\frac{ct}{\lambda},\\ \eta^* &= \frac{\eta}{d_1},\quad p^*=\frac{d_1^2 p}{c\lambda\mu},\quad \theta=\frac{T-T_0}{T_1-T_0},\quad \phi=\frac{C-C_0}{C_1-C_0}\end{aligned}$$

equations (2.1) – (2.8) after removing the asterisks become

$$\operatorname{Re}\left(\delta\frac{\partial u}{\partial t}+u\delta\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2.11}$$

$$\operatorname{Re}\delta\left(\delta\frac{\partial v}{\partial t}+u\delta\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{2.12}$$

$$\begin{aligned}\operatorname{Re}\left(\delta\frac{\partial \theta}{\partial t}+u\delta\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right)&=\frac{1}{\Pr}\left(\delta^2\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+Ec\left[4\left(\delta\frac{\partial u}{\partial x}\right)^2+\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]\\ &+Nb\left(\delta^2\frac{\partial\phi}{\partial x}\frac{\partial\theta}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\theta}{\partial y}\right)+Nt\left[\left(\delta\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial x}\right)^2\right],\end{aligned} \tag{2.13}$$

$$\operatorname{Re}Sc\left(\delta\frac{\partial \phi}{\partial t}+u\delta\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}\right)=\left(\delta^2\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)+\frac{Nt}{Nb}\left(\delta^2\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right). \tag{2.14}$$

The boundary conditions are given by

$$u = 0, \quad \theta = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\}, \quad \phi = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\}, \quad \text{at} \quad y = \pm\eta \tag{2.15}$$

$$\left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t}\right] \eta = \left(\delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \mathrm{Re}\left(\delta \frac{\partial u}{\partial t} + \delta u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) \quad \text{at} \quad y = \pm\eta. \tag{2.16}$$

The dimensionless form of η is

$$\eta(x, t) = [1 + \epsilon \sin 2\pi\,(x - t)]. \tag{2.17}$$

If $\psi(x, y, t)$ is the stream function then the above non-dimensional systems (2.11) – (2.16) become

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\delta \frac{\partial \psi}{\partial x}, \tag{2.18}$$

$$\mathrm{Re}\left(\delta \frac{\partial^2 \psi}{\partial t \partial y} + \delta \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \delta \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2}\right) = -\frac{\partial p}{\partial x} + \left(\delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right), \tag{2.19}$$

$$\mathrm{Re}\,\delta\left(-\delta^2 \frac{\partial^2 \psi}{\partial t \partial x} - \delta^2 \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} + \delta^2 \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y \partial x}\right) = -\frac{\partial p}{\partial y} - \delta\left(\delta^3 \frac{\partial^3 \psi}{\partial x^3} + \delta \frac{\partial^3 \psi}{\partial y^2 \partial x}\right), \tag{2.20}$$

$$\begin{aligned}\mathrm{Re}\left(\delta \frac{\partial \theta}{\partial t} + \delta \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \delta \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right) &= \frac{1}{\mathrm{Pr}}\left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + Ec\left[4\delta \frac{\partial^2 \psi}{\partial x \partial y} + \left(-\delta^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)\right] \\ &+ Nb\left(\delta^2 \frac{\partial \phi}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y}\right) + Nt\left[\left(\delta \frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial x}\right)^2\right],\end{aligned} \tag{2.21}$$

$$\mathrm{Re}\,Sc\left(\delta \frac{\partial \phi}{\partial t} + \delta \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \delta \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y}\right) = \left(\delta^2 \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + \frac{Nt}{Nb}\left(\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right), \tag{2.22}$$

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{at} \quad y = \pm\eta, \tag{2.23}$$

$$\begin{aligned}&\left[E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x \partial t^2} + E_3 \frac{\partial^2}{\partial x \partial t}\right] \eta = \left(\delta^2 \frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3}\right) - \\ &\mathrm{Re}\left(\delta \frac{\partial^2 \psi}{\partial t \partial y} + \delta \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \delta \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2}\right) \quad \text{at} \quad y = \pm\eta.\end{aligned} \tag{2.24}$$

Note that the continuity equation (2.4) is satisfied identically, Here ϵ $(= a/d_1)$ is the amplitude ratio, δ $(= d_1/\lambda)$ the wave number, Nb $(= \tau D_B(C_1 - C_0)/\nu)$ the Brownian motion parameter, Nt $(= \tau D_T(T_1 - T_0)/T_m\nu)$ the thermophoresis parameter, Re $(= c\rho d_1/\mu)$ depicts the Reynolds number, $\Pr = \nu/\alpha$ the Prandtl number, Sc $(= \nu/D_B)$ the Schmidt number, Ec $(= c^2/c_f(T_1 - T_0))$ the Eckert number and E_1 $(= -\tau d_1^3/\lambda^3\mu c)$, E_2 $(= m_1 c d_1^3/\lambda^3\mu)$ and E_3 $(= d d_1^3/\lambda^2\mu)$ represent the non-dimensional elasticity parameters.

Now applying long wavelength and low Reynolds number approximations on Eqs. (2.19) – (2.22) and eliminating pressure gradient from the resulting momentum equations, we have

$$\frac{\partial^4\psi}{\partial y^4} = 0, \tag{2.25}$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} + Nb\frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y} + Nt\left(\frac{\partial\theta}{\partial y}\right)^2 + Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 = 0, \tag{2.26}$$

$$\frac{\partial^2\phi}{\partial y^2} + \frac{Nt}{Nb}\frac{\partial^2\theta}{\partial y^2} = 0. \tag{2.27}$$

The boundary conditions (2.9), (2.23) and (2.24) become

$$\frac{\partial\psi}{\partial y} = 0 \quad \text{at} \quad y = \pm\eta,$$

$$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial x\partial t}\right]\eta = \frac{\partial^3\psi}{\partial y^3} \quad \text{at} \quad y = \pm\eta, \tag{2.28}$$

$$\theta = \left\{\begin{array}{c} 1 \\ 0 \end{array}\right\}, \quad \phi = \left\{\begin{array}{c} 1 \\ 0 \end{array}\right\}, \quad \text{at} \quad y = \pm\eta. \tag{2.29}$$

Exact solution of Eq. (2.25) with boundary conditions (2.28) has the following form

$$\psi = \frac{4\pi^3\epsilon}{3}\left\{\frac{E_3}{2\pi}\sin 2\pi(x-t) - (E_1 + E_2)\cos 2\pi(x-t)\right\} y(y^2 - 3\eta^2). \tag{2.30}$$

Now putting ψ from Eq. (2.30) into Eqs. (2.26) and (2.27), the resulting equations. can be solved for temperature and concentration functions.

2.4 Methods of solution

2.4.1 Homotopy Solution

For the HAM solution of Eqs. (2.26) and (2.28) subject to the boundary conditions (2.29), we choose the following initial approximation for the functions θ and ϕ as

$$\theta_0 = \frac{1}{2}\left(1+\frac{y}{\eta}\right), \qquad \phi_0 = \frac{1}{2}\left(1+\frac{y}{\eta}\right). \tag{2.31}$$

We choose the auxiliary linear operators

$$\mathcal{L}_\theta(\theta) = \frac{d^2\theta}{dy^2}, \qquad \mathcal{L}_\phi(\phi) = \frac{d^2\phi}{dy^2}. \tag{2.32}$$

Zeroth order deformation problem

The problem at the zeroth order are given by

$$(1-q)\mathcal{L}_\theta[\Theta(y;q) - \theta_0(\eta)] = q\hbar_\theta \mathcal{N}_\theta[\Theta(y;q), \Phi(y;q)], \tag{2.33}$$

$$(1-q)\mathcal{L}_\phi[\Phi(y;q) - \phi_0(\eta)] = q\hbar_\phi \mathcal{N}_\phi[\Theta(y;q), \Phi(y;q)], \tag{2.34}$$

$$\Theta(y;q) = \begin{Bmatrix}1\\0\end{Bmatrix}, \quad \Phi(y;q) = \begin{Bmatrix}1\\0\end{Bmatrix}, \quad \text{at} \quad y = \pm\eta, \tag{2.35}$$

$$\mathcal{N}_\theta\left[\Theta(y;q), \Phi(y;q)\right] = \frac{1}{\Pr}\frac{\partial^2\Theta}{\partial y^2} + Nb\frac{\partial\Theta}{\partial y}\frac{\partial\Phi}{\partial y} + Nt\left(\frac{\partial\Theta}{\partial y}\right)^2 + Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2, \tag{2.36}$$

$$\mathcal{N}_\phi[\Theta(y;q), \Phi(y;q)] = \frac{\partial^2\Phi}{\partial y^2} + \frac{Nt}{Nb}\frac{\partial^2\Theta}{\partial y^2}. \tag{2.37}$$

Here $q \in [0,1]$ is the embedding parameter, $\hbar_\theta$ and $\hbar_\phi$ the non-zero auxiliary parameters. For $q=0$ and $q=1$, we respectively have

$$\Theta(y;0) = \theta_0(y), \quad \Theta(y;1) = \theta(y), \qquad \Phi(y;0) = \phi_0(y), \quad \Phi(y;1) = \phi(y). \tag{2.38}$$

As q increases from 0 to 1, $\Theta(y;q)$, $\Phi(y;q)$ vary from the initial guesses $\theta_0(y)$, $\phi_0(y)$ to the exact solution $\theta(y)$, $\phi(y)$. By Taylor's theorem and Eq. (2.38), one can write

$$\theta(y;q) = \theta_0(y) + \sum_{m=1}^{\infty} \theta_m(y) q^m, \qquad \phi(y;q) = \phi_0(y) + \sum_{m=1}^{\infty} \phi_m(y) q^m, \tag{2.39}$$

where

$$\theta_m(y) = \frac{1}{m!} \frac{\partial^m \Theta(y;q)}{\partial q^m}\bigg|_{q=0}, \qquad \phi_m(y) = \frac{1}{m!} \frac{\partial^m \Phi(y;q)}{\partial q^m}\bigg|_{q=0}. \tag{2.40}$$

The convergence of two series in Eq. (2.39) depends on the auxiliary parameter $\hbar$. Assume that $\hbar$ is chosen in such a way that the two series in Eq. (2.39) are convergent at $q = 1$ then due to Eq. (2.39), we get

$$\theta(y) = \theta_0(y) + \sum_{m=1}^{\infty} \theta_m(y), \qquad \phi(y) = \phi_0(y) + \sum_{m=1}^{\infty} \phi_m(y). \tag{2.41}$$

m-th order deformation equation

Differentiating the zeroth order deformation Eqs. (2.33) and (2.34) with respect to embedding parameter m-times, dividing by $m!$ and finally setting $q = 0$, we get the m^{th} order deformation problem as follows:

$$\mathcal{L}_\theta \left[\theta_m(y) - \chi_m \theta_{m-1}(y)\right] = \hbar_\theta \mathcal{R}_m^\theta(y), \tag{2.42}$$

$$\mathcal{L}_\phi \left[\phi_m(y) - \chi_m \phi_{m-1}(y)\right] = \hbar_\phi \mathcal{R}_m^\phi(y), \tag{2.43}$$

$$\theta_m(y) = 0, \quad \phi_m(y) = 0, \quad \text{at } y = \pm\eta, \tag{2.44}$$

$$\begin{aligned} \mathcal{R}_m^\theta(y) &= \frac{\partial^2 \theta_{m-1}}{\partial y^2} + \Pr \sum_{i=0}^{m-1} \left[Nb \frac{\partial \theta_{m-i-1}}{\partial y} \frac{\partial \phi_i}{\partial y} + Nt \frac{\partial \theta_{m-i-1}}{\partial y} \frac{\partial \theta_i}{\partial y} \right] \\ & + \Pr Ec \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 (1 - \chi_m), \end{aligned} \tag{2.45}$$

$$\mathcal{R}_m^\varphi(y) = \frac{\partial^2 \phi_{m-1}}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta_{m-1}}{\partial y^2}, \tag{2.46}$$

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \tag{2.47}$$

2.4.2 Convergence of homotopy solutions

Homotopy solutions for the resulting boundary value problems for θ and ϕ are obtained. The auxiliary parameters $\hbar_\theta$, $\hbar_\phi$ in the homotopy solutions are found to be; $-1.4 \leq \hbar_\theta \leq -0.1$ and $-2.1 \leq \hbar_\phi \leq -0.9$ (see Fig 2.1).

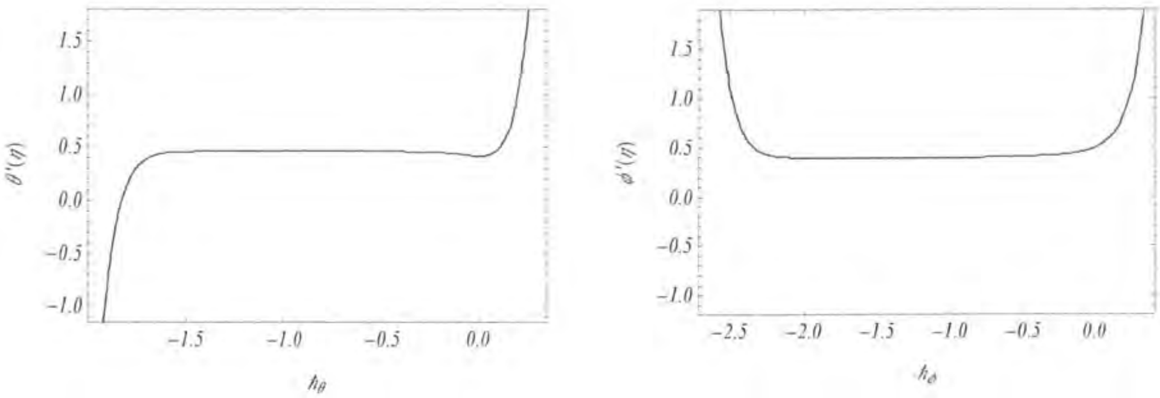


Fig **2.1:** $\hbar$-curves for the functions θ and ϕ

2.4.3 Numerical solution

A numerical solution for resulting problem is found through built-in function for BVP via shooting method through command $NDSolve$ of the software **Mathematica**. Fig. 2.2 indicates very good agreement of HAM and numerical solutions

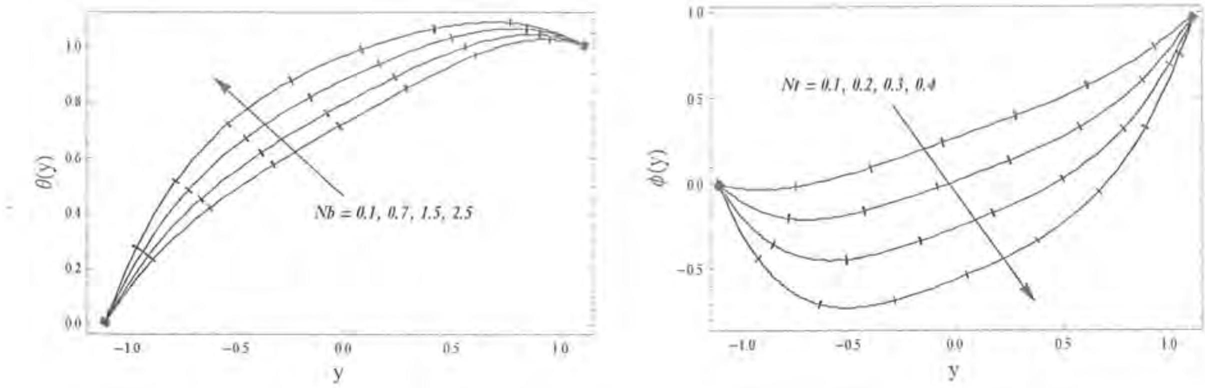


Fig 2.2: Comparison of HAM and numerical solutions. Here solid lines show the HAM solutions.

2.5 Results and discussion

The purpose of this section is to describe the influence of pertinent parameters on temperature θ, concentration ϕ and heat transfer coefficient Z.

The temperature distribution for various embedded parameters is examined in the Figs. 2.3 – 2.6. In Fig. 2.3 we examine the effects of Brownian motion parameter Nb and thermophoresis parameter Nt. We can see that by increasing the value of Nb and Nt the temperature increases. As the Brownian motion and thermophoresis effects intensify this correspond to the effective movement of nanoparticles from the wall to the fluid which result an increase in temperature. Fig. 2.4 shows that for increasing values of Pr the temperature increases. Similar behavior is noticed for increasing values of Ec (Fig. 2.5). Fig. 2.6 depicts that the temperature increases with the increase in E_1 and E_2. However it decreases when we increase E_3.

Figs. 2.7 and 2.8 represent the behavior of Brownian motion parameter Nb and elasticity parameters E_1, E_2 and E_3 on the concentration respectively. It is observed from Fig. 2.7 that the concentration decreases when Brownian motion Nb increases. Fig. 2.8 shows that concentration decreases with an increase in E_3. However it increases for an increase in E_1 and E_2.

Figs. 2.9 – 2.12 are prepared in order to study the role of different parameters on the heat transfer coefficient. Figs. 2.9 and 2.10 shows that the heat transfer coefficient decreases when Pr, Nb and Nt increase. In this case we neglect the viscous dissipation effect ($Ec = 0$). On the other side if we consider the viscous dissipation effect ($Ec = 1$) as shown in Figs 2.11 and 2.12 then we can see the heat transfer coefficient increases by increasing Pr, Nb and Nt.

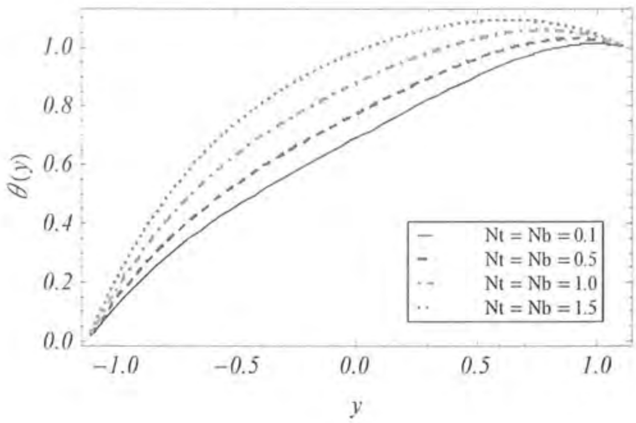


Fig. 2.3. Effect of Nb, Nt on θ.

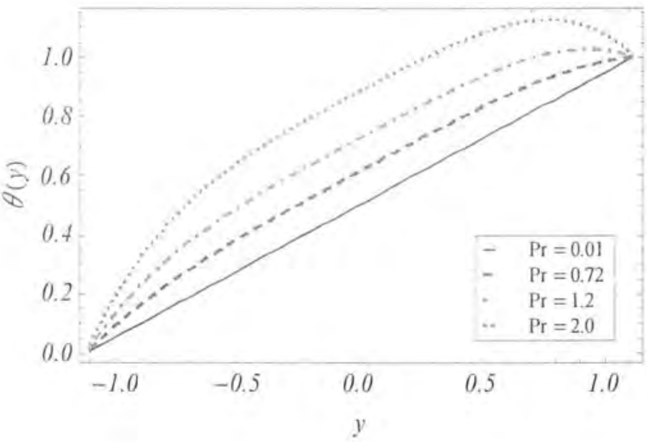


Fig. 2.4. Effect of Pr on θ.

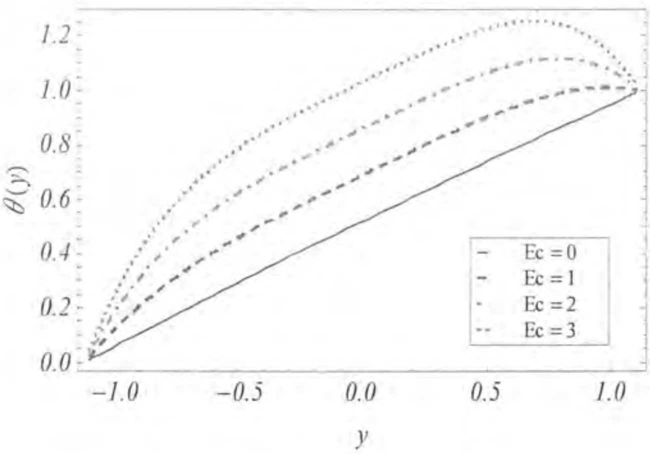


Fig. 2.5. Effect of Ec on θ.

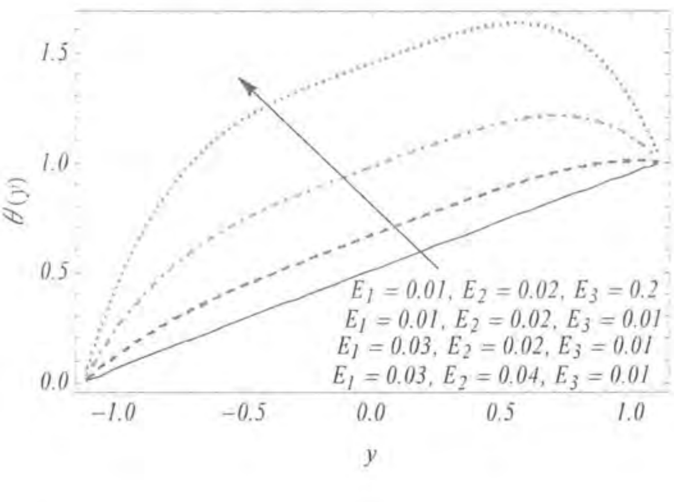


Fig. 2.6. Effect of parameters of wall properties on θ.

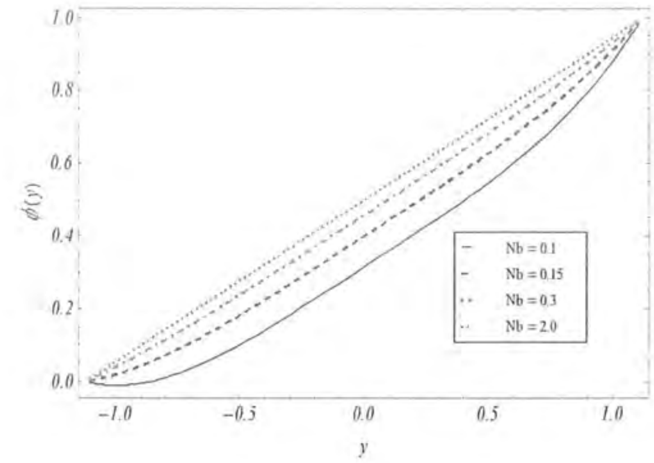


Fig. 2.7. Effect of Nb on ϕ.

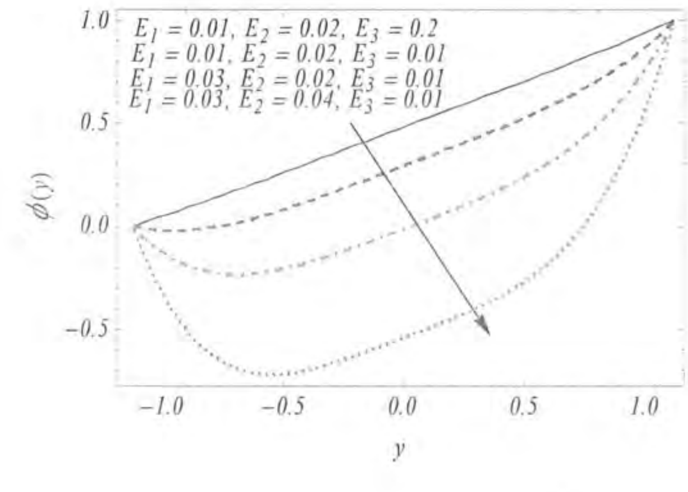


Fig. 2.8. Effect of parameters of wall properties on ϕ.

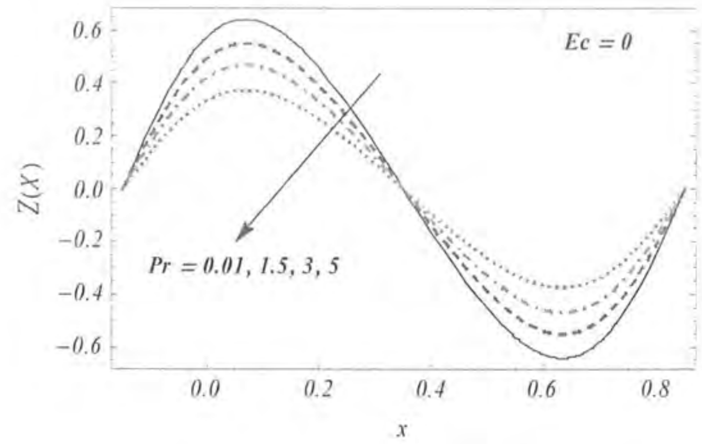


Fig. 2.9. Effect of Pr on Z.

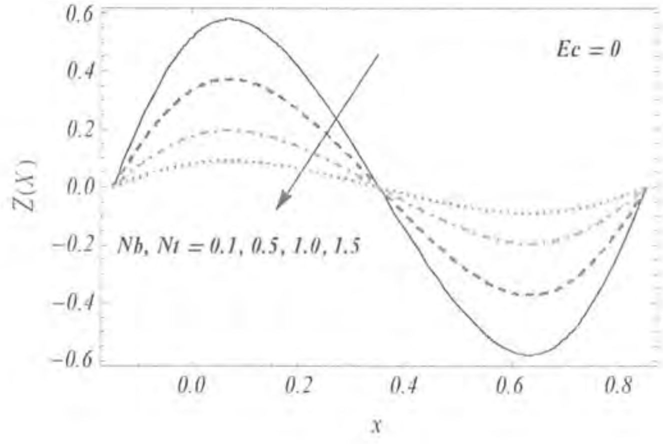


Fig. 2.10. Effect of Nb, Nt on Z.

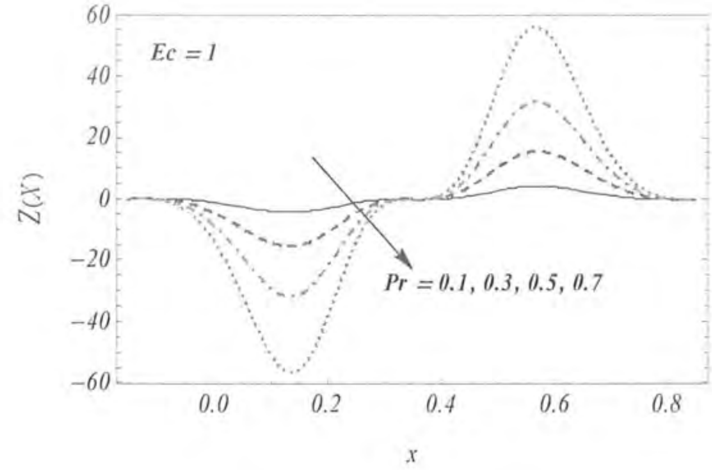


Fig. 2.11. Effect of Pr on Z.

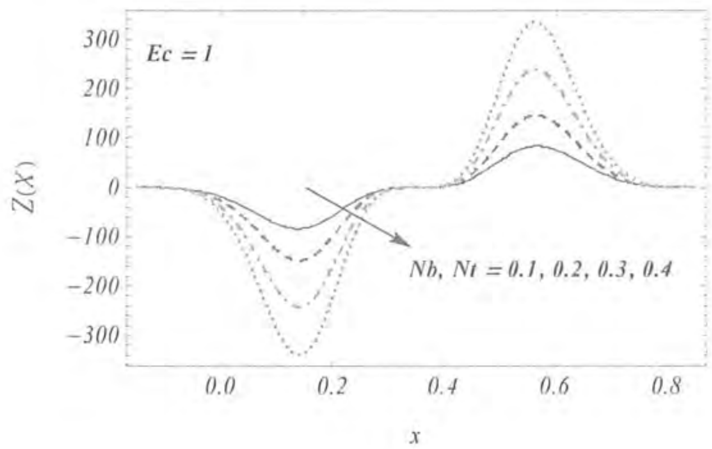


Fig. 2.12. Effect of Nb, Nt on Z.

Chapter 3

Simultaneous effects of slip and Joule heating on MHD peristaltic motion of nanofluid

3.1 Introduction

This chapter explores the effects of MHD on the peristaltic motion of an incompressible nanofluid in a channel with wall properties. Slip and joule heating effects are also taken into account. Mathematical model is formulated using long wavelength and low Reynolds number assumptions. Both numerical and analytic solutions are constructed and compared. The effects of various embedded parameters on the velocity profile, temperature, concentration and heat transfer coefficient have been pointed out.

3.2 Mathematical formulation

Consider the peristaltic flow of an incompressible nanofluid in a channel of uniform thickness $2d_1$. The coordinate axes are chosen in such a way that the $x-axis$ is along the length of the channel whereas the $y-axis$ is normal to the $x-axis$. A uniform magnetic field $\mathbf{B}_0$ is applied in the $y-direction$. Flow is generated due to propagation of peristaltic waves along the channel walls. Mathematically, the wall shapes can be defined as

$$y=\eta(x,t)=+\left[d_1+a\sin\frac{2\pi}{\lambda}(x-ct)\right] \qquad \text{at} \quad \text{upper wall}, \tag{3.1}$$

$$y=\eta(x,t)=-\left[d_1+a\sin\frac{2\pi}{\lambda}(x-ct)\right] \qquad \text{at} \quad \text{lower wall}, \tag{3.2}$$

where c is the wave speed, a and λ are the wave amplitude and wavelength respectively.

3.3 Problem formulation

The governing equations for the flow analysis under consideration are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{3.3}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho_f}u\ , \tag{3.4}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3.5}$$

$$\begin{aligned}\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}&=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\nu}{c_f}\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]\\&+\tau D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\tau\frac{D_T}{T_m}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\sigma B_o^2}{\rho_f c_f}u^2,\end{aligned} \tag{3.6}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{3.7}$$

The relevant boundary conditions are

$$u\pm\beta_1\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)=0,\quad T\pm\beta_2\frac{\partial T}{\partial y}=\begin{Bmatrix}T_1\\T_0\end{Bmatrix},\quad C\pm\beta_3\frac{\partial C}{\partial y}=\begin{Bmatrix}C_1\\C_0\end{Bmatrix}\text{ at } y=\pm\eta, \tag{3.8}$$

$$\left[-\tau_1\frac{\partial^3}{\partial x^3}+m_1\frac{\partial^3}{\partial x\partial t^2}+d\frac{\partial^2}{\partial t\partial x}\right]\eta=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho_f\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\sigma B_o^2 u\ \text{ at } y=\pm\eta. \tag{3.9}$$

in which u , v designate the velocities in the $x-$ and $y-$ directions respectively, σ the electric conductivity of the fluid, $\mathbf{B}_0$ the applied magnetic field and last term on the right-hand side of Eq. (3.6) represents Joule heating effect. Here β_1 denotes the velocity slip parameter, β_2 the thermal slip parameter and β_3 the concentration slip parameter.

Defining

$$\begin{aligned} u^* &= \frac{u}{c},\ v^* = \frac{v}{c},\ x^* = \frac{x}{\lambda},\ y^* = \frac{y}{d_1},\ \beta_i^* = \frac{\beta_i}{d_1}\ (i = 1-3), \\ t^* &= \frac{ct}{\lambda},\ \eta^* = \frac{\eta}{d_1},\ p^* = \frac{d_1^2 p}{c\lambda\mu},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \phi = \frac{C - C_0}{C_1 - C_0} \end{aligned}$$

the equations (3.1) – (3.9) after dropping asterisks yields

$$\mathrm{Re}\left(\delta\frac{\partial u}{\partial t} + u\delta\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - M^2 u, \tag{3.10}$$

$$\mathrm{Re}\,\delta\left(\delta\frac{\partial v}{\partial t} + u\delta\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{3.11}$$

$$\begin{aligned} \mathrm{Re}\left(\delta\frac{\partial \theta}{\partial t} + u\delta\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) &= \frac{1}{\mathrm{Pr}}\left(\delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + Ec\left[4\left(\delta\frac{\partial u}{\partial x}\right)^2 + \left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \\ &+ Nb\left(\delta^2\frac{\partial \phi}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y}\right) + Nt\left[\left(\delta\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right] + EcM^2u^2, \end{aligned} \tag{3.12}$$

$$\mathrm{Re}\,Sc\left(\delta\frac{\partial \phi}{\partial t} + u\delta\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y}\right) = \left(\delta^2\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + \frac{Nt}{Nb}\left(\delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right). \tag{3.13}$$

The boundary conditions become

$$u \pm \beta_1\left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = 0,\ \theta \pm \beta_2\frac{\partial \theta}{\partial y} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix},\ \phi \pm \beta_3\frac{\partial \phi}{\partial y} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ at } y = \pm\eta, \tag{3.14}$$

$$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x \partial t^2} + E_3\frac{\partial^2}{\partial x \partial t}\right]\eta = \left[\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \mathrm{Re}\left(\delta\frac{\partial u}{\partial t} + u\delta\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - M^2 u \text{ at } y = \pm\eta. \tag{3.15}$$

The dimensionless form of η is

$$\eta(x,t) = [1 + \epsilon\sin 2\pi(x - t)]. \tag{3.16}$$

If $\psi(x,y,t)$ is the stream function, then

$$u=\frac{\partial\psi}{\partial y},\ v=-\delta\frac{\partial\psi}{\partial x}. \tag{3.17}$$

Note that Eq. (3.3) is satisfied automatically. Making use of long wavelength and low Reynolds number approximations, Eqs. (3.10) – (3.15) become

$$\frac{\partial^4\psi}{\partial y^4}-M^2\frac{\partial^2\psi}{\partial y^2}=0, \tag{3.18}$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Nb\frac{\partial\theta}{\partial y}\frac{\partial\phi}{\partial y}+Nt\left(\frac{\partial\theta}{\partial y}\right)^2+Ec\left[\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+M^2\left(\frac{\partial\psi}{\partial y}\right)^2\right]=0, \tag{3.19}$$

$$\frac{\partial^2\phi}{\partial y^2}+\frac{Nt}{Nb}\frac{\partial^2\theta}{\partial y^2}=0. \tag{3.20}$$

The boundary conditions are

$$\begin{aligned}\frac{\partial\psi}{\partial y}\pm\beta_1\frac{\partial^2\psi}{\partial y^2} &= 0 \text{ at } y=\pm\eta,\\ \left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right]\eta &= \frac{\partial^3\psi}{\partial y^3}-M^2\frac{\partial\psi}{\partial y} \text{ at } y=\pm\eta,\end{aligned} \tag{3.21}$$

$$\theta\pm\beta_2\frac{\partial\theta}{\partial y}=\begin{Bmatrix}1\\0\end{Bmatrix},\ \phi\pm\beta_3\frac{\partial\phi}{\partial y}=\begin{Bmatrix}1\\0\end{Bmatrix} \text{ at } y=\pm\eta. \tag{3.22}$$

Here $\Pr\ (=\nu/\alpha)$ is the Prandtl number, $Nb\ (=\tau D_B(C_1-C_0)/\nu)$ the Brownian motion parameter, $Nt\ (=\tau D_T(T_1-T_0)/T_m\nu)$ the thermophoresis parameter, $M\ (=\sqrt{\frac{\sigma}{\mu}}B_0d_1)$ the Hartman number, $Ec\ \left(=c^2/c_f\,(T_1-T_0)\right)$ the Eckert number and $E_1\ \left(=-\tau d_1^3/\lambda^3\mu c\right)$, $E_2\ (=m_1cd_1^3/\lambda^3\mu)$ and $E_3\ \left(=dd_1^3/\lambda^2\mu\right)$ represent the non-dimensional elasticity parameters. Exact solution of Eq. (3.18) with boundary conditions (3.21) is

$$\psi=\frac{8\epsilon\pi^3[(E_1+E_2)\cos 2\pi(x-t)-\frac{E_3}{2\pi}\sin 2\pi(x-t)]}{M^2}\times\left[\frac{\sinh My}{M(\cos M\eta+M\beta_1\sinh M\eta)}-y\right]. \tag{3.23}$$

After substituting ψ from Eq. (3.23) into the Eqs. (3.19) and (3.20), the resulting equations can be solved for temperature and concentration functions.

3.4 Methods of solution

3.4.1 Homotopy solution

For the HAM solution of Eqs. (3.19) and (3.20) subject to boundary conditions (3.22), we choose the following initial approximation for the functions θ and ϕ as

$$\theta_0=\frac{1}{2}\left(1+\frac{y}{\eta+\beta_2}\right),\qquad \phi_0=\frac{1}{2}\left(1+\frac{y}{\eta+\beta_3}\right). \tag{3.24}$$

We choose the auxiliary linear operators

$$\mathcal{L}_\theta(\theta)=\frac{d^2\theta}{dy^2},\qquad \mathcal{L}_\phi(\phi)=\frac{d^2\phi}{dy^2}. \tag{3.25}$$

Zeroth order deformation problem

The problems at the zeroth order are given by

$$(1-q)\mathcal{L}_\theta[\Theta(y;q)-\theta_0(\eta)] = q\hbar_\theta\mathcal{N}_\theta[\Theta(y;q),\Phi(y;q)], \tag{3.26}$$

$$(1-q)\mathcal{L}_\phi[\Phi(y;q)-\phi_0(\eta)] = q\hbar_\phi\mathcal{N}_\phi[\Theta(y;q),\Phi(y;q)], \tag{3.27}$$

$$\Theta(y;q)\pm\beta_2\frac{\partial\Theta(y;q)}{\partial y}=\begin{Bmatrix}1\\0\end{Bmatrix},\quad \Phi(y;q)\pm\beta_3\frac{\partial\Phi(y;q)}{\partial y}=\begin{Bmatrix}1\\0\end{Bmatrix},\quad \text{at } y=\pm\eta, \tag{3.28}$$

$$\mathcal{N}_\theta\left[\Theta(y;q),\Phi(y;q)\right]=\frac{1}{\Pr}\frac{\partial^2\Theta}{\partial y^2}+Nb\frac{\partial\Theta}{\partial y}\frac{\partial\Phi}{\partial y}+Nt\left(\frac{\partial\Theta}{\partial y}\right)^2+Ec\left[\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+M^2\left(\frac{\partial\psi}{\partial y}\right)^2\right], \tag{3.29}$$

$$\mathcal{N}_\phi[\Theta(y;q),\Phi(y;q)]=\frac{\partial^2\Phi}{\partial y^2}+\frac{Nt}{Nb}\frac{\partial^2\Theta}{\partial y^2}. \tag{3.30}$$

Note that $q\in[0,1]$ is the embedding parameter and $\hbar_\theta$ and $\hbar_\phi$ the non-zero auxiliary parameters. For $q=0$ and $q=1$, we respectively have

$$\Theta(y;0)=\theta_0(y),\quad \Theta(y;1)=\theta(y),\qquad \Phi(y;0)=\phi_0(y),\quad \Phi(y;1)=\phi(y). \tag{3.31}$$

As q increases from 0 to 1, $\Theta(y;q),\Phi(y;q)$ varies from the initial guesses $\theta_0(y),\phi_0(y)$ to the exact solutions $\theta(y), \phi(y)$. By Taylor's theorem and Eq. (3.31), one can write

$$\theta(y;q) = \theta_0(y) + \sum_{m=1}^{\infty} \theta_m(y) q^m, \qquad \phi(y;q) = \phi_0(y) + \sum_{m=1}^{\infty} \phi_m(y) q^m, \tag{3.32}$$

where

$$\theta_m(y) = \frac{1}{m!} \frac{\partial^m \Theta(y;q)}{\partial q^m}\bigg|_{q=0}, \qquad \phi_m(y) = \frac{1}{m!} \frac{\partial^m \Phi(y;q)}{\partial q^m}\bigg|_{q=0}. \tag{3.33}$$

The convergence of the two series in Eq. (3.32) depends on the auxiliary parameter $\hbar$. Assume that $\hbar$ is chosen in such a way that the two series in Eq. (3.32) are convergent at $q = 1$ then due to Eq. (3.32), we get

$$\theta(y) = \theta_0(y) + \sum_{m=1}^{\infty} \theta_m(y), \qquad \phi(y) = \phi_0(y) + \sum_{m=1}^{\infty} \phi_m(y). \tag{3.34}$$

m-th order deformation problems

The resulting problems have are

$$\mathcal{L}_\theta \left[\theta_m(y) - \chi_m \theta_{m-1}(y)\right] = \hbar_\theta \mathcal{R}_m^\theta(y), \tag{3.35}$$

$$\mathcal{L}_\phi \left[\phi_m(y) - \chi_m \phi_{m-1}(y)\right] = \hbar_\phi \mathcal{R}_m^\phi(y), \tag{3.36}$$

$$\theta_m(y) \pm \beta_2 \frac{\partial \theta_m(y)}{\partial y} = 0, \quad \phi_m(y) \pm \beta_3 \frac{\partial \phi_m(y)}{\partial y} = 0, \quad \text{at } y = \pm\eta, \tag{3.37}$$

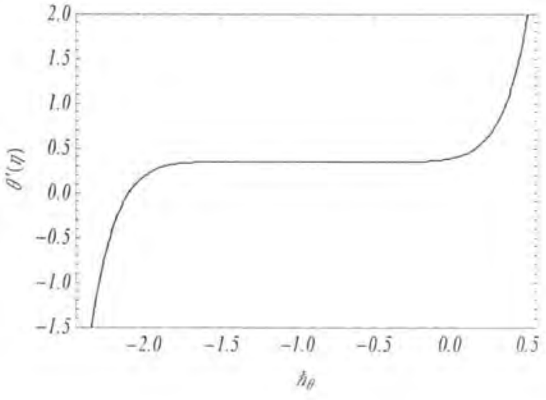
$$\begin{aligned} \mathcal{R}_m^\theta(y) &= \frac{\partial^2 \theta_{m-1}}{\partial y^2} + \Pr \sum_{i=0}^{m-1} \left[Nb \frac{\partial \theta_{m-i-1}}{\partial y} \frac{\partial \phi_i}{\partial y} + Nt \frac{\partial \theta_{m-i-1}}{\partial y} \frac{\partial \theta_i}{\partial y} \right] \\ &\quad + \Pr Ec \left[\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + M^2 \left(\frac{\partial \psi}{\partial y} \right)^2 \right] (1 - \chi_m), \end{aligned} \tag{3.38}$$

$$\mathcal{R}_m^\phi(y) = \frac{\partial^2 \phi_{m-1}}{\partial y^2} + \frac{Nt}{Nb} \frac{\partial^2 \theta_{m-1}}{\partial y^2}. \tag{3.39}$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{3.40}$$

3.4.2 Convergence of homotopy solution

Homotopy solutions for equations (3.19) and (3.20) and boundary condition (3.22) are obtained. The auxiliary parameters $\hbar_\theta$ and $\hbar_\phi$ in the homotopy solutions are determind; $-1.6 \leq \hbar_\theta \leq -0.1$ and $-1.4 \leq \hbar_\phi \leq -0.5$ (Fig. 3.1).



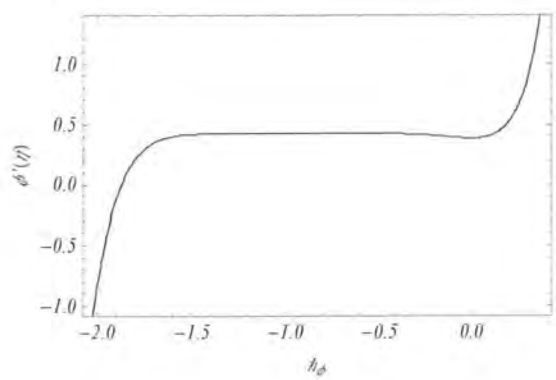


Fig 3.1: $\hbar$-curves for the θ and ϕ

3.4.3 Numerical method

The incoming problems (3.19) and (3.20) are also solved numerically by built in routine for solving nonlinear boundary value problem via shooting method through command $NDSolve$ of the software **Mathematica**.

3.4.4 Comparison of numerical and homotopy solutions

In this section, a comparative study is made between the numerical and HAM solutions. For such objective Table 3.1 is prepared. This table indicates very good agreement between the numerical and HAM solutions.

Parameters					HAM solutions		Numerical solutions	
Nb	Nt	M	β_2	β_3	$\theta_y\big\|_{y=\eta}$	$\phi_y\big\|_{y=\eta}$	$\theta_y\big\|_{y=\eta}$	$\phi_y\big\|_{y=\eta}$
0.10	0.10	0.10	0.10	0.10	-0.062291	0.883443	-0.062441	0.883759
0.20	0.10	0.10	0.10	0.10	-0.075279	0.653441	-0.075478	0.653759
0.10	**0.20**	0.10	0.10	0.10	-0.075279	1.382601	-0.075478	1.382931
0.10	**0.30**	0.10	0.10	0.10	-0.087831	1.906503	-0.088054	1.906800
0.10	0.10	**0.20**	0.10	0.10	-0.058954	0.880209	-0.059146	0.880463
0.10	0.10	**0.30**	0.10	0.10	-0.053386	0.874609	-0.053587	0.874904
0.10	0.10	0.10	**0.20**	0.10	-0.090731	0.880021	-0.090937	0.880251
0.10	0.10	0.10	**0.25**	0.10	-0.103431	0.878549	-0.103661	0.878753
0.10	0.10	0.10	0.10	**0.20**	-0.061235	0.852205	-0.061461	0.852454
0.10	0.10	0.10	0.10	**0.25**	-0.060865	0.838231	-0.061023	0.838494

Table 3.1: Comparison of HAM and numerical solutions for different values of parameters.

3.5 Graphical results and discussion

The aim of this section is to describe the variation of embedded parameters on the velocity profile, temperature, concentration and heat transfer coefficient.

The velocity profile for various parameters of interest are displayed in Figs. 3.2 – 3.5. In Fig. 3.2 we note that the velocity increases by increasing the value of velocity slip parameter β_1. Effect of Hartman number M on the velocity distribution can be seen through Fig. 3.3. It reveals that the velocity decreases with an increase of M. Fig. 3.4 shows that velocity increases when amplitude ratio ϵ increases. In Fig. 3.5, the effect of wall parameters E_1, E_2, E_3 on the velocity is analyzed. This shows that velocity increases when E_1 and E_2 are increased and it decreases when E_3 is increased. Moreover, it is also observed that the velocity profile is parabolic for fixed values of parameters and its magnitude is maximum near the center of the channel.

Figs. 3.6 – 3.12 plot the temperature profiles for different values of parameters. Fig. 3.6 depicts the behavior of temperature for different values of Brownian motion parameter Nb and thermophoresis parameter Nt. It is observed that the temperature increases for increasing values of Nb and Nt. The Prandtl number Pr and Eckert number Ec effects on the temperature are captured in Figs. 3.7 and 3.8 respectively. A strong viscous dissipation effects correspond to an increase in the temperature when Pr and Ec are increased. The variation of elastic parameters E_1 , E_2 and E_3 are shown in Fig. 3.9. For increasing the value of E_1 and E_2 the temperature increases and it decreases when E_3 increases. By Fig. 3.10 we note that the temperature increases when there is an increase in the amplitude ratio ϵ. Fig. 3.11 elucidates that increasing the value of thermal slip parameter β_2 the temperature profile increases. Fig. 3.12 is prepared to examine the effects of temperature profile for different values of Hartman number M. It shows that the temperature decreases by increasing the value of M.

Figs. 3.13 – 3.17 are plotted to investigate the physical parameters on the concentration. From Fig. 3.13 we capture the effect of Brownian motion parameter Nb on concentration field. It is observed that the concentration decreases when Nb increases. Fig. 3.14 shows that concentration decreases with an increase in E_3 whereas it increases for an increase in E_1 and E_2. In Figs. 3.15 and 3.16 we observe that by increasing the value of concentration slip parameter β_3 and amplitude ratio ϵ the concentration decreases. Fig. 3.17 shows that with an increase of

Hartman number M the concentration field increases.

Figs. $3.18-3.23$ represent the behavior of involved parameters in heat transfer coefficient Z. As expected the heat transfer coefficient has oscillatory behavior. In Figs. 3.18 and 3.19 heat transfer coefficient is a decreasing function of Pr, Nb and Nt when the viscous dissipation is neglected. However in Figs. 3.20 and 3.21 for strong viscous dissipation effects the rate of heat transfer coefficient at the wall increases with the increase in Pr, Nb and Nt. Fig. 3.22 indicates that the heat transfer coefficient is an increasing function of amplitude raio ϵ. In Fig. 3.23 we can see that by increasing the value of Hartman number M, the heat transfer coefficient decreases.

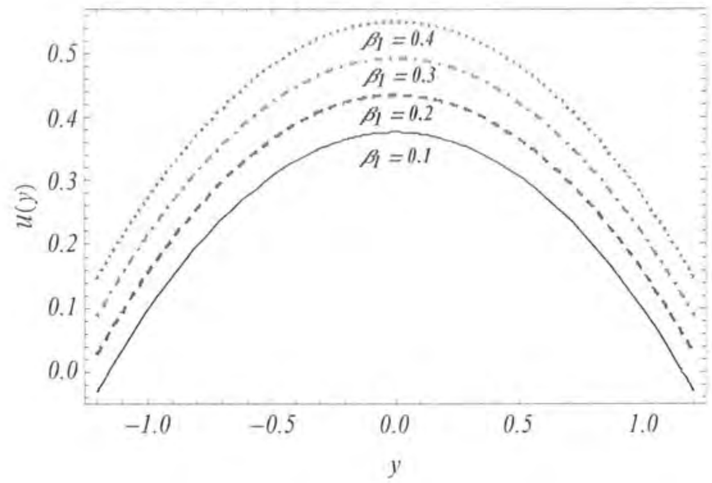


Fig. 3.2. Effect of $\beta_1$on u.

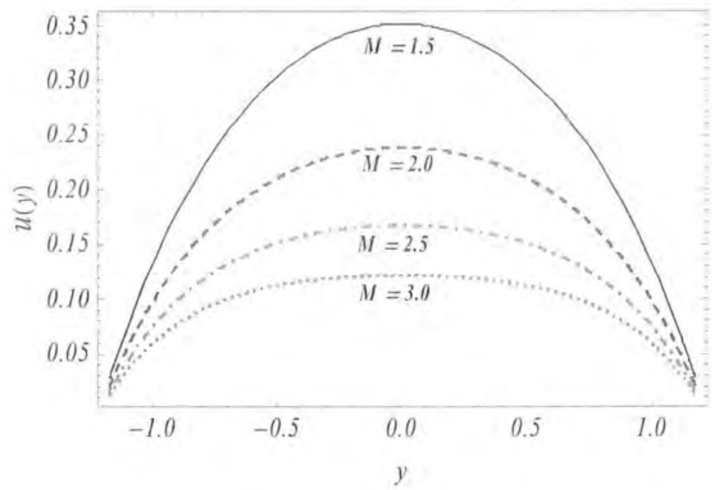


Fig. 3.3. Effect of M on u.

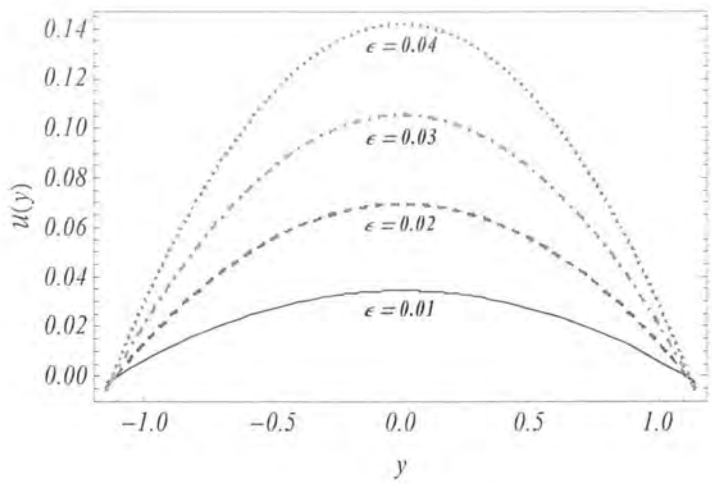


Fig. 3.4. Effect of ϵ on u.

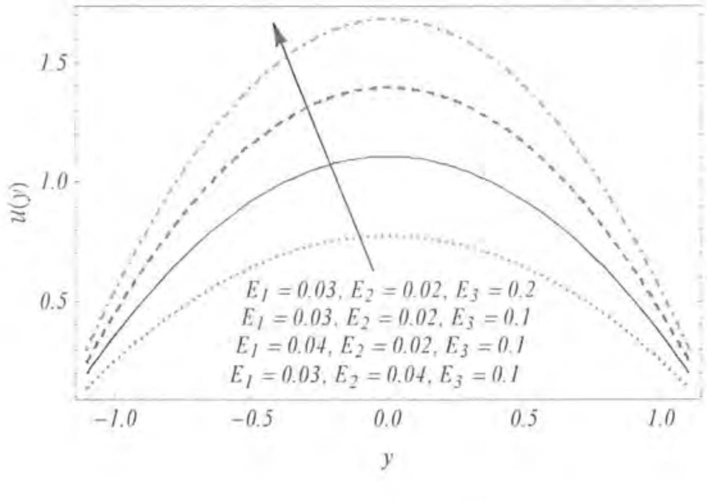


Fig. 3.5. Effect of parameters of wall properties on u.

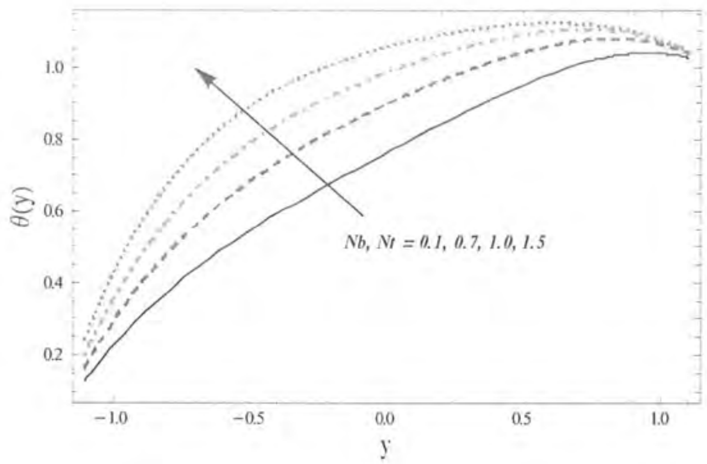


Fig. 3.6. Effect of Nb, Nt on θ

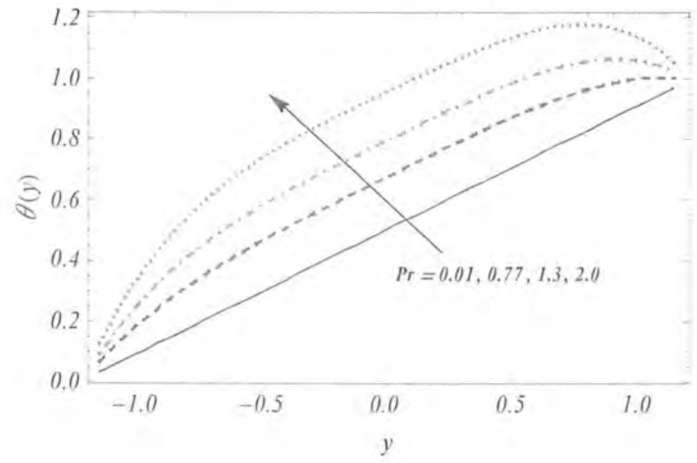


Fig. 3.7. Effect of Pr on θ.

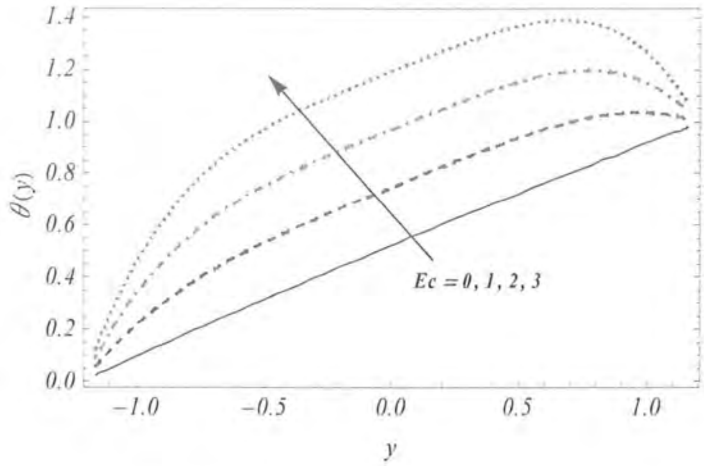


Fig. 3.8. Effect of Ec on θ.

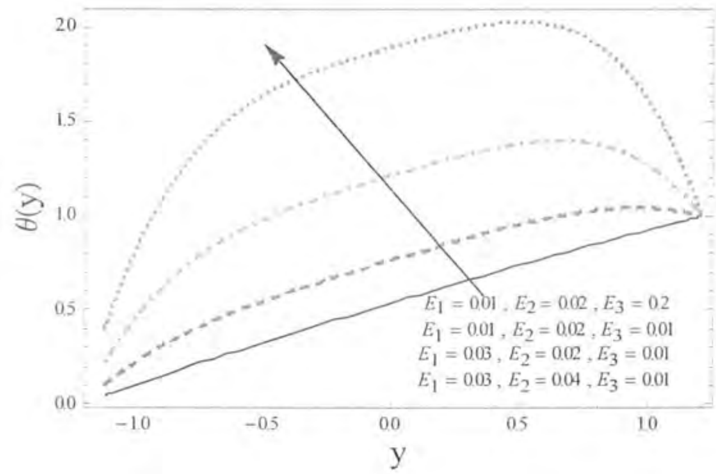


Fig. 3.9. Effect of parameters of wall properties on θ.

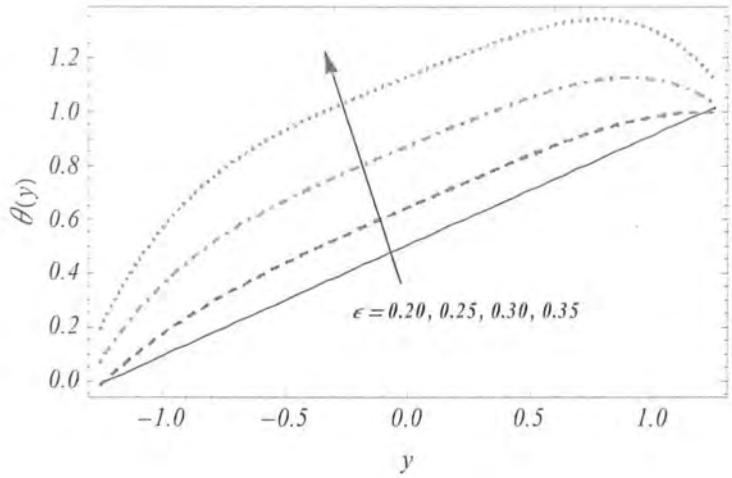


Fig. 3.10. Effect of ϵ on θ.

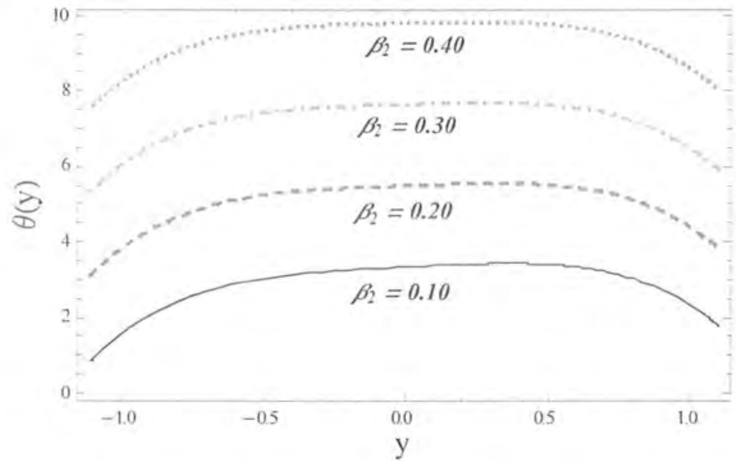


Fig. 3.11. Effect of β_2 on θ.

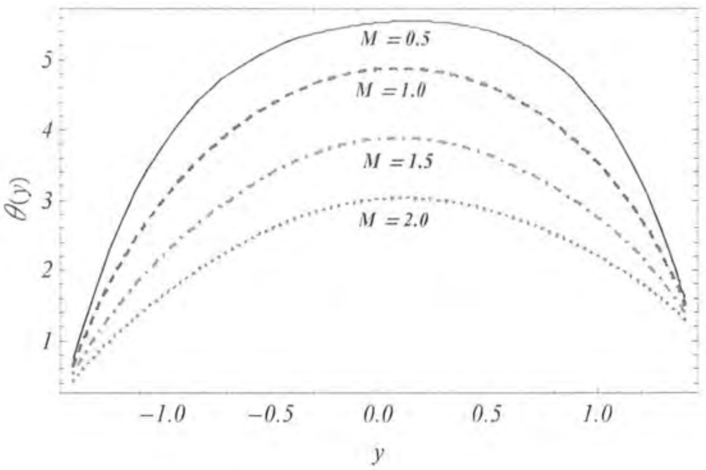


Fig. 3.12. Effect of M on θ.

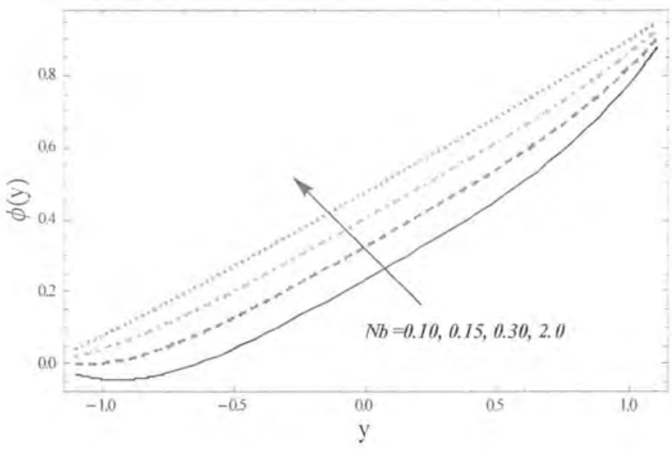


Fig. 3.13. Effect of Nb 0n ϕ.

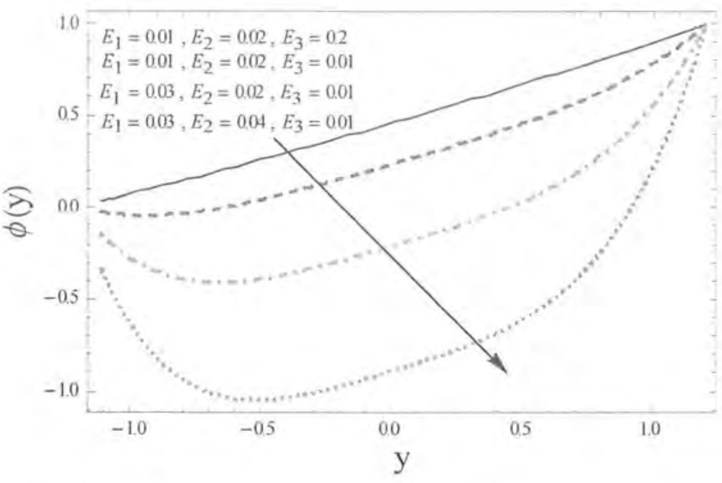


Fig. 3.14. Effect of parameters of wall properties on ϕ.

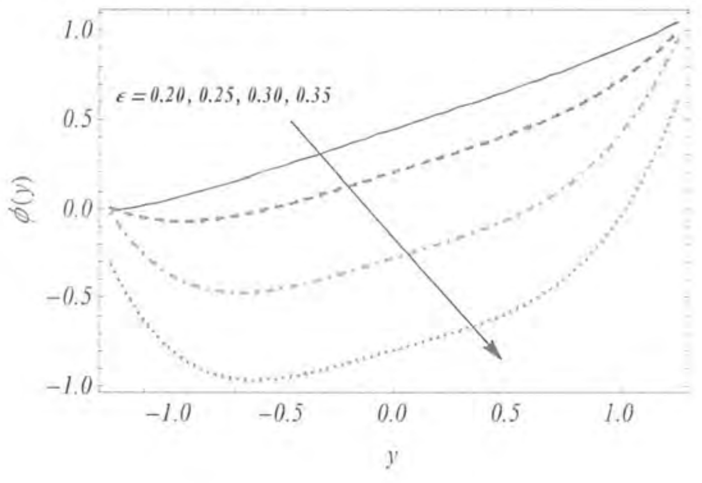


Fig. 3.15. Effect of ϵ on ϕ.

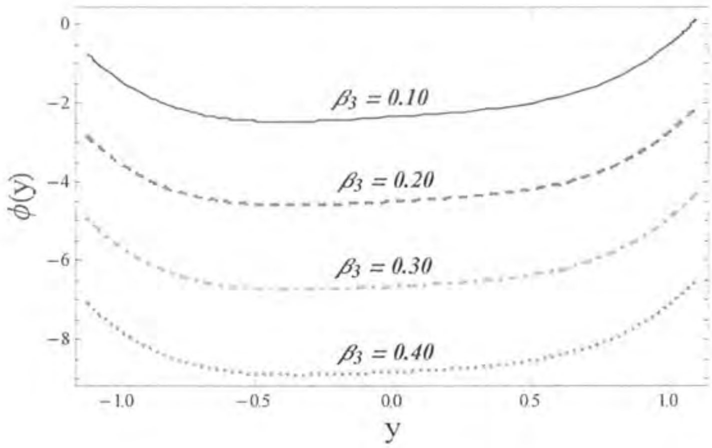


Fig. 3.16. Effect of β_3 on ϕ.

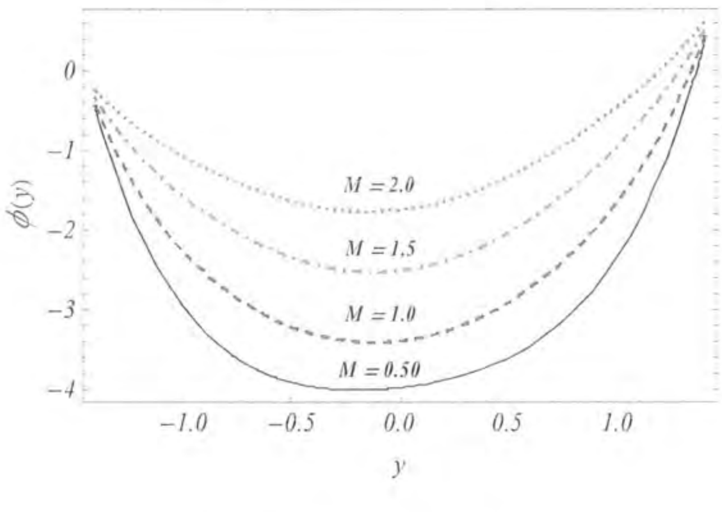


Fig. 3.17. Effect of M on ϕ.

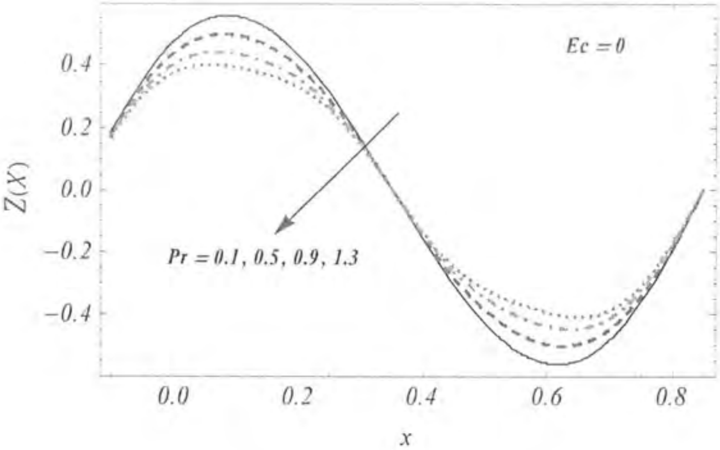


Fig. 3.18. Effect of Pr on Z.

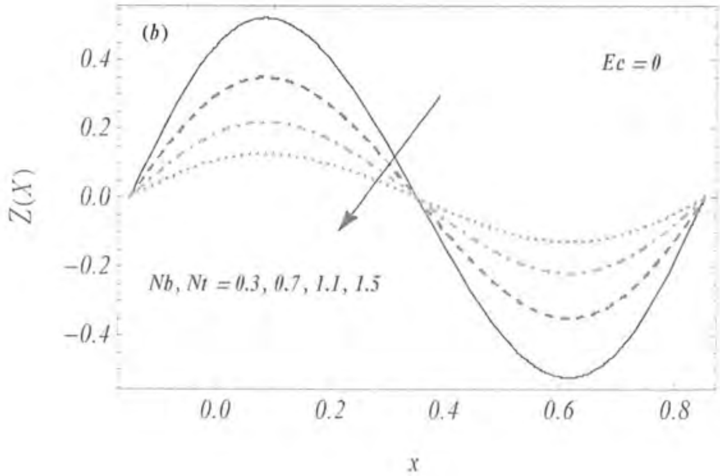


Fig. 3.19. Effect of Nb, Nt on Z.

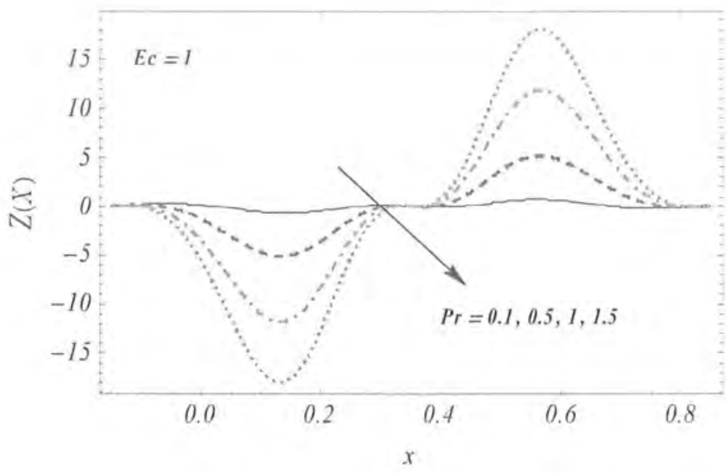


Fig. 3.20. Effect of Pr on Z.

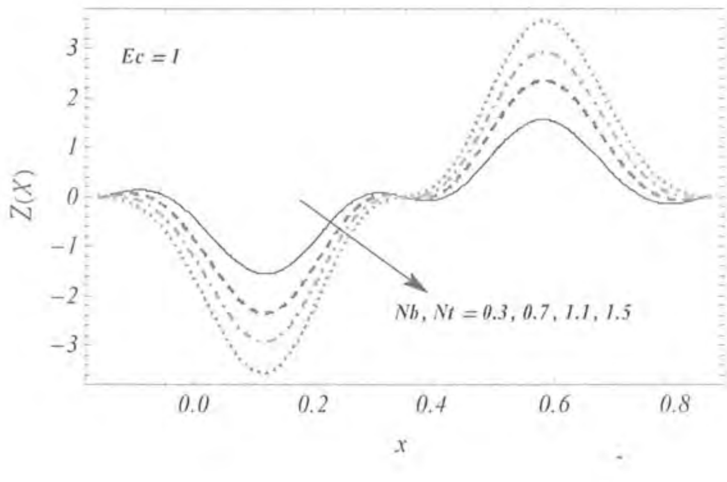


Fig. 3.21. Effect of Nb, Nt on Z.

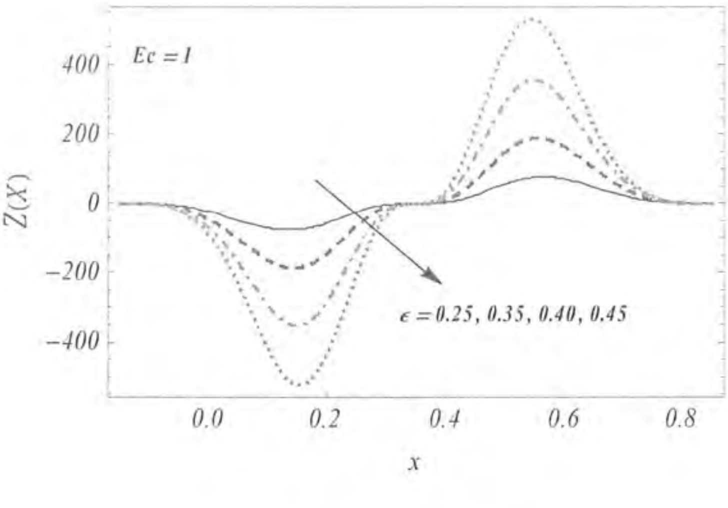


Fig. 3.22. Effect of ϵ on Z.

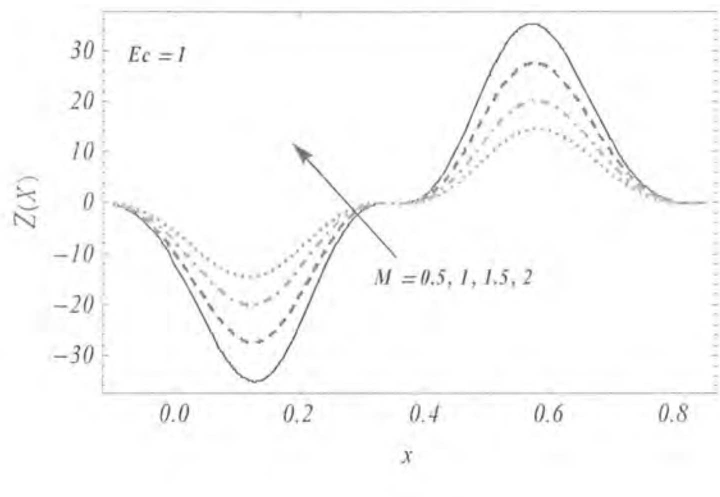


Fig. 3.23. Effect of M on Z.

3.6 Conclusions

Here, the simultaneous effects of slip and wall properties on MHD peristaltic motion of nanofluid are discussed. Joule heating effects are also present. The main points are listed below.

- The velocity field increases by increasing the velocity slip parameter β_1.
- By increasing E_1, E_2 and ϵ, the velocity profile increases and increasing E_3 and M it decreases.
- Temperature increases with an increase in Brownian motion parameter Nb.
- Temperature increases for increasing values of thermal slip parameter β_2.
- Temperature and concentration increase by increasing the elastic parameters E_1 and E_2, and decrease for E_3.
- The temperature profile decreases with an increase in Hartman number M and concentration field increases when M increases.
- The role of concentration slip parameter β_3 on concentration field is quite opposite to that of M.
- The absolute value of heat transfer coefficient decreases with an increase of M.

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