Allocative Efficiency in the Manufacturing Sector of Pakistan

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Certificate

We accept the work contained in this dissertation titled "Allocative Efficiency in the Manufacturing Sector of Pakistan" as conforming to the required standard for partial fulfilment of the Degree of Master of Philosophy in Economics.

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Abstract

This study investigates the nature of allocative inefficiencies in Pakistan's large scale manufacturing sector, using pooled provincial annual time series data of Pakistan from 1969-70 to 1990-91. The analysis in this study is based on translog cost function and three variants to the base model, which incorporate technological change and allocative inefficiencies. Using the system's approach, we estimate parameters of four models, each consisting of the cost function and three share equations, to conduct different tests of hypotheses and to compute elasticities and returns to scale. Our results indicate that there is strong evidence of allocative inefficiencies and technological change in large scale manufacturing sector of Pakistan. In the presence of technological change, pair wise relative price efficiency tests suggest that raw materials, capital, and energy are over-utilized relative to labor. The over-utilization of raw material is most severe, relative to labor and energy. Capital found to be over-utilized relative to energy. The estimates of own price elasticity show that capital is most responsive factor to change in its price, while all other factors show inelastic demand pattern. Moreover, the estimates of Allen partial elasticities of substitution, and Morishima elasticities of substitution reveal that there is no evidence of energy/capital complementarity in the large scale manufacturing sector of Pakistan. The results of this study suggests that the long-run economic growth could be achieved through adjustment policies that correct input market price distortions and promote shift toward efficient factor proportions.

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Acronyms

AES	Allen Partial Elasticities of Substitution
BTU	British Thermal Unit
CES	Constant Elasticity of Substitution
CMIs	Census of Manufacturing Industries
COLS	Corrected Ordinary Least Square
DEA	Data Envelopment Analysis
E_{ii}	Own price Elasticities
E_{ij}	Cross price Elasticities
iid	Independently and Identically Distributed
IZEF	Iterative Zellner Efficient Estimator
LDCs	Less Developed Countries
MES	Morishima Elasticities of Substitution
MLE	Maximum Likelihood Estimators
MOLS	Modified Ordinary Least Square
MRTS	Marginal Rate of Technical Substitution
NCES	Nested CES
OLS	Ordinary Least Square
SE	Scale Economies
SUR	Seemingly Unrelated Regressions
TC	Total Cost
TFP	Total Factor Productivity
UOP	Unit Output Profit Function
VES	Variable Elasticity of Substitution Function

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Mahmmod-ul-Hasan Khan

To

My Loving Parents

Chapter

1 Introduction

1.1 Background

Lack of effective competition in factor markets produces allocative or price inefficiencies in the manufacturing sectors of the developing countries, like Pakistan. Such inefficiencies are common due to distortions in factor markets leading to the use of inappropriate factor proportions (Lau and Yotopoulos [1971,1972], Yotopoulos and Lau [1973], Kumbhakar and Bhattacharyya [1992], Moussa and Jones [1991], Burki *et al.* [1997]). However, it is shown that due to abundance of labor, relative to other factors "greater labor-intensity in less developed countries' (LDCs) manufacturing sector is feasible and efficient" (White [1978]). Pakistan is also one of the countries where labor is abundant but capital and raw material are scarce. But Burki *et al.* [1997] show that distortions in the factor markets in the manufacturing sector of India and Pakistan do produce allocative inefficiencies which lead to over- or under-utilization of factor inputs relative to their factor endowments. This finding greatly undermines the validity of estimates of elasticities of demand and substitution which are based on classical assumption that factor markets are perfectly competitive in Pakistan, (Kemal [1981,1982], Kazi *et al.*[1976], Battese and Malik [1987,1988], Malik *et al.* [1989], Zahid *et al.*[1992], Battese *et al.*[1993], Khan and Rafiq [1993], Mahmood [1989, 1992], and Idrees [1997]).

As noted above, Pakistan is characterized by abundant labor and scarcity of capital, industrial raw materials and entrepreneurship. At the time of independence, Pakistan identified the process of industrialization as a prerequisite for economic development and rapid economic growth. Therefore, Pakistan followed the policy of import-substituting industrialization in the 1950s and the 1960s. Though this policy was successful in achieving high economic growth rates, but it had a pro capital and raw material bias. For example, this policy was promoted through a policy of low rates of interest, heavy protection in the guise of infant industry argument, overvalued exchange rates, fiscal concessions, import licensing system and agricultural raw material prices set below the world market prices. Further the capital-intensive technologies were also supplemented by the government guarantees regarding the supply of imported raw materials. Such policies resulted into a marked bias in favor of capital-intensive large scale industries with excessive use of capital and imported raw material. As a result, there was little expansion in the demand for labor. Public investments were never made on the criterion of relative factor endowments or employment generation (Ahmed and Amjad, [1984]). Other reasons to adopt capital-intensive technology were the policy maker's sense of being-up-to-date and that the most modern technologies required less administrative efforts in labor supervision (Little *et al.* [1970]). These policies may have produced factor price distortions in Pakistan's large scale manufacturing. Whether or not these factor price distortions led to allocative inefficiencies is an empirical question.

This study investigates the nature of allocative inefficiencies in Pakistan's large scale manufacturing by using the flexible translog cost function and three extensions to the basic model aimed at modeling allocative inefficiencies and technological change biases. We use pooled provincial annual time series data of Pakistan from 1969-70 to 1990-91. The study goes on to estimate own and cross price elasticities of demand, and Allen and Morishima elasticities of substitution.

1.2 Statement of Objectives

The principal objectives of the study are to investigate,

- the nature and degree of allocative efficiencies in Pakistan's large scale manufacturing sector,
- technological change bias with and without imposing relative price efficiency,
- substitution and complementary possibilities between different inputs at the aggregate level,
- returns to scale and economies of scale, and
- to make policy recommendations.

1.3 Organization of the Study

The rest of the study is structured as follows. The theoretical underpinnings of the literature on efficiency measurement techniques are reviewed in chapter 2. It also includes the review of previous studies on allocative efficiencies and production relations in Pakistan. Chapter 3 presents the theoretical framework leading to the empirical specifications used in this study. Essentially, it contains four different types of models based on modifications to the translog cost function aimed at modeling allocative inefficiency and technological change. The description of data and estimation techniques are described in chapter 4. In chapter 5, we present and analyze our empirical results based on simple translog cost function and discuss own and cross and substitution elasticities and returns to scale and compare these results with the existing studies on Pakistan. Chapter 6 contains the results based on translog cost function when we allow it to incorporate technological change under the maintained hypothesis of relative price efficiency. The nature of allocative inefficiencies are analyzed in chapter 7. Chapter 8 presents results of the model when allocative efficiency and technological change are simultaneously included. Chapter 9 gives concluding remarks and policy implications.

Chapter

2

Review of Literature

This chapter reviews the literature on efficiency and its measurement of efficiency. The chapter begins with description of the theoretical underpinnings of efficiency. Then we discuss different measures of efficiency, that is technical and allocative efficiencies. This is followed by estimating techniques of measures and empirical studies on allocative efficiency. Empirical studies on Pakistan's large scale manufacturing sector are discussed in the end of this chapter.

2.1 Theoretical Underpinnings

It is usual to describe the performance of a production unit as more or less "efficient" or more or less "productive". The productivity of a production unit means the ratio of its output to its inputs. The variation in productivity is attributed to production technology, the efficiency of the production process, and the environment in which production takes place.

Efficiency of a production unit is simply a comparison between observed and desired values of its output and factor inputs, or it is the ratio of observed to maximum potential output obtainable from given factor inputs, or the ratio of minimum potential to observed factor inputs required to produce given output, or some combination of the two.

Economic Efficiency can be decomposed into two components: technical and allocative efficiency. The technical or physical efficiency refers to the ability of a production unit to avoid waste or producing as much output as possible with the use of given factor inputs or by using as little factor inputs as possible to produce given output. Thus, technical efficiency is an output orientation or have an input-conserving orientation. The allocative or price efficiency refers to the ability of a production unit to combine inputs and output in optimal proportions in the light of prevailing prices.[1]

A formal definition of efficiency is "[a] producer is said to be technically efficient if an increase in any output requires a reduction in at least one other output or an increase in at least one input, and if a reduction in any input requires an increase in at least one other input or a reduction in at least one output. Thus a technically inefficient producer could produce the same output with less of at least one input or could use the same inputs to produce more of at least one output."[2]

A production unit is said to be price efficient if it maximizes its profits. Profit maximization implies equalization of the value of the marginal product of each variable input to its price. To explain it in more detail, we consider a production unit employing n inputs, $X = (X_1, X_2, \ldots, X_n)'$ available at fixed prices $P = (P_1, P_2, \ldots, P_n)' \geq 0$ to produce a single output "Q", that can be sold at fixed price $P > 0$. The production function $F(X)$ provides the efficient transformation of factor inputs into output, or shows the maximum output obtainable from various factor inputs.

Another equivalent representation of efficient production technology under certain regulatory conditions, is provided by the cost function.

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[1] For details, see Lovell [1993].

[2] See, Koopmans [1951].

$$C(Q, P) \equiv Min_x \left[P^{/} X \mid F(X) \geq Q , X \geq 0 \right] \tag{2.1}$$

which shows the minimum expenditure required to produce output Q at input prices P. By means of Shephard's Lemma we can obtain a vector of cost minimizing input demands.

To explain efficient transformation of factor inputs into output, we suppose that the production unit is observed at a production plan (Q^0, X^0) . This plan is said to be technically efficient if $Q^0 = F(X^0)$ and technically inefficient if $Q^0 \leq F(X^0)$. (while $Q^0 \geq F(X^0)$ is assumed to be technically impossible). One measure of technical inefficiency of this plan is provided by the ratio $0 \leq Q^0 / F(X^0) \leq 1$. Technical inefficiency may be due to excessive input usage, which is costly and so $P^{/} X^0 \geq C(Q^0, P)$ i.e., cost is not minimum.

The observed plan (Q^0, X^0) is said to be allocatively efficient if $F_i(X) / F_j(X) = P_i / P_j$, and allocatively inefficient if $F_i(X) / F_j(X) \neq P_i / P_j$, assuming if "F" to be differentiable. Allocative inefficiency results from employing inputs in the wrong proportion, which is costly, so $P^{/} X^0 > C(Q^0, P)$.

It is clear from above that observed expenditure $P^{\prime} X^0$ coincides with minimum cost $C(Q^0, P)$ *if and only if* the production unit is both technically and allocatively efficient. If $P^{\prime} X^0 > C(Q^0, P)$, this may be due to technical inefficiency, or allocative inefficiency, or some combination of the two.

Now we explain the allocative and technical inefficiency with the help of a figure.[3] For this purpose, consider a production unit using two inputs X_1 and X_2 and producing output Q^0 and its production function is $Q^0 = f(X_1, X_2)$. Assume that there is a constant returns to scale, so we write the production function as $1 = f(X_1 / Q^0, X_2 / Q^0)$. The efficient production technology is characterized by an isoquant and is denoted by *SS* in figure (2.1). The line *PP* shows the observed price ratio. The technically and allocatively efficient point for given output is "*E*". Assume that point "*A*" in fig. (2.1) represents $(X_1 / Q^0, X_2 / Q^0)$. Point "*A*" cannot lie below the efficient isoquant *SS* by definition. Then the ratio *OB/OA* measures the technical inefficiency, that is the ratio of factor inputs needed to produce Q^0 to factor inputs actually used to produce Q^0. The cost of production "*C*" are the same as that of the allocatively efficient point "*E*". Therefore, the ratio *OC/OB* measures the allocative inefficiency. Finally, the ratio *OC/OA* measures total efficiency.

2.2 Measures of Efficiency[4]

The existing approaches to efficiency are not so clear conceptually and empirically. However, we will start from conventional variants of efficiency by classifying them into economic efficiency, price or allocative efficiency, and technical efficiency. We will also point out the ambiguities attached with these variants. The very first and simplest measure of economic efficiency is the partial productivity index. The major drawback of this approach is that this approach ignores the presence of other

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[3] See, Green [1993]

[4] The discussion in this section is mainly drawn from Farrell [1957], Lau and Yotopoulos [1971, 1972], and Yotopoulos and Lau [1973].

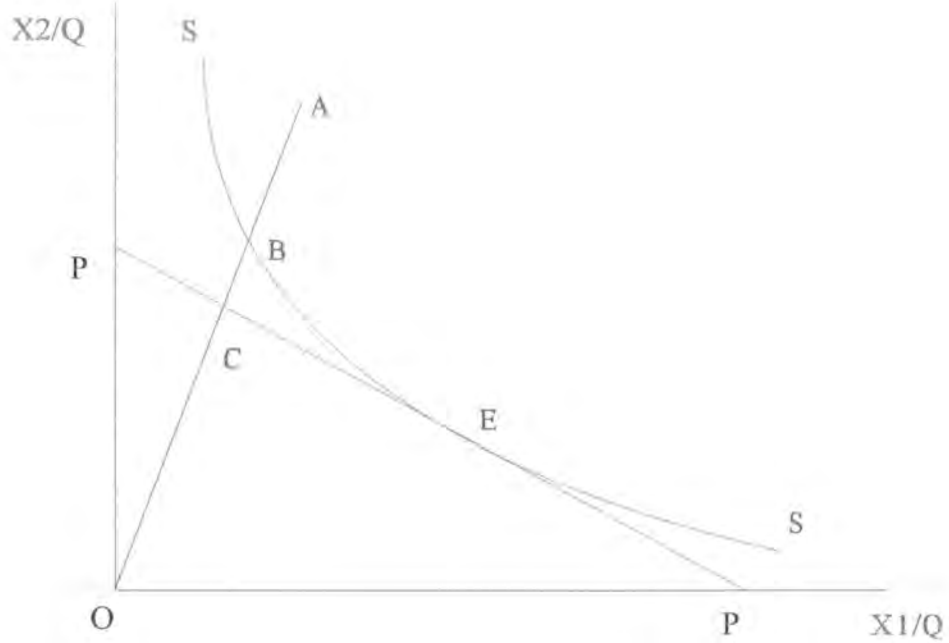

Fig. 2.1: Measurement of Technical and Allocative Efficiency

factors of production, which affect average and marginal productivity. Therefore, this approach is the most naive measure of economic efficiency. A more sophisticated approach is to construct index based on weighted averages of inputs (relative prices or relative factor shares can be used as weights) which is compared to output. In simple, the constructed index is an output cost ratio. This approach is also not free from problems, but involves the usual problems of index numbers.

The conventional measure of price or allocative efficiency traditionally is an index of marginal productivity and of opportunity cost. This approach runs into a number of problems. *First*, its usefulness is doubtful when one compares different groups of production units, even after allowing for differences in production functions and input prices. This is so because of absolute concept. A production unit is said to be price efficient, if the ratios of the marginal products to opportunity costs are equal to one. Therefore, the direct comparison among the production units that satisfy this equality to the different degree is almost impossible. *Second*, it does not take into account the possible differences in the initial endowment of fixed factors.

Technical efficiency is purely an engineering concept. The traditional measurement of technical efficiency concentrates on the neutral displacement of production function either between groups of production units or over time. A more sophisticated approach to measure technical efficiency has been purposed by Farrell [1957]. Farrell derived the pessimistic unit-isoquant under the assumption of constant returns to scale, i.e., isoquant which envelops the observations lying between the pessimistic isoquant and the origin. By using this approach efficiency index is constructed by measuring the distance between a specific point and the pessimistic isoquant. This approach is also not free of problems. This approach not only ignores the effect of relative prices but also has the additional disadvantage that arise when one attempts to describe a stochastic universe by a deterministic process. The pessimistic isoquant is extremely sensitive to outliers.

In sum, the problems with the existing approaches to efficiency measures dictate the minimum requirements that a new concept of relative economic efficiency should meet, if it is to be fruitful at all. *First*, it should account for differences in technical efficiency, i.e., the production units produce different quantities of output from a given set of factor inputs. *Second*, it should take into account the component of price efficiency which implies the different production units succeed to varying degrees in profit maximizing, i.e., in equating the values of marginal product of each variable factor of production to its prices. *Third*, it should also account that the production units operate at different sets of market prices. The profit maximization conditions yield actual profit as a function of input prices. It is very clear that the two production units of equal technical efficiency which have successfully maximized profit would still have different values of profit as long as they face different prices.

2.3 Techniques for Efficiency Measurement

The production theory is based on efficient subsets of production sets, on value such as minimum cost function and maximizing profit or revenue function and related properties of production theories which yield cost minimizing input demand functions, revenue or profit maximizing output supply and input demand functions. Therefore, in production theory emphasis is placed on efficient production and its consequences and in literature the term "frontier" is applied to these bounding functions. We have different techniques to analyze the nature of frontier. Mainly we have two broad techniques, one is mathematical programming technique and the other one is econometrics technique.[5]

The mathematical programming technique constructs the production frontiers and measures efficiency relative to that constructed frontier. This technique frequently goes by the descriptive title of data envelopment analysis (DEA). It envelops the data set as tightly as possible and also imposes three restrictions, prior to solving envelop problem. This technique has certain problems. The mathematical programming approach is non-stochastic and lumps noise and inefficiency together and calls that combination inefficiency. This approach also imposes three restrictions on the frontier technology. The restrictions are constant returns to scale, strong disposability of inputs and outputs, and convexity of the set of feasible input output combinations.

In sharp contrast to the mathematical programming techniques are the econometrics techniques that can be used to investigate the structure of economic frontier and measurement of economic relative efficiency to these frontiers. This approach attempts to separate the effect of noise from the effect of inefficiency. The conventional econometrics techniques also need some modifications.[6]

The literature on frontiers technology is classified according to the way the frontier is specified and estimated. *First,* the frontier may be specified as a parametric function of inputs or it may not. *Second,* an explicit statistical model of relationship between observed output and the frontier may be specified. *Finally,* the frontier itself

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[5] See, lovell [1993].

[6] For details, see Lovell [1993].

may be specified to be either deterministic or random. Now we provide an over view of these frontiers.

Deterministic Non-Parametric Frontier

The seminal work on frontier and efficiency measurement is done by Farrell [1957], in which he provided definitions and a computational framework for both technical and allocative inefficiency. Farrell's approach is non-parametric because he simply uses the linear programming technique to construct the free disposal convex hull of the observed input-output ratios. This procedure does not base on any explicit model of the frontier or of the relationship of observations to the frontier. The major advantage of this procedure is that it does not impose any functional form on the data. The principal disadvantage of this approach is its crude assumption of constant returns to scale, which is restrictive, and it is cumbersome task to extend it to non-constant returns to scale technology. Another disadvantage of this approach is that it constructs the frontier from a supporting subset of observations from the sample and is, therefore, susceptible to extreme observations and measurement error.

Deterministic Parametric Frontier

This approach is also proposed by Farrell and has proved more fruitful than the first one. It computes a parametric convex hull of the observed input-output ratios. He recommended a Cobb-Douglas form for this purpose. With this approach, we gain the advantage of being able to express frontier in a simple mathematical form. The major advantages of this approach are the ability to characterize frontier technology in a simple mathematical form and the ability to accommodate non-constant returns to scale technology. The disadvantage of this approach is that it imposes structure on the frontier that may be unwarranted. This approach also imposes a limitation on the number of observations that can be technically efficient. Another problem with this approach is that the estimates of parameters which it produces have no statistical properties. That is, mathematical programming procedure produces estimates without standard errors, *t*-statistics, etc., because no assumptions are made about the disturbances. This approach specifies the homogeneous Cobb-Douglas production function, which is restrictive. However this restrictive specification has been relaxed by Forsund and Jansen [1977] and Forsund and Hjalmorsson [1979a].

The Deterministic Statistical Frontier

The recommended Cobb-Douglas functional form in the previous section can be made amenable to statistical analysis by making some assumptions about the disturbance term. So we can write the model as

$$Y = f(x)\, e^{-u} \tag{2.2}$$

Taking logs on both sides, it becomes

$$ln\, Y = ln\, f(x) - u \tag{2.3}$$

Where $u \geq 0$ and thus $0 \leq e^{-u} \leq 1$. It is assumed that u is independently and identically distributed (iid) and " x " is exogenous. Any number of distributions for u could be specified.

The problem with this approach is the distribution of u. Afrait [1972] was first to purpose a two-parameter beta distribution for e^{-u} , and suggested the maximum likelihood method to estimate the model. The maximum likelihood estimates are sensitive to the functional form of u. Therefore, different assumed distributions led to different estimates. Another problem with maximum likelihood estimates is the range of dependent variable which depends on the parameters to be estimated. This violates the conditions involved to prove the general theorem that maximum likelihood estimators are consistent and asymptotically efficient. So the statistical properties of the maximum likelihood estimators should be reconsidered.[7] In this regard Greene [1980a] proved that the desirable properties of MLE hold if the density of u satisfies the following conditions.

a) The density of u is zero at $u=0$.

b) As u approaches to zero the derivative of the density of u with respect to its parameters also approaches to zero.

The Gamma distribution satisfies this criterion and is potentially useful.

There is another method of estimation based on the results of ordinary least squares. In literature, it is known as corrected ordinary least square or simply COLS. This method was proposed by Winsten [1957]. While, Richmond [1974] introduced the Modified or Displaced ordinary least square method, or MOLS. In these cases, the linear Cobb-Douglas specification, given in equation (2.3), can be written as

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[7] For details, see Forsund *et al.* [1980].

$$lnY = (\alpha - u) + \sum_{i=1}^{n} \alpha_i \ln x_i - (v - u) \qquad (2.4)$$

Now, error term has zero mean. In this case, error term satisfies all the classic assumptions of the error term except for normality. In both COLS and MOLS the above equation can be simply estimated by using OLS method. In COLS the obtained estimates can be used to correct the down-ward biases in constant term by shifting it up until all the corrected residuals are non positive and at least one is zero. While in case of MOLS the estimated OLS constant term is corrected by shifting it up until it is consistent estimate of $(\alpha - u)$.

The problem with MOLS method is that even after correcting for the constant term, some residuals may still have wrong signs. So that these observations end up above the estimated frontier. It makes the MOLS awkward for computing the technical efficiency of individual observations. Another problem with MOLS technique is that the correction to the constant term is not independent of the distribution assumed for u.

The Stochastic Frontier

The deterministic frontier ignores the very real possibility that the performance of production units may be affected by factors entirely outside its control as well as factors under its control. This technique combines the effects of exogenous shocks, both fortunate and unfortunate, together with the effects of measurement error and inefficiency into a single one-sided error term and label the mixture "inefficiency". This procedure is somewhat questionable. This conclusion is reinforced if one takes into account the statistical noise that every empirical relationship contains. The standard interpretation is that i) there may be measurement error, and ii) the equation may not be correctly specified.

The notion of stochastic frontier model is that the error term is the sum of two components. One is symmetric component that permits random variation of the frontier across production units and captures the effect of measurement error and the other is statistical noise. A stochastic production frontier model can be written as

$$Y = f(x)\ e^{(v-u)} \tag{2.5}$$

Where v has a symmetric distribution, which captures the random effect of measurement error and exogenous shocks. Technical inefficiency relative to the stochastic production frontier is then captured by the one-sided error component e^{-u} , $u \geq 0$. The condition $u \geq 0$ ensures that all observations lie on or beneath the stochastic production frontier. One can estimate the stochastic frontier by using MOLS or MLE. The major disadvantage of this approach was that the results obtained from MLE or MOLS contained both noise and inefficiency. One could not decompose the error term into two components. However, the method proposed by Jondrow *et al.* [1982] solved this problem. He gave a method that could be used to decompose the composed error into technical inefficiency and the random error. A major problem with the stochastic frontier is that the assumption of distribution for the asymmetric error is purely arbitrary. Assuming different distributions for the one-sided error produces different estimates for efficiency of individual units.

Duality Considerations

The estimation of production frontier provides information on technical inefficiency but not on the allocative inefficiency. It is well known that the production function and cost function defines the technology equally. Whether cost or production function is to be estimated depends on the availability of data. Estimation of cost function is generated by cost minimization with given output and requires data on input prices but not input quantities. Cost function also yields information on the extra cost of technical and allocative inefficiency. Cost function may also be deterministic or stochastic. Forsund and Jansen [1977] have estimated a deterministic Cobb-Douglas cost frontier, while Schmidt and Lovell [1979] have estimated a stochastic Cobb-Douglas frontier. Greene [1980b] estimated a deterministic translog cost frontier along with share equations.

Nonfrontier Efficiency Measurement Models

The above stated techniques are enveloping techniques. In these techniques technical efficiency is measured in terms of distance to a production frontier and economic efficiency in terms of distance to an appropriate economic frontier. Allocative efficiency is then measured as a ratio of economic efficiency to technical efficiency. These measures do not completely solve the problem because they differ mainly in techniques they employ to construct frontiers and to measure distance.

There is a literature in which efficiency is measured without explicit use of frontier.[8] In this literature, they do not attempt to envelop data or to relate efficiency with the distance from an enveloping surface. The main focus of this literature is on allocative efficiency. Allocative inefficiency is modeled by allowing producers to make their decisions about input demands and output supplies to the shadow prices. Technical inefficiency is modeled by shifting the intercepts of output supplies and input demands. This parameterizing inefficiency leads to actual output supply and input demand equations. The nonfrontier efficiency measurement models serve the two purposes most frequently. Thus the allocative inefficiency in the unconstrained pursuit of cost minimization suggests allocative efficiency is a more complicated environment, and the difference in shadow prices from observed prices provides the basis for hypothesis tests. The econometrics method is used for estimation.

2.4 Empirical Studies on Allocative Efficiency

The first model in this approach is due to Hopper [1965], who estimated a Cobb-Douglas production function to calculate the value of marginal product for each input. He used OLS method and found that subsistence agriculture in India to attain a higher degree of allocative efficiency. He also made two comparisons: the value of an input's marginal product across outputs, and the value of an input's marginal product with its prices. In each comparison equality implies allocative efficiency and the sign and magnitude of any inequality indicate the direction and severity of the allocative inefficiency. Hopper's work was heavily criticized.

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[8] For more details, see Forsund *et al.* [1980] and Lovell [1993].

Lau and Yotopoulos [1971,1972] and Yotopoulos and Lau [1973] used the nonfrontier efficiency models to examine price efficiency. They simply parametrized Hopper's comparisons, with inequalities being replaced with parameters to be estimated. Lau and Yotopoulos [1971,1972] and Yotopoulos and Lau [1973] used aggregate data from six Indian states to test relative economic efficiency in Indian agriculture. For this purpose, unit-output price (UOP) profit function was used due to the following advantages. *First,* from UOP profit function one can easily derive the firm's supply functions by applying Shephard-Uzawa-McFadden Lemma. *Second,* one can easily obtain the supply function and the factor demand functions by starting with an arbitrary UOP profit function, which is decreasing and convex in normalized prices of variable inputs and increasing in fixed inputs. *Third,* by starting with a UOP profit function, one can assure by duality that the resulting system of supply and factor demand functions is obtainable from the maximization of a convex production function subject to given fixed inputs under competitive market. *Fourth*, the derived demand functions and the supply functions are only functions of the normalized input prices and quantities of fixed inputs, i.e., the variables that are normally considered to be determined independently of the firms behavior.[9] They found that small farms are more efficient than large farms. The rate of return on fixed capital and land are larger than on the large farms. As price efficiency is concerned, both farms are equally price efficient. So the small and large farms have maximized their profits. They also found the existence of constant returns to scale in Indian agriculture. The problem with Lau and Yotopoulos [1971,1972] and Yotopoulos and Lau [1973] was that they used restrictive Cobb-Douglas functional form.

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[9] For details, see Lau and Yotopoulos [1972].

Another problem is that they ignored the effect of heteroscedasticity that may have produced potential bias due to the use of grouped data.

Narasimhan and Fabrycy [1974] measured relative efficiency of organized industries in India. Their analysis was based on Cobb-Douglas production function, constant elasticity of substitution and homothetic isoquants. For technical change, they used time trend and analysis of covariance. They found that Indian industry was subject to constant returns to scale. When technological change was constrained to be constant over time, the changes were found to be statistically insignificant. However, when analysis of covariance was used, highly significant changes were observed over time. The major problem with this study was the use of restricted functional form to measure efficiency. Their model did not incorporate the parameters to measure allocative efficiency clearly. Another problem was that they used a limited data set of only ten years, i.e., 1949-58.

Toda [1976] estimated a cost function with single output, two inputs capital and labor, when cost is not minimum. He formulated the cost function on the basis of the generalized Leontief cost function. The author estimated it for the annual time series data of the Soviet manufacturing industry for the period 1958-71. According to his estimates, the shadow rental wage ratio differ significantly from the observed rental wage ratio for the three years. He also found that the estimated growth of total factor productivity was higher if the index is measured in shadow prices than if it is measured in actual prices.

The analysis by Toda [1976] contains several limitations. *First*, the cost function is constructed with the output level and factor prices as independent variables. But in the case of Soviet Union, level of output is assigned to an enterprise as a target and the profitability is also assigned as target. So the study used those figures that are the result of over or under fulfilment of the assigned target. *Second,* the prices of intermediate materials were not included in the cost function and the gross output was used as a variable. *Third*, the author made a very rigid assumption about the constancy of the disparity between the observed price ratio and the shadow price ratio. Hence, due to the above problems, results were not so reliable.[10]

Trosper [1978] measured the American-Indian relative ranching efficiency using restricted Cobb-Douglas production function and the assumption of constant returns to scale. For estimation, he normalized the production function with the amount of land. He used the same method to measure relative economic efficiency and other types of efficiency, that was used by Lau and Yotopoulos [1971].The author found that the Indians and the Americans were both relative and price efficient. Controlling for land tenure, capital, and labor, Indian and Americans appeared equal in price and technical efficiency. The problem with their study was that they used the individual level data for a small number of persons. So it is irresponsible to recommend policy implications based on the limited data. Another problem was the use of restricted functional form with rigid assumption of constant return to scale.

Atkinson and Halvorsen [1980] used a shadow price approach to test a relative and absolute price efficiency in regulated utilities. To estimate the system of profit and factor demand equations, the authors used data of 38 steam electric plants for the year 1973. The hypothesis of relative price efficiency was rejected, which implies that the regulated electric utilities do not minimize costs. The empirical evidence shows that input

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[10] For details, see Toda [1976].

choice in regulated utilities is affected significantly. There is also a rigid assumption that all the electric plants are equally technically efficient.

Lovell and Sickles [1983] used a parametric approach to test efficiency hypothesis in joint production. For this purpose, they used the generalized Leontief profit function. Technical inefficiency was modeled by adjusting the intercept so as to permit a divergence between actual and profit maximizing output supplies and input demands. Allocative inefficiency was modeled by assuming that the production units adjust output supplies and input demands to the wrong price ratios. The input demand equations and output supply equations were estimated by seemingly unrelated regressions (SUR). They found that technical inefficiency can be non-neutral and allocative inefficiency can be non-consistent. This study was also not free from problems. They assumed that the market is competitive and scale inefficiency is the major source of inefficiency (or the divergence between output prices and marginal revenues). Another problem with this study was that they assumed that all prices are known with certainty.

Atkinson and Halvorsen [1984] applied parametric efficiency tests and found economies of scale and input demand in USA electric power generation. They estimated a cost function with share equations based on translog cost function. They found that there was relative inefficiency and because of this cost of the firms increased by 3.8 percent at the mean of the data. They also found that the estimates of scale economies were higher with shadow costs than the actual costs. The major problem with this study was that they ignored the effects of technological change and the distortions were assumed to be input-specific but not firm specific.

Moussa and Jones [1991] estimated a Cobb-Douglas profit function with factor demand functions to measure efficiency and farm size in Egypt. They used the same model as used by Lau and Yotopoulos [1971,1972] with proper modifications to accommodate some observed problems. They found that the economic efficiency was same for top and bottom observations of the data. They also found that both farm sizes had same price efficiency. Their empirical evidence showed that both the small and large farms were not able to allocate their resources successfully in the profit maximizing sense. The problem with this study was that they used a restricted Cobb-Douglas profit function.

Kumbhakar and Bhattacharyya [1992] estimated a translog profit function, with some restrictions, to measure price distortions and resource use efficiency in the Indian agriculture sector. They found that small and large farms were equally technically efficient and there was a positive relationship between education and technical efficiency. The empirical evidence also showed that the efficiency improvement with higher education was smaller for large farms as compared to small farms. They used the years of schooling for education, which assumed the constant effect of education for each year.

Bhattacharyya *et al.* [1994] used the generalized cost function, with a number of prior restrictions on it, to examine the effect of ownership on the relative efficiency of public and private water utilities. For estimation they used the data on cost behavior of 225 public and 32 private water utilities. The empirical results provide evidence that public water utilities are more efficient than the private water utilities on average but are more usually dispressed between the best and worst practices.

Burki *et al.* [1997] estimated a profit function based on translog functional form to test allocative efficiency in the manufacturing sectors of Pakistan and India. They used time series data to estimate the model. The empirical results provide the evidence that the manufacturing sector in India and Pakistan employ factors of production sub-optimally because the ratio of effective and actual factor prices diverge significantly. That is, manufacturing sector fails to minimize costs in both countries. They further investigated that the over-utilization of capital is far greater for Pakistan than for India, but the difference is not statistically significant. In their study it is implicitly assumed that both countries are equally technically efficient, which is a weaker assumption.

Burki and Terrel [1998] measure production efficiency of small firms in Pakistan. For this purpose they used the data envelopment analysis method to measure efficiency. They used the survey data of 153 small manufacturing firms from nine industries. The results provide evidence that on average the sampled firms can raise output from 6 to 29 percent by improving their overall technical efficiency. They also investigated that the primary source of the scale inefficiency is their operation at less than the optimal level of production. They further analyzed that primary school education and functional literacy improves the efficiency of small firms. In this study it is implicitly assumed that the market is competitive. So the existing inefficiency is not due to price inefficiency but is due to scale inefficiency. In Pakistan the factor market is not perfect so there may be allocative inefficiency.

In the present study we use a shadow price approach, originally suggested by Lau and Yotopoulos [1971,1972] and extended by Atkinson and Halvorsen [1984,1998] . In this study, we estimate the cost function based on the translog cost function. We have tried to avoid the problems associated with the approach. We have modified the model to take into account not only the allocative efficiency but also technological change.

2.5 Studies on Pakistan's Large Scale Manufacturing

The literature on production relations in Pakistan's large scale manufacturing sector is confined to the estimation of restrictive functional forms such as the Cobb-Douglas. Constant elasticity of substitution (CES), Variable elasticity of substitution (VES) and Nested CES. It is well known that these functional forms impose unitary or constant elasticities of substitution, which make them restrictive. More recently, few studies have also employed more flexible functional forms. It is interesting to note that studies on efficiency are restricted to measure technological bias, or effects of protection on the efficiency of the firms. Price or allocative inefficiencies are quite common in the manufacturing sector of developing countries. There is no reason to assume that price inefficiency is not present in Pakistan's large scale manufacturing. Because, Burki, Khan, and Bratsberg [1997] have found the presence of allocative inefficiency in the manufacturing sectors of both Pakistan and India. Given their evidence, the estimates of substitution elasticities, based on functional forms that do not control for allocative inefficiency are expected to be biased. Such biases may also result if we detect biases of technological changes in the data and fail to control for it while estimating elasticities.

In this backdrop, we present a brief overview of the existing studies to identify the gaps that could be filled by the present study.

Kazi *et al.* [1976] have estimated the CES production function with time series and cross sectional data to examine production relations in the manufacturing industries of Pakistan. The study provides evidence that the time series estimates have comparatively lower elasticity values than the cross sectional data. Similarly, Kemal

[1981,1982] examined the substitution elasticities between capital and labor in the manufacturing sector of Pakistan. He also used the CES and VES production functions and found that the elasticities of substitution between capital and labor was 0.58 and 0.67, respectively.

Battese and Malik [1987] used CES production function for the estimation of elasticities of substitution by using data on selected manufacturing industries of Pakistan for the years 1969-70 to 1976-77. Their estimates of elasticities between capital and labor were 1.31 and 1.02 under the assumptions of constant return to scale, and variable return to scale, respectively. Later, Battese and Malik [1988] estimated elasticities of substitution for CES and VES production functions by using firm level data for food processing industries in Pakistan. The estimates of elasticities under constant return to scale were 0.82 and 0.79 for CES and VES production functions, respectively.

Khan [1989] has utilized the nested CES production function to calculate the elasticities of substitution between three inputs capital, labor and energy. The two level or Nested CES production function was estimated for the manufacturing sector of Pakistan covering the time period from 1959-60 to 1982-83. Elasticity of substitution between capital and energy was 0.175, while the elasticity of substitution between capital and labor was 0.48. Regarding technical progress he found that manufacturing sector experienced disembodied technical progress at the rate of 3.7 percent per annum.

Malik *et al.* [1989] analyzed the production relations in the large scale textile manufacturing sector of Pakistan. The authors used the provinces vide pooled data from 1969-70 to 1980-81, to estimate the CES and VES production functions. Comparison of these two results showed that under the assumption of constant return to scale CES production function adequately explain the underlying production structure.

Zahid *et al.* [1992] estimated the elasticities of substitution in Pakistan 's large scale manufacturing sector to determine the potential for switching to relatively more labor intensive production techniques. For this purpose they used the CES production function. They found that the manufacturing sector in Pakistan appears to be characterized by varying return to scales and elasticities of substitution for all industries are between zero and one, with the exception of drugs and pharmaceuticals industries. For the majority of the industries , the value of return to scale parameter is between 0.75 and one.

Battese *et al.* [1993] have estimated the elasticities of substitution between factor inputs by utilizing the CES production function under the assumption of constant return to scale. The elasticity of substitution between capital and labor was highest for the year 1975-76, that is 2.22. The estimate of elasticity of substitution for the pooled data from 1969-70 to 1986-87 was 1.31.

Khan and Rafiq [1993] have estimated the Nested CES production function to estimate substitution possibilities among labor, capital, imported raw materials and bank credit in the manufacturing sector of Pakistan. The study covered the time period from 1972-73 to 1990-91. The authors found that the elasticity of substitution between capital and labor was 0.62, while the elasticity of substitution between the extended working capital and labor is less than one, i. e. -0.663.

All the above studies utilized the Cobb-Douglas or CES or VES or nested CES production functions, which have limited power to explain the substitution possibilities between the factor of production. In case of Cobb-Douglas production function, elasticities of substitution is one and in its original form it is assumed that the production function is subject to constant returns to scale which is a restrictive assumption. Therefore, in case of Cobb-Douglas production function the researcher knows the values of some parameters before estimation and thus remains little to estimate. Both CES and VES production functions are nonlinear and cannot be estimated by linear regressions. CES production function has a constant value of Allen Elasticity of substitution equal to σ for all levels of output and inputs. A somewhat surprising and useful discovery regarding the estimate of σ is that there is a systemic variation in σ which is solely due to the choice of equation to be estimated.[11] In CES production function, the elasticity of substitution is invariant with respect to changes in capital labor ratio. However in case of VES production function elasticity of substitution varies with the change in capital labor ratio. From the above discussion, we conclude that all the above production functions are subject to very rigid assumptions and provide limited information about the substitution possibilities.

Mahmmod [1989] used translog cost function to estimate derived demand for factors and their elasticities of substitution. He found that energy and capital were good complements to each other and raw material was turned out to be substitute to all other factor inputs, while capital and labor are good-substitute to each other.

Naqvi and Kemal [1989] Compared the level of efficiency in public and private enterprises producing similar goods. The study showed that some public enterprises face losses while most of them made sufficiently large profits and their high rates of profits are not due to high rates of protection, but because, the average rate of effective

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[11] See, Barten [1969].

protection was for public sector industries were lower than that for industries in the private sector. Their strongest result was that "there is nothing inherently good or bad about the public sector, or even about the private sector for that matter."

Shamim and Annice [1991] examined the efficiency analysis of projects in the Economy of Pakistan. They attempted to evaluate the efficiency of large scale industrial projects in the public sector. The empirical evidence showed that the industries are not performing as well as they showed.

Mahmood [1992] has examined the effects of change in government pricing policy and external price shocks on factor demand by utilizing the concept of elasticities of factor substitution and price elasticities of factor demand. To derive the estimates of the elasticities a tronslog cost function is used covering the time period 1954-55 to 1985-86. In this study the estimates of scale economies showed that the large scale manufacturing sector of Pakistan has realized growing economies of scale with industrialization. The study also showed that non-production workers were highly substitute with energy and capital. Energy and capital used to be complementary factors become low substitutes, while the price elasticities of factors demands are relatively inelastic.

Idrees [1997] has estimated the translog and Generalized cost functions. The study used the pooled data for the Provence of Punjab and Sindh for the years 1969-70 to 1990-91.The estimates of the Allen Elasticities of substitution for Translog and Generalized cost functions were very interesting. He found capital, labor, and energy all were substitute of each other in case of Generalized cost function, while labor and capital were complements to each other in case of translog cost function. Energy and capital were good-substitute, when he used Generalized cost function and were weak substitute in case of translog cost function. In both cases there was no evidence of energy / capital complementarity.

The studies used the flexible functional forms such as translog or Generalized Lointief assumed the classical assumptions regarding the factor markets. Most of the studies are confined to measure elasticities of substitutions and technological biases. As we have already mentioned that there is no reason to assume that factor prices are not distort in the LDCs and also in Pakistan. In present study we will start with flexible functional farms, i.e., translog cost function, and then we will make three modifications in this model. In first attempt we will incorporate technological biases in translog cost function. In second attempt we will derive an actual cost function based on translog cost functional form as a function of shadow prices or in other words, we will modify the translog cost function to accommodate the relative price inefficiencies. In final case we will combine the above two modifications to obtain more reliable information than the simple flexible functional form.

Chapter

3

Theoretical Framework

This chapter lays out the theoretical framework to estimate allocative inefficiencies in Pakistan's large scale manufacturing along with technological change. We start with the simple translog cost function assuming that the classical assumption of perfect competition holds and that there is no allocative inefficiencies and technological change. Then we incorporate these two assumptions in the translog cost function one by one, to analyze the effects of each separately. Finally, we modify the translog cost function to take into account not only allocative inefficiencies but also technological changes.

3.1 The Translog Cost Function

We use the translog cost function which provides a convenient second-order approximation to an arbitrary twice differentiable cost function.[1] It also allows the scale economies to vary with the level of output. For a translog cost function, we denote factor prices by P_i and their respective quantities by X_i and the level of output by Q, while total cost denoted by C equates $\sum P_i X_i$. Symbolically, translog cost function is written as

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[1] For details, see Christensen, Jorgenson and Lau [1971, 1973], Diewert [1974], and Lau [1974].

$$\begin{aligned}\ln C &= \alpha_0 + \gamma_Q \ln(Q) + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\ &+ \sum_i \alpha_i \ln(p_i) + \sum_i \gamma_{iQ} \ln(Q)\ln(p_i) \\ &+ \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln(p_i)\ln(p_j) \\ &\qquad \text{for all } i, j = 1, 2, \ldots, n.\end{aligned} \tag{3.1}$$

Where *i* and *j* denote factors.

Symmetry on the cross price effect implies

$$\gamma_{ij} = \gamma_{ji} \tag{3.2}$$

The restrictions required for theoretical consistency of the cost function and other structural properties are also imposed. For example, linear homogeneity in factor prices implies that holding output as constant, total cost should increase proportionally when all factor prices increase. Symbolically, it imposes the following restrictions on the parameters

$$\begin{aligned}&\sum_i \alpha_i = 1 \qquad\qquad \sum_i \gamma_{iQ} = 0 \\ &\sum_i \gamma_{ij} = \sum_j \gamma_{ji} = \sum_i \sum_j \gamma_{ij} = 0\end{aligned} \tag{3.3}$$

Cost minimizing factor demands are obtained from the cost function by using the Shephard's Lemma and transforming the resulting expression into share equations. We know that

$$\frac{\partial \ln C}{\partial \ln p_i} = \frac{\partial C}{\partial P_i}\,\frac{P_i}{C}$$

By Shephard's Lemma, $\partial C / \partial P_i = X_i$. Substituting it in the above expression, we obtain

$$\frac{\partial \ln C}{\partial \ln P_i} = \frac{P_i X_i}{C} = S_i$$

Which is our share equation. Thus, form (3.1) it implies

$$\frac{\partial \ln C}{\partial \ln P_i} = S_i = \alpha_i + \sum_j \gamma_{ij} \ln(P_j) + \sum_i \gamma_{iQ} \ln(Q). \tag{3.4}$$

The restrictions of linear homogeneity in prices (3.3) ensure that the input demand functions are homogeneous of degree zero in input prices while the cost function is homogeneous of degree one. We will estimate the total cost Eq. (3.1) along with the cost share Eqs. (3.4) with linear homogeneity in prices (3.3) imposed.

The flexible cost function does not satisfy *a priori* the properties of monotonicity and concavity in factor prices and thus they need to be tested. Therefore, we will test monotonicity and concavity conditions.

A cost function is said to be monotonically increasing in prices if

$$C(Q, P_i) > C(Q, \overline{P_i}) \qquad \text{where} \quad P_i > \overline{P}_i.$$

It implies that $\partial C / \partial P_j > 0$. We can write this condition as

$$\frac{\partial C}{\partial P_j} = \frac{\partial \ln C}{\partial \ln P_j} . \frac{C}{P_j} > 0 \tag{3.5}$$

We know from above that $\partial \ln C \ / \ \partial \ln P_j = S_j$. Substituting this expression in (3.5) we get an equivalent expression as

$$\frac{\partial C}{\partial P_j} = \frac{S_j \, C}{P_j} \quad > 0 \tag{3.6}$$

From (3.6) it appears that monotonicity in prices holds *if and only if* cost shares are positive, since total cost (*C*) and factor prices (P_i) are positive by definition.

Similarly, monotonicity in output requires that the partial derivative of total cost function with respect to output is positive. We can translate this condition for translog cost function as

$$\frac{\partial \ln C}{\partial \ln Q} = \gamma_Q + \gamma_{QQ} \ \ln(Q) + \sum_i \gamma_{iQ} \ \ln(P_i) > 0 \tag{3.7}$$

The monotonicity conditions in Eqs. (3.6) and (3.7) may or may not hold in our data set. If these conditions do not hold then we will impose them.

The curvature condition for the cost function requires that the Hessian matrix, *H*, of the second order partial derivatives with respect to factor prices should be negative semi-definite. The symmetric Hessian matrix, *H*, has $\partial^2 C \ / \ \partial P_i^2$ as diagonal elements and $\partial^2 C \ / \ \partial P_i \, \partial P_j \ \ (i \neq j)$ as off-diagonal elements. In matrix form the curvature condition for four inputs is written as[2]

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[2] For a proof, see Appendix.

$$H = \begin{bmatrix} \gamma_{EE} + \alpha_E^2 - \alpha_E & \gamma_{EK} + \alpha_E \alpha_K & \gamma_{EL} + \alpha_E \alpha_L & \gamma_{EM} + \alpha_M \alpha_M \\ \gamma_{EK} + \alpha_E \alpha_K & \gamma_{KK} + \alpha_K^2 - \alpha_K & \gamma_{KL} + \alpha_K \alpha_L & \gamma_{EM} + \alpha \, E \alpha_M \\ \gamma_{EL} + \alpha_E \alpha_M & \gamma_{KL} + \alpha_K \alpha_L & \gamma_{LL} + \alpha_L^2 - \alpha_L & \gamma_{LM} + \alpha_L \alpha_M \\ \gamma_{EM} + \alpha_E \alpha_M & \gamma_{KM} + \alpha_K \alpha_M & \gamma_{LM} + \alpha_L \alpha_M & \gamma_{MM} + \alpha_M^2 - \alpha_M \end{bmatrix}$$

The necessary and sufficient condition for concavity is that the Hessian matrix must alternate in signs as $H_{11} \leq 0$, $H_{22} \geq 0$, $H_{33} \leq 0$ and $H_{44} \geq 0$.We will test and impose curvature condition in case of violation.

3.2 Specification of Formulas

The returns to scale measure the relationship between total cost and output along the expansion path. The elasticity of total cost with respect to output is given by

$$\frac{\partial \ln C}{\partial \ln Q} = \gamma_Q + \gamma_{QQ} \ln(Q) + \sum_i \gamma_{iQ} \ln(P_i) > 0 \tag{3.8}$$

The scale economies are defined as unity minus the elasticity of total cost with respect to output.

$$SE = 1 - \frac{\partial \ln C}{\partial \ln Q} \tag{3.9}$$

Scale economies are independent of factor prices if and only if the production function is homothetic. The production function can be restricted to be homothetic if and only if the cost function can be written as a separable function in prices and output.

The homotheticity restrictions for a translog cost function are,

$$\gamma_{iQ} = 0 \tag{3.10}$$

The production function can be further restricted to be homogeneous of a constant degree if and only if the elasticity of total cost with respect to output is constant. This condition requires that in addition to linear homogeneity in prices and output, and homotheticity, the following must hold.

$$\gamma_{iQ} = 0\,, \quad \gamma_{QQ} = 0 \tag{3.11}$$

Elasticities of Demand and Substitution

The own price elasticity of demand for input *i* with respect to its market price can be calculated as

$$E_{ii} = \frac{S_i\,(S_i - 1) + \gamma_{ii}}{S_i} \tag{3.12}$$

The cross price elasticity of demand is defined as

$$E_{ij} = \frac{S_i\,S_j + \gamma_{ij}}{S_i} \quad \text{for all } i \neq j \tag{3.13}$$

The Allen partial elasticities of substitution are defined as[3]

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[3] For further details on own price and Allen partial elasticities of substitution, see Binswanger [1974].

$$AES = \sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i\, S_j} \quad \text{for all} \quad i \neq j \tag{3.14}$$

Morishima Elasticities of Substitution

Blackorby and Russell [1989] have shown that Allen partial elasticities of substitution do not preserve the salient properties of the Hicksian notion. In particular, they note that Allen partial elasticity of substitution (i) is not a measure of the "ease" of substitution, or curvature of isoquant, (ii) provides no information about relative factor shares for which the elasticity of substitution was originally defined, and (iii) cannot be interpreted as a derivative of a quantity ratio with respect to a price ratio. In short, AES are completely uninformative. Blackorby and Russell [1989] present an alternative concept called the Morishima elasticities of substitution (MES)[4] given by

$$MES = \eta_{ij} = E_{ji} - E_{ii} \tag{3.15}$$

Where E_{ji} and E_{ii} are cross and own price elasticities. We will also measure Morishima elasticities and compare them with AES.

3.3 Modeling Technological Change

Technological change refers to the change in the production process due to the application of scientific knowledge. These changes in the production process can be analyzed in various ways. In our case, the effects of technological change can be

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[4] For further details on Morishima elasticities, see Blackorby and Russell [1989].

expressed in terms of reduction in the cost of production. A very simple way to introduce disembodied technological change in a cost structure is to make parameters of the cost function dependent on time. In this way we can analyze the technological changes, speed of technological change, presence of factor bias and the changing pattern of substitution elasticities. We use the translog cost function incorporating the biases of technological change. Therefore we define the cost function as $C = C(Q, P, T)$, where *C, Q, P* and *T* denote the cost, output, vector of factor prices and time, respectively. Symbolically, a translog cost function incorporating the effects of disembodied technological progress is written as

$$
\begin{aligned}
\ln C = {} & \alpha_0 + \gamma_Q \ln(Q) + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\
& + \sum_i \alpha_i \ln(p_i) + \sum_i \gamma_{iQ} \ln(Q) \ln(p_i) \\
& + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln(p_i) \ln(p_j) + \alpha_t T \\
& + \frac{1}{2}\alpha_{tt} T^2 + \sum_i \alpha_{it} \ln(P_i) T + \gamma_{Qt} \ln(Q) T
\end{aligned}
\tag{3.16}
$$

Where *i* and *j* denote factors, and *T* denotes time. To impose linear homogeneity in prices we add a restriction $\sum_i \alpha_{it} = 0$ to (3.3) above, while the symmetry condition remains as in (3.2) above.

Now the share equations become

$$
\frac{\partial \ln C}{\partial \ln P_i} = S_i = \alpha_i + \alpha_{it} T + \sum_j \gamma_{ij} \ln(P_j) + \sum_i \gamma_{iQ} \ln(Q) \tag{3.17}
$$

The formulas for elasticities remains unchanged as stated in Eqs. (3.12) through (3.15). In addition, the inclusion of time as a variable allows us to comment on the rate of technological change and total factor productivity (TFP). Differentiating (3.16) with respect to time gives

$$\frac{d \ln C}{d T} = \sum_i \frac{\partial \ln C}{\partial \ln P_i} \frac{d \ln P_i}{d T} + \frac{\partial \ln C}{\partial \ln Q} \frac{d \ln Q}{d T} + \frac{\partial \ln C}{\partial T} \tag{3.18}$$

By substituting the Shephard's Lemma $X_i = \partial C(Q, P, T) / \partial P_i$, and $S_i = \partial \ln C / \partial \ln P_i$, in the Eqs (3.18), and after rearranging

$$- \frac{\partial \ln C}{\partial T} = \sum_i S_i \frac{d \ln p_i}{d T} + \frac{\partial \ln C}{\partial \ln Q} \frac{d \ln Q}{d T} - \frac{d \ln C}{d T} \tag{3.19}$$

Where $- \partial \ln C / \partial T$ is defined as the *dual rate of technological change*. Eq. (3.19) shows that the dual rate equals an index of the rate of change in factor prices plus a scale effect minus the rate of change of total cost.

Note that the primal and dual rates of technological change are equal if and only if technology exhibits constant returns to scale and in that case the dual rate of technological change is also called the total factor productivity growth. In our multiple-input production process, technological change can affect the input productivity and factor utilization differentially. This observation will lead to the distinction between *neutral* and *biased* technological change.

3.4 Modeling Allocative Efficiency

First we use a translog cost function where we implicitly assume that there is no technological progress and manufacturing sector makes its decisions on market prices.

A translog cost function also assumes that manufacturing sector minimizes costs subject to an output constraint only. While a translog cost function incorporating the technological change also assume that the manufacturing sector minimize cost subject to output and market prices are imposed as a maintained hypothesis. Now we propose a generalized cost function based on a translog cost function. Here the manufacturing sector is assumed to base their production decision on unobservable shadow prices which reflect the effects of regulation on the effective prices of input. In other words here we not only take into account the output constraint but the constraints imposed by the regulatory environment also. The existence of the regulatory environment constraint may result in failure of cost minimize condition. We attempt to incorporate an explicit presentation of regulatory environment by using simple parametric approximation of the shadow prices as originally suggested by Lau and Yotopoulos [1971] in a different context and by Atkinson and Halvorsen [1984] in the same context. The shadow price for input j, P_j^*, is approximated by

$$P_j^* = k_j p_j \tag{3.20}$$

Where the factor of proportionality, k_j is input specific. Expression (3.20) can be treated as a first order Taylor's series expansion of an arbitrary shadow price function, $g_j(p_j)$,which has the properties $g(0) = 0$ and $\partial g_j(p_j) / \partial p_j \geq 0$. Using this shadow price approach, we derive a cost function which makes it possible to minimize costs subject to shadow and market prices.

Shadow and Actual Cost Functions

Production units are assumed to choose inputs in such a way that minimize their shadow costs, $\sum_i (k_i p_i) X_i$, for the given level of output. Production unit's total shadow cost function is defined as

$$C^S = C^S(KP, Q) \tag{3.21}$$

Where KP is a vector of input specific shadow prices. We can derive the actual input demand functions from the shadow cost function by applying the Shephard's Lemma

$$\frac{\partial C^S}{\partial k_i P_i} = X_i \tag{3.22}$$

Now production unit's total cost is defined as

$$C^A = \sum_i P_i X_i \tag{3.23}$$

To express the total actual cost as a function of shadow cost, we substitute the value of X_i from Eq. (3.22) in Eq. (3.23) and get

$$C^A = \sum_i P_i \frac{\partial C^S}{\partial k_i P_i} \tag{3.24}$$

By specifying an appropriate functional form for the shadow cost function, we derive a parametric expression for the production unit's total actual costs.

The subsequent notation in the above equation can be simplified by defining M_i^S, as the shadow cost share of input i

$$M_i^S \equiv \frac{k_i P_i X_i}{C^S} \tag{3.25}$$

rearrange the above equation as

$$X_i = M_i^S C^S (k_i P_i)^{-1} \tag{3.26}$$

Substituting (3.26) in Eq. (3.23), total actual cost is

$$C^A = C^S \sum_i k_i^{-1} M_i^S \tag{3.27}$$

Taking the logarithm on both sides, we obtain

$$\ln C^A = \ln C^S + \ln \sum_i k_i^{-1} M_i^S \tag{3.28}$$

We can write the translog cost function as shadow cost function as

$$\begin{aligned} \ln C^S = {} & \alpha_0 + \alpha_Q \ln (Q) + \frac{1}{2} \gamma_{QQ} (\ln Q)^2 \\ & + \sum_i \gamma_{iQ} \ln Q \ln (k_i P_i) + \sum_i \alpha_i \ln (k_i P_i) \\ & + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln (k_i P_i) \ln (k_j P_j) \end{aligned} \tag{3.29}$$

where C^S denotes shadow costs and $\gamma_{ij} = \gamma_{ji}$. Holding output as constant, total shadow cost should increase proportionally when all shadow prices increase proportionally. To impose linear homogeneity in prices we have same restrictions on the parameters as given in Eq. (3.3).

To obtain the shadow cost shares M_i^S , we differentiate Eq. (3.29) with respect to shadow prices and get

$$\begin{aligned} \frac{\partial lnC^S}{\partial \ln(k_iP_i)} &= \frac{k_iP_i}{C^S}\frac{\partial C^S}{\partial k_iP_i} = \frac{k_iP_iX_i}{C^S} = M_i^S \\ M_i^S &= \alpha_i + \sum_i \gamma_{ij}\ln(k_j P_j) + \gamma_{iQ}\ln(Q) \end{aligned} \tag{3.30}$$

substituting (3.29) and (3.30) in (3.28) yields the total actual cost function

$$\begin{aligned} \ln C^A &= \alpha_0 + \alpha_Q \ln(Q) + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\ &+ \sum_i \gamma_{iQ}\ln Q \ln(k_i P_i) + \sum_i \alpha_i \ln(k_i P_i) \\ &+ \frac{1}{2}\sum_i \sum_j \gamma_{ij}\ln(k_i P_i)\ln(k_j P_j) + \\ &+ \ln\left[\sum_i k_i^{-1}(\alpha_i + \sum_j \gamma_{ij}\ln(K_j P_j) + \gamma_{iQ}\ln Q)\right] \end{aligned} \tag{3.31}$$

Note that if $k_i = k_j$ for all *i, j,* the total actual cost function (3.31) reduces to its total shadow cost function (3.29), which in turn is equivalent to the translog cost function.

Additional degree of freedom can be obtained by estimating the actual cost share equations together with its total actual cost function. The actual cost share for input *i* is

$$M_i^A = \frac{P_i X_i}{C^A} \tag{3.32}$$

Substituting in Eq. (3.32) for X_i and C^A from Eqs. (3.26) and (3.27) respectively, we get

$$M_i^A = \frac{M_i^S k_i^{-1}}{\sum_i M_i^S k_i^{-1}} \tag{3.33}$$

Substituting in Eq. (3.33) for M_i^S from Eq. (3.30) we get

$$M_i^A = \frac{[\alpha_i + \sum_j \gamma_{ij} \ln(k_j P_j) + \gamma_{iQ} \ln(Q)]\, k_i^{-1}}{\sum_i [\alpha_i + \sum_j \gamma_{ij} \ln(k_j P_j) + \gamma_{iQ} \ln(Q)]\, k_i^{-1}} \tag{3.34}$$

Equations (3.31) and (3.34) will be estimated by appending a classical error term to each equation to reflect errors in shadow cost minimizing behavior.

Parametric Efficiency Tests

Relative price efficiency exists if marginal rates of technical substitution (MRTS) are equated to the corresponding ratios of market prices for inputs, and absolute price efficiency exists if the value of marginal product for each input is equated to the input's market price. Relative price efficiency with respect to all pairs of inputs implies that output is produced at minimum cost, while absolute price efficiency implies both cost minimization and production at the efficient level of output.

In our model relative price efficiency with respect to inputs *i* and *j* exists if and only if $k_i = k_j$ and absolute price efficiency exists if and only if $k_i = 1$ for all *i*. Total actual cost and cost share equations are homogeneous of degree zero in the k_i's. Therefore, the absolute value of k_i's cannot be estimated and consistent with economic theory. This means that the generalized cost function cannot be used to test for absolute price efficiency. However, the relative values of k_i's can be estimated after an appropriate normalization. Therefore, the generalized cost function can be used to test for relative price efficiency.

Economies of Scale

Returns to scale are appropriately measured by the relationship between total cost and output along the expansion path. In our case, two concepts of returns to scale are relevant. The relationship between total actual cost and output corresponds to the usual definition of returns to scale. On the other hand, the relationship between total shadow cost and output is relevant concept for evaluating the optimal scale, because it is total shadow cost rather than total actual cost that manufacturing sector is assumed to minimize.

The elasticity of total actual cost with respect to output is given by

$$\frac{\partial \ln C^A}{\partial \ln Q} = \gamma_Q + \gamma_{QQ} \ln (Q) + \sum_i \gamma_{iQ} \ln (k_i P_i) + \frac{\sum_i \gamma_{iQ} k_i^{-1}}{\sum_i M_i^S k_i^{-1}} \quad (3.35)$$

The formula for the elasticity of total shadow cost with respect to output differs in that it does not include the last term in Eq. (3.35). Since this term may be positive or negative, therefore the elasticity for total shadow cost may be either smaller or larger than the elasticity for total actual cost.

Scale economies, SE, are defined as unity minus the elasticity of total costs with respect to output.

$$SE^{A} \equiv 1.0 - \frac{\partial \ln C^{A}}{\partial \ln Q}, \qquad SE^{S} \equiv 1.0 - \frac{\partial \ln C^{S}}{\partial \ln Q} \tag{3.36}$$

Multiplication by 100 yields estimates of scale economies expressed in percentages.

The expressions for own and cross price elasticities, and for the elasticities of substitution are the same as described in Eqs.(3.12) through (3.15).

3.5 Modeling Allocative Efficiency and Technological Change

In previous model we incorporate the parametric tests for Allocative inefficiency but we assume that there is no biases of technological change. Technological changes in a production process comes from the application of scientific knowledge. Through improved methods of utilizing existing resources such that a higher output rate per unit of input is obtained, often referred to as technological change. We incorporate such changes in our model by introducing a time variable. As for Allocative inefficiency is concerned, we use the same method as discussed under the heading " Modeling Allocative Efficiency ." To introduce technological change in this model we make the following modifications.

Shadow and Actual Cost Functions with Technological change

To incorporate the technological change in total shadow cost function we modify the Eq. (3.21) as

$$C^S = C^S(KP, Q, T) \tag{3.37}$$

Where KP is a vector of input specific shadow prices, Q dentes the output and T denotes the time. Eqs. (3.22) through (3.28) are same. The translog shadow cost function incorporating the effects of technological change can be written as

$$\begin{aligned} \ln C^S &= \alpha_0 + \alpha_Q \ln(Q) + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\ &+ \sum_i \gamma_{iQ} \ln Q \ln(k_i P_i) + \sum_i \alpha_i \ln(k_i P_i) \\ &+ \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln(k_i P_i) \ln(k_j P_j) + \alpha_t T \\ &+ \frac{1}{2}\alpha_{tt} T^2 + \sum_i \alpha_{it} \ln(k_i P_i) T + \gamma_{Qt} \ln(Q) T \\ & \qquad i, j = E, K, L, M. \end{aligned} \tag{3.38}$$

Where $\gamma_{ij} = \gamma_{ji}$. To impose linear homogeneity in prices we add a restriction $\sum_i \alpha_{it} = 0$ to (3.3) above while the symmetry condition remains same as in (3.2) above.

Now the shadow share equations become

$$\frac{\partial \ln C^S}{\partial \ln(k_i P_i)} = M_i^S = \alpha_i + \sum_i \alpha_{it} T + \sum_i \gamma_{ij} \ln(k_j P_j) + \gamma_{iQ} \ln(Q) \tag{3.39}$$

Now substituting the equations (3.38) and (3.39) for shadow cost function and shadow share in equation (3.28), which will yields the total actual cost function

$$
\begin{aligned}
\ln C^A = {} & \alpha_0 + \alpha_Q \ln(Q) + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\
& + \sum_i \gamma_{iQ} \ln Q \ln(k_i P_i) + \sum_i \alpha_i \ln(k_i P_i) \\
& + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln(k_i P_i) \ln(k_j P_j) + \alpha_t T \\
& + \frac{1}{2}\alpha_{tt} T^2 + \sum_i \alpha_{it} \ln(k_i P_i) T + \gamma_{Qt} \ln(Q) T \\
& + \ln\left[\sum_i k_i^{-1}(\alpha_i + \alpha_{it} T + \sum_j \gamma_{ij} \ln(K_j P_j) + \gamma_{iQ} \ln Q)\right]
\end{aligned}
\tag{3.40}
$$

Note if $k_i = k_j$ for all *i, j,* the actual total cost function (3.40), reduces to its total shadow cost function (3.38).

The actual cost share equations related with actual total cost function (3.40) are obtained by substituting the shadow cost share Eqs. (3.39) in equation (3.33) as

$$
M_i^A = \frac{[\alpha_i + \alpha_{it} T + \sum_j \gamma_{ij} \ln(k_j P_j) + \gamma_{iQ} \ln(Q)]\, k_i^{-1}}{\sum_i [\alpha_i + \alpha_{it} T + \sum_j \gamma_{ij} \ln(k_j P_j) + \gamma_{iQ} \ln(Q)]\, k_i^{-1}}
\tag{3.41}
$$

We will estimate Eqs. (3.40) and (3.41) that is total actual cost equation and actual cost share equations respectively by appending a classical error term to each equation to reflect error in shadow cost minimizing behavior.

All the parametric efficiency tests are same as discussed in the previous model. Economies of scale will change due to change in the elasticity of total actual cost with respect to output as

$$\frac{\partial \ln C^A}{\partial \ln Q} = \gamma_Q + \gamma_{Qt} T + \gamma_{QQ} \ln(Q) + \sum_i \gamma_{iQ} \ln(k_i P_i) + \frac{\sum_i \gamma_{iQ} k_i^{-1}}{\sum_i M_i^S k_i^{-1}} \quad (3.42)$$

$$\text{for all} \quad i = E, K, L, M.$$

Scale economies and elasticities formulas for actual and shadow costs functions are same as in equation (3.36), while the formulas for elasticities remains unchanged as stated in Eqs.(3.12) through (3.15). In addition to these we will also differentiate the total shadow cost function with respect to time as in Eq. (3.19) to obtain the dual rate of technological change. Under the assumption of constant return to scale the dual rate of technological change will become the total factor productivity.

Chapter

4

Data and the Estimation Procedure

The primary data source for Pakistan's large scale manufacturing industries is the periodical *Census of Manufacturing Industries* (CMIs). We use the data from CMIs, while supplementary information is obtained from various issues of the *Economic Survey* (Government of Pakistan [1997]) and *Monthly Statistical Bulletin.* We pool the provincial level data of Punjab and Sindh, which represents more than 80% of total manufacturing industries of Pakistan. We implicitly assume by pooling provincial data that firms in both provinces are characterized by similar production technologies.[1]

4.1 Construction of Variables

Data on prices and quantities of energy, capital, labor, and raw material are used to obtain total cost of production. We use the Divisia quantity and price index, where aggregation was needed, due to its desirable properties.[2]

We calculate the energy price index by aggregating various energy sources used

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[1] Battese and Malik [1987], Malik [1989], Khawaja [1991], and Idrees [1997] have also used pooled data.

[2] Divisia indices have several desirable properties. One, they are chain-linked Laspeyres indices, that is, for each year the current prices are used as a base in estimating the rate of growth to the next year. Two, they are also chained Paasche and Fisher indices. Three, they are symmetric in prices and quantities. Four, reversing the time and comuting the indices backwards, will obtain the same result. Last, if we form indices of subgroups of the prices and quantities and combine them by using Divisia mehtod, the resulting index is same as aggregating in one step from the original series. For more details, see Jorgenson and Griliches [1971], and Diewert [1976].

in manufacturing industries. They includes fire wood, coal, coke, charcoal, kerosene oil, petrol, furnace oil, diesel oil, natural gas and electricity. The measurement scale for the variety of energy sources have always posed difficulty for researchers to bring them on a common scale. The construction of British Thermal Units (BTU) is also a step in that direction. However, it has been found that Divisia energy index is superior due to some obvious problems in using the BTU index.[3]

The CMIs only report quantities used and values of various energy sources. We obtain prices by dividing values with the respective quantities. Electricity data is available on electricity purchased and electricity generated. Due to incomplete information on the values of generated electricity, we multiply the quantity units the price of purchased electricity to obtain value. The price of purchased electricity is obtained by dividing the cost of purchased electricity with its quantity.

The stock of capital is calculated by the perpetual inventory method while the *user cost of capital* is calculated as

$$P_K = P_M (r + \delta - \pi_M) \tag{4.1}$$

where P_K is the user cost of capital, P_M is the price index of capital goods, r is the real rate of interest, δ is the depreciation rate and π_M is the rate of growth in the price index of capital. Therefore, the user cost of capital is an increasing function of the price of capital, real rate of interest and the depreciation rate while a decreasing function of the appreciation in the value of capital.

To calculate the user cost of capital, supplementary data from *Monthly Statistical*

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[3] See, Nguyen, [1987].

Bulletin on price of the machinery from was also employed, which implicitly assume that this price is same for both Punjab and Sindh. The interest rate is the average scheduled bank rate on long-term advances for manufacturing sector, reported in *Monthly Statistical Bulletin*. The depreciation rate is calculated by dividing total depreciation with the value of fixed assets at the beginning of the year.

The capital stock series is calculated from a gross investment series by using a perpetual inventory method, and a constant rate of depreciation. To calculate the gross investment series, we take the difference between value of fixed assets[4] at the end of the year and at the beginning of the year, and then add the amount of depreciation in it. The data on value of fixed assets at the beginning and at the end of the year, and the amount of depreciation is given in CMIs. The constant rate of depreciation is obtained by taking the average of the ratio of total depreciation amount with the value of fixed assets at the beginning of the year. Now, let GI be the gross investment, K_t be the capital stock in time period t, and δ be the rate of depreciation. Then the series for capital stock is computed as

$$K_t = (1 - \delta) K_{t-1} + GI_{t-1} \qquad (4.2)$$

We use the value of fixed assets at the beginning of the year 1969-70 as the benchmark value for calculation of the capital stock series.

Total employment cost and average daily employment is readily available in CMI's. Total employment cost includes wages and salaries, other cash payments, non

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[4] Value of fixed assets includes the value of land and building, plant and machinery, transport equipment and fixed assets which are expected to have productive life of more than one year, and are used by the manufacturing industries.

cash benefits and the amount paid to contract labor. The amount paid to contract workers was not included in total cost of employment in CMI's for the years 1969-70 and 1970-71. Therefore, total cost of employment for these years are adjusted on the assumption that the wages paid to contract workers are the same as the regular workers. We also adjust average daily employment for the years 1969-70 and 1970-71. Then wages are calculated by dividing total employment cost by average number of workers employed daily and converting it into an index gives the price of labor.

Data on raw material cost is directly available in CMI. The raw material cost includes cost on indigenous and imported raw material. The raw material price is taken from *Economic Survey* , which is converted into price index .

We compute real output by dividing value of production by the whole scale price index. The data on value of production[5] is from CMI while the data on wholesale price index is taken from various issues of the *Monthly Statistical Bulletin.*

Total cost (TC) is obtained by summing up the value of energy (V_1), the value of capital stock (V_2), total employment cost (V_3), and the value of raw material (V_4), as

$$TC = V_1 + V_2 + V_3 + V_4 \tag{4.3}$$

The input shares are obtained by dividing the cost of each input with total costs (TC) and denoted by M_i .

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[5] Value of production consists of the value of finished products, receipts for work done for others, receipts for repairs and maintenance, value of the sales of semi-finished products and by products, wastes and used goods, value of electricity sold, value of sales of goods purchased for resale, the net increase in the value of work in process and value of fixed assets produced by the establishment for its own use. Valuation is made at ex-factory prices which includes indirect taxes and exclude transport cost outside the factory gate.

$$M_i = \frac{V_i}{TC} \qquad for\ \ all \qquad i = E,\ K,\ L,\ M. \qquad (4.4)$$

4.2 The Estimation Procedure

We estimate parameters of the cost functions given in chapter 3 along with their share equations in equation system. We use the Iterative Zellner Efficient (IZEF) method proposed by Berndt and Christenson [1973a]. The first step in this procedure is to use the least square to estimate the system of equations and construct a consistent estimate of covariance matrix from the least square residuals. The estimator is computed by using the estimated covariance matrix in first step, and new covariance matrix estimate is constructed. This procedure continues to iterate from estimates of parameters to estimate the covariance matrix until convergence is achieved. In other words this procedure continues to iterate until the parameter estimates change less than a specified tolerance level, i.e., 0.001 in this study.

Since the cost shares satisfy the adding-up restriction, i.e., $\sum S_i = 1$, it means that the error sum to zero. Because of this, the error covariance matrix of the system of share equations becomes singular. While the SUR estimators require the inverse of the error covariance matrix, which will not exist for the share equation subsystem. To solve the problem of singularity, one of the share equations is dropped. We drop the share equation of raw material and recover its parameters with the help of adding-up restrictions. Because the IZEF estimates converge to MLEs, and the MLEs estimates are unique, it implies that the IZEF estimates are invariant to which equation is dropped.

Chapter

5

Estimating Elasticities Using the Translog Cost Function

In this chapter we present empirical results of simple translog cost function, without incorporating technological change and allocative inefficiencies. These results will enable us to obtain the estimates on returns to scale, and elasticities of demand and substitution under the assumption that the production units attain relative price efficiency and that there is no technological change bias. The results obtained will be comparable with some previous studies.

5.1 Results of the Translog Cost Function

The system of equations for simple translog cost function, Eq. (3.1), along with three out of four share equations, Eqs. (3.4), constrained to satisfy linear homogeneity in prices and symmetry, was estimated using iterative Zellner efficient (IZEF) method. Table 5.1 reports the estimated parameters along with their asymptotic *t*-statistics. Most of the estimated parameters are statistically significant at the conventional level.

The duality between the cost and production functions is satisfied only if symmetry, and linear homogeneity in prices, monotonicity and concavity conditions hold. All the factor shares evaluated at each observation were found to be positive, which implies that the monotonicity in prices is satisfied by our data.[1] Similarly, monotonicity

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[1] The values of factor shares at the mean of the data were 0.045, 0.209, 0.082, and 0.664 for energy, capital, labor and raw materials, respectively.

Table 5.1: ***Parameter Estimates Using the Translog Cost Function***

Parameter	Estimate	Asymptotic t-statistics
α_0	52.950	5.31^{*}
α_E	-0.221	-2.11^{*}
γ_{EE}	-0.019	-1.94^{**}
γ_{EK}	0.020	1.12
γ_{EL}	0.013	2.55^{*}
γ_{EM}	-0.014	-0.60
α_K	0.889	3.31^{*}
γ_{KK}	-0.229	-3.58^{*}
γ_{KL}	0.049	3.56^{*}
γ_{KM}	0.160	2.05^{*}
α_L	0.793	6.86^{*}
γ_{LL}	0.027	2.53^{*}
γ_{LM}	-0.088	-4.77^{*}
α_M	-0.467	-1.38
γ_{MM}	-0.058	-0.58
γ_Q	-5.060	-4.43^{*}
γ_{QQ}	0.346	5.28^{*}
γ_{EQ}	0.015	2.56^{*}
γ_{KQ}	-0.040	-2.58^{*}
γ_{LQ}	-0.041	-6.15^{*}
γ_{MQ}	0.066	3.37^{*}
χ^2	326.550	---
N	32	---

Note: * indicates significant at the 5% level and ** indicates significant at the 10% level.

in output, Eq. (3.7), also holds since the partial derivative, $\partial \ln C / \partial \ln Q$, was positive, i.e., 0.987.

The concavity of the cost in input prices was checked by determining, if the principal minors of the Hessian matrix are of the correct sign and find that this condition holds, since the Hessian matrix was negative semi-definite.[2]

Homotheticity and Homogeneity

The translog cost function does not impose homotheticity on the structure of production and is equivalent to homothetic production function if it can be written as a separable function in output and factor prices. It is well known that all homogeneous functions are homothetic while the converse is not true. The homotheticity of the production function was tested by imposing condition Eq. (3.10) on the cost function, but was rejected because the computed χ^2 test statistic, 27.578, was greater than the critical value of 13.28 at 0.01 level.[3] It follows that the underlying production function is not homogeneous. The rejection of homotheticity implies that there exists a systematic bias in favor of certain inputs as the scale of productivity expands. The coefficient of price-output variable (γ_{iQ}) can be used to study changes in input intensities as the level of output increases. It measures the change in the cost share of input *i* with respect to 1% increase in output while the prices of all other inputs are held constant. The value of γ_{iQ} would be positive (negative) if the intensity of input *i* increase (decreases) with the level

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[2] The calculated values of the principal minors of the Hessian Matrix are $H_{11} = 0$, $H_{22} = 1.1485$, $H_{33} = -1.0015$, and $H_{44} = 0.2574$.

[3] The computed test statistic is -2 ln λ, where $\lambda$is the ratio of the maximum value of log-likelihood function of the restricted system to the likelihood of the unrestricted system of equations.

of output. The estimates of the parameters reveal that the intensity of energy and raw material increases with the level of output while the intensity of labor and capital decreases with the level of output. These factor intensities change as shown below, when the model is modified to incorporate technological change, allocative inefficiency, or both.

5.2 Returns to Scale and Scale Economies

The translog cost function as specified in Eq. (3.1) is highly general, non homothetic function, which implies that the returns to scale are not constrained *a priori*. The returns to scale are computed as the inverse of the partial derivative of the cost function with respect to output and written as

$$\theta = (\partial \ln C / \partial \ln Q)^{-1} = \frac{1}{\partial \ln C \ / \ \partial \ln Q} \qquad (5.1)$$

The expression in the denominator is the same as given in Eq. (3.7). The estimate of the returns to scale at the mean of the data is 1.0127, which is statistically not different from one. We also estimate scale economies (*SE*) at the mean of the data by utilizing Eq. (3.9) and find that the scale economies are 0.0125 or 1.25 %. These results on scale economies are consistent with Mahmood [1992]. These results imply that with industrialization Pakistan's large scale manufacturing sector has realized very little economies of scale. How these results change when we incorporate technological change and allocative inefficiency or both will be explored in the following chapters.

5.3 Elasticities of Demand and Substitution

We use the parameter estimates of the translog cost function reported in a table 5.1 to estimate the own and cross price elasticities, and the elasticities of substitution. The estimates of the own and cross price elasticities of demand at the mean of the data, along with their asymptotic *t*-statistic are presented in a table 5.2. All the own price elasticities of demand are statistically different from zero at the 1% level. The elasticities for energy and capital are greater than one, while the other two elasticities are less then one. These results show that the demand for capital is most responsive to its price with an estimated own price elasticity of -1.89, while the raw material demand is least responsive with own price elasticity of -0.42.

Most of the cross price elasticities are statistically different from zero at the 0.05 or 0.10 level. It can be seen from the table 5.2 that all the cross price elasticities of demand are less than one except for capital /raw material. The results indicate that energy demand is inelastic or not much responsive to the price of other inputs. These results suggest that an increase in the price of energy (for instance, due to elimination of the subsidy on energy) would increase the demand for capital and labor, but an increase in the demand for labor will be less than the increase in capital. In other words, increase in price of energy generates fewer jobs than the use of capital. Increase in price of capital would generate fewer jobs as the cross price elasticity of demand between capital/labor is inelastic but statistically differ from zero. On the other hand, increase in price of labor will increase the demand for capital in a greater proportion. Similarly, interesting

Table 5.2: ***Matrix of Own and Cross Price Elasticities of Demand***

Variables	*Estimates of Elasticities*			
	Energy	Capital	Labor	Raw material
Energy	-1.365	0.653	0.367	0.345
	(-57.57)*	(1.63)**	(3.27)*	(0.65)
Capital	0.141	-1.89	0.315	1.432
	(1.68)**	(-28.10)*	(5.07)*	(3.61)*
Labor	0.201	0.801	-0.60	-0.41
	(3.28)*	(4.70)*	(-142.9)*	(-1.86)**
Raw Material	0.023	0.451	-0.05	-0.42
	(0.65)	(3.97)*	(-1.89)*	(-36.88)*

Note: Figures in parentheses are asymptotic t-values.
* indicates significant at the 5% level and ** indicates significant at the 10% level.

interpretation is attached with other elasticities, but they are also inelastic except capital/raw material.

To analyze substitution and complementary possibilities between the pairs of inputs, we use AES and MES, which serve the purpose well. The estimates of AES and MES are presented in a table 5.3 along with their asymptotic t-statistics. As can be seen from a panel (a) of the table 5.3 that the magnitudes of elasticities are generally higher as compared with other developing countries. The results show that energy and capital are good substitutes. Similarly, energy/labor and capital/labor are also substitutes. The economists do not seem to be in agreement on the issue of energy/capital complementarity. The empirical literature does not provide clear evidence on this debate. For example, Mahmood [1989], Chishti and Mahmud [1991] found that in Pakistan's large scale manufacturing energy and capital are strong complements to each other, while

Table 5.3: ***Allen and Morishima Elasticities of Substitution***

Elasticities	Estimates	Asymptotic t-statistic
(a) Allen Elasticities of Substitution		
σ_{EK}	3.126	1.67**
σ_{EL}	4.459	3.27*
σ_{EM}	0.520	0.65
σ_{KL}	3.834	5.07*
σ_{KM}	2.157	3.42*
σ_{LM}	-0.612	-1.89**
(b) Morishima Elasticities of Substitution		
η_{EK}	1.511	17.15*
η_{KE}	2.028	21.28*
η_{EL}	1.566	24.37*
η_{LE}	0.962	8.56*
η_{EM}	1.388	34.51*
η_{ME}	0.769	1.44
η_{KL}	2.689	17.29*
η_{LK}	0.911	14.56*
η_{KM}	2.338	13.78*
η_{MK}	1.855	4.54*
η_{LM}	0.545	20.24*
η_{ML}	0.017	0.08

Note: * indicates significant at the 5% level and ** indicates significant at the 10% level.

Khan [1989], Mahmood [1992], and Idrees [1997] found that energy and capital are substitutes consistent with evidence in the later studies, we find that energy and capital are good substitutes to each other when we use simple translog cost function. What will happen to this energy/capital complementarity in case of MES when we incorporate technological change, allocative inefficiency, and both will be explored in chapters 6 through 8. It is interesting to note that our results are consistent with the previous studies in respect of signs but not in magnitude.[4]

As discussed in chapter 3, the estimates on AES do not serve the purpose for which these are calculated. An alternative, but more relevant, concept of elasticities of substitution is MES. We report estimates of the MES with their asymptotic *t*-statistics in the panel (b) of a table 5.3. Most MES are statistically different from zero at the 0.05 or 0.10 level. In sharp contrast to the AES, the MES are asymmetric. Due to asymmetric nature of MES, the estimates of MES provide more information on substitution and complementarity possibilities than the AES because in case of MES we can also analyze more clearly which variable is more sensitive to change in price. The magnitudes of the MES show that the AES reveal over flexibility in the manufacturing sector of Pakistan. The MES estimates suggest that the elasticity of substitution between capital/energy is greater than the elasticity of substitution between energy/capital and positive signed is attached. So again there is no possibility of energy/capital complementarity in case of MES. The estimates also indicate that raw materials turn out to be a substitute with all other inputs. These results are in agreement with Hudson and Jorgenson [1974], and Mahmood [1989] and show that with the use of more energy, capital, and labor,

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[4] see Malik *et al.* [1989], Mahmood [1989].

manufacturing sector can economize on the use of raw material. Similarly, elasticity of substitution between capital/labor is greater than labor/capital. Therefore, it means that the change in price of capital will change the demand for energy and labor relatively more, while the converse is not true.

The above discussion provides the evidence that the results of the translog cost function are mostly consistent with the previous studies. However, we do not think that these results are reliable for policy recommendations or for other purposes, because this functional form does not probe the possibility of the existence of technological change and the relative price efficiency. Therefore, in the following chapters we relax these assumptions to evaluate the existence or otherwise of these assumptions.

Chapter

6

Technological Change and Substitutability

In chapter 5, we adopted the translog cost function with the assumption that relative price efficiency holds, and there is no technological change over time. In this chapter, we relax the assumption of technological change and evaluate its impact on our results in chapter 5. The technological change is an important factor that may be effecting production technology in Pakistan's large scale manufacturing sector.

6.1 Results with Technological Change

To allow for technological change, we estimate Eqs. (3.16) and (3.17), which are constrained to satisfy symmetry and linear homogeneity in prices. The parameter estimates are reported in table 6.1 along with their asymptotic *t*-statistic. As before, most of the parameters are statistically significant at the 0.05 or 0.10 level. The monotonicity condition was satisfied at each observation since all the estimated variable factors were positive.[1] The cost function was also montonically increasing in output , as the elasticity of total cost with respect to output was 1.0747. The concavity condition also holds as the principal minors of the Hessian matrix alternate in signs.[2]

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[1] The estimated factor shares at the mean of the data are $SE = 0.04506$, $SK = 0.20928$, $SL = 0.08221$, and $SM = 0.66344$.

[2] The computed principal minors of the Hessian matrix are, $H_{11} = -1.328$, $H_{22} = 7.4177$, $H_{33} = -0.6763$, and $H_{44} = 0$.

Table 6.1: ***Results with Technological Change***

Parameter	Estimate	Asymptotic *t*-statistics	Parameter	Estimate	Asymptotic *t*-statistics
α_0	-132.69	1.36	α_{Lt}	0.001	0.35
α_t	-0.809	-0.75	γ_{LL}	-0.001	-0.05
α_{tt}	-0.001	-0.09	γ_{LM}	-0.06	-2.39*
α_E	1.061	6.64*	α_M	-1.96	-2.36*
α_{Et}	0.007	7.83*	α_{Mt}	-0.01	-2.35*
γ_{EE}	-0.006	-1.05	γ_{MM}	-0.22	-3.19*
γ_{EK}	-0.012	-1.02	γ_Q	16.61	1.40
γ_{EL}	0.009	1.28*	γ_{QQ}	- 0.92	-1.28
γ_{EM}	0.009	0.62	γ_{Qt}	0.046	0.71
α_K	1.333	1.83**	γ_{EQ}	-0.062	-6.48*
α_{Kt}	0.003	0.82	γ_{KQ}	-0.067	-1.52
γ_{KK}	-0.305	-6.26*	γ_{LQ}	-0.026	-2.44*
γ_{KL}	0.049	2.94*	γ_{MQ}	0.156	3.14*
γ_{KM}	0.268	4.79*	χ^2	356.32	-
α_L	0.541	2.98*	N	32	-

Note: *indicates significant at the 5% and ** indicates significant at the 10 % level.

The homotheticity condition was rejected because the computed χ^2 statistic, 39.438, was greater than the critical value of 13.28 at the 0.01 level. Thus, there is a systematic bias in favor of certain inputs when scale of production changes. We find that the direction of factor intensity is changed when we introduce technological change in the cost structure. The intensity of raw material increases with the level of output, while the intensity of energy, capital, and labor decreases with the level of output.

6.2 Tests of Hypotheses

We conduct a number of tests to evaluate various characteristics of the production technology. These includes technological change, cost share neturality test and returns to scale and scale economies.[3]

Technological Change

To test the technological change we use the log-likelihood ratio test. The hypothesis of no technological change is rejected, because the computed test statistic 59.53 is greater than the critical value of 18.48 at 0.01 level. The rejection of this hypothesis implies that technological change take place over the period of our analysis. To further investigate the pattern of technological change, we use the time trend parameters reported in table 6.1. The time trend parameters α_t and α_{tt} indicate the direction and the rate of the shift in the cost function, independent of prices and quantity. The estimate of α_t is -0.8091, indicating that the cost function is shifting inward

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[3] For more details on technilogy tests, see Toft and Bjorndal [1997].

independent of change in factor prices and output, although no statistical significance is attached to this time shift. The parameter α_{tt} is also not statistically different from zero.

Due to rejection of this hypothesis, Eq. (3.19) can be interpreted as the dual rate of technological change. The dual rate of technological change at the mean of the data is -0.0076 or -0.76%, which is approximately close to one. The estimates show that the manufacturing sector of Pakistan did experience disembodied technological progress at the rate of 0.76% per annum. It means that if all prices and output are constant, the cost of production would be decreasing at the rate of 0.76 percentage per year. Hence during the period of our analysis, very small technical progress has occurred. This result is in agreement with the study of Khan [1989], who found that rate of technological progress was 3.7 percent per year, and in disagreement with Idrees [1997] who found that cost of production was increasing at 0.04% per annum. We will further evaluate these results, when allocative inefficiency is incorporated along with technological change on our model.

Cost Share Neutrality Test

In evaluating technological change, based on the cost function, it is common to distinguish between technological change that influences the optimal choice of factor combination used in production and technological change, that is factor share neutral.[4] The effect of technological change on the factor shares are given by

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[4] See, Jorgenson [1986].

$$\frac{\partial S_i}{\partial T} = \alpha_{it} \qquad (6.1)$$

Cost share neutrality therefore implies $\alpha_{it} = 0$ for all factors. We test the cost share neutrality by log-likelihood ratio test. We find that the hypothesis of share neutral technical change is rejected because the computed χ^2 test statistic 44.1251 is greater than the tabulated value of the Chi-square at 0.01 level, i.e., 13.28. Therefore, technological change has also affected the cost factor shares. The parameters α_{it} indicates that the technological change is raw material saving while energy, capital, and labor using.

Returns to Scale and Scale Economies

The returns to scale are computed as the inverse of the partial derivative of the total costs, Eq. (3.16), with respect to output, or with the help of Eq. (5.1). The estimated returns to scale is 0.931. It means large scale manufacturing sector of Pakistan is subject to decreasing returns to scale. We have also calculated the scale economies at the mean of the data by using formula given in Eq. (3.9), which is -0.0745 or -7.45 %. The negative scale economies imply the operation of the manufacturing sector on the upward sloping portion of their long run cost function. These results indicate diseconomies of scale against the earlier results of economies of scale. However, we may further expect changes in these results due to allocative inefficiency.

6.3 Elasticities of Demand and Substitution

The estimated parameters of Eqs. (3.16) and (3.17) are used to estimate own, cross price, and substitution elasticities and are reported in table 6.2. Three out of four own price elasticities of demand are statistically different from zero. The signs and magnitudes of the own price elasticities follow the same pattern as in chapter 5. We find that the demand for capital is most responsive to its price with an estimate of 2.2463, followed by energy with an estimate of 1.0836. The own price elasticity for raw material is inelastic, i.e., 0.6674, indicating that the demand for raw material is less elastic to change in price.

All of the cross price elasticities show inelastic pattern except capital/raw material. The demand for energy, labor, and raw material is inelastic to the change in price of all other factor inputs.

Table 6.2: ***Matrix of Own and Cross Price Elasticities of Demand***

Variables	***Estimates of Elasticities***			
	Energy	Capital	Labor	Raw Material
Energy	-1.08 (-1.67)**	-0.05 (-0.05)	0.28 (0.37)	0.85 (0.94)
Capital	-0.01 (-0.06)	-2.25 (-30.25)*	0.31 (4.06)*	1.94 (2.04)**
Labor	0.15 (0.82)	0.80 (3.94)*	-0.93 (-582.12)*	-0.03 (-0.03)
Raw Material	0.06 (0.35)	0.61 (1.17)	-0.01 (-0.03)	-0.67 (-0.51)

Note: Figures in parentheses are asymptotic *t*- values.
* indicates significant at the 5% and ** indicates significant at the 10% level.

To study the changing pattern of substitution elasticities due to technological change, we calculated the AES and MES at the mean values of output and factor prices, allowing variation due to changes in parameters. The estimates of the AES and MES are reported in table 6.3 with their asymptotic *t*-statistics. The capital and labor are good substitutes of each other and the magnitude of elasticity is almost the same as in case of simple translog cost function in chapter 5. The AES between energy and capital show interesting results. These two inputs are good substitute of each other at the point of approximation in chapter 5, but here these two inputs are complements of each other. However, they are not statistically significant. It implies that due to the inclusion of technological change, energy and capital turn out to be complements to each other. The estimates indicate that under the *ceteris paribus* assumption, higher priced energy will dampen not only the demand for energy but also the demand for capital. Labor and raw material are complements to each other, while all other inputs are substitutes.

Most of the MES elasticities are statistically different from zero at the 0.05 or 0.10 level. MES estimates provide no evidence about the complementarity of the energy and capital. The magnitudes and signs of MES follow the same pattern as in case of simple translog cost function discussed in chapter 5. Estimates of MES reveal that the elasticities of substitution between capital and labor is high, followed by capital and raw material. The variation in the estimates of MES is due to technological change. The comparison between AES and MES in these two models provide evidence that the estimates of AES are more sensitive to the inclusion of technological change than the MES.

From above discussion, we observe that inclusion of technological change has not only affected the direction of factor intensity but also estimates of scale economies and the returns to scale. The pattern of technological change has an interesting effect on the

Table 6.3: ***Allen and Morishima Elasticities of Substitution***

Elasticities	Estimates	Asymptotic t-statistic
	(a) Allen Elasticities of Substitution	
σ_{EK}	-0.22	-0.05
σ_{EL}	3.37	0.37
σ_{EM}	1.29	0.96
σ_{KL}	3.83	4.06*
σ_{KM}	2.93	1.17
σ_{LM}	-0.04	-0.03
	(b) Morishima Elasticities of Substitution	
η_{EK}	1.07	2.18*
η_{KE}	2.20	2.30*
η_{EL}	1.24	2.51*
η_{LE}	1.20	1.60**
η_{EM}	1.14	2.34
η_{ME}	1.52	1.50
η_{KL}	3.05	15.41*
η_{LK}	1.24	16.03*
η_{KM}	2.86	5.40*
η_{MK}	2.61	5.19*
η_{LM}	0.92	7.92*
η_{ML}	0.64	1.23

Note: * indicates significant at the 5% and ** indicates significant at the 10% level.

substitution and complementarity possibilities. Energy and capital turn out to be complements in case of AES, but in case of simple translog cost function these two inputs are substitutes.

Chapter

7

Allocative Efficiency and Substitution Possibilities

Inefficiency in resource allocation is quite common in the manufacturing sector of developing countries. Lack of effective competition in factor market produces allocative inefficiencies, which tends to reduce the profitability of the production units beneath their full potential. This chapter uses a modified translog cost function, which incorporates allocative inefficiency. The cost function is then used to conduct a number of tests to analyze various characteristics of the production technology and to estimate elasticities and substitution possibilities.

7.1 Results with Allocative Efficiency

The system of equations in Eqs. (3.31) and (3.34) are estimated where symmetry (3.2), and linear homogeneity in prices (3.4), are imposed. The estimated parameters along with their asymptotic *t*-statistics are reported in table 7.1.

The shadow cost function corresponds to a well behaved production function only if it is monotonically increasing in prices and concave in shadow prices. The monotonicity condition is satisfied for each observations, as all the calculated variable shadow factor shares are positive.[1] However, the concavity condition in input prices is not satisfied as the principal minors of the Hessian matrix have wrong

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[1] The cost shares at the mean of the data are $SE = 0.0167$, $SK = 0.0999$, $SL = 0.1741$, and $SM = 0.7093$.

Table 7.1: ***Results with Allocative Efficiency***

Parameter	Estimate	Asymptotic *t*-statistics
α_0	60.102	5.13*
α_E	0.121	1.58
γ_{EE}	-0.009	-1.37
γ_{EK}	-0.001	-0.04
γ_{EL}	0.027	1.82**
γ_{EM}	-0.018	-1.27
α_K	1.093	2.35*
γ_{KK}	-0.163	-2.10*
γ_{KL}	0.112	2.31*
γ_{KM}	0.051	0.93
α_L	1.648	2.71*
γ_{LL}	0.057	1.83**
γ_{LM}	-0.196	-2.44*
α_M	-1.861	-1.81**
γ_{MM}	0.162	1.72**
γ_Q	-5.811	-4.28*
γ_{QQ}	0.384	4.88*
γ_{EQ}	-0.008	-1.50
γ_{KQ}	-0.070	-2.48*
γ_{LQ}	-0.080	-2.30*
γ_{MQ}	0.157	2.54*
K_E	0.173	2.37*
K_K	0.224	3.22*
K_L	1.000	-
K_M	0.504	1.37
χ^2	332.710	-
N	32	-

Note: * indicates significant at the 5% and ** indicates significant at the 10% level.

sign.[2] This does not imply that statistical test for curvature condition must be rejected. The failure of the cost function to be concave in input prices can be interpreted as a violation of the cost minimizing assumption underlying the development of the cost function model.[3]

7.2 Relative Price Efficiency

Relative price efficiency with respect to all inputs is attained if and only if all K's are equal, i.e., $K_E = K_K = K_L = K_M$. The actual cost and cost shares are homogeneous of degree zero in K's, therefore, we cannot estimate the values of K for each input. Estimation of their relative values require that the value of the K's be normalized. The estimated relative values of K's , and all other parameters are invariant to the choice of which K_i is normalized and the value chosen for it.[4] We normalize the value of K_L to equate one, i.e., K_L =1. With this normalization, the restriction for relative price efficiency with respect to all inputs become $K_E = K_K = K_M = 1$. We test the hypothesis of relative price efficiency and rejected it because the computed value of χ^2 test statistic is 12.326, which is greater than the critical value of the χ^2 (11.34) at the 1% level of significance.[5] It implies that the manufacturing sector of Pakistan does not minimize costs subject to market prices and there is evidence of allocative inefficiency. In other words,

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[2] The evaluated minors of the Hessian matrix are, H_{11}=8.015, H_{22} = -0.173, H_{33} = 0, and H_{44}= -1.50.

[3] For further details, see Capalblo and Antle (1988).

[4] For further details, see Atkinson and Halvorsen [1984].

[5] Relative price efficiency with respect to all inputs was also rejected by Atkinson and Halvorsen [1980, 1984], Burki *at el.* [1997].

it is inappropriate to make the assumption that relative price efficiency is attained.

It can be seen from table 7.1 that the estimated values of K's statistically differ from zero. To investigate it further, we test the relative price efficiency between each pair of inputs using the hypothesis that $K_i = K_j$, where $i, j = E, K, L, M$. The hypotheses tests of pair wise relative price efficiency are presented in table 7.2. The table 7.2 shows that $K_E \neq K_L$ and $K_K \neq K_L$, while all the other inputs are relatively equally price inefficient. Our results also indicate that relative to labor energy is the most inefficiently utilized factor of production followed by capital and raw material. We also find that energy is inefficiently utilized relative to capital and raw material but their difference is statistically insignificant.[6]

Table 7.2: ***Relative Efficiency Test for Each Pair of Inputs.***

Hypothesis	Ratio of Relative efficiency	*t*-statistics	Hypothesis	Ratio of Relative efficiency	*t*-statistics
$K_E = K_K$	0.7756	0.7595	$K_L = K_K$	4.4706	11.1721
$K_L = K_E$	5.7643	11.276	$K_K = K_M$	0.4439	0.7849
$K_E = K_M$	0.3443	0.8965	$K_L = K_M$	1.9847	1.3420

The effects of relative price inefficiency on cost of production can be evaluated by comparing actual total cost with the cost when relative price efficiency has been attained. The efficient level of cost is estimated by imposing restrictions $K_E = K_K = K_L = K_M = 1$, in Eq. (3.31). A comparison with the fitted total actual cost indicates that over

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[6] Burki *et al.* (1997) found that capital and raw material is over-utilized relative to labor, while labor is over-utilized as compared to energy. Their results also showed that energy is over-utilized relative to raw material.

the period of our analysis, relative price inefficiency increases total cost by 0.2136% per annum. It implies that allocative inefficiency increase the cost of production or reduce the profitability of the production units beneath their full potential.

7.3 Returns to Scale and Scale Economies

Two types of scale economies are relevant: one is the actual scale economies and the other is the shadow scale economies. The estimates of scale economies with respect to actual cost, (SE^A), at the mean of the data is 15.89 %, while the same with respect to shadow cost, (SE^S), is 10.09 %. These results indicate that large scale manufacturing sector of Pakistan has realized the economies of scale with industrialization. A comparison with simple translog cost function clearly indicates that manufacturing sector realizes greater economies of scale if the production units make their decisions on the basis of shadow prices rather than the market prices.

We calculated actual returns to scale and the shadow returns to scale. The returns to scale are computed as the inverse of partial derivative of shadow costs and actual costs with respect to output as specified in Eq.(5.1). The estimated shadow and actual returns to scale are 1.1122 and 1.1889, respectively. The values for returns to scale suggest that the manufacturing sector of Pakistan is subject to increasing returns or decreasing costs. A comparison with the calculated returns to scale in case of simple translog cost function reveals that returns to scale have bigger values if allocative inefficiencies are taken into account.

7.4 Elasticities of Demand and Substitution

The estimates of own and cross price elasticities of demand at the mean of the data, and their asymptotic *t*-statistic are reported in table 7.3. All the own price elasticities are of the correct sign. The own price elasticities of capital and energy show an elastic response, while labor and raw material have inelastic demand patterns. The demand for capital is most responsive to variation in price while the raw material is least responsive to change in price. Most of cross price elasticities are inelastic except energy/labor, capital/labor, and capital/ raw material. There is significant change in the magnitudes of elasticities if compared with a model that does not control for allocative inefficiency. For instance, the value of own price elasticity of energy and capital increases, while for labor and raw material decreases. Similarly, estimates of cross price elasticities also reveal that

Table 7.3: ***Matrix of Own and Cross Price Elasticities of Demand***

Variables	***Estimates of Elasticities***			
	Energy	Capital	Labor	Raw material
Energy	-1.52 (-0.56)	0.08 (0.14)	1.81 (0.24)	-0.37 (-0.07)
Capital	0.01 (0.11)	-2.53 (-1.72)**	1.29 (1.05)	1.22 (0.74)
Labor	0.17 (0.96)	0.74 (1.88)**	-0.50 (-11.80)*	-0.41 (-0.75)
Raw material	-0.01 (-0.09)	0.17 (1.15)	-0.10 (-0.35)	-0.06 (-0.09)

Note: Figures in parentheses are asymptotic *t*- values.
* indicates significant at the 5% and ** indicates significant at the 10% level.

the values of some cross price elasticities increased and while other decreased, if compared with simple model.

The AES and MES, estimated at the mean of the data, are reported in table 7.4. The AES provide evidence that energy/raw material and labor/raw material show comlplimentary relation, while all other pairs are substitutes. As expected Energy/labor and capital/labor are good substitute to each other. Similarly, we find that energy/capital pair is also substitutes to each other. It implies that even after controlling for allocative or price inefficiencies there is no possibility of energy/capital complementarity from our data set.

Consistent with the AES, the results of MES indicate that capital/labor, labor/energy, and capital/energy are good substitutes to each others. We also find that change in price of capital effects the demand for other inputs in a greater proportion than other inputs effecting the demand for capital. This is indicated by the higher magnitude of MES. The MES estimates also provide no evidence about energy and capital complementarity.

A comparison with estimates of AES when relative price efficiency is imposed, provides evidence that energy and raw material turn out to be complements when allocative inefficiency is imposed. However, all other estimates of AES have similar signs and as in MES.

A comparison of MES estimates with models when allocative inefficiency is not imposed reveals that the magnitude of MES are bigger cases where relative price efficiency is not imposed. The MES estimates also provide an interesting interpretation about energy/raw material and labor/raw material pairs. The results indicate that energy

Table 7.4: ***Allen and Morishima Elasticities of Substitution***

	When Relative Price Efficiency is not imposed		When Relative Price Efficiency is Imposed	
Elasticities	Estimates	Asymptotic t-statistic	Estimates	Asymptotic t-statistic
(a) Allen Elasticities of Substitution				
σ_{EK}	0.80	0.15	3.13	1.67**
σ_{EL}	10.39	0.24	4.46	3.27*
σ_{EM}	-0.52	-0.06	0.52	0.65
σ_{KL}	7.42	0.93	3.83	5.07*
σ_{KM}	1.73	1.24	2.16	3.42*
σ_{LM}	-0.58	-0.37	-0.61	-1.89**
(b) Morishima Elasticities of Substitution				
η_{EK}	1.53	0.58	1.51	17.15*
η_{KE}	2.61	2.02	2.03	21.28*
η_{EL}	1.69	0.66	1.57	24.37*
η_{LE}	2.31	0.31	0.96	8.56*
η_{EM}	1.51	0.57	1.39	34.51*
η_{ME}	-0.31	-0.06	0.77	1.44
η_{KL}	3.27	2.06*	2.69	17.29*
η_{LK}	1.79	1.48	0.91	14.56*
η_{KM}	2.70	1.94**	2.34	13.78*
η_{MK}	1.29	1.17	1.86	4.54*
η_{LM}	0.40	1.46	0.55	20.24*
η_{ML}	-0.35	-0.86	0.02	0.08

Note: * indicates significant at the 5% and ** indicates significant at the 10% level.

and raw material are substitutes if we analyze the effect of change in price of energy on raw material but becomes complement if we analyze the effect of change in price of raw material on the demand for energy. Similar is the case of labor and raw material.

In sum, the above discussion shows that the hypothesis of relative price efficiency is rejected, which implies that it is inappropriate to assume relative price efficiency as a maintained hypothesis. The rejection of allocative efficiency as a maintained hypothesis also implies variation in the estimates of scale economies, returns to scale and elasticities of demand and substitution. It appears from these results that one could expect further variation in technology tests when technological change and allocative inefficiency are simultaneously imposed. These aspects are investigated in chapter 8.

Chapter

8

Allocative Efficiency and Technological Change

The evidence of technological change (chapter 6) and the presence of allocative inefficiency (chapter 7) suggest a reconsideration of the hypotheses of allocative efficiency. In this chapter, therefore, we extended the previous analysis by simultaneously incorporating technological change and allocative inefficiencies.

8.1 Parameter Estimates

The estimated parameters of the modified translog cost function along with share equations, Eqs. (3.40) and (3.41), constrained to satisfy symmetry and linear homogeneity in prices, are reported in table 8.1 together with their asymptotic *t*- statistics. Most of the estimated parameters are significant at the conventional levels.

The duality between the cost function and the underlying production function holds only if symmetry, linear homogeneity in prices, montonicity and concavity conditions hold. The monotonicity condition was satisfied for each observation since all the estimated factor shares were positive.[1] The cost function was also monotonically increasing in output since Eq. (3.7) was satisfied. That value of the partial derivative of the total actual cost function and total shadow cost function with respect to output was 0.9057 and 0.4840, respectively.

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[1] The calculated factor shares at the mean of the data are $SE = 0.1062$, $SK = 0.1559$, $SL = 0.5524$, and $SM = 0.1854$.

Table 8.1: *Results with Allocative Efficiency and Technological Change*

Parameter	Estimate	Asymptotic t-statistics	Parameter	Estimate	Asymptotic t-statistics
α_0	-81.80	-0.87	γ_{LM}	-0.086	-2.97*
α_t	-0.66	-0.62	α_M	-2.141	-2.36*
α_{tt}	-0.001	-0.13	α_{Mt}	-0.006	-1.60**
α_E	1.98	1.99**	γ_{MM}	0.003	0.10
α_{Et}	0.014	1.89**	γ_Q	10.94	0.96
γ_{EE}	-0.006	-0.35	γ_{QQ}	-0.608	-0.88
γ_{EK}	-0.015	-1.14	γ_{Qt}	0.038	0.58
γ_{EL}	0.030	1.12	γ_{EQ}	-0.120	-2.04*
γ_{EM}	-0.009	-0.80	γ_{KQ}	-0.053	-1.79**
α_K	0.846	1.68	γ_{LQ}	0.024	0.33
α_{Kt}	-0.003	-1.11	γ_{MQ}	0.149	2.49*
γ_{KK}	-0.241	-5.58*	K_E	0.348	1.52
γ_{KL}	0.165	4.11*	K_K	0.110	2.68*
γ_{KM}	0.091	1.88**	K_L	1.000	-
α_L	0.315	0.28	K_M	0.041	1.41
α_{Lt}	-0.004	-0.74	χ^2	366.06	-
γ_{LL}	-0.110	-1.56	N	32	-

Note: * indicates significant at the 5% and ** indicates significant at the 10% level.

Concavity in shadow prices was checked by determining if the principal minors of the Hessian matrix were of correct sign. We find that the curvature condition holds since the values are alternate in signs.[2]

8.2 Technology Tests

We undertake a number of tests to analyze various characteristics of the production technology. These include, the pattern of technological change, cost share neutrality test, relative price efficiency, returns to scale and scale economies.

Test of Technological Change

Technological change may be an important factor affecting production technology. Thus factor usage and factor mix are important in analyzing production structure. In table 8.1, the estimated parameters of time trend, α_t and α_{tt}, indicate the direction and the rate of change of shift in the cost function independent of factor prices and quantity. In table 8.1, α_t =-0.6619 indicating that the cost function was shifting inward independent of changes in factor prices and output, although this shift is not statistically significant. The parameter α_{tt} =-0.0008 was also statistically insignificant. We also calculated the dual rate of technological change given in Eq. (3.19) and find that the average dual rate of technological change was 0.0063 or 0.63 %, which is close to one. It means that if taking all prices and output constant, the cost of production was

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[2] The computed principal minors of the Hessian matrix are $H_{11} = -1.31$, $H_{22} = 9.85$, $H_{33} = -0.58$, and $H_{44} = 0$.

increasing at 0.63 % per annum.[3] It implies that during the period of analysis, no technological progress occurred when technological change and allocative inefficiency were modeled simultaneously.

The Cost Share Neutrality Test

We further analyze technological change to distinguish its effect on optimal choice of factor combinations and on factor share neutrality. We evaluate this effect by taking partial derivative of cost shares with respect to time, as specified in Eq. (6.1), which implies that factor shares are natural over time, or α_{it} =0 for all factor inputs. We use the log-likelihood ratio test, which indicate that the hypothesis of share neutrality was rejected. The computed χ^2 test statistic 139.38 was greater than the critical value of 18.28 at the 0.01 level these results indicates that technological change has effected the factor shares. The parameter estimates of α_{it}, provide evidence that technological change was capital-, labor- and raw material-saving and energy-using.

8.3 Relative Price Efficiency

Relative price efficiency with respect to all inputs is attained if and only if $K_E = K_K = K_L = K_M$. The absolute value of *K*'s cannot be estimated as the total actual cost and actual cost shares, Eqs. (3.40) and (3.41), are homogeneous of degree zero in *K*'s. Therefore, we can estimate their relative values only, which requires that the value of one *K* must be normalized. The estimated relative values of *K*'s , as well as all of other

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[3] Idrees [1997] found that holding prices and output constant, the cost of production was increasing at the rate of 0.04 percent per annum due to technological changes.

parameters are invariant, not only to the choice of which K_i to be normalized, but also to the value chosen for it.[4] We normalize the value of $K_L = 1$. With this normalization, the restrictions for relative price efficiency, with respect to all inputs, become $K_E = K_K = K_L = K_M = 1$. The hypothesis of relative price efficiency was rejected. The computed χ^2 test statistic 19.49 was greater than the critical value of 11.34 at 0.01 level. It indicates that large scale manufacturing sector of Pakistan employs factor inputs sub-optimally because the ratio of shadow and actual factor prices diverge significantly. In other words, manufacturing sector fails to minimize cost of production. Thus, it implies that the manufacturing sector of Pakistan is subject to relative price inefficiency. We further investigate price inefficiency by testing the relative price efficiency between each pair of inputs, using the hypothesis $K_i = K_j$, where $i, j = E, K, L, M$. These hypothesis were tested by using t-test. The results are presented in table 8.2. The columns for ratio of relative efficiency illustrate the direction of relative allocative inefficiency of factor use because we know that $F_L / F_K = K_L P_L / K_K P_K$. To illustrate when $K_L / K_K = 9.08$, this implies that $F_L / F_K > P_L / P_K$ i.e., that the marginal rate of technical substitution is greater then the ratio of input prices. In other words, capital is over-utilized relative to labor. Tests of pairwise efficiency suggest that input mix inefficiency takes the form of over-utilization of raw material, capital and energy as compared to labor. The results also indicate that raw material is also over-utilized as compared with capital and energy, while in case of energy and capital, capital is over-utilized then energy.[5] Our results indicate

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[4] See, Atkinson and Halvorsen [1984].

[5] Burki *et al.* [1997] found that capital and raw material are over utilized as compare to labor. Raw material is also over-utilized as compare to energy.

that raw material is most over-utilized factor while labor is most under-utilized factor of production. Furthermore, a comparison of capital relative to raw material indicates that manufacturing sector uses too much of raw material. The implications of our results are that the raw material are used most inefficiently while capital is the second most inefficiently used factor of production. This pattern of input-use was damaging for the most abundant resource, labor. The harmful effects of this pattern of input use are reflected in the form of high rates of unemployment and under employment of labor.

Table 8.2: ***Relative Efficiency Test for Each Pair of Inputs***

Hypothesis	Ratio of Relative Efficiency	t-statistics	Hypothesis	Ratio of Relative Efficiency	t-statistics
$K_K = K_E$	0.3163	1.0716	$K_L = K_K$	9.0765	21.650
$K_L = K_E$	2.8707	2.8510	$K_K = K_M$	2.6756	1.671
$K_E = K_M$	8.2168	1.3510	$K_L = K_M$	24.285	32.810

8.4 Returns to Scale and Scale Economies

Two types of scale economies are relevant for our purpose: actual scale economies, and the shadow scale economies. The estimate of scale economies with respect to actual cost, SE^A, at the mean of the data is 0.0943 or 9.43 %, while the estimate of scale economies with respect to shadow cost, SE^S, at the mean of the data is 0.516 or 51.6%. Hence, the estimates of scale economies with respect to shadow cost are larger than the estimates of actual scale economies.[6] It indicates that the manufacturing sector

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[6] Atkinson and Halvorsen [1984] also found the same type of results about scale economies.

of Pakistan has realized the economies of scale with industrialization. We also calculated two types of returns to scale: which are actual returns to scale and shadow returns to scale. The returns to scale were computed by using Eq. (5.1), and are 1.1041 for actual total cost and 2.066 for shadow cost function, respectively. Both of the estimates suggest that the large scale manufacturing sector of Pakistan is subject to increasing return to scale, or decreasing costs.

8.5 Elasticities of Demand and Substitution

The estimates of own price elasticities and cross price elasticities of demand at the mean of the data along with their asymptotic *t*-statistics, are reported in table 8.3. All the own price elasticities have expected signs. The own price elasticity of capital is elastic, while all other elasticities are inelastic. It means that capital is more responsive to change in price. These estimates also indicate that labor is least responsive to a change in its price.

Our results also reveal that with the exception of cross price elasticities between capital and labor, all other elasticities show inelastic pattern. In other words, increasing the price of capital would generate more jobs because substitution between capital and labor is high. Most of the elasticities are statistically not different from zero.

The AES, and MES at the mean of the data are reported in table 8.4. AES between capital and labor is high, i.e., 2.91, which suggests that capital and labor are good substitutes of each other. The estimate of AES between energy/capital is 0.0674, which provides the evidence that their is no possibility of complementarity between these two inputs.

Table 8.3: ***Matrix of Own and Cross Price Elasticities***

Variables	*Elasticity Estimates for*			
	Energy	Capital	Labor	Raw material
Energy	-0.948 (-0.58)	0.011 (0.01)	0.836 (0.28)	0.101 (0.08)
Capital	0.007 (0.01)	-2.390 (-1.95)**	1.611 (2.08)*	0.772 (0.51)
Labor	0.161 (0.15)	0.455 (2.62)*	-0.646 (-3.44)*	0.031 (0.03)
Raw material	0.058 (0.05)	0.65 (0.27)	0.091 (0.04)	-0.80 (-0.92)

Note: Figures in parentheses are asymptotic t- values.
* indicates significant at the 5% and ** indicates significant at 10% level.

As can be seen from the table 8.4 penal (b) , almost half of the MES are statistically different from zero at 0.05 or 0.10 levels. The results reveal that labor and capital are good substitutes. The change in price of capital change the demand for labor proportionateley more while the converse is not true. In other words, increase in price of capital will generate more jobs. The estimates also provide evidence that change in capital will affect energy, labor, and raw material in a greater proportion than the change in latter will affecting capital. As in previous chapters, we do not find energy/capital complementarity since the estimate of MES has a positive sign.

A comparison between AES and MES shows that the estimates of MES provide more information about the substitution possibilities. Due to asymmetric nature of MES, we can also obtain information on which factor's price change will provide desirable results.

Table 8.4: ***Allen and Morishima Elasticities of Substitution***

	When Relative Price Efficiency is not imposed		When Relative Price Efficiency is imposed	
Elasticities	Estimates	Asymptotic t-statistic	Estimates	Asymptotic t-statistic
	(a) Allen Elasticities of Substitution			
σ_{EK}	0.067	0.01	-0.223	-0.05
σ_{EL}	1.514	0.29	3.366	0.37
σ_{EM}	0.546	0.11	1.287	0.96
σ_{KL}	2.916	2.13*	3.825	4.06*
σ_{KM}	4.162	0.30	2.927	1.17
σ_{LM}	0.165	0.04	-0.038	-0.03
	(b) Morishima Elasticities of Substitution			
η_{EK}	0.955	1.88**	1.073	2.18*
η_{KE}	2.400	1.61**	2.200	2.30*
η_{EL}	1.109	2.04*	1.235	2.51*
η_{LE}	1.482	0.52	1.204	1.60**
η_{EM}	1.006	1.68**	1.142	2.34*
η_{ME}	0.899	1.11	1.521	1.50
η_{KL}	2.845	2.60*	3.047	15.40*
η_{LK}	2.257	3.06*	1.242	16.01*
η_{KM}	3.039	1.51	2.859	5.40*
η_{MK}	1.570	2.07*	2.609	5.19*
η_{LM}	0.737	0.32	0.924	7.92*
η_{ML}	0.828	16.8*	0.642	1.23*

Note: * indicates significant at the 5% and ** indicates significant at the 10% level.

8.6 Comparison with other studies

In the area of large scale manufacturing sector of Pakistan, the study by Kazi [1976], Battese and Malik [1987] found that the elasticity between capital/labor is greater then one and these two inputs are good substitutes of each other. While the other studies on production relations, i.e., Kemal [1981], Battese and Malik [1988] estimated the elasticity of substitution between capital/labor is less than one. Mahmood (1989) calculated the elasticity of substitution between capital/labor is 2.42. In our study the estimates of AES between capital/labor ranges from 2.9164 to 7.4159, while the estimates of MES between capital/labor ranges from 2.6887 to 3.2692 in all of the four models. In previous studies, there is a little bet debate on energy/capital complementarity. Mahmood [1989] found that the energy/capital are good complements to each other with estimate -4.031, while in [1992] he found that energy/capital were complements between 1954-55 to 1970-71 and become substitutes to each other between 1975-76 to 1985-86. In our study the estimates of AES shows that energy/capital are substitute in three of four models. But in case of MES energy/capital are substitute to each other in all of the four models, the estimates ranges from 0.9551 to 1.5293. Therefore our results provide evidence that there is no possibility of energy/capital complementarity over the analysis period.

Chapter

9

Conclusion and Policy Implications

This study attempts to investigate the nature of allocative inefficiencies in Pakistan's large scale manufacturing sector using pooled provincial time series data, and the translog cost function. We have explored the nature of allocative inefficiencies by employing different models, which include or exclude the biases of technological change. In particular, we have estimated four models generated by modifications to the translog cost function, to comment on substitutability between different inputs, and the scale economies. In the first model, we simply take the translog cost function without introducing biases of technological change and allocative inefficiencies, and compare results on substitution possibilities with the existing studies. Our data set yields results quite consistent with existing studies. In the second model, we introduce technological change bias and evaluate its effect on the estimates of elasticities, relative to the first model. We find that technological change affects the magnitudes of the elasticities of demand and substitution, and reduces the cost of production by 0.76% under *ceteris paribus* assumption. In the third model, we incorporate allocative or price inefficiencies in our basic model to investigate if the assumption of perfect competition in factor markets hold. And, the last model investigates the possibility of the simultaneous existence of allocative inefficiencies and technological change in our data set.

Our tests of hypothesis whether the classical assumption of perfect competition in input market holds was rejected. This finding is consistent with Burki *et al.* [1997]

who used Pakistan's aggregate time series data and the translog profit function to test the same hypothesis. However, we focus primarily on our results from third and forth models which allow the possibility of non-competitive factor markets.

We have estimated the models by using pooled provincial level data of Sindh and Punjab, which represents more than 80% of total manufacturing industries of Pakistan. The system of equations for the cost function and input shares were estimated using iterative Zellner-efficient technique. Our data satisfied monotonicity and concavity conditions for the estimated cost functions.

The relative price efficiency between each pair of inputs provides the evidence that energy, capital, and raw material are over-utilized as compared to labor while capital is over-utilized relative to raw material. The estimates of own price elasticities reveal that capital and energy have elastic behavior, while the remaining two have inelastic pattern of demand. The Allen elasticities of substitution (AES) reveal that energy/labor and capital/energy are good-substitutes to each others. On the other hand, the estimates of Morishima elasticities of substitution (MES) provide interesting results about energy/raw material and labor/raw material in the sense that these two are complements in case of AES, while the MES reveal that if price of raw material increases the demand for labor decreases under *ceteris paribus* assumption. Similarly, when the price of labor increases, the demand for raw material also increases. This interpretation also applies to energy/raw material.

Our results provide strong indication of inefficiencies in the allocation of resources. In particular, raw material, capital, and energy are over-utilized as compared to labor when technological biases are taken into account. This over-utilization of raw material is more severe, relative to labor. Raw material is also over-utilized relative to capital and energy. Capital is over-utilized relative to energy. These results are hardly surprising given the structure of factor markets and past governmental policies towards manufacturing sector of Pakistan. It is well known that the government adopted a development strategy that supported capital-intensive investment together with government guarantees for the supplies of raw material.

The rejection of cost minimizing restrictions in our results imply that the use of neoclassical cost function, which imposes cost minimization as a maintained hypothesis, is inappropriate in this application. The homotheticity restrictions on the underlying production technologies were also rejected. The manufacturing sector of Pakistan has realized -0.76 % dual rate of technological change at the mean of the data when relative price efficiency is imposed. It implies that when all prices and output are taken as constant, the cost of production decreases by 0.76 % per annum due to tecchnological change. Incorporating the relative price inefficiency, the dual rate of technological change reduces to 0.63 % per annum at the mean of the data. It means that holding prices and output as constant, cost of production increases at 0.63 % per annum due to technological change. The factor share neutrality test was also rejected in both the cases.

The estimates of own-price elasticities show that at the given output, demand for energy, labor, and raw material is inelastic, while the demand for capital is highly elastic (with an estimate of -2.39). It means that capital is more responsive to changes in its price under the *ceteris paribus* assumption.

We have argued that the estimates of the MES are better as compared to the AES. The results indicate that all factor inputs are substitutes to each other. Capital and labor is good substitute to each other but change in price of capital would affect the demand for labor in greater proportion while the converse is not true. Energy and capital also turn out to be substitutes. The estimates of elasticities of substitution also indicate that changes in price of capital affect all other factors of production in greater proportion while converse is not true. Raw material is found to be substitute to all other inputs, which indicates that with the use of more energy, capital and labor, manufacturing sector can economize on the use of raw material. A comparison of the results of elasticities of substitution obtained from the modified cost function and its counterpart, when relative price efficiency is imposed, indicates that the use of latter results have substantial biases in the estimation of elasticities of substitution. The estimates of MES indicate that the manufacturing sector of Pakistan is relatively flexible against the notion that the developing countries are relatively rigid.

Two types of returns to scale are computed in this study. The results indicate that the manufacturing sector of Pakistan is subject to increasing returns to scale with respect to actual costs and shadow costs. The results also show that if the manufacturing sector of Pakistan makes decision on the basis of shadow prices then the degree of returns to scale would be higher. We have also calculated two types of scale economies: actual scale economies and shadow scale economies. The estimates of scale economies are higher in case of shadow costs than in actual costs.

Several policy implications flow out from our results.

- *First*, economic policies in Pakistan have always supported the use of raw material and capital (see chapter 1). The distortion parameters in our final model suggest that raw material is most inefficiently utilized factor followed by capital and energy relative to labor. The under-utilization of labor is also pronounced from the low elasticity in Pakistan's large scale manufacturing. Therefore, steps should be taken to rule out price distortions in the factor markets, and an appropriate change in relative prices is recommended to reduce capital-intensity in the large scale manufacturing sector. This is also the appeal of measured aimed at reducing poverty in Pakistan.

- *Secondly*, the finding that demand for labor is inelastic suggests that a policy of wage change will not affect the demand for labor. On the other hand, if the government sets the wage floor above the prevailing wage rate, there would be no drastic reduction in the demand for labor. The own price elasticity of capital is greater than one, which indicates that government can affect the demand for capital by using appropriate pricing policy for capital.
- *Thirdly*, the cross price elasticities between labor and capital show inelastic behavior. However, the relation between capital and labor is elastic which implies that changes in price of capital will generate more opportunities for labor. Since capital and labor are good-substitutes, an increase in price of capital would have a favorable effect on employment.
- *Fourthly*, our results indicate that the share of labor is decreasing due to technological change but increasing with the level of output. Though the net effect is negative but its magnitude is very small. A striking feature of this result is that the intensity of labor is decreasing due to technological change. Hence, steps should be taken in this regard. For example, government can provide an incentive package for the use of more labor intensive techniques. Unfortunately

they are readily available.

- *Fifthly*, it appears from the above that the removal of price distortions and adoption of labor-intensive techniques would be fruitful, if supplemented by appropriate indigenous technology development. For this purpose, government can change the relative prices and can provide appropriate facilities or if possible subsidies for the technological development. Adoption of labor-intensive techniques, by altering the relative factor prices, may be a long run phenomenon. However, this would increase the social welfare in the short run through optimal allocation of resources.

In sum, it should be emphasized that the observed deviations from optimal factor use are costly for the manufacturing sector. Therefore, long run economic growth could be achieved through adjustment policies that correct input market price distortions and promote shift towards efficient factor proportions. The results in this study can be extended in several different ways. In this study we incorporate allocative inefficiencies along with technological change. One of the extension could be modify the model to take into account the price distortion in output markets. With this modification, one can also comment on static and dynamic equilibrium (Atkinson and Halvorsen [1998]). The model can also be modified to incorporate technical efficiency using multi-output cost function. In addition, more information can be gained if the researcher disaggregates the energy input into two or more components.

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Appendix

Curvature conditions

The curvature condition for the cost function requires that the Hessian matrix, H, of the second order partial derivatives with respect to factor prices be negative semi-definite. The symmetric Hessian matrix, H, has $\partial^2 C / \partial P_i^2$ as diagonal elements and $\partial^2 C / \partial P_i \partial P_j \quad i \neq j$ as the off-diagonal elements. The second order partial derivatives of the translog cost function with respect to *ith* factor price yields the expression $\frac{\partial^2 \ln C}{(\partial \ln P_i)^2} = \gamma_{ii}$. This expression can be written as

$$\frac{\partial S_i}{\partial \ln P_i} = \gamma_{ii} \quad \text{where} \quad S_i = \frac{\partial \ln C}{\partial \ln P_i} \tag{A.1}$$

and

$$\gamma_{ii} = \frac{\partial S_i}{\partial P_i} * P_i \tag{A.2}$$

Since

$$S_i = \frac{\partial \ln C}{\partial \ln p_i} = \frac{\partial C / C}{\partial P_i / P_i} = \frac{\partial C}{\partial P_i} * \frac{P_i}{C} \tag{A.3}$$

Now by substitute the Eq. (A.3) in Eq. (A.2), and some manipulations, we can write

$$\gamma_{ii} = P_i * \frac{\partial\left(\partial C / \partial P_i * p_i / C\right)}{\partial P_i}$$

or

$$\gamma_{ii} = P_i \left[\frac{\partial^2 C}{\partial P_i} \frac{P_i}{C} + \frac{\partial C}{\partial P_i} \frac{\partial (P_i / C)}{\partial P_i} \right]$$

or

$$\gamma_{ii} = P_i \left[\frac{\partial^2 C}{\partial P_i^2} * \frac{P_i}{C} + \frac{\partial C}{\partial P_i} \left(\frac{C\,(\partial P_i / \partial P_i) - P_i (\partial C / \partial P_i)}{C^2} \right) \right]$$

or

$$\gamma_{ii} = \frac{\partial^2 C}{\partial P_i^2} \frac{P_i^2}{C} + \frac{\partial C}{\partial P_i} \frac{P_i}{C} - \frac{P_i^2}{C^2} \left(\frac{\partial C}{\partial P_i} \right)^2 \qquad \text{(A.4)}$$

Since $\partial C / \partial P_i = X_i$, substituting it in Eq. (A.4) we get

$$\gamma_{ii} = \left(\left(\frac{\partial^2 C}{\partial P_i^2} \frac{P_i^2}{C}\right) + \frac{P_i X_i}{C} - \frac{P_i^2 X_i^2}{C^2} \right) \qquad \text{(A.5)}$$

At the point of approximation, i.e., that is when all factor prices and output are indexed to 1.0. The shares become $S_i = \alpha_i > 0$. Now we use this approximation in equation (A.5) as

$$\gamma_{ii} = \left(\frac{\partial^2 C}{\partial P_i^2} \frac{P_i^2}{C} \right) + \alpha_i - \alpha_i^2 \qquad \text{where} \quad \frac{P_i X_i}{C} = S_i = \alpha_i \tag{A.6}$$

We can rearrange equation (A.6) to isolate the diagonal elements of required Hessian matrix ,as follows

$$\frac{\partial^2 C}{\partial P_i^2} = \frac{C}{P_i^2} \left(\gamma_{ii} + \alpha_i^2 - \alpha_i \right) \tag{A.7}$$

In equation (A.7) costs (C) and factor prices (P_i) are nonnegative, therefore the Hessian matrix contains the following elements on the diagonal.

$$\gamma_{ii} + \alpha_i^2 - \alpha_i \qquad \text{for all} \quad i = E, K, L, M, \tag{A.8}$$

Similarly, we derive the off-diagonal elements which are

$$\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{C}{P_i P_j} * \left(\gamma_{ij} + \alpha_i \alpha_j \right) \qquad \text{for all} \quad i \neq j \tag{A.9}$$

In equation (A.9), the cost (C) and factor prices P_i and P_j are nonnegative, therefore, the Hessian matrix contains the following off diagonal elements.

$$\gamma_{ij} + \alpha_i \alpha_j \qquad \text{for all} \quad i \neq j \tag{A.10}$$

The sign of each element in the Hessian matrix is determined by Eqs. (A.8) and (A.10). Thus, we can evaluate a modified Hessian matrix as

$$H = \begin{bmatrix} \gamma_{EE} + \alpha_E^2 - \alpha_E & \gamma_{EK} + \alpha_E \alpha_K & \gamma_{EL} + \alpha_E \alpha_L & \gamma_{EM} + \alpha_M \alpha_M \\ \gamma_{EK} + \alpha_E \alpha_K & \gamma_{KK} + \alpha_K^2 - \alpha_K & \gamma_{KL} + \alpha_K \alpha_L & \gamma_{EM} + \alpha\ E \alpha_M \\ \gamma_{EL} + \alpha_E \alpha_M & \gamma_{KL} + \alpha_K \alpha_L & \gamma_{LL} + \alpha_L^2 - \alpha_L & \gamma_{LM} + \alpha_L \alpha_M \\ \gamma_{EM} + \alpha_E \alpha_M & \gamma_{KM} + \alpha_K \alpha_M & \gamma_{LM} + \alpha_L \alpha_M & \gamma_{MM} + \alpha_M^2 - \alpha_M \end{bmatrix}$$

The necessary and sufficient condition for concavity is that, the Hessian matrix must alternate in signs as $H_{11} \leq 0$, $H_{22} \geq 0$, $H_{33} \leq 0$ and $H_{44} \geq 0$.